Computer Algebra Independent Integration Tests

Summer 2023 edition

3-Logarithms/60-3.2.2-f+g-x-^m-h+i-x-^q-A+B-log-e-a+b-x-over-c+d-x-^n-^p

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [263]. This is test number [60].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (263)	0.00 (0)
Mathematica	94.68 (249)	5.32 (14)
Maxima	68.44 (180)	31.56 (83)
Maple	61.22 (161)	38.78 (102)
Fricas	59.32 (156)	40.68 (107)
Mupad	48.29 (127)	51.71 (136)
Giac	47.53 (125)	52.47 (138)
Sympy	20.53 (54)	79.47 (209)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

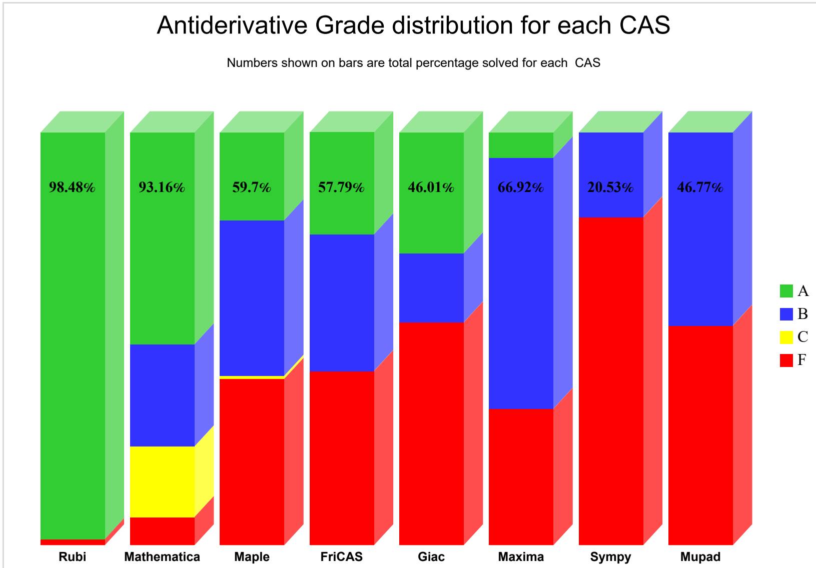
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

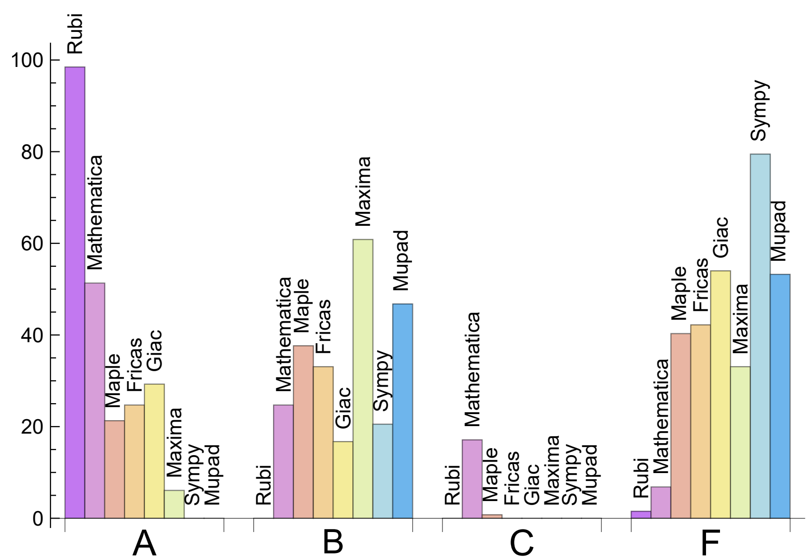
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.479	0.000	0.000	1.521
Mathematica	51.331	24.715	17.110	6.844
Giac	29.278	16.730	0.000	53.992
Fricas	24.715	33.080	0.000	42.205
Maple	21.293	37.643	0.760	40.304
Maxima	6.084	60.837	0.000	33.080
Mupad	0.000	46.768	0.000	53.232
Sympy	0.000	20.532	0.000	79.468

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	14	100.00	0.00	0.00
Maxima	83	100.00	0.00	0.00
Maple	102	100.00	0.00	0.00
Fricas	107	100.00	0.00	0.00
Mupad	136	0.00	100.00	0.00
Giac	138	92.75	5.80	1.45
Sympy	209	22.01	73.21	4.78

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.24
Fricas	0.36
Maxima	0.39
Mathematica	0.85
Mupad	3.87
Maple	15.62
Sympy	21.50
Giac	27.40

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	309.75	1.00	269.00	1.00
Fricas	660.65	2.43	438.00	2.09
Maple	718.80	2.66	509.00	2.19
Mathematica	878.90	2.60	374.00	1.26
Giac	942.44	4.02	394.00	1.58
Sympy	948.35	4.84	869.50	4.35
Mupad	1048.30	3.36	638.00	2.70
Maxima	2290.62	7.49	1392.00	4.91

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

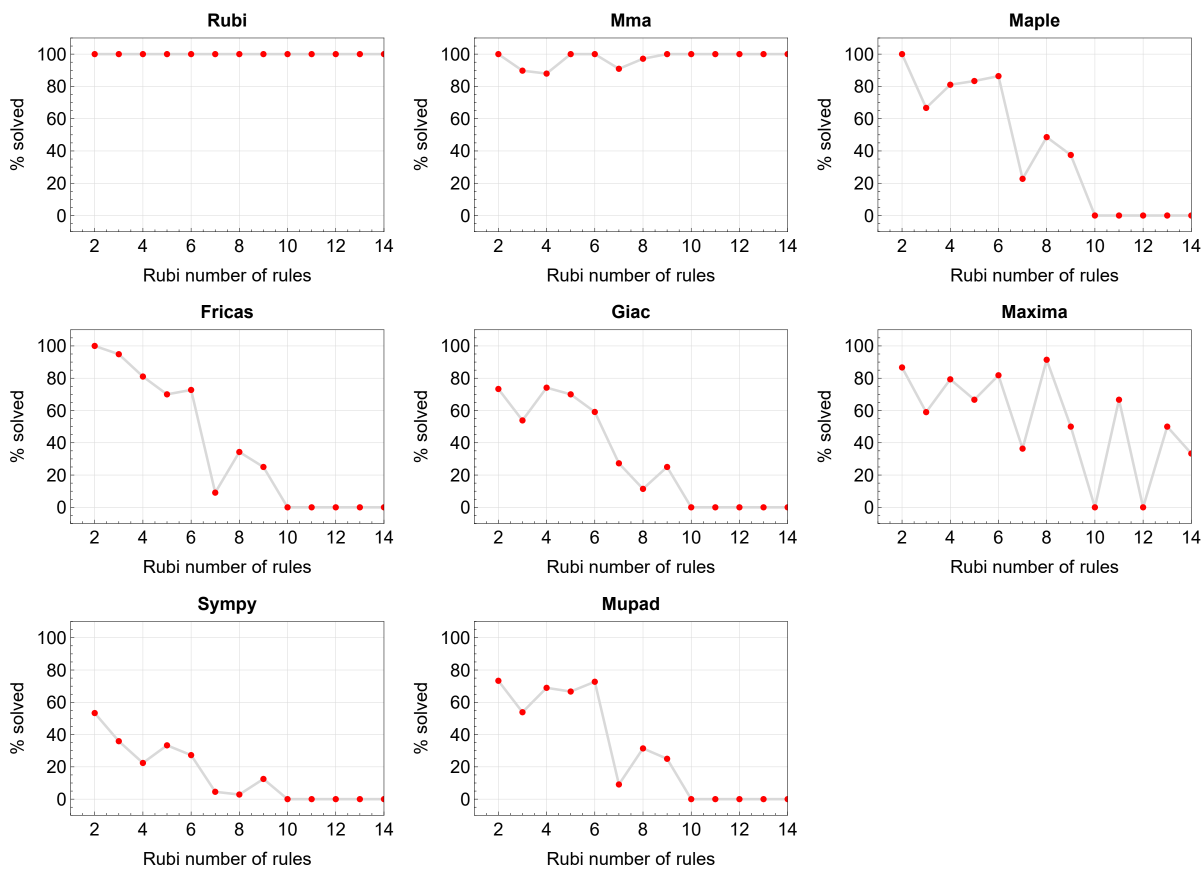


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

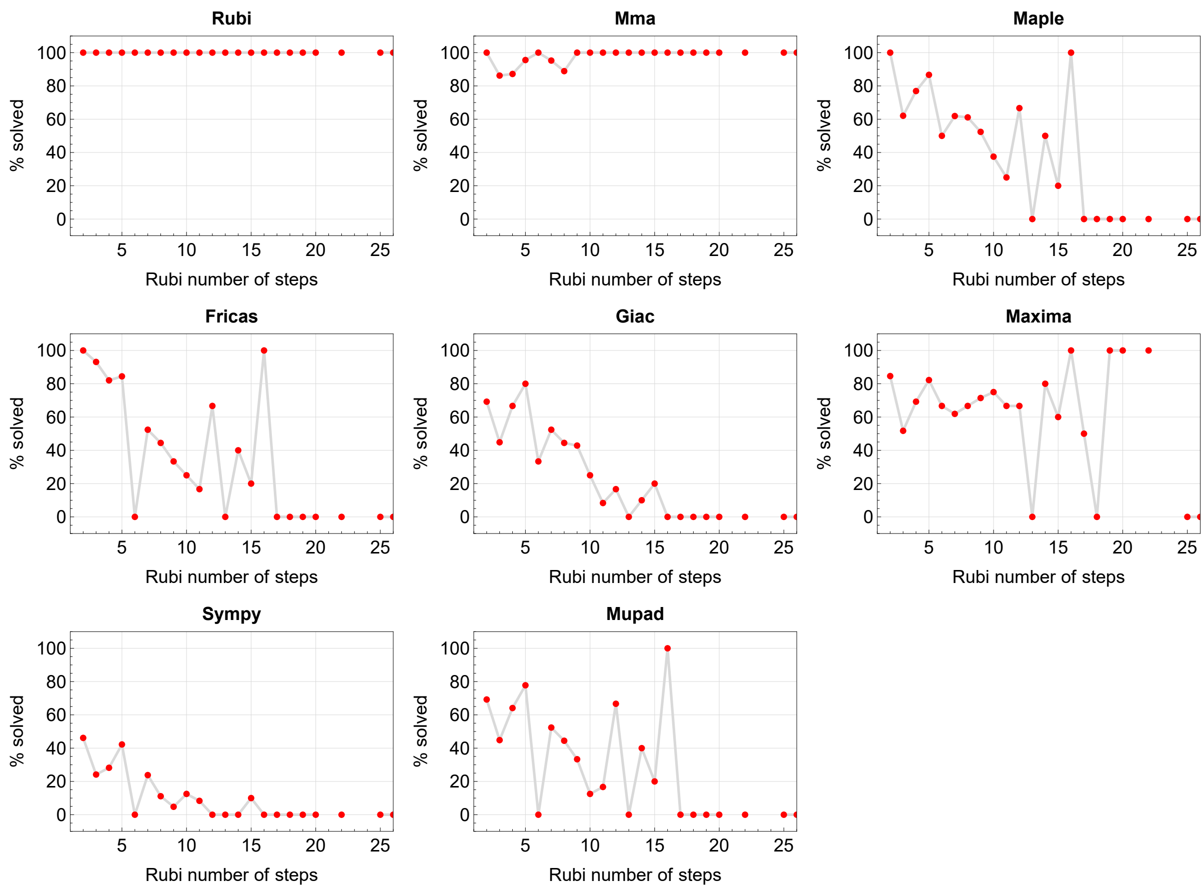


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

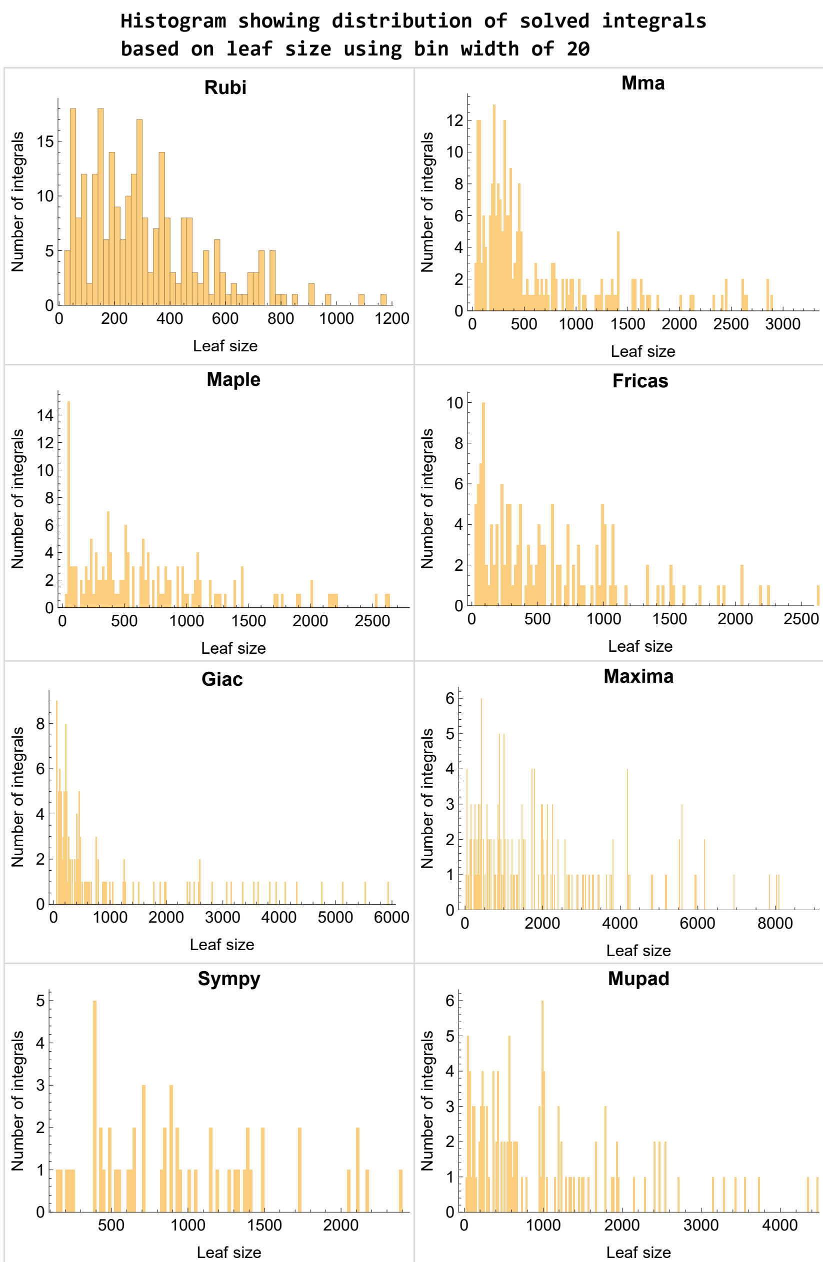


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

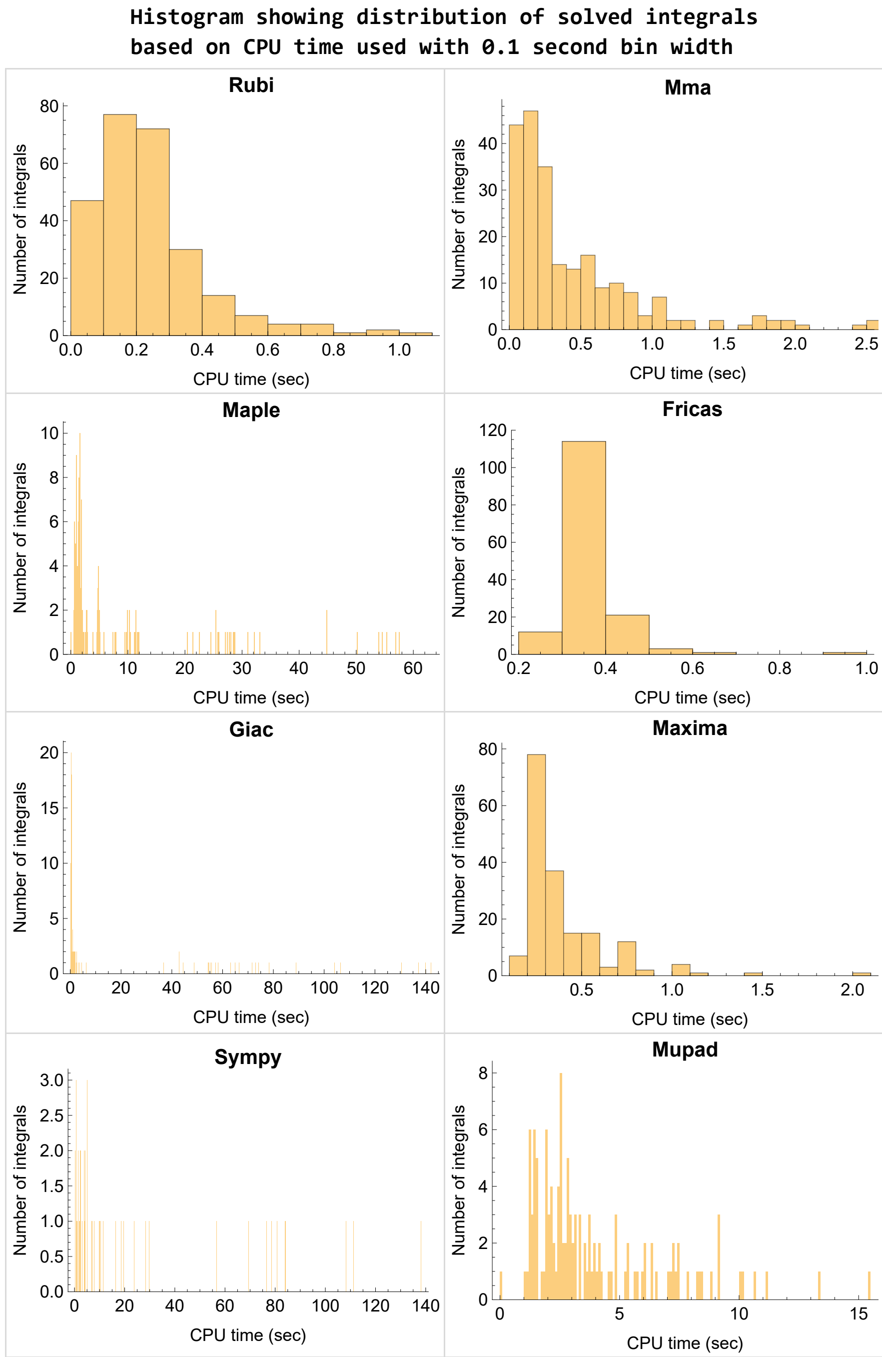


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

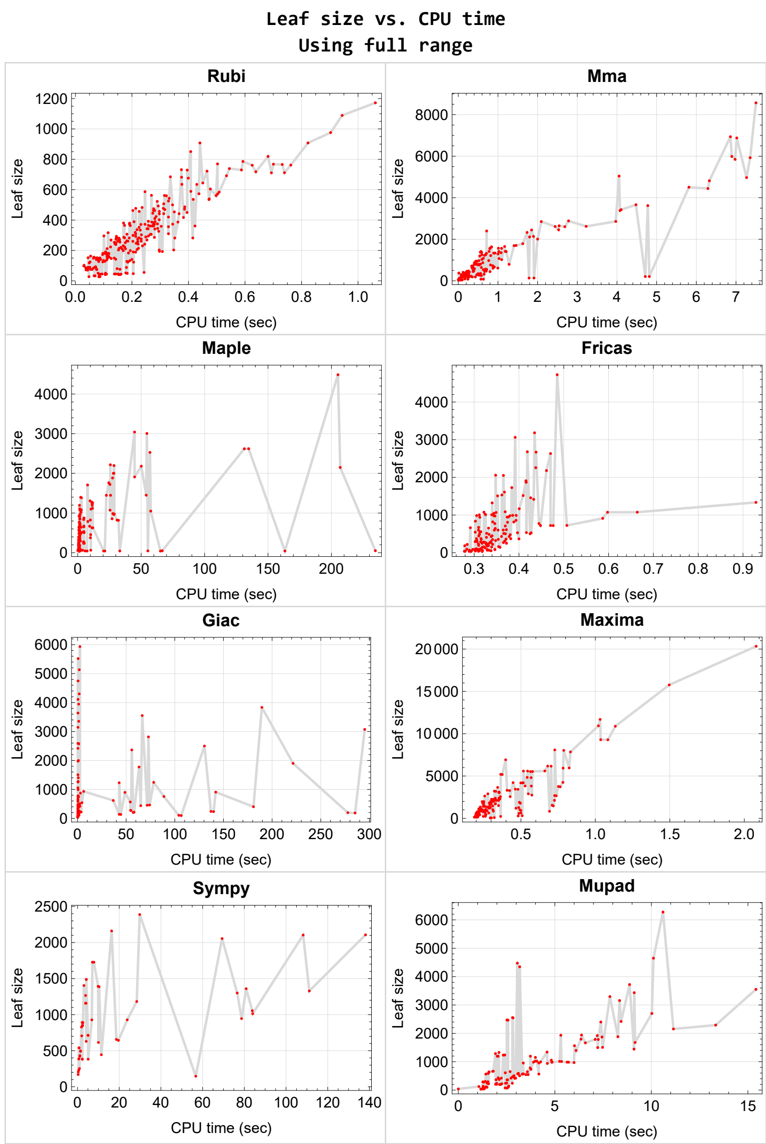


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{253, 254, 262, 263\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple $\{252, 261\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

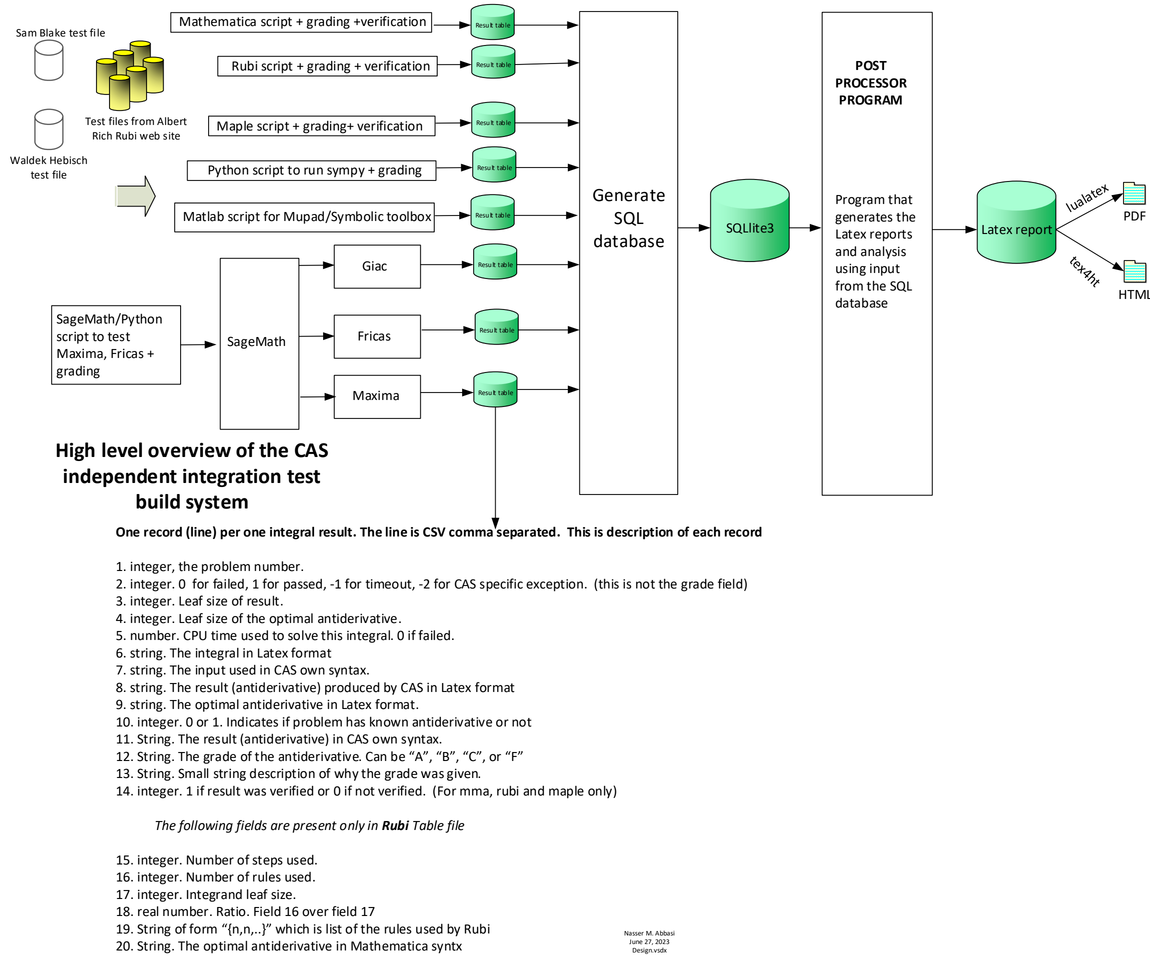
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 255, 256, 257, 258, 259, 260, 261 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 34, 39, 40, 41, 42, 47, 48, 50, 55, 56, 58, 65, 66, 67, 75, 76, 77, 84, 85, 87, 88, 89, 90, 91, 96, 97, 98, 99, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 143, 144, 145, 146, 151, 152, 154, 159, 160, 162, 169, 170, 171, 179, 180, 181, 186, 187, 189, 190, 212, 213, 214, 218, 219, 220, 224, 225, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 246, 247, 248 }

B grade { 7, 17, 18, 28, 29, 30, 49, 57, 59, 60, 64, 68, 69, 70, 74, 78, 79, 80, 86, 92, 93, 94, 100, 101, 114, 124, 125, 153, 161, 163, 164, 168, 172, 173, 174, 178, 182, 183, 184, 185, 188, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 251, 252, 255, 256, 257, 258, 260, 261 }

C grade { 35, 36, 37, 38, 43, 44, 45, 46, 51, 52, 53, 54, 61, 62, 63, 71, 72, 73, 81, 82, 83, 95, 102, 103, 139, 140, 141, 142, 147, 148, 149, 150, 155, 156, 157, 158, 165, 166, 167, 175, 176, 177, 197, 204, 205 }

F normal fail { 210, 211, 215, 216, 217, 221, 222, 223, 226, 227, 240, 249, 250, 259 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 3, 4, 7, 8, 9, 11, 12, 13, 18, 19, 30, 34, 35, 36, 37, 38, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 95, 96, 103, 139, 140, 146, 147, 154, 197, 224, 225, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 245, 255, 256, 257, 258 }

B grade { 1, 2, 5, 6, 10, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 39, 40, 49, 61, 62, 63, 71, 72, 73, 81, 82, 83, 87, 88, 89, 90, 91, 97, 98, 99, 102, 104, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 124, 125, 126, 127, 128, 129, 130, 141, 142, 148, 149, 150, 153, 155, 156, 157, 158, 165, 166, 167, 175, 176, 177, 190, 191, 192, 193, 198, 199, 200, 201, 204, 205, 206, 207, 208, 209, 213, 214, 219, 220 }

C grade { 252, 261 }

F normal fail { 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 84, 85, 86, 92, 93, 94, 100, 101, 112, 113, 121, 122, 123, 131, 132, 133, 134, 135, 136, 137, 138, 143, 144, 145, 151, 152, 159, 160, 161, 162, 163, 164, 168, 169, 170, 171, 172, 173, 174, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 194, 195, 196, 202, 203, 210, 211, 212, 215, 216, 217, 218, 221, 222, 223, 226, 227, 236, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 259, 260 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 3, 4, 10, 11, 12, 22, 35, 36, 37, 38, 42, 43, 44, 45, 50, 51, 52, 89, 90, 91, 95, 96, 97, 98, 103, 104, 105, 139, 140, 141, 146, 147, 148, 154, 155, 197, 198, 215, 216, 220, 221, 222, 224, 225, 230, 231, 232, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 246, 247, 248, 255, 256, 257, 258 }

B grade { 1, 2, 7, 8, 9, 13, 17, 18, 19, 20, 21, 23, 28, 29, 30, 46, 49, 53, 54, 61, 62, 63, 71, 72, 73, 81, 82, 83, 88, 99, 102, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 124, 125, 126, 127, 128, 129, 130, 142, 149, 150, 153, 156, 157, 158, 165, 166, 167, 175, 176, 177, 190, 191, 192, 193, 199, 200, 201, 204, 205, 206, 207, 208, 209, 212, 213, 214, 217, 218, 219, 223, 228, 229, 233 }

C grade { }

F normal fail { 5, 6, 14, 15, 16, 24, 25, 26, 27, 31, 32, 33, 34, 39, 40, 41, 47, 48, 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 84, 85, 86, 87, 92, 93, 94, 100, 101, 112, 113, 121, 122, 123, 131, 132, 133, 134, 135, 136, 137, 138, 143, 144, 145, 151, 152, 159, 160, 161, 162, 163, 164, 168, 169, 170, 171, 172, 173, 174, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 194, 195, 196, 202, 203, 210, 211, 226, 227, 240, 249, 250, 251, 252, 259, 260, 261 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 4, 5, 14, 33, 42, 50, 111, 112, 146, 154, 231, 232, 237, 238, 239, 245 }

B grade { 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 49, 51, 52, 53, 54, 55, 56, 57, 58, 61, 62, 63, 64, 65, 66, 67, 71, 72, 73, 74, 75, 76, 77, 81, 82, 83, 88, 89, 90, 91, 95, 96, 97, 98, 99, 102, 103, 104, 105, 106, 107, 108, 109, 110, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 139, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 153, 155, 156, 157, 158, 159, 160, 161, 162, 165, 166, 167, 168, 169, 170, 171, 175, 176, 177, 178, 179, 180, 181, 190, 191, 192, 193, 197, 198, 199, 200, 201, 204, 205, 206, 207, 208, 209, 228, 229, 230, 233, 255, 256, 257, 258 }

C grade { }

F normal fail { 6, 16, 27, 34, 41, 48, 59, 60, 68, 69, 70, 78, 79, 80, 84, 85, 86, 87, 92, 93, 94, 100, 101, 113, 123, 134, 138, 145, 152, 163, 164, 172, 173, 174, 182, 183, 184, 185, 186, 187, 188, 189, 194, 195, 196, 202, 203, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 259, 260, 261 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 7, 8, 9, 17, 18, 19, 28, 29, 30, 37, 38, 42, 43, 44, 45, 46, 49, 50, 51, 61, 62, 63, 71, 72, 73, 81, 82, 83, 90, 91, 95, 96, 97, 98, 99, 102, 103, 104, 114, 115, 116, 124, 125, 126, 139, 141, 142, 146, 147, 148, 149, 150, 153, 154, 155, 165, 166, 167, 175, 176, 177, 192, 197, 198, 199, 204, 205, 206, 224, 225, 231, 234, 235, 236, 237, 238, 239 }

B grade { 1, 2, 3, 4, 10, 11, 12, 13, 20, 21, 22, 23, 31, 32, 33, 34, 35, 39, 40, 41, 88, 108, 109, 110, 111, 117, 118, 119, 120, 127, 128, 129, 130, 135, 136, 137, 138, 143, 144, 145, 190, 232, 233, 245 }

C grade { }

F normal fail { 5, 6, 14, 15, 16, 24, 25, 26, 27, 36, 47, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 84, 85, 86, 87, 89, 92, 93, 94, 100, 101, 105, 106, 107, 112, 113, 121, 122, 123, 131, 132, 133, 134, 140, 151, 152, 156, 157, 158, 159, 160, 161, 162, 163, 164, 170, 171, 172, 173, 174, 180, 181, 182, 183, 184, 186, 187, 188, 189, 191, 194, 195, 196, 202, 203, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 228, 229, 230, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 255, 256, 257, 258, 259, 260, 261 }

F(-1) timedout fail { 168, 169, 178, 179, 185, 193, 200, 201 }

F(-2) exception fail { 226, 227 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 28, 29, 30, 35, 36, 37, 38, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 61, 62, 63, 71, 72, 73, 81, 82, 83, 88, 89, 90, 91, 95, 96, 97, 98, 99, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 124, 125, 126, 127, 128, 129, 130, 139, 140, 141, 142, 146, 147, 148, 149, 150, 153, 154, 155, 156, 157, 158, 165, 166, 167, 175, 176, 177, 190, 191, 192, 193, 197, 198, 200, 201, 204, 205, 206, 207, 208, 209, 228, 229, 230, 231, 232, 233, 237, 238, 239, 245 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 6, 14, 15, 16, 24, 25, 26, 27, 31, 32, 33, 34, 39, 40, 41, 47, 48, 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 84, 85, 86, 87, 92, 93, 94, 100, 101, 112, 113, 121, 122, 123, 131, 132, 133, 134, 135, 136, 137, 138, 143, 144, 145, 151, 152, 159, 160, 161, 162, 163, 164, 168, 169, 170, 171, 172, 173, 174, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 194, 195, 196, 199, 202, 203, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 255, 256, 257, 258, 259, 260, 261 }

F(-2) exception fail { }

Sympy

A grade { }

B grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 35, 36, 37, 38, 42, 43, 44, 49, 50, 51, 53, 61, 62, 71, 72, 88, 89, 90, 91, 95, 96, 97, 102, 103, 104, 109, 110, 111, 114, 119, 120, 130, 146, 153, 154, 245 }

C grade { }

F normal fail { 5, 14, 16, 24, 26, 27, 31, 32, 33, 34, 47, 48, 59, 68, 70, 78, 80, 84, 85, 86, 87, 100, 101, 112, 121, 123, 131, 134, 135, 136, 137, 138, 139, 152, 163, 165, 174, 175, 188, 189, 190, 197, 203, 204, 205, 246 }

F(-1) timedout fail { 6, 15, 25, 28, 29, 30, 39, 40, 41, 45, 46, 52, 54, 55, 56, 57, 58, 60, 63, 64, 65, 66, 67, 69, 73, 74, 75, 76, 77, 79, 81, 82, 83, 92, 93, 94, 98, 99, 105, 106, 107, 108, 113, 115, 116, 117, 118, 122, 124, 125, 126, 127, 128, 129, 132, 133, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 155, 156, 157, 158, 159, 160, 161, 162, 164, 166, 167, 168, 169, 170, 171, 172, 173, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 206, 207, 208, 209, 210, 211, 217, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263 }

F(-2) exception fail { 212, 213, 214, 215, 216, 218, 219, 220, 221, 222 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	261	574	1022	504	1158	3637	1195
N.S.	1	1.00	1.23	2.71	4.82	2.38	5.46	17.16	5.64
time (sec)	N/A	0.112	0.155	0.957	0.241	0.370	4.095	0.506	2.014

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	217	396	671	370	850	2418	638
N.S.	1	1.00	1.21	2.20	3.73	2.06	4.72	13.43	3.54
time (sec)	N/A	0.118	0.111	0.800	0.216	0.382	2.478	0.457	1.589

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	181	206	361	226	498	1254	282
N.S.	1	1.00	1.29	1.47	2.58	1.61	3.56	8.96	2.01
time (sec)	N/A	0.074	0.164	0.634	0.206	0.327	1.444	0.497	1.312

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	70	108	144	127	253	627	126
N.S.	1	1.00	0.86	1.33	1.78	1.57	3.12	7.74	1.56
time (sec)	N/A	0.040	0.025	0.559	0.207	0.325	0.943	0.407	1.068

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	164	417	241	0	0	0	0
N.S.	1	1.00	1.23	3.14	1.81	0.00	0.00	0.00	0.00
time (sec)	N/A	0.193	0.078	1.491	0.262	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	175	370	0	0	0	0	0
N.S.	1	1.00	1.23	2.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.143	0.096	1.378	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	208	153	570	177	384	134	197
N.S.	1	1.00	2.45	1.80	6.71	2.08	4.52	1.58	2.32
time (sec)	N/A	0.065	0.101	0.810	0.219	0.330	2.273	0.427	2.139

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	187	279	933	363	629	270	361
N.S.	1	1.00	1.08	1.61	5.39	2.10	3.64	1.56	2.09
time (sec)	N/A	0.096	0.237	0.953	0.240	0.371	4.169	0.531	2.523

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	210	390	1386	602	928	426	590
N.S.	1	1.00	0.78	1.45	5.15	2.24	3.45	1.58	2.19
time (sec)	N/A	0.126	0.277	1.506	0.261	0.365	6.868	0.532	3.114

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	423	429	925	1789	724	1727	4746	2473
N.S.	1	1.00	1.01	2.19	4.23	1.71	4.08	11.22	5.85
time (sec)	N/A	0.285	0.229	1.185	0.244	0.507	7.850	0.597	2.519

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	362	636	1200	534	1266	3144	1287
N.S.	1	1.00	1.07	1.89	3.56	1.58	3.76	9.33	3.82
time (sec)	N/A	0.218	0.164	1.003	0.228	0.417	3.866	0.517	1.925

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	216	396	671	367	850	1975	636
N.S.	1	1.00	0.90	1.66	2.81	1.54	3.56	8.26	2.66
time (sec)	N/A	0.143	0.133	0.812	0.216	0.400	2.347	0.483	1.588

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	97	206	280	223	491	1056	290
N.S.	1	1.00	0.82	1.75	2.37	1.89	4.16	8.95	2.46
time (sec)	N/A	0.049	0.029	0.672	0.204	0.341	1.434	0.436	1.272

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	252	698	518	0	0	0	0
N.S.	1	1.00	0.91	2.53	1.88	0.00	0.00	0.00	0.00
time (sec)	N/A	0.233	0.132	1.570	0.261	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	221	568	992	0	0	0	0
N.S.	1	1.00	0.89	2.30	4.02	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	0.140	1.556	0.266	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	244	522	0	0	0	0	0
N.S.	1	1.00	1.06	2.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.237	0.187	1.489	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	315	185	1515	271	614	140	423
N.S.	1	1.00	3.54	2.08	17.02	3.04	6.90	1.57	4.75
time (sec)	N/A	0.070	0.204	1.013	0.252	0.335	10.028	0.503	2.731

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	454	319	2218	510	928	282	647
N.S.	1	1.00	2.51	1.76	12.25	2.82	5.13	1.56	3.57
time (sec)	N/A	0.104	0.240	1.591	0.304	0.366	23.848	0.549	3.578

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	344	430	3029	807	1300	444	941
N.S.	1	1.00	1.22	1.53	10.78	2.87	4.63	1.58	3.35
time (sec)	N/A	0.144	0.499	1.701	0.355	0.374	76.593	0.548	4.614

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	457	586	1194	2637	912	2161	5520	4347
N.S.	1	1.00	1.28	2.61	5.77	2.00	4.73	12.08	9.51
time (sec)	N/A	0.297	0.408	1.438	0.257	0.587	16.359	0.669	3.180

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	429	925	1789	722	1727	4110	2465
N.S.	1	1.00	1.16	2.49	4.82	1.95	4.65	11.08	6.64
time (sec)	N/A	0.232	0.216	1.157	0.250	0.470	7.138	0.551	2.573

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	261	568	1022	502	1158	2589	1192
N.S.	1	1.00	0.96	2.10	3.77	1.85	4.27	9.55	4.40
time (sec)	N/A	0.157	0.135	0.886	0.311	0.425	3.835	0.477	2.063

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	120	314	439	322	706	1506	566
N.S.	1	1.00	0.81	2.11	2.95	2.16	4.74	10.11	3.80
time (sec)	N/A	0.064	0.040	0.738	0.265	0.348	2.024	0.473	1.429

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	352	1086	850	0	0	0	0
N.S.	1	1.00	0.99	3.05	2.39	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	0.194	1.806	0.315	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	373	373	374	863	1501	0	0	0	0
N.S.	1	1.00	1.00	2.31	4.02	0.00	0.00	0.00	0.00
time (sec)	N/A	0.280	0.244	1.658	0.349	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	314	732	2302	0	0	0	0
N.S.	1	1.00	0.91	2.12	6.67	0.00	0.00	0.00	0.00
time (sec)	N/A	0.291	0.231	1.608	0.359	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	308	685	0	0	0	0	0
N.S.	1	1.00	0.99	2.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	0.261	1.609	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	427	185	3107	355	0	140	780
N.S.	1	1.00	4.80	2.08	34.91	3.99	0.00	1.57	8.76
time (sec)	N/A	0.072	0.278	1.575	0.353	0.354	0.000	0.547	3.723

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	608	357	4218	644	0	282	1053
N.S.	1	1.00	3.36	1.97	23.30	3.56	0.00	1.56	5.82
time (sec)	N/A	0.107	0.354	1.602	0.447	0.347	0.000	0.546	4.818

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	642	481	5524	991	0	444	1396
N.S.	1	1.00	2.28	1.71	19.66	3.53	0.00	1.58	4.97
time (sec)	N/A	0.148	0.581	2.259	0.564	0.394	0.000	0.612	6.099

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	354	1105	790	0	0	3552	0
N.S.	1	1.00	1.40	4.38	3.13	0.00	0.00	14.10	0.00
time (sec)	N/A	0.220	0.184	1.646	0.260	0.000	0.000	66.529	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	254	657	477	0	0	2364	0
N.S.	1	1.00	1.28	3.32	2.41	0.00	0.00	11.94	0.00
time (sec)	N/A	0.169	0.123	1.773	0.263	0.000	0.000	55.791	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	162	332	221	0	0	1230	0
N.S.	1	1.00	1.30	2.66	1.77	0.00	0.00	9.84	0.00
time (sec)	N/A	0.097	0.065	1.343	0.247	0.000	0.000	42.824	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	95	146	0	0	0	617	0
N.S.	1	1.00	1.25	1.92	0.00	0.00	0.00	8.12	0.00
time (sec)	N/A	0.158	0.024	1.370	0.000	0.000	0.000	36.747	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	207	72	172	60	170	104	69
N.S.	1	1.00	4.70	1.64	3.91	1.36	3.86	2.36	1.57
time (sec)	N/A	0.088	0.078	0.622	0.210	0.319	0.310	0.324	2.516

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	292	230	424	144	386	0	241
N.S.	1	1.00	1.69	1.33	2.45	0.83	2.23	0.00	1.39
time (sec)	N/A	0.114	0.176	0.977	0.217	0.358	0.618	0.000	2.446

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	418	365	885	349	889	139	545
N.S.	1	1.00	1.64	1.43	3.47	1.37	3.49	0.55	2.14
time (sec)	N/A	0.158	0.223	1.257	0.243	0.341	2.588	42.817	3.504

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	373	373	492	500	1469	611	1392	270	970
N.S.	1	1.00	1.32	1.34	3.94	1.64	3.73	0.72	2.60
time (sec)	N/A	0.191	0.386	1.634	0.310	0.372	9.881	55.044	5.995

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	359	846	1341	0	0	2814	0
N.S.	1	1.00	1.05	2.48	3.93	0.00	0.00	8.25	0.00
time (sec)	N/A	0.281	0.257	1.674	0.275	0.000	0.000	72.934	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	239	532	886	0	0	1774	0
N.S.	1	1.00	0.92	2.05	3.41	0.00	0.00	6.82	0.00
time (sec)	N/A	0.203	0.145	1.488	0.273	0.000	0.000	63.252	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	175	295	0	0	0	895	0
N.S.	1	1.00	1.09	1.84	0.00	0.00	0.00	5.59	0.00
time (sec)	N/A	0.114	0.102	1.475	0.000	0.000	0.000	48.777	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	104	82	134	88	231	115	106
N.S.	1	1.00	1.06	0.84	1.37	0.90	2.36	1.17	1.08
time (sec)	N/A	0.030	0.031	0.595	0.200	0.346	0.613	0.398	1.440

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	292	231	421	151	386	233	247
N.S.	1	1.00	1.87	1.48	2.70	0.97	2.47	1.49	1.58
time (sec)	N/A	0.112	0.164	0.901	0.223	0.339	0.601	0.434	2.503

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	324	366	859	334	828	133	415
N.S.	1	1.00	1.24	1.40	3.29	1.28	3.17	0.51	1.59
time (sec)	N/A	0.151	0.245	1.136	0.238	0.306	1.900	44.472	2.927

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	453	506	1721	664	0	276	984
N.S.	1	1.00	1.24	1.39	4.73	1.82	0.00	0.76	2.70
time (sec)	N/A	0.194	0.373	2.875	0.330	0.292	0.000	54.532	5.636

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	457	520	640	2560	1019	0	436	1679
N.S.	1	1.00	1.14	1.40	5.60	2.23	0.00	0.95	3.67
time (sec)	N/A	0.226	0.705	2.645	0.428	0.325	0.000	64.990	9.157

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	317	681	2037	0	0	0	0
N.S.	1	1.00	0.88	1.89	5.64	0.00	0.00	0.00	0.00
time (sec)	N/A	0.277	0.254	1.598	0.316	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	245	463	0	0	0	0	0
N.S.	1	1.00	0.98	1.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.177	0.196	1.561	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	207	174	567	185	382	132	198
N.S.	1	1.00	2.44	2.05	6.67	2.18	4.49	1.55	2.33
time (sec)	N/A	0.045	0.102	0.806	0.217	0.279	2.324	0.405	2.197

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	111	221	255	221	422	236	208
N.S.	1	1.00	0.77	1.53	1.77	1.53	2.93	1.64	1.44
time (sec)	N/A	0.078	0.072	0.771	0.209	0.285	1.084	0.394	1.982

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	418	367	885	355	889	442	545
N.S.	1	1.00	1.72	1.51	3.64	1.46	3.66	1.82	2.24
time (sec)	N/A	0.140	0.267	1.217	0.255	0.316	2.505	0.406	3.614

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	452	506	1721	672	0	0	983
N.S.	1	1.00	1.24	1.39	4.72	1.84	0.00	0.00	2.69
time (sec)	N/A	0.174	0.378	2.830	0.327	0.310	0.000	0.000	5.707

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	533	649	2380	1011	2106	0	1443
N.S.	1	1.00	1.15	1.40	5.14	2.18	4.55	0.00	3.12
time (sec)	N/A	0.203	0.631	2.085	0.352	0.326	138.117	0.000	9.101

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	563	563	637	787	3816	1509	0	0	2291
N.S.	1	1.00	1.13	1.40	6.78	2.68	0.00	0.00	4.07
time (sec)	N/A	0.269	0.887	7.993	0.570	0.352	0.000	0.000	13.330

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	539	539	905	0	3186	0	0	0	0
N.S.	1	1.00	1.68	0.00	5.91	0.00	0.00	0.00	0.00
time (sec)	N/A	0.474	0.449	0.000	0.331	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	450	450	680	0	2243	0	0	0	0
N.S.	1	1.00	1.51	0.00	4.98	0.00	0.00	0.00	0.00
time (sec)	N/A	0.395	0.342	0.000	0.307	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	343	343	869	0	1252	0	0	0	0
N.S.	1	1.00	2.53	0.00	3.65	0.00	0.00	0.00	0.00
time (sec)	N/A	0.257	0.405	0.000	0.295	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	203	203	205	0	633	0	0	0	0
N.S.	1	1.00	1.01	0.00	3.12	0.00	0.00	0.00	0.00
time (sec)	N/A	0.153	0.119	0.000	0.284	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	286	286	1214	0	0	0	0	0	0
N.S.	1	1.00	4.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.279	0.818	0.000	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	241	241	1407	0	0	0	0	0	0
N.S.	1	1.00	5.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.245	1.207	0.000	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	765	330	1987	289	714	206	469
N.S.	1	1.00	5.43	2.34	14.09	2.05	5.06	1.46	3.33
time (sec)	N/A	0.089	0.527	0.797	0.304	0.315	5.020	0.452	2.812

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	1032	643	3282	601	1387	455	955
N.S.	1	1.00	3.60	2.24	11.44	2.09	4.83	1.59	3.33
time (sec)	N/A	0.171	0.596	0.988	0.424	0.310	10.342	0.511	4.173

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	445	445	1255	926	4808	985	0	744	1870
N.S.	1	1.00	2.82	2.08	10.80	2.21	0.00	1.67	4.20
time (sec)	N/A	0.274	0.687	1.609	0.568	0.306	0.000	0.557	7.441

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	711	711	1559	0	5178	0	0	0	0
N.S.	1	1.00	2.19	0.00	7.28	0.00	0.00	0.00	0.00
time (sec)	N/A	0.694	0.794	0.000	0.374	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	761	761	1194	0	3656	0	0	0	0
N.S.	1	1.00	1.57	0.00	4.80	0.00	0.00	0.00	0.00
time (sec)	N/A	0.626	0.541	0.000	0.344	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	589	589	677	0	2259	0	0	0	0
N.S.	1	1.00	1.15	0.00	3.84	0.00	0.00	0.00	0.00
time (sec)	N/A	0.408	0.356	0.000	0.318	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	334	334	287	0	1202	0	0	0	0
N.S.	1	1.00	0.86	0.00	3.60	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	0.129	0.000	0.285	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	535	535	2615	0	0	0	0	0	0
N.S.	1	1.00	4.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.473	2.442	0.000	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	442	442	2649	0	0	0	0	0	0
N.S.	1	1.00	5.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.367	2.542	0.000	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	387	387	3426	0	0	0	0	0	0
N.S.	1	1.00	8.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.363	4.105	0.000	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	1352	367	5532	444	1182	219	1153
N.S.	1	1.00	9.20	2.50	37.63	3.02	8.04	1.49	7.84
time (sec)	N/A	0.127	1.080	1.049	0.578	0.310	28.408	0.520	3.993

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	1703	685	8031	837	2055	479	1940
N.S.	1	1.00	5.70	2.29	26.86	2.80	6.87	1.60	6.49
time (sec)	N/A	0.204	1.456	1.638	0.788	0.303	69.337	0.577	6.382

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	2010	968	10880	1323	0	780	3434
N.S.	1	1.00	4.34	2.09	23.50	2.86	0.00	1.68	7.42
time (sec)	N/A	0.280	1.999	1.812	1.134	0.340	0.000	0.566	9.112

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1089	1089	2330	0	6921	0	0	0	0
N.S.	1	1.00	2.14	0.00	6.36	0.00	0.00	0.00	0.00
time (sec)	N/A	0.944	1.735	0.000	0.397	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	908	908	1555	0	5196	0	0	0	0
N.S.	1	1.00	1.71	0.00	5.72	0.00	0.00	0.00	0.00
time (sec)	N/A	0.823	0.798	0.000	0.364	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	730	730	901	0	3218	0	0	0	0
N.S.	1	1.00	1.23	0.00	4.41	0.00	0.00	0.00	0.00
time (sec)	N/A	0.587	0.444	0.000	0.322	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	420	420	389	0	1789	0	0	0	0
N.S.	1	1.00	0.93	0.00	4.26	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	0.191	0.000	0.287	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	712	712	5044	0	0	0	0	0	0
N.S.	1	1.00	7.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.740	4.053	0.000	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	692	692	4506	0	0	0	0	0	0
N.S.	1	1.00	6.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.535	5.813	0.000	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	604	604	5989	0	0	0	0	0	0
N.S.	1	1.00	9.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.478	6.894	0.000	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	2401	367	11688	559	0	218	1565
N.S.	1	1.00	16.33	2.50	79.51	3.80	0.00	1.48	10.65
time (sec)	N/A	0.113	0.715	1.646	1.032	0.315	0.000	0.644	6.018

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	2456	722	15765	1045	0	479	3720
N.S.	1	1.00	8.21	2.41	52.73	3.49	0.00	1.60	12.44
time (sec)	N/A	0.204	2.529	1.857	1.496	0.344	0.000	0.641	8.871

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	2606	1019	20330	1610	0	780	6275
N.S.	1	1.00	5.63	2.20	43.91	3.48	0.00	1.68	13.55
time (sec)	N/A	0.276	2.686	2.524	2.080	0.368	0.000	0.665	10.601

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	718	718	1020	0	0	0	0	0	0
N.S.	1	1.00	1.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.638	1.093	0.000	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	536	536	727	0	0	0	0	0	0
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.417	0.869	0.000	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	283	283	1227	0	0	0	0	0	0
N.S.	1	1.00	4.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.225	0.662	0.000	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	252	338	0	0	0	0	0
N.S.	1	1.00	1.98	2.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.082	0.580	1.407	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	79	106	397	87	206	132	96
N.S.	1	1.00	1.80	2.41	9.02	1.98	4.68	3.00	2.18
time (sec)	N/A	0.110	0.242	0.664	0.222	0.310	0.376	0.467	2.579

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	186	400	1008	231	541	0	419
N.S.	1	1.00	1.02	2.19	5.51	1.26	2.96	0.00	2.29
time (sec)	N/A	0.188	0.430	0.984	0.262	0.310	0.743	0.000	2.971

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	318	767	2115	540	1488	211	981
N.S.	1	1.00	0.93	2.24	6.17	1.57	4.34	0.62	2.86
time (sec)	N/A	0.253	0.650	1.307	0.350	0.313	4.236	58.157	4.841

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	507	507	442	1076	3434	940	2388	451	1882
N.S.	1	1.00	0.87	2.12	6.77	1.85	4.71	0.89	3.71
time (sec)	N/A	0.313	0.800	1.839	0.480	0.313	29.821	71.588	8.266

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	722	722	4443	0	0	0	0	0	0
N.S.	1	1.00	6.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.466	6.293	0.000	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	469	469	2622	0	0	0	0	0	0
N.S.	1	1.00	5.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.316	3.223	0.000	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	261	261	1412	0	0	0	0	0	0
N.S.	1	1.00	5.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.186	1.022	0.000	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	315	182	416	155	432	173	222
N.S.	1	1.00	2.07	1.20	2.74	1.02	2.84	1.14	1.46
time (sec)	N/A	0.045	0.240	0.677	0.216	0.321	1.140	0.400	2.256

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	187	398	1004	236	539	339	423
N.S.	1	1.00	0.87	1.86	4.69	1.10	2.52	1.58	1.98
time (sec)	N/A	0.155	0.352	0.982	0.261	0.332	0.725	0.430	2.923

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	307	766	1995	515	1404	203	731
N.S.	1	1.00	0.84	2.10	5.47	1.41	3.85	0.56	2.00
time (sec)	N/A	0.258	0.518	1.220	0.315	0.328	3.080	57.239	3.762

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	523	523	466	1080	4187	1005	0	460	1497
N.S.	1	1.00	0.89	2.07	8.01	1.92	0.00	0.88	2.86
time (sec)	N/A	0.290	0.703	2.713	0.511	0.312	0.000	74.128	7.222

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	682	682	613	1386	6160	1534	0	754	2701
N.S.	1	1.00	0.90	2.03	9.03	2.25	0.00	1.11	3.96
time (sec)	N/A	0.376	0.997	2.935	0.703	0.362	0.000	88.805	10.025

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	635	635	5929	0	0	0	0	0	0
N.S.	1	1.00	9.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.377	7.357	0.000	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	410	410	3384	0	0	0	0	0	0
N.S.	1	1.00	8.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.281	4.077	0.000	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	767	344	1966	295	712	198	474
N.S.	1	1.00	5.44	2.44	13.94	2.09	5.05	1.40	3.36
time (sec)	N/A	0.075	0.497	0.902	0.295	0.302	5.086	0.456	2.856

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	444	483	848	373	892	371	507
N.S.	1	1.00	1.50	1.63	2.86	1.26	3.01	1.25	1.71
time (sec)	N/A	0.101	0.227	0.973	0.228	0.305	2.210	0.438	2.881

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	290	769	2116	545	1488	672	984
N.S.	1	1.00	0.77	2.05	5.64	1.45	3.97	1.79	2.62
time (sec)	N/A	0.290	0.466	1.368	0.337	0.302	4.201	0.470	4.861

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	525	525	453	1080	4188	1008	0	0	1505
N.S.	1	1.00	0.86	2.06	7.98	1.92	0.00	0.00	2.87
time (sec)	N/A	0.329	0.665	2.782	0.499	0.339	0.000	0.000	7.474

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	685	685	611	1396	5583	1517	0	0	2155
N.S.	1	1.00	0.89	2.04	8.15	2.21	0.00	0.00	3.15
time (sec)	N/A	0.336	0.877	2.337	0.545	0.409	0.000	0.000	11.142

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	851	851	793	1708	9282	2257	0	0	3550
N.S.	1	1.00	0.93	2.01	10.91	2.65	0.00	0.00	4.17
time (sec)	N/A	0.408	1.281	7.752	1.084	0.437	0.000	0.000	15.415

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	269	1195	1118	720	0	3945	1237
N.S.	1	1.00	1.21	5.36	5.01	3.23	0.00	17.69	5.55
time (sec)	N/A	0.124	0.161	11.437	0.217	0.476	0.000	1.234	2.406

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	225	876	740	529	1013	2571	663
N.S.	1	1.00	1.18	4.61	3.89	2.78	5.33	13.53	3.49
time (sec)	N/A	0.107	0.121	5.002	0.207	0.427	84.025	1.317	1.813

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	189	507	393	309	643	1276	295
N.S.	1	1.00	1.27	3.40	2.64	2.07	4.32	8.56	1.98
time (sec)	N/A	0.074	0.182	1.716	0.197	0.368	19.632	0.659	1.533

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	74	250	156	162	382	580	134
N.S.	1	1.00	0.86	2.91	1.81	1.88	4.44	6.74	1.56
time (sec)	N/A	0.036	0.014	0.685	0.195	0.354	5.086	0.462	1.290

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	172	0	276	0	0	0	0
N.S.	1	1.00	1.22	0.00	1.96	0.00	0.00	0.00	0.00
time (sec)	N/A	0.200	0.084	0.000	0.482	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	150	189	0	0	0	0	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.138	0.102	0.000	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	216	237	582	250	1360	100	204
N.S.	1	1.00	2.43	2.66	6.54	2.81	15.28	1.12	2.29
time (sec)	N/A	0.051	0.103	5.162	0.204	0.372	80.892	0.784	1.984

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	196	455	945	478	0	234	374
N.S.	1	1.00	1.08	2.51	5.22	2.64	0.00	1.29	2.07
time (sec)	N/A	0.097	0.278	10.203	0.217	0.381	0.000	1.077	2.117

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	220	986	1398	773	0	394	610
N.S.	1	1.00	0.78	3.51	4.98	2.75	0.00	1.40	2.17
time (sec)	N/A	0.154	0.322	27.713	0.247	0.346	0.000	1.363	2.435

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	442	442	441	1761	1978	1074	0	5133	2555
N.S.	1	1.00	1.00	3.98	4.48	2.43	0.00	11.61	5.78
time (sec)	N/A	0.313	0.256	24.523	0.240	0.598	0.000	1.803	2.807

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	374	1274	1336	774	0	3352	1328
N.S.	1	1.00	1.06	3.62	3.80	2.20	0.00	9.52	3.77
time (sec)	N/A	0.225	0.180	11.129	0.227	0.445	0.000	1.271	2.106

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	224	837	740	530	1054	1997	661
N.S.	1	1.00	0.90	3.35	2.96	2.12	4.22	7.99	2.64
time (sec)	N/A	0.143	0.129	4.756	0.204	0.397	83.821	1.003	1.784

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	101	463	309	297	656	990	303
N.S.	1	1.00	0.81	3.73	2.49	2.40	5.29	7.98	2.44
time (sec)	N/A	0.052	0.019	1.958	0.196	0.338	18.637	0.647	1.401

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	289	289	264	0	580	0	0	0	0
N.S.	1	1.00	0.91	0.00	2.01	0.00	0.00	0.00	0.00
time (sec)	N/A	0.249	0.130	0.000	0.487	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	259	259	233	0	1190	0	0	0	0
N.S.	1	1.00	0.90	0.00	4.59	0.00	0.00	0.00	0.00
time (sec)	N/A	0.221	0.141	0.000	0.467	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	242	242	258	0	0	0	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	0.198	0.000	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	329	376	1544	409	0	108	421
N.S.	1	1.00	3.54	4.04	16.60	4.40	0.00	1.16	4.53
time (sec)	N/A	0.071	0.192	9.999	0.250	0.327	0.000	1.554	2.226

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	474	855	2247	710	0	250	652
N.S.	1	1.00	2.51	4.52	11.89	3.76	0.00	1.32	3.45
time (sec)	N/A	0.106	0.258	27.074	0.302	0.370	0.000	1.978	2.683

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	357	1049	3058	1087	0	418	954
N.S.	1	1.00	1.22	3.58	10.44	3.71	0.00	1.43	3.26
time (sec)	N/A	0.145	0.560	57.556	0.351	0.369	0.000	2.578	3.365

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	477	477	631	2179	2901	1336	0	5934	4476
N.S.	1	1.00	1.32	4.57	6.08	2.80	0.00	12.44	9.38
time (sec)	N/A	0.311	0.369	50.102	0.278	0.929	0.000	2.503	3.060

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	441	1721	1978	1075	0	4300	2547
N.S.	1	1.00	1.14	4.45	5.11	2.78	0.00	11.11	6.58
time (sec)	N/A	0.270	0.223	25.411	0.251	0.664	0.000	1.781	2.835

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	269	1119	1118	721	0	2584	1234
N.S.	1	1.00	0.95	3.95	3.95	2.55	0.00	9.13	4.36
time (sec)	N/A	0.174	0.142	11.618	0.227	0.448	0.000	1.179	2.328

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	124	652	479	429	945	1402	588
N.S.	1	1.00	0.79	4.18	3.07	2.75	6.06	8.99	3.77
time (sec)	N/A	0.066	0.029	4.810	0.207	0.378	78.682	0.801	1.552

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	373	373	368	0	935	0	0	0	0
N.S.	1	1.00	0.99	0.00	2.51	0.00	0.00	0.00	0.00
time (sec)	N/A	0.344	0.182	0.000	0.499	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	390	390	394	0	1785	0	0	0	0
N.S.	1	1.00	1.01	0.00	4.58	0.00	0.00	0.00	0.00
time (sec)	N/A	0.282	0.240	0.000	0.496	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	361	361	331	0	2746	0	0	0	0
N.S.	1	1.00	0.92	0.00	7.61	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	0.230	0.000	0.573	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	326	326	326	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.295	0.254	0.000	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	269	269	370	0	1003	0	0	3832	0
N.S.	1	1.00	1.38	0.00	3.73	0.00	0.00	14.25	0.00
time (sec)	N/A	0.253	0.182	0.000	0.494	0.000	0.000	189.494	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	211	211	266	0	627	0	0	2499	0
N.S.	1	1.00	1.26	0.00	2.97	0.00	0.00	11.84	0.00
time (sec)	N/A	0.177	0.121	0.000	0.500	0.000	0.000	130.459	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	170	0	306	0	0	1242	0
N.S.	1	1.00	1.27	0.00	2.28	0.00	0.00	9.27	0.00
time (sec)	N/A	0.100	0.079	0.000	0.509	0.000	0.000	78.291	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	101	0	0	0	0	566	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	7.08	0.00
time (sec)	N/A	0.148	0.025	0.000	0.000	0.000	0.000	54.287	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	219	84	175	74	0	92	76
N.S.	1	1.00	4.38	1.68	3.50	1.48	0.00	1.84	1.52
time (sec)	N/A	0.080	0.074	1.456	0.203	0.317	0.000	0.426	2.428

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	304	269	427	194	0	0	239
N.S.	1	1.00	1.68	1.49	2.36	1.07	0.00	0.00	1.32
time (sec)	N/A	0.114	0.169	4.793	0.216	0.310	0.000	0.000	2.798

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	434	628	888	483	0	104	573
N.S.	1	1.00	1.63	2.36	3.34	1.82	0.00	0.39	2.15
time (sec)	N/A	0.154	0.216	11.918	0.252	0.312	0.000	104.068	3.080

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	518	1072	1472	859	0	232	986
N.S.	1	1.00	1.33	2.76	3.78	2.21	0.00	0.60	2.53
time (sec)	N/A	0.208	0.383	25.467	0.290	0.393	0.000	139.847	3.893

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	359	359	375	0	1892	0	0	3072	0
N.S.	1	1.00	1.04	0.00	5.27	0.00	0.00	8.56	0.00
time (sec)	N/A	0.304	0.249	0.000	0.490	0.000	0.000	295.165	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	275	275	252	0	1273	0	0	1898	0
N.S.	1	1.00	0.92	0.00	4.63	0.00	0.00	6.90	0.00
time (sec)	N/A	0.220	0.144	0.000	0.485	0.000	0.000	221.565	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	168	168	183	0	0	0	0	906	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	5.39	0.00
time (sec)	N/A	0.116	0.113	0.000	0.000	0.000	0.000	142.033	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	114	81	136	105	444	91	113
N.S.	1	1.00	1.12	0.79	1.33	1.03	4.35	0.89	1.11
time (sec)	N/A	0.031	0.031	2.079	0.187	0.305	11.438	0.557	1.437

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	304	269	424	196	0	208	241
N.S.	1	1.00	1.83	1.62	2.55	1.18	0.00	1.25	1.45
time (sec)	N/A	0.112	0.159	4.760	0.207	0.328	0.000	1.046	1.491

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	342	666	862	450	0	96	432
N.S.	1	1.00	1.25	2.44	3.16	1.65	0.00	0.35	1.58
time (sec)	N/A	0.144	0.252	9.417	0.219	0.340	0.000	106.593	2.063

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	478	977	1724	946	0	238	1016
N.S.	1	1.00	1.26	2.57	4.54	2.49	0.00	0.63	2.67
time (sec)	N/A	0.192	0.385	28.773	0.290	0.374	0.000	137.129	3.973

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	477	477	549	1910	2563	1458	0	401	1665
N.S.	1	1.00	1.15	4.00	5.37	3.06	0.00	0.84	3.49
time (sec)	N/A	0.214	0.717	44.855	0.362	0.426	0.000	180.849	6.581

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	382	382	334	0	2894	0	0	0	0
N.S.	1	1.00	0.87	0.00	7.58	0.00	0.00	0.00	0.00
time (sec)	N/A	0.277	0.246	0.000	0.550	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	263	263	259	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.182	0.202	0.000	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	215	237	578	250	1328	101	205
N.S.	1	1.00	2.42	2.66	6.49	2.81	14.92	1.13	2.30
time (sec)	N/A	0.045	0.102	4.635	0.198	0.318	111.169	0.804	1.930

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	115	272	259	266	2103	207	221
N.S.	1	1.00	0.76	1.80	1.72	1.76	13.93	1.37	1.46
time (sec)	N/A	0.075	0.075	4.592	0.196	0.338	108.219	0.770	1.570

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	434	628	888	487	0	418	573
N.S.	1	1.00	1.71	2.47	3.50	1.92	0.00	1.65	2.26
time (sec)	N/A	0.137	0.211	11.281	0.232	0.333	0.000	1.516	3.368

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	477	962	1724	949	0	0	1018
N.S.	1	1.00	1.25	2.52	4.52	2.49	0.00	0.00	2.67
time (sec)	N/A	0.170	0.386	28.546	0.300	0.377	0.000	0.000	4.070

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	483	483	561	1450	2383	1416	0	0	1341
N.S.	1	1.00	1.16	3.00	4.93	2.93	0.00	0.00	2.78
time (sec)	N/A	0.236	0.683	53.995	0.328	0.433	0.000	0.000	4.597

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	587	587	671	2149	3819	2181	0	0	2400
N.S.	1	1.00	1.14	3.66	6.51	3.72	0.00	0.00	4.09
time (sec)	N/A	0.246	0.909	206.922	0.528	0.461	0.000	0.000	7.382

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	584	584	949	0	3764	0	0	0	0
N.S.	1	1.00	1.62	0.00	6.45	0.00	0.00	0.00	0.00
time (sec)	N/A	0.509	0.470	0.000	0.752	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	487	487	716	0	2691	0	0	0	0
N.S.	1	1.00	1.47	0.00	5.53	0.00	0.00	0.00	0.00
time (sec)	N/A	0.387	0.339	0.000	0.726	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	372	372	937	0	1542	0	0	0	0
N.S.	1	1.00	2.52	0.00	4.15	0.00	0.00	0.00	0.00
time (sec)	N/A	0.246	0.440	0.000	0.710	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	220	220	216	0	825	0	0	0	0
N.S.	1	1.00	0.98	0.00	3.75	0.00	0.00	0.00	0.00
time (sec)	N/A	0.161	0.075	0.000	0.692	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	306	306	1372	0	0	0	0	0	0
N.S.	1	1.00	4.48	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.265	0.588	0.000	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	261	261	1564	0	0	0	0	0	0
N.S.	1	1.00	5.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.233	1.092	0.000	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	801	530	2017	600	0	195	561
N.S.	1	1.00	5.30	3.51	13.36	3.97	0.00	1.29	3.72
time (sec)	N/A	0.087	0.502	4.830	0.315	0.345	0.000	1.705	3.313

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	1082	1099	3312	1167	0	501	993
N.S.	1	1.00	3.52	3.58	10.79	3.80	0.00	1.63	3.23
time (sec)	N/A	0.182	0.631	10.491	0.407	0.401	0.000	2.306	4.264

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	475	475	1319	2197	4838	1868	0	871	1794
N.S.	1	1.00	2.78	4.63	10.19	3.93	0.00	1.83	3.78
time (sec)	N/A	0.277	0.709	28.621	0.542	0.416	0.000	3.065	6.355

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	766	766	1634	0	5952	0	0	0	0
N.S.	1	1.00	2.13	0.00	7.77	0.00	0.00	0.00	0.00
time (sec)	N/A	0.732	0.837	0.000	0.824	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	819	819	1254	0	4247	0	0	0	0
N.S.	1	1.00	1.53	0.00	5.19	0.00	0.00	0.00	0.00
time (sec)	N/A	0.682	0.595	0.000	0.782	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	635	635	713	0	2662	0	0	0	0
N.S.	1	1.00	1.12	0.00	4.19	0.00	0.00	0.00	0.00
time (sec)	N/A	0.429	0.365	0.000	0.737	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	361	361	303	0	1473	0	0	0	0
N.S.	1	1.00	0.84	0.00	4.08	0.00	0.00	0.00	0.00
time (sec)	N/A	0.265	0.073	0.000	0.718	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	572	572	2852	0	0	0	0	0	0
N.S.	1	1.00	4.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.503	2.092	0.000	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	472	472	2885	0	0	0	0	0	0
N.S.	1	1.00	6.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.386	2.779	0.000	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	417	417	3662	0	0	0	0	0	0
N.S.	1	1.00	8.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.382	4.478	0.000	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	1418	825	5588	975	0	216	1195
N.S.	1	1.00	9.03	5.25	35.59	6.21	0.00	1.38	7.61
time (sec)	N/A	0.119	1.184	9.980	0.517	0.346	0.000	3.773	3.734

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	1787	1884	8087	1729	0	541	1934
N.S.	1	1.00	5.60	5.91	25.35	5.42	0.00	1.70	6.06
time (sec)	N/A	0.212	1.625	27.470	0.727	0.384	0.000	4.546	5.310

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	493	493	2112	2528	10936	2633	0	931	3296
N.S.	1	1.00	4.28	5.13	22.18	5.34	0.00	1.89	6.69
time (sec)	N/A	0.293	1.798	56.947	1.021	0.470	0.000	6.304	7.852

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1172	1172	2448	0	7845	0	0	0	0
N.S.	1	1.00	2.09	0.00	6.69	0.00	0.00	0.00	0.00
time (sec)	N/A	1.061	1.847	0.000	0.833	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	976	976	1627	0	5931	0	0	0	0
N.S.	1	1.00	1.67	0.00	6.08	0.00	0.00	0.00	0.00
time (sec)	N/A	0.903	0.837	0.000	0.784	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	786	786	945	0	3724	0	0	0	0
N.S.	1	1.00	1.20	0.00	4.74	0.00	0.00	0.00	0.00
time (sec)	N/A	0.593	0.441	0.000	0.764	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	454	454	409	0	2129	0	0	0	0
N.S.	1	1.00	0.90	0.00	4.69	0.00	0.00	0.00	0.00
time (sec)	N/A	0.350	0.121	0.000	0.725	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	762	762	4969	0	0	0	0	0	0
N.S.	1	1.00	6.52	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.762	7.267	0.000	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	739	739	4818	0	0	0	0	0	0
N.S.	1	1.00	6.52	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.545	6.329	0.000	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	644	644	6938	0	0	0	0	0	0
N.S.	1	1.00	10.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.451	6.860	0.000	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	561	561	8570	0	0	0	0	0	0
N.S.	1	1.00	15.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.498	7.503	0.000	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	768	768	1072	0	0	0	0	0	0
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.701	1.026	0.000	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	573	573	763	0	0	0	0	0	0
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.437	0.813	0.000	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	303	303	1385	0	0	0	0	0	0
N.S.	1	1.00	4.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.234	0.556	0.000	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	137	268	0	0	0	0	0	0
N.S.	1	1.00	1.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.081	0.522	0.000	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	90	118	407	149	0	167	122
N.S.	1	1.00	1.80	2.36	8.14	2.98	0.00	3.34	2.44
time (sec)	N/A	0.106	0.166	1.438	0.219	0.319	0.000	0.552	2.600

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	793	509	1018	428	0	0	361
N.S.	1	1.00	3.98	2.56	5.12	2.15	0.00	0.00	1.81
time (sec)	N/A	0.194	0.449	4.862	0.266	0.338	0.000	0.000	2.554

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	975	1259	2126	1068	0	196	1011
N.S.	1	1.00	2.64	3.41	5.76	2.89	0.00	0.53	2.74
time (sec)	N/A	0.276	0.767	11.703	0.337	0.349	0.000	277.816	5.248

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	543	1295	2216	3445	1910	0	0	1921
N.S.	1	1.00	2.38	4.08	6.34	3.52	0.00	0.00	3.54
time (sec)	N/A	0.332	1.001	25.753	0.462	0.416	0.000	0.000	7.198

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	770	770	5850	0	0	0	0	0	0
N.S.	1	1.00	7.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.503	6.978	0.000	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	500	500	2859	0	0	0	0	0	0
N.S.	1	1.00	5.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.344	3.969	0.000	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	282	282	1570	0	0	0	0	0	0
N.S.	1	1.00	5.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.190	1.010	0.000	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	331	294	428	263	0	175	237
N.S.	1	1.00	2.03	1.80	2.63	1.61	0.00	1.07	1.45
time (sec)	N/A	0.054	0.235	2.163	0.205	0.326	0.000	0.994	1.988

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	789	509	1014	432	0	368	365
N.S.	1	1.00	3.42	2.20	4.39	1.87	0.00	1.59	1.58
time (sec)	N/A	0.169	0.403	5.056	0.265	0.361	0.000	0.900	1.956

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	392	870	1307	2006	983	0	185	0
N.S.	1	1.00	2.22	3.33	5.12	2.51	0.00	0.47	0.00
time (sec)	N/A	0.258	0.617	9.718	0.309	0.380	0.000	285.296	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	560	560	1340	2001	4198	2052	0	0	1784
N.S.	1	1.00	2.39	3.57	7.50	3.66	0.00	0.00	3.19
time (sec)	N/A	0.322	0.925	28.097	0.508	0.365	0.000	0.000	7.250

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	729	729	1695	3041	6171	3183	0	0	3157
N.S.	1	1.00	2.33	4.17	8.47	4.37	0.00	0.00	4.33
time (sec)	N/A	0.396	1.405	44.816	0.680	0.435	0.000	0.000	8.352

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	676	676	6878	0	0	0	0	0	0
N.S.	1	1.00	10.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.399	7.019	0.000	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	441	441	3622	0	0	0	0	0	0
N.S.	1	1.00	8.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.297	4.776	0.000	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	803	530	1995	600	0	195	565
N.S.	1	1.00	5.32	3.51	13.21	3.97	0.00	1.29	3.74
time (sec)	N/A	0.078	0.558	4.680	0.306	0.353	0.000	1.913	4.172

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	464	672	861	654	0	407	505
N.S.	1	1.00	1.46	2.12	2.72	2.06	0.00	1.28	1.59
time (sec)	N/A	0.116	0.222	4.859	0.239	0.332	0.000	1.088	3.175

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	971	1236	2126	1076	0	747	1007
N.S.	1	1.00	2.42	3.07	5.29	2.68	0.00	1.86	2.50
time (sec)	N/A	0.297	0.519	11.421	0.331	0.322	0.000	1.195	5.346

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	562	562	1334	2000	4199	2057	0	0	1785
N.S.	1	1.00	2.37	3.56	7.47	3.66	0.00	0.00	3.18
time (sec)	N/A	0.315	0.892	28.472	0.516	0.348	0.000	0.000	7.089

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	732	732	1653	3006	5594	3062	0	0	2419
N.S.	1	1.00	2.26	4.11	7.64	4.18	0.00	0.00	3.30
time (sec)	N/A	0.375	1.168	54.526	0.661	0.392	0.000	0.000	8.431

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	908	908	2138	4486	9293	4725	0	0	4649
N.S.	1	1.00	2.35	4.94	10.23	5.20	0.00	0.00	5.12
time (sec)	N/A	0.441	1.896	205.155	1.036	0.485	0.000	0.000	10.108

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	189	189	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	190	190	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.205	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	292	292	206	0	0	2680	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	9.18	0.00	0.00	0.00
time (sec)	N/A	0.214	4.714	0.000	0.000	0.419	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	134	2620	0	991	0	0	0
N.S.	1	1.00	0.64	12.48	0.00	4.72	0.00	0.00	0.00
time (sec)	N/A	0.166	1.908	134.763	0.000	0.386	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	78	817	0	274	0	0	0
N.S.	1	1.00	0.61	6.38	0.00	2.14	0.00	0.00	0.00
time (sec)	N/A	0.104	0.634	32.119	0.000	0.324	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	125	125	0	0	0	98	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	0.186	0.000	0.000	0.000	0.306	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	206	206	0	0	0	293	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.218	0.000	0.000	0.000	0.307	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	295	295	0	0	0	818	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	2.77	0.00	0.00	0.00
time (sec)	N/A	0.258	0.000	0.000	0.000	0.341	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	309	309	206	0	0	2669	0	0	0
N.S.	1	1.00	0.67	0.00	0.00	8.64	0.00	0.00	0.00
time (sec)	N/A	0.213	4.815	0.000	0.000	0.438	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	134	2619	0	983	0	0	0
N.S.	1	1.00	0.60	11.74	0.00	4.41	0.00	0.00	0.00
time (sec)	N/A	0.174	1.784	131.303	0.000	0.344	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	78	821	0	269	0	0	0
N.S.	1	1.00	0.57	5.99	0.00	1.96	0.00	0.00	0.00
time (sec)	N/A	0.111	0.571	31.007	0.000	0.365	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	128	128	0	0	0	93	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.182	0.000	0.000	0.000	0.284	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	214	214	0	0	0	284	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	1.33	0.00	0.00	0.00
time (sec)	N/A	0.223	0.000	0.000	0.000	0.321	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	306	306	0	0	0	815	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	2.66	0.00	0.00	0.00
time (sec)	N/A	0.268	0.000	0.000	0.000	0.389	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	40	43	0	65	0	41	0
N.S.	1	1.00	0.98	1.05	0.00	1.59	0.00	1.00	0.00
time (sec)	N/A	0.084	0.022	7.372	0.000	0.333	0.000	0.293	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	40	63	0	65	0	41	0
N.S.	1	1.00	0.98	1.54	0.00	1.59	0.00	1.00	0.00
time (sec)	N/A	0.106	0.002	10.279	0.000	0.304	0.000	0.299	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	193	193	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.308	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	194	194	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.297	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	44	766	375	0	0	141
N.S.	1	1.00	0.96	0.98	17.02	8.33	0.00	0.00	3.13
time (sec)	N/A	0.138	0.016	33.157	0.247	0.315	0.000	0.000	2.613

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	44	387	188	0	0	100
N.S.	1	1.00	0.96	0.98	8.60	4.18	0.00	0.00	2.22
time (sec)	N/A	0.143	0.014	5.816	0.219	0.332	0.000	0.000	1.487

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	62	151	72	0	0	71
N.S.	1	1.00	0.96	1.38	3.36	1.60	0.00	0.00	1.58
time (sec)	N/A	0.095	0.015	3.843	0.210	0.310	0.000	0.000	1.363

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	39	43	49	45	0	41	40
N.S.	1	1.00	0.95	1.05	1.20	1.10	0.00	1.00	0.98
time (sec)	N/A	0.153	0.061	21.342	0.295	0.303	0.000	0.302	1.293

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	41	43	81	86	0	99	42
N.S.	1	1.00	0.95	1.00	1.88	2.00	0.00	2.30	0.98
time (sec)	N/A	0.153	0.015	65.195	0.321	0.329	0.000	0.286	1.251

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	44	220	238	0	321	72
N.S.	1	1.00	0.96	0.98	4.89	5.29	0.00	7.13	1.60
time (sec)	N/A	0.151	0.017	163.308	0.364	0.327	0.000	0.292	1.284

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	47	51	0	81	0	49	0
N.S.	1	1.00	0.96	1.04	0.00	1.65	0.00	1.00	0.00
time (sec)	N/A	0.183	0.022	234.622	0.000	0.346	0.000	0.292	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	51	57	0	85	0	53	0
N.S.	1	1.00	0.93	1.04	0.00	1.55	0.00	0.96	0.00
time (sec)	N/A	0.243	0.034	0.023	0.000	0.301	0.000	0.299	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	52	52	50	0	0	83	0	51	0
N.S.	1	1.00	0.96	0.00	0.00	1.60	0.00	0.98	0.00
time (sec)	N/A	0.178	0.003	0.000	0.000	0.306	0.000	0.283	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	39	43	49	45	0	41	40
N.S.	1	1.00	0.95	1.05	1.20	1.10	0.00	1.00	0.98
time (sec)	N/A	0.158	0.007	20.474	0.295	0.294	0.000	0.274	0.002

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	43	49	53	51	0	45	44
N.S.	1	1.00	0.91	1.04	1.13	1.09	0.00	0.96	0.94
time (sec)	N/A	0.205	0.100	66.617	0.300	0.338	0.000	0.275	1.341

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	42	46	51	48	0	43	42
N.S.	1	1.00	0.95	1.05	1.16	1.09	0.00	0.98	0.95
time (sec)	N/A	0.161	0.008	55.309	0.297	0.312	0.000	0.291	1.204

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.181	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	110	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	1.47	0.00	0.00	0.00
time (sec)	N/A	0.075	0.023	0.000	0.000	0.315	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	88	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	0.076	0.018	0.000	0.000	0.308	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	66	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.054	0.016	0.000	0.000	0.290	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	72	0	0	40	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.038	0.067	0.000	0.000	0.289	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	34	35	37	34	146	88	33
N.S.	1	1.00	1.03	1.06	1.12	1.03	4.42	2.67	1.00
time (sec)	N/A	0.066	0.053	1.105	0.301	0.293	56.715	0.486	1.167

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	71	0	0	40	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.043	0.058	0.000	0.000	0.280	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	66	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.059	0.013	0.000	0.000	0.329	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	88	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	0.075	0.015	0.000	0.000	0.297	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	361	361	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.423	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	282	282	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.352	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	203	203	1415	0	0	0	0	0	0
N.S.	1	1.00	6.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.297	0.776	0.000	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	123	123	304	1447	0	0	0	0	0
N.S.	1	1.00	2.47	11.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.185	0.257	22.578	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	43	43	45	43	45	76	0	45	45
N.S.	1	1.00	1.05	1.00	1.05	1.77	0.00	1.05	1.05
time (sec)	N/A	0.167	0.169	0.228	0.342	0.304	0.000	0.315	1.238

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	43	43	45	43	506	145	0	45	45
N.S.	1	1.00	1.05	1.00	11.77	3.37	0.00	1.05	1.05
time (sec)	N/A	0.156	0.328	0.026	0.378	0.306	0.000	0.342	1.288

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	298	42	357	32	0	0	0
N.S.	1	1.00	9.03	1.27	10.82	0.97	0.00	0.00	0.00
time (sec)	N/A	0.070	0.130	1.342	0.204	0.318	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	375	46	344	38	0	0	0
N.S.	1	1.00	13.89	1.70	12.74	1.41	0.00	0.00	0.00
time (sec)	N/A	0.048	0.224	1.898	0.201	0.310	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	375	46	336	38	0	0	0
N.S.	1	1.00	13.89	1.70	12.44	1.41	0.00	0.00	0.00
time (sec)	N/A	0.083	0.170	1.823	0.203	0.278	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	375	46	343	38	0	0	0
N.S.	1	1.00	13.89	1.70	12.70	1.41	0.00	0.00	0.00
time (sec)	N/A	0.087	0.013	1.861	0.201	0.289	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	282	282	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.415	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	203	203	1415	0	0	0	0	0	0
N.S.	1	1.00	6.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.348	0.573	0.000	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	123	123	303	1447	0	0	0	0	0
N.S.	1	1.00	2.46	11.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.206	0.178	25.905	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	53	51	53	76	0	53	53
N.S.	1	1.00	1.04	1.00	1.04	1.49	0.00	1.04	1.04
time (sec)	N/A	0.201	0.088	0.394	0.345	0.346	0.000	0.307	1.127

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	53	51	506	145	0	53	53
N.S.	1	1.00	1.04	1.00	9.92	2.84	0.00	1.04	1.04
time (sec)	N/A	0.185	0.139	0.033	0.408	0.290	0.000	0.346	1.213

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [74] had the largest ratio of [.3332999999999999985]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.00	38	0.105
2	A	5	4	1.00	38	0.105
3	A	5	4	1.00	36	0.111
4	A	4	3	1.00	28	0.107
5	A	6	6	1.00	38	0.158
6	A	5	5	1.00	38	0.132
7	A	2	2	1.00	38	0.053
8	A	5	4	1.00	38	0.105
9	A	5	5	1.00	38	0.132
10	A	5	5	1.00	40	0.125
11	A	5	5	1.00	40	0.125
12	A	5	5	1.00	38	0.132
13	A	4	3	1.00	30	0.100
14	A	10	8	1.00	40	0.200
15	A	8	8	1.00	40	0.200
16	A	7	5	1.00	40	0.125
17	A	2	2	1.00	40	0.050
18	A	5	4	1.00	40	0.100
19	A	5	5	1.00	40	0.125
20	A	5	5	1.00	40	0.125
21	A	5	5	1.00	40	0.125
22	A	5	5	1.00	38	0.132
23	A	4	3	1.00	30	0.100
24	A	14	8	1.00	40	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	11	9	1.00	40	0.225
26	A	9	8	1.00	40	0.200
27	A	9	5	1.00	40	0.125
28	A	2	2	1.00	40	0.050
29	A	5	4	1.00	40	0.100
30	A	5	5	1.00	40	0.125
31	A	6	4	1.00	40	0.100
32	A	5	4	1.00	40	0.100
33	A	4	4	1.00	38	0.105
34	A	5	5	1.00	30	0.167
35	A	2	2	1.00	40	0.050
36	A	5	5	1.00	40	0.125
37	A	7	7	1.00	40	0.175
38	A	8	6	1.00	40	0.150
39	A	9	7	1.00	40	0.175
40	A	8	7	1.00	40	0.175
41	A	7	6	1.00	38	0.158
42	A	3	2	1.00	30	0.067
43	A	5	4	1.00	40	0.100
44	A	4	4	1.00	40	0.100
45	A	8	6	1.00	40	0.150
46	A	4	4	1.00	40	0.100
47	A	9	8	1.00	40	0.200
48	A	8	7	1.00	40	0.175
49	A	2	2	1.00	38	0.053
50	A	4	3	1.00	30	0.100
51	A	4	4	1.00	40	0.100
52	A	4	4	1.00	40	0.100
53	A	5	4	1.00	40	0.100
54	A	8	6	1.00	40	0.150
55	A	11	8	1.00	40	0.200
56	A	10	8	1.00	40	0.200
57	A	9	8	1.00	38	0.210
58	A	7	7	1.00	30	0.233
59	A	8	8	1.00	40	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	7	7	1.00	40	0.175
61	A	3	3	1.00	40	0.075
62	A	7	4	1.00	40	0.100
63	A	9	4	1.00	40	0.100
64	A	17	13	1.00	42	0.310
65	A	15	11	1.00	42	0.262
66	A	14	11	1.00	40	0.275
67	A	11	8	1.00	32	0.250
68	A	15	11	1.00	42	0.262
69	A	11	10	1.00	42	0.238
70	A	10	7	1.00	42	0.167
71	A	3	3	1.00	42	0.071
72	A	7	4	1.00	42	0.095
73	A	9	4	1.00	42	0.095
74	A	22	14	1.00	42	0.333
75	A	20	11	1.00	42	0.262
76	A	19	11	1.00	40	0.275
77	A	15	8	1.00	32	0.250
78	A	26	12	1.00	42	0.286
79	A	17	14	1.00	42	0.333
80	A	13	10	1.00	42	0.238
81	A	3	3	1.00	42	0.071
82	A	7	4	1.00	42	0.095
83	A	9	4	1.00	42	0.095
84	A	25	13	1.00	42	0.310
85	A	15	12	1.00	42	0.286
86	A	9	7	1.00	40	0.175
87	A	4	4	1.00	32	0.125
88	A	3	3	1.00	42	0.071
89	A	7	6	1.00	42	0.143
90	A	9	6	1.00	42	0.143
91	A	11	6	1.00	42	0.143
92	A	18	14	1.00	42	0.333
93	A	12	9	1.00	42	0.214
94	A	9	7	1.00	40	0.175

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	4	3	1.00	32	0.094
96	A	7	6	1.00	42	0.143
97	A	10	8	1.00	42	0.190
98	A	12	8	1.00	42	0.190
99	A	14	8	1.00	42	0.190
100	A	14	11	1.00	42	0.262
101	A	11	9	1.00	42	0.214
102	A	3	3	1.00	40	0.075
103	A	8	6	1.00	32	0.188
104	A	15	9	1.00	42	0.214
105	A	12	8	1.00	42	0.190
106	A	14	8	1.00	42	0.190
107	A	16	8	1.00	42	0.190
108	A	5	4	1.00	41	0.098
109	A	5	4	1.00	41	0.098
110	A	5	4	1.00	39	0.103
111	A	4	3	1.00	31	0.097
112	A	6	6	1.00	41	0.146
113	A	5	5	1.00	41	0.122
114	A	2	2	1.00	41	0.049
115	A	5	4	1.00	41	0.098
116	A	5	5	1.00	41	0.122
117	A	5	5	1.00	43	0.116
118	A	5	5	1.00	43	0.116
119	A	5	5	1.00	41	0.122
120	A	4	3	1.00	33	0.091
121	A	10	8	1.00	43	0.186
122	A	8	8	1.00	43	0.186
123	A	7	5	1.00	43	0.116
124	A	2	2	1.00	43	0.047
125	A	5	4	1.00	43	0.093
126	A	5	5	1.00	43	0.116
127	A	5	5	1.00	43	0.116
128	A	5	5	1.00	43	0.116
129	A	5	5	1.00	41	0.122

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	4	3	1.00	33	0.091
131	A	14	8	1.00	43	0.186
132	A	11	9	1.00	43	0.209
133	A	9	8	1.00	43	0.186
134	A	9	5	1.00	43	0.116
135	A	6	4	1.00	43	0.093
136	A	5	4	1.00	43	0.093
137	A	4	4	1.00	41	0.098
138	A	5	5	1.00	33	0.152
139	A	2	2	1.00	43	0.047
140	A	5	5	1.00	43	0.116
141	A	7	7	1.00	43	0.163
142	A	8	6	1.00	43	0.140
143	A	9	7	1.00	43	0.163
144	A	8	7	1.00	43	0.163
145	A	7	6	1.00	41	0.146
146	A	3	2	1.00	33	0.061
147	A	5	4	1.00	43	0.093
148	A	4	4	1.00	43	0.093
149	A	8	6	1.00	43	0.140
150	A	4	4	1.00	43	0.093
151	A	9	8	1.00	43	0.186
152	A	8	7	1.00	43	0.163
153	A	2	2	1.00	41	0.049
154	A	4	3	1.00	33	0.091
155	A	4	4	1.00	43	0.093
156	A	4	4	1.00	43	0.093
157	A	5	4	1.00	43	0.093
158	A	8	6	1.00	43	0.140
159	A	11	8	1.00	43	0.186
160	A	10	8	1.00	43	0.186
161	A	9	8	1.00	41	0.195
162	A	7	7	1.00	33	0.212
163	A	8	8	1.00	43	0.186
164	A	7	7	1.00	43	0.163

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	3	3	1.00	43	0.070
166	A	7	4	1.00	43	0.093
167	A	9	4	1.00	43	0.093
168	A	17	13	1.00	45	0.289
169	A	15	11	1.00	45	0.244
170	A	14	11	1.00	43	0.256
171	A	11	8	1.00	35	0.229
172	A	15	11	1.00	45	0.244
173	A	11	10	1.00	45	0.222
174	A	10	7	1.00	45	0.156
175	A	3	3	1.00	45	0.067
176	A	7	4	1.00	45	0.089
177	A	9	4	1.00	45	0.089
178	A	22	14	1.00	45	0.311
179	A	20	11	1.00	45	0.244
180	A	19	11	1.00	43	0.256
181	A	15	8	1.00	35	0.229
182	A	26	12	1.00	45	0.267
183	A	17	14	1.00	45	0.311
184	A	13	10	1.00	45	0.222
185	A	13	7	1.00	45	0.156
186	A	25	13	1.00	45	0.289
187	A	15	12	1.00	45	0.267
188	A	9	7	1.00	43	0.163
189	A	4	4	1.00	35	0.114
190	A	3	3	1.00	45	0.067
191	A	7	6	1.00	45	0.133
192	A	9	6	1.00	45	0.133
193	A	11	6	1.00	45	0.133
194	A	18	14	1.00	45	0.311
195	A	12	9	1.00	45	0.200
196	A	9	7	1.00	43	0.163
197	A	4	3	1.00	35	0.086
198	A	7	6	1.00	45	0.133
199	A	10	8	1.00	45	0.178

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	12	8	1.00	45	0.178
201	A	14	8	1.00	45	0.178
202	A	14	11	1.00	45	0.244
203	A	11	9	1.00	45	0.200
204	A	3	3	1.00	43	0.070
205	A	8	6	1.00	35	0.171
206	A	15	9	1.00	45	0.200
207	A	12	8	1.00	45	0.178
208	A	14	8	1.00	45	0.178
209	A	16	8	1.00	45	0.178
210	A	3	3	1.00	49	0.061
211	A	3	3	1.00	49	0.061
212	A	4	3	1.00	49	0.061
213	A	3	3	1.00	49	0.061
214	A	2	2	1.00	47	0.043
215	A	3	3	1.00	49	0.061
216	A	4	4	1.00	49	0.082
217	A	5	4	1.00	49	0.082
218	A	4	3	1.00	49	0.061
219	A	3	3	1.00	49	0.061
220	A	2	2	1.00	47	0.043
221	A	3	3	1.00	49	0.061
222	A	4	4	1.00	49	0.082
223	A	5	4	1.00	49	0.082
224	A	3	3	1.00	35	0.086
225	A	4	4	1.00	42	0.095
226	A	4	4	1.00	50	0.080
227	A	4	4	1.00	50	0.080
228	A	4	4	1.00	40	0.100
229	A	4	4	1.00	40	0.100
230	A	3	3	1.00	38	0.079
231	A	4	4	1.00	40	0.100
232	A	4	4	1.00	40	0.100
233	A	4	4	1.00	40	0.100
234	A	4	4	1.00	40	0.100

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	4	4	1.00	46	0.087
236	A	5	5	1.00	50	0.100
237	A	4	4	1.00	40	0.100
238	A	4	4	1.00	46	0.087
239	A	5	5	1.00	50	0.100
240	A	4	4	1.00	40	0.100
241	A	3	3	1.00	35	0.086
242	A	3	3	1.00	35	0.086
243	A	3	3	1.00	33	0.091
244	A	3	3	1.00	28	0.107
245	A	3	3	1.00	35	0.086
246	A	3	3	1.00	28	0.107
247	A	3	3	1.00	33	0.091
248	A	3	3	1.00	35	0.086
249	A	8	7	1.00	43	0.163
250	A	7	7	1.00	43	0.163
251	A	6	6	1.00	43	0.140
252	A	5	5	1.00	41	0.122
253	N/A	0	0	1.00	43	0.000
254	N/A	0	0	1.00	43	0.000
255	A	2	2	1.00	40	0.050
256	A	2	2	1.00	38	0.053
257	A	3	3	1.00	33	0.091
258	A	3	3	1.00	38	0.079
259	A	8	8	1.00	51	0.157
260	A	7	7	1.00	51	0.137
261	A	6	6	1.00	49	0.122
262	N/A	0	0	1.00	51	0.000
263	N/A	0	0	1.00	51	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 212

$$\begin{aligned}\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx\\ =-\frac{B(bc-ad)^4g^3ix}{20bd^3}+\frac{B(bc-ad)^3g^3i(a+bx)^2}{40b^2d^2}\\ -\frac{B(bc-ad)^2g^3i(a+bx)^3}{60b^2d}+\frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5b}\\ +\frac{(bc-ad)g^3i(a+bx)^4\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^2}+\frac{B(bc-ad)^5g^3i\log(c+dx)}{20b^2d^4}\end{aligned}$$

```
[Out] -1/20*B*(-a*d+b*c)^4*g^3*i*x/b/d^3+1/40*B*(-a*d+b*c)^3*g^3*i*(b*x+a)^2/b^2/
d^2-1/60*B*(-a*d+b*c)^2*g^3*i*(b*x+a)^3/b^2/d+1/5*g^3*i*(b*x+a)^4*(d*x+c)*(
A+B*ln(e*(b*x+a)/(d*x+c)))/b+1/20*(-a*d+b*c)*g^3*i*(b*x+a)^4*(A-B+B*ln(e*(b
*x+a)/(d*x+c)))/b^2+1/20*B*(-a*d+b*c)^5*g^3*i*ln(d*x+c)/b^2/d^4
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used

$= \{2560, 2548, 21, 45\}$

$$\int (ag + bgx)^3 (ci + dix) \left(A + B \log \left(\frac{e(a + bx)}{c + dx} \right) \right) dx$$
$$= \frac{g^3 i (a + bx)^4 (bc - ad) \left(B \log \left(\frac{e(a+bx)}{c+dx} \right) + A - B \right)}{20b^2} + \frac{g^3 i (a + bx)^4 (c + dx) \left(B \log \left(\frac{e(a+bx)}{c+dx} \right) + A \right)}{5b} + \frac{Bg^3 i (bc - ad)^5 \log(c + dx)}{20b^2 d^4} + \frac{Bg^3 i (a + bx)^2 (bc - ad)^3}{40b^2 d^2} - \frac{Bg^3 i (a + bx)^3 (bc - ad)^2}{60b^2 d} - \frac{Bg^3 ix (bc - ad)^4}{20bd^3}$$

```
[In] Int[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -1/20*(B*(b*c - a*d)^4*g^3*i*x)/(b*d^3) + (B*(b*c - a*d)^3*g^3*i*(a + b*x)^
2)/(40*b^2*d^2) - (B*(b*c - a*d)^2*g^3*i*(a + b*x)^3)/(60*b^2*d) + (g^3*i*(
a + b*x)^4*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*b) + ((b*c -
a*d)*g^3*i*(a + b*x)^4*(A - B + B*Log[(e*(a + b*x))/(c + d*x)]))/(20*b^2) +
 (B*(b*c - a*d)^5*g^3*i*Log[c + d*x])/(20*b^2*d^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rule 2560

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Sim
```

```
p[(f + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*
(m + 2))), x] + Dist[i*((b*c - a*d)/(b*d*(m + 2))), Int[(f + g*x)^m*(A - B*
n + B*Log[e*((a + b*x)^n/(c + d*x)^n)]), x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, i, A, B, m, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d,
 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IGtQ[m, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g^3 i(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5b} + \frac{((bc-ad)i)\int (ag+bgx)^3\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx}{5b}\\
&= \frac{g^3 i(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5b} + \frac{(bc-ad)g^3 i(a+bx)^4\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^2} - \frac{\left(B(bc-ad)^2 i\right)\int \frac{(ag+bgx)^4}{(a+bx)(c+dx)}dx}{20b^2 g}\\
&= \frac{g^3 i(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5b} + \frac{(bc-ad)g^3 i(a+bx)^4\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^2} - \frac{\left(B(bc-ad)^2 g^3 i\right)\int \frac{(a+bx)^3}{c+dx}dx}{20b^2}\\
&= \frac{g^3 i(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5b} + \frac{(bc-ad)g^3 i(a+bx)^4\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^2} - \frac{\left(B(bc-ad)^2 g^3 i\right)\int \left(\frac{b(bc-ad)^2}{d^3} - \frac{b(bc-ad)(a+bx)}{d^2} + \frac{b(a+bx)^2}{d} + \frac{(-bc+ad)^3}{d^3(c+dx)}\right)dx}{20b^2}\\
&= -\frac{B(bc-ad)^4 g^3 ix}{20bd^3} + \frac{B(bc-ad)^3 g^3 i(a+bx)^2}{40b^2d^2} - \frac{B(bc-ad)^2 g^3 i(a+bx)^3}{60b^2 d} + \frac{g^3 i(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5b} + \frac{(bc-ad)g^3 i(a+bx)^4\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^2} + \frac{B(bc-ad)^5 g^3 i\log(c+dx)}{20b^2d^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.23

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{g^3i\left(30(bc-ad)(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+24d(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-\frac{5B(bc-ad)^2(6bd(bc}{}\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]
),x]
```

```
[Out] (g^3*i*(30*(b*c - a*d)*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2
4*d*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (5*B*(b*c - a*d)^2*(
6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^
3 - 6*(b*c - a*d)^3*Log[c + d*x]))/d^4 + (2*B*(b*c - a*d)*(12*b*d*(b*c - a*
d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c - a*d)*(a + b*x)^3 -
3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]))/d^4))/(120*b^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(200) = 400$.

Time = 0.96 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.71

method	result
risch	$\frac{i\,g^3b^3Ac\,x^4}{4}+\frac{i\,g^3b^2dBa\,x^4}{20}-\frac{i\,g^3b^3Bc\,x^4}{20}+i\,g^3bdA\,a^2x^3+\frac{11i\,g^3bdB\,a^2x^3}{60}-\frac{i\,g^3b^3B\,c^2x^3}{60d}+\frac{i\,g^3dA\,a^3x}{2}$
parallelrisch	Expression too large to display
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/4*i*g^3*b^3*A*c*x^4+1/20*i*g^3*b^2*d*B*a*x^4-1/20*i*g^3*b^3*B*c*x^4+i*g^3
*b*d*A*a^2*x^3+11/60*i*g^3*b*d*B*a^2*x^3-1/60*i*g^3*b^3/d*B*c^2*x^3+1/2*i*g
^3*d*A*a^3*x^2+9/40*i*g^3*d*B*a^3*x^2+1/40*i*g^3*b^3/d^2*B*c^3*x^2+i*g^3*b^
2*A*a*c*x^3-1/6*i*g^3*b^2*B*a*c*x^3+3/2*i*g^3*b*A*a^2*c*x^2-1/8*i*g^3*b*B*a
^2*c*x^2-1/8*i*g^3*b^2/d*B*a*c^2*x^2+i*g^3*A*a^3*c*x+1/20*i*g^3/b*d*B*a^4*x
-1/20*i*g^3*b^3/d^3*B*c^4*x-1/2*i*g^3/d*B*ln(-d*x-c)*a^3*c^2+1/4*i*g^3*B*a^
3*c*x-1/2*i*g^3*b/d*B*a^2*c^2*x+1/4*i*g^3*b^2/d^2*B*a*c^3*x+1/2*i*g^3*b/d^2
*B*ln(-d*x-c)*a^2*c^3-1/4*i*g^3*b^2/d^3*B*ln(-d*x-c)*a*c^4+1/20*i*g^3*b^3/d
^4*B*ln(-d*x-c)*c^5-1/20*i*g^3/b^2*d*B*ln(b*x+a)*a^5+1/4*i*g^3/b*B*ln(b*x+a
```

```
)*a^4*c+3/4*i*g^3*b^2*d*A*a*x^4+1/5*i*g^3*b^3*d*A*x^5+1/20*i*g^3*B*x*(4*b^3
*d*x^4+15*a*b^2*d*x^3+5*b^3*c*x^3+20*a^2*b*d*x^2+20*a*b^2*c*x^2+10*a^3*d*x+
30*a^2*b*c*x+20*a^3*c)*ln(e*(b*x+a)/(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(200) = 400$.

Time = 0.37 (sec) , antiderivative size = 504, normalized size of antiderivative = 2.38

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{24\,Ab^5d^5g^3ix^5+6\,((5\,A-B)b^5cd^4+(15\,A+B)ab^4d^5)g^3ix^4-2\,(Bb^5c^2d^3-10\,(6\,A-B)ab^4cd^4-(60\,A+$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorit
hm="fricas")
```

```
[Out] 1/120*(24*A*b^5*d^5*g^3*i*x^5 + 6*((5*A - B)*b^5*c*d^4 + (15*A + B)*a*b^4*d
^5)*g^3*i*x^4 - 2*(B*b^5*c^2*d^3 - 10*(6*A - B)*a*b^4*c*d^4 - (60*A + 11*B)
*a^2*b^3*d^5)*g^3*i*x^3 + 3*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 5*(12*A -
B)*a^2*b^3*c*d^4 + (20*A + 9*B)*a^3*b^2*d^5)*g^3*i*x^2 - 6*(B*b^5*c^4*d - 5
*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3*c^2*d^3 - 5*(4*A + B)*a^3*b^2*c*d^4 - B*a^4
*b*d^5)*g^3*i*x + 6*(5*B*a^4*b*c*d^4 - B*a^5*d^5)*g^3*i*log(b*x + a) + 6*(B
*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2*d^3)*g
^3*i*log(d*x + c) + 6*(4*B*b^5*d^5*g^3*i*x^5 + 20*B*a^3*b^2*c*d^4*g^3*i*x +
 5*(B*b^5*c*d^4 + 3*B*a*b^4*d^5)*g^3*i*x^4 + 20*(B*a*b^4*c*d^4 + B*a^2*b^3*
d^5)*g^3*i*x^3 + 10*(3*B*a^2*b^3*c*d^4 + B*a^3*b^2*d^5)*g^3*i*x^2)*log((b*e
*x + a*e)/(d*x + c)))/(b^2*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1158 vs. $2(194) = 388$.

Time = 4.09 (sec) , antiderivative size = 1158, normalized size of antiderivative = 5.46

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{Ab^3dg^3ix^5}{5}$$

$$-\frac{Ba^4g^3i(ad-5bc)\log\left(x+\frac{Ba^5cd^4g^3i+\frac{Ba^5d^4g^3i(ad-5bc)}{b}-15Ba^4bc^2d^3g^3i-Ba^4cd^3g^3i(ad-5bc)+10Ba^3b^2c^3d^2g^3i-5Ba^2b^3c^4dg^3}{Ba^5d^5g^3i-5Ba^4bcd^4g^3i-10Ba^3b^2c^2d^3g^3i+10Ba^2b^3c^3d^2g^3i-5Bab^4c^4dg^3i+Bb^5c^5g^3i}\right.}{20b^2}$$

$$-\frac{Bc^2g^3i(10a^3d^3-10a^2bcd^2+5ab^2c^2d-b^3c^3)\log\left(x+\frac{Ba^5cd^4g^3i-15Ba^4bc^2d^3g^3i+10Ba^3b^2c^3d^2g^3i-5Ba^2b^3c^4dg^3i+B}{Ba^5d^5g^3i-5Ba^4bcd^4g^3i-}\right.}{20d^4}$$

$$+x^4\cdot\left(\frac{3Aab^2dg^3i}{4}+\frac{Ab^3cg^3i}{4}+\frac{Bab^2dg^3i}{20}-\frac{Bb^3cg^3i}{20}\right)$$

$$+x^3\left(Aa^2bdg^3i+Aab^2cg^3i+\frac{11Ba^2bdg^3i}{60}-\frac{Bab^2cg^3i}{6}-\frac{Bb^3c^2g^3i}{60d}\right)$$

$$+x^2\left(\frac{Aa^3dg^3i}{2}+\frac{3Aa^2bcg^3i}{2}+\frac{9Ba^3dg^3i}{40}-\frac{Ba^2bcg^3i}{8}-\frac{Bab^2c^2g^3i}{8d}+\frac{Bb^3c^3g^3i}{40d^2}\right)$$

$$+x\left(Aa^3cg^3i+\frac{Ba^4dg^3i}{20b}+\frac{Ba^3cg^3i}{4}-\frac{Ba^2bc^2g^3i}{2d}+\frac{Bab^2c^3g^3i}{4d^2}-\frac{Bb^3c^4g^3i}{20d^3}\right)$$

$$+\left(Ba^3cg^3ix+\frac{Ba^3dg^3ix^2}{2}+\frac{3Ba^2bcg^3ix^2}{2}+Ba^2bdg^3ix^3+Bab^2cg^3ix^3+\frac{3Bab^2dg^3ix^4}{4}+\frac{Bb^3cg^3ix^4}{4}+\frac{Bb^3dg^3ix^5}{5}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

```
[In] integrate((b*g*x+a*g)**3*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b**3*d*g**3*i*x**5/5 - B*a**4*g**3*i*(a*d - 5*b*c)*log(x + (B*a**5*c*d**4
*g**3*i + B*a**5*d**4*g**3*i*(a*d - 5*b*c)/b - 15*B*a**4*b*c**2*d**3*g**3*i
 - B*a**4*c*d**3*g**3*i*(a*d - 5*b*c) + 10*B*a**3*b**2*c**3*d**2*g**3*i - 5
*B*a**2*b**3*c**4*d*g**3*i + B*a*b**4*c**5*g**3*i)/(B*a**5*d**5*g**3*i - 5*
B*a**4*b*c*d**4*g**3*i - 10*B*a**3*b**2*c**2*d**3*g**3*i + 10*B*a**2*b**3*c
**3*d**2*g**3*i - 5*B*a*b**4*c**4*d*g**3*i + B*b**5*c**5*g**3*i))/(20*b**2)
 - B*c**2*g**3*i*(10*a**3*d**3 - 10*a**2*b*c*d**2 + 5*a*b**2*c**2*d - b**3*
c**3)*log(x + (B*a**5*c*d**4*g**3*i - 15*B*a**4*b*c**2*d**3*g**3*i + 10*B*a
**3*b**2*c**3*d**2*g**3*i - 5*B*a**2*b**3*c**4*d*g**3*i + B*a*b**4*c**5*g**
3*i + B*a*b*c**2*g**3*i*(10*a**3*d**3 - 10*a**2*b*c*d**2 + 5*a*b**2*c**2*d
- b**3*c**3) - B*b**2*c**3*g**3*i*(10*a**3*d**3 - 10*a**2*b*c*d**2 + 5*a*b*
*2*c**2*d - b**3*c**3)/d)/(B*a**5*d**5*g**3*i - 5*B*a**4*b*c*d**4*g**3*i -
10*B*a**3*b**2*c**2*d**3*g**3*i + 10*B*a**2*b**3*c**3*d**2*g**3*i - 5*B*a*b
**4*c**4*d*g**3*i + B*b**5*c**5*g**3*i))/(20*d**4) + x**4*(3*A*a*b**2*d*g**
3*i/4 + A*b**3*c*g**3*i/4 + B*a*b**2*d*g**3*i/20 - B*b**3*c*g**3*i/20) + x*
```

```
*3*(A*a**2*b*d*g**3*i + A*a*b**2*c*g**3*i + 11*B*a**2*b*d*g**3*i/60 - B*a*b
**2*c*g**3*i/6 - B*b**3*c**2*g**3*i/(60*d)) + x**2*(A*a**3*d*g**3*i/2 + 3*A
*a**2*b*c*g**3*i/2 + 9*B*a**3*d*g**3*i/40 - B*a**2*b*c*g**3*i/8 - B*a*b**2*
c**2*g**3*i/(8*d) + B*b**3*c**3*g**3*i/(40*d**2)) + x*(A*a**3*c*g**3*i + B*
a**4*d*g**3*i/(20*b) + B*a**3*c*g**3*i/4 - B*a**2*b*c**2*g**3*i/(2*d) + B*a
*b**2*c**3*g**3*i/(4*d**2) - B*b**3*c**4*g**3*i/(20*d**3)) + (B*a**3*c*g**3
*i*x + B*a**3*d*g**3*i*x**2/2 + 3*B*a**2*b*c*g**3*i*x**2/2 + B*a**2*b*d*g**
3*i*x**3 + B*a*b**2*c*g**3*i*x**3 + 3*B*a*b**2*d*g**3*i*x**4/4 + B*b**3*c*g
**3*i*x**4/4 + B*b**3*d*g**3*i*x**5/5)*log(e*(a + b*x)/(c + d*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1022 vs. $2(200) = 400$.

Time = 0.24 (sec) , antiderivative size = 1022, normalized size of antiderivative = 4.82

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$= \frac{1}{5}Ab^3dg^3ix^5 + \frac{1}{4}Ab^3cg^3ix^4 + \frac{3}{4}Aab^2dg^3ix^4 + Aab^2cg^3ix^3 + Aa^2bdg^3ix^3 + \frac{3}{2}Aa^2bcg^3ix^2$$
$$+ \frac{1}{2}Aa^3dg^3ix^2 + \left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Ba^3cg^3i$$
$$+\frac{3}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Ba^2bcg^3i$$
$$+\frac{1}{2}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2c\cdots}{b^2d^2}\right.$$
$$+\frac{1}{24}\left(6x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6a^4\log(bx+a)}{b^4}+\frac{6c^4\log(dx+c)}{d^4}-\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3c^2d\cdots}{b^3\cdots}\right.$$
$$+\frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Ba^3dg^3i$$
$$+\frac{1}{2}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2c\cdots}{b^2d^2}\right.$$
$$+\frac{1}{8}\left(6x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6a^4\log(bx+a)}{b^4}+\frac{6c^4\log(dx+c)}{d^4}-\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3c^2d\cdots}{b^3c\cdots}\right.$$
$$+\frac{1}{60}\left(12x^5\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{12a^5\log(bx+a)}{b^5}-\frac{12c^5\log(dx+c)}{d^5}-\frac{3(b^4cd^3-ab^3d^4)x^4-4(b^4\cdots}{\cdots}\right.$$
$$+Aa^3cg^3ix$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorit
hm="maxima")
```

```
[Out] 1/5*A*b^3*d*g^3*i*x^5 + 1/4*A*b^3*c*g^3*i*x^4 + 3/4*A*a*b^2*d*g^3*i*x^4 + A
*a*b^2*c*g^3*i*x^3 + A*a^2*b*d*g^3*i*x^3 + 3/2*A*a^2*b*c*g^3*i*x^2 + 1/2*A*
a^3*d*g^3*i*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/
```

```
b - c*log(d*x + c)/d)*B*a^3*c*g^3*i + 3/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d
*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d
))*B*a^2*b*c*g^3*i + 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^
3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*
(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*b^2*c*g^3*i + 1/24*(6*x^4*log(b*e*x/(
d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4
 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*
c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*c*g^3*i + 1/2*(x^2*log(b*e*x/(d*x + c) +
 a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)
*x/(b*d))*B*a^3*d*g^3*i + 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) +
 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2
 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a^2*b*d*g^3*i + 1/8*(6*x^4*log(b*e
*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)
/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(
b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*a*b^2*d*g^3*i + 1/60*(12*x^5*log(b*e*x/(
d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d
^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6
*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^3*d
*g^3*i + A*a^3*c*g^3*i*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3637 vs. $2(200) = 400$.

Time = 0.51 (sec) , antiderivative size = 3637, normalized size of antiderivative = 17.16

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorit
hm="giac")
```

```
[Out] -1/120*(6*(B*b^9*c^6*e^6*g^3*i - 6*B*a*b^8*c^5*d*e^6*g^3*i + 15*B*a^2*b^7*c
^4*d^2*e^6*g^3*i - 20*B*a^3*b^6*c^3*d^3*e^6*g^3*i + 15*B*a^4*b^5*c^2*d^4*e^
6*g^3*i - 6*B*a^5*b^4*c*d^5*e^6*g^3*i + B*a^6*b^3*d^6*e^6*g^3*i - 5*(b*e*x
+ a*e)*B*b^8*c^6*d*e^5*g^3*i/(d*x + c) + 30*(b*e*x + a*e)*B*a*b^7*c^5*d^2*e
^5*g^3*i/(d*x + c) - 75*(b*e*x + a*e)*B*a^2*b^6*c^4*d^3*e^5*g^3*i/(d*x + c)
 + 100*(b*e*x + a*e)*B*a^3*b^5*c^3*d^4*e^5*g^3*i/(d*x + c) - 75*(b*e*x + a*
e)*B*a^4*b^4*c^2*d^5*e^5*g^3*i/(d*x + c) + 30*(b*e*x + a*e)*B*a^5*b^3*c*d^6
*e^5*g^3*i/(d*x + c) - 5*(b*e*x + a*e)*B*a^6*b^2*d^7*e^5*g^3*i/(d*x + c) +
10*(b*e*x + a*e)^2*B*b^7*c^6*d^2*e^4*g^3*i/(d*x + c)^2 - 60*(b*e*x + a*e)^2
*B*a*b^6*c^5*d^3*e^4*g^3*i/(d*x + c)^2 + 150*(b*e*x + a*e)^2*B*a^2*b^5*c^4*
d^4*e^4*g^3*i/(d*x + c)^2 - 200*(b*e*x + a*e)^2*B*a^3*b^4*c^3*d^5*e^4*g^3*i
/(d*x + c)^2 + 150*(b*e*x + a*e)^2*B*a^4*b^3*c^2*d^6*e^4*g^3*i/(d*x + c)^2
- 60*(b*e*x + a*e)^2*B*a^5*b^2*c*d^7*e^4*g^3*i/(d*x + c)^2 + 10*(b*e*x + a*
e)^2*B*a^6*b*d^8*e^4*g^3*i/(d*x + c)^2 - 10*(b*e*x + a*e)^3*B*b^6*c^6*d^3*e
```

^3*g^3*i/(d*x + c)^3 + 60*(b*e*x + a*e)^3*B*a*b^5*c^5*d^4*e^3*g^3*i/(d*x + c)^3 - 150*(b*e*x + a*e)^3*B*a^2*b^4*c^4*d^5*e^3*g^3*i/(d*x + c)^3 + 200*(b*e*x + a*e)^3*B*a^3*b^3*c^3*d^6*e^3*g^3*i/(d*x + c)^3 - 150*(b*e*x + a*e)^3*B*a^4*b^2*c^2*d^7*e^3*g^3*i/(d*x + c)^3 + 60*(b*e*x + a*e)^3*B*a^5*b*c*d^8*e^3*g^3*i/(d*x + c)^3 - 10*(b*e*x + a*e)^3*B*a^6*d^9*e^3*g^3*i/(d*x + c)^3)*log((b*e*x + a*e)/(d*x + c))/(b^5*d^4*e^5 - 5*(b*e*x + a*e)*b^4*d^5*e^4/(d*x + c) + 10*(b*e*x + a*e)^2*b^3*d^6*e^3/(d*x + c)^2 - 10*(b*e*x + a*e)^3*b^2*d^7*e^2/(d*x + c)^3 + 5*(b*e*x + a*e)^4*b*d^8*e/(d*x + c)^4 - (b*e*x + a*e)^5*d^9/(d*x + c)^5) + (6*A*b^10*c^6*e^6*g^3*i + 5*B*b^10*c^6*e^6*g^3*i - 36*A*a*b^9*c^5*d*e^6*g^3*i - 30*B*a*b^9*c^5*d*e^6*g^3*i + 90*A*a^2*b^8*c^4*d^2*e^6*g^3*i + 75*B*a^2*b^8*c^4*d^2*e^6*g^3*i - 120*A*a^3*b^7*c^3*d^3*e^6*g^3*i - 100*B*a^3*b^7*c^3*d^3*e^6*g^3*i + 90*A*a^4*b^6*c^2*d^4*e^6*g^3*i + 75*B*a^4*b^6*c^2*d^4*e^6*g^3*i - 36*A*a^5*b^5*c*d^5*e^6*g^3*i - 30*B*a^5*b^5*c*d^5*e^6*g^3*i + 6*A*a^6*b^4*d^6*e^6*g^3*i + 5*B*a^6*b^4*d^6*e^6*g^3*i - 30*(b*e*x + a*e)*A*b^9*c^6*d*e^5*g^3*i/(d*x + c) - 19*(b*e*x + a*e)*B*b^9*c^6*d*e^5*g^3*i/(d*x + c) + 180*(b*e*x + a*e)*A*a*b^8*c^5*d^2*e^5*g^3*i/(d*x + c) + 114*(b*e*x + a*e)*B*a*b^8*c^5*d^2*e^5*g^3*i/(d*x + c) - 450*(b*e*x + a*e)*A*a^2*b^7*c^4*d^3*e^5*g^3*i/(d*x + c) - 285*(b*e*x + a*e)*B*a^2*b^7*c^4*d^3*e^5*g^3*i/(d*x + c) + 600*(b*e*x + a*e)*A*a^3*b^6*c^3*d^4*e^5*g^3*i/(d*x + c) + 380*(b*e*x + a*e)*B*a^3*b^6*c^3*d^4*e^5*g^3*i/(d*x + c) - 450*(b*e*x + a*e)*A*a^4*b^5*c^2*d^5*e^5*g^3*i/(d*x + c) - 285*(b*e*x + a*e)*B*a^4*b^5*c^2*d^5*e^5*g^3*i/(d*x + c) + 180*(b*e*x + a*e)*A*a^5*b^4*c*d^6*e^5*g^3*i/(d*x + c) + 114*(b*e*x + a*e)*B*a^5*b^4*c*d^6*e^5*g^3*i/(d*x + c) - 30*(b*e*x + a*e)*A*a^6*b^3*d^7*e^5*g^3*i/(d*x + c) - 19*(b*e*x + a*e)*B*a^6*b^3*d^7*e^5*g^3*i/(d*x + c) + 60*(b*e*x + a*e)^2*A*b^8*c^6*d^2*e^4*g^3*i/(d*x + c)^2 + 23*(b*e*x + a*e)^2*B*b^8*c^6*d^2*e^4*g^3*i/(d*x + c)^2 - 360*(b*e*x + a*e)^2*A*a*b^7*c^5*d^3*e^4*g^3*i/(d*x + c)^2 - 138*(b*e*x + a*e)^2*B*a*b^7*c^5*d^3*e^4*g^3*i/(d*x + c)^2 + 900*(b*e*x + a*e)^2*A*a^2*b^6*c^4*d^4*e^4*g^3*i/(d*x + c)^2 + 345*(b*e*x + a*e)^2*B*a^2*b^6*c^4*d^4*e^4*g^3*i/(d*x + c)^2 - 1200*(b*e*x + a*e)^2*A*a^3*b^5*c^3*d^5*e^4*g^3*i/(d*x + c)^2 - 460*(b*e*x + a*e)^2*B*a^3*b^5*c^3*d^5*e^4*g^3*i/(d*x + c)^2 + 900*(b*e*x + a*e)^2*A*a^4*b^4*c^2*d^6*e^4*g^3*i/(d*x + c)^2 + 345*(b*e*x + a*e)^2*B*a^4*b^4*c^2*d^6*e^4*g^3*i/(d*x + c)^2 - 360*(b*e*x + a*e)^2*A*a^5*b^3*c*d^7*e^4*g^3*i/(d*x + c)^2 - 138*(b*e*x + a*e)^2*B*a^5*b^3*c*d^7*e^4*g^3*i/(d*x + c)^2 + 60*(b*e*x + a*e)^2*A*a^6*b^2*d^8*e^4*g^3*i/(d*x + c)^2 + 23*(b*e*x + a*e)^2*B*a^6*b^2*d^8*e^4*g^3*i/(d*x + c)^2 - 60*(b*e*x + a*e)^3*A*b^7*c^6*d^3*e^3*g^3*i/(d*x + c)^3 - 3*(b*e*x + a*e)^3*B*b^7*c^6*d^3*e^3*g^3*i/(d*x + c)^3 + 360*(b*e*x + a*e)^3*A*a*b^6*c^5*d^4*e^3*g^3*i/(d*x + c)^3 + 18*(b*e*x + a*e)^3*B*a*b^6*c^5*d^4*e^3*g^3*i/(d*x + c)^3 - 900*(b*e*x + a*e)^3*A*a^2*b^5*c^4*d^5*e^3*g^3*i/(d*x + c)^3 - 45*(b*e*x + a*e)^3*B*a^2*b^5*c^4*d^5*e^3*g^3*i/(d*x + c)^3 + 1200*(b*e*x + a*e)^3*A*a^3*b^4*c^3*d^6*e^3*g^3*i/(d*x + c)^3 + 60*(b*e*x + a*e)^3*B*a^3*b^4*c^3*d^6*e^3*g^3*i/(d*x + c)^3 - 900*(b*e*x + a*e)^3*A*a^4*b^3*c^2*d^7*e^3*g^3*i/(d*x + c)^3 - 45*(b*e*x + a*e)^3*B*a^4*b^3*c^2*d^7*e^3*g^3*i/(d*x + c)^3 + 360*(b*e*x + a*e)^3*A*a^5*b^2*c*d^8*e^3*g^3*i/(d*x + c)^3 + 18*(b*e*x + a*e)^3*B*a^5*b^2*c*d^8*e^3*

```
g^3*i/(d*x + c)^3 - 60*(b*e*x + a*e)^3*A*a^6*b*d^9*e^3*g^3*i/(d*x + c)^3 -
3*(b*e*x + a*e)^3*B*a^6*b*d^9*e^3*g^3*i/(d*x + c)^3 - 6*(b*e*x + a*e)^4*B*b
^6*c^6*d^4*e^2*g^3*i/(d*x + c)^4 + 36*(b*e*x + a*e)^4*B*a*b^5*c^5*d^5*e^2*g
^3*i/(d*x + c)^4 - 90*(b*e*x + a*e)^4*B*a^2*b^4*c^4*d^6*e^2*g^3*i/(d*x + c)
^4 + 120*(b*e*x + a*e)^4*B*a^3*b^3*c^3*d^7*e^2*g^3*i/(d*x + c)^4 - 90*(b*e*
x + a*e)^4*B*a^4*b^2*c^2*d^8*e^2*g^3*i/(d*x + c)^4 + 36*(b*e*x + a*e)^4*B*a
^5*b*c*d^9*e^2*g^3*i/(d*x + c)^4 - 6*(b*e*x + a*e)^4*B*a^6*d^10*e^2*g^3*i/(
d*x + c)^4)/(b^6*d^4*e^5 - 5*(b*e*x + a*e)*b^5*d^5*e^4/(d*x + c) + 10*(b*e*
x + a*e)^2*b^4*d^6*e^3/(d*x + c)^2 - 10*(b*e*x + a*e)^3*b^3*d^7*e^2/(d*x +
c)^3 + 5*(b*e*x + a*e)^4*b^2*d^8*e/(d*x + c)^4 - (b*e*x + a*e)^5*b*d^9/(d*x
 + c)^5) + 6*(B*b^6*c^6*e*g^3*i - 6*B*a*b^5*c^5*d*e*g^3*i + 15*B*a^2*b^4*c^
4*d^2*e*g^3*i - 20*B*a^3*b^3*c^3*d^3*e*g^3*i + 15*B*a^4*b^2*c^2*d^4*e*g^3*i
 - 6*B*a^5*b*c*d^5*e*g^3*i + B*a^6*d^6*e*g^3*i)*log(-b*e + (b*e*x + a*e)*d/
(d*x + c))/(b^2*d^4) - 6*(B*b^6*c^6*e*g^3*i - 6*B*a*b^5*c^5*d*e*g^3*i + 15*
B*a^2*b^4*c^4*d^2*e*g^3*i - 20*B*a^3*b^3*c^3*d^3*e*g^3*i + 15*B*a^4*b^2*c^2
*d^4*e*g^3*i - 6*B*a^5*b*c*d^5*e*g^3*i + B*a^6*d^6*e*g^3*i)*log((b*e*x + a*
e)/(d*x + c))/(b^2*d^4))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e -
 a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 1195, normalized size of antiderivative = 5.64

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

```
[In] int((a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x*((a*c*(((20*a*d + 20*b*c)*((b^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d - B*b*
c))/5 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/20))/(20*b*d) - (b*g^3*i*(24*A*a^2*
d^2 + 4*A*b^2*c^2 + 3*B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d - 2*B*a*b*c*d))/
(4*d) + A*a*b^2*c*g^3*i))/(b*d) - ((20*a*d + 20*b*c)*(((20*a*d + 20*b*c)*((
(20*a*d + 20*b*c)*((b^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d - B*b*c))/5 - (A
*b^2*g^3*i*(20*a*d + 20*b*c))/20))/(20*b*d) - (b*g^3*i*(24*A*a^2*d^2 + 4*A*
b^2*c^2 + 3*B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d - 2*B*a*b*c*d))/(4*d) + A*
a*b^2*c*g^3*i))/(20*b*d) - (a*c*((b^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d -
B*b*c))/5 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/20))/(b*d) + (a*g^3*i*(4*A*a^2*
d^2 + 4*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 12*A*a*b*c*d))/d))/(20*b*d) + (
a^2*g^3*i*(2*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^2 - 3*B*b^2*c^2 + 16*A*a*b*
c*d + 2*B*a*b*c*d))/(2*b*d)) + x^4*((b^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d
 - B*b*c))/20 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/80) - x^3*(((20*a*d + 20*b*
c)*((b^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d - B*b*c))/5 - (A*b^2*g^3*i*(20*
a*d + 20*b*c))/20))/(60*b*d) - (b*g^3*i*(24*A*a^2*d^2 + 4*A*b^2*c^2 + 3*B*a
^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d - 2*B*a*b*c*d))/(12*d) + (A*a*b^2*c*g^3*i
)/3) + x^2*(((20*a*d + 20*b*c)*(((20*a*d + 20*b*c)*((b^2*g^3*i*(20*A*a*d +
```

```
10*A*b*c + B*a*d - B*b*c))/5 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/20))/(20*b*d
) - (b*g^3*i*(24*A*a^2*d^2 + 4*A*b^2*c^2 + 3*B*a^2*d^2 - B*b^2*c^2 + 32*A*a
*b*c*d - 2*B*a*b*c*d))/(4*d) + A*a*b^2*c*g^3*i))/(40*b*d) - (a*c*((b^2*g^3*
i*(20*A*a*d + 10*A*b*c + B*a*d - B*b*c))/5 - (A*b^2*g^3*i*(20*a*d + 20*b*c)
)/20))/(2*b*d) + (a*g^3*i*(4*A*a^2*d^2 + 4*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^
2 + 12*A*a*b*c*d))/(2*d)) + log((e*(a + b*x))/(c + d*x))*((B*a^2*g^3*i*x^2*
(a*d + 3*b*c))/2 + (B*b^2*g^3*i*x^4*(3*a*d + b*c))/4 + B*a^3*c*g^3*i*x + (B
*b^3*d*g^3*i*x^5)/5 + B*a*b*g^3*i*x^3*(a*d + b*c)) - (log(a + b*x)*(B*a^5*d
*g^3*i - 5*B*a^4*b*c*g^3*i))/(20*b^2) + (log(c + d*x)*(B*b^3*c^5*g^3*i - 10
*B*a^3*c^2*d^3*g^3*i - 5*B*a*b^2*c^4*d*g^3*i + 10*B*a^2*b*c^3*d^2*g^3*i))/(
20*d^4) + (A*b^3*d*g^3*i*x^5)/5
```

3.2 $\int (ag+bgx)^2(ci+dix) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) \, dx$

Optimal result

Integrand size = 38, antiderivative size = 180

$$\begin{aligned}
&\int (ag+bgx)^2(ci+dix) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) \, dx \\
&= \frac{B(bc-ad)^3g^2ix}{12bd^2} - \frac{B(bc-ad)^2g^2i(a+bx)^2}{24b^2d} \\
&\quad + \frac{g^2i(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b} \\
&\quad + \frac{(bc-ad)g^2i(a+bx)^3\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2} - \frac{B(bc-ad)^4g^2i\log(c+dx)}{12b^2d^3}
\end{aligned}$$

```
[Out] 1/12*B*(-a*d+b*c)^3*g^2*i*x/b/d^2-1/24*B*(-a*d+b*c)^2*g^2*i*(b*x+a)^2/b^2/d
+1/4*g^2*i*(b*x+a)^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b+1/12*(-a*d+b*c)*
g^2*i*(b*x+a)^3*(A-B+B*ln(e*(b*x+a)/(d*x+c)))/b^2-1/12*B*(-a*d+b*c)^4*g^2*i
*ln(d*x+c)/b^2/d^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used

$= \{2560, 2548, 21, 45\}$

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{g^2i(a+bx)^3(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{12b^2}+\frac{g^2i(a+bx)^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}-\frac{Bg^2i(bc-ad)^4\log(c+dx)}{12b^2d^3}-\frac{Bg^2i(a+bx)^2(bc-ad)^2}{24b^2d}+\frac{Bg^2ix(bc-ad)^3}{12bd^2}$$

```
[In] Int[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (B*(b*c - a*d)^3*g^2*i*x)/(12*b*d^2) - (B*(b*c - a*d)^2*g^2*i*(a + b*x)^2)/
(24*b^2*d) + (g^2*i*(a + b*x)^3*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(4*b) + ((b*c - a*d)*g^2*i*(a + b*x)^3*(A - B + B*Log[(e*(a + b*x))/(c
 + d*x)]))/(12*b^2) - (B*(b*c - a*d)^4*g^2*i*Log[c + d*x])/(12*b^2*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rule 2560

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Sim
p[(f + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*
```

```
(m + 2))), x] + Dist[i*((b*c - a*d)/(b*d*(m + 2))), Int[(f + g*x)^m*(A - B*
n + B*Log[e*((a + b*x)^n/(c + d*x)^n)]), x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, i, A, B, m, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d,
 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IGtQ[m, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g^2 i(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b} \\
&\quad + \frac{((bc-ad)i)\int(ag+bgx)^2\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx}{4b} \\
&= \frac{g^2 i(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b} \\
&\quad + \frac{(bc-ad)g^2 i(a+bx)^3\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2} \\
&\quad - \frac{\left(B(bc-ad)^2 i\right)\int\frac{(ag+bgx)^3}{(a+bx)(c+dx)}dx}{12b^2 g} \\
&= \frac{g^2 i(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b} \\
&\quad + \frac{(bc-ad)g^2 i(a+bx)^3\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2} \\
&\quad - \frac{\left(B(bc-ad)^2 g^2 i\right)\int\frac{(a+bx)^2}{c+dx}dx}{12b^2} \\
&= \frac{g^2 i(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b} \\
&\quad + \frac{(bc-ad)g^2 i(a+bx)^3\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2} \\
&\quad - \frac{\left(B(bc-ad)^2 g^2 i\right)\int\left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx}{12b^2} \\
&= \frac{B(bc-ad)^3 g^2 ix}{12bd^2} - \frac{B(bc-ad)^2 g^2 i(a+bx)^2}{24b^2 d} + \frac{g^2 i(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b} \\
&\quad + \frac{(bc-ad)g^2 i(a+bx)^3\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2} - \frac{B(bc-ad)^4 g^2 i\log(c+dx)}{12b^2 d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.21

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{g^2i\left(8(bc-ad)(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+6d(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{4B(bc-ad)^2(2bd(bc-ad)\ldots}{}\right.}{24b^2}$$

```
[In] Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]
),x]
```

```
[Out] (g^2*i*(8*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*
d*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (4*B*(b*c - a*d)^2*(2*
b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]))/d^3 -
(B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 +
2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/d^3))/(24*b^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(170) = 340.

Time = 0.80 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.20

method	result
risch	$\frac{i\,g^2Bx\left(3b^2d\,x^3+8abd\,x^2+4b^2c\,x^2+6a^2dx+12abcx+12a^2c\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{12}+\frac{i\,g^2b^2dA\,x^4}{4}+\frac{2i\,g^2bdAa\,x^3}{3}+\frac{i\,g^2b^2Ac\,x\ldots}{3}$
parallelrisch	$\frac{16A\,x^3a\,b^3d^4g^2i+8A\,x^3b^4c\,d^3g^2i-36A\,a^3bc\,d^3g^2i+6B\,x^4\ln\left(\frac{e(bx+a)}{dx+c}\right)b^4d^4g^2i+16B\,x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^3d^4g^2i+7Ba\,b^3c^3\ldots}{}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/12*i*g^2*B*x*(3*b^2*d*x^3+8*a*b*d*x^2+4*b^2*c*x^2+6*a^2*d*x+12*a*b*c*x+12
*a^2*c)*ln(e*(b*x+a)/(d*x+c))+1/4*i*g^2*b^2*d*A*x^4+2/3*i*g^2*b*d*A*a*x^3+1
/3*i*g^2*b^2*A*c*x^3+1/12*i*g^2*b*d*B*a*x^3-1/12*i*g^2*b^2*B*c*x^3+1/2*i*g^
2*d*A*a^2*x^2+i*g^2*b*A*a*c*x^2+5/24*i*g^2*d*B*a^2*x^2-1/6*i*g^2*b*B*a*c*x^
2-1/24*i*g^2*b^2/d*B*c^2*x^2+i*g^2*A*a^2*c*x-1/12*i*g^2/b^2*d*B*ln(b*x+a)*a
^4+1/3*i*g^2/b*B*ln(b*x+a)*a^3*c-1/2*i*g^2/d*B*ln(-d*x-c)*a^2*c^2+1/3*i*g^2
*b/d^2*B*ln(-d*x-c)*a*c^3-1/12*i*g^2*b^2/d^3*B*ln(-d*x-c)*c^4+1/12*i*g^2/b*
d*B*a^3*x+1/6*i*g^2*B*a^2*c*x-1/3*i*g^2*b/d*B*a*c^2*x+1/12*i*g^2*b^2/d^2*B*
c^3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(170) = 340$.

Time = 0.38 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.06

$$\int (ag + bgx)^2(ci + dix)\left(A + B\log\left(\frac{e(a + bx)}{c + dx}\right)\right)\,dx$$
$$= \frac{6\,Ab^4d^4g^2ix^4 + 2\,((4\,A - B)b^4cd^3 + (8\,A + B)ab^3d^4)g^2ix^3 - (Bb^4c^2d^2 - 4\,(6\,A - B)ab^3cd^3 - (12\,A + 5\,B}{}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorit
hm="fricas")
```

```
[Out] 1/24*(6*A*b^4*d^4*g^2*i*x^4 + 2*((4*A - B)*b^4*c*d^3 + (8*A + B)*a*b^3*d^4)
*g^2*i*x^3 - (B*b^4*c^2*d^2 - 4*(6*A - B)*a*b^3*c*d^3 - (12*A + 5*B)*a^2*b^
2*d^4)*g^2*i*x^2 + 2*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 2*(6*A + B)*a^2*b^2
*c*d^3 + B*a^3*b*d^4)*g^2*i*x + 2*(4*B*a^3*b*c*d^3 - B*a^4*d^4)*g^2*i*log(b
*x + a) - 2*(B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2)*g^2*i*log(d
*x + c) + 2*(3*B*b^4*d^4*g^2*i*x^4 + 12*B*a^2*b^2*c*d^3*g^2*i*x + 4*(B*b^4*
c*d^3 + 2*B*a*b^3*d^4)*g^2*i*x^3 + 6*(2*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*g^2*
i*x^2)*log((b*e*x + a*e)/(d*x + c)))/(b^2*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 850 vs. $2(163) = 326$.

Time = 2.48 (sec) , antiderivative size = 850, normalized size of antiderivative = 4.72

$$\int (ag + bgx)^2(ci + dix)\left(A + B\log\left(\frac{e(a + bx)}{c + dx}\right)\right)\,dx = \frac{Ab^2dg^2ix^4}{4}$$
$$- \frac{Ba^3g^2i(ad - 4bc)\log\left(x + \frac{Ba^4cd^3g^2i + \frac{Ba^4d^3g^2i(ad-4bc)}{b} - 10Ba^3bc^2d^2g^2i - Ba^3cd^2g^2i(ad-4bc) + 4Ba^2b^2c^3dg^2i - Bab^3c^4g^2i}{Ba^4d^4g^2i - 4Ba^3bcd^3g^2i - 6Ba^2b^2c^2d^2g^2i + 4Bab^3c^3dg^2i - Bb^4c^4g^2i}\right)}{12b^2}$$
$$- \frac{Bc^2g^2i(6a^2d^2 - 4abcd + b^2c^2)\log\left(x + \frac{Ba^4cd^3g^2i - 10Ba^3bc^2d^2g^2i + 4Ba^2b^2c^3dg^2i - Bab^3c^4g^2i + Babc^2g^2i(6a^2d^2 - 4abcd + b^2}{Ba^4d^4g^2i - 4Ba^3bcd^3g^2i - 6Ba^2b^2c^2d^2g^2i + 4Bab^3c^3dg^2i - B}\right.}{12d^3}$$
$$+ x^3\cdot\left(\frac{2Aabdg^2i}{3} + \frac{Ab^2cg^2i}{3} + \frac{Babdg^2i}{12} - \frac{Bb^2cg^2i}{12}\right)$$
$$+ x^2\left(\frac{Aa^2dg^2i}{2} + Aabcg^2i + \frac{5Ba^2dg^2i}{24} - \frac{Babcg^2i}{6} - \frac{Bb^2c^2g^2i}{24d}\right)$$
$$+ x\left(Aa^2cg^2i + \frac{Ba^3dg^2i}{12b} + \frac{Ba^2cg^2i}{6} - \frac{Babc^2g^2i}{3d} + \frac{Bb^2c^3g^2i}{12d^2}\right) + \left(Ba^2cg^2ix + \frac{Ba^2dg^2ix^2}{2}\right.$$
$$\left. + Babcg^2ix^2 + \frac{2Babdg^2ix^3}{3} + \frac{Bb^2cg^2ix^3}{3} + \frac{Bb^2dg^2ix^4}{4}\right)\log\left(\frac{e(a + bx)}{c + dx}\right)$$

```
[In] integrate((b*g*x+a*g)**2*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b**2*d*g**2*i*x**4/4 - B*a**3*g**2*i*(a*d - 4*b*c)*log(x + (B*a**4*c*d**3
*g**2*i + B*a**4*d**3*g**2*i*(a*d - 4*b*c)/b - 10*B*a**3*b*c**2*d**2*g**2*i
 - B*a**3*c*d**2*g**2*i*(a*d - 4*b*c) + 4*B*a**2*b**2*c**3*d*g**2*i - B*a*b
**3*c**4*g**2*i)/(B*a**4*d**4*g**2*i - 4*B*a**3*b*c*d**3*g**2*i - 6*B*a**2*
b**2*c**2*d**2*g**2*i + 4*B*a*b**3*c**3*d*g**2*i - B*b**4*c**4*g**2*i))/(12
*b**2) - B*c**2*g**2*i*(6*a**2*d**2 - 4*a*b*c*d + b**2*c**2)*log(x + (B*a**
4*c*d**3*g**2*i - 10*B*a**3*b*c**2*d**2*g**2*i + 4*B*a**2*b**2*c**3*d*g**2*
i - B*a*b**3*c**4*g**2*i + B*a*b*c**2*g**2*i*(6*a**2*d**2 - 4*a*b*c*d + b**
2*c**2) - B*b**2*c**3*g**2*i*(6*a**2*d**2 - 4*a*b*c*d + b**2*c**2)/d)/(B*a*
*4*d**4*g**2*i - 4*B*a**3*b*c*d**3*g**2*i - 6*B*a**2*b**2*c**2*d**2*g**2*i
+ 4*B*a*b**3*c**3*d*g**2*i - B*b**4*c**4*g**2*i))/(12*d**3) + x**3*(2*A*a*b
*d*g**2*i/3 + A*b**2*c*g**2*i/3 + B*a*b*d*g**2*i/12 - B*b**2*c*g**2*i/12) +
 x**2*(A*a**2*d*g**2*i/2 + A*a*b*c*g**2*i + 5*B*a**2*d*g**2*i/24 - B*a*b*c*
g**2*i/6 - B*b**2*c**2*g**2*i/(24*d)) + x*(A*a**2*c*g**2*i + B*a**3*d*g**2*
i/(12*b) + B*a**2*c*g**2*i/6 - B*a*b*c**2*g**2*i/(3*d) + B*b**2*c**3*g**2*i
/(12*d**2)) + (B*a**2*c*g**2*i*x + B*a**2*d*g**2*i*x**2/2 + B*a*b*c*g**2*i*
x**2 + 2*B*a*b*d*g**2*i*x**3/3 + B*b**2*c*g**2*i*x**3/3 + B*b**2*d*g**2*i*x
**4/4)*log(e*(a + b*x)/(c + d*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(170) = 340$.

Time = 0.22 (sec) , antiderivative size = 671, normalized size of antiderivative = 3.73

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{1}{4}\,Ab^2dg^2ix^4 + \frac{1}{3}\,Ab^2cg^2ix^3 + \frac{2}{3}\,Aabdg^2ix^3 + Aabcg^2ix^2 + \frac{1}{2}\,Aa^2dg^2ix^2$$
$$+ \left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Ba^2cg^2i$$
$$+ \left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Babcg^2i$$
$$+\frac{1}{6}\left(2\,x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2c}{b^2d^2}\right.$$
$$+\frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Ba^2dg^2i$$
$$+\frac{1}{3}\left(2\,x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2c}{b^2d^2}\right.$$
$$+\frac{1}{24}\left(6\,x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6\,a^4\log(bx+a)}{b^4}+\frac{6\,c^4\log(dx+c)}{d^4}-\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2d}{b^3}\right.$$
$$+ Aa^2cg^2ix$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorit
hm="maxima")
```

```
[Out] 1/4*A*b^2*d*g^2*i*x^4 + 1/3*A*b^2*c*g^2*i*x^3 + 2/3*A*a*b*d*g^2*i*x^3 + A*a
*b*c*g^2*i*x^2 + 1/2*A*a^2*d*g^2*i*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x
+ c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*a^2*c*g^2*i + (x^2*log(b*e*x
/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 -
 (b*c - a*d)*x/(b*d))*B*a*b*c*g^2*i + 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*e/
(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d -
a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*c*g^2*i + 1/2*(x^2
*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x
+ c)/d^2 - (b*c - a*d)*x/(b*d))*B*a^2*d*g^2*i + 1/3*(2*x^3*log(b*e*x/(d*x +
 c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((
b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*b*d*g^2*i
+ 1/24*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4
 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d -
 a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^2*d*g^2*i + A*a^2
*c*g^2*i*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2418 vs. $2(170) = 340$.

Time = 0.46 (sec) , antiderivative size = 2418, normalized size of antiderivative = 13.43

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorit
hm="giac")
```

```
[Out] 1/24*(2*(B*b^7*c^5*e^5*g^2*i - 5*B*a*b^6*c^4*d*e^5*g^2*i + 10*B*a^2*b^5*c^3
*d^2*e^5*g^2*i - 10*B*a^3*b^4*c^2*d^3*e^5*g^2*i + 5*B*a^4*b^3*c*d^4*e^5*g^2
*i - B*a^5*b^2*d^5*e^5*g^2*i - 4*(b*e*x + a*e)*B*b^6*c^5*d*e^4*g^2*i/(d*x +
 c) + 20*(b*e*x + a*e)*B*a*b^5*c^4*d^2*e^4*g^2*i/(d*x + c) - 40*(b*e*x + a*
e)*B*a^2*b^4*c^3*d^3*e^4*g^2*i/(d*x + c) + 40*(b*e*x + a*e)*B*a^3*b^3*c^2*d
^4*e^4*g^2*i/(d*x + c) - 20*(b*e*x + a*e)*B*a^4*b^2*c*d^5*e^4*g^2*i/(d*x +
c) + 4*(b*e*x + a*e)*B*a^5*b*d^6*e^4*g^2*i/(d*x + c) + 6*(b*e*x + a*e)^2*B*
b^5*c^5*d^2*e^3*g^2*i/(d*x + c)^2 - 30*(b*e*x + a*e)^2*B*a*b^4*c^4*d^3*e^3*
g^2*i/(d*x + c)^2 + 60*(b*e*x + a*e)^2*B*a^2*b^3*c^3*d^4*e^3*g^2*i/(d*x + c
)^2 - 60*(b*e*x + a*e)^2*B*a^3*b^2*c^2*d^5*e^3*g^2*i/(d*x + c)^2 + 30*(b*e*
x + a*e)^2*B*a^4*b*c*d^6*e^3*g^2*i/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a^5*d^
7*e^3*g^2*i/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/(b^4*d^3*e^4 - 4*(b*e
*x + a*e)*b^3*d^4*e^3/(d*x + c) + 6*(b*e*x + a*e)^2*b^2*d^5*e^2/(d*x + c)^2
 - 4*(b*e*x + a*e)^3*b*d^6*e/(d*x + c)^3 + (b*e*x + a*e)^4*d^7/(d*x + c)^4)
 + (2*A*b^8*c^5*e^5*g^2*i + B*b^8*c^5*e^5*g^2*i - 10*A*a*b^7*c^4*d*e^5*g^2*
```

```
i - 5*B*a*b^7*c^4*d*e^5*g^2*i + 20*A*a^2*b^6*c^3*d^2*e^5*g^2*i + 10*B*a^2*b
^6*c^3*d^2*e^5*g^2*i - 20*A*a^3*b^5*c^2*d^3*e^5*g^2*i - 10*B*a^3*b^5*c^2*d^
3*e^5*g^2*i + 10*A*a^4*b^4*c*d^4*e^5*g^2*i + 5*B*a^4*b^4*c*d^4*e^5*g^2*i -
2*A*a^5*b^3*d^5*e^5*g^2*i - B*a^5*b^3*d^5*e^5*g^2*i - 8*(b*e*x + a*e)*A*b^7
*c^5*d*e^4*g^2*i/(d*x + c) - 2*(b*e*x + a*e)*B*b^7*c^5*d*e^4*g^2*i/(d*x + c
) + 40*(b*e*x + a*e)*A*a*b^6*c^4*d^2*e^4*g^2*i/(d*x + c) + 10*(b*e*x + a*e)
*B*a*b^6*c^4*d^2*e^4*g^2*i/(d*x + c) - 80*(b*e*x + a*e)*A*a^2*b^5*c^3*d^3*e
^4*g^2*i/(d*x + c) - 20*(b*e*x + a*e)*B*a^2*b^5*c^3*d^3*e^4*g^2*i/(d*x + c)
 + 80*(b*e*x + a*e)*A*a^3*b^4*c^2*d^4*e^4*g^2*i/(d*x + c) + 20*(b*e*x + a*e
)*B*a^3*b^4*c^2*d^4*e^4*g^2*i/(d*x + c) - 40*(b*e*x + a*e)*A*a^4*b^3*c*d^5*
e^4*g^2*i/(d*x + c) - 10*(b*e*x + a*e)*B*a^4*b^3*c*d^5*e^4*g^2*i/(d*x + c)
+ 8*(b*e*x + a*e)*A*a^5*b^2*d^6*e^4*g^2*i/(d*x + c) + 2*(b*e*x + a*e)*B*a^5
*b^2*d^6*e^4*g^2*i/(d*x + c) + 12*(b*e*x + a*e)^2*A*b^6*c^5*d^2*e^3*g^2*i/(
d*x + c)^2 - (b*e*x + a*e)^2*B*b^6*c^5*d^2*e^3*g^2*i/(d*x + c)^2 - 60*(b*e*
x + a*e)^2*A*a*b^5*c^4*d^3*e^3*g^2*i/(d*x + c)^2 + 5*(b*e*x + a*e)^2*B*a*b^
5*c^4*d^3*e^3*g^2*i/(d*x + c)^2 + 120*(b*e*x + a*e)^2*A*a^2*b^4*c^3*d^4*e^3
*g^2*i/(d*x + c)^2 - 10*(b*e*x + a*e)^2*B*a^2*b^4*c^3*d^4*e^3*g^2*i/(d*x +
c)^2 - 120*(b*e*x + a*e)^2*A*a^3*b^3*c^2*d^5*e^3*g^2*i/(d*x + c)^2 + 10*(b*
e*x + a*e)^2*B*a^3*b^3*c^2*d^5*e^3*g^2*i/(d*x + c)^2 + 60*(b*e*x + a*e)^2*A
*a^4*b^2*c*d^6*e^3*g^2*i/(d*x + c)^2 - 5*(b*e*x + a*e)^2*B*a^4*b^2*c*d^6*e^
3*g^2*i/(d*x + c)^2 - 12*(b*e*x + a*e)^2*A*a^5*b*d^7*e^3*g^2*i/(d*x + c)^2
+ (b*e*x + a*e)^2*B*a^5*b*d^7*e^3*g^2*i/(d*x + c)^2 + 2*(b*e*x + a*e)^3*B*b
^5*c^5*d^3*e^2*g^2*i/(d*x + c)^3 - 10*(b*e*x + a*e)^3*B*a*b^4*c^4*d^4*e^2*g
^2*i/(d*x + c)^3 + 20*(b*e*x + a*e)^3*B*a^2*b^3*c^3*d^5*e^2*g^2*i/(d*x + c)
^3 - 20*(b*e*x + a*e)^3*B*a^3*b^2*c^2*d^6*e^2*g^2*i/(d*x + c)^3 + 10*(b*e*x
 + a*e)^3*B*a^4*b*c*d^7*e^2*g^2*i/(d*x + c)^3 - 2*(b*e*x + a*e)^3*B*a^5*d^8
*e^2*g^2*i/(d*x + c)^3)/(b^5*d^3*e^4 - 4*(b*e*x + a*e)*b^4*d^4*e^3/(d*x + c
) + 6*(b*e*x + a*e)^2*b^3*d^5*e^2/(d*x + c)^2 - 4*(b*e*x + a*e)^3*b^2*d^6*e
/(d*x + c)^3 + (b*e*x + a*e)^4*b*d^7/(d*x + c)^4) + 2*(B*b^5*c^5*e*g^2*i -
5*B*a*b^4*c^4*d*e*g^2*i + 10*B*a^2*b^3*c^3*d^2*e*g^2*i - 10*B*a^3*b^2*c^2*d
^3*e*g^2*i + 5*B*a^4*b*c*d^4*e*g^2*i - B*a^5*d^5*e*g^2*i)*log(-b*e + (b*e*x
 + a*e)*d/(d*x + c))/(b^2*d^3) - 2*(B*b^5*c^5*e*g^2*i - 5*B*a*b^4*c^4*d*e*g
^2*i + 10*B*a^2*b^3*c^3*d^2*e*g^2*i - 10*B*a^3*b^2*c^2*d^3*e*g^2*i + 5*B*a^
4*b*c*d^4*e*g^2*i - B*a^5*d^5*e*g^2*i)*log((b*e*x + a*e)/(d*x + c))/(b^2*d^
3))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 638, normalized size of antiderivative = 3.54

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$

$$= x^3\left(\frac{b\,g^2\,i\,(12\,A\,a\,d+8\,A\,b\,c+B\,a\,d-B\,b\,c)}{12}-\frac{A\,b\,g^2\,i\,(12\,a\,d+12\,b\,c)}{36}\right)$$

$$-x^2\left(\frac{\left(\frac{b\,g^2\,i\,(12\,A\,a\,d+8\,A\,b\,c+B\,a\,d-B\,b\,c)}{4}-\frac{A\,b\,g^2\,i\,(12\,a\,d+12\,b\,c)}{12}\right)\,(12\,a\,d+12\,b\,c)}{24\,b\,d}\right.$$

$$\left.-\frac{g^2\,i\,(9\,A\,a^2\,d^2+3\,A\,b^2\,c^2+2\,B\,a^2\,d^2-B\,b^2\,c^2+18\,A\,a\,b\,c\,d-B\,a\,b\,c\,d)}{6\,d}+\frac{A\,a\,b\,c\,g^2\,i}{2}\right)$$

$$+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(B\,a^2\,c\,g^2\,i\,x+\frac{B\,a\,g^2\,i\,x^2\,(a\,d+2\,b\,c)}{2}+\frac{B\,b\,g^2\,i\,x^3\,(2\,a\,d+b\,c)}{3}\right.$$

$$\left.+\frac{B\,b^2\,d\,g^2\,i\,x^4}{4}\right)$$

$$+x\left(\frac{(12\,a\,d+12\,b\,c)\,\left(\frac{\left(\frac{b\,g^2\,i\,(12\,A\,a\,d+8\,A\,b\,c+B\,a\,d-B\,b\,c)}{4}-\frac{A\,b\,g^2\,i\,(12\,a\,d+12\,b\,c)}{12}\right)\,(12\,a\,d+12\,b\,c)}{12\,b\,d}-\frac{g^2\,i\,(9\,A\,a^2\,d^2+3\,A\,b^2\,c^2+2\,B\,a^2\,\cdots}{3\cdots}\right.}{12\,b\,d}\right.$$

$$-\frac{a\,c\,\left(\frac{b\,g^2\,i\,(12\,A\,a\,d+8\,A\,b\,c+B\,a\,d-B\,b\,c)}{4}-\frac{A\,b\,g^2\,i\,(12\,a\,d+12\,b\,c)}{12}\right)}{b\,d}$$

$$\left.+\frac{a\,g^2\,i\,(2\,A\,a^2\,d^2+6\,A\,b^2\,c^2+B\,a^2\,d^2-2\,B\,b^2\,c^2+12\,A\,a\,b\,c\,d+B\,a\,b\,c\,d)}{2\,b\,d}\right)$$

$$-\frac{\ln(c+d\,x)\,(6\,B\,i\,a^2\,c^2\,d^2\,g^2-4\,B\,i\,a\,b\,c^3\,d\,g^2+B\,i\,b^2\,c^4\,g^2)}{12\,d^3}$$

$$-\frac{\ln(a+b\,x)\,(B\,a^4\,d\,g^2\,i-4\,B\,a^3\,b\,c\,g^2\,i)}{12\,b^2}+\frac{A\,b^2\,d\,g^2\,i\,x^4}{4}$$

```
[In] int((a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x^3*((b*g^2*i*(12*A*a*d + 8*A*b*c + B*a*d - B*b*c))/12 - (A*b*g^2*i*(12*a*d
 + 12*b*c))/36) - x^2*((((b*g^2*i*(12*A*a*d + 8*A*b*c + B*a*d - B*b*c))/4 -
 (A*b*g^2*i*(12*a*d + 12*b*c))/12)*(12*a*d + 12*b*c))/(24*b*d) - (g^2*i*(9*
A*a^2*d^2 + 3*A*b^2*c^2 + 2*B*a^2*d^2 - B*b^2*c^2 + 18*A*a*b*c*d - B*a*b*c*
d))/(6*d) + (A*a*b*c*g^2*i)/2) + log((e*(a + b*x))/(c + d*x))*(B*a^2*c*g^2*
```

```
i*x + (B*a*g^2*i*x^2*(a*d + 2*b*c))/2 + (B*b*g^2*i*x^3*(2*a*d + b*c))/3 + (
B*b^2*d*g^2*i*x^4)/4) + x*(((12*a*d + 12*b*c)*((((b*g^2*i*(12*A*a*d + 8*A*b
*c + B*a*d - B*b*c))/4 - (A*b*g^2*i*(12*a*d + 12*b*c))/12)*(12*a*d + 12*b*c
))/(12*b*d) - (g^2*i*(9*A*a^2*d^2 + 3*A*b^2*c^2 + 2*B*a^2*d^2 - B*b^2*c^2 +
 18*A*a*b*c*d - B*a*b*c*d))/(3*d) + A*a*b*c*g^2*i))/(12*b*d) - (a*c*((b*g^2
*i*(12*A*a*d + 8*A*b*c + B*a*d - B*b*c))/4 - (A*b*g^2*i*(12*a*d + 12*b*c))/
12))/(b*d) + (a*g^2*i*(2*A*a^2*d^2 + 6*A*b^2*c^2 + B*a^2*d^2 - 2*B*b^2*c^2
+ 12*A*a*b*c*d + B*a*b*c*d))/(2*b*d)) - (log(c + d*x)*(B*b^2*c^4*g^2*i + 6*
B*a^2*c^2*d^2*g^2*i - 4*B*a*b*c^3*d*g^2*i))/(12*d^3) - (log(a + b*x)*(B*a^4
*d*g^2*i - 4*B*a^3*b*c*g^2*i))/(12*b^2) + (A*b^2*d*g^2*i*x^4)/4
```

3.3 $\int (ag+bgx)(ci+dix) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

Optimal result

Integrand size = 36, antiderivative size = 140

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = -\frac{B(bc-ad)^2gix}{6bd}+\frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b} + \frac{(bc-ad)gi(a+bx)^2\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^2}+\frac{B(bc-ad)^3gi\log(c+dx)}{6b^2d^2}$$

```
[Out] -1/6*B*(-a*d+b*c)^2*g*i*x/b/d+1/3*g*i*(b*x+a)^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(
d*x+c)))/b+1/6*(-a*d+b*c)*g*i*(b*x+a)^2*(A-B+B*ln(e*(b*x+a)/(d*x+c)))/b^2+1
/6*B*(-a*d+b*c)^3*g*i*ln(d*x+c)/b^2/d^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2560, 2548, 21, 45}

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{gi(a+bx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{6b^2} + \frac{gi(a+bx)^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b} + \frac{Bgi(bc-ad)^3\log(c+dx)}{6b^2d^2}-\frac{Bgix(bc-ad)^2}{6bd}$$

```
[In] Int[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -1/6*(B*(b*c - a*d)^2*g*i*x)/(b*d) + (g*i*(a + b*x)^2*(c + d*x)*(A + B*Log[
(e*(a + b*x))/(c + d*x)]))/(3*b) + ((b*c - a*d)*g*i*(a + b*x)^2*(A - B + B*
Log[(e*(a + b*x))/(c + d*x)]))/(6*b^2) + (B*(b*c - a*d)^3*g*i*Log[c + d*x])
/(6*b^2*d^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rule 2560

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Sim
p[(f + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*
(m + 2))), x] + Dist[i*((b*c - a*d)/(b*d*(m + 2))), Int[(f + g*x)^m*(A - B*
n + B*Log[e*((a + b*x)^n/(c + d*x)^n)]), x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, i, A, B, m, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d,
 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IGtQ[m, -2]
```

Rubi steps

$$\text{integral} = \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b} + \frac{((bc-ad)i)\int(ag+bgx)\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx}{3b}$$

$$= \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b} + \frac{(bc-ad)gi(a+bx)^2\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^2} - \frac{(B(bc-ad)^2i)\int\frac{(ag+bgx)^2}{(a+bx)(c+dx)}\,dx}{6b^2g}$$

$$= \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b} + \frac{(bc-ad)gi(a+bx)^2\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^2} - \frac{(B(bc-ad)^2gi)\int\frac{a+bx}{c+dx}\,dx}{6b^2}$$

$$= \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b} + \frac{(bc-ad)gi(a+bx)^2\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^2} - \frac{(B(bc-ad)^2gi)\int\left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx}{6b^2}$$

$$= -\frac{B(bc-ad)^2gix}{6bd} + \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b} + \frac{(bc-ad)gi(a+bx)^2\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^2} + \frac{B(bc-ad)^3gi\log(c+dx)}{6b^2d^2}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.29

$$\int(ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{gi\left(-a^2Bd^2(3bc+ad)\log(a+bx)+b\left(dx(a^2Bd^2-b^2Bc(c+dx)+Ab^2dx(3c+2dx)+abd(6Ac+3Adx\right.\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),
x]
```

```
[Out] (g*i*(-(a^2*B*d^2*(3*b*c + a*d)*Log[a + b*x]) + b*(d*x*(a^2*B*d^2 - b^2*B*c
*(c + d*x) + A*b^2*d*x*(3*c + 2*d*x) + a*b*d*(6*A*c + 3*A*d*x + B*d*x)) + B
*d^2*(6*a^2*c + 3*a*b*x*(2*c + d*x) + b^2*x^2*(3*c + 2*d*x))*Log[(e*(a + b*
x))/(c + d*x)] + B*c*(b^2*c^2 - 3*a*b*c*d + 6*a^2*d^2)*Log[c + d*x])))/(6*b
^2*d^2)
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.47

method	result
risch	$\frac{giBx(2bd\,x^2+3xad+3bcx+6ca)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{6}+\frac{igbdA\,x^3}{3}+\frac{igdAa\,x^2}{2}+\frac{igbAc\,x^2}{2}+\frac{igdBa\,x^2}{6}-\frac{igbBc\,x^2}{6}+ig\ldots$
parallelrisch	$\frac{-B\ln(bx+a)a^3d^3gi+B\ln(bx+a)b^3c^3gi+2A\,x^3b^3d^3gi-B\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3c^3gi+3B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3c\,d^2gi+6Axa\,b^2c\,d^2g\ldots}{\ldots}$
parts	$Agi\left(xca+\frac{(ad+cb)x^2}{2}+\frac{bd\,x^3}{3}\right)-\frac{Bgi(ad-cb)^2e^2\left(d^2(ad-cb)\left(-\frac{\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}{2e^2b^2d}-\frac{1}{2ebd\left(\left(\frac{be}{d}+\frac{(ad-cb)}{d(dx+c}\ldots\right.\right.}\ldots\right.\right.}{\ldots}$
derivativedivides	$-\frac{e(ad-cb)\left(A\,d^2egi(a^2d^2-2abcd+b^2c^2)\left(-\frac{1}{2d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}+\frac{be}{3d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^3}\right)+B\,d^2egi(a^2d^2\ldots\right.}{\ldots}$
default	$-\frac{e(ad-cb)\left(A\,d^2egi(a^2d^2-2abcd+b^2c^2)\left(-\frac{1}{2d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}+\frac{be}{3d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^3}\right)+B\,d^2egi(a^2d^2\ldots\right.}{\ldots}$

```
[In] int((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/6*g*i*B*x*(2*b*d*x^2+3*a*d*x+3*b*c*x+6*a*c)*ln(e*(b*x+a)/(d*x+c))+1/3*i*g
*b*d*A*x^3+1/2*i*g*d*A*a*x^2+1/2*i*g*b*A*c*x^2+1/6*i*g*d*B*a*x^2-1/6*i*g*b*
B*c*x^2+i*g*A*a*c*x-1/6*i*g/b^2*d*B*ln(b*x+a)*a^3+1/2*i*g/b*B*ln(b*x+a)*a^2
*c-1/2*i*g/d*B*ln(-d*x-c)*a*c^2+1/6*i*g*b/d^2*B*ln(-d*x-c)*c^3+1/6*i*g/b*d*
B*a^2*x-1/6*i*g*b/d*B*c^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.61

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{2\,Ab^3d^3gix^3+((3\,A-B)b^3cd^2+(3\,A+B)ab^2d^3)gix^2-(Bb^3c^2d-6\,Aab^2cd^2-Ba^2bd^3)gix+(3\,Ba^2bcd^2\ldots}{\ldots}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm
="fricas")
```

```
[Out] 1/6*(2*A*b^3*d^3*g*i*x^3 + ((3*A - B)*b^3*c*d^2 + (3*A + B)*a*b^2*d^3)*g*i*
x^2 - (B*b^3*c^2*d - 6*A*a*b^2*c*d^2 - B*a^2*b*d^3)*g*i*x + (3*B*a^2*b*c*d^
2 - B*a^3*d^3)*g*i*log(b*x + a) + (B*b^3*c^3 - 3*B*a*b^2*c^2*d)*g*i*log(d*x
 + c) + (2*B*b^3*d^3*g*i*x^3 + 6*B*a*b^2*c*d^2*g*i*x + 3*(B*b^3*c*d^2 + B*a
*b^2*d^3)*g*i*x^2)*log((b*e*x + a*e)/(d*x + c)))/(b^2*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(126) = 252$.

Time = 1.44 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.56

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$= \frac{Abdgix^3}{3} - \frac{Ba^2gi(ad-3bc)\log\left(x+\frac{Ba^3cd^2gi+\frac{Ba^3d^2gi(ad-3bc)}{b}-6Ba^2bc^2dgi-Ba^2cdgi(ad-3bc)+Bab^2c^3gi}{Ba^3d^3gi-3Ba^2bcd^2gi-3Bab^2c^2dgi+Bb^3c^3gi}\right)}{6b^2} - \frac{Bc^2gi(3ad-bc)\log\left(x+\frac{Ba^3cd^2gi-6Ba^2bc^2dgi+Bab^2c^3gi+Babc^2gi(3ad-bc)-\frac{Bb^2c^3gi(3ad-bc)}{d}}{Ba^3d^3gi-3Ba^2bcd^2gi-3Bab^2c^2dgi+Bb^3c^3gi}\right)}{6d^2} + x^2\left(\frac{Aadgi}{2}+\frac{Abcgi}{2}+\frac{Badgi}{6}-\frac{Bbcgi}{6}\right) + x\left(Aacgi+\frac{Ba^2dgi}{6b}-\frac{Bbc^2gi}{6d}\right) + \left(Bacgix+\frac{Badgix^2}{2}+\frac{Bbcgix^2}{2}+\frac{Bbdgix^3}{3}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b*d*g*i*x**3/3 - B*a**2*g*i*(a*d - 3*b*c)*log(x + (B*a**3*c*d**2*g*i + B*
a**3*d**2*g*i*(a*d - 3*b*c)/b - 6*B*a**2*b*c**2*d*g*i - B*a**2*c*d*g*i*(a*d
 - 3*b*c) + B*a*b**2*c**3*g*i)/(B*a**3*d**3*g*i - 3*B*a**2*b*c*d**2*g*i - 3
*B*a*b**2*c**2*d*g*i + B*b**3*c**3*g*i))/(6*b**2) - B*c**2*g*i*(3*a*d - b*c
)*log(x + (B*a**3*c*d**2*g*i - 6*B*a**2*b*c**2*d*g*i + B*a*b**2*c**3*g*i +
B*a*b*c**2*g*i*(3*a*d - b*c) - B*b**2*c**3*g*i*(3*a*d - b*c)/d)/(B*a**3*d**
3*g*i - 3*B*a**2*b*c*d**2*g*i - 3*B*a*b**2*c**2*d*g*i + B*b**3*c**3*g*i))/(
6*d**2) + x**2*(A*a*d*g*i/2 + A*b*c*g*i/2 + B*a*d*g*i/6 - B*b*c*g*i/6) + x*
(A*a*c*g*i + B*a**2*d*g*i/(6*b) - B*b*c**2*g*i/(6*d)) + (B*a*c*g*i*x + B*a*
d*g*i*x**2/2 + B*b*c*g*i*x**2/2 + B*b*d*g*i*x**3/3)*log(e*(a + b*x)/(c + d*
x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(132) = 264.

Time = 0.21 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.58

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{1}{3}Abdgix^3+\frac{1}{2}Abcgix^2 + \frac{1}{2}Aadgix^2+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bacgi + \frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bbcgi + \frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Badgi + \frac{1}{6}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2c}{b^2d^2}\right. + Aacgix$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm
="maxima")
```

```
[Out] 1/3*A*b*d*g*i*x^3 + 1/2*A*b*c*g*i*x^2 + 1/2*A*a*d*g*i*x^2 + (x*log(b*e*x/(d
*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*a*c*g*i +
 1/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2
*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*b*c*g*i + 1/2*(x^2*log(b*e*x/(d*
x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*
c - a*d)*x/(b*d))*B*a*d*g*i + 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c
)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)
*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b*d*g*i + A*a*c*g*i*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1254 vs. 2(132) = 264.

Time = 0.50 (sec) , antiderivative size = 1254, normalized size of antiderivative = 8.96

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm
="giac")
```

```
[Out] -1/6*((B*b^5*c^4*e^4*g*i - 4*B*a*b^4*c^3*d*e^4*g*i + 6*B*a^2*b^3*c^2*d^2*e^
4*g*i - 4*B*a^3*b^2*c*d^3*e^4*g*i + B*a^4*b*d^4*e^4*g*i - 3*(b*e*x + a*e)*B
*b^4*c^4*d*e^3*g*i/(d*x + c) + 12*(b*e*x + a*e)*B*a*b^3*c^3*d^2*e^3*g*i/(d*
```

```
x + c) - 18*(b*e*x + a*e)*B*a^2*b^2*c^2*d^3*e^3*g*i/(d*x + c) + 12*(b*e*x +
 a*e)*B*a^3*b*c*d^4*e^3*g*i/(d*x + c) - 3*(b*e*x + a*e)*B*a^4*d^5*e^3*g*i/(
d*x + c))*log((b*e*x + a*e)/(d*x + c))/(b^3*d^2*e^3 - 3*(b*e*x + a*e)*b^2*d
^3*e^2/(d*x + c) + 3*(b*e*x + a*e)^2*b*d^4*e/(d*x + c)^2 - (b*e*x + a*e)^3*
d^5/(d*x + c)^3) + (A*b^6*c^4*e^4*g*i - 4*A*a*b^5*c^3*d*e^4*g*i + 6*A*a^2*b
^4*c^2*d^2*e^4*g*i - 4*A*a^3*b^3*c*d^3*e^4*g*i + A*a^4*b^2*d^4*e^4*g*i - 3*
(b*e*x + a*e)*A*b^5*c^4*d*e^3*g*i/(d*x + c) + (b*e*x + a*e)*B*b^5*c^4*d*e^3
*g*i/(d*x + c) + 12*(b*e*x + a*e)*A*a*b^4*c^3*d^2*e^3*g*i/(d*x + c) - 4*(b*
e*x + a*e)*B*a*b^4*c^3*d^2*e^3*g*i/(d*x + c) - 18*(b*e*x + a*e)*A*a^2*b^3*c
^2*d^3*e^3*g*i/(d*x + c) + 6*(b*e*x + a*e)*B*a^2*b^3*c^2*d^3*e^3*g*i/(d*x +
 c) + 12*(b*e*x + a*e)*A*a^3*b^2*c*d^4*e^3*g*i/(d*x + c) - 4*(b*e*x + a*e)*
B*a^3*b^2*c*d^4*e^3*g*i/(d*x + c) - 3*(b*e*x + a*e)*A*a^4*b*d^5*e^3*g*i/(d*
x + c) + (b*e*x + a*e)*B*a^4*b*d^5*e^3*g*i/(d*x + c) - (b*e*x + a*e)^2*B*b^
4*c^4*d^2*e^2*g*i/(d*x + c)^2 + 4*(b*e*x + a*e)^2*B*a*b^3*c^3*d^3*e^2*g*i/(
d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a^2*b^2*c^2*d^4*e^2*g*i/(d*x + c)^2 + 4*(b
*e*x + a*e)^2*B*a^3*b*c*d^5*e^2*g*i/(d*x + c)^2 - (b*e*x + a*e)^2*B*a^4*d^6
*e^2*g*i/(d*x + c)^2)/(b^4*d^2*e^3 - 3*(b*e*x + a*e)*b^3*d^3*e^2/(d*x + c)
+ 3*(b*e*x + a*e)^2*b^2*d^4*e/(d*x + c)^2 - (b*e*x + a*e)^3*b*d^5/(d*x + c)
^3) + (B*b^4*c^4*e*g*i - 4*B*a*b^3*c^3*d*e*g*i + 6*B*a^2*b^2*c^2*d^2*e*g*i
- 4*B*a^3*b*c*d^3*e*g*i + B*a^4*d^4*e*g*i)*log(-b*e + (b*e*x + a*e)*d/(d*x
+ c))/(b^2*d^2) - (B*b^4*c^4*e*g*i - 4*B*a*b^3*c^3*d*e*g*i + 6*B*a^2*b^2*c^
2*d^2*e*g*i - 4*B*a^3*b*c*d^3*e*g*i + B*a^4*d^4*e*g*i)*log((b*e*x + a*e)/(d
*x + c))/(b^2*d^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*
e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.01

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= x^2\left(\frac{g\,i\,(6\,A\,a\,d+6\,A\,b\,c+B\,a\,d-B\,b\,c)}{6}-\frac{A\,g\,i\,(6\,a\,d+6\,b\,c)}{12}\right)$$
$$-x\left(\frac{\left(\frac{g\,i\,(6\,A\,a\,d+6\,A\,b\,c+B\,a\,d-B\,b\,c)}{3}-\frac{A\,g\,i\,(6\,a\,d+6\,b\,c)}{6}\right)(6\,a\,d+6\,b\,c)}{6\,b\,d}+A\,a\,c\,g\,i\right.$$
$$\left.-\frac{g\,i\,(2\,A\,a^2\,d^2+2\,A\,b^2\,c^2+B\,a^2\,d^2-B\,b^2\,c^2+8\,A\,a\,b\,c\,d)}{2\,b\,d}\right)$$
$$+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(\frac{B\,b\,d\,g\,i\,x^3}{3}+\frac{B\,g\,i\,(a\,d+b\,c)\,x^2}{2}+B\,a\,c\,g\,i\,x\right)$$
$$-\frac{\ln(a+b\,x)\,(B\,a^3\,d\,g\,i-3\,B\,a^2\,b\,c\,g\,i)}{6\,b^2}$$
$$+\frac{\ln(c+d\,x)\,(B\,b\,c^3\,g\,i-3\,B\,a\,c^2\,d\,g\,i)}{6\,d^2}+\frac{A\,b\,d\,g\,i\,x^3}{3}$$

```
[In] int((a*g + b*g*x)*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x^2*((g*i*(6*A*a*d + 6*A*b*c + B*a*d - B*b*c))/6 - (A*g*i*(6*a*d + 6*b*c))/
12) - x*((((g*i*(6*A*a*d + 6*A*b*c + B*a*d - B*b*c))/3 - (A*g*i*(6*a*d + 6*
b*c))/6)*(6*a*d + 6*b*c))/(6*b*d) + A*a*c*g*i - (g*i*(2*A*a^2*d^2 + 2*A*b^2
*c^2 + B*a^2*d^2 - B*b^2*c^2 + 8*A*a*b*c*d))/(2*b*d)) + log((e*(a + b*x))/(
c + d*x))*((B*g*i*x^2*(a*d + b*c))/2 + (B*b*d*g*i*x^3)/3 + B*a*c*g*i*x) - (
log(a + b*x)*(B*a^3*d*g*i - 3*B*a^2*b*c*g*i))/(6*b^2) + (log(c + d*x)*(B*b*
c^3*g*i - 3*B*a*c^2*d*g*i))/(6*d^2) + (A*b*d*g*i*x^3)/3
```

3.4 $\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 81

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=-\frac{B(bc-ad)ix}{2b}-\frac{B(bc-ad)^2 i\log(a+bx)}{2b^2d}+\frac{i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d}$$

```
[Out] -1/2*B*(-a*d+b*c)*i*x/b-1/2*B*(-a*d+b*c)^2*i*ln(b*x+a)/b^2/d+1/2*i*(d*x+c)^
2*(A+B*ln(e*(b*x+a)/(d*x+c)))/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2548, 21, 45}

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\frac{i(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d}-\frac{Bi(bc-ad)^2\log(a+bx)}{2b^2d}-\frac{Bix(bc-ad)}{2b}$$

```
[In] Int[(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -1/2*(B*(b*c - a*d)*i*x)/b - (B*(b*c - a*d)^2*i*Log[a + b*x])/(2*b^2*d) + (
i*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d}-\frac{(B(bc-ad))\int\frac{(ci+dix)^2}{(a+bx)(c+dx)}\,dx}{2di}\\
&= \frac{i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d}-\frac{(B(bc-ad)i)\int\frac{c+dx}{a+bx}\,dx}{2d}\\
&= \frac{i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d}-\frac{(B(bc-ad)i)\int\left(\frac{d}{b}+\frac{bc-ad}{b(a+bx)}\right)dx}{2d}\\
&= -\frac{B(bc-ad)ix}{2b}-\frac{B(bc-ad)^2i\log(a+bx)}{2b^2d}+\frac{i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx
=\frac{i\left(-\frac{B(bc-ad)(bdx+(bc-ad)\log(a+bx))}{b^2}+(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\right)}{2d}$$

```
[In] Integrate[(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (i*(-((B*(b*c - a*d)*(b*d*x + (b*c - a*d)*Log[a + b*x]))/b^2) + (c + d*x)^2
*(A + B*Log[(e*(a + b*x))/(c + d*x)])))/(2*d)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.33

method	result
risch	$\frac{iBx(dx+2c)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2}+\frac{idA\,x^2}{2}+iAcx-\frac{B\,c^2 i\ln(-dx-c)}{2d}-\frac{idB\ln(bx+a)a^2}{2b^2}+\frac{iB\ln(bx+a)ac}{b}+\frac{idBa}{2b}$
parallelrisch	$\frac{B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^2d^2i+A\,x^2b^2d^2i+2Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^2cdi+2Ax\,b^2cdi-B\ln(bx+a)a^2d^2i+2B\ln(bx+a)abcdi-B\ln(ba}{2b^2d}$
parts	$Ai\left(\frac{1}{2}d\,x^2+xc\right)-Bi(ad-cb)^2e^2\left(-\frac{\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}{2e^2b^2d}-\frac{1}{2ebd\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}+\frac{\ln(}{}\right.$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{Adei(ad-cb)}{2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}-B\,d^2ei(ad-cb)\left(\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{2b^2e^2d}-\frac{1}{2bed\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{\ln}{}\right.\right.}{d^2}$
default	$-\frac{e(ad-cb)\left(-\frac{Adei(ad-cb)}{2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}-B\,d^2ei(ad-cb)\left(\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{2b^2e^2d}-\frac{1}{2bed\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{\ln}{}\right.\right.}{d^2}$

```
[In] int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*i*B*x*(d*x+2*c)*ln(e*(b*x+a)/(d*x+c))+1/2*i*d*A*x^2+i*A*c*x-1/2*B*c^2*i
/d*ln(-d*x-c)-1/2*i/b^2*d*B*ln(b*x+a)*a^2+i/b*B*ln(b*x+a)*a*c+1/2*i/b*d*B*a
*x-1/2*i*B*c*x
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.57

$$\int(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{Ab^2d^2ix^2-Bb^2c^2i\log(dx+c)+((2\,A-B)b^2cd+Babd^2)ix+(2\,Babcd-Ba^2d^2)i\log(bx+a)+(Bb^2d}{2\,b^2d}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/2*(A*b^2*d^2*i*x^2 - B*b^2*c^2*i*log(d*x + c) + ((2*A - B)*b^2*c*d + B*a*
b*d^2)*i*x + (2*B*a*b*c*d - B*a^2*d^2)*i*log(b*x + a) + (B*b^2*d^2*i*x^2 +
2*B*b^2*c*d*i*x)*log((b*e*x + a*e)/(d*x + c)))/(b^2*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(68) = 136$.

Time = 0.94 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.12

$$\int (ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{Adix^2}{2} - \frac{Bai(ad-2bc)\log\left(x + \frac{Ba^2cdi + \frac{Ba^2di(ad-2bc)}{b} - 3Babc^2i - Baci(ad-2bc)}{Ba^2d^2i - 2Babcdi - Bb^2c^2i}\right)}{2b^2} - \frac{Bc^2i\log\left(x + \frac{Ba^2cdi - 2Babc^2i - \frac{Bb^2c^3i}{d}}{Ba^2d^2i - 2Babcdi - Bb^2c^2i}\right)}{2d} + x\left(Aci + \frac{Badi}{2b} - \frac{Bci}{2}\right) + \left(Bcix + \frac{Bdix^2}{2}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*d*i*x**2/2 - B*a*i*(a*d - 2*b*c)*log(x + (B*a**2*c*d*i + B*a**2*d*i*(a*d
- 2*b*c)/b - 3*B*a*b*c**2*i - B*a*c*i*(a*d - 2*b*c))/(B*a**2*d**2*i - 2*B*a
*b*c*d*i - B*b**2*c**2*i))/(2*b**2) - B*c**2*i*log(x + (B*a**2*c*d*i - 2*B*
a*b*c**2*i - B*b**2*c**3*i/d)/(B*a**2*d**2*i - 2*B*a*b*c*d*i - B*b**2*c**2*
i))/(2*d) + x*(A*c*i + B*a*d*i/(2*b) - B*c*i/2) + (B*c*i*x + B*d*i*x**2/2)*
log(e*(a + b*x)/(c + d*x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.78

$$\int (ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{1}{2}Adix^2 + \left(x\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) + \frac{a\log(bx+a)}{b} - \frac{c\log(dx+c)}{d}\right)Bci + \frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) - \frac{a^2\log(bx+a)}{b^2} + \frac{c^2\log(dx+c)}{d^2} - \frac{(bc-ad)x}{bd}\right)Bdi + Acix$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/2*A*d*i*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b
- c*log(d*x + c)/d)*B*c*i + 1/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) -
 a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*d*i +
 A*c*i*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(75) = 150$.

Time = 0.41 (sec) , antiderivative size = 627, normalized size of antiderivative = 7.74

$$\int (ci + dix) \left(A + B \log\left(\frac{e(a + bx)}{c + dx}\right)\right) dx$$
$$= \frac{1}{2} \left(\frac{(Bb^3c^3e^3i - 3\,Bab^2c^2de^3i + 3\,Ba^2bcd^2e^3i - Ba^3d^3e^3i) \log\left(\frac{bex+ae}{dx+c}\right)}{b^2de^2 - \frac{2\,(bex+ae)bd^2e}{dx+c} + \frac{(bex+ae)^2d^3}{(dx+c)^2}} + \frac{Ab^4c^3e^3i - Bb^4c^3e^3i - 3\,Aab^3c^2 \cdots}{\cdots} \right.$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] 1/2*((B*b^3*c^3*e^3*i - 3*B*a*b^2*c^2*d*e^3*i + 3*B*a^2*b*c*d^2*e^3*i - B*a
^3*d^3*e^3*i)*log((b*e*x + a*e)/(d*x + c))/(b^2*d*e^2 - 2*(b*e*x + a*e)*b*d
^2*e/(d*x + c) + (b*e*x + a*e)^2*d^3/(d*x + c)^2) + (A*b^4*c^3*e^3*i - B*b^
4*c^3*e^3*i - 3*A*a*b^3*c^2*d*e^3*i + 3*B*a*b^3*c^2*d*e^3*i + 3*A*a^2*b^2*c
*d^2*e^3*i - 3*B*a^2*b^2*c*d^2*e^3*i - A*a^3*b*d^3*e^3*i + B*a^3*b*d^3*e^3*
i + (b*e*x + a*e)*B*b^3*c^3*d*e^2*i/(d*x + c) - 3*(b*e*x + a*e)*B*a*b^2*c^2
*d^2*e^2*i/(d*x + c) + 3*(b*e*x + a*e)*B*a^2*b*c*d^3*e^2*i/(d*x + c) - (b*e
*x + a*e)*B*a^3*d^4*e^2*i/(d*x + c))/(b^3*d*e^2 - 2*(b*e*x + a*e)*b^2*d^2*e
/(d*x + c) + (b*e*x + a*e)^2*b*d^3/(d*x + c)^2) + (B*b^3*c^3*e*i - 3*B*a*b^
2*c^2*d*e*i + 3*B*a^2*b*c*d^2*e*i - B*a^3*d^3*e*i)*log(-b*e + (b*e*x + a*e)
*d/(d*x + c))/(b^2*d) - (B*b^3*c^3*e*i - 3*B*a*b^2*c^2*d*e*i + 3*B*a^2*b*c*
d^2*e*i - B*a^3*d^3*e*i)*log((b*e*x + a*e)/(d*x + c))/(b^2*d))*(b*c/((b*c*e
 - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.56

$$\int (ci + dix) \left(A + B \log\left(\frac{e(a + bx)}{c + dx}\right)\right) dx = x \left(\frac{i\,(2\,A\,a\,d + 4\,A\,b\,c + B\,a\,d - B\,b\,c)}{2\,b} - \frac{A\,i\,(2\,a\,d + 2\,b\,c)}{2\,b}\right) + \ln\left(\frac{e\,(a + b\,x)}{c + d\,x}\right) \left(\frac{B\,d\,i\,x^2}{2} + B\,c\,i\,x\right) - \frac{\ln(a + b\,x)\,\left(B\,a^2\,d\,i - 2\,B\,a\,b\,c\,i\right)}{2\,b^2} + \frac{A\,d\,i\,x^2}{2} - \frac{B\,c^2\,i\,\ln(c + d\,x)}{2\,d}$$

```
[In] int((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x*((i*(2*A*a*d + 4*A*b*c + B*a*d - B*b*c))/(2*b) - (A*i*(2*a*d + 2*b*c))/(2
*b)) + log((e*(a + b*x))/(c + d*x))*((B*d*i*x^2)/2 + B*c*i*x) - (log(a + b*
x)*(B*a^2*d*i - 2*B*a*b*c*i))/(2*b^2) + (A*d*i*x^2)/2 - (B*c^2*i*log(c + d*
x))/(2*d)
```

3.5 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 133

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = \frac{i(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg} - \frac{(bc-ad)i\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g} + \frac{B(bc-ad)i\,\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{b^2g}$$

```
[Out] i*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g-(-a*d+b*c)*i*ln((a*d-b*c)/d/(b*x+
a))*(A-B+B*ln(e*(b*x+a)/(d*x+c)))/b^2/g+B*(-a*d+b*c)*i*polylog(2,1+(-a*d+b*
c)/d/(b*x+a))/b^2/g
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2560, 2542, 2458, 2378, 2370, 2352}

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = -\frac{i(bc-ad)\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{b^2g} + \frac{i(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg} + \frac{Bi(bc-ad)\,\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{b^2g}$$

```
[In] Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x),x]
```

```
[Out] (i*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b*g) - ((b*c - a*d)*i*L
og[-((b*c - a*d)/(d*(a + b*x)))]*(A - B + B*Log[(e*(a + b*x))/(c + d*x)]))/
(b^2*g) + (B*(b*c - a*d)*i*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b^2*
g)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2542

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
 - a*d)/g), Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
 - a*d, 0] && EqQ[b*f - a*g, 0]
```

Rule 2560

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Sim
p[(f + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*
(m + 2))), x] + Dist[i*((b*c - a*d)/(b*d*(m + 2))), Int[(f + g*x)^m*(A - B*
n + B*Log[e*((a + b*x)^n/(c + d*x)^n)]), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
g, h, i, A, B, m, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d,
0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IGtQ[m, -2]
```

Rubi steps

$$\text{integral} = \frac{i(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg} + \frac{((bc-ad)i)\int \frac{A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx}\,dx}{b}$$

$$= \frac{i(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg} - \frac{(bc-ad)i\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g} + \frac{(B(bc-ad)^2i)\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx}{b^2g}$$

$$= \frac{i(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg} - \frac{(bc-ad)i\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g} + \frac{(B(bc-ad)^2i)\,\text{Subst}\left(\int \frac{\log\left(\frac{-bc+ad}{dx}\right)}{x\left(\frac{bc-ad}{b}+\frac{dx}{b}\right)}\,dx, x, a+bx\right)}{b^3g}$$

$$= \frac{i(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg} - \frac{(bc-ad)i\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g} - \frac{(B(bc-ad)^2i)\,\text{Subst}\left(\int \frac{\log\left(\frac{(-bc+ad)x}{d}\right)}{\left(\frac{bc-ad}{b}+\frac{d}{bx}\right)x}\,dx, x, \frac{1}{a+bx}\right)}{b^3g}$$

$$= \frac{i(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg} - \frac{(bc-ad)i\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g} - \frac{(B(bc-ad)^2i)\,\text{Subst}\left(\int \frac{\log\left(\frac{(-bc+ad)x}{d}\right)}{\frac{d}{b}+\frac{(bc-ad)x}{b}}\,dx, x, \frac{1}{a+bx}\right)}{b^3g}$$

$$= \frac{i(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg} - \frac{(bc-ad)i\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g} + \frac{B(bc-ad)i\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.23

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$= \frac{i\Big((-bBc+aBd)\log^2(a+bx)+2\Big(Abdx+Bd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+(-bBc+aBd)\log(c+dx)\Big)+2(bc}{2b^2g}$$

```
[In] Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x
),x]
```

```
[Out] (i*((-(b*B*c) + a*B*d)*Log[a + b*x]^2 + 2*(A*b*d*x + B*d*(a + b*x)*Log[(e*(
a + b*x))/(c + d*x)] + (-(b*B*c) + a*B*d)*Log[c + d*x]) + 2*(b*c - a*d)*Log
[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)] + B*Log[(b*(c + d*x))/(b*c -
a*d)]) + 2*B*(b*c - a*d)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/(2*b^2*
g)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(132) = 264$.

Time = 1.49 (sec) , antiderivative size = 417, normalized size of antiderivative = 3.14

method	result
parts	$\frac{iA\left(\frac{xd}{b}+\frac{(-ad+cb)\ln(bx+a)}{b^2}\right)}{g} - \frac{iB(ad-cb)^2e^2\left(\frac{d^3\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)b^2e^2} - \frac{d^4\left(\frac{\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be}{be}\right)}{d}+\frac{\ln\left(\frac{be}{d}+\frac{(ad-}{d(d}\right.\cdots}{\cdots}\right.\cdots}{(ad-cb)b^2e^2\cdots}\right.\cdots}{\cdots}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{i\,d^2eA\left(\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^2e^2}+\frac{1}{be\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{b^2e^2}\right)}{g}+\frac{i\,d^2eB\left(-\frac{d\left(\frac{\operatorname{dilog}\left(-\frac{\left(\cdots\right.}{\cdots}\right.\cdots}{\cdots}\right.\cdots}{\cdots}\right.\cdots}{\cdots}\right.\cdots}{\cdots}$
default	$-\frac{e(ad-cb)\left(\frac{i\,d^2eA\left(\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^2e^2}+\frac{1}{be\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{b^2e^2}\right)}{g}+\frac{i\,d^2eB\left(-\frac{d\left(\frac{\operatorname{dilog}\left(-\frac{\left(\cdots\right.}{\cdots}\right.\cdots}{\cdots}\right.\cdots}{\cdots}\right.\cdots}{\cdots}\right.\cdots}{\cdots}$
risch	Expression too large to display

```
[In] int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x,method=_RETURNVER
BOSE)
```

```
[Out] i*A/g*(x*d/b+(-a*d+b*c)/b^2*ln(b*x+a))-i*B/g/d^3*(a*d-b*c)^2*e^2*(1/2/(a*d-
b*c)*d^3/b^2/e^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/(a*d-b*c)*d^4/b^2/e^2*
(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d
/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)+1/(a*d-b*c)*d^4
/b/e*(1/b/e/d*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)-ln(b*e/d+(a*d-b*c)*e/
d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/((b*e/d+(a*d-b*c)*e/d/(d*x+c))
*d-b*e)))
```

Fricas [F]

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag + bgx}\,dx = \int \frac{(dix + ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)}{bgx + ag}\,dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorithm
="fricas")
```

```
[Out] integral((A*d*i*x + A*c*i + (B*d*i*x + B*c*i)*log((b*e*x + a*e)/(d*x + c)))
/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag + bgx}\,dx = \frac{i\left(\int \frac{Ac}{a+bx}\,dx + \int \frac{Adx}{a+bx}\,dx + \int \frac{Bc\log\left(\frac{ae}{c+dx} + \frac{bex}{c+dx}\right)}{a+bx}\,dx + \int \frac{Bdx\log\left(\frac{ae}{c+dx} + \frac{bex}{c+dx}\right)}{a+bx}\,dx\right)}{g}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x)
```

```
[Out] i*(Integral(A*c/(a + b*x), x) + Integral(A*d*x/(a + b*x), x) + Integral(B*c
*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) + Integral(B*d*x*log(a*
e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x))/g
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.81

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag + bgx}\,dx = Adi\left(\frac{x}{bg} - \frac{a\log(bx+a)}{b^2g}\right) + \frac{Aci\log(bgx+ag)}{bg} - \frac{Bci\log(dx+c)}{bg} + \frac{(bci-adi)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right) + \mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{b^2g} + \frac{2\,Bbdix\log(e) + (bci-adi)B\log(bx+a)^2 + 2\,(Bbdix + (bci\log(e) - (i\log(e)-i)ad)B)\log(bx+a) \cdots}{2\,b^2g}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorithm
="maxima")
```

```
[Out] A*d*i*(x/(b*g) - a*log(b*x + a)/(b^2*g)) + A*c*i*log(b*g*x + a*g)/(b*g) - B
*c*i*log(d*x + c)/(b*g) + (b*c*i - a*d*i)*(log(b*x + a)*log((b*d*x + a*d)/(
b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(b^2*g) + 1/2*(2*B*b
*d*i*x*log(e) + (b*c*i - a*d*i)*B*log(b*x + a)^2 + 2*(B*b*d*i*x + (b*c*i*lo
g(e) - (i*log(e) - i)*a*d)*B)*log(b*x + a) - 2*(B*b*d*i*x + (b*c*i - a*d*i)
*B*log(b*x + a))*log(d*x + c))/(b^2*g)
```

Giac [F]

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag + bgx}\,dx = \int \frac{(dix + ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)}{bgx + ag}\,dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorithm
="giac")
```

```
[Out] integrate((d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag + bgx}\,dx = \int \frac{(c\,i + d\,i\,x)\,\left(A + B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{a\,g + b\,g\,x}\,dx$$

```
[In] int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x),x)
```

```
[Out] int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x), x)
```

3.6 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 142

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = -\frac{Bi(c+dx)}{bg^2(a+bx)} - \frac{i(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)} - \frac{di\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} + \frac{Bdi\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}$$

```
[Out] -B*i*(d*x+c)/b/g^2/(b*x+a)-i*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g^2/(b*x
+a)-d*i*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^2/g^2+B*d*i
*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/g^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2562, 2380, 2341, 2379, 2438}

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = -\frac{di\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^2g^2} - \frac{i(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg^2(a+bx)} + \frac{Bdi\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} - \frac{Bi(c+dx)}{bg^2(a+bx)}$$

```
[In] Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^2,x]
```

```
[Out] -((B*i*(c + d*x))/(b*g^2*(a + b*x))) - (i*(c + d*x)*(A + B*Log[(e*(a + b*x)
)/(c + d*x)]))/(b*g^2*(a + b*x)) - (d*i*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^2*g^2) + (B*d*i*PolyLog[2, (b*(c
 + d*x))/(d*(a + b*x))])/(b^2*g^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{i\text{Subst}\left(\int \frac{A+B\log(ex)}{x^2(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^2}$$

$$= \frac{i\text{Subst}\left(\int \frac{A+B\log(ex)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^2} + \frac{(di)\text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^2}$$

$$= -\frac{Bi(c+dx)}{bg^2(a+bx)} - \frac{i(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)} - \frac{di\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} + \frac{(Bdi)\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2}$$

$$= -\frac{Bi(c+dx)}{bg^2(a+bx)} - \frac{i(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)} - \frac{di\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} + \frac{Bdi\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.23

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$$
$$= \frac{i\left(-2(A+B)(bc-ad) - Bd(a+bx)\log^2(a+bx) + 2(-bBc+aBd)\log\left(\frac{e(a+bx)}{c+dx}\right) + 2Bd(a+bx)\log(c+\right.}{2b}$$

```
[In] Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x
)^2,x]
```

```
[Out] (i*(-2*(A + B)*(b*c - a*d) - B*d*(a + b*x)*Log[a + b*x]^2 + 2*(-(b*B*c) + a
*B*d)*Log[(e*(a + b*x))/(c + d*x)] + 2*B*d*(a + b*x)*Log[c + d*x] + 2*d*(a
+ b*x)*Log[a + b*x]*(A - B + B*Log[(e*(a + b*x))/(c + d*x)] + B*Log[(b*(c +
 d*x))/(b*c - a*d)]) + 2*B*d*(a + b*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a
*d)]))/(2*b^2*g^2*(a + b*x))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(142) = 284$.

Time = 1.38 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.61

method	result
parts	$\frac{iA\left(\frac{d\ln(bx+a)}{b^2}-\frac{-ad+cb}{b^2(bx+a)}\right)}{g^2}-\frac{iB(ad-cb)^2e^2\left(-\frac{d^3\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^2be}-\frac{d^4\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^2b^2e^2}+\frac{d^5}{}\right.}{g^2d^3}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{i\,d^2eA\left(-\frac{d\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{b^2e^2}-\frac{1}{be\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^2e^2}\right)}{(ad-cb)g^2}-\frac{i\,d^2eB\left(\frac{d\ln\left(\frac{be}{d}+\frac{(ad-}{d(dx}\right.}{2b^2e^2}\right.}{}\right.}{}$
default	$-\frac{e(ad-cb)\left(-\frac{i\,d^2eA\left(-\frac{d\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{b^2e^2}-\frac{1}{be\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^2e^2}\right)}{(ad-cb)g^2}-\frac{i\,d^2eB\left(\frac{d\ln\left(\frac{be}{d}+\frac{(ad-}{d(dx}\right.}{2b^2e^2}\right.}{}\right.}{}$
risch	Expression too large to display

```
[In] int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x,method=_RETURNV
ERBOSE)
```

```
[Out] i*A/g^2*(d/b^2*ln(b*x+a)-(-a*d+b*c)/b^2/(b*x+a))-i*B/g^2/d^3*(a*d-b*c)^2*e^
2*(-1/(a*d-b*c)^2*d^3/b/e*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b
*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-1/2/(a*d-b*c)^2*d^4/b^2/e
^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+1/(a*d-b*c)^2*d^5/b^2/e^2*(dilog(-((b*
e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln
(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d))
```

Fricas [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algorit
hm="fricas")
```

```
[Out] integral((A*d*i*x + A*c*i + (B*d*i*x + B*c*i)*log((b*e*x + a*e)/(d*x + c)))
/(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx=\text{Timed out}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algorit
hm="maxima")
```

```
[Out] -B*d*i*(((b*x + a)*log(b*x + a) + a)*log(d*x + c)/(b^3*g^2*x + a*b^2*g^2) -
 integrate((b^2*d*x^2*log(e) + a^2*d + (b^2*c*log(e) + a*b*d)*x + (2*b^2*d*
```

```
x^2 + a^2*d + (b^2*c + 2*a*b*d)*x)*log(b*x + a))/(b^4*d*g^2*x^3 + a^2*b^2*c
*g^2 + (b^4*c*g^2 + 2*a*b^3*d*g^2)*x^2 + (2*a*b^3*c*g^2 + a^2*b^2*d*g^2)*x)
, x)) + A*d*i*(a/(b^3*g^2*x + a*b^2*g^2) + log(b*x + a)/(b^2*g^2)) - B*c*i*
(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2) + 1/(b^2*g^2*x
+ a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/((b^2*c
- a*b*d)*g^2)) - A*c*i/(b^2*g^2*x + a*b*g^2)
```

Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algorit
hm="giac")
[Out] integrate((d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g)^2,
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{{(a\,g+b\,g\,x)}^2}\,dx$$

```
[In] int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^2,x)
[Out] int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^2, x
)
```

3.7 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 85

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = -\frac{Bi(c+dx)^2}{4(bc-ad)g^3(a+bx)^2} - \frac{i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)g^3(a+bx)^2}$$

```
[Out] -1/4*B*i*(d*x+c)^2/(-a*d+b*c)/g^3/(b*x+a)^2-1/2*i*(d*x+c)^2*(A+B*ln(e*(b*x+
a)/(d*x+c)))/(-a*d+b*c)/g^3/(b*x+a)^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2562, 2341}

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = -\frac{i(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^3(a+bx)^2(bc-ad)} - \frac{Bi(c+dx)^2}{4g^3(a+bx)^2(bc-ad)}$$

```
[In] Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^3,x]
```

```
[Out] -1/4*(B*i*(c + d*x)^2)/((b*c - a*d)*g^3*(a + b*x)^2) - (i*(c + d*x)^2*(A +
B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*c - a*d)*g^3*(a + b*x)^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{i\text{Subst}\left(\int \frac{A+B\log(ex)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^3}$$

$$= -\frac{Bi(c+dx)^2}{4(bc-ad)g^3(a+bx)^2} - \frac{i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)g^3(a+bx)^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 208 vs. 2(85) = 170.

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.45

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$$

$$= \frac{i\left(-\frac{(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2(a+bx)^2} - \frac{d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2(a+bx)} - \frac{Bd\left(\frac{1}{a+bx}+\frac{d\log(a+bx)}{bc-ad}-\frac{d\log(c+dx)}{bc-ad}\right)}{b^2} - \frac{B\left(\frac{bc-ad}{(a+bx)^2}-\frac{2d}{a+bx}-\frac{2d^2\log(a+bx)}{bc-ad}\right.}{4b^2}}{g^3}$$

```
[In] Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x
)^3,x]
```

```
[Out] (i*(-1/2*((b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^2*(a + b*x)^
2) - (d*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^2*(a + b*x)) - (B*d*((a +
b*x)^(-1) + (d*Log[a + b*x])/(b*c - a*d) - (d*Log[c + d*x])/(b*c - a*d)))/b
^2 - (B*((b*c - a*d)/(a + b*x)^2 - (2*d)/(a + b*x) - (2*d^2*Log[a + b*x])/(
b*c - a*d) + (2*d^2*Log[c + d*x])/(b*c - a*d)))/(4*b^2)))/g^3
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.80

method	result
parts	$\frac{iA\left(-\frac{-ad+cb}{2b^2(bx+a)^2}-\frac{d}{b^2(bx+a)}\right)}{g^3}-\frac{iB\,e^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}\right)}{g^3(ad-cb)}$
norman	$\frac{\frac{Bcdix\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(ad-cb)}-\frac{2Aadi+2Abci+Badi+Bbci}{4g\,b^2}-\frac{(2Adi+Bdi)x}{2gb}+\frac{Bi\,c^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2g(ad-cb)}+\frac{B\,d^2i\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2(ad-cb)g}}{(bx+a)^2g^2}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{i\,d^2eA}{2(ad-cb)^2g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{i\,d^2eB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}\right)}{(ad-cb)^2g^3}\right)}{d^2}$
default	$-\frac{e(ad-cb)\left(-\frac{i\,d^2eA}{2(ad-cb)^2g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{i\,d^2eB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}\right)}{(ad-cb)^2g^3}\right)}{d^2}$
parallelrisch	$-\frac{-2A\,c^2i\,b^4d+2A\,a^2b^2d^3i+B\,a^2b^2d^3i-B\,b^4c^2di-4Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^4c\,d^2i-2B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^4d^3i+4Axa\,b^3d^3i-4Ax}{4g^3(bx+a)^2(ad-cb)b^4d}$
risch	$-\frac{Bi(2bdx+ad+cb)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2g^3(bx+a)^2b^2}-\frac{i(2B\ln(dx+c)b^2d^2x^2-2B\ln(-bx-a)b^2d^2x^2+4B\ln(dx+c)ab\,d^2x-4B\ln(-bx-a)a}{}$

```
[In] int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x,method=_RETURNV
ERBOSE)
```

```
[Out] i*A/g^3*(-1/2*(-a*d+b*c)/b^2/(b*x+a)^2-d/b^2/(b*x+a))-i*B/g^3/(a*d-b*c)*e^2
*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/
(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(81) = 162.

Time = 0.33 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.08

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx =$$

$$-\frac{2\left(((2A+B)b^2cd-(2A+B)abd^2)ix+((2A+B)b^2c^2-(2A+B)a^2d^2)i+2\left(Bb^2d^2ix^2+2Bb^2cdix+\right.\right.}{4\left((b^5c-ab^4d)g^3x^2+2\left(ab^4c-a^2b^3d\right)g^3x+\left(a^2b^3c-a^3b^2d\right)g^3\right)}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algorit
hm="fricas")
```

```
[Out] -1/4*(2*((2*A + B)*b^2*c*d - (2*A + B)*a*b*d^2)*i*x + ((2*A + B)*b^2*c^2 -
(2*A + B)*a^2*d^2)*i + 2*(B*b^2*d^2*i*x^2 + 2*B*b^2*c*d*i*x + B*b^2*c^2*i)*
log((b*e*x + a*e)/(d*x + c)))/((b^5*c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2
*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(73) = 146$.

Time = 2.27 (sec) , antiderivative size = 384, normalized size of antiderivative = 4.52

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$$
$$= -\frac{Bd^2i\log\left(x+\frac{-\frac{Ba^2d^4i}{ad-bc}+\frac{2Babcd^3i}{ad-bc}+Bad^3i-\frac{Bb^2c^2d^2i}{ad-bc}+Bbcd^2i}{2Bbd^3i}\right)}{2b^2g^3\,(ad-bc)}$$
$$+\frac{Bd^2i\log\left(x+\frac{\frac{Ba^2d^4i}{ad-bc}-\frac{2Babcd^3i}{ad-bc}+Bad^3i+\frac{Bb^2c^2d^2i}{ad-bc}+Bbcd^2i}{2Bbd^3i}\right)}{2b^2g^3\,(ad-bc)}$$
$$+\frac{-2Aadi-2Abci-Badi-Bbci+x(-4Abdi-2Bbdi)}{4a^2b^2g^3+8ab^3g^3x+4b^4g^3x^2}$$
$$+\frac{(-Badi-Bbci-2Bbdix)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2a^2b^2g^3+4ab^3g^3x+2b^4g^3x^2}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3,x)
```

```
[Out] -B*d**2*i*log(x + (-B*a**2*d**4*i/(a*d - b*c) + 2*B*a*b*c*d**3*i/(a*d - b*c
) + B*a*d**3*i - B*b**2*c**2*d**2*i/(a*d - b*c) + B*b*c*d**2*i)/(2*B*b*d**3
*i))/(2*b**2*g**3*(a*d - b*c)) + B*d**2*i*log(x + (B*a**2*d**4*i/(a*d - b*c
) - 2*B*a*b*c*d**3*i/(a*d - b*c) + B*a*d**3*i + B*b**2*c**2*d**2*i/(a*d - b
*c) + B*b*c*d**2*i)/(2*B*b*d**3*i))/(2*b**2*g**3*(a*d - b*c)) + (-2*A*a*d*i
 - 2*A*b*c*i - B*a*d*i - B*b*c*i + x*(-4*A*b*d*i - 2*B*b*d*i))/(4*a**2*b**2
*g**3 + 8*a*b**3*g**3*x + 4*b**4*g**3*x**2) + (-B*a*d*i - B*b*c*i - 2*B*b*d
*i*x)*log(e*(a + b*x)/(c + d*x))/(2*a**2*b**2*g**3 + 4*a*b**3*g**3*x + 2*b*
*4*g**3*x**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(81) = 162$.

Time = 0.22 (sec) , antiderivative size = 570, normalized size of antiderivative = 6.71

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx =$$

$$-\frac{1}{4}\,Bdi\left(\frac{2\,(2\,bx+a)\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{b^4g^3x^2+2\,ab^3g^3x+a^2b^2g^3}+\frac{3\,abc-a^2d+2\,(2\,b^2c-abd)x}{(b^5c-ab^4d)g^3x^2+2\,(ab^4c-a^2b^3d)g^3x+(a^2b^3c-a^3b^2d)g^3}+\ldots\right.$$

$$+\frac{1}{4}\,Bci\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}-\frac{2\,\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{b^3g^3x^2+2\,ab^2g^3x+a^2bg^3}+\ldots\right.$$

$$-\frac{(2\,bx+a)Adi}{2\,(b^4g^3x^2+2\,ab^3g^3x+a^2b^2g^3)}-\frac{Aci}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algorit
hm="maxima")
```

```
[Out] -1/4*B*d*i*(2*(2*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^4*g^3*x^2
 + 2*a*b^3*g^3*x + a^2*b^2*g^3) + (3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*x)
/((b^5*c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c -
a^3*b^2*d)*g^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*c*d
+ a^2*b^2*d^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a*b^3*
c*d + a^2*b^2*d^2)*g^3)) + 1/4*B*c*i*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b
^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3)
- 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2
*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*
d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*(2*b*x +
a)*A*d*i/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) - 1/2*A*c*i/(b^3*g^3*x
^2 + 2*a*b^2*g^3*x + a^2*b*g^3)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.58

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx =$$

$$-\frac{1}{4}\left(\frac{2\,(dx+c)^2Be^3i\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^2g^3}+\frac{(2\,Ae^3i+Be^3i)(dx+c)^2}{(bex+ae)^2g^3}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-\ldots}\right.$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algorit
hm="giac")
```

```
[Out] -1/4*(2*(d*x + c)^2*B*e^3*i*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^2*g
^3) + (2*A*e^3*i + B*e^3*i)*(d*x + c)^2/((b*e*x + a*e)^2*g^3))*(b*c/((b*c*e
 - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.32

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = -\frac{x\,(2\,A\,b\,d\,i+B\,b\,d\,i)+A\,a\,d\,i+A\,b\,c\,i+\frac{B\,a\,d\,i}{2}+\frac{B\,b\,c\,i}{2}}{2\,a^2\,b^2\,g^3+4\,a\,b^3\,g^3\,x+2\,b^4\,g^3\,x^2} - \frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{B\,c\,i}{2\,b^2\,g^3}+\frac{B\,a\,d\,i}{2\,b^3\,g^3}+\frac{B\,d\,i\,x}{b^2\,g^3}\right)}{2\,a\,x+b\,x^2+\frac{a^2}{b}} - \frac{B\,d^2\,i\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,1\mathrm{i}}{b^2\,g^3\,\left(a\,d-b\,c\right)}$$

```
[In] int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^3,x)
```

```
[Out] - (x*(2*A*b*d*i + B*b*d*i) + A*a*d*i + A*b*c*i + (B*a*d*i)/2 + (B*b*c*i)/2)
/(2*a^2*b^2*g^3 + 2*b^4*g^3*x^2 + 4*a*b^3*g^3*x) - (log((e*(a + b*x))/(c +
d*x))*((B*c*i)/(2*b^2*g^3) + (B*a*d*i)/(2*b^3*g^3) + (B*d*i*x)/(b^2*g^3)))/
(2*a*x + b*x^2 + a^2/b) - (B*d^2*i*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1
i)*1i)/(b^2*g^3*(a*d - b*c))
```

3.8 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 173

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = \frac{Bdi(c+dx)^2}{4(bc-ad)^2g^4(a+bx)^2} - \frac{bBi(c+dx)^3}{9(bc-ad)^2g^4(a+bx)^3} + \frac{di(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^2g^4(a+bx)^2} - \frac{bi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^2g^4(a+bx)^3}$$

```
[Out] 1/4*B*d*i*(d*x+c)^2/(-a*d+b*c)^2/g^4/(b*x+a)^2-1/9*b*B*i*(d*x+c)^3/(-a*d+b*
c)^2/g^4/(b*x+a)^3+1/2*d*i*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)
^2/g^4/(b*x+a)^2-1/3*b*i*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^2
/g^4/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used

$= \{2562, 45, 2372, 12\}$

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = -\frac{bi(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3g^4(a+bx)^3(bc-ad)^2} + \frac{di(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^4(a+bx)^2(bc-ad)^2} - \frac{bBi(c+dx)^3}{9g^4(a+bx)^3(bc-ad)^2} + \frac{Bdi(c+dx)^2}{4g^4(a+bx)^2(bc-ad)^2}$$

```
[In] Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^4,x]
```

```
[Out] (B*d*i*(c + d*x)^2)/(4*(b*c - a*d)^2*g^4*(a + b*x)^2) - (b*B*i*(c + d*x)^3)
/(9*(b*c - a*d)^2*g^4*(a + b*x)^3) + (d*i*(c + d*x)^2*(A + B*Log[(e*(a + b*
x))/(c + d*x)]))/(2*(b*c - a*d)^2*g^4*(a + b*x)^2) - (b*i*(c + d*x)^3*(A +
B*Log[(e*(a + b*x))/(c + d*x)]))/(3*(b*c - a*d)^2*g^4*(a + b*x)^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
```

```
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex))}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^4} \\
&= \frac{di(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^2g^4(a+bx)^2} - \frac{bi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^2g^4(a+bx)^3} \\
&\quad - \frac{(Bi)\text{Subst}\left(\int \frac{-2b+3dx}{6x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^4} \\
&= \frac{di(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^2g^4(a+bx)^2} - \frac{bi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^2g^4(a+bx)^3} \\
&\quad - \frac{(Bi)\text{Subst}\left(\int \frac{-2b+3dx}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{6(bc-ad)^2g^4} \\
&= \frac{di(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^2g^4(a+bx)^2} - \frac{bi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^2g^4(a+bx)^3} \\
&\quad - \frac{(Bi)\text{Subst}\left(\int \left(-\frac{2b}{x^4}+\frac{3d}{x^3}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{6(bc-ad)^2g^4} \\
&= \frac{Bdi(c+dx)^2}{4(bc-ad)^2g^4(a+bx)^2} - \frac{bBi(c+dx)^3}{9(bc-ad)^2g^4(a+bx)^3} \\
&\quad + \frac{di(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^2g^4(a+bx)^2} - \frac{bi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^2g^4(a+bx)^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.08

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = -\frac{i\left(\frac{12Abc}{(a+bx)^3}+\frac{4bBc}{(a+bx)^3}-\frac{12aAd}{(a+bx)^3}-\frac{4aBd}{(a+bx)^3}+\frac{18Ad}{(a+bx)^2}+\frac{3Bd}{(a+bx)^2}-\frac{6Bd^2}{(bc-ad)(a+bx)}-\frac{6Bd^3\log(a+bx)}{(bc-ad)^2}+\frac{6B(2bc+ad+3bdx)\,\mathrm{l}}{(a+bx)^{\cdot}}\right.}{36b^2g^4}$$

```
[In] Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x
)^4,x]
```

```
[Out] -1/36*(i*((12*A*b*c)/(a + b*x)^3 + (4*b*B*c)/(a + b*x)^3 - (12*a*A*d)/(a +
b*x)^3 - (4*a*B*d)/(a + b*x)^3 + (18*A*d)/(a + b*x)^2 + (3*B*d)/(a + b*x)^2
 - (6*B*d^2)/((b*c - a*d)*(a + b*x)) - (6*B*d^3*Log[a + b*x])/(b*c - a*d)^2
 + (6*B*(2*b*c + a*d + 3*b*d*x)*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x)^3 +
 (6*B*d^3*Log[c + d*x])/(b*c - a*d)^2))/(b^2*g^4)
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.61

method	result
parts	$\frac{iA\left(-\frac{-ad+cb}{3b^2(bx+a)^3}-\frac{d}{2b^2(bx+a)^2}\right)}{g^4}-\frac{iB(ad-cb)^2e^2\left(\frac{d^4\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}\right)}{(ad-cb)^4}-\frac{d^3be\left(-\frac{\ln\left(\frac{be}{d}+\cdots\right)}{3\left(\frac{be}{d}+\cdots\right)}\cdots\right.}{\cdots}\right.}{g^4d^3}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{i\,d^2e^2Ab}{3(ad-cb)^3g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{i\,d^3eA}{2(ad-cb)^3g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{i\,d^2e^2Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\cdots\right)}\cdots\right.}{(ad-cb)^3g^4}\cdots\right.}{d^2}$
default	$-\frac{e(ad-cb)\left(\frac{i\,d^2e^2Ab}{3(ad-cb)^3g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{i\,d^3eA}{2(ad-cb)^3g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{i\,d^2e^2Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\cdots\right)}\cdots\right.}{(ad-cb)^3g^4}\cdots\right.}{d^2}$
norman	$\frac{-\frac{6A\,a^2d^2i+6Aabcdi-12A\,b^2c^2i+3B\,a^2d^2i+5Babcdi-4B\,b^2c^2i}{36b^2g(ad-cb)}-\frac{\left(6Aa\,d^2i-6Abcdi+3Ba\,d^2i-Bbcdi\right)x}{12g(ad-cb)b}+\frac{B\,d^2ib\,x^3}{18a(ad-cb)g}+\frac{Bi\,c^2(3ac\cdots}{6\left(a^2d\cdots\right.}\cdots}{(bx+a)^3g^3}$
risch	$-\frac{Bi(3bdx+ad+2cb)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{6(bx+a)^3b^2g^4}-\frac{\left(-6B\ln(-bx-a)b^3d^3x^3+6B\ln(dx+c)b^3d^3x^3-18B\ln(-bx-a)a\,b^2d^3x^2+18B\ln\cdots\right.}{\cdots}$
parallelrisch	$-\frac{-36Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^5c\,d^3i-6B\,x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)b^6d^4i+6B\,x^2a\,b^5d^4i-6B\,x^2b^6c\,d^3i+18Ax\,a^2b^4d^4i+18Ax\,b^6c^2d^2i+\cdots}{\cdots}$

```
[In] int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x,method=_RETURNV
ERBOSE)
```

```
[Out] i*A/g^4*(-1/3*(-a*d+b*c)/b^2/(b*x+a)^3-1/2*d/b^2/(b*x+a)^2)-i*B/g^4/d^3*(a*
d-b*c)^2*e^2*(d^4/(a*d-b*c)^4*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/
d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-d^3/(a*d-b*c)
^4*b*e*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(165) = 330.

Time = 0.37 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.10

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$$
$$=\frac{6\,(Bb^3cd^2-Bab^2d^3)ix^2-3\,((6\,A+B)b^3c^2d-6\,(2\,A+B)ab^2cd^2+(6\,A+5\,B)a^2bd^3)ix-(4\,(3\,A+B)b^3c}{36\,((b^7c^2-2\,ab^6cd+a^2b^5d^2)g^4x^3+3\,(ab^6c^2-2\,a^2b^5c}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algorit
hm="fricas")
```

```
[Out] 1/36*(6*(B*b^3*c*d^2 - B*a*b^2*d^3)*i*x^2 - 3*((6*A + B)*b^3*c^2*d - 6*(2*A
 + B)*a*b^2*c*d^2 + (6*A + 5*B)*a^2*b*d^3)*i*x - (4*(3*A + B)*b^3*c^3 - 9*(
2*A + B)*a*b^2*c^2*d + (6*A + 5*B)*a^3*d^3)*i + 6*(B*b^3*d^3*i*x^3 + 3*B*a*
b^2*d^3*i*x^2 - 3*(B*b^3*c^2*d - 2*B*a*b^2*c*d^2)*i*x - (2*B*b^3*c^3 - 3*B*
a*b^2*c^2*d)*i)*log((b*e*x + a*e)/(d*x + c)))/((b^7*c^2 - 2*a*b^6*c*d + a^2
*b^5*d^2)*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3
*(a^2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b
^3*c*d + a^5*b^2*d^2)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. 2(158) = 316.

Time = 4.17 (sec) , antiderivative size = 629, normalized size of antiderivative = 3.64

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$$
$$=-\frac{Bd^3i\log\left(x+\frac{-\frac{Ba^3d^6i}{(ad-bc)^2}+\frac{3Ba^2bcd^5i}{(ad-bc)^2}-\frac{3Bab^2c^2d^4i}{(ad-bc)^2}+Bad^4i+\frac{Bb^3c^3d^3i}{(ad-bc)^2}+Bbcd^3i}{2Bbd^4i}\right)}{6b^2g^4\,(ad-bc)^2}$$
$$+\frac{Bd^3i\log\left(x+\frac{\frac{Ba^3d^6i}{(ad-bc)^2}-\frac{3Ba^2bcd^5i}{(ad-bc)^2}+\frac{3Bab^2c^2d^4i}{(ad-bc)^2}+Bad^4i-\frac{Bb^3c^3d^3i}{(ad-bc)^2}+Bbcd^3i}{2Bbd^4i}\right)}{6b^2g^4\,(ad-bc)^2}$$
$$+\frac{(-Badi-2Bbci-3Bbdix)\log\left(\frac{e(a+bx)}{c+dx}\right)}{6a^3b^2g^4+18a^2b^3g^4x+18ab^4g^4x^2+6b^5g^4x^3}$$
$$+\frac{-6Aa^2d^2i-6Aabcdi+12Ab^2c^2i-5Ba^2d^2i-5Babcdi+4Bb^2c^2i-6Bb^2d^2ix^2+x(-18Aabd^2i+18Ab}{36a^4b^2dg^4-36a^3b^3cg^4+x^3\cdot(36ab^5dg^4-36b^6cg^4)+x^2\cdot(108a^2b^4dg^4-108ab^5cg^4)+x\,(108a^3b}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4,x)
```

```
[Out] -B*d**3*i*log(x + (-B*a**3*d**6*i/(a*d - b*c)**2 + 3*B*a**2*b*c*d**5*i/(a*d
 - b*c)**2 - 3*B*a*b**2*c**2*d**4*i/(a*d - b*c)**2 + B*a*d**4*i + B*b**3*c*
*3*d**3*i/(a*d - b*c)**2 + B*b*c*d**3*i)/(2*B*b*d**4*i))/(6*b**2*g**4*(a*d
- b*c)**2) + B*d**3*i*log(x + (B*a**3*d**6*i/(a*d - b*c)**2 - 3*B*a**2*b*c*
d**5*i/(a*d - b*c)**2 + 3*B*a*b**2*c**2*d**4*i/(a*d - b*c)**2 + B*a*d**4*i
- B*b**3*c**3*d**3*i/(a*d - b*c)**2 + B*b*c*d**3*i)/(2*B*b*d**4*i))/(6*b**2
*g**4*(a*d - b*c)**2) + (-B*a*d*i - 2*B*b*c*i - 3*B*b*d*i*x)*log(e*(a + b*x
)/(c + d*x))/(6*a**3*b**2*g**4 + 18*a**2*b**3*g**4*x + 18*a*b**4*g**4*x**2
+ 6*b**5*g**4*x**3) + (-6*A*a**2*d**2*i - 6*A*a*b*c*d*i + 12*A*b**2*c**2*i
- 5*B*a**2*d**2*i - 5*B*a*b*c*d*i + 4*B*b**2*c**2*i - 6*B*b**2*d**2*i*x**2
+ x*(-18*A*a*b*d**2*i + 18*A*b**2*c*d*i - 15*B*a*b*d**2*i + 3*B*b**2*c*d*i)
)/(36*a**4*b**2*d*g**4 - 36*a**3*b**3*c*g**4 + x**3*(36*a*b**5*d*g**4 - 36*
b**6*c*g**4) + x**2*(108*a**2*b**4*d*g**4 - 108*a*b**5*c*g**4) + x*(108*a**
3*b**3*d*g**4 - 108*a**2*b**4*c*g**4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(165) = 330$.

Time = 0.24 (sec) , antiderivative size = 933, normalized size of antiderivative = 5.39

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx =$$
$$-\frac{1}{36}\,Bdi\left(\frac{6\,(3\,bx+a)\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{b^5g^4x^3+3\,ab^4g^4x^2+3\,a^2b^3g^4x+a^3b^2g^4}+\frac{5\,ab^2c^2-22\,a^2bcd+5\,a^3}{(b^7c^2-2\,ab^6cd+a^2b^5d^2)g^4x^3+3\,(ab^6c^2-2\,a^2b^5c}\right.$$
$$-\frac{1}{18}\,Bci\left(\frac{6\,b^2d^2x^2+2\,b^2c^2-7\,abcd+11\,a^2d^2-3\,(b^2cd-5\,abd^2)}{(b^6c^2-2\,ab^5cd+a^2b^4d^2)g^4x^3+3\,(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2)g^4x^2+3\,(a^2b^4c^2-2\,a^3b^3cd+a^4b}\right.$$
$$-\frac{(3\,bx+a)Adi}{6\,(b^5g^4x^3+3\,ab^4g^4x^2+3\,a^2b^3g^4x+a^3b^2g^4)}$$
$$-\frac{Aci}{3\,(b^4g^4x^3+3\,ab^3g^4x^2+3\,a^2b^2g^4x+a^3bg^4)}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algorit
hm="maxima")
```

```
[Out] -1/36*B*d*i*(6*(3*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^5*g^4*x^
3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) + (5*a*b^2*c^2 - 22*a^
2*b*c*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c^2 - 16*a*b
^2*c*d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3 + 3
*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 - 2*a^3
*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)
*g^4) - 6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^
```

```
2*b^3*c*d^2 - a^3*b^2*d^3)*g^4) + 6*(3*b*c*d^2 - a*d^3)*log(d*x + c)/((b^5*
c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4)) - 1/18*B*c*i*((6
*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)
*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b
^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^
2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*log(b*e*x/(d*
x + c) + a*e/(d*x + c))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x +
a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2
 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b
^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/6*(3*b*x + a)*A*d*i/(b^5*g^4*x^3 + 3*a*b^4*
g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - 1/3*A*c*i/(b^4*g^4*x^3 + 3*a*b^3
*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Giac [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.56

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = -\frac{1}{36}\left(\frac{6\left(2\,Bbe^4i-\frac{3\,(bex+ae)Bde^3i}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^3bcg^4}{(dx+c)^3}-\frac{(bex+ae)^3adg^4}{(dx+c)^3}}+\frac{12\,Abe^4i+4\,Bbe^4i-\frac{18\,(bex+ae)Ade^3i}{dx+c}-\frac{9\,(bex+ae)Bde^3i}{dx+c}}{\frac{(bex+ae)^3bcg^4}{(dx+c)^3}-\frac{(bex+ae)^3adg^4}{(dx+c)^3}}\right)\Big($$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algorit
hm="giac")
```

```
[Out] -1/36*(6*(2*B*b*e^4*i - 3*(b*e*x + a*e)*B*d*e^3*i/(d*x + c))*log((b*e*x + a
*e)/(d*x + c))/((b*e*x + a*e)^3*b*c*g^4/(d*x + c)^3 - (b*e*x + a*e)^3*a*d*g
^4/(d*x + c)^3) + (12*A*b*e^4*i + 4*B*b*e^4*i - 18*(b*e*x + a*e)*A*d*e^3*i/
(d*x + c) - 9*(b*e*x + a*e)*B*d*e^3*i/(d*x + c))/((b*e*x + a*e)^3*b*c*g^4/(
d*x + c)^3 - (b*e*x + a*e)^3*a*d*g^4/(d*x + c)^3))*(b*c/((b*c*e - a*d*e)*(b
*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.09

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = -\frac{\frac{6\,A\,a^2\,d^2\,i-12\,A\,b^2\,c^2\,i+5\,B\,a^2\,d^2\,i-4\,B\,b^2\,c^2\,i+6\,A\,a\,b\,c\,d\,i+5\,B\,a\,b\,c\,d\,i}{6\,(a\,d-b\,c)}+\frac{x\,(6\,A\,a\,b\,d^2\,i+5\,B\,a\,b\,d^2\,i-6\,A\,b^2\,c\,d\,i-B\,b^2\,c\,d\,i)}{2\,(a\,d-b\,c)}+\frac{B\,b^2\,d^2\,i\,x^2}{a\,d-b\,c}}{6\,a^3\,b^2\,g^4+18\,a^2\,b^3\,g^4\,x+18\,a\,b^4\,g^4\,x^2+6\,b^5\,g^4\,x^3} - \frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{B\,c\,i}{3\,b^2\,g^4}+\frac{B\,a\,d\,i}{6\,b^3\,g^4}+\frac{B\,d\,i\,x}{2\,b^2\,g^4}\right)}{3\,a^2\,x+\frac{a^3}{b}+b^2\,x^3+3\,a\,b\,x^2} - \frac{B\,d^3\,i\,\mathrm{atanh}\left(\frac{6\,b^4\,c^2\,g^4-6\,a^2\,b^2\,d^2\,g^4}{6\,b^2\,g^4\,{(a\,d-b\,c)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{3\,b^2\,g^4\,{(a\,d-b\,c)}^2}$$

```
[In] int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^4,x)
```

```
[Out] - ((6*A*a^2*d^2*i - 12*A*b^2*c^2*i + 5*B*a^2*d^2*i - 4*B*b^2*c^2*i + 6*A*a*
b*c*d*i + 5*B*a*b*c*d*i)/(6*(a*d - b*c)) + (x*(6*A*a*b*d^2*i + 5*B*a*b*d^2*
i - 6*A*b^2*c*d*i - B*b^2*c*d*i))/(2*(a*d - b*c)) + (B*b^2*d^2*i*x^2)/(a*d
- b*c))/(6*a^3*b^2*g^4 + 6*b^5*g^4*x^3 + 18*a^2*b^3*g^4*x + 18*a*b^4*g^4*x^
2) - (log((e*(a + b*x))/(c + d*x))*((B*c*i)/(3*b^2*g^4) + (B*a*d*i)/(6*b^3*
g^4) + (B*d*i*x)/(2*b^2*g^4)))/(3*a^2*x + a^3/b + b^2*x^3 + 3*a*b*x^2) - (B
*d^3*i*atanh((6*b^4*c^2*g^4 - 6*a^2*b^2*d^2*g^4)/(6*b^2*g^4*(a*d - b*c)^2)
- (2*b*d*x)/(a*d - b*c)))/(3*b^2*g^4*(a*d - b*c)^2)
```

3.9 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 269

$$\begin{aligned}\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = &-\frac{Bd^2i(c+dx)^2}{4(bc-ad)^3g^5(a+bx)^2}+\frac{2bBdi(c+dx)^3}{9(bc-ad)^3g^5(a+bx)^3}\\&-\frac{b^2Bi(c+dx)^4}{16(bc-ad)^3g^5(a+bx)^4}\\&-\frac{d^2i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^5(a+bx)^2}\\&+\frac{2bdi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^3g^5(a+bx)^3}\\&-\frac{b^2i(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^3g^5(a+bx)^4}\end{aligned}$$

```
[Out] -1/4*B*d^2*i*(d*x+c)^2/(-a*d+b*c)^3/g^5/(b*x+a)^2+2/9*b*B*d*i*(d*x+c)^3/(-a
*d+b*c)^3/g^5/(b*x+a)^3-1/16*b^2*B*i*(d*x+c)^4/(-a*d+b*c)^3/g^5/(b*x+a)^4-1
/2*d^2*i*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^5/(b*x+a)^2+2
/3*b*d*i*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^5/(b*x+a)^3-1
/4*b^2*i*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^5/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2562, 45, 2372, 12, 14}

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx=-\frac{b^2i(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4g^5(a+bx)^4(bc-ad)^3}-\frac{d^2i(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^5(a+bx)^2(bc-ad)^3}+\frac{2bdi(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3g^5(a+bx)^3(bc-ad)^3}-\frac{b^2Bi(c+dx)^4}{16g^5(a+bx)^4(bc-ad)^3}-\frac{Bd^2i(c+dx)^2}{4g^5(a+bx)^2(bc-ad)^3}+\frac{2bBdi(c+dx)^3}{9g^5(a+bx)^3(bc-ad)^3}$$

```
[In] Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^5,x]
```

```
[Out] -1/4*(B*d^2*i*(c + d*x)^2)/((b*c - a*d)^3*g^5*(a + b*x)^2) + (2*b*B*d*i*(c
+ d*x)^3)/(9*(b*c - a*d)^3*g^5*(a + b*x)^3) - (b^2*B*i*(c + d*x)^4)/(16*(b*
c - a*d)^3*g^5*(a + b*x)^4) - (d^2*i*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(
c + d*x)]))/(2*(b*c - a*d)^3*g^5*(a + b*x)^2) + (2*b*d*i*(c + d*x)^3*(A + B
*Log[(e*(a + b*x))/(c + d*x)]))/(3*(b*c - a*d)^3*g^5*(a + b*x)^3) - (b^2*i*
(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*(b*c - a*d)^3*g^5*(a +
 b*x)^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{i\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex))}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^5}$$

$$= -\frac{d^2i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^5(a+bx)^2} + \frac{2bdi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^3g^5(a+bx)^3} - \frac{b^2i(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^3g^5(a+bx)^4} - \frac{(Bi)\text{Subst}\left(\int \frac{-3b^2+8bdx-6d^2x^2}{12x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^5}$$

$$= -\frac{d^2i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^5(a+bx)^2} + \frac{2bdi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^3g^5(a+bx)^3} - \frac{b^2i(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^3g^5(a+bx)^4} - \frac{(Bi)\text{Subst}\left(\int \frac{-3b^2+8bdx-6d^2x^2}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{12(bc-ad)^3g^5}$$

$$= -\frac{d^2i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^5(a+bx)^2} + \frac{2bdi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^3g^5(a+bx)^3} - \frac{b^2i(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^3g^5(a+bx)^4} - \frac{(Bi)\text{Subst}\left(\int \left(-\frac{3b^2}{x^5}+\frac{8bd}{x^4}-\frac{6d^2}{x^3}\right)dx, x, \frac{a+bx}{c+dx}\right)}{12(bc-ad)^3g^5}$$

$$= -\frac{Bd^2i(c+dx)^2}{4(bc-ad)^3g^5(a+bx)^2} + \frac{2bBdi(c+dx)^3}{9(bc-ad)^3g^5(a+bx)^3} - \frac{b^2Bi(c+dx)^4}{16(bc-ad)^3g^5(a+bx)^4} - \frac{d^2i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^5(a+bx)^2} + \frac{2bdi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^3g^5(a+bx)^3} - \frac{b^2i(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^3g^5(a+bx)^4}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.78

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = -\frac{i\left(\frac{36Abc}{(a+bx)^4} + \frac{9bBc}{(a+bx)^4} - \frac{36aAd}{(a+bx)^4} - \frac{9aBd}{(a+bx)^4} + \frac{48Ad}{(a+bx)^3} + \frac{4Bd}{(a+bx)^3} - \frac{6Bd^2}{(bc-ad)(a+bx)^2} + \frac{12Bd^3}{(bc-ad)^2(a+bx)} + \frac{12Bd^4\log(a+bx}{(bc-ad)^3}\right.}{144b^2g^5}$$

```
[In] Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x
)^5,x]
```

```
[Out] -1/144*(i*((36*A*b*c)/(a + b*x)^4 + (9*b*B*c)/(a + b*x)^4 - (36*a*A*d)/(a +
 b*x)^4 - (9*a*B*d)/(a + b*x)^4 + (48*A*d)/(a + b*x)^3 + (4*B*d)/(a + b*x)^
3 - (6*B*d^2)/((b*c - a*d)*(a + b*x)^2) + (12*B*d^3)/((b*c - a*d)^2*(a + b*
x)) + (12*B*d^4*Log[a + b*x])/(b*c - a*d)^3 + (12*B*(3*b*c + a*d + 4*b*d*x)
*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x)^4 - (12*B*d^4*Log[c + d*x])/(b*c -
 a*d)^3))/(b^2*g^5)
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.45

method	result
parts	$\frac{iA\left(-\frac{d}{3b^2(bx+a)^3}-\frac{-ad+cb}{4b^2(bx+a)^4}\right)}{g^5}-\frac{iB(ad-cb)^2e^2\left(\frac{d^5\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}\right)}{(ad-cb)^5}-\frac{2d^4be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad}{d(\cdots}\right)}{3\left(\frac{be}{d}+\frac{(ad-}{d(d\cdots}\right)}\cdots\right.}{\cdots}\cdots\right.}{g^5\cdots}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{i\,d^2e^3A\,b^2}{4(ad-cb)^4g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}+\frac{2i\,d^3e^2Ab}{3(ad-cb)^4g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{i\,d^4eA}{2(ad-cb)^4g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{i\,d^2e^3B\,b^2\left(\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
default	$-\frac{e(ad-cb)\left(-\frac{i\,d^2e^3A\,b^2}{4(ad-cb)^4g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}+\frac{2i\,d^3e^2Ab}{3(ad-cb)^4g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{i\,d^4eA}{2(ad-cb)^4g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{i\,d^2e^3B\,b^2\left(\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
risch	$-\frac{iB(4bdx+ad+3cb)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{12(bx+a)^4b^2g^5}-\frac{\left(-48Ba\,b^3c\,d^3x^2-72B\,a^2b^2c\,d^3x+24Ba\,b^3c^2d^2x-72A\,a^2b^2c^2d^2+96Aa\,b^3c^3d-48\cdots\right.}{\cdots}$
norman	$\frac{\left(12A\,a^2c\,d^2i-24Aab\,c^2di+12A\,b^2c^3i+6B\,a^2c\,d^2i-8Bab\,c^2di+3B\,b^2c^3i\right)x}{12ga\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{\left(12A\,a^3d^3i+12A\,a^2bc\,d^2i-60Aa\,b^2c^2di+36A\,b^3c^3i+6B\,a\cdots\right.}{24g\,a^2\left(a^2d^2-2abc\cdots\right.}$
parallelrisch	$\frac{-144Ax\,a^5b^3c^5i+72Bx\,a^8c^2d^3i-36Bx\,a^5b^3c^5i+72B\ln\left(\frac{e(bx+a)}{dx+c}\right)a^8c^3d^2i+36B\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6b^2c^5i+12B\,x^4\ln\left(\frac{e(bx+a)}{dx+c}\cdots\right.}{\cdots}$

```
[In] int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x,method=_RETURNV
ERBOSE)
```

```
[Out] i*A/g^5*(-1/3*d/b^2/(b*x+a)^3-1/4*(-a*d+b*c)/b^2/(b*x+a)^4)-i*B/g^5/d^3*(a*
d-b*c)^2*e^2*(d^5/(a*d-b*c)^5*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/
d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-2*d^4/(a*d-b*
c)^5*b*e*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+
c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)+d^3/(a*d-b*c)^5*e^2*b^2*(-1/4/(b*e
/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*
d-b*c)*e/d/(d*x+c))^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(257) = 514$.

Time = 0.36 (sec) , antiderivative size = 602, normalized size of antiderivative = 2.24

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag + bgx)^5}\,dx = -\frac{12\,(Bb^4cd^3 - Bab^3d^4)ix^3 - 6\,(Bb^4c^2d^2 - 8\,Bab^3cd^3 + 7\,Ba^2b^2d^4)ix^2 + 4\,((12\,A + B)b^4c^3d - 6\,(6\,A + B}{144\,((b^9c^3 - 3\,ab^8c^2d + 3\,a^2b}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algorit
hm="fricas")
```

```
[Out] -1/144*(12*(B*b^4*c*d^3 - B*a*b^3*d^4)*i*x^3 - 6*(B*b^4*c^2*d^2 - 8*B*a*b^3
*c*d^3 + 7*B*a^2*b^2*d^4)*i*x^2 + 4*((12*A + B)*b^4*c^3*d - 6*(6*A + B)*a*b
^3*c^2*d^2 + 18*(2*A + B)*a^2*b^2*c*d^3 - (12*A + 13*B)*a^3*b*d^4)*i*x + (9
*(4*A + B)*b^4*c^4 - 32*(3*A + B)*a*b^3*c^3*d + 36*(2*A + B)*a^2*b^2*c^2*d^
2 - (12*A + 13*B)*a^4*d^4)*i + 12*(B*b^4*d^4*i*x^4 + 4*B*a*b^3*d^4*i*x^3 +
6*B*a^2*b^2*d^4*i*x^2 + 4*(B*b^4*c^3*d - 3*B*a*b^3*c^2*d^2 + 3*B*a^2*b^2*c*
d^3)*i*x + (3*B*b^4*c^4 - 8*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2)*i)*log((b*
e*x + a*e)/(d*x + c)))/((b^9*c^3 - 3*a*b^8*c^2*d + 3*a^2*b^7*c*d^2 - a^3*b^
6*d^3)*g^5*x^4 + 4*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*a^3*b^6*c*d^2 - a^4*b^5
*d^3)*g^5*x^3 + 6*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*a^4*b^5*c*d^2 - a^5*b^
4*d^3)*g^5*x^2 + 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2 - a^6*b
^3*d^3)*g^5*x + (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d^2 - a^7*b^2*
d^3)*g^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 928 vs. $2(252) = 504$.

Time = 6.87 (sec) , antiderivative size = 928, normalized size of antiderivative = 3.45

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$$
$$= -\frac{Bd^4i\log\left(x+\frac{-\frac{Ba^4d^8i}{(ad-bc)^3}+\frac{4Ba^3bcd^7i}{(ad-bc)^3}-\frac{6Ba^2b^2c^2d^6i}{(ad-bc)^3}+\frac{4Bab^3c^3d^5i}{(ad-bc)^3}+Bad^5i-\frac{Bb^4c^4d^4i}{(ad-bc)^3}+Bbcd^4i}{2Bbd^5i}\right)}{12b^2g^5(ad-bc)^3}$$
$$+\frac{Bd^4i\log\left(x+\frac{\frac{Ba^4d^8i}{(ad-bc)^3}-\frac{4Ba^3bcd^7i}{(ad-bc)^3}+\frac{6Ba^2b^2c^2d^6i}{(ad-bc)^3}-\frac{4Bab^3c^3d^5i}{(ad-bc)^3}+Bad^5i+\frac{Bb^4c^4d^4i}{(ad-bc)^3}+Bbcd^4i}{2Bbd^5i}\right)}{12b^2g^5(ad-bc)^3}$$
$$+\frac{(-Badi-3Bbci-4Bbdix)\log\left(\frac{e(a+bx)}{c+dx}\right)}{12a^4b^2g^5+48a^3b^3g^5x+72a^2b^4g^5x^2+48ab^5g^5x^3+12b^6g^5x^4}$$
$$+\frac{-12Aa^3d^3i-12Aa^2bcd^2i+60Aab^2c^2di-36Ab^3c^3i-13Ba^3d^3i-13Ba^2bcd^2i+23Bab^2c^2di-9Bb^3}{144a^6b^2d^2g^5-288a^5b^3cdg^5+144a^4b^4c^2g^5+x^4\cdot(144a^2b^6d^2g^5-288ab^7cdg^5+144b^8c^2g^5)+x^3\cdot(576a^3b^5}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**5,x)
```

```
[Out] -B*d**4*i*log(x + (-B*a**4*d**8*i/(a*d - b*c)**3 + 4*B*a**3*b*c*d**7*i/(a*d
 - b*c)**3 - 6*B*a**2*b**2*c**2*d**6*i/(a*d - b*c)**3 + 4*B*a*b**3*c**3*d**
5*i/(a*d - b*c)**3 + B*a*d**5*i - B*b**4*c**4*d**4*i/(a*d - b*c)**3 + B*b*c
*d**4*i)/(2*B*b*d**5*i))/(12*b**2*g**5*(a*d - b*c)**3) + B*d**4*i*log(x + (
B*a**4*d**8*i/(a*d - b*c)**3 - 4*B*a**3*b*c*d**7*i/(a*d - b*c)**3 + 6*B*a**
2*b**2*c**2*d**6*i/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**5*i/(a*d - b*c)**3 +
 B*a*d**5*i + B*b**4*c**4*d**4*i/(a*d - b*c)**3 + B*b*c*d**4*i)/(2*B*b*d**5
*i))/(12*b**2*g**5*(a*d - b*c)**3) + (-B*a*d*i - 3*B*b*c*i - 4*B*b*d*i*x)*l
og(e*(a + b*x)/(c + d*x))/(12*a**4*b**2*g**5 + 48*a**3*b**3*g**5*x + 72*a**
2*b**4*g**5*x**2 + 48*a*b**5*g**5*x**3 + 12*b**6*g**5*x**4) + (-12*A*a**3*d
**3*i - 12*A*a**2*b*c*d**2*i + 60*A*a*b**2*c**2*d*i - 36*A*b**3*c**3*i - 13
*B*a**3*d**3*i - 13*B*a**2*b*c*d**2*i + 23*B*a*b**2*c**2*d*i - 9*B*b**3*c**
3*i - 12*B*b**3*d**3*i*x**3 + x**2*(-42*B*a*b**2*d**3*i + 6*B*b**3*c*d**2*i
) + x*(-48*A*a**2*b*d**3*i + 96*A*a*b**2*c*d**2*i - 48*A*b**3*c**2*d*i - 52
*B*a**2*b*d**3*i + 20*B*a*b**2*c*d**2*i - 4*B*b**3*c**2*d*i))/(144*a**6*b**
2*d**2*g**5 - 288*a**5*b**3*c*d*g**5 + 144*a**4*b**4*c**2*g**5 + x**4*(144*
a**2*b**6*d**2*g**5 - 288*a*b**7*c*d*g**5 + 144*b**8*c**2*g**5) + x**3*(576
*a**3*b**5*d**2*g**5 - 1152*a**2*b**6*c*d*g**5 + 576*a*b**7*c**2*g**5) + x*
*2*(864*a**4*b**4*d**2*g**5 - 1728*a**3*b**5*c*d*g**5 + 864*a**2*b**6*c**2*
g**5) + x*(576*a**5*b**3*d**2*g**5 - 1152*a**4*b**4*c*d*g**5 + 576*a**3*b**
5*c**2*g**5))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1386 vs. $2(257) = 514$.

Time = 0.26 (sec) , antiderivative size = 1386, normalized size of antiderivative = 5.15

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag + bgx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algorit
hm="maxima")
```

```
[Out] -1/144*B*d*i*(12*(4*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^6*g^5*
x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g^5)
+ (7*a*b^3*c^3 - 33*a^2*b^2*c^2*d + 75*a^3*b*c*d^2 - 13*a^4*d^3 + 12*(4*b^4
*c*d^2 - a*b^3*d^3)*x^3 - 6*(4*b^4*c^2*d - 29*a*b^3*c*d^2 + 7*a^2*b^2*d^3)*
x^2 + 4*(4*b^4*c^3 - 21*a*b^3*c^2*d + 57*a^2*b^2*c*d^2 - 13*a^3*b*d^3)*x)/(
(b^9*c^3 - 3*a*b^8*c^2*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*g^5*x^4 + 4*(a*b^
8*c^3 - 3*a^2*b^7*c^2*d + 3*a^3*b^6*c*d^2 - a^4*b^5*d^3)*g^5*x^3 + 6*(a^2*b
^7*c^3 - 3*a^3*b^6*c^2*d + 3*a^4*b^5*c*d^2 - a^5*b^4*d^3)*g^5*x^2 + 4*(a^3*
b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2 - a^6*b^3*d^3)*g^5*x + (a^4*b^5
*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d^2 - a^7*b^2*d^3)*g^5) + 12*(4*b*c*d^
3 - a*d^4)*log(b*x + a)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a
^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5) - 12*(4*b*c*d^3 - a*d^4)*log(d*x + c)/((b^
6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*
g^5)) + 1/48*B*c*i*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b
*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*
b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 -
a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a
^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 -
a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 -
 a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^
7*b*d^3)*g^5) - 12*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^5*g^5*x^4 + 4*a*
b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log
(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 +
 a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^
3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/12*(4*b*x + a)*A*d*i/(b^
6*g^5*x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2
*g^5) - 1/4*A*c*i/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^
3*b^2*g^5*x + a^4*b*g^5)
```

Giac [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.58

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx =$$
$$-\frac{1}{144}\left(\frac{12\left(3\,Bb^2e^5i-\frac{8\,(bex+ae)Bbde^4i}{dx+c}+\frac{6\,(bex+ae)^2Bd^2e^3i}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^4b^2c^2g^5}{(dx+c)^4}-\frac{2\,(bex+ae)^4abcdg^5}{(dx+c)^4}+\frac{(bex+ae)^4a^2d^2g^5}{(dx+c)^4}}+\frac{36\,Ab^2e^5i+9\,Bb^2e^5i-\frac{96\,(bex+ae)A}{dx+c}}{\frac{(bex+ae)^4b^2c^2}{(dx+c)^4}}\right.$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algorit
hm="giac")
```

```
[Out] -1/144*(12*(3*B*b^2*e^5*i - 8*(b*e*x + a*e)*B*b*d*e^4*i/(d*x + c) + 6*(b*e*
x + a*e)^2*B*d^2*e^3*i/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/((b*e*x +
a*e)^4*b^2*c^2*g^5/(d*x + c)^4 - 2*(b*e*x + a*e)^4*a*b*c*d*g^5/(d*x + c)^4
+ (b*e*x + a*e)^4*a^2*d^2*g^5/(d*x + c)^4) + (36*A*b^2*e^5*i + 9*B*b^2*e^5*
i - 96*(b*e*x + a*e)*A*b*d*e^4*i/(d*x + c) - 32*(b*e*x + a*e)*B*b*d*e^4*i/(
d*x + c) + 72*(b*e*x + a*e)^2*A*d^2*e^3*i/(d*x + c)^2 + 36*(b*e*x + a*e)^2*
B*d^2*e^3*i/(d*x + c)^2)/((b*e*x + a*e)^4*b^2*c^2*g^5/(d*x + c)^4 - 2*(b*e*
x + a*e)^4*a*b*c*d*g^5/(d*x + c)^4 + (b*e*x + a*e)^4*a^2*d^2*g^5/(d*x + c)^
4))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 3.11 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.19

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$$
$$=\frac{B\,d^4\,i\,\mathrm{atanh}\left(\frac{12\,a^3\,b^2\,d^3\,g^5-12\,a^2\,b^3\,c\,d^2\,g^5-12\,a\,b^4\,c^2\,d\,g^5+12\,b^5\,c^3\,g^5}{12\,b^2\,g^5\,(a\,d-b\,c)^3}+\frac{2\,b\,d\,x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^3}\right)}{6\,b^2\,g^5\,(a\,d-b\,c)^3}$$
$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{B\,c\,i}{4\,b^2\,g^5}+\frac{B\,a\,d\,i}{12\,b^3\,g^5}+\frac{B\,d\,i\,x}{3\,b^2\,g^5}\right)}{4\,a^3\,x+\frac{a^4}{b}+b^3\,x^4+6\,a^2\,b\,x^2+4\,a\,b^2\,x^3}$$
$$-\frac{\frac{12\,A\,a^3\,d^3\,i+36\,A\,b^3\,c^3\,i+13\,B\,a^3\,d^3\,i+9\,B\,b^3\,c^3\,i-60\,A\,a\,b^2\,c^2\,d\,i+12\,A\,a^2\,b\,c\,d^2\,i-23\,B\,a\,b^2\,c^2\,d\,i+13\,B\,a^2\,b\,c\,d^2\,i}{12\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\frac{x\,(12\,A\,a^2\,b\,d^3\,i+13\,B\,a^2\,b}{}}{12\,a^4\,b^2\,g^5+48\,a^3\,b^3\,g^5\,x+72\,a^2\,b^4\,g^5\,x^2+}$$

```
[In] int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^5,x)
```

```
[Out] (B*d^4*i*atanh((12*b^5*c^3*g^5 + 12*a^3*b^2*d^3*g^5 - 12*a*b^4*c^2*d*g^5 -
12*a^2*b^3*c*d^2*g^5)/(12*b^2*g^5*(a*d - b*c)^3) + (2*b*d*x*(a^2*d^2 + b^2*
```

```
c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(6*b^2*g^5*(a*d - b*c)^3) - (log((e*(a +
b*x))/(c + d*x))*((B*c*i)/(4*b^2*g^5) + (B*a*d*i)/(12*b^3*g^5) + (B*d*i*x)/
(3*b^2*g^5)))/(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3) - ((1
2*A*a^3*d^3*i + 36*A*b^3*c^3*i + 13*B*a^3*d^3*i + 9*B*b^3*c^3*i - 60*A*a*b^
2*c^2*d*i + 12*A*a^2*b*c*d^2*i - 23*B*a*b^2*c^2*d*i + 13*B*a^2*b*c*d^2*i)/(
12*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(12*A*a^2*b*d^3*i + 13*B*a^2*b*d^3
*i + 12*A*b^3*c^2*d*i + B*b^3*c^2*d*i - 24*A*a*b^2*c*d^2*i - 5*B*a*b^2*c*d^
2*i))/(3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (d*x^2*(B*b^3*c*d*i - 7*B*a*b^2
*d^2*i))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B*b^3*d^3*i*x^3)/(a^2*d^2 +
 b^2*c^2 - 2*a*b*c*d))/(12*a^4*b^2*g^5 + 12*b^6*g^5*x^4 + 48*a^3*b^3*g^5*x
+ 48*a*b^5*g^5*x^3 + 72*a^2*b^4*g^5*x^2)
```

3.10 $\int (ag+bgx)^3(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

Optimal result

Integrand size = 40, antiderivative size = 423

$$\begin{aligned}
&\int (ag+bgx)^3(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx \\
&= \frac{B(bc-ad)^5g^3i^2x}{60b^2d^3}+\frac{B(bc-ad)^4g^3i^2(c+dx)^2}{120bd^4}-\frac{19B(bc-ad)^3g^3i^2(c+dx)^3}{180d^4} \\
&+\frac{13bB(bc-ad)^2g^3i^2(c+dx)^4}{120d^4}-\frac{b^2B(bc-ad)g^3i^2(c+dx)^5}{30d^4} \\
&+\frac{B(bc-ad)^6g^3i^2\log\left(\frac{a+bx}{c+dx}\right)}{60b^3d^4}-\frac{(bc-ad)^3g^3i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^4} \\
&+\frac{3b(bc-ad)^2g^3i^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^4} \\
&-\frac{3b^2(bc-ad)g^3i^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^4} \\
&+\frac{b^3g^3i^2(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^4}+\frac{B(bc-ad)^6g^3i^2\log(c+dx)}{60b^3d^4}
\end{aligned}$$

```
[Out] 1/60*B*(-a*d+b*c)^5*g^3*i^2*x/b^2/d^3+1/120*B*(-a*d+b*c)^4*g^3*i^2*(d*x+c)^
2/b/d^4-19/180*B*(-a*d+b*c)^3*g^3*i^2*(d*x+c)^3/d^4+13/120*b*B*(-a*d+b*c)^2
*g^3*i^2*(d*x+c)^4/d^4-1/30*b^2*B*(-a*d+b*c)*g^3*i^2*(d*x+c)^5/d^4+1/60*B*(
-a*d+b*c)^6*g^3*i^2*ln((b*x+a)/(d*x+c))/b^3/d^4-1/3*(-a*d+b*c)^3*g^3*i^2*(d
*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4+3/4*b*(-a*d+b*c)^2*g^3*i^2*(d*x+c)^
4*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4-3/5*b^2*(-a*d+b*c)*g^3*i^2*(d*x+c)^5*(A+B
*ln(e*(b*x+a)/(d*x+c)))/d^4+1/6*b^3*g^3*i^2*(d*x+c)^6*(A+B*ln(e*(b*x+a)/(d*
x+c)))/d^4+1/60*B*(-a*d+b*c)^6*g^3*i^2*ln(d*x+c)/b^3/d^4
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2562, 45, 2382, 12, 1634}

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{b^3g^3i^2(c+dx)^6\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6d^4}-\frac{3b^2g^3i^2(c+dx)^5(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^4}-\frac{g^3i^2(c+dx)^3(bc-ad)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^4}+\frac{3bg^3i^2(c+dx)^4(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^4}+\frac{Bg^3i^2(bc-ad)^6\log\left(\frac{a+bx}{c+dx}\right)}{60b^3d^4}+\frac{Bg^3i^2(bc-ad)^6\log(c+dx)}{60b^3d^4}-\frac{b^2Bg^3i^2(c+dx)^5(bc-ad)}{30d^4}+\frac{Bg^3i^2x(bc-ad)^5}{60b^2d^3}+\frac{Bg^3i^2(c+dx)^2(bc-ad)^4}{120bd^4}-\frac{19Bg^3i^2(c+dx)^3(bc-ad)^3}{180d^4}+\frac{13bBg^3i^2(c+dx)^4(bc-ad)^2}{120d^4}$$

```
[In] Int[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (B*(b*c - a*d)^5*g^3*i^2*x)/(60*b^2*d^3) + (B*(b*c - a*d)^4*g^3*i^2*(c + d*
x)^2)/(120*b*d^4) - (19*B*(b*c - a*d)^3*g^3*i^2*(c + d*x)^3)/(180*d^4) + (1
3*b*B*(b*c - a*d)^2*g^3*i^2*(c + d*x)^4)/(120*d^4) - (b^2*B*(b*c - a*d)*g^3
*i^2*(c + d*x)^5)/(30*d^4) + (B*(b*c - a*d)^6*g^3*i^2*Log[(a + b*x)/(c + d*
x)])/(60*b^3*d^4) - ((b*c - a*d)^3*g^3*i^2*(c + d*x)^3*(A + B*Log[(e*(a + b
*x))/(c + d*x)]))/(3*d^4) + (3*b*(b*c - a*d)^2*g^3*i^2*(c + d*x)^4*(A + B*L
og[(e*(a + b*x))/(c + d*x)]))/(4*d^4) - (3*b^2*(b*c - a*d)*g^3*i^2*(c + d*x
)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*d^4) + (b^3*g^3*i^2*(c + d*x)^
6*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(6*d^4) + (B*(b*c - a*d)^6*g^3*i^2*
Log[c + d*x])/(60*b^3*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((bc-ad)^6 g^3 i^2\right) \operatorname{Subst}\left(\int \frac{x^3(A+B\log(ex))}{(b-dx)^7}\,dx, x, \frac{a+bx}{c+dx}\right)\\
&= -\frac{(bc-ad)^3 g^3 i^2 (c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^4}\\
&\quad+\frac{3b(bc-ad)^2 g^3 i^2 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^4}\\
&\quad-\frac{3b^2(bc-ad) g^3 i^2 (c+dx)^5 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^4}\\
&\quad+\frac{b^3 g^3 i^2 (c+dx)^6 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^4}\\
&\quad-\left(B(bc-ad)^6 g^3 i^2\right) \operatorname{Subst}\left(\int \frac{-b^3+6b^2dx-15bd^2x^2+20d^3x^3}{60d^4x(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)
\end{aligned}$$

$$= -\frac{(bc-ad)^3 g^3 i^2 (c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^4} + \frac{3b(bc-ad)^2 g^3 i^2 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^4} - \frac{3b^2(bc-ad) g^3 i^2 (c+dx)^5 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^4} + \frac{b^3 g^3 i^2 (c+dx)^6 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^4} - \frac{(B(bc-ad)^6 g^3 i^2)\,\text{Subst}\left(\int \frac{-b^3+6b^2dx-15bd^2x^2+20d^3x^3}{x(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{60d^4}$$

$$= -\frac{(bc-ad)^3 g^3 i^2 (c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^4} + \frac{3b(bc-ad)^2 g^3 i^2 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^4} - \frac{3b^2(bc-ad) g^3 i^2 (c+dx)^5 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^4} + \frac{b^3 g^3 i^2 (c+dx)^6 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^4} - \frac{(B(bc-ad)^6 g^3 i^2)\,\text{Subst}\left(\int \left(-\frac{1}{b^3x} + \frac{10b^2d}{(b-dx)^6} - \frac{26bd}{(b-dx)^5} + \frac{19d}{(b-dx)^4} - \frac{d}{b(b-dx)^3} - \frac{d}{b^2(b-dx)^2} - \frac{d}{b^3(b-dx)}\right)\right.}{60d^4}$$

$$= \frac{B(bc-ad)^5 g^3 i^2 x}{60b^2d^3} + \frac{B(bc-ad)^4 g^3 i^2 (c+dx)^2}{120bd^4} - \frac{19B(bc-ad)^3 g^3 i^2 (c+dx)^3}{180d^4} + \frac{13bB(bc-ad)^2 g^3 i^2 (c+dx)^4}{120d^4} - \frac{b^2B(bc-ad) g^3 i^2 (c+dx)^5}{30d^4} + \frac{B(bc-ad)^6 g^3 i^2 \log\left(\frac{a+bx}{c+dx}\right)}{60b^3d^4} - \frac{(bc-ad)^3 g^3 i^2 (c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^4} + \frac{3b(bc-ad)^2 g^3 i^2 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^4} - \frac{3b^2(bc-ad) g^3 i^2 (c+dx)^5 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^4} + \frac{b^3 g^3 i^2 (c+dx)^6 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^4} + \frac{B(bc-ad)^6 g^3 i^2 \log(c+dx)}{60b^3d^4}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.01

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{g^3i^2\left(90d^4(bc-ad)^2(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+144d^5(bc-ad)(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+6\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x
)]),x]
```

```
[Out] (g^3*i^2*(90*d^4*(b*c - a*d)^2*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*
x)]) + 144*d^5*(b*c - a*d)*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])
 + 60*d^6*(a + b*x)^6*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 15*B*(b*c - a*
d)^3*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a +
 b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^2*(12*b*d*(b*c -
 a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c - a*d)*(a + b*x)^3
 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]) - B*(b*c - a*d)*(60*b
*d*(b*c - a*d)^4*x + 30*d^2*(-(b*c) + a*d)^3*(a + b*x)^2 + 20*d^3*(b*c - a*
d)^2*(a + b*x)^3 + 15*d^4*(-(b*c) + a*d)*(a + b*x)^4 + 12*d^5*(a + b*x)^5 -
 60*(b*c - a*d)^5*Log[c + d*x])))/(360*b^3*d^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. 2(401) = 802.

Time = 1.18 (sec) , antiderivative size = 925, normalized size of antiderivative = 2.19

method	result
risch	$\frac{i^2g^3B\,a^3c^2x}{12}-\frac{i^2g^3bB\,a^2c^3x}{4d}+\frac{i^2g^3b^2Ba\,c^4x}{10d^2}+\frac{i^2g^3bB\ln(dx+c)a^2c^4}{4d^2}+\frac{7i^2g^3bdB\,a^2c\,x^3}{60}-\frac{13i^2g^3b^2Ba\,c^2x^3}{60}+$
parallelrisch	Expression too large to display
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETUR
NVERBOSE)
```

```
[Out] 1/12*i^2*g^3*B*a^3*c^2*x-1/4*i^2*g^3*b/d*B*a^2*c^3*x+1/10*i^2*g^3*b^2/d^2*B
*a*c^4*x+1/4*i^2*g^3*b/d^2*B*ln(d*x+c)*a^2*c^4+7/60*i^2*g^3*b*d*B*a^2*c*x^3
-13/60*i^2*g^3*b^2*B*a*c^2*x^3+i^2*g^3*d*A*a^3*c*x^2+3/2*i^2*g^3*b*A*a^2*c^
2*x^2+17/60*i^2*g^3*d*B*a^3*c*x^2-1/4*i^2*g^3*b*B*a^2*c^2*x^2-1/20*i^2*g^3*
b^2/d*B*a*c^3*x^2+i^2*g^3*A*a^3*c^2*x+1/10*i^2*g^3/b*d*B*a^4*c*x-1/60*i^2*g
```

```
^3*b^3/d^3*B*c^5*x-1/3*i^2*g^3/d*B*ln(d*x+c)*a^3*c^3+1/4*i^2*g^3/b*B*ln(-b*
x-a)*a^4*c^2+1/60*i^2*g^3*b^3/d^4*B*ln(d*x+c)*c^6+1/60*i^2*g^3/b^3*d^2*B*ln
(-b*x-a)*a^6+3/5*i^2*g^3*b^2*d^2*A*a*x^5+2/5*i^2*g^3*b^3*d*A*c*x^5+1/30*i^2
*g^3*b^2*d^2*B*a*x^5-1/30*i^2*g^3*b^3*d*B*c*x^5+3/4*i^2*g^3*b*d^2*A*a^2*x^4
+1/4*i^2*g^3*b^3*A*c^2*x^4+13/120*i^2*g^3*b*d^2*B*a^2*x^4-7/120*i^2*g^3*b^3
*B*c^2*x^4+1/3*i^2*g^3*d^2*A*a^3*x^3+19/180*i^2*g^3*d^2*B*a^3*x^3-1/180*i^2
*g^3*b^3/d*B*c^3*x^3+1/120*i^2*g^3/b*d^2*B*a^4*x^2+1/120*i^2*g^3*b^3/d^2*B*
c^4*x^2-1/60*i^2*g^3/b^2*d^2*B*a^5*x-1/10*i^2*g^3*b^2/d^3*B*ln(d*x+c)*a*c^5
-1/10*i^2*g^3/b^2*d*B*ln(-b*x-a)*a^5*c+1/6*i^2*g^3*b^3*d^2*A*x^6+3/2*i^2*g^
3*b^2*d*A*a*c*x^4-1/20*i^2*g^3*b^2*d*B*a*c*x^4+2*i^2*g^3*b*d*A*a^2*c*x^3+i^
2*g^3*b^2*A*a*c^2*x^3+1/60*i^2*g^3*B*x*(10*b^3*d^2*x^5+36*a*b^2*d^2*x^4+24*
b^3*c*d*x^4+45*a^2*b*d^2*x^3+90*a*b^2*c*d*x^3+15*b^3*c^2*x^3+20*a^3*d^2*x^2
+120*a^2*b*c*d*x^2+60*a*b^2*c^2*x^2+60*a^3*c*d*x+90*a^2*b*c^2*x+60*a^3*c^2)
*ln(e*(b*x+a)/(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 724, normalized size of antiderivative = 1.71

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{60\,Ab^6d^6g^3i^2x^6+12\,((12\,A-B)b^6cd^5+(18\,A+B)ab^5d^6)g^3i^2x^5+3\,((30\,A-7\,B)b^6c^2d^4+6\,(30\,A-B)a}{}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algor
ithm="fricas")
```

```
[Out] 1/360*(60*A*b^6*d^6*g^3*i^2*x^6 + 12*((12*A - B)*b^6*c*d^5 + (18*A + B)*a*b
^5*d^6)*g^3*i^2*x^5 + 3*((30*A - 7*B)*b^6*c^2*d^4 + 6*(30*A - B)*a*b^5*c*d^
5 + (90*A + 13*B)*a^2*b^4*d^6)*g^3*i^2*x^4 - 2*(B*b^6*c^3*d^3 - 3*(60*A - 1
3*B)*a*b^5*c^2*d^4 - 3*(120*A + 7*B)*a^2*b^4*c*d^5 - (60*A + 19*B)*a^3*b^3*
d^6)*g^3*i^2*x^3 + 3*(B*b^6*c^4*d^2 - 6*B*a*b^5*c^3*d^3 + 30*(6*A - B)*a^2*
b^4*c^2*d^4 + 2*(60*A + 17*B)*a^3*b^3*c*d^5 + B*a^4*b^2*d^6)*g^3*i^2*x^2 -
6*(B*b^6*c^5*d - 6*B*a*b^5*c^4*d^2 + 15*B*a^2*b^4*c^3*d^3 - 5*(12*A + B)*a^
3*b^3*c^2*d^4 - 6*B*a^4*b^2*c*d^5 + B*a^5*b*d^6)*g^3*i^2*x + 6*(15*B*a^4*b^
2*c^2*d^4 - 6*B*a^5*b*c*d^5 + B*a^6*d^6)*g^3*i^2*log(b*x + a) + 6*(B*b^6*c^
6 - 6*B*a*b^5*c^5*d + 15*B*a^2*b^4*c^4*d^2 - 20*B*a^3*b^3*c^3*d^3)*g^3*i^2*
log(d*x + c) + 6*(10*B*b^6*d^6*g^3*i^2*x^6 + 60*B*a^3*b^3*c^2*d^4*g^3*i^2*x
 + 12*(2*B*b^6*c*d^5 + 3*B*a*b^5*d^6)*g^3*i^2*x^5 + 15*(B*b^6*c^2*d^4 + 6*B
*a*b^5*c*d^5 + 3*B*a^2*b^4*d^6)*g^3*i^2*x^4 + 20*(3*B*a*b^5*c^2*d^4 + 6*B*a
^2*b^4*c*d^5 + B*a^3*b^3*d^6)*g^3*i^2*x^3 + 30*(3*B*a^2*b^4*c^2*d^4 + 2*B*a
^3*b^3*c*d^5)*g^3*i^2*x^2)*log((b*e*x + a*e)/(d*x + c)))/(b^3*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1727 vs. 2(398) = 796.

Time = 7.85 (sec) , antiderivative size = 1727, normalized size of antiderivative = 4.08

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)**3*(d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b**3*d**2*g**3*i**2*x**6/6 + B*a**4*g**3*i**2*(a**2*d**2 - 6*a*b*c*d + 15
*b**2*c**2)*log(x + (B*a**6*c*d**5*g**3*i**2 - 6*B*a**5*b*c**2*d**4*g**3*i*
*2 + B*a**5*d**4*g**3*i**2*(a**2*d**2 - 6*a*b*c*d + 15*b**2*c**2)/b + 35*B*
a**4*b**2*c**3*d**3*g**3*i**2 - B*a**4*c*d**3*g**3*i**2*(a**2*d**2 - 6*a*b*
c*d + 15*b**2*c**2) - 15*B*a**3*b**3*c**4*d**2*g**3*i**2 + 6*B*a**2*b**4*c*
*5*d*g**3*i**2 - B*a*b**5*c**6*g**3*i**2)/(B*a**6*d**6*g**3*i**2 - 6*B*a**5
*b*c*d**5*g**3*i**2 + 15*B*a**4*b**2*c**2*d**4*g**3*i**2 + 20*B*a**3*b**3*c
**3*d**3*g**3*i**2 - 15*B*a**2*b**4*c**4*d**2*g**3*i**2 + 6*B*a*b**5*c**5*d
*g**3*i**2 - B*b**6*c**6*g**3*i**2))/(60*b**3) - B*c**3*g**3*i**2*(20*a**3*
d**3 - 15*a**2*b*c*d**2 + 6*a*b**2*c**2*d - b**3*c**3)*log(x + (B*a**6*c*d*
*5*g**3*i**2 - 6*B*a**5*b*c**2*d**4*g**3*i**2 + 35*B*a**4*b**2*c**3*d**3*g*
*3*i**2 - 15*B*a**3*b**3*c**4*d**2*g**3*i**2 + 6*B*a**2*b**4*c**5*d*g**3*i*
*2 - B*a*b**5*c**6*g**3*i**2 - B*a*b**2*c**3*g**3*i**2*(20*a**3*d**3 - 15*a
**2*b*c*d**2 + 6*a*b**2*c**2*d - b**3*c**3) + B*b**3*c**4*g**3*i**2*(20*a**
3*d**3 - 15*a**2*b*c*d**2 + 6*a*b**2*c**2*d - b**3*c**3)/d)/(B*a**6*d**6*g*
*3*i**2 - 6*B*a**5*b*c*d**5*g**3*i**2 + 15*B*a**4*b**2*c**2*d**4*g**3*i**2
+ 20*B*a**3*b**3*c**3*d**3*g**3*i**2 - 15*B*a**2*b**4*c**4*d**2*g**3*i**2 +
 6*B*a*b**5*c**5*d*g**3*i**2 - B*b**6*c**6*g**3*i**2))/(60*d**4) + x**5*(3*
A*a*b**2*d**2*g**3*i**2/5 + 2*A*b**3*c*d*g**3*i**2/5 + B*a*b**2*d**2*g**3*i
**2/30 - B*b**3*c*d*g**3*i**2/30) + x**4*(3*A*a**2*b*d**2*g**3*i**2/4 + 3*A
*a*b**2*c*d*g**3*i**2/2 + A*b**3*c**2*g**3*i**2/4 + 13*B*a**2*b*d**2*g**3*i
**2/120 - B*a*b**2*c*d*g**3*i**2/20 - 7*B*b**3*c**2*g**3*i**2/120) + x**3*(
A*a**3*d**2*g**3*i**2/3 + 2*A*a**2*b*c*d*g**3*i**2 + A*a*b**2*c**2*g**3*i**
2 + 19*B*a**3*d**2*g**3*i**2/180 + 7*B*a**2*b*c*d*g**3*i**2/60 - 13*B*a*b**
2*c**2*g**3*i**2/60 - B*b**3*c**3*g**3*i**2/(180*d)) + x**2*(A*a**3*c*d*g**
3*i**2 + 3*A*a**2*b*c**2*g**3*i**2/2 + B*a**4*d**2*g**3*i**2/(120*b) + 17*B
*a**3*c*d*g**3*i**2/60 - B*a**2*b*c**2*g**3*i**2/4 - B*a*b**2*c**3*g**3*i**
2/(20*d) + B*b**3*c**4*g**3*i**2/(120*d**2)) + x*(A*a**3*c**2*g**3*i**2 - B
*a**5*d**2*g**3*i**2/(60*b**2) + B*a**4*c*d*g**3*i**2/(10*b) + B*a**3*c**2*
g**3*i**2/12 - B*a**2*b*c**3*g**3*i**2/(4*d) + B*a*b**2*c**4*g**3*i**2/(10*
d**2) - B*b**3*c**5*g**3*i**2/(60*d**3)) + (B*a**3*c**2*g**3*i**2*x + B*a**
3*c*d*g**3*i**2*x**2 + B*a**3*d**2*g**3*i**2*x**3/3 + 3*B*a**2*b*c**2*g**3*
i**2*x**2/2 + 2*B*a**2*b*c*d*g**3*i**2*x**3 + 3*B*a**2*b*d**2*g**3*i**2*x**
4/4 + B*a*b**2*c**2*g**3*i**2*x**3 + 3*B*a*b**2*c*d*g**3*i**2*x**4/2 + 3*B*
a*b**2*d**2*g**3*i**2*x**5/5 + B*b**3*c**2*g**3*i**2*x**4/4 + 2*B*b**3*c*d*
g**3*i**2*x**5/5 + B*b**3*d**2*g**3*i**2*x**6/6)*log(e*(a + b*x)/(c + d*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1789 vs. 2(401) = 802.

Time = 0.24 (sec) , antiderivative size = 1789, normalized size of antiderivative = 4.23

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algor
ithm="maxima")
```

```
[Out] 1/6*A*b^3*d^2*g^3*i^2*x^6 + 2/5*A*b^3*c*d*g^3*i^2*x^5 + 3/5*A*a*b^2*d^2*g^3
*i^2*x^5 + 1/4*A*b^3*c^2*g^3*i^2*x^4 + 3/2*A*a*b^2*c*d*g^3*i^2*x^4 + 3/4*A*
a^2*b*d^2*g^3*i^2*x^4 + A*a*b^2*c^2*g^3*i^2*x^3 + 2*A*a^2*b*c*d*g^3*i^2*x^3
 + 1/3*A*a^3*d^2*g^3*i^2*x^3 + 3/2*A*a^2*b*c^2*g^3*i^2*x^2 + A*a^3*c*d*g^3*
i^2*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*lo
g(d*x + c)/d)*B*a^3*c^2*g^3*i^2 + 3/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x +
 c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B
*a^2*b*c^2*g^3*i^2 + 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^
3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*
(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*b^2*c^2*g^3*i^2 + 1/24*(6*x^4*log(b*e
*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)
/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(
b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*c^2*g^3*i^2 + (x^2*log(b*e*x/(d*x +
c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c -
a*d)*x/(b*d))*B*a^3*c*d*g^3*i^2 + (2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c
)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)
*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a^2*b*c*d*g^3*i^2 + 1/4*(6*x^4
*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(
d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x
^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*a*b^2*c*d*g^3*i^2 + 1/30*(12*x^5
*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*lo
g(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*
d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d
^4))*B*b^3*c*d*g^3*i^2 + 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) +
2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2
- 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a^3*d^2*g^3*i^2 + 1/8*(6*x^4*log(b*
e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c
)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*
(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*a^2*b*d^2*g^3*i^2 + 1/20*(12*x^5*log(b*
e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x +
 c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^
3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*
a*b^2*d^2*g^3*i^2 + 1/360*(60*x^6*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 60
*a^6*log(b*x + a)/b^6 + 60*c^6*log(d*x + c)/d^6 - (12*(b^5*c*d^4 - a*b^4*d^
```

```
5)*x^5 - 15*(b^5*c^2*d^3 - a^2*b^3*d^5)*x^4 + 20*(b^5*c^3*d^2 - a^3*b^2*d^5
)*x^3 - 30*(b^5*c^4*d - a^4*b*d^5)*x^2 + 60*(b^5*c^5 - a^5*d^5)*x)/(b^5*d^5
))*B*b^3*d^2*g^3*i^2 + A*a^3*c^2*g^3*i^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4746 vs. $2(401) = 802$.

Time = 0.60 (sec) , antiderivative size = 4746, normalized size of antiderivative = 11.22

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algor
ithm="giac")
```

```
[Out] -1/360*(6*(B*b^10*c^7*e^7*g^3*i^2 - 7*B*a*b^9*c^6*d*e^7*g^3*i^2 + 21*B*a^2*
b^8*c^5*d^2*e^7*g^3*i^2 - 35*B*a^3*b^7*c^4*d^3*e^7*g^3*i^2 + 35*B*a^4*b^6*c
^3*d^4*e^7*g^3*i^2 - 21*B*a^5*b^5*c^2*d^5*e^7*g^3*i^2 + 7*B*a^6*b^4*c*d^6*e
^7*g^3*i^2 - B*a^7*b^3*d^7*e^7*g^3*i^2 - 6*(b*e*x + a*e)*B*b^9*c^7*d*e^6*g^
3*i^2/(d*x + c) + 42*(b*e*x + a*e)*B*a*b^8*c^6*d^2*e^6*g^3*i^2/(d*x + c) -
126*(b*e*x + a*e)*B*a^2*b^7*c^5*d^3*e^6*g^3*i^2/(d*x + c) + 210*(b*e*x + a*
e)*B*a^3*b^6*c^4*d^4*e^6*g^3*i^2/(d*x + c) - 210*(b*e*x + a*e)*B*a^4*b^5*c^
3*d^5*e^6*g^3*i^2/(d*x + c) + 126*(b*e*x + a*e)*B*a^5*b^4*c^2*d^6*e^6*g^3*i
^2/(d*x + c) - 42*(b*e*x + a*e)*B*a^6*b^3*c*d^7*e^6*g^3*i^2/(d*x + c) + 6*(
b*e*x + a*e)*B*a^7*b^2*d^8*e^6*g^3*i^2/(d*x + c) + 15*(b*e*x + a*e)^2*B*b^8
*c^7*d^2*e^5*g^3*i^2/(d*x + c)^2 - 105*(b*e*x + a*e)^2*B*a*b^7*c^6*d^3*e^5*
g^3*i^2/(d*x + c)^2 + 315*(b*e*x + a*e)^2*B*a^2*b^6*c^5*d^4*e^5*g^3*i^2/(d*
x + c)^2 - 525*(b*e*x + a*e)^2*B*a^3*b^5*c^4*d^5*e^5*g^3*i^2/(d*x + c)^2 +
525*(b*e*x + a*e)^2*B*a^4*b^4*c^3*d^6*e^5*g^3*i^2/(d*x + c)^2 - 315*(b*e*x
+ a*e)^2*B*a^5*b^3*c^2*d^7*e^5*g^3*i^2/(d*x + c)^2 + 105*(b*e*x + a*e)^2*B*
a^6*b^2*c*d^8*e^5*g^3*i^2/(d*x + c)^2 - 15*(b*e*x + a*e)^2*B*a^7*b*d^9*e^5*
g^3*i^2/(d*x + c)^2 - 20*(b*e*x + a*e)^3*B*b^7*c^7*d^3*e^4*g^3*i^2/(d*x + c
)^3 + 140*(b*e*x + a*e)^3*B*a*b^6*c^6*d^4*e^4*g^3*i^2/(d*x + c)^3 - 420*(b*
e*x + a*e)^3*B*a^2*b^5*c^5*d^5*e^4*g^3*i^2/(d*x + c)^3 + 700*(b*e*x + a*e)^
3*B*a^3*b^4*c^4*d^6*e^4*g^3*i^2/(d*x + c)^3 - 700*(b*e*x + a*e)^3*B*a^4*b^3
*c^3*d^7*e^4*g^3*i^2/(d*x + c)^3 + 420*(b*e*x + a*e)^3*B*a^5*b^2*c^2*d^8*e^
4*g^3*i^2/(d*x + c)^3 - 140*(b*e*x + a*e)^3*B*a^6*b*c*d^9*e^4*g^3*i^2/(d*x
+ c)^3 + 20*(b*e*x + a*e)^3*B*a^7*d^10*e^4*g^3*i^2/(d*x + c)^3)*log((b*e*x
+ a*e)/(d*x + c))/(b^6*d^4*e^6 - 6*(b*e*x + a*e)*b^5*d^5*e^5/(d*x + c) + 15
*(b*e*x + a*e)^2*b^4*d^6*e^4/(d*x + c)^2 - 20*(b*e*x + a*e)^3*b^3*d^7*e^3/(
d*x + c)^3 + 15*(b*e*x + a*e)^4*b^2*d^8*e^2/(d*x + c)^4 - 6*(b*e*x + a*e)^5
*b*d^9*e/(d*x + c)^5 + (b*e*x + a*e)^6*d^10/(d*x + c)^6) + (6*A*b^12*c^7*e^
7*g^3*i^2 + 2*B*b^12*c^7*e^7*g^3*i^2 - 42*A*a*b^11*c^6*d*e^7*g^3*i^2 - 14*B
*a*b^11*c^6*d*e^7*g^3*i^2 + 126*A*a^2*b^10*c^5*d^2*e^7*g^3*i^2 + 42*B*a^2*b
^10*c^5*d^2*e^7*g^3*i^2 - 210*A*a^3*b^9*c^4*d^3*e^7*g^3*i^2 - 70*B*a^3*b^9*
```

c^4*d^3*e^7*g^3*i^2 + 210*A*a^4*b^8*c^3*d^4*e^7*g^3*i^2 + 70*B*a^4*b^8*c^3*
d^4*e^7*g^3*i^2 - 126*A*a^5*b^7*c^2*d^5*e^7*g^3*i^2 - 42*B*a^5*b^7*c^2*d^5*
e^7*g^3*i^2 + 42*A*a^6*b^6*c*d^6*e^7*g^3*i^2 + 14*B*a^6*b^6*c*d^6*e^7*g^3*i
^2 - 6*A*a^7*b^5*d^7*e^7*g^3*i^2 - 2*B*a^7*b^5*d^7*e^7*g^3*i^2 - 36*(b*e*x
+ a*e)*A*b^11*c^7*d*e^6*g^3*i^2/(d*x + c) - 6*(b*e*x + a*e)*B*b^11*c^7*d*e^
6*g^3*i^2/(d*x + c) + 252*(b*e*x + a*e)*A*a*b^10*c^6*d^2*e^6*g^3*i^2/(d*x +
c) + 42*(b*e*x + a*e)*B*a*b^10*c^6*d^2*e^6*g^3*i^2/(d*x + c) - 756*(b*e*x
+ a*e)*A*a^2*b^9*c^5*d^3*e^6*g^3*i^2/(d*x + c) - 126*(b*e*x + a*e)*B*a^2*b^
9*c^5*d^3*e^6*g^3*i^2/(d*x + c) + 1260*(b*e*x + a*e)*A*a^3*b^8*c^4*d^4*e^6*
g^3*i^2/(d*x + c) + 210*(b*e*x + a*e)*B*a^3*b^8*c^4*d^4*e^6*g^3*i^2/(d*x +
c) - 1260*(b*e*x + a*e)*A*a^4*b^7*c^3*d^5*e^6*g^3*i^2/(d*x + c) - 210*(b*e*
x + a*e)*B*a^4*b^7*c^3*d^5*e^6*g^3*i^2/(d*x + c) + 756*(b*e*x + a*e)*A*a^5*
b^6*c^2*d^6*e^6*g^3*i^2/(d*x + c) + 126*(b*e*x + a*e)*B*a^5*b^6*c^2*d^6*e^6
*g^3*i^2/(d*x + c) - 252*(b*e*x + a*e)*A*a^6*b^5*c*d^7*e^6*g^3*i^2/(d*x + c
) - 42*(b*e*x + a*e)*B*a^6*b^5*c*d^7*e^6*g^3*i^2/(d*x + c) + 36*(b*e*x + a*
e)*A*a^7*b^4*d^8*e^6*g^3*i^2/(d*x + c) + 6*(b*e*x + a*e)*B*a^7*b^4*d^8*e^6*
g^3*i^2/(d*x + c) + 90*(b*e*x + a*e)^2*A*b^10*c^7*d^2*e^5*g^3*i^2/(d*x + c)
^2 - 3*(b*e*x + a*e)^2*B*b^10*c^7*d^2*e^5*g^3*i^2/(d*x + c)^2 - 630*(b*e*x
+ a*e)^2*A*a*b^9*c^6*d^3*e^5*g^3*i^2/(d*x + c)^2 + 21*(b*e*x + a*e)^2*B*a*b
^9*c^6*d^3*e^5*g^3*i^2/(d*x + c)^2 + 1890*(b*e*x + a*e)^2*A*a^2*b^8*c^5*d^4
*e^5*g^3*i^2/(d*x + c)^2 - 63*(b*e*x + a*e)^2*B*a^2*b^8*c^5*d^4*e^5*g^3*i^2
/(d*x + c)^2 - 3150*(b*e*x + a*e)^2*A*a^3*b^7*c^4*d^5*e^5*g^3*i^2/(d*x + c)
^2 + 105*(b*e*x + a*e)^2*B*a^3*b^7*c^4*d^5*e^5*g^3*i^2/(d*x + c)^2 + 3150*(
b*e*x + a*e)^2*A*a^4*b^6*c^3*d^6*e^5*g^3*i^2/(d*x + c)^2 - 105*(b*e*x + a*e
)^2*B*a^4*b^6*c^3*d^6*e^5*g^3*i^2/(d*x + c)^2 - 1890*(b*e*x + a*e)^2*A*a^5*
b^5*c^2*d^7*e^5*g^3*i^2/(d*x + c)^2 + 63*(b*e*x + a*e)^2*B*a^5*b^5*c^2*d^7*
e^5*g^3*i^2/(d*x + c)^2 + 630*(b*e*x + a*e)^2*A*a^6*b^4*c*d^8*e^5*g^3*i^2/(
d*x + c)^2 - 21*(b*e*x + a*e)^2*B*a^6*b^4*c*d^8*e^5*g^3*i^2/(d*x + c)^2 - 9
0*(b*e*x + a*e)^2*A*a^7*b^3*d^9*e^5*g^3*i^2/(d*x + c)^2 + 3*(b*e*x + a*e)^2
*B*a^7*b^3*d^9*e^5*g^3*i^2/(d*x + c)^2 - 120*(b*e*x + a*e)^3*A*b^9*c^7*d^3*
e^4*g^3*i^2/(d*x + c)^3 + 34*(b*e*x + a*e)^3*B*b^9*c^7*d^3*e^4*g^3*i^2/(d*x
+ c)^3 + 840*(b*e*x + a*e)^3*A*a*b^8*c^6*d^4*e^4*g^3*i^2/(d*x + c)^3 - 238
*(b*e*x + a*e)^3*B*a*b^8*c^6*d^4*e^4*g^3*i^2/(d*x + c)^3 - 2520*(b*e*x + a*
e)^3*A*a^2*b^7*c^5*d^5*e^4*g^3*i^2/(d*x + c)^3 + 714*(b*e*x + a*e)^3*B*a^2*
b^7*c^5*d^5*e^4*g^3*i^2/(d*x + c)^3 + 4200*(b*e*x + a*e)^3*A*a^3*b^6*c^4*d^
6*e^4*g^3*i^2/(d*x + c)^3 - 1190*(b*e*x + a*e)^3*B*a^3*b^6*c^4*d^6*e^4*g^3*
i^2/(d*x + c)^3 - 4200*(b*e*x + a*e)^3*A*a^4*b^5*c^3*d^7*e^4*g^3*i^2/(d*x +
c)^3 + 1190*(b*e*x + a*e)^3*B*a^4*b^5*c^3*d^7*e^4*g^3*i^2/(d*x + c)^3 + 25
20*(b*e*x + a*e)^3*A*a^5*b^4*c^2*d^8*e^4*g^3*i^2/(d*x + c)^3 - 714*(b*e*x +
a*e)^3*B*a^5*b^4*c^2*d^8*e^4*g^3*i^2/(d*x + c)^3 - 840*(b*e*x + a*e)^3*A*a
^6*b^3*c*d^9*e^4*g^3*i^2/(d*x + c)^3 + 238*(b*e*x + a*e)^3*B*a^6*b^3*c*d^9*
e^4*g^3*i^2/(d*x + c)^3 + 120*(b*e*x + a*e)^3*A*a^7*b^2*d^10*e^4*g^3*i^2/(d
x + c)^3 - 34(b*e*x + a*e)^3*B*a^7*b^2*d^10*e^4*g^3*i^2/(d*x + c)^3 - 33*
(b*e*x + a*e)^4*B*b^8*c^7*d^4*e^3*g^3*i^2/(d*x + c)^4 + 231*(b*e*x + a*e)^4
*B*a*b^7*c^6*d^5*e^3*g^3*i^2/(d*x + c)^4 - 693*(b*e*x + a*e)^4*B*a^2*b^6*c^

```
5*d^6*e^3*g^3*i^2/(d*x + c)^4 + 1155*(b*e*x + a*e)^4*B*a^3*b^5*c^4*d^7*e^3*
g^3*i^2/(d*x + c)^4 - 1155*(b*e*x + a*e)^4*B*a^4*b^4*c^3*d^8*e^3*g^3*i^2/(d
*x + c)^4 + 693*(b*e*x + a*e)^4*B*a^5*b^3*c^2*d^9*e^3*g^3*i^2/(d*x + c)^4 -
 231*(b*e*x + a*e)^4*B*a^6*b^2*c*d^10*e^3*g^3*i^2/(d*x + c)^4 + 33*(b*e*x +
 a*e)^4*B*a^7*b*d^11*e^3*g^3*i^2/(d*x + c)^4 + 6*(b*e*x + a*e)^5*B*b^7*c^7*
d^5*e^2*g^3*i^2/(d*x + c)^5 - 42*(b*e*x + a*e)^5*B*a*b^6*c^6*d^6*e^2*g^3*i^
2/(d*x + c)^5 + 126*(b*e*x + a*e)^5*B*a^2*b^5*c^5*d^7*e^2*g^3*i^2/(d*x + c)
^5 - 210*(b*e*x + a*e)^5*B*a^3*b^4*c^4*d^8*e^2*g^3*i^2/(d*x + c)^5 + 210*(b
*e*x + a*e)^5*B*a^4*b^3*c^3*d^9*e^2*g^3*i^2/(d*x + c)^5 - 126*(b*e*x + a*e)
^5*B*a^5*b^2*c^2*d^10*e^2*g^3*i^2/(d*x + c)^5 + 42*(b*e*x + a*e)^5*B*a^6*b*
c*d^11*e^2*g^3*i^2/(d*x + c)^5 - 6*(b*e*x + a*e)^5*B*a^7*d^12*e^2*g^3*i^2/(
d*x + c)^5)/(b^8*d^4*e^6 - 6*(b*e*x + a*e)*b^7*d^5*e^5/(d*x + c) + 15*(b*e*
x + a*e)^2*b^6*d^6*e^4/(d*x + c)^2 - 20*(b*e*x + a*e)^3*b^5*d^7*e^3/(d*x +
c)^3 + 15*(b*e*x + a*e)^4*b^4*d^8*e^2/(d*x + c)^4 - 6*(b*e*x + a*e)^5*b^3*d
^9*e/(d*x + c)^5 + (b*e*x + a*e)^6*b^2*d^10/(d*x + c)^6) + 6*(B*b^7*c^7*e*g
^3*i^2 - 7*B*a*b^6*c^6*d*e*g^3*i^2 + 21*B*a^2*b^5*c^5*d^2*e*g^3*i^2 - 35*B*
a^3*b^4*c^4*d^3*e*g^3*i^2 + 35*B*a^4*b^3*c^3*d^4*e*g^3*i^2 - 21*B*a^5*b^2*c
^2*d^5*e*g^3*i^2 + 7*B*a^6*b*c*d^6*e*g^3*i^2 - B*a^7*d^7*e*g^3*i^2)*log(-b*
e + (b*e*x + a*e)*d/(d*x + c))/(b^3*d^4) - 6*(B*b^7*c^7*e*g^3*i^2 - 7*B*a*b
^6*c^6*d*e*g^3*i^2 + 21*B*a^2*b^5*c^5*d^2*e*g^3*i^2 - 35*B*a^3*b^4*c^4*d^3*
e*g^3*i^2 + 35*B*a^4*b^3*c^3*d^4*e*g^3*i^2 - 21*B*a^5*b^2*c^2*d^5*e*g^3*i^2
 + 7*B*a^6*b*c*d^6*e*g^3*i^2 - B*a^7*d^7*e*g^3*i^2)*log((b*e*x + a*e)/(d*x
+ c))/(b^3*d^4))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*
(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 2473, normalized size of antiderivative = 5.85

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

```
[In] int((a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x^3*((g^3*i^2*(16*A*a^3*d^3 + 4*A*b^3*c^3 + 3*B*a^3*d^3 - B*b^3*c^3 + 48*A*
a*b^2*c^2*d + 72*A*a^2*b*c*d^2 - 5*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2))/(12*d)
 + ((60*a*d + 60*b*c)*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d - B*b*
c))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*b*d)
 - (b*g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*d^2 - 2*B*b^2*c^2 + 60
*A*a*b*c*d - B*a*b*c*d))/5 + A*a*b^2*c*d*g^3*i^2))/(180*b*d) - (a*c*((b^2*d
*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d - B*b*c))/6 - (A*b^2*d*g^3*i^2*(60*a*
d + 60*b*c))/60))/(3*b*d)) - x^4*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B
*a*d - B*b*c))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c
))/(240*b*d) - (b*g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*d^2 - 2*B*
b^2*c^2 + 60*A*a*b*c*d - B*a*b*c*d))/20 + (A*a*b^2*c*d*g^3*i^2)/4) + x^2*((
```

```
a*c*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d - B*b*c))/6 - (A*b^2*d*g
^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*b*d) - (b*g^3*i^2*(30*
A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*d^2 - 2*B*b^2*c^2 + 60*A*a*b*c*d - B*a*b
*c*d))/5 + A*a*b^2*c*d*g^3*i^2))/(2*b*d) - ((60*a*d + 60*b*c)*((g^3*i^2*(16
*A*a^3*d^3 + 4*A*b^3*c^3 + 3*B*a^3*d^3 - B*b^3*c^3 + 48*A*a*b^2*c^2*d + 72*
A*a^2*b*c*d^2 - 5*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2))/(4*d) + ((60*a*d + 60*b
*c)*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d - B*b*c))/6 - (A*b^2*d*g
^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*b*d) - (b*g^3*i^2*(30*
A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*d^2 - 2*B*b^2*c^2 + 60*A*a*b*c*d - B*a*b
*c*d))/5 + A*a*b^2*c*d*g^3*i^2))/(60*b*d) - (a*c*((b^2*d*g^3*i^2*(24*A*a*d
 + 18*A*b*c + B*a*d - B*b*c))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60))/(
b*d)))/(120*b*d) + (a*g^3*i^2*(3*A*a^3*d^3 + 12*A*b^3*c^3 + B*a^3*d^3 - 3*B
*b^3*c^3 + 54*A*a*b^2*c^2*d + 36*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d + 5*B*a^2*
b*c*d^2))/(6*b*d)) + log((e*(a + b*x))/(c + d*x))*(B*a^3*c^2*g^3*i^2*x + (B
*a*g^3*i^2*x^3*(a^2*d^2 + 3*b^2*c^2 + 6*a*b*c*d))/3 + (B*b*g^3*i^2*x^4*(3*a
^2*d^2 + b^2*c^2 + 6*a*b*c*d))/4 + (B*b^3*d^2*g^3*i^2*x^6)/6 + (B*a^2*c*g^3
*i^2*x^2*(2*a*d + 3*b*c))/2 + (B*b^2*d*g^3*i^2*x^5*(3*a*d + 2*b*c))/5) + x^
5*((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d - B*b*c))/30 - (A*b^2*d*g^3*
i^2*(60*a*d + 60*b*c))/300) - x*(((60*a*d + 60*b*c)*((a*c*((((b^2*d*g^3*i^2
*(24*A*a*d + 18*A*b*c + B*a*d - B*b*c))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b
*c))/60)*(60*a*d + 60*b*c))/(60*b*d) - (b*g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*
c^2 + 3*B*a^2*d^2 - 2*B*b^2*c^2 + 60*A*a*b*c*d - B*a*b*c*d))/5 + A*a*b^2*c*
d*g^3*i^2))/(b*d) - ((60*a*d + 60*b*c)*((g^3*i^2*(16*A*a^3*d^3 + 4*A*b^3*c^
3 + 3*B*a^3*d^3 - B*b^3*c^3 + 48*A*a*b^2*c^2*d + 72*A*a^2*b*c*d^2 - 5*B*a*b
^2*c^2*d + 3*B*a^2*b*c*d^2))/(4*d) + ((60*a*d + 60*b*c)*((((b^2*d*g^3*i^2*(
24*A*a*d + 18*A*b*c + B*a*d - B*b*c))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c
))/60)*(60*a*d + 60*b*c))/(60*b*d) - (b*g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^
2 + 3*B*a^2*d^2 - 2*B*b^2*c^2 + 60*A*a*b*c*d - B*a*b*c*d))/5 + A*a*b^2*c*d*
g^3*i^2))/(60*b*d) - (a*c*((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d - B*
b*c))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60))/(b*d)))/(60*b*d) + (a*g^
3*i^2*(3*A*a^3*d^3 + 12*A*b^3*c^3 + B*a^3*d^3 - 3*B*b^3*c^3 + 54*A*a*b^2*c^
2*d + 36*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d + 5*B*a^2*b*c*d^2))/(3*b*d)))/(60*
b*d) + (a*c*((g^3*i^2*(16*A*a^3*d^3 + 4*A*b^3*c^3 + 3*B*a^3*d^3 - B*b^3*c^3
 + 48*A*a*b^2*c^2*d + 72*A*a^2*b*c*d^2 - 5*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)
)/(4*d) + ((60*a*d + 60*b*c)*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d
 - B*b*c))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(
60*b*d) - (b*g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*d^2 - 2*B*b^2*c
^2 + 60*A*a*b*c*d - B*a*b*c*d))/5 + A*a*b^2*c*d*g^3*i^2))/(60*b*d) - (a*c*(
(b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d - B*b*c))/6 - (A*b^2*d*g^3*i^2*
(60*a*d + 60*b*c))/60))/(b*d)))/(b*d) - (a^2*c*g^3*i^2*(6*A*a^2*d^2 + 12*A*
b^2*c^2 + 2*B*a^2*d^2 - 3*B*b^2*c^2 + 24*A*a*b*c*d + B*a*b*c*d))/(2*b*d)) +
 (log(a + b*x)*(B*a^6*d^2*g^3*i^2 + 15*B*a^4*b^2*c^2*g^3*i^2 - 6*B*a^5*b*c*
d*g^3*i^2))/(60*b^3) + (log(c + d*x)*(B*b^3*c^6*g^3*i^2 - 20*B*a^3*c^3*d^3*
g^3*i^2 - 6*B*a*b^2*c^5*d*g^3*i^2 + 15*B*a^2*b*c^4*d^2*g^3*i^2))/(60*d^4) +
 (A*b^3*d^2*g^3*i^2*x^6)/6
```

3.11 $\int (ag+bgx)^2(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

Optimal result

Integrand size = 40, antiderivative size = 337

$$\begin{aligned}
&\int (ag+bgx)^2(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx \\
&= -\frac{B(bc-ad)^4g^2i^2x}{30b^2d^2} - \frac{B(bc-ad)^3g^2i^2(c+dx)^2}{60bd^3} + \frac{B(bc-ad)^2g^2i^2(c+dx)^3}{10d^3} \\
&\quad - \frac{bB(bc-ad)g^2i^2(c+dx)^4}{20d^3} - \frac{B(bc-ad)^5g^2i^2\log\left(\frac{a+bx}{c+dx}\right)}{30b^3d^3} \\
&\quad + \frac{(bc-ad)^2g^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^3} \\
&\quad - \frac{b(bc-ad)g^2i^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^3} \\
&\quad + \frac{b^2g^2i^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^3} - \frac{B(bc-ad)^5g^2i^2\log(c+dx)}{30b^3d^3}
\end{aligned}$$

```
[Out] -1/30*B*(-a*d+b*c)^4*g^2*i^2*x/b^2/d^2-1/60*B*(-a*d+b*c)^3*g^2*i^2*(d*x+c)^
2/b/d^3+1/10*B*(-a*d+b*c)^2*g^2*i^2*(d*x+c)^3/d^3-1/20*b*B*(-a*d+b*c)*g^2*i
^2*(d*x+c)^4/d^3-1/30*B*(-a*d+b*c)^5*g^2*i^2*ln((b*x+a)/(d*x+c))/b^3/d^3+1/
3*(-a*d+b*c)^2*g^2*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3-1/2*b*(-a*
d+b*c)*g^2*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3+1/5*b^2*g^2*i^2*(d
*x+c)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3-1/30*B*(-a*d+b*c)^5*g^2*i^2*ln(d*x+
c)/b^3/d^3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2562, 45, 2382, 12, 907}

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{b^2g^2i^2(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^3}+\frac{g^2i^2(c+dx)^3(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^3}-\frac{bg^2i^2(c+dx)^4(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d^3}-\frac{Bg^2i^2(bc-ad)^5\log\left(\frac{a+bx}{c+dx}\right)}{30b^3d^3}-\frac{Bg^2i^2(bc-ad)^5\log(c+dx)}{30b^3d^3}-\frac{Bg^2i^2x(bc-ad)^4}{30b^2d^2}-\frac{Bg^2i^2(c+dx)^2(bc-ad)^3}{60bd^3}+\frac{Bg^2i^2(c+dx)^3(bc-ad)^2}{10d^3}-\frac{bBg^2i^2(c+dx)^4(bc-ad)}{20d^3}$$

```
[In] Int[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -1/30*(B*(b*c - a*d)^4*g^2*i^2*x)/(b^2*d^2) - (B*(b*c - a*d)^3*g^2*i^2*(c +
 d*x)^2)/(60*b*d^3) + (B*(b*c - a*d)^2*g^2*i^2*(c + d*x)^3)/(10*d^3) - (b*B
*(b*c - a*d)*g^2*i^2*(c + d*x)^4)/(20*d^3) - (B*(b*c - a*d)^5*g^2*i^2*Log[(
a + b*x)/(c + d*x)])/(30*b^3*d^3) + ((b*c - a*d)^2*g^2*i^2*(c + d*x)^3*(A +
 B*Log[(e*(a + b*x))/(c + d*x)]))/(3*d^3) - (b*(b*c - a*d)*g^2*i^2*(c + d*x
)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d^3) + (b^2*g^2*i^2*(c + d*x)^
5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*d^3) - (B*(b*c - a*d)^5*g^2*i^2*
Log[c + d*x])/(30*b^3*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
```

```
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))
)
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((bc-ad)^5g^2i^2\right)\text{Subst}\left(\int \frac{x^2(A+B\log(ex))}{(b-dx)^6}\,dx,x,\frac{a+bx}{c+dx}\right)\\
&= \frac{(bc-ad)^2g^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^3}\\
&\quad-\frac{b(bc-ad)g^2i^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^3}\\
&\quad+\frac{b^2g^2i^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^3}\\
&\quad-\left(B(bc-ad)^5g^2i^2\right)\text{Subst}\left(\int \frac{b^2-5bdx+10d^2x^2}{30d^3x(b-dx)^5}\,dx,x,\frac{a+bx}{c+dx}\right)\\
&= \frac{(bc-ad)^2g^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^3}\\
&\quad-\frac{b(bc-ad)g^2i^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^3}\\
&\quad+\frac{b^2g^2i^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^3}\\
&\quad-\frac{\left(B(bc-ad)^5g^2i^2\right)\text{Subst}\left(\int \frac{b^2-5bdx+10d^2x^2}{x(b-dx)^5}\,dx,x,\frac{a+bx}{c+dx}\right)}{30d^3}
\end{aligned}$$

$$= \frac{(bc-ad)^2g^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^3} - \frac{b(bc-ad)g^2i^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^3} + \frac{b^2g^2i^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^3} - \frac{(B(bc-ad)^5g^2i^2)\operatorname{Subst}\left(\int\left(\frac{1}{b^3x}+\frac{6bd}{(b-dx)^5}-\frac{9d}{(b-dx)^4}+\frac{d}{b(b-dx)^3}+\frac{d}{b^2(b-dx)^2}+\frac{d}{b^3(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{30d^3}$$

$$= -\frac{B(bc-ad)^4g^2i^2x}{30b^2d^2} - \frac{B(bc-ad)^3g^2i^2(c+dx)^2}{60bd^3} + \frac{B(bc-ad)^2g^2i^2(c+dx)^3}{10d^3} - \frac{bB(bc-ad)g^2i^2(c+dx)^4}{20d^3} - \frac{B(bc-ad)^5g^2i^2\log\left(\frac{a+bx}{c+dx}\right)}{30b^3d^3} + \frac{(bc-ad)^2g^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^3} - \frac{b(bc-ad)g^2i^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^3} + \frac{b^2g^2i^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^3} - \frac{B(bc-ad)^5g^2i^2\log(c+dx)}{30b^3d^3}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.07

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{g^2i^2\left(20d^3(bc-ad)^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+30d^4(bc-ad)(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x
)]),x]
```

```
[Out] (g^2*i^2*(20*d^3*(b*c - a*d)^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*
x)]) + 30*d^4*(b*c - a*d)*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])
+ 12*d^5*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 10*B*(b*c - a*d
)^3*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x])
- 5*B*(b*c - a*d)^2*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)
^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + B*(b*c - a*d)*(12*
b*d*(b*c - a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c - a*d)*(
a + b*x)^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]))/(60*b^3*d
^3)
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.89

method	result
risch	$\frac{i^2 g^2 b^2 d A c\,x^4}{2}+\frac{i^2 g^2 b\,d^2 B a\,x^4}{20}-\frac{i^2 g^2 b^2 d B c\,x^4}{20}+\frac{i^2 g^2 d^2 A\,a^2 x^3}{3}+\frac{i^2 g^2 b^2 A\,c^2 x^3}{3}+\frac{i^2 g^2 d^2 B\,a^2 x^3}{10}-\frac{i^2 g^2 b^2 B\,c^2}{10}$
parallelrisch	Expression too large to display
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETUR
NVERBOSE)
```

```
[Out] 1/2*i^2*g^2*b^2*d*A*c*x^4+1/20*i^2*g^2*b*d^2*B*a*x^4-1/20*i^2*g^2*b^2*d*B*c
*x^4+1/3*i^2*g^2*d^2*A*a^2*x^3+1/3*i^2*g^2*b^2*A*c^2*x^3+1/10*i^2*g^2*d^2*B
*a^2*x^3-1/10*i^2*g^2*b^2*B*c^2*x^3+1/60*i^2*g^2/b*d^2*B*a^3*x^2-1/60*i^2*g
^2*b^2/d*B*c^3*x^2-1/30*i^2*g^2/b^2*d^2*B*a^4*x+1/30*i^2*g^2*b^2/d^2*B*c^4*
x+1/30*i^2*g^2/b^3*d^2*B*ln(-b*x-a)*a^5-1/30*i^2*g^2*b^2/d^3*B*ln(d*x+c)*c^
5+1/3*i^2*g^2/b*B*ln(-b*x-a)*a^3*c^2+1/2*i^2*g^2*b*d^2*A*a*x^4-1/3*i^2*g^2/
d*B*ln(d*x+c)*a^2*c^3+1/30*i^2*g^2*B*x*(6*b^2*d^2*x^4+15*a*b*d^2*x^3+15*b^2
*c*d*x^3+10*a^2*d^2*x^2+40*a*b*c*d*x^2+10*b^2*c^2*x^2+30*a^2*c*d*x+30*a*b*c
^2*x+30*a^2*c^2)*ln(e*(b*x+a)/(d*x+c))+4/3*i^2*g^2*b*d*A*a*c*x^3+i^2*g^2*d*
A*a^2*c*x^2+i^2*g^2*b*A*a*c^2*x^2+1/4*i^2*g^2*d*B*a^2*c*x^2-1/4*i^2*g^2*b*B
*a*c^2*x^2+i^2*g^2*A*a^2*c^2*x+1/6*i^2*g^2/b*d*B*a^3*c*x-1/6*i^2*g^2*b/d*B*
a*c^3*x-1/6*i^2*g^2/b^2*d*B*ln(-b*x-a)*a^4*c+1/6*i^2*g^2*b/d^2*B*ln(d*x+c)*
a*c^4+1/5*i^2*g^2*b^2*d^2*A*x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.58

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{12\,Ab^5d^5g^2i^2x^5+3\,((10\,A-B)b^5cd^4+(10\,A+B)ab^4d^5)g^2i^2x^4+2\,((10\,A-3\,B)b^5c^2d^3+40\,Aab^4cd^4+(1}{}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algor
ithm="fricas")
```

```
[Out] 1/60*(12*A*b^5*d^5*g^2*i^2*x^5 + 3*((10*A - B)*b^5*c*d^4 + (10*A + B)*a*b^4
*d^5)*g^2*i^2*x^4 + 2*((10*A - 3*B)*b^5*c^2*d^3 + 40*A*a*b^4*c*d^4 + (10*A
+ 3*B)*a^2*b^3*d^5)*g^2*i^2*x^3 - (B*b^5*c^3*d^2 - 15*(4*A - B)*a*b^4*c^2*d
```

```
^3 - 15*(4*A + B)*a^2*b^3*c*d^4 - B*a^3*b^2*d^5)*g^2*i^2*x^2 + 2*(B*b^5*c^4
*d - 5*B*a*b^4*c^3*d^2 + 30*A*a^2*b^3*c^2*d^3 + 5*B*a^3*b^2*c*d^4 - B*a^4*b
*d^5)*g^2*i^2*x + 2*(10*B*a^3*b^2*c^2*d^3 - 5*B*a^4*b*c*d^4 + B*a^5*d^5)*g^
2*i^2*log(b*x + a) - 2*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2)
*g^2*i^2*log(d*x + c) + 2*(6*B*b^5*d^5*g^2*i^2*x^5 + 30*B*a^2*b^3*c^2*d^3*g
^2*i^2*x + 15*(B*b^5*c*d^4 + B*a*b^4*d^5)*g^2*i^2*x^4 + 10*(B*b^5*c^2*d^3 +
 4*B*a*b^4*c*d^4 + B*a^2*b^3*d^5)*g^2*i^2*x^3 + 30*(B*a*b^4*c^2*d^3 + B*a^2
*b^3*c*d^4)*g^2*i^2*x^2)*log((b*e*x + a*e)/(d*x + c)))/(b^3*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1266 vs. $2(311) = 622$.

Time = 3.87 (sec) , antiderivative size = 1266, normalized size of antiderivative = 3.76

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\frac{Ab^2d^2g^2i^2x^5}{5}$$
$$+\frac{Ba^3g^2i^2(a^2d^2-5abcd+10b^2c^2)\log\left(x+\frac{Ba^5cd^4g^2i^2-5Ba^4bc^2d^3g^2i^2+\frac{Ba^4d^3g^2i^2\left(a^2d^2-5abcd+10b^2c^2\right)}{b}+20Ba^3b^2c^3d^2g^2i^2-}{Ba^5d^5g^2i^2-5Ba^4bcd^4g^2i^2+10Ba^3b^2c^2d^3g^2i^2+10Ba^2b}\right.}{30b^3}$$
$$-\frac{Bc^3g^2i^2\cdot(10a^2d^2-5abcd+b^2c^2)\log\left(x+\frac{Ba^5cd^4g^2i^2-5Ba^4bc^2d^3g^2i^2+20Ba^3b^2c^3d^2g^2i^2-5Ba^2b^3c^4dg^2i^2+Bab^4c^5g^2i^2-}{Ba^5d^5g^2i^2-5Ba^4bcd^4g^2i^2+10Ba^3b^2c^2d^3g^2i^2+10B}\right.}{30d^3}$$
$$+x^4\left(\frac{Aabd^2g^2i^2}{2}+\frac{Ab^2cdg^2i^2}{2}+\frac{Babd^2g^2i^2}{20}-\frac{Bb^2cdg^2i^2}{20}\right)$$
$$+x^3\left(\frac{Aa^2d^2g^2i^2}{3}+\frac{4Aabcdg^2i^2}{3}+\frac{Ab^2c^2g^2i^2}{3}+\frac{Ba^2d^2g^2i^2}{10}-\frac{Bb^2c^2g^2i^2}{10}\right)$$
$$+x^2\left(Aa^2cdg^2i^2+Aabc^2g^2i^2+\frac{Ba^3d^2g^2i^2}{60b}+\frac{Ba^2cdg^2i^2}{4}-\frac{Babc^2g^2i^2}{4}-\frac{Bb^2c^3g^2i^2}{60d}\right)$$
$$+x\left(Aa^2c^2g^2i^2-\frac{Ba^4d^2g^2i^2}{30b^2}+\frac{Ba^3cdg^2i^2}{6b}-\frac{Babc^3g^2i^2}{6d}+\frac{Bb^2c^4g^2i^2}{30d^2}\right)$$
$$+\left(Ba^2c^2g^2i^2x+Ba^2cdg^2i^2x^2+\frac{Ba^2d^2g^2i^2x^3}{3}+Babc^2g^2i^2x^2+\frac{4Babcdg^2i^2x^3}{3}\right.$$
$$\left.+\frac{Babd^2g^2i^2x^4}{2}+\frac{Bb^2c^2g^2i^2x^3}{3}+\frac{Bb^2cdg^2i^2x^4}{2}+\frac{Bb^2d^2g^2i^2x^5}{5}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

```
[In] integrate((b*g*x+a*g)**2*(d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b**2*d**2*g**2*i**2*x**5/5 + B*a**3*g**2*i**2*(a**2*d**2 - 5*a*b*c*d + 10
*b**2*c**2)*log(x + (B*a**5*c*d**4*g**2*i**2 - 5*B*a**4*b*c**2*d**3*g**2*i*
*2 + B*a**4*d**3*g**2*i**2*(a**2*d**2 - 5*a*b*c*d + 10*b**2*c**2)/b + 20*B*
a**3*b**2*c**3*d**2*g**2*i**2 - B*a**3*c*d**2*g**2*i**2*(a**2*d**2 - 5*a*b*
c*d + 10*b**2*c**2) - 5*B*a**2*b**3*c**4*d*g**2*i**2 + B*a*b**4*c**5*g**2*i
```

```
**2)/(B*a**5*d**5*g**2*i**2 - 5*B*a**4*b*c*d**4*g**2*i**2 + 10*B*a**3*b**2*
c**2*d**3*g**2*i**2 + 10*B*a**2*b**3*c**3*d**2*g**2*i**2 - 5*B*a*b**4*c**4*
d*g**2*i**2 + B*b**5*c**5*g**2*i**2))/(30*b**3) - B*c**3*g**2*i**2*(10*a**2
*d**2 - 5*a*b*c*d + b**2*c**2)*log(x + (B*a**5*c*d**4*g**2*i**2 - 5*B*a**4*
b*c**2*d**3*g**2*i**2 + 20*B*a**3*b**2*c**3*d**2*g**2*i**2 - 5*B*a**2*b**3*
c**4*d*g**2*i**2 + B*a*b**4*c**5*g**2*i**2 - B*a*b**2*c**3*g**2*i**2*(10*a*
*2*d**2 - 5*a*b*c*d + b**2*c**2) + B*b**3*c**4*g**2*i**2*(10*a**2*d**2 - 5*
a*b*c*d + b**2*c**2)/d)/(B*a**5*d**5*g**2*i**2 - 5*B*a**4*b*c*d**4*g**2*i**
2 + 10*B*a**3*b**2*c**2*d**3*g**2*i**2 + 10*B*a**2*b**3*c**3*d**2*g**2*i**2
 - 5*B*a*b**4*c**4*d*g**2*i**2 + B*b**5*c**5*g**2*i**2))/(30*d**3) + x**4*(
A*a*b*d**2*g**2*i**2/2 + A*b**2*c*d*g**2*i**2/2 + B*a*b*d**2*g**2*i**2/20 -
 B*b**2*c*d*g**2*i**2/20) + x**3*(A*a**2*d**2*g**2*i**2/3 + 4*A*a*b*c*d*g**
2*i**2/3 + A*b**2*c**2*g**2*i**2/3 + B*a**2*d**2*g**2*i**2/10 - B*b**2*c**2
*g**2*i**2/10) + x**2*(A*a**2*c*d*g**2*i**2 + A*a*b*c**2*g**2*i**2 + B*a**3
*d**2*g**2*i**2/(60*b) + B*a**2*c*d*g**2*i**2/4 - B*a*b*c**2*g**2*i**2/4 -
B*b**2*c**3*g**2*i**2/(60*d)) + x*(A*a**2*c**2*g**2*i**2 - B*a**4*d**2*g**2
*i**2/(30*b**2) + B*a**3*c*d*g**2*i**2/(6*b) - B*a*b*c**3*g**2*i**2/(6*d) +
 B*b**2*c**4*g**2*i**2/(30*d**2)) + (B*a**2*c**2*g**2*i**2*x + B*a**2*c*d*g
**2*i**2*x**2 + B*a**2*d**2*g**2*i**2*x**3/3 + B*a*b*c**2*g**2*i**2*x**2 +
4*B*a*b*c*d*g**2*i**2*x**3/3 + B*a*b*d**2*g**2*i**2*x**4/2 + B*b**2*c**2*g*
*2*i**2*x**3/3 + B*b**2*c*d*g**2*i**2*x**4/2 + B*b**2*d**2*g**2*i**2*x**5/5
)*log(e*(a + b*x)/(c + d*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1200 vs. $2(319) = 638$.

Time = 0.23 (sec) , antiderivative size = 1200, normalized size of antiderivative = 3.56

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algor
ithm="maxima")
```

```
[Out] 1/5*A*b^2*d^2*g^2*i^2*x^5 + 1/2*A*b^2*c*d*g^2*i^2*x^4 + 1/2*A*a*b*d^2*g^2*i
^2*x^4 + 1/3*A*b^2*c^2*g^2*i^2*x^3 + 4/3*A*a*b*c*d*g^2*i^2*x^3 + 1/3*A*a^2*
d^2*g^2*i^2*x^3 + A*a*b*c^2*g^2*i^2*x^2 + A*a^2*c*d*g^2*i^2*x^2 + (x*log(b*
e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*a^2
*c^2*g^2*i^2 + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)
/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a*b*c^2*g^2*i^2 + 1/6*
(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^
3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b
^2*d^2))*B*b^2*c^2*g^2*i^2 + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^
2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a^2*c*d*
g^2*i^2 + 2/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x +
```

```
a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 -
a^2*d^2)*x)/(b^2*d^2))*B*a*b*c*d*g^2*i^2 + 1/12*(6*x^4*log(b*e*x/(d*x + c)
+ a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^
3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3
*d^3)*x)/(b^3*d^3))*B*b^2*c*d*g^2*i^2 + 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*
e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d
- a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a^2*d^2*g^2*i^2 + 1/
12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6
*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2
*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*a*b*d^2*g^2*i^2 + 1/60*
(12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12
*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a
^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)
/(b^4*d^4))*B*b^2*d^2*g^2*i^2 + A*a^2*c^2*g^2*i^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3144 vs. 2(319) = 638.

Time = 0.52 (sec) , antiderivative size = 3144, normalized size of antiderivative = 9.33

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algor
ithm="giac")
```

```
[Out] 1/60*(2*(B*b^8*c^6*e^6*g^2*i^2 - 6*B*a*b^7*c^5*d*e^6*g^2*i^2 + 15*B*a^2*b^6
*c^4*d^2*e^6*g^2*i^2 - 20*B*a^3*b^5*c^3*d^3*e^6*g^2*i^2 + 15*B*a^4*b^4*c^2*
d^4*e^6*g^2*i^2 - 6*B*a^5*b^3*c*d^5*e^6*g^2*i^2 + B*a^6*b^2*d^6*e^6*g^2*i^2
 - 5*(b*e*x + a*e)*B*b^7*c^6*d*e^5*g^2*i^2/(d*x + c) + 30*(b*e*x + a*e)*B*a
*b^6*c^5*d^2*e^5*g^2*i^2/(d*x + c) - 75*(b*e*x + a*e)*B*a^2*b^5*c^4*d^3*e^5
*g^2*i^2/(d*x + c) + 100*(b*e*x + a*e)*B*a^3*b^4*c^3*d^4*e^5*g^2*i^2/(d*x +
 c) - 75*(b*e*x + a*e)*B*a^4*b^3*c^2*d^5*e^5*g^2*i^2/(d*x + c) + 30*(b*e*x
+ a*e)*B*a^5*b^2*c*d^6*e^5*g^2*i^2/(d*x + c) - 5*(b*e*x + a*e)*B*a^6*b*d^7*
e^5*g^2*i^2/(d*x + c) + 10*(b*e*x + a*e)^2*B*b^6*c^6*d^2*e^4*g^2*i^2/(d*x +
 c)^2 - 60*(b*e*x + a*e)^2*B*a*b^5*c^5*d^3*e^4*g^2*i^2/(d*x + c)^2 + 150*(b
*e*x + a*e)^2*B*a^2*b^4*c^4*d^4*e^4*g^2*i^2/(d*x + c)^2 - 200*(b*e*x + a*e)
^2*B*a^3*b^3*c^3*d^5*e^4*g^2*i^2/(d*x + c)^2 + 150*(b*e*x + a*e)^2*B*a^4*b^
2*c^2*d^6*e^4*g^2*i^2/(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*a^5*b*c*d^7*e^4*g^
2*i^2/(d*x + c)^2 + 10*(b*e*x + a*e)^2*B*a^6*d^8*e^4*g^2*i^2/(d*x + c)^2)*l
og((b*e*x + a*e)/(d*x + c))/(b^5*d^3*e^5 - 5*(b*e*x + a*e)*b^4*d^4*e^4/(d*x
 + c) + 10*(b*e*x + a*e)^2*b^3*d^5*e^3/(d*x + c)^2 - 10*(b*e*x + a*e)^3*b^2
*d^6*e^2/(d*x + c)^3 + 5*(b*e*x + a*e)^4*b*d^7*e/(d*x + c)^4 - (b*e*x + a*e
)^5*d^8/(d*x + c)^5) + (2*A*b^10*c^6*e^6*g^2*i^2 - 12*A*a*b^9*c^5*d*e^6*g^2
*i^2 + 30*A*a^2*b^8*c^4*d^2*e^6*g^2*i^2 - 40*A*a^3*b^7*c^3*d^3*e^6*g^2*i^2
```

+ 30*A*a^4*b^6*c^2*d^4*e^6*g^2*i^2 - 12*A*a^5*b^5*c*d^5*e^6*g^2*i^2 + 2*A*a
^6*b^4*d^6*e^6*g^2*i^2 - 10*(b*e*x + a*e)*A*b^9*c^6*d*e^5*g^2*i^2/(d*x + c)
 + 2*(b*e*x + a*e)*B*b^9*c^6*d*e^5*g^2*i^2/(d*x + c) + 60*(b*e*x + a*e)*A*a
*b^8*c^5*d^2*e^5*g^2*i^2/(d*x + c) - 12*(b*e*x + a*e)*B*a*b^8*c^5*d^2*e^5*g
^2*i^2/(d*x + c) - 150*(b*e*x + a*e)*A*a^2*b^7*c^4*d^3*e^5*g^2*i^2/(d*x + c
) + 30*(b*e*x + a*e)*B*a^2*b^7*c^4*d^3*e^5*g^2*i^2/(d*x + c) + 200*(b*e*x +
 a*e)*A*a^3*b^6*c^3*d^4*e^5*g^2*i^2/(d*x + c) - 40*(b*e*x + a*e)*B*a^3*b^6*
c^3*d^4*e^5*g^2*i^2/(d*x + c) - 150*(b*e*x + a*e)*A*a^4*b^5*c^2*d^5*e^5*g^2
*i^2/(d*x + c) + 30*(b*e*x + a*e)*B*a^4*b^5*c^2*d^5*e^5*g^2*i^2/(d*x + c) +
 60*(b*e*x + a*e)*A*a^5*b^4*c*d^6*e^5*g^2*i^2/(d*x + c) - 12*(b*e*x + a*e)*
B*a^5*b^4*c*d^6*e^5*g^2*i^2/(d*x + c) - 10*(b*e*x + a*e)*A*a^6*b^3*d^7*e^5*
g^2*i^2/(d*x + c) + 2*(b*e*x + a*e)*B*a^6*b^3*d^7*e^5*g^2*i^2/(d*x + c) + 2
0*(b*e*x + a*e)^2*A*b^8*c^6*d^2*e^4*g^2*i^2/(d*x + c)^2 - 9*(b*e*x + a*e)^2
*B*b^8*c^6*d^2*e^4*g^2*i^2/(d*x + c)^2 - 120*(b*e*x + a*e)^2*A*a*b^7*c^5*d^
3*e^4*g^2*i^2/(d*x + c)^2 + 54*(b*e*x + a*e)^2*B*a*b^7*c^5*d^3*e^4*g^2*i^2/
(d*x + c)^2 + 300*(b*e*x + a*e)^2*A*a^2*b^6*c^4*d^4*e^4*g^2*i^2/(d*x + c)^2
 - 135*(b*e*x + a*e)^2*B*a^2*b^6*c^4*d^4*e^4*g^2*i^2/(d*x + c)^2 - 400*(b*e
*x + a*e)^2*A*a^3*b^5*c^3*d^5*e^4*g^2*i^2/(d*x + c)^2 + 180*(b*e*x + a*e)^2
*B*a^3*b^5*c^3*d^5*e^4*g^2*i^2/(d*x + c)^2 + 300*(b*e*x + a*e)^2*A*a^4*b^4*
c^2*d^6*e^4*g^2*i^2/(d*x + c)^2 - 135*(b*e*x + a*e)^2*B*a^4*b^4*c^2*d^6*e^4
*g^2*i^2/(d*x + c)^2 - 120*(b*e*x + a*e)^2*A*a^5*b^3*c*d^7*e^4*g^2*i^2/(d*x
 + c)^2 + 54*(b*e*x + a*e)^2*B*a^5*b^3*c*d^7*e^4*g^2*i^2/(d*x + c)^2 + 20*(
b*e*x + a*e)^2*A*a^6*b^2*d^8*e^4*g^2*i^2/(d*x + c)^2 - 9*(b*e*x + a*e)^2*B*
a^6*b^2*d^8*e^4*g^2*i^2/(d*x + c)^2 + 9*(b*e*x + a*e)^3*B*b^7*c^6*d^3*e^3*g
^2*i^2/(d*x + c)^3 - 54*(b*e*x + a*e)^3*B*a*b^6*c^5*d^4*e^3*g^2*i^2/(d*x +
c)^3 + 135*(b*e*x + a*e)^3*B*a^2*b^5*c^4*d^5*e^3*g^2*i^2/(d*x + c)^3 - 180*
(b*e*x + a*e)^3*B*a^3*b^4*c^3*d^6*e^3*g^2*i^2/(d*x + c)^3 + 135*(b*e*x + a*
e)^3*B*a^4*b^3*c^2*d^7*e^3*g^2*i^2/(d*x + c)^3 - 54*(b*e*x + a*e)^3*B*a^5*b
^2*c*d^8*e^3*g^2*i^2/(d*x + c)^3 + 9*(b*e*x + a*e)^3*B*a^6*b*d^9*e^3*g^2*i^
2/(d*x + c)^3 - 2*(b*e*x + a*e)^4*B*b^6*c^6*d^4*e^2*g^2*i^2/(d*x + c)^4 + 1
2*(b*e*x + a*e)^4*B*a*b^5*c^5*d^5*e^2*g^2*i^2/(d*x + c)^4 - 30*(b*e*x + a*e
)^4*B*a^2*b^4*c^4*d^6*e^2*g^2*i^2/(d*x + c)^4 + 40*(b*e*x + a*e)^4*B*a^3*b^
3*c^3*d^7*e^2*g^2*i^2/(d*x + c)^4 - 30*(b*e*x + a*e)^4*B*a^4*b^2*c^2*d^8*e^
2*g^2*i^2/(d*x + c)^4 + 12*(b*e*x + a*e)^4*B*a^5*b*c*d^9*e^2*g^2*i^2/(d*x +
 c)^4 - 2*(b*e*x + a*e)^4*B*a^6*d^10*e^2*g^2*i^2/(d*x + c)^4)/(b^7*d^3*e^5
- 5*(b*e*x + a*e)*b^6*d^4*e^4/(d*x + c) + 10*(b*e*x + a*e)^2*b^5*d^5*e^3/(d
x + c)^2 - 10(b*e*x + a*e)^3*b^4*d^6*e^2/(d*x + c)^3 + 5*(b*e*x + a*e)^4*
b^3*d^7*e/(d*x + c)^4 - (b*e*x + a*e)^5*b^2*d^8/(d*x + c)^5) + 2*(B*b^6*c^6
*e*g^2*i^2 - 6*B*a*b^5*c^5*d*e*g^2*i^2 + 15*B*a^2*b^4*c^4*d^2*e*g^2*i^2 - 2
0*B*a^3*b^3*c^3*d^3*e*g^2*i^2 + 15*B*a^4*b^2*c^2*d^4*e*g^2*i^2 - 6*B*a^5*b*
c*d^5*e*g^2*i^2 + B*a^6*d^6*e*g^2*i^2)*log(-b*e + (b*e*x + a*e)*d/(d*x + c)
)/(b^3*d^3) - 2*(B*b^6*c^6*e*g^2*i^2 - 6*B*a*b^5*c^5*d*e*g^2*i^2 + 15*B*a^2
*b^4*c^4*d^2*e*g^2*i^2 - 20*B*a^3*b^3*c^3*d^3*e*g^2*i^2 + 15*B*a^4*b^2*c^2*
d^4*e*g^2*i^2 - 6*B*a^5*b*c*d^5*e*g^2*i^2 + B*a^6*d^6*e*g^2*i^2)*log((b*e*x
 + a*e)/(d*x + c))/(b^3*d^3))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*

```
c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 1287, normalized size of antiderivative = 3.82

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

```
[In] int((a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] log((e*(a + b*x))/(c + d*x))*((B*g^2*i^2*x^3*(a^2*d^2 + b^2*c^2 + 4*a*b*c*d
))/3 + B*a^2*c^2*g^2*i^2*x + (B*b^2*d^2*g^2*i^2*x^5)/5 + B*a*c*g^2*i^2*x^2*
(a*d + b*c) + (B*b*d*g^2*i^2*x^4*(a*d + b*c))/2) - x^3*(((30*a*d + 30*b*c)*
((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d - B*b*c))/5 - (A*b*d*g^2*i^2*(30
*a*d + 30*b*c))/30))/(90*b*d) - (g^2*i^2*(6*A*a^2*d^2 + 6*A*b^2*c^2 + B*a^2
*d^2 - B*b^2*c^2 + 18*A*a*b*c*d))/6 + (A*a*b*c*d*g^2*i^2)/3) + x*((a*c*(((3
0*a*d + 30*b*c)*((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d - B*b*c))/5 - (A
*b*d*g^2*i^2*(30*a*d + 30*b*c))/30))/(30*b*d) - (g^2*i^2*(6*A*a^2*d^2 + 6*A
*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 18*A*a*b*c*d))/2 + A*a*b*c*d*g^2*i^2))/(
b*d) - ((30*a*d + 30*b*c)*(((30*a*d + 30*b*c)*(((30*a*d + 30*b*c)*((b*d*g^2
*i^2*(15*A*a*d + 15*A*b*c + B*a*d - B*b*c))/5 - (A*b*d*g^2*i^2*(30*a*d + 30
*b*c))/30))/(30*b*d) - (g^2*i^2*(6*A*a^2*d^2 + 6*A*b^2*c^2 + B*a^2*d^2 - B*
b^2*c^2 + 18*A*a*b*c*d))/2 + A*a*b*c*d*g^2*i^2))/(30*b*d) + (g^2*i^2*(3*A*a
^3*d^3 + 3*A*b^3*c^3 + B*a^3*d^3 - B*b^3*c^3 + 27*A*a*b^2*c^2*d + 27*A*a^2*
b*c*d^2 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2))/(3*b*d) - (a*c*((b*d*g^2*i^2*
(15*A*a*d + 15*A*b*c + B*a*d - B*b*c))/5 - (A*b*d*g^2*i^2*(30*a*d + 30*b*c)
)/30))/(b*d)))/(30*b*d) + (a*c*g^2*i^2*(3*A*a^2*d^2 + 3*A*b^2*c^2 + B*a^2*d
^2 - B*b^2*c^2 + 9*A*a*b*c*d))/(b*d)) + x^2*(((30*a*d + 30*b*c)*(((30*a*d +
 30*b*c)*((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d - B*b*c))/5 - (A*b*d*g^
2*i^2*(30*a*d + 30*b*c))/30))/(30*b*d) - (g^2*i^2*(6*A*a^2*d^2 + 6*A*b^2*c^
2 + B*a^2*d^2 - B*b^2*c^2 + 18*A*a*b*c*d))/2 + A*a*b*c*d*g^2*i^2))/(60*b*d)
 + (g^2*i^2*(3*A*a^3*d^3 + 3*A*b^3*c^3 + B*a^3*d^3 - B*b^3*c^3 + 27*A*a*b^2
*c^2*d + 27*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2))/(6*b*d) - (
a*c*((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d - B*b*c))/5 - (A*b*d*g^2*i^2
*(30*a*d + 30*b*c))/30))/(2*b*d)) + x^4*((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c
+ B*a*d - B*b*c))/20 - (A*b*d*g^2*i^2*(30*a*d + 30*b*c))/120) + (log(a + b*
x)*(B*a^5*d^2*g^2*i^2 + 10*B*a^3*b^2*c^2*g^2*i^2 - 5*B*a^4*b*c*d*g^2*i^2))/
(30*b^3) - (log(c + d*x)*(B*b^2*c^5*g^2*i^2 + 10*B*a^2*c^3*d^2*g^2*i^2 - 5*
B*a*b*c^4*d*g^2*i^2))/(30*d^3) + (A*b^2*d^2*g^2*i^2*x^5)/5
```

3.12 $\int (ag+bgx)(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

Optimal result

Integrand size = 38, antiderivative size = 239

$$\int (ag+bgx)(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = \frac{B(bc-ad)^3gi^2x}{12b^2d} + \frac{B(bc-ad)^2gi^2(c+dx)^2}{24bd^2} - \frac{B(bc-ad)gi^2(c+dx)^3}{12d^2} + \frac{B(bc-ad)^4gi^2\log\left(\frac{a+bx}{c+dx}\right)}{12b^3d^2} - \frac{(bc-ad)gi^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^2} + \frac{bgi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2} + \frac{B(bc-ad)^4gi^2\log(c+dx)}{12b^3d^2}$$

```
[Out] 1/12*B*(-a*d+b*c)^3*g*i^2*x/b^2/d+1/24*B*(-a*d+b*c)^2*g*i^2*(d*x+c)^2/b/d^2
-1/12*B*(-a*d+b*c)*g*i^2*(d*x+c)^3/d^2+1/12*B*(-a*d+b*c)^4*g*i^2*ln((b*x+a)
/(d*x+c))/b^3/d^2-1/3*(-a*d+b*c)*g*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c))
)/d^2+1/4*b*g*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^2+1/12*B*(-a*d+b*
c)^4*g*i^2*ln(d*x+c)/b^3/d^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used

= {2562, 45, 2382, 12, 78}

$$\int (ag+bgx)(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= -\frac{gi^2(c+dx)^3(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^2} + \frac{bgi^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^2} + \frac{Bgi^2(bc-ad)^4\log\left(\frac{a+bx}{c+dx}\right)}{12b^3d^2} + \frac{Bgi^2(bc-ad)^4\log(c+dx)}{12b^3d^2} + \frac{Bgi^2x(bc-ad)^3}{12b^2d} + \frac{Bgi^2(c+dx)^2(bc-ad)^2}{24bd^2} - \frac{Bgi^2(c+dx)^3(bc-ad)}{12d^2}$$

```
[In] Int[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (B*(b*c - a*d)^3*g*i^2*x)/(12*b^2*d) + (B*(b*c - a*d)^2*g*i^2*(c + d*x)^2)/
(24*b*d^2) - (B*(b*c - a*d)*g*i^2*(c + d*x)^3)/(12*d^2) + (B*(b*c - a*d)^4*
g*i^2*Log[(a + b*x)/(c + d*x)])/(12*b^3*d^2) - ((b*c - a*d)*g*i^2*(c + d*x)
^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*d^2) + (b*g*i^2*(c + d*x)^4*(A
+ B*Log[(e*(a + b*x))/(c + d*x)]))/(4*d^2) + (B*(b*c - a*d)^4*g*i^2*Log[c +
 d*x])/(12*b^3*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
```

```
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left((bc-ad)^4 gi^2\right) \text{Subst}\left(\int \frac{x(A+B\log(ex))}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right) \\
&= -\frac{(bc-ad)gi^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^2} \\
&\quad + \frac{bgi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2} \\
&\quad - \left(B(bc-ad)^4 gi^2\right) \text{Subst}\left(\int \frac{-b+4dx}{12d^2x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right) \\
&= -\frac{(bc-ad)gi^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^2} \\
&\quad + \frac{bgi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2} \\
&\quad - \frac{\left(B(bc-ad)^4 gi^2\right) \text{Subst}\left(\int \frac{-b+4dx}{x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{12d^2} \\
&= -\frac{(bc-ad)gi^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^2} + \frac{bgi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2} \\
&\quad - \frac{\left(B(bc-ad)^4 gi^2\right) \text{Subst}\left(\int \left(-\frac{1}{b^3x} + \frac{3d}{(b-dx)^4} - \frac{d}{b(b-dx)^3} - \frac{d}{b^2(b-dx)^2} - \frac{d}{b^3(b-dx)}\right) dx, x, \frac{a+bx}{c+dx}\right)}{12d^2} \\
&= \frac{B(bc-ad)^3 gi^2 x}{12b^2 d} + \frac{B(bc-ad)^2 gi^2(c+dx)^2}{24bd^2} - \frac{B(bc-ad)gi^2(c+dx)^3}{12d^2} \\
&\quad + \frac{B(bc-ad)^4 gi^2 \log\left(\frac{a+bx}{c+dx}\right)}{12b^3d^2} - \frac{(bc-ad)gi^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^2} \\
&\quad + \frac{bgi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2} + \frac{B(bc-ad)^4 gi^2\log(c+dx)}{12b^3d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.90

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{gi^2\left(\frac{4B(bc-ad)^2(2bd(bc-ad)x+b^2(c+dx)^2+2(bc-ad)^2\log(a+bx))}{b^3}-\frac{B(bc-ad)(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad)}{b^3}\right.}{24d^2}$$

```
[In] Integrate[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
),x]
```

```
[Out] (g*i^2*((4*B*(b*c - a*d)^2*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c
- a*d)^2*Log[a + b*x]))/b^3 - (B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*b^2
*(b*c - a*d)*(c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]
))/b^3 - 8*(b*c - a*d)*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6
*b*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])))/(24*d^2)
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.66

method	result
risch	$\frac{g\,i^2 Bx\left(3b\,d^2x^3+4a\,d^2x^2+8bcd\,x^2+12acdx+6b\,c^2x+12a\,c^2\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{12}+\frac{i^2gb\,d^2A\,x^4}{4}+\frac{i^2g\,d^2Aa\,x^3}{3}+\frac{2i^2gbdA}{3}$
parallelrisch	$\frac{-2B\ln\left(\frac{e(bx+a)}{dx+c}\right)b^4c^4g\,i^2-36Aa\,b^3c^3dg\,i^2+2B\ln(bx+a)a^4d^4g\,i^2+2B\ln(bx+a)b^4c^4g\,i^2+6A\,x^4b^4d^4g\,i^2+4B\,x^2a\,b^3c\,d^3}{}$
parts	$Ag\,i^2\left(\frac{b\,d^2x^4}{4}+\frac{(a\,d^2+2bcd)x^3}{3}+\frac{(2acd+b\,c^2)x^2}{2}+xa\,c^2\right)-\frac{Bg\,i^2(ad-cb)^3e^3\left(b\,d^3e(ad-cb)\left(\frac{}{8b^2e^2d\left(\left(\frac{be}{d}+\right.\right.}\right.\right.}{}$
derivativedivides	$-\frac{e(ad-cb)\left(-A\,d^2e^2g\,i^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\left(\frac{be}{4d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^4}-\frac{1}{3d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^3}\right.\right.}{}$
default	$-\frac{e(ad-cb)\left(-A\,d^2e^2g\,i^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\left(\frac{be}{4d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^4}-\frac{1}{3d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^3}\right.\right.}{}$

```
[In] int((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/12*g*i^2*B*x*(3*b*d^2*x^3+4*a*d^2*x^2+8*b*c*d*x^2+12*a*c*d*x+6*b*c^2*x+12
*a*c^2)*ln(e*(b*x+a)/(d*x+c))+1/4*i^2*g*b*d^2*A*x^4+1/3*i^2*g*d^2*A*a*x^3+2
/3*i^2*g*b*d*A*c*x^3+1/12*i^2*g*d^2*B*a*x^3-1/12*i^2*g*b*d*B*c*x^3+i^2*g*d*
A*a*c*x^2+1/2*i^2*g*b*A*c^2*x^2+1/24*i^2*g/b*d^2*B*a^2*x^2+1/6*i^2*g*d*B*a*
c*x^2-5/24*i^2*g*b*B*c^2*x^2+i^2*g*A*a*c^2*x-1/3*i^2*g/d*B*ln(d*x+c)*a*c^3+
1/12*i^2*g*b/d^2*B*ln(d*x+c)*c^4+1/12*i^2*g/b^3*d^2*B*ln(-b*x-a)*a^4-1/3*i^
2*g/b^2*d*B*ln(-b*x-a)*a^3*c+1/2*i^2*g/b*B*ln(-b*x-a)*a^2*c^2-1/12*i^2*g/b^
2*d^2*B*a^3*x+1/3*i^2*g/b*d*B*a^2*c*x-1/6*i^2*g*B*a*c^2*x-1/12*i^2*g*b/d*B*
c^3*x
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.54

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{6\,Ab^4d^4gi^2x^4+2\left((8\,A-B)b^4cd^3+(4\,A+B)ab^3d^4\right)gi^2x^3+\left((12\,A-5\,B)b^4c^2d^2+4\,(6\,A+B)ab^3cd^3+B\right.}{}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorit
hm="fricas")
```

```
[Out] 1/24*(6*A*b^4*d^4*g*i^2*x^4 + 2*((8*A - B)*b^4*c*d^3 + (4*A + B)*a*b^3*d^4)
*g*i^2*x^3 + ((12*A - 5*B)*b^4*c^2*d^2 + 4*(6*A + B)*a*b^3*c*d^3 + B*a^2*b^
2*d^4)*g*i^2*x^2 - 2*(B*b^4*c^3*d - 2*(6*A - B)*a*b^3*c^2*d^2 - 4*B*a^2*b^2
*c*d^3 + B*a^3*b*d^4)*g*i^2*x + 2*(6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3 +
B*a^4*d^4)*g*i^2*log(b*x + a) + 2*(B*b^4*c^4 - 4*B*a*b^3*c^3*d)*g*i^2*log(d
*x + c) + 2*(3*B*b^4*d^4*g*i^2*x^4 + 12*B*a*b^3*c^2*d^2*g*i^2*x + 4*(2*B*b^
4*c*d^3 + B*a*b^3*d^4)*g*i^2*x^3 + 6*(B*b^4*c^2*d^2 + 2*B*a*b^3*c*d^3)*g*i^
2*x^2)*log((b*e*x + a*e)/(d*x + c)))/(b^3*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 850 vs. $2(221) = 442$.

Time = 2.35 (sec) , antiderivative size = 850, normalized size of antiderivative = 3.56

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\frac{Abd^2gi^2x^4}{4}+\frac{Ba^2gi^2(a^2d^2-4abcd+6b^2c^2)\log\left(x+\frac{Ba^4cd^3gi^2-4Ba^3bc^2d^2gi^2+\frac{Ba^3d^2gi^2(a^2d^2-4abcd+6b^2c^2)}{b}+10Ba^2b^2c^3dgi^2-Ba^2cdgi\cdots}{Ba^4d^4gi^2-4Ba^3bcd^3gi^2+6Ba^2b^2c^2d^2gi^2+4Bab^3c^3dgi^2-\cdots}\right)}{12b^3}-\frac{Bc^3gi^2\cdot(4ad-bc)\log\left(x+\frac{Ba^4cd^3gi^2-4Ba^3bc^2d^2gi^2+10Ba^2b^2c^3dgi^2-Bab^3c^4gi^2-Bab^2c^3gi^2\cdot(4ad-bc)+\frac{Bb^3c^4gi^2\cdot(4ad-bc)}{d}}{Ba^4d^4gi^2-4Ba^3bcd^3gi^2+6Ba^2b^2c^2d^2gi^2+4Bab^3c^3dgi^2-Bb^4c^4gi^2}\right)}{12d^2}+x^3\left(\frac{Aad^2gi^2}{3}+\frac{2Abcdgi^2}{3}+\frac{Bad^2gi^2}{12}-\frac{Bbcdgi^2}{12}\right)+x^2\left(Aacdgi^2+\frac{Abc^2gi^2}{2}+\frac{Ba^2d^2gi^2}{24b}+\frac{Bacdgi^2}{6}-\frac{5Bbc^2gi^2}{24}\right)+x\left(Aac^2gi^2-\frac{Ba^3d^2gi^2}{12b^2}+\frac{Ba^2cdgi^2}{3b}-\frac{Bac^2gi^2}{6}-\frac{Bbc^3gi^2}{12d}\right)+\left(Bac^2gi^2x+Bacdgi^2x^2+\frac{Bad^2gi^2x^3}{3}+\frac{Bbc^2gi^2x^2}{2}+\frac{2Bbcdgi^2x^3}{3}+\frac{Bbd^2gi^2x^4}{4}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b*d**2*g*i**2*x**4/4 + B*a**2*g*i**2*(a**2*d**2 - 4*a*b*c*d + 6*b**2*c**2
)*log(x + (B*a**4*c*d**3*g*i**2 - 4*B*a**3*b*c**2*d**2*g*i**2 + B*a**3*d**2
*g*i**2*(a**2*d**2 - 4*a*b*c*d + 6*b**2*c**2)/b + 10*B*a**2*b**2*c**3*d*g*i
**2 - B*a**2*c*d*g*i**2*(a**2*d**2 - 4*a*b*c*d + 6*b**2*c**2) - B*a*b**3*c*
*4*g*i**2)/(B*a**4*d**4*g*i**2 - 4*B*a**3*b*c*d**3*g*i**2 + 6*B*a**2*b**2*c
**2*d**2*g*i**2 + 4*B*a*b**3*c**3*d*g*i**2 - B*b**4*c**4*g*i**2))/(12*b**3)
 - B*c**3*g*i**2*(4*a*d - b*c)*log(x + (B*a**4*c*d**3*g*i**2 - 4*B*a**3*b*c
**2*d**2*g*i**2 + 10*B*a**2*b**2*c**3*d*g*i**2 - B*a*b**3*c**4*g*i**2 - B*a
*b**2*c**3*g*i**2*(4*a*d - b*c) + B*b**3*c**4*g*i**2*(4*a*d - b*c)/d)/(B*a*
*4*d**4*g*i**2 - 4*B*a**3*b*c*d**3*g*i**2 + 6*B*a**2*b**2*c**2*d**2*g*i**2
+ 4*B*a*b**3*c**3*d*g*i**2 - B*b**4*c**4*g*i**2))/(12*d**2) + x**3*(A*a*d**
2*g*i**2/3 + 2*A*b*c*d*g*i**2/3 + B*a*d**2*g*i**2/12 - B*b*c*d*g*i**2/12) +
 x**2*(A*a*c*d*g*i**2 + A*b*c**2*g*i**2/2 + B*a**2*d**2*g*i**2/(24*b) + B*a
*c*d*g*i**2/6 - 5*B*b*c**2*g*i**2/24) + x*(A*a*c**2*g*i**2 - B*a**3*d**2*g*
i**2/(12*b**2) + B*a**2*c*d*g*i**2/(3*b) - B*a*c**2*g*i**2/6 - B*b*c**3*g*i
**2/(12*d)) + (B*a*c**2*g*i**2*x + B*a*c*d*g*i**2*x**2 + B*a*d**2*g*i**2*x*
*3/3 + B*b*c**2*g*i**2*x**2/2 + 2*B*b*c*d*g*i**2*x**3/3 + B*b*d**2*g*i**2*x
**4/4)*log(e*(a + b*x)/(c + d*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. 2(225) = 450.

Time = 0.22 (sec) , antiderivative size = 671, normalized size of antiderivative = 2.81

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$= \frac{1}{4}Abd^2gi^2x^4+\frac{2}{3}Abcdgi^2x^3+\frac{1}{3}Aad^2gi^2x^3+\frac{1}{2}Abc^2gi^2x^2+Aacdgi^2x^2$$
$$+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bac^2gi^2$$
$$+\frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bbc^2gi^2$$
$$+\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bacdgi^2$$
$$+\frac{1}{3}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2c$$
$$+\frac{1}{6}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2c$$
$$+\frac{1}{24}\left(6x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6a^4\log(bx+a)}{b^4}+\frac{6c^4\log(dx+c)}{d^4}-\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3c^2d$$
$$+Aac^2gi^2x$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorit
hm="maxima")
```

```
[Out] 1/4*A*b*d^2*g*i^2*x^4 + 2/3*A*b*c*d*g*i^2*x^3 + 1/3*A*a*d^2*g*i^2*x^3 + 1/2
*A*b*c^2*g*i^2*x^2 + A*a*c*d*g*i^2*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x
+ c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*a*c^2*g*i^2 + 1/2*(x^2*log(b
*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d
^2 - (b*c - a*d)*x/(b*d))*B*b*c^2*g*i^2 + (x^2*log(b*e*x/(d*x + c) + a*e/(d
*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d
))*B*a*c*d*g*i^2 + 1/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*
log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b
^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b*c*d*g*i^2 + 1/6*(2*x^3*log(b*e*x/(d*x +
 c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((
b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*d^2*g*i^2
+ 1/24*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4
 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d -
 a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b*d^2*g*i^2 + A*a*c
^2*g*i^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1975 vs. 2(225) = 450.

Time = 0.48 (sec) , antiderivative size = 1975, normalized size of antiderivative = 8.26

$$\int (ag+bgx)(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorit
hm="giac")
```

```
[Out] -1/24*(2*(B*b^6*c^5*e^5*g*i^2 - 5*B*a*b^5*c^4*d*e^5*g*i^2 + 10*B*a^2*b^4*c^
3*d^2*e^5*g*i^2 - 10*B*a^3*b^3*c^2*d^3*e^5*g*i^2 + 5*B*a^4*b^2*c*d^4*e^5*g*
i^2 - B*a^5*b*d^5*e^5*g*i^2 - 4*(b*e*x + a*e)*B*b^5*c^5*d*e^4*g*i^2/(d*x +
c) + 20*(b*e*x + a*e)*B*a*b^4*c^4*d^2*e^4*g*i^2/(d*x + c) - 40*(b*e*x + a*e
)*B*a^2*b^3*c^3*d^3*e^4*g*i^2/(d*x + c) + 40*(b*e*x + a*e)*B*a^3*b^2*c^2*d^
4*e^4*g*i^2/(d*x + c) - 20*(b*e*x + a*e)*B*a^4*b*c*d^5*e^4*g*i^2/(d*x + c)
+ 4*(b*e*x + a*e)*B*a^5*d^6*e^4*g*i^2/(d*x + c))*log((b*e*x + a*e)/(d*x + c
))/(b^4*d^2*e^4 - 4*(b*e*x + a*e)*b^3*d^3*e^3/(d*x + c) + 6*(b*e*x + a*e)^2
*b^2*d^4*e^2/(d*x + c)^2 - 4*(b*e*x + a*e)^3*b*d^5*e/(d*x + c)^3 + (b*e*x +
 a*e)^4*d^6/(d*x + c)^4) + (2*A*b^8*c^5*e^5*g*i^2 - B*b^8*c^5*e^5*g*i^2 - 1
0*A*a*b^7*c^4*d*e^5*g*i^2 + 5*B*a*b^7*c^4*d*e^5*g*i^2 + 20*A*a^2*b^6*c^3*d^
2*e^5*g*i^2 - 10*B*a^2*b^6*c^3*d^2*e^5*g*i^2 - 20*A*a^3*b^5*c^2*d^3*e^5*g*i
^2 + 10*B*a^3*b^5*c^2*d^3*e^5*g*i^2 + 10*A*a^4*b^4*c*d^4*e^5*g*i^2 - 5*B*a^
4*b^4*c*d^4*e^5*g*i^2 - 2*A*a^5*b^3*d^5*e^5*g*i^2 + B*a^5*b^3*d^5*e^5*g*i^2
 - 8*(b*e*x + a*e)*A*b^7*c^5*d*e^4*g*i^2/(d*x + c) + 6*(b*e*x + a*e)*B*b^7*
c^5*d*e^4*g*i^2/(d*x + c) + 40*(b*e*x + a*e)*A*a*b^6*c^4*d^2*e^4*g*i^2/(d*x
 + c) - 30*(b*e*x + a*e)*B*a*b^6*c^4*d^2*e^4*g*i^2/(d*x + c) - 80*(b*e*x +
a*e)*A*a^2*b^5*c^3*d^3*e^4*g*i^2/(d*x + c) + 60*(b*e*x + a*e)*B*a^2*b^5*c^3
*d^3*e^4*g*i^2/(d*x + c) + 80*(b*e*x + a*e)*A*a^3*b^4*c^2*d^4*e^4*g*i^2/(d*
x + c) - 60*(b*e*x + a*e)*B*a^3*b^4*c^2*d^4*e^4*g*i^2/(d*x + c) - 40*(b*e*x
 + a*e)*A*a^4*b^3*c*d^5*e^4*g*i^2/(d*x + c) + 30*(b*e*x + a*e)*B*a^4*b^3*c*
d^5*e^4*g*i^2/(d*x + c) + 8*(b*e*x + a*e)*A*a^5*b^2*d^6*e^4*g*i^2/(d*x + c)
 - 6*(b*e*x + a*e)*B*a^5*b^2*d^6*e^4*g*i^2/(d*x + c) - 7*(b*e*x + a*e)^2*B*
b^6*c^5*d^2*e^3*g*i^2/(d*x + c)^2 + 35*(b*e*x + a*e)^2*B*a*b^5*c^4*d^3*e^3*
g*i^2/(d*x + c)^2 - 70*(b*e*x + a*e)^2*B*a^2*b^4*c^3*d^4*e^3*g*i^2/(d*x + c
)^2 + 70*(b*e*x + a*e)^2*B*a^3*b^3*c^2*d^5*e^3*g*i^2/(d*x + c)^2 - 35*(b*e*
x + a*e)^2*B*a^4*b^2*c*d^6*e^3*g*i^2/(d*x + c)^2 + 7*(b*e*x + a*e)^2*B*a^5*
b*d^7*e^3*g*i^2/(d*x + c)^2 + 2*(b*e*x + a*e)^3*B*b^5*c^5*d^3*e^2*g*i^2/(d*
x + c)^3 - 10*(b*e*x + a*e)^3*B*a*b^4*c^4*d^4*e^2*g*i^2/(d*x + c)^3 + 20*(b
*e*x + a*e)^3*B*a^2*b^3*c^3*d^5*e^2*g*i^2/(d*x + c)^3 - 20*(b*e*x + a*e)^3*
B*a^3*b^2*c^2*d^6*e^2*g*i^2/(d*x + c)^3 + 10*(b*e*x + a*e)^3*B*a^4*b*c*d^7*
e^2*g*i^2/(d*x + c)^3 - 2*(b*e*x + a*e)^3*B*a^5*d^8*e^2*g*i^2/(d*x + c)^3)/
(b^6*d^2*e^4 - 4*(b*e*x + a*e)*b^5*d^3*e^3/(d*x + c) + 6*(b*e*x + a*e)^2*b^
4*d^4*e^2/(d*x + c)^2 - 4*(b*e*x + a*e)^3*b^3*d^5*e/(d*x + c)^3 + (b*e*x +
```

```
a*e)^4*b^2*d^6/(d*x + c)^4) + 2*(B*b^5*c^5*e*g*i^2 - 5*B*a*b^4*c^4*d*e*g*i^
2 + 10*B*a^2*b^3*c^3*d^2*e*g*i^2 - 10*B*a^3*b^2*c^2*d^3*e*g*i^2 + 5*B*a^4*b
*c*d^4*e*g*i^2 - B*a^5*d^5*e*g*i^2)*log(-b*e + (b*e*x + a*e)*d/(d*x + c))/(
b^3*d^2) - 2*(B*b^5*c^5*e*g*i^2 - 5*B*a*b^4*c^4*d*e*g*i^2 + 10*B*a^2*b^3*c^
3*d^2*e*g*i^2 - 10*B*a^3*b^2*c^2*d^3*e*g*i^2 + 5*B*a^4*b*c*d^4*e*g*i^2 - B*
a^5*d^5*e*g*i^2)*log((b*e*x + a*e)/(d*x + c))/(b^3*d^2))*(b*c/((b*c*e - a*d
*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 636, normalized size of antiderivative = 2.66

$$
\begin{aligned}
&\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx \\
&= x^3\left(\frac{d\,g\,i^2\,(8\,A\,a\,d+12\,A\,b\,c+B\,a\,d-B\,b\,c)}{12}-\frac{A\,d\,g\,i^2\,(12\,a\,d+12\,b\,c)}{36}\right) \\
&\quad - x^2\left(\frac{\left(\frac{d\,g\,i^2\,(8\,A\,a\,d+12\,A\,b\,c+B\,a\,d-B\,b\,c)}{4}-\frac{A\,d\,g\,i^2\,(12\,a\,d+12\,b\,c)}{12}\right)(12\,a\,d+12\,b\,c)}{24\,b\,d}\right. \\
&\quad\left. -\frac{g\,i^2\,(3\,A\,a^2\,d^2+9\,A\,b^2\,c^2+B\,a^2\,d^2-2\,B\,b^2\,c^2+18\,A\,a\,b\,c\,d+B\,a\,b\,c\,d)}{6\,b}+\frac{A\,a\,c\,d\,g\,i^2}{2}\right) \\
&\quad + \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(B\,a\,c^2\,g\,i^2\,x+\frac{B\,c\,g\,i^2\,x^2\,(2\,a\,d+b\,c)}{2}+\frac{B\,d\,g\,i^2\,x^3\,(a\,d+2\,b\,c)}{3}\right. \\
&\quad\left. +\frac{B\,b\,d^2\,g\,i^2\,x^4}{4}\right) \\
&\quad + x\left(\frac{(12\,a\,d+12\,b\,c)\left(\frac{\left(\frac{d\,g\,i^2\,(8\,A\,a\,d+12\,A\,b\,c+B\,a\,d-B\,b\,c)}{4}-\frac{A\,d\,g\,i^2\,(12\,a\,d+12\,b\,c)}{12}\right)(12\,a\,d+12\,b\,c)}{12\,b\,d}-\frac{g\,i^2\,(3\,A\,a^2\,d^2+9\,A\,b^2\,c^2+B\,a^2\,d^2-\cdots}{3\,b\cdots}\right.}{12\,b\,d}\right. \\
&\quad -\frac{a\,c\left(\frac{d\,g\,i^2\,(8\,A\,a\,d+12\,A\,b\,c+B\,a\,d-B\,b\,c)}{4}-\frac{A\,d\,g\,i^2\,(12\,a\,d+12\,b\,c)}{12}\right)}{b\,d} \\
&\quad\left. +\frac{c\,g\,i^2\,(6\,A\,a^2\,d^2+2\,A\,b^2\,c^2+2\,B\,a^2\,d^2-B\,b^2\,c^2+12\,A\,a\,b\,c\,d-B\,a\,b\,c\,d)}{2\,b\,d}\right) \\
&\quad + \frac{\ln(a+b\,x)\,(B\,g\,a^4\,d^2\,i^2-4\,B\,g\,a^3\,b\,c\,d\,i^2+6\,B\,g\,a^2\,b^2\,c^2\,i^2)}{12\,b^3} \\
&\quad + \frac{\ln(c+d\,x)\,(B\,b\,c^4\,g\,i^2-4\,B\,a\,c^3\,d\,g\,i^2)}{12\,d^2}+\frac{A\,b\,d^2\,g\,i^2\,x^4}{4}
\end{aligned}
$$

```
[In] int((a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x^3*((d*g*i^2*(8*A*a*d + 12*A*b*c + B*a*d - B*b*c))/12 - (A*d*g*i^2*(12*a*d
 + 12*b*c))/36) - x^2*((((d*g*i^2*(8*A*a*d + 12*A*b*c + B*a*d - B*b*c))/4 -
 (A*d*g*i^2*(12*a*d + 12*b*c))/12)*(12*a*d + 12*b*c))/(24*b*d) - (g*i^2*(3*
A*a^2*d^2 + 9*A*b^2*c^2 + B*a^2*d^2 - 2*B*b^2*c^2 + 18*A*a*b*c*d + B*a*b*c*
d))/(6*b) + (A*a*c*d*g*i^2)/2) + log((e*(a + b*x))/(c + d*x))*(B*a*c^2*g*i^
2*x + (B*c*g*i^2*x^2*(2*a*d + b*c))/2 + (B*d*g*i^2*x^3*(a*d + 2*b*c))/3 + (
B*b*d^2*g*i^2*x^4)/4) + x*(((12*a*d + 12*b*c)*((((d*g*i^2*(8*A*a*d + 12*A*b
*c + B*a*d - B*b*c))/4 - (A*d*g*i^2*(12*a*d + 12*b*c))/12)*(12*a*d + 12*b*c
))/(12*b*d) - (g*i^2*(3*A*a^2*d^2 + 9*A*b^2*c^2 + B*a^2*d^2 - 2*B*b^2*c^2 +
 18*A*a*b*c*d + B*a*b*c*d))/(3*b) + A*a*c*d*g*i^2))/(12*b*d) - (a*c*((d*g*i
^2*(8*A*a*d + 12*A*b*c + B*a*d - B*b*c))/4 - (A*d*g*i^2*(12*a*d + 12*b*c))/
12))/(b*d) + (c*g*i^2*(6*A*a^2*d^2 + 2*A*b^2*c^2 + 2*B*a^2*d^2 - B*b^2*c^2
+ 12*A*a*b*c*d - B*a*b*c*d))/(2*b*d)) + (log(a + b*x)*(B*a^4*d^2*g*i^2 + 6*
B*a^2*b^2*c^2*g*i^2 - 4*B*a^3*b*c*d*g*i^2))/(12*b^3) + (log(c + d*x)*(B*b*c
^4*g*i^2 - 4*B*a*c^3*d*g*i^2))/(12*d^2) + (A*b*d^2*g*i^2*x^4)/4
```

3.13 $\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 118

$$\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=-\frac{B(bc-ad)^2i^2x}{3b^2}-\frac{B(bc-ad)i^2(c+dx)^2}{6bd}-\frac{B(bc-ad)^3i^2\log(a+bx)}{3b^3d}+\frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d}$$

```
[Out] -1/3*B*(-a*d+b*c)^2*i^2*x/b^2-1/6*B*(-a*d+b*c)*i^2*(d*x+c)^2/b/d-1/3*B*(-a*
d+b*c)^3*i^2*ln(b*x+a)/b^3/d+1/3*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/
d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2548, 21, 45}

$$\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\frac{i^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d}-\frac{Bi^2(bc-ad)^3\log(a+bx)}{3b^3d}-\frac{Bi^2x(bc-ad)^2}{3b^2}-\frac{Bi^2(c+dx)^2(bc-ad)}{6bd}$$

```
[In] Int[(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -1/3*(B*(b*c - a*d)^2*i^2*x)/b^2 - (B*(b*c - a*d)*i^2*(c + d*x)^2)/(6*b*d)
- (B*(b*c - a*d)^3*i^2*Log[a + b*x])/(3*b^3*d) + (i^2*(c + d*x)^3*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]))/(3*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d}-\frac{(B(bc-ad))\int\frac{(ci+dix)^3}{(a+bx)(c+dx)}\,dx}{3di}\\
&= \frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d}-\frac{(B(bc-ad)i^2)\int\frac{(c+dx)^2}{a+bx}\,dx}{3d}\\
&= \frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d}-\frac{(B(bc-ad)i^2)\int\left(\frac{d(bc-ad)}{b^2}+\frac{(bc-ad)^2}{b^2(a+bx)}+\frac{d(c+dx)}{b}\right)dx}{3d}\\
&= -\frac{B(bc-ad)^2i^2x}{3b^2}-\frac{B(bc-ad)i^2(c+dx)^2}{6bd}\\
&\quad-\frac{B(bc-ad)^3i^2\log(a+bx)}{3b^3d}+\frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82

$$\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{i^2\left(-\frac{B(bc-ad)\left(2bd(bc-ad)x+b^2(c+dx)^2+2(bc-ad)^2\log(a+bx)\right)}{2b^3}+(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\right)}{3d}$$

```
[In] Integrate[(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (i^2*(-1/2*(B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c -
 a*d)^2*Log[a + b*x]))/b^3 + (c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)
])))/(3*d)
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.75

method	result
risch	$\frac{i^2(dx+c)^3B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{3d}+\frac{i^2d^2Ax^3}{3}+i^2dAcx^2+\frac{i^2d^2Bax^2}{6b}-\frac{i^2dBcx^2}{6}+i^2Ac^2x+\frac{i^2d^2B\ln(bx+a)a^3}{3b^3}$ …
parts	$\frac{Ai^2(dx+c)^3}{3d}-Bi^2(ad-cb)^3e^3\left(-\frac{1}{6bed\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)^2}+\frac{1}{3b^2e^2d\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}+\frac{\ln\left(\left(\right.\right.}{}\right.$ …
parallelrisch	$\frac{-5Ba^2bcd^2i^2+2Ba^3d^3i^2+4Bb^3c^3i^2+6Axb^3c^2di^2-2Bxa^2bd^3i^2-4Bxb^3c^2di^2+2Bx^3\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3d^3i^2+6Ax^2b^3c\,d}{}$ …
derivativedivides	$-\frac{e(ad-cb)\left(\frac{Ade^2i^2\left(a^2d^2-2abcd+b^2c^2\right)}{3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^3}+Bd^2e^2i^2\left(a^2d^2-2abcd+b^2c^2\right)\left(\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{3b^3e^3d}-\frac{1}{3b^2e^2d\left(be-\left(\frac{be}{d}+\frac{(a}{d(}\right.\right.}\right.\right.}{}$ …
default	$-\frac{e(ad-cb)\left(\frac{Ade^2i^2\left(a^2d^2-2abcd+b^2c^2\right)}{3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^3}+Bd^2e^2i^2\left(a^2d^2-2abcd+b^2c^2\right)\left(\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{3b^3e^3d}-\frac{1}{3b^2e^2d\left(be-\left(\frac{be}{d}+\frac{(a}{d(}\right.\right.}\right.\right.}{}$ …

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*i^2*(d*x+c)^3*B/d*ln(e*(b*x+a)/(d*x+c))+1/3*i^2*d^2*A*x^3+i^2*d*A*c*x^2
+1/6*i^2/b*d^2*B*a*x^2-1/6*i^2*d*B*c*x^2+i^2*A*c^2*x+1/3*i^2/b^3*d^2*B*ln(b
*x+a)*a^3-i^2/b^2*d*B*ln(b*x+a)*a^2*c+i^2/b*B*ln(b*x+a)*a*c^2-1/3*i^2/d*B*l
n(b*x+a)*c^3-1/3*i^2/b^2*d^2*B*a^2*x+i^2/b*d*B*a*c*x-2/3*i^2*B*c^2*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(110) = 220$.

Time = 0.34 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.89

$$\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{2\,Ab^3d^3i^2x^3 - 2\,Bb^3c^3i^2\log(dx+c) + ((6\,A-B)b^3cd^2 + Bab^2d^3)i^2x^2 + 2\,((3\,A-2\,B)b^3c^2d + 3\,Bab^2cd^2}{}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/6*(2*A*b^3*d^3*i^2*x^3 - 2*B*b^3*c^3*i^2*log(d*x + c) + ((6*A - B)*b^3*c*
d^2 + B*a*b^2*d^3)*i^2*x^2 + 2*((3*A - 2*B)*b^3*c^2*d + 3*B*a*b^2*c*d^2 - B
*a^2*b*d^3)*i^2*x + 2*(3*B*a*b^2*c^2*d - 3*B*a^2*b*c*d^2 + B*a^3*d^3)*i^2*l
og(b*x + a) + 2*(B*b^3*d^3*i^2*x^3 + 3*B*b^3*c*d^2*i^2*x^2 + 3*B*b^3*c^2*d*
i^2*x)*log((b*e*x + a*e)/(d*x + c)))/(b^3*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(100) = 200$.

Time = 1.43 (sec) , antiderivative size = 491, normalized size of antiderivative = 4.16

$$\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{Ad^2i^2x^3}{3}$$
$$+ \frac{Bai^2(a^2d^2-3abcd+3b^2c^2)\log\left(x + \frac{Ba^3cd^2i^2 - 3Ba^2bc^2di^2 + \frac{Ba^2di^2(a^2d^2-3abcd+3b^2c^2)}{b} + 4Bab^2c^3i^2 - Baci^2(a^2d^2-3abcd+3}{Ba^3d^3i^2 - 3Ba^2bcd^2i^2 + 3Bab^2c^2di^2 + Bb^3c^3i^2}\right.}{3b^3}$$
$$- \frac{Bc^3i^2\log\left(x + \frac{Ba^3cd^2i^2 - 3Ba^2bc^2di^2 + 3Bab^2c^3i^2 + \frac{Bb^3c^4i^2}{d}}{Ba^3d^3i^2 - 3Ba^2bcd^2i^2 + 3Bab^2c^2di^2 + Bb^3c^3i^2}\right)}{3d}$$
$$+ x^2\left(Acdi^2 + \frac{Bad^2i^2}{6b} - \frac{Bcdi^2}{6}\right) + x\left(Ac^2i^2 - \frac{Ba^2d^2i^2}{3b^2} + \frac{Bacdi^2}{b} - \frac{2Bc^2i^2}{3}\right)$$
$$+ \left(Bc^2i^2x + Bcdi^2x^2 + \frac{Bd^2i^2x^3}{3}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*d**2*i**2*x**3/3 + B*a*i**2*(a**2*d**2 - 3*a*b*c*d + 3*b**2*c**2)*log(x +
 (B*a**3*c*d**2*i**2 - 3*B*a**2*b*c**2*d*i**2 + B*a**2*d*i**2*(a**2*d**2 -
3*a*b*c*d + 3*b**2*c**2)/b + 4*B*a*b**2*c**3*i**2 - B*a*c*i**2*(a**2*d**2 -
 3*a*b*c*d + 3*b**2*c**2))/(B*a**3*d**3*i**2 - 3*B*a**2*b*c*d**2*i**2 + 3*B
*a*b**2*c**2*d*i**2 + B*b**3*c**3*i**2))/(3*b**3) - B*c**3*i**2*log(x + (B*
```

```
a**3*c*d**2*i**2 - 3*B*a**2*b*c**2*d*i**2 + 3*B*a*b**2*c**3*i**2 + B*b**3*c
**4*i**2/d)/(B*a**3*d**3*i**2 - 3*B*a**2*b*c*d**2*i**2 + 3*B*a*b**2*c**2*d*
i**2 + B*b**3*c**3*i**2))/(3*d) + x**2*(A*c*d*i**2 + B*a*d**2*i**2/(6*b) -
B*c*d*i**2/6) + x*(A*c**2*i**2 - B*a**2*d**2*i**2/(3*b**2) + B*a*c*d*i**2/b
 - 2*B*c**2*i**2/3) + (B*c**2*i**2*x + B*c*d*i**2*x**2 + B*d**2*i**2*x**3/3
)*log(e*(a + b*x)/(c + d*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(110) = 220$.

Time = 0.20 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.37

$$\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{1}{3}Ad^2i^2x^3+Acdi^2x^2+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bc^2i^2$$
$$+\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bcdi^2$$
$$+\frac{1}{6}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2d^2)x}{b^2d^2}\right)Bd^2i^2$$
$$+Ac^2i^2x$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/3*A*d^2*i^2*x^3 + A*c*d*i^2*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c))
 + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*c^2*i^2 + (x^2*log(b*e*x/(d*x + c
) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a
*d)*x/(b*d))*B*c*d*i^2 + 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) +
2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2
- 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*d^2*i^2 + A*c^2*i^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1056 vs. $2(110) = 220$.

Time = 0.44 (sec) , antiderivative size = 1056, normalized size of antiderivative = 8.95

$$\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{1}{6}\left(\frac{2\left(Bb^4c^4e^4i^2-4Bab^3c^3de^4i^2+6Ba^2b^2c^2d^2e^4i^2-4Ba^3bcd^3e^4i^2+Ba^4d^4e^4i^2\right)\log\left(\frac{bex+ae}{dx+c}\right)}{b^3de^3-\frac{3(bex+ae)b^2d^2e^2}{dx+c}+\frac{3(bex+ae)^2bd^3e}{(dx+c)^2}-\frac{(bex+ae)^3d^4}{(dx+c)^3}}+\frac{2Ab^6c^4e^4\cdots}{\cdots}\right.$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] 1/6*(2*(B*b^4*c^4*e^4*i^2 - 4*B*a*b^3*c^3*d*e^4*i^2 + 6*B*a^2*b^2*c^2*d^2*e
^4*i^2 - 4*B*a^3*b*c*d^3*e^4*i^2 + B*a^4*d^4*e^4*i^2)*log((b*e*x + a*e)/(d*
x + c))/(b^3*d*e^3 - 3*(b*e*x + a*e)*b^2*d^2*e^2/(d*x + c) + 3*(b*e*x + a*e
)^2*b*d^3*e/(d*x + c)^2 - (b*e*x + a*e)^3*d^4/(d*x + c)^3) + (2*A*b^6*c^4*e
^4*i^2 - 3*B*b^6*c^4*e^4*i^2 - 8*A*a*b^5*c^3*d*e^4*i^2 + 12*B*a*b^5*c^3*d*e
^4*i^2 + 12*A*a^2*b^4*c^2*d^2*e^4*i^2 - 18*B*a^2*b^4*c^2*d^2*e^4*i^2 - 8*A*
a^3*b^3*c*d^3*e^4*i^2 + 12*B*a^3*b^3*c*d^3*e^4*i^2 + 2*A*a^4*b^2*d^4*e^4*i^
2 - 3*B*a^4*b^2*d^4*e^4*i^2 + 5*(b*e*x + a*e)*B*b^5*c^4*d*e^3*i^2/(d*x + c)
 - 20*(b*e*x + a*e)*B*a*b^4*c^3*d^2*e^3*i^2/(d*x + c) + 30*(b*e*x + a*e)*B*
a^2*b^3*c^2*d^3*e^3*i^2/(d*x + c) - 20*(b*e*x + a*e)*B*a^3*b^2*c*d^4*e^3*i^
2/(d*x + c) + 5*(b*e*x + a*e)*B*a^4*b*d^5*e^3*i^2/(d*x + c) - 2*(b*e*x + a*
e)^2*B*b^4*c^4*d^2*e^2*i^2/(d*x + c)^2 + 8*(b*e*x + a*e)^2*B*a*b^3*c^3*d^3*
e^2*i^2/(d*x + c)^2 - 12*(b*e*x + a*e)^2*B*a^2*b^2*c^2*d^4*e^2*i^2/(d*x + c
)^2 + 8*(b*e*x + a*e)^2*B*a^3*b*c*d^5*e^2*i^2/(d*x + c)^2 - 2*(b*e*x + a*e)
^2*B*a^4*d^6*e^2*i^2/(d*x + c)^2)/(b^5*d*e^3 - 3*(b*e*x + a*e)*b^4*d^2*e^2/
(d*x + c) + 3*(b*e*x + a*e)^2*b^3*d^3*e/(d*x + c)^2 - (b*e*x + a*e)^3*b^2*d
^4/(d*x + c)^3) + 2*(B*b^4*c^4*e*i^2 - 4*B*a*b^3*c^3*d*e*i^2 + 6*B*a^2*b^2*
c^2*d^2*e*i^2 - 4*B*a^3*b*c*d^3*e*i^2 + B*a^4*d^4*e*i^2)*log(-b*e + (b*e*x
+ a*e)*d/(d*x + c))/(b^3*d) - 2*(B*b^4*c^4*e*i^2 - 4*B*a*b^3*c^3*d*e*i^2 +
6*B*a^2*b^2*c^2*d^2*e*i^2 - 4*B*a^3*b*c*d^3*e*i^2 + B*a^4*d^4*e*i^2)*log((b
*e*x + a*e)/(d*x + c))/(b^3*d))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((
b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.46

$$\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=x^2\left(\frac{d\,i^2\,(3\,A\,a\,d+9\,A\,b\,c+B\,a\,d-B\,b\,c)}{6\,b}-\frac{A\,d\,i^2\,(3\,a\,d+3\,b\,c)}{6\,b}\right)-x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{d\,i^2\,(3\,A\,a\,d+9\,A\,b\,c+B\,a\,d-B\,b\,c)}{3\,b}-\frac{A\,d\,i^2\,(3\,a\,d+3\,b\,c)}{3\,b}\right)}{3\,b\,d}-\frac{c\,i^2\,(3\,A\,a\,d+3\,A\,b\,c+B\,a\,d-B\,b\,c)}{b}+\frac{A\,a\,c\,d\,i^2}{b}\right)+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(B\,c^2\,i^2\,x+B\,c\,d\,i^2\,x^2+\frac{B\,d^2\,i^2\,x^3}{3}\right)+\frac{\ln(a+b\,x)\,(B\,a^3\,d^2\,i^2-3\,B\,a^2\,b\,c\,d\,i^2+3\,B\,a\,b^2\,c^2\,i^2)}{3\,b^3}+\frac{A\,d^2\,i^2\,x^3}{3}-\frac{B\,c^3\,i^2\,\ln(c+d\,x)}{3\,d}$$

```
[In] int((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x^2*((d*i^2*(3*A*a*d + 9*A*b*c + B*a*d - B*b*c))/(6*b) - (A*d*i^2*(3*a*d +
3*b*c))/(6*b)) - x*(((3*a*d + 3*b*c)*((d*i^2*(3*A*a*d + 9*A*b*c + B*a*d - B
*b*c))/(3*b) - (A*d*i^2*(3*a*d + 3*b*c))/(3*b)))/(3*b*d) - (c*i^2*(3*A*a*d
+ 3*A*b*c + B*a*d - B*b*c))/b + (A*a*c*d*i^2)/b) + log((e*(a + b*x))/(c + d
*x))*((B*d^2*i^2*x^3)/3 + B*c^2*i^2*x + B*c*d*i^2*x^2) + (log(a + b*x)*(B*a
^3*d^2*i^2 + 3*B*a*b^2*c^2*i^2 - 3*B*a^2*b*c*d*i^2))/(3*b^3) + (A*d^2*i^2*x
^3)/3 - (B*c^3*i^2*log(c + d*x))/(3*d)
```

3.14 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 276

$$\begin{aligned}\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx &= -\frac{Bd(bc-ad)i^2x}{2b^2g}-\frac{B(bc-ad)^2i^2\log\left(\frac{a+bx}{c+dx}\right)}{2b^3g}\\ &\quad+\frac{d(bc-ad)i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g}+\frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg}\\ &\quad-\frac{3B(bc-ad)^2i^2\log(c+dx)}{2b^3g}-\frac{(bc-ad)^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}\\ &\quad+\frac{B(bc-ad)^2i^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}\end{aligned}$$

```
[Out] -1/2*B*d*(-a*d+b*c)*i^2*x/b^2/g-1/2*B*(-a*d+b*c)^2*i^2*ln((b*x+a)/(d*x+c))/
b^3/g+d*(-a*d+b*c)*i^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/g+1/2*i^2*(d
*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g-3/2*B*(-a*d+b*c)^2*i^2*ln(d*x+c)/b^
3/g-(-a*d+b*c)^2*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/
b^3/g+B*(-a*d+b*c)^2*i^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2562, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$=\frac{di^2(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3g}$$
$$-\frac{i^2(bc-ad)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3g}$$
$$+\frac{i^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2bg}+\frac{Bi^2(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}$$
$$-\frac{Bi^2(bc-ad)^2\log\left(\frac{a+bx}{c+dx}\right)}{2b^3g}-\frac{3Bi^2(bc-ad)^2\log(c+dx)}{2b^3g}-\frac{Bdi^2x(bc-ad)}{2b^2g}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x),x]
```

```
[Out] -1/2*(B*d*(b*c - a*d)*i^2*x)/(b^2*g) - (B*(b*c - a*d)^2*i^2*Log[(a + b*x)/(
c + d*x)])/(2*b^3*g) + (d*(b*c - a*d)*i^2*(a + b*x)*(A + B*Log[(e*(a + b*x)
)/(c + d*x)]))/(b^3*g) + (i^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(2*b*g) - (3*B*(b*c - a*d)^2*i^2*Log[c + d*x])/(2*b^3*g) - ((b*c - a*d
)^2*i^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a +
b*x))])/(b^3*g) + (B*(b*c - a*d)^2*i^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x
))])/(b^3*g)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
```

```
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)^2 i^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{g}$$

$$= \frac{((bc-ad)^2 i^2) \operatorname{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg} + \frac{(d(bc-ad)^2 i^2) \operatorname{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg}$$

$$= \frac{i^2(c+dx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg} + \frac{((bc-ad)^2 i^2) \operatorname{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2 g} - \frac{(B(bc-ad)^2 i^2) \operatorname{Subst}\left(\int \frac{1}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{2bg} + \frac{(d(bc-ad)^2 i^2) \operatorname{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2 g}$$

$$= \frac{d(bc-ad)i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3 g} + \frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg} - \frac{(bc-ad)^2 i^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3 g} + \frac{(B(bc-ad)^2 i^2)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3 g} - \frac{(B(bc-ad)^2 i^2)\operatorname{Subst}\left(\int \left(\frac{1}{b^2 x}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{2bg} - \frac{(Bd(bc-ad)^2 i^2)\operatorname{Subst}\left(\int \frac{1}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3 g}$$

$$= -\frac{Bd(bc-ad)i^2 x}{2b^2 g} - \frac{B(bc-ad)^2 i^2 \log\left(\frac{a+bx}{c+dx}\right)}{2b^3 g} + \frac{d(bc-ad)i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3 g} + \frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg} - \frac{3B(bc-ad)^2 i^2 \log(c+dx)}{2b^3 g} - \frac{(bc-ad)^2 i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3 g} + \frac{B(bc-ad)^2 i^2 \operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3 g}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.91

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$= \frac{i^2\Big(2Abd(bc-ad)x - B(bc-ad)(bdx+(bc-ad)\log(a+bx)) + 2Bd(bc-ad)(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right) + b^2}{}$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g
*x),x]
```

```
[Out] (i^2*(2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*(b*d*x + (b*c - a*d)*Log[a + b*
x]) + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + b^2*(c + d
*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*(b*c - a*d)^2*Log[g*(a + b*x
)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*(b*c - a*d)^2*Log[c + d*x] +
B*(b*c - a*d)^2*(-(Log[g*(a + b*x)]*(Log[g*(a + b*x)] - 2*Log[(b*(c + d*x))
/(b*c - a*d)])) + 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(2*b^3*g)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. 2(268) = 536.

Time = 1.57 (sec) , antiderivative size = 698, normalized size of antiderivative = 2.53

method	result
parts	$\frac{i^2 A\left(\frac{d\left(\frac{1}{2} b d x^2-x a d+2 b c x\right)}{b^2}+\frac{\left(a^2 d^2-2 a b c d+b^2 c^2\right)\ln(b x+a)}{b^3}\right)}{g}-\frac{i^2 B(ad-cb)^3 e^3\left(-\frac{d^4\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)b^3e^3}+\frac{d^5\left(\frac{\mathrm{dilog}\left(-\frac{\left(\frac{b}{}\right.}{}\right.}{}\right.}{}\right.}{}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{A d^2 e^2 i^2 (ad-cb)\left(\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^3e^3}+\frac{1}{b^2e^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{b^3e^3}+\frac{}{2be\left(be-\left(\frac{be}{d}\right.\right.}\right)}{g}\right.}{}$
default	$-\frac{e(ad-cb)\left(-\frac{A d^2 e^2 i^2 (ad-cb)\left(\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^3e^3}+\frac{1}{b^2e^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{b^3e^3}+\frac{}{2be\left(be-\left(\frac{be}{d}\right.\right.}\right)}{g}\right.}{}$
risch	Expression too large to display

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x,method=_RETURNV
ERBOSE)
```

```
[Out] i^2*A/g*(d/b^2*(1/2*b*d*x^2-x*a*d+2*b*c*x)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3*
ln(b*x+a))-i^2*B/g/d^4*(a*d-b*c)^3*e^3*(-1/2/(a*d-b*c)*d^4/b^3/e^3*ln(b*e/d
+(a*d-b*c)*e/d/(d*x+c))^2+1/(a*d-b*c)*d^5/b^3/e^3*(dilog(-((b*e/d+(a*d-b*c)
*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*
d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)-1/(a*d-b*c)*d^5/b^2/e^2*(1/b/e/d*ln((b*e
/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*
d-b*c)*e/d/(d*x+c))/b/e/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e))+1/(a*d-b*c)*
d^5/b/e*(-1/2/e^2/b^2/d*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)-1/2/e/b/d/(
(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)+1/2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b
*e/d+(a*d-b*c)*e/d/(d*x+c))*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-2*b*e)/e^2/b^2
/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)^2))
```

Fricas [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorit
hm="fricas")
```

```
[Out] integral((A*d^2*i^2*x^2 + 2*A*c*d*i^2*x + A*c^2*i^2 + (B*d^2*i^2*x^2 + 2*B*
c*d*i^2*x + B*c^2*i^2)*log((b*e*x + a*e)/(d*x + c)))/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$=\frac{i^2\left(\int \frac{Ac^2}{a+bx}\,dx+\int \frac{Ad^2x^2}{a+bx}\,dx+\int \frac{Bc^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx+\int \frac{2Acdx}{a+bx}\,dx+\int \frac{Bd^2x^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx+\int \frac{2Bcdx\log\left(\right.}{a}\right.}{g}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x)
```

```
[Out] i**2*(Integral(A*c**2/(a + b*x), x) + Integral(A*d**2*x**2/(a + b*x), x) +
Integral(B*c**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) + Integr
al(2*A*c*d*x/(a + b*x), x) + Integral(B*d**2*x**2*log(a*e/(c + d*x) + b*e*x
/(c + d*x))/(a + b*x), x) + Integral(2*B*c*d*x*log(a*e/(c + d*x) + b*e*x/(c
 + d*x))/(a + b*x), x))/g
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.88

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$= 2\,Acdi^2\left(\frac{x}{bg}-\frac{a\log(bx+a)}{b^2g}\right)+\frac{1}{2}\,Ad^2i^2\left(\frac{2\,a^2\log(bx+a)}{b^3g}+\frac{bx^2-2\,ax}{b^2g}\right)$$
$$+\frac{Ac^2i^2\log(bgx+ag)}{bg}-\frac{(3\,bc^2i^2-2\,acdi^2)B\log(dx+c)}{2\,b^2g}$$
$$+\frac{(b^2c^2i^2-2\,abcdi^2+a^2d^2i^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{b^3g}$$
$$+\frac{Bb^2d^2i^2x^2\log(e)+(b^2c^2i^2-2\,abcdi^2+a^2d^2i^2)B\log(bx+a)^2+((4\,i^2\log(e)-i^2)b^2cd-(2\,i^2\log(e)-}{}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorit
hm="maxima")
```

```
[Out] 2*A*c*d*i^2*(x/(b*g) - a*log(b*x + a)/(b^2*g)) + 1/2*A*d^2*i^2*(2*a^2*log(b
*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A*c^2*i^2*log(b*g*x + a*g)/(b*
g) - 1/2*(3*b*c^2*i^2 - 2*a*c*d*i^2)*B*log(d*x + c)/(b^2*g) + (b^2*c^2*i^2
- 2*a*b*c*d*i^2 + a^2*d^2*i^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d)
+ 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(b^3*g) + 1/2*(B*b^2*d^2*i^2*x^
2*log(e) + (b^2*c^2*i^2 - 2*a*b*c*d*i^2 + a^2*d^2*i^2)*B*log(b*x + a)^2 + (
(4*i^2*log(e) - i^2)*b^2*c*d - (2*i^2*log(e) - i^2)*a*b*d^2)*B*x + (B*b^2*d
^2*i^2*x^2 + 2*(2*b^2*c*d*i^2 - a*b*d^2*i^2)*B*x + (2*b^2*c^2*i^2*log(e) -
4*(i^2*log(e) - i^2)*a*b*c*d + (2*i^2*log(e) - 3*i^2)*a^2*d^2)*B)*log(b*x +
 a) - (B*b^2*d^2*i^2*x^2 + 2*(2*b^2*c*d*i^2 - a*b*d^2*i^2)*B*x + 2*(b^2*c^2
*i^2 - 2*a*b*c*d*i^2 + a^2*d^2*i^2)*B*log(b*x + a))*log(d*x + c))/(b^3*g)
```

Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorit
hm="giac")
```

```
[Out] integrate((d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = \int \frac{(c\,i+d\,i\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{a\,g+b\,g\,x}\,dx$$

```
[In] int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x),x)
[Out] int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x), x
)
```

3.15 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 247

$$
\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx \\
&= -\frac{B(bc-ad)i^2(c+dx)}{b^2g^2(a+bx)}+\frac{d^2i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^2} \\
&\quad -\frac{(bc-ad)i^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g^2(a+bx)}-\frac{Bd(bc-ad)i^2\log(c+dx)}{b^3g^2} \\
&\quad -\frac{2d(bc-ad)i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} \\
&\quad +\frac{2Bd(bc-ad)i^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}
\end{aligned}
$$

```
[Out] -B*(-a*d+b*c)*i^2*(d*x+c)/b^2/g^2/(b*x+a)+d^2*i^2*(b*x+a)*(A+B*ln(e*(b*x+a)
/(d*x+c)))/b^3/g^2-(-a*d+b*c)*i^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/g
^2/(b*x+a)-B*d*(-a*d+b*c)*i^2*ln(d*x+c)/b^3/g^2-2*d*(-a*d+b*c)*i^2*(A+B*ln(
e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g^2+2*B*d*(-a*d+b*c)*i^2*
polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2562, 46, 2393, 2341, 2351, 31, 2379, 2438}

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$$
$$=\frac{d^2i^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3g^2}$$
$$-\frac{2di^2(bc-ad)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3g^2}$$
$$-\frac{i^2(c+dx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^2g^2(a+bx)}+\frac{2Bdi^2(bc-ad)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}$$
$$-\frac{Bdi^2(bc-ad)\log(c+dx)}{b^3g^2}-\frac{Bi^2(c+dx)(bc-ad)}{b^2g^2(a+bx)}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^2,
x]
```

```
[Out] -((B*(b*c - a*d)*i^2*(c + d*x))/(b^2*g^2*(a + b*x))) + (d^2*i^2*(a + b*x)*(
A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^3*g^2) - ((b*c - a*d)*i^2*(c + d*x)
*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^2*g^2*(a + b*x)) - (B*d*(b*c - a*
d)*i^2*Log[c + d*x])/(b^3*g^2) - (2*d*(b*c - a*d)*i^2*(A + B*Log[(e*(a + b*
x))/(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^3*g^2) + (2*B*d*(b
*c - a*d)*i^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b^3*g^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)i^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^2(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^2}$$

$$= \frac{((bc-ad)i^2)\,\text{Subst}\left(\int \left(\frac{A+B\log(ex)}{b^2x^2}+\frac{d^2(A+B\log(ex))}{b^2(b-dx)^2}+\frac{2d(A+B\log(ex))}{b^2x(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{g^2}$$

$$= \frac{((bc-ad)i^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2} + \frac{(2d(bc-ad)i^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2} + \frac{(d^2(bc-ad)i^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2}$$

$$= -\frac{B(bc-ad)i^2(c+dx)}{b^2g^2(a+bx)} + \frac{d^2i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^2} - \frac{(bc-ad)i^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g^2(a+bx)} - \frac{2d(bc-ad)i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} + \frac{(2Bd(bc-ad)i^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^2} - \frac{(Bd^2(bc-ad)i^2)\,\text{Subst}\left(\int \frac{1}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^2}$$

$$= -\frac{B(bc-ad)i^2(c+dx)}{b^2g^2(a+bx)} + \frac{d^2i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^2} - \frac{(bc-ad)i^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g^2(a+bx)} - \frac{Bd(bc-ad)i^2\log(c+dx)}{b^3g^2} - \frac{2d(bc-ad)i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} + \frac{2Bd(bc-ad)i^2\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.89

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$$

$$= \frac{i^2\left(Abd^2x - \frac{B(bc-ad)^2}{a+bx} + Bd(-bc+ad)\log(a+bx) + Bd^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right) - \frac{(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{a+bx}\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g
*x)^2,x]
```

```
[Out] (i^2*(A*b*d^2*x - (B*(b*c - a*d)^2)/(a + b*x) + B*d*(-(b*c) + a*d)*Log[a +
b*x] + B*d^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - ((b*c - a*d)^2*(A + B
*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) + 2*d*(b*c - a*d)*Log[a + b*x]*(A
 + B*Log[(e*(a + b*x))/(c + d*x)]) + B*d*(-(b*c) + a*d)*(Log[a + b*x]*(Log[
a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(
-(b*c) + a*d)])))/(b^3*g^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(247) = 494$.

Time = 1.56 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.30

method	result
parts	$\frac{i^2 A\left(\frac{x d^2}{b^2}-\frac{2d(ad-cb)\ln(bx+a)}{b^3}-\frac{a^2d^2-2abcd+b^2c^2}{b^3(bx+a)}\right)}{g^2} - \frac{i^2 B(ad-cb)^3 e^3\left(\frac{d^4\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^2b^2e^2}+\frac{d^5 \cdots}{\cdots}\cdots\right)}{\cdots}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{i^2d^2e^2A\left(\frac{d}{b^2e^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}-\frac{2d\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{b^3e^3}-\frac{1}{b^2e^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{2d\ln\left(\frac{be}{d}+\frac{(ad-\cdots}{d(d\cdots}\right)}{b^3e^3}\cdots\right)}{g^2}\cdots\right)}{\cdots}$
default	$-\frac{e(ad-cb)\left(\frac{i^2d^2e^2A\left(\frac{d}{b^2e^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}-\frac{2d\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{b^3e^3}-\frac{1}{b^2e^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{2d\ln\left(\frac{be}{d}+\frac{(ad-\cdots}{d(d\cdots}\right)}{b^3e^3}\cdots\right)}{g^2}\cdots\right)}{\cdots}$
risch	Expression too large to display

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x,method=_RETUR
NVERBOSE)
```

```
[Out] i^2*A/g^2*(x*d^2/b^2-2/b^3*d*(a*d-b*c)*ln(b*x+a)-1/b^3*(a^2*d^2-2*a*b*c*d+b
^2*c^2)/(b*x+a))-i^2*B/g^2/d^4*(a*d-b*c)^3*e^3*(1/(a*d-b*c)^2*d^4/b^2/e^2*(
-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(
a*d-b*c)*e/d/(d*x+c)))+1/(a*d-b*c)^2*d^5/b^3/e^3*ln(b*e/d+(a*d-b*c)*e/d/(d*
x+c))^2-2/(a*d-b*c)^2*d^6/b^3/e^3*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-
b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+
c))*d-b*e)/b/e)/d)+1/(a*d-b*c)^2*d^6/b^2/e^2*(1/b/e/d*ln((b*e/d+(a*d-b*c)*e
/d/(d*x+c))*d-b*e)-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*
x+c))/b/e/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)))
```

Fricas [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algor
ithm="fricas")
```

```
[Out] integral((A*d^2*i^2*x^2 + 2*A*c*d*i^2*x + A*c^2*i^2 + (B*d^2*i^2*x^2 + 2*B*
c*d*i^2*x + B*c^2*i^2)*log((b*e*x + a*e)/(d*x + c)))/(b^2*g^2*x^2 + 2*a*b*g
^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx=\text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. 2(246) = 492.

Time = 0.27 (sec) , antiderivative size = 992, normalized size of antiderivative = 4.02

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$$
$$= -A\left(\frac{a^2}{b^4g^2x+ab^3g^2}-\frac{x}{b^2g^2}+\frac{2\,a\log(bx+a)}{b^3g^2}\right)d^2i^2$$
$$+2\,Acdi^2\left(\frac{a}{b^3g^2x+ab^2g^2}+\frac{\log(bx+a)}{b^2g^2}\right)$$
$$-Bc^2i^2\left(\frac{\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{b^2g^2x+abg^2}+\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{(b^2c-abd)g^2}-\frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)$$
$$-\frac{Ac^2i^2}{b^2g^2x+abg^2}-\frac{(b^2c^2di^2+abcd^2i^2-a^2d^3i^2)B\log(dx+c)}{b^4cg^2-ab^3dg^2}$$
$$+\frac{(b^3cd^2i^2\log(e)-ab^2d^3i^2\log(e))Bx^2+(ab^2cd^2i^2\log(e)-a^2bd^3i^2\log(e))Bx+((b^3c^2di^2-2\,ab^2cd^2i^2+\ldots}{}$$
$$+\frac{2\,(bcdi^2-ad^2i^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{b^3g^2}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algor
ithm="maxima")
```

```
[Out] -A*(a^2/(b^4*g^2*x + a*b^3*g^2) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*g^2))
*d^2*i^2 + 2*A*c*d*i^2*(a/(b^3*g^2*x + a*b^2*g^2) + log(b*x + a)/(b^2*g^2))
 - B*c^2*i^2*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2) +
1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x
+ c)/((b^2*c - a*b*d)*g^2)) - A*c^2*i^2/(b^2*g^2*x + a*b*g^2) - (b^2*c^2*d*
i^2 + a*b*c*d^2*i^2 - a^2*d^3*i^2)*B*log(d*x + c)/(b^4*c*g^2 - a*b^3*d*g^2)
 + ((b^3*c*d^2*i^2*log(e) - a*b^2*d^3*i^2*log(e))*B*x^2 + (a*b^2*c*d^2*i^2*
log(e) - a^2*b*d^3*i^2*log(e))*B*x + ((b^3*c^2*d*i^2 - 2*a*b^2*c*d^2*i^2 +
a^2*b*d^3*i^2)*B*x + (a*b^2*c^2*d*i^2 - 2*a^2*b*c*d^2*i^2 + a^3*d^3*i^2)*B)
*log(b*x + a)^2 + (2*(i^2*log(e) + i^2)*a*b^2*c^2*d - 3*(i^2*log(e) + i^2)*
a^2*b*c*d^2 + (i^2*log(e) + i^2)*a^3*d^3)*B + ((b^3*c*d^2*i^2 - a*b^2*d^3*i
^2)*B*x^2 + (2*b^3*c^2*d*i^2*log(e) - 4*(i^2*log(e) - i^2)*a*b^2*c*d^2 + (2
*i^2*log(e) - 3*i^2)*a^2*b*d^3)*B*x - (4*a^2*b*c*d^2*i^2*log(e) - 2*(i^2*lo
g(e) + i^2)*a*b^2*c^2*d - (2*i^2*log(e) - i^2)*a^3*d^3)*B)*log(b*x + a) - (
(b^3*c*d^2*i^2 - a*b^2*d^3*i^2)*B*x^2 + (a*b^2*c*d^2*i^2 - a^2*b*d^3*i^2)*B
*x + (2*a*b^2*c^2*d*i^2 - 3*a^2*b*c*d^2*i^2 + a^3*d^3*i^2)*B + 2*((b^3*c^2*
d*i^2 - 2*a*b^2*c*d^2*i^2 + a^2*b*d^3*i^2)*B*x + (a*b^2*c^2*d*i^2 - 2*a^2*b
*c*d^2*i^2 + a^3*d^3*i^2)*B)*log(b*x + a))*log(d*x + c))/(a*b^4*c*g^2 - a^2
*b^3*d*g^2 + (b^5*c*g^2 - a*b^4*d*g^2)*x) + 2*(b*c*d*i^2 - a*d^2*i^2)*(log(
```

```
b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a
*d)))*B/(b^3*g^2)
```

Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algor
ithm="giac")
```

```
[Out] integrate((d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g)^
2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(c\,i+d\,i\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{(a\,g+b\,g\,x)^2}\,dx$$

```
[In] int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^2,
x)
```

```
[Out] int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^2,
 x)
```

3.16 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 230

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = -\frac{Bdi^2(c+dx)}{b^2g^3(a+bx)} - \frac{Bi^2(c+dx)^2}{4bg^3(a+bx)^2} - \frac{di^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g^3(a+bx)} - \frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a+bx)^2} - \frac{d^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} + \frac{Bd^2i^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3}$$

```
[Out] -B*d*i^2*(d*x+c)/b^2/g^3/(b*x+a)-1/4*B*i^2*(d*x+c)^2/b/g^3/(b*x+a)^2-d*i^2*
(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/g^3/(b*x+a)-1/2*i^2*(d*x+c)^2*(A+B*
ln(e*(b*x+a)/(d*x+c)))/b/g^3/(b*x+a)^2-d^2*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*
ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g^3+B*d^2*i^2*polylog(2,b*(d*x+c)/d/(b*x+a))/
b^3/g^3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2562, 2380, 2341, 2379, 2438}

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = -\frac{d^2i^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3g^3} - \frac{di^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^2g^3(a+bx)} - \frac{i^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2bg^3(a+bx)^2} + \frac{Bd^2i^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} - \frac{Bdi^2(c+dx)}{b^2g^3(a+bx)} - \frac{Bi^2(c+dx)^2}{4bg^3(a+bx)^2}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^3,
x]
```

```
[Out] -((B*d*i^2*(c + d*x))/(b^2*g^3*(a + b*x))) - (B*i^2*(c + d*x)^2)/(4*b*g^3*(
a + b*x)^2) - (d*i^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^2*g
^3*(a + b*x)) - (i^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*b
*g^3*(a + b*x)^2) - (d^2*i^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (
b*(c + d*x))/(d*(a + b*x))])/(b^3*g^3) + (B*d^2*i^2*PolyLog[2, (b*(c + d*x)
)/(d*(a + b*x))])/(b^3*g^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
```

```
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i^2\text{Subst}\left(\int \frac{A+B\log(ex)}{x^3(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^3} \\
&= \frac{i^2\text{Subst}\left(\int \frac{A+B\log(ex)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^3} + \frac{(di^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^2(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^3} \\
&= -\frac{Bi^2(c+dx)^2}{4bg^3(a+bx)^2} - \frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a+bx)^2} \\
&\quad + \frac{(di^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^3} + \frac{(d^2i^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^3} \\
&= -\frac{Bdi^2(c+dx)}{b^2g^3(a+bx)} - \frac{Bi^2(c+dx)^2}{4bg^3(a+bx)^2} - \frac{di^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g^3(a+bx)} \\
&\quad - \frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a+bx)^2} \\
&\quad - \frac{d^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} \\
&\quad + \frac{(Bd^2i^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^3}
\end{aligned}$$

$$= -\frac{Bdi^2(c+dx)}{b^2g^3(a+bx)} - \frac{Bi^2(c+dx)^2}{4bg^3(a+bx)^2} - \frac{di^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g^3(a+bx)} - \frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a+bx)^2} - \frac{d^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} + \frac{Bd^2i^2\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.06

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = \frac{i^2\left(-\frac{B(bc-ad)^2}{(a+bx)^2} + \frac{6Bd(-bc+ad)}{a+bx} - 6Bd^2\log(a+bx) - \frac{2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2} + \frac{8d(-bc+ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{a+bx} + \right.}{}$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g
*x)^3,x]

[Out] (i^2*(-((B*(b*c - a*d)^2)/(a + b*x)^2) + (6*B*d*(-(b*c) + a*d))/(a + b*x) -
 6*B*d^2*Log[a + b*x] - (2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)
]))/(a + b*x)^2 + (8*d*(-(b*c) + a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))
/(a + b*x) + 4*d^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*B*
d^2*Log[c + d*x] - 2*B*d^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x)
)/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(4*b^3*g^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(226) = 452$.

Time = 1.49 (sec) , antiderivative size = 522, normalized size of antiderivative = 2.27

method	result
parts	$\frac{i^2 A\left(\frac{d^2\ln(bx+a)}{b^3}-\frac{a^2d^2-2abcd+b^2c^2}{2b^3(bx+a)^2}+\frac{2d(ad-cb)}{b^3(bx+a)}\right)}{g^3}-\frac{i^2B(ad-cb)^3e^3\left(-\frac{d^5\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^3b^2e^2}-\frac{d^6\ln}{}\right.}{}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{i^2d^2e^2A\left(-\frac{1}{2be\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{d^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^3e^3}-\frac{d}{b^2e^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{d^2\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{b^3e^3}\right.}{(ad-cb)g^3}\right.}{}$
default	$-\frac{e(ad-cb)\left(-\frac{i^2d^2e^2A\left(-\frac{1}{2be\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{d^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^3e^3}-\frac{d}{b^2e^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{d^2\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{b^3e^3}\right.}{(ad-cb)g^3}\right.}{}$
risch	Expression too large to display

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x,method=_RETUR
NVERBOSE)
```

```
[Out] i^2*A/g^3*(d^2/b^3*ln(b*x+a)-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^2+
2/b^3*d*(a*d-b*c)/(b*x+a))-i^2*B/g^3/d^4*(a*d-b*c)^3*e^3*(-1/(a*d-b*c)^3*d^
5/b^2/e^2*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))
-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-1/2/(a*d-b*c)^3*d^6/b^3/e^3*ln(b*e/d+(a*d
-b*c)*e/d/(d*x+c))^2+1/(a*d-b*c)^3*d^7/b^3/e^3*(dilog(-((b*e/d+(a*d-b*c)*e/
d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b
*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)-1/(a*d-b*c)^3*d^4/b/e*(-1/2/(b*e/d+(a*d-b*c
)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^2))
```

Fricas [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^3}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algor
ithm="fricas")
```

```
[Out] integral((A*d^2*i^2*x^2 + 2*A*c*d*i^2*x + A*c^2*i^2 + (B*d^2*i^2*x^2 + 2*B*
c*d*i^2*x + B*c^2*i^2)*log((b*e*x + a*e)/(d*x + c)))/(b^3*g^3*x^3 + 3*a*b^2
*g^3*x^2 + 3*a^2*b*g^3*x + a^3*g^3), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$$
$$=\frac{i^2\left(\int \frac{Ac^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{Ad^2x^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{Bc^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{2Acdx}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\right.}{g^3}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3,x)
```

```
[Out] i**2*(Integral(A*c**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) +
 Integral(A*d**2*x**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) +
 Integral(B*c**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x +
3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*c*d*x/(a**3 + 3*a**2*b*x + 3*
a*b**2*x**2 + b**3*x**3), x) + Integral(B*d**2*x**2*log(a*e/(c + d*x) + b*e
*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integra
l(2*B*c*d*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b
**2*x**2 + b**3*x**3), x))/g**3
```

Maxima [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^3}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algor
ithm="maxima")
```

```
[Out] -1/2*B*d^2*i^2*((4*a*b*x + 3*a^2 + 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a)
)*log(d*x + c)/(b^5*g^3*x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) - 2*integrate(1/
2*(2*b^3*d*x^3*log(e) + 7*a^2*b*d*x + 3*a^3*d + 2*(b^3*c*log(e) + 2*a*b^2*d
)*x^2 + 2*(2*b^3*d*x^3 + 3*a^2*b*d*x + a^3*d + (b^3*c + 3*a*b^2*d)*x^2)*log
(b*x + a))/(b^6*d*g^3*x^4 + a^3*b^3*c*g^3 + (b^6*c*g^3 + 3*a*b^5*d*g^3)*x^3
 + 3*(a*b^5*c*g^3 + a^2*b^4*d*g^3)*x^2 + (3*a^2*b^4*c*g^3 + a^3*b^3*d*g^3)*
x), x)) - 1/2*B*c*d*i^2*(2*(2*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))
/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) + (3*a*b*c - a^2*d + 2*(2*b^2*
c - a*b*d)*x)/((b^5*c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x +
(a^2*b^3*c - a^3*b^2*d)*g^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 -
 2*a*b^3*c*d + a^2*b^2*d^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c
^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3)) + 1/2*A*d^2*i^2*((4*a*b*x + 3*a^2)/(b
^5*g^3*x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(b^3*g^3)) + 1/4
*B*c^2*i^2*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c
 - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) - 2*log(b*e*x/(d*x + c) +
a*e/(d*x + c))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x +
a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2
 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - (2*b*x + a)*A*c*d*i^2/(b^4*g^3*x^2 + 2*
a*b^3*g^3*x + a^2*b^2*g^3) - 1/2*A*c^2*i^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a
^2*b*g^3)
```

Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^3}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algor
ithm="giac")
```

```
[Out] integrate((d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g)^
3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = \int \frac{(c\,i+d\,i\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{{(a\,g+b\,g\,x)}^3}\,dx$$

```
[In] int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^3,
x)
[Out] int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^3,
 x)
```

3.17 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 89

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx=-\frac{Bi^2(c+dx)^3}{9(bc-ad)g^4(a+bx)^3}-\frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)g^4(a+bx)^3}$$

```
[Out] -1/9*B*i^2*(d*x+c)^3/(-a*d+b*c)/g^4/(b*x+a)^3-1/3*i^2*(d*x+c)^3*(A+B*ln(e*(
b*x+a)/(d*x+c)))/(-a*d+b*c)/g^4/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2562, 2341}

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx=-\frac{i^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3g^4(a+bx)^3(bc-ad)}-\frac{Bi^2(c+dx)^3}{9g^4(a+bx)^3(bc-ad)}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^4,
x]
```

```
[Out] -1/9*(B*i^2*(c + d*x)^3)/((b*c - a*d)*g^4*(a + b*x)^3) - (i^2*(c + d*x)^3*(
A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*(b*c - a*d)*g^4*(a + b*x)^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{i^2\text{Subst}\left(\int \frac{A+B\log(ex)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^4}$$

$$= -\frac{Bi^2(c+dx)^3}{9(bc-ad)g^4(a+bx)^3} - \frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)g^4(a+bx)^3}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 315 vs. 2(89) = 178.

Time = 0.20 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.54

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx =$$

$$-\frac{i^2\left(3Ab^3c^3+b^3Bc^3-3a^3Ad^3-a^3Bd^3+9Ab^3c^2dx+3b^3Bc^2dx-9a^2Abd^3x-3a^2bBd^3x+9Ab^3cd^2x^2\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g
*x)^4,x]
```

```
[Out] -1/9*(i^2*(3*A*b^3*c^3 + b^3*B*c^3 - 3*a^3*A*d^3 - a^3*B*d^3 + 9*A*b^3*c^2*
d*x + 3*b^3*B*c^2*d*x - 9*a^2*A*b*d^3*x - 3*a^2*b*B*d^3*x + 9*A*b^3*c*d^2*x
^2 + 3*b^3*B*c*d^2*x^2 - 9*a*A*b^2*d^3*x^2 - 3*a*b^2*B*d^3*x^2 + 3*B*d^3*(a
 + b*x)^3*Log[a + b*x] + 3*B*(b*c - a*d)*(a^2*d^2 + a*b*d*(c + 3*d*x) + b^2
*(c^2 + 3*c*d*x + 3*d^2*x^2))*Log[(e*(a + b*x))/(c + d*x)] - 3*a^3*B*d^3*Lo
g[c + d*x] - 9*a^2*b*B*d^3*x*Log[c + d*x] - 9*a*b^2*B*d^3*x^2*Log[c + d*x]
- 3*b^3*B*d^3*x^3*Log[c + d*x]))/(b^3*(b*c - a*d)*g^4*(a + b*x)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(85) = 170$.

Time = 1.01 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.08

method	result
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{i^2d^2e^2A}{3(ad-cb)^2g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{i^2d^2e^2B\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}\right)}{(ad-cb)^2g^4}\right)}{d^2}$
default	$-\frac{e(ad-cb)\left(-\frac{i^2d^2e^2A}{3(ad-cb)^2g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{i^2d^2e^2B\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}\right)}{(ad-cb)^2g^4}\right)}{d^2}$
parts	$\frac{i^2A\left(-\frac{a^2d^2-2abcd+b^2c^2}{3b^3(bx+a)^3}+\frac{d(ad-cb)}{b^3(bx+a)^2}-\frac{d^2}{b^3(bx+a)}\right)}{g^4}-\frac{i^2Be^3\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}\right)}{g^4(ad-cb)}$
norman	$\frac{\frac{Bcd^2i^2x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(ad-cb)}+\frac{Bc^2di^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(ad-cb)}-\frac{3Aacdi^2+3Abc^2i^2+Bacdi^2+Bbc^2i^2}{9gb^2}+\frac{\left(3i^2Ad^2+Bd^2i^2\right)x^3}{9ag}-\frac{\left(3Acdi^2+Bcd\right.}{3gb}}{(bx+a)^3g^3}$
parallelrisch	$-\frac{-9Bx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^5cd^3i^2-9Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^5c^2d^2i^2-3Bx^3\ln\left(\frac{e(bx+a)}{dx+c}\right)b^5d^4i^2+9Ax^2ab^4d^4i^2-9Ax^2b^5cd^3i^2+3}{}$
risch	$-\frac{Bi^2\left(3d^2x^2b^2+3abd^2x+3b^2cdx+a^2d^2+abcd+b^2c^2\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{3(bx+a)^3g^4b^3}-\frac{i^2\left(-3B\ln(-bx-a)b^3d^3x^3+3B\ln(dx+c)b^3d^3x^3-\right.}{}$

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x,method=_RETUR
NVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(-1/3*i^2*d^2*e^2/(a*d-b*c)^2/g^4*A/(b*e/d+(a*d-b*c)*e/d
/(d*x+c))^3+i^2*d^2*e^2/(a*d-b*c)^2/g^4*B*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c
))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(85) = 170$.

Time = 0.34 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.04

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx =$$

$$-\frac{3\left((3A+B)b^3cd^2-(3A+B)ab^2d^3\right)i^2x^2+3\left((3A+B)b^3c^2d-(3A+B)a^2bd^3\right)i^2x+\left((3A+B)b^3c^3-\right.}{9\left((b^7c-ab^6d)g^4x^3+3(ab^6c-a^2b^5d)g^4x^2+3(a^2b^5\right.}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algor
ithm="fricas")
```

```
[Out] -1/9*(3*((3*A + B)*b^3*c*d^2 - (3*A + B)*a*b^2*d^3)*i^2*x^2 + 3*((3*A + B)*
b^3*c^2*d - (3*A + B)*a^2*b*d^3)*i^2*x + ((3*A + B)*b^3*c^3 - (3*A + B)*a^3
*d^3)*i^2 + 3*(B*b^3*d^3*i^2*x^3 + 3*B*b^3*c*d^2*i^2*x^2 + 3*B*b^3*c^2*d*i^
2*x + B*b^3*c^3*i^2)*log((b*e*x + a*e)/(d*x + c)))/((b^7*c - a*b^6*d)*g^4*x
^3 + 3*(a*b^6*c - a^2*b^5*d)*g^4*x^2 + 3*(a^2*b^5*c - a^3*b^4*d)*g^4*x + (a
^3*b^4*c - a^4*b^3*d)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(76) = 152$.

Time = 10.03 (sec) , antiderivative size = 614, normalized size of antiderivative = 6.90

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$$
$$= -\frac{Bd^3i^2\log\left(x+\frac{-\frac{Ba^2d^5i^2}{ad-bc}+\frac{2Babcd^4i^2}{ad-bc}+Bad^4i^2-\frac{Bb^2c^2d^3i^2}{ad-bc}+Bbcd^3i^2}{2Bbd^4i^2}\right)}{3b^3g^4(ad-bc)}$$
$$+\frac{Bd^3i^2\log\left(x+\frac{\frac{Ba^2d^5i^2}{ad-bc}-\frac{2Babcd^4i^2}{ad-bc}+Bad^4i^2+\frac{Bb^2c^2d^3i^2}{ad-bc}+Bbcd^3i^2}{2Bbd^4i^2}\right)}{3b^3g^4(ad-bc)}$$
$$+\frac{-3Aa^2d^2i^2-3Aabcdi^2-3Ab^2c^2i^2-Ba^2d^2i^2-Babcdi^2-Bb^2c^2i^2+x^2(-9Ab^2d^2i^2-3Bb^2d^2i^2)+x(\cdots}{9a^3b^3g^4+27a^2b^4g^4x+27ab^5g^4x^2+9b^6g^4x^3}$$
$$+\frac{(-Ba^2d^2i^2-Babcdi^2-3Babd^2i^2x-Bb^2c^2i^2-3Bb^2cdi^2x-3Bb^2d^2i^2x^2)\log\left(\frac{e(a+bx)}{c+dx}\right)}{3a^3b^3g^4+9a^2b^4g^4x+9ab^5g^4x^2+3b^6g^4x^3}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4,x)
```

```
[Out] -B*d**3*i**2*log(x + (-B*a**2*d**5*i**2/(a*d - b*c) + 2*B*a*b*c*d**4*i**2/(
a*d - b*c) + B*a*d**4*i**2 - B*b**2*c**2*d**3*i**2/(a*d - b*c) + B*b*c*d**3
*i**2)/(2*B*b*d**4*i**2))/(3*b**3*g**4*(a*d - b*c)) + B*d**3*i**2*log(x + (
B*a**2*d**5*i**2/(a*d - b*c) - 2*B*a*b*c*d**4*i**2/(a*d - b*c) + B*a*d**4*i
**2 + B*b**2*c**2*d**3*i**2/(a*d - b*c) + B*b*c*d**3*i**2)/(2*B*b*d**4*i**2
))/(3*b**3*g**4*(a*d - b*c)) + (-3*A*a**2*d**2*i**2 - 3*A*a*b*c*d*i**2 - 3*
A*b**2*c**2*i**2 - B*a**2*d**2*i**2 - B*a*b*c*d*i**2 - B*b**2*c**2*i**2 + x
**2*(-9*A*b**2*d**2*i**2 - 3*B*b**2*d**2*i**2) + x*(-9*A*a*b*d**2*i**2 - 9*
A*b**2*c*d*i**2 - 3*B*a*b*d**2*i**2 - 3*B*b**2*c*d*i**2))/(9*a**3*b**3*g**4
 + 27*a**2*b**4*g**4*x + 27*a*b**5*g**4*x**2 + 9*b**6*g**4*x**3) + (-B*a**2
*d**2*i**2 - B*a*b*c*d*i**2 - 3*B*a*b*d**2*i**2*x - B*b**2*c**2*i**2 - 3*B*
b**2*c*d*i**2*x - 3*B*b**2*d**2*i**2*x**2)*log(e*(a + b*x)/(c + d*x))/(3*a*
*3*b**3*g**4 + 9*a**2*b**4*g**4*x + 9*a*b**5*g**4*x**2 + 3*b**6*g**4*x**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1515 vs. 2(85) = 170.

Time = 0.25 (sec) , antiderivative size = 1515, normalized size of antiderivative = 17.02

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algor
ithm="maxima")
```

```
[Out] -1/18*B*d^2*i^2*(6*(3*b^2*x^2 + 3*a*b*x + a^2)*log(b*e*x/(d*x + c) + a*e/(d
*x + c))/(b^6*g^4*x^3 + 3*a*b^5*g^4*x^2 + 3*a^2*b^4*g^4*x + a^3*b^3*g^4) +
(11*a^2*b^2*c^2 - 7*a^3*b*c*d + 2*a^4*d^2 + 6*(3*b^4*c^2 - 3*a*b^3*c*d + a^
2*b^2*d^2)*x^2 + 3*(9*a*b^3*c^2 - 7*a^2*b^2*c*d + 2*a^3*b*d^2)*x)/((b^8*c^2
 - 2*a*b^7*c*d + a^2*b^6*d^2)*g^4*x^3 + 3*(a*b^7*c^2 - 2*a^2*b^6*c*d + a^3*
b^5*d^2)*g^4*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d + a^4*b^4*d^2)*g^4*x + (a
^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2)*g^4) + 6*(3*b^2*c^2*d - 3*a*b*c*d
^2 + a^2*d^3)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^
3*b^3*d^3)*g^4) - 6*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(d*x + c)/((b^
6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4)) - 1/18*B*c*d*i
^2*(6*(3*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^5*g^4*x^3 + 3*a*b
^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) + (5*a*b^2*c^2 - 22*a^2*b*c*d +
 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c^2 - 16*a*b^2*c*d +
5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3 + 3*(a*b^6*c
^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*c*d
+ a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4) - 6
*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d
^2 - a^3*b^2*d^3)*g^4) + 6*(3*b*c*d^2 - a*d^3)*log(d*x + c)/((b^5*c^3 - 3*a
*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4)) - 1/18*B*c^2*i^2*((6*b^2*
d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/(
(b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*
d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^
4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*log(b*e*x/(d*x + c
) + a*e/(d*x + c))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b
*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^
3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*
d^2 - a^3*b*d^3)*g^4)) - 1/3*(3*b*x + a)*A*c*d*i^2/(b^5*g^4*x^3 + 3*a*b^4*g
^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - 1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*A
*d^2*i^2/(b^6*g^4*x^3 + 3*a*b^5*g^4*x^2 + 3*a^2*b^4*g^4*x + a^3*b^3*g^4) -
1/3*A*c^2*i^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.57

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = -\frac{1}{9}\left(\frac{3\,(dx+c)^3Be^4i^2\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^3g^4}+\frac{(3\,Ae^4i^2+Be^4i^2)(dx+c)^3}{(bex+ae)^3g^4}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algor
ithm="giac")
```

```
[Out] -1/9*(3*(d*x + c)^3*B*e^4*i^2*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^3
*g^4) + (3*A*e^4*i^2 + B*e^4*i^2)*(d*x + c)^3/((b*e*x + a*e)^3*g^4))*(b*c/(
(b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 423, normalized size of antiderivative = 4.75

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = -\frac{x^2\,(3\,A\,b^2\,d^2\,i^2+B\,b^2\,d^2\,i^2)+x\,(3\,A\,a\,b\,d^2\,i^2+B\,a\,b\,d^2\,i^2+3\,A\,b^2\,c\,d\,i^2+B\,b^2\,c\,d\,i^2)+A\,a^2\,d^2\,i^2+A\,b^2\,c^2\,i^2+\dots}{3\,a^3\,b^3\,g^4+9\,a^2\,b^4\,g^4\,x+9\,a\,b^5\,g^4\,x^2+3\,b^6\,g^4\,x^3} - \frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(a\,\left(\frac{B\,a\,d^2\,i^2}{3\,b^4\,g^4}+\frac{B\,c\,d\,i^2}{3\,b^3\,g^4}\right)+x\,\left(b\,\left(\frac{B\,a\,d^2\,i^2}{3\,b^4\,g^4}+\frac{B\,c\,d\,i^2}{3\,b^3\,g^4}\right)+\frac{2\,B\,a\,d^2\,i^2}{3\,b^3\,g^4}+\frac{2\,B\,c\,d\,i^2}{3\,b^2\,g^4}\right)+\frac{B\,c^2\,i^2}{3\,b^2\,g^4}+\frac{B\,d^2\,i^2\,x^2}{b^2\,g^4}\right)}{3\,a^2\,x+\frac{a^3}{b}+b^2\,x^3+3\,a\,b\,x^2} - \frac{B\,d^3\,i^2\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{3\,b^3\,g^4\,\left(a\,d-b\,c\right)}$$

```
[In] int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^4,
x)
```

```
[Out] - (x^2*(3*A*b^2*d^2*i^2 + B*b^2*d^2*i^2) + x*(3*A*a*b*d^2*i^2 + B*a*b*d^2*i
^2 + 3*A*b^2*c*d*i^2 + B*b^2*c*d*i^2) + A*a^2*d^2*i^2 + A*b^2*c^2*i^2 + (B*
a^2*d^2*i^2)/3 + (B*b^2*c^2*i^2)/3 + A*a*b*c*d*i^2 + (B*a*b*c*d*i^2)/3)/(3*
a^3*b^3*g^4 + 3*b^6*g^4*x^3 + 9*a^2*b^4*g^4*x + 9*a*b^5*g^4*x^2) - (log((e*
(a + b*x))/(c + d*x))*(a*((B*a*d^2*i^2)/(3*b^4*g^4) + (B*c*d*i^2)/(3*b^3*g^
4)) + x*(b*((B*a*d^2*i^2)/(3*b^4*g^4) + (B*c*d*i^2)/(3*b^3*g^4)) + (2*B*a*d
^2*i^2)/(3*b^3*g^4) + (2*B*c*d*i^2)/(3*b^2*g^4)) + (B*c^2*i^2)/(3*b^2*g^4)
+ (B*d^2*i^2*x^2)/(b^2*g^4)))/(3*a^2*x + a^3/b + b^2*x^3 + 3*a*b*x^2) - (B*
d^3*i^2*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(3*b^3*g^4*(a*d - b*
c))
```

3.18 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 181

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = \frac{Bdi^2(c+dx)^3}{9(bc-ad)^2g^5(a+bx)^3} - \frac{bBi^2(c+dx)^4}{16(bc-ad)^2g^5(a+bx)^4} + \frac{di^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^2g^5(a+bx)^3} - \frac{bi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^2g^5(a+bx)^4}$$

```
[Out] 1/9*B*d*i^2*(d*x+c)^3/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/16*b*B*i^2*(d*x+c)^4/(-a
*d+b*c)^2/g^5/(b*x+a)^4+1/3*d*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a
*d+b*c)^2/g^5/(b*x+a)^3-1/4*b*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a
*d+b*c)^2/g^5/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used

= {2562, 45, 2372, 12}

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = -\frac{bi^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4g^5(a+bx)^4(bc-ad)^2} + \frac{di^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3g^5(a+bx)^3(bc-ad)^2} - \frac{bBi^2(c+dx)^4}{16g^5(a+bx)^4(bc-ad)^2} + \frac{Bdi^2(c+dx)^3}{9g^5(a+bx)^3(bc-ad)^2}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^5,
x]
```

```
[Out] (B*d*i^2*(c + d*x)^3)/(9*(b*c - a*d)^2*g^5*(a + b*x)^3) - (b*B*i^2*(c + d*x
)^4)/(16*(b*c - a*d)^2*g^5*(a + b*x)^4) + (d*i^2*(c + d*x)^3*(A + B*Log[(e*
(a + b*x))/(c + d*x)]))/(3*(b*c - a*d)^2*g^5*(a + b*x)^3) - (b*i^2*(c + d*x
)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*(b*c - a*d)^2*g^5*(a + b*x)^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
```

```
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{i^2\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex))}{x^5} \, dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 g^5}$$

$$= \frac{di^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^2 g^5(a+bx)^3} - \frac{bi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^2 g^5(a+bx)^4} - \frac{(Bi^2)\,\text{Subst}\left(\int \frac{-3b+4dx}{12x^5} \, dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 g^5}$$

$$= \frac{di^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^2 g^5(a+bx)^3} - \frac{bi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^2 g^5(a+bx)^4} - \frac{(Bi^2)\,\text{Subst}\left(\int \frac{-3b+4dx}{x^5} \, dx, x, \frac{a+bx}{c+dx}\right)}{12(bc-ad)^2 g^5}$$

$$= \frac{di^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^2 g^5(a+bx)^3} - \frac{bi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^2 g^5(a+bx)^4} - \frac{(Bi^2)\,\text{Subst}\left(\int \left(-\frac{3b}{x^5}+\frac{4d}{x^4}\right) \, dx, x, \frac{a+bx}{c+dx}\right)}{12(bc-ad)^2 g^5}$$

$$= \frac{Bdi^2(c+dx)^3}{9(bc-ad)^2 g^5(a+bx)^3} - \frac{bBi^2(c+dx)^4}{16(bc-ad)^2 g^5(a+bx)^4} + \frac{di^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^2 g^5(a+bx)^3} - \frac{bi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^2 g^5(a+bx)^4}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 454 vs. $2(181) = 362$.

Time = 0.24 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.51

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5} \, dx =$$

$$-\frac{i^2\left(36Ab^4c^4+9b^4Bc^4-48aAb^3c^3d-16ab^3Bc^3d+12a^4Ad^4+7a^4Bd^4+96Ab^4c^3dx+20b^4Bc^3dx-144\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g
*x)^5,x]
```

```
[Out] -1/144*(i^2*(36*A*b^4*c^4 + 9*b^4*B*c^4 - 48*a*A*b^3*c^3*d - 16*a*b^3*B*c^3
*d + 12*a^4*A*d^4 + 7*a^4*B*d^4 + 96*A*b^4*c^3*d*x + 20*b^4*B*c^3*d*x - 144
*a*A*b^3*c^2*d^2*x - 48*a*b^3*B*c^2*d^2*x + 48*a^3*A*b*d^4*x + 28*a^3*b*B*d
^4*x + 72*A*b^4*c^2*d^2*x^2 + 6*b^4*B*c^2*d^2*x^2 - 144*a*A*b^3*c*d^3*x^2 -
 48*a*b^3*B*c*d^3*x^2 + 72*a^2*A*b^2*d^4*x^2 + 42*a^2*b^2*B*d^4*x^2 - 12*b^
4*B*c*d^3*x^3 + 12*a*b^3*B*d^4*x^3 - 12*B*d^4*(a + b*x)^4*Log[a + b*x] + 12
*B*(b*c - a*d)^2*(a^2*d^2 + 2*a*b*d*(c + 2*d*x) + b^2*(3*c^2 + 8*c*d*x + 6*
d^2*x^2))*Log[(e*(a + b*x))/(c + d*x)] + 12*a^4*B*d^4*Log[c + d*x] + 48*a^3
*b*B*d^4*x*Log[c + d*x] + 72*a^2*b^2*B*d^4*x^2*Log[c + d*x] + 48*a*b^3*B*d^
4*x^3*Log[c + d*x] + 12*b^4*B*d^4*x^4*Log[c + d*x]))/(b^3*(b*c - a*d)^2*g^5
*(a + b*x)^4)
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.76

method	result
parts	$\frac{i^2 A\left(\frac{2d(ad-cb)}{3b^3(bx+a)^3}-\frac{a^2d^2-2abcd+b^2c^2}{4b^3(bx+a)^4}-\frac{d^2}{2b^3(bx+a)^2}\right)}{g^5}-\frac{i^2B(ad-cb)^3e^3\left(\frac{d^5\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}\right)}{(ad-cb)^5}\right.}{g^5d^4}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{i^2d^2e^3Ab}{4(ad-cb)^3g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{i^2d^3e^2A}{3(ad-cb)^3g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{i^2d^2e^3Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{1}{16\left(\frac{be}{d}+\frac{(a}{d(}\right.}\right.}{(ad-cb)^3g^5}\right.}{d^2}$
default	$-\frac{e(ad-cb)\left(\frac{i^2d^2e^3Ab}{4(ad-cb)^3g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{i^2d^3e^2A}{3(ad-cb)^3g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{i^2d^2e^3Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{1}{16\left(\frac{be}{d}+\frac{(a}{d(}\right.}\right.}{(ad-cb)^3g^5}\right.}{d^2}$
risch	$-\frac{i^2B\left(6d^2x^2b^2+4abd^2x+8b^2cdx+a^2d^2+2abcd+3b^2c^2\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{12(bx+a)^4g^5b^3}-\frac{(-48Bab^3cd^3x^2-48Bab^3c^2d^2x-48Aab^3c^3}{}$
norman	$\frac{\frac{\left(12Aac^2di^2-12Abc^3i^2+4Bac^2di^2-3Bbc^3i^2\right)x}{12ga(ad-cb)}+\frac{\left(24Aa^2cd^2i^2+12Aabc^2di^2-36Ab^2c^3i^2+8Ba^2cd^2i^2+7Babc^2di^2-9Bb^2c^3i^{}\right.}{24ga^2(ad-cb)}}{}$
parallelrisch	$12Ax^4a^6bcd^4i^2-48Ax^4a^3b^4c^4di^2+7Bx^4a^6bcd^4i^2-16Bx^4a^3b^4c^4di^2+48Bx^3\ln\left(\frac{e(bx+a)}{dx+c}\right)a^7cd^4i^2-192Ax^3a^4b^3c^4d$

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x,method=_RETUR
```

```
NVERBOSE)
```

```
[Out] i^2*A/g^5*(2/3*d*(a*d-b*c)/b^3/(b*x+a)^3-1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^
3/(b*x+a)^4-1/2*d^2/b^3/(b*x+a)^2)-i^2*B/g^5/d^4*(a*d-b*c)^3*e^3*(d^5/(a*d-
b*c)^5*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-d^4/(a*d-b*c)^5*b*e*(-1/4/(b*e/d+(a*
d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)
*e/d/(d*x+c))^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(173) = 346$.

Time = 0.37 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.82

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = \frac{12\,(Bb^4cd^3-Bab^3d^4)i^2x^3-6\,((12\,A+B)b^4c^2d^2-8\,(3\,A+B)ab^3cd^3+(12\,A+7\,B)a^2b^2d^4)i^2x^2-4\,((24\,A}{144\,((b^9c^2-}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algor
ithm="fricas")
```

```
[Out] 1/144*(12*(B*b^4*c*d^3 - B*a*b^3*d^4)*i^2*x^3 - 6*((12*A + B)*b^4*c^2*d^2 -
 8*(3*A + B)*a*b^3*c*d^3 + (12*A + 7*B)*a^2*b^2*d^4)*i^2*x^2 - 4*((24*A + 5
*B)*b^4*c^3*d - 12*(3*A + B)*a*b^3*c^2*d^2 + (12*A + 7*B)*a^3*b*d^4)*i^2*x
- (9*(4*A + B)*b^4*c^4 - 16*(3*A + B)*a*b^3*c^3*d + (12*A + 7*B)*a^4*d^4)*i
^2 + 12*(B*b^4*d^4*i^2*x^4 + 4*B*a*b^3*d^4*i^2*x^3 - 6*(B*b^4*c^2*d^2 - 2*B
*a*b^3*c*d^3)*i^2*x^2 - 4*(2*B*b^4*c^3*d - 3*B*a*b^3*c^2*d^2)*i^2*x - (3*B*
b^4*c^4 - 4*B*a*b^3*c^3*d)*i^2)*log((b*e*x + a*e)/(d*x + c)))/((b^9*c^2 - 2
*a*b^8*c*d + a^2*b^7*d^2)*g^5*x^4 + 4*(a*b^8*c^2 - 2*a^2*b^7*c*d + a^3*b^6*
d^2)*g^5*x^3 + 6*(a^2*b^7*c^2 - 2*a^3*b^6*c*d + a^4*b^5*d^2)*g^5*x^2 + 4*(a
^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b^4*d^2)*g^5*x + (a^4*b^5*c^2 - 2*a^5*b^4*
c*d + a^6*b^3*d^2)*g^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 928 vs. $2(165) = 330$.

Time = 23.85 (sec) , antiderivative size = 928, normalized size of antiderivative = 5.13

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$$
$$= -\frac{Bd^4i^2\log\left(x+\frac{-\frac{Ba^3d^7i^2}{(ad-bc)^2}+\frac{3Ba^2bcd^6i^2}{(ad-bc)^2}-\frac{3Bab^2c^2d^5i^2}{(ad-bc)^2}+Bad^5i^2+\frac{Bb^3c^3d^4i^2}{(ad-bc)^2}+Bbcd^4i^2}{2Bbd^5i^2}\right)}{12b^3g^5(ad-bc)^2}$$
$$+\frac{Bd^4i^2\log\left(x+\frac{\frac{Ba^3d^7i^2}{(ad-bc)^2}-\frac{3Ba^2bcd^6i^2}{(ad-bc)^2}+\frac{3Bab^2c^2d^5i^2}{(ad-bc)^2}+Bad^5i^2-\frac{Bb^3c^3d^4i^2}{(ad-bc)^2}+Bbcd^4i^2}{2Bbd^5i^2}\right)}{12b^3g^5(ad-bc)^2}$$
$$+\frac{-12Aa^3d^3i^2-12Aa^2bcd^2i^2-12Aab^2c^2di^2+36Ab^3c^3i^2-7Ba^3d^3i^2-7Ba^2bcd^2i^2-7Bab^2c^2di^2+9Bb}{144a^5b^3dg^5-144a^4b^4cg^5+x^4\cdot(144ab^7dg}$$
$$+\frac{(-Ba^2d^2i^2-2Babcdi^2-4Babd^2i^2x-3Bb^2c^2i^2-8Bb^2cdi^2x-6Bb^2d^2i^2x^2)\log\left(\frac{e(a+bx)}{c+dx}\right)}{12a^4b^3g^5+48a^3b^4g^5x+72a^2b^5g^5x^2+48ab^6g^5x^3+12b^7g^5x^4}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**5,x)
```

```
[Out] -B*d**4*i**2*log(x + (-B*a**3*d**7*i**2/(a*d - b*c)**2 + 3*B*a**2*b*c*d**6*
i**2/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**5*i**2/(a*d - b*c)**2 + B*a*d**5*i
**2 + B*b**3*c**3*d**4*i**2/(a*d - b*c)**2 + B*b*c*d**4*i**2)/(2*B*b*d**5*i
**2))/(12*b**3*g**5*(a*d - b*c)**2) + B*d**4*i**2*log(x + (B*a**3*d**7*i**2
/(a*d - b*c)**2 - 3*B*a**2*b*c*d**6*i**2/(a*d - b*c)**2 + 3*B*a*b**2*c**2*d
**5*i**2/(a*d - b*c)**2 + B*a*d**5*i**2 - B*b**3*c**3*d**4*i**2/(a*d - b*c)
**2 + B*b*c*d**4*i**2)/(2*B*b*d**5*i**2))/(12*b**3*g**5*(a*d - b*c)**2) + (
-12*A*a**3*d**3*i**2 - 12*A*a**2*b*c*d**2*i**2 - 12*A*a*b**2*c**2*d*i**2 +
36*A*b**3*c**3*i**2 - 7*B*a**3*d**3*i**2 - 7*B*a**2*b*c*d**2*i**2 - 7*B*a*b
**2*c**2*d*i**2 + 9*B*b**3*c**3*i**2 - 12*B*b**3*d**3*i**2*x**3 + x**2*(-72
*A*a*b**2*d**3*i**2 + 72*A*b**3*c*d**2*i**2 - 42*B*a*b**2*d**3*i**2 + 6*B*b
**3*c*d**2*i**2) + x*(-48*A*a**2*b*d**3*i**2 - 48*A*a*b**2*c*d**2*i**2 + 96
*A*b**3*c**2*d*i**2 - 28*B*a**2*b*d**3*i**2 - 28*B*a*b**2*c*d**2*i**2 + 20*
B*b**3*c**2*d*i**2))/(144*a**5*b**3*d*g**5 - 144*a**4*b**4*c*g**5 + x**4*(1
44*a*b**7*d*g**5 - 144*b**8*c*g**5) + x**3*(576*a**2*b**6*d*g**5 - 576*a*b*
*7*c*g**5) + x**2*(864*a**3*b**5*d*g**5 - 864*a**2*b**6*c*g**5) + x*(576*a*
*4*b**4*d*g**5 - 576*a**3*b**5*c*g**5)) + (-B*a**2*d**2*i**2 - 2*B*a*b*c*d*
i**2 - 4*B*a*b*d**2*i**2*x - 3*B*b**2*c**2*i**2 - 8*B*b**2*c*d*i**2*x - 6*B
*b**2*d**2*i**2*x**2)*log(e*(a + b*x)/(c + d*x))/(12*a**4*b**3*g**5 + 48*a*
*3*b**4*g**5*x + 72*a**2*b**5*g**5*x**2 + 48*a*b**6*g**5*x**3 + 12*b**7*g**
5*x**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2218 vs. $2(173) = 346$.

Time = 0.30 (sec) , antiderivative size = 2218, normalized size of antiderivative = 12.25

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algor
ithm="maxima")
```

```
[Out] -1/144*B*d^2*i^2*(12*(6*b^2*x^2 + 4*a*b*x + a^2)*log(b*e*x/(d*x + c) + a*e/
(d*x + c))/(b^7*g^5*x^4 + 4*a*b^6*g^5*x^3 + 6*a^2*b^5*g^5*x^2 + 4*a^3*b^4*g
^5*x + a^4*b^3*g^5) + (13*a^2*b^3*c^3 - 75*a^3*b^2*c^2*d + 33*a^4*b*c*d^2 -
 7*a^5*d^3 - 12*(6*b^5*c^2*d - 4*a*b^4*c*d^2 + a^2*b^3*d^3)*x^3 + 6*(6*b^5*
c^3 - 46*a*b^4*c^2*d + 29*a^2*b^3*c*d^2 - 7*a^3*b^2*d^3)*x^2 + 4*(10*a*b^4*
c^3 - 63*a^2*b^3*c^2*d + 33*a^3*b^2*c*d^2 - 7*a^4*b*d^3)*x)/((b^10*c^3 - 3*
a*b^9*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7*d^3)*g^5*x^4 + 4*(a*b^9*c^3 - 3*a^2
*b^8*c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*d^3)*g^5*x^3 + 6*(a^2*b^8*c^3 - 3*a^
3*b^7*c^2*d + 3*a^4*b^6*c*d^2 - a^5*b^5*d^3)*g^5*x^2 + 4*(a^3*b^7*c^3 - 3*a
^4*b^6*c^2*d + 3*a^5*b^5*c*d^2 - a^6*b^4*d^3)*g^5*x + (a^4*b^6*c^3 - 3*a^5*
b^5*c^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d^3)*g^5) - 12*(6*b^2*c^2*d^2 - 4*a*b
*c*d^3 + a^2*d^4)*log(b*x + a)/((b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^
2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4)*g^5) + 12*(6*b^2*c^2*d^2 - 4*a*b*c*d^3 +
 a^2*d^4)*log(d*x + c)/((b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^
3*b^4*c*d^3 + a^4*b^3*d^4)*g^5)) - 1/72*B*c*d*i^2*(12*(4*b*x + a)*log(b*e*x
/(d*x + c) + a*e/(d*x + c))/(b^6*g^5*x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*
x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g^5) + (7*a*b^3*c^3 - 33*a^2*b^2*c^2*d + 75
*a^3*b*c*d^2 - 13*a^4*d^3 + 12*(4*b^4*c*d^2 - a*b^3*d^3)*x^3 - 6*(4*b^4*c^2
*d - 29*a*b^3*c*d^2 + 7*a^2*b^2*d^3)*x^2 + 4*(4*b^4*c^3 - 21*a*b^3*c^2*d +
57*a^2*b^2*c*d^2 - 13*a^3*b*d^3)*x)/((b^9*c^3 - 3*a*b^8*c^2*d + 3*a^2*b^7*c
*d^2 - a^3*b^6*d^3)*g^5*x^4 + 4*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*a^3*b^6*c*
d^2 - a^4*b^5*d^3)*g^5*x^3 + 6*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*a^4*b^5*c
*d^2 - a^5*b^4*d^3)*g^5*x^2 + 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*
c*d^2 - a^6*b^3*d^3)*g^5*x + (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d
^2 - a^7*b^2*d^3)*g^5) + 12*(4*b*c*d^3 - a*d^4)*log(b*x + a)/((b^6*c^4 - 4*
a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5) - 12*
(4*b*c*d^3 - a*d^4)*log(d*x + c)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*
d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5)) + 1/48*B*c^2*i^2*((12*b^3*d^3*x^
3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2
 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8
*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^
3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c
^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*
```

```
c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3
 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) - 12*log(b*e*x/(d*x
+ c) + a*e/(d*x + c))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 +
4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*
d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x
+ c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*
b*d^4)*g^5)) - 1/6*(4*b*x + a)*A*c*d*i^2/(b^6*g^5*x^4 + 4*a*b^5*g^5*x^3 + 6
*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g^5) - 1/12*(6*b^2*x^2 + 4*a*b
*x + a^2)*A*d^2*i^2/(b^7*g^5*x^4 + 4*a*b^6*g^5*x^3 + 6*a^2*b^5*g^5*x^2 + 4*
a^3*b^4*g^5*x + a^4*b^3*g^5) - 1/4*A*c^2*i^2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3
 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

Giac [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.56

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = -\frac{1}{144}\left(\frac{12\left(3\,Bbe^5i^2-\frac{4\,(bex+ae)Bde^4i^2}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^4bcg^5}{(dx+c)^4}-\frac{(bex+ae)^4adg^5}{(dx+c)^4}}+\frac{36\,Abe^5i^2+9\,Bbe^5i^2-\frac{48\,(bex+ae)Ade^4i^2}{dx+c}-\frac{16\,(bex+ae)}{dx+c}}{\frac{(bex+ae)^4bcg^5}{(dx+c)^4}-\frac{(bex+ae)^4adg^5}{(dx+c)^4}}\right.$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algor
ithm="giac")
```

```
[Out] -1/144*(12*(3*B*b*e^5*i^2 - 4*(b*e*x + a*e)*B*d*e^4*i^2/(d*x + c))*log((b*e
*x + a*e)/(d*x + c))/((b*e*x + a*e)^4*b*c*g^5/(d*x + c)^4 - (b*e*x + a*e)^4
*a*d*g^5/(d*x + c)^4) + (36*A*b*e^5*i^2 + 9*B*b*e^5*i^2 - 48*(b*e*x + a*e)*
A*d*e^4*i^2/(d*x + c) - 16*(b*e*x + a*e)*B*d*e^4*i^2/(d*x + c))/((b*e*x + a
*e)^4*b*c*g^5/(d*x + c)^4 - (b*e*x + a*e)^4*a*d*g^5/(d*x + c)^4))*(b*c/((b*
c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 3.58 (sec) , antiderivative size = 647, normalized size of antiderivative = 3.57

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = -\frac{\frac{12\,A\,a^3\,d^3\,i^2-36\,A\,b^3\,c^3\,i^2+7\,B\,a^3\,d^3\,i^2-9\,B\,b^3\,c^3\,i^2+12\,A\,a\,b^2\,c^2\,d\,i^2+12\,A\,a^2\,b\,c\,d^2\,i^2+7\,B\,a\,b^2\,c^2\,d\,i^2+7\,B\,a^2\,b\,c\,d^2\,i^2}{12\,(a\,d-b\,c)}+\frac{x^2\,(12\,A\,a\,b^2\,d^3\,i^2+7\cdots}{\cdots}}{12\,a^4\,b^3\,g^5+48\,a^3\,b^4\,g^5\,x+7\cdots} - \frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(a\,\left(\frac{B\,a\,d^2\,i^2}{12\,b^4\,g^5}+\frac{B\,c\,d\,i^2}{6\,b^3\,g^5}\right)+x\,\left(b\,\left(\frac{B\,a\,d^2\,i^2}{12\,b^4\,g^5}+\frac{B\,c\,d\,i^2}{6\,b^3\,g^5}\right)+\frac{B\,a\,d^2\,i^2}{4\,b^3\,g^5}+\frac{B\,c\,d\,i^2}{2\,b^2\,g^5}\right)+\frac{B\,c^2\,i^2}{4\,b^2\,g^5}+\frac{B\,d^2\,i^2\,x^2}{2\,b^2\,g^5}\right)}{4\,a^3\,x+\frac{a^4}{b}+b^3\,x^4+6\,a^2\,b\,x^2+4\,a\,b^2\,x^3} - \frac{B\,d^4\,i^2\,\mathrm{atanh}\left(\frac{12\,b^5\,c^2\,g^5-12\,a^2\,b^3\,d^2\,g^5}{12\,b^3\,g^5\,{(a\,d-b\,c)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{6\,b^3\,g^5\,{(a\,d-b\,c)}^2}$$

```
[In] int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^5,
x)

[Out] - ((12*A*a^3*d^3*i^2 - 36*A*b^3*c^3*i^2 + 7*B*a^3*d^3*i^2 - 9*B*b^3*c^3*i^2
 + 12*A*a*b^2*c^2*d*i^2 + 12*A*a^2*b*c*d^2*i^2 + 7*B*a*b^2*c^2*d*i^2 + 7*B*
a^2*b*c*d^2*i^2)/(12*(a*d - b*c)) + (x^2*(12*A*a*b^2*d^3*i^2 + 7*B*a*b^2*d^
3*i^2 - 12*A*b^3*c*d^2*i^2 - B*b^3*c*d^2*i^2))/(2*(a*d - b*c)) + (x*(12*A*a
^2*b*d^3*i^2 + 7*B*a^2*b*d^3*i^2 - 24*A*b^3*c^2*d*i^2 - 5*B*b^3*c^2*d*i^2 +
 12*A*a*b^2*c*d^2*i^2 + 7*B*a*b^2*c*d^2*i^2))/(3*(a*d - b*c)) + (B*b^3*d^3*
i^2*x^3)/(a*d - b*c))/(12*a^4*b^3*g^5 + 12*b^7*g^5*x^4 + 48*a^3*b^4*g^5*x +
 48*a*b^6*g^5*x^3 + 72*a^2*b^5*g^5*x^2) - (log((e*(a + b*x))/(c + d*x))*(a*
((B*a*d^2*i^2)/(12*b^4*g^5) + (B*c*d*i^2)/(6*b^3*g^5)) + x*(b*((B*a*d^2*i^2
)/(12*b^4*g^5) + (B*c*d*i^2)/(6*b^3*g^5)) + (B*a*d^2*i^2)/(4*b^3*g^5) + (B*
c*d*i^2)/(2*b^2*g^5)) + (B*c^2*i^2)/(4*b^2*g^5) + (B*d^2*i^2*x^2)/(2*b^2*g^
5)))/(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3) - (B*d^4*i^2*a
tanh((12*b^5*c^2*g^5 - 12*a^2*b^3*d^2*g^5)/(12*b^3*g^5*(a*d - b*c)^2) - (2*
b*d*x)/(a*d - b*c)))/(6*b^3*g^5*(a*d - b*c)^2)
```

3.19 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 281

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = -\frac{Bd^2i^2(c+dx)^3}{9(bc-ad)^3g^6(a+bx)^3} + \frac{bBdi^2(c+dx)^4}{8(bc-ad)^3g^6(a+bx)^4} - \frac{b^2Bi^2(c+dx)^5}{25(bc-ad)^3g^6(a+bx)^5} - \frac{d^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^3g^6(a+bx)^3} + \frac{bdi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^6(a+bx)^4} - \frac{b^2i^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5(bc-ad)^3g^6(a+bx)^5}$$

```
[Out] -1/9*B*d^2*i^2*(d*x+c)^3/(-a*d+b*c)^3/g^6/(b*x+a)^3+1/8*b*B*d*i^2*(d*x+c)^4
/(-a*d+b*c)^3/g^6/(b*x+a)^4-1/25*b^2*B*i^2*(d*x+c)^5/(-a*d+b*c)^3/g^6/(b*x+
a)^5-1/3*d^2*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^6/(b*
x+a)^3+1/2*b*d*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^6/(
b*x+a)^4-1/5*b^2*i^2*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^6
/(b*x+a)^5
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2562, 45, 2372, 12, 14}

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = -\frac{b^2i^2(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5g^6(a+bx)^5(bc-ad)^3} - \frac{d^2i^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3g^6(a+bx)^3(bc-ad)^3} + \frac{bdi^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^6(a+bx)^4(bc-ad)^3} - \frac{b^2Bi^2(c+dx)^5}{25g^6(a+bx)^5(bc-ad)^3} - \frac{Bd^2i^2(c+dx)^3}{9g^6(a+bx)^3(bc-ad)^3} + \frac{bBdi^2(c+dx)^4}{8g^6(a+bx)^4(bc-ad)^3}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^6,
x]
```

```
[Out] -1/9*(B*d^2*i^2*(c + d*x)^3)/((b*c - a*d)^3*g^6*(a + b*x)^3) + (b*B*d*i^2*(
c + d*x)^4)/(8*(b*c - a*d)^3*g^6*(a + b*x)^4) - (b^2*B*i^2*(c + d*x)^5)/(25
*(b*c - a*d)^3*g^6*(a + b*x)^5) - (d^2*i^2*(c + d*x)^3*(A + B*Log[(e*(a + b
*x))/(c + d*x)]))/(3*(b*c - a*d)^3*g^6*(a + b*x)^3) + (b*d*i^2*(c + d*x)^4*
(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*c - a*d)^3*g^6*(a + b*x)^4) - (
b^2*i^2*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*(b*c - a*d)^3*
g^6*(a + b*x)^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{i^2\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex))}{x^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^6}$$

$$= -\frac{d^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^3g^6(a+bx)^3} + \frac{bdi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^6(a+bx)^4} - \frac{b^2i^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5(bc-ad)^3g^6(a+bx)^5} - \frac{(Bi^2)\,\text{Subst}\left(\int \frac{-6b^2+15bdx-10d^2x^2}{30x^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^6}$$

$$= -\frac{d^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^3g^6(a+bx)^3} + \frac{bdi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^6(a+bx)^4} - \frac{b^2i^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5(bc-ad)^3g^6(a+bx)^5} - \frac{(Bi^2)\,\text{Subst}\left(\int \frac{-6b^2+15bdx-10d^2x^2}{x^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{30(bc-ad)^3g^6}$$

$$= -\frac{d^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^3g^6(a+bx)^3} + \frac{bdi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^6(a+bx)^4} - \frac{b^2i^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5(bc-ad)^3g^6(a+bx)^5} - \frac{(Bi^2)\,\text{Subst}\left(\int \left(-\frac{6b^2}{x^6}+\frac{15bd}{x^5}-\frac{10d^2}{x^4}\right)dx, x, \frac{a+bx}{c+dx}\right)}{30(bc-ad)^3g^6}$$

$$= -\frac{Bd^2i^2(c+dx)^3}{9(bc-ad)^3g^6(a+bx)^3} + \frac{bBdi^2(c+dx)^4}{8(bc-ad)^3g^6(a+bx)^4} - \frac{b^2Bi^2(c+dx)^5}{25(bc-ad)^3g^6(a+bx)^5} - \frac{d^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^3g^6(a+bx)^3} + \frac{bdi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^6(a+bx)^4} - \frac{b^2i^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5(bc-ad)^3g^6(a+bx)^5}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.22

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$$

$$= \frac{i^2\left(-\frac{360Ab^2c^2}{(a+bx)^5} - \frac{72b^2Bc^2}{(a+bx)^5} + \frac{720aAbcd}{(a+bx)^5} + \frac{144abBcd}{(a+bx)^5} - \frac{360a^2Ad^2}{(a+bx)^5} - \frac{72a^2Bd^2}{(a+bx)^5} - \frac{900Abcd}{(a+bx)^4} - \frac{135bBcd}{(a+bx)^4} + \frac{900aAd^2}{(a+bx)^4} + \frac{135aBd^2}{(a+bx)^4} - \right.}{}$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g
*x)^6,x]
```

```
[Out] (i^2*((-360*A*b^2*c^2)/(a + b*x)^5 - (72*b^2*B*c^2)/(a + b*x)^5 + (720*a*A*
b*c*d)/(a + b*x)^5 + (144*a*b*B*c*d)/(a + b*x)^5 - (360*a^2*A*d^2)/(a + b*x
)^5 - (72*a^2*B*d^2)/(a + b*x)^5 - (900*A*b*c*d)/(a + b*x)^4 - (135*b*B*c*d
)/(a + b*x)^4 + (900*a*A*d^2)/(a + b*x)^4 + (135*a*B*d^2)/(a + b*x)^4 - (60
0*A*d^2)/(a + b*x)^3 - (20*B*d^2)/(a + b*x)^3 + (30*B*d^3)/((b*c - a*d)*(a
+ b*x)^2) - (60*B*d^4)/((b*c - a*d)^2*(a + b*x)) - (60*B*d^5*Log[a + b*x])/
(b*c - a*d)^3 - (60*B*(a^2*d^2 + a*b*d*(3*c + 5*d*x) + b^2*(6*c^2 + 15*c*d*
x + 10*d^2*x^2))*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x)^5 + (60*B*d^5*Log[
c + d*x])/(b*c - a*d)^3))/(1800*b^3*g^6)
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.53

method	result
parts	$\frac{i^2 A\left(-\frac{d^2}{3b^3(bx+a)^3}+\frac{d(ad-cb)}{2b^3(bx+a)^4}-\frac{a^2d^2-2abcd+b^2c^2}{5b^3(bx+a)^5}\right)}{g^6}-\frac{i^2B(ad-cb)^3e^3\left(\frac{d^6\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}\cdots\right)}{(ad-cb)^6}\cdots\right)}{\cdots}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{i^2d^2e^4Ab^2}{5(ad-cb)^4g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}+\frac{i^2d^3e^3Ab}{2(ad-cb)^4g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{i^2d^4e^2A}{3(ad-cb)^4g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{i^2d^2e^4Bb^2\cdots}{\cdots}\right)}{\cdots}$
default	$-\frac{e(ad-cb)\left(-\frac{i^2d^2e^4Ab^2}{5(ad-cb)^4g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}+\frac{i^2d^3e^3Ab}{2(ad-cb)^4g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{i^2d^4e^2A}{3(ad-cb)^4g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{i^2d^2e^4Bb^2\cdots}{\cdots}\right)}{\cdots}$
risch	$-\frac{i^2B\left(10d^2x^2b^2+5ab\,d^2x+15b^2cdx+a^2d^2+3abcd+6b^2c^2\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{30(bx+a)^5g^6b^3}-\frac{\left(60B\ln(dx+c)a^5d^5-72B\,b^5c^5+60Ba\,b^4d^5\cdots\right.}{\cdots}$
parallelrisch	$-\frac{-1800A\,x^2a^2b^7c\,d^5i^2+1800A\,x^2a\,b^8c^2d^4i^2-600B\,x^2a^2b^7c\,d^5i^2-1800Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^2b^7c^2d^4i^2-300B\,x^4\ln\left(\frac{e(bx+\cdots}{dx+\cdots}\right)\cdots}{\cdots}$
norman	Expression too large to display

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x,method=_RETUR
NVERBOSE)
```

```
[Out] i^2*A/g^6*(-1/3*d^2/b^3/(b*x+a)^3+1/2*d*(a*d-b*c)/b^3/(b*x+a)^4-1/5*(a^2*d^
2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^5)-i^2*B/g^6/d^4*(a*d-b*c)^3*e^3*(d^6/(a*d
-b*c)^6*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c
))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-2*d^5/(a*d-b*c)^6*b*e*(-1/4/(b*e/d+
(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b
*c)*e/d/(d*x+c))^4)+d^4/(a*d-b*c)^6*e^2*b^2*(-1/5/(b*e/d+(a*d-b*c)*e/d/(d*x
+c))^5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/25/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 807 vs. 2(269) = 538.

Time = 0.37 (sec) , antiderivative size = 807, normalized size of antiderivative = 2.87

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = -\frac{60\,(Bb^5cd^4-Bab^4d^5)i^2x^4-30\,(Bb^5c^2d^3-10\,Bab^4cd^4+9\,Ba^2b^3d^5)i^2x^3+10\,(2\,(30\,A+B)b^5c^3d^2-15\,(}{}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x, algor
ithm="fricas")
```

```
[Out] -1/1800*(60*(B*b^5*c*d^4 - B*a*b^4*d^5)*i^2*x^4 - 30*(B*b^5*c^2*d^3 - 10*B*
a*b^4*c*d^4 + 9*B*a^2*b^3*d^5)*i^2*x^3 + 10*(2*(30*A + B)*b^5*c^3*d^2 - 15*
(12*A + B)*a*b^4*c^2*d^3 + 60*(3*A + B)*a^2*b^3*c*d^4 - (60*A + 47*B)*a^3*b
^2*d^5)*i^2*x^2 + 5*(9*(20*A + 3*B)*b^5*c^4*d - 20*(24*A + 5*B)*a*b^4*c^3*d
^2 + 120*(3*A + B)*a^2*b^3*c^2*d^3 - (60*A + 47*B)*a^4*b*d^5)*i^2*x + (72*(
5*A + B)*b^5*c^5 - 225*(4*A + B)*a*b^4*c^4*d + 200*(3*A + B)*a^2*b^3*c^3*d^
2 - (60*A + 47*B)*a^5*d^5)*i^2 + 60*(B*b^5*d^5*i^2*x^5 + 5*B*a*b^4*d^5*i^2*
x^4 + 10*B*a^2*b^3*d^5*i^2*x^3 + 10*(B*b^5*c^3*d^2 - 3*B*a*b^4*c^2*d^3 + 3*
B*a^2*b^3*c*d^4)*i^2*x^2 + 5*(3*B*b^5*c^4*d - 8*B*a*b^4*c^3*d^2 + 6*B*a^2*b
^3*c^2*d^3)*i^2*x + (6*B*b^5*c^5 - 15*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2)
*i^2)*log((b*e*x + a*e)/(d*x + c)))/((b^11*c^3 - 3*a*b^10*c^2*d + 3*a^2*b^9
*c*d^2 - a^3*b^8*d^3)*g^6*x^5 + 5*(a*b^10*c^3 - 3*a^2*b^9*c^2*d + 3*a^3*b^8
*c*d^2 - a^4*b^7*d^3)*g^6*x^4 + 10*(a^2*b^9*c^3 - 3*a^3*b^8*c^2*d + 3*a^4*b
^7*c*d^2 - a^5*b^6*d^3)*g^6*x^3 + 10*(a^3*b^8*c^3 - 3*a^4*b^7*c^2*d + 3*a^5
*b^6*c*d^2 - a^6*b^5*d^3)*g^6*x^2 + 5*(a^4*b^7*c^3 - 3*a^5*b^6*c^2*d + 3*a^
6*b^5*c*d^2 - a^7*b^4*d^3)*g^6*x + (a^5*b^6*c^3 - 3*a^6*b^5*c^2*d + 3*a^7*b
^4*c*d^2 - a^8*b^3*d^3)*g^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1300 vs. 2(258) = 516.

Time = 76.59 (sec) , antiderivative size = 1300, normalized size of antiderivative = 4.63

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$$
$$= -\frac{Bd^5i^2\log\left(x+\frac{-\frac{Ba^4d^9i^2}{(ad-bc)^3}+\frac{4Ba^3bcd^8i^2}{(ad-bc)^3}-\frac{6Ba^2b^2c^2d^7i^2}{(ad-bc)^3}+\frac{4Bab^3c^3d^6i^2}{(ad-bc)^3}+Bad^6i^2-\frac{Bb^4c^4d^5i^2}{(ad-bc)^3}+Bbcd^5i^2}{2Bbd^6i^2}\right)}{30b^3g^6(ad-bc)^3}$$
$$+\frac{Bd^5i^2\log\left(x+\frac{\frac{Ba^4d^9i^2}{(ad-bc)^3}-\frac{4Ba^3bcd^8i^2}{(ad-bc)^3}+\frac{6Ba^2b^2c^2d^7i^2}{(ad-bc)^3}-\frac{4Bab^3c^3d^6i^2}{(ad-bc)^3}+Bad^6i^2+\frac{Bb^4c^4d^5i^2}{(ad-bc)^3}+Bbcd^5i^2}{2Bbd^6i^2}\right)}{30b^3g^6(ad-bc)^3}$$
$$+\frac{-60Aa^4d^4i^2-60Aa^3bcd^3i^2-60Aa^2b^2c^2d^2i^2+540Aab^3c^3di^2-360Ab^4c^4i^2-47Ba^4d^4i^2-47Ba^3bcd^3i\cdots}{1800a^7b^3d^2g^6-3600a^6b^4cdg^6+1800a^5b^5\cdots}$$
$$+\frac{(-Ba^2d^2i^2-3Babcdi^2-5Babd^2i^2x-6Bb^2c^2i^2-15Bb^2cdi^2x-10Bb^2d^2i^2x^2)\log\left(\frac{e(a+bx)}{c+dx}\right)}{30a^5b^3g^6+150a^4b^4g^6x+300a^3b^5g^6x^2+300a^2b^6g^6x^3+150ab^7g^6x^4+30b^8g^6x^5}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**6,x)
```

```
[Out] -B*d**5*i**2*log(x + (-B*a**4*d**9*i**2/(a*d - b*c)**3 + 4*B*a**3*b*c*d**8*
i**2/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**7*i**2/(a*d - b*c)**3 + 4*B*a*b
**3*c**3*d**6*i**2/(a*d - b*c)**3 + B*a*d**6*i**2 - B*b**4*c**4*d**5*i**2/(
a*d - b*c)**3 + B*b*c*d**5*i**2)/(2*B*b*d**6*i**2))/(30*b**3*g**6*(a*d - b*
c)**3) + B*d**5*i**2*log(x + (B*a**4*d**9*i**2/(a*d - b*c)**3 - 4*B*a**3*b*
c*d**8*i**2/(a*d - b*c)**3 + 6*B*a**2*b**2*c**2*d**7*i**2/(a*d - b*c)**3 -
4*B*a*b**3*c**3*d**6*i**2/(a*d - b*c)**3 + B*a*d**6*i**2 + B*b**4*c**4*d**5
*i**2/(a*d - b*c)**3 + B*b*c*d**5*i**2)/(2*B*b*d**6*i**2))/(30*b**3*g**6*(a
*d - b*c)**3) + (-60*A*a**4*d**4*i**2 - 60*A*a**3*b*c*d**3*i**2 - 60*A*a**2
*b**2*c**2*d**2*i**2 + 540*A*a*b**3*c**3*d*i**2 - 360*A*b**4*c**4*i**2 - 47
*B*a**4*d**4*i**2 - 47*B*a**3*b*c*d**3*i**2 - 47*B*a**2*b**2*c**2*d**2*i**2
 + 153*B*a*b**3*c**3*d*i**2 - 72*B*b**4*c**4*i**2 - 60*B*b**4*d**4*i**2*x**
4 + x**3*(-270*B*a*b**3*d**4*i**2 + 30*B*b**4*c*d**3*i**2) + x**2*(-600*A*a
**2*b**2*d**4*i**2 + 1200*A*a*b**3*c*d**3*i**2 - 600*A*b**4*c**2*d**2*i**2
- 470*B*a**2*b**2*d**4*i**2 + 130*B*a*b**3*c*d**3*i**2 - 20*B*b**4*c**2*d**
2*i**2) + x*(-300*A*a**3*b*d**4*i**2 - 300*A*a**2*b**2*c*d**3*i**2 + 1500*A
*a*b**3*c**2*d**2*i**2 - 900*A*b**4*c**3*d*i**2 - 235*B*a**3*b*d**4*i**2 -
235*B*a**2*b**2*c*d**3*i**2 + 365*B*a*b**3*c**2*d**2*i**2 - 135*B*b**4*c**3
*d*i**2))/(1800*a**7*b**3*d**2*g**6 - 3600*a**6*b**4*c*d*g**6 + 1800*a**5*b
**5*c**2*g**6 + x**5*(1800*a**2*b**8*d**2*g**6 - 3600*a*b**9*c*d*g**6 + 180
0*b**10*c**2*g**6) + x**4*(9000*a**3*b**7*d**2*g**6 - 18000*a**2*b**8*c*d*g
**6 + 9000*a*b**9*c**2*g**6) + x**3*(18000*a**4*b**6*d**2*g**6 - 36000*a**3
```

```
*b**7*c*d*g**6 + 18000*a**2*b**8*c**2*g**6) + x**2*(18000*a**5*b**5*d**2*g*
*6 - 36000*a**4*b**6*c*d*g**6 + 18000*a**3*b**7*c**2*g**6) + x*(9000*a**6*b
**4*d**2*g**6 - 18000*a**5*b**5*c*d*g**6 + 9000*a**4*b**6*c**2*g**6)) + (-B
*a**2*d**2*i**2 - 3*B*a*b*c*d*i**2 - 5*B*a*b*d**2*i**2*x - 6*B*b**2*c**2*i*
*2 - 15*B*b**2*c*d*i**2*x - 10*B*b**2*d**2*i**2*x**2)*log(e*(a + b*x)/(c +
d*x))/(30*a**5*b**3*g**6 + 150*a**4*b**4*g**6*x + 300*a**3*b**5*g**6*x**2 +
 300*a**2*b**6*g**6*x**3 + 150*a*b**7*g**6*x**4 + 30*b**8*g**6*x**5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3029 vs. $2(269) = 538$.

Time = 0.35 (sec) , antiderivative size = 3029, normalized size of antiderivative = 10.78

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x, algor
ithm="maxima")
```

```
[Out] -1/1800*B*d^2*i^2*(60*(10*b^2*x^2 + 5*a*b*x + a^2)*log(b*e*x/(d*x + c) + a*
e/(d*x + c))/(b^8*g^6*x^5 + 5*a*b^7*g^6*x^4 + 10*a^2*b^6*g^6*x^3 + 10*a^3*b
^5*g^6*x^2 + 5*a^4*b^4*g^6*x + a^5*b^3*g^6) + (47*a^2*b^4*c^4 - 278*a^3*b^3
*c^3*d + 822*a^4*b^2*c^2*d^2 - 278*a^5*b*c*d^3 + 47*a^6*d^4 + 60*(10*b^6*c^
2*d^2 - 5*a*b^5*c*d^3 + a^2*b^4*d^4)*x^4 - 30*(10*b^6*c^3*d - 95*a*b^5*c^2*
d^2 + 46*a^2*b^4*c*d^3 - 9*a^3*b^3*d^4)*x^3 + 10*(20*b^6*c^4 - 140*a*b^5*c^
3*d + 537*a^2*b^4*c^2*d^2 - 248*a^3*b^3*c*d^3 + 47*a^4*b^2*d^4)*x^2 + 5*(35
*a*b^5*c^4 - 218*a^2*b^4*c^3*d + 702*a^3*b^3*c^2*d^2 - 278*a^4*b^2*c*d^3 +
47*a^5*b*d^4)*x)/((b^12*c^4 - 4*a*b^11*c^3*d + 6*a^2*b^10*c^2*d^2 - 4*a^3*b
^9*c*d^3 + a^4*b^8*d^4)*g^6*x^5 + 5*(a*b^11*c^4 - 4*a^2*b^10*c^3*d + 6*a^3*
b^9*c^2*d^2 - 4*a^4*b^8*c*d^3 + a^5*b^7*d^4)*g^6*x^4 + 10*(a^2*b^10*c^4 - 4
*a^3*b^9*c^3*d + 6*a^4*b^8*c^2*d^2 - 4*a^5*b^7*c*d^3 + a^6*b^6*d^4)*g^6*x^3
 + 10*(a^3*b^9*c^4 - 4*a^4*b^8*c^3*d + 6*a^5*b^7*c^2*d^2 - 4*a^6*b^6*c*d^3
+ a^7*b^5*d^4)*g^6*x^2 + 5*(a^4*b^8*c^4 - 4*a^5*b^7*c^3*d + 6*a^6*b^6*c^2*d
^2 - 4*a^7*b^5*c*d^3 + a^8*b^4*d^4)*g^6*x + (a^5*b^7*c^4 - 4*a^6*b^6*c^3*d
+ 6*a^7*b^5*c^2*d^2 - 4*a^8*b^4*c*d^3 + a^9*b^3*d^4)*g^6) + 60*(10*b^2*c^2*
d^3 - 5*a*b*c*d^4 + a^2*d^5)*log(b*x + a)/((b^8*c^5 - 5*a*b^7*c^4*d + 10*a^
2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 + 5*a^4*b^4*c*d^4 - a^5*b^3*d^5)*g^6) -
60*(10*b^2*c^2*d^3 - 5*a*b*c*d^4 + a^2*d^5)*log(d*x + c)/((b^8*c^5 - 5*a*b^
7*c^4*d + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 + 5*a^4*b^4*c*d^4 - a^5*b
^3*d^5)*g^6)) - 1/600*B*c*d*i^2*(60*(5*b*x + a)*log(b*e*x/(d*x + c) + a*e/(
d*x + c))/(b^7*g^6*x^5 + 5*a*b^6*g^6*x^4 + 10*a^2*b^5*g^6*x^3 + 10*a^3*b^4*
g^6*x^2 + 5*a^4*b^3*g^6*x + a^5*b^2*g^6) + (27*a*b^4*c^4 - 148*a^2*b^3*c^3*
d + 352*a^3*b^2*c^2*d^2 - 548*a^4*b*c*d^3 + 77*a^5*d^4 - 60*(5*b^5*c*d^3 -
a*b^4*d^4)*x^4 + 30*(5*b^5*c^2*d^2 - 46*a*b^4*c*d^3 + 9*a^2*b^3*d^4)*x^3 -
```

10*(10*b^5*c^3*d - 67*a*b^4*c^2*d^2 + 248*a^2*b^3*c*d^3 - 47*a^3*b^2*d^4)*x^2 + 5*(15*b^5*c^4 - 88*a*b^4*c^3*d + 232*a^2*b^3*c^2*d^2 - 428*a^3*b^2*c*d^3 + 77*a^4*b*d^4)*x)/((b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4)*g^6*x^5 + 5*(a*b^10*c^4 - 4*a^2*b^9*c^3*d + 6*a^3*b^8*c^2*d^2 - 4*a^4*b^7*c*d^3 + a^5*b^6*d^4)*g^6*x^4 + 10*(a^2*b^9*c^4 - 4*a^3*b^8*c^3*d + 6*a^4*b^7*c^2*d^2 - 4*a^5*b^6*c*d^3 + a^6*b^5*d^4)*g^6*x^3 + 10*(a^3*b^8*c^4 - 4*a^4*b^7*c^3*d + 6*a^5*b^6*c^2*d^2 - 4*a^6*b^5*c*d^3 + a^7*b^4*d^4)*g^6*x^2 + 5*(a^4*b^7*c^4 - 4*a^5*b^6*c^3*d + 6*a^6*b^5*c^2*d^2 - 4*a^7*b^4*c*d^3 + a^8*b^3*d^4)*g^6*x + (a^5*b^6*c^4 - 4*a^6*b^5*c^3*d + 6*a^7*b^4*c^2*d^2 - 4*a^8*b^3*c*d^3 + a^9*b^2*d^4)*g^6) - 60*(5*b*c*d^4 - a*d^5)*log(b*x + a)/((b^7*c^5 - 5*a*b^6*c^4*d + 10*a^2*b^5*c^3*d^2 - 10*a^3*b^4*c^2*d^3 + 5*a^4*b^3*c*d^4 - a^5*b^2*d^5)*g^6) + 60*(5*b*c*d^4 - a*d^5)*log(d*x + c)/((b^7*c^5 - 5*a*b^6*c^4*d + 10*a^2*b^5*c^3*d^2 - 10*a^3*b^4*c^2*d^3 + 5*a^4*b^3*c*d^4 - a^5*b^2*d^5)*g^6)) - 1/300*B*c^2*i^2*((60*b^4*d^4*x^4 + 12*b^4*c^4 - 63*a*b^3*c^3*d + 137*a^2*b^2*c^2*d^2 - 163*a^3*b*c*d^3 + 137*a^4*d^4 - 30*(b^4*c*d^3 - 9*a*b^3*d^4)*x^3 + 10*(2*b^4*c^2*d^2 - 13*a*b^3*c*d^3 + 47*a^2*b^2*d^4)*x^2 - 5*(3*b^4*c^3*d - 17*a*b^3*c^2*d^2 + 43*a^2*b^2*c*d^3 - 77*a^3*b*d^4)*x)/((b^10*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*g^6*x^5 + 5*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 + a^5*b^5*d^4)*g^6*x^4 + 10*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2 - 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*g^6*x^3 + 10*(a^3*b^7*c^4 - 4*a^4*b^6*c^3*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*g^6*x^2 + 5*(a^4*b^6*c^4 - 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d^4)*g^6*x + (a^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4*a^8*b^2*c*d^3 + a^9*b*d^4)*g^6) + 60*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^6*g^6*x^5 + 5*a*b^5*g^6*x^4 + 10*a^2*b^4*g^6*x^3 + 10*a^3*b^3*g^6*x^2 + 5*a^4*b^2*g^6*x + a^5*b*g^6) + 60*d^5*log(b*x + a)/((b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5)*g^6) - 60*d^5*log(d*x + c)/((b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5)*g^6)) - 1/10*(5*b*x + a)*A*c*d*i^2/(b^7*g^6*x^5 + 5*a*b^6*g^6*x^4 + 10*a^2*b^5*g^6*x^3 + 10*a^3*b^4*g^6*x^2 + 5*a^4*b^3*g^6*x + a^5*b^2*g^6) - 1/30*(10*b^2*x^2 + 5*a*b*x + a^2)*A*d^2*i^2/(b^8*g^6*x^5 + 5*a*b^7*g^6*x^4 + 10*a^2*b^6*g^6*x^3 + 10*a^3*b^5*g^6*x^2 + 5*a^4*b^4*g^6*x + a^5*b^3*g^6) - 1/5*A*c^2*i^2/(b^6*g^6*x^5 + 5*a*b^5*g^6*x^4 + 10*a^2*b^4*g^6*x^3 + 10*a^3*b^3*g^6*x^2 + 5*a^4*b^2*g^6*x + a^5*b*g^6)

Giac [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.58

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx =$$
$$-\frac{1}{1800}\left(\frac{60\left(6\,Bb^2e^6i^2-\frac{15\,(bex+ae)Bbde^5i^2}{dx+c}+\frac{10\,(bex+ae)^2Bd^2e^4i^2}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^5b^2c^2g^6}{(dx+c)^5}-\frac{2\,(bex+ae)^5abcdg^6}{(dx+c)^5}+\frac{(bex+ae)^5a^2d^2g^6}{(dx+c)^5}}+\frac{360\,Ab^2e^6i^2+72\,Bb^2e^6i^2-}{}\right.$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x, algor
ithm="giac")
```

```
[Out] -1/1800*(60*(6*B*b^2*e^6*i^2 - 15*(b*e*x + a*e)*B*b*d*e^5*i^2/(d*x + c) + 1
0*(b*e*x + a*e)^2*B*d^2*e^4*i^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/(
(b*e*x + a*e)^5*b^2*c^2*g^6/(d*x + c)^5 - 2*(b*e*x + a*e)^5*a*b*c*d*g^6/(d*
x + c)^5 + (b*e*x + a*e)^5*a^2*d^2*g^6/(d*x + c)^5) + (360*A*b^2*e^6*i^2 +
72*B*b^2*e^6*i^2 - 900*(b*e*x + a*e)*A*b*d*e^5*i^2/(d*x + c) - 225*(b*e*x +
 a*e)*B*b*d*e^5*i^2/(d*x + c) + 600*(b*e*x + a*e)^2*A*d^2*e^4*i^2/(d*x + c)
^2 + 200*(b*e*x + a*e)^2*B*d^2*e^4*i^2/(d*x + c)^2)/((b*e*x + a*e)^5*b^2*c^
2*g^6/(d*x + c)^5 - 2*(b*e*x + a*e)^5*a*b*c*d*g^6/(d*x + c)^5 + (b*e*x + a*
e)^5*a^2*d^2*g^6/(d*x + c)^5))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b
*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 4.61 (sec) , antiderivative size = 941, normalized size of antiderivative = 3.35

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$$
$$=\frac{B\,d^5\,i^2\,\mathrm{atanh}\left(\frac{30\,a^3\,b^3\,d^3\,g^6-30\,a^2\,b^4\,c\,d^2\,g^6-30\,a\,b^5\,c^2\,d\,g^6+30\,b^6\,c^3\,g^6}{30\,b^3\,g^6\,(a\,d-b\,c)^3}+\frac{2\,b\,d\,x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^3}\right)}{15\,b^3\,g^6\,(a\,d-b\,c)^3}$$
$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(a\left(\frac{B\,a\,d^2\,i^2}{30\,b^4\,g^6}+\frac{B\,c\,d\,i^2}{10\,b^3\,g^6}\right)+x\left(b\left(\frac{B\,a\,d^2\,i^2}{30\,b^4\,g^6}+\frac{B\,c\,d\,i^2}{10\,b^3\,g^6}\right)+\frac{2\,B\,a\,d^2\,i^2}{15\,b^3\,g^6}+\frac{2\,B\,c\,d\,i^2}{5\,b^2\,g^6}\right)+\frac{B\,c^2\,i^2}{5\,b^2\,g^6}+\frac{B\,d^2\,i^2\,x^2}{3\,b^2\,g^6}\right)}{5\,a^4\,x+\frac{a^5}{b}+b^4\,x^5+10\,a^3\,b\,x^2+5\,a\,b^3\,x^4+10\,a^2\,b^2\,x^3}$$
$$-\frac{60\,A\,a^4\,d^4\,i^2+360\,A\,b^4\,c^4\,i^2+47\,B\,a^4\,d^4\,i^2+72\,B\,b^4\,c^4\,i^2+60\,A\,a^2\,b^2\,c^2\,d^2\,i^2+47\,B\,a^2\,b^2\,c^2\,d^2\,i^2-540\,A\,a\,b^3\,c^3\,d\,i^2+60\,A\,a^3\,b\,c\,d^3\,i^2-153\,B\,a\,b^3\,c}{60\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}$$

```
[In] int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^6,
x)
```

```
[Out] (B*d^5*i^2*atanh((30*b^6*c^3*g^6 + 30*a^3*b^3*d^3*g^6 - 30*a*b^5*c^2*d*g^6
- 30*a^2*b^4*c*d^2*g^6)/(30*b^3*g^6*(a*d - b*c)^3) + (2*b*d*x*(a^2*d^2 + b^
2*c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(15*b^3*g^6*(a*d - b*c)^3) - (log((e*(a
 + b*x))/(c + d*x))*(a*((B*a*d^2*i^2)/(30*b^4*g^6) + (B*c*d*i^2)/(10*b^3*g^
6)) + x*(b*((B*a*d^2*i^2)/(30*b^4*g^6) + (B*c*d*i^2)/(10*b^3*g^6)) + (2*B*a
*d^2*i^2)/(15*b^3*g^6) + (2*B*c*d*i^2)/(5*b^2*g^6)) + (B*c^2*i^2)/(5*b^2*g^
6) + (B*d^2*i^2*x^2)/(3*b^2*g^6)))/(5*a^4*x + a^5/b + b^4*x^5 + 10*a^3*b*x^
2 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3) - ((60*A*a^4*d^4*i^2 + 360*A*b^4*c^4*i^2
+ 47*B*a^4*d^4*i^2 + 72*B*b^4*c^4*i^2 + 60*A*a^2*b^2*c^2*d^2*i^2 + 47*B*a^2
*b^2*c^2*d^2*i^2 - 540*A*a*b^3*c^3*d*i^2 + 60*A*a^3*b*c*d^3*i^2 - 153*B*a*b
^3*c^3*d*i^2 + 47*B*a^3*b*c*d^3*i^2)/(60*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) +
 (x^2*(60*A*a^2*b^2*d^4*i^2 + 47*B*a^2*b^2*d^4*i^2 + 60*A*b^4*c^2*d^2*i^2 +
 2*B*b^4*c^2*d^2*i^2 - 120*A*a*b^3*c*d^3*i^2 - 13*B*a*b^3*c*d^3*i^2))/(6*(a
^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(60*A*a^3*b*d^4*i^2 + 47*B*a^3*b*d^4*i^
2 + 180*A*b^4*c^3*d*i^2 + 27*B*b^4*c^3*d*i^2 - 300*A*a*b^3*c^2*d^2*i^2 + 60
*A*a^2*b^2*c*d^3*i^2 - 73*B*a*b^3*c^2*d^2*i^2 + 47*B*a^2*b^2*c*d^3*i^2))/(1
2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (d*x^3*(9*B*a*b^3*d^3*i^2 - B*b^4*c*d^
2*i^2))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B*b^4*d^4*i^2*x^4)/(a^2*d^2
+ b^2*c^2 - 2*a*b*c*d))/(30*a^5*b^3*g^6 + 30*b^8*g^6*x^5 + 150*a^4*b^4*g^6*
x + 150*a*b^7*g^6*x^4 + 300*a^3*b^5*g^6*x^2 + 300*a^2*b^6*g^6*x^3)
```

3.20 $\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

Optimal result

Integrand size = 40, antiderivative size = 457

$$\begin{aligned}
&\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx \\
&= \frac{B(bc-ad)^6g^3i^3x}{140b^3d^3} + \frac{B(bc-ad)^5g^3i^3(c+dx)^2}{280b^2d^4} + \frac{B(bc-ad)^4g^3i^3(c+dx)^3}{420bd^4} \\
&\quad - \frac{17B(bc-ad)^3g^3i^3(c+dx)^4}{280d^4} + \frac{bB(bc-ad)^2g^3i^3(c+dx)^5}{14d^4} - \frac{b^2B(bc-ad)g^3i^3(c+dx)^6}{42d^4} \\
&\quad + \frac{B(bc-ad)^7g^3i^3\log\left(\frac{a+bx}{c+dx}\right)}{140b^4d^4} - \frac{(bc-ad)^3g^3i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^4} \\
&\quad + \frac{3b(bc-ad)^2g^3i^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^4} \\
&\quad - \frac{b^2(bc-ad)g^3i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^4} \\
&\quad + \frac{b^3g^3i^3(c+dx)^7\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{7d^4} + \frac{B(bc-ad)^7g^3i^3\log(c+dx)}{140b^4d^4}
\end{aligned}$$

```
[Out] 1/140*B*(-a*d+b*c)^6*g^3*i^3*x/b^3/d^3+1/280*B*(-a*d+b*c)^5*g^3*i^3*(d*x+c)
^2/b^2/d^4+1/420*B*(-a*d+b*c)^4*g^3*i^3*(d*x+c)^3/b/d^4-17/280*B*(-a*d+b*c)
^3*g^3*i^3*(d*x+c)^4/d^4+1/14*b*B*(-a*d+b*c)^2*g^3*i^3*(d*x+c)^5/d^4-1/42*b
^2*B*(-a*d+b*c)*g^3*i^3*(d*x+c)^6/d^4+1/140*B*(-a*d+b*c)^7*g^3*i^3*ln((b*x+
a)/(d*x+c))/b^4/d^4-1/4*(-a*d+b*c)^3*g^3*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d
*x+c)))/d^4+3/5*b*(-a*d+b*c)^2*g^3*i^3*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d*x+c))
)/d^4-1/2*b^2*(-a*d+b*c)*g^3*i^3*(d*x+c)^6*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4+
1/7*b^3*g^3*i^3*(d*x+c)^7*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4+1/140*B*(-a*d+b*c
)^7*g^3*i^3*ln(d*x+c)/b^4/d^4
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2562, 45, 2382, 12, 1634}

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{b^3g^3i^3(c+dx)^7\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{7d^4}-\frac{b^2g^3i^3(c+dx)^6(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d^4}$$
$$-\frac{g^3i^3(c+dx)^4(bc-ad)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^4}$$
$$+\frac{3bg^3i^3(c+dx)^5(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^4}+\frac{Bg^3i^3(bc-ad)^7\log\left(\frac{a+bx}{c+dx}\right)}{140b^4d^4}$$
$$+\frac{Bg^3i^3(bc-ad)^7\log(c+dx)}{140b^4d^4}+\frac{Bg^3i^3x(bc-ad)^6}{140b^3d^3}+\frac{Bg^3i^3(c+dx)^2(bc-ad)^5}{280b^2d^4}$$
$$-\frac{b^2Bg^3i^3(c+dx)^6(bc-ad)}{42d^4}+\frac{Bg^3i^3(c+dx)^3(bc-ad)^4}{420bd^4}$$
$$-\frac{17Bg^3i^3(c+dx)^4(bc-ad)^3}{280d^4}+\frac{bBg^3i^3(c+dx)^5(bc-ad)^2}{14d^4}$$

```
[In] Int[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (B*(b*c - a*d)^6*g^3*i^3*x)/(140*b^3*d^3) + (B*(b*c - a*d)^5*g^3*i^3*(c + d
*x)^2)/(280*b^2*d^4) + (B*(b*c - a*d)^4*g^3*i^3*(c + d*x)^3)/(420*b*d^4) -
(17*B*(b*c - a*d)^3*g^3*i^3*(c + d*x)^4)/(280*d^4) + (b*B*(b*c - a*d)^2*g^3
*i^3*(c + d*x)^5)/(14*d^4) - (b^2*B*(b*c - a*d)*g^3*i^3*(c + d*x)^6)/(42*d^
4) + (B*(b*c - a*d)^7*g^3*i^3*Log[(a + b*x)/(c + d*x)])/(140*b^4*d^4) - ((b
*c - a*d)^3*g^3*i^3*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*d^
4) + (3*b*(b*c - a*d)^2*g^3*i^3*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d
*x)]))/(5*d^4) - (b^2*(b*c - a*d)*g^3*i^3*(c + d*x)^6*(A + B*Log[(e*(a + b*
x))/(c + d*x)]))/(2*d^4) + (b^3*g^3*i^3*(c + d*x)^7*(A + B*Log[(e*(a + b*x)
)/(c + d*x)]))/(7*d^4) + (B*(b*c - a*d)^7*g^3*i^3*Log[c + d*x])/(140*b^4*d^
4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((bc-ad)^7 g^3 i^3\right) \text{Subst}\left(\int \frac{x^3(A+B\log(ex))}{(b-dx)^8}\,dx, x, \frac{a+bx}{c+dx}\right)\\
&= -\frac{(bc-ad)^3 g^3 i^3 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^4}\\
&\quad+\frac{3b(bc-ad)^2 g^3 i^3 (c+dx)^5 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^4}\\
&\quad-\frac{b^2(bc-ad) g^3 i^3 (c+dx)^6 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^4}\\
&\quad+\frac{b^3 g^3 i^3 (c+dx)^7 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{7d^4}\\
&\quad-\left(B(bc-ad)^7 g^3 i^3\right) \text{Subst}\left(\int \frac{-b^3+7b^2dx-21bd^2x^2+35d^3x^3}{140d^4x(b-dx)^7}\,dx, x, \frac{a+bx}{c+dx}\right)
\end{aligned}$$

$$= -\frac{(bc-ad)^3 g^3 i^3 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^4} + \frac{3b(bc-ad)^2 g^3 i^3 (c+dx)^5 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^4} - \frac{b^2(bc-ad) g^3 i^3 (c+dx)^6 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^4} + \frac{b^3 g^3 i^3 (c+dx)^7 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{7d^4} - \frac{(B(bc-ad)^7 g^3 i^3)\operatorname{Subst}\left(\int \frac{-b^3+7b^2dx-21bd^2x^2+35d^3x^3}{x(b-dx)^7}\,dx, x, \frac{a+bx}{c+dx}\right)}{140d^4}$$

$$= -\frac{(bc-ad)^3 g^3 i^3 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^4} + \frac{3b(bc-ad)^2 g^3 i^3 (c+dx)^5 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^4} - \frac{b^2(bc-ad) g^3 i^3 (c+dx)^6 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^4} + \frac{b^3 g^3 i^3 (c+dx)^7 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{7d^4} - \frac{(B(bc-ad)^7 g^3 i^3)\operatorname{Subst}\left(\int \left(-\frac{1}{b^4x} + \frac{20b^2d}{(b-dx)^7} - \frac{50bd}{(b-dx)^6} + \frac{34d}{(b-dx)^5} - \frac{d}{b(b-dx)^4} - \frac{d}{b^2(b-dx)^3} - \frac{d}{b^3(b-dx)^2}\right.\right.}{140d^4}$$

$$= \frac{B(bc-ad)^6 g^3 i^3 x}{140b^3d^3} + \frac{B(bc-ad)^5 g^3 i^3 (c+dx)^2}{280b^2d^4} + \frac{B(bc-ad)^4 g^3 i^3 (c+dx)^3}{420bd^4} - \frac{17B(bc-ad)^3 g^3 i^3 (c+dx)^4}{280d^4} + \frac{bB(bc-ad)^2 g^3 i^3 (c+dx)^5}{14d^4} - \frac{b^2B(bc-ad) g^3 i^3 (c+dx)^6}{42d^4} + \frac{B(bc-ad)^7 g^3 i^3 \log\left(\frac{a+bx}{c+dx}\right)}{140b^4d^4} - \frac{(bc-ad)^3 g^3 i^3 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^4} + \frac{3b(bc-ad)^2 g^3 i^3 (c+dx)^5 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^4} - \frac{b^2(bc-ad) g^3 i^3 (c+dx)^6 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^4} + \frac{b^3 g^3 i^3 (c+dx)^7 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{7d^4} + \frac{B(bc-ad)^7 g^3 i^3 \log(c+dx)}{140b^4d^4}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.28

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{g^3i^3\left(\frac{120b^2Bc(bc-ad)^5x}{d^3}-\frac{126bB(bc-ad)^6x}{d^3}+\frac{120abB(-bc+ad)^5x}{d^2}-\frac{60bBc(bc-ad)^4(a+bx)^2}{d^2}+\frac{60aB(bc-ad)^4(a+bx)^2}{d}+\frac{63B(bc-ad)}{d^2}\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x
)]),x]
```

```
[Out] (g^3*i^3*((120*b^2*B*c*(b*c - a*d)^5*x)/d^3 - (126*b*B*(b*c - a*d)^6*x)/d^3
 + (120*a*b*B*(-(b*c) + a*d)^5*x)/d^2 - (60*b*B*c*(b*c - a*d)^4*(a + b*x)^2
)/d^2 + (60*a*B*(b*c - a*d)^4*(a + b*x)^2)/d + (63*B*(b*c - a*d)^5*(a + b*x
)^2)/d^2 + (40*b*B*c*(b*c - a*d)^3*(a + b*x)^3)/d - (42*B*(b*c - a*d)^4*(a
+ b*x)^3)/d + 40*a*B*(-(b*c) + a*d)^3*(a + b*x)^3 - 30*b*B*c*(b*c - a*d)^2*
(a + b*x)^4 + 30*a*B*d*(b*c - a*d)^2*(a + b*x)^4 + 21*B*(-(b*c) + a*d)^3*(a
 + b*x)^4 + 24*b*B*c*d*(b*c - a*d)*(a + b*x)^5 - 84*B*d*(b*c - a*d)^2*(a +
b*x)^5 + 24*a*B*d^2*(-(b*c) + a*d)*(a + b*x)^5 - 20*b*B*c*d^2*(a + b*x)^6 +
 20*a*B*d^3*(a + b*x)^6 + 210*(b*c - a*d)^3*(a + b*x)^4*(A + B*Log[(e*(a +
b*x))/(c + d*x)]) + 504*d*(b*c - a*d)^2*(a + b*x)^5*(A + B*Log[(e*(a + b*x)
)/(c + d*x)]) + 420*d^2*(b*c - a*d)*(a + b*x)^6*(A + B*Log[(e*(a + b*x))/(c
 + d*x)]) + 120*d^3*(a + b*x)^7*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (120
*b*B*c*(b*c - a*d)^6*Log[c + d*x])/d^4 + (120*a*B*(b*c - a*d)^6*Log[c + d*x
])/d^3 + (126*B*(b*c - a*d)^7*Log[c + d*x])/d^4))/(840*b^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1193 vs. 2(433) = 866.

Time = 1.44 (sec) , antiderivative size = 1194, normalized size of antiderivative = 2.61

method	result	size
risch	Expression too large to display	1194
parallelrisch	Expression too large to display	1958
parts	Expression too large to display	2513
derivativedivides	Expression too large to display	2536
default	Expression too large to display	2536

```
[In] int((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETUR
NVERBOSE)
```

```
[Out] 3/10*i^3*g^3*d*B*a^3*c^2*x^2-1/40*i^3*g^3/d*b^2*B*a*c^4*x^2+i^3*g^3*A*a^3*c
^3*x+i^3*g^3*d^2*A*a^3*c*x^3+3*i^3*g^3*d*b*A*a^2*c^2*x^3+1/2*i^3*g^3*d^3*b^
```

```
2*A*a*x^6+1/2*i^3*g^3*d^2*b^3*A*c*x^6+1/42*i^3*g^3*d^3*b^2*B*a*x^6-1/42*i^3
*g^3*d^2*b^3*B*c*x^6+3/5*i^3*g^3*d^3*b*A*a^2*x^5+3/5*i^3*g^3*d*b^3*A*c^2*x^
5+1/14*i^3*g^3*d^3*b*B*a^2*x^5+7/30*i^3*g^3*d^2*B*a^3*c*x^3-1/14*i^3*g^3*d*
b^3*B*c^2*x^5+1/4*i^3*g^3*d^3*A*a^3*x^4+1/4*i^3*g^3*b^3*A*c^3*x^4+17/280*i^
3*g^3*d^3*B*a^3*x^4-17/280*i^3*g^3*b^3*B*c^3*x^4+9/5*i^3*g^3*d^2*b^2*A*a*c*
x^5+9/4*i^3*g^3*d^2*b*A*a^2*c*x^4+9/4*i^3*g^3*d*b^2*A*a*c^2*x^4+7/40*i^3*g^
3*d^2*b*B*a^2*c*x^4-7/40*i^3*g^3*d*b^2*B*a*c^2*x^4+i^3*g^3*b^2*A*a*c^3*x^3+
1/420*i^3*g^3*d^3/b*B*a^4*x^3-1/420*i^3*g^3/d*b^3*B*c^4*x^3-1/280*i^3*g^3*d
^3/b^2*B*a^5*x^2+1/280*i^3*g^3/d^2*b^3*B*c^5*x^2+1/140*i^3*g^3*d^3/b^3*B*a^
6*x-1/140*i^3*g^3/d^3*b^3*B*c^6*x+1/140*i^3*g^3/d^4*b^3*B*ln(-d*x-c)*c^7-1/
140*i^3*g^3*d^3/b^4*B*ln(b*x+a)*a^7-1/4*i^3*g^3/d*B*ln(-d*x-c)*a^3*c^4+1/4*
i^3*g^3/b*B*ln(b*x+a)*a^4*c^3-3/10*i^3*g^3*b*B*a^2*c^3*x^2-7/30*i^3*g^3*b^2
*B*a*c^3*x^3+3/2*i^3*g^3*d*A*a^3*c^2*x^2+3/2*i^3*g^3*b*A*a^2*c^3*x^2+1/40*i
^3*g^3*d^2/b*B*a^4*c*x^2-1/20*i^3*g^3*d^2/b^2*B*a^5*c*x+3/20*i^3*g^3*d/b*B*
a^4*c^2*x-3/20*i^3*g^3/d*b*B*a^2*c^4*x+1/20*i^3*g^3/d^2*b^2*B*a*c^5*x+3/20*
i^3*g^3/d^2*b*B*ln(-d*x-c)*a^2*c^5+1/20*i^3*g^3*d^2/b^3*B*ln(b*x+a)*a^6*c-3
/20*i^3*g^3*d/b^2*B*ln(b*x+a)*a^5*c^2-1/20*i^3*g^3/d^3*b^2*B*ln(-d*x-c)*a*c
^6+1/140*i^3*g^3*B*x*(20*b^3*d^3*x^6+70*a*b^2*d^3*x^5+70*b^3*c*d^2*x^5+84*a
^2*b*d^3*x^4+252*a*b^2*c*d^2*x^4+84*b^3*c^2*d*x^4+35*a^3*d^3*x^3+315*a^2*b*
c*d^2*x^3+315*a*b^2*c^2*d*x^3+35*b^3*c^3*x^3+140*a^3*c*d^2*x^2+420*a^2*b*c^
2*d*x^2+140*a*b^2*c^3*x^2+210*a^3*c^2*d*x+210*a^2*b*c^3*x+140*a^3*c^3)*ln(e
*(b*x+a)/(d*x+c))+1/7*i^3*g^3*d^3*b^3*A*x^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(433) = 866$.

Time = 0.59 (sec) , antiderivative size = 912, normalized size of antiderivative = 2.00

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{120\,Ab^7d^7g^3i^3x^7+20\left((21\,A-B)b^7cd^6+(21\,A+B)ab^6d^7\right)g^3i^3x^6+12\left((42\,A-5\,B)b^7c^2d^5+126\,Aab^6cd\right.}{}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algor
ithm="fricas")
```

```
[Out] 1/840*(120*A*b^7*d^7*g^3*i^3*x^7 + 20*((21*A - B)*b^7*c*d^6 + (21*A + B)*a*
b^6*d^7)*g^3*i^3*x^6 + 12*((42*A - 5*B)*b^7*c^2*d^5 + 126*A*a*b^6*c*d^6 + (
42*A + 5*B)*a^2*b^5*d^7)*g^3*i^3*x^5 + 3*((70*A - 17*B)*b^7*c^3*d^4 + 7*(90
*A - 7*B)*a*b^6*c^2*d^5 + 7*(90*A + 7*B)*a^2*b^5*c*d^6 + (70*A + 17*B)*a^3*
b^4*d^7)*g^3*i^3*x^4 - 2*(B*b^7*c^4*d^3 - 14*(30*A - 7*B)*a*b^6*c^3*d^4 - 1
260*A*a^2*b^5*c^2*d^5 - 14*(30*A + 7*B)*a^3*b^4*c*d^6 - B*a^4*b^3*d^7)*g^3*
i^3*x^3 + 3*(B*b^7*c^5*d^2 - 7*B*a*b^6*c^4*d^3 + 84*(5*A - B)*a^2*b^5*c^3*d
^4 + 84*(5*A + B)*a^3*b^4*c^2*d^5 + 7*B*a^4*b^3*c*d^6 - B*a^5*b^2*d^7)*g^3*
```

```
i^3*x^2 - 6*(B*b^7*c^6*d - 7*B*a*b^6*c^5*d^2 + 21*B*a^2*b^5*c^4*d^3 - 140*A
*a^3*b^4*c^3*d^4 - 21*B*a^4*b^3*c^2*d^5 + 7*B*a^5*b^2*c*d^6 - B*a^6*b*d^7)*
g^3*i^3*x + 6*(35*B*a^4*b^3*c^3*d^4 - 21*B*a^5*b^2*c^2*d^5 + 7*B*a^6*b*c*d^
6 - B*a^7*d^7)*g^3*i^3*log(b*x + a) + 6*(B*b^7*c^7 - 7*B*a*b^6*c^6*d + 21*B
*a^2*b^5*c^5*d^2 - 35*B*a^3*b^4*c^4*d^3)*g^3*i^3*log(d*x + c) + 6*(20*B*b^7
*d^7*g^3*i^3*x^7 + 140*B*a^3*b^4*c^3*d^4*g^3*i^3*x + 70*(B*b^7*c*d^6 + B*a*
b^6*d^7)*g^3*i^3*x^6 + 84*(B*b^7*c^2*d^5 + 3*B*a*b^6*c*d^6 + B*a^2*b^5*d^7)
*g^3*i^3*x^5 + 35*(B*b^7*c^3*d^4 + 9*B*a*b^6*c^2*d^5 + 9*B*a^2*b^5*c*d^6 +
B*a^3*b^4*d^7)*g^3*i^3*x^4 + 140*(B*a*b^6*c^3*d^4 + 3*B*a^2*b^5*c^2*d^5 + B
*a^3*b^4*c*d^6)*g^3*i^3*x^3 + 210*(B*a^2*b^5*c^3*d^4 + B*a^3*b^4*c^2*d^5)*g
^3*i^3*x^2)*log((b*e*x + a*e)/(d*x + c)))/(b^4*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2161 vs. $2(427) = 854$.

Time = 16.36 (sec) , antiderivative size = 2161, normalized size of antiderivative = 4.73

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)**3*(d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b**3*d**3*g**3*i**3*x**7/7 - B*a**4*g**3*i**3*(a**3*d**3 - 7*a**2*b*c*d**
2 + 21*a*b**2*c**2*d - 35*b**3*c**3)*log(x + (B*a**7*c*d**6*g**3*i**3 - 7*B
*a**6*b*c**2*d**5*g**3*i**3 + 21*B*a**5*b**2*c**3*d**4*g**3*i**3 + B*a**5*d
**4*g**3*i**3*(a**3*d**3 - 7*a**2*b*c*d**2 + 21*a*b**2*c**2*d - 35*b**3*c**
3)/b - 70*B*a**4*b**3*c**4*d**3*g**3*i**3 - B*a**4*c*d**3*g**3*i**3*(a**3*d
**3 - 7*a**2*b*c*d**2 + 21*a*b**2*c**2*d - 35*b**3*c**3) + 21*B*a**3*b**4*c
**5*d**2*g**3*i**3 - 7*B*a**2*b**5*c**6*d*g**3*i**3 + B*a*b**6*c**7*g**3*i*
*3)/(B*a**7*d**7*g**3*i**3 - 7*B*a**6*b*c*d**6*g**3*i**3 + 21*B*a**5*b**2*c
**2*d**5*g**3*i**3 - 35*B*a**4*b**3*c**3*d**4*g**3*i**3 - 35*B*a**3*b**4*c*
*4*d**3*g**3*i**3 + 21*B*a**2*b**5*c**5*d**2*g**3*i**3 - 7*B*a*b**6*c**6*d*
g**3*i**3 + B*b**7*c**7*g**3*i**3))/(140*b**4) - B*c**4*g**3*i**3*(35*a**3*
d**3 - 21*a**2*b*c*d**2 + 7*a*b**2*c**2*d - b**3*c**3)*log(x + (B*a**7*c*d*
*6*g**3*i**3 - 7*B*a**6*b*c**2*d**5*g**3*i**3 + 21*B*a**5*b**2*c**3*d**4*g*
*3*i**3 - 70*B*a**4*b**3*c**4*d**3*g**3*i**3 + 21*B*a**3*b**4*c**5*d**2*g**
3*i**3 - 7*B*a**2*b**5*c**6*d*g**3*i**3 + B*a*b**6*c**7*g**3*i**3 + B*a*b**
3*c**4*g**3*i**3*(35*a**3*d**3 - 21*a**2*b*c*d**2 + 7*a*b**2*c**2*d - b**3*
c**3) - B*b**4*c**5*g**3*i**3*(35*a**3*d**3 - 21*a**2*b*c*d**2 + 7*a*b**2*c
**2*d - b**3*c**3)/d)/(B*a**7*d**7*g**3*i**3 - 7*B*a**6*b*c*d**6*g**3*i**3
+ 21*B*a**5*b**2*c**2*d**5*g**3*i**3 - 35*B*a**4*b**3*c**3*d**4*g**3*i**3 -
 35*B*a**3*b**4*c**4*d**3*g**3*i**3 + 21*B*a**2*b**5*c**5*d**2*g**3*i**3 -
7*B*a*b**6*c**6*d*g**3*i**3 + B*b**7*c**7*g**3*i**3))/(140*d**4) + x**6*(A*
a*b**2*d**3*g**3*i**3/2 + A*b**3*c*d**2*g**3*i**3/2 + B*a*b**2*d**3*g**3*i*
*3/42 - B*b**3*c*d**2*g**3*i**3/42) + x**5*(3*A*a**2*b*d**3*g**3*i**3/5 + 9
```

```
*A*a*b**2*c*d**2*g**3*i**3/5 + 3*A*b**3*c**2*d*g**3*i**3/5 + B*a**2*b*d**3*
g**3*i**3/14 - B*b**3*c**2*d*g**3*i**3/14) + x**4*(A*a**3*d**3*g**3*i**3/4
+ 9*A*a**2*b*c*d**2*g**3*i**3/4 + 9*A*a*b**2*c**2*d*g**3*i**3/4 + A*b**3*c*
*3*g**3*i**3/4 + 17*B*a**3*d**3*g**3*i**3/280 + 7*B*a**2*b*c*d**2*g**3*i**3
/40 - 7*B*a*b**2*c**2*d*g**3*i**3/40 - 17*B*b**3*c**3*g**3*i**3/280) + x**3
*(A*a**3*c*d**2*g**3*i**3 + 3*A*a**2*b*c**2*d*g**3*i**3 + A*a*b**2*c**3*g**
3*i**3 + B*a**4*d**3*g**3*i**3/(420*b) + 7*B*a**3*c*d**2*g**3*i**3/30 - 7*B
*a*b**2*c**3*g**3*i**3/30 - B*b**3*c**4*g**3*i**3/(420*d)) + x**2*(3*A*a**3
*c**2*d*g**3*i**3/2 + 3*A*a**2*b*c**3*g**3*i**3/2 - B*a**5*d**3*g**3*i**3/(
280*b**2) + B*a**4*c*d**2*g**3*i**3/(40*b) + 3*B*a**3*c**2*d*g**3*i**3/10 -
 3*B*a**2*b*c**3*g**3*i**3/10 - B*a*b**2*c**4*g**3*i**3/(40*d) + B*b**3*c**
5*g**3*i**3/(280*d**2)) + x*(A*a**3*c**3*g**3*i**3 + B*a**6*d**3*g**3*i**3/
(140*b**3) - B*a**5*c*d**2*g**3*i**3/(20*b**2) + 3*B*a**4*c**2*d*g**3*i**3/
(20*b) - 3*B*a**2*b*c**4*g**3*i**3/(20*d) + B*a*b**2*c**5*g**3*i**3/(20*d**
2) - B*b**3*c**6*g**3*i**3/(140*d**3)) + (B*a**3*c**3*g**3*i**3*x + 3*B*a**
3*c**2*d*g**3*i**3*x**2/2 + B*a**3*c*d**2*g**3*i**3*x**3 + B*a**3*d**3*g**3
*i**3*x**4/4 + 3*B*a**2*b*c**3*g**3*i**3*x**2/2 + 3*B*a**2*b*c**2*d*g**3*i*
*3*x**3 + 9*B*a**2*b*c*d**2*g**3*i**3*x**4/4 + 3*B*a**2*b*d**3*g**3*i**3*x*
*5/5 + B*a*b**2*c**3*g**3*i**3*x**3 + 9*B*a*b**2*c**2*d*g**3*i**3*x**4/4 +
9*B*a*b**2*c*d**2*g**3*i**3*x**5/5 + B*a*b**2*d**3*g**3*i**3*x**6/2 + B*b**
3*c**3*g**3*i**3*x**4/4 + 3*B*b**3*c**2*d*g**3*i**3*x**5/5 + B*b**3*c*d**2*
g**3*i**3*x**6/2 + B*b**3*d**3*g**3*i**3*x**7/7)*log(e*(a + b*x)/(c + d*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2637 vs. 2(433) = 866.

Time = 0.26 (sec) , antiderivative size = 2637, normalized size of antiderivative = 5.77

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algor
ithm="maxima")
```

```
[Out] 1/7*A*b^3*d^3*g^3*i^3*x^7 + 1/2*A*b^3*c*d^2*g^3*i^3*x^6 + 1/2*A*a*b^2*d^3*g
^3*i^3*x^6 + 3/5*A*b^3*c^2*d*g^3*i^3*x^5 + 9/5*A*a*b^2*c*d^2*g^3*i^3*x^5 +
3/5*A*a^2*b*d^3*g^3*i^3*x^5 + 1/4*A*b^3*c^3*g^3*i^3*x^4 + 9/4*A*a*b^2*c^2*d
*g^3*i^3*x^4 + 9/4*A*a^2*b*c*d^2*g^3*i^3*x^4 + 1/4*A*a^3*d^3*g^3*i^3*x^4 +
A*a*b^2*c^3*g^3*i^3*x^3 + 3*A*a^2*b*c^2*d*g^3*i^3*x^3 + A*a^3*c*d^2*g^3*i^3
*x^3 + 3/2*A*a^2*b*c^3*g^3*i^3*x^2 + 3/2*A*a^3*c^2*d*g^3*i^3*x^2 + (x*log(b
*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*a^
3*c^3*g^3*i^3 + 3/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x
 + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a^2*b*c^3*g^3*i^3
 + 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3
 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2
```

)*x)/(b^2*d^2))*B*a*b^2*c^3*g^3*i^3 + 1/24*(6*x^4*log(b*e*x/(d*x + c) + a*e
/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d
^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)
*x)/(b^3*d^3))*B*b^3*c^3*g^3*i^3 + 3/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x
+ c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*
B*a^3*c^2*d*g^3*i^3 + 3/2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a
^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2
*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a^2*b*c^2*d*g^3*i^3 + 3/8*(6*x^4*log(b
*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x +
c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6
*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*a*b^2*c^2*d*g^3*i^3 + 1/20*(12*x^5*log
(b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*
x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)
x^3 + 6(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))
*B*b^3*c^2*d*g^3*i^3 + 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*
a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 -
2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a^3*c*d^2*g^3*i^3 + 3/8*(6*x^4*log(b*
e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c
)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*
(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*a^2*b*c*d^2*g^3*i^3 + 3/20*(12*x^5*log(
b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x
 + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*
x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*
B*a*b^2*c*d^2*g^3*i^3 + 1/120*(60*x^6*log(b*e*x/(d*x + c) + a*e/(d*x + c))
- 60*a^6*log(b*x + a)/b^6 + 60*c^6*log(d*x + c)/d^6 - (12*(b^5*c*d^4 - a*b^
4*d^5)*x^5 - 15*(b^5*c^2*d^3 - a^2*b^3*d^5)*x^4 + 20*(b^5*c^3*d^2 - a^3*b^2
*d^5)*x^3 - 30*(b^5*c^4*d - a^4*b*d^5)*x^2 + 60*(b^5*c^5 - a^5*d^5)*x)/(b^5
*d^5))*B*b^3*c*d^2*g^3*i^3 + 1/24*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c
)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^
2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*
d^3))*B*a^3*d^3*g^3*i^3 + 1/20*(12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c))
 + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^
3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*
x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*a^2*b*d^3*g^3*i^3 + 1/120*(60*
x^6*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 60*a^6*log(b*x + a)/b^6 + 60*c^6
*log(d*x + c)/d^6 - (12*(b^5*c*d^4 - a*b^4*d^5)*x^5 - 15*(b^5*c^2*d^3 - a^2
*b^3*d^5)*x^4 + 20*(b^5*c^3*d^2 - a^3*b^2*d^5)*x^3 - 30*(b^5*c^4*d - a^4*b*
d^5)*x^2 + 60*(b^5*c^5 - a^5*d^5)*x)/(b^5*d^5))*B*a*b^2*d^3*g^3*i^3 + 1/420
*(60*x^7*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 60*a^7*log(b*x + a)/b^7 - 6
0*c^7*log(d*x + c)/d^7 - (10*(b^6*c*d^5 - a*b^5*d^6)*x^6 - 12*(b^6*c^2*d^4
- a^2*b^4*d^6)*x^5 + 15*(b^6*c^3*d^3 - a^3*b^3*d^6)*x^4 - 20*(b^6*c^4*d^2 -
 a^4*b^2*d^6)*x^3 + 30*(b^6*c^5*d - a^5*b*d^6)*x^2 - 60*(b^6*c^6 - a^6*d^6)
*x)/(b^6*d^6))*B*b^3*d^3*g^3*i^3 + A*a^3*c^3*g^3*i^3*x

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5520 vs. 2(433) = 866.

Time = 0.67 (sec) , antiderivative size = 5520, normalized size of antiderivative = 12.08

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algor
ithm="giac")
```

```
[Out] -1/840*(6*(B*b^11*c^8*e^8*g^3*i^3 - 8*B*a*b^10*c^7*d*e^8*g^3*i^3 + 28*B*a^2
*b^9*c^6*d^2*e^8*g^3*i^3 - 56*B*a^3*b^8*c^5*d^3*e^8*g^3*i^3 + 70*B*a^4*b^7*
c^4*d^4*e^8*g^3*i^3 - 56*B*a^5*b^6*c^3*d^5*e^8*g^3*i^3 + 28*B*a^6*b^5*c^2*d
^6*e^8*g^3*i^3 - 8*B*a^7*b^4*c*d^7*e^8*g^3*i^3 + B*a^8*b^3*d^8*e^8*g^3*i^3
- 7*(b*e*x + a*e)*B*b^10*c^8*d*e^7*g^3*i^3/(d*x + c) + 56*(b*e*x + a*e)*B*a
*b^9*c^7*d^2*e^7*g^3*i^3/(d*x + c) - 196*(b*e*x + a*e)*B*a^2*b^8*c^6*d^3*e^
7*g^3*i^3/(d*x + c) + 392*(b*e*x + a*e)*B*a^3*b^7*c^5*d^4*e^7*g^3*i^3/(d*x
+ c) - 490*(b*e*x + a*e)*B*a^4*b^6*c^4*d^5*e^7*g^3*i^3/(d*x + c) + 392*(b*e
*x + a*e)*B*a^5*b^5*c^3*d^6*e^7*g^3*i^3/(d*x + c) - 196*(b*e*x + a*e)*B*a^6
*b^4*c^2*d^7*e^7*g^3*i^3/(d*x + c) + 56*(b*e*x + a*e)*B*a^7*b^3*c*d^8*e^7*g
^3*i^3/(d*x + c) - 7*(b*e*x + a*e)*B*a^8*b^2*d^9*e^7*g^3*i^3/(d*x + c) + 21
*(b*e*x + a*e)^2*B*b^9*c^8*d^2*e^6*g^3*i^3/(d*x + c)^2 - 168*(b*e*x + a*e)^
2*B*a*b^8*c^7*d^3*e^6*g^3*i^3/(d*x + c)^2 + 588*(b*e*x + a*e)^2*B*a^2*b^7*c
^6*d^4*e^6*g^3*i^3/(d*x + c)^2 - 1176*(b*e*x + a*e)^2*B*a^3*b^6*c^5*d^5*e^6
*g^3*i^3/(d*x + c)^2 + 1470*(b*e*x + a*e)^2*B*a^4*b^5*c^4*d^6*e^6*g^3*i^3/(
d*x + c)^2 - 1176*(b*e*x + a*e)^2*B*a^5*b^4*c^3*d^7*e^6*g^3*i^3/(d*x + c)^2
 + 588*(b*e*x + a*e)^2*B*a^6*b^3*c^2*d^8*e^6*g^3*i^3/(d*x + c)^2 - 168*(b*e
*x + a*e)^2*B*a^7*b^2*c*d^9*e^6*g^3*i^3/(d*x + c)^2 + 21*(b*e*x + a*e)^2*B*
a^8*b*d^10*e^6*g^3*i^3/(d*x + c)^2 - 35*(b*e*x + a*e)^3*B*b^8*c^8*d^3*e^5*g
^3*i^3/(d*x + c)^3 + 280*(b*e*x + a*e)^3*B*a*b^7*c^7*d^4*e^5*g^3*i^3/(d*x +
 c)^3 - 980*(b*e*x + a*e)^3*B*a^2*b^6*c^6*d^5*e^5*g^3*i^3/(d*x + c)^3 + 196
0*(b*e*x + a*e)^3*B*a^3*b^5*c^5*d^6*e^5*g^3*i^3/(d*x + c)^3 - 2450*(b*e*x +
 a*e)^3*B*a^4*b^4*c^4*d^7*e^5*g^3*i^3/(d*x + c)^3 + 1960*(b*e*x + a*e)^3*B*
a^5*b^3*c^3*d^8*e^5*g^3*i^3/(d*x + c)^3 - 980*(b*e*x + a*e)^3*B*a^6*b^2*c^2
*d^9*e^5*g^3*i^3/(d*x + c)^3 + 280*(b*e*x + a*e)^3*B*a^7*b*c*d^10*e^5*g^3*i
^3/(d*x + c)^3 - 35*(b*e*x + a*e)^3*B*a^8*d^11*e^5*g^3*i^3/(d*x + c)^3)*log
((b*e*x + a*e)/(d*x + c))/(b^7*d^4*e^7 - 7*(b*e*x + a*e)*b^6*d^5*e^6/(d*x +
 c) + 21*(b*e*x + a*e)^2*b^5*d^6*e^5/(d*x + c)^2 - 35*(b*e*x + a*e)^3*b^4*d
^7*e^4/(d*x + c)^3 + 35*(b*e*x + a*e)^4*b^3*d^8*e^3/(d*x + c)^4 - 21*(b*e*x
 + a*e)^5*b^2*d^9*e^2/(d*x + c)^5 + 7*(b*e*x + a*e)^6*b*d^10*e/(d*x + c)^6
- (b*e*x + a*e)^7*d^11/(d*x + c)^7) + (6*A*b^14*c^8*e^8*g^3*i^3 - 48*A*a*b^
13*c^7*d*e^8*g^3*i^3 + 168*A*a^2*b^12*c^6*d^2*e^8*g^3*i^3 - 336*A*a^3*b^11*
c^5*d^3*e^8*g^3*i^3 + 420*A*a^4*b^10*c^4*d^4*e^8*g^3*i^3 - 336*A*a^5*b^9*c^
3*d^5*e^8*g^3*i^3 + 168*A*a^6*b^8*c^2*d^6*e^8*g^3*i^3 - 48*A*a^7*b^7*c*d^7*
```

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e^8*g^3*i^3 + 6*A*a^8*b^6*d^8*e^8*g^3*i^3 - 42*(b*e*x + a*e)*A*b^13*c^8*d*e
^7*g^3*i^3/(d*x + c) + 6*(b*e*x + a*e)*B*b^13*c^8*d*e^7*g^3*i^3/(d*x + c) +
 336*(b*e*x + a*e)*A*a*b^12*c^7*d^2*e^7*g^3*i^3/(d*x + c) - 48*(b*e*x + a*e
)*B*a*b^12*c^7*d^2*e^7*g^3*i^3/(d*x + c) - 1176*(b*e*x + a*e)*A*a^2*b^11*c^
6*d^3*e^7*g^3*i^3/(d*x + c) + 168*(b*e*x + a*e)*B*a^2*b^11*c^6*d^3*e^7*g^3*
i^3/(d*x + c) + 2352*(b*e*x + a*e)*A*a^3*b^10*c^5*d^4*e^7*g^3*i^3/(d*x + c)
 - 336*(b*e*x + a*e)*B*a^3*b^10*c^5*d^4*e^7*g^3*i^3/(d*x + c) - 2940*(b*e*x
 + a*e)*A*a^4*b^9*c^4*d^5*e^7*g^3*i^3/(d*x + c) + 420*(b*e*x + a*e)*B*a^4*b
^9*c^4*d^5*e^7*g^3*i^3/(d*x + c) + 2352*(b*e*x + a*e)*A*a^5*b^8*c^3*d^6*e^7
*g^3*i^3/(d*x + c) - 336*(b*e*x + a*e)*B*a^5*b^8*c^3*d^6*e^7*g^3*i^3/(d*x +
 c) - 1176*(b*e*x + a*e)*A*a^6*b^7*c^2*d^7*e^7*g^3*i^3/(d*x + c) + 168*(b*e
*x + a*e)*B*a^6*b^7*c^2*d^7*e^7*g^3*i^3/(d*x + c) + 336*(b*e*x + a*e)*A*a^7
*b^6*c*d^8*e^7*g^3*i^3/(d*x + c) - 48*(b*e*x + a*e)*B*a^7*b^6*c*d^8*e^7*g^3
*i^3/(d*x + c) - 42*(b*e*x + a*e)*A*a^8*b^5*d^9*e^7*g^3*i^3/(d*x + c) + 6*(
b*e*x + a*e)*B*a^8*b^5*d^9*e^7*g^3*i^3/(d*x + c) + 126*(b*e*x + a*e)^2*A*b^
12*c^8*d^2*e^6*g^3*i^3/(d*x + c)^2 - 39*(b*e*x + a*e)^2*B*b^12*c^8*d^2*e^6*
g^3*i^3/(d*x + c)^2 - 1008*(b*e*x + a*e)^2*A*a*b^11*c^7*d^3*e^6*g^3*i^3/(d*
x + c)^2 + 312*(b*e*x + a*e)^2*B*a*b^11*c^7*d^3*e^6*g^3*i^3/(d*x + c)^2 + 3
528*(b*e*x + a*e)^2*A*a^2*b^10*c^6*d^4*e^6*g^3*i^3/(d*x + c)^2 - 1092*(b*e*
x + a*e)^2*B*a^2*b^10*c^6*d^4*e^6*g^3*i^3/(d*x + c)^2 - 7056*(b*e*x + a*e)^
2*A*a^3*b^9*c^5*d^5*e^6*g^3*i^3/(d*x + c)^2 + 2184*(b*e*x + a*e)^2*B*a^3*b^
9*c^5*d^5*e^6*g^3*i^3/(d*x + c)^2 + 8820*(b*e*x + a*e)^2*A*a^4*b^8*c^4*d^6*
e^6*g^3*i^3/(d*x + c)^2 - 2730*(b*e*x + a*e)^2*B*a^4*b^8*c^4*d^6*e^6*g^3*i^
3/(d*x + c)^2 - 7056*(b*e*x + a*e)^2*A*a^5*b^7*c^3*d^7*e^6*g^3*i^3/(d*x + c
)^2 + 2184*(b*e*x + a*e)^2*B*a^5*b^7*c^3*d^7*e^6*g^3*i^3/(d*x + c)^2 + 3528
*(b*e*x + a*e)^2*A*a^6*b^6*c^2*d^8*e^6*g^3*i^3/(d*x + c)^2 - 1092*(b*e*x +
a*e)^2*B*a^6*b^6*c^2*d^8*e^6*g^3*i^3/(d*x + c)^2 - 1008*(b*e*x + a*e)^2*A*a
^7*b^5*c*d^9*e^6*g^3*i^3/(d*x + c)^2 + 312*(b*e*x + a*e)^2*B*a^7*b^5*c*d^9*
e^6*g^3*i^3/(d*x + c)^2 + 126*(b*e*x + a*e)^2*A*a^8*b^4*d^10*e^6*g^3*i^3/(d
*x + c)^2 - 39*(b*e*x + a*e)^2*B*a^8*b^4*d^10*e^6*g^3*i^3/(d*x + c)^2 - 210
*(b*e*x + a*e)^3*A*b^11*c^8*d^3*e^5*g^3*i^3/(d*x + c)^3 + 107*(b*e*x + a*e)
^3*B*b^11*c^8*d^3*e^5*g^3*i^3/(d*x + c)^3 + 1680*(b*e*x + a*e)^3*A*a*b^10*c
^7*d^4*e^5*g^3*i^3/(d*x + c)^3 - 856*(b*e*x + a*e)^3*B*a*b^10*c^7*d^4*e^5*g
^3*i^3/(d*x + c)^3 - 5880*(b*e*x + a*e)^3*A*a^2*b^9*c^6*d^5*e^5*g^3*i^3/(d*
x + c)^3 + 2996*(b*e*x + a*e)^3*B*a^2*b^9*c^6*d^5*e^5*g^3*i^3/(d*x + c)^3 +
 11760*(b*e*x + a*e)^3*A*a^3*b^8*c^5*d^6*e^5*g^3*i^3/(d*x + c)^3 - 5992*(b*
e*x + a*e)^3*B*a^3*b^8*c^5*d^6*e^5*g^3*i^3/(d*x + c)^3 - 14700*(b*e*x + a*e
)^3*A*a^4*b^7*c^4*d^7*e^5*g^3*i^3/(d*x + c)^3 + 7490*(b*e*x + a*e)^3*B*a^4*
b^7*c^4*d^7*e^5*g^3*i^3/(d*x + c)^3 + 11760*(b*e*x + a*e)^3*A*a^5*b^6*c^3*d
^8*e^5*g^3*i^3/(d*x + c)^3 - 5992*(b*e*x + a*e)^3*B*a^5*b^6*c^3*d^8*e^5*g^3
*i^3/(d*x + c)^3 - 5880*(b*e*x + a*e)^3*A*a^6*b^5*c^2*d^9*e^5*g^3*i^3/(d*x
+ c)^3 + 2996*(b*e*x + a*e)^3*B*a^6*b^5*c^2*d^9*e^5*g^3*i^3/(d*x + c)^3 + 1
680*(b*e*x + a*e)^3*A*a^7*b^4*c*d^10*e^5*g^3*i^3/(d*x + c)^3 - 856*(b*e*x +
 a*e)^3*B*a^7*b^4*c*d^10*e^5*g^3*i^3/(d*x + c)^3 - 210*(b*e*x + a*e)^3*A*a^
8*b^3*d^11*e^5*g^3*i^3/(d*x + c)^3 + 107*(b*e*x + a*e)^3*B*a^8*b^3*d^11*e^5
```

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*g^3*i^3/(d*x + c)^3 - 107*(b*e*x + a*e)^4*B*b^10*c^8*d^4*e^4*g^3*i^3/(d*x
+ c)^4 + 856*(b*e*x + a*e)^4*B*a*b^9*c^7*d^5*e^4*g^3*i^3/(d*x + c)^4 - 2996
*(b*e*x + a*e)^4*B*a^2*b^8*c^6*d^6*e^4*g^3*i^3/(d*x + c)^4 + 5992*(b*e*x +
a*e)^4*B*a^3*b^7*c^5*d^7*e^4*g^3*i^3/(d*x + c)^4 - 7490*(b*e*x + a*e)^4*B*a
^4*b^6*c^4*d^8*e^4*g^3*i^3/(d*x + c)^4 + 5992*(b*e*x + a*e)^4*B*a^5*b^5*c^3
*d^9*e^4*g^3*i^3/(d*x + c)^4 - 2996*(b*e*x + a*e)^4*B*a^6*b^4*c^2*d^10*e^4*
g^3*i^3/(d*x + c)^4 + 856*(b*e*x + a*e)^4*B*a^7*b^3*c*d^11*e^4*g^3*i^3/(d*x
 + c)^4 - 107*(b*e*x + a*e)^4*B*a^8*b^2*d^12*e^4*g^3*i^3/(d*x + c)^4 + 39*(
b*e*x + a*e)^5*B*b^9*c^8*d^5*e^3*g^3*i^3/(d*x + c)^5 - 312*(b*e*x + a*e)^5*
B*a*b^8*c^7*d^6*e^3*g^3*i^3/(d*x + c)^5 + 1092*(b*e*x + a*e)^5*B*a^2*b^7*c^
6*d^7*e^3*g^3*i^3/(d*x + c)^5 - 2184*(b*e*x + a*e)^5*B*a^3*b^6*c^5*d^8*e^3*
g^3*i^3/(d*x + c)^5 + 2730*(b*e*x + a*e)^5*B*a^4*b^5*c^4*d^9*e^3*g^3*i^3/(d
*x + c)^5 - 2184*(b*e*x + a*e)^5*B*a^5*b^4*c^3*d^10*e^3*g^3*i^3/(d*x + c)^5
 + 1092*(b*e*x + a*e)^5*B*a^6*b^3*c^2*d^11*e^3*g^3*i^3/(d*x + c)^5 - 312*(b
*e*x + a*e)^5*B*a^7*b^2*c*d^12*e^3*g^3*i^3/(d*x + c)^5 + 39*(b*e*x + a*e)^5
*B*a^8*b*d^13*e^3*g^3*i^3/(d*x + c)^5 - 6*(b*e*x + a*e)^6*B*b^8*c^8*d^6*e^2
*g^3*i^3/(d*x + c)^6 + 48*(b*e*x + a*e)^6*B*a*b^7*c^7*d^7*e^2*g^3*i^3/(d*x
+ c)^6 - 168*(b*e*x + a*e)^6*B*a^2*b^6*c^6*d^8*e^2*g^3*i^3/(d*x + c)^6 + 33
6*(b*e*x + a*e)^6*B*a^3*b^5*c^5*d^9*e^2*g^3*i^3/(d*x + c)^6 - 420*(b*e*x +
a*e)^6*B*a^4*b^4*c^4*d^10*e^2*g^3*i^3/(d*x + c)^6 + 336*(b*e*x + a*e)^6*B*a
^5*b^3*c^3*d^11*e^2*g^3*i^3/(d*x + c)^6 - 168*(b*e*x + a*e)^6*B*a^6*b^2*c^2
*d^12*e^2*g^3*i^3/(d*x + c)^6 + 48*(b*e*x + a*e)^6*B*a^7*b*c*d^13*e^2*g^3*i
^3/(d*x + c)^6 - 6*(b*e*x + a*e)^6*B*a^8*d^14*e^2*g^3*i^3/(d*x + c)^6)/(b^1
0*d^4*e^7 - 7*(b*e*x + a*e)*b^9*d^5*e^6/(d*x + c) + 21*(b*e*x + a*e)^2*b^8*
d^6*e^5/(d*x + c)^2 - 35*(b*e*x + a*e)^3*b^7*d^7*e^4/(d*x + c)^3 + 35*(b*e*
x + a*e)^4*b^6*d^8*e^3/(d*x + c)^4 - 21*(b*e*x + a*e)^5*b^5*d^9*e^2/(d*x +
c)^5 + 7*(b*e*x + a*e)^6*b^4*d^10*e/(d*x + c)^6 - (b*e*x + a*e)^7*b^3*d^11/
(d*x + c)^7) + 6*(B*b^8*c^8*e*g^3*i^3 - 8*B*a*b^7*c^7*d*e*g^3*i^3 + 28*B*a^
2*b^6*c^6*d^2*e*g^3*i^3 - 56*B*a^3*b^5*c^5*d^3*e*g^3*i^3 + 70*B*a^4*b^4*c^4
*d^4*e*g^3*i^3 - 56*B*a^5*b^3*c^3*d^5*e*g^3*i^3 + 28*B*a^6*b^2*c^2*d^6*e*g^
3*i^3 - 8*B*a^7*b*c*d^7*e*g^3*i^3 + B*a^8*d^8*e*g^3*i^3)*log(-b*e + (b*e*x
+ a*e)*d/(d*x + c))/(b^4*d^4) - 6*(B*b^8*c^8*e*g^3*i^3 - 8*B*a*b^7*c^7*d*e*
g^3*i^3 + 28*B*a^2*b^6*c^6*d^2*e*g^3*i^3 - 56*B*a^3*b^5*c^5*d^3*e*g^3*i^3 +
 70*B*a^4*b^4*c^4*d^4*e*g^3*i^3 - 56*B*a^5*b^3*c^3*d^5*e*g^3*i^3 + 28*B*a^6
*b^2*c^2*d^6*e*g^3*i^3 - 8*B*a^7*b*c*d^7*e*g^3*i^3 + B*a^8*d^8*e*g^3*i^3)*l
og((b*e*x + a*e)/(d*x + c))/(b^4*d^4))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) -
 a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 3.18 (sec) , antiderivative size = 4347, normalized size of antiderivative = 9.51

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

```
[In] int((a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x*(((140*a*d + 140*b*c)*(((140*a*d + 140*b*c)*((a*c*((((b^2*d^2*g^3*i^3*(28
*A*a*d + 28*A*b*c + B*a*d - B*b*c))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b
*c))/140)*(140*a*d + 140*b*c))/(140*b*d) - (b*d*g^3*i^3*(12*A*a^2*d^2 + 12*
A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d))/2 + A*a*b^2*c*d^2*g^3*i^
3))/(b*d) - ((140*a*d + 140*b*c)*((g^3*i^3*(20*A*a^3*d^3 + 20*A*b^3*c^3 + 3
*B*a^3*d^3 - 3*B*b^3*c^3 + 120*A*a*b^2*c^2*d + 120*A*a^2*b*c*d^2 - 6*B*a*b^
2*c^2*d + 6*B*a^2*b*c*d^2))/5 + ((140*a*d + 140*b*c)*((((b^2*d^2*g^3*i^3*(2
8*A*a*d + 28*A*b*c + B*a*d - B*b*c))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*
b*c))/140)*(140*a*d + 140*b*c))/(140*b*d) - (b*d*g^3*i^3*(12*A*a^2*d^2 + 12
*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d))/2 + A*a*b^2*c*d^2*g^3*i
^3))/(140*b*d) - (a*c*((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d - B*b*
c))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140))/(b*d)))/(140*b*d) + (
g^3*i^3*(4*A*a^4*d^4 + 4*A*b^4*c^4 + B*a^4*d^4 - B*b^4*c^4 + 144*A*a^2*b^2*
c^2*d^2 + 64*A*a*b^3*c^3*d + 64*A*a^3*b*c*d^3 - 8*B*a*b^3*c^3*d + 8*B*a^3*b
*c*d^3))/(4*b*d)))/(140*b*d) + (a*c*((g^3*i^3*(20*A*a^3*d^3 + 20*A*b^3*c^3
+ 3*B*a^3*d^3 - 3*B*b^3*c^3 + 120*A*a*b^2*c^2*d + 120*A*a^2*b*c*d^2 - 6*B*a
*b^2*c^2*d + 6*B*a^2*b*c*d^2))/5 + ((140*a*d + 140*b*c)*((((b^2*d^2*g^3*i^3
*(28*A*a*d + 28*A*b*c + B*a*d - B*b*c))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 1
40*b*c))/140)*(140*a*d + 140*b*c))/(140*b*d) - (b*d*g^3*i^3*(12*A*a^2*d^2 +
 12*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d))/2 + A*a*b^2*c*d^2*g^
3*i^3))/(140*b*d) - (a*c*((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d - B
*b*c))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140))/(b*d)))/(b*d) - (a
*c*g^3*i^3*(4*A*a^3*d^3 + 4*A*b^3*c^3 + B*a^3*d^3 - B*b^3*c^3 + 24*A*a*b^2*
c^2*d + 24*A*a^2*b*c*d^2 - 2*B*a*b^2*c^2*d + 2*B*a^2*b*c*d^2))/(b*d)))/(140
*b*d) - (a*c*((a*c*((((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d - B*b*c
))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140)*(140*a*d + 140*b*c))/(1
40*b*d) - (b*d*g^3*i^3*(12*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2
 + 32*A*a*b*c*d))/2 + A*a*b^2*c*d^2*g^3*i^3))/(b*d) - ((140*a*d + 140*b*c)*
((g^3*i^3*(20*A*a^3*d^3 + 20*A*b^3*c^3 + 3*B*a^3*d^3 - 3*B*b^3*c^3 + 120*A*
a*b^2*c^2*d + 120*A*a^2*b*c*d^2 - 6*B*a*b^2*c^2*d + 6*B*a^2*b*c*d^2))/5 + (
(140*a*d + 140*b*c)*((((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d - B*b*
c))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140)*(140*a*d + 140*b*c))/(
140*b*d) - (b*d*g^3*i^3*(12*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^
2 + 32*A*a*b*c*d))/2 + A*a*b^2*c*d^2*g^3*i^3))/(140*b*d) - (a*c*((b^2*d^2*g
^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d - B*b*c))/7 - (A*b^2*d^2*g^3*i^3*(140*a
*d + 140*b*c))/140))/(b*d)))/(140*b*d) + (g^3*i^3*(4*A*a^4*d^4 + 4*A*b^4*c^
```

4 + B*a^4*d^4 - B*b^4*c^4 + 144*A*a^2*b^2*c^2*d^2 + 64*A*a*b^3*c^3*d + 64*A
*a^3*b*c*d^3 - 8*B*a*b^3*c^3*d + 8*B*a^3*b*c*d^3))/(4*b*d)))/(b*d) + (a^2*c
^2*g^3*i^3*(12*A*a^2*d^2 + 12*A*b^2*c^2 + 3*B*a^2*d^2 - 3*B*b^2*c^2 + 32*A*
a*b*c*d))/(2*b*d)) + x^6*((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d - B
*b*c))/42 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/840) + x^3*((a*c*((((b^
2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d - B*b*c))/7 - (A*b^2*d^2*g^3*i^3
*(140*a*d + 140*b*c))/140)*(140*a*d + 140*b*c))/(140*b*d) - (b*d*g^3*i^3*(1
2*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d))/2 + A*a
*b^2*c*d^2*g^3*i^3))/(3*b*d) - ((140*a*d + 140*b*c)*((g^3*i^3*(20*A*a^3*d^3
 + 20*A*b^3*c^3 + 3*B*a^3*d^3 - 3*B*b^3*c^3 + 120*A*a*b^2*c^2*d + 120*A*a^2
*b*c*d^2 - 6*B*a*b^2*c^2*d + 6*B*a^2*b*c*d^2))/5 + ((140*a*d + 140*b*c)*(((
(b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d - B*b*c))/7 - (A*b^2*d^2*g^3*
i^3*(140*a*d + 140*b*c))/140)*(140*a*d + 140*b*c))/(140*b*d) - (b*d*g^3*i^3
*(12*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d))/2 +
A*a*b^2*c*d^2*g^3*i^3))/(140*b*d) - (a*c*((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A
*b*c + B*a*d - B*b*c))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140))/(b
*d)))/(420*b*d) + (g^3*i^3*(4*A*a^4*d^4 + 4*A*b^4*c^4 + B*a^4*d^4 - B*b^4*c
^4 + 144*A*a^2*b^2*c^2*d^2 + 64*A*a*b^3*c^3*d + 64*A*a^3*b*c*d^3 - 8*B*a*b^
3*c^3*d + 8*B*a^3*b*c*d^3))/(12*b*d)) - x^2*(((140*a*d + 140*b*c)*((a*c*(((
(b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d - B*b*c))/7 - (A*b^2*d^2*g^3*
i^3*(140*a*d + 140*b*c))/140)*(140*a*d + 140*b*c))/(140*b*d) - (b*d*g^3*i^3
*(12*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d))/2 +
A*a*b^2*c*d^2*g^3*i^3))/(b*d) - ((140*a*d + 140*b*c)*((g^3*i^3*(20*A*a^3*d^
3 + 20*A*b^3*c^3 + 3*B*a^3*d^3 - 3*B*b^3*c^3 + 120*A*a*b^2*c^2*d + 120*A*a^
2*b*c*d^2 - 6*B*a*b^2*c^2*d + 6*B*a^2*b*c*d^2))/5 + ((140*a*d + 140*b*c)*((
((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d - B*b*c))/7 - (A*b^2*d^2*g^3
i^3(140*a*d + 140*b*c))/140)*(140*a*d + 140*b*c))/(140*b*d) - (b*d*g^3*i^
3*(12*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d))/2 +
 A*a*b^2*c*d^2*g^3*i^3))/(140*b*d) - (a*c*((b^2*d^2*g^3*i^3*(28*A*a*d + 28*
A*b*c + B*a*d - B*b*c))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140))/(
b*d)))/(140*b*d) + (g^3*i^3*(4*A*a^4*d^4 + 4*A*b^4*c^4 + B*a^4*d^4 - B*b^4*
c^4 + 144*A*a^2*b^2*c^2*d^2 + 64*A*a*b^3*c^3*d + 64*A*a^3*b*c*d^3 - 8*B*a*b
^3*c^3*d + 8*B*a^3*b*c*d^3))/(4*b*d)))/(280*b*d) + (a*c*((g^3*i^3*(20*A*a^3
*d^3 + 20*A*b^3*c^3 + 3*B*a^3*d^3 - 3*B*b^3*c^3 + 120*A*a*b^2*c^2*d + 120*A
*a^2*b*c*d^2 - 6*B*a*b^2*c^2*d + 6*B*a^2*b*c*d^2))/5 + ((140*a*d + 140*b*c)
*((((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d - B*b*c))/7 - (A*b^2*d^2*
g^3*i^3*(140*a*d + 140*b*c))/140)*(140*a*d + 140*b*c))/(140*b*d) - (b*d*g^3
i^3(12*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d))/
2 + A*a*b^2*c*d^2*g^3*i^3))/(140*b*d) - (a*c*((b^2*d^2*g^3*i^3*(28*A*a*d +
28*A*b*c + B*a*d - B*b*c))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140)
)/(b*d)))/(2*b*d) - (a*c*g^3*i^3*(4*A*a^3*d^3 + 4*A*b^3*c^3 + B*a^3*d^3 - B
*b^3*c^3 + 24*A*a*b^2*c^2*d + 24*A*a^2*b*c*d^2 - 2*B*a*b^2*c^2*d + 2*B*a^2*
b*c*d^2))/(2*b*d)) - x^5*((((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d -
 B*b*c))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140)*(140*a*d + 140*b*
c))/(700*b*d) - (b*d*g^3*i^3*(12*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^2 - B*b

```
^2*c^2 + 32*A*a*b*c*d))/10 + (A*a*b^2*c*d^2*g^3*i^3)/5) + log((e*(a + b*x))
/(c + d*x))*((B*g^3*i^3*x^4*(a^3*d^3 + b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*
d^2))/4 + B*a^3*c^3*g^3*i^3*x + (B*b^3*d^3*g^3*i^3*x^7)/7 + (3*B*a^2*c^2*g^
3*i^3*x^2*(a*d + b*c))/2 + (B*b^2*d^2*g^3*i^3*x^6*(a*d + b*c))/2 + B*a*c*g^
3*i^3*x^3*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d) + (3*B*b*d*g^3*i^3*x^5*(a^2*d^2 +
 b^2*c^2 + 3*a*b*c*d))/5) + x^4*((g^3*i^3*(20*A*a^3*d^3 + 20*A*b^3*c^3 + 3*
B*a^3*d^3 - 3*B*b^3*c^3 + 120*A*a*b^2*c^2*d + 120*A*a^2*b*c*d^2 - 6*B*a*b^2
*c^2*d + 6*B*a^2*b*c*d^2))/20 + ((140*a*d + 140*b*c)*((((b^2*d^2*g^3*i^3*(2
8*A*a*d + 28*A*b*c + B*a*d - B*b*c))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*
b*c))/140)*(140*a*d + 140*b*c))/(140*b*d) - (b*d*g^3*i^3*(12*A*a^2*d^2 + 12
*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d))/2 + A*a*b^2*c*d^2*g^3*i
^3))/(560*b*d) - (a*c*((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d - B*b*
c))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140))/(4*b*d)) - (log(a + b
*x)*(B*a^7*d^3*g^3*i^3 - 35*B*a^4*b^3*c^3*g^3*i^3 - 7*B*a^6*b*c*d^2*g^3*i^3
 + 21*B*a^5*b^2*c^2*d*g^3*i^3))/(140*b^4) + (log(c + d*x)*(B*b^3*c^7*g^3*i^
3 - 35*B*a^3*c^4*d^3*g^3*i^3 - 7*B*a*b^2*c^6*d*g^3*i^3 + 21*B*a^2*b*c^5*d^2
*g^3*i^3))/(140*d^4) + (A*b^3*d^3*g^3*i^3*x^7)/7
```

3.21 $\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

Optimal result

Integrand size = 40, antiderivative size = 371

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = -\frac{B(bc-ad)^5g^2i^3x}{60b^3d^2}-\frac{B(bc-ad)^4g^2i^3(c+dx)^2}{120b^2d^3}-\frac{B(bc-ad)^3g^2i^3(c+dx)^3}{180bd^3}+\frac{7B(bc-ad)^2g^2i^3(c+dx)^4}{120d^3}-\frac{bB(bc-ad)g^2i^3(c+dx)^5}{30d^3}-\frac{B(bc-ad)^6g^2i^3\log\left(\frac{a+bx}{c+dx}\right)}{60b^4d^3}+\frac{(bc-ad)^2g^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^3}-\frac{2b(bc-ad)g^2i^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^3}+\frac{b^2g^2i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^3}-\frac{B(bc-ad)^6g^2i^3\log(c+dx)}{60b^4d^3}$$

```
[Out] -1/60*B*(-a*d+b*c)^5*g^2*i^3*x/b^3/d^2-1/120*B*(-a*d+b*c)^4*g^2*i^3*(d*x+c)
^2/b^2/d^3-1/180*B*(-a*d+b*c)^3*g^2*i^3*(d*x+c)^3/b/d^3+7/120*B*(-a*d+b*c)^
2*g^2*i^3*(d*x+c)^4/d^3-1/30*b*B*(-a*d+b*c)*g^2*i^3*(d*x+c)^5/d^3-1/60*B*(-
a*d+b*c)^6*g^2*i^3*ln((b*x+a)/(d*x+c))/b^4/d^3+1/4*(-a*d+b*c)^2*g^2*i^3*(d*
x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3-2/5*b*(-a*d+b*c)*g^2*i^3*(d*x+c)^5*(
A+B*ln(e*(b*x+a)/(d*x+c)))/d^3+1/6*b^2*g^2*i^3*(d*x+c)^6*(A+B*ln(e*(b*x+a)/
(d*x+c)))/d^3-1/60*B*(-a*d+b*c)^6*g^2*i^3*ln(d*x+c)/b^4/d^3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2562, 45, 2382, 12, 907}

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{b^2g^2i^3(c+dx)^6\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6d^3}+\frac{g^2i^3(c+dx)^4(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^3}-\frac{2bg^2i^3(c+dx)^5(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^3}-\frac{Bg^2i^3(bc-ad)^6\log\left(\frac{a+bx}{c+dx}\right)}{60b^4d^3}-\frac{Bg^2i^3(bc-ad)^6\log(c+dx)}{60b^4d^3}-\frac{Bg^2i^3x(bc-ad)^5}{60b^3d^2}-\frac{Bg^2i^3(c+dx)^2(bc-ad)^4}{120b^2d^3}-\frac{Bg^2i^3(c+dx)^3(bc-ad)^3}{180bd^3}+\frac{7Bg^2i^3(c+dx)^4(bc-ad)^2}{120d^3}-\frac{bBg^2i^3(c+dx)^5(bc-ad)}{30d^3}$$

```
[In] Int[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -1/60*(B*(b*c - a*d)^5*g^2*i^3*x)/(b^3*d^2) - (B*(b*c - a*d)^4*g^2*i^3*(c +
 d*x)^2)/(120*b^2*d^3) - (B*(b*c - a*d)^3*g^2*i^3*(c + d*x)^3)/(180*b*d^3)
+ (7*B*(b*c - a*d)^2*g^2*i^3*(c + d*x)^4)/(120*d^3) - (b*B*(b*c - a*d)*g^2*
i^3*(c + d*x)^5)/(30*d^3) - (B*(b*c - a*d)^6*g^2*i^3*Log[(a + b*x)/(c + d*x
)])/(60*b^4*d^3) + ((b*c - a*d)^2*g^2*i^3*(c + d*x)^4*(A + B*Log[(e*(a + b*
x))/(c + d*x)]))/(4*d^3) - (2*b*(b*c - a*d)*g^2*i^3*(c + d*x)^5*(A + B*Log[
(e*(a + b*x))/(c + d*x)]))/(5*d^3) + (b^2*g^2*i^3*(c + d*x)^6*(A + B*Log[(e
*(a + b*x))/(c + d*x)]))/(6*d^3) - (B*(b*c - a*d)^6*g^2*i^3*Log[c + d*x])/(
60*b^4*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
```

```
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left((bc-ad)^6 g^2 i^3\right) \operatorname{Subst}\left(\int \frac{x^2(A+B\log(ex))}{(b-dx)^7}\,dx, x, \frac{a+bx}{c+dx}\right) \\
&= \frac{(bc-ad)^2 g^2 i^3 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^3} \\
&\quad - \frac{2b(bc-ad) g^2 i^3 (c+dx)^5 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^3} \\
&\quad + \frac{b^2 g^2 i^3 (c+dx)^6 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^3} \\
&\quad - \left(B(bc-ad)^6 g^2 i^3\right) \operatorname{Subst}\left(\int \frac{b^2-6bdx+15d^2x^2}{60d^3 x(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right) \\
&= \frac{(bc-ad)^2 g^2 i^3 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^3} \\
&\quad - \frac{2b(bc-ad) g^2 i^3 (c+dx)^5 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^3} \\
&\quad + \frac{b^2 g^2 i^3 (c+dx)^6 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^3} \\
&\quad - \frac{\left(B(bc-ad)^6 g^2 i^3\right) \operatorname{Subst}\left(\int \frac{b^2-6bdx+15d^2x^2}{x(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{60d^3}
\end{aligned}
$$

$$= \frac{(bc-ad)^2 g^2 i^3 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^3} - \frac{2b(bc-ad)g^2 i^3 (c+dx)^5 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^3} + \frac{b^2 g^2 i^3 (c+dx)^6 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^3} - \frac{(B(bc-ad)^6 g^2 i^3)\,\text{Subst}\left(\int \left(\frac{1}{b^4x} + \frac{10bd}{(b-dx)^6} - \frac{14d}{(b-dx)^5} + \frac{d}{b(b-dx)^4} + \frac{d}{b^2(b-dx)^3} + \frac{d}{b^3(b-dx)^2} + \frac{d}{b^4(b-dx)}\right) d}{60d^3}$$

$$= -\frac{B(bc-ad)^5 g^2 i^3 x}{60b^3d^2} - \frac{B(bc-ad)^4 g^2 i^3 (c+dx)^2}{120b^2d^3} - \frac{B(bc-ad)^3 g^2 i^3 (c+dx)^3}{180bd^3} + \frac{7B(bc-ad)^2 g^2 i^3 (c+dx)^4}{120d^3} - \frac{bB(bc-ad)g^2 i^3 (c+dx)^5}{30d^3} - \frac{B(bc-ad)^6 g^2 i^3 \log\left(\frac{a+bx}{c+dx}\right)}{60b^4d^3} + \frac{(bc-ad)^2 g^2 i^3 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^3} - \frac{2b(bc-ad)g^2 i^3 (c+dx)^5 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^3} + \frac{b^2 g^2 i^3 (c+dx)^6 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^3} - \frac{B(bc-ad)^6 g^2 i^3 \log(c+dx)}{60b^4d^3}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.16

$$\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = \frac{g^2i^3\Big(-15B(bc-ad)^3\left(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad)^3\log(a+bx)\right)+}{}$$

```
[In] Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x
)]),x]
```

```
[Out] (g^2*i^3*(-15*B*(b*c - a*d)^3*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c
 + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]) + 12*B*(b*c -
 a*d)^2*(12*b*d*(b*c - a*d)^3*x + 6*b^2*(b*c - a*d)^2*(c + d*x)^2 + 4*b^3*(
b*c - a*d)*(c + d*x)^3 + 3*b^4*(c + d*x)^4 + 12*(b*c - a*d)^4*Log[a + b*x])
 - B*(b*c - a*d)*(60*b*d*(b*c - a*d)^4*x + 30*b^2*(b*c - a*d)^3*(c + d*x)^2
 + 20*b^3*(b*c - a*d)^2*(c + d*x)^3 + 15*b^4*(b*c - a*d)*(c + d*x)^4 + 12*b
^5*(c + d*x)^5 + 60*(b*c - a*d)^5*Log[a + b*x]) + 90*b^4*(b*c - a*d)^2*(c +
 d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 144*b^5*(b*c - a*d)*(c + d*x
)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 60*b^6*(c + d*x)^6*(A + B*Log[(e
*(a + b*x))/(c + d*x)])))/(360*b^4*d^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(351) = 702$.

Time = 1.16 (sec) , antiderivative size = 925, normalized size of antiderivative = 2.49

method	result
risch	$-\frac{i^3 g^2 b^2 d^2 B c\,x^5}{30}+\frac{i^3 g^2 d^3 A\,a^2 x^4}{4}+\frac{3 i^3 g^2 b^2 d A\,c^2 x^4}{4}+\frac{7 i^3 g^2 d^3 B\,a^2 x^4}{120}-\frac{13 i^3 g^2 b^2 d B\,c^2 x^4}{120}+\frac{i^3 g^2 b^2 A\,c^3 x^3}{3}+$
parallelrisch	Expression too large to display
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETUR
NVERBOSE)
```

```
[Out] -1/30*i^3*g^2*b^2*d^2*B*c*x^5+1/4*i^3*g^2*d^3*A*a^2*x^4+3/4*i^3*g^2*b^2*d*A
*c^2*x^4+7/120*i^3*g^2*d^3*B*a^2*x^4-13/120*i^3*g^2*b^2*d*B*c^2*x^4+1/3*i^3
*g^2*b^2*A*c^3*x^3+1/180*i^3*g^2/b*d^3*B*a^3*x^3-19/180*i^3*g^2*b^2*B*c^3*x
^3-1/120*i^3*g^2/b^2*d^3*B*a^4*x^2-1/120*i^3*g^2*b^2/d*B*c^4*x^2+3/2*i^3*g^
2*b*d^2*A*a*c*x^4+1/20*i^3*g^2*b*d^2*B*a*c*x^4+i^3*g^2*d^2*A*a^2*c*x^3+2*i^
3*g^2*b*d*A*a*c^2*x^3+13/60*i^3*g^2*d^2*B*a^2*c*x^3-7/60*i^3*g^2*b*d*B*a*c^
2*x^3+3/2*i^3*g^2*d*A*a^2*c^2*x^2+i^3*g^2*b*A*a*c^3*x^2+1/20*i^3*g^2/b*d^2*
B*a^3*c*x^2+1/4*i^3*g^2*d*B*a^2*c^2*x^2-17/60*i^3*g^2*b*B*a*c^3*x^2+i^3*g^2
*A*a^2*c^3*x-1/10*i^3*g^2/b^2*d^2*B*a^4*c*x+1/4*i^3*g^2/b*d*B*a^3*c^2*x-1/1
2*i^3*g^2*B*a^2*c^3*x-1/10*i^3*g^2*b/d*B*a*c^4*x+1/10*i^3*g^2*b/d^2*B*ln(-d
*x-c)*a*c^5+1/10*i^3*g^2/b^3*d^2*B*ln(b*x+a)*a^5*c-1/4*i^3*g^2/b^2*d*B*ln(b
*x+a)*a^4*c^2+2/5*i^3*g^2*b*d^3*A*a*x^5+3/5*i^3*g^2*b^2*d^2*A*c*x^5+1/30*i^
3*g^2*b*d^3*B*a*x^5+1/60*i^3*g^2/b^3*d^3*B*a^5*x+1/60*i^3*g^2*b^2/d^2*B*c^5
*x-1/60*i^3*g^2*b^2/d^3*B*ln(-d*x-c)*c^6-1/60*i^3*g^2/b^4*d^3*B*ln(b*x+a)*a
^6-1/4*i^3*g^2/d*B*ln(-d*x-c)*a^2*c^4+1/3*i^3*g^2/b*B*ln(b*x+a)*a^3*c^3+1/6
0*i^3*g^2*B*x*(10*b^2*d^3*x^5+24*a*b*d^3*x^4+36*b^2*c*d^2*x^4+15*a^2*d^3*x^
3+90*a*b*c*d^2*x^3+45*b^2*c^2*d*x^3+60*a^2*c*d^2*x^2+120*a*b*c^2*d*x^2+20*b
^2*c^3*x^2+90*a^2*c^2*d*x+60*a*b*c^3*x+60*a^2*c^3)*ln(e*(b*x+a)/(d*x+c))+1/
6*i^3*g^2*b^2*d^3*A*x^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 722 vs. 2(351) = 702.

Time = 0.47 (sec) , antiderivative size = 722, normalized size of antiderivative = 1.95

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{60\,Ab^6d^6g^2i^3x^6+12\,((18\,A-B)b^6cd^5+(12\,A+B)ab^5d^6)g^2i^3x^5+3\,((90\,A-13\,B)b^6c^2d^4+6\,(30\,A+B)c}{}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algor
ithm="fricas")
```

```
[Out] 1/360*(60*A*b^6*d^6*g^2*i^3*x^6 + 12*((18*A - B)*b^6*c*d^5 + (12*A + B)*a*b
^5*d^6)*g^2*i^3*x^5 + 3*((90*A - 13*B)*b^6*c^2*d^4 + 6*(30*A + B)*a*b^5*c*d
^5 + (30*A + 7*B)*a^2*b^4*d^6)*g^2*i^3*x^4 + 2*((60*A - 19*B)*b^6*c^3*d^3 +
 3*(120*A - 7*B)*a*b^5*c^2*d^4 + 3*(60*A + 13*B)*a^2*b^4*c*d^5 + B*a^3*b^3*
d^6)*g^2*i^3*x^3 - 3*(B*b^6*c^4*d^2 - 2*(60*A - 17*B)*a*b^5*c^3*d^3 - 30*(6
*A + B)*a^2*b^4*c^2*d^4 - 6*B*a^3*b^3*c*d^5 + B*a^4*b^2*d^6)*g^2*i^3*x^2 +
6*(B*b^6*c^5*d - 6*B*a*b^5*c^4*d^2 + 5*(12*A - B)*a^2*b^4*c^3*d^3 + 15*B*a^
3*b^3*c^2*d^4 - 6*B*a^4*b^2*c*d^5 + B*a^5*b*d^6)*g^2*i^3*x + 6*(20*B*a^3*b^
3*c^3*d^3 - 15*B*a^4*b^2*c^2*d^4 + 6*B*a^5*b*c*d^5 - B*a^6*d^6)*g^2*i^3*log
(b*x + a) - 6*(B*b^6*c^6 - 6*B*a*b^5*c^5*d + 15*B*a^2*b^4*c^4*d^2)*g^2*i^3*
log(d*x + c) + 6*(10*B*b^6*d^6*g^2*i^3*x^6 + 60*B*a^2*b^4*c^3*d^3*g^2*i^3*x
 + 12*(3*B*b^6*c*d^5 + 2*B*a*b^5*d^6)*g^2*i^3*x^5 + 15*(3*B*b^6*c^2*d^4 + 6
*B*a*b^5*c*d^5 + B*a^2*b^4*d^6)*g^2*i^3*x^4 + 20*(B*b^6*c^3*d^3 + 6*B*a*b^5
*c^2*d^4 + 3*B*a^2*b^4*c*d^5)*g^2*i^3*x^3 + 30*(2*B*a*b^5*c^3*d^3 + 3*B*a^2
*b^4*c^2*d^4)*g^2*i^3*x^2)*log((b*e*x + a*e)/(d*x + c)))/(b^4*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1727 vs. 2(347) = 694.

Time = 7.14 (sec) , antiderivative size = 1727, normalized size of antiderivative = 4.65

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)**2*(d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b**2*d**3*g**2*i**3*x**6/6 - B*a**3*g**2*i**3*(a**3*d**3 - 6*a**2*b*c*d**
2 + 15*a*b**2*c**2*d - 20*b**3*c**3)*log(x + (B*a**6*c*d**5*g**2*i**3 - 6*B
*a**5*b*c**2*d**4*g**2*i**3 + 15*B*a**4*b**2*c**3*d**3*g**2*i**3 + B*a**4*d
**3*g**2*i**3*(a**3*d**3 - 6*a**2*b*c*d**2 + 15*a*b**2*c**2*d - 20*b**3*c**
3)/b - 35*B*a**3*b**3*c**4*d**2*g**2*i**3 - B*a**3*c*d**2*g**2*i**3*(a**3*d
```

```
**3 - 6*a**2*b*c*d**2 + 15*a*b**2*c**2*d - 20*b**3*c**3) + 6*B*a**2*b**4*c*
*5*d*g**2*i**3 - B*a*b**5*c**6*g**2*i**3)/(B*a**6*d**6*g**2*i**3 - 6*B*a**5
*b*c*d**5*g**2*i**3 + 15*B*a**4*b**2*c**2*d**4*g**2*i**3 - 20*B*a**3*b**3*c
**3*d**3*g**2*i**3 - 15*B*a**2*b**4*c**4*d**2*g**2*i**3 + 6*B*a*b**5*c**5*d
*g**2*i**3 - B*b**6*c**6*g**2*i**3))/(60*b**4) - B*c**4*g**2*i**3*(15*a**2*
d**2 - 6*a*b*c*d + b**2*c**2)*log(x + (B*a**6*c*d**5*g**2*i**3 - 6*B*a**5*b
*c**2*d**4*g**2*i**3 + 15*B*a**4*b**2*c**3*d**3*g**2*i**3 - 35*B*a**3*b**3*
c**4*d**2*g**2*i**3 + 6*B*a**2*b**4*c**5*d*g**2*i**3 - B*a*b**5*c**6*g**2*i
**3 + B*a*b**3*c**4*g**2*i**3*(15*a**2*d**2 - 6*a*b*c*d + b**2*c**2) - B*b*
*4*c**5*g**2*i**3*(15*a**2*d**2 - 6*a*b*c*d + b**2*c**2)/d)/(B*a**6*d**6*g*
*2*i**3 - 6*B*a**5*b*c*d**5*g**2*i**3 + 15*B*a**4*b**2*c**2*d**4*g**2*i**3
- 20*B*a**3*b**3*c**3*d**3*g**2*i**3 - 15*B*a**2*b**4*c**4*d**2*g**2*i**3 +
 6*B*a*b**5*c**5*d*g**2*i**3 - B*b**6*c**6*g**2*i**3))/(60*d**3) + x**5*(2*
A*a*b*d**3*g**2*i**3/5 + 3*A*b**2*c*d**2*g**2*i**3/5 + B*a*b*d**3*g**2*i**3
/30 - B*b**2*c*d**2*g**2*i**3/30) + x**4*(A*a**2*d**3*g**2*i**3/4 + 3*A*a*b
*c*d**2*g**2*i**3/2 + 3*A*b**2*c**2*d*g**2*i**3/4 + 7*B*a**2*d**3*g**2*i**3
/120 + B*a*b*c*d**2*g**2*i**3/20 - 13*B*b**2*c**2*d*g**2*i**3/120) + x**3*(
A*a**2*c*d**2*g**2*i**3 + 2*A*a*b*c**2*d*g**2*i**3 + A*b**2*c**3*g**2*i**3/
3 + B*a**3*d**3*g**2*i**3/(180*b) + 13*B*a**2*c*d**2*g**2*i**3/60 - 7*B*a*b
*c**2*d*g**2*i**3/60 - 19*B*b**2*c**3*g**2*i**3/180) + x**2*(3*A*a**2*c**2*
d*g**2*i**3/2 + A*a*b*c**3*g**2*i**3 - B*a**4*d**3*g**2*i**3/(120*b**2) + B
*a**3*c*d**2*g**2*i**3/(20*b) + B*a**2*c**2*d*g**2*i**3/4 - 17*B*a*b*c**3*g
**2*i**3/60 - B*b**2*c**4*g**2*i**3/(120*d)) + x*(A*a**2*c**3*g**2*i**3 + B
*a**5*d**3*g**2*i**3/(60*b**3) - B*a**4*c*d**2*g**2*i**3/(10*b**2) + B*a**3
*c**2*d*g**2*i**3/(4*b) - B*a**2*c**3*g**2*i**3/12 - B*a*b*c**4*g**2*i**3/(
10*d) + B*b**2*c**5*g**2*i**3/(60*d**2)) + (B*a**2*c**3*g**2*i**3*x + 3*B*a
**2*c**2*d*g**2*i**3*x**2/2 + B*a**2*c*d**2*g**2*i**3*x**3 + B*a**2*d**3*g*
*2*i**3*x**4/4 + B*a*b*c**3*g**2*i**3*x**2 + 2*B*a*b*c**2*d*g**2*i**3*x**3
+ 3*B*a*b*c*d**2*g**2*i**3*x**4/2 + 2*B*a*b*d**3*g**2*i**3*x**5/5 + B*b**2*
c**3*g**2*i**3*x**3/3 + 3*B*b**2*c**2*d*g**2*i**3*x**4/4 + 3*B*b**2*c*d**2*
g**2*i**3*x**5/5 + B*b**2*d**3*g**2*i**3*x**6/6)*log(e*(a + b*x)/(c + d*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1789 vs. $2(351) = 702$.

Time = 0.25 (sec) , antiderivative size = 1789, normalized size of antiderivative = 4.82

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algor
ithm="maxima")
```

```
[Out] 1/6*A*b^2*d^3*g^2*i^3*x^6 + 3/5*A*b^2*c*d^2*g^2*i^3*x^5 + 2/5*A*a*b*d^3*g^2
*i^3*x^5 + 3/4*A*b^2*c^2*d*g^2*i^3*x^4 + 3/2*A*a*b*c*d^2*g^2*i^3*x^4 + 1/4*
```

```
A*a^2*d^3*g^2*i^3*x^4 + 1/3*A*b^2*c^3*g^2*i^3*x^3 + 2*A*a*b*c^2*d*g^2*i^3*x
^3 + A*a^2*c*d^2*g^2*i^3*x^3 + A*a*b*c^3*g^2*i^3*x^2 + 3/2*A*a^2*c^2*d*g^2*
i^3*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*lo
g(d*x + c)/d)*B*a^2*c^3*g^2*i^3 + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))
 - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a*b
*c^3*g^2*i^3 + 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(
b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c
^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*c^3*g^2*i^3 + 3/2*(x^2*log(b*e*x/(d*x + c
) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a
*d)*x/(b*d))*B*a^2*c^2*d*g^2*i^3 + (2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x +
c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2
)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*b*c^2*d*g^2*i^3 + 1/8*(6*x^
4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log
(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*
x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^2*c^2*d*g^2*i^3 + 1/2*(2*x^3*
log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d
*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)
)*B*a^2*c*d^2*g^2*i^3 + 1/4*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6
*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)
*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*
B*a*b*c*d^2*g^2*i^3 + 1/20*(12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 1
2*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^
4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2
- 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^2*c*d^2*g^2*i^3 + 1/24*(6*x^4*lo
g(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x
 + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2
+ 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*a^2*d^3*g^2*i^3 + 1/30*(12*x^5*log(
b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x
 + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*
x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*
B*a*b*d^3*g^2*i^3 + 1/360*(60*x^6*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 60
*a^6*log(b*x + a)/b^6 + 60*c^6*log(d*x + c)/d^6 - (12*(b^5*c*d^4 - a*b^4*d^
5)*x^5 - 15*(b^5*c^2*d^3 - a^2*b^3*d^5)*x^4 + 20*(b^5*c^3*d^2 - a^3*b^2*d^5
)*x^3 - 30*(b^5*c^4*d - a^4*b*d^5)*x^2 + 60*(b^5*c^5 - a^5*d^5)*x)/(b^5*d^5
))*B*b^2*d^3*g^2*i^3 + A*a^2*c^3*g^2*i^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4110 vs. $2(351) = 702$.

Time = 0.55 (sec) , antiderivative size = 4110, normalized size of antiderivative = 11.08

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algor
```

ithm="giac")

[Out] 1/360*(6*(B*b^9*c^7*e^7*g^2*i^3 - 7*B*a*b^8*c^6*d*e^7*g^2*i^3 + 21*B*a^2*b^7*c^5*d^2*e^7*g^2*i^3 - 35*B*a^3*b^6*c^4*d^3*e^7*g^2*i^3 + 35*B*a^4*b^5*c^3*d^4*e^7*g^2*i^3 - 21*B*a^5*b^4*c^2*d^5*e^7*g^2*i^3 + 7*B*a^6*b^3*c*d^6*e^7*g^2*i^3 - B*a^7*b^2*d^7*e^7*g^2*i^3 - 6*(b*e*x + a*e)*B*b^8*c^7*d*e^6*g^2*i^3/(d*x + c) + 42*(b*e*x + a*e)*B*a*b^7*c^6*d^2*e^6*g^2*i^3/(d*x + c) - 126*(b*e*x + a*e)*B*a^2*b^6*c^5*d^3*e^6*g^2*i^3/(d*x + c) + 210*(b*e*x + a*e)*B*a^3*b^5*c^4*d^4*e^6*g^2*i^3/(d*x + c) - 210*(b*e*x + a*e)*B*a^4*b^4*c^3*d^5*e^6*g^2*i^3/(d*x + c) + 126*(b*e*x + a*e)*B*a^5*b^3*c^2*d^6*e^6*g^2*i^3/(d*x + c) - 42*(b*e*x + a*e)*B*a^6*b^2*c*d^7*e^6*g^2*i^3/(d*x + c) + 6*(b*e*x + a*e)*B*a^7*b*d^8*e^6*g^2*i^3/(d*x + c) + 15*(b*e*x + a*e)^2*B*b^7*c^7*d^2*e^5*g^2*i^3/(d*x + c)^2 - 105*(b*e*x + a*e)^2*B*a*b^6*c^6*d^3*e^5*g^2*i^3/(d*x + c)^2 + 315*(b*e*x + a*e)^2*B*a^2*b^5*c^5*d^4*e^5*g^2*i^3/(d*x + c)^2 - 525*(b*e*x + a*e)^2*B*a^3*b^4*c^4*d^5*e^5*g^2*i^3/(d*x + c)^2 + 525*(b*e*x + a*e)^2*B*a^4*b^3*c^3*d^6*e^5*g^2*i^3/(d*x + c)^2 - 315*(b*e*x + a*e)^2*B*a^5*b^2*c^2*d^7*e^5*g^2*i^3/(d*x + c)^2 + 105*(b*e*x + a*e)^2*B*a^6*b*c*d^8*e^5*g^2*i^3/(d*x + c)^2 - 15*(b*e*x + a*e)^2*B*a^7*d^9*e^5*g^2*i^3/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/(b^6*d^3*e^6 - 6*(b*e*x + a*e)*b^5*d^4*e^5/(d*x + c) + 15*(b*e*x + a*e)^2*b^4*d^5*e^4/(d*x + c)^2 - 20*(b*e*x + a*e)^3*b^3*d^6*e^3/(d*x + c)^3 + 15*(b*e*x + a*e)^4*b^2*d^7*e^2/(d*x + c)^4 - 6*(b*e*x + a*e)^5*b*d^8*e/(d*x + c)^5 + (b*e*x + a*e)^6*d^9/(d*x + c)^6) + (6*A*b^12*c^7*e^7*g^2*i^3 - 2*B*b^12*c^7*e^7*g^2*i^3 - 42*A*a*b^11*c^6*d*e^7*g^2*i^3 + 14*B*a*b^11*c^6*d*e^7*g^2*i^3 + 126*A*a^2*b^10*c^5*d^2*e^7*g^2*i^3 - 42*B*a^2*b^10*c^5*d^2*e^7*g^2*i^3 - 210*A*a^3*b^9*c^4*d^3*e^7*g^2*i^3 + 70*B*a^3*b^9*c^4*d^3*e^7*g^2*i^3 + 210*A*a^4*b^8*c^3*d^4*e^7*g^2*i^3 - 70*B*a^4*b^8*c^3*d^4*e^7*g^2*i^3 - 126*A*a^5*b^7*c^2*d^5*e^7*g^2*i^3 + 42*B*a^5*b^7*c^2*d^5*e^7*g^2*i^3 + 42*A*a^6*b^6*c*d^6*e^7*g^2*i^3 - 14*B*a^6*b^6*c*d^6*e^7*g^2*i^3 - 6*A*a^7*b^5*d^7*e^7*g^2*i^3 + 2*B*a^7*b^5*d^7*e^7*g^2*i^3 - 36*(b*e*x + a*e)*A*b^11*c^7*d*e^6*g^2*i^3/(d*x + c) + 18*(b*e*x + a*e)*B*b^11*c^7*d*e^6*g^2*i^3/(d*x + c) + 252*(b*e*x + a*e)*A*a*b^10*c^6*d^2*e^6*g^2*i^3/(d*x + c) - 126*(b*e*x + a*e)*B*a*b^10*c^6*d^2*e^6*g^2*i^3/(d*x + c) - 756*(b*e*x + a*e)*A*a^2*b^9*c^5*d^3*e^6*g^2*i^3/(d*x + c) + 378*(b*e*x + a*e)*B*a^2*b^9*c^5*d^3*e^6*g^2*i^3/(d*x + c) + 1260*(b*e*x + a*e)*A*a^3*b^8*c^4*d^4*e^6*g^2*i^3/(d*x + c) - 630*(b*e*x + a*e)*B*a^3*b^8*c^4*d^4*e^6*g^2*i^3/(d*x + c) - 1260*(b*e*x + a*e)*A*a^4*b^7*c^3*d^5*e^6*g^2*i^3/(d*x + c) + 630*(b*e*x + a*e)*B*a^4*b^7*c^3*d^5*e^6*g^2*i^3/(d*x + c) + 756*(b*e*x + a*e)*A*a^5*b^6*c^2*d^6*e^6*g^2*i^3/(d*x + c) - 378*(b*e*x + a*e)*B*a^5*b^6*c^2*d^6*e^6*g^2*i^3/(d*x + c) - 252*(b*e*x + a*e)*A*a^6*b^5*c*d^7*e^6*g^2*i^3/(d*x + c) + 126*(b*e*x + a*e)*B*a^6*b^5*c*d^7*e^6*g^2*i^3/(d*x + c) + 36*(b*e*x + a*e)*A*a^7*b^4*d^8*e^6*g^2*i^3/(d*x + c) - 18*(b*e*x + a*e)*B*a^7*b^4*d^8*e^6*g^2*i^3/(d*x + c) + 90*(b*e*x + a*e)^2*A*b^10*c^7*d^2*e^5*g^2*i^3/(d*x + c)^2 - 63*(b*e*x + a*e)^2*B*b^10*c^7*d^2*e^5*g^2*i^3/(d*x + c)^2 - 630*(b*e*x + a*e)^2*A*a*b^9*c^6*d^3*e^5*g^2*i^3/(d*x + c)^2 + 441*(b*e*x + a*e)^2*B*a*b^9*c^6*d^3*e^5*g^2*i^3/(d*x + c)^2 + 1890*(b*e

```
*x + a*e)^2*A*a^2*b^8*c^5*d^4*e^5*g^2*i^3/(d*x + c)^2 - 1323*(b*e*x + a*e)^
2*B*a^2*b^8*c^5*d^4*e^5*g^2*i^3/(d*x + c)^2 - 3150*(b*e*x + a*e)^2*A*a^3*b^
7*c^4*d^5*e^5*g^2*i^3/(d*x + c)^2 + 2205*(b*e*x + a*e)^2*B*a^3*b^7*c^4*d^5*
e^5*g^2*i^3/(d*x + c)^2 + 3150*(b*e*x + a*e)^2*A*a^4*b^6*c^3*d^6*e^5*g^2*i^
3/(d*x + c)^2 - 2205*(b*e*x + a*e)^2*B*a^4*b^6*c^3*d^6*e^5*g^2*i^3/(d*x + c
)^2 - 1890*(b*e*x + a*e)^2*A*a^5*b^5*c^2*d^7*e^5*g^2*i^3/(d*x + c)^2 + 1323
*(b*e*x + a*e)^2*B*a^5*b^5*c^2*d^7*e^5*g^2*i^3/(d*x + c)^2 + 630*(b*e*x + a
*e)^2*A*a^6*b^4*c*d^8*e^5*g^2*i^3/(d*x + c)^2 - 441*(b*e*x + a*e)^2*B*a^6*b
^4*c*d^8*e^5*g^2*i^3/(d*x + c)^2 - 90*(b*e*x + a*e)^2*A*a^7*b^3*d^9*e^5*g^2
*i^3/(d*x + c)^2 + 63*(b*e*x + a*e)^2*B*a^7*b^3*d^9*e^5*g^2*i^3/(d*x + c)^2
 + 74*(b*e*x + a*e)^3*B*b^9*c^7*d^3*e^4*g^2*i^3/(d*x + c)^3 - 518*(b*e*x +
a*e)^3*B*a*b^8*c^6*d^4*e^4*g^2*i^3/(d*x + c)^3 + 1554*(b*e*x + a*e)^3*B*a^2
*b^7*c^5*d^5*e^4*g^2*i^3/(d*x + c)^3 - 2590*(b*e*x + a*e)^3*B*a^3*b^6*c^4*d
^6*e^4*g^2*i^3/(d*x + c)^3 + 2590*(b*e*x + a*e)^3*B*a^4*b^5*c^3*d^7*e^4*g^2
*i^3/(d*x + c)^3 - 1554*(b*e*x + a*e)^3*B*a^5*b^4*c^2*d^8*e^4*g^2*i^3/(d*x
+ c)^3 + 518*(b*e*x + a*e)^3*B*a^6*b^3*c*d^9*e^4*g^2*i^3/(d*x + c)^3 - 74*(
b*e*x + a*e)^3*B*a^7*b^2*d^10*e^4*g^2*i^3/(d*x + c)^3 - 33*(b*e*x + a*e)^4*
B*b^8*c^7*d^4*e^3*g^2*i^3/(d*x + c)^4 + 231*(b*e*x + a*e)^4*B*a*b^7*c^6*d^5
*e^3*g^2*i^3/(d*x + c)^4 - 693*(b*e*x + a*e)^4*B*a^2*b^6*c^5*d^6*e^3*g^2*i^
3/(d*x + c)^4 + 1155*(b*e*x + a*e)^4*B*a^3*b^5*c^4*d^7*e^3*g^2*i^3/(d*x + c
)^4 - 1155*(b*e*x + a*e)^4*B*a^4*b^4*c^3*d^8*e^3*g^2*i^3/(d*x + c)^4 + 693*
(b*e*x + a*e)^4*B*a^5*b^3*c^2*d^9*e^3*g^2*i^3/(d*x + c)^4 - 231*(b*e*x + a*
e)^4*B*a^6*b^2*c*d^10*e^3*g^2*i^3/(d*x + c)^4 + 33*(b*e*x + a*e)^4*B*a^7*b*
d^11*e^3*g^2*i^3/(d*x + c)^4 + 6*(b*e*x + a*e)^5*B*b^7*c^7*d^5*e^2*g^2*i^3/
(d*x + c)^5 - 42*(b*e*x + a*e)^5*B*a*b^6*c^6*d^6*e^2*g^2*i^3/(d*x + c)^5 +
126*(b*e*x + a*e)^5*B*a^2*b^5*c^5*d^7*e^2*g^2*i^3/(d*x + c)^5 - 210*(b*e*x
+ a*e)^5*B*a^3*b^4*c^4*d^8*e^2*g^2*i^3/(d*x + c)^5 + 210*(b*e*x + a*e)^5*B*
a^4*b^3*c^3*d^9*e^2*g^2*i^3/(d*x + c)^5 - 126*(b*e*x + a*e)^5*B*a^5*b^2*c^2
*d^10*e^2*g^2*i^3/(d*x + c)^5 + 42*(b*e*x + a*e)^5*B*a^6*b*c*d^11*e^2*g^2*i
^3/(d*x + c)^5 - 6*(b*e*x + a*e)^5*B*a^7*d^12*e^2*g^2*i^3/(d*x + c)^5)/(b^9
*d^3*e^6 - 6*(b*e*x + a*e)*b^8*d^4*e^5/(d*x + c) + 15*(b*e*x + a*e)^2*b^7*d
^5*e^4/(d*x + c)^2 - 20*(b*e*x + a*e)^3*b^6*d^6*e^3/(d*x + c)^3 + 15*(b*e*x
 + a*e)^4*b^5*d^7*e^2/(d*x + c)^4 - 6*(b*e*x + a*e)^5*b^4*d^8*e/(d*x + c)^5
 + (b*e*x + a*e)^6*b^3*d^9/(d*x + c)^6) + 6*(B*b^7*c^7*e*g^2*i^3 - 7*B*a*b^
6*c^6*d*e*g^2*i^3 + 21*B*a^2*b^5*c^5*d^2*e*g^2*i^3 - 35*B*a^3*b^4*c^4*d^3*e
*g^2*i^3 + 35*B*a^4*b^3*c^3*d^4*e*g^2*i^3 - 21*B*a^5*b^2*c^2*d^5*e*g^2*i^3
+ 7*B*a^6*b*c*d^6*e*g^2*i^3 - B*a^7*d^7*e*g^2*i^3)*log(-b*e + (b*e*x + a*e)
*d/(d*x + c))/(b^4*d^3) - 6*(B*b^7*c^7*e*g^2*i^3 - 7*B*a*b^6*c^6*d*e*g^2*i^
3 + 21*B*a^2*b^5*c^5*d^2*e*g^2*i^3 - 35*B*a^3*b^4*c^4*d^3*e*g^2*i^3 + 35*B*
a^4*b^3*c^3*d^4*e*g^2*i^3 - 21*B*a^5*b^2*c^2*d^5*e*g^2*i^3 + 7*B*a^6*b*c*d^
6*e*g^2*i^3 - B*a^7*d^7*e*g^2*i^3)*log((b*e*x + a*e)/(d*x + c))/(b^4*d^3))*
(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 2465, normalized size of antiderivative = 6.64

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

```
[In] int((a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x^3*((g^2*i^3*(4*A*a^3*d^3 + 16*A*b^3*c^3 + B*a^3*d^3 - 3*B*b^3*c^3 + 72*A*
a*b^2*c^2*d + 48*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d + 5*B*a^2*b*c*d^2))/(12*b)
 + ((60*a*d + 60*b*c)*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d - B*b*
c))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*b*d)
 - (d*g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*d^2 - 3*B*b^2*c^2 + 60
*A*a*b*c*d + B*a*b*c*d))/5 + A*a*b*c*d^2*g^2*i^3))/(180*b*d) - (a*c*((b*d^2
*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d - B*b*c))/6 - (A*b*d^2*g^2*i^3*(60*a*
d + 60*b*c))/60))/(3*b*d)) - x^4*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B
*a*d - B*b*c))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c
))/(240*b*d) - (d*g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*d^2 - 3*B*
b^2*c^2 + 60*A*a*b*c*d + B*a*b*c*d))/20 + (A*a*b*c*d^2*g^2*i^3)/4) + x^2*((
a*c*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d - B*b*c))/6 - (A*b*d^2*g
^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*b*d) - (d*g^2*i^3*(15*
A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*d^2 - 3*B*b^2*c^2 + 60*A*a*b*c*d + B*a*b
*c*d))/5 + A*a*b*c*d^2*g^2*i^3))/(2*b*d) - ((60*a*d + 60*b*c)*((g^2*i^3*(4*
A*a^3*d^3 + 16*A*b^3*c^3 + B*a^3*d^3 - 3*B*b^3*c^3 + 72*A*a*b^2*c^2*d + 48*
A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d + 5*B*a^2*b*c*d^2))/(4*b) + ((60*a*d + 60*b
*c)*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d - B*b*c))/6 - (A*b*d^2*g
^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*b*d) - (d*g^2*i^3*(15*
A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*d^2 - 3*B*b^2*c^2 + 60*A*a*b*c*d + B*a*b
*c*d))/5 + A*a*b*c*d^2*g^2*i^3))/(60*b*d) - (a*c*((b*d^2*g^2*i^3*(18*A*a*d
+ 24*A*b*c + B*a*d - B*b*c))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60))/(
b*d)))/(120*b*d) + (c*g^2*i^3*(12*A*a^3*d^3 + 3*A*b^3*c^3 + 3*B*a^3*d^3 - B
*b^3*c^3 + 36*A*a*b^2*c^2*d + 54*A*a^2*b*c*d^2 - 5*B*a*b^2*c^2*d + 3*B*a^2*
b*c*d^2))/(6*b*d)) + log((e*(a + b*x))/(c + d*x))*(B*a^2*c^3*g^2*i^3*x + (B
*c*g^2*i^3*x^3*(3*a^2*d^2 + b^2*c^2 + 6*a*b*c*d))/3 + (B*d*g^2*i^3*x^4*(a^2
*d^2 + 3*b^2*c^2 + 6*a*b*c*d))/4 + (B*b^2*d^3*g^2*i^3*x^6)/6 + (B*a*c^2*g^2
*i^3*x^2*(3*a*d + 2*b*c))/2 + (B*b*d^2*g^2*i^3*x^5*(2*a*d + 3*b*c))/5) + x^
5*((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d - B*b*c))/30 - (A*b*d^2*g^2*
i^3*(60*a*d + 60*b*c))/300) - x*(((60*a*d + 60*b*c)*((a*c*((((b*d^2*g^2*i^3
*(18*A*a*d + 24*A*b*c + B*a*d - B*b*c))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b
*c))/60)*(60*a*d + 60*b*c))/(60*b*d) - (d*g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*
c^2 + 2*B*a^2*d^2 - 3*B*b^2*c^2 + 60*A*a*b*c*d + B*a*b*c*d))/5 + A*a*b*c*d^
2*g^2*i^3))/(b*d) - ((60*a*d + 60*b*c)*((g^2*i^3*(4*A*a^3*d^3 + 16*A*b^3*c^
3 + B*a^3*d^3 - 3*B*b^3*c^3 + 72*A*a*b^2*c^2*d + 48*A*a^2*b*c*d^2 - 3*B*a*b
^2*c^2*d + 5*B*a^2*b*c*d^2))/(4*b) + ((60*a*d + 60*b*c)*((((b*d^2*g^2*i^3*(
```

```
18*A*a*d + 24*A*b*c + B*a*d - B*b*c))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c
))/60)*(60*a*d + 60*b*c))/(60*b*d) - (d*g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^
2 + 2*B*a^2*d^2 - 3*B*b^2*c^2 + 60*A*a*b*c*d + B*a*b*c*d))/5 + A*a*b*c*d^2*
g^2*i^3))/(60*b*d) - (a*c*((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d - B*
b*c))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60))/(b*d)))/(60*b*d) + (c*g^
2*i^3*(12*A*a^3*d^3 + 3*A*b^3*c^3 + 3*B*a^3*d^3 - B*b^3*c^3 + 36*A*a*b^2*c^
2*d + 54*A*a^2*b*c*d^2 - 5*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2))/(3*b*d)))/(60*
b*d) + (a*c*((g^2*i^3*(4*A*a^3*d^3 + 16*A*b^3*c^3 + B*a^3*d^3 - 3*B*b^3*c^3
 + 72*A*a*b^2*c^2*d + 48*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d + 5*B*a^2*b*c*d^2)
)/(4*b) + ((60*a*d + 60*b*c)*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d
 - B*b*c))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(
60*b*d) - (d*g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*d^2 - 3*B*b^2*c
^2 + 60*A*a*b*c*d + B*a*b*c*d))/5 + A*a*b*c*d^2*g^2*i^3))/(60*b*d) - (a*c*(
(b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d - B*b*c))/6 - (A*b*d^2*g^2*i^3*
(60*a*d + 60*b*c))/60))/(b*d)))/(b*d) - (a*c^2*g^2*i^3*(12*A*a^2*d^2 + 6*A*
b^2*c^2 + 3*B*a^2*d^2 - 2*B*b^2*c^2 + 24*A*a*b*c*d - B*a*b*c*d))/(2*b*d)) -
 (log(c + d*x)*(B*b^2*c^6*g^2*i^3 + 15*B*a^2*c^4*d^2*g^2*i^3 - 6*B*a*b*c^5*
d*g^2*i^3))/(60*d^3) - (log(a + b*x)*(B*a^6*d^3*g^2*i^3 - 20*B*a^3*b^3*c^3*
g^2*i^3 - 6*B*a^5*b*c*d^2*g^2*i^3 + 15*B*a^4*b^2*c^2*d*g^2*i^3))/(60*b^4) +
 (A*b^2*d^3*g^2*i^3*x^6)/6
```

3.22 $\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

Optimal result

Integrand size = 38, antiderivative size = 271

$$\begin{aligned}
&\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx \\
&= \frac{B(bc-ad)^4 gi^3 x}{20b^3 d} + \frac{B(bc-ad)^3 gi^3 (c+dx)^2}{40b^2 d^2} + \frac{B(bc-ad)^2 gi^3 (c+dx)^3}{60bd^2} \\
&\quad - \frac{B(bc-ad) gi^3 (c+dx)^4}{20d^2} + \frac{B(bc-ad)^5 gi^3 \log\left(\frac{a+bx}{c+dx}\right)}{20b^4 d^2} \\
&\quad - \frac{(bc-ad) gi^3 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2} \\
&\quad + \frac{bgi^3 (c+dx)^5 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^2} + \frac{B(bc-ad)^5 gi^3 \log(c+dx)}{20b^4 d^2}
\end{aligned}$$

```
[Out] 1/20*B*(-a*d+b*c)^4*g*i^3*x/b^3/d+1/40*B*(-a*d+b*c)^3*g*i^3*(d*x+c)^2/b^2/d
^2+1/60*B*(-a*d+b*c)^2*g*i^3*(d*x+c)^3/b/d^2-1/20*B*(-a*d+b*c)*g*i^3*(d*x+c
)^4/d^2+1/20*B*(-a*d+b*c)^5*g*i^3*ln((b*x+a)/(d*x+c))/b^4/d^2-1/4*(-a*d+b*c
)*g*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^2+1/5*b*g*i^3*(d*x+c)^5*(A+
B*ln(e*(b*x+a)/(d*x+c)))/d^2+1/20*B*(-a*d+b*c)^5*g*i^3*ln(d*x+c)/b^4/d^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2562, 45, 2382, 12, 78}

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=-\frac{gi^3(c+dx)^4(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^2}+\frac{bgi^3(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^2}+\frac{Bgi^3(bc-ad)^5\log\left(\frac{a+bx}{c+dx}\right)}{20b^4d^2}+\frac{Bgi^3(bc-ad)^5\log(c+dx)}{20b^4d^2}+\frac{Bgi^3x(bc-ad)^4}{20b^3d}+\frac{Bgi^3(c+dx)^2(bc-ad)^3}{40b^2d^2}+\frac{Bgi^3(c+dx)^3(bc-ad)^2}{60bd^2}-\frac{Bgi^3(c+dx)^4(bc-ad)}{20d^2}$$

```
[In] Int[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] (B*(b*c - a*d)^4*g*i^3*x)/(20*b^3*d) + (B*(b*c - a*d)^3*g*i^3*(c + d*x)^2)/
(40*b^2*d^2) + (B*(b*c - a*d)^2*g*i^3*(c + d*x)^3)/(60*b*d^2) - (B*(b*c - a
*d)*g*i^3*(c + d*x)^4)/(20*d^2) + (B*(b*c - a*d)^5*g*i^3*Log[(a + b*x)/(c +
 d*x)])/(20*b^4*d^2) - ((b*c - a*d)*g*i^3*(c + d*x)^4*(A + B*Log[(e*(a + b*
x))/(c + d*x)]))/(4*d^2) + (b*g*i^3*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c
 + d*x)]))/(5*d^2) + (B*(b*c - a*d)^5*g*i^3*Log[c + d*x])/(20*b^4*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((bc-ad)^5 gi^3\right) \text{Subst}\left(\int \frac{x(A+B\log(ex))}{(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)\\
&= -\frac{(bc-ad)gi^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2}\\
&\quad+\frac{bgi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^2}\\
&\quad-\left(B(bc-ad)^5 gi^3\right) \text{Subst}\left(\int \frac{-b+5dx}{20d^2x(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)\\
&= -\frac{(bc-ad)gi^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2}\\
&\quad+\frac{bgi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^2}\\
&\quad-\frac{\left(B(bc-ad)^5 gi^3\right) \text{Subst}\left(\int \frac{-b+5dx}{x(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{20d^2}\\
&= -\frac{(bc-ad)gi^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2}+\frac{bgi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^2}\\
&\quad-\frac{\left(B(bc-ad)^5 gi^3\right) \text{Subst}\left(\int \left(-\frac{1}{b^4x}+\frac{4d}{(b-dx)^5}-\frac{d}{b(b-dx)^4}-\frac{d}{b^2(b-dx)^3}-\frac{d}{b^3(b-dx)^2}-\frac{d}{b^4(b-dx)}\right)dx, x, \frac{a+}{c+}}{20d^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{B(bc-ad)^4 gi^3 x}{20b^3 d} + \frac{B(bc-ad)^3 gi^3 (c+dx)^2}{40b^2 d^2} + \frac{B(bc-ad)^2 gi^3 (c+dx)^3}{60bd^2} \\
&- \frac{B(bc-ad) gi^3 (c+dx)^4}{20d^2} + \frac{B(bc-ad)^5 gi^3 \log\left(\frac{a+bx}{c+dx}\right)}{20b^4 d^2} \\
&- \frac{(bc-ad) gi^3 (c+dx)^4 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2} \\
&+ \frac{bgi^3 (c+dx)^5 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^2} + \frac{B(bc-ad)^5 gi^3 \log(c+dx)}{20b^4 d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.96

$$
\int (ag+bgx)(ci+dix)^3 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx
$$

$$
= \frac{gi^3 \left(\frac{5B(bc-ad)^2 (6bd(bc-ad)^2 x + 3b^2(bc-ad)(c+dx)^2 + 2b^3(c+dx)^3 + 6(bc-ad)^3 \log(a+bx))}{b^4} - \frac{2B(bc-ad)(12bd(bc-ad)^3 x + 6b^2(bc-ad)^2(c+dx)\right.}{}
$$

```
[In] Integrate[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]
),x]
```

```
[Out] (g*i^3*((5*B*(b*c - a*d)^2*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c +
d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]))/b^4 - (2*B*(b*c
 - a*d)*(12*b*d*(b*c - a*d)^3*x + 6*b^2*(b*c - a*d)^2*(c + d*x)^2 + 4*b^3*(
b*c - a*d)*(c + d*x)^3 + 3*b^4*(c + d*x)^4 + 12*(b*c - a*d)^4*Log[a + b*x])
)/b^4 - 30*(b*c - a*d)*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2
4*b*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])))/(120*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. 2(255) = 510.

Time = 0.89 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.10

method	result
risch	$\frac{i^3 g\, d^3 B\, a^4 x}{20 b^3} - \frac{i^3 g b B\, c^4 x}{20 d} - \frac{i^3 g\, d^3 B \ln(bx+a) a^5}{20 b^4} + \frac{i^3 g b B \ln(-dx-c) c^5}{20 d^2} + \frac{i^3 g\, d^3 A a\, x^4}{4} + \frac{3 i^3 g b\, d^2 A c\, x^4}{4} + \frac{i^3 g\, d^3}{2}$
parallelrisch	$\frac{-6 B\, a^5 d^5 g\, i^3 + 6 B\, b^5 c^5 g\, i^3 + 27 B\, a^4 b c\, d^4 g\, i^3 - 60 B \ln(bx+a) a^3 b^2 c^2 d^3 g\, i^3 + 60 B \ln(bx+a) a^2 b^3 c^3 d^2 g\, i^3 - 30 B \ln(bx+a) a\, b^4 c}{}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

```
[In] int((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/20*i^3*g/b^3*d^3*B*a^4*x-1/20*i^3*g*b/d*B*c^4*x-1/20*i^3*g/b^4*d^3*B*ln(b
*x+a)*a^5+1/20*i^3*g*b/d^2*B*ln(-d*x-c)*c^5+1/4*i^3*g*d^3*A*a*x^4+3/4*i^3*g
*b*d^2*A*c*x^4+1/20*i^3*g*d^3*B*a*x^4-1/20*i^3*g*b*d^2*B*c*x^4+i^3*g*b*d*A*
c^2*x^3+1/60*i^3*g/b*d^3*B*a^2*x^3-11/60*i^3*g*b*d*B*c^2*x^3+1/2*i^3*g*b*A*
c^3*x^2-1/40*i^3*g/b^2*d^3*B*a^3*x^2-9/40*i^3*g*b*B*c^3*x^2+i^3*g*d^2*A*a*c
*x^3+1/6*i^3*g*d^2*B*a*c*x^3+3/2*i^3*g*d*A*a*c^2*x^2+1/8*i^3*g/b*d^2*B*a^2*
c*x^2+1/8*i^3*g*d*B*a*c^2*x^2+i^3*g*A*a*c^3*x-1/4*i^3*g/b^2*d^2*B*a^3*c*x+1
/2*i^3*g/b*d*B*a^2*c^2*x-1/4*i^3*g*B*a*c^3*x+1/4*i^3*g/b^3*d^2*B*ln(b*x+a)*
a^4*c-1/2*i^3*g/b^2*d*B*ln(b*x+a)*a^3*c^2+1/2*i^3*g/b*B*ln(b*x+a)*a^2*c^3-1
/4*i^3*g/d*B*ln(-d*x-c)*a*c^4+1/5*i^3*g*b*d^3*A*x^5+1/20*g*i^3*B*x*(4*b*d^3
*x^4+5*a*d^3*x^3+15*b*c*d^2*x^3+20*a*c*d^2*x^2+20*b*c^2*d*x^2+30*a*c^2*d*x+
10*b*c^3*x+20*a*c^3)*ln(e*(b*x+a)/(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.85

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{24\,Ab^5d^5gi^3x^5+6\,((15\,A-B)b^5cd^4+(5\,A+B)ab^4d^5)gi^3x^4+2\,((60\,A-11\,B)b^5c^2d^3+10\,(6\,A+B)ab^4c\cdots}{\cdots}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorit
hm="fricas")
```

```
[Out] 1/120*(24*A*b^5*d^5*g*i^3*x^5 + 6*((15*A - B)*b^5*c*d^4 + (5*A + B)*a*b^4*d
^5)*g*i^3*x^4 + 2*((60*A - 11*B)*b^5*c^2*d^3 + 10*(6*A + B)*a*b^4*c*d^4 + B
*a^2*b^3*d^5)*g*i^3*x^3 + 3*((20*A - 9*B)*b^5*c^3*d^2 + 5*(12*A + B)*a*b^4*
c^2*d^3 + 5*B*a^2*b^3*c*d^4 - B*a^3*b^2*d^5)*g*i^3*x^2 - 6*(B*b^5*c^4*d - 5
*(4*A - B)*a*b^4*c^3*d^2 - 10*B*a^2*b^3*c^2*d^3 + 5*B*a^3*b^2*c*d^4 - B*a^4
*b*d^5)*g*i^3*x + 6*(10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*
b*c*d^4 - B*a^5*d^5)*g*i^3*log(b*x + a) + 6*(B*b^5*c^5 - 5*B*a*b^4*c^4*d)*g
*i^3*log(d*x + c) + 6*(4*B*b^5*d^5*g*i^3*x^5 + 20*B*a*b^4*c^3*d^2*g*i^3*x +
 5*(3*B*b^5*c*d^4 + B*a*b^4*d^5)*g*i^3*x^4 + 20*(B*b^5*c^2*d^3 + B*a*b^4*c*
d^4)*g*i^3*x^3 + 10*(B*b^5*c^3*d^2 + 3*B*a*b^4*c^2*d^3)*g*i^3*x^2)*log((b*e
*x + a*e)/(d*x + c)))/(b^4*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1158 vs. $2(252) = 504$.

Time = 3.84 (sec) , antiderivative size = 1158, normalized size of antiderivative = 4.27

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\frac{Abd^3gi^3x^5}{5}$$

$$-\frac{Ba^2gi^3(a^3d^3-5a^2bcd^2+10ab^2c^2d-10b^3c^3)\log\left(x+\frac{Ba^5cd^4gi^3-5Ba^4bc^2d^3gi^3+10Ba^3b^2c^3d^2gi^3+\frac{Ba^3d^2gi^3\left(a^3d^3-5a^2b\cdots\right.}{\cdots}}{Ba^5d^5gi^3-5Ba^4bcd^4gi^3+1\cdots}\right.}{20b^4}$$

$$-\frac{Bc^4gi^3\cdot(5ad-bc)\log\left(x+\frac{Ba^5cd^4gi^3-5Ba^4bc^2d^3gi^3+10Ba^3b^2c^3d^2gi^3-15Ba^2b^3c^4dgi^3+Bab^4c^5gi^3+Bab^3c^4gi^3\cdot(5ad-bc)-\frac{Bb^4\cdots}{\cdots}}{Ba^5d^5gi^3-5Ba^4bcd^4gi^3+10Ba^3b^2c^2d^3gi^3-10Ba^2b^3c^3d^2gi^3-5Bab^4c^4dgi^3+Bb^5c^5gi^3}\right.}{20d^2}$$

$$+x^4\left(\frac{Aad^3gi^3}{4}+\frac{3Abcd^2gi^3}{4}+\frac{Bad^3gi^3}{20}-\frac{Bbcd^2gi^3}{20}\right)$$

$$+x^3\left(Aacd^2gi^3+Abc^2dgi^3+\frac{Ba^2d^3gi^3}{60b}+\frac{Bacd^2gi^3}{6}-\frac{11Bbc^2dgi^3}{60}\right)+x^2$$

$$\cdot\left(\frac{3Aac^2dgi^3}{2}+\frac{Abc^3gi^3}{2}-\frac{Ba^3d^3gi^3}{40b^2}+\frac{Ba^2cd^2gi^3}{8b}+\frac{Bac^2dgi^3}{8}-\frac{9Bbc^3gi^3}{40}\right)$$

$$+x\left(Aac^3gi^3+\frac{Ba^4d^3gi^3}{20b^3}-\frac{Ba^3cd^2gi^3}{4b^2}+\frac{Ba^2c^2dgi^3}{2b}-\frac{Bac^3gi^3}{4}-\frac{Bbc^4gi^3}{20d}\right)$$

$$+\left(Bac^3gi^3x+\frac{3Bac^2dgi^3x^2}{2}+Bacd^2gi^3x^3+\frac{Bad^3gi^3x^4}{4}+\frac{Bbc^3gi^3x^2}{2}+Bbc^2dgi^3x^3\right.$$

$$\left.+\frac{3Bbcd^2gi^3x^4}{4}+\frac{Bbd^3gi^3x^5}{5}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b*d**3*g*i**3*x**5/5 - B*a**2*g*i**3*(a**3*d**3 - 5*a**2*b*c*d**2 + 10*a*
b**2*c**2*d - 10*b**3*c**3)*log(x + (B*a**5*c*d**4*g*i**3 - 5*B*a**4*b*c**2
*d**3*g*i**3 + 10*B*a**3*b**2*c**3*d**2*g*i**3 + B*a**3*d**2*g*i**3*(a**3*d
**3 - 5*a**2*b*c*d**2 + 10*a*b**2*c**2*d - 10*b**3*c**3)/b - 15*B*a**2*b**3
*c**4*d*g*i**3 - B*a**2*c*d*g*i**3*(a**3*d**3 - 5*a**2*b*c*d**2 + 10*a*b**2
*c**2*d - 10*b**3*c**3) + B*a*b**4*c**5*g*i**3)/(B*a**5*d**5*g*i**3 - 5*B*a
**4*b*c*d**4*g*i**3 + 10*B*a**3*b**2*c**2*d**3*g*i**3 - 10*B*a**2*b**3*c**3
*d**2*g*i**3 - 5*B*a*b**4*c**4*d*g*i**3 + B*b**5*c**5*g*i**3))/(20*b**4) -
B*c**4*g*i**3*(5*a*d - b*c)*log(x + (B*a**5*c*d**4*g*i**3 - 5*B*a**4*b*c**2
*d**3*g*i**3 + 10*B*a**3*b**2*c**3*d**2*g*i**3 - 15*B*a**2*b**3*c**4*d*g*i*
*3 + B*a*b**4*c**5*g*i**3 + B*a*b**3*c**4*g*i**3*(5*a*d - b*c) - B*b**4*c**
5*g*i**3*(5*a*d - b*c)/d)/(B*a**5*d**5*g*i**3 - 5*B*a**4*b*c*d**4*g*i**3 +
10*B*a**3*b**2*c**2*d**3*g*i**3 - 10*B*a**2*b**3*c**3*d**2*g*i**3 - 5*B*a*b
**4*c**4*d*g*i**3 + B*b**5*c**5*g*i**3))/(20*d**2) + x**4*(A*a*d**3*g*i**3/
4 + 3*A*b*c*d**2*g*i**3/4 + B*a*d**3*g*i**3/20 - B*b*c*d**2*g*i**3/20) + x*
```

```
*3*(A*a*c*d**2*g*i**3 + A*b*c**2*d*g*i**3 + B*a**2*d**3*g*i**3/(60*b) + B*a
*c*d**2*g*i**3/6 - 11*B*b*c**2*d*g*i**3/60) + x**2*(3*A*a*c**2*d*g*i**3/2 +
 A*b*c**3*g*i**3/2 - B*a**3*d**3*g*i**3/(40*b**2) + B*a**2*c*d**2*g*i**3/(8
*b) + B*a*c**2*d*g*i**3/8 - 9*B*b*c**3*g*i**3/40) + x*(A*a*c**3*g*i**3 + B*
a**4*d**3*g*i**3/(20*b**3) - B*a**3*c*d**2*g*i**3/(4*b**2) + B*a**2*c**2*d*
g*i**3/(2*b) - B*a*c**3*g*i**3/4 - B*b*c**4*g*i**3/(20*d)) + (B*a*c**3*g*i*
*3*x + 3*B*a*c**2*d*g*i**3*x**2/2 + B*a*c*d**2*g*i**3*x**3 + B*a*d**3*g*i**
3*x**4/4 + B*b*c**3*g*i**3*x**2/2 + B*b*c**2*d*g*i**3*x**3 + 3*B*b*c*d**2*g
*i**3*x**4/4 + B*b*d**3*g*i**3*x**5/5)*log(e*(a + b*x)/(c + d*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1022 vs. $2(255) = 510$.

Time = 0.31 (sec) , antiderivative size = 1022, normalized size of antiderivative = 3.77

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{1}{5}Abd^3gi^3x^5 + \frac{3}{4}Abcd^2gi^3x^4 + \frac{1}{4}Aad^3gi^3x^4 + Abc^2dgi^3x^3 + Aacd^2gi^3x^3 + \frac{1}{2}Abc^3gi^3x^2$$
$$+ \frac{3}{2}Aac^2dgi^3x^2 + \left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right) + \frac{a\log(bx+a)}{b} - \frac{c\log(dx+c)}{d}\right)Bac^3gi^3$$
$$+\frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right) - \frac{a^2\log(bx+a)}{b^2} + \frac{c^2\log(dx+c)}{d^2} - \frac{(bc-ad)x}{bd}\right)Bbc^3gi^3$$
$$+\frac{3}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right) - \frac{a^2\log(bx+a)}{b^2} + \frac{c^2\log(dx+c)}{d^2} - \frac{(bc-ad)x}{bd}\right)Bac^2dgi^3$$
$$+\frac{1}{2}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right) + \frac{2a^3\log(bx+a)}{b^3} - \frac{2c^3\log(dx+c)}{d^3} - \frac{(b^2cd-abd^2)x^2 - 2(b^2c^2-a}{b^2d^2}\right.$$
$$+\frac{1}{2}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right) + \frac{2a^3\log(bx+a)}{b^3} - \frac{2c^3\log(dx+c)}{d^3} - \frac{(b^2cd-abd^2)x^2 - 2(b^2c^2-a}{b^2d^2}\right.$$
$$+\frac{1}{8}\left(6x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right) - \frac{6a^4\log(bx+a)}{b^4} + \frac{6c^4\log(dx+c)}{d^4} - \frac{2(b^3cd^2-ab^2d^3)x^3 - 3(b^3c^2c}{b^3}\right.$$
$$+\frac{1}{24}\left(6x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right) - \frac{6a^4\log(bx+a)}{b^4} + \frac{6c^4\log(dx+c)}{d^4} - \frac{2(b^3cd^2-ab^2d^3)x^3 - 3(b^3c^2}{}\right.$$
$$+\frac{1}{60}\left(12x^5\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right) + \frac{12a^5\log(bx+a)}{b^5} - \frac{12c^5\log(dx+c)}{d^5} - \frac{3(b^4cd^3-ab^3d^4)x^4 - 4(}{}\right.$$
$$+ Aac^3gi^3x$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorit
hm="maxima")
```

```
[Out] 1/5*A*b*d^3*g*i^3*x^5 + 3/4*A*b*c*d^2*g*i^3*x^4 + 1/4*A*a*d^3*g*i^3*x^4 + A
*b*c^2*d*g*i^3*x^3 + A*a*c*d^2*g*i^3*x^3 + 1/2*A*b*c^3*g*i^3*x^2 + 3/2*A*a*
c^2*d*g*i^3*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/
```

```
b - c*log(d*x + c)/d)*B*a*c^3*g*i^3 + 1/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d
*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d
))*B*b*c^3*g*i^3 + 3/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(
b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a*c^2*d*g*i^3
+ 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3
- 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)
*x)/(b^2*d^2))*B*b*c^2*d*g*i^3 + 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x
+ c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d
^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*c*d^2*g*i^3 + 1/8*(6*x^4*
log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d
*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^
2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b*c*d^2*g*i^3 + 1/24*(6*x^4*log(b
*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x +
c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6
*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*a*d^3*g*i^3 + 1/60*(12*x^5*log(b*e*x/(
d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d
^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6
*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b*d^3
*g*i^3 + A*a*c^3*g*i^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2589 vs. $2(255) = 510$.

Time = 0.48 (sec) , antiderivative size = 2589, normalized size of antiderivative = 9.55

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorit
hm="giac")
```

```
[Out] -1/120*(6*(B*b^7*c^6*e^6*g*i^3 - 6*B*a*b^6*c^5*d*e^6*g*i^3 + 15*B*a^2*b^5*c
^4*d^2*e^6*g*i^3 - 20*B*a^3*b^4*c^3*d^3*e^6*g*i^3 + 15*B*a^4*b^3*c^2*d^4*e^
6*g*i^3 - 6*B*a^5*b^2*c*d^5*e^6*g*i^3 + B*a^6*b*d^6*e^6*g*i^3 - 5*(b*e*x +
a*e)*B*b^6*c^6*d*e^5*g*i^3/(d*x + c) + 30*(b*e*x + a*e)*B*a*b^5*c^5*d^2*e^5
*g*i^3/(d*x + c) - 75*(b*e*x + a*e)*B*a^2*b^4*c^4*d^3*e^5*g*i^3/(d*x + c) +
 100*(b*e*x + a*e)*B*a^3*b^3*c^3*d^4*e^5*g*i^3/(d*x + c) - 75*(b*e*x + a*e)
*B*a^4*b^2*c^2*d^5*e^5*g*i^3/(d*x + c) + 30*(b*e*x + a*e)*B*a^5*b*c*d^6*e^5
*g*i^3/(d*x + c) - 5*(b*e*x + a*e)*B*a^6*d^7*e^5*g*i^3/(d*x + c))*log((b*e*
x + a*e)/(d*x + c))/(b^5*d^2*e^5 - 5*(b*e*x + a*e)*b^4*d^3*e^4/(d*x + c) +
10*(b*e*x + a*e)^2*b^3*d^4*e^3/(d*x + c)^2 - 10*(b*e*x + a*e)^3*b^2*d^5*e^2
/(d*x + c)^3 + 5*(b*e*x + a*e)^4*b*d^6*e/(d*x + c)^4 - (b*e*x + a*e)^5*d^7/
(d*x + c)^5) + (6*A*b^10*c^6*e^6*g*i^3 - 5*B*b^10*c^6*e^6*g*i^3 - 36*A*a*b^
9*c^5*d*e^6*g*i^3 + 30*B*a*b^9*c^5*d*e^6*g*i^3 + 90*A*a^2*b^8*c^4*d^2*e^6*g
*i^3 - 75*B*a^2*b^8*c^4*d^2*e^6*g*i^3 - 120*A*a^3*b^7*c^3*d^3*e^6*g*i^3 + 1
```

00*B*a^3*b^7*c^3*d^3*e^6*g*i^3 + 90*A*a^4*b^6*c^2*d^4*e^6*g*i^3 - 75*B*a^4*b^6*c^2*d^4*e^6*g*i^3 - 36*A*a^5*b^5*c*d^5*e^6*g*i^3 + 30*B*a^5*b^5*c*d^5*e^6*g*i^3 + 6*A*a^6*b^4*d^6*e^6*g*i^3 - 5*B*a^6*b^4*d^6*e^6*g*i^3 - 30*(b*e*x + a*e)*A*b^9*c^6*d*e^5*g*i^3/(d*x + c) + 31*(b*e*x + a*e)*B*b^9*c^6*d*e^5*g*i^3/(d*x + c) + 180*(b*e*x + a*e)*A*a*b^8*c^5*d^2*e^5*g*i^3/(d*x + c) - 186*(b*e*x + a*e)*B*a*b^8*c^5*d^2*e^5*g*i^3/(d*x + c) - 450*(b*e*x + a*e)*A*a^2*b^7*c^4*d^3*e^5*g*i^3/(d*x + c) + 465*(b*e*x + a*e)*B*a^2*b^7*c^4*d^3*e^5*g*i^3/(d*x + c) + 600*(b*e*x + a*e)*A*a^3*b^6*c^3*d^4*e^5*g*i^3/(d*x + c) - 620*(b*e*x + a*e)*B*a^3*b^6*c^3*d^4*e^5*g*i^3/(d*x + c) - 450*(b*e*x + a*e)*A*a^4*b^5*c^2*d^5*e^5*g*i^3/(d*x + c) + 465*(b*e*x + a*e)*B*a^4*b^5*c^2*d^5*e^5*g*i^3/(d*x + c) + 180*(b*e*x + a*e)*A*a^5*b^4*c*d^6*e^5*g*i^3/(d*x + c) - 186*(b*e*x + a*e)*B*a^5*b^4*c*d^6*e^5*g*i^3/(d*x + c) - 30*(b*e*x + a*e)*A*a^6*b^3*d^7*e^5*g*i^3/(d*x + c) + 31*(b*e*x + a*e)*B*a^6*b^3*d^7*e^5*g*i^3/(d*x + c) - 47*(b*e*x + a*e)^2*B*b^8*c^6*d^2*e^4*g*i^3/(d*x + c)^2 + 282*(b*e*x + a*e)^2*B*a*b^7*c^5*d^3*e^4*g*i^3/(d*x + c)^2 - 705*(b*e*x + a*e)^2*B*a^2*b^6*c^4*d^4*e^4*g*i^3/(d*x + c)^2 + 940*(b*e*x + a*e)^2*B*a^3*b^5*c^3*d^5*e^4*g*i^3/(d*x + c)^2 - 705*(b*e*x + a*e)^2*B*a^4*b^4*c^2*d^6*e^4*g*i^3/(d*x + c)^2 + 282*(b*e*x + a*e)^2*B*a^5*b^3*c*d^7*e^4*g*i^3/(d*x + c)^2 - 47*(b*e*x + a*e)^2*B*a^6*b^2*d^8*e^4*g*i^3/(d*x + c)^2 + 27*(b*e*x + a*e)^3*B*b^7*c^6*d^3*e^3*g*i^3/(d*x + c)^3 - 162*(b*e*x + a*e)^3*B*a*b^6*c^5*d^4*e^3*g*i^3/(d*x + c)^3 + 405*(b*e*x + a*e)^3*B*a^2*b^5*c^4*d^5*e^3*g*i^3/(d*x + c)^3 - 540*(b*e*x + a*e)^3*B*a^3*b^4*c^3*d^6*e^3*g*i^3/(d*x + c)^3 + 405*(b*e*x + a*e)^3*B*a^4*b^3*c^2*d^7*e^3*g*i^3/(d*x + c)^3 - 162*(b*e*x + a*e)^3*B*a^5*b^2*c*d^8*e^3*g*i^3/(d*x + c)^3 + 27*(b*e*x + a*e)^3*B*a^6*b*d^9*e^3*g*i^3/(d*x + c)^3 - 6*(b*e*x + a*e)^4*B*b^6*c^6*d^4*e^2*g*i^3/(d*x + c)^4 + 36*(b*e*x + a*e)^4*B*a*b^5*c^5*d^5*e^2*g*i^3/(d*x + c)^4 - 90*(b*e*x + a*e)^4*B*a^2*b^4*c^4*d^6*e^2*g*i^3/(d*x + c)^4 + 120*(b*e*x + a*e)^4*B*a^3*b^3*c^3*d^7*e^2*g*i^3/(d*x + c)^4 - 90*(b*e*x + a*e)^4*B*a^4*b^2*c^2*d^8*e^2*g*i^3/(d*x + c)^4 + 36*(b*e*x + a*e)^4*B*a^5*b*c*d^9*e^2*g*i^3/(d*x + c)^4 - 6*(b*e*x + a*e)^4*B*a^6*d^10*e^2*g*i^3/(d*x + c)^4)/(b^8*d^2*e^5 - 5*(b*e*x + a*e)*b^7*d^3*e^4/(d*x + c) + 10*(b*e*x + a*e)^2*b^6*d^4*e^3/(d*x + c)^2 - 10*(b*e*x + a*e)^3*b^5*d^5*e^2/(d*x + c)^3 + 5*(b*e*x + a*e)^4*b^4*d^6*e/(d*x + c)^4 - (b*e*x + a*e)^5*b^3*d^7/(d*x + c)^5) + 6*(B*b^6*c^6*e*g*i^3 - 6*B*a*b^5*c^5*d*e*g*i^3 + 15*B*a^2*b^4*c^4*d^2*e*g*i^3 - 20*B*a^3*b^3*c^3*d^3*e*g*i^3 + 15*B*a^4*b^2*c^2*d^4*e*g*i^3 - 6*B*a^5*b*c*d^5*e*g*i^3 + B*a^6*d^6*e*g*i^3)*log(-b*e + (b*e*x + a*e)*d/(d*x + c))/(b^4*d^2) - 6*(B*b^6*c^6*e*g*i^3 - 6*B*a*b^5*c^5*d*e*g*i^3 + 15*B*a^2*b^4*c^4*d^2*e*g*i^3 - 20*B*a^3*b^3*c^3*d^3*e*g*i^3 + 15*B*a^4*b^2*c^2*d^4*e*g*i^3 - 6*B*a^5*b*c*d^5*e*g*i^3 + B*a^6*d^6*e*g*i^3)*log((b*e*x + a*e)/(d*x + c))/(b^4*d^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))

Mupad [B] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 1192, normalized size of antiderivative = 4.40

$$\int (ag + bgx)(ci + dix)^3 \left(A + B \log \left(\frac{e(a + bx)}{c + dx}\right)\right) dx = \text{Too large to display}$$

```
[In] int((a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))),x)
[Out] x^4*((d^2*g*i^3*(10*A*a*d + 20*A*b*c + B*a*d - B*b*c))/20 - (A*d^2*g*i^3*(2
0*a*d + 20*b*c))/80) + x*((a*c*(((20*a*d + 20*b*c)*((d^2*g*i^3*(10*A*a*d +
20*A*b*c + B*a*d - B*b*c))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c))/20))/(20*b*d
) - (d*g*i^3*(4*A*a^2*d^2 + 24*A*b^2*c^2 + B*a^2*d^2 - 3*B*b^2*c^2 + 32*A*a
*b*c*d + 2*B*a*b*c*d))/(4*b) + A*a*c*d^2*g*i^3))/(b*d) - ((20*a*d + 20*b*c)
*(((20*a*d + 20*b*c)*(((20*a*d + 20*b*c)*((d^2*g*i^3*(10*A*a*d + 20*A*b*c +
 B*a*d - B*b*c))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c))/20))/(20*b*d) - (d*g*i
^3*(4*A*a^2*d^2 + 24*A*b^2*c^2 + B*a^2*d^2 - 3*B*b^2*c^2 + 32*A*a*b*c*d + 2
*B*a*b*c*d))/(4*b) + A*a*c*d^2*g*i^3))/(20*b*d) - (a*c*((d^2*g*i^3*(10*A*a*
d + 20*A*b*c + B*a*d - B*b*c))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c))/20))/(b*
d) + (c*g*i^3*(4*A*a^2*d^2 + 4*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 12*A*a*b
*c*d))/b))/(20*b*d) + (c^2*g*i^3*(12*A*a^2*d^2 + 2*A*b^2*c^2 + 3*B*a^2*d^2
- B*b^2*c^2 + 16*A*a*b*c*d - 2*B*a*b*c*d))/(2*b*d)) - x^3*(((20*a*d + 20*b*
c)*((d^2*g*i^3*(10*A*a*d + 20*A*b*c + B*a*d - B*b*c))/5 - (A*d^2*g*i^3*(20*
a*d + 20*b*c))/20))/(60*b*d) - (d*g*i^3*(4*A*a^2*d^2 + 24*A*b^2*c^2 + B*a^2
*d^2 - 3*B*b^2*c^2 + 32*A*a*b*c*d + 2*B*a*b*c*d))/(12*b) + (A*a*c*d^2*g*i^3
)/3) + x^2*(((20*a*d + 20*b*c)*(((20*a*d + 20*b*c)*((d^2*g*i^3*(10*A*a*d +
20*A*b*c + B*a*d - B*b*c))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c))/20))/(20*b*d
) - (d*g*i^3*(4*A*a^2*d^2 + 24*A*b^2*c^2 + B*a^2*d^2 - 3*B*b^2*c^2 + 32*A*a
*b*c*d + 2*B*a*b*c*d))/(4*b) + A*a*c*d^2*g*i^3))/(40*b*d) - (a*c*((d^2*g*i^
3*(10*A*a*d + 20*A*b*c + B*a*d - B*b*c))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c)
)/20))/(2*b*d) + (c*g*i^3*(4*A*a^2*d^2 + 4*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^
2 + 12*A*a*b*c*d))/(2*b)) + log((e*(a + b*x))/(c + d*x))*((B*c^2*g*i^3*x^2*
(3*a*d + b*c))/2 + (B*d^2*g*i^3*x^4*(a*d + 3*b*c))/4 + B*a*c^3*g*i^3*x + (B
*b*d^3*g*i^3*x^5)/5 + B*c*d*g*i^3*x^3*(a*d + b*c)) + (log(c + d*x)*(B*b*c^5
*g*i^3 - 5*B*a*c^4*d*g*i^3))/(20*d^2) - (log(a + b*x)*(B*a^5*d^3*g*i^3 - 10
*B*a^2*b^3*c^3*g*i^3 - 5*B*a^4*b*c*d^2*g*i^3 + 10*B*a^3*b^2*c^2*d*g*i^3))/(
20*b^4) + (A*b*d^3*g*i^3*x^5)/5
```

3.23 $\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

Optimal result

Integrand size = 30, antiderivative size = 149

$$\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = -\frac{B(bc-ad)^3 i^3 x}{4b^3} - \frac{B(bc-ad)^2 i^3 (c+dx)^2}{8b^2 d} - \frac{B(bc-ad) i^3 (c+dx)^3}{12bd} - \frac{B(bc-ad)^4 i^3 \log(a+bx)}{4b^4 d} + \frac{i^3 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d}$$

```
[Out] -1/4*B*(-a*d+b*c)^3*i^3*x/b^3-1/8*B*(-a*d+b*c)^2*i^3*(d*x+c)^2/b^2/d-1/12*B
*(-a*d+b*c)*i^3*(d*x+c)^3/b/d-1/4*B*(-a*d+b*c)^4*i^3*ln(b*x+a)/b^4/d+1/4*i^
3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2548, 21, 45}

$$\int (ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{i^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d} - \frac{Bi^3(bc-ad)^4\log(a+bx)}{4b^4d} - \frac{Bi^3x(bc-ad)^3}{4b^3} - \frac{Bi^3(c+dx)^2(bc-ad)^2}{8b^2d} - \frac{Bi^3(c+dx)^3(bc-ad)}{12bd}$$

```
[In] Int[(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -1/4*(B*(b*c - a*d)^3*i^3*x)/b^3 - (B*(b*c - a*d)^2*i^3*(c + d*x)^2)/(8*b^2
*d) - (B*(b*c - a*d)*i^3*(c + d*x)^3)/(12*b*d) - (B*(b*c - a*d)^4*i^3*Log[a
 + b*x])/(4*b^4*d) + (i^3*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))
/(4*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\text{integral} = \frac{i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d} - \frac{(B(bc-ad))\int\frac{(ci+dix)^4}{(a+bx)(c+dx)}\,dx}{4di}$$

$$= \frac{i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d} - \frac{(B(bc-ad)i^3)\int\frac{(c+dx)^3}{a+bx}\,dx}{4d}$$

$$= \frac{i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d} - \frac{(B(bc-ad)i^3)\int\left(\frac{d(bc-ad)^2}{b^3}+\frac{(bc-ad)^3}{b^3(a+bx)}+\frac{d(bc-ad)(c+dx)}{b^2}+\frac{d(c+dx)^2}{b}\right)dx}{4d}$$

$$= -\frac{B(bc-ad)^3i^3x}{4b^3} - \frac{B(bc-ad)^2i^3(c+dx)^2}{8b^2d} - \frac{B(bc-ad)i^3(c+dx)^3}{12bd} - \frac{B(bc-ad)^4i^3\log(a+bx)}{4b^4d} + \frac{i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$\int(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{i^3\left(-\frac{B(bc-ad)\left(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad)^3\log(a+bx)\right)}{6b^4}+(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\right)}{4d}$$

```
[In] Integrate[(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
[Out] (i^3*(-1/6*(B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c + d
*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]))/b^4 + (c + d*x)^
4*(A + B*Log[(e*(a + b*x))/(c + d*x)])))/(4*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(139) = 278.

Time = 0.74 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.11

method	result
risch	$\frac{i^3(dx+c)^4 B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{4d}+\frac{i^3d^3Ax^4}{4}+i^3d^2Acx^3+\frac{i^3d^3Bax^3}{12b}-\frac{i^3d^2Bcx^3}{12}+\frac{3i^3dAc^2x^2}{2}-\frac{i^3d^3Ba^2x^2}{8b^2}+$
parts	$\frac{Ai^3(dx+c)^4}{4d}-Bi^3(ad-cb)^4e^4\left(\frac{1}{8b^2e^2d\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)^2}-\frac{1}{12bed\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)^3}-\frac{\ln\left(\left(\frac{b}{}\right.\right.}{}\right.$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{Ade^3i^3\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{4\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^4}-Bd^2e^3i^3\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)\left(\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)}{d(dx+c)}\right.\right.}{4b^4e^4d}\right.\right.}{}$
default	$-\frac{e(ad-cb)\left(-\frac{Ade^3i^3\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{4\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^4}-Bd^2e^3i^3\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)\left(\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)}{d(dx+c)}\right.\right.}{4b^4e^4d}\right.\right.}{}$
parallelrisch	$\frac{24Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^4c^3di^3+24Bx^3\ln\left(\frac{e(bx+a)}{dx+c}\right)b^4cd^3i^3+21Ba^3bcd^3i^3+36Bx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^4c^2d^2i^3+12Bx^2ab^3cd^3i^3}{}$

```
[In] int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*i^3*(d*x+c)^4*B/d*ln(e*(b*x+a)/(d*x+c))+1/4*i^3*d^3*A*x^4+i^3*d^2*A*c*x
^3+1/12*i^3/b*d^3*B*a*x^3-1/12*i^3*d^2*B*c*x^3+3/2*i^3*d*A*c^2*x^2-1/8*i^3/
b^2*d^3*B*a^2*x^2+1/2*i^3/b*d^2*B*a*c*x^2-3/8*i^3*d*B*c^2*x^2+i^3*A*c^3*x-1
/4*i^3/b^4*d^3*B*ln(b*x+a)*a^4+i^3/b^3*d^2*B*ln(b*x+a)*a^3*c-3/2*i^3/b^2*d*
B*ln(b*x+a)*a^2*c^2+i^3/b*B*ln(b*x+a)*a*c^3-1/4*i^3/d*B*ln(b*x+a)*c^4+1/4*i
^3/b^3*d^3*B*a^3*x-i^3/b^2*d^2*B*a^2*c*x+3/2*i^3/b*d*B*a*c^2*x-3/4*i^3*B*c^
3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(139) = 278$.

Time = 0.35 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.16

$$\int (ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{6\,Ab^4d^4i^3x^4-6\,Bb^4c^4i^3\log{(dx+c)}+2\left((12\,A-B)b^4cd^3+Bab^3d^4\right)i^3x^3+3\left(3\,(4\,A-B)b^4c^2d^2+4\,Bab^3c\right.}{}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/24*(6*A*b^4*d^4*i^3*x^4 - 6*B*b^4*c^4*i^3*log(d*x + c) + 2*((12*A - B)*b^
4*c*d^3 + B*a*b^3*d^4)*i^3*x^3 + 3*(3*(4*A - B)*b^4*c^2*d^2 + 4*B*a*b^3*c*d
^3 - B*a^2*b^2*d^4)*i^3*x^2 + 6*((4*A - 3*B)*b^4*c^3*d + 6*B*a*b^3*c^2*d^2
- 4*B*a^2*b^2*c*d^3 + B*a^3*b*d^4)*i^3*x + 6*(4*B*a*b^3*c^3*d - 6*B*a^2*b^2
*c^2*d^2 + 4*B*a^3*b*c*d^3 - B*a^4*d^4)*i^3*log(b*x + a) + 6*(B*b^4*d^4*i^3
*x^4 + 4*B*b^4*c*d^3*i^3*x^3 + 6*B*b^4*c^2*d^2*i^3*x^2 + 4*B*b^4*c^3*d*i^3*
x)*log((b*e*x + a*e)/(d*x + c)))/(b^4*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(128) = 256$.

Time = 2.02 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.74

$$\int (ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\frac{Ad^3i^3x^4}{4}$$

$$-\frac{Bai^3(ad-2bc)(a^2d^2-2abcd+2b^2c^2)\log\left(x+\frac{Ba^4cd^3i^3-4Ba^3bc^2d^2i^3+6Ba^2b^2c^3di^3+\frac{Ba^2di^3(ad-2bc)\left(a^2d^2-2abcd+2b^2c\right.}{b}}{Ba^4d^4i^3-4Ba^3bcd^3i^3+6Ba^2b^2c^2d^2i^3-4}\right.}{4b^4}$$

$$-\frac{Bc^4i^3\log\left(x+\frac{Ba^4cd^3i^3-4Ba^3bc^2d^2i^3+6Ba^2b^2c^3di^3-4Bab^3c^4i^3-\frac{Bb^4c^5i^3}{d}}{Ba^4d^4i^3-4Ba^3bcd^3i^3+6Ba^2b^2c^2d^2i^3-4Bab^3c^3di^3-Bb^4c^4i^3}\right)}{4d}$$

$$+x^3\left(Acd^2i^3+\frac{Bad^3i^3}{12b}-\frac{Bcd^2i^3}{12}\right)+x^2\cdot\left(\frac{3Ac^2di^3}{2}-\frac{Ba^2d^3i^3}{8b^2}+\frac{Bacd^2i^3}{2b}-\frac{3Bc^2di^3}{8}\right)$$

$$+x\left(Ac^3i^3+\frac{Ba^3d^3i^3}{4b^3}-\frac{Ba^2cd^2i^3}{b^2}+\frac{3Bac^2di^3}{2b}-\frac{3Bc^3i^3}{4}\right)$$

$$+\left(Bc^3i^3x+\frac{3Bc^2di^3x^2}{2}+Bcd^2i^3x^3+\frac{Bd^3i^3x^4}{4}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

```
[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*d**3*i**3*x**4/4 - B*a*i**3*(a*d - 2*b*c)*(a**2*d**2 - 2*a*b*c*d + 2*b**2
*c**2)*log(x + (B*a**4*c*d**3*i**3 - 4*B*a**3*b*c**2*d**2*i**3 + 6*B*a**2*b
**2*c**3*d*i**3 + B*a**2*d*i**3*(a*d - 2*b*c)*(a**2*d**2 - 2*a*b*c*d + 2*b*
*2*c**2)/b - 5*B*a*b**3*c**4*i**3 - B*a*c*i**3*(a*d - 2*b*c)*(a**2*d**2 - 2
*a*b*c*d + 2*b**2*c**2))/(B*a**4*d**4*i**3 - 4*B*a**3*b*c*d**3*i**3 + 6*B*a
**2*b**2*c**2*d**2*i**3 - 4*B*a*b**3*c**3*d*i**3 - B*b**4*c**4*i**3))/(4*b*
*4) - B*c**4*i**3*log(x + (B*a**4*c*d**3*i**3 - 4*B*a**3*b*c**2*d**2*i**3 +
 6*B*a**2*b**2*c**3*d*i**3 - 4*B*a*b**3*c**4*i**3 - B*b**4*c**5*i**3/d)/(B*
a**4*d**4*i**3 - 4*B*a**3*b*c*d**3*i**3 + 6*B*a**2*b**2*c**2*d**2*i**3 - 4*
B*a*b**3*c**3*d*i**3 - B*b**4*c**4*i**3))/(4*d) + x**3*(A*c*d**2*i**3 + B*a
*d**3*i**3/(12*b) - B*c*d**2*i**3/12) + x**2*(3*A*c**2*d*i**3/2 - B*a**2*d*
*3*i**3/(8*b**2) + B*a*c*d**2*i**3/(2*b) - 3*B*c**2*d*i**3/8) + x*(A*c**3*i
**3 + B*a**3*d**3*i**3/(4*b**3) - B*a**2*c*d**2*i**3/b**2 + 3*B*a*c**2*d*i*
*3/(2*b) - 3*B*c**3*i**3/4) + (B*c**3*i**3*x + 3*B*c**2*d*i**3*x**2/2 + B*c
*d**2*i**3*x**3 + B*d**3*i**3*x**4/4)*log(e*(a + b*x)/(c + d*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(139) = 278$.

Time = 0.27 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.95

$$\int (ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{1}{4}Ad^3i^3x^4 + Acd^2i^3x^3 + \frac{3}{2}Ac^2di^3x^2 + \left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bc^3i^3 + \frac{3}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bc^2di^3 + \frac{1}{2}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2c\ldots}{b^2d^2}\right. + \frac{1}{24}\left(6x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6a^4\log(bx+a)}{b^4}+\frac{6c^4\log(dx+c)}{d^4}-\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3c^2d\ldots}{b^3\ldots}\right. + Ac^3i^3x$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/4*A*d^3*i^3*x^4 + A*c*d^2*i^3*x^3 + 3/2*A*c^2*d*i^3*x^2 + (x*log(b*e*x/(d
*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*c^3*i^3 +
 3/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2
*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*c^2*d*i^3 + 1/2*(2*x^3*log(b*e*x
/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d
^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*c*d^2
*i^3 + 1/24*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a
)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^
2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*d^3*i^3 + A*c^
3*i^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1506 vs. $2(139) = 278$.

Time = 0.47 (sec) , antiderivative size = 1506, normalized size of antiderivative = 10.11

$$\int (ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] 1/24*(6*(B*b^5*c^5*e^5*i^3 - 5*B*a*b^4*c^4*d*e^5*i^3 + 10*B*a^2*b^3*c^3*d^2
*e^5*i^3 - 10*B*a^3*b^2*c^2*d^3*e^5*i^3 + 5*B*a^4*b*c*d^4*e^5*i^3 - B*a^5*d
^5*e^5*i^3)*log((b*e*x + a*e)/(d*x + c))/(b^4*d*e^4 - 4*(b*e*x + a*e)*b^3*d
```

```
^2*e^3/(d*x + c) + 6*(b*e*x + a*e)^2*b^2*d^3*e^2/(d*x + c)^2 - 4*(b*e*x + a
*e)^3*b*d^4*e/(d*x + c)^3 + (b*e*x + a*e)^4*d^5/(d*x + c)^4) + (6*A*b^8*c^5
*e^5*i^3 - 11*B*b^8*c^5*e^5*i^3 - 30*A*a*b^7*c^4*d*e^5*i^3 + 55*B*a*b^7*c^4
*d*e^5*i^3 + 60*A*a^2*b^6*c^3*d^2*e^5*i^3 - 110*B*a^2*b^6*c^3*d^2*e^5*i^3 -
 60*A*a^3*b^5*c^2*d^3*e^5*i^3 + 110*B*a^3*b^5*c^2*d^3*e^5*i^3 + 30*A*a^4*b^
4*c*d^4*e^5*i^3 - 55*B*a^4*b^4*c*d^4*e^5*i^3 - 6*A*a^5*b^3*d^5*e^5*i^3 + 11
*B*a^5*b^3*d^5*e^5*i^3 + 26*(b*e*x + a*e)*B*b^7*c^5*d*e^4*i^3/(d*x + c) - 1
30*(b*e*x + a*e)*B*a*b^6*c^4*d^2*e^4*i^3/(d*x + c) + 260*(b*e*x + a*e)*B*a^
2*b^5*c^3*d^3*e^4*i^3/(d*x + c) - 260*(b*e*x + a*e)*B*a^3*b^4*c^2*d^4*e^4*i
^3/(d*x + c) + 130*(b*e*x + a*e)*B*a^4*b^3*c*d^5*e^4*i^3/(d*x + c) - 26*(b*
e*x + a*e)*B*a^5*b^2*d^6*e^4*i^3/(d*x + c) - 21*(b*e*x + a*e)^2*B*b^6*c^5*d
^2*e^3*i^3/(d*x + c)^2 + 105*(b*e*x + a*e)^2*B*a*b^5*c^4*d^3*e^3*i^3/(d*x +
 c)^2 - 210*(b*e*x + a*e)^2*B*a^2*b^4*c^3*d^4*e^3*i^3/(d*x + c)^2 + 210*(b*
e*x + a*e)^2*B*a^3*b^3*c^2*d^5*e^3*i^3/(d*x + c)^2 - 105*(b*e*x + a*e)^2*B*
a^4*b^2*c*d^6*e^3*i^3/(d*x + c)^2 + 21*(b*e*x + a*e)^2*B*a^5*b*d^7*e^3*i^3/
(d*x + c)^2 + 6*(b*e*x + a*e)^3*B*b^5*c^5*d^3*e^2*i^3/(d*x + c)^3 - 30*(b*e
*x + a*e)^3*B*a*b^4*c^4*d^4*e^2*i^3/(d*x + c)^3 + 60*(b*e*x + a*e)^3*B*a^2*
b^3*c^3*d^5*e^2*i^3/(d*x + c)^3 - 60*(b*e*x + a*e)^3*B*a^3*b^2*c^2*d^6*e^2*
i^3/(d*x + c)^3 + 30*(b*e*x + a*e)^3*B*a^4*b*c*d^7*e^2*i^3/(d*x + c)^3 - 6*
(b*e*x + a*e)^3*B*a^5*d^8*e^2*i^3/(d*x + c)^3)/(b^7*d*e^4 - 4*(b*e*x + a*e)
*b^6*d^2*e^3/(d*x + c) + 6*(b*e*x + a*e)^2*b^5*d^3*e^2/(d*x + c)^2 - 4*(b*e
*x + a*e)^3*b^4*d^4*e/(d*x + c)^3 + (b*e*x + a*e)^4*b^3*d^5/(d*x + c)^4) +
6*(B*b^5*c^5*e*i^3 - 5*B*a*b^4*c^4*d*e*i^3 + 10*B*a^2*b^3*c^3*d^2*e*i^3 - 1
0*B*a^3*b^2*c^2*d^3*e*i^3 + 5*B*a^4*b*c*d^4*e*i^3 - B*a^5*d^5*e*i^3)*log(-b
*e + (b*e*x + a*e)*d/(d*x + c))/(b^4*d) - 6*(B*b^5*c^5*e*i^3 - 5*B*a*b^4*c^
4*d*e*i^3 + 10*B*a^2*b^3*c^3*d^2*e*i^3 - 10*B*a^3*b^2*c^2*d^3*e*i^3 + 5*B*a
^4*b*c*d^4*e*i^3 - B*a^5*d^5*e*i^3)*log((b*e*x + a*e)/(d*x + c))/(b^4*d))*(
b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 566, normalized size of antiderivative = 3.80

$$\int (ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$

$$= x\left(\frac{(4\,a\,d+4\,b\,c)\left(\frac{\left(\frac{d^2\,i^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d-B\,b\,c)}{4\,b}-\frac{A\,d^2\,i^3\,(4\,a\,d+4\,b\,c)}{4\,b}\right)(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{c\,d\,i^3\,(4\,A\,a\,d+6\,A\,b\,c+B\,a\,d-B\,b\,c)}{b}+\frac{A\,a\,c\,d^2\,i^3}{b}\right)}{4\,b\,d}+\frac{c^2\,i^3\,(12\,A\,a\,d+8\,A\,b\,c+3\,B\,a\,d-3\,B\,b\,c)}{2\,b}-\frac{a\,c\left(\frac{d^2\,i^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d-B\,b\,c)}{4\,b}-\frac{A\,d^2\,i^3\,(4\,a\,d+4\,b\,c)}{4\,b}\right)}{b\,d}\right)$$

$$-x^2\left(\frac{\left(\frac{d^2\,i^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d-B\,b\,c)}{4\,b}-\frac{A\,d^2\,i^3\,(4\,a\,d+4\,b\,c)}{4\,b}\right)(4\,a\,d+4\,b\,c)}{8\,b\,d}-\frac{c\,d\,i^3\,(4\,A\,a\,d+6\,A\,b\,c+B\,a\,d-B\,b\,c)}{2\,b}+\frac{A\,a\,c\,d^2\,i^3}{2\,b}\right)$$

$$+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(B\,c^3\,i^3\,x+\frac{3\,B\,c^2\,d\,i^3\,x^2}{2}+B\,c\,d^2\,i^3\,x^3+\frac{B\,d^3\,i^3\,x^4}{4}\right)$$

$$+x^3\left(\frac{d^2\,i^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d-B\,b\,c)}{12\,b}-\frac{A\,d^2\,i^3\,(4\,a\,d+4\,b\,c)}{12\,b}\right)$$

$$-\frac{\ln(a+b\,x)\,(B\,a^4\,d^3\,i^3-4\,B\,a^3\,b\,c\,d^2\,i^3+6\,B\,a^2\,b^2\,c^2\,d\,i^3-4\,B\,a\,b^3\,c^3\,i^3)}{4\,b^4}$$

$$+\frac{A\,d^3\,i^3\,x^4}{4}-\frac{B\,c^4\,i^3\,\ln(c+d\,x)}{4\,d}$$

```
[In] int((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

```
[Out] x*(((4*a*d + 4*b*c)*((((d^2*i^3*(4*A*a*d + 16*A*b*c + B*a*d - B*b*c))/(4*b)
 - (A*d^2*i^3*(4*a*d + 4*b*c))/(4*b))*(4*a*d + 4*b*c))/(4*b*d) - (c*d*i^3*(
4*A*a*d + 6*A*b*c + B*a*d - B*b*c))/b + (A*a*c*d^2*i^3)/b))/(4*b*d) + (c^2*
i^3*(12*A*a*d + 8*A*b*c + 3*B*a*d - 3*B*b*c))/(2*b) - (a*c*((d^2*i^3*(4*A*a
*d + 16*A*b*c + B*a*d - B*b*c))/(4*b) - (A*d^2*i^3*(4*a*d + 4*b*c))/(4*b)))
/(b*d)) - x^2*((((d^2*i^3*(4*A*a*d + 16*A*b*c + B*a*d - B*b*c))/(4*b) - (A*
d^2*i^3*(4*a*d + 4*b*c))/(4*b))*(4*a*d + 4*b*c))/(8*b*d) - (c*d*i^3*(4*A*a*
d + 6*A*b*c + B*a*d - B*b*c))/(2*b) + (A*a*c*d^2*i^3)/(2*b)) + log((e*(a +
```

```
b*x))/(c + d*x))*((B*d^3*i^3*x^4)/4 + B*c^3*i^3*x + (3*B*c^2*d*i^3*x^2)/2 +
 B*c*d^2*i^3*x^3) + x^3*((d^2*i^3*(4*A*a*d + 16*A*b*c + B*a*d - B*b*c))/(12
*b) - (A*d^2*i^3*(4*a*d + 4*b*c))/(12*b)) - (log(a + b*x)*(B*a^4*d^3*i^3 -
4*B*a*b^3*c^3*i^3 + 6*B*a^2*b^2*c^2*d*i^3 - 4*B*a^3*b*c*d^2*i^3))/(4*b^4) +
 (A*d^3*i^3*x^4)/4 - (B*c^4*i^3*log(c + d*x))/(4*d)
```

3.24 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 356

$$\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx \\
&= -\frac{5Bd(bc-ad)^2i^3x}{6b^3g} - \frac{B(bc-ad)i^3(c+dx)^2}{6b^2g} - \frac{5B(bc-ad)^3i^3\log\left(\frac{a+bx}{c+dx}\right)}{6b^4g} \\
&\quad + \frac{d(bc-ad)^2i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g} \\
&\quad + \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g} + \frac{i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bg} \\
&\quad - \frac{11B(bc-ad)^3i^3\log(c+dx)}{6b^4g} - \frac{(bc-ad)^3i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g} \\
&\quad + \frac{B(bc-ad)^3i^3\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}
\end{aligned}$$

```
[Out] -5/6*B*d*(-a*d+b*c)^2*i^3*x/b^3/g-1/6*B*(-a*d+b*c)*i^3*(d*x+c)^2/b^2/g-5/6*
B*(-a*d+b*c)^3*i^3*ln((b*x+a)/(d*x+c))/b^4/g+d*(-a*d+b*c)^2*i^3*(b*x+a)*(A+
B*ln(e*(b*x+a)/(d*x+c)))/b^4/g+1/2*(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+
a)/(d*x+c)))/b^2/g+1/3*i^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g-11/6*B
*(-a*d+b*c)^3*i^3*ln(d*x+c)/b^4/g-(-a*d+b*c)^3*i^3*(A+B*ln(e*(b*x+a)/(d*x+c
)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g+B*(-a*d+b*c)^3*i^3*polylog(2,b*(d*x+c)/
d/(b*x+a))/b^4/g
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2562, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$= \frac{di^3(a+bx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4g} - \frac{i^3(bc-ad)^3\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4g} + \frac{i^3(c+dx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b^2g} + \frac{i^3(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3bg} + \frac{Bi^3(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g} - \frac{5Bi^3(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{6b^4g} - \frac{11Bi^3(bc-ad)^3\log(c+dx)}{6b^4g} - \frac{5Bdi^3x(bc-ad)^2}{6b^3g} - \frac{Bi^3(c+dx)^2(bc-ad)}{6b^2g}$$

```
[In] Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x),x]
```

```
[Out] (-5*B*d*(b*c - a*d)^2*i^3*x)/(6*b^3*g) - (B*(b*c - a*d)*i^3*(c + d*x)^2)/(6
*b^2*g) - (5*B*(b*c - a*d)^3*i^3*Log[(a + b*x)/(c + d*x)])/(6*b^4*g) + (d*(
b*c - a*d)^2*i^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^4*g) +
((b*c - a*d)*i^3*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*b^2*g
) + (i^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b*g) - (11*B*
(b*c - a*d)^3*i^3*Log[c + d*x])/(6*b^4*g) - ((b*c - a*d)^3*i^3*(A + B*Log[(
e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^4*g) + (B
*(b*c - a*d)^3*i^3*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b^4*g)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\left((bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{g}$$

$$= \frac{\left((bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg} + \frac{\left(d(bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg}$$

$$= \frac{i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bg} + \frac{\left((bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2 g} - \frac{\left(B(bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{1}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3bg} + \frac{\left(d(bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2 g}$$

$$= \frac{(bc-ad) i^3 (c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2 g} + \frac{i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bg} + \frac{\left((bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3 g} - \frac{\left(B(bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{1}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b^2 g} - \frac{\left(B(bc-ad)^3 i^3\right) \text{Subst}\left(\int \left(\frac{1}{b^3 x} + \frac{d}{b(b-dx)^3} + \frac{d}{b^2(b-dx)^2} + \frac{d}{b^3(b-dx)}\right) dx, x, \frac{a+bx}{c+dx}\right)}{3bg} + \frac{\left(d(bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3 g}$$

$$= -\frac{Bd(bc-ad)^2 i^3 x}{3b^3 g} - \frac{B(bc-ad)i^3(c+dx)^2}{6b^2 g} - \frac{B(bc-ad)^3 i^3 \log\left(\frac{a+bx}{c+dx}\right)}{3b^4 g} + \frac{d(bc-ad)^2 i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4 g} + \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2 g} + \frac{i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bg} - \frac{B(bc-ad)^3 i^3 \log(c+dx)}{3b^4 g} - \frac{(bc-ad)^3 i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4 g} + \frac{(B(bc-ad)^3 i^3)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4 g} - \frac{(B(bc-ad)^3 i^3)\operatorname{Subst}\left(\int \left(\frac{1}{b^2 x}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{2b^2 g} - \frac{(Bd(bc-ad)^3 i^3)\operatorname{Subst}\left(\int \frac{1}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4 g}$$

$$= -\frac{5Bd(bc-ad)^2 i^3 x}{6b^3 g} - \frac{B(bc-ad)i^3(c+dx)^2}{6b^2 g} - \frac{5B(bc-ad)^3 i^3 \log\left(\frac{a+bx}{c+dx}\right)}{6b^4 g} + \frac{d(bc-ad)^2 i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4 g} + \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2 g} + \frac{i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bg} - \frac{11B(bc-ad)^3 i^3 \log(c+dx)}{6b^4 g} - \frac{(bc-ad)^3 i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4 g} + \frac{B(bc-ad)^3 i^3 \operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4 g}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.99

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$=\frac{i^3\Big(6Abd(bc-ad)^2x-3B(bc-ad)^2(bdx+(bc-ad)\log(a+bx))-B(bc-ad)\left(2bd(bc-ad)x+b^2(c+\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g
*x),x]
```

```
[Out] (i^3*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*(b*d*x + (b*c - a*d)*Log[
a + b*x]) - B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c -
 a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(c
+ d*x)] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
) + 2*b^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*(b*c - a*d)^
3*Log[g*(a + b*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(b*c - a*d)^3
*Log[c + d*x] - 3*B*(b*c - a*d)^3*(Log[g*(a + b*x)]*(Log[g*(a + b*x)] - 2*L
og[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]
)))/(6*b^4*g)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. 2(344) = 688.

Time = 1.81 (sec) , antiderivative size = 1086, normalized size of antiderivative = 3.05

method	result	size
parts	Expression too large to display	1086
derivativedivides	Expression too large to display	1178
default	Expression too large to display	1178
risch	Expression too large to display	5361

```
[In] int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x,method=_RETURNV
ERBOSE)
```

```
[Out] i^3*A/g*(d/b^3*(1/3*b^2*d^2*x^3-1/2*a*b*d^2*x^2+3/2*b^2*c*d*x^2+a^2*d^2*x-3
*a*b*c*d*x+3*b^2*c^2*x)+(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4*
ln(b*x+a))-i^3*B/g/d^5*(a*d-b*c)^4*e^4*(1/(a*d-b*c)*d^6/b^3/e^3*(1/b/e/d*ln
((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/
d+(a*d-b*c)*e/d/(d*x+c))/b/e/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e))-1/(a*d-
b*c)*d^6/b^2/e^2*(-1/2/e^2/b^2/d*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)-1/
2/e/b/d/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)+1/2*ln(b*e/d+(a*d-b*c)*e/d/(d
```

```
*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-2*b*e
)/e^2/b^2/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)^2)+1/2/(a*d-b*c)*d^5/b^4/e^
4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+1/(a*d-b*c)*d^6/b/e*(-1/6/b/e/d/((b*e/d
+(a*d-b*c)*e/d/(d*x+c))*d-b*e)^2+1/3/b^2/e^2/d/((b*e/d+(a*d-b*c)*e/d/(d*x+c
))*d-b*e)+1/3/b^3/e^3/d*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)-1/3*ln(b*e/
d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(3*e^2*b^2-3*(b*e/d+
(a*d-b*c)*e/d/(d*x+c))*d*b*e+d^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)/b^3/e^3/(
(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)^3)-1/(a*d-b*c)*d^6/b^4/e^4*(dilog(-((b
*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*l
n(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d))
```

Fricas [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorit
hm="fricas")
```

```
[Out] integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3 +
 (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log((b*e
*x + a*e)/(d*x + c)))/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$=\frac{i^3\left(\int \frac{Ac^3}{a+bx}\,dx+\int \frac{Ad^3x^3}{a+bx}\,dx+\int \frac{Bc^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx+\int \frac{3Acd^2x^2}{a+bx}\,dx+\int \frac{3Ac^2dx}{a+bx}\,dx+\int \frac{Bd^3x^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx$$
$$g$$

```
[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x)
```

```
[Out] i**3*(Integral(A*c**3/(a + b*x), x) + Integral(A*d**3*x**3/(a + b*x), x) +
Integral(B*c**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) + Integr
al(3*A*c*d**2*x**2/(a + b*x), x) + Integral(3*A*c**2*d*x/(a + b*x), x) + In
tegral(B*d**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) + Int
egral(3*B*c*d**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) +
Integral(3*B*c**2*d*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x))/g
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 850 vs. $2(343) = 686$.

Time = 0.31 (sec) , antiderivative size = 850, normalized size of antiderivative = 2.39

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$= 3\,Ac^2di^3\left(\frac{x}{bg}-\frac{a\log(bx+a)}{b^2g}\right)-\frac{1}{6}\,Ad^3i^3\left(\frac{6\,a^3\log(bx+a)}{b^4g}-\frac{2\,b^2x^3-3\,abx^2+6\,a^2x}{b^3g}\right)$$
$$+\frac{3}{2}\,Acd^2i^3\left(\frac{2\,a^2\log(bx+a)}{b^3g}+\frac{bx^2-2\,ax}{b^2g}\right)+\frac{Ac^3i^3\log(bgx+ag)}{bg}$$
$$-\frac{(11\,b^2c^3i^3-15\,abc^2di^3+6\,a^2cd^2i^3)B\log(dx+c)}{6\,b^3g}$$
$$+\frac{(b^3c^3i^3-3\,ab^2c^2di^3+3\,a^2bcd^2i^3-a^3d^3i^3)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{b^4g}$$
$$+\frac{2\,Bb^3d^3i^3x^3\log(e)+((9\,i^3\log(e)-i^3)b^3cd^2-(3\,i^3\log(e)-i^3)ab^2d^3)Bx^2+3\,(b^3c^3i^3-3\,ab^2c^2di^3+3\,a}{}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorit
hm="maxima")
```

```
[Out] 3*A*c^2*d*i^3*(x/(b*g) - a*log(b*x + a)/(b^2*g)) - 1/6*A*d^3*i^3*(6*a^3*log
(b*x + a)/(b^4*g) - (2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/(b^3*g)) + 3/2*A*c*d^
2*i^3*(2*a^2*log(b*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A*c^3*i^3*lo
g(b*g*x + a*g)/(b*g) - 1/6*(11*b^2*c^3*i^3 - 15*a*b*c^2*d*i^3 + 6*a^2*c*d^2
*i^3)*B*log(d*x + c)/(b^3*g) + (b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^2*b*c
*d^2*i^3 - a^3*d^3*i^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) +
dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(b^4*g) + 1/6*(2*B*b^3*d^3*i^3*x^3*log
(e) + ((9*i^3*log(e) - i^3)*b^3*c*d^2 - (3*i^3*log(e) - i^3)*a*b^2*d^3)*B*x
^2 + 3*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^2*b*c*d^2*i^3 - a^3*d^3*i^3)*
B*log(b*x + a)^2 + ((18*i^3*log(e) - 7*i^3)*b^3*c^2*d - 6*(3*i^3*log(e) - 2
*i^3)*a*b^2*c*d^2 + (6*i^3*log(e) - 5*i^3)*a^2*b*d^3)*B*x + (2*B*b^3*d^3*i^
3*x^3 + 3*(3*b^3*c*d^2*i^3 - a*b^2*d^3*i^3)*B*x^2 + 6*(3*b^3*c^2*d*i^3 - 3*
a*b^2*c*d^2*i^3 + a^2*b*d^3*i^3)*B*x + (6*b^3*c^3*i^3*log(e) - 18*(i^3*log(
e) - i^3)*a*b^2*c^2*d + 9*(2*i^3*log(e) - 3*i^3)*a^2*b*c*d^2 - (6*i^3*log(e
) - 11*i^3)*a^3*d^3)*B)*log(b*x + a) - (2*B*b^3*d^3*i^3*x^3 + 3*(3*b^3*c*d^
2*i^3 - a*b^2*d^3*i^3)*B*x^2 + 6*(3*b^3*c^2*d*i^3 - 3*a*b^2*c*d^2*i^3 + a^2
*b*d^3*i^3)*B*x + 6*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^2*b*c*d^2*i^3 -
a^3*d^3*i^3)*B*log(b*x + a))*log(d*x + c))/(b^4*g)
```

Giac [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = \int \frac{(dix+ci)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorit
hm="giac")
```

```
[Out] integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = \int \frac{(c\,i+d\,i\,x)^3 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{a\,g+b\,g\,x}\,dx$$

```
[In] int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x),x)
```

```
[Out] int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x), x
)
```

3.25 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 373

$$
\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx \\
&= -\frac{Bd^2(bc-ad)i^3x}{2b^3g^2}-\frac{B(bc-ad)^2i^3(c+dx)}{b^3g^2(a+bx)}-\frac{Bd(bc-ad)^2i^3\log\left(\frac{a+bx}{c+dx}\right)}{2b^4g^2} \\
&\quad+\frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^2} \\
&\quad-\frac{(bc-ad)^2i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^2(a+bx)}+\frac{di^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g^2} \\
&\quad-\frac{5Bd(bc-ad)^2i^3\log(c+dx)}{2b^4g^2}-\frac{3d(bc-ad)^2i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} \\
&\quad+\frac{3Bd(bc-ad)^2i^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}
\end{aligned}
$$

```
[Out] -1/2*B*d^2*(-a*d+b*c)*i^3*x/b^3/g^2-B*(-a*d+b*c)^2*i^3*(d*x+c)/b^3/g^2/(b*x
+a)-1/2*B*d*(-a*d+b*c)^2*i^3*ln((b*x+a)/(d*x+c))/b^4/g^2+2*d^2*(-a*d+b*c)*i
^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/g^2-(-a*d+b*c)^2*i^3*(d*x+c)*(A+
B*ln(e*(b*x+a)/(d*x+c)))/b^3/g^2/(b*x+a)+1/2*d*i^3*(d*x+c)^2*(A+B*ln(e*(b*x
+a)/(d*x+c)))/b^2/g^2-5/2*B*d*(-a*d+b*c)^2*i^3*ln(d*x+c)/b^4/g^2-3*d*(-a*d+
b*c)^2*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^2+3*
B*d*(-a*d+b*c)^2*i^3*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {2562, 46, 2393, 2341, 2356, 2351, 31, 2379, 2438}

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$$
$$=\frac{2d^2i^3(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4g^2}$$
$$-\frac{3di^3(bc-ad)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4g^2}$$
$$-\frac{i^3(c+dx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3g^2(a+bx)}+\frac{di^3(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b^2g^2}$$
$$+\frac{3Bdi^3(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}-\frac{Bdi^3(bc-ad)^2\log\left(\frac{a+bx}{c+dx}\right)}{2b^4g^2}$$
$$-\frac{5Bdi^3(bc-ad)^2\log(c+dx)}{2b^4g^2}-\frac{Bd^2i^3x(bc-ad)}{2b^3g^2}-\frac{Bi^3(c+dx)(bc-ad)^2}{b^3g^2(a+bx)}$$

```
[In] Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^2,
x]
```

```
[Out] -1/2*(B*d^2*(b*c - a*d)*i^3*x)/(b^3*g^2) - (B*(b*c - a*d)^2*i^3*(c + d*x))/
(b^3*g^2*(a + b*x)) - (B*d*(b*c - a*d)^2*i^3*Log[(a + b*x)/(c + d*x)])/(2*b
^4*g^2) + (2*d^2*(b*c - a*d)*i^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*
x)]))/(b^4*g^2) - ((b*c - a*d)^2*i^3*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c
+ d*x)]))/(b^3*g^2*(a + b*x)) + (d*i^3*(c + d*x)^2*(A + B*Log[(e*(a + b*x))
/(c + d*x)]))/(2*b^2*g^2) - (5*B*d*(b*c - a*d)^2*i^3*Log[c + d*x])/(2*b^4*g
^2) - (3*d*(b*c - a*d)^2*i^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (
b*(c + d*x))/(d*(a + b*x))])/(b^4*g^2) + (3*B*d*(b*c - a*d)^2*i^3*PolyLog[2
, (b*(c + d*x))/(d*(a + b*x))])/(b^4*g^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
 n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\left((bc-ad)^2 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex)}{x^2(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^2}$$

$$= \frac{\left((bc-ad)^2 i^3\right) \text{Subst}\left(\int \left(\frac{A+B\log(ex)}{b^3x^2} + \frac{d^2(A+B\log(ex))}{b^2(b-dx)^3} + \frac{2d^2(A+B\log(ex))}{b^3(b-dx)^2} + \frac{3d(A+B\log(ex))}{b^3x(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{g^2}$$

$$= \frac{\left((bc-ad)^2 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^2} + \frac{\left(3d(bc-ad)^2 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^2} + \frac{\left(2d^2(bc-ad)^2 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^2} + \frac{\left(d^2(bc-ad)^2 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2}$$

$$= -\frac{B(bc-ad)^2 i^3 (c+dx)}{b^3 g^2 (a+bx)} + \frac{2d^2 (bc-ad) i^3 (a+bx) \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4 g^2}$$
$$- \frac{(bc-ad)^2 i^3 (c+dx) \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3 g^2 (a+bx)}$$
$$+ \frac{d i^3 (c+dx)^2 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2 g^2}$$
$$- \frac{3d(bc-ad)^2 i^3 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log\left(1 - \frac{b(c+dx)}{d(a+bx)}\right)}{b^4 g^2}$$
$$+ \frac{(3Bd(bc-ad)^2 i^3) \operatorname{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)}{x}\, dx, x, \frac{a+bx}{c+dx}\right)}{b^4 g^2}$$
$$- \frac{(Bd(bc-ad)^2 i^3) \operatorname{Subst}\left(\int \frac{1}{x(b-dx)^2}\, dx, x, \frac{a+bx}{c+dx}\right)}{2b^2 g^2}$$
$$- \frac{(2Bd^2 (bc-ad)^2 i^3) \operatorname{Subst}\left(\int \frac{1}{b-dx}\, dx, x, \frac{a+bx}{c+dx}\right)}{b^4 g^2}$$

$$= -\frac{B(bc-ad)^2 i^3 (c+dx)}{b^3 g^2 (a+bx)} + \frac{2d^2 (bc-ad) i^3 (a+bx) \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4 g^2}$$
$$- \frac{(bc-ad)^2 i^3 (c+dx) \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3 g^2 (a+bx)}$$
$$+ \frac{d i^3 (c+dx)^2 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2 g^2} - \frac{2Bd(bc-ad)^2 i^3 \log(c+dx)}{b^4 g^2}$$
$$- \frac{3d(bc-ad)^2 i^3 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log\left(1 - \frac{b(c+dx)}{d(a+bx)}\right)}{b^4 g^2}$$
$$+ \frac{3Bd(bc-ad)^2 i^3 \operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4 g^2}$$
$$- \frac{(Bd(bc-ad)^2 i^3) \operatorname{Subst}\left(\int \left(\frac{1}{b^2 x} + \frac{d}{b(b-dx)^2} + \frac{d}{b^2(b-dx)}\right) dx, x, \frac{a+bx}{c+dx}\right)}{2b^2 g^2}$$

$$= -\frac{Bd^2(bc-ad)i^3x}{2b^3g^2} - \frac{B(bc-ad)^2i^3(c+dx)}{b^3g^2(a+bx)} - \frac{Bd(bc-ad)^2i^3\log\left(\frac{a+bx}{c+dx}\right)}{2b^4g^2}$$
$$+ \frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^2}$$
$$- \frac{(bc-ad)^2i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^2(a+bx)}$$
$$+ \frac{di^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g^2} - \frac{5Bd(bc-ad)^2i^3\log(c+dx)}{2b^4g^2}$$
$$- \frac{3d(bc-ad)^2i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$+ \frac{3Bd(bc-ad)^2i^3\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.00

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$$
$$= \frac{i^3\left(2Abd^2(3bc-2ad)x - bBd^2(bc-ad)x - \frac{2B(bc-ad)^3}{a+bx} - a^2Bd^3\log(a+bx) - 2Bd(bc-ad)^2\log(a+bx) + \right.}{}$$

```
[In] Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g
*x)^2,x]
```

```
[Out] (i^3*(2*A*b*d^2*(3*b*c - 2*a*d)*x - b*B*d^2*(b*c - a*d)*x - (2*B*(b*c - a*d
)^3)/(a + b*x) - a^2*B*d^3*Log[a + b*x] - 2*B*d*(b*c - a*d)^2*Log[a + b*x]
+ 2*B*d^2*(3*b*c - 2*a*d)*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + b^2*d^3*
x^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (2*(b*c - a*d)^3*(A + B*Log[(e*(
a + b*x))/(c + d*x)]))/(a + b*x) + 6*d*(b*c - a*d)^2*Log[a + b*x]*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]) + b^2*B*c^2*d*Log[c + d*x] + 2*B*d*(b*c - a*d)^
2*Log[c + d*x] - 2*B*d*(-(b*c) + a*d)*(-3*b*c + 2*a*d)*Log[c + d*x] - 3*B*d
*(b*c - a*d)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d
)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(2*b^4*g^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 862 vs. 2(365) = 730.

Time = 1.66 (sec) , antiderivative size = 863, normalized size of antiderivative = 2.31

method	result
parts	$\frac{i^3 A\left(\frac{d^2\left(\frac{1}{2}bd\,x^2-2xad+3bcx\right)}{b^3}+\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)\ln(bx+a)}{b^4}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{b^4(bx+a)}\right)}{g^2}-\frac{i^3B(ad-cb)^4e^4\left(-\cdots\right.}{}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{A\,d^2e^3i^3(ad-cb)\left(-\frac{3d\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{b^4e^4}+\frac{2d}{b^3e^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{d}{2b^2e^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right.}\cdots\right.}{g^2}\right.}{}$
default	$-\frac{e(ad-cb)\left(-\frac{A\,d^2e^3i^3(ad-cb)\left(-\frac{3d\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{b^4e^4}+\frac{2d}{b^3e^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{d}{2b^2e^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right.}\cdots\right.}{g^2}\right.}{}$
risch	Expression too large to display

```
[In] int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x,method=_RETUR
NVERBOSE)
```

```
[Out] i^3*A/g^2*(d^2/b^3*(1/2*b*d*x^2-2*x*a*d+3*b*c*x)+3/b^4*d*(a^2*d^2-2*a*b*c*d
+b^2*c^2)*ln(b*x+a)-1/b^4*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/(b
*x+a))-i^3*B/g^2/d^5*(a*d-b*c)^4*e^4*(-2/(a*d-b*c)^2*d^7/b^3/e^3*(1/b/e/d*l
n((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e
/d+(a*d-b*c)*e/d/(d*x+c))/b/e/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e))+1/(a*d
-b*c)^2*d^7/b^2/e^2*(-1/2/e^2/b^2/d*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)
-1/2/e/b/d/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)+1/2*ln(b*e/d+(a*d-b*c)*e/d
/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-2*
b*e)/e^2/b^2/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)^2)-1/(a*d-b*c)^2*d^5/b^3
/e^3*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b
*e/d+(a*d-b*c)*e/d/(d*x+c)))-3/2/(a*d-b*c)^2*d^6/b^4/e^4*ln(b*e/d+(a*d-b*c)
*e/d/(d*x+c))^2+3/(a*d-b*c)^2*d^7/b^4/e^4*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*
x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e
/d/(d*x+c))*d-b*e)/b/e)/d))
```

Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algor
ithm="fricas")
```

```
[Out] integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3 +
 (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log((b*e
*x + a*e)/(d*x + c)))/(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx=\text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1501 vs. $2(364) = 728$.

Time = 0.35 (sec) , antiderivative size = 1501, normalized size of antiderivative = 4.02

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algor
ithm="maxima")
```

```
[Out] -3*A*(a^2/(b^4*g^2*x + a*b^3*g^2) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*g^2
))*c*d^2*i^3 + 1/2*(2*a^3/(b^5*g^2*x + a*b^4*g^2) + 6*a^2*log(b*x + a)/(b^4
*g^2) + (b*x^2 - 4*a*x)/(b^3*g^2))*A*d^3*i^3 + 3*A*c^2*d*i^3*(a/(b^3*g^2*x
+ a*b^2*g^2) + log(b*x + a)/(b^2*g^2)) - B*c^3*i^3*(log(b*e*x/(d*x + c) + a
*e/(d*x + c))/(b^2*g^2*x + a*b*g^2) + 1/(b^2*g^2*x + a*b*g^2) + d*log(b*x +
 a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - A*c^3*i
^3/(b^2*g^2*x + a*b*g^2) - 1/2*(5*b^3*c^3*d*i^3 - 3*a*b^2*c^2*d^2*i^3 - 2*a
^2*b*c*d^3*i^3 + 2*a^3*d^4*i^3)*B*log(d*x + c)/(b^5*c*g^2 - a*b^4*d*g^2) +
1/2*((b^4*c*d^3*i^3*log(e) - a*b^3*d^4*i^3*log(e))*B*x^3 + ((6*i^3*log(e) -
 i^3)*b^4*c^2*d^2 - (9*i^3*log(e) - 2*i^3)*a*b^3*c*d^3 + (3*i^3*log(e) - i^
3)*a^2*b^2*d^4)*B*x^2 + ((6*i^3*log(e) - i^3)*a*b^3*c^2*d^2 - 2*(5*i^3*log(
e) - i^3)*a^2*b^2*c*d^3 + (4*i^3*log(e) - i^3)*a^3*b*d^4)*B*x + 3*((b^4*c^3
*d*i^3 - 3*a*b^3*c^2*d^2*i^3 + 3*a^2*b^2*c*d^3*i^3 - a^3*b*d^4*i^3)*B*x + (
a*b^3*c^3*d*i^3 - 3*a^2*b^2*c^2*d^2*i^3 + 3*a^3*b*c*d^3*i^3 - a^4*d^4*i^3)*
B)*log(b*x + a)^2 + 2*(3*(i^3*log(e) + i^3)*a*b^3*c^3*d - 6*(i^3*log(e) + i
^3)*a^2*b^2*c^2*d^2 + 4*(i^3*log(e) + i^3)*a^3*b*c*d^3 - (i^3*log(e) + i^3)
*a^4*d^4)*B + ((b^4*c*d^3*i^3 - a*b^3*d^4*i^3)*B*x^3 + 3*(2*b^4*c^2*d^2*i^3
 - 3*a*b^3*c*d^3*i^3 + a^2*b^2*d^4*i^3)*B*x^2 + (6*b^4*c^3*d*i^3*log(e) - 1
8*(i^3*log(e) - i^3)*a*b^3*c^2*d^2 + 9*(2*i^3*log(e) - 3*i^3)*a^2*b^2*c*d^3
 - (6*i^3*log(e) - 11*i^3)*a^3*b*d^4)*B*x - (18*a^2*b^2*c^2*d^2*i^3*log(e)
- 6*(i^3*log(e) + i^3)*a*b^3*c^3*d - 9*(2*i^3*log(e) - i^3)*a^3*b*c*d^3 + (
6*i^3*log(e) - 5*i^3)*a^4*d^4)*B)*log(b*x + a) - ((b^4*c*d^3*i^3 - a*b^3*d^
4*i^3)*B*x^3 + 3*(2*b^4*c^2*d^2*i^3 - 3*a*b^3*c*d^3*i^3 + a^2*b^2*d^4*i^3)*
B*x^2 + 2*(3*a*b^3*c^2*d^2*i^3 - 5*a^2*b^2*c*d^3*i^3 + 2*a^3*b*d^4*i^3)*B*x
 + 2*(3*a*b^3*c^3*d*i^3 - 6*a^2*b^2*c^2*d^2*i^3 + 4*a^3*b*c*d^3*i^3 - a^4*d
^4*i^3)*B + 6*((b^4*c^3*d*i^3 - 3*a*b^3*c^2*d^2*i^3 + 3*a^2*b^2*c*d^3*i^3 -
 a^3*b*d^4*i^3)*B*x + (a*b^3*c^3*d*i^3 - 3*a^2*b^2*c^2*d^2*i^3 + 3*a^3*b*c*
d^3*i^3 - a^4*d^4*i^3)*B)*log(b*x + a))*log(d*x + c))/(a*b^5*c*g^2 - a^2*b^
4*d*g^2 + (b^6*c*g^2 - a*b^5*d*g^2)*x) + 3*(b^2*c^2*d*i^3 - 2*a*b*c*d^2*i^3
 + a^2*d^3*i^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(
b*d*x + a*d)/(b*c - a*d)))*B/(b^4*g^2)
```

Giac [F]

$$\int \frac{(ci + dix)^3 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag + bgx)^2}\, dx = \int \frac{(dix + ci)^3 \left(B \log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)}{(bgx + ag)^2}\, dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algor
ithm="giac")
```

```
[Out] integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g)^
2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci + dix)^3 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag + bgx)^2}\, dx = \int \frac{(c\,i + d\,i\,x)^3 \left(A + B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{(a\,g + b\,g\,x)^2}\, dx$$

```
[In] int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^2,
x)
```

```
[Out] int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^2,
 x)
```

3.26 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 345

$$\begin{aligned}\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx &= -\frac{2Bd(bc-ad)i^3(c+dx)}{b^3g^3(a+bx)}-\frac{B(bc-ad)i^3(c+dx)^2}{4b^2g^3(a+bx)^2}\\ &+\frac{d^3i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^3}-\frac{2d(bc-ad)i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^3(a+bx)}\\ &-\frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g^3(a+bx)^2}-\frac{Bd^2(bc-ad)i^3\log(c+dx)}{b^4g^3}\\ &-\frac{3d^2(bc-ad)i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}\\ &+\frac{3Bd^2(bc-ad)i^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}\end{aligned}$$

```
[Out] -2*B*d*(-a*d+b*c)*i^3*(d*x+c)/b^3/g^3/(b*x+a)-1/4*B*(-a*d+b*c)*i^3*(d*x+c)^
2/b^2/g^3/(b*x+a)^2+d^3*i^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/g^3-2*d
*(-a*d+b*c)*i^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/g^3/(b*x+a)-1/2*(-a
*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/g^3/(b*x+a)^2-B*d^2*(
-a*d+b*c)*i^3*ln(d*x+c)/b^4/g^3-3*d^2*(-a*d+b*c)*i^3*(A+B*ln(e*(b*x+a)/(d*x
+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^3+3*B*d^2*(-a*d+b*c)*i^3*polylog(2,b*
(d*x+c)/d/(b*x+a))/b^4/g^3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2562, 46, 2393, 2341, 2351, 31, 2379, 2438}

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$$
$$= \frac{d^3i^3(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4g^3} - \frac{3d^2i^3(bc-ad)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4g^3} - \frac{2di^3(c+dx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3g^3(a+bx)} - \frac{i^3(c+dx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b^2g^3(a+bx)^2} + \frac{3Bd^2i^3(bc-ad)\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} - \frac{Bd^2i^3(bc-ad)\log(c+dx)}{b^4g^3} - \frac{2Bdi^3(c+dx)(bc-ad)}{b^3g^3(a+bx)} - \frac{Bi^3(c+dx)^2(bc-ad)}{4b^2g^3(a+bx)^2}$$

```
[In] Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^3,
x]
```

```
[Out] (-2*B*d*(b*c - a*d)*i^3*(c + d*x))/(b^3*g^3*(a + b*x)) - (B*(b*c - a*d)*i^3
*(c + d*x)^2)/(4*b^2*g^3*(a + b*x)^2) + (d^3*i^3*(a + b*x)*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]))/(b^4*g^3) - (2*d*(b*c - a*d)*i^3*(c + d*x)*(A + B*Log
[(e*(a + b*x))/(c + d*x)]))/(b^3*g^3*(a + b*x)) - ((b*c - a*d)*i^3*(c + d*x
)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*b^2*g^3*(a + b*x)^2) - (B*d^2*
(b*c - a*d)*i^3*Log[c + d*x])/(b^4*g^3) - (3*d^2*(b*c - a*d)*i^3*(A + B*Log
[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^4*g^3)
+ (3*B*d^2*(b*c - a*d)*i^3*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b^4*g^
3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
 n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^3(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^3}$$

$$= \frac{((bc-ad)i^3)\,\text{Subst}\left(\int \left(\frac{A+B\log(ex)}{b^2x^3} + \frac{2d(A+B\log(ex))}{b^3x^2} + \frac{d^3(A+B\log(ex))}{b^3(b-dx)^2} + \frac{3d^2(A+B\log(ex))}{b^3x(b-dx)}\right) dx, x, \frac{a+bx}{c+dx}\right)}{g^3}$$

$$= \frac{((bc-ad)i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^3} + \frac{(2d(bc-ad)i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^3} + \frac{(3d^2(bc-ad)i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^3} + \frac{(d^3(bc-ad)i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^3}$$

$$= -\frac{2Bd(bc-ad)i^3(c+dx)}{b^3g^3(a+bx)} - \frac{B(bc-ad)i^3(c+dx)^2}{4b^2g^3(a+bx)^2} + \frac{d^3i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^3} - \frac{2d(bc-ad)i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^3(a+bx)} - \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g^3(a+bx)^2} - \frac{3d^2(bc-ad)i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} + \frac{(3Bd^2(bc-ad)i^3)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4g^3} - \frac{(Bd^3(bc-ad)i^3)\,\text{Subst}\left(\int \frac{1}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4g^3}$$

$$= -\frac{2Bd(bc-ad)i^3(c+dx)}{b^3g^3(a+bx)} - \frac{B(bc-ad)i^3(c+dx)^2}{4b^2g^3(a+bx)^2} + \frac{d^3i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^3} - \frac{2d(bc-ad)i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^3(a+bx)} - \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g^3(a+bx)^2} - \frac{Bd^2(bc-ad)i^3\log(c+dx)}{b^4g^3} - \frac{3d^2(bc-ad)i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} + \frac{3Bd^2(bc-ad)i^3\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.91

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = \frac{i^3\left(4Abd^3x - \frac{B(bc-ad)^3}{(a+bx)^2} - \frac{10Bd(bc-ad)^2}{a+bx} + 10Bd^2(-bc+ad)\log(a+bx) + 4Bd^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right) - \frac{2(bc-\cdots}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g
*x)^3,x]

[Out] (i^3*(4*A*b*d^3*x - (B*(b*c - a*d)^3)/(a + b*x)^2 - (10*B*d*(b*c - a*d)^2)/
(a + b*x) + 10*B*d^2*(-(b*c) + a*d)*Log[a + b*x] + 4*B*d^3*(a + b*x)*Log[(e
*(a + b*x))/(c + d*x)] - (2*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(a + b*x)^2 - (12*d*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
)/(a + b*x) + 12*d^2*(b*c - a*d)*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c +
 d*x)]) + 6*B*d^2*(b*c - a*d)*Log[c + d*x] + 6*B*d^2*(-(b*c) + a*d)*(Log[a
+ b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*
(a + b*x))/(-(b*c) + a*d)])))/(4*b^4*g^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(341) = 682$.

Time = 1.61 (sec) , antiderivative size = 732, normalized size of antiderivative = 2.12

method	result
parts	$\frac{i^3 A\left(\frac{x\,d^3}{b^3}-\frac{3d^2(ad-cb)\ln(bx+a)}{b^4}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{2b^4(bx+a)^2}-\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)}{b^4(bx+a)}\right)}{g^3}-\frac{i^3B(ad-cb)^4e^4\left(\frac{d^8\left(\frac{\ln\left(\left(\right.\right.}{}\right.}{}\right.}{}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{i^3d^2e^3A\left(-\frac{1}{2b^2e^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{3d^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^4e^4}-\frac{2d}{b^3e^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^2}{b^3e^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)c\right.}\right.}{g^3}\right.}{}$
default	$-\frac{e(ad-cb)\left(\frac{i^3d^2e^3A\left(-\frac{1}{2b^2e^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{3d^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^4e^4}-\frac{2d}{b^3e^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^2}{b^3e^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)c\right.}\right.}{g^3}\right.}{}$
risch	Expression too large to display

```
[In] int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x,method=_RETUR
NVERBOSE)
```

```
[Out] i^3*A/g^3*(x*d^3/b^3-3/b^4*d^2*(a*d-b*c)*ln(b*x+a)-1/2/b^4*(-a^3*d^3+3*a^2*
b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/(b*x+a)^2-3/b^4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2
)/(b*x+a))-i^3*B/g^3/d^5*(a*d-b*c)^4*e^4*(1/(a*d-b*c)^3*d^8/b^3/e^3*(1/b/e/
d*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(
b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e))+2/(
a*d-b*c)^3*d^6/b^3/e^3*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)
*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+3/2/(a*d-b*c)^3*d^7/b^4/e^4*
ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-3/(a*d-b*c)^3*d^8/b^4/e^4*(dilog(-((b*e/d
+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-(
(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)+1/(a*d-b*c)^3*d^5/b^2/e^2*(-1/
2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/
d+(a*d-b*c)*e/d/(d*x+c))^2))
```

Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^3}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algor
ithm="fricas")
```

```
[Out] integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3 +
 (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log((b*e
*x + a*e)/(d*x + c)))/(b^3*g^3*x^3 + 3*a*b^2*g^3*x^2 + 3*a^2*b*g^3*x + a^3*
g^3), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$$
$$=\frac{i^3\left(\int \frac{Ac^3}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{Ad^3x^3}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{Bc^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{3Acd^2x^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\right.}{}$$

```
[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3,x)
```

```
[Out] i**3*(Integral(A*c**3/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) +
 Integral(A*d**3*x**3/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) +
```

```
 Integral(B*c**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x +
3*a*b**2*x**2 + b**3*x**3), x) + Integral(3*A*c*d**2*x**2/(a**3 + 3*a**2*b*
x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(3*A*c**2*d*x/(a**3 + 3*a**2*b
*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(B*d**3*x**3*log(a*e/(c + d*x
) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) +
Integral(3*B*c*d**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**
2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(3*B*c**2*d*x*log(a*e/(c +
 d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x
))/g**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2302 vs. $2(340) = 680$.

Time = 0.36 (sec) , antiderivative size = 2302, normalized size of antiderivative = 6.67

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algor
ithm="maxima")
```

```
[Out] -3/4*B*c^2*d*i^3*(2*(2*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^4*g
^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) + (3*a*b*c - a^2*d + 2*(2*b^2*c - a*b
*d)*x)/((b^5*c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^
3*c - a^3*b^2*d)*g^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^
3*c*d + a^2*b^2*d^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*
a*b^3*c*d + a^2*b^2*d^2)*g^3)) - 1/2*A*d^3*i^3*((6*a^2*b*x + 5*a^3)/(b^6*g^
3*x^2 + 2*a*b^5*g^3*x + a^2*b^4*g^3) - 2*x/(b^3*g^3) + 6*a*log(b*x + a)/(b^
4*g^3)) + 3/2*A*c*d^2*i^3*((4*a*b*x + 3*a^2)/(b^5*g^3*x^2 + 2*a*b^4*g^3*x +
 a^2*b^3*g^3) + 2*log(b*x + a)/(b^3*g^3)) + 1/4*B*c^3*i^3*((2*b*d*x - b*c +
 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b
^2*c - a^3*b*d)*g^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^3*g^3*x^2
+ 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d +
 a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*
g^3)) - 3/2*(2*b*x + a)*A*c^2*d*i^3/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*
g^3) - 1/2*A*c^3*i^3/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*(2*b^3
*c^3*d^2*i^3 + 8*a*b^2*c^2*d^3*i^3 - 13*a^2*b*c*d^4*i^3 + 5*a^3*d^5*i^3)*B*
log(d*x + c)/(b^6*c^2*g^3 - 2*a*b^5*c*d*g^3 + a^2*b^4*d^2*g^3) + 1/4*(4*(b^
5*c^2*d^3*i^3*log(e) - 2*a*b^4*c*d^4*i^3*log(e) + a^2*b^3*d^5*i^3*log(e))*B
*x^3 + 8*(a*b^4*c^2*d^3*i^3*log(e) - 2*a^2*b^3*c*d^4*i^3*log(e) + a^3*b^2*d
^5*i^3*log(e))*B*x^2 + 2*(12*(i^3*log(e) + i^3)*a*b^4*c^3*d^2 - (28*i^3*log
(e) + 27*i^3)*a^2*b^3*c^2*d^3 + 20*(i^3*log(e) + i^3)*a^3*b^2*c*d^4 - (4*i^
3*log(e) + 5*i^3)*a^4*b*d^5)*B*x + 6*((b^5*c^3*d^2*i^3 - 3*a*b^4*c^2*d^3*i^
3 + 3*a^2*b^3*c*d^4*i^3 - a^3*b^2*d^5*i^3)*B*x^2 + 2*(a*b^4*c^3*d^2*i^3 - 3
```

```
*a^2*b^3*c^2*d^3*i^3 + 3*a^3*b^2*c*d^4*i^3 - a^4*b*d^5*i^3)*B*x + (a^2*b^3*
c^3*d^2*i^3 - 3*a^3*b^2*c^2*d^3*i^3 + 3*a^4*b*c*d^4*i^3 - a^5*d^5*i^3)*B)*l
og(b*x + a)^2 + (3*(6*i^3*log(e) + 7*i^3)*a^2*b^3*c^3*d^2 - (46*i^3*log(e)
+ 47*i^3)*a^3*b^2*c^2*d^3 + (38*i^3*log(e) + 35*i^3)*a^4*b*c*d^4 - (10*i^3*
log(e) + 9*i^3)*a^5*d^5)*B + 2*(2*(b^5*c^2*d^3*i^3 - 2*a*b^4*c*d^4*i^3 + a^
2*b^3*d^5*i^3)*B*x^3 + (6*b^5*c^3*d^2*i^3*log(e) - 18*(i^3*log(e) - i^3)*a*
b^4*c^2*d^3 + 9*(2*i^3*log(e) - 3*i^3)*a^2*b^3*c*d^4 - (6*i^3*log(e) - 11*i
^3)*a^3*b^2*d^5)*B*x^2 - 2*(18*a^2*b^3*c^2*d^3*i^3*log(e) - 6*(i^3*log(e) +
 i^3)*a*b^4*c^3*d^2 - 9*(2*i^3*log(e) - i^3)*a^3*b^2*c*d^4 + (6*i^3*log(e)
- 5*i^3)*a^4*b*d^5)*B*x + (18*a^4*b*c*d^4*i^3*log(e) + 3*(2*i^3*log(e) + 3*
i^3)*a^2*b^3*c^3*d^2 - 9*(2*i^3*log(e) + i^3)*a^3*b^2*c^2*d^3 - 2*(3*i^3*lo
g(e) - i^3)*a^5*d^5)*B)*log(b*x + a) - 2*(2*(b^5*c^2*d^3*i^3 - 2*a*b^4*c*d^
4*i^3 + a^2*b^3*d^5*i^3)*B*x^3 + 4*(a*b^4*c^2*d^3*i^3 - 2*a^2*b^3*c*d^4*i^3
 + a^3*b^2*d^5*i^3)*B*x^2 + 4*(3*a*b^4*c^3*d^2*i^3 - 7*a^2*b^3*c^2*d^3*i^3
+ 5*a^3*b^2*c*d^4*i^3 - a^4*b*d^5*i^3)*B*x + (9*a^2*b^3*c^3*d^2*i^3 - 23*a^
3*b^2*c^2*d^3*i^3 + 19*a^4*b*c*d^4*i^3 - 5*a^5*d^5*i^3)*B + 6*((b^5*c^3*d^2
*i^3 - 3*a*b^4*c^2*d^3*i^3 + 3*a^2*b^3*c*d^4*i^3 - a^3*b^2*d^5*i^3)*B*x^2 +
 2*(a*b^4*c^3*d^2*i^3 - 3*a^2*b^3*c^2*d^3*i^3 + 3*a^3*b^2*c*d^4*i^3 - a^4*b
*d^5*i^3)*B*x + (a^2*b^3*c^3*d^2*i^3 - 3*a^3*b^2*c^2*d^3*i^3 + 3*a^4*b*c*d^
4*i^3 - a^5*d^5*i^3)*B)*log(b*x + a))*log(d*x + c))/(a^2*b^6*c^2*g^3 - 2*a^
3*b^5*c*d*g^3 + a^4*b^4*d^2*g^3 + (b^8*c^2*g^3 - 2*a*b^7*c*d*g^3 + a^2*b^6*
d^2*g^3)*x^2 + 2*(a*b^7*c^2*g^3 - 2*a^2*b^6*c*d*g^3 + a^3*b^5*d^2*g^3)*x) +
 3*(b*c*d^2*i^3 - a*d^3*i^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) +
1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(b^4*g^3)
```

Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^3}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algor
ithm="giac")
```

```
[Out] integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g)^
3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx=\int \frac{{\left(c\,i+d\,i\,x\right)}^3\,\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}{{\left(a\,g+b\,g\,x\right)}^3}\,dx$$

```
[In] int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^3,
x)
[Out] int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^3,
 x)
```

3.27 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 310

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx=-\frac{Bd^2i^3(c+dx)}{b^3g^4(a+bx)}-\frac{Bdi^3(c+dx)^2}{4b^2g^4(a+bx)^2}-\frac{Bi^3(c+dx)^3}{9bg^4(a+bx)^3}-\frac{d^2i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^4(a+bx)}-\frac{di^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g^4(a+bx)^2}-\frac{i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bg^4(a+bx)^3}-\frac{d^3i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4}+\frac{Bd^3i^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4}$$

```
[Out] -B*d^2*i^3*(d*x+c)/b^3/g^4/(b*x+a)-1/4*B*d*i^3*(d*x+c)^2/b^2/g^4/(b*x+a)^2-
1/9*B*i^3*(d*x+c)^3/b/g^4/(b*x+a)^3-d^2*i^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+
c)))/b^3/g^4/(b*x+a)-1/2*d*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/g^
4/(b*x+a)^2-1/3*i^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g^4/(b*x+a)^3-d
^3*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^4+B*d^3*
i^3*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^4
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2562, 2380, 2341, 2379, 2438}

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = -\frac{d^3i^3\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4g^4} - \frac{d^2i^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3g^4(a+bx)} - \frac{di^3(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b^2g^4(a+bx)^2} - \frac{i^3(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3bg^4(a+bx)^3} + \frac{Bd^3i^3\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4} - \frac{Bd^2i^3(c+dx)}{b^3g^4(a+bx)} - \frac{Bdi^3(c+dx)^2}{4b^2g^4(a+bx)^2} - \frac{Bi^3(c+dx)^3}{9bg^4(a+bx)^3}$$

```
[In] Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^4,
x]

[Out] -((B*d^2*i^3*(c + d*x))/(b^3*g^4*(a + b*x))) - (B*d*i^3*(c + d*x)^2)/(4*b^2
*g^4*(a + b*x)^2) - (B*i^3*(c + d*x)^3)/(9*b*g^4*(a + b*x)^3) - (d^2*i^3*(c
 + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^3*g^4*(a + b*x)) - (d*i^3*
(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*b^2*g^4*(a + b*x)^2) -
 (i^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b*g^4*(a + b*x)^
3) - (d^3*i^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x))/(d
*(a + b*x))])/(b^4*g^4) + (B*d^3*i^3*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))
])/(b^4*g^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
```

```
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i^3\text{Subst}\left(\int \frac{A+B\log(ex)}{x^4(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^4}\\
&= \frac{i^3\text{Subst}\left(\int \frac{A+B\log(ex)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^4} + \frac{(di^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^3(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^4}\\
&= -\frac{Bi^3(c+dx)^3}{9bg^4(a+bx)^3} - \frac{i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bg^4(a+bx)^3}\\
&\quad + \frac{(di^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^4} + \frac{(d^2i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^2(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^4}\\
&= -\frac{Bdi^3(c+dx)^2}{4b^2g^4(a+bx)^2} - \frac{Bi^3(c+dx)^3}{9bg^4(a+bx)^3} - \frac{di^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g^4(a+bx)^2}\\
&\quad - \frac{i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bg^4(a+bx)^3} + \frac{(d^2i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^4}\\
&\quad + \frac{(d^3i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^4}
\end{aligned}$$

$$= -\frac{Bd^2i^3(c+dx)}{b^3g^4(a+bx)} - \frac{Bdi^3(c+dx)^2}{4b^2g^4(a+bx)^2} - \frac{Bi^3(c+dx)^3}{9bg^4(a+bx)^3} - \frac{d^2i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^4(a+bx)} - \frac{di^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g^4(a+bx)^2} - \frac{i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bg^4(a+bx)^3} - \frac{d^3i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4} + \frac{(Bd^3i^3)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}dx, x, \frac{a+bx}{c+dx}\right)}{b^4g^4}$$

$$= -\frac{Bd^2i^3(c+dx)}{b^3g^4(a+bx)} - \frac{Bdi^3(c+dx)^2}{4b^2g^4(a+bx)^2} - \frac{Bi^3(c+dx)^3}{9bg^4(a+bx)^3} - \frac{d^2i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^4(a+bx)} - \frac{di^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g^4(a+bx)^2} - \frac{i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bg^4(a+bx)^3} - \frac{d^3i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4} + \frac{Bd^3i^3\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.99

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = \frac{i^3\left(-\frac{4B(bc-ad)^3}{(a+bx)^3} - \frac{21Bd(bc-ad)^2}{(a+bx)^2} + \frac{66Bd^2(-bc+ad)}{a+bx} - 66Bd^3\log(a+bx) - \frac{12(bc-ad)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3} - \frac{54d(bc-ad)^2}{}\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g
*x)^4,x]
```

```
[Out] (i^3*((-4*B*(b*c - a*d)^3)/(a + b*x)^3 - (21*B*d*(b*c - a*d)^2)/(a + b*x)^2
 + (66*B*d^2*(-(b*c) + a*d))/(a + b*x) - 66*B*d^3*Log[a + b*x] - (12*(b*c -
 a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x)^3 - (54*d*(b*c - a*
d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x)^2 + (108*d^2*(-(b*c) +
 a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) + 36*d^3*Log[a + b*x]
*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 66*B*d^3*Log[c + d*x] - 18*B*d^3*(L
og[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2
, (d*(a + b*x))/(-(b*c) + a*d)])))/(36*b^4*g^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(302) = 604$.

Time $= 1.61$ (sec) , antiderivative size $= 685$, normalized size of antiderivative $= 2.21$

method	result
parts	$\frac{i^3 A\left(-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{3b^4(bx+a)^3}+\frac{d^3\ln(bx+a)}{b^4}-\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)}{2b^4(bx+a)^2}+\frac{3d^2(ad-cb)}{b^4(bx+a)}\right)}{g^4}-\frac{i^3B(ad-cb)^4e^4\left(-\frac{d^5\left(-\ldots\right)}{\ldots}\right.}{\ldots}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{i^3d^2e^3A\left(-\frac{1}{3be\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{d^3\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^4e^4}-\frac{d^2}{b^3e^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{d}{2b^2e^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{d^3}{\ldots}\right.}{(ad-cb)g^4}\right.}{\ldots}$
default	$-\frac{e(ad-cb)\left(-\frac{i^3d^2e^3A\left(-\frac{1}{3be\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{d^3\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^4e^4}-\frac{d^2}{b^3e^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{d}{2b^2e^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{d^3}{\ldots}\right.}{(ad-cb)g^4}\right.}{\ldots}$
risch	Expression too large to display

```
[In] int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x,method=_RETUR
NVERBOSE)
```

```
[Out] i^3*A/g^4*(-1/3*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^
3+d^3/b^4*ln(b*x+a)-3/2*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/(b*x+a)^2+3/b^4*d
^2*(a*d-b*c)/(b*x+a))-i^3*B/g^4/d^5*(a*d-b*c)^4*e^4*(-1/(a*d-b*c)^4*d^5/b/e
*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/
(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-1/(a*d-b*c)^4*d^7/b^3/e^3*(-1/(b*e/d+(a*d-
b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d
*x+c)))-1/2/(a*d-b*c)^4*d^8/b^4/e^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+1/(a*
d-b*c)^4*d^9/b^4/e^4*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/
e)/d)-1/(a*d-b*c)^4*d^6/b^2/e^2*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))
```

Fricas [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^4}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algor
ithm="fricas")
```

```
[Out] integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3 +
 (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log((b*e
*x + a*e)/(d*x + c)))/(b^4*g^4*x^4 + 4*a*b^3*g^4*x^3 + 6*a^2*b^2*g^4*x^2 +
4*a^3*b*g^4*x + a^4*g^4), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$$
$$=\frac{i^3\left(\int \frac{Ac^3}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx+\int \frac{Ad^3x^3}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx+\int \frac{Bc^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx+\right.}{}$$

```
[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4,x)
```

```
[Out] i**3*(Integral(A*c**3/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3
 + b**4*x**4), x) + Integral(A*d**3*x**3/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x
**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(B*c**3*log(a*e/(c + d*x) +
```

```
b*e*x/(c + d*x))/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b*
*4*x**4), x) + Integral(3*A*c*d**2*x**2/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x*
*2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(3*A*c**2*d*x/(a**4 + 4*a**3*
b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(B*d**3*x
**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x
**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(3*B*c*d**2*x**2*log(a*e/(c
+ d*x) + b*e*x/(c + d*x))/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*
x**3 + b**4*x**4), x) + Integral(3*B*c**2*d*x*log(a*e/(c + d*x) + b*e*x/(c
+ d*x))/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4),
 x))/g**4
```

Maxima [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^4}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algor
ithm="maxima")
```

```
[Out] -1/6*B*d^3*i^3*((18*a*b^2*x^2 + 27*a^2*b*x + 11*a^3 + 6*(b^3*x^3 + 3*a*b^2*
x^2 + 3*a^2*b*x + a^3)*log(b*x + a))*log(d*x + c)/(b^7*g^4*x^3 + 3*a*b^6*g^
4*x^2 + 3*a^2*b^5*g^4*x + a^3*b^4*g^4) - 6*integrate(1/6*(6*b^4*d*x^4*log(e
) + 45*a^2*b^2*d*x^2 + 38*a^3*b*d*x + 11*a^4*d + 6*(b^4*c*log(e) + 3*a*b^3*
d)*x^3 + 6*(2*b^4*d*x^4 + 6*a^2*b^2*d*x^2 + 4*a^3*b*d*x + a^4*d + (b^4*c +
4*a*b^3*d)*x^3)*log(b*x + a))/(b^8*d*g^4*x^5 + a^4*b^4*c*g^4 + (b^8*c*g^4 +
 4*a*b^7*d*g^4)*x^4 + 2*(2*a*b^7*c*g^4 + 3*a^2*b^6*d*g^4)*x^3 + 2*(3*a^2*b^
6*c*g^4 + 2*a^3*b^5*d*g^4)*x^2 + (4*a^3*b^5*c*g^4 + a^4*b^4*d*g^4)*x), x))
- 1/6*B*c*d^2*i^3*(6*(3*b^2*x^2 + 3*a*b*x + a^2)*log(b*e*x/(d*x + c) + a*e/
(d*x + c))/(b^6*g^4*x^3 + 3*a*b^5*g^4*x^2 + 3*a^2*b^4*g^4*x + a^3*b^3*g^4)
+ (11*a^2*b^2*c^2 - 7*a^3*b*c*d + 2*a^4*d^2 + 6*(3*b^4*c^2 - 3*a*b^3*c*d +
a^2*b^2*d^2)*x^2 + 3*(9*a*b^3*c^2 - 7*a^2*b^2*c*d + 2*a^3*b*d^2)*x)/((b^8*c
^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*g^4*x^3 + 3*(a*b^7*c^2 - 2*a^2*b^6*c*d + a^
3*b^5*d^2)*g^4*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d + a^4*b^4*d^2)*g^4*x +
(a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2)*g^4) + 6*(3*b^2*c^2*d - 3*a*b*c
*d^2 + a^2*d^3)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 -
a^3*b^3*d^3)*g^4) - 6*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(d*x + c)/((
b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4)) - 1/12*B*c^2
*d*i^3*(6*(3*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^5*g^4*x^3 + 3
*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) + (5*a*b^2*c^2 - 22*a^2*b*c
*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c^2 - 16*a*b^2*c*
d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3 + 3*(a*b
^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*
c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4)
```

```
 - 6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3
*c*d^2 - a^3*b^2*d^3)*g^4) + 6*(3*b*c*d^2 - a*d^3)*log(d*x + c)/((b^5*c^3 -
 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4)) - 1/18*B*c^3*i^3*((6*
b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*
x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^
4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2
)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*log(b*e*x/(d*x
 + c) + a*e/(d*x + c))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a
^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2
- a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^
2*c*d^2 - a^3*b*d^3)*g^4)) + 1/6*A*d^3*i^3*((18*a*b^2*x^2 + 27*a^2*b*x + 11
*a^3)/(b^7*g^4*x^3 + 3*a*b^6*g^4*x^2 + 3*a^2*b^5*g^4*x + a^3*b^4*g^4) + 6*l
og(b*x + a)/(b^4*g^4)) - 1/2*(3*b*x + a)*A*c^2*d*i^3/(b^5*g^4*x^3 + 3*a*b^4
*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - (3*b^2*x^2 + 3*a*b*x + a^2)*A*c
*d^2*i^3/(b^6*g^4*x^3 + 3*a*b^5*g^4*x^2 + 3*a^2*b^4*g^4*x + a^3*b^3*g^4) -
1/3*A*c^3*i^3/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^4}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algor
ithm="giac")
```

```
[Out] integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g)^
4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx=\int \frac{{\left(c\,i+d\,i\,x\right)}^3\,\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}{{\left(a\,g+b\,g\,x\right)}^4}\,dx$$

```
[In] int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^4,
x)
```

```
[Out] int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^4,
 x)
```

3.28 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 89

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = -\frac{Bi^3(c+dx)^4}{16(bc-ad)g^5(a+bx)^4} - \frac{i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)g^5(a+bx)^4}$$

```
[Out] -1/16*B*i^3*(d*x+c)^4/(-a*d+b*c)/g^5/(b*x+a)^4-1/4*i^3*(d*x+c)^4*(A+B*ln(e*
(b*x+a)/(d*x+c)))/(-a*d+b*c)/g^5/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2562, 2341}

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = -\frac{i^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4g^5(a+bx)^4(bc-ad)} - \frac{Bi^3(c+dx)^4}{16g^5(a+bx)^4(bc-ad)}$$

```
[In] Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^5,
x]
```

```
[Out] -1/16*(B*i^3*(c + d*x)^4)/((b*c - a*d)*g^5*(a + b*x)^4) - (i^3*(c + d*x)^4*
(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*(b*c - a*d)*g^5*(a + b*x)^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{i^3\text{Subst}\left(\int \frac{A+B\log(ex)}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^5}$$
$$= -\frac{Bi^3(c+dx)^4}{16(bc-ad)g^5(a+bx)^4} - \frac{i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)g^5(a+bx)^4}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 427 vs. 2(89) = 178.

Time = 0.28 (sec) , antiderivative size = 427, normalized size of antiderivative = 4.80

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx =$$
$$-\frac{i^3\left(4Ab^4c^4+b^4Bc^4-4a^4Ad^4-a^4Bd^4+16Ab^4c^3dx+4b^4Bc^3dx-16a^3Abd^4x-4a^3bBd^4x+24Ab^4c^2d\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g
*x)^5,x]
```

```
[Out] -1/16*(i^3*(4*A*b^4*c^4 + b^4*B*c^4 - 4*a^4*A*d^4 - a^4*B*d^4 + 16*A*b^4*c^
3*d*x + 4*b^4*B*c^3*d*x - 16*a^3*A*b*d^4*x - 4*a^3*b*B*d^4*x + 24*A*b^4*c^2
*d^2*x^2 + 6*b^4*B*c^2*d^2*x^2 - 24*a^2*A*b^2*d^4*x^2 - 6*a^2*b^2*B*d^4*x^2
 + 16*A*b^4*c*d^3*x^3 + 4*b^4*B*c*d^3*x^3 - 16*a*A*b^3*d^4*x^3 - 4*a*b^3*B*
d^4*x^3 + 4*B*d^4*(a + b*x)^4*Log[a + b*x] + 4*B*(-(a^4*d^4) - 4*a^3*b*d^4*
```

```
x - 6*a^2*b^2*d^4*x^2 - 4*a*b^3*d^4*x^3 + b^4*c*(c^3 + 4*c^2*d*x + 6*c*d^2*
x^2 + 4*d^3*x^3))*Log[(e*(a + b*x))/(c + d*x)] - 4*a^4*B*d^4*Log[c + d*x] -
 16*a^3*b*B*d^4*x*Log[c + d*x] - 24*a^2*b^2*B*d^4*x^2*Log[c + d*x] - 16*a*b
^3*B*d^4*x^3*Log[c + d*x] - 4*b^4*B*d^4*x^4*Log[c + d*x]))/(b^4*(b*c - a*d)
*g^5*(a + b*x)^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(85) = 170$.

Time = 1.58 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.08

method	result
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{i^3 d^2 e^3 A}{4(ad-cb)^2 g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}+\frac{i^3 d^2 e^3 B\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{1}{16\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}\right)}{(ad-cb)^2 g^5}\right)}{d^2}$
default	$-\frac{e(ad-cb)\left(-\frac{i^3 d^2 e^3 A}{4(ad-cb)^2 g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}+\frac{i^3 d^2 e^3 B\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{1}{16\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}\right)}{(ad-cb)^2 g^5}\right)}{d^2}$
parts	$\frac{i^3 A\left(-\frac{d\left(a^2d^2-2abcd+b^2c^2\right)}{b^4(bx+a)^3}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{4b^4(bx+a)^4}+\frac{3d^2(ad-cb)}{2b^4(bx+a)^2}-\frac{d^3}{b^4(bx+a)}\right)}{g^5}-\frac{i^3 B\,e^4\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}\right.}{g^5(ad-c}$
parallelrisch	$\frac{4B\,x^4\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6 c\,d^4 i^3+16B\,x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6c^2d^3i^3+24B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6c^3d^2i^3+16Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6c^4d\,i^3+4}{}$
norman	$\frac{B c\,d^3 i^3 x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)}{(ad-cb)g}+\frac{B\,c^3 d\,i^3 x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(ad-cb)}+\frac{\left(4A\,c^3i^3+B\,c^3i^3\right)x}{4ga}+\frac{3\left(4Aa\,c^2d\,i^3+4Ab\,c^3i^3+Ba\,c^2d\,i^3+Bb\,c^3i^3\right)x^2}{8g\,a^2}+\frac{(4A\,a^2}{}$
risch	$-\frac{i^3B\left(4d^3x^3b^3+6a\,b^2d^3x^2+6b^3c\,d^2x^2+4a^2b\,d^3x+4a\,b^2c\,d^2x+4b^3c^2dx+a^3d^3+a^2bc\,d^2+a\,b^2c^2d+b^3c^3\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{4(bx+a)^4g^5b^4}-$

```
[In] int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x,method=_RETUR
NVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(-1/4*i^3*d^2*e^3/(a*d-b*c)^2/g^5*A/(b*e/d+(a*d-b*c)*e/d
/(d*x+c))^4+i^3*d^2*e^3/(a*d-b*c)^2/g^5*B*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c
))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(85) = 170.

Time = 0.35 (sec) , antiderivative size = 355, normalized size of antiderivative = 3.99

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = -\frac{4\left((4A+B)b^4cd^3-(4A+B)ab^3d^4\right)i^3x^3+6\left((4A+B)b^4c^2d^2-(4A+B)a^2b^2d^4\right)i^3x^2+4\left((4A+B)b\right.}{16\left((b^9c-ab^8d)g^5x^4+4\left(ab^8c-\right.\right.}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algor
ithm="fricas")
```

```
[Out] -1/16*(4*((4*A + B)*b^4*c*d^3 - (4*A + B)*a*b^3*d^4)*i^3*x^3 + 6*((4*A + B)
*b^4*c^2*d^2 - (4*A + B)*a^2*b^2*d^4)*i^3*x^2 + 4*((4*A + B)*b^4*c^3*d - (4
*A + B)*a^3*b*d^4)*i^3*x + ((4*A + B)*b^4*c^4 - (4*A + B)*a^4*d^4)*i^3 + 4*
(B*b^4*d^4*i^3*x^4 + 4*B*b^4*c*d^3*i^3*x^3 + 6*B*b^4*c^2*d^2*i^3*x^2 + 4*B*
b^4*c^3*d*i^3*x + B*b^4*c^4*i^3)*log((b*e*x + a*e)/(d*x + c)))/((b^9*c - a*
b^8*d)*g^5*x^4 + 4*(a*b^8*c - a^2*b^7*d)*g^5*x^3 + 6*(a^2*b^7*c - a^3*b^6*d
)*g^5*x^2 + 4*(a^3*b^6*c - a^4*b^5*d)*g^5*x + (a^4*b^5*c - a^5*b^4*d)*g^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**5,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3107 vs. 2(85) = 170.

Time = 0.35 (sec) , antiderivative size = 3107, normalized size of antiderivative = 34.91

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algor
ithm="maxima")
```

```
[Out] -1/48*B*d^3*i^3*(12*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)*log(b*e*x/(
d*x + c) + a*e/(d*x + c))/(b^8*g^5*x^4 + 4*a*b^7*g^5*x^3 + 6*a^2*b^6*g^5*x^
2 + 4*a^3*b^5*g^5*x + a^4*b^4*g^5) + (25*a^3*b^3*c^3 - 23*a^4*b^2*c^2*d + 1
3*a^5*b*c*d^2 - 3*a^6*d^3 + 12*(4*b^6*c^3 - 6*a*b^5*c^2*d + 4*a^2*b^4*c*d^2
 - a^3*b^3*d^3)*x^3 + 6*(18*a*b^5*c^3 - 22*a^2*b^4*c^2*d + 13*a^3*b^3*c*d^2
 - 3*a^4*b^2*d^3)*x^2 + 4*(22*a^2*b^4*c^3 - 23*a^3*b^3*c^2*d + 13*a^4*b^2*c
*d^2 - 3*a^5*b*d^3)*x)/((b^11*c^3 - 3*a*b^10*c^2*d + 3*a^2*b^9*c*d^2 - a^3*
b^8*d^3)*g^5*x^4 + 4*(a*b^10*c^3 - 3*a^2*b^9*c^2*d + 3*a^3*b^8*c*d^2 - a^4*
b^7*d^3)*g^5*x^3 + 6*(a^2*b^9*c^3 - 3*a^3*b^8*c^2*d + 3*a^4*b^7*c*d^2 - a^5
*b^6*d^3)*g^5*x^2 + 4*(a^3*b^8*c^3 - 3*a^4*b^7*c^2*d + 3*a^5*b^6*c*d^2 - a^
6*b^5*d^3)*g^5*x + (a^4*b^7*c^3 - 3*a^5*b^6*c^2*d + 3*a^6*b^5*c*d^2 - a^7*b
^4*d^3)*g^5) + 12*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3 - a^3*d^4)
*log(b*x + a)/((b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d
^3 + a^4*b^4*d^4)*g^5) - 12*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3
- a^3*d^4)*log(d*x + c)/((b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a
^3*b^5*c*d^3 + a^4*b^4*d^4)*g^5)) - 1/48*B*c*d^2*i^3*(12*(6*b^2*x^2 + 4*a*b
*x + a^2)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^7*g^5*x^4 + 4*a*b^6*g^5*x
^3 + 6*a^2*b^5*g^5*x^2 + 4*a^3*b^4*g^5*x + a^4*b^3*g^5) + (13*a^2*b^3*c^3 -
 75*a^3*b^2*c^2*d + 33*a^4*b*c*d^2 - 7*a^5*d^3 - 12*(6*b^5*c^2*d - 4*a*b^4*
c*d^2 + a^2*b^3*d^3)*x^3 + 6*(6*b^5*c^3 - 46*a*b^4*c^2*d + 29*a^2*b^3*c*d^2
 - 7*a^3*b^2*d^3)*x^2 + 4*(10*a*b^4*c^3 - 63*a^2*b^3*c^2*d + 33*a^3*b^2*c*d
^2 - 7*a^4*b*d^3)*x)/((b^10*c^3 - 3*a*b^9*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7
*d^3)*g^5*x^4 + 4*(a*b^9*c^3 - 3*a^2*b^8*c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*
d^3)*g^5*x^3 + 6*(a^2*b^8*c^3 - 3*a^3*b^7*c^2*d + 3*a^4*b^6*c*d^2 - a^5*b^5
*d^3)*g^5*x^2 + 4*(a^3*b^7*c^3 - 3*a^4*b^6*c^2*d + 3*a^5*b^5*c*d^2 - a^6*b^
4*d^3)*g^5*x + (a^4*b^6*c^3 - 3*a^5*b^5*c^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d
^3)*g^5) - 12*(6*b^2*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*log(b*x + a)/((b^7*c^
4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4)*g^5)
 + 12*(6*b^2*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*log(d*x + c)/((b^7*c^4 - 4*a*
b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4)*g^5)) - 1/48
*B*c^2*d*i^3*(12*(4*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^6*g^5*
x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g^5)
+ (7*a*b^3*c^3 - 33*a^2*b^2*c^2*d + 75*a^3*b*c*d^2 - 13*a^4*d^3 + 12*(4*b^4
*c*d^2 - a*b^3*d^3)*x^3 - 6*(4*b^4*c^2*d - 29*a*b^3*c*d^2 + 7*a^2*b^2*d^3)*
x^2 + 4*(4*b^4*c^3 - 21*a*b^3*c^2*d + 57*a^2*b^2*c*d^2 - 13*a^3*b*d^3)*x)/(
(b^9*c^3 - 3*a*b^8*c^2*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*g^5*x^4 + 4*(a*b^
8*c^3 - 3*a^2*b^7*c^2*d + 3*a^3*b^6*c*d^2 - a^4*b^5*d^3)*g^5*x^3 + 6*(a^2*b
^7*c^3 - 3*a^3*b^6*c^2*d + 3*a^4*b^5*c*d^2 - a^5*b^4*d^3)*g^5*x^2 + 4*(a^3*
b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2 - a^6*b^3*d^3)*g^5*x + (a^4*b^5
*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d^2 - a^7*b^2*d^3)*g^5) + 12*(4*b*c*d^
3 - a*d^4)*log(b*x + a)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a
^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5) - 12*(4*b*c*d^3 - a*d^4)*log(d*x + c)/((b^
6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*
g^5)) + 1/48*B*c^3*i^3*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a
^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d -
```

```
5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^
2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2
 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^
2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d
^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2
- a^7*b*d^3)*g^5) - 12*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^5*g^5*x^4 +
4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4
*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d
^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^
2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*(4*b*x + a)*A*c^2*
d*i^3/(b^6*g^5*x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x
+ a^4*b^2*g^5) - 1/4*(6*b^2*x^2 + 4*a*b*x + a^2)*A*c*d^2*i^3/(b^7*g^5*x^4 +
 4*a*b^6*g^5*x^3 + 6*a^2*b^5*g^5*x^2 + 4*a^3*b^4*g^5*x + a^4*b^3*g^5) - 1/4
*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)*A*d^3*i^3/(b^8*g^5*x^4 + 4*a*b
^7*g^5*x^3 + 6*a^2*b^6*g^5*x^2 + 4*a^3*b^5*g^5*x + a^4*b^4*g^5) - 1/4*A*c^3
*i^3/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x +
 a^4*b*g^5)
```

Giac [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.57

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = -\frac{1}{16}\left(\frac{4\,(dx+c)^4Be^5i^3\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^4g^5}+\frac{(4\,Ae^5i^3+Be^5i^3)(dx+c)^4}{(bex+ae)^4g^5}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{a}{(bce-ade}\right.$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algor
ithm="giac")
```

```
[Out] -1/16*(4*(d*x + c)^4*B*e^5*i^3*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^
4*g^5) + (4*A*e^5*i^3 + B*e^5*i^3)*(d*x + c)^4/((b*e*x + a*e)^4*g^5))*(b*c/
((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 780, normalized size of antiderivative = 8.76

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{x^3\,(4\,A\,b^3\,d^3\,i^3+B\,b^3\,d^3\,i^3)+x^2\left(6\,A\,a\,b^2\,d^3\,i^3+\frac{3\,B\,a\,b^2\,d^3\,i^3}{2}+6\,A\,b^3\,c\,d^2\,i^3+\frac{3\,B\,b^3\,c\,d^2\,i^3}{2}\right)+x\,(4\,A\,a^2\,b\,d^3\,i \cdots}{\cdots}$$

$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(x^2\left(b\left(b\left(\frac{B\,a\,d^3\,i^3}{4\,b^5\,g^5}+\frac{B\,c\,d^2\,i^3}{4\,b^4\,g^5}\right)+\frac{B\,a\,d^3\,i^3}{2\,b^4\,g^5}+\frac{B\,c\,d^2\,i^3}{2\,b^3\,g^5}\right)+\frac{3\,B\,a\,d^3\,i^3}{4\,b^3\,g^5}+\frac{3\,B\,c\,d^2\,i^3}{4\,b^2\,g^5}\right)+x\left(b\left(a\left(\frac{B\,a\,d^3\,i^3}{4\,b^5\,g^5} \cdots\right.\right.\right.\right.}{4\,a \cdots}$$

$$-\frac{B\,d^4\,i^3\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,1\mathrm{i}}{2\,b^4\,g^5\,\left(a\,d-b\,c\right)}$$

```
[In] int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^5,
x)

[Out] - (x^3*(4*A*b^3*d^3*i^3 + B*b^3*d^3*i^3) + x^2*(6*A*a*b^2*d^3*i^3 + (3*B*a*
b^2*d^3*i^3)/2 + 6*A*b^3*c*d^2*i^3 + (3*B*b^3*c*d^2*i^3)/2) + x*(4*A*a^2*b*
d^3*i^3 + B*a^2*b*d^3*i^3 + 4*A*b^3*c^2*d*i^3 + B*b^3*c^2*d*i^3 + 4*A*a*b^2
*c*d^2*i^3 + B*a*b^2*c*d^2*i^3) + A*a^3*d^3*i^3 + A*b^3*c^3*i^3 + (B*a^3*d^
3*i^3)/4 + (B*b^3*c^3*i^3)/4 + A*a*b^2*c^2*d*i^3 + A*a^2*b*c*d^2*i^3 + (B*a
*b^2*c^2*d*i^3)/4 + (B*a^2*b*c*d^2*i^3)/4)/(4*a^4*b^4*g^5 + 4*b^8*g^5*x^4 +
 16*a^3*b^5*g^5*x + 16*a*b^7*g^5*x^3 + 24*a^2*b^6*g^5*x^2) - (log((e*(a + b
*x))/(c + d*x))*(x^2*(b*(b*((B*a*d^3*i^3)/(4*b^5*g^5) + (B*c*d^2*i^3)/(4*b^
4*g^5)) + (B*a*d^3*i^3)/(2*b^4*g^5) + (B*c*d^2*i^3)/(2*b^3*g^5)) + (3*B*a*d
^3*i^3)/(4*b^3*g^5) + (3*B*c*d^2*i^3)/(4*b^2*g^5)) + x*(b*(a*((B*a*d^3*i^3)
/(4*b^5*g^5) + (B*c*d^2*i^3)/(4*b^4*g^5)) + (B*c^2*d*i^3)/(4*b^3*g^5)) + a*
(b*((B*a*d^3*i^3)/(4*b^5*g^5) + (B*c*d^2*i^3)/(4*b^4*g^5)) + (B*a*d^3*i^3)/
(2*b^4*g^5) + (B*c*d^2*i^3)/(2*b^3*g^5)) + (3*B*c^2*d*i^3)/(4*b^2*g^5)) + a
*(a*((B*a*d^3*i^3)/(4*b^5*g^5) + (B*c*d^2*i^3)/(4*b^4*g^5)) + (B*c^2*d*i^3)
/(4*b^3*g^5)) + (B*c^3*i^3)/(4*b^2*g^5) + (B*d^3*i^3*x^3)/(b^2*g^5)))/(4*a^
3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3) - (B*d^4*i^3*atan((b*c*2
i + b*d*x*2i)/(a*d - b*c) + 1i)*1i)/(2*b^4*g^5*(a*d - b*c))
```

3.29 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 181

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = \frac{Bdi^3(c+dx)^4}{16(bc-ad)^2g^6(a+bx)^4} - \frac{bBi^3(c+dx)^5}{25(bc-ad)^2g^6(a+bx)^5} + \frac{di^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^2g^6(a+bx)^4} - \frac{bi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5(bc-ad)^2g^6(a+bx)^5}$$

```
[Out] 1/16*B*d*i^3*(d*x+c)^4/(-a*d+b*c)^2/g^6/(b*x+a)^4-1/25*b*B*i^3*(d*x+c)^5/(-
a*d+b*c)^2/g^6/(b*x+a)^5+1/4*d*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-
a*d+b*c)^2/g^6/(b*x+a)^4-1/5*b*i^3*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-
a*d+b*c)^2/g^6/(b*x+a)^5
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used

$= \{2562, 45, 2372, 12\}$

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = -\frac{bi^3(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5g^6(a+bx)^5(bc-ad)^2} + \frac{di^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4g^6(a+bx)^4(bc-ad)^2} - \frac{bBi^3(c+dx)^5}{25g^6(a+bx)^5(bc-ad)^2} + \frac{Bdi^3(c+dx)^4}{16g^6(a+bx)^4(bc-ad)^2}$$

```
[In] Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^6,
x]
```

```
[Out] (B*d*i^3*(c + d*x)^4)/(16*(b*c - a*d)^2*g^6*(a + b*x)^4) - (b*B*i^3*(c + d*
x)^5)/(25*(b*c - a*d)^2*g^6*(a + b*x)^5) + (d*i^3*(c + d*x)^4*(A + B*Log[(e
*(a + b*x))/(c + d*x)]))/(4*(b*c - a*d)^2*g^6*(a + b*x)^4) - (b*i^3*(c + d*
x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*(b*c - a*d)^2*g^6*(a + b*x)^5
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
```

```
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{i^3\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex))}{x^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^6}$$

$$= \frac{di^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^2g^6(a+bx)^4} - \frac{bi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5(bc-ad)^2g^6(a+bx)^5} - \frac{(Bi^3)\,\text{Subst}\left(\int \frac{-4b+5dx}{20x^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^6}$$

$$= \frac{di^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^2g^6(a+bx)^4} - \frac{bi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5(bc-ad)^2g^6(a+bx)^5} - \frac{(Bi^3)\,\text{Subst}\left(\int \frac{-4b+5dx}{x^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{20(bc-ad)^2g^6}$$

$$= \frac{di^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^2g^6(a+bx)^4} - \frac{bi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5(bc-ad)^2g^6(a+bx)^5} - \frac{(Bi^3)\,\text{Subst}\left(\int \left(-\frac{4b}{x^6}+\frac{5d}{x^5}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{20(bc-ad)^2g^6}$$

$$= \frac{Bdi^3(c+dx)^4}{16(bc-ad)^2g^6(a+bx)^4} - \frac{bBi^3(c+dx)^5}{25(bc-ad)^2g^6(a+bx)^5} + \frac{di^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^2g^6(a+bx)^4} - \frac{bi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5(bc-ad)^2g^6(a+bx)^5}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 608 vs. $2(181) = 362$.

Time = 0.35 (sec) , antiderivative size = 608, normalized size of antiderivative = 3.36

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx =$$

$$-\frac{i^3\left(80Ab^5c^5+16b^5Bc^5-100aAb^4c^4d-25ab^4Bc^4d+20a^5Ad^5+9a^5Bd^5+300Ab^5c^4dx+55b^5Bc^4dx-\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g
*x)^6,x]
```

```
[Out] -1/400*(i^3*(80*A*b^5*c^5 + 16*b^5*B*c^5 - 100*a*A*b^4*c^4*d - 25*a*b^4*B*c
^4*d + 20*a^5*A*d^5 + 9*a^5*B*d^5 + 300*A*b^5*c^4*d*x + 55*b^5*B*c^4*d*x -
400*a*A*b^4*c^3*d^2*x - 100*a*b^4*B*c^3*d^2*x + 100*a^4*A*b*d^5*x + 45*a^4*
b*B*d^5*x + 400*A*b^5*c^3*d^2*x^2 + 60*b^5*B*c^3*d^2*x^2 - 600*a*A*b^4*c^2*
d^3*x^2 - 150*a*b^4*B*c^2*d^3*x^2 + 200*a^3*A*b^2*d^5*x^2 + 90*a^3*b^2*B*d^
5*x^2 + 200*A*b^5*c^2*d^3*x^3 + 10*b^5*B*c^2*d^3*x^3 - 400*a*A*b^4*c*d^4*x^
3 - 100*a*b^4*B*c*d^4*x^3 + 200*a^2*A*b^3*d^5*x^3 + 90*a^2*b^3*B*d^5*x^3 -
20*b^5*B*c*d^4*x^4 + 20*a*b^4*B*d^5*x^4 - 20*B*d^5*(a + b*x)^5*Log[a + b*x]
 + 20*B*(b*c - a*d)^2*(a^3*d^3 + a^2*b*d^2*(2*c + 5*d*x) + a*b^2*d*(3*c^2 +
 10*c*d*x + 10*d^2*x^2) + b^3*(4*c^3 + 15*c^2*d*x + 20*c*d^2*x^2 + 10*d^3*x
^3))*Log[(e*(a + b*x))/(c + d*x)] + 20*a^5*B*d^5*Log[c + d*x] + 100*a^4*b*B
*d^5*x*Log[c + d*x] + 200*a^3*b^2*B*d^5*x^2*Log[c + d*x] + 200*a^2*b^3*B*d^
5*x^3*Log[c + d*x] + 100*a*b^4*B*d^5*x^4*Log[c + d*x] + 20*b^5*B*d^5*x^5*Lo
g[c + d*x]))/(b^4*(b*c - a*d)^2*g^6*(a + b*x)^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(173) = 346$.

Time = 1.60 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.97

method	result
derivativedivides	$-\frac{e(ad-cb)\left(\frac{i^3d^2e^4Ab}{5(ad-cb)^3g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}-\frac{i^3d^3e^3A}{4(ad-cb)^3g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{i^3d^2e^4Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}-\frac{1}{25\left(\frac{be}{d}+\frac{(a}{d(}\cdots\right.}\cdots\right.}{(ad-cb)^3g^6}\cdots\right.}{d^2}$
default	$-\frac{e(ad-cb)\left(\frac{i^3d^2e^4Ab}{5(ad-cb)^3g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}-\frac{i^3d^3e^3A}{4(ad-cb)^3g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{i^3d^2e^4Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}-\frac{1}{25\left(\frac{be}{d}+\frac{(a}{d(}\cdots\right.}\cdots\right.}{(ad-cb)^3g^6}\cdots\right.}{d^2}$
parts	$\frac{i^3A\left(\frac{d^2(ad-cb)}{b^4(bx+a)^3}-\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)}{4b^4(bx+a)^4}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{5b^4(bx+a)^5}-\frac{d^3}{2b^4(bx+a)^2}\right)}{g^6}-\frac{i^3B(ad-cb)^4e^4\left(d^6\left(-\frac{\ln\left(\cdots\right)}{4\left(\cdots\right)}\cdots\right.\right.}{\cdots}$
risch	$-\frac{i^3B\left(10d^3x^3b^3+10a\,b^2d^3x^2+20b^3c\,d^2x^2+5a^2b\,d^3x+10a\,b^2c\,d^2x+15b^3c^2dx+a^3d^3+2a^2bc\,d^2+3a\,b^2c^2d+4b^3c^3\right)\ln\left(\frac{e(b\cdots}{d\cdots}\right)}{20(bx+a)^5g^6b^4}\cdots$
parallelrisch	$-\frac{-150B\,x^2a\,b^7c^2d^4i^3+300Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^8c^4d^2i^3-400Axa\,b^7c^3d^3i^3-100Bxa\,b^7c^3d^3i^3-100B\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^7c^4d\cdots}{\cdots}$
norman	$\frac{\frac{\left(20Aa\,c^3d\,i^3-20Ab\,c^4i^3+5Ba\,c^3d\,i^3-4Bb\,c^4i^3\right)x}{20ga(ad-cb)}+\frac{\left(60A\,a^2c^2d^2i^3+20Aab\,c^3d\,i^3-80A\,b^2c^4i^3+15B\,a^2c^2d^2i^3+9Bab\,c^3d\,i^3-16B\,b^2\cdots\right.}{40g\,a^2(ad-cb)}\cdots}{\cdots}$

```
[In] int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x,method=_RETUR
NVERBOSE)
[Out] -1/d^2*e*(a*d-b*c)*(1/5*i^3*d^2*e^4/(a*d-b*c)^3/g^6*A*b/(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^5-1/4*i^3*d^3*e^3/(a*d-b*c)^3/g^6*A/(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^4-i^3*d^2*e^4/(a*d-b*c)^3/g^6*B*b*(-1/5/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/25/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5)+i^3*d^
3*e^3/(a*d-b*c)^3/g^6*B*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d
-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(173) = 346$.

Time = 0.35 (sec) , antiderivative size = 644, normalized size of antiderivative = 3.56

$$\int\frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = \frac{20\left(Bb^5cd^4-Bab^4d^5\right)i^3x^4-10\left((20A+B)b^5c^2d^3-10(4A+B)ab^4cd^4+(20A+9B)a^2b^3d^5\right)i^3x^3-10\,(\cdots}{\cdots}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x, algor
ithm="fricas")
```

```
[Out] 1/400*(20*(B*b^5*c*d^4 - B*a*b^4*d^5)*i^3*x^4 - 10*((20*A + B)*b^5*c^2*d^3
- 10*(4*A + B)*a*b^4*c*d^4 + (20*A + 9*B)*a^2*b^3*d^5)*i^3*x^3 - 10*(2*(20*
A + 3*B)*b^5*c^3*d^2 - 15*(4*A + B)*a*b^4*c^2*d^3 + (20*A + 9*B)*a^3*b^2*d^
5)*i^3*x^2 - 5*((60*A + 11*B)*b^5*c^4*d - 20*(4*A + B)*a*b^4*c^3*d^2 + (20*
A + 9*B)*a^4*b*d^5)*i^3*x - (16*(5*A + B)*b^5*c^5 - 25*(4*A + B)*a*b^4*c^4*
d + (20*A + 9*B)*a^5*d^5)*i^3 + 20*(B*b^5*d^5*i^3*x^5 + 5*B*a*b^4*d^5*i^3*x
^4 - 10*(B*b^5*c^2*d^3 - 2*B*a*b^4*c*d^4)*i^3*x^3 - 10*(2*B*b^5*c^3*d^2 - 3
*B*a*b^4*c^2*d^3)*i^3*x^2 - 5*(3*B*b^5*c^4*d - 4*B*a*b^4*c^3*d^2)*i^3*x - (
4*B*b^5*c^5 - 5*B*a*b^4*c^4*d)*i^3)*log((b*e*x + a*e)/(d*x + c)))/((b^11*c^
2 - 2*a*b^10*c*d + a^2*b^9*d^2)*g^6*x^5 + 5*(a*b^10*c^2 - 2*a^2*b^9*c*d + a
^3*b^8*d^2)*g^6*x^4 + 10*(a^2*b^9*c^2 - 2*a^3*b^8*c*d + a^4*b^7*d^2)*g^6*x^
3 + 10*(a^3*b^8*c^2 - 2*a^4*b^7*c*d + a^5*b^6*d^2)*g^6*x^2 + 5*(a^4*b^7*c^2
 - 2*a^5*b^6*c*d + a^6*b^5*d^2)*g^6*x + (a^5*b^6*c^2 - 2*a^6*b^5*c*d + a^7*
b^4*d^2)*g^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**6,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4218 vs. $2(173) = 346$.

Time = 0.45 (sec) , antiderivative size = 4218, normalized size of antiderivative = 23.30

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x, algor
ithm="maxima")
```

```
[Out] -1/1200*B*d^3*i^3*(60*(10*b^3*x^3 + 10*a*b^2*x^2 + 5*a^2*b*x + a^3)*log(b*e
*x/(d*x + c) + a*e/(d*x + c))/(b^9*g^6*x^5 + 5*a*b^8*g^6*x^4 + 10*a^2*b^7*g
^6*x^3 + 10*a^3*b^6*g^6*x^2 + 5*a^4*b^5*g^6*x + a^5*b^4*g^6) + (77*a^3*b^4*
c^4 - 548*a^4*b^3*c^3*d + 352*a^5*b^2*c^2*d^2 - 148*a^6*b*c*d^3 + 27*a^7*d^
4 - 60*(10*b^7*c^3*d - 10*a*b^6*c^2*d^2 + 5*a^2*b^5*c*d^3 - a^3*b^4*d^4)*x^
```

4 + 30*(10*b^7*c^4 - 100*a*b^6*c^3*d + 95*a^2*b^5*c^2*d^2 - 46*a^3*b^4*c*d^
3 + 9*a^4*b^3*d^4)*x^3 + 10*(50*a*b^6*c^4 - 410*a^2*b^5*c^3*d + 337*a^3*b^4
*c^2*d^2 - 148*a^4*b^3*c*d^3 + 27*a^5*b^2*d^4)*x^2 + 5*(65*a^2*b^5*c^4 - 48
8*a^3*b^4*c^3*d + 352*a^4*b^3*c^2*d^2 - 148*a^5*b^2*c*d^3 + 27*a^6*b*d^4)*x
)/((b^13*c^4 - 4*a*b^12*c^3*d + 6*a^2*b^11*c^2*d^2 - 4*a^3*b^10*c*d^3 + a^4
*b^9*d^4)*g^6*x^5 + 5*(a*b^12*c^4 - 4*a^2*b^11*c^3*d + 6*a^3*b^10*c^2*d^2 -
4*a^4*b^9*c*d^3 + a^5*b^8*d^4)*g^6*x^4 + 10*(a^2*b^11*c^4 - 4*a^3*b^10*c^3
*d + 6*a^4*b^9*c^2*d^2 - 4*a^5*b^8*c*d^3 + a^6*b^7*d^4)*g^6*x^3 + 10*(a^3*b
^10*c^4 - 4*a^4*b^9*c^3*d + 6*a^5*b^8*c^2*d^2 - 4*a^6*b^7*c*d^3 + a^7*b^6*d
^4)*g^6*x^2 + 5*(a^4*b^9*c^4 - 4*a^5*b^8*c^3*d + 6*a^6*b^7*c^2*d^2 - 4*a^7*
b^6*c*d^3 + a^8*b^5*d^4)*g^6*x + (a^5*b^8*c^4 - 4*a^6*b^7*c^3*d + 6*a^7*b^6
*c^2*d^2 - 4*a^8*b^5*c*d^3 + a^9*b^4*d^4)*g^6) - 60*(10*b^3*c^3*d^2 - 10*a*
b^2*c^2*d^3 + 5*a^2*b*c*d^4 - a^3*d^5)*log(b*x + a)/((b^9*c^5 - 5*a*b^8*c^4
*d + 10*a^2*b^7*c^3*d^2 - 10*a^3*b^6*c^2*d^3 + 5*a^4*b^5*c*d^4 - a^5*b^4*d^
5)*g^6) + 60*(10*b^3*c^3*d^2 - 10*a*b^2*c^2*d^3 + 5*a^2*b*c*d^4 - a^3*d^5)*
log(d*x + c)/((b^9*c^5 - 5*a*b^8*c^4*d + 10*a^2*b^7*c^3*d^2 - 10*a^3*b^6*c^
2*d^3 + 5*a^4*b^5*c*d^4 - a^5*b^4*d^5)*g^6)) - 1/600*B*c*d^2*i^3*(60*(10*b^
2*x^2 + 5*a*b*x + a^2)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^8*g^6*x^5 +
5*a*b^7*g^6*x^4 + 10*a^2*b^6*g^6*x^3 + 10*a^3*b^5*g^6*x^2 + 5*a^4*b^4*g^6*x
+ a^5*b^3*g^6) + (47*a^2*b^4*c^4 - 278*a^3*b^3*c^3*d + 822*a^4*b^2*c^2*d^2
- 278*a^5*b*c*d^3 + 47*a^6*d^4 + 60*(10*b^6*c^2*d^2 - 5*a*b^5*c*d^3 + a^2*
b^4*d^4)*x^4 - 30*(10*b^6*c^3*d - 95*a*b^5*c^2*d^2 + 46*a^2*b^4*c*d^3 - 9*a
^3*b^3*d^4)*x^3 + 10*(20*b^6*c^4 - 140*a*b^5*c^3*d + 537*a^2*b^4*c^2*d^2 -
248*a^3*b^3*c*d^3 + 47*a^4*b^2*d^4)*x^2 + 5*(35*a*b^5*c^4 - 218*a^2*b^4*c^3
*d + 702*a^3*b^3*c^2*d^2 - 278*a^4*b^2*c*d^3 + 47*a^5*b*d^4)*x)/((b^12*c^4
- 4*a*b^11*c^3*d + 6*a^2*b^10*c^2*d^2 - 4*a^3*b^9*c*d^3 + a^4*b^8*d^4)*g^6*
x^5 + 5*(a*b^11*c^4 - 4*a^2*b^10*c^3*d + 6*a^3*b^9*c^2*d^2 - 4*a^4*b^8*c*d^
3 + a^5*b^7*d^4)*g^6*x^4 + 10*(a^2*b^10*c^4 - 4*a^3*b^9*c^3*d + 6*a^4*b^8*c
^2*d^2 - 4*a^5*b^7*c*d^3 + a^6*b^6*d^4)*g^6*x^3 + 10*(a^3*b^9*c^4 - 4*a^4*b
^8*c^3*d + 6*a^5*b^7*c^2*d^2 - 4*a^6*b^6*c*d^3 + a^7*b^5*d^4)*g^6*x^2 + 5*(
a^4*b^8*c^4 - 4*a^5*b^7*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^7*b^5*c*d^3 + a^8*b
^4*d^4)*g^6*x + (a^5*b^7*c^4 - 4*a^6*b^6*c^3*d + 6*a^7*b^5*c^2*d^2 - 4*a^8*
b^4*c*d^3 + a^9*b^3*d^4)*g^6) + 60*(10*b^2*c^2*d^3 - 5*a*b*c*d^4 + a^2*d^5)
*log(b*x + a)/((b^8*c^5 - 5*a*b^7*c^4*d + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c
^2*d^3 + 5*a^4*b^4*c*d^4 - a^5*b^3*d^5)*g^6) - 60*(10*b^2*c^2*d^3 - 5*a*b*c
*d^4 + a^2*d^5)*log(d*x + c)/((b^8*c^5 - 5*a*b^7*c^4*d + 10*a^2*b^6*c^3*d^2
- 10*a^3*b^5*c^2*d^3 + 5*a^4*b^4*c*d^4 - a^5*b^3*d^5)*g^6)) - 1/400*B*c^2*
d*i^3*(60*(5*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^7*g^6*x^5 + 5
*a*b^6*g^6*x^4 + 10*a^2*b^5*g^6*x^3 + 10*a^3*b^4*g^6*x^2 + 5*a^4*b^3*g^6*x
+ a^5*b^2*g^6) + (27*a*b^4*c^4 - 148*a^2*b^3*c^3*d + 352*a^3*b^2*c^2*d^2 -
548*a^4*b*c*d^3 + 77*a^5*d^4 - 60*(5*b^5*c*d^3 - a*b^4*d^4)*x^4 + 30*(5*b^5
*c^2*d^2 - 46*a*b^4*c*d^3 + 9*a^2*b^3*d^4)*x^3 - 10*(10*b^5*c^3*d - 67*a*b^
4*c^2*d^2 + 248*a^2*b^3*c*d^3 - 47*a^3*b^2*d^4)*x^2 + 5*(15*b^5*c^4 - 88*a*
b^4*c^3*d + 232*a^2*b^3*c^2*d^2 - 428*a^3*b^2*c*d^3 + 77*a^4*b*d^4)*x)/((b^
11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4

```
)*g^6*x^5 + 5*(a*b^10*c^4 - 4*a^2*b^9*c^3*d + 6*a^3*b^8*c^2*d^2 - 4*a^4*b^7
*c*d^3 + a^5*b^6*d^4)*g^6*x^4 + 10*(a^2*b^9*c^4 - 4*a^3*b^8*c^3*d + 6*a^4*b
^7*c^2*d^2 - 4*a^5*b^6*c*d^3 + a^6*b^5*d^4)*g^6*x^3 + 10*(a^3*b^8*c^4 - 4*a
^4*b^7*c^3*d + 6*a^5*b^6*c^2*d^2 - 4*a^6*b^5*c*d^3 + a^7*b^4*d^4)*g^6*x^2 +
 5*(a^4*b^7*c^4 - 4*a^5*b^6*c^3*d + 6*a^6*b^5*c^2*d^2 - 4*a^7*b^4*c*d^3 + a
^8*b^3*d^4)*g^6*x + (a^5*b^6*c^4 - 4*a^6*b^5*c^3*d + 6*a^7*b^4*c^2*d^2 - 4*
a^8*b^3*c*d^3 + a^9*b^2*d^4)*g^6) - 60*(5*b*c*d^4 - a*d^5)*log(b*x + a)/((b
^7*c^5 - 5*a*b^6*c^4*d + 10*a^2*b^5*c^3*d^2 - 10*a^3*b^4*c^2*d^3 + 5*a^4*b^
3*c*d^4 - a^5*b^2*d^5)*g^6) + 60*(5*b*c*d^4 - a*d^5)*log(d*x + c)/((b^7*c^5
 - 5*a*b^6*c^4*d + 10*a^2*b^5*c^3*d^2 - 10*a^3*b^4*c^2*d^3 + 5*a^4*b^3*c*d^
4 - a^5*b^2*d^5)*g^6)) - 1/300*B*c^3*i^3*((60*b^4*d^4*x^4 + 12*b^4*c^4 - 63
*a*b^3*c^3*d + 137*a^2*b^2*c^2*d^2 - 163*a^3*b*c*d^3 + 137*a^4*d^4 - 30*(b^
4*c*d^3 - 9*a*b^3*d^4)*x^3 + 10*(2*b^4*c^2*d^2 - 13*a*b^3*c*d^3 + 47*a^2*b^
2*d^4)*x^2 - 5*(3*b^4*c^3*d - 17*a*b^3*c^2*d^2 + 43*a^2*b^2*c*d^3 - 77*a^3*
b*d^4)*x)/((b^10*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3
+ a^4*b^6*d^4)*g^6*x^5 + 5*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2
 - 4*a^4*b^6*c*d^3 + a^5*b^5*d^4)*g^6*x^4 + 10*(a^2*b^8*c^4 - 4*a^3*b^7*c^3
*d + 6*a^4*b^6*c^2*d^2 - 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*g^6*x^3 + 10*(a^3*b
^7*c^4 - 4*a^4*b^6*c^3*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^
4)*g^6*x^2 + 5*(a^4*b^6*c^4 - 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b
^3*c*d^3 + a^8*b^2*d^4)*g^6*x + (a^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*
c^2*d^2 - 4*a^8*b^2*c*d^3 + a^9*b*d^4)*g^6) + 60*log(b*e*x/(d*x + c) + a*e/
(d*x + c))/(b^6*g^6*x^5 + 5*a*b^5*g^6*x^4 + 10*a^2*b^4*g^6*x^3 + 10*a^3*b^3
*g^6*x^2 + 5*a^4*b^2*g^6*x + a^5*b*g^6) + 60*d^5*log(b*x + a)/((b^6*c^5 - 5
*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 -
a^5*b*d^5)*g^6) - 60*d^5*log(d*x + c)/((b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^
4*c^3*d^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5)*g^6)) - 3/20*
(5*b*x + a)*A*c^2*d*i^3/(b^7*g^6*x^5 + 5*a*b^6*g^6*x^4 + 10*a^2*b^5*g^6*x^3
 + 10*a^3*b^4*g^6*x^2 + 5*a^4*b^3*g^6*x + a^5*b^2*g^6) - 1/10*(10*b^2*x^2 +
 5*a*b*x + a^2)*A*c*d^2*i^3/(b^8*g^6*x^5 + 5*a*b^7*g^6*x^4 + 10*a^2*b^6*g^6
*x^3 + 10*a^3*b^5*g^6*x^2 + 5*a^4*b^4*g^6*x + a^5*b^3*g^6) - 1/20*(10*b^3*x
^3 + 10*a*b^2*x^2 + 5*a^2*b*x + a^3)*A*d^3*i^3/(b^9*g^6*x^5 + 5*a*b^8*g^6*x
^4 + 10*a^2*b^7*g^6*x^3 + 10*a^3*b^6*g^6*x^2 + 5*a^4*b^5*g^6*x + a^5*b^4*g^
6) - 1/5*A*c^3*i^3/(b^6*g^6*x^5 + 5*a*b^5*g^6*x^4 + 10*a^2*b^4*g^6*x^3 + 10
*a^3*b^3*g^6*x^2 + 5*a^4*b^2*g^6*x + a^5*b*g^6)
```

Giac [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.56

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = -\frac{1}{400}\left(\frac{20\left(4\,Bbe^6i^3-\frac{5\,(bex+ae)Bde^5i^3}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^5bcg^6}{(dx+c)^5}-\frac{(bex+ae)^5adg^6}{(dx+c)^5}}+\frac{80\,Abe^6i^3+16\,Bbe^6i^3-\frac{100\,(bex+ae)Ade^5i^3}{dx+c}-\frac{25\,(bex+\cdots}{dx\cdots}}{\frac{(bex+ae)^5bcg^6}{(dx+c)^5}-\frac{(bex+ae)^5adg^6}{(dx+c)^5}}\right.$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x, algor
ithm="giac")
```

```
[Out] -1/400*(20*(4*B*b*e^6*i^3 - 5*(b*e*x + a*e)*B*d*e^5*i^3/(d*x + c))*log((b*e
*x + a*e)/(d*x + c))/((b*e*x + a*e)^5*b*c*g^6/(d*x + c)^5 - (b*e*x + a*e)^5
*a*d*g^6/(d*x + c)^5) + (80*A*b*e^6*i^3 + 16*B*b*e^6*i^3 - 100*(b*e*x + a*e
)*A*d*e^5*i^3/(d*x + c) - 25*(b*e*x + a*e)*B*d*e^5*i^3/(d*x + c))/((b*e*x +
 a*e)^5*b*c*g^6/(d*x + c)^5 - (b*e*x + a*e)^5*a*d*g^6/(d*x + c)^5))*(b*c/((
b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 4.82 (sec) , antiderivative size = 1053, normalized size of antiderivative = 5.82

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = -\frac{\frac{20\,A\,a^4\,d^4\,i^3-80\,A\,b^4\,c^4\,i^3+9\,B\,a^4\,d^4\,i^3-16\,B\,b^4\,c^4\,i^3+20\,A\,a^2\,b^2\,c^2\,d^2\,i^3+9\,B\,a^2\,b^2\,c^2\,d^2\,i^3+20\,A\,a\,b^3\,c^3\,d\,i^3+20\,A\,a^3\,b\,c\,d^3\,i^3+9\,B\,a\,b^3\,c^3\,d\,i^3\cdots}{20\,(a\,d-b\,c)}\cdots}{\cdots} - \frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(x^2\,\left(b\,\left(b\,\left(\frac{B\,a\,d^3\,i^3}{20\,b^5\,g^6}+\frac{B\,c\,d^2\,i^3}{10\,b^4\,g^6}\right)+\frac{3\,B\,a\,d^3\,i^3}{20\,b^4\,g^6}+\frac{3\,B\,c\,d^2\,i^3}{10\,b^3\,g^6}\right)+\frac{3\,B\,a\,d^3\,i^3}{10\,b^3\,g^6}+\frac{3\,B\,c\,d^2\,i^3}{5\,b^2\,g^6}\right)+x\,\left(b\,\left(a\,\left(\frac{B\,a\cdots}{20\,\cdots}\cdots}{5\,a^4\cdots} - \frac{B\,d^5\,i^3\,\mathrm{atanh}\left(\frac{20\,b^6\,c^2\,g^6-20\,a^2\,b^4\,d^2\,g^6}{20\,b^4\,g^6\,(a\,d-b\,c)^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{10\,b^4\,g^6\,(a\,d-b\,c)^2}$$

```
[In] int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^6,
x)
```

```
[Out] - ((20*A*a^4*d^4*i^3 - 80*A*b^4*c^4*i^3 + 9*B*a^4*d^4*i^3 - 16*B*b^4*c^4*i^
3 + 20*A*a^2*b^2*c^2*d^2*i^3 + 9*B*a^2*b^2*c^2*d^2*i^3 + 20*A*a*b^3*c^3*d*i
^3 + 20*A*a^3*b*c*d^3*i^3 + 9*B*a*b^3*c^3*d*i^3 + 9*B*a^3*b*c*d^3*i^3)/(20*
(a*d - b*c)) + (x^2*(20*A*a^2*b^2*d^4*i^3 + 9*B*a^2*b^2*d^4*i^3 - 40*A*b^4*
```

```
c^2*d^2*i^3 - 6*B*b^4*c^2*d^2*i^3 + 20*A*a*b^3*c*d^3*i^3 + 9*B*a*b^3*c*d^3*
i^3))/(2*(a*d - b*c)) + (x*(20*A*a^3*b*d^4*i^3 + 9*B*a^3*b*d^4*i^3 - 60*A*b
^4*c^3*d*i^3 - 11*B*b^4*c^3*d*i^3 + 20*A*a*b^3*c^2*d^2*i^3 + 20*A*a^2*b^2*c
*d^3*i^3 + 9*B*a*b^3*c^2*d^2*i^3 + 9*B*a^2*b^2*c*d^3*i^3))/(4*(a*d - b*c))
+ (x^3*(20*A*a*b^3*d^4*i^3 + 9*B*a*b^3*d^4*i^3 - 20*A*b^4*c*d^3*i^3 - B*b^4
*c*d^3*i^3))/(2*(a*d - b*c)) + (B*b^4*d^4*i^3*x^4)/(a*d - b*c))/(20*a^5*b^4
*g^6 + 20*b^9*g^6*x^5 + 100*a^4*b^5*g^6*x + 100*a*b^8*g^6*x^4 + 200*a^3*b^6
*g^6*x^2 + 200*a^2*b^7*g^6*x^3) - (log((e*(a + b*x))/(c + d*x))*(x^2*(b*(b*
((B*a*d^3*i^3)/(20*b^5*g^6) + (B*c*d^2*i^3)/(10*b^4*g^6)) + (3*B*a*d^3*i^3)
/(20*b^4*g^6) + (3*B*c*d^2*i^3)/(10*b^3*g^6)) + (3*B*a*d^3*i^3)/(10*b^3*g^6
) + (3*B*c*d^2*i^3)/(5*b^2*g^6)) + x*(b*(a*((B*a*d^3*i^3)/(20*b^5*g^6) + (B
*c*d^2*i^3)/(10*b^4*g^6)) + (3*B*c^2*d*i^3)/(20*b^3*g^6)) + a*(b*((B*a*d^3*
i^3)/(20*b^5*g^6) + (B*c*d^2*i^3)/(10*b^4*g^6)) + (3*B*a*d^3*i^3)/(20*b^4*g
^6) + (3*B*c*d^2*i^3)/(10*b^3*g^6)) + (3*B*c^2*d*i^3)/(5*b^2*g^6)) + a*(a*(
(B*a*d^3*i^3)/(20*b^5*g^6) + (B*c*d^2*i^3)/(10*b^4*g^6)) + (3*B*c^2*d*i^3)/
(20*b^3*g^6)) + (B*c^3*i^3)/(5*b^2*g^6) + (B*d^3*i^3*x^3)/(2*b^2*g^6)))/(5*
a^4*x + a^5/b + b^4*x^5 + 10*a^3*b*x^2 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3) - (B
*d^5*i^3*atanh((20*b^6*c^2*g^6 - 20*a^2*b^4*d^2*g^6)/(20*b^4*g^6*(a*d - b*c
)^2) - (2*b*d*x)/(a*d - b*c)))/(10*b^4*g^6*(a*d - b*c)^2)
```

3.30 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 281

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx = -\frac{Bd^2i^3(c+dx)^4}{16(bc-ad)^3g^7(a+bx)^4} + \frac{2bBdi^3(c+dx)^5}{25(bc-ad)^3g^7(a+bx)^5} - \frac{b^2Bi^3(c+dx)^6}{36(bc-ad)^3g^7(a+bx)^6} - \frac{d^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^3g^7(a+bx)^4} + \frac{2bdi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5(bc-ad)^3g^7(a+bx)^5} - \frac{b^2i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6(bc-ad)^3g^7(a+bx)^6}$$

```
[Out] -1/16*B*d^2*i^3*(d*x+c)^4/(-a*d+b*c)^3/g^7/(b*x+a)^4+2/25*b*B*d*i^3*(d*x+c)
^5/(-a*d+b*c)^3/g^7/(b*x+a)^5-1/36*b^2*B*i^3*(d*x+c)^6/(-a*d+b*c)^3/g^7/(b*
x+a)^6-1/4*d^2*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^7/(
b*x+a)^4+2/5*b*d*i^3*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^7
/(b*x+a)^5-1/6*b^2*i^3*(d*x+c)^6*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g
^7/(b*x+a)^6
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2562, 45, 2372, 12, 14}

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx=-\frac{b^2i^3(c+dx)^6\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6g^7(a+bx)^6(bc-ad)^3}-\frac{d^2i^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4g^7(a+bx)^4(bc-ad)^3}+\frac{2bdi^3(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5g^7(a+bx)^5(bc-ad)^3}-\frac{b^2Bi^3(c+dx)^6}{36g^7(a+bx)^6(bc-ad)^3}-\frac{Bd^2i^3(c+dx)^4}{16g^7(a+bx)^4(bc-ad)^3}+\frac{2bBdi^3(c+dx)^5}{25g^7(a+bx)^5(bc-ad)^3}$$

```
[In] Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^7,
x]
```

```
[Out] -1/16*(B*d^2*i^3*(c + d*x)^4)/((b*c - a*d)^3*g^7*(a + b*x)^4) + (2*b*B*d*i^
3*(c + d*x)^5)/(25*(b*c - a*d)^3*g^7*(a + b*x)^5) - (b^2*B*i^3*(c + d*x)^6)
/(36*(b*c - a*d)^3*g^7*(a + b*x)^6) - (d^2*i^3*(c + d*x)^4*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]))/(4*(b*c - a*d)^3*g^7*(a + b*x)^4) + (2*b*d*i^3*(c + d
*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*(b*c - a*d)^3*g^7*(a + b*x)^
5) - (b^2*i^3*(c + d*x)^6*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(6*(b*c - a
*d)^3*g^7*(a + b*x)^6)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i^3\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex))}{x^7}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^7}\\
&= -\frac{d^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^3g^7(a+bx)^4}+\frac{2bdi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5(bc-ad)^3g^7(a+bx)^5}\\
&\quad-\frac{b^2i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6(bc-ad)^3g^7(a+bx)^6}-\frac{(Bi^3)\,\text{Subst}\left(\int \frac{-10b^2+24bdx-15d^2x^2}{60x^7}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^7}\\
&= -\frac{d^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^3g^7(a+bx)^4}+\frac{2bdi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5(bc-ad)^3g^7(a+bx)^5}\\
&\quad-\frac{b^2i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6(bc-ad)^3g^7(a+bx)^6}-\frac{(Bi^3)\,\text{Subst}\left(\int \frac{-10b^2+24bdx-15d^2x^2}{x^7}\,dx,x,\frac{a+bx}{c+dx}\right)}{60(bc-ad)^3g^7}\\
&= -\frac{d^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^3g^7(a+bx)^4}+\frac{2bdi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5(bc-ad)^3g^7(a+bx)^5}\\
&\quad-\frac{b^2i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6(bc-ad)^3g^7(a+bx)^6}\\
&\quad-\frac{(Bi^3)\,\text{Subst}\left(\int \left(-\frac{10b^2}{x^7}+\frac{24bd}{x^6}-\frac{15d^2}{x^5}\right)dx,x,\frac{a+bx}{c+dx}\right)}{60(bc-ad)^3g^7}
\end{aligned}$$

$$= -\frac{Bd^2i^3(c+dx)^4}{16(bc-ad)^3g^7(a+bx)^4} + \frac{2bBdi^3(c+dx)^5}{25(bc-ad)^3g^7(a+bx)^5} - \frac{b^2Bi^3(c+dx)^6}{36(bc-ad)^3g^7(a+bx)^6} - \frac{d^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^3g^7(a+bx)^4} + \frac{2bdi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5(bc-ad)^3g^7(a+bx)^5} - \frac{b^2i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6(bc-ad)^3g^7(a+bx)^6}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 642 vs. $2(281) = 562$.

Time = 0.58 (sec) , antiderivative size = 642, normalized size of antiderivative = 2.28

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx = \frac{i^3\left(-100B(bc-ad)^6+432aBd(-bc+ad)^5-432bBd(bc-ad)^5x+540aBd^2(bc-ad)^4(a+bx)+120Bd(\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g
*x)^7,x]
```

```
[Out] (i^3*(-100*B*(b*c - a*d)^6 + 432*a*B*d*(-(b*c) + a*d)^5 - 432*b*B*d*(b*c -
a*d)^5*x + 540*a*B*d^2*(b*c - a*d)^4*(a + b*x) + 120*B*d*(b*c - a*d)^5*(a +
 b*x) + 540*b*B*d^2*(b*c - a*d)^4*x*(a + b*x) - 825*B*d^2*(b*c - a*d)^4*(a
+ b*x)^2 + 720*a*B*d^3*(-(b*c) + a*d)^3*(a + b*x)^2 - 720*b*B*d^3*(b*c - a*
d)^3*x*(a + b*x)^2 + 1080*a*B*d^4*(b*c - a*d)^2*(a + b*x)^3 + 700*B*d^3*(b*
c - a*d)^3*(a + b*x)^3 + 1080*b*B*d^4*(b*c - a*d)^2*x*(a + b*x)^3 - 1050*B*
d^4*(b*c - a*d)^2*(a + b*x)^4 + 2160*a*B*d^5*(-(b*c) + a*d)*(a + b*x)^4 - 2
160*b*B*d^5*(b*c - a*d)*x*(a + b*x)^4 + 2100*B*d^5*(b*c - a*d)*(a + b*x)^5
- 2160*a*B*d^6*(a + b*x)^5*Log[a + b*x] - 2160*b*B*d^6*x*(a + b*x)^5*Log[a
+ b*x] + 2100*B*d^6*(a + b*x)^6*Log[a + b*x] - 600*(b*c - a*d)^6*(A + B*Log
[(e*(a + b*x))/(c + d*x)]) + 2160*d*(-(b*c) + a*d)^5*(a + b*x)*(A + B*Log[(
e*(a + b*x))/(c + d*x)]) - 2700*d^2*(b*c - a*d)^4*(a + b*x)^2*(A + B*Log[(e
*(a + b*x))/(c + d*x)]) + 1200*d^3*(-(b*c) + a*d)^3*(a + b*x)^3*(A + B*Log[
(e*(a + b*x))/(c + d*x)]) + 2160*a*B*d^6*(a + b*x)^5*Log[c + d*x] + 2160*b*
B*d^6*x*(a + b*x)^5*Log[c + d*x] - 2100*B*d^6*(a + b*x)^6*Log[c + d*x]))/(3
600*b^4*(b*c - a*d)^3*g^7*(a + b*x)^6)
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.71

method	result
parts	$\frac{i^3A\left(-\frac{d^3}{3b^4(bx+a)^3}+\frac{3d^2(ad-cb)}{4b^4(bx+a)^4}-\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)}{5b^4(bx+a)^5}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{6b^4(bx+a)^6}\right)}{g^7}-\frac{i^3B(ad-cb)^4e^4\left(d^7\left(-\ldots\right.\right.}{\ldots}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{i^3d^2e^5A\,b^2}{6(ad-cb)^4g^7\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^6}+\frac{2i^3d^3e^4Ab}{5(ad-cb)^4g^7\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}-\frac{i^3d^4e^3A}{4(ad-cb)^4g^7\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}+\frac{i^3d^2e^5B\,b^2\ldots}{\ldots}\right.}{\ldots}$
default	$-\frac{e(ad-cb)\left(-\frac{i^3d^2e^5A\,b^2}{6(ad-cb)^4g^7\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^6}+\frac{2i^3d^3e^4Ab}{5(ad-cb)^4g^7\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}-\frac{i^3d^4e^3A}{4(ad-cb)^4g^7\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}+\frac{i^3d^2e^5B\,b^2\ldots}{\ldots}\right.}{\ldots}$
risch	$-\frac{i^3B\left(20d^3x^3b^3+15a\,b^2d^3x^2+45b^3c\,d^2x^2+6a^2b\,d^3x+18a\,b^2c\,d^2x+36b^3c^2dx+a^3d^3+3a^2bc\,d^2+6a\,b^2c^2d+10b^3c^3\right)\ln\left(\frac{e\ldots}{\ldots}\right.}{60(bx+a)^6g^7b^4}$
parallelrisch	Expression too large to display
norman	Expression too large to display

```
[In] int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^7,x,method=_RETUR
NVERBOSE)
```

```
[Out] i^3*A/g^7*(-1/3*d^3/b^4/(b*x+a)^3+3/4*d^2*(a*d-b*c)/b^4/(b*x+a)^4-3/5*d*(a^
2*d^2-2*a*b*c*d+b^2*c^2)/b^4/(b*x+a)^5-1/6*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*
c^2*d+b^3*c^3)/b^4/(b*x+a)^6)-i^3*B/g^7/d^5*(a*d-b*c)^4*e^4*(d^7/(a*d-b*c)^
7*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/1
6/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4)-2*d^6/(a*d-b*c)^7*b*e*(-1/5/(b*e/d+(a*d-
b*c)*e/d/(d*x+c))^5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/25/(b*e/d+(a*d-b*c)*e
/d/(d*x+c))^5)+d^5/(a*d-b*c)^7*e^2*b^2*(-1/6/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^
6*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/36/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^6))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 991 vs. 2(269) = 538.

Time = 0.39 (sec) , antiderivative size = 991, normalized size of antiderivative = 3.53

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx = -\frac{60\,(Bb^6cd^5-Bab^5d^6)i^3x^5-30\,(Bb^6c^2d^4-12\,Bab^5cd^5+11\,Ba^2b^4d^6)i^3x^4+20\,((60\,A+B)b^6c^3d^3-9\,(2}{}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^7,x, algor
ithm="fricas")
```

```
[Out] -1/3600*(60*(B*b^6*c*d^5 - B*a*b^5*d^6)*i^3*x^5 - 30*(B*b^6*c^2*d^4 - 12*B*
a*b^5*c*d^5 + 11*B*a^2*b^4*d^6)*i^3*x^4 + 20*((60*A + B)*b^6*c^3*d^3 - 9*(2
0*A + B)*a*b^5*c^2*d^4 + 45*(4*A + B)*a^2*b^4*c*d^5 - (60*A + 37*B)*a^3*b^3
*d^6)*i^3*x^3 + 15*((180*A + 19*B)*b^6*c^4*d^2 - 24*(20*A + 3*B)*a*b^5*c^3*
d^3 + 90*(4*A + B)*a^2*b^4*c^2*d^4 - (60*A + 37*B)*a^4*b^2*d^6)*i^3*x^2 + 6
*(4*(90*A + 13*B)*b^6*c^5*d - 15*(60*A + 11*B)*a*b^5*c^4*d^2 + 150*(4*A + B
)*a^2*b^4*c^3*d^3 - (60*A + 37*B)*a^5*b*d^6)*i^3*x + (100*(6*A + B)*b^6*c^6
 - 288*(5*A + B)*a*b^5*c^5*d + 225*(4*A + B)*a^2*b^4*c^4*d^2 - (60*A + 37*B
)*a^6*d^6)*i^3 + 60*(B*b^6*d^6*i^3*x^6 + 6*B*a*b^5*d^6*i^3*x^5 + 15*B*a^2*b
^4*d^6*i^3*x^4 + 20*(B*b^6*c^3*d^3 - 3*B*a*b^5*c^2*d^4 + 3*B*a^2*b^4*c*d^5)
*i^3*x^3 + 15*(3*B*b^6*c^4*d^2 - 8*B*a*b^5*c^3*d^3 + 6*B*a^2*b^4*c^2*d^4)*i
^3*x^2 + 6*(6*B*b^6*c^5*d - 15*B*a*b^5*c^4*d^2 + 10*B*a^2*b^4*c^3*d^3)*i^3*
x + (10*B*b^6*c^6 - 24*B*a*b^5*c^5*d + 15*B*a^2*b^4*c^4*d^2)*i^3)*log((b*e*
x + a*e)/(d*x + c)))/((b^13*c^3 - 3*a*b^12*c^2*d + 3*a^2*b^11*c*d^2 - a^3*b
^10*d^3)*g^7*x^6 + 6*(a*b^12*c^3 - 3*a^2*b^11*c^2*d + 3*a^3*b^10*c*d^2 - a^
4*b^9*d^3)*g^7*x^5 + 15*(a^2*b^11*c^3 - 3*a^3*b^10*c^2*d + 3*a^4*b^9*c*d^2
- a^5*b^8*d^3)*g^7*x^4 + 20*(a^3*b^10*c^3 - 3*a^4*b^9*c^2*d + 3*a^5*b^8*c*d
^2 - a^6*b^7*d^3)*g^7*x^3 + 15*(a^4*b^9*c^3 - 3*a^5*b^8*c^2*d + 3*a^6*b^7*c
*d^2 - a^7*b^6*d^3)*g^7*x^2 + 6*(a^5*b^8*c^3 - 3*a^6*b^7*c^2*d + 3*a^7*b^6*
c*d^2 - a^8*b^5*d^3)*g^7*x + (a^6*b^7*c^3 - 3*a^7*b^6*c^2*d + 3*a^8*b^5*c*d
^2 - a^9*b^4*d^3)*g^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**7,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5524 vs. $2(269) = 538$.

Time = 0.56 (sec) , antiderivative size = 5524, normalized size of antiderivative = 19.66

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^7,x, algor
ithm="maxima")
```

```
[Out] -1/3600*B*d^3*i^3*(60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)*log(b*e
*x/(d*x + c) + a*e/(d*x + c))/(b^10*g^7*x^6 + 6*a*b^9*g^7*x^5 + 15*a^2*b^8*
g^7*x^4 + 20*a^3*b^7*g^7*x^3 + 15*a^4*b^6*g^7*x^2 + 6*a^5*b^5*g^7*x + a^6*b
^4*g^7) + (57*a^3*b^5*c^5 - 405*a^4*b^4*c^4*d + 1470*a^5*b^3*c^3*d^2 - 730*
a^6*b^2*c^2*d^3 + 245*a^7*b*c*d^4 - 37*a^8*d^5 + 60*(20*b^8*c^3*d^2 - 15*a*
b^7*c^2*d^3 + 6*a^2*b^6*c*d^4 - a^3*b^5*d^5)*x^5 - 30*(20*b^8*c^4*d - 235*a
*b^7*c^3*d^2 + 171*a^2*b^6*c^2*d^3 - 67*a^3*b^5*c*d^4 + 11*a^4*b^4*d^5)*x^4
 + 20*(20*b^8*c^5 - 175*a*b^7*c^4*d + 866*a^2*b^6*c^3*d^2 - 604*a^3*b^5*c^2
*d^3 + 230*a^4*b^4*c*d^4 - 37*a^5*b^3*d^5)*x^3 + 15*(35*a*b^7*c^5 - 271*a^2
*b^6*c^4*d + 1128*a^3*b^5*c^3*d^2 - 700*a^4*b^4*c^2*d^3 + 245*a^5*b^3*c*d^4
 - 37*a^6*b^2*d^5)*x^2 + 6*(47*a^2*b^6*c^5 - 345*a^3*b^5*c^4*d + 1320*a^4*b
^4*c^3*d^2 - 730*a^5*b^3*c^2*d^3 + 245*a^6*b^2*c*d^4 - 37*a^7*b*d^5)*x)/((b
^15*c^5 - 5*a*b^14*c^4*d + 10*a^2*b^13*c^3*d^2 - 10*a^3*b^12*c^2*d^3 + 5*a^
4*b^11*c*d^4 - a^5*b^10*d^5)*g^7*x^6 + 6*(a*b^14*c^5 - 5*a^2*b^13*c^4*d + 1
0*a^3*b^12*c^3*d^2 - 10*a^4*b^11*c^2*d^3 + 5*a^5*b^10*c*d^4 - a^6*b^9*d^5)*
g^7*x^5 + 15*(a^2*b^13*c^5 - 5*a^3*b^12*c^4*d + 10*a^4*b^11*c^3*d^2 - 10*a^
5*b^10*c^2*d^3 + 5*a^6*b^9*c*d^4 - a^7*b^8*d^5)*g^7*x^4 + 20*(a^3*b^12*c^5
 - 5*a^4*b^11*c^4*d + 10*a^5*b^10*c^3*d^2 - 10*a^6*b^9*c^2*d^3 + 5*a^7*b^8*c
*d^4 - a^8*b^7*d^5)*g^7*x^3 + 15*(a^4*b^11*c^5 - 5*a^5*b^10*c^4*d + 10*a^6*
b^9*c^3*d^2 - 10*a^7*b^8*c^2*d^3 + 5*a^8*b^7*c*d^4 - a^9*b^6*d^5)*g^7*x^2 +
 6*(a^5*b^10*c^5 - 5*a^6*b^9*c^4*d + 10*a^7*b^8*c^3*d^2 - 10*a^8*b^7*c^2*d^
3 + 5*a^9*b^6*c*d^4 - a^10*b^5*d^5)*g^7*x + (a^6*b^9*c^5 - 5*a^7*b^8*c^4*d
 + 10*a^8*b^7*c^3*d^2 - 10*a^9*b^6*c^2*d^3 + 5*a^10*b^5*c*d^4 - a^11*b^4*d^5
)*g^7) + 60*(20*b^3*c^3*d^3 - 15*a*b^2*c^2*d^4 + 6*a^2*b*c*d^5 - a^3*d^6)*l
og(b*x + a)/((b^10*c^6 - 6*a*b^9*c^5*d + 15*a^2*b^8*c^4*d^2 - 20*a^3*b^7*c^
3*d^3 + 15*a^4*b^6*c^2*d^4 - 6*a^5*b^5*c*d^5 + a^6*b^4*d^6)*g^7) - 60*(20*b
^3*c^3*d^3 - 15*a*b^2*c^2*d^4 + 6*a^2*b*c*d^5 - a^3*d^6)*log(d*x + c)/((b^1
0*c^6 - 6*a*b^9*c^5*d + 15*a^2*b^8*c^4*d^2 - 20*a^3*b^7*c^3*d^3 + 15*a^4*b^
6*c^2*d^4 - 6*a^5*b^5*c*d^5 + a^6*b^4*d^6)*g^7)) - 1/1200*B*c*d^2*i^3*(60*(
15*b^2*x^2 + 6*a*b*x + a^2)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^9*g^7*x
^6 + 6*a*b^8*g^7*x^5 + 15*a^2*b^7*g^7*x^4 + 20*a^3*b^6*g^7*x^3 + 15*a^4*b^5
*g^7*x^2 + 6*a^5*b^4*g^7*x + a^6*b^3*g^7) + (37*a^2*b^5*c^5 - 245*a^3*b^4*c
^4*d + 730*a^4*b^3*c^3*d^2 - 1470*a^5*b^2*c^2*d^3 + 405*a^6*b*c*d^4 - 57*a^
```

7*d^5 - 60*(15*b^7*c^2*d^3 - 6*a*b^6*c*d^4 + a^2*b^5*d^5)*x^5 + 30*(15*b^7*
c^3*d^2 - 171*a*b^6*c^2*d^3 + 67*a^2*b^5*c*d^4 - 11*a^3*b^4*d^5)*x^4 - 20*(
15*b^7*c^4*d - 126*a*b^6*c^3*d^2 + 604*a^2*b^5*c^2*d^3 - 230*a^3*b^4*c*d^4
+ 37*a^4*b^3*d^5)*x^3 + 15*(15*b^7*c^5 - 111*a*b^6*c^4*d + 388*a^2*b^5*c^3*
d^2 - 1000*a^3*b^4*c^2*d^3 + 365*a^4*b^3*c*d^4 - 57*a^5*b^2*d^5)*x^2 + 6*(2
7*a*b^6*c^5 - 185*a^2*b^5*c^4*d + 580*a^3*b^4*c^3*d^2 - 1270*a^4*b^3*c^2*d^
3 + 405*a^5*b^2*c*d^4 - 57*a^6*b*d^5)*x)/((b^14*c^5 - 5*a*b^13*c^4*d + 10*a
^2*b^12*c^3*d^2 - 10*a^3*b^11*c^2*d^3 + 5*a^4*b^10*c*d^4 - a^5*b^9*d^5)*g^7
x^6 + 6(a*b^13*c^5 - 5*a^2*b^12*c^4*d + 10*a^3*b^11*c^3*d^2 - 10*a^4*b^10
*c^2*d^3 + 5*a^5*b^9*c*d^4 - a^6*b^8*d^5)*g^7*x^5 + 15*(a^2*b^12*c^5 - 5*a^
3*b^11*c^4*d + 10*a^4*b^10*c^3*d^2 - 10*a^5*b^9*c^2*d^3 + 5*a^6*b^8*c*d^4 -
a^7*b^7*d^5)*g^7*x^4 + 20*(a^3*b^11*c^5 - 5*a^4*b^10*c^4*d + 10*a^5*b^9*c^
3*d^2 - 10*a^6*b^8*c^2*d^3 + 5*a^7*b^7*c*d^4 - a^8*b^6*d^5)*g^7*x^3 + 15*(a
^4*b^10*c^5 - 5*a^5*b^9*c^4*d + 10*a^6*b^8*c^3*d^2 - 10*a^7*b^7*c^2*d^3 + 5
*a^8*b^6*c*d^4 - a^9*b^5*d^5)*g^7*x^2 + 6*(a^5*b^9*c^5 - 5*a^6*b^8*c^4*d +
10*a^7*b^7*c^3*d^2 - 10*a^8*b^6*c^2*d^3 + 5*a^9*b^5*c*d^4 - a^10*b^4*d^5)*g
^7*x + (a^6*b^8*c^5 - 5*a^7*b^7*c^4*d + 10*a^8*b^6*c^3*d^2 - 10*a^9*b^5*c^2
*d^3 + 5*a^10*b^4*c*d^4 - a^11*b^3*d^5)*g^7) - 60*(15*b^2*c^2*d^4 - 6*a*b*c
*d^5 + a^2*d^6)*log(b*x + a)/((b^9*c^6 - 6*a*b^8*c^5*d + 15*a^2*b^7*c^4*d^2
- 20*a^3*b^6*c^3*d^3 + 15*a^4*b^5*c^2*d^4 - 6*a^5*b^4*c*d^5 + a^6*b^3*d^6)
g^7) + 60(15*b^2*c^2*d^4 - 6*a*b*c*d^5 + a^2*d^6)*log(d*x + c)/((b^9*c^6
- 6*a*b^8*c^5*d + 15*a^2*b^7*c^4*d^2 - 20*a^3*b^6*c^3*d^3 + 15*a^4*b^5*c^2*
d^4 - 6*a^5*b^4*c*d^5 + a^6*b^3*d^6)*g^7)) - 1/600*B*c^2*d*i^3*(60*(6*b*x +
a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^8*g^7*x^6 + 6*a*b^7*g^7*x^5 + 1
5*a^2*b^6*g^7*x^4 + 20*a^3*b^5*g^7*x^3 + 15*a^4*b^4*g^7*x^2 + 6*a^5*b^3*g^7
*x + a^6*b^2*g^7) + (22*a*b^5*c^5 - 140*a^2*b^4*c^4*d + 385*a^3*b^3*c^3*d^2
- 615*a^4*b^2*c^2*d^3 + 735*a^5*b*c*d^4 - 87*a^6*d^5 + 60*(6*b^6*c*d^4 - a
*b^5*d^5)*x^5 - 30*(6*b^6*c^2*d^3 - 67*a*b^5*c*d^4 + 11*a^2*b^4*d^5)*x^4 +
20*(6*b^6*c^3*d^2 - 49*a*b^5*c^2*d^3 + 230*a^2*b^4*c*d^4 - 37*a^3*b^3*d^5)*
x^3 - 15*(6*b^6*c^4*d - 43*a*b^5*c^3*d^2 + 145*a^2*b^4*c^2*d^3 - 365*a^3*b^
3*c*d^4 + 57*a^4*b^2*d^5)*x^2 + 6*(12*b^6*c^5 - 80*a*b^5*c^4*d + 235*a^2*b^
4*c^3*d^2 - 415*a^3*b^3*c^2*d^3 + 585*a^4*b^2*c*d^4 - 87*a^5*b*d^5)*x)/((b^
13*c^5 - 5*a*b^12*c^4*d + 10*a^2*b^11*c^3*d^2 - 10*a^3*b^10*c^2*d^3 + 5*a^4
*b^9*c*d^4 - a^5*b^8*d^5)*g^7*x^6 + 6*(a*b^12*c^5 - 5*a^2*b^11*c^4*d + 10*a
^3*b^10*c^3*d^2 - 10*a^4*b^9*c^2*d^3 + 5*a^5*b^8*c*d^4 - a^6*b^7*d^5)*g^7*x
^5 + 15*(a^2*b^11*c^5 - 5*a^3*b^10*c^4*d + 10*a^4*b^9*c^3*d^2 - 10*a^5*b^8*
c^2*d^3 + 5*a^6*b^7*c*d^4 - a^7*b^6*d^5)*g^7*x^4 + 20*(a^3*b^10*c^5 - 5*a^4
*b^9*c^4*d + 10*a^5*b^8*c^3*d^2 - 10*a^6*b^7*c^2*d^3 + 5*a^7*b^6*c*d^4 - a^
8*b^5*d^5)*g^7*x^3 + 15*(a^4*b^9*c^5 - 5*a^5*b^8*c^4*d + 10*a^6*b^7*c^3*d^2
- 10*a^7*b^6*c^2*d^3 + 5*a^8*b^5*c*d^4 - a^9*b^4*d^5)*g^7*x^2 + 6*(a^5*b^8
*c^5 - 5*a^6*b^7*c^4*d + 10*a^7*b^6*c^3*d^2 - 10*a^8*b^5*c^2*d^3 + 5*a^9*b^
4*c*d^4 - a^10*b^3*d^5)*g^7*x + (a^6*b^7*c^5 - 5*a^7*b^6*c^4*d + 10*a^8*b^5
*c^3*d^2 - 10*a^9*b^4*c^2*d^3 + 5*a^10*b^3*c*d^4 - a^11*b^2*d^5)*g^7) + 60*
(6*b*c*d^5 - a*d^6)*log(b*x + a)/((b^8*c^6 - 6*a*b^7*c^5*d + 15*a^2*b^6*c^4
*d^2 - 20*a^3*b^5*c^3*d^3 + 15*a^4*b^4*c^2*d^4 - 6*a^5*b^3*c*d^5 + a^6*b^2*

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d^6)*g^7) - 60*(6*b*c*d^5 - a*d^6)*log(d*x + c)/((b^8*c^6 - 6*a*b^7*c^5*d +
 15*a^2*b^6*c^4*d^2 - 20*a^3*b^5*c^3*d^3 + 15*a^4*b^4*c^2*d^4 - 6*a^5*b^3*c
*d^5 + a^6*b^2*d^6)*g^7)) + 1/360*B*c^3*i^3*((60*b^5*d^5*x^5 - 10*b^5*c^5 +
 62*a*b^4*c^4*d - 163*a^2*b^3*c^3*d^2 + 237*a^3*b^2*c^2*d^3 - 213*a^4*b*c*d
^4 + 147*a^5*d^5 - 30*(b^5*c*d^4 - 11*a*b^4*d^5)*x^4 + 20*(b^5*c^2*d^3 - 8*
a*b^4*c*d^4 + 37*a^2*b^3*d^5)*x^3 - 15*(b^5*c^3*d^2 - 7*a*b^4*c^2*d^3 + 23*
a^2*b^3*c*d^4 - 57*a^3*b^2*d^5)*x^2 + 6*(2*b^5*c^4*d - 13*a*b^4*c^3*d^2 + 3
7*a^2*b^3*c^2*d^3 - 63*a^3*b^2*c*d^4 + 87*a^4*b*d^5)*x)/((b^12*c^5 - 5*a*b^
11*c^4*d + 10*a^2*b^10*c^3*d^2 - 10*a^3*b^9*c^2*d^3 + 5*a^4*b^8*c*d^4 - a^5
*b^7*d^5)*g^7*x^6 + 6*(a*b^11*c^5 - 5*a^2*b^10*c^4*d + 10*a^3*b^9*c^3*d^2 -
 10*a^4*b^8*c^2*d^3 + 5*a^5*b^7*c*d^4 - a^6*b^6*d^5)*g^7*x^5 + 15*(a^2*b^10
*c^5 - 5*a^3*b^9*c^4*d + 10*a^4*b^8*c^3*d^2 - 10*a^5*b^7*c^2*d^3 + 5*a^6*b^
6*c*d^4 - a^7*b^5*d^5)*g^7*x^4 + 20*(a^3*b^9*c^5 - 5*a^4*b^8*c^4*d + 10*a^5
*b^7*c^3*d^2 - 10*a^6*b^6*c^2*d^3 + 5*a^7*b^5*c*d^4 - a^8*b^4*d^5)*g^7*x^3
+ 15*(a^4*b^8*c^5 - 5*a^5*b^7*c^4*d + 10*a^6*b^6*c^3*d^2 - 10*a^7*b^5*c^2*d
^3 + 5*a^8*b^4*c*d^4 - a^9*b^3*d^5)*g^7*x^2 + 6*(a^5*b^7*c^5 - 5*a^6*b^6*c^
4*d + 10*a^7*b^5*c^3*d^2 - 10*a^8*b^4*c^2*d^3 + 5*a^9*b^3*c*d^4 - a^10*b^2*
d^5)*g^7*x + (a^6*b^6*c^5 - 5*a^7*b^5*c^4*d + 10*a^8*b^4*c^3*d^2 - 10*a^9*b
^3*c^2*d^3 + 5*a^10*b^2*c*d^4 - a^11*b*d^5)*g^7) - 60*log(b*e*x/(d*x + c) +
 a*e/(d*x + c))/(b^7*g^7*x^6 + 6*a*b^6*g^7*x^5 + 15*a^2*b^5*g^7*x^4 + 20*a^
3*b^4*g^7*x^3 + 15*a^4*b^3*g^7*x^2 + 6*a^5*b^2*g^7*x + a^6*b*g^7) + 60*d^6*
log(b*x + a)/((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^
3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*g^7) - 60*d^6*log
(d*x + c)/((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d
^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*g^7)) - 1/10*(6*b*x
+ a)*A*c^2*d*i^3/(b^8*g^7*x^6 + 6*a*b^7*g^7*x^5 + 15*a^2*b^6*g^7*x^4 + 20*a
^3*b^5*g^7*x^3 + 15*a^4*b^4*g^7*x^2 + 6*a^5*b^3*g^7*x + a^6*b^2*g^7) - 1/20
*(15*b^2*x^2 + 6*a*b*x + a^2)*A*c*d^2*i^3/(b^9*g^7*x^6 + 6*a*b^8*g^7*x^5 +
15*a^2*b^7*g^7*x^4 + 20*a^3*b^6*g^7*x^3 + 15*a^4*b^5*g^7*x^2 + 6*a^5*b^4*g^
7*x + a^6*b^3*g^7) - 1/60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)*A*d
^3*i^3/(b^10*g^7*x^6 + 6*a*b^9*g^7*x^5 + 15*a^2*b^8*g^7*x^4 + 20*a^3*b^7*g^
7*x^3 + 15*a^4*b^6*g^7*x^2 + 6*a^5*b^5*g^7*x + a^6*b^4*g^7) - 1/6*A*c^3*i^3
/(b^7*g^7*x^6 + 6*a*b^6*g^7*x^5 + 15*a^2*b^5*g^7*x^4 + 20*a^3*b^4*g^7*x^3 +
 15*a^4*b^3*g^7*x^2 + 6*a^5*b^2*g^7*x + a^6*b*g^7)
```

Giac [A] (verification not implemented)

none

Time = 0.61 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.58

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx = -\frac{1}{3600}\left(\frac{60\left(10\,Bb^2e^7i^3-\frac{24\,(bex+ae)Bbde^6i^3}{dx+c}+\frac{15\,(bex+ae)^2Bd^2e^5i^3}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^6b^2c^2g^7}{(dx+c)^6}-\frac{2\,(bex+ae)^6abcdg^7}{(dx+c)^6}+\frac{(bex+ae)^6a^2d^2g^7}{(dx+c)^6}}+\frac{600\,Ab^2e^7i^3+100\,Bb^2e^7i^3 -}{}\right.$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^7,x, algor
ithm="giac")
```

```
[Out] -1/3600*(60*(10*B*b^2*e^7*i^3 - 24*(b*e*x + a*e)*B*b*d*e^6*i^3/(d*x + c) +
15*(b*e*x + a*e)^2*B*d^2*e^5*i^3/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/
((b*e*x + a*e)^6*b^2*c^2*g^7/(d*x + c)^6 - 2*(b*e*x + a*e)^6*a*b*c*d*g^7/(d
*x + c)^6 + (b*e*x + a*e)^6*a^2*d^2*g^7/(d*x + c)^6) + (600*A*b^2*e^7*i^3 +
 100*B*b^2*e^7*i^3 - 1440*(b*e*x + a*e)*A*b*d*e^6*i^3/(d*x + c) - 288*(b*e*
x + a*e)*B*b*d*e^6*i^3/(d*x + c) + 900*(b*e*x + a*e)^2*A*d^2*e^5*i^3/(d*x +
 c)^2 + 225*(b*e*x + a*e)^2*B*d^2*e^5*i^3/(d*x + c)^2)/((b*e*x + a*e)^6*b^2
*c^2*g^7/(d*x + c)^6 - 2*(b*e*x + a*e)^6*a*b*c*d*g^7/(d*x + c)^6 + (b*e*x +
 a*e)^6*a^2*d^2*g^7/(d*x + c)^6))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/
((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 6.10 (sec) , antiderivative size = 1396, normalized size of antiderivative = 4.97

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx$$

$$= \frac{B\,d^6\,i^3\,\mathrm{atanh}\left(\frac{60\,a^3\,b^4\,d^3\,g^7-60\,a^2\,b^5\,c\,d^2\,g^7-60\,a\,b^6\,c^2\,d\,g^7+60\,b^7\,c^3\,g^7}{60\,b^4\,g^7\,(a\,d-b\,c)^3}+\frac{2\,b\,d\,x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^3}\right)}{30\,b^4\,g^7\,(a\,d-b\,c)^3} - \frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(x^2\,\left(b\,\left(b\,\left(\frac{B\,a\,d^3\,i^3}{60\,b^5\,g^7}+\frac{B\,c\,d^2\,i^3}{20\,b^4\,g^7}\right)+\frac{B\,a\,d^3\,i^3}{15\,b^4\,g^7}+\frac{B\,c\,d^2\,i^3}{5\,b^3\,g^7}\right)+\frac{B\,a\,d^3\,i^3}{6\,b^3\,g^7}+\frac{B\,c\,d^2\,i^3}{2\,b^2\,g^7}\right)+x\,\left(b\,\left(a\,\left(\frac{B\,a\,d^3\,i^3}{60\,b^5\,g^7}+\right.\right.\right.\right.}{6\,a^5\,x+\frac{a^6}{b}+b^5\,x^6} - \frac{60\,A\,a^5\,d^5\,i^3+600\,A\,b^5\,c^5\,i^3+37\,B\,a^5\,d^5\,i^3+100\,B\,b^5\,c^5\,i^3+60\,A\,a^2\,b^3\,c^3\,d^2\,i^3+60\,A\,a^3\,b^2\,c^2\,d^3\,i^3+37\,B\,a^2\,b^3\,c^3\,d^2\,i^3+37\,B\,a^3\,b^2\,c^2\,d^3\,i^3-840\,A\,a}{60\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}$$

```
[In] int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^7,
x)
```

```
[Out] (B*d^6*i^3*atanh((60*b^7*c^3*g^7 + 60*a^3*b^4*d^3*g^7 - 60*a*b^6*c^2*d*g^7
- 60*a^2*b^5*c*d^2*g^7)/(60*b^4*g^7*(a*d - b*c)^3) + (2*b*d*x*(a^2*d^2 + b^
2*c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(30*b^4*g^7*(a*d - b*c)^3) - (log((e*(a
```

```
 + b*x))/(c + d*x))*(x^2*(b*(b*((B*a*d^3*i^3)/(60*b^5*g^7) + (B*c*d^2*i^3)/
(20*b^4*g^7)) + (B*a*d^3*i^3)/(15*b^4*g^7) + (B*c*d^2*i^3)/(5*b^3*g^7)) + (
B*a*d^3*i^3)/(6*b^3*g^7) + (B*c*d^2*i^3)/(2*b^2*g^7)) + x*(b*(a*((B*a*d^3*i
^3)/(60*b^5*g^7) + (B*c*d^2*i^3)/(20*b^4*g^7)) + (B*c^2*d*i^3)/(10*b^3*g^7)
) + a*(b*((B*a*d^3*i^3)/(60*b^5*g^7) + (B*c*d^2*i^3)/(20*b^4*g^7)) + (B*a*d
^3*i^3)/(15*b^4*g^7) + (B*c*d^2*i^3)/(5*b^3*g^7)) + (B*c^2*d*i^3)/(2*b^2*g^
7)) + a*(a*((B*a*d^3*i^3)/(60*b^5*g^7) + (B*c*d^2*i^3)/(20*b^4*g^7)) + (B*c
^2*d*i^3)/(10*b^3*g^7)) + (B*c^3*i^3)/(6*b^2*g^7) + (B*d^3*i^3*x^3)/(3*b^2*
g^7)))/(6*a^5*x + a^6/b + b^5*x^6 + 15*a^4*b*x^2 + 6*a*b^4*x^5 + 20*a^3*b^2
*x^3 + 15*a^2*b^3*x^4) - ((60*A*a^5*d^5*i^3 + 600*A*b^5*c^5*i^3 + 37*B*a^5*
d^5*i^3 + 100*B*b^5*c^5*i^3 + 60*A*a^2*b^3*c^3*d^2*i^3 + 60*A*a^3*b^2*c^2*d
^3*i^3 + 37*B*a^2*b^3*c^3*d^2*i^3 + 37*B*a^3*b^2*c^2*d^3*i^3 - 840*A*a*b^4*
c^4*d*i^3 + 60*A*a^4*b*c*d^4*i^3 - 188*B*a*b^4*c^4*d*i^3 + 37*B*a^4*b*c*d^4
*i^3)/(60*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x^2*(60*A*a^3*b^2*d^5*i^3 + 3
7*B*a^3*b^2*d^5*i^3 + 180*A*b^5*c^3*d^2*i^3 + 19*B*b^5*c^3*d^2*i^3 - 300*A*
a*b^4*c^2*d^3*i^3 + 60*A*a^2*b^3*c*d^4*i^3 - 53*B*a*b^4*c^2*d^3*i^3 + 37*B*
a^2*b^3*c*d^4*i^3))/(4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(60*A*a^4*b*d^
5*i^3 + 37*B*a^4*b*d^5*i^3 + 360*A*b^5*c^4*d*i^3 + 52*B*b^5*c^4*d*i^3 - 540
*A*a*b^4*c^3*d^2*i^3 + 60*A*a^3*b^2*c*d^4*i^3 - 113*B*a*b^4*c^3*d^2*i^3 + 3
7*B*a^3*b^2*c*d^4*i^3 + 60*A*a^2*b^3*c^2*d^3*i^3 + 37*B*a^2*b^3*c^2*d^3*i^3
))/(10*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x^3*(60*A*a^2*b^3*d^5*i^3 + 37*B
*a^2*b^3*d^5*i^3 + 60*A*b^5*c^2*d^3*i^3 + B*b^5*c^2*d^3*i^3 - 120*A*a*b^4*c
*d^4*i^3 - 8*B*a*b^4*c*d^4*i^3))/(3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (d*x
^4*(11*B*a*b^4*d^4*i^3 - B*b^5*c*d^3*i^3))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*
d)) + (B*b^5*d^5*i^3*x^5)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(60*a^6*b^4*g^7
+ 60*b^10*g^7*x^6 + 360*a^5*b^5*g^7*x + 360*a*b^9*g^7*x^5 + 900*a^4*b^6*g^7
*x^2 + 1200*a^3*b^7*g^7*x^3 + 900*a^2*b^8*g^7*x^4)
```

3.31 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 252

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3di} - \frac{(bc-ad)g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^2i} + \frac{(bc-ad)^2g^3(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^3i} + \frac{(bc-ad)^3g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^4i} + \frac{B(bc-ad)^3g^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}$$

```
[Out] 1/3*g^3*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i-1/6*(-a*d+b*c)*g^3*(b*x+a
)^2*(3*A+B+3*B*ln(e*(b*x+a)/(d*x+c)))/d^2/i+1/6*(-a*d+b*c)^2*g^3*(b*x+a)*(6
*A+5*B+6*B*ln(e*(b*x+a)/(d*x+c)))/d^3/i+1/6*(-a*d+b*c)^3*g^3*ln((-a*d+b*c)/
b/(d*x+c))*(6*A+11*B+6*B*ln(e*(b*x+a)/(d*x+c)))/d^4/i+B*(-a*d+b*c)^3*g^3*po
lylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2562, 2384, 2354, 2438}

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \frac{g^3(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A+11B\right)}{6d^4i} + \frac{g^3(a+bx)(bc-ad)^2\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A+5B\right)}{6d^3i} - \frac{g^3(a+bx)^2(bc-ad)\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{6d^2i} + \frac{g^3(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3di} + \frac{Bg^3(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}$$

```
[In] Int[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x),x]
```

```
[Out] (g^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*d*i) - ((b*c - a*
d)*g^3*(a + b*x)^2*(3*A + B + 3*B*Log[(e*(a + b*x))/(c + d*x)]))/(6*d^2*i)
+ ((b*c - a*d)^2*g^3*(a + b*x)*(6*A + 5*B + 6*B*Log[(e*(a + b*x))/(c + d*x)
]))/(6*d^3*i) + ((b*c - a*d)^3*g^3*Log[(b*c - a*d)/(b*(c + d*x))]*(6*A + 11
*B + 6*B*Log[(e*(a + b*x))/(c + d*x)]))/(6*d^4*i) + (B*(b*c - a*d)^3*g^3*Po
lyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^4*i)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)^3g^3)\,\text{Subst}\left(\int \frac{x^3(A+B\log(ex))}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{i}$$

$$= \frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3di} - \frac{((bc-ad)^3g^3)\,\text{Subst}\left(\int \frac{x^2(3A+B+3B\log(ex))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3di}$$

$$= \frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3di} - \frac{(bc-ad)g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^2i} + \frac{((bc-ad)^3g^3)\,\text{Subst}\left(\int \frac{x(3B+2(3A+B)+6B\log(ex))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{6d^2i}$$

$$= \frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3di} - \frac{(bc-ad)g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^2i} + \frac{(bc-ad)^2g^3(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^3i} - \frac{((bc-ad)^3g^3)\,\text{Subst}\left(\int \frac{9B+2(3A+B)+6B\log(ex)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{6d^3i}$$

$$= \frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3di} - \frac{(bc-ad)g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^2i} + \frac{(bc-ad)^2g^3(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^3i} + \frac{(bc-ad)^3g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^4i} - \frac{(B(bc-ad)^3g^3)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^4i}$$

$$= \frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3di} - \frac{(bc-ad)g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^2i} + \frac{(bc-ad)^2g^3(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^3i} + \frac{(bc-ad)^3g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^4i} + \frac{B(bc-ad)^3g^3\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.40

$$\int\frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \frac{g^3\left(6Abd(bc-ad)^2x+6Bd(bc-ad)^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+3d^2(-bc+ad)(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.\right.}{}$$

```
[In] Integrate[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i
*x),x]
```

```
[Out] (g^3*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b
*x))/(c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x))
/(c + d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B
*(b*c - a*d)^3*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a +
```

```
b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (-(b*c
) + a*d)*Log[c + d*x]) - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)
])*Log[i*(c + d*x)] + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d
)] - Log[i*(c + d*x)])*Log[i*(c + d*x)] + 2*PolyLog[2, (b*(c + d*x))/(b*c -
a*d)])))/(6*d^4*i)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1104 vs. 2(244) = 488.

Time = 1.65 (sec) , antiderivative size = 1105, normalized size of antiderivative = 4.38

method	result	size
derivativedivides	Expression too large to display	1105
default	Expression too large to display	1105
parts	Expression too large to display	1116
risch	Expression too large to display	3731

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x,method=_RETURNV
ERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(A*d^2*g^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/e/i*(-3/2*b^2/d^4
*e^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2+1/d^4*ln(b*e-(b*e/d+(a*d-b*c)*
e/d/(d*x+c))*d)+1/3*b^3*e^3/d^4/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^3+3*b
*e/d^4/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d))+B*d^2*g^3*(a^2*d^2-2*a*b*c*d+
b^2*c^2)/e/i*(3*b^2/d^3*e^2*(-1/2/b^2/e^2/d*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*
x+c))*d)+1/2/b/e/d/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)-1/2*ln(b*e/d+(a*d-
b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(2*b*e-(b*e/d+(a*d-b*c)*e/d
/(d*x+c))*d)/b^2/e^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2)+1/d^3*(dilog(
-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c
))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)+b^3*e^3/d^3*(1/3/b^3/e
^3/d*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)-1/3/b^2/e^2/d/(b*e-(b*e/d+(a*d
-b*c)*e/d/(d*x+c))*d)-1/6/b/e/d/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2+1/3
*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(3*e^2*b^2-3
*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d*b*e+d^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)/b
^3/e^3/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^3)+3*b*e/d^3*(1/b/e/d*ln(b*e-(
b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d
-b*c)*e/d/(d*x+c))/b/e/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d))))
```

Fricas [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \int \frac{(bgx+ag)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{dix+ci}\,dx$$

[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorit
hm="fricas")

[Out] integral((A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3 +
(B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log((b*e
*x + a*e)/(d*x + c)))/(d*i*x + c*i), x)

Sympy [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \frac{g^3\left(\int \frac{Aa^3}{c+dx}\,dx + \int \frac{Ab^3x^3}{c+dx}\,dx + \int \frac{Ba^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx + \int \frac{3Aab^2x^2}{c+dx}\,dx + \int \frac{3Aa^2bx}{c+dx}\,dx + \int \frac{Bb^3x^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\right.}{i}$$

[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x)

[Out] g**3*(Integral(A*a**3/(c + d*x), x) + Integral(A*b**3*x**3/(c + d*x), x) +
Integral(B*a**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x) + Integr
al(3*A*a*b**2*x**2/(c + d*x), x) + Integral(3*A*a**2*b*x/(c + d*x), x) + In
tegral(B*b**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x) + Int
egral(3*B*a*b**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x) +
Integral(3*B*a**2*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x))/i

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 790 vs. $2(243) = 486$.

Time = 0.26 (sec) , antiderivative size = 790, normalized size of antiderivative = 3.13

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$$
$$= 3\,Aa^2bg^3\left(\frac{x}{di}-\frac{c\log(dx+c)}{d^2i}\right)-\frac{1}{6}\,Ab^3g^3\left(\frac{6\,c^3\log(dx+c)}{d^4i}-\frac{2\,d^2x^3-3\,cdx^2+6\,c^2x}{d^3i}\right)$$
$$+\frac{3}{2}\,Aab^2g^3\left(\frac{2\,c^2\log(dx+c)}{d^3i}+\frac{dx^2-2\,cx}{d^2i}\right)+\frac{Aa^3g^3\log(dix+ci)}{di}$$
$$-\frac{(b^3c^3g^3-3\,ab^2c^2dg^3+3\,a^2bcd^2g^3-a^3d^3g^3)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{d^4i}$$
$$+\frac{(6\,a^3d^3g^3\log(e)-(6\,g^3\log(e)+11\,g^3)b^3c^3+9\,(2\,g^3\log(e)+3\,g^3)ab^2c^2d-18\,(g^3\log(e)+g^3)a^2bcd^2)B}{6\,d^4i}$$
$$+\frac{2\,Bb^3d^3g^3x^3\log(e)-((3\,g^3\log(e)+g^3)b^3cd^2-(9\,g^3\log(e)+g^3)ab^2d^3)Bx^2+3\,(b^3c^3g^3-3\,ab^2c^2dg^3+}{}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorit
hm="maxima")
```

```
[Out] 3*A*a^2*b*g^3*(x/(d*i) - c*log(d*x + c)/(d^2*i)) - 1/6*A*b^3*g^3*(6*c^3*log
(d*x + c)/(d^4*i) - (2*d^2*x^3 - 3*c*d*x^2 + 6*c^2*x)/(d^3*i)) + 3/2*A*a*b^
2*g^3*(2*c^2*log(d*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A*a^3*g^3*lo
g(d*i*x + c*i)/(d*i) - (b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3
 - a^3*d^3*g^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(
b*d*x + a*d)/(b*c - a*d)))*B/(d^4*i) + 1/6*(6*a^3*d^3*g^3*log(e) - (6*g^3*l
og(e) + 11*g^3)*b^3*c^3 + 9*(2*g^3*log(e) + 3*g^3)*a*b^2*c^2*d - 18*(g^3*lo
g(e) + g^3)*a^2*b*c*d^2)*B*log(d*x + c)/(d^4*i) + 1/6*(2*B*b^3*d^3*g^3*x^3*
log(e) - ((3*g^3*log(e) + g^3)*b^3*c*d^2 - (9*g^3*log(e) + g^3)*a*b^2*d^3)*
B*x^2 + 3*(b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 - a^3*d^3*g^
3)*B*log(d*x + c)^2 + ((6*g^3*log(e) + 5*g^3)*b^3*c^2*d - 6*(3*g^3*log(e) +
 2*g^3)*a*b^2*c*d^2 + (18*g^3*log(e) + 7*g^3)*a^2*b*d^3)*B*x + (2*B*b^3*d^3
*g^3*x^3 - 3*(b^3*c*d^2*g^3 - 3*a*b^2*d^3*g^3)*B*x^2 + 6*(b^3*c^2*d*g^3 - 3
*a*b^2*c*d^2*g^3 + 3*a^2*b*d^3*g^3)*B*x + (6*a*b^2*c^2*d*g^3 - 15*a^2*b*c*d
^2*g^3 + 11*a^3*d^3*g^3)*B)*log(b*x + a) - (2*B*b^3*d^3*g^3*x^3 - 3*(b^3*c*
d^2*g^3 - 3*a*b^2*d^3*g^3)*B*x^2 + 6*(b^3*c^2*d*g^3 - 3*a*b^2*c*d^2*g^3 + 3
*a^2*b*d^3*g^3)*B*x)*log(d*x + c))/(d^4*i)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3552 vs. $2(243) = 486$.

Time = 66.53 (sec) , antiderivative size = 3552, normalized size of antiderivative = 14.10

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorit
hm="giac")
```

```
[Out] -1/120*(6*(B*b^9*c^6*e^6*g^3 - 6*B*a*b^8*c^5*d*e^6*g^3 + 15*B*a^2*b^7*c^4*d
^2*e^6*g^3 - 20*B*a^3*b^6*c^3*d^3*e^6*g^3 + 15*B*a^4*b^5*c^2*d^4*e^6*g^3 -
6*B*a^5*b^4*c*d^5*e^6*g^3 + B*a^6*b^3*d^6*e^6*g^3 - 5*(b*e*x + a*e)*B*b^8*c
^6*d*e^5*g^3/(d*x + c) + 30*(b*e*x + a*e)*B*a*b^7*c^5*d^2*e^5*g^3/(d*x + c)
 - 75*(b*e*x + a*e)*B*a^2*b^6*c^4*d^3*e^5*g^3/(d*x + c) + 100*(b*e*x + a*e)
*B*a^3*b^5*c^3*d^4*e^5*g^3/(d*x + c) - 75*(b*e*x + a*e)*B*a^4*b^4*c^2*d^5*e
^5*g^3/(d*x + c) + 30*(b*e*x + a*e)*B*a^5*b^3*c*d^6*e^5*g^3/(d*x + c) - 5*(
b*e*x + a*e)*B*a^6*b^2*d^7*e^5*g^3/(d*x + c) + 10*(b*e*x + a*e)^2*B*b^7*c^6
*d^2*e^4*g^3/(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*a*b^6*c^5*d^3*e^4*g^3/(d*x
+ c)^2 + 150*(b*e*x + a*e)^2*B*a^2*b^5*c^4*d^4*e^4*g^3/(d*x + c)^2 - 200*(b
*e*x + a*e)^2*B*a^3*b^4*c^3*d^5*e^4*g^3/(d*x + c)^2 + 150*(b*e*x + a*e)^2*B
*a^4*b^3*c^2*d^6*e^4*g^3/(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*a^5*b^2*c*d^7*e
^4*g^3/(d*x + c)^2 + 10*(b*e*x + a*e)^2*B*a^6*b*d^8*e^4*g^3/(d*x + c)^2 - 1
0*(b*e*x + a*e)^3*B*b^6*c^6*d^3*e^3*g^3/(d*x + c)^3 + 60*(b*e*x + a*e)^3*B*
a*b^5*c^5*d^4*e^3*g^3/(d*x + c)^3 - 150*(b*e*x + a*e)^3*B*a^2*b^4*c^4*d^5*e
^3*g^3/(d*x + c)^3 + 200*(b*e*x + a*e)^3*B*a^3*b^3*c^3*d^6*e^3*g^3/(d*x + c
)^3 - 150*(b*e*x + a*e)^3*B*a^4*b^2*c^2*d^7*e^3*g^3/(d*x + c)^3 + 60*(b*e*x
 + a*e)^3*B*a^5*b*c*d^8*e^3*g^3/(d*x + c)^3 - 10*(b*e*x + a*e)^3*B*a^6*d^9*
e^3*g^3/(d*x + c)^3)*log((b*e*x + a*e)/(d*x + c))/(b^5*d^4*e^5*i - 5*(b*e*x
 + a*e)*b^4*d^5*e^4*i/(d*x + c) + 10*(b*e*x + a*e)^2*b^3*d^6*e^3*i/(d*x + c
)^2 - 10*(b*e*x + a*e)^3*b^2*d^7*e^2*i/(d*x + c)^3 + 5*(b*e*x + a*e)^4*b*d^
8*e*i/(d*x + c)^4 - (b*e*x + a*e)^5*d^9*i/(d*x + c)^5) + (6*A*b^10*c^6*e^6*
g^3 + 5*B*b^10*c^6*e^6*g^3 - 36*A*a*b^9*c^5*d*e^6*g^3 - 30*B*a*b^9*c^5*d*e^
6*g^3 + 90*A*a^2*b^8*c^4*d^2*e^6*g^3 + 75*B*a^2*b^8*c^4*d^2*e^6*g^3 - 120*A
*a^3*b^7*c^3*d^3*e^6*g^3 - 100*B*a^3*b^7*c^3*d^3*e^6*g^3 + 90*A*a^4*b^6*c^2
*d^4*e^6*g^3 + 75*B*a^4*b^6*c^2*d^4*e^6*g^3 - 36*A*a^5*b^5*c*d^5*e^6*g^3 -
30*B*a^5*b^5*c*d^5*e^6*g^3 + 6*A*a^6*b^4*d^6*e^6*g^3 + 5*B*a^6*b^4*d^6*e^6*
g^3 - 30*(b*e*x + a*e)*A*b^9*c^6*d*e^5*g^3/(d*x + c) - 19*(b*e*x + a*e)*B*b
^9*c^6*d*e^5*g^3/(d*x + c) + 180*(b*e*x + a*e)*A*a*b^8*c^5*d^2*e^5*g^3/(d*x
 + c) + 114*(b*e*x + a*e)*B*a*b^8*c^5*d^2*e^5*g^3/(d*x + c) - 450*(b*e*x +
a*e)*A*a^2*b^7*c^4*d^3*e^5*g^3/(d*x + c) - 285*(b*e*x + a*e)*B*a^2*b^7*c^4*
d^3*e^5*g^3/(d*x + c) + 600*(b*e*x + a*e)*A*a^3*b^6*c^3*d^4*e^5*g^3/(d*x +
c) + 380*(b*e*x + a*e)*B*a^3*b^6*c^3*d^4*e^5*g^3/(d*x + c) - 450*(b*e*x + a
```

*e)*A*a^4*b^5*c^2*d^5*e^5*g^3/(d*x + c) - 285*(b*e*x + a*e)*B*a^4*b^5*c^2*d
^5*e^5*g^3/(d*x + c) + 180*(b*e*x + a*e)*A*a^5*b^4*c*d^6*e^5*g^3/(d*x + c)
+ 114*(b*e*x + a*e)*B*a^5*b^4*c*d^6*e^5*g^3/(d*x + c) - 30*(b*e*x + a*e)*A*
a^6*b^3*d^7*e^5*g^3/(d*x + c) - 19*(b*e*x + a*e)*B*a^6*b^3*d^7*e^5*g^3/(d*x
 + c) + 60*(b*e*x + a*e)^2*A*b^8*c^6*d^2*e^4*g^3/(d*x + c)^2 + 23*(b*e*x +
a*e)^2*B*b^8*c^6*d^2*e^4*g^3/(d*x + c)^2 - 360*(b*e*x + a*e)^2*A*a*b^7*c^5*
d^3*e^4*g^3/(d*x + c)^2 - 138*(b*e*x + a*e)^2*B*a*b^7*c^5*d^3*e^4*g^3/(d*x
+ c)^2 + 900*(b*e*x + a*e)^2*A*a^2*b^6*c^4*d^4*e^4*g^3/(d*x + c)^2 + 345*(b
*e*x + a*e)^2*B*a^2*b^6*c^4*d^4*e^4*g^3/(d*x + c)^2 - 1200*(b*e*x + a*e)^2*
A*a^3*b^5*c^3*d^5*e^4*g^3/(d*x + c)^2 - 460*(b*e*x + a*e)^2*B*a^3*b^5*c^3*d
^5*e^4*g^3/(d*x + c)^2 + 900*(b*e*x + a*e)^2*A*a^4*b^4*c^2*d^6*e^4*g^3/(d*x
 + c)^2 + 345*(b*e*x + a*e)^2*B*a^4*b^4*c^2*d^6*e^4*g^3/(d*x + c)^2 - 360*(
b*e*x + a*e)^2*A*a^5*b^3*c*d^7*e^4*g^3/(d*x + c)^2 - 138*(b*e*x + a*e)^2*B*
a^5*b^3*c*d^7*e^4*g^3/(d*x + c)^2 + 60*(b*e*x + a*e)^2*A*a^6*b^2*d^8*e^4*g^
3/(d*x + c)^2 + 23*(b*e*x + a*e)^2*B*a^6*b^2*d^8*e^4*g^3/(d*x + c)^2 - 60*(
b*e*x + a*e)^3*A*b^7*c^6*d^3*e^3*g^3/(d*x + c)^3 - 3*(b*e*x + a*e)^3*B*b^7*
c^6*d^3*e^3*g^3/(d*x + c)^3 + 360*(b*e*x + a*e)^3*A*a*b^6*c^5*d^4*e^3*g^3/(
d*x + c)^3 + 18*(b*e*x + a*e)^3*B*a*b^6*c^5*d^4*e^3*g^3/(d*x + c)^3 - 900*(
b*e*x + a*e)^3*A*a^2*b^5*c^4*d^5*e^3*g^3/(d*x + c)^3 - 45*(b*e*x + a*e)^3*B
*a^2*b^5*c^4*d^5*e^3*g^3/(d*x + c)^3 + 1200*(b*e*x + a*e)^3*A*a^3*b^4*c^3*d
^6*e^3*g^3/(d*x + c)^3 + 60*(b*e*x + a*e)^3*B*a^3*b^4*c^3*d^6*e^3*g^3/(d*x
+ c)^3 - 900*(b*e*x + a*e)^3*A*a^4*b^3*c^2*d^7*e^3*g^3/(d*x + c)^3 - 45*(b*
e*x + a*e)^3*B*a^4*b^3*c^2*d^7*e^3*g^3/(d*x + c)^3 + 360*(b*e*x + a*e)^3*A*
a^5*b^2*c*d^8*e^3*g^3/(d*x + c)^3 + 18*(b*e*x + a*e)^3*B*a^5*b^2*c*d^8*e^3*
g^3/(d*x + c)^3 - 60*(b*e*x + a*e)^3*A*a^6*b*d^9*e^3*g^3/(d*x + c)^3 - 3*(b
*e*x + a*e)^3*B*a^6*b*d^9*e^3*g^3/(d*x + c)^3 - 6*(b*e*x + a*e)^4*B*b^6*c^6
*d^4*e^2*g^3/(d*x + c)^4 + 36*(b*e*x + a*e)^4*B*a*b^5*c^5*d^5*e^2*g^3/(d*x
+ c)^4 - 90*(b*e*x + a*e)^4*B*a^2*b^4*c^4*d^6*e^2*g^3/(d*x + c)^4 + 120*(b*
e*x + a*e)^4*B*a^3*b^3*c^3*d^7*e^2*g^3/(d*x + c)^4 - 90*(b*e*x + a*e)^4*B*a
^4*b^2*c^2*d^8*e^2*g^3/(d*x + c)^4 + 36*(b*e*x + a*e)^4*B*a^5*b*c*d^9*e^2*g
^3/(d*x + c)^4 - 6*(b*e*x + a*e)^4*B*a^6*d^10*e^2*g^3/(d*x + c)^4)/(b^6*d^4
*e^5*i - 5*(b*e*x + a*e)*b^5*d^5*e^4*i/(d*x + c) + 10*(b*e*x + a*e)^2*b^4*d
^6*e^3*i/(d*x + c)^2 - 10*(b*e*x + a*e)^3*b^3*d^7*e^2*i/(d*x + c)^3 + 5*(b*
e*x + a*e)^4*b^2*d^8*e*i/(d*x + c)^4 - (b*e*x + a*e)^5*b*d^9*i/(d*x + c)^5)
 + 6*(B*b^6*c^6*e*g^3 - 6*B*a*b^5*c^5*d*e*g^3 + 15*B*a^2*b^4*c^4*d^2*e*g^3
- 20*B*a^3*b^3*c^3*d^3*e*g^3 + 15*B*a^4*b^2*c^2*d^4*e*g^3 - 6*B*a^5*b*c*d^5
*e*g^3 + B*a^6*d^6*e*g^3)*log(-b*e + (b*e*x + a*e)*d/(d*x + c))/(b^2*d^4*i)
 - 6*(B*b^6*c^6*e*g^3 - 6*B*a*b^5*c^5*d*e*g^3 + 15*B*a^2*b^4*c^4*d^2*e*g^3
- 20*B*a^3*b^3*c^3*d^3*e*g^3 + 15*B*a^4*b^2*c^2*d^4*e*g^3 - 6*B*a^5*b*c*d^5
*e*g^3 + B*a^6*d^6*e*g^3)*log((b*e*x + a*e)/(d*x + c))/(b^2*d^4*i))*(b*c/((
b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx=\int \frac{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{c\,i+d\,i\,x}\,dx$$

```
[In] int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x),x)
[Out] int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x), x
)
```

3.32 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 198

$$\begin{aligned}\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx &= \frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di} - \frac{(bc-ad)g^2(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i} \\ &\quad - \frac{(bc-ad)^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^3i} \\ &\quad - \frac{B(bc-ad)^2g^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}\end{aligned}$$

```
[Out] 1/2*g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i-1/2*(-a*d+b*c)*g^2*(b*x+a
)*(2*A+B+2*B*ln(e*(b*x+a)/(d*x+c)))/d^2/i-1/2*(-a*d+b*c)^2*g^2*ln((-a*d+b*c
)/b/(d*x+c))*(2*A+3*B+2*B*ln(e*(b*x+a)/(d*x+c)))/d^3/i-B*(-a*d+b*c)^2*g^2*p
olylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used

= {2562, 2384, 2354, 2438}

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = -\frac{g^2(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+3B\right)}{2d^3i} - \frac{g^2(a+bx)(bc-ad)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{2d^2i} + \frac{g^2(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2di} - \frac{Bg^2(bc-ad)^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}$$

```
[In] Int[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x),x]
```

```
[Out] (g^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d*i) - ((b*c - a*
d)*g^2*(a + b*x)*(2*A + B + 2*B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d^2*i) -
((b*c - a*d)^2*g^2*Log[(b*c - a*d)/(b*(c + d*x))]*(2*A + 3*B + 2*B*Log[(e*(
a + b*x))/(c + d*x)]))/(2*d^3*i) - (B*(b*c - a*d)^2*g^2*PolyLog[2, (d*(a +
b*x))/(b*(c + d*x))])/(d^3*i)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
```

```
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left((bc-ad)^2g^2\right)\text{Subst}\left(\int \frac{x^2(A+B\log(ex))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{i}\\
&= \frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di} - \frac{\left((bc-ad)^2g^2\right)\text{Subst}\left(\int \frac{x(2A+B+2B\log(ex))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{2di}\\
&= \frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di}\\
&\quad - \frac{(bc-ad)g^2(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i}\\
&\quad + \frac{\left((bc-ad)^2g^2\right)\text{Subst}\left(\int \frac{2A+3B+2B\log(ex)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{2d^2i}\\
&= \frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di}\\
&\quad - \frac{(bc-ad)g^2(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i}\\
&\quad - \frac{(bc-ad)^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^3i}\\
&\quad + \frac{\left(B(bc-ad)^2g^2\right)\text{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i}\\
&= \frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di} - \frac{(bc-ad)g^2(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i}\\
&\quad - \frac{(bc-ad)^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^3i} - \frac{B(bc-ad)^2g^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.28

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$$
$$=\frac{g^2\left(-2Abd(bc-ad)x+2Bd(-bc+ad)(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+d^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+2B(\right.}{}$$

```
[In] Integrate[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i
*x),x]
```

```
[Out] (g^2*(-2*A*b*d*(b*c - a*d)*x + 2*B*d*(-(b*c) + a*d)*(a + b*x)*Log[(e*(a + b
*x))/(c + d*x)] + d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*
B*(b*c - a*d)^2*Log[c + d*x] - B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log[c
+ d*x]) + 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[i*(c + d
*x)] - B*(b*c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[i*(c + d
*x)])*Log[i*(c + d*x)] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(2*d^3*
i)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. 2(192) = 384.

Time = 1.77 (sec) , antiderivative size = 657, normalized size of antiderivative = 3.32

method	result
parts	$\frac{g^2 A\left(\frac{b\left(\frac{1}{2} b d x^2+2 x a d-b c x\right)}{d^2}+\frac{\left(a^2 d^2-2 a b c d+b^2 c^2\right) \ln (d x+c)}{d^3}\right)}{i}-\frac{g^2 B\left(\frac{e^2 b^2\left(a^2 d^2-2 a b c d+b^2 c^2\right)\left(-\frac{\ln\left(\left(\frac{b e}{d}+\frac{(a d-c b) e}{d(d x+c)}\right) d-b e\right)}{2 e^2 b^2 d} \ldots\right.}{\ldots}\ldots\right.}{\ldots}$
derivativedivides	$-\frac{e(a d-c b)\left(-\frac{A d^2 g^2(a d-c b)\left(-\frac{\ln\left(b e-\left(\frac{b e}{d}+\frac{(a d-c b) e}{d(d x+c)}\right) d\right)}{d^3}-\frac{2 b e}{d^3\left(b e-\left(\frac{b e}{d}+\frac{(a d-c b) e}{d(d x+c)}\right) d\right)}+\frac{b^2 e^2}{2 d^3\left(b e-\left(\frac{b e}{d}+\frac{(a d-c b) e}{d(d x+c)}\right) d\right)^2}\right)}{e i}-\ldots\right.}{\ldots}$
default	$-\frac{e(a d-c b)\left(-\frac{A d^2 g^2(a d-c b)\left(-\frac{\ln\left(b e-\left(\frac{b e}{d}+\frac{(a d-c b) e}{d(d x+c)}\right) d\right)}{d^3}-\frac{2 b e}{d^3\left(b e-\left(\frac{b e}{d}+\frac{(a d-c b) e}{d(d x+c)}\right) d\right)}+\frac{b^2 e^2}{2 d^3\left(b e-\left(\frac{b e}{d}+\frac{(a d-c b) e}{d(d x+c)}\right) d\right)^2}\right)}{e i}-\ldots\right.}{\ldots}$
risch	Expression too large to display

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x,method=_RETURNV
ERBOSE)
```

```
[Out] g^2*A/i*(b/d^2*(1/2*b*d*x^2+2*x*a*d-b*c*x)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3*
ln(d*x+c))-g^2*B/i/d*(e^2*b^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d*(-1/2/e^2/b^2/d
*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)-1/2/e/b/d/((b*e/d+(a*d-b*c)*e/d/(d
*x+c))*d-b*e)+1/2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x
+c))*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-2*b*e)/e^2/b^2/((b*e/d+(a*d-b*c)*e/d/
(d*x+c))*d-b*e)^2)+2*b*e*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d*(1/b/e/d*ln((b*e/d+(
a*d-b*c)*e/d/(d*x+c))*d-b*e)-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*
c)*e/d/(d*x+c))/b/e/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e))+1/d*(a^2*d^2-2*a
*b*c*d+b^2*c^2)*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e
```

```
/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)
)
```

Fricas [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{dix+ci}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorit
hm="fricas")
```

```
[Out] integral((A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*B*
a*b*g^2*x + B*a^2*g^2)*log((b*e*x + a*e)/(d*x + c)))/(d*i*x + c*i), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$$
$$= \frac{g^2\left(\int \frac{Aa^2}{c+dx}\,dx + \int \frac{Ab^2x^2}{c+dx}\,dx + \int \frac{Ba^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx + \int \frac{2Aabx}{c+dx}\,dx + \int \frac{Bb^2x^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx + \int \frac{2Babx\log\left(\right.}{c}\right.}{i}$$

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x)
```

```
[Out] g**2*(Integral(A*a**2/(c + d*x), x) + Integral(A*b**2*x**2/(c + d*x), x) +
Integral(B*a**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x) + Integr
al(2*A*a*b*x/(c + d*x), x) + Integral(B*b**2*x**2*log(a*e/(c + d*x) + b*e*x
/(c + d*x))/(c + d*x), x) + Integral(2*B*a*b*x*log(a*e/(c + d*x) + b*e*x/(c
 + d*x))/(c + d*x), x))/i
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(191) = 382$.

Time = 0.26 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.41

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = 2\,Aabg^2\left(\frac{x}{di}-\frac{c\log(dx+c)}{d^2i}\right)$$
$$+\frac{1}{2}\,Ab^2g^2\left(\frac{2\,c^2\log(dx+c)}{d^3i}+\frac{dx^2-2\,cx}{d^2i}\right)+\frac{Aa^2g^2\log(dix+ci)}{di}$$
$$+\frac{(b^2c^2g^2-2\,abcdg^2+a^2d^2g^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{d^3i}$$
$$+\frac{(2\,a^2d^2g^2\log(e)+(2\,g^2\log(e)+3\,g^2)b^2c^2-4\,(g^2\log(e)+g^2)abcd)B\log(dx+c)}{2\,d^3i}$$
$$+\frac{Bb^2d^2g^2x^2\log(e)-(b^2c^2g^2-2\,abcdg^2+a^2d^2g^2)B\log(dx+c)^2-((2\,g^2\log(e)+g^2)b^2cd-(4\,g^2\log(e)}{}$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorit
hm="maxima")
```

```
[Out] 2*A*a*b*g^2*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + 1/2*A*b^2*g^2*(2*c^2*log(d
*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A*a^2*g^2*log(d*i*x + c*i)/(d*
i) + (b^2*c^2*g^2 - 2*a*b*c*d*g^2 + a^2*d^2*g^2)*(log(b*x + a)*log((b*d*x +
 a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(d^3*i) + 1/2
*(2*a^2*d^2*g^2*log(e) + (2*g^2*log(e) + 3*g^2)*b^2*c^2 - 4*(g^2*log(e) + g
^2)*a*b*c*d)*B*log(d*x + c)/(d^3*i) + 1/2*(B*b^2*d^2*g^2*x^2*log(e) - (b^2*
c^2*g^2 - 2*a*b*c*d*g^2 + a^2*d^2*g^2)*B*log(d*x + c)^2 - ((2*g^2*log(e) +
g^2)*b^2*c*d - (4*g^2*log(e) + g^2)*a*b*d^2)*B*x + (B*b^2*d^2*g^2*x^2 - 2*(
b^2*c*d*g^2 - 2*a*b*d^2*g^2)*B*x - (2*a*b*c*d*g^2 - 3*a^2*d^2*g^2)*B)*log(b
*x + a) - (B*b^2*d^2*g^2*x^2 - 2*(b^2*c*d*g^2 - 2*a*b*d^2*g^2)*B*x)*log(d*x
 + c))/(d^3*i)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2364 vs. $2(191) = 382$.

Time = 55.79 (sec) , antiderivative size = 2364, normalized size of antiderivative = 11.94

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorit
hm="giac")
```

```
[Out] 1/24*(2*(B*b^7*c^5*e^5*g^2 - 5*B*a*b^6*c^4*d*e^5*g^2 + 10*B*a^2*b^5*c^3*d^2
*e^5*g^2 - 10*B*a^3*b^4*c^2*d^3*e^5*g^2 + 5*B*a^4*b^3*c*d^4*e^5*g^2 - B*a^5
*b^2*d^5*e^5*g^2 - 4*(b*e*x + a*e)*B*b^6*c^5*d*e^4*g^2/(d*x + c) + 20*(b*e*
x + a*e)*B*a*b^5*c^4*d^2*e^4*g^2/(d*x + c) - 40*(b*e*x + a*e)*B*a^2*b^4*c^3
```

```
*d^3*e^4*g^2/(d*x + c) + 40*(b*e*x + a*e)*B*a^3*b^3*c^2*d^4*e^4*g^2/(d*x +
c) - 20*(b*e*x + a*e)*B*a^4*b^2*c*d^5*e^4*g^2/(d*x + c) + 4*(b*e*x + a*e)*B
*a^5*b*d^6*e^4*g^2/(d*x + c) + 6*(b*e*x + a*e)^2*B*b^5*c^5*d^2*e^3*g^2/(d*x
 + c)^2 - 30*(b*e*x + a*e)^2*B*a*b^4*c^4*d^3*e^3*g^2/(d*x + c)^2 + 60*(b*e*
x + a*e)^2*B*a^2*b^3*c^3*d^4*e^3*g^2/(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*a^3
*b^2*c^2*d^5*e^3*g^2/(d*x + c)^2 + 30*(b*e*x + a*e)^2*B*a^4*b*c*d^6*e^3*g^2
/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a^5*d^7*e^3*g^2/(d*x + c)^2)*log((b*e*x
+ a*e)/(d*x + c))/(b^4*d^3*e^4*i - 4*(b*e*x + a*e)*b^3*d^4*e^3*i/(d*x + c)
+ 6*(b*e*x + a*e)^2*b^2*d^5*e^2*i/(d*x + c)^2 - 4*(b*e*x + a*e)^3*b*d^6*e*i
/(d*x + c)^3 + (b*e*x + a*e)^4*d^7*i/(d*x + c)^4) + (2*A*b^8*c^5*e^5*g^2 +
B*b^8*c^5*e^5*g^2 - 10*A*a*b^7*c^4*d*e^5*g^2 - 5*B*a*b^7*c^4*d*e^5*g^2 + 20
*A*a^2*b^6*c^3*d^2*e^5*g^2 + 10*B*a^2*b^6*c^3*d^2*e^5*g^2 - 20*A*a^3*b^5*c^
2*d^3*e^5*g^2 - 10*B*a^3*b^5*c^2*d^3*e^5*g^2 + 10*A*a^4*b^4*c*d^4*e^5*g^2 +
 5*B*a^4*b^4*c*d^4*e^5*g^2 - 2*A*a^5*b^3*d^5*e^5*g^2 - B*a^5*b^3*d^5*e^5*g^
2 - 8*(b*e*x + a*e)*A*b^7*c^5*d*e^4*g^2/(d*x + c) - 2*(b*e*x + a*e)*B*b^7*c
^5*d*e^4*g^2/(d*x + c) + 40*(b*e*x + a*e)*A*a*b^6*c^4*d^2*e^4*g^2/(d*x + c)
 + 10*(b*e*x + a*e)*B*a*b^6*c^4*d^2*e^4*g^2/(d*x + c) - 80*(b*e*x + a*e)*A*
a^2*b^5*c^3*d^3*e^4*g^2/(d*x + c) - 20*(b*e*x + a*e)*B*a^2*b^5*c^3*d^3*e^4*
g^2/(d*x + c) + 80*(b*e*x + a*e)*A*a^3*b^4*c^2*d^4*e^4*g^2/(d*x + c) + 20*(
b*e*x + a*e)*B*a^3*b^4*c^2*d^4*e^4*g^2/(d*x + c) - 40*(b*e*x + a*e)*A*a^4*b
^3*c*d^5*e^4*g^2/(d*x + c) - 10*(b*e*x + a*e)*B*a^4*b^3*c*d^5*e^4*g^2/(d*x
+ c) + 8*(b*e*x + a*e)*A*a^5*b^2*d^6*e^4*g^2/(d*x + c) + 2*(b*e*x + a*e)*B*
a^5*b^2*d^6*e^4*g^2/(d*x + c) + 12*(b*e*x + a*e)^2*A*b^6*c^5*d^2*e^3*g^2/(d
*x + c)^2 - (b*e*x + a*e)^2*B*b^6*c^5*d^2*e^3*g^2/(d*x + c)^2 - 60*(b*e*x +
 a*e)^2*A*a*b^5*c^4*d^3*e^3*g^2/(d*x + c)^2 + 5*(b*e*x + a*e)^2*B*a*b^5*c^4
*d^3*e^3*g^2/(d*x + c)^2 + 120*(b*e*x + a*e)^2*A*a^2*b^4*c^3*d^4*e^3*g^2/(d
*x + c)^2 - 10*(b*e*x + a*e)^2*B*a^2*b^4*c^3*d^4*e^3*g^2/(d*x + c)^2 - 120*
(b*e*x + a*e)^2*A*a^3*b^3*c^2*d^5*e^3*g^2/(d*x + c)^2 + 10*(b*e*x + a*e)^2*
B*a^3*b^3*c^2*d^5*e^3*g^2/(d*x + c)^2 + 60*(b*e*x + a*e)^2*A*a^4*b^2*c*d^6*
e^3*g^2/(d*x + c)^2 - 5*(b*e*x + a*e)^2*B*a^4*b^2*c*d^6*e^3*g^2/(d*x + c)^2
 - 12*(b*e*x + a*e)^2*A*a^5*b*d^7*e^3*g^2/(d*x + c)^2 + (b*e*x + a*e)^2*B*a
^5*b*d^7*e^3*g^2/(d*x + c)^2 + 2*(b*e*x + a*e)^3*B*b^5*c^5*d^3*e^2*g^2/(d*x
 + c)^3 - 10*(b*e*x + a*e)^3*B*a*b^4*c^4*d^4*e^2*g^2/(d*x + c)^3 + 20*(b*e*
x + a*e)^3*B*a^2*b^3*c^3*d^5*e^2*g^2/(d*x + c)^3 - 20*(b*e*x + a*e)^3*B*a^3
*b^2*c^2*d^6*e^2*g^2/(d*x + c)^3 + 10*(b*e*x + a*e)^3*B*a^4*b*c*d^7*e^2*g^2
/(d*x + c)^3 - 2*(b*e*x + a*e)^3*B*a^5*d^8*e^2*g^2/(d*x + c)^3)/(b^5*d^3*e^
4*i - 4*(b*e*x + a*e)*b^4*d^4*e^3*i/(d*x + c) + 6*(b*e*x + a*e)^2*b^3*d^5*e
^2*i/(d*x + c)^2 - 4*(b*e*x + a*e)^3*b^2*d^6*e*i/(d*x + c)^3 + (b*e*x + a*e
)^4*b*d^7*i/(d*x + c)^4) + 2*(B*b^5*c^5*e*g^2 - 5*B*a*b^4*c^4*d*e*g^2 + 10*
B*a^2*b^3*c^3*d^2*e*g^2 - 10*B*a^3*b^2*c^2*d^3*e*g^2 + 5*B*a^4*b*c*d^4*e*g^
2 - B*a^5*d^5*e*g^2)*log(-b*e + (b*e*x + a*e)*d/(d*x + c))/(b^2*d^3*i) - 2*
(B*b^5*c^5*e*g^2 - 5*B*a*b^4*c^4*d*e*g^2 + 10*B*a^2*b^3*c^3*d^2*e*g^2 - 10*
B*a^3*b^2*c^2*d^3*e*g^2 + 5*B*a^4*b*c*d^4*e*g^2 - B*a^5*d^5*e*g^2)*log((b*e
*x + a*e)/(d*x + c))/(b^2*d^3*i))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/
((b*c*e - a*d*e)*(b*c - a*d)))^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx=\int \frac{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{c\,i+d\,i\,x}\,dx$$

```
[In] int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x),x)
[Out] int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x), x
)
```

3.33 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 125

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \frac{g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di}+\frac{(bc-ad)g\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2i}+\frac{B(bc-ad)g\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}$$

```
[Out] g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i+(-a*d+b*c)*g*ln((-a*d+b*c)/b/(d*x
+c))*(A+B+B*ln(e*(b*x+a)/(d*x+c)))/d^2/i+B*(-a*d+b*c)*g*polylog(2,d*(b*x+a)
/b/(d*x+c))/d^2/i
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2562, 2384, 2354, 2438}

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \frac{g(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A+B\right)}{d^2i}+\frac{g(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{di}+\frac{Bg(bc-ad)\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}$$

```
[In] Int[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x),x]
```

```
[Out] (g*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d*i) + ((b*c - a*d)*g*L
og[(b*c - a*d)/(b*(c + d*x))]*(A + B + B*Log[(e*(a + b*x))/(c + d*x)]))/(d^
2*i) + (B*(b*c - a*d)*g*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^2*i)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)g)\text{Subst}\left(\int \frac{x(A+B\log(ex))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{i}$$

$$= \frac{g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di} - \frac{((bc-ad)g)\text{Subst}\left(\int \frac{A+B+B\log(ex)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{di}$$

$$= \frac{g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di} + \frac{(bc-ad)g\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2i} - \frac{(B(bc-ad)g)\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^2i}$$

$$= \frac{g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di} + \frac{(bc-ad)g\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2i} + \frac{B(bc-ad)g\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.30

$$\int\frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$$
$$= \frac{g\left(2Abdx+2Bd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-2B(bc-ad)\log(c+dx)-2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\right.}{2d^2i}$$

```
[In] Integrate[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x
),x]
```

```
[Out] (g*(2*A*b*d*x + 2*B*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 2*B*(b*c - a
*d)*Log[c + d*x] - 2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c
 + d*x] + B*(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x
])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(2*d^2*i)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(125) = 250.

Time = 1.34 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.66

method	result
parts	$\frac{gA\left(\frac{bx}{d}+\frac{(ad-cb)\ln(dx+c)}{d^2}\right)}{i}-\frac{gB\left(be(ad-cb)\left(\frac{\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}{bed}-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}\right)+(ad-cb\right.}{id}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{gAb}{i\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{gA\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{ei}+\frac{gB\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{ei}+\frac{gdB\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right.}{ei\left(be-\left(\frac{be}{d}+\right.\right.}\right.}{d^2}$
default	$-\frac{e(ad-cb)\left(\frac{gAb}{i\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{gA\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{ei}+\frac{gB\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{ei}+\frac{gdB\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right.}{ei\left(be-\left(\frac{be}{d}+\right.\right.}\right.}{d^2}$
risch	$\frac{gAbx}{id}+\frac{gA\ln(dx+c)a}{id}-\frac{gA\ln(dx+c)cb}{i\,d^2}-\frac{gB\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)a}{id}+\frac{gBb\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)c}{i\,d^2}+$

[In] int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x,method=_RETURNVERBOSE)

[Out] g*A/i*(b*x/d+(a*d-b*c)/d^2*ln(d*x+c))-g*B/i/d*(b*e*(a*d-b*c)*(1/b/e/d*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e))+(a*d-b*c)*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d))

Fricas [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx=\int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{dix+ci}\,dx$$

[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorithm="fricas")

[Out] integral((A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((b*e*x + a*e)/(d*x + c)))/(d*i*x + c*i), x)

Sympy [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \frac{g\left(\int \frac{Aa}{c+dx}\,dx+\int \frac{Abx}{c+dx}\,dx+\int \frac{Ba\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx+\int \frac{Bbx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx\right)}{i}$$

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x)
```

```
[Out] g*(Integral(A*a/(c + d*x), x) + Integral(A*b*x/(c + d*x), x) + Integral(B*a
*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x) + Integral(B*b*x*log(a*
e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x))/i
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.77

$$\begin{aligned}\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx &= Abg\left(\frac{x}{di}-\frac{c\log(dx+c)}{d^2i}\right) \\ &+ \frac{Aag\log(dix+ci)}{di}-\frac{(bcg-adg)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{d^2i} \\ &+ \frac{(adg\log(e)-(g\log(e)+g)bc)B\log(dx+c)}{d^2i} \\ &- \frac{2\,Bbdgx\log(dx+c)-2\,Bbdgx\log(e)-(bcg-adg)B\log(dx+c)^2-2\,(Bbdgx+Badg)\log(bx+a)}{2\,d^2i}\end{aligned}$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorithm
="maxima")
```

```
[Out] A*b*g*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + A*a*g*log(d*i*x + c*i)/(d*i) - (
b*c*g - a*d*g)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b
*d*x + a*d)/(b*c - a*d)))*B/(d^2*i) + (a*d*g*log(e) - (g*log(e) + g)*b*c)*B
*log(d*x + c)/(d^2*i) - 1/2*(2*B*b*d*g*x*log(d*x + c) - 2*B*b*d*g*x*log(e)
- (b*c*g - a*d*g)*B*log(d*x + c)^2 - 2*(B*b*d*g*x + B*a*d*g)*log(b*x + a))/
(d^2*i)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1230 vs. $2(124) = 248$.

Time = 42.82 (sec) , antiderivative size = 1230, normalized size of antiderivative = 9.84

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorithm
="giac")
```

```
[Out] -1/6*((B*b^5*c^4*e^4*g - 4*B*a*b^4*c^3*d*e^4*g + 6*B*a^2*b^3*c^2*d^2*e^4*g
- 4*B*a^3*b^2*c*d^3*e^4*g + B*a^4*b*d^4*e^4*g - 3*(b*e*x + a*e)*B*b^4*c^4*d
*e^3*g/(d*x + c) + 12*(b*e*x + a*e)*B*a*b^3*c^3*d^2*e^3*g/(d*x + c) - 18*(b
*e*x + a*e)*B*a^2*b^2*c^2*d^3*e^3*g/(d*x + c) + 12*(b*e*x + a*e)*B*a^3*b*c*
d^4*e^3*g/(d*x + c) - 3*(b*e*x + a*e)*B*a^4*d^5*e^3*g/(d*x + c))*log((b*e*x
 + a*e)/(d*x + c))/(b^3*d^2*e^3*i - 3*(b*e*x + a*e)*b^2*d^3*e^2*i/(d*x + c)
 + 3*(b*e*x + a*e)^2*b*d^4*e*i/(d*x + c)^2 - (b*e*x + a*e)^3*d^5*i/(d*x + c
)^3) + (A*b^6*c^4*e^4*g - 4*A*a*b^5*c^3*d*e^4*g + 6*A*a^2*b^4*c^2*d^2*e^4*g
 - 4*A*a^3*b^3*c*d^3*e^4*g + A*a^4*b^2*d^4*e^4*g - 3*(b*e*x + a*e)*A*b^5*c^
4*d*e^3*g/(d*x + c) + (b*e*x + a*e)*B*b^5*c^4*d*e^3*g/(d*x + c) + 12*(b*e*x
 + a*e)*A*a*b^4*c^3*d^2*e^3*g/(d*x + c) - 4*(b*e*x + a*e)*B*a*b^4*c^3*d^2*e
^3*g/(d*x + c) - 18*(b*e*x + a*e)*A*a^2*b^3*c^2*d^3*e^3*g/(d*x + c) + 6*(b*
e*x + a*e)*B*a^2*b^3*c^2*d^3*e^3*g/(d*x + c) + 12*(b*e*x + a*e)*A*a^3*b^2*c
*d^4*e^3*g/(d*x + c) - 4*(b*e*x + a*e)*B*a^3*b^2*c*d^4*e^3*g/(d*x + c) - 3*
(b*e*x + a*e)*A*a^4*b*d^5*e^3*g/(d*x + c) + (b*e*x + a*e)*B*a^4*b*d^5*e^3*g
/(d*x + c) - (b*e*x + a*e)^2*B*b^4*c^4*d^2*e^2*g/(d*x + c)^2 + 4*(b*e*x + a
*e)^2*B*a*b^3*c^3*d^3*e^2*g/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a^2*b^2*c^2*d
^4*e^2*g/(d*x + c)^2 + 4*(b*e*x + a*e)^2*B*a^3*b*c*d^5*e^2*g/(d*x + c)^2 -
(b*e*x + a*e)^2*B*a^4*d^6*e^2*g/(d*x + c)^2)/(b^4*d^2*e^3*i - 3*(b*e*x + a*
e)*b^3*d^3*e^2*i/(d*x + c) + 3*(b*e*x + a*e)^2*b^2*d^4*e*i/(d*x + c)^2 - (b
*e*x + a*e)^3*b*d^5*i/(d*x + c)^3) + (B*b^4*c^4*e*g - 4*B*a*b^3*c^3*d*e*g +
 6*B*a^2*b^2*c^2*d^2*e*g - 4*B*a^3*b*c*d^3*e*g + B*a^4*d^4*e*g)*log(-b*e +
(b*e*x + a*e)*d/(d*x + c))/(b^2*d^2*i) - (B*b^4*c^4*e*g - 4*B*a*b^3*c^3*d*e
*g + 6*B*a^2*b^2*c^2*d^2*e*g - 4*B*a^3*b*c*d^3*e*g + B*a^4*d^4*e*g)*log((b*
e*x + a*e)/(d*x + c))/(b^2*d^2*i))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d
/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx=\int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{c\,i+d\,i\,x}\,dx$$

```
[In] int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x),x)
[Out] int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x), x)
```

3.34 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci+dix}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 76

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci+dix}\,dx=-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di}-\frac{B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{di}$$

```
[Out] -ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i-B*polylog(2,d*(b*
x+a)/b/(d*x+c))/d/i
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2544, 2458, 2378, 2370, 2352}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci+dix}\,dx=-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{di}-\frac{B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{di}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(c*i + d*i*x),x]
```

```
[Out] -((Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d*
i)) - (B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d*i)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2544

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c +
 d*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Dist[B*n*((b*c
- a*d)/g), Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && EqQ[d*f - c*g, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di}+\frac{(B(bc-ad))\int\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}\,dx}{di}\\
&= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di}+\frac{(B(bc-ad))\text{Subst}\left(\int\frac{\log\left(\frac{bc-ad}{bx}\right)}{x\left(\frac{-bc+ad}{d}+\frac{bx}{d}\right)}\,dx,x,c+dx\right)}{d^2i}\\
&= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di}-\frac{(B(bc-ad))\text{Subst}\left(\int\frac{\log\left(\frac{(bc-ad)x}{b}\right)}{\left(\frac{-bc+ad}{d}+\frac{b}{dx}\right)x}\,dx,x,\frac{1}{c+dx}\right)}{d^2i}\\
&= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di}-\frac{(B(bc-ad))\text{Subst}\left(\int\frac{\log\left(\frac{(bc-ad)x}{b}\right)}{\frac{b}{d}+\frac{(-bc+ad)x}{d}}\,dx,x,\frac{1}{c+dx}\right)}{d^2i}\\
&= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di}-\frac{B\,\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{di}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.25

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci+dix}\,dx$$
$$=\frac{\log(i(c+dx))\left(2A-2B\log\left(\frac{d(a+bx)}{-bc+ad}\right)+2B\log\left(\frac{e(a+bx)}{c+dx}\right)+B\log(i(c+dx))\right)-2B\,\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{2di}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(c*i + d*i*x),x]
```

```
[Out] (Log[i*(c + d*x)]*(2*A - 2*B*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*B*Log[(e
*(a + b*x))/(c + d*x)] + B*Log[i*(c + d*x)]) - 2*B*PolyLog[2, (b*(c + d*x))
/(b*c - a*d)])/(2*d*i)
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.92

method	result
parts	$\frac{A\ln(dx+c)}{id}+\frac{B\left(-\frac{\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be}{be}\right)}{d}-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be}{be}\right)}{d}\right)}{i}$
risch	$\frac{A\ln(dx+c)}{id}-\frac{B\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be}{be}\right)}{id}-\frac{B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be}{be}\right)}{id}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{dA\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{ie(ad-cb)}-\frac{d^2B\left(-\frac{\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be}{be}\right)}{d}-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(-\frac{\left(\frac{be}{d}+\frac{(ad-c}{d(dx+}\right.}{be}\right.}{d}\right.}{ie(ad-cb)}\right.}{d^2}$
default	$-\frac{e(ad-cb)\left(\frac{dA\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{ie(ad-cb)}-\frac{d^2B\left(-\frac{\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be}{be}\right)}{d}-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(-\frac{\left(\frac{be}{d}+\frac{(ad-c}{d(dx+}\right.}{be}\right.}{d}\right.}{ie(ad-cb)}\right.}{d^2}$

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x,method=_RETURNVERBOSE)
```

```
[Out] A/i*ln(d*x+c)/d+B/i*(-dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d-l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/
e)/d)
```

Fricas [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci+dix}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{dix+ci}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorithm="fricas")
```

```
[Out] integral((B*log((b*e*x + a*e)/(d*x + c)) + A)/(d*i*x + c*i), x)
```

Sympy [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci+dix}\,dx=\frac{\int \frac{A}{c+dx}\,dx+\int \frac{B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx}{i}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x)
```

```
[Out] (Integral(A/(c + d*x), x) + Integral(B*log(a*e/(c + d*x) + b*e*x/(c + d*x))
/(c + d*x), x))/i
```

Maxima [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci+dix}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{dix+ci}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorithm="maxima")
```

```
[Out] -1/2*B*(log(d*x + c)^2/(d*i) - 2*integrate((log(b*x + a) + log(e))/(d*i*x +
 c*i), x)) + A*log(d*i*x + c*i)/(d*i)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(75) = 150$.

Time = 36.75 (sec) , antiderivative size = 617, normalized size of antiderivative = 8.12

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci+dix}\,dx$$
$$=\frac{1}{2}\left(\frac{(Bb^3c^3e^3-3\,Bab^2c^2de^3+3\,Ba^2bcd^2e^3-Ba^3d^3e^3)\log\left(\frac{bex+ae}{dx+c}\right)}{b^2de^2i-\frac{2\,(bex+ae)bd^2ei}{dx+c}+\frac{(bex+ae)^2d^3i}{(dx+c)^2}}+\frac{Ab^4c^3e^3-Bb^4c^3e^3-3\,Aab^3c^2de^3+}{}\right.$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorithm="giac")
```

```
[Out] 1/2*((B*b^3*c^3*e^3 - 3*B*a*b^2*c^2*d*e^3 + 3*B*a^2*b*c*d^2*e^3 - B*a^3*d^3
*e^3)*log((b*e*x + a*e)/(d*x + c))/(b^2*d*e^2*i - 2*(b*e*x + a*e)*b*d^2*e*i
/(d*x + c) + (b*e*x + a*e)^2*d^3*i/(d*x + c)^2) + (A*b^4*c^3*e^3 - B*b^4*c^
3*e^3 - 3*A*a*b^3*c^2*d*e^3 + 3*B*a*b^3*c^2*d*e^3 + 3*A*a^2*b^2*c*d^2*e^3 -
 3*B*a^2*b^2*c*d^2*e^3 - A*a^3*b*d^3*e^3 + B*a^3*b*d^3*e^3 + (b*e*x + a*e)*
B*b^3*c^3*d*e^2/(d*x + c) - 3*(b*e*x + a*e)*B*a*b^2*c^2*d^2*e^2/(d*x + c) +
 3*(b*e*x + a*e)*B*a^2*b*c*d^3*e^2/(d*x + c) - (b*e*x + a*e)*B*a^3*d^4*e^2/
```

```
(d*x + c))/(b^3*d*e^2*i - 2*(b*e*x + a*e)*b^2*d^2*e*i/(d*x + c) + (b*e*x +
a*e)^2*b*d^3*i/(d*x + c)^2) + (B*b^3*c^3*e - 3*B*a*b^2*c^2*d*e + 3*B*a^2*b*
c*d^2*e - B*a^3*d^3*e)*log(-b*e + (b*e*x + a*e)*d/(d*x + c))/(b^2*d*i) - (B
*b^3*c^3*e - 3*B*a*b^2*c^2*d*e + 3*B*a^2*b*c*d^2*e - B*a^3*d^3*e)*log((b*e*
x + a*e)/(d*x + c))/(b^2*d*i))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b
*c*e - a*d*e)*(b*c - a*d)))^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{ci + dix}\,dx = \int \frac{A + B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{c\,i + d\,i\,x}\,dx$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(c*i + d*i*x),x)
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))/(c*i + d*i*x), x)
```

3.35 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 44

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx=\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2B(bc-ad)gi}$$

```
[Out] 1/2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/B/(-a*d+b*c)/g/i
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2562, 2338}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx=\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2Bgi(bc-ad)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)*(c*i + d*i*x)),x]
```

```
[Out] (A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(2*B*(b*c - a*d)*g*i)
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
```

```
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{A+B\log(ex)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)gi}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2B(bc-ad)gi}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 207, normalized size of antiderivative = 4.70

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx$$
$$= \frac{2A\log(a+bx)-B\log^2(a+bx)+2B\log(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-2A\log(c+dx)+2B\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log(c$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)*(c*i + d*i*x)
),x]
```

```
[Out] (2*A*Log[a + b*x] - B*Log[a + b*x]^2 + 2*B*Log[a + b*x]*Log[(e*(a + b*x))/(
c + d*x)] - 2*A*Log[c + d*x] + 2*B*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c
+ d*x] - 2*B*Log[(e*(a + b*x))/(c + d*x)]*Log[c + d*x] - B*Log[c + d*x]^2 +
 2*B*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 2*B*PolyLog[2, (d*(a + b
*x))/(-(b*c) + a*d)] + 2*B*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(2*(b*c -
 a*d)*g*i)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.64

method	result	size
norman	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)^{2}}{2gi(ad-cb)}-\frac{A\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi(ad-cb)}$	72
parallelrisch	$-\frac{a^{2}c^{2}B\ln\left(\frac{e(bx+a)}{dx+c}\right)^{2}+2A\ln\left(\frac{e(bx+a)}{dx+c}\right)a^{2}c^{2}}{2a^{2}c^{2}gi(ad-cb)}$	75
parts	$\frac{A\left(\frac{\ln(dx+c)}{ad-cb}-\frac{\ln(bx+a)}{ad-cb}\right)}{gi}-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)^{2}}{2gi(ad-cb)}$	82
risch	$\frac{A\ln(dx+c)}{gi(ad-cb)}-\frac{A\ln(bx+a)}{gi(ad-cb)}-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)^{2}}{2gi(ad-cb)}$	87
derivativedivides	$-\frac{e(ad-cb)\left(\frac{d^{2}A\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei(ad-cb)^{2}g}+\frac{d^{2}B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}}{2ei(ad-cb)^{2}g}\right)}{d^{2}}$	123
default	$-\frac{e(ad-cb)\left(\frac{d^{2}A\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei(ad-cb)^{2}g}+\frac{d^{2}B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}}{2ei(ad-cb)^{2}g}\right)}{d^{2}}$	123

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/2*B/g/i/(a*d-b*c)*ln(e*(b*x+a)/(d*x+c))^2-A/g/i/(a*d-b*c)*ln(e*(b*x+a)/(
d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx=\frac{B\log\left(\frac{bex+ae}{dx+c}\right)^{2}+2\,A\log\left(\frac{bex+ae}{dx+c}\right)}{2\,(bc-ad)gi}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i),x, algorithm
="fricas")
```

```
[Out] 1/2*(B*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*log((b*e*x + a*e)/(d*x + c)))/(
(b*c - a*d)*g*i)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(31) = 62$.

Time = 0.31 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.86

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx = A\left(\frac{\log\left(x+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{gi\,(ad-bc)} - \frac{\log\left(x+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{gi\,(ad-bc)}\right) - \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2adgi-2bcgi}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i),x)
```

```
[Out] A*(log(x + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**
2/(a*d - b*c) + b*c)/(2*b*d))/(g*i*(a*d - b*c)) - log(x + (a**2*d**2/(a*d -
 b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d))
/(g*i*(a*d - b*c))) - B*log(e*(a + b*x)/(c + d*x))**2/(2*a*d*g*i - 2*b*c*g*
i)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(42) = 84$.

Time = 0.21 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.91

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx &= B\left(\frac{\log(bx+a)}{(bc-ad)gi} - \frac{\log(dx+c)}{(bc-ad)gi}\right)\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right) \\ &\quad + A\left(\frac{\log(bx+a)}{(bc-ad)gi} - \frac{\log(dx+c)}{(bc-ad)gi}\right) \\ &\quad - \frac{(\log(bx+a)^2 - 2\log(bx+a)\log(dx+c) + \log(dx+c)^2)B}{2(bcgi-adgi)}\end{aligned}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i),x, algorithm
="maxima")
```

```
[Out] B*(log(b*x + a)/((b*c - a*d)*g*i) - log(d*x + c)/((b*c - a*d)*g*i))*log(b*e
*x/(d*x + c) + a*e/(d*x + c)) + A*(log(b*x + a)/((b*c - a*d)*g*i) - log(d*x
 + c)/((b*c - a*d)*g*i)) - 1/2*(log(b*x + a)^2 - 2*log(b*x + a)*log(d*x + c
) + log(d*x + c)^2)*B/(b*c*g*i - a*d*g*i)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(42) = 84$.

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.36

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)(ci + dix)}\,dx = \frac{\left(Be\log\left(\frac{bex+ae}{dx+c}\right)^2 + 2\,Ae\log\left(\frac{bex+ae}{dx+c}\right)\right)\left(\frac{bc}{(bce-ade)(bc-ad)} - \frac{ad}{(bce-ade)(bc-ad)}\right)}{2\,gi}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i),x, algorithm
="giac")
```

```
[Out] 1/2*(B*e*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*e*log((b*e*x + a*e)/(d*x + c)
))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/
(g*i)
```

Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.57

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)(ci + dix)}\,dx = -\frac{B\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2 - A\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c} + 1\mathrm{i}\right)\,4\mathrm{i}}{2\,g\,i\,(a\,d-b\,c)}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)*(c*i + d*i*x)),x)
```

```
[Out] -(B*log((e*(a + b*x))/(c + d*x))^2 - A*atan((b*c*2i + b*d*x*2i)/(a*d - b*c)
 + 1i)*4i)/(2*g*i*(a*d - b*c))
```

3.36 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 173

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{bB(c+dx)}{(bc-ad)^2g^2i(a+bx)} + \frac{Bd\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^2g^2i} - \frac{b(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^2i(a+bx)} - \frac{d\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^2i}$$

```
[Out] -b*B*(d*x+c)/(-a*d+b*c)^2/g^2/i/(b*x+a)+1/2*B*d*ln((b*x+a)/(d*x+c))^2/(-a*d
+b*c)^2/g^2/i-b*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^2/g^2/i/(b*x
+a)-d*ln((b*x+a)/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^2/g^2/i
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2562, 45, 2372, 14, 2338}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2i(bc-ad)^2} - \frac{b(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2i(a+bx)(bc-ad)^2} - \frac{bB(c+dx)}{g^2i(a+bx)(bc-ad)^2} + \frac{Bd\log^2\left(\frac{a+bx}{c+dx}\right)}{2g^2i(bc-ad)^2}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^2*(c*i + d*i*x)),x]
[Out] -((b*B*(c + d*x))/((b*c - a*d)^2*g^2*i*(a + b*x))) + (B*d*Log[(a + b*x)/(c
+ d*x)]^2)/(2*(b*c - a*d)^2*g^2*i) - (b*(c + d*x)*(A + B*Log[(e*(a + b*x))/
(c + d*x)]))/((b*c - a*d)^2*g^2*i*(a + b*x)) - (d*Log[(a + b*x)/(c + d*x)]*
(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^2*g^2*i)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex))}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^2i}$$

$$= -\frac{b(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^2i(a+bx)} - \frac{d\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^2i} - \frac{B\text{Subst}\left(\int \frac{-b-dx\log(x)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^2i}$$

$$= -\frac{b(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^2i(a+bx)} - \frac{d\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^2i} - \frac{B\text{Subst}\left(\int \left(-\frac{b}{x^2}-\frac{d\log(x)}{x}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^2i}$$

$$= -\frac{bB(c+dx)}{(bc-ad)^2g^2i(a+bx)} - \frac{b(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^2i(a+bx)} - \frac{d\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^2i} + \frac{(Bd)\text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^2i}$$

$$= -\frac{bB(c+dx)}{(bc-ad)^2g^2i(a+bx)} + \frac{Bd\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^2g^2i} - \frac{b(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^2i(a+bx)} - \frac{d\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^2i}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.69

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+2d(a+bx)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2d(a+bx)\left(A+\right.}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^2*(c*i + d*i*
x)),x]
```

```
[Out] -1/2*(2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*d*(a + b*x)*Lo
g[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*d*(a + b*x)*(A + B*Log[
(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 2*B*(b*c - a*d + d*(a + b*x)*Log[a
 + b*x] - d*(a + b*x)*Log[c + d*x]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a +
b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*
c) + a*d)]) + B*d*(a + b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c +
 d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d
)^2*g^2*i*(a + b*x))
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.33

method	result
parts	$\frac{A\left(\frac{d\ln(dx+c)}{(ad-cb)^2}+\frac{1}{(bx+a)(ad-cb)}-\frac{d\ln(bx+a)}{(ad-cb)^2}\right)}{g^2 i}-\frac{B\left(\frac{d^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^2}-\frac{dbe\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^2}\right)}{g^2 i d}$
norman	$\frac{-\frac{(Aad+Bbc)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{Bad\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{2gi\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{b(Ad+Bd)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{(A+B)bx}{gia(ad-cb)}-\frac{bBdx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{2gi\left(a^2d^2-2abcd+b^2c^2\right)}}{g(bx+a)}$
parallelrisch	$-\frac{2Axa^3bc^2d+2Bxa^3bc^2d+B\ln\left(\frac{e(bx+a)}{dx+c}\right)^2a^4c^2d-2Axa^2b^2c^3+2A\ln\left(\frac{e(bx+a)}{dx+c}\right)a^4c^2d-2Bxa^2b^2c^3+2B\ln\left(\frac{e(bx+a)}{dx+c}\right)c}{2ig^2(bx+a)(a^2d^2-2abcd+b^2c^2)c^2c}$
risch	$\frac{Ad\ln(dx+c)}{g^2i(ad-cb)^2}+\frac{A}{g^2i(bx+a)(ad-cb)}-\frac{Ad\ln(bx+a)}{g^2i(ad-cb)^2}-\frac{Bd\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2g^2i(ad-cb)^2}-\frac{Bbe\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{g^2i(ad-cb)^2\left(\frac{be}{d}+\frac{ea}{dx+c}-\frac{ecb}{d(dx+c)}\right)}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{d^2Ab}{i(ad-cb)^3g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^3A\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei(ad-cb)^3g^2}-\frac{d^2Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{i(ad-cb)^3g^2}+\frac{d^3B\ln(}{2ei}\right.}{d^2}$
default	$-\frac{e(ad-cb)\left(\frac{d^2Ab}{i(ad-cb)^3g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^3A\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei(ad-cb)^3g^2}-\frac{d^2Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{i(ad-cb)^3g^2}+\frac{d^3B\ln(}{2ei}\right.}{d^2}$

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i),x,method=_RETURNV
ERBOSE)
```

```
[Out] A/g^2/i*(d/(a*d-b*c)^2*ln(d*x+c)+1/(b*x+a)/(a*d-b*c)-d/(a*d-b*c)^2*ln(b*x+a
))-B/g^2/i/d*(1/2*d^2/(a*d-b*c)^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-d/(a*d-
b*c)^2*b*e*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.83

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{2\,(A+B)bc-2\,(A+B)ad+(Bbdx+Bad)\log\left(\frac{bex+ae}{dx+c}\right)^2+2\,((A+B)bdx+Bbc+Aad)\log\left(\frac{bex+ae}{dx+c}\right)}{2\,((b^3c^2-2\,ab^2cd+a^2bd^2)g^2ix+(ab^2c^2-2\,a^2bcd+a^3d^2)g^2i)}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i),x, algorit
hm="fricas")
```

```
[Out] -1/2*(2*(A + B)*b*c - 2*(A + B)*a*d + (B*b*d*x + B*a*d)*log((b*e*x + a*e)/(
d*x + c))^2 + 2*((A + B)*b*d*x + B*b*c + A*a*d)*log((b*e*x + a*e)/(d*x + c)
))/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^2*i*x + (a*b^2*c^2 - 2*a^2*b*c*d
+ a^3*d^2)*g^2*i)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(144) = 288.

Time = 0.62 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.23

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{Bd\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2a^2d^2g^2i-4abcdg^2i+2b^2c^2g^2i} + \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{a^2dg^2i-abcg^2i+abdg^2ix-b^2cg^2ix} + (A+B)\left(\frac{d\log\left(x+\frac{-\frac{a^3d^4}{(ad-bc)^2}+\frac{3a^2bcd^3}{(ad-bc)^2}-\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2+\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{g^2i\,(ad-bc)^2} - \frac{d\log\left(x+\frac{\frac{a^3d^4}{(ad-bc)^2}-\frac{3a^2bcd^3}{(ad-bc)^2}+\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2-\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{g^2i\,(ad-bc)^2} + \frac{1}{a^2dg^2i-abcg^2i+x\,(abdg^2i-b^2cg^2i)}\right)$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2/(d*i*x+c*i),x)
```

```
[Out] -B*d*log(e*(a + b*x)/(c + d*x))**2/(2*a**2*d**2*g**2*i - 4*a*b*c*d*g**2*i +
 2*b**2*c**2*g**2*i) + B*log(e*(a + b*x)/(c + d*x))/(a**2*d*g**2*i - a*b*c*
g**2*i + a*b*d*g**2*i*x - b**2*c*g**2*i*x) + (A + B)*(d*log(x + (-a**3*d**4
/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*d - b*c)**2 - 3*a*b**2*c**2*d**2/(a*d
- b*c)**2 + a*d**2 + b**3*c**3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(g**2*
i*(a*d - b*c)**2) - d*log(x + (a**3*d**4/(a*d - b*c)**2 - 3*a**2*b*c*d**3/(
a*d - b*c)**2 + 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 - b**3*c**3*d/(a
*d - b*c)**2 + b*c*d)/(2*b*d**2))/(g**2*i*(a*d - b*c)**2) + 1/(a**2*d*g**2*
i - a*b*c*g**2*i + x*(a*b*d*g**2*i - b**2*c*g**2*i)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(171) = 342$.

Time = 0.22 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.45

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx =$$
$$-B\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i}+\frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}-\frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)\log\left(\frac{b}{dx}\right.$$
$$\left.+\frac{ae}{dx+c}\right)$$
$$-A\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i}+\frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}-\frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)$$
$$+\frac{\left((bdx+ad)\log(bx+a)^2+(bdx+ad)\log(dx+c)^2-2\,bc+2\,ad-2\,(bdx+ad)\log(bx+a)+2\,(bdx+\right.}{2\,(ab^2c^2g^2i-2\,a^2bcdg^2i+a^3d^2g^2i+(b^3c^2g^2i-2\,ab^2cdg^2i+a^2bd^2}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i),x, algorit
hm="maxima")
```

```
[Out] -B*(1/((b^2*c - a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/((
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*c*
d + a^2*d^2)*g^2*i))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - A*(1/((b^2*c -
a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g^2
*i)) + 1/2*((b*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2
*b*c + 2*a*d - 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d
)*log(b*x + a))*log(d*x + c))*B/(a*b^2*c^2*g^2*i - 2*a^2*b*c*d*g^2*i + a^3*
d^2*g^2*i + (b^3*c^2*g^2*i - 2*a*b^2*c*d*g^2*i + a^2*b*d^2*g^2*i)*x)
```

Giac [F]

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx = \int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right) + A}{(bgx+ag)^2(dix+ci)}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i),x, algorit
hm="giac")
```

```
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)/((b*g*x + a*g)^2*(d*i*x + c*i)
), x)
```

Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.39

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx = \frac{A+B}{(a\,d-b\,c)\,\left(a\,g^2\,i+b\,g^2\,i\,x\right)} - \frac{B\,d\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2}{2\,g^2\,i\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)} + \frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,(a\,d-b\,c)}{b\,d\,g^2\,i\,\left(\frac{x}{d}+\frac{a}{b\,d}\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)} + \frac{d\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{a^2\,d^2\,g^2\,i-b^2\,c^2\,g^2\,i}{g^2\,i\,(a\,d-b\,c)}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(A+B)\,2\mathrm{i}}{g^2\,i\,{(a\,d-b\,c)}^2}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^2*(c*i + d*i*x)),x)
```

```
[Out] (A + B)/((a*d - b*c)*(a*g^2*i + b*g^2*i*x)) + (d*atan(((2*b*d*x + (a^2*d^2*
g^2*i - b^2*c^2*g^2*i)/(g^2*i*(a*d - b*c)))*1i)/(a*d - b*c))*(A + B)*2i)/(g
^2*i*(a*d - b*c)^2) - (B*d*log((e*(a + b*x))/(c + d*x))^2)/(2*g^2*i*(a^2*d^
2 + b^2*c^2 - 2*a*b*c*d)) + (B*log((e*(a + b*x))/(c + d*x))*(a*d - b*c))/(b
*d*g^2*i*(x/d + a/(b*d))*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
```

3.37 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 255

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx = -\frac{B(c+dx)^2\left(b-\frac{4d(a+bx)}{c+dx}\right)^2}{4(bc-ad)^3g^3i(a+bx)^2} - \frac{Bd^2\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^3g^3i} + \frac{2bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2} + \frac{d^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^3i}$$

```
[Out] -1/4*B*(d*x+c)^2*(b-4*d*(b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g^3/i/(b*x+a)^2-1/2
*B*d^2*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g^3/i+2*b*d*(d*x+c)*(A+B*ln(e*(b*
x+a)/(d*x+c)))/(-a*d+b*c)^3/g^3/i/(b*x+a)-1/2*b^2*(d*x+c)^2*(A+B*ln(e*(b*x+
a)/(d*x+c)))/(-a*d+b*c)^3/g^3/i/(b*x+a)^2+d^2*ln((b*x+a)/(d*x+c))*(A+B*ln(e
*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^3/i
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {2562, 45, 2372, 12, 14, 37, 2338}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx=-\frac{b^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^3i(a+bx)^2(bc-ad)^3}+\frac{d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^3i(bc-ad)^3}+\frac{2bd(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^3i(a+bx)(bc-ad)^3}-\frac{Bd^2\log^2\left(\frac{a+bx}{c+dx}\right)}{2g^3i(bc-ad)^3}-\frac{B(c+dx)^2\left(b-\frac{4d(a+bx)}{c+dx}\right)^2}{4g^3i(a+bx)^2(bc-ad)^3}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^3*(c*i + d*i*x)),x]
```

```
[Out] -1/4*(B*(c + d*x)^2*(b - (4*d*(a + b*x))/(c + d*x))^2)/((b*c - a*d)^3*g^3*i
*(a + b*x)^2) - (B*d^2*Log[(a + b*x)/(c + d*x)]^2)/(2*(b*c - a*d)^3*g^3*i)
+ (2*b*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^3*g^3
*i*(a + b*x)) - (b^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(
b*c - a*d)^3*g^3*i*(a + b*x)^2) + (d^2*Log[(a + b*x)/(c + d*x)]*(A + B*Log[
(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^3*g^3*i)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex))}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i}$$

$$= \frac{2bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2} + \frac{d^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^3i} - \frac{B\text{Subst}\left(\int \frac{-b^2+4bdx+2d^2x^2\log(x)}{2x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i}$$

$$= \frac{2bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2} + \frac{d^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^3i} - \frac{B\text{Subst}\left(\int \frac{-b^2+4bdx+2d^2x^2\log(x)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{2(bc-ad)^3g^3i}$$

$$= \frac{2bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2} + \frac{d^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^3i} - \frac{B\text{Subst}\left(\int\left(-\frac{b(b-4dx)}{x^3}+\frac{2d^2\log(x)}{x}\right)dx, x, \frac{a+bx}{c+dx}\right)}{2(bc-ad)^3g^3i}$$

$$= \frac{2bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2} + \frac{d^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^3i} + \frac{(bB)\text{Subst}\left(\int\frac{b-4dx}{x^3}dx, x, \frac{a+bx}{c+dx}\right)}{2(bc-ad)^3g^3i} - \frac{(Bd^2)\,\text{Subst}\left(\int\frac{\log(x)}{x}dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i}$$

$$= -\frac{B(c+dx)^2\left(b-\frac{4d(a+bx)}{c+dx}\right)^2}{4(bc-ad)^3g^3i(a+bx)^2} - \frac{Bd^2\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^3g^3i} + \frac{2bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2} + \frac{d^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^3i}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.22 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.64

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx = \frac{-2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+4d(bc-ad)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+4d^2(a+bx)^2\log(a-}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^3*(c*i + d*i*
x)),x]
[Out] (-2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*d*(b*c - a*d)*(a
 + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*d^2*(a + b*x)^2*Log[a + b*
x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 4*d^2*(a + b*x)^2*(A + B*Log[(e*(
a + b*x))/(c + d*x)])*Log[c + d*x] + 4*B*d*(a + b*x)*(b*c - a*d + d*(a + b*
x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*((b*c - a*d)^2 + 2*d*(-(b*c
) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log
[c + d*x]) - 2*B*d^2*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c
+ d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*B*d
^2*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c
+ d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(4*(b*c - a*d)^3*g^3*i*(
a + b*x)^2)
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.43

method	result
parts	$\frac{A\left(\frac{d^2\ln(dx+c)}{(ad-cb)^3}+\frac{1}{2(ad-cb)(bx+a)^2}+\frac{d}{(ad-cb)^2(bx+a)}-\frac{d^2\ln(bx+a)}{(ad-cb)^3}\right)}{g^3 i}-\frac{B\left(\frac{d^3\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^3}-\frac{2d^2be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)}{d(dx+c}\right.}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right.}{(ad-c}\right.}{}$
risch	$\frac{Ad^2\ln(dx+c)}{g^3 i(ad-cb)^3}+\frac{A}{2g^3 i(ad-cb)(bx+a)^2}+\frac{Ad}{g^3 i(ad-cb)^2(bx+a)}-\frac{Ad^2\ln(bx+a)}{g^3 i(ad-cb)^3}-\frac{Bd^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2g^3 i(ad-cb)^3}-\frac{}{g^3 i(}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^2eAb^2}{2i(ad-cb)^4g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{2d^3Ab}{i(ad-cb)^4g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^4A\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei(ad-cb)^4g^3}+\frac{d^2eBb^2\left(-\frac{\ln\left(\frac{be}{d}+\right.}{2\left(\frac{be}{d}+\right.}\right.}{}\right.}{d^2}$
default	$-\frac{e(ad-cb)\left(-\frac{d^2eAb^2}{2i(ad-cb)^4g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{2d^3Ab}{i(ad-cb)^4g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^4A\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei(ad-cb)^4g^3}+\frac{d^2eBb^2\left(-\frac{\ln\left(\frac{be}{d}+\right.}{2\left(\frac{be}{d}+\right.}\right.}{}\right.}{d^2}$
parallelrisch	$-\frac{2Bx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2a^4b^2c^2d^2+4Ax^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a^4b^2c^2d^2+6Bx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a^4b^2c^2d^2+4Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2a^5bc^2d^2}{}$
norman	$\frac{\frac{6Aab^2d-2Ab^3c+7Bab^2d-Bb^3c}{4gi(ad-cb)^2b^2}-\frac{\left(2Aa^2d^2+4Babcd-Bb^2c^2\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2ig\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}+\frac{\left(2Ab^2d+3Bb^2d\right)x}{2ig\left(a^2d^2-2abcd+b^2c^2\right)b}-\frac{Ba^2d^2\ln\left(\frac{e(bx+}{dx+}\right.}{2ig\left(a^3d^3-3a^2bcd^2+3a\right.}}{}$

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i),x,method=_RETURNV
ERBOSE)
```

```
[Out] A/g^3/i*(d^2/(a*d-b*c)^3*ln(d*x+c)+1/2/(a*d-b*c)/(b*x+a)^2+d/(a*d-b*c)^2/(b
*x+a)-d^2/(a*d-b*c)^3*ln(b*x+a))-B/g^3/i/d*(1/2*d^3/(a*d-b*c)^3*ln(b*e/d+(a
*d-b*c)*e/d/(d*x+c))^2-2*d^2/(a*d-b*c)^3*b*e*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+
c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+d/(a*d
-b*c)^3*e^2*b^2*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/
d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.37

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx = -\frac{(2\,A+B)b^2c^2-8\,(A+B)abcd+(6\,A+7\,B)a^2d^2-2\,(Bb^2d^2x^2+2\,Babd^2x+Ba^2d^2)\log\left(\frac{bex+ae}{dx+c}\right)^2-}{4\,((b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3)g^3ix^2+2\,(a}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i),x, algorit
hm="fricas")
```

```
[Out] -1/4*((2*A + B)*b^2*c^2 - 8*(A + B)*a*b*c*d + (6*A + 7*B)*a^2*d^2 - 2*(B*b^
2*d^2*x^2 + 2*B*a*b*d^2*x + B*a^2*d^2)*log((b*e*x + a*e)/(d*x + c))^2 - 2*(
(2*A + 3*B)*b^2*c*d - (2*A + 3*B)*a*b*d^2)*x - 2*((2*A + 3*B)*b^2*d^2*x^2 -
 B*b^2*c^2 + 4*B*a*b*c*d + 2*A*a^2*d^2 + 2*(B*b^2*c*d + 2*(A + B)*a*b*d^2)*
x)*log((b*e*x + a*e)/(d*x + c)))/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^
2 - a^3*b^2*d^3)*g^3*i*x^2 + 2*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d
^2 - a^4*b*d^3)*g^3*i*x + (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 -
a^5*d^3)*g^3*i)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(221) = 442$.

Time = 2.59 (sec) , antiderivative size = 889, normalized size of antiderivative = 3.49

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx = -\frac{Bd^2\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2a^3d^3g^3i-6a^2bcd^2g^3i+6ab^2c^2dg^3i-2b^3c^3g^3i}$$
$$+\frac{d^2\cdot(2A+3B)\log\left(x+\frac{2Aad^3+2Abcd^2+3Bad^3+3Bbcd^2-\frac{a^4d^6\cdot(2A+3B)}{(ad-bc)^3}+\frac{4a^3bcd^5\cdot(2A+3B)}{(ad-bc)^3}-\frac{6a^2b^2c^2d^4\cdot(2A+3B)}{(ad-bc)^3}+\frac{4ab^3c^3d^3\cdot(2A+3}{(ad-bc)^3}}{4Abd^3+6Bbd^3}\right.}{2g^3i\,(ad-bc)^3}$$
$$-\frac{d^2\cdot(2A+3B)\log\left(x+\frac{2Aad^3+2Abcd^2+3Bad^3+3Bbcd^2+\frac{a^4d^6\cdot(2A+3B)}{(ad-bc)^3}-\frac{4a^3bcd^5\cdot(2A+3B)}{(ad-bc)^3}+\frac{6a^2b^2c^2d^4\cdot(2A+3B)}{(ad-bc)^3}-\frac{4ab^3c^3d^3\cdot(2A+3}{(ad-bc)^3}}{4Abd^3+6Bbd^3}\right.}{2g^3i\,(ad-bc)^3}$$
$$+\frac{(3Bad-Bbc+2Bbdx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2a^4d^2g^3i-4a^3bcdg^3i+4a^3bd^2g^3ix+2a^2b^2c^2g^3i-8a^2b^2cdg^3ix+2a^2b^2d^2g^3ix^2+4ab^3c^2g^3ix-4ab^3cdg}$$
$$+\frac{6Aad-2Abc+7Bad-Bbc+x(4Abd+6Bbd)}{4a^4d^2g^3i-8a^3bcdg^3i+4a^2b^2c^2g^3i+x^2\cdot(4a^2b^2d^2g^3i-8ab^3cdg^3i+4b^4c^2g^3i)+x\,(8a^3bd^2g^3i-16a^2b^2c}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3/(d*i*x+c*i),x)
```

```
[Out] -B*d**2*log(e*(a + b*x)/(c + d*x))**2/(2*a**3*d**3*g**3*i - 6*a**2*b*c*d**2
*g**3*i + 6*a*b**2*c**2*d*g**3*i - 2*b**3*c**3*g**3*i) + d**2*(2*A + 3*B)*l
og(x + (2*A*a*d**3 + 2*A*b*c*d**2 + 3*B*a*d**3 + 3*B*b*c*d**2 - a**4*d**6*(
2*A + 3*B)/(a*d - b*c)**3 + 4*a**3*b*c*d**5*(2*A + 3*B)/(a*d - b*c)**3 - 6*
a**2*b**2*c**2*d**4*(2*A + 3*B)/(a*d - b*c)**3 + 4*a*b**3*c**3*d**3*(2*A +
3*B)/(a*d - b*c)**3 - b**4*c**4*d**2*(2*A + 3*B)/(a*d - b*c)**3)/(4*A*b*d**
3 + 6*B*b*d**3))/(2*g**3*i*(a*d - b*c)**3) - d**2*(2*A + 3*B)*log(x + (2*A*
a*d**3 + 2*A*b*c*d**2 + 3*B*a*d**3 + 3*B*b*c*d**2 + a**4*d**6*(2*A + 3*B)/(
a*d - b*c)**3 - 4*a**3*b*c*d**5*(2*A + 3*B)/(a*d - b*c)**3 + 6*a**2*b**2*c*
*2*d**4*(2*A + 3*B)/(a*d - b*c)**3 - 4*a*b**3*c**3*d**3*(2*A + 3*B)/(a*d -
b*c)**3 + b**4*c**4*d**2*(2*A + 3*B)/(a*d - b*c)**3)/(4*A*b*d**3 + 6*B*b*d*
*3))/(2*g**3*i*(a*d - b*c)**3) + (3*B*a*d - B*b*c + 2*B*b*d*x)*log(e*(a + b
*x)/(c + d*x))/(2*a**4*d**2*g**3*i - 4*a**3*b*c*d*g**3*i + 4*a**3*b*d**2*g*
*3*i*x + 2*a**2*b**2*c**2*g**3*i - 8*a**2*b**2*c*d*g**3*i*x + 2*a**2*b**2*d
**2*g**3*i*x**2 + 4*a*b**3*c**2*g**3*i*x - 4*a*b**3*c*d*g**3*i*x**2 + 2*b**
4*c**2*g**3*i*x**2) + (6*A*a*d - 2*A*b*c + 7*B*a*d - B*b*c + x*(4*A*b*d + 6
*B*b*d))/(4*a**4*d**2*g**3*i - 8*a**3*b*c*d*g**3*i + 4*a**2*b**2*c**2*g**3*
i + x**2*(4*a**2*b**2*d**2*g**3*i - 8*a*b**3*c*d*g**3*i + 4*b**4*c**2*g**3*
i) + x*(8*a**3*b*d**2*g**3*i - 16*a**2*b**2*c*d*g**3*i + 8*a*b**3*c**2*g**3
*i))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 885 vs. $2(249) = 498$.

Time = 0.24 (sec) , antiderivative size = 885, normalized size of antiderivative = 3.47

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx$$
$$= \frac{1}{2}B\Bigg(\frac{2\,bdx - bc + 3\,ad}{(b^4c^2 - 2\,ab^3cd + a^2b^2d^2)g^3ix^2 + 2\,(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2)g^3ix + (a^2b^2c^2 - 2\,a^3bcd + a^4d^2)g^3i} + \frac{\ }{(}$$
$$+ \frac{ae}{dx+c}\Bigg)$$
$$+ \frac{1}{2}A\Bigg(\frac{2\,bdx - bc + 3\,ad}{(b^4c^2 - 2\,ab^3cd + a^2b^2d^2)g^3ix^2 + 2\,(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2)g^3ix + (a^2b^2c^2 - 2\,a^3bcd + a^4d^2)g^3i} +$$
$$- \frac{\left(b^2c^2 - 8\,abcd + 7\,a^2d^2 + 2\,(b^2d^2x^2 + 2\,abd^2x + a^2d^2)\log\left(bx+a\right)^2 + 2\,(b^2d^2x^2 + 2\,abd^2x + a^2d^2)\log\left(dx\right.\right.}{4\,(a^2b^3c^3g^3i - 3\,a^3b^2c^2dg^3i + 3\,a^4bcd^2g^3i - a^5d^3g^3i + (}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i),x, algorit
hm="maxima")
```

```
[Out] 1/2*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3*i
*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i*x + (a^2*b^2*c^2 - 2
```

```
*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2*d
 + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c)/((b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i))*log(b*e*x/(d*x + c) + a*e/(d*x +
c)) + 1/2*A*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)
*g^3*i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i*x + (a^2*b^2*c
^2 - 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c)/((b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i)) - 1/4*(b^2*c^2 - 8*a*b*c*d
+ 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^2 + 2*(b
^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2)*
x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*x^2
 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*
x + a))*log(d*x + c))*B/(a^2*b^3*c^3*g^3*i - 3*a^3*b^2*c^2*d*g^3*i + 3*a^4*
b*c*d^2*g^3*i - a^5*d^3*g^3*i + (b^5*c^3*g^3*i - 3*a*b^4*c^2*d*g^3*i + 3*a^
2*b^3*c*d^2*g^3*i - a^3*b^2*d^3*g^3*i)*x^2 + 2*(a*b^4*c^3*g^3*i - 3*a^2*b^3
*c^2*d*g^3*i + 3*a^3*b^2*c*d^2*g^3*i - a^4*b*d^3*g^3*i)*x)
```

Giac [A] (verification not implemented)

none

Time = 42.82 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.55

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx = -\frac{1}{4}\left(\frac{2\,(dx+c)^2Be^3\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^2g^3i}+\frac{(2\,Ae^3+Be^3)(dx+c)^2}{(bex+ae)^2g^3i}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i),x, algorit
hm="giac")
```

```
[Out] -1/4*(2*(d*x + c)^2*B*e^3*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^2*g^3
*i) + (2*A*e^3 + B*e^3)*(d*x + c)^2/((b*e*x + a*e)^2*g^3*i))*(b*c/((b*c*e -
 a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

Mupad [B] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 545, normalized size of antiderivative = 2.14

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx = \frac{3\,A\,a\,d}{2\,g^3\,i\,(a\,d-b\,c)^2\,(a+b\,x)^2} - \frac{B\,d^2\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2}{2\,g^3\,i\,(a\,d-b\,c)^3} - \frac{A\,b\,c}{2\,g^3\,i\,(a\,d-b\,c)^2\,(a+b\,x)^2} + \frac{7\,B\,a\,d}{4\,g^3\,i\,(a\,d-b\,c)^2\,(a+b\,x)^2} - \frac{B\,b\,c}{4\,g^3\,i\,(a\,d-b\,c)^2\,(a+b\,x)^2} + \frac{3\,B\,a^2\,d^2\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,g^3\,i\,(a\,d-b\,c)^3\,(a+b\,x)^2} + \frac{B\,b^2\,c^2\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,g^3\,i\,(a\,d-b\,c)^3\,(a+b\,x)^2} + \frac{A\,b\,d\,x}{g^3\,i\,(a\,d-b\,c)^2\,(a+b\,x)^2} + \frac{3\,B\,b\,d\,x}{2\,g^3\,i\,(a\,d-b\,c)^2\,(a+b\,x)^2} + \frac{B\,a\,b\,d^2\,x\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^3\,i\,(a\,d-b\,c)^3\,(a+b\,x)^2} - \frac{B\,b^2\,c\,d\,x\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^3\,i\,(a\,d-b\,c)^3\,(a+b\,x)^2} - \frac{2\,B\,a\,b\,c\,d\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^3\,i\,(a\,d-b\,c)^3\,(a+b\,x)^2} + \frac{A\,d^2\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{g^3\,i\,(a\,d-b\,c)^3} + \frac{B\,d^2\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,3\mathrm{i}}{g^3\,i\,(a\,d-b\,c)^3}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^3*(c*i + d*i*x)),x)
```

```
[Out] (A*d^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(g^3*i*(a*d - b*c
)^3) + (B*d^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*3i)/(g^3*i*(a*
d - b*c)^3) - (B*d^2*log((e*(a + b*x))/(c + d*x))^2)/(2*g^3*i*(a*d - b*c)^3
) + (3*A*a*d)/(2*g^3*i*(a*d - b*c)^2*(a + b*x)^2) - (A*b*c)/(2*g^3*i*(a*d -
 b*c)^2*(a + b*x)^2) + (7*B*a*d)/(4*g^3*i*(a*d - b*c)^2*(a + b*x)^2) - (B*b
*c)/(4*g^3*i*(a*d - b*c)^2*(a + b*x)^2) + (3*B*a^2*d^2*log((e*(a + b*x))/(c
 + d*x)))/(2*g^3*i*(a*d - b*c)^3*(a + b*x)^2) + (B*b^2*c^2*log((e*(a + b*x)
)/(c + d*x)))/(2*g^3*i*(a*d - b*c)^3*(a + b*x)^2) + (A*b*d*x)/(g^3*i*(a*d -
 b*c)^2*(a + b*x)^2) + (3*B*b*d*x)/(2*g^3*i*(a*d - b*c)^2*(a + b*x)^2) + (B
*a*b*d^2*x*log((e*(a + b*x))/(c + d*x)))/(g^3*i*(a*d - b*c)^3*(a + b*x)^2)
- (B*b^2*c*d*x*log((e*(a + b*x))/(c + d*x)))/(g^3*i*(a*d - b*c)^3*(a + b*x)
^2) - (2*B*a*b*c*d*log((e*(a + b*x))/(c + d*x)))/(g^3*i*(a*d - b*c)^3*(a +
b*x)^2)
```

3.38 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 373

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx = -\frac{3bBd^2(c+dx)}{(bc-ad)^4g^4i(a+bx)}+\frac{3b^2Bd(c+dx)^2}{4(bc-ad)^4g^4i(a+bx)^2} - \frac{b^3B(c+dx)^3}{9(bc-ad)^4g^4i(a+bx)^3}+\frac{Bd^3\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^4i} - \frac{3bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^4i(a+bx)} + \frac{3b^2d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2} - \frac{b^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^4g^4i(a+bx)^3} - \frac{d^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^4i}$$

```
[Out] -3*b*B*d^2*(d*x+c)/(-a*d+b*c)^4/g^4/i/(b*x+a)+3/4*b^2*B*d*(d*x+c)^2/(-a*d+b
*c)^4/g^4/i/(b*x+a)^2-1/9*b^3*B*(d*x+c)^3/(-a*d+b*c)^4/g^4/i/(b*x+a)^3+1/2*
B*d^3*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^4/g^4/i-3*b*d^2*(d*x+c)*(A+B*ln(e*(b
*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^4/i/(b*x+a)+3/2*b^2*d*(d*x+c)^2*(A+B*ln(e*(b
*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^4/i/(b*x+a)^2-1/3*b^3*(d*x+c)^3*(A+B*ln(e*(b
*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^4/i/(b*x+a)^3-d^3*ln((b*x+a)/(d*x+c))*(A+B*l
n(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^4/i
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2562, 45, 2372, 12, 14, 2338}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx = -\frac{b^3(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3g^4i(a+bx)^3(bc-ad)^4} + \frac{3b^2d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^4i(a+bx)^2(bc-ad)^4} - \frac{d^3\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^4i(bc-ad)^4} - \frac{3bd^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^4i(a+bx)(bc-ad)^4} - \frac{b^3B(c+dx)^3}{9g^4i(a+bx)^3(bc-ad)^4} + \frac{3b^2Bd(c+dx)^2}{4g^4i(a+bx)^2(bc-ad)^4} + \frac{Bd^3\log^2\left(\frac{a+bx}{c+dx}\right)}{2g^4i(bc-ad)^4} - \frac{3bBd^2(c+dx)}{g^4i(a+bx)(bc-ad)^4}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^4*(c*i + d*i*x)),x]
```

```
[Out] (-3*b*B*d^2*(c + d*x))/((b*c - a*d)^4*g^4*i*(a + b*x)) + (3*b^2*B*d*(c + d*
x)^2)/(4*(b*c - a*d)^4*g^4*i*(a + b*x)^2) - (b^3*B*(c + d*x)^3)/(9*(b*c - a
*d)^4*g^4*i*(a + b*x)^3) + (B*d^3*Log[(a + b*x)/(c + d*x)]^2)/(2*(b*c - a*d
)^4*g^4*i) - (3*b*d^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c
 - a*d)^4*g^4*i*(a + b*x)) + (3*b^2*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/
(c + d*x)]))/(2*(b*c - a*d)^4*g^4*i*(a + b*x)^2) - (b^3*(c + d*x)^3*(A + B*
Log[(e*(a + b*x))/(c + d*x)]))/(3*(b*c - a*d)^4*g^4*i*(a + b*x)^3) - (d^3*L
og[(a + b*x)/(c + d*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^
4*g^4*i)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)^3(A+B\log(ex))}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i}\\
&= -\frac{3bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^4i(a+bx)} + \frac{3b^2d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2}\\
&\quad - \frac{b^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^4g^4i(a+bx)^3} - \frac{d^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^4i}\\
&\quad - \frac{B\text{Subst}\left(\int \frac{-2b^3+9b^2dx-18bd^2x^2-6d^3x^3\log(x)}{6x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i}
\end{aligned}$$

$$= -\frac{3bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^4i(a+bx)} + \frac{3b^2d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2} - \frac{b^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^4g^4i(a+bx)^3} - \frac{d^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^4i} - \frac{B\text{Subst}\left(\int \frac{-2b^3+9b^2dx-18bd^2x^2-6d^3x^3\log(x)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{6(bc-ad)^4g^4i}$$

$$= -\frac{3bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^4i(a+bx)} + \frac{3b^2d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2} - \frac{b^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^4g^4i(a+bx)^3} - \frac{d^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^4i} - \frac{B\text{Subst}\left(\int \left(-\frac{b(2b^2-9bdx+18d^2x^2)}{x^4} - \frac{6d^3\log(x)}{x}\right)dx, x, \frac{a+bx}{c+dx}\right)}{6(bc-ad)^4g^4i}$$

$$= -\frac{3bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^4i(a+bx)} + \frac{3b^2d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2} - \frac{b^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^4g^4i(a+bx)^3} - \frac{d^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^4i} + \frac{(bB)\text{Subst}\left(\int \frac{2b^2-9bdx+18d^2x^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{6(bc-ad)^4g^4i} + \frac{(Bd^3)\,\text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i}$$

$$= \frac{Bd^3\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^4i} - \frac{3bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^4i(a+bx)} + \frac{3b^2d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2} - \frac{b^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^4g^4i(a+bx)^3} - \frac{d^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^4i} + \frac{(bB)\text{Subst}\left(\int \left(\frac{2b^2}{x^4} - \frac{9bd}{x^3} + \frac{18d^2}{x^2}\right)dx, x, \frac{a+bx}{c+dx}\right)}{6(bc-ad)^4g^4i}$$

$$
= -\frac{3bBd^2(c+dx)}{(bc-ad)^4g^4i(a+bx)} + \frac{3b^2Bd(c+dx)^2}{4(bc-ad)^4g^4i(a+bx)^2} - \frac{b^3B(c+dx)^3}{9(bc-ad)^4g^4i(a+bx)^3} + \frac{Bd^3\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^4i} - \frac{3bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^4i(a+bx)} + \frac{3b^2d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2} - \frac{b^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^4g^4i(a+bx)^3} - \frac{d^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^4i}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.39 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.32

$$
\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx
$$

$$
= \frac{-\frac{12A(bc-ad)^3}{(a+bx)^3} - \frac{4B(bc-ad)^3}{(a+bx)^3} + \frac{18Ad(bc-ad)^2}{(a+bx)^2} + \frac{15Bd(bc-ad)^2}{(a+bx)^2} + \frac{36Ad^2(-bc+ad)}{a+bx} + \frac{66Bd^2(-bc+ad)}{a+bx} - 36Ad^3\log(a+bx)}{}
$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^4*(c*i + d*i*
x)),x]
```

```
[Out] ((-12*A*(b*c - a*d)^3)/(a + b*x)^3 - (4*B*(b*c - a*d)^3)/(a + b*x)^3 + (18*
A*d*(b*c - a*d)^2)/(a + b*x)^2 + (15*B*d*(b*c - a*d)^2)/(a + b*x)^2 + (36*A
*d^2*(-(b*c) + a*d))/(a + b*x) + (66*B*d^2*(-(b*c) + a*d))/(a + b*x) - 36*A
*d^3*Log[a + b*x] - 66*B*d^3*Log[a + b*x] + 18*B*d^3*Log[a + b*x]^2 - (12*B
*(b*c - a*d)^3*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x)^3 + (18*B*d*(b*c - a
*d)^2*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x)^2 + (36*B*d^2*(-(b*c) + a*d)*
Log[(e*(a + b*x))/(c + d*x)])/(a + b*x) - 36*B*d^3*Log[a + b*x]*Log[(e*(a +
 b*x))/(c + d*x)] + 36*A*d^3*Log[c + d*x] + 66*B*d^3*Log[c + d*x] - 36*B*d^
3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] + 36*B*d^3*Log[(e*(a + b*x
))/(c + d*x)]*Log[c + d*x] + 18*B*d^3*Log[c + d*x]^2 - 36*B*d^3*Log[a + b*x
]*Log[(b*(c + d*x))/(b*c - a*d)] - 36*B*d^3*PolyLog[2, (d*(a + b*x))/(-(b*c
) + a*d)] - 36*B*d^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(36*(b*c - a*d)
^4*g^4*i)
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.34

method	result
parts	$\frac{A\left(\frac{d^3\ln(dx+c)}{(ad-cb)^4}+\frac{1}{3(ad-cb)(bx+a)^3}+\frac{d}{2(ad-cb)^2(bx+a)^2}+\frac{d^2}{(ad-cb)^3(bx+a)}-\frac{d^3\ln(bx+a)}{(ad-cb)^4}\right)}{g^4 i}-\frac{B\left(\frac{d^4\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^4}-\frac{3d^3 \cdots}{\cdots}\cdots\right.}{\cdots}$
risch	$\frac{A\,d^3\ln(dx+c)}{g^4 i(ad-cb)^4}+\frac{A}{3g^4 i(ad-cb)(bx+a)^3}+\frac{Ad}{2g^4 i(ad-cb)^2(bx+a)^2}+\frac{A\,d^2}{g^4 i(ad-cb)^3(bx+a)}-\frac{A\,d^3\ln(bx+a)}{g^4 i(ad-cb)^4}-\frac{B\,d^3\ln \cdots}{2 \cdots}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{d^2e^2A\,b^3}{3i(ad-cb)^5g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{3d^3eA\,b^2}{2i(ad-cb)^5g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{3d^4Ab}{i(ad-cb)^5g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^5A\ln\left(\frac{be}{d}+\frac{(a \cdots}{d \cdots}\right.}{ei(ad-cb) \cdots}\cdots\right.}{\cdots}$
default	$-\frac{e(ad-cb)\left(\frac{d^2e^2A\,b^3}{3i(ad-cb)^5g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{3d^3eA\,b^2}{2i(ad-cb)^5g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{3d^4Ab}{i(ad-cb)^5g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^5A\ln\left(\frac{be}{d}+\frac{(a \cdots}{d \cdots}\right.}{ei(ad-cb) \cdots}\cdots\right.}{\cdots}$
parallelrisch	$-\frac{-288A\,x^2a^5b^3c^3d^2+162A\,x^2a^4b^4c^4d-12A\,x^3a^2b^6c^5-4B\,x^3a^2b^6c^5-36A\,x^2a^3b^5c^5+189B\,x^2a^6b^2c^2d^3-258B\,x^2a^5b^3c^3 \cdots}{\cdots}$
norman	Expression too large to display

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i),x,method=_RETURNV
ERBOSE)
```

```
[Out] A/g^4/i*(d^3/(a*d-b*c)^4*ln(d*x+c)+1/3/(a*d-b*c)/(b*x+a)^3+1/2*d/(a*d-b*c)^
2/(b*x+a)^2+d^2/(a*d-b*c)^3/(b*x+a)-d^3/(a*d-b*c)^4*ln(b*x+a))-B/g^4/i/d*(1
/2*d^4/(a*d-b*c)^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-3*d^3/(a*d-b*c)^4*b*e*
(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+
(a*d-b*c)*e/d/(d*x+c)))+3*d^2/(a*d-b*c)^4*b^2*e^2*(-1/2/(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+
c))^2)-d/(a*d-b*c)^4*b^3*e^3*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d
+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.64

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx = -\frac{4\,(3\,A+B)b^3c^3-27\,(2\,A+B)ab^2c^2d+108\,(A+B)a^2bcd^2-(66\,A+85\,B)a^3d^3+6\,((6\,A+11\,B)b^3cd}{36\,((b^7}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i),x, algorit
hm="fricas")
```

```
[Out] -1/36*(4*(3*A + B)*b^3*c^3 - 27*(2*A + B)*a*b^2*c^2*d + 108*(A + B)*a^2*b*c
*d^2 - (66*A + 85*B)*a^3*d^3 + 6*((6*A + 11*B)*b^3*c*d^2 - (6*A + 11*B)*a*b
^2*d^3)*x^2 + 18*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3*x^2 + 3*B*a^2*b*d^3*x + B*a
^3*d^3)*log((b*e*x + a*e)/(d*x + c))^2 - 3*((6*A + 5*B)*b^3*c^2*d - 18*(2*A
 + 3*B)*a*b^2*c*d^2 + (30*A + 49*B)*a^2*b*d^3)*x + 6*((6*A + 11*B)*b^3*d^3*
x^3 + 2*B*b^3*c^3 - 9*B*a*b^2*c^2*d + 18*B*a^2*b*c*d^2 + 6*A*a^3*d^3 + 3*(2
*B*b^3*c*d^2 + 3*(2*A + 3*B)*a*b^2*d^3)*x^2 - 3*(B*b^3*c^2*d - 6*B*a*b^2*c*
d^2 - 6*(A + B)*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^7*c^4 - 4*a
*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4)*g^4*i*x^3 +
 3*(a*b^6*c^4 - 4*a^2*b^5*c^3*d + 6*a^3*b^4*c^2*d^2 - 4*a^4*b^3*c*d^3 + a^5
*b^2*d^4)*g^4*i*x^2 + 3*(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 6*a^4*b^3*c^2*d^2
- 4*a^5*b^2*c*d^3 + a^6*b*d^4)*g^4*i*x + (a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6
*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)*g^4*i)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1392 vs. $2(332) = 664$.

Time = 9.88 (sec) , antiderivative size = 1392, normalized size of antiderivative = 3.73

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4/(d*i*x+c*i),x)
```

```
[Out] -B*d**3*log(e*(a + b*x)/(c + d*x))**2/(2*a**4*d**4*g**4*i - 8*a**3*b*c*d**3
*g**4*i + 12*a**2*b**2*c**2*d**2*g**4*i - 8*a*b**3*c**3*d*g**4*i + 2*b**4*c
**4*g**4*i) + d**3*(6*A + 11*B)*log(x + (6*A*a*d**4 + 6*A*b*c*d**3 + 11*B*a
*d**4 + 11*B*b*c*d**3 - a**5*d**8*(6*A + 11*B)/(a*d - b*c)**4 + 5*a**4*b*c*
d**7*(6*A + 11*B)/(a*d - b*c)**4 - 10*a**3*b**2*c**2*d**6*(6*A + 11*B)/(a*d
 - b*c)**4 + 10*a**2*b**3*c**3*d**5*(6*A + 11*B)/(a*d - b*c)**4 - 5*a*b**4*
```

```
c**4*d**4*(6*A + 11*B)/(a*d - b*c)**4 + b**5*c**5*d**3*(6*A + 11*B)/(a*d -
b*c)**4)/(12*A*b*d**4 + 22*B*b*d**4))/(6*g**4*i*(a*d - b*c)**4) - d**3*(6*A
 + 11*B)*log(x + (6*A*a*d**4 + 6*A*b*c*d**3 + 11*B*a*d**4 + 11*B*b*c*d**3 +
 a**5*d**8*(6*A + 11*B)/(a*d - b*c)**4 - 5*a**4*b*c*d**7*(6*A + 11*B)/(a*d
- b*c)**4 + 10*a**3*b**2*c**2*d**6*(6*A + 11*B)/(a*d - b*c)**4 - 10*a**2*b*
*3*c**3*d**5*(6*A + 11*B)/(a*d - b*c)**4 + 5*a*b**4*c**4*d**4*(6*A + 11*B)/
(a*d - b*c)**4 - b**5*c**5*d**3*(6*A + 11*B)/(a*d - b*c)**4)/(12*A*b*d**4 +
 22*B*b*d**4))/(6*g**4*i*(a*d - b*c)**4) + (11*B*a**2*d**2 - 7*B*a*b*c*d +
15*B*a*b*d**2*x + 2*B*b**2*c**2 - 3*B*b**2*c*d*x + 6*B*b**2*d**2*x**2)*log(
e*(a + b*x)/(c + d*x))/(6*a**6*d**3*g**4*i - 18*a**5*b*c*d**2*g**4*i + 18*a
**5*b*d**3*g**4*i*x + 18*a**4*b**2*c**2*d*g**4*i - 54*a**4*b**2*c*d**2*g**4
*i*x + 18*a**4*b**2*d**3*g**4*i*x**2 - 6*a**3*b**3*c**3*g**4*i + 54*a**3*b*
*3*c**2*d*g**4*i*x - 54*a**3*b**3*c*d**2*g**4*i*x**2 + 6*a**3*b**3*d**3*g**
4*i*x**3 - 18*a**2*b**4*c**3*g**4*i*x + 54*a**2*b**4*c**2*d*g**4*i*x**2 - 1
8*a**2*b**4*c*d**2*g**4*i*x**3 - 18*a*b**5*c**3*g**4*i*x**2 + 18*a*b**5*c**
2*d*g**4*i*x**3 - 6*b**6*c**3*g**4*i*x**3) + (66*A*a**2*d**2 - 42*A*a*b*c*d
 + 12*A*b**2*c**2 + 85*B*a**2*d**2 - 23*B*a*b*c*d + 4*B*b**2*c**2 + x**2*(3
6*A*b**2*d**2 + 66*B*b**2*d**2) + x*(90*A*a*b*d**2 - 18*A*b**2*c*d + 147*B*
a*b*d**2 - 15*B*b**2*c*d))/(36*a**6*d**3*g**4*i - 108*a**5*b*c*d**2*g**4*i
+ 108*a**4*b**2*c**2*d*g**4*i - 36*a**3*b**3*c**3*g**4*i + x**3*(36*a**3*b*
*3*d**3*g**4*i - 108*a**2*b**4*c*d**2*g**4*i + 108*a*b**5*c**2*d*g**4*i - 3
6*b**6*c**3*g**4*i) + x**2*(108*a**4*b**2*d**3*g**4*i - 324*a**3*b**3*c*d**
2*g**4*i + 324*a**2*b**4*c**2*d*g**4*i - 108*a*b**5*c**3*g**4*i) + x*(108*a
**5*b*d**3*g**4*i - 324*a**4*b**2*c*d**2*g**4*i + 324*a**3*b**3*c**2*d*g**4
*i - 108*a**2*b**4*c**3*g**4*i))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1469 vs. $2(363) = 726$.

Time = 0.31 (sec) , antiderivative size = 1469, normalized size of antiderivative = 3.94

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i),x, algorit
hm="maxima")
```

```
[Out] -1/6*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
5*a*b*d^2)*x)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^
4*i*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*g
^4*i*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*
g^4*i*x + (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*g^4*i)
+ 6*d^3*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4)*g^4*i) - 6*d^3*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d +
```

```
6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i))*log(b*e*x/(d*x + c) +
a*e/(d*x + c)) - 1/6*A*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2
 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 -
 a^3*b^3*d^3)*g^4*i*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2
- a^4*b^2*d^3)*g^4*i*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d
^2 - a^5*b*d^3)*g^4*i*x + (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 -
a^6*d^3)*g^4*i) + 6*d^3*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*
c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i) - 6*d^3*log(d*x + c)/((b^4*c^4 -
4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i)) - 1/36
*(4*b^3*c^3 - 27*a*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2
 - a*b^2*d^3)*x^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3
*d^3)*log(b*x + a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x +
a^3*d^3)*log(d*x + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x
 + 66*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a
) - 6*(11*b^3*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*
(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log
(d*x + c))*B/(a^3*b^4*c^4*g^4*i - 4*a^4*b^3*c^3*d*g^4*i + 6*a^5*b^2*c^2*d^2
*g^4*i - 4*a^6*b*c*d^3*g^4*i + a^7*d^4*g^4*i + (b^7*c^4*g^4*i - 4*a*b^6*c^3
*d*g^4*i + 6*a^2*b^5*c^2*d^2*g^4*i - 4*a^3*b^4*c*d^3*g^4*i + a^4*b^3*d^4*g^
4*i)*x^3 + 3*(a*b^6*c^4*g^4*i - 4*a^2*b^5*c^3*d*g^4*i + 6*a^3*b^4*c^2*d^2*g
^4*i - 4*a^4*b^3*c*d^3*g^4*i + a^5*b^2*d^4*g^4*i)*x^2 + 3*(a^2*b^5*c^4*g^4*
i - 4*a^3*b^4*c^3*d*g^4*i + 6*a^4*b^3*c^2*d^2*g^4*i - 4*a^5*b^2*c*d^3*g^4*i
 + a^6*b*d^4*g^4*i)*x)
```

Giac [A] (verification not implemented)

none

Time = 55.04 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.72

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx = -\frac{1}{36}\left(\frac{6\left(2\,Bbe^4-\frac{3\,(bex+ae)Bde^3}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^3bcg^4i}{(dx+c)^3}-\frac{(bex+ae)^3adg^4i}{(dx+c)^3}}+\frac{12\,Abe^4+4\,Bbe^4-\frac{18\,(bex+ae)Ade^3}{dx+c}-\frac{9\,(bex+ae)Bde^3}{dx+c}}{\frac{(bex+ae)^3bcg^4i}{(dx+c)^3}-\frac{(bex+ae)^3adg^4i}{(dx+c)^3}}\right)\left(\frac{}{(bc}\right.$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i),x, algorit
hm="giac")
```

```
[Out] -1/36*(6*(2*B*b*e^4 - 3*(b*e*x + a*e)*B*d*e^3/(d*x + c))*log((b*e*x + a*e)/
(d*x + c))/((b*e*x + a*e)^3*b*c*g^4*i/(d*x + c)^3 - (b*e*x + a*e)^3*a*d*g^4
*i/(d*x + c)^3) + (12*A*b*e^4 + 4*B*b*e^4 - 18*(b*e*x + a*e)*A*d*e^3/(d*x +
 c) - 9*(b*e*x + a*e)*B*d*e^3/(d*x + c))/((b*e*x + a*e)^3*b*c*g^4*i/(d*x +
c)^3 - (b*e*x + a*e)^3*a*d*g^4*i/(d*x + c)^3))*(b*c/((b*c*e - a*d*e)*(b*c -
 a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

Mupad [B] (verification not implemented)

Time = 5.99 (sec) , antiderivative size = 970, normalized size of antiderivative = 2.60

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx = \frac{11\,A\,a^2\,d^2}{6\,g^4\,i\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^3}-\frac{B\,d^3\,{\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}^2}{2\,g^4\,i\,{\left(a\,d-b\,c\right)}^4}+\frac{A\,b^2\,c^2}{3\,g^4\,i\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^3}+\frac{85\,B\,a^2\,d^2}{36\,g^4\,i\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^3}+\frac{B\,b^2\,c^2}{9\,g^4\,i\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^3}+\frac{11\,B\,a^3\,d^3\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{6\,g^4\,i\,{\left(a\,d-b\,c\right)}^4\,{\left(a+b\,x\right)}^3}-\frac{B\,b^3\,c^3\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{3\,g^4\,i\,{\left(a\,d-b\,c\right)}^4\,{\left(a+b\,x\right)}^3}+\frac{A\,b^2\,d^2\,x^2}{g^4\,i\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^3}+\frac{11\,B\,b^2\,d^2\,x^2}{6\,g^4\,i\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^3}-\frac{7\,A\,a\,b\,c\,d}{6\,g^4\,i\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^3}-\frac{23\,B\,a\,b\,c\,d}{36\,g^4\,i\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^3}+\frac{5\,A\,a\,b\,d^2\,x}{2\,g^4\,i\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^3}+\frac{49\,B\,a\,b\,d^2\,x}{12\,g^4\,i\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^3}-\frac{A\,b^2\,c\,d\,x}{2\,g^4\,i\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^3}-\frac{5\,B\,b^2\,c\,d\,x}{12\,g^4\,i\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^3}+\frac{3\,B\,a\,b^2\,c^2\,d\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{2\,g^4\,i\,{\left(a\,d-b\,c\right)}^4\,{\left(a+b\,x\right)}^3}-\frac{3\,B\,a^2\,b\,c\,d^2\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{g^4\,i\,{\left(a\,d-b\,c\right)}^4\,{\left(a+b\,x\right)}^3}+\frac{5\,B\,a^2\,b\,d^3\,x\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{2\,g^4\,i\,{\left(a\,d-b\,c\right)}^4\,{\left(a+b\,x\right)}^3}+\frac{B\,b^3\,c^2\,d\,x\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{2\,g^4\,i\,{\left(a\,d-b\,c\right)}^4\,{\left(a+b\,x\right)}^3}+\frac{B\,a\,b^2\,d^3\,x^2\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{g^4\,i\,{\left(a\,d-b\,c\right)}^4\,{\left(a+b\,x\right)}^3}-\frac{B\,b^3\,c\,d^2\,x^2\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{g^4\,i\,{\left(a\,d-b\,c\right)}^4\,{\left(a+b\,x\right)}^3}-\frac{3\,B\,a\,b^2\,c\,d^2\,x\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{g^4\,i\,{\left(a\,d-b\,c\right)}^4\,{\left(a+b\,x\right)}^3}+\frac{A\,d^3\,\mathrm{atan}\left(\frac{a\,d\,1{}\mathrm{i}+b\,c\,1{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}\right)\,2{}\mathrm{i}}{g^4\,i\,{\left(a\,d-b\,c\right)}^4}+\frac{B\,d^3\,\mathrm{atan}\left(\frac{a\,d\,1{}\mathrm{i}+b\,c\,1{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}\right)\,11{}\mathrm{i}}{3\,g^4\,i\,{\left(a\,d-b\,c\right)}^4}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^4*(c*i + d*i*x)),x)
```

```
[Out] (A*d^3*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(g^4*i*(a*d - b*c
)^4) + (B*d^3*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*11i)/(3*g^4*i*
(a*d - b*c)^4) - (B*d^3*log((e*(a + b*x))/(c + d*x))^2)/(2*g^4*i*(a*d - b*c
)^4) + (11*A*a^2*d^2)/(6*g^4*i*(a*d - b*c)^3*(a + b*x)^3) + (A*b^2*c^2)/(3*
```

```
g^4*i*(a*d - b*c)^3*(a + b*x)^3) + (85*B*a^2*d^2)/(36*g^4*i*(a*d - b*c)^3*(
a + b*x)^3) + (B*b^2*c^2)/(9*g^4*i*(a*d - b*c)^3*(a + b*x)^3) + (11*B*a^3*d
^3*log((e*(a + b*x))/(c + d*x)))/(6*g^4*i*(a*d - b*c)^4*(a + b*x)^3) - (B*b
^3*c^3*log((e*(a + b*x))/(c + d*x)))/(3*g^4*i*(a*d - b*c)^4*(a + b*x)^3) +
(A*b^2*d^2*x^2)/(g^4*i*(a*d - b*c)^3*(a + b*x)^3) + (11*B*b^2*d^2*x^2)/(6*g
^4*i*(a*d - b*c)^3*(a + b*x)^3) - (7*A*a*b*c*d)/(6*g^4*i*(a*d - b*c)^3*(a +
 b*x)^3) - (23*B*a*b*c*d)/(36*g^4*i*(a*d - b*c)^3*(a + b*x)^3) + (5*A*a*b*d
^2*x)/(2*g^4*i*(a*d - b*c)^3*(a + b*x)^3) + (49*B*a*b*d^2*x)/(12*g^4*i*(a*d
 - b*c)^3*(a + b*x)^3) - (A*b^2*c*d*x)/(2*g^4*i*(a*d - b*c)^3*(a + b*x)^3)
- (5*B*b^2*c*d*x)/(12*g^4*i*(a*d - b*c)^3*(a + b*x)^3) + (3*B*a*b^2*c^2*d*l
og((e*(a + b*x))/(c + d*x)))/(2*g^4*i*(a*d - b*c)^4*(a + b*x)^3) - (3*B*a^2
*b*c*d^2*log((e*(a + b*x))/(c + d*x)))/(g^4*i*(a*d - b*c)^4*(a + b*x)^3) +
(5*B*a^2*b*d^3*x*log((e*(a + b*x))/(c + d*x)))/(2*g^4*i*(a*d - b*c)^4*(a +
b*x)^3) + (B*b^3*c^2*d*x*log((e*(a + b*x))/(c + d*x)))/(2*g^4*i*(a*d - b*c)
^4*(a + b*x)^3) + (B*a*b^2*d^3*x^2*log((e*(a + b*x))/(c + d*x)))/(g^4*i*(a*
d - b*c)^4*(a + b*x)^3) - (B*b^3*c*d^2*x^2*log((e*(a + b*x))/(c + d*x)))/(g
^4*i*(a*d - b*c)^4*(a + b*x)^3) - (3*B*a*b^2*c*d^2*x*log((e*(a + b*x))/(c +
 d*x)))/(g^4*i*(a*d - b*c)^4*(a + b*x)^3)
```

3.39 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 341

$$\begin{aligned}
&\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx \\
&= \frac{3B(bc-ad)^2g^3(a+bx)}{d^3i^2(c+dx)} - \frac{(6A+5B)(bc-ad)^2g^3(a+bx)}{2d^3i^2(c+dx)} \\
&\quad - \frac{3B(bc-ad)^2g^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^3i^2(c+dx)} + \frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di^2(c+dx)} \\
&\quad - \frac{(bc-ad)g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i^2(c+dx)} \\
&\quad - \frac{b(bc-ad)^2g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^4i^2} \\
&\quad - \frac{3bB(bc-ad)^2g^3\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}
\end{aligned}$$

```
[Out] 3*B*(-a*d+b*c)^2*g^3*(b*x+a)/d^3/i^2/(d*x+c)-1/2*(6*A+5*B)*(-a*d+b*c)^2*g^3
*(b*x+a)/d^3/i^2/(d*x+c)-3*B*(-a*d+b*c)^2*g^3*(b*x+a)*ln(e*(b*x+a)/(d*x+c))
/d^3/i^2/(d*x+c)+1/2*g^3*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i^2/(d*x+c
)-1/2*(-a*d+b*c)*g^3*(b*x+a)^2*(3*A+B+3*B*ln(e*(b*x+a)/(d*x+c)))/d^2/i^2/(d
*x+c)-1/2*b*(-a*d+b*c)^2*g^3*ln((-a*d+b*c)/b/(d*x+c))*(6*A+5*B+6*B*ln(e*(b*
x+a)/(d*x+c)))/d^4/i^2-3*b*B*(-a*d+b*c)^2*g^3*polylog(2,d*(b*x+a)/b/(d*x+c)
)/d^4/i^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {2562, 2384, 45, 2393, 2332, 2354, 2438}

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$$
$$= -\frac{bg^3(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A+5B\right)}{2d^4i^2}$$
$$-\frac{g^3(6A+5B)(a+bx)(bc-ad)^2}{2d^3i^2(c+dx)}-\frac{g^3(a+bx)^2(bc-ad)\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d^2i^2(c+dx)}$$
$$+\frac{g^3(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2di^2(c+dx)}-\frac{3bBg^3(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}$$
$$-\frac{3Bg^3(a+bx)(bc-ad)^2\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^3i^2(c+dx)}+\frac{3Bg^3(a+bx)(bc-ad)^2}{d^3i^2(c+dx)}$$

```
[In] Int[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x)^2,
x]
```

```
[Out] (3*B*(b*c - a*d)^2*g^3*(a + b*x))/(d^3*i^2*(c + d*x)) - ((6*A + 5*B)*(b*c -
 a*d)^2*g^3*(a + b*x))/(2*d^3*i^2*(c + d*x)) - (3*B*(b*c - a*d)^2*g^3*(a +
b*x)*Log[(e*(a + b*x))/(c + d*x)])/(d^3*i^2*(c + d*x)) + (g^3*(a + b*x)^3*(
A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d*i^2*(c + d*x)) - ((b*c - a*d)*g^3
*(a + b*x)^2*(3*A + B + 3*B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d^2*i^2*(c +
d*x)) - (b*(b*c - a*d)^2*g^3*Log[(b*c - a*d)/(b*(c + d*x))]*(6*A + 5*B + 6*
B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d^4*i^2) - (3*b*B*(b*c - a*d)^2*g^3*Pol
yLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^4*i^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)^2g^3)\,\text{Subst}\left(\int \frac{x^3(A+B\log(ex))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{i^2}$$

$$= \frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di^2(c+dx)} - \frac{((bc-ad)^2g^3)\,\text{Subst}\left(\int \frac{x^2(3A+B+3B\log(ex))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{2di^2}$$

$$=\frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di^2(c+dx)}-\frac{(bc-ad)g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i^2(c+dx)}+\frac{((bc-ad)^2g^3)\,\text{Subst}\left(\int\frac{x(3B+2(3A+B)+6B\log(ex))}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{2d^2i^2}$$

$$=\frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di^2(c+dx)}-\frac{(bc-ad)g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i^2(c+dx)}+\frac{((bc-ad)^2g^3)\,\text{Subst}\left(\int\left(-\frac{3B+2(3A+B)+6B\log(ex)}{d}-\frac{b(3B+2(3A+B)+6B\log(ex))}{d(-b+dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{2d^2i^2}$$

$$=\frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di^2(c+dx)}-\frac{(bc-ad)g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i^2(c+dx)}-\frac{((bc-ad)^2g^3)\,\text{Subst}\left(\int(3B+2(3A+B)+6B\log(ex))\,dx,x,\frac{a+bx}{c+dx}\right)}{2d^3i^2}-\frac{(b(bc-ad)^2g^3)\,\text{Subst}\left(\int\frac{3B+2(3A+B)+6B\log(ex)}{-b+dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{2d^3i^2}$$

$$=-\frac{(6A+5B)(bc-ad)^2g^3(a+bx)}{2d^3i^2(c+dx)}+\frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di^2(c+dx)}-\frac{(bc-ad)g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i^2(c+dx)}-\frac{b(bc-ad)^2g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^4i^2}+\frac{(3bB(bc-ad)^2g^3)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^4i^2}-\frac{(3B(bc-ad)^2g^3)\,\text{Subst}\left(\int\log(ex)\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i^2}$$

$$= \frac{3B(bc-ad)^2g^3(a+bx)}{d^3i^2(c+dx)} - \frac{(6A+5B)(bc-ad)^2g^3(a+bx)}{2d^3i^2(c+dx)} - \frac{3B(bc-ad)^2g^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^3i^2(c+dx)} + \frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di^2(c+dx)} - \frac{(bc-ad)g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i^2(c+dx)} - \frac{b(bc-ad)^2g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^4i^2} - \frac{3bB(bc-ad)^2g^3\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.05

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \frac{g^3\left(-2Ab^2d(2bc-3ad)x-2bBd(2bc-3ad)(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+b^3d^2x^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{2(bc-c}{}\right.}{}$$

```
[In] Integrate[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i
*x)^2,x]
```

```
[Out] (g^3*(-2*A*b^2*d*(2*b*c - 3*a*d)*x - 2*b*B*d*(2*b*c - 3*a*d)*(a + b*x)*Log[
(e*(a + b*x))/(c + d*x)] + b^3*d^2*x^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
 + (2*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x) + 2*b*B
*(2*b*c - 3*a*d)*(b*c - a*d)*Log[c + d*x] + 6*b*(b*c - a*d)^2*(A + B*Log[(e
*(a + b*x))/(c + d*x)])*Log[c + d*x] - 2*B*(b*c - a*d)^2*((b*c - a*d)/(c +
d*x) + b*Log[a + b*x] - b*Log[c + d*x]) + b*B*(-(a^2*d^2*Log[a + b*x]) + b*
(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - 3*b*B*(b*c - a*d)^2*((2*Log[(d
*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*
(c + d*x))/(b*c - a*d)])))/(2*d^4*i^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 845 vs. $2(333) = 666$.

Time = 1.67 (sec) , antiderivative size = 846, normalized size of antiderivative = 2.48

method	result
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{A\,d^2 g^3 (ad-cb)\left(-\frac{-\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}{d^3}+\frac{b^3e^3}{2d^4\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}-\frac{3be\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{d^4}-\frac{3b}{d^4\left(be-\left(\frac{be}{d}+\right.\right.}\right.}{e^2i^2}\right.}{}$
default	$-\frac{e(ad-cb)\left(-\frac{A\,d^2 g^3 (ad-cb)\left(-\frac{-\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}{d^3}+\frac{b^3e^3}{2d^4\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}-\frac{3be\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{d^4}-\frac{3b}{d^4\left(be-\left(\frac{be}{d}+\right.\right.}\right.}{e^2i^2}\right.}{}$
parts	$\frac{g^3A\left(\frac{b^2\left(\frac{1}{2}bd\,x^2+3xad-2bcx\right)}{d^3}+\frac{3b\left(a^2d^2-2abcd+b^2c^2\right)\ln(dx+c)}{d^4}-\frac{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}{d^4(dx+c)}\right)}{i^2}-\frac{g^3B\left(\frac{\left(a^3d^3-3a^2bc\,d^2-\right.}{}\right.}{}$
risch	Expression too large to display

[In] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x,method=_RETURNVERBOSE)

[Out] -1/d^2*e*(a*d-b*c)*(-A*d^2*g^3*(a*d-b*c)/e^2/i^2*(-(b*e/d+(a*d-b*c)*e/d/(d*x+c))/d^3+1/2*b^3*e^3/d^4/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2-3/d^4*b*e*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)-3/d^4*b^2*e^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d))-B*d^2*g^3*(a*d-b*c)/e^2/i^2*(-1/d^3*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)-b^3*e^3/d^3*(-1/2/b^2/e^2/d*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)+1/2/b/e/d/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)-1/2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(2*b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)/b^2/e^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2)-3/d^3*b*e*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)-3/d^3*b^2*e^2*(1/b/e/d*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d))))

Fricas [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^2}\,dx$$

[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algorithm="fricas")

[Out] integral((A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3 + (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log((b*e*x + a*e)/(d*x + c)))/(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \text{Timed out}$$

[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**2,x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1341 vs. 2(332) = 664.

Time = 0.27 (sec) , antiderivative size = 1341, normalized size of antiderivative = 3.93

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algor
ithm="maxima")
```

```
[Out] 1/2*(2*c^3/(d^5*i^2*x + c*d^4*i^2) + 6*c^2*log(d*x + c)/(d^4*i^2) + (d*x^2
- 4*c*x)/(d^3*i^2))*A*b^3*g^3 - 3*A*a*b^2*(c^2/(d^4*i^2*x + c*d^3*i^2) - x/
(d^2*i^2) + 2*c*log(d*x + c)/(d^3*i^2))*g^3 + 3*A*a^2*b*g^3*(c/(d^3*i^2*x +
 c*d^2*i^2) + log(d*x + c)/(d^2*i^2)) - B*a^3*g^3*(log(b*e*x/(d*x + c) + a*
e/(d*x + c))/(d^2*i^2*x + c*d*i^2) - 1/(d^2*i^2*x + c*d*i^2) - b*log(b*x +
a)/((b*c*d - a*d^2)*i^2) + b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) - A*a^3*g^
3/(d^2*i^2*x + c*d*i^2) - 1/2*(6*a^3*b*d^3*g^3*log(e) - (6*g^3*log(e) + 7*g
^3)*b^4*c^3 + (18*g^3*log(e) + 17*g^3)*a*b^3*c^2*d - 6*(3*g^3*log(e) + 2*g^
3)*a^2*b^2*c*d^2)*B*log(d*x + c)/(b*c*d^4*i^2 - a*d^5*i^2) + 1/2*((b^4*c*d^
3*g^3*log(e) - a*b^3*d^4*g^3*log(e))*B*x^3 - ((3*g^3*log(e) + g^3)*b^4*c^2*
d^2 - (9*g^3*log(e) + 2*g^3)*a*b^3*c*d^3 + (6*g^3*log(e) + g^3)*a^2*b^2*d^4
)*B*x^2 - ((4*g^3*log(e) + g^3)*b^4*c^3*d - 2*(5*g^3*log(e) + g^3)*a*b^3*c^
2*d^2 + (6*g^3*log(e) + g^3)*a^2*b^2*c*d^3)*B*x - 3*((b^4*c^3*d*g^3 - 3*a*b
^3*c^2*d^2*g^3 + 3*a^2*b^2*c*d^3*g^3 - a^3*b*d^4*g^3)*B*x + (b^4*c^4*g^3 -
3*a*b^3*c^3*d*g^3 + 3*a^2*b^2*c^2*d^2*g^3 - a^3*b*c*d^3*g^3)*B)*log(d*x + c
)^2 + 2*((g^3*log(e) - g^3)*b^4*c^4 - 4*(g^3*log(e) - g^3)*a*b^3*c^3*d + 6*
(g^3*log(e) - g^3)*a^2*b^2*c^2*d^2 - 3*(g^3*log(e) - g^3)*a^3*b*c*d^3)*B +
((b^4*c*d^3*g^3 - a*b^3*d^4*g^3)*B*x^3 - 3*(b^4*c^2*d^2*g^3 - 3*a*b^3*c*d^3
*g^3 + 2*a^2*b^2*d^4*g^3)*B*x^2 - (6*b^4*c^3*d*g^3 - 12*a*b^3*c^2*d^2*g^3 +
 3*a^2*b^2*c*d^3*g^3 + 5*a^3*b*d^4*g^3)*B*x - (6*a*b^3*c^3*d*g^3 - 15*a^2*b
^2*c^2*d^2*g^3 + 11*a^3*b*c*d^3*g^3)*B)*log(b*x + a) - ((b^4*c*d^3*g^3 - a*
b^3*d^4*g^3)*B*x^3 - 3*(b^4*c^2*d^2*g^3 - 3*a*b^3*c*d^3*g^3 + 2*a^2*b^2*d^4
*g^3)*B*x^2 - 2*(2*b^4*c^3*d*g^3 - 5*a*b^3*c^2*d^2*g^3 + 3*a^2*b^2*c*d^3*g^
3)*B*x + 2*(b^4*c^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 3*a^3
*b*c*d^3*g^3)*B)*log(d*x + c))/(b*c^2*d^4*i^2 - a*c*d^5*i^2 + (b*c*d^5*i^2
- a*d^6*i^2)*x) + 3*(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3)*(log(b*
x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d
)))*B/(d^4*i^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2814 vs. 2(332) = 664.

Time = 72.93 (sec) , antiderivative size = 2814, normalized size of antiderivative = 8.25

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algor
ithm="giac")
```

```
[Out] -1/24*(6*(B*b^8*c^5*e^5*g^3 - 5*B*a*b^7*c^4*d*e^5*g^3 + 10*B*a^2*b^6*c^3*d^
2*e^5*g^3 - 10*B*a^3*b^5*c^2*d^3*e^5*g^3 + 5*B*a^4*b^4*c*d^4*e^5*g^3 - B*a^
5*b^3*d^5*e^5*g^3 - 4*(b*e*x + a*e)*B*b^7*c^5*d*e^4*g^3/(d*x + c) + 20*(b*e
*x + a*e)*B*a*b^6*c^4*d^2*e^4*g^3/(d*x + c) - 40*(b*e*x + a*e)*B*a^2*b^5*c^
3*d^3*e^4*g^3/(d*x + c) + 40*(b*e*x + a*e)*B*a^3*b^4*c^2*d^4*e^4*g^3/(d*x +
 c) - 20*(b*e*x + a*e)*B*a^4*b^3*c*d^5*e^4*g^3/(d*x + c) + 4*(b*e*x + a*e)*
B*a^5*b^2*d^6*e^4*g^3/(d*x + c) + 6*(b*e*x + a*e)^2*B*b^6*c^5*d^2*e^3*g^3/(
d*x + c)^2 - 30*(b*e*x + a*e)^2*B*a*b^5*c^4*d^3*e^3*g^3/(d*x + c)^2 + 60*(b
*e*x + a*e)^2*B*a^2*b^4*c^3*d^4*e^3*g^3/(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*
a^3*b^3*c^2*d^5*e^3*g^3/(d*x + c)^2 + 30*(b*e*x + a*e)^2*B*a^4*b^2*c*d^6*e^
3*g^3/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a^5*b*d^7*e^3*g^3/(d*x + c)^2 - 4*(
b*e*x + a*e)^3*B*b^5*c^5*d^3*e^2*g^3/(d*x + c)^3 + 20*(b*e*x + a*e)^3*B*a*b
^4*c^4*d^4*e^2*g^3/(d*x + c)^3 - 40*(b*e*x + a*e)^3*B*a^2*b^3*c^3*d^5*e^2*g
^3/(d*x + c)^3 + 40*(b*e*x + a*e)^3*B*a^3*b^2*c^2*d^6*e^2*g^3/(d*x + c)^3 -
 20*(b*e*x + a*e)^3*B*a^4*b*c*d^7*e^2*g^3/(d*x + c)^3 + 4*(b*e*x + a*e)^3*B
*a^5*d^8*e^2*g^3/(d*x + c)^3)*log((b*e*x + a*e)/(d*x + c))/(b^4*d^4*e^4*i^2
 - 4*(b*e*x + a*e)*b^3*d^5*e^3*i^2/(d*x + c) + 6*(b*e*x + a*e)^2*b^2*d^6*e^
2*i^2/(d*x + c)^2 - 4*(b*e*x + a*e)^3*b*d^7*e*i^2/(d*x + c)^3 + (b*e*x + a*
e)^4*d^8*i^2/(d*x + c)^4) + (6*A*b^8*c^5*e^5*g^3 + 11*B*b^8*c^5*e^5*g^3 - 3
0*A*a*b^7*c^4*d*e^5*g^3 - 55*B*a*b^7*c^4*d*e^5*g^3 + 60*A*a^2*b^6*c^3*d^2*e
^5*g^3 + 110*B*a^2*b^6*c^3*d^2*e^5*g^3 - 60*A*a^3*b^5*c^2*d^3*e^5*g^3 - 110
*B*a^3*b^5*c^2*d^3*e^5*g^3 + 30*A*a^4*b^4*c*d^4*e^5*g^3 + 55*B*a^4*b^4*c*d^
4*e^5*g^3 - 6*A*a^5*b^3*d^5*e^5*g^3 - 11*B*a^5*b^3*d^5*e^5*g^3 - 24*(b*e*x
+ a*e)*A*b^7*c^5*d*e^4*g^3/(d*x + c) - 38*(b*e*x + a*e)*B*b^7*c^5*d*e^4*g^3
/(d*x + c) + 120*(b*e*x + a*e)*A*a*b^6*c^4*d^2*e^4*g^3/(d*x + c) + 190*(b*e
*x + a*e)*B*a*b^6*c^4*d^2*e^4*g^3/(d*x + c) - 240*(b*e*x + a*e)*A*a^2*b^5*c
^3*d^3*e^4*g^3/(d*x + c) - 380*(b*e*x + a*e)*B*a^2*b^5*c^3*d^3*e^4*g^3/(d*x
 + c) + 240*(b*e*x + a*e)*A*a^3*b^4*c^2*d^4*e^4*g^3/(d*x + c) + 380*(b*e*x
+ a*e)*B*a^3*b^4*c^2*d^4*e^4*g^3/(d*x + c) - 120*(b*e*x + a*e)*A*a^4*b^3*c*
d^5*e^4*g^3/(d*x + c) - 190*(b*e*x + a*e)*B*a^4*b^3*c*d^5*e^4*g^3/(d*x + c)
 + 24*(b*e*x + a*e)*A*a^5*b^2*d^6*e^4*g^3/(d*x + c) + 38*(b*e*x + a*e)*B*a^
5*b^2*d^6*e^4*g^3/(d*x + c) + 36*(b*e*x + a*e)^2*A*b^6*c^5*d^2*e^3*g^3/(d*x
 + c)^2 + 45*(b*e*x + a*e)^2*B*b^6*c^5*d^2*e^3*g^3/(d*x + c)^2 - 180*(b*e*x
```

```
 + a*e)^2*A*a*b^5*c^4*d^3*e^3*g^3/(d*x + c)^2 - 225*(b*e*x + a*e)^2*B*a*b^5
*c^4*d^3*e^3*g^3/(d*x + c)^2 + 360*(b*e*x + a*e)^2*A*a^2*b^4*c^3*d^4*e^3*g^
3/(d*x + c)^2 + 450*(b*e*x + a*e)^2*B*a^2*b^4*c^3*d^4*e^3*g^3/(d*x + c)^2 -
 360*(b*e*x + a*e)^2*A*a^3*b^3*c^2*d^5*e^3*g^3/(d*x + c)^2 - 450*(b*e*x + a
*e)^2*B*a^3*b^3*c^2*d^5*e^3*g^3/(d*x + c)^2 + 180*(b*e*x + a*e)^2*A*a^4*b^2
*c*d^6*e^3*g^3/(d*x + c)^2 + 225*(b*e*x + a*e)^2*B*a^4*b^2*c*d^6*e^3*g^3/(d
*x + c)^2 - 36*(b*e*x + a*e)^2*A*a^5*b*d^7*e^3*g^3/(d*x + c)^2 - 45*(b*e*x
+ a*e)^2*B*a^5*b*d^7*e^3*g^3/(d*x + c)^2 - 24*(b*e*x + a*e)^3*A*b^5*c^5*d^3
*e^2*g^3/(d*x + c)^3 - 18*(b*e*x + a*e)^3*B*b^5*c^5*d^3*e^2*g^3/(d*x + c)^3
 + 120*(b*e*x + a*e)^3*A*a*b^4*c^4*d^4*e^2*g^3/(d*x + c)^3 + 90*(b*e*x + a*
e)^3*B*a*b^4*c^4*d^4*e^2*g^3/(d*x + c)^3 - 240*(b*e*x + a*e)^3*A*a^2*b^3*c^
3*d^5*e^2*g^3/(d*x + c)^3 - 180*(b*e*x + a*e)^3*B*a^2*b^3*c^3*d^5*e^2*g^3/(
d*x + c)^3 + 240*(b*e*x + a*e)^3*A*a^3*b^2*c^2*d^6*e^2*g^3/(d*x + c)^3 + 18
0*(b*e*x + a*e)^3*B*a^3*b^2*c^2*d^6*e^2*g^3/(d*x + c)^3 - 120*(b*e*x + a*e)
^3*A*a^4*b*c*d^7*e^2*g^3/(d*x + c)^3 - 90*(b*e*x + a*e)^3*B*a^4*b*c*d^7*e^2
*g^3/(d*x + c)^3 + 24*(b*e*x + a*e)^3*A*a^5*d^8*e^2*g^3/(d*x + c)^3 + 18*(b
*e*x + a*e)^3*B*a^5*d^8*e^2*g^3/(d*x + c)^3)/(b^4*d^4*e^4*i^2 - 4*(b*e*x +
a*e)*b^3*d^5*e^3*i^2/(d*x + c) + 6*(b*e*x + a*e)^2*b^2*d^6*e^2*i^2/(d*x + c
)^2 - 4*(b*e*x + a*e)^3*b*d^7*e*i^2/(d*x + c)^3 + (b*e*x + a*e)^4*d^8*i^2/(
d*x + c)^4) + 6*(B*b^5*c^5*e*g^3 - 5*B*a*b^4*c^4*d*e*g^3 + 10*B*a^2*b^3*c^3
*d^2*e*g^3 - 10*B*a^3*b^2*c^2*d^3*e*g^3 + 5*B*a^4*b*c*d^4*e*g^3 - B*a^5*d^5
*e*g^3)*log(-b*e + (b*e*x + a*e)*d/(d*x + c))/(b*d^4*i^2) - 6*(B*b^5*c^5*e*
g^3 - 5*B*a*b^4*c^4*d*e*g^3 + 10*B*a^2*b^3*c^3*d^2*e*g^3 - 10*B*a^3*b^2*c^2
*d^3*e*g^3 + 5*B*a^4*b*c*d^4*e*g^3 - B*a^5*d^5*e*g^3)*log((b*e*x + a*e)/(d*
x + c))/(b*d^4*i^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d
*e)*(b*c - a*d)))^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx=\int \frac{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{(c\,i+d\,i\,x)^2}\,dx$$

```
[In] int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^2,
x)

[Out] int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^2,
 x)
```

3.40 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 260

$$\begin{aligned}\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx &= -\frac{2B(bc-ad)g^2(a+bx)}{d^2i^2(c+dx)}+\frac{(2A+B)(bc-ad)g^2(a+bx)}{d^2i^2(c+dx)}\\ &\quad+\frac{2B(bc-ad)g^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^2i^2(c+dx)}+\frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di^2(c+dx)}\\ &\quad+\frac{b(bc-ad)g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i^2}\\ &\quad+\frac{2bB(bc-ad)g^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2}\end{aligned}$$

```
[Out] -2*B*(-a*d+b*c)*g^2*(b*x+a)/d^2/i^2/(d*x+c)+(2*A+B)*(-a*d+b*c)*g^2*(b*x+a)/
d^2/i^2/(d*x+c)+2*B*(-a*d+b*c)*g^2*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/d^2/i^2/(d
*x+c)+g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i^2/(d*x+c)+b*(-a*d+b*c)*
g^2*ln((-a*d+b*c)/b/(d*x+c))*(2*A+B+2*B*ln(e*(b*x+a)/(d*x+c)))/d^3/i^2+2*b*
B*(-a*d+b*c)*g^2*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {2562, 2384, 45, 2393, 2332, 2354, 2438}

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$$
$$=\frac{bg^2(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{d^3i^2}+\frac{g^2(2A+B)(a+bx)(bc-ad)}{d^2i^2(c+dx)}$$
$$+\frac{g^2(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{di^2(c+dx)}+\frac{2bBg^2(bc-ad)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2}$$
$$+\frac{2Bg^2(a+bx)(bc-ad)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^2i^2(c+dx)}-\frac{2Bg^2(a+bx)(bc-ad)}{d^2i^2(c+dx)}$$

```
[In] Int[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x)^2,
x]
```

```
[Out] (-2*B*(b*c - a*d)*g^2*(a + b*x))/(d^2*i^2*(c + d*x)) + ((2*A + B)*(b*c - a*
d)*g^2*(a + b*x))/(d^2*i^2*(c + d*x)) + (2*B*(b*c - a*d)*g^2*(a + b*x)*Log[
(e*(a + b*x))/(c + d*x)])/(d^2*i^2*(c + d*x)) + (g^2*(a + b*x)^2*(A + B*Log
[(e*(a + b*x))/(c + d*x)]))/(d*i^2*(c + d*x)) + (b*(b*c - a*d)*g^2*Log[(b*c
 - a*d)/(b*(c + d*x))]*(2*A + B + 2*B*Log[(e*(a + b*x))/(c + d*x)]))/(d^3*i
^2) + (2*b*B*(b*c - a*d)*g^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^3*
i^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
```

```
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)g^2)\,\text{Subst}\left(\int \frac{x^2(A+B\log(ex))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{i^2}$$

$$= \frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di^2(c+dx)} - \frac{((bc-ad)g^2)\,\text{Subst}\left(\int \frac{x(2A+B+2B\log(ex))}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{di^2}$$

$$= \frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di^2(c+dx)} - \frac{((bc-ad)g^2)\,\text{Subst}\left(\int \left(-\frac{2A+B+2B\log(ex)}{d} - \frac{b(2A+B+2B\log(ex))}{d(-b+dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{di^2}$$

$$
\begin{aligned}
&= \frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di^2(c+dx)} \\
&+ \frac{((bc-ad)g^2)\operatorname{Subst}\left(\int(2A+B+2B\log(ex))\,dx,x,\frac{a+bx}{c+dx}\right)}{d^2i^2} \\
&+ \frac{(b(bc-ad)g^2)\operatorname{Subst}\left(\int\frac{2A+B+2B\log(ex)}{-b+dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^2i^2} \\
&= \frac{(2A+B)(bc-ad)g^2(a+bx)}{d^2i^2(c+dx)} + \frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di^2(c+dx)} \\
&+ \frac{b(bc-ad)g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i^2} \\
&- \frac{(2bB(bc-ad)g^2)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i^2} \\
&+ \frac{(2B(bc-ad)g^2)\operatorname{Subst}\left(\int\log(ex)\,dx,x,\frac{a+bx}{c+dx}\right)}{d^2i^2} \\
&= -\frac{2B(bc-ad)g^2(a+bx)}{d^2i^2(c+dx)} + \frac{(2A+B)(bc-ad)g^2(a+bx)}{d^2i^2(c+dx)} \\
&+ \frac{2B(bc-ad)g^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^2i^2(c+dx)} + \frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di^2(c+dx)} \\
&+ \frac{b(bc-ad)g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i^2} \\
&+ \frac{2bB(bc-ad)g^2\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.92

$$
\int\frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx
$$

$$
= \frac{g^2\left(Ab^2dx+\frac{B(bc-ad)^2}{c+dx}+bB(bc-ad)\log(a+bx)+bBd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-\frac{(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{c+dx}-\right.}{}
$$

```
[In] Integrate[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i
*x)^2,x]
```

```
[Out] (g^2*(A*b^2*d*x + (B*(b*c - a*d)^2)/(c + d*x) + b*B*(b*c - a*d)*Log[a + b*x
] + b*B*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - ((b*c - a*d)^2*(A + B*Lo
```

```
g[(e*(a + b*x))/(c + d*x)]))/(c + d*x) - 2*b*B*(b*c - a*d)*Log[c + d*x] - 2
*b*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + b*B*(b*c
 - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x]
+ 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(d^3*i^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(260) = 520$.

Time = 1.49 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.05

method	result
parts	$\frac{g^2A\left(\frac{x b^2}{d^2}+\frac{2b(ad-cb)\ln(dx+c)}{d^3}-\frac{a^2d^2-2abcd+b^2c^2}{d^3(dx+c)}\right)}{i^2}-\frac{g^2B\left(\frac{\left(a^2d^2-2abcd+b^2c^2\right)\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad}{d(d}\right.}{d^2}\right.}{}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{g^2d^2A\left(\frac{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}{d^2}+\frac{b^2e^2}{d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{2be\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{d^3}\right)}{e^2i^2}+\frac{g^2d^2B\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{}\right.}{}\right.}{}$
default	$-\frac{e(ad-cb)\left(\frac{g^2d^2A\left(\frac{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}{d^2}+\frac{b^2e^2}{d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{2be\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{d^3}\right)}{e^2i^2}+\frac{g^2d^2B\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{}\right.}{}\right.}{}$
risch	Expression too large to display

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x,method=_RETUR
NVERBOSE)
```

```
[Out] g^2*A/i^2*(x*b^2/d^2+2*b/d^3*(a*d-b*c)*ln(d*x+c)-1/d^3*(a^2*d^2-2*a*b*c*d+b
^2*c^2)/(d*x+c))-g^2*B/i^2/(a*d-b*c)/e*(1/d^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(
(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d
/(d*x+c)-b*e/d)+2*b/d^2*e*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(dilog(-((b*e/d+(a*d-
b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d
+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)+e^2*b^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/
d^2*(1/b/e/d*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)-ln(b*e/d+(a*d-b*c)*e/d
/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*
d-b*e)))
```

Fricas [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algor
ithm="fricas")
```

```
[Out] integral((A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*B*
a*b*g^2*x + B*a^2*g^2)*log((b*e*x + a*e)/(d*x + c)))/(d^2*i^2*x^2 + 2*c*d*i
^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 886 vs. $2(259) = 518$.

Time = 0.27 (sec) , antiderivative size = 886, normalized size of antiderivative = 3.41

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$$
$$= -Ab^2\left(\frac{c^2}{d^4i^2x+cd^3i^2}-\frac{x}{d^2i^2}+\frac{2\,c\log(dx+c)}{d^3i^2}\right)g^2$$
$$+2\,Aabg^2\left(\frac{c}{d^3i^2x+cd^2i^2}+\frac{\log(dx+c)}{d^2i^2}\right)$$
$$-Ba^2g^2\left(\frac{\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{d^2i^2x+cdi^2}-\frac{1}{d^2i^2x+cdi^2}-\frac{b\log(bx+a)}{(bcd-ad^2)i^2}+\frac{b\log(dx+c)}{(bcd-ad^2)i^2}\right)$$
$$-\frac{Aa^2g^2}{d^2i^2x+cdi^2}$$
$$-\frac{(2\,a^2bd^2g^2\log(e)+2\,(g^2\log(e)+g^2)b^3c^2-(4\,g^2\log(e)+3\,g^2)ab^2cd)B\log(dx+c)}{bcd^3i^2-ad^4i^2}$$
$$+\frac{(b^3cd^2g^2\log(e)-ab^2d^3g^2\log(e))Bx^2+(b^3c^2dg^2\log(e)-ab^2cd^2g^2\log(e))Bx+((b^3c^2dg^2-2\,ab^2cd^2g^2-}{}$$
$$-\frac{2\,(b^2cg^2-abdg^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{d^3i^2}$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algor
ithm="maxima")
```

```
[Out] -A*b^2*(c^2/(d^4*i^2*x + c*d^3*i^2) - x/(d^2*i^2) + 2*c*log(d*x + c)/(d^3*i
^2))*g^2 + 2*A*a*b*g^2*(c/(d^3*i^2*x + c*d^2*i^2) + log(d*x + c)/(d^2*i^2))
 - B*a^2*g^2*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^2*i^2*x + c*d*i^2) -
1/(d^2*i^2*x + c*d*i^2) - b*log(b*x + a)/((b*c*d - a*d^2)*i^2) + b*log(d*x
+ c)/((b*c*d - a*d^2)*i^2)) - A*a^2*g^2/(d^2*i^2*x + c*d*i^2) - (2*a^2*b*d^
2*g^2*log(e) + 2*(g^2*log(e) + g^2)*b^3*c^2 - (4*g^2*log(e) + 3*g^2)*a*b^2*
c*d)*B*log(d*x + c)/(b*c*d^3*i^2 - a*d^4*i^2) + ((b^3*c*d^2*g^2*log(e) - a*
b^2*d^3*g^2*log(e))*B*x^2 + (b^3*c^2*d*g^2*log(e) - a*b^2*c*d^2*g^2*log(e))
*B*x + ((b^3*c^2*d*g^2 - 2*a*b^2*c*d^2*g^2 + a^2*b*d^3*g^2)*B*x + (b^3*c^3*
g^2 - 2*a*b^2*c^2*d*g^2 + a^2*b*c*d^2*g^2)*B)*log(d*x + c)^2 - ((g^2*log(e)
 - g^2)*b^3*c^3 - 3*(g^2*log(e) - g^2)*a*b^2*c^2*d + 2*(g^2*log(e) - g^2)*a
^2*b*c*d^2)*B + ((b^3*c*d^2*g^2 - a*b^2*d^3*g^2)*B*x^2 + (2*b^3*c^2*d*g^2 -
 2*a*b^2*c*d^2*g^2 - a^2*b*d^3*g^2)*B*x + (2*a*b^2*c^2*d*g^2 - 3*a^2*b*c*d^
2*g^2)*B)*log(b*x + a) - ((b^3*c*d^2*g^2 - a*b^2*d^3*g^2)*B*x^2 + (b^3*c^2*
d*g^2 - a*b^2*c*d^2*g^2)*B*x - (b^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 2*a^2*b*c
*d^2*g^2)*B)*log(d*x + c))/(b*c^2*d^3*i^2 - a*c*d^4*i^2 + (b*c*d^4*i^2 - a*
d^5*i^2)*x) - 2*(b^2*c*g^2 - a*b*d*g^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*
c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(d^3*i^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1774 vs. 2(259) = 518.

Time = 63.25 (sec) , antiderivative size = 1774, normalized size of antiderivative = 6.82

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algor
ithm="giac")
```

```
[Out] 1/6*(2*(B*b^6*c^4*e^4*g^2 - 4*B*a*b^5*c^3*d*e^4*g^2 + 6*B*a^2*b^4*c^2*d^2*e
^4*g^2 - 4*B*a^3*b^3*c*d^3*e^4*g^2 + B*a^4*b^2*d^4*e^4*g^2 - 3*(b*e*x + a*e
)*B*b^5*c^4*d*e^3*g^2/(d*x + c) + 12*(b*e*x + a*e)*B*a*b^4*c^3*d^2*e^3*g^2/
(d*x + c) - 18*(b*e*x + a*e)*B*a^2*b^3*c^2*d^3*e^3*g^2/(d*x + c) + 12*(b*e*
x + a*e)*B*a^3*b^2*c*d^4*e^3*g^2/(d*x + c) - 3*(b*e*x + a*e)*B*a^4*b*d^5*e^
3*g^2/(d*x + c) + 3*(b*e*x + a*e)^2*B*b^4*c^4*d^2*e^2*g^2/(d*x + c)^2 - 12*
(b*e*x + a*e)^2*B*a*b^3*c^3*d^3*e^2*g^2/(d*x + c)^2 + 18*(b*e*x + a*e)^2*B*
a^2*b^2*c^2*d^4*e^2*g^2/(d*x + c)^2 - 12*(b*e*x + a*e)^2*B*a^3*b*c*d^5*e^2*
g^2/(d*x + c)^2 + 3*(b*e*x + a*e)^2*B*a^4*d^6*e^2*g^2/(d*x + c)^2)*log((b*e
*x + a*e)/(d*x + c))/(b^3*d^3*e^3*i^2 - 3*(b*e*x + a*e)*b^2*d^4*e^2*i^2/(d*
x + c) + 3*(b*e*x + a*e)^2*b*d^5*e*i^2/(d*x + c)^2 - (b*e*x + a*e)^3*d^6*i^
2/(d*x + c)^3) + (2*A*b^6*c^4*e^4*g^2 + 3*B*b^6*c^4*e^4*g^2 - 8*A*a*b^5*c^3
*d*e^4*g^2 - 12*B*a*b^5*c^3*d*e^4*g^2 + 12*A*a^2*b^4*c^2*d^2*e^4*g^2 + 18*B
*a^2*b^4*c^2*d^2*e^4*g^2 - 8*A*a^3*b^3*c*d^3*e^4*g^2 - 12*B*a^3*b^3*c*d^3*e
^4*g^2 + 2*A*a^4*b^2*d^4*e^4*g^2 + 3*B*a^4*b^2*d^4*e^4*g^2 - 6*(b*e*x + a*e
)*A*b^5*c^4*d*e^3*g^2/(d*x + c) - 7*(b*e*x + a*e)*B*b^5*c^4*d*e^3*g^2/(d*x
+ c) + 24*(b*e*x + a*e)*A*a*b^4*c^3*d^2*e^3*g^2/(d*x + c) + 28*(b*e*x + a*e
)*B*a*b^4*c^3*d^2*e^3*g^2/(d*x + c) - 36*(b*e*x + a*e)*A*a^2*b^3*c^2*d^3*e^
3*g^2/(d*x + c) - 42*(b*e*x + a*e)*B*a^2*b^3*c^2*d^3*e^3*g^2/(d*x + c) + 24
*(b*e*x + a*e)*A*a^3*b^2*c*d^4*e^3*g^2/(d*x + c) + 28*(b*e*x + a*e)*B*a^3*b
^2*c*d^4*e^3*g^2/(d*x + c) - 6*(b*e*x + a*e)*A*a^4*b*d^5*e^3*g^2/(d*x + c)
- 7*(b*e*x + a*e)*B*a^4*b*d^5*e^3*g^2/(d*x + c) + 6*(b*e*x + a*e)^2*A*b^4*c
^4*d^2*e^2*g^2/(d*x + c)^2 + 4*(b*e*x + a*e)^2*B*b^4*c^4*d^2*e^2*g^2/(d*x +
 c)^2 - 24*(b*e*x + a*e)^2*A*a*b^3*c^3*d^3*e^2*g^2/(d*x + c)^2 - 16*(b*e*x
+ a*e)^2*B*a*b^3*c^3*d^3*e^2*g^2/(d*x + c)^2 + 36*(b*e*x + a*e)^2*A*a^2*b^2
*c^2*d^4*e^2*g^2/(d*x + c)^2 + 24*(b*e*x + a*e)^2*B*a^2*b^2*c^2*d^4*e^2*g^2
/(d*x + c)^2 - 24*(b*e*x + a*e)^2*A*a^3*b*c*d^5*e^2*g^2/(d*x + c)^2 - 16*(b
*e*x + a*e)^2*B*a^3*b*c*d^5*e^2*g^2/(d*x + c)^2 + 6*(b*e*x + a*e)^2*A*a^4*d
^6*e^2*g^2/(d*x + c)^2 + 4*(b*e*x + a*e)^2*B*a^4*d^6*e^2*g^2/(d*x + c)^2)/(
b^3*d^3*e^3*i^2 - 3*(b*e*x + a*e)*b^2*d^4*e^2*i^2/(d*x + c) + 3*(b*e*x + a*
e)^2*b*d^5*e*i^2/(d*x + c)^2 - (b*e*x + a*e)^3*d^6*i^2/(d*x + c)^3) + 2*(B*
b^4*c^4*e*g^2 - 4*B*a*b^3*c^3*d*e*g^2 + 6*B*a^2*b^2*c^2*d^2*e*g^2 - 4*B*a^3
*b*c*d^3*e*g^2 + B*a^4*d^4*e*g^2)*log(-b*e + (b*e*x + a*e)*d/(d*x + c))/(b*
```

```
d^3*i^2) - 2*(B*b^4*c^4*e*g^2 - 4*B*a*b^3*c^3*d*e*g^2 + 6*B*a^2*b^2*c^2*d^2
*e*g^2 - 4*B*a^3*b*c*d^3*e*g^2 + B*a^4*d^4*e*g^2)*log((b*e*x + a*e)/(d*x +
c))/(b*d^3*i^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*
(b*c - a*d)))^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx=\int \frac{{\left(a\,g+b\,g\,x\right)}^2\,\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}{{\left(c\,i+d\,i\,x\right)}^2}\,dx$$

```
[In] int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^2,
x)

[Out] int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^2,
 x)
```

3.41 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 160

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = -\frac{Ag(a+bx)}{di^2(c+dx)}+\frac{Bg(a+bx)}{di^2(c+dx)} - \frac{Bg(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{di^2(c+dx)} - \frac{bg\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2i^2} - \frac{bBg\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2}$$

```
[Out] -A*g*(b*x+a)/d/i^2/(d*x+c)+B*g*(b*x+a)/d/i^2/(d*x+c)-B*g*(b*x+a)*ln(e*(b*x+
a)/(d*x+c))/d/i^2/(d*x+c)-b*g*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d
*x+c)))/d^2/i^2-b*B*g*polylog(2,d*(b*x+a)/b/(d*x+c))/d^2/i^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used

= {2562, 45, 2393, 2332, 2354, 2438}

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = -\frac{bg\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^2i^2} - \frac{Ag(a+bx)}{di^2(c+dx)} - \frac{bBg\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2} - \frac{Bg(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{di^2(c+dx)} + \frac{Bg(a+bx)}{di^2(c+dx)}$$

```
[In] Int[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x)^2,x]
```

```
[Out] -((A*g*(a + b*x))/(d*i^2*(c + d*x))) + (B*g*(a + b*x))/(d*i^2*(c + d*x)) -
(B*g*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(d*i^2*(c + d*x)) - (b*g*Log[(
b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d^2*i^2) -
 (b*B*g*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^2*i^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{g\text{Subst}\left(\int \frac{x(A+B\log(ex))}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{i^2} \\
&= \frac{g\text{Subst}\left(\int \left(-\frac{A+B\log(ex)}{d} - \frac{b(A+B\log(ex))}{d(-b+dx)}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{i^2} \\
&= -\frac{g\text{Subst}\left(\int (A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{di^2} - \frac{(bg)\text{Subst}\left(\int \frac{A+B\log(ex)}{-b+dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{di^2} \\
&= -\frac{Ag(a+bx)}{di^2(c+dx)} - \frac{bg\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2i^2} \\
&\quad + \frac{(bBg)\text{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^2} - \frac{(Bg)\text{Subst}\left(\int \log(ex)\,dx, x, \frac{a+bx}{c+dx}\right)}{di^2} \\
&= -\frac{Ag(a+bx)}{di^2(c+dx)} + \frac{Bg(a+bx)}{di^2(c+dx)} - \frac{Bg(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{di^2(c+dx)} \\
&\quad - \frac{bg\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2i^2} - \frac{bBg\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.09

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$$
$$= \frac{g\left(\frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{c+dx}+2b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(c+dx)-2B\left(\frac{bc-ad}{c+dx}+b\log(a+bx)-b\log(c+\right.\right.}{2d^2i^2}$$

```
[In] Integrate[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x
)^2,x]
```

```
[Out] (g*((2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x) + 2*b*(A
 + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 2*B*((b*c - a*d)/(c + d*x
) + b*Log[a + b*x] - b*Log[c + d*x]) - b*B*((2*Log[(d*(a + b*x))/(-(b*c) +
a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)
])))/(2*d^2*i^2)
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.84

method	result
parts	$\frac{gA\left(\frac{b\ln(dx+c)}{d^2}-\frac{ad-cb}{d^2(dx+c)}\right)}{i^2}-\frac{gB\left(\frac{(ad-cb)\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad-cb)e}{d(dx+c)}-\frac{be}{d}\right)}{d}+\frac{be(ad-cb)\left(\frac{\operatorname{dilog}\left(-\frac{(\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)}{i^2(ad-cb)e\cdots}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{g\,d^2A\left(-\frac{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}{d}-\frac{be\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{d^2}\right)}{(ad-cb)e^2i^2}-\frac{g\,d^2B\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad\cdots}{d(\cdots}}{d}\cdots\right)}{\cdots}\right)}{d^2\cdots}$
default	$-\frac{e(ad-cb)\left(-\frac{g\,d^2A\left(-\frac{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}{d}-\frac{be\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{d^2}\right)}{(ad-cb)e^2i^2}-\frac{g\,d^2B\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad\cdots}{d(\cdots}}{d}\cdots\right)}{\cdots}\right)}{d^2\cdots}$
risch	$\frac{gAb\ln(dx+c)}{i^2d^2}-\frac{gAa}{i^2d(dx+c)}+\frac{gAcb}{i^2d^2(dx+c)}-\frac{gB\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)ba}{i^2(ad-cb)d}+\frac{gB\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)b^2c}{i^2(ad-cb)d^2}-\frac{gB\ln\left(\frac{be}{d}+\cdots\right.}{i^2(ad-c\cdots}$

```
[In] int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x,method=_RETURNV
ERBOSE)
```

```
[Out] g*A/i^2*(b/d^2*ln(d*x+c)-(a*d-b*c)/d^2/(d*x+c))-g*B/i^2/(a*d-b*c)/e*((a*d-b
*c)/d*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b
*c)*e/d/(d*x+c)-b*e/d)+b*e*(a*d-b*c)/d*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c
))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/
(d*x+c))*d-b*e)/b/e)/d))
```

Fricas [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algorit
hm="fricas")
```

```
[Out] integral((A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((b*e*x + a*e)/(d*x + c)))
/(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algorit
hm="maxima")
```

```
[Out] -1/2*B*b*g*(((d*x + c)*log(d*x + c)^2 + 2*c*log(d*x + c))/(d^3*i^2*x + c*d^
2*i^2) - 2*integrate((d*x*log(b*x + a) + d*x*log(e) + c)/(d^3*i^2*x^2 + 2*c
*d^2*i^2*x + c^2*d*i^2), x)) + A*b*g*(c/(d^3*i^2*x + c*d^2*i^2) + log(d*x +
 c)/(d^2*i^2)) - B*a*g*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^2*i^2*x + c
*d*i^2) - 1/(d^2*i^2*x + c*d*i^2) - b*log(b*x + a)/((b*c*d - a*d^2)*i^2) +
b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) - A*a*g/(d^2*i^2*x + c*d*i^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 895 vs. $2(159) = 318$.

Time = 48.78 (sec) , antiderivative size = 895, normalized size of antiderivative = 5.59

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx =$$

$$-\frac{1}{2}\left(\frac{\left(Bb^4c^3e^3g - 3\,Bab^3c^2de^3g + 3\,Ba^2b^2cd^2e^3g - Ba^3bd^3e^3g - \frac{2\,(bex+ae)Bb^3c^3de^2g}{dx+c} + \frac{6\,(bex+ae)Bab^2c^2d^2e^2g}{dx+c}\right.}{b^2d^2e^2i^2 - \frac{2\,(bex+ae)bd^3ei^2}{dx+c} + \frac{(bex+ae)^2d^4i^2}{(dx+c)^2}}\right.$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algorit
hm="giac")
```

```
[Out] -1/2*((B*b^4*c^3*e^3*g - 3*B*a*b^3*c^2*d*e^3*g + 3*B*a^2*b^2*c*d^2*e^3*g -
B*a^3*b*d^3*e^3*g - 2*(b*e*x + a*e)*B*b^3*c^3*d*e^2*g/(d*x + c) + 6*(b*e*x
+ a*e)*B*a*b^2*c^2*d^2*e^2*g/(d*x + c) - 6*(b*e*x + a*e)*B*a^2*b*c*d^3*e^2*
g/(d*x + c) + 2*(b*e*x + a*e)*B*a^3*d^4*e^2*g/(d*x + c))*log((b*e*x + a*e)/
(d*x + c))/(b^2*d^2*e^2*i^2 - 2*(b*e*x + a*e)*b*d^3*e*i^2/(d*x + c) + (b*e*
x + a*e)^2*d^4*i^2/(d*x + c)^2) + (A*b^4*c^3*e^3*g + B*b^4*c^3*e^3*g - 3*A*
a*b^3*c^2*d*e^3*g - 3*B*a*b^3*c^2*d*e^3*g + 3*A*a^2*b^2*c*d^2*e^3*g + 3*B*a
^2*b^2*c*d^2*e^3*g - A*a^3*b*d^3*e^3*g - B*a^3*b*d^3*e^3*g - 2*(b*e*x + a*e
)*A*b^3*c^3*d*e^2*g/(d*x + c) - (b*e*x + a*e)*B*b^3*c^3*d*e^2*g/(d*x + c) +
 6*(b*e*x + a*e)*A*a*b^2*c^2*d^2*e^2*g/(d*x + c) + 3*(b*e*x + a*e)*B*a*b^2*
c^2*d^2*e^2*g/(d*x + c) - 6*(b*e*x + a*e)*A*a^2*b*c*d^3*e^2*g/(d*x + c) - 3
*(b*e*x + a*e)*B*a^2*b*c*d^3*e^2*g/(d*x + c) + 2*(b*e*x + a*e)*A*a^3*d^4*e^
2*g/(d*x + c) + (b*e*x + a*e)*B*a^3*d^4*e^2*g/(d*x + c))/(b^2*d^2*e^2*i^2 -
 2*(b*e*x + a*e)*b*d^3*e*i^2/(d*x + c) + (b*e*x + a*e)^2*d^4*i^2/(d*x + c)^
2) + (B*b^3*c^3*e*g - 3*B*a*b^2*c^2*d*e*g + 3*B*a^2*b*c*d^2*e*g - B*a^3*d^3
*e*g)*log(-b*e + (b*e*x + a*e)*d/(d*x + c))/(b*d^2*i^2) - (B*b^3*c^3*e*g -
3*B*a*b^2*c^2*d*e*g + 3*B*a^2*b*c*d^2*e*g - B*a^3*d^3*e*g)*log((b*e*x + a*e
)/(d*x + c))/(b*d^2*i^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e
- a*d*e)*(b*c - a*d)))^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{{(c\,i+d\,i\,x)}^2}\,dx$$

```
[In] int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^2,x)
```

```
[Out] int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^2, x
)
```

3.42 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 98

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx = \frac{A(a+bx)}{(bc-ad)i^2(c+dx)} - \frac{B(a+bx)}{(bc-ad)i^2(c+dx)} + \frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)i^2(c+dx)}$$

```
[Out] A*(b*x+a)/(-a*d+b*c)/i^2/(d*x+c)-B*(b*x+a)/(-a*d+b*c)/i^2/(d*x+c)+B*(b*x+a)
*ln(e*(b*x+a)/(d*x+c))/(-a*d+b*c)/i^2/(d*x+c)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2552, 2332}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx = \frac{A(a+bx)}{i^2(c+dx)(bc-ad)} + \frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{i^2(c+dx)(bc-ad)} - \frac{B(a+bx)}{i^2(c+dx)(bc-ad)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(c*i + d*i*x)^2,x]
```

```
[Out] (A*(a + b*x))/((b*c - a*d)*i^2*(c + d*x)) - (B*(a + b*x))/((b*c - a*d)*i^2*
(c + d*x)) + (B*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/((b*c - a*d)*i^2*(c
 + d*x))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int (A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}\\ &= \frac{A(a+bx)}{(bc-ad)i^2(c+dx)} + \frac{B\text{Subst}\left(\int \log(ex)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}\\ &= \frac{A(a+bx)}{(bc-ad)i^2(c+dx)} - \frac{B(a+bx)}{(bc-ad)i^2(c+dx)} + \frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)i^2(c+dx)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx$$
$$= \frac{Abc - bBc - aAd + aBd - bB(c+dx)\log(a+bx) + B(bc-ad)\log\left(\frac{e(a+bx)}{c+dx}\right) + bBc\log(c+dx) + bBdx}{d(-bc+ad)i^2(c+dx)}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(c*i + d*i*x)^2,x]
```

```
[Out] (A*b*c - b*B*c - a*A*d + a*B*d - b*B*(c + d*x)*Log[a + b*x] + B*(b*c - a*d)
*Log[(e*(a + b*x))/(c + d*x)] + b*B*c*Log[c + d*x] + b*B*d*x*Log[c + d*x])/
(d*(-(b*c) + a*d)*i^2*(c + d*x))
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.84

method	result	size
parts	$-\frac{A}{i^{2}\left(dx+c\right)d}-\frac{B\left(\frac{e\left(bx+a\right)\ln\left(\frac{e\left(bx+a\right)}{dx+c}\right)}{dx+c}-\frac{e\left(bx+a\right)}{dx+c}\right)}{i^{2}e\left(ad-cb\right)}$	82
norman	$\frac{\frac{\left(-B+A\right)x}{ic}-\frac{aB\ln\left(\frac{e\left(bx+a\right)}{dx+c}\right)}{\left(ad-cb\right)i}-\frac{Bbx\ln\left(\frac{e\left(bx+a\right)}{dx+c}\right)}{\left(ad-cb\right)i}}{i\left(dx+c\right)}$	91
parallelrisch	$-\frac{Bx\ln\left(\frac{e\left(bx+a\right)}{dx+c}\right)b^{2}d^{3}+B\ln\left(\frac{e\left(bx+a\right)}{dx+c}\right)ab\,d^{3}-Bab\,d^{3}+B\,b^{2}c\,d^{2}+Aab\,d^{3}-A\,b^{2}c\,d^{2}}{i^{2}\left(dx+c\right)b\,d^{3}\left(ad-cb\right)}$	110
risch	$-\frac{B\ln\left(\frac{e\left(bx+a\right)}{dx+c}\right)}{d\,i^{2}\left(dx+c\right)}-\frac{B\ln\left(bx+a\right)bdx-B\ln\left(-dx-c\right)bdx+B\ln\left(bx+a\right)bc-B\ln\left(-dx-c\right)bc+Aad-Abc-Bad+Bbc}{i^{2}\left(dx+c\right)d\left(ad-cb\right)}$	127
derivativedivides	$-\frac{e\left(ad-cb\right)\left(\frac{d^{2}A\left(\frac{be}{d}+\frac{\left(ad-cb\right)e}{d\left(dx+c\right)}\right)}{\left(ad-cb\right)^{2}e^{2}i^{2}}+\frac{d^{2}B\left(\left(\frac{be}{d}+\frac{\left(ad-cb\right)e}{d\left(dx+c\right)}\right)\ln\left(\frac{be}{d}+\frac{\left(ad-cb\right)e}{d\left(dx+c\right)}\right)-\frac{\left(ad-cb\right)e}{d\left(dx+c\right)}-\frac{be}{d}\right)}{\left(ad-cb\right)^{2}e^{2}i^{2}}\right)}{d^{2}}$	170
default	$-\frac{e\left(ad-cb\right)\left(\frac{d^{2}A\left(\frac{be}{d}+\frac{\left(ad-cb\right)e}{d\left(dx+c\right)}\right)}{\left(ad-cb\right)^{2}e^{2}i^{2}}+\frac{d^{2}B\left(\left(\frac{be}{d}+\frac{\left(ad-cb\right)e}{d\left(dx+c\right)}\right)\ln\left(\frac{be}{d}+\frac{\left(ad-cb\right)e}{d\left(dx+c\right)}\right)-\frac{\left(ad-cb\right)e}{d\left(dx+c\right)}-\frac{be}{d}\right)}{\left(ad-cb\right)^{2}e^{2}i^{2}}\right)}{d^{2}}$	170

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x,method=_RETURNVERBOSE)
[Out] -A/i^2/(d*x+c)/d-B/i^2/e/(a*d-b*c)*(e*(b*x+a)/(d*x+c)*ln(e*(b*x+a)/(d*x+c))
-e*(b*x+a)/(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.90

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx=-\frac{(A-B)bc-(A-B)ad-(Bbdx+Bad)\log\left(\frac{bex+ae}{dx+c}\right)}{(bcd^2-ad^3)i^2x+(bc^2d-acd^2)i^2}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algorithm="fricas")
[Out] -((A - B)*b*c - (A - B)*a*d - (B*b*d*x + B*a*d)*log((b*e*x + a*e)/(d*x + c)
))/((b*c*d^2 - a*d^3)*i^2*x + (b*c^2*d - a*c*d^2)*i^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(78) = 156$.

Time = 0.61 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.36

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx = \frac{Bb\log\left(x + \frac{-\frac{Ba^2bd^2}{ad-bc} + \frac{2Bab^2cd}{ad-bc} + Babd - \frac{Bb^3c^2}{ad-bc} + Bb^2c}{2Bb^2d}\right)}{di^2\,(ad-bc)} - \frac{Bb\log\left(x + \frac{\frac{Ba^2bd^2}{ad-bc} - \frac{2Bab^2cd}{ad-bc} + Babd + \frac{Bb^3c^2}{ad-bc} + Bb^2c}{2Bb^2d}\right)}{di^2\,(ad-bc)} - \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{cdi^2 + d^2i^2x} + \frac{-A+B}{cdi^2 + d^2i^2x}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**2,x)
```

```
[Out] B*b*log(x + (-B*a**2*b*d**2/(a*d - b*c) + 2*B*a*b**2*c*d/(a*d - b*c) + B*a*
b*d - B*b**3*c**2/(a*d - b*c) + B*b**2*c)/(2*B*b**2*d))/(d*i**2*(a*d - b*c)
) - B*b*log(x + (B*a**2*b*d**2/(a*d - b*c) - 2*B*a*b**2*c*d/(a*d - b*c) + B
*a*b*d + B*b**3*c**2/(a*d - b*c) + B*b**2*c)/(2*B*b**2*d))/(d*i**2*(a*d - b
*c)) - B*log(e*(a + b*x)/(c + d*x))/(c*d*i**2 + d**2*i**2*x) + (-A + B)/(c*
d*i**2 + d**2*i**2*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.37

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx = -B\left(\frac{\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right)}{d^2i^2x + cdi^2} - \frac{1}{d^2i^2x + cdi^2} - \frac{b\log(bx+a)}{(bcd-ad^2)i^2} + \frac{b\log(dx+c)}{(bcd-ad^2)i^2}\right) - \frac{A}{d^2i^2x + cdi^2}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algorithm="maxima")
```

```
[Out] -B*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^2*i^2*x + c*d*i^2) - 1/(d^2*i^2
*x + c*d*i^2) - b*log(b*x + a)/((b*c*d - a*d^2)*i^2) + b*log(d*x + c)/((b*c
*d - a*d^2)*i^2)) - A/(d^2*i^2*x + c*d*i^2)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.17

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx = \left(\frac{bc}{(bce-ade)(bc-ad)} - \frac{ad}{(bce-ade)(bc-ad)}\right)\left(\frac{(bex+ae)B\log\left(\frac{bex+ae}{dx+c}\right)}{(dx+c)i^2} + \frac{(bex+ae)(A-B)}{(dx+c)i^2}\right)$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algorithm="giac")
```

```
[Out] (b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))*((b
*e*x + a*e)*B*log((b*e*x + a*e)/(d*x + c))/((d*x + c)*i^2) + (b*e*x + a*e)*
(A - B)/((d*x + c)*i^2))
```

Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx = -\frac{A-B}{x\,d^2\,i^2 + c\,d\,i^2} - \frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{d^2\,i^2\,\left(x+\frac{c}{d}\right)} + \frac{B\,b\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{d\,i^2\,\left(a\,d-b\,c\right)}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(c*i + d*i*x)^2,x)
```

```
[Out] (B*b*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(d*i^2*(a*d - b*c)) - (
B*log((e*(a + b*x))/(c + d*x)))/(d^2*i^2*(x + c/d)) - (A - B)/(d^2*i^2*x +
c*d*i^2)
```

3.43 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 156

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx = -\frac{Ad(a+bx)}{(bc-ad)^2gi^2(c+dx)}+\frac{Bd(a+bx)}{(bc-ad)^2gi^2(c+dx)} - \frac{Bd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^2gi^2(c+dx)}+\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2B(bc-ad)^2gi^2}$$

```
[Out] -A*d*(b*x+a)/(-a*d+b*c)^2/g/i^2/(d*x+c)+B*d*(b*x+a)/(-a*d+b*c)^2/g/i^2/(d*x
+c)-B*d*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(-a*d+b*c)^2/g/i^2/(d*x+c)+1/2*b*(A+B
*ln(e*(b*x+a)/(d*x+c)))^2/B/(-a*d+b*c)^2/g/i^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2562, 2388, 2338, 2332}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx = \frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2Bgi^2(bc-ad)^2}-\frac{Ad(a+bx)}{gi^2(c+dx)(bc-ad)^2} - \frac{Bd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{gi^2(c+dx)(bc-ad)^2}+\frac{Bd(a+bx)}{gi^2(c+dx)(bc-ad)^2}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)*(c*i + d*i*x)^2),x]
```

```
[Out] -((A*d*(a + b*x))/((b*c - a*d)^2*g*i^2*(c + d*x))) + (B*d*(a + b*x))/((b*c
- a*d)^2*g*i^2*(c + d*x)) - (B*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/((
```

```
b*c - a*d)^2*g*i^2*(c + d*x)) + (b*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/
(2*B*(b*c - a*d)^2*g*i^2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex))}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 gi^2}\\
&= \frac{b\text{Subst}\left(\int \frac{A+B\log(ex)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 gi^2} - \frac{d\text{Subst}\left(\int (A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 gi^2}\\
&= -\frac{Ad(a+bx)}{(bc-ad)^2 gi^2 (c+dx)} + \frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2B(bc-ad)^2 gi^2} - \frac{(Bd)\text{Subst}\left(\int \log(ex)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 gi^2}\\
&= -\frac{Ad(a+bx)}{(bc-ad)^2 gi^2 (c+dx)} + \frac{Bd(a+bx)}{(bc-ad)^2 gi^2 (c+dx)}\\
&\quad - \frac{Bd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^2 gi^2 (c+dx)} + \frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2B(bc-ad)^2 gi^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.87

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+2b(c+dx)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2b(c+dx)\left(A+B\log\right.}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)*(c*i + d*i*x)
^2),x]
```

```
[Out] (2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*b*(c + d*x)*Log[a +
 b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b*(c + d*x)*(A + B*Log[(e*(a
 + b*x))/(c + d*x)])*Log[c + d*x] - 2*B*(b*c - a*d + b*(c + d*x)*Log[a + b*
x] - b*(c + d*x)*Log[c + d*x]) - b*B*(c + d*x)*(Log[a + b*x]*(Log[a + b*x]
- 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) +
a*d)]) + b*B*(c + d*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x]
)*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(2*(b*c - a*d)^2
*g*i^2*(c + d*x))
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.48

method	result
parts	$\frac{A\left(-\frac{1}{(ad-cb)(dx+c)}-\frac{b\ln(dx+c)}{(ad-cb)^2}+\frac{b\ln(bx+a)}{(ad-cb)^2}\right)}{g\,i^2} - \frac{B\left(\frac{d\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad-cb)e}{d(dx+c)}-\frac{be}{d}\right)}{ad-cb}-\frac{be\ln\left(\frac{be}{d}+\frac{(ad}{d(d}\right.}{2(ad-c}\right.}{g\,i^2(ad-cb)e}$
parallelrisch	$\frac{2Bab^2d^4-2Bb^3cd^3-2Aab^2d^4+2Ab^3cd^3+Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2b^3d^4+2Ax\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3d^4-2Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3d^4+B\ln(}{2i^2g(dx+c)b^2d^3(a^2d^2-2abcd+b^2c^2)}$
norman	$\frac{\frac{(Abc-Bad)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{d(Ab-Bb)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{(-B+A)dx}{gic(ad-cb)}+\frac{Bbc\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{2gi\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{bBdx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{2gi\left(a^2d^2-2abcd+b^2c^2\right)}}{i(dx+c)}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^2Ab\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^2(ad-cb)^3g}+\frac{d^3A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^2(ad-cb)^3g}-\frac{d^2Bb\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2e\,i^2(ad-cb)^3g}+\frac{d^3B\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ac}{d(}\right.\right.}{e^2i^2(ad-cb)}\right.}{d^2}$
default	$-\frac{e(ad-cb)\left(-\frac{d^2Ab\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^2(ad-cb)^3g}+\frac{d^3A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^2(ad-cb)^3g}-\frac{d^2Bb\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2e\,i^2(ad-cb)^3g}+\frac{d^3B\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ac}{d(}\right.\right.}{e^2i^2(ad-cb)}\right.}{d^2}$
risch	$-\frac{A}{g\,i^2(ad-cb)(dx+c)}-\frac{Ab\ln(dx+c)}{g\,i^2(ad-cb)^2}+\frac{Ab\ln(bx+a)}{g\,i^2(ad-cb)^2}-\frac{B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)b}{g\,i^2(ad-cb)^2}-\frac{Bd\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)a}{g\,i^2(ad-cb)^2(dx+c)}+\frac{B\ln(}{g\,i^2(}$

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^2,x,method=_RETURNV
ERBOSE)
```

```
[Out] A/g/i^2*(-1/(a*d-b*c)/(d*x+c)-b/(a*d-b*c)^2*ln(d*x+c)+b/(a*d-b*c)^2*ln(b*x+
a))-B/g/i^2/(a*d-b*c)/e*(d/(a*d-b*c)*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/
d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)-1/2*b*e/(a*d-b*c)*ln(
b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.97

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx$$
$$=\frac{2\,(A-B)bc-2\,(A-B)ad+(Bbdx+Bbc)\log\left(\frac{bex+ae}{dx+c}\right)^2+2\,((A-B)bdx+Abc-Bad)\log\left(\frac{bex+ae}{dx+c}\right)}{2\,((b^2c^2d-2\,abcd^2+a^2d^3)gi^2x+(b^2c^3-2\,abc^2d+a^2cd^2)gi^2)}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^2,x, algorit
hm="fricas")
```

```
[Out] 1/2*(2*(A - B)*b*c - 2*(A - B)*a*d + (B*b*d*x + B*b*c)*log((b*e*x + a*e)/(d
*x + c))^2 + 2*((A - B)*b*d*x + A*b*c - B*a*d)*log((b*e*x + a*e)/(d*x + c))
)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g*i^2*x + (b^2*c^3 - 2*a*b*c^2*d + a
^2*c*d^2)*g*i^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(131) = 262$.

Time = 0.60 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.47

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx = \frac{Bb\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2a^2d^2gi^2 - 4abcdgi^2 + 2b^2c^2gi^2} - \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{acdgi^2 + ad^2gi^2x - bc^2gi^2 - bcdgi^2x} + (A-B)\left(-\frac{b\log\left(x + \frac{-\frac{a^3bd^3}{(ad-bc)^2} + \frac{3a^2b^2cd^2}{(ad-bc)^2} - \frac{3ab^3c^2d}{(ad-bc)^2} + abd + \frac{b^4c^3}{(ad-bc)^2} + b^2c}{2b^2d}\right)}{gi^2(ad-bc)^2} + \frac{b\log\left(x + \frac{\frac{a^3bd^3}{(ad-bc)^2} - \frac{3a^2b^2cd^2}{(ad-bc)^2} + \frac{3ab^3c^2d}{(ad-bc)^2} + abd - \frac{b^4c^3}{(ad-bc)^2} + b^2c}{2b^2d}\right)}{gi^2(ad-bc)^2} - \frac{1}{acdgi^2 - bc^2gi^2 + x(ad^2gi^2 - bcdgi^2)}\right)$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)**2,x)
```

```
[Out] B*b*log(e*(a + b*x)/(c + d*x))**2/(2*a**2*d**2*g*i**2 - 4*a*b*c*d*g*i**2 +
2*b**2*c**2*g*i**2) - B*log(e*(a + b*x)/(c + d*x))/(a*c*d*g*i**2 + a*d**2*g
*i**2*x - b*c**2*g*i**2 - b*c*d*g*i**2*x) + (A - B)*(-b*log(x + (-a**3*b*d*
*3/(a*d - b*c)**2 + 3*a**2*b**2*c*d**2/(a*d - b*c)**2 - 3*a*b**3*c**2*d/(a*
d - b*c)**2 + a*b*d + b**4*c**3/(a*d - b*c)**2 + b**2*c)/(2*b**2*d))/(g*i**
2*(a*d - b*c)**2) + b*log(x + (a**3*b*d**3/(a*d - b*c)**2 - 3*a**2*b**2*c*d
**2/(a*d - b*c)**2 + 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d - b**4*c**3/(a*
d - b*c)**2 + b**2*c)/(2*b**2*d))/(g*i**2*(a*d - b*c)**2) - 1/(a*c*d*g*i**2
 - b*c**2*g*i**2 + x*(a*d**2*g*i**2 - b*c*d*g*i**2)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. 2(154) = 308.

Time = 0.22 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.70

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx$$
$$=B\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2}+\frac{b\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}-\frac{b\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}\right)\log\left(\frac{bex}{dx+}\right.$$
$$\left.+\frac{ae}{dx+c}\right)$$
$$+A\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2}+\frac{b\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}-\frac{b\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}\right)$$
$$-\frac{\left((bdx+bc)\log(bx+a)^2+(bdx+bc)\log(dx+c)^2+2\,bc-2\,ad+2\,(bdx+bc)\log(bx+a)-2\,(bdx+b\right.}{2\,(b^2c^3gi^2-2\,abc^2dgi^2+a^2cd^2gi^2+(b^2c^2dgi^2-2\,abcd^2gi^2+a^2d^3g}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^2,x, algorit
hm="maxima")
```

```
[Out] B*(1/((b*c*d - a*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*g*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + A*(1/((b*c*d - a
*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g*i^
2)) - 1/2*((b*d*x + b*c)*log(b*x + a)^2 + (b*d*x + b*c)*log(d*x + c)^2 + 2*
b*c - 2*a*d + 2*(b*d*x + b*c)*log(b*x + a) - 2*(b*d*x + b*c + (b*d*x + b*c)
*log(b*x + a))*log(d*x + c))*B/(b^2*c^3*g*i^2 - 2*a*b*c^2*d*g*i^2 + a^2*c*d
^2*g*i^2 + (b^2*c^2*d*g*i^2 - 2*a*b*c*d^2*g*i^2 + a^2*d^3*g*i^2)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.49

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx$$
$$=\frac{1}{2}\left(\frac{Bbe\log\left(\frac{bex+ae}{dx+c}\right)^2}{bcgi^2-adgi^2}+\frac{2\,Abe\log\left(\frac{bex+ae}{dx+c}\right)}{bcgi^2-adgi^2}-\frac{2\,(bex+ae)Bd\log\left(\frac{bex+ae}{dx+c}\right)}{(bcgi^2-adgi^2)(dx+c)}-\frac{2\,(bex+ae)(Ad-Bd)}{(bcgi^2-adgi^2)(dx+c)}\right)\left(\frac{}{(bc}\right.$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^2,x, algorit
hm="giac")
```

```
[Out] 1/2*(B*b*e*log((b*e*x + a*e)/(d*x + c))^2/(b*c*g*i^2 - a*d*g*i^2) + 2*A*b*e
*log((b*e*x + a*e)/(d*x + c))/(b*c*g*i^2 - a*d*g*i^2) - 2*(b*e*x + a*e)*B*d
*log((b*e*x + a*e)/(d*x + c))/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)) - 2*(b*e*
x + a*e)*(A*d - B*d)/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)))*(b*c/((b*c*e - a*
d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.58

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx = \frac{B\,b\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2}{2\,g\,i^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)} - \frac{A-B}{\left(a\,d-b\,c\right)\,\left(c\,g\,i^2+d\,g\,i^2\,x\right)} - \frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(a\,d-b\,c\right)}{b\,d\,g\,i^2\,\left(\frac{x}{b}+\frac{c}{b\,d}\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)} - \frac{b\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{a^2\,d^2\,g\,i^2-b^2\,c^2\,g\,i^2}{g\,i^2\,(a\,d-b\,c)}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,\left(A-B\right)\,2\mathrm{i}}{g\,i^2\,{\left(a\,d-b\,c\right)}^2}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)*(c*i + d*i*x)^2),x)
```

```
[Out] (B*b*log((e*(a + b*x))/(c + d*x))^2)/(2*g*i^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*
d)) - (b*atan(((2*b*d*x + (a^2*d^2*g*i^2 - b^2*c^2*g*i^2)/(g*i^2*(a*d - b*c
)))*1i)/(a*d - b*c))*(A - B)*2i)/(g*i^2*(a*d - b*c)^2) - (A - B)/((a*d - b*
c)*(c*g*i^2 + d*g*i^2*x)) - (B*log((e*(a + b*x))/(c + d*x))*(a*d - b*c))/(b
*d*g*i^2*(x/b + c/(b*d))*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
```

3.44 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 261

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx = &-\frac{Bd^2(a+bx)}{(bc-ad)^3g^2i^2(c+dx)}-\frac{b^2B(c+dx)}{(bc-ad)^3g^2i^2(a+bx)} \\ &+\frac{bBd\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}+\frac{d^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^2i^2(c+dx)} \\ &-\frac{b^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^2i^2(a+bx)} \\ &-\frac{2bd\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^2i^2}\end{aligned}$$

```
[Out] -B*d^2*(b*x+a)/(-a*d+b*c)^3/g^2/i^2/(d*x+c)-b^2*B*(d*x+c)/(-a*d+b*c)^3/g^2/
i^2/(b*x+a)+b*B*d*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g^2/i^2+d^2*(b*x+a)*(A
+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^2/i^2/(d*x+c)-b^2*(d*x+c)*(A+B*ln(
e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^2/i^2/(b*x+a)-2*b*d*ln((b*x+a)/(d*x+c))*
(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^2/i^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2562, 45, 2372, 2338}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx = -\frac{b^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2i^2(a+bx)(bc-ad)^3} + \frac{d^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2i^2(c+dx)(bc-ad)^3} - \frac{2bd\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2i^2(bc-ad)^3} - \frac{b^2B(c+dx)}{g^2i^2(a+bx)(bc-ad)^3} - \frac{Bd^2(a+bx)}{g^2i^2(c+dx)(bc-ad)^3} + \frac{bBd\log^2\left(\frac{a+bx}{c+dx}\right)}{g^2i^2(bc-ad)^3}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^2*(c*i + d*i*x)^2),
x]
```

```
[Out] -((B*d^2*(a + b*x))/((b*c - a*d)^3*g^2*i^2*(c + d*x))) - (b^2*B*(c + d*x))/
((b*c - a*d)^3*g^2*i^2*(a + b*x)) + (b*B*d*Log[(a + b*x)/(c + d*x)]^2)/((b*
c - a*d)^3*g^2*i^2) + (d^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/
((b*c - a*d)^3*g^2*i^2*(c + d*x)) - (b^2*(c + d*x)*(A + B*Log[(e*(a + b*x))
/(c + d*x)]))/((b*c - a*d)^3*g^2*i^2*(a + b*x)) - (2*b*d*Log[(a + b*x)/(c +
 d*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^3*g^2*i^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
```

```
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex))}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}$$

$$= \frac{d^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^2i^2(c+dx)} - \frac{b^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^2i^2(a+bx)} - \frac{2bd\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^2i^2} - \frac{B\text{Subst}\left(\int\left(d^2-\frac{b^2}{x^2}-\frac{2bd\log(x)}{x}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}$$

$$= -\frac{Bd^2(a+bx)}{(bc-ad)^3g^2i^2(c+dx)} - \frac{b^2B(c+dx)}{(bc-ad)^3g^2i^2(a+bx)} + \frac{d^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^2i^2(c+dx)} - \frac{b^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^2i^2(a+bx)} - \frac{2bd\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^2i^2} + \frac{(2bBd)\text{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}$$

$$= -\frac{Bd^2(a+bx)}{(bc-ad)^3g^2i^2(c+dx)} - \frac{b^2B(c+dx)}{(bc-ad)^3g^2i^2(a+bx)} + \frac{bBd\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2} + \frac{d^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^2i^2(c+dx)} - \frac{b^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^2i^2(a+bx)} - \frac{2bd\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^2i^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.24

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx$$
$$= \frac{-\frac{b^2Bc}{a+bx}+\frac{abBd}{a+bx}+\frac{bBcd}{c+dx}-\frac{aBd^2}{c+dx}-\frac{b(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{a+bx}+\frac{d(-bc+ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{c+dx}-2bd\log(a+bx)\left(A\right.}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^2*(c*i + d*i*
x)^2),x]
```

```
[Out] (-((b^2*B*c)/(a + b*x)) + (a*b*B*d)/(a + b*x) + (b*B*c*d)/(c + d*x) - (a*B*
d^2)/(c + d*x) - (b*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a +
b*x) + (d*(-(b*c) + a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x) -
2*b*d*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*b*d*(A + B*Log[
(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + b*B*d*(Log[a + b*x]*(Log[a + b*x]
- 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) +
a*d)]) - b*B*d*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c
+ d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)^3*g^2*i^2)
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.40

method	result
parts	$\frac{A\left(-\frac{d}{(ad-cb)^2(dx+c)}-\frac{2db\ln(dx+c)}{(ad-cb)^3}-\frac{b}{(ad-cb)^2(bx+a)}+\frac{2db\ln(bx+a)}{(ad-cb)^3}\right)}{g^2i^2}-\frac{B\left(\frac{d^2\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad-cb)e}{d(dx+}\right.}{(ad-cb)^2}\right.}{}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^2Ab^2}{i^2(ad-cb)^4g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{2d^3Ab\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei^2(ad-cb)^4g^2}+\frac{d^4A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^2(ad-cb)^4g^2}+\frac{d^2Bb^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right.}{i^2(ad-cb)^4}\right.}{d^2}$
default	$-\frac{e(ad-cb)\left(-\frac{d^2Ab^2}{i^2(ad-cb)^4g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{2d^3Ab\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei^2(ad-cb)^4g^2}+\frac{d^4A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^2(ad-cb)^4g^2}+\frac{d^2Bb^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right.}{i^2(ad-cb)^4}\right.}{d^2}$
risch	$-\frac{Ad}{g^2i^2(ad-cb)^2(dx+c)}-\frac{2Adb\ln(dx+c)}{g^2i^2(ad-cb)^3}-\frac{Ab}{g^2i^2(ad-cb)^2(bx+a)}+\frac{2Adb\ln(bx+a)}{g^2i^2(ad-cb)^3}-\frac{Bd\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)b}{g^2i^2(ad-cb)^3}-$
norman	$\frac{\left(2Aabcd-Ba^2d^2+Bb^2c^2\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}+\frac{\left(2Aabd^2+2Ab^2cd-2Babd^2+2Bb^2cd\right)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}+\frac{2b^2d^2Ax^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b\right.}$
parallelrisch	$2Ax\ln\left(\frac{e(bx+a)}{dx+c}\right)a^4bc^3d^2+2Ax\ln\left(\frac{e(bx+a)}{dx+c}\right)a^3b^2c^4d-2Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^4bc^3d^2+2Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^3b^2c^4d+Bx^2\ln\left(\frac{e(}{d}\right.$

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x,method=_RETUR
NVERBOSE)
```

```
[Out] A/g^2/i^2*(-d/(a*d-b*c)^2/(d*x+c)-2*d/(a*d-b*c)^3*b*ln(d*x+c)-b/(a*d-b*c)^2
/(b*x+a)+2*d/(a*d-b*c)^3*b*ln(b*x+a))-B/g^2/i^2/(a*d-b*c)/e*(d^2/(a*d-b*c)^
2*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*
e/d/(d*x+c)-b*e/d)-1/(a*d-b*c)^2*b*d*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+1/
(a*d-b*c)^2*e^2*b^2*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/
d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.28

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx =$$

$$-\frac{(A+B)b^2c^2-2\,Babcd-(A-B)a^2d^2+(Bb^2d^2x^2+Babcd+(Bb^2cd+Babd^2)x)\log\left(\frac{bex+ae}{dx+c}\right)^2+2\,(Ab}{(b^4c^3d-3\,ab^3c^2d^2+3\,a^2b^2cd^3-a^3bd^4)g^2i^2x^2+(b^4c^4-2\,ab^3c^3d+}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, algor
ithm="fricas")
```

```
[Out] -((A + B)*b^2*c^2 - 2*B*a*b*c*d - (A - B)*a^2*d^2 + (B*b^2*d^2*x^2 + B*a*b*
c*d + (B*b^2*c*d + B*a*b*d^2)*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*b^2*
c*d - A*a*b*d^2)*x + (2*A*b^2*d^2*x^2 + B*b^2*c^2 + 2*A*a*b*c*d - B*a^2*d^2
 + 2*((A + B)*b^2*c*d + (A - B)*a*b*d^2)*x)*log((b*e*x + a*e)/(d*x + c)))/(
(b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*g^2*i^2*x^2 + (
b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*g^2*i^2*x + (a*b^3*c^4 -
 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*g^2*i^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(235) = 470$.

Time = 1.90 (sec) , antiderivative size = 828, normalized size of antiderivative = 3.17

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx$$
$$= -\frac{2Abd\log\left(x+\frac{-\frac{2Aa^4bd^5}{(ad-bc)^3}+\frac{8Aa^3b^2cd^4}{(ad-bc)^3}-\frac{12Aa^2b^3c^2d^3}{(ad-bc)^3}+\frac{8Aab^4c^3d^2}{(ad-bc)^3}+2Aabd^2-\frac{2Ab^5c^4d}{(ad-bc)^3}+2Ab^2cd}{4Ab^2d^2}\right)}{g^2i^2\left(ad-bc\right)^3}$$
$$+\frac{2Abd\log\left(x+\frac{\frac{2Aa^4bd^5}{(ad-bc)^3}-\frac{8Aa^3b^2cd^4}{(ad-bc)^3}+\frac{12Aa^2b^3c^2d^3}{(ad-bc)^3}-\frac{8Aab^4c^3d^2}{(ad-bc)^3}+2Aabd^2+\frac{2Ab^5c^4d}{(ad-bc)^3}+2Ab^2cd}{4Ab^2d^2}\right)}{g^2i^2\left(ad-bc\right)^3}$$
$$+\frac{Bbd\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{a^3d^3g^2i^2-3a^2bcd^2g^2i^2+3ab^2c^2dg^2i^2-b^3c^3g^2i^2}$$
$$+\frac{(-Bad-Bbc-2Bbdx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{a^3cd^2g^2i^2+a^3d^3g^2i^2x-2a^2bc^2dg^2i^2-a^2bcd^2g^2i^2x+a^2bd^3g^2i^2x^2+ab^2c^3g^2i^2-ab^2c^2dg^2i^2x-2ab^2cd^2}$$
$$-\frac{Aad+Abc+2Abdx-Bad+Bbc}{a^3cd^2g^2i^2-2a^2bc^2dg^2i^2+ab^2c^3g^2i^2+x^2\left(a^2bd^3g^2i^2-2ab^2cd^2g^2i^2+b^3c^2dg^2i^2\right)+x\left(a^3d^3g^2i^2-a^2bcd^2\right)}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2/(d*i*x+c*i)**2,x)
```

```
[Out] -2*A*b*d*log(x + (-2*A*a**4*b*d**5/(a*d - b*c)**3 + 8*A*a**3*b**2*c*d**4/(a
*d - b*c)**3 - 12*A*a**2*b**3*c**2*d**3/(a*d - b*c)**3 + 8*A*a*b**4*c**3*d*
*2/(a*d - b*c)**3 + 2*A*a*b*d**2 - 2*A*b**5*c**4*d/(a*d - b*c)**3 + 2*A*b**
2*c*d)/(4*A*b**2*d**2))/(g**2*i**2*(a*d - b*c)**3) + 2*A*b*d*log(x + (2*A*a
**4*b*d**5/(a*d - b*c)**3 - 8*A*a**3*b**2*c*d**4/(a*d - b*c)**3 + 12*A*a**2
*b**3*c**2*d**3/(a*d - b*c)**3 - 8*A*a*b**4*c**3*d**2/(a*d - b*c)**3 + 2*A*
a*b*d**2 + 2*A*b**5*c**4*d/(a*d - b*c)**3 + 2*A*b**2*c*d)/(4*A*b**2*d**2))/
(g**2*i**2*(a*d - b*c)**3) + B*b*d*log(e*(a + b*x)/(c + d*x))**2/(a**3*d**3
```

```
*g**2*i**2 - 3*a**2*b*c*d**2*g**2*i**2 + 3*a*b**2*c**2*d*g**2*i**2 - b**3*c
**3*g**2*i**2) + (-B*a*d - B*b*c - 2*B*b*d*x)*log(e*(a + b*x)/(c + d*x))/(a
**3*c*d**2*g**2*i**2 + a**3*d**3*g**2*i**2*x - 2*a**2*b*c**2*d*g**2*i**2 -
a**2*b*c*d**2*g**2*i**2*x + a**2*b*d**3*g**2*i**2*x**2 + a*b**2*c**3*g**2*i
**2 - a*b**2*c**2*d*g**2*i**2*x - 2*a*b**2*c*d**2*g**2*i**2*x**2 + b**3*c**
3*g**2*i**2*x + b**3*c**2*d*g**2*i**2*x**2) - (A*a*d + A*b*c + 2*A*b*d*x -
B*a*d + B*b*c)/(a**3*c*d**2*g**2*i**2 - 2*a**2*b*c**2*d*g**2*i**2 + a*b**2*
c**3*g**2*i**2 + x**2*(a**2*b*d**3*g**2*i**2 - 2*a*b**2*c*d**2*g**2*i**2 +
b**3*c**2*d*g**2*i**2) + x*(a**3*d**3*g**2*i**2 - a**2*b*c*d**2*g**2*i**2 -
 a*b**2*c**2*d*g**2*i**2 + b**3*c**3*g**2*i**2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(261) = 522$.

Time = 0.24 (sec) , antiderivative size = 859, normalized size of antiderivative = 3.29

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx =$$
$$-B\left(\frac{2\,bdx + bc + ad}{(b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)g^2i^2x^2 + (b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3)g^2i^2x + (ab^2c^3 - 2\,a^2bc^2d + a^3cd^2\right.$$
$$\left. + \frac{ae}{dx+c}\right)$$
$$-A\left(\frac{2\,bdx + bc + ad}{(b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)g^2i^2x^2 + (b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3)g^2i^2x + (ab^2c^3 - 2\,a^2bc^2d + a^3cd^2\right.$$
$$-\frac{\left(b^2c^2 - 2\,abcd + a^2d^2 - (b^2d^2x^2 + abcd + (b^2cd + abd^2)x)\log(bx+a)^2 + 2\,(b^2d^2x^2 + abcd + (b^2cd + abd\right.}{ab^3c^4g^2i^2 - 3\,a^2b^2c^3dg^2i^2 + 3\,a^3bc^2d^2g^2i^2 - a^4cd^3g^2i^2 + (b^4c^3dg^2i^2 - 3\,ab^3c^2d^2g^2i^2 + 3\,a^2b^2cd^3g^2}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, algor
ithm="maxima")
```

```
[Out] -B*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*g^2*i^2*
x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*g^2*i^2*x + (a*b^2*c^
3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2) + 2*b*d*log(b*x + a)/((b^3*c^3 - 3*
a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2) - 2*b*d*log(d*x + c)/((b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2))*log(b*e*x/(d*x + c
) + a*e/(d*x + c)) - A*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2*a*b^2*c*d^2 +
 a^2*b*d^3)*g^2*i^2*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*g
^2*i^2*x + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2) + 2*b*d*log(b*x
 + a)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2) - 2*b*d
*log(d*x + c)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2)
) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*
b*d^2)*x)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x
)*log(b*x + a)*log(d*x + c) - (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*
```

```
x)*log(d*x + c)^2)*B/(a*b^3*c^4*g^2*i^2 - 3*a^2*b^2*c^3*d*g^2*i^2 + 3*a^3*b
*c^2*d^2*g^2*i^2 - a^4*c*d^3*g^2*i^2 + (b^4*c^3*d*g^2*i^2 - 3*a*b^3*c^2*d^2
*g^2*i^2 + 3*a^2*b^2*c*d^3*g^2*i^2 - a^3*b*d^4*g^2*i^2)*x^2 + (b^4*c^4*g^2*
i^2 - 2*a*b^3*c^3*d*g^2*i^2 + 2*a^3*b*c*d^3*g^2*i^2 - a^4*d^4*g^2*i^2)*x)
```

Giac [A] (verification not implemented)

none

Time = 44.47 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.51

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx = -\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)^2\left(\frac{(dx+c)Be^2\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)g^2i^2}+\frac{(Ae^2+Be^2)(dx+c)}{(bex+ae)g^2i^2}\right)$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, algor
ithm="giac")
```

```
[Out] -(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2*
((d*x + c)*B*e^2*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)*g^2*i^2) + (A*
e^2 + B*e^2)*(d*x + c)/((b*e*x + a*e)*g^2*i^2))
```

Mupad [B] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.59

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx = \frac{B\,b\,d\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2}{g^2\,i^2\,(a\,d-b\,c)^3} - \frac{A\,a\,d}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{A\,b\,c}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} + \frac{B\,a\,d}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{B\,b\,c}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{2\,A\,b\,d\,x}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{B\,a\,d\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{B\,b\,c\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{2\,B\,b\,d\,x\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{A\,b\,d\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,4\mathrm{i}}{g^2\,i^2\,(a\,d-b\,c)^3}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^2*(c*i + d*i*x)^2),
x)
```

```
[Out] (B*b*d*log((e*(a + b*x))/(c + d*x))^2)/(g^2*i^2*(a*d - b*c)^3) - (A*b*d*ata
n((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*4i)/(g^2*i^2*(a*d - b*c)^3) - (
A*a*d)/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (A*b*c)/(g^2*i^2*(a*d
- b*c)^2*(a + b*x)*(c + d*x)) + (B*a*d)/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c
 + d*x)) - (B*b*c)/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (2*A*b*d*x
)/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (B*a*d*log((e*(a + b*x))/(c
 + d*x)))/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (B*b*c*log((e*(a +
b*x))/(c + d*x)))/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (2*B*b*d*x*
log((e*(a + b*x))/(c + d*x)))/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x))
```

3.45 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 364

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx = \frac{Bd^3(a+bx)}{(bc-ad)^4g^3i^2(c+dx)}+\frac{3b^2Bd(c+dx)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3B(c+dx)^2}{4(bc-ad)^4g^3i^2(a+bx)^2}-\frac{3bBd^2\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^3i^2} - \frac{d^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(c+dx)} + \frac{3b^2d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} + \frac{3bd^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2}$$

```
[Out] B*d^3*(b*x+a)/(-a*d+b*c)^4/g^3/i^2/(d*x+c)+3*b^2*B*d*(d*x+c)/(-a*d+b*c)^4/g
^3/i^2/(b*x+a)-1/4*b^3*B*(d*x+c)^2/(-a*d+b*c)^4/g^3/i^2/(b*x+a)^2-3/2*b*B*d
^2*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^4/g^3/i^2-d^3*(b*x+a)*(A+B*ln(e*(b*x+a)
/(d*x+c)))/(-a*d+b*c)^4/g^3/i^2/(d*x+c)+3*b^2*d*(d*x+c)*(A+B*ln(e*(b*x+a)/(
d*x+c)))/(-a*d+b*c)^4/g^3/i^2/(b*x+a)-1/2*b^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(
d*x+c)))/(-a*d+b*c)^4/g^3/i^2/(b*x+a)^2+3*b*d^2*ln((b*x+a)/(d*x+c))*(A+B*ln
(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^3/i^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2562, 45, 2372, 12, 14, 2338}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx=-\frac{b^3(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^3i^2(a+bx)^2(bc-ad)^4}+\frac{3b^2d(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^3i^2(a+bx)(bc-ad)^4}-\frac{d^3(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^3i^2(c+dx)(bc-ad)^4}+\frac{3bd^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^3i^2(bc-ad)^4}-\frac{b^3B(c+dx)^2}{4g^3i^2(a+bx)^2(bc-ad)^4}+\frac{3b^2Bd(c+dx)}{g^3i^2(a+bx)(bc-ad)^4}+\frac{Bd^3(a+bx)}{g^3i^2(c+dx)(bc-ad)^4}-\frac{3bBd^2\log^2\left(\frac{a+bx}{c+dx}\right)}{2g^3i^2(bc-ad)^4}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^3*(c*i + d*i*x)^2),
x]
```

```
[Out] (B*d^3*(a + b*x))/((b*c - a*d)^4*g^3*i^2*(c + d*x)) + (3*b^2*B*d*(c + d*x))
/((b*c - a*d)^4*g^3*i^2*(a + b*x)) - (b^3*B*(c + d*x)^2)/(4*(b*c - a*d)^4*g
^3*i^2*(a + b*x)^2) - (3*b*B*d^2*Log[(a + b*x)/(c + d*x)]^2)/(2*(b*c - a*d)
^4*g^3*i^2) - (d^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c -
a*d)^4*g^3*i^2*(c + d*x)) + (3*b^2*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c
+ d*x)]))/((b*c - a*d)^4*g^3*i^2*(a + b*x)) - (b^3*(c + d*x)^2*(A + B*Log[(
e*(a + b*x))/(c + d*x)]))/(2*(b*c - a*d)^4*g^3*i^2*(a + b*x)^2) + (3*b*d^2*
Log[(a + b*x)/(c + d*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)
^4*g^3*i^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)^3(A+B\log(ex))}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2} \\
&= -\frac{d^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(c+dx)} + \frac{3b^2d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} \\
&\quad - \frac{b^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} + \frac{3bd^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2} \\
&\quad - \frac{B\text{Subst}\left(\int \frac{-b^3+6b^2dx-2d^3x^3+6bd^2x^2\log(x)}{2x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2}
\end{aligned}$$

$$= -\frac{d^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(c+dx)}+\frac{3b^2d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2}+\frac{3bd^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2} - \frac{B\text{Subst}\left(\int\frac{-b^3+6b^2dx-2d^3x^3+6bd^2x^2\log(x)}{x^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^3i^2}$$

$$= -\frac{d^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(c+dx)}+\frac{3b^2d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2}+\frac{3bd^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2} - \frac{B\text{Subst}\left(\int\left(\frac{-b^3+6b^2dx-2d^3x^3}{x^3}+\frac{6bd^2\log(x)}{x}\right)dx,x,\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^3i^2}$$

$$= -\frac{d^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(c+dx)}+\frac{3b^2d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2}+\frac{3bd^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2} - \frac{B\text{Subst}\left(\int\frac{-b^3+6b^2dx-2d^3x^3}{x^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^3i^2}-\frac{(3bBd^2)\,\text{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2}$$

$$= -\frac{3bBd^2\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^3i^2}-\frac{d^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(c+dx)} + \frac{3b^2d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(a+bx)}-\frac{b^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} + \frac{3bd^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2} - \frac{B\text{Subst}\left(\int\left(-2d^3-\frac{b^3}{x^3}+\frac{6b^2d}{x^2}\right)dx,x,\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^3i^2}$$

$$= \frac{Bd^3(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} + \frac{3b^2Bd(c+dx)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3B(c+dx)^2}{4(bc-ad)^4g^3i^2(a+bx)^2} - \frac{3bBd^2\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^3i^2} - \frac{d^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(c+dx)} + \frac{3b^2d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} + \frac{3bd^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.37 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.24

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx$$

$$= \frac{-\frac{bB(bc-ad)^2}{(a+bx)^2} + \frac{8b^2Bcd}{a+bx} - \frac{8abBd^2}{a+bx} + \frac{2bBd(bc-ad)}{a+bx} - \frac{4bBcd^2}{c+dx} + \frac{4aBd^3}{c+dx} + 6bBd^2\log(a+bx) - \frac{2b(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2}}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^3*(c*i + d*i*
x)^2),x]

[Out] (-((b*B*(b*c - a*d)^2)/(a + b*x)^2) + (8*b^2*B*c*d)/(a + b*x) - (8*a*b*B*d^
2)/(a + b*x) + (2*b*B*d*(b*c - a*d))/(a + b*x) - (4*b*B*c*d^2)/(c + d*x) +
(4*a*B*d^3)/(c + d*x) + 6*b*B*d^2*Log[a + b*x] - (2*b*(b*c - a*d)^2*(A + B*
Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x)^2 + (8*b*d*(b*c - a*d)*(A + B*Log[
(e*(a + b*x))/(c + d*x)]))/(a + b*x) + (4*d^2*(b*c - a*d)*(A + B*Log[(e*(a
+ b*x))/(c + d*x)]))/(c + d*x) + 12*b*d^2*Log[a + b*x]*(A + B*Log[(e*(a + b
*x))/(c + d*x)]) - 6*b*B*d^2*Log[c + d*x] - 12*b*d^2*(A + B*Log[(e*(a + b*x
))/(c + d*x)])*Log[c + d*x] - 6*b*B*d^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log
[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])
+ 6*b*B*d^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d
*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(4*(b*c - a*d)^4*g^3*i^2)
```

Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.39

method	result
parts	$\frac{A\left(-\frac{d^{2}}{(ad-cb)^{3}(dx+c)}-\frac{3d^{2}b\ln(dx+c)}{(ad-cb)^{4}}-\frac{b}{2(ad-cb)^{2}(bx+a)^{2}}+\frac{3d^{2}b\ln(bx+a)}{(ad-cb)^{4}}-\frac{2bd}{(ad-cb)^{3}(bx+a)}\right)}{g^{3}i^{2}}-\frac{B\left(\frac{d^{3}\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{d^{2}eAb^{3}}{2i^{2}(ad-cb)^{5}g^{3}\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}}-\frac{3d^{3}Ab^{2}}{i^{2}(ad-cb)^{5}g^{3}\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{3d^{4}Ab\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^{2}(ad-cb)^{5}g^{3}}+\frac{d^{5}A\left(\frac{be}{d}+\frac{(ad-cb)}{d(dx+c}\right.}{e^{2}i^{2}(ad-cb)^{5}g^{3}}\cdots\right.}{\cdots}$
default	$-\frac{e(ad-cb)\left(\frac{d^{2}eAb^{3}}{2i^{2}(ad-cb)^{5}g^{3}\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}}-\frac{3d^{3}Ab^{2}}{i^{2}(ad-cb)^{5}g^{3}\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{3d^{4}Ab\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^{2}(ad-cb)^{5}g^{3}}+\frac{d^{5}A\left(\frac{be}{d}+\frac{(ad-cb)}{d(dx+c}\right.}{e^{2}i^{2}(ad-cb)^{5}g^{3}}\cdots\right.}{\cdots}$
risch	$-\frac{A\,d^{2}}{g^{3}i^{2}(ad-cb)^{3}(dx+c)}-\frac{3A\,d^{2}b\ln(dx+c)}{g^{3}i^{2}(ad-cb)^{4}}-\frac{Ab}{2g^{3}i^{2}(ad-cb)^{2}(bx+a)^{2}}+\frac{3A\,d^{2}b\ln(bx+a)}{g^{3}i^{2}(ad-cb)^{4}}-\frac{2Abd}{g^{3}i^{2}(ad-cb)^{3}(bx+a)}\cdots$
parallelrisch	$\frac{6Bx^{3}\ln\left(\frac{e(bx+a)}{dx+c}\right)^{2}b^{7}d^{6}+12Ax^{3}\ln\left(\frac{e(bx+a)}{dx+c}\right)b^{7}d^{6}+6Bx^{3}\ln\left(\frac{e(bx+a)}{dx+c}\right)b^{7}d^{6}-12Ax^{2}ab^{6}d^{6}+12Ax^{2}b^{7}cd^{5}-6Bx^{2}ab^{6}d^{6}\cdots}{\cdots}$
norman	Expression too large to display

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x,method=_RETUR
NVERBOSE)
```

```
[Out] A/g^3/i^2*(-d^2/(a*d-b*c)^3/(d*x+c)-3*d^2/(a*d-b*c)^4*b*ln(d*x+c)-1/2*b/(a*
d-b*c)^2/(b*x+a)^2+3*d^2/(a*d-b*c)^4*b*ln(b*x+a)-2*b/(a*d-b*c)^3*d/(b*x+a))
-B/g^3/i^2/(a*d-b*c)/e*(d^3/(a*d-b*c)^3*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b
*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)-3/2/(a*d-b*c)^3*b*
d^2*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+3/(a*d-b*c)^3*b^2*d*e^2*(-1/(b*e/d+
(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e
/d/(d*x+c)))-1/(a*d-b*c)^3*b^3*e^3*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln
(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.82

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx = -\frac{(2A+B)b^3c^3-12(A+B)ab^2c^2d+3(2A+5B)a^2bcd^2+4(A-B)a^3d^3-6((2A+B)b^3cd^2-(2A}{4((b^6c^4d-4ab}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, algor
ithm="fricas")
```

```
[Out] -1/4*((2*A + B)*b^3*c^3 - 12*(A + B)*a*b^2*c^2*d + 3*(2*A + 5*B)*a^2*b*c*d^
2 + 4*(A - B)*a^3*d^3 - 6*((2*A + B)*b^3*c*d^2 - (2*A + B)*a*b^2*d^3)*x^2 -
 6*(B*b^3*d^3*x^3 + B*a^2*b*c*d^2 + (B*b^3*c*d^2 + 2*B*a*b^2*d^3)*x^2 + (2*
B*a*b^2*c*d^2 + B*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c))^2 - 3*((2*A +
3*B)*b^3*c^2*d + 2*(2*A - B)*a*b^2*c*d^2 - (6*A + B)*a^2*b*d^3)*x - 2*(3*(2
*A + B)*b^3*d^3*x^3 - B*b^3*c^3 + 6*B*a*b^2*c^2*d + 6*A*a^2*b*c*d^2 - 2*B*a
^3*d^3 + 3*((2*A + 3*B)*b^3*c*d^2 + 4*A*a*b^2*d^3)*x^2 + 3*(B*b^3*c^2*d + 4
*(A + B)*a*b^2*c*d^2 + 2*(A - B)*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c))
)/((b^6*c^4*d - 4*a*b^5*c^3*d^2 + 6*a^2*b^4*c^2*d^3 - 4*a^3*b^3*c*d^4 + a^4
*b^2*d^5)*g^3*i^2*x^3 + (b^6*c^5 - 2*a*b^5*c^4*d - 2*a^2*b^4*c^3*d^2 + 8*a^
3*b^3*c^2*d^3 - 7*a^4*b^2*c*d^4 + 2*a^5*b*d^5)*g^3*i^2*x^2 + (2*a*b^5*c^5 -
 7*a^2*b^4*c^4*d + 8*a^3*b^3*c^3*d^2 - 2*a^4*b^2*c^2*d^3 - 2*a^5*b*c*d^4 +
a^6*d^5)*g^3*i^2*x + (a^2*b^4*c^5 - 4*a^3*b^3*c^4*d + 6*a^4*b^2*c^3*d^2 - 4
*a^5*b*c^2*d^3 + a^6*c*d^4)*g^3*i^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3/(d*i*x+c*i)**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1721 vs. $2(358) = 716$.

Time = 0.33 (sec) , antiderivative size = 1721, normalized size of antiderivative = 4.73

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, algor
ithm="maxima")
```

```
[Out] 1/2*B*((6*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a*
b*d^2)*x)/((b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*g^
3*i^2*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 -
2*a^4*b*d^4)*g^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d
^2 + a^4*b*c*d^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*
a^4*b*c^2*d^2 - a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x + a)/((b^4*c^4 - 4*a*
b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2) - 6*b*d^2
*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*g^3*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 1/2*A*((6*b^2*
d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a*b*d^2)*x)/((b^
5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*g^3*i^2*x^3 + (b
^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*g
^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 + a^4*b*c*d
^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b*c^2*d^2
- a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*
a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2) - 6*b*d^2*log(d*x + c)/
((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^
3*i^2)) - 1/4*(b^3*c^3 - 12*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3 - 6*(b
^3*c*d^2 - a*b^2*d^3)*x^2 + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a
*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a)^2 + 6*(b^3*d^3*
x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*
d^3)*x)*log(d*x + c)^2 - 3*(3*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b*d^3)*x - 6*
(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2
 + a^2*b*d^3)*x)*log(b*x + a) + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 +
 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x - 2*(b^3*d^3*x^3 + a^2*b*
c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(
b*x + a))*log(d*x + c))*B/(a^2*b^4*c^5*g^3*i^2 - 4*a^3*b^3*c^4*d*g^3*i^2 +
6*a^4*b^2*c^3*d^2*g^3*i^2 - 4*a^5*b*c^2*d^3*g^3*i^2 + a^6*c*d^4*g^3*i^2 + (
b^6*c^4*d*g^3*i^2 - 4*a*b^5*c^3*d^2*g^3*i^2 + 6*a^2*b^4*c^2*d^3*g^3*i^2 - 4
*a^3*b^3*c*d^4*g^3*i^2 + a^4*b^2*d^5*g^3*i^2)*x^3 + (b^6*c^5*g^3*i^2 - 2*a*
b^5*c^4*d*g^3*i^2 - 2*a^2*b^4*c^3*d^2*g^3*i^2 + 8*a^3*b^3*c^2*d^3*g^3*i^2 -
 7*a^4*b^2*c*d^4*g^3*i^2 + 2*a^5*b*d^5*g^3*i^2)*x^2 + (2*a*b^5*c^5*g^3*i^2
- 7*a^2*b^4*c^4*d*g^3*i^2 + 8*a^3*b^3*c^3*d^2*g^3*i^2 - 2*a^4*b^2*c^2*d^3*g
^3*i^2 - 2*a^5*b*c*d^4*g^3*i^2 + a^6*d^5*g^3*i^2)*x)
```

Giac [A] (verification not implemented)

none

Time = 54.53 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.76

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx =$$
$$-\frac{1}{4}\left(\frac{2\left(Bbe^3-\frac{2\,(bex+ae)Bde^2}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^2bcg^3i^2}{(dx+c)^2}-\frac{(bex+ae)^2adg^3i^2}{(dx+c)^2}}+\frac{2\,Abe^3+Bbe^3-\frac{4\,(bex+ae)Ade^2}{dx+c}-\frac{4\,(bex+ae)Bde^2}{dx+c}}{\frac{(bex+ae)^2bcg^3i^2}{(dx+c)^2}-\frac{(bex+ae)^2adg^3i^2}{(dx+c)^2}}\right)\left(\frac{}{(bce-ad}\right.$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, algor
ithm="giac")
```

```
[Out] -1/4*(2*(B*b*e^3 - 2*(b*e*x + a*e)*B*d*e^2/(d*x + c))*log((b*e*x + a*e)/(d*
x + c))/((b*e*x + a*e)^2*b*c*g^3*i^2/(d*x + c)^2 - (b*e*x + a*e)^2*a*d*g^3*
i^2/(d*x + c)^2) + (2*A*b*e^3 + B*b*e^3 - 4*(b*e*x + a*e)*A*d*e^2/(d*x + c)
 - 4*(b*e*x + a*e)*B*d*e^2/(d*x + c))/((b*e*x + a*e)^2*b*c*g^3*i^2/(d*x + c
)^2 - (b*e*x + a*e)^2*a*d*g^3*i^2/(d*x + c)^2))*(b*c/((b*c*e - a*d*e)*(b*c
 - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

Mupad [B] (verification not implemented)

Time = 5.64 (sec) , antiderivative size = 984, normalized size of antiderivative = 2.70

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx = \frac{3\,B\,b\,d^2\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2}{2\,g^3\,i^2\,{\left(a\,d-b\,c\right)}^4} - \frac{A\,a^2\,d^2}{g^3\,i^2\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}$$
$$+\frac{A\,b^2\,c^2}{2\,g^3\,i^2\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}$$
$$+\frac{B\,a^2\,d^2}{g^3\,i^2\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}$$
$$+\frac{B\,b^2\,c^2}{4\,g^3\,i^2\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}$$
$$-\frac{B\,a\,d\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^3\,i^2\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}$$
$$-\frac{B\,b\,c\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,g^3\,i^2\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}$$
$$-\frac{3\,A\,b^2\,d^2\,x^2}{g^3\,i^2\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}$$
$$-\frac{3\,B\,b^2\,d^2\,x^2}{2\,g^3\,i^2\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}$$
$$-\frac{5\,A\,a\,b\,c\,d}{2\,g^3\,i^2\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}$$
$$-\frac{11\,B\,a\,b\,c\,d}{4\,g^3\,i^2\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}$$
$$-\frac{3\,B\,b\,d\,x\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,g^3\,i^2\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}$$
$$-\frac{3\,B\,b^2\,d^2\,x^2\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^3\,i^2\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}$$
$$-\frac{9\,A\,a\,b\,d^2\,x}{2\,g^3\,i^2\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}$$
$$-\frac{3\,B\,a\,b\,d^2\,x}{4\,g^3\,i^2\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}$$
$$-\frac{3\,A\,b^2\,c\,d\,x}{2\,g^3\,i^2\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}$$
$$-\frac{9\,B\,b^2\,c\,d\,x}{4\,g^3\,i^2\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}$$
$$-\frac{3\,B\,a\,b\,c\,d\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^3\,i^2\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}$$
$$-\frac{3\,B\,a\,b\,d^2\,x\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^3\,i^2\,{\left(a\,d-b\,c\right)}^3\,{\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}$$
$$3\,B\,b^2\,c\,d\,x\,\ln\left(\frac{e\,(a+b\,x)}{\ldots}\right)$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^3*(c*i + d*i*x)^2),
x)
```

```
[Out] (3*B*b*d^2*log((e*(a + b*x))/(c + d*x))^2)/(2*g^3*i^2*(a*d - b*c)^4) - (A*b
*d^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*6i)/(g^3*i^2*(a*d - b*c
)^4) - (B*b*d^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*3i)/(g^3*i^2
*(a*d - b*c)^4) - (A*a^2*d^2)/(g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x))
 + (A*b^2*c^2)/(2*g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) + (B*a^2*d^2
)/(g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) + (B*b^2*c^2)/(4*g^3*i^2*(a
*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (B*a*d*log((e*(a + b*x))/(c + d*x)))/(
g^3*i^2*(a*d - b*c)^2*(a + b*x)^2*(c + d*x)) - (B*b*c*log((e*(a + b*x))/(c
+ d*x)))/(2*g^3*i^2*(a*d - b*c)^2*(a + b*x)^2*(c + d*x)) - (3*A*b^2*d^2*x^2
)/(g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (3*B*b^2*d^2*x^2)/(2*g^3*
i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (5*A*a*b*c*d)/(2*g^3*i^2*(a*d -
b*c)^3*(a + b*x)^2*(c + d*x)) - (11*B*a*b*c*d)/(4*g^3*i^2*(a*d - b*c)^3*(a
+ b*x)^2*(c + d*x)) - (3*B*b*d*x*log((e*(a + b*x))/(c + d*x)))/(2*g^3*i^2*(
a*d - b*c)^2*(a + b*x)^2*(c + d*x)) - (3*B*b^2*d^2*x^2*log((e*(a + b*x))/(c
 + d*x)))/(g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (9*A*a*b*d^2*x)/(
2*g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (3*B*a*b*d^2*x)/(4*g^3*i^2
*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (3*A*b^2*c*d*x)/(2*g^3*i^2*(a*d - b
*c)^3*(a + b*x)^2*(c + d*x)) - (9*B*b^2*c*d*x)/(4*g^3*i^2*(a*d - b*c)^3*(a
+ b*x)^2*(c + d*x)) - (3*B*a*b*c*d*log((e*(a + b*x))/(c + d*x)))/(g^3*i^2*(
a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (3*B*a*b*d^2*x*log((e*(a + b*x))/(c +
 d*x)))/(g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (3*B*b^2*c*d*x*log(
(e*(a + b*x))/(c + d*x)))/(g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x))
```

3.46 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 457

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = -\frac{Bd^4(a+bx)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{6b^2Bd^2(c+dx)}{(bc-ad)^5g^4i^2(a+bx)} + \frac{b^3Bd(c+dx)^2}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{b^4B(c+dx)^3}{9(bc-ad)^5g^4i^2(a+bx)^3} + \frac{2bBd^3\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2} + \frac{d^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{6b^2d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(a+bx)} + \frac{2b^3d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{b^4(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^5g^4i^2(a+bx)^3} - \frac{4bd^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2}$$

```
[Out] -B*d^4*(b*x+a)/(-a*d+b*c)^5/g^4/i^2/(d*x+c)-6*b^2*B*d^2*(d*x+c)/(-a*d+b*c)^
5/g^4/i^2/(b*x+a)+b^3*B*d*(d*x+c)^2/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-1/9*b^4*
B*(d*x+c)^3/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^3+2*b*B*d^3*ln((b*x+a)/(d*x+c))^2/
(-a*d+b*c)^5/g^4/i^2+d^4*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^5/g
^4/i^2/(d*x+c)-6*b^2*d^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^5/g
^4/i^2/(b*x+a)+2*b^3*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^5/g
```

```
^4/i^2/(b*x+a)^2-1/3*b^4*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^5
/g^4/i^2/(b*x+a)^3-4*b*d^3*ln((b*x+a)/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/
(-a*d+b*c)^5/g^4/i^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2562, 45, 2372, 2338}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = -\frac{b^4(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3g^4i^2(a+bx)^3(bc-ad)^5} + \frac{2b^3d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^4i^2(a+bx)^2(bc-ad)^5} - \frac{6b^2d^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^4i^2(a+bx)(bc-ad)^5} + \frac{d^4(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^4i^2(c+dx)(bc-ad)^5} - \frac{4bd^3\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^4i^2(bc-ad)^5} - \frac{b^4B(c+dx)^3}{9g^4i^2(a+bx)^3(bc-ad)^5} + \frac{b^3Bd(c+dx)^2}{g^4i^2(a+bx)^2(bc-ad)^5} - \frac{6b^2Bd^2(c+dx)}{g^4i^2(a+bx)(bc-ad)^5} - \frac{Bd^4(a+bx)}{g^4i^2(c+dx)(bc-ad)^5} + \frac{2bBd^3\log^2\left(\frac{a+bx}{c+dx}\right)}{g^4i^2(bc-ad)^5}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^4*(c*i + d*i*x)^2),
x]

[Out] -((B*d^4*(a + b*x))/((b*c - a*d)^5*g^4*i^2*(c + d*x))) - (6*b^2*B*d^2*(c +
d*x))/((b*c - a*d)^5*g^4*i^2*(a + b*x)) + (b^3*B*d*(c + d*x)^2)/((b*c - a*d
)^5*g^4*i^2*(a + b*x)^2) - (b^4*B*(c + d*x)^3)/(9*(b*c - a*d)^5*g^4*i^2*(a
+ b*x)^3) + (2*b*B*d^3*Log[(a + b*x)/(c + d*x)]^2)/((b*c - a*d)^5*g^4*i^2)
+ (d^4*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^5*g^4*i
^2*(c + d*x)) - (6*b^2*d^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/
((b*c - a*d)^5*g^4*i^2*(a + b*x)) + (2*b^3*d*(c + d*x)^2*(A + B*Log[(e*(a +
 b*x))/(c + d*x)]))/((b*c - a*d)^5*g^4*i^2*(a + b*x)^2) - (b^4*(c + d*x)^3*
(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*(b*c - a*d)^5*g^4*i^2*(a + b*x)^3)
 - (4*b*d^3*Log[(a + b*x)/(c + d*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/
((b*c - a*d)^5*g^4*i^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)^4(A+B\log(ex))}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2} \\
&= \frac{d^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{6b^2d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(a+bx)} \\
&\quad + \frac{2b^3d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{b^4(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^5g^4i^2(a+bx)^3} \\
&\quad - \frac{4bd^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2} \\
&\quad - \frac{B\text{Subst}\left(\int \left(d^4-\frac{b^4}{3x^4}+\frac{2b^3d}{x^3}-\frac{6b^2d^2}{x^2}-\frac{4bd^3\log(x)}{x}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2}
\end{aligned}$$

$$= -\frac{Bd^4(a+bx)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{6b^2Bd^2(c+dx)}{(bc-ad)^5g^4i^2(a+bx)} + \frac{b^3Bd(c+dx)^2}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{b^4B(c+dx)^3}{9(bc-ad)^5g^4i^2(a+bx)^3} + \frac{d^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{6b^2d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(a+bx)} + \frac{2b^3d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{b^4(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^5g^4i^2(a+bx)^3} - \frac{4bd^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2} + \frac{(4bBd^3)\,\mathrm{Subst}\left(\int\frac{\log(x)}{x}dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2}$$

$$= -\frac{Bd^4(a+bx)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{6b^2Bd^2(c+dx)}{(bc-ad)^5g^4i^2(a+bx)} + \frac{b^3Bd(c+dx)^2}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{b^4B(c+dx)^3}{9(bc-ad)^5g^4i^2(a+bx)^3} + \frac{2bBd^3\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2} + \frac{d^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{6b^2d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(a+bx)} + \frac{2b^3d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{b^4(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^5g^4i^2(a+bx)^3} - \frac{4bd^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.71 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.14

$$\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}dx =$$

$$-\frac{\frac{bB(bc-ad)^3}{(a+bx)^3} - \frac{6bBd(bc-ad)^2}{(a+bx)^2} + \frac{27b^2Bcd^2}{a+bx} - \frac{27abBd^3}{a+bx} + \frac{12bBd^2(bc-ad)}{a+bx} - \frac{9bBcd^3}{c+dx} + \frac{9aBd^4}{c+dx} + 30bBd^3\log(a+bx) + \frac{3b}{}}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^4*(c*i + d*i*
x)^2),x]
```

```
[Out] -1/9*((b*B*(b*c - a*d)^3)/(a + b*x)^3 - (6*b*B*d*(b*c - a*d)^2)/(a + b*x)^2
 + (27*b^2*B*c*d^2)/(a + b*x) - (27*a*b*B*d^3)/(a + b*x) + (12*b*B*d^2*(b*c
 - a*d))/(a + b*x) - (9*b*B*c*d^3)/(c + d*x) + (9*a*B*d^4)/(c + d*x) + 30*b
*B*d^3*Log[a + b*x] + (3*b*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)
]))/(a + b*x)^3 - (9*b*d*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
```

```
)/(a + b*x)^2 + (27*b*d^2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))
/(a + b*x) - (9*d^3*(-(b*c) + a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c
 + d*x) + 36*b*d^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 30*b
*B*d^3*Log[c + d*x] - 36*b*d^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c +
 d*x] - 18*b*B*d^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c -
 a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 18*b*B*d^3*((2*Log[
(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (
b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)^5*g^4*i^2)
```

Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.40

method	result
parts	$\frac{A\left(-\frac{d^3}{(ad-cb)^4(dx+c)}-\frac{4d^3b\ln(dx+c)}{(ad-cb)^5}-\frac{b}{3(ad-cb)^2(bx+a)^3}+\frac{4d^3b\ln(bx+a)}{(ad-cb)^5}-\frac{3b\,d^2}{(ad-cb)^4(bx+a)}-\frac{bd}{(ad-cb)^3(bx+a)^2}\right)}{g^4i^2}-\frac{B\left(\frac{d^4\left(\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^2e^2A\,b^4}{3i^2(ad-cb)^6g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{2d^3eA\,b^3}{i^2(ad-cb)^6g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{6d^4A\,b^2}{i^2(ad-cb)^6g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{4d^5Ab\ln\left(\cdots\right.}{e\,i^2(\cdots}\cdots\right.}{\cdots}$
default	$-\frac{e(ad-cb)\left(-\frac{d^2e^2A\,b^4}{3i^2(ad-cb)^6g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{2d^3eA\,b^3}{i^2(ad-cb)^6g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{6d^4A\,b^2}{i^2(ad-cb)^6g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{4d^5Ab\ln\left(\cdots\right.}{e\,i^2(\cdots}\cdots\right.}{\cdots}$
risch	$-\frac{A\,d^3}{g^4i^2(ad-cb)^4(dx+c)}-\frac{4A\,d^3b\ln(dx+c)}{g^4i^2(ad-cb)^5}-\frac{Ab}{3g^4i^2(ad-cb)^2(bx+a)^3}+\frac{4A\,d^3b\ln(bx+a)}{g^4i^2(ad-cb)^5}-\frac{3Ab\,d^2}{g^4i^2(ad-cb)^4(bx+a)}\cdots$
parallelrisch	Expression too large to display
norman	Expression too large to display

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x,method=_RETUR
NVERBOSE)
```

```
[Out] A/g^4/i^2*(-d^3/(a*d-b*c)^4/(d*x+c)-4*d^3/(a*d-b*c)^5*b*ln(d*x+c)-1/3*b/(a*
d-b*c)^2/(b*x+a)^3+4*d^3/(a*d-b*c)^5*b*ln(b*x+a)-3*b/(a*d-b*c)^4*d^2/(b*x+a
)-b/(a*d-b*c)^3*d/(b*x+a)^2)-B/g^4/i^2/(a*d-b*c)/e*(d^4/(a*d-b*c)^4*((b*e/d
+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+
c)-b*e/d)-2/(a*d-b*c)^4*b*d^3*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+6/(a*d-b*
c)^4*b^2*d^2*e^2*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(
```

```
d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-4/(a*d-b*c)^4*b^3*d*e^3*(-1/2/(b*e
/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d
-b*c)*e/d/(d*x+c))^2)+1/(a*d-b*c)^4*b^4*e^4*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x
+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1019 vs. 2(453) = 906.

Time = 0.32 (sec) , antiderivative size = 1019, normalized size of antiderivative = 2.23

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = -\frac{(3A+B)b^4c^4-9(2A+B)ab^3c^3d+54(A+B)a^2b^2c^2d^2-5(6A+11B)a^3bcd^3-9(A-B)a^4d^4+6}{}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, algor
ithm="fricas")
```

```
[Out] -1/9*((3*A + B)*b^4*c^4 - 9*(2*A + B)*a*b^3*c^3*d + 54*(A + B)*a^2*b^2*c^2*
d^2 - 5*(6*A + 11*B)*a^3*b*c*d^3 - 9*(A - B)*a^4*d^4 + 6*((6*A + 5*B)*b^4*c
*d^3 - (6*A + 5*B)*a*b^3*d^4)*x^3 + 3*((6*A + 11*B)*b^4*c^2*d^2 + 8*(3*A +
B)*a*b^3*c*d^3 - (30*A + 19*B)*a^2*b^2*d^4)*x^2 + 18*(B*b^4*d^4*x^4 + B*a^3
*b*c*d^3 + (B*b^4*c*d^3 + 3*B*a*b^3*d^4)*x^3 + 3*(B*a*b^3*c*d^3 + B*a^2*b^2
*d^4)*x^2 + (3*B*a^2*b^2*c*d^3 + B*a^3*b*d^4)*x)*log((b*e*x + a*e)/(d*x + c
))^2 - ((6*A + 5*B)*b^4*c^3*d - 27*(2*A + 3*B)*a*b^3*c^2*d^2 - 3*(6*A - 19*
B)*a^2*b^2*c*d^3 + (66*A + 19*B)*a^3*b*d^4)*x + 3*(2*(6*A + 5*B)*b^4*d^4*x^
4 + B*b^4*c^4 - 6*B*a*b^3*c^3*d + 18*B*a^2*b^2*c^2*d^2 + 12*A*a^3*b*c*d^3 -
 3*B*a^4*d^4 + 2*((6*A + 11*B)*b^4*c*d^3 + 9*(2*A + B)*a*b^3*d^4)*x^3 + 6*(
B*b^4*c^2*d^2 + 3*(2*A + 3*B)*a*b^3*c*d^3 + 6*A*a^2*b^2*d^4)*x^2 - 2*(B*b^4
*c^3*d - 9*B*a*b^3*c^2*d^2 - 18*(A + B)*a^2*b^2*c*d^3 - 6*(A - B)*a^3*b*d^4
)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^8*c^5*d - 5*a*b^7*c^4*d^2 + 10*a^2*b
^6*c^3*d^3 - 10*a^3*b^5*c^2*d^4 + 5*a^4*b^4*c*d^5 - a^5*b^3*d^6)*g^4*i^2*x^
4 + (b^8*c^6 - 2*a*b^7*c^5*d - 5*a^2*b^6*c^4*d^2 + 20*a^3*b^5*c^3*d^3 - 25*
a^4*b^4*c^2*d^4 + 14*a^5*b^3*c*d^5 - 3*a^6*b^2*d^6)*g^4*i^2*x^3 + 3*(a*b^7*
c^6 - 4*a^2*b^6*c^5*d + 5*a^3*b^5*c^4*d^2 - 5*a^5*b^3*c^2*d^4 + 4*a^6*b^2*c
*d^5 - a^7*b*d^6)*g^4*i^2*x^2 + (3*a^2*b^6*c^6 - 14*a^3*b^5*c^5*d + 25*a^4*
b^4*c^4*d^2 - 20*a^5*b^3*c^3*d^3 + 5*a^6*b^2*c^2*d^4 + 2*a^7*b*c*d^5 - a^8*
d^6)*g^4*i^2*x + (a^3*b^5*c^6 - 5*a^4*b^4*c^5*d + 10*a^5*b^3*c^4*d^2 - 10*a
^6*b^2*c^3*d^3 + 5*a^7*b*c^2*d^4 - a^8*c*d^5)*g^4*i^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4/(d*i*x+c*i)**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2560 vs. $2(453) = 906$.

Time = 0.43 (sec) , antiderivative size = 2560, normalized size of antiderivative = 5.60

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, algor
ithm="maxima")
[Out] -1/3*B*((12*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*d + 13*a^2*b*c*d^2 + 3*a^3*
d^3 + 6*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*c^2*d - 8*a*b^2*c*d^2 - 11*a
^2*b*d^3)*x)/((b^7*c^4*d - 4*a*b^6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*
c*d^4 + a^4*b^3*d^5)*g^4*i^2*x^4 + (b^7*c^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d
^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a^5*b^2*d^5)*g^4*i^2*x^3 + 3
*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^4*b^3*c^2*d^3 - 3*a
^5*b^2*c*d^4 + a^6*b*d^5)*g^4*i^2*x^2 + (3*a^2*b^5*c^5 - 11*a^3*b^4*c^4*d +
 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)*g^4*i^2*x
+ (a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^
7*c*d^4)*g^4*i^2) + 12*b*d^3*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^
2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2) - 12
*b*d^3*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3
*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2))*log(b*e*x/(d*x + c) + a*e
/(d*x + c)) - 1/3*A*((12*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*d + 13*a^2*b*c
*d^2 + 3*a^3*d^3 + 6*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*c^2*d - 8*a*b^2
*c*d^2 - 11*a^2*b*d^3)*x)/((b^7*c^4*d - 4*a*b^6*c^3*d^2 + 6*a^2*b^5*c^2*d^3
 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*g^4*i^2*x^4 + (b^7*c^5 - a*b^6*c^4*d - 6*
a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a^5*b^2*d^5)*g^
4*i^2*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^4*b^3*
c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*g^4*i^2*x^2 + (3*a^2*b^5*c^5 - 11*a^
3*b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^
5)*g^4*i^2*x + (a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b
*c^2*d^3 + a^7*c*d^4)*g^4*i^2) + 12*b*d^3*log(b*x + a)/((b^5*c^5 - 5*a*b^4*
```

```
c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*
g^4*i^2) - 12*b*d^3*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3
*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2)) - 1/9*(b^4*c
^4 - 9*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 - 55*a^3*b*c*d^3 + 9*a^4*d^4 + 30*(
b^4*c*d^3 - a*b^3*d^4)*x^3 + 3*(11*b^4*c^2*d^2 + 8*a*b^3*c*d^3 - 19*a^2*b^2
*d^4)*x^2 - 18*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 +
 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b
*x + a)^2 - 18*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 +
 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(d
*x + c)^2 - (5*b^4*c^3*d - 81*a*b^3*c^2*d^2 + 57*a^2*b^2*c*d^3 + 19*a^3*b*d
^4)*x + 30*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(
a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x +
 a) - 6*(5*b^4*d^4*x^4 + 5*a^3*b*c*d^3 + 5*(b^4*c*d^3 + 3*a*b^3*d^4)*x^3 +
15*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 5*(3*a^2*b^2*c*d^3 + a^3*b*d^4)*x - 6*
(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3
 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a))*log(d*
x + c))*B/(a^3*b^5*c^6*g^4*i^2 - 5*a^4*b^4*c^5*d*g^4*i^2 + 10*a^5*b^3*c^4*d
^2*g^4*i^2 - 10*a^6*b^2*c^3*d^3*g^4*i^2 + 5*a^7*b*c^2*d^4*g^4*i^2 - a^8*c*d
^5*g^4*i^2 + (b^8*c^5*d*g^4*i^2 - 5*a*b^7*c^4*d^2*g^4*i^2 + 10*a^2*b^6*c^3*
d^3*g^4*i^2 - 10*a^3*b^5*c^2*d^4*g^4*i^2 + 5*a^4*b^4*c*d^5*g^4*i^2 - a^5*b^
3*d^6*g^4*i^2)*x^4 + (b^8*c^6*g^4*i^2 - 2*a*b^7*c^5*d*g^4*i^2 - 5*a^2*b^6*c
^4*d^2*g^4*i^2 + 20*a^3*b^5*c^3*d^3*g^4*i^2 - 25*a^4*b^4*c^2*d^4*g^4*i^2 +
14*a^5*b^3*c*d^5*g^4*i^2 - 3*a^6*b^2*d^6*g^4*i^2)*x^3 + 3*(a*b^7*c^6*g^4*i^
2 - 4*a^2*b^6*c^5*d*g^4*i^2 + 5*a^3*b^5*c^4*d^2*g^4*i^2 - 5*a^5*b^3*c^2*d^4
*g^4*i^2 + 4*a^6*b^2*c*d^5*g^4*i^2 - a^7*b*d^6*g^4*i^2)*x^2 + (3*a^2*b^6*c^
6*g^4*i^2 - 14*a^3*b^5*c^5*d*g^4*i^2 + 25*a^4*b^4*c^4*d^2*g^4*i^2 - 20*a^5*
b^3*c^3*d^3*g^4*i^2 + 5*a^6*b^2*c^2*d^4*g^4*i^2 + 2*a^7*b*c*d^5*g^4*i^2 - a
^8*d^6*g^4*i^2)*x)
```

Giac [A] (verification not implemented)

none

Time = 64.99 (sec) , antiderivative size = 436, normalized size of antiderivative = 0.95

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = -\frac{1}{18}\left(\frac{6\left(Bb^2e^4-\frac{3(bex+ae)Bbde^3}{dx+c}+\frac{3(bex+ae)^2Bd^2e^2}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^3b^2c^2g^4i^2}{(dx+c)^3}-\frac{2(bex+ae)^3abcdg^4i^2}{(dx+c)^3}+\frac{(bex+ae)^3a^2d^2g^4i^2}{(dx+c)^3}}+\frac{6Ab^2e^4+2Bb^2e^4-\frac{18(bex+ae)Abde^3}{dx+c}-9}{\frac{(bex+ae)^3b^2c^2g^4i^2}{(dx+c)^3}-2(be}\right.$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, algor
ithm="giac")
```

```
[Out] -1/18*(6*(B*b^2*e^4 - 3*(b*e*x + a*e)*B*b*d*e^3/(d*x + c) + 3*(b*e*x + a*e)
^2*B*d^2*e^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^3*b^2
*c^2*g^4*i^2/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4*i^2/(d*x + c)^3 +
(b*e*x + a*e)^3*a^2*d^2*g^4*i^2/(d*x + c)^3) + (6*A*b^2*e^4 + 2*B*b^2*e^4 -
 18*(b*e*x + a*e)*A*b*d*e^3/(d*x + c) - 9*(b*e*x + a*e)*B*b*d*e^3/(d*x + c)
 + 18*(b*e*x + a*e)^2*A*d^2*e^2/(d*x + c)^2 + 18*(b*e*x + a*e)^2*B*d^2*e^2/
(d*x + c)^2)/((b*e*x + a*e)^3*b^2*c^2*g^4*i^2/(d*x + c)^3 - 2*(b*e*x + a*e)
^3*a*b*c*d*g^4*i^2/(d*x + c)^3 + (b*e*x + a*e)^3*a^2*d^2*g^4*i^2/(d*x + c)^
3))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
^2
```

Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 1679, normalized size of antiderivative = 3.67

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^4*(c*i + d*i*x)^2),
x)
```

```
[Out] (2*B*b*d^3*log((e*(a + b*x))/(c + d*x))^2)/(g^4*i^2*(a*d - b*c)^2*(a^3*d^3
- b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (log((e*(a + b*x))/(c + d*x))
*(x*((4*B)/(3*g^4*i^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (4*B*b*d^3*(((2*a^
2*d^2 + b^2*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2))*(a*d +
b*c) + (a*c*(a*d - b*c))/d^2))/(g^4*i^2*(a*d - b*c)^2*(a^3*d^3 - b^3*c^3 +
3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) + (B*(3*a*d + b*c))/(3*g^4*i^2*(a^2*b*d^3
+ b^3*c^2*d - 2*a*b^2*c*d^2)) + (4*B*b^2*d^2*x^3)/(g^4*i^2*(a*d - b*c)*(a^3
*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (4*B*b*d^3*x^2*(b*d*((2*
a^2*d^2 + b^2*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)) + ((a
*d + b*c)*(a*d - b*c))/d^2))/(g^4*i^2*(a*d - b*c)^2*(a^3*d^3 - b^3*c^3 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (4*B*a*b*c*d^3*((2*a^2*d^2 + b^2*c^2 - 3*a*
b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)))/(g^4*i^2*(a*d - b*c)^2*(a^3*
d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))))/(b^2*x^4 + (a^3*c)/(b*d)
+ (x*(a^3*d + 3*a^2*b*c))/(b*d) + (x^3*(b^3*c + 3*a*b^2*d))/(b*d) + (x^2*(3
*a*b^2*c + 3*a^2*b*d))/(b*d)) - (b*d^3*atan((b*d^3*((a^5*d^5*g^4*i^2 + b^5*
c^5*g^4*i^2 - 3*a*b^4*c^4*d*g^4*i^2 - 3*a^4*b*c*d^4*g^4*i^2 + 2*a^2*b^3*c^3
*d^2*g^4*i^2 + 2*a^3*b^2*c^2*d^3*g^4*i^2)/(a^4*d^4*g^4*i^2 + b^4*c^4*g^4*i^
2 - 4*a*b^3*c^3*d*g^4*i^2 - 4*a^3*b*c*d^3*g^4*i^2 + 6*a^2*b^2*c^2*d^2*g^4*i
^2) + 2*b*d*x)*(6*A + 5*B)*(a^4*d^4*g^4*i^2 + b^4*c^4*g^4*i^2 - 4*a*b^3*c^3
*d*g^4*i^2 - 4*a^3*b*c*d^3*g^4*i^2 + 6*a^2*b^2*c^2*d^2*g^4*i^2)*2i)/(g^4*i^
2*(a*d - b*c)^5*(12*A*b*d^3 + 10*B*b*d^3)))*(6*A + 5*B)*4i)/(3*g^4*i^2*(a*d
 - b*c)^5) - ((9*A*a^3*d^3 + 3*A*b^3*c^3 - 9*B*a^3*d^3 + B*b^3*c^3 - 15*A*a
*b^2*c^2*d + 39*A*a^2*b*c*d^2 - 8*B*a*b^2*c^2*d + 46*B*a^2*b*c*d^2)/(3*(a*d
```

```
- b*c)) + (x*(66*A*a^2*b*d^3 + 19*B*a^2*b*d^3 - 6*A*b^3*c^2*d - 5*B*b^3*c^
2*d + 48*A*a*b^2*c*d^2 + 76*B*a*b^2*c*d^2))/(3*(a*d - b*c)) + (x^2*(30*A*a*
b^2*d^3 + 19*B*a*b^2*d^3 + 6*A*b^3*c*d^2 + 11*B*b^3*c*d^2))/(a*d - b*c) + (
2*x^3*(6*A*b^3*d^3 + 5*B*b^3*d^3))/(a*d - b*c))/(x*(3*a^6*d^4*g^4*i^2 - 9*a
^2*b^4*c^4*g^4*i^2 + 24*a^3*b^3*c^3*d*g^4*i^2 - 18*a^4*b^2*c^2*d^2*g^4*i^2)
- x^2*(9*a*b^5*c^4*g^4*i^2 - 9*a^5*b*d^4*g^4*i^2 - 18*a^2*b^4*c^3*d*g^4*i^
2 + 18*a^4*b^2*c*d^3*g^4*i^2) - x^3*(3*b^6*c^4*g^4*i^2 - 9*a^4*b^2*d^4*g^4*
i^2 + 24*a^3*b^3*c*d^3*g^4*i^2 - 18*a^2*b^4*c^2*d^2*g^4*i^2) + x^4*(3*a^3*b
^3*d^4*g^4*i^2 - 3*b^6*c^3*d*g^4*i^2 + 9*a*b^5*c^2*d^2*g^4*i^2 - 9*a^2*b^4*
c*d^3*g^4*i^2) - 3*a^3*b^3*c^4*g^4*i^2 + 3*a^6*c*d^3*g^4*i^2 + 9*a^4*b^2*c^
3*d*g^4*i^2 - 9*a^5*b*c^2*d^2*g^4*i^2)
```

3.47 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 361

$$\begin{aligned}\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx &= -\frac{3B(bc-ad)g^3(a+bx)^2}{4d^2i^3(c+dx)^2}-\frac{3bB(bc-ad)g^3(a+bx)}{d^3i^3(c+dx)}+\frac{b(3A+B)(bc-ad)g^3(a+bx)}{d^3i^3(c+dx)}\\
&\quad+\frac{3bB(bc-ad)g^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^3i^3(c+dx)}+\frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di^3(c+dx)^2}\\
&\quad+\frac{(bc-ad)g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i^3(c+dx)^2}\\
&\quad+\frac{b^2(bc-ad)g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4i^3}\\
&\quad+\frac{3b^2B(bc-ad)g^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}\end{aligned}$$

```
[Out] -3/4*B*(-a*d+b*c)*g^3*(b*x+a)^2/d^2/i^3/(d*x+c)^2-3*b*B*(-a*d+b*c)*g^3*(b*x
+a)/d^3/i^3/(d*x+c)+b*(3*A+B)*(-a*d+b*c)*g^3*(b*x+a)/d^3/i^3/(d*x+c)+3*b*B*
(-a*d+b*c)*g^3*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/d^3/i^3/(d*x+c)+g^3*(b*x+a)^3*
(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i^3/(d*x+c)^2+1/2*(-a*d+b*c)*g^3*(b*x+a)^2*(3
*A+B+3*B*ln(e*(b*x+a)/(d*x+c)))/d^2/i^3/(d*x+c)^2+b^2*(-a*d+b*c)*g^3*ln((-a
*d+b*c)/b/(d*x+c))*(3*A+B+3*B*ln(e*(b*x+a)/(d*x+c)))/d^4/i^3+3*b^2*B*(-a*d+
b*c)*g^3*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i^3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2562, 2384, 45, 2393, 2332, 2341, 2354, 2438}

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$$
$$= \frac{b^2g^3(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{d^4i^3}$$
$$+\frac{bg^3(3A+B)(a+bx)(bc-ad)}{d^3i^3(c+dx)}+\frac{g^3(a+bx)^2(bc-ad)\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d^2i^3(c+dx)^2}$$
$$+\frac{g^3(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{di^3(c+dx)^2}+\frac{3b^2Bg^3(bc-ad)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}$$
$$+\frac{3bBg^3(a+bx)(bc-ad)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^3i^3(c+dx)}$$
$$-\frac{3bBg^3(a+bx)(bc-ad)}{d^3i^3(c+dx)}-\frac{3Bg^3(a+bx)^2(bc-ad)}{4d^2i^3(c+dx)^2}$$

```
[In] Int[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x)^3,
x]
```

```
[Out] (-3*B*(b*c - a*d)*g^3*(a + b*x)^2)/(4*d^2*i^3*(c + d*x)^2) - (3*b*B*(b*c -
a*d)*g^3*(a + b*x))/(d^3*i^3*(c + d*x)) + (b*(3*A + B)*(b*c - a*d)*g^3*(a +
 b*x))/(d^3*i^3*(c + d*x)) + (3*b*B*(b*c - a*d)*g^3*(a + b*x)*Log[(e*(a + b
*x))/(c + d*x)])/(d^3*i^3*(c + d*x)) + (g^3*(a + b*x)^3*(A + B*Log[(e*(a +
b*x))/(c + d*x)]))/(d*i^3*(c + d*x)^2) + ((b*c - a*d)*g^3*(a + b*x)^2*(3*A
+ B + 3*B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d^2*i^3*(c + d*x)^2) + (b^2*(b*
c - a*d)*g^3*Log[(b*c - a*d)/(b*(c + d*x))]*(3*A + B + 3*B*Log[(e*(a + b*x)
)/(c + d*x)]))/(d^4*i^3) + (3*b^2*B*(b*c - a*d)*g^3*PolyLog[2, (d*(a + b*x)
)/(b*(c + d*x))])/(d^4*i^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)g^3)\,\text{Subst}\left(\int \frac{x^3(A+B\log(ex))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{i^3}$$

$$= \frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di^3(c+dx)^2} - \frac{((bc-ad)g^3)\,\text{Subst}\left(\int \frac{x^2(3A+B+3B\log(ex))}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{di^3}$$

$$= \frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di^3(c+dx)^2} - \frac{((bc-ad)g^3)\,\text{Subst}\left(\int \left(-\frac{b(3A+B+3B\log(ex))}{d^2} - \frac{x(3A+B+3B\log(ex))}{d} - \frac{b^2(3A+B+3B\log(ex))}{d^2(-b+dx)}\right) dx, x, \frac{a+bx}{c+dx}\right)}{di^3}$$

$$= \frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di^3(c+dx)^2} + \frac{(b(bc-ad)g^3)\,\text{Subst}\left(\int (3A+B+3B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^3} + \frac{(b^2(bc-ad)g^3)\,\text{Subst}\left(\int \frac{3A+B+3B\log(ex)}{-b+dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^3} + \frac{((bc-ad)g^3)\,\text{Subst}\left(\int x(3A+B+3B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^3}$$

$$= -\frac{3B(bc-ad)g^3(a+bx)^2}{4d^2i^3(c+dx)^2} + \frac{b(3A+B)(bc-ad)g^3(a+bx)}{d^3i^3(c+dx)} + \frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di^3(c+dx)^2} + \frac{(bc-ad)g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i^3(c+dx)^2} + \frac{b^2(bc-ad)g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4i^3} - \frac{(3b^2B(bc-ad)g^3)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4i^3} + \frac{(3bB(bc-ad)g^3)\,\text{Subst}\left(\int \log(ex)\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^3}$$

$$= -\frac{3B(bc-ad)g^3(a+bx)^2}{4d^2i^3(c+dx)^2} - \frac{3bB(bc-ad)g^3(a+bx)}{d^3i^3(c+dx)} + \frac{b(3A+B)(bc-ad)g^3(a+bx)}{d^3i^3(c+dx)} + \frac{3bB(bc-ad)g^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^3i^3(c+dx)} + \frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di^3(c+dx)^2} + \frac{(bc-ad)g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i^3(c+dx)^2} + \frac{b^2(bc-ad)g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4i^3} + \frac{3b^2B(bc-ad)g^3\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.88

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$$

$$= \frac{g^3\left(4Ab^3dx - \frac{B(bc-ad)^3}{(c+dx)^2} + \frac{10bB(bc-ad)^2}{c+dx} + 10b^2B(bc-ad)\log(a+bx) + 4b^2Bd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right) + \frac{2(bc-a\cdots}{}\right.}{}$$

```
[In] Integrate[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i
*x)^3,x]
```

```
[Out] (g^3*(4*A*b^3*d*x - (B*(b*c - a*d)^3)/(c + d*x)^2 + (10*b*B*(b*c - a*d)^2)/
(c + d*x) + 10*b^2*B*(b*c - a*d)*Log[a + b*x] + 4*b^2*B*d*(a + b*x)*Log[(e*
(a + b*x))/(c + d*x)] + (2*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)
]))/(c + d*x)^2 - (12*b*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))
/(c + d*x) - 14*b^2*B*(b*c - a*d)*Log[c + d*x] - 12*b^2*(b*c - a*d)*(A + B*
Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 6*b^2*B*(b*c - a*d)*((2*Log[(d
*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*
(c + d*x))/(b*c - a*d)])))/(4*d^4*i^3)
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.89

method	result
derivativedivides	$-\frac{e(ad-cb)\left(\frac{g^3 d^2 A\left(\frac{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)be+\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2 d}{2}}{d^3}+\frac{b^3e^3}{d^4\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{3b^2e^2\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{d^4}\cdots\right)}{e^3 i^3}\cdots\right)}{\cdots}$
default	$-\frac{e(ad-cb)\left(\frac{g^3 d^2 A\left(\frac{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)be+\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2 d}{2}}{d^3}+\frac{b^3e^3}{d^4\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{3b^2e^2\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{d^4}\cdots\right)}{e^3 i^3}\cdots\right)}{\cdots}$
parts	$\frac{g^3 A\left(\frac{x b^3}{d^3}+\frac{3b^2(ad-cb)\ln(dx+c)}{d^4}-\frac{3b\left(a^2d^2-2abcd+b^2c^2\right)}{d^4(dx+c)}-\frac{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}{2d^4(dx+c)^2}\right)}{i^3}-\frac{g^3 B d\left(\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2\cdots\right)}{\cdots}\cdots\right)}{\cdots}$
risch	Expression too large to display

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x,method=_RETUR
NVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(g^3*d^2/e^3/i^3*A*(1/d^3*(2*(b*e/d+(a*d-b*c)*e/d/(d*x+c
))*b*e+1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*d)+b^3*e^3/d^4/(b*e-(b*e/d+(a*d-
b*c)*e/d/(d*x+c))*d)+3/d^4*b^2*e^2*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d))
+g^3*d^2/e^3/i^3*B*(1/d^2*(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*
d-b*c)*e/d/(d*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+2*b*e/d^3*((b*e/d+
(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c
)-b*e/d)+b^3*e^3/d^3*(1/b/e/d*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)+ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/(b*e-(b*e/d+(a
*d-b*c)*e/d/(d*x+c))*d))+3/d^3*b^2*e^2*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c
))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/
(d*x+c))*d-b*e)/b/e)/d)))
```

Fricas [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^3}\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algor
ithm="fricas")
```

```
[Out] integral((A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3 +
 (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log((b*e
*x + a*e)/(d*x + c)))/(d^3*i^3*x^3 + 3*c*d^2*i^3*x^2 + 3*c^2*d*i^3*x + c^3*
i^3), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$$
$$=\frac{g^3\left(\int \frac{Aa^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{Ab^3x^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{Ba^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{3Aab^2x^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\right.}{}$$

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**3,x)
```

```
[Out] g**3*(Integral(A*a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) +
 Integral(A*b**3*x**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) +
 Integral(B*a**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x +
3*c*d**2*x**2 + d**3*x**3), x) + Integral(3*A*a*b**2*x**2/(c**3 + 3*c**2*d*
```

```
x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(3*A*a**2*b*x/(c**3 + 3*c**2*d
*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(B*b**3*x**3*log(a*e/(c + d*x
) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) +
Integral(3*B*a*b**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**
2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(3*B*a**2*b*x*log(a*e/(c +
 d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x
))/i**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2037 vs. $2(356) = 712$.

Time = 0.32 (sec) , antiderivative size = 2037, normalized size of antiderivative = 5.64

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algor
ithm="maxima")
```

```
[Out] -3/4*B*a^2*b*g^3*(2*(2*d*x + c)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^4*i
^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - (b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d
^2)*x)/((b*c*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*
d^2 - a*c^2*d^3)*i^3) - 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*
a*b*c*d^3 + a^2*d^4)*i^3) + 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2
- 2*a*b*c*d^3 + a^2*d^4)*i^3)) - 1/2*A*b^3*g^3*((6*c^2*d*x + 5*c^3)/(d^6*i^
3*x^2 + 2*c*d^5*i^3*x + c^2*d^4*i^3) - 2*x/(d^3*i^3) + 6*c*log(d*x + c)/(d^
4*i^3)) + 1/4*B*a^3*g^3*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2
 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) - 2*log(b*e*x
/(d*x + c) + a*e/(d*x + c))/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) + 2*b
^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x +
 c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) + 3/2*A*a*b^2*g^3*((4*c*d*x
+ 3*c^2)/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) + 2*log(d*x + c)/(d^3*
i^3)) - 3/2*(2*d*x + c)*A*a^2*b*g^3/(d^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*
i^3) - 1/2*A*a^3*g^3/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) + 1/2*(6*a^3
*b^2*d^3*g^3*log(e) - (6*g^3*log(e) + 7*g^3)*b^5*c^3 + (18*g^3*log(e) + 19*
g^3)*a*b^4*c^2*d - 2*(9*g^3*log(e) + 7*g^3)*a^2*b^3*c*d^2)*B*log(d*x + c)/(
b^2*c^2*d^4*i^3 - 2*a*b*c*d^5*i^3 + a^2*d^6*i^3) + 1/4*(4*(b^5*c^2*d^3*g^3*
log(e) - 2*a*b^4*c*d^4*g^3*log(e) + a^2*b^3*d^5*g^3*log(e))*B*x^3 + 8*(b^5*
c^3*d^2*g^3*log(e) - 2*a*b^4*c^2*d^3*g^3*log(e) + a^2*b^3*c*d^4*g^3*log(e))
*B*x^2 - 2*((4*g^3*log(e) - 5*g^3)*b^5*c^4*d - 20*(g^3*log(e) - g^3)*a*b^4*
c^3*d^2 + (28*g^3*log(e) - 27*g^3)*a^2*b^3*c^2*d^3 - 12*(g^3*log(e) - g^3)*
a^3*b^2*c*d^4)*B*x + 6*((b^5*c^3*d^2*g^3 - 3*a*b^4*c^2*d^3*g^3 + 3*a^2*b^3*
c*d^4*g^3 - a^3*b^2*d^5*g^3)*B*x^2 + 2*(b^5*c^4*d*g^3 - 3*a*b^4*c^3*d^2*g^3
 + 3*a^2*b^3*c^2*d^3*g^3 - a^3*b^2*c*d^4*g^3)*B*x + (b^5*c^5*g^3 - 3*a*b^4*
```

```
c^4*d*g^3 + 3*a^2*b^3*c^3*d^2*g^3 - a^3*b^2*c^2*d^3*g^3)*B)*log(d*x + c)^2
- ((10*g^3*log(e) - 9*g^3)*b^5*c^5 - (38*g^3*log(e) - 35*g^3)*a*b^4*c^4*d +
 (46*g^3*log(e) - 47*g^3)*a^2*b^3*c^3*d^2 - 3*(6*g^3*log(e) - 7*g^3)*a^3*b^
2*c^2*d^3)*B + 2*(2*(b^5*c^2*d^3*g^3 - 2*a*b^4*c*d^4*g^3 + a^2*b^3*d^5*g^3)
*B*x^3 + (9*b^5*c^3*d^2*g^3 - 21*a*b^4*c^2*d^3*g^3 + 12*a^2*b^3*c*d^4*g^3 +
 2*a^3*b^2*d^5*g^3)*B*x^2 + 2*(3*b^5*c^4*d*g^3 - 3*a*b^4*c^3*d^2*g^3 - 6*a^
2*b^3*c^2*d^3*g^3 + 8*a^3*b^2*c*d^4*g^3)*B*x + (6*a*b^4*c^4*d*g^3 - 15*a^2*
b^3*c^3*d^2*g^3 + 11*a^3*b^2*c^2*d^3*g^3)*B)*log(b*x + a) - 2*(2*(b^5*c^2*d
^3*g^3 - 2*a*b^4*c*d^4*g^3 + a^2*b^3*d^5*g^3)*B*x^3 + 4*(b^5*c^3*d^2*g^3 -
2*a*b^4*c^2*d^3*g^3 + a^2*b^3*c*d^4*g^3)*B*x^2 - 4*(b^5*c^4*d*g^3 - 5*a*b^4
*c^3*d^2*g^3 + 7*a^2*b^3*c^2*d^3*g^3 - 3*a^3*b^2*c*d^4*g^3)*B*x - (5*b^5*c^
5*g^3 - 19*a*b^4*c^4*d*g^3 + 23*a^2*b^3*c^3*d^2*g^3 - 9*a^3*b^2*c^2*d^3*g^3
)*B)*log(d*x + c))/(b^2*c^4*d^4*i^3 - 2*a*b*c^3*d^5*i^3 + a^2*c^2*d^6*i^3 +
 (b^2*c^2*d^6*i^3 - 2*a*b*c*d^7*i^3 + a^2*d^8*i^3)*x^2 + 2*(b^2*c^3*d^5*i^3
 - 2*a*b*c^2*d^6*i^3 + a^2*c*d^7*i^3)*x) - 3*(b^3*c*g^3 - a*b^2*d*g^3)*(log
(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c -
a*d)))*B/(d^4*i^3)
```

Giac [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^3}\,dx$$

[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algorithm="giac")

[Out] integrate((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)/(d*i*x + c*i)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx=\int \frac{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{(c\,i+d\,i\,x)^3}\,dx$$

[In] int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^3, x)

[Out] int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^3, x)

3.48 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 251

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = \frac{Bg^2(a+bx)^2}{4di^3(c+dx)^2}-\frac{Abg^2(a+bx)}{d^2i^3(c+dx)}+\frac{bBg^2(a+bx)}{d^2i^3(c+dx)}-\frac{bBg^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^2i^3(c+dx)}-\frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di^3(c+dx)^2}-\frac{b^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i^3}-\frac{b^2Bg^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3}$$

```
[Out] 1/4*B*g^2*(b*x+a)^2/d/i^3/(d*x+c)^2-A*b*g^2*(b*x+a)/d^2/i^3/(d*x+c)+b*B*g^2
*(b*x+a)/d^2/i^3/(d*x+c)-b*B*g^2*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/d^2/i^3/(d*x
+c)-1/2*g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i^3/(d*x+c)^2-b^2*g^2*l
n((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3/i^3-b^2*B*g^2*polyl
og(2,d*(b*x+a)/b/(d*x+c))/d^3/i^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {2562, 45, 2393, 2332, 2341, 2354, 2438}

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}dx = -\frac{b^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^3i^3} - \frac{g^2(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2di^3(c+dx)^2} - \frac{Abg^2(a+bx)}{d^2i^3(c+dx)} - \frac{b^2Bg^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3} - \frac{bBg^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^2i^3(c+dx)} + \frac{bBg^2(a+bx)}{d^2i^3(c+dx)} + \frac{Bg^2(a+bx)^2}{4di^3(c+dx)^2}$$

```
[In] Int[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x)^3,
x]
```

```
[Out] (B*g^2*(a + b*x)^2)/(4*d*i^3*(c + d*x)^2) - (A*b*g^2*(a + b*x))/(d^2*i^3*(c
 + d*x)) + (b*B*g^2*(a + b*x))/(d^2*i^3*(c + d*x)) - (b*B*g^2*(a + b*x)*Log
[(e*(a + b*x))/(c + d*x)])/(d^2*i^3*(c + d*x)) - (g^2*(a + b*x)^2*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]))/(2*d*i^3*(c + d*x)^2) - (b^2*g^2*Log[(b*c - a*
d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d^3*i^3) - (b^2*B*
g^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^3*i^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g^2 \text{Subst}\left(\int \frac{x^2(A+B\log(ex))}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{i^3} \\
&= \frac{g^2 \text{Subst}\left(\int \left(-\frac{b(A+B\log(ex))}{d^2} - \frac{x(A+B\log(ex))}{d} - \frac{b^2(A+B\log(ex))}{d^2(-b+dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{i^3} \\
&= -\frac{(bg^2)\,\text{Subst}\left(\int (A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2 i^3} \\
&\quad - \frac{(b^2 g^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{-b+dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2 i^3} \\
&\quad - \frac{g^2 \text{Subst}\left(\int x(A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{d i^3}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{Bg^2(a+bx)^2}{4di^3(c+dx)^2} - \frac{Abg^2(a+bx)}{d^2i^3(c+dx)} - \frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di^3(c+dx)^2} \\
&\quad - \frac{b^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i^3} \\
&\quad + \frac{(b^2Bg^2)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^3} - \frac{(bBg^2)\,\mathrm{Subst}\left(\int\log(ex)\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^3} \\
&= \frac{Bg^2(a+bx)^2}{4di^3(c+dx)^2} - \frac{Abg^2(a+bx)}{d^2i^3(c+dx)} + \frac{bBg^2(a+bx)}{d^2i^3(c+dx)} \\
&\quad - \frac{bBg^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^2i^3(c+dx)} - \frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di^3(c+dx)^2} \\
&\quad - \frac{b^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i^3} - \frac{b^2Bg^2\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.98

$$
\int\frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx
= \frac{g^2\left(\frac{B(bc-ad)^2}{(c+dx)^2} - \frac{6bB(bc-ad)}{c+dx} - 6b^2B\log(a+bx) - \frac{2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2} + \frac{8b(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{c+dx} + 6b^2\right.}{}
$$

```
[In] Integrate[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i
*x)^3,x]
```

```
[Out] (g^2*((B*(b*c - a*d)^2)/(c + d*x)^2 - (6*b*B*(b*c - a*d))/(c + d*x) - 6*b^2
*B*Log[a + b*x] - (2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c
 + d*x)^2 + (8*b*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x
) + 6*b^2*B*Log[c + d*x] + 4*b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c
 + d*x] - 2*b^2*B*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log
[c + d*x] + 2*PolyLog[2, (b*c + d*x))/(b*c - a*d)])))/(4*d^3*i^3)
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.84

method	result
parts	$\frac{g^2 A\left(\frac{b^2\ln(dx+c)}{d^3}-\frac{2b(ad-cb)}{d^3(dx+c)}-\frac{a^2d^2-2abcd+b^2c^2}{2d^3(dx+c)^2}\right)}{i^3} - \frac{g^2Bd\left(\frac{\left(a^2d^2-2abcd+b^2c^2\right)\left(d\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right.}{2}\right.\right.}{}\right.}{}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{g^2d^2A\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)be+\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2d}{2}}{d^2}-\frac{b^2e^2\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{d^3}\right)}{(ad-cb)e^3i^3}-\frac{g^2d^2B\left(-\frac{\left(\frac{be}{d}+\frac{(ad}{d(d}\right.}{}\right.}{}\right.}{}$
default	$-\frac{e(ad-cb)\left(-\frac{g^2d^2A\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)be+\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2d}{2}}{d^2}-\frac{b^2e^2\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{d^3}\right)}{(ad-cb)e^3i^3}-\frac{g^2d^2B\left(-\frac{\left(\frac{be}{d}+\frac{(ad}{d(d}\right.}{}\right.}{}\right.}{}$
risch	Expression too large to display

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x,method=_RETUR
NVERBOSE)
```

```
[Out] g^2*A/i^3*(b^2/d^3*ln(d*x+c)-2*b/d^3*(a*d-b*c)/(d*x+c)-1/2*(a^2*d^2-2*a*b*c
*d+b^2*c^2)/d^3/(d*x+c)^2)-g^2*B/i^3*d/(a*d-b*c)^2/e^2*((a^2*d^2-2*a*b*c*d+
b^2*c^2)/d^3*(d*(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d
/(d*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+b*e*((b*e/d+(a*d-b*c)*e/d/(d
*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d))+e^2*b^
2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-
b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+
c))*d-b*e)/b/e)/d))
```

Fricas [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^3}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algor
ithm="fricas")
```

```
[Out] integral((A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*B*
a*b*g^2*x + B*a^2*g^2)*log((b*e*x + a*e)/(d*x + c)))/(d^3*i^3*x^3 + 3*c*d^2
*i^3*x^2 + 3*c^2*d*i^3*x + c^3*i^3), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$$
$$=\frac{g^2\left(\int \frac{Aa^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{Ab^2x^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{Ba^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{2Aabx}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,d\right.}{i^3}$$

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**3,x)
```

```
[Out] g**2*(Integral(A*a**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) +
 Integral(A*b**2*x**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) +
 Integral(B*a**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x +
3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*a*b*x/(c**3 + 3*c**2*d*x + 3*
c*d**2*x**2 + d**3*x**3), x) + Integral(B*b**2*x**2*log(a*e/(c + d*x) + b*e
*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integra
l(2*B*a*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d
**2*x**2 + d**3*x**3), x))/i**3
```

Maxima [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^3}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algor
ithm="maxima")
```

```
[Out] -1/2*B*a*b*g^2*(2*(2*d*x + c)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^4*i^3
*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - (b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2
)*x)/((b*c*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^
2 - a*c^2*d^3)*i^3) - 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*
b*c*d^3 + a^2*d^4)*i^3) + 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 -
2*a*b*c*d^3 + a^2*d^4)*i^3)) + 1/4*B*a^2*g^2*((2*b*d*x + 3*b*c - a*d)/((b*c
*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^
2)*i^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^3*i^3*x^2 + 2*c*d^2*i^3
*x + c^2*d*i^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i
^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) + 1/2*A
*b^2*g^2*((4*c*d*x + 3*c^2)/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) + 2
*log(d*x + c)/(d^3*i^3)) - 1/2*B*b^2*g^2*(((d^2*x^2 + 2*c*d*x + c^2)*log(d*
x + c)^2 + (4*c*d*x + 3*c^2)*log(d*x + c))/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c
^2*d^3*i^3) - 2*integrate(1/2*(2*d^2*x^2*log(b*x + a) + 2*d^2*x^2*log(e) +
4*c*d*x + 3*c^2)/(d^5*i^3*x^3 + 3*c*d^4*i^3*x^2 + 3*c^2*d^3*i^3*x + c^3*d^2
*i^3), x)) - (2*d*x + c)*A*a*b*g^2/(d^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i
^3) - 1/2*A*a^2*g^2/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3)
```

Giac [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^3}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algor
ithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)/(d*i*x + c*i)^
3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx=\int \frac{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{{(c\,i+d\,i\,x)}^3}\,dx$$

```
[In] int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^3,
x)
```

```
[Out] int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^3,
 x)
```

3.49 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 85

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = -\frac{Bg(a+bx)^2}{4(bc-ad)i^3(c+dx)^2} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)i^3(c+dx)^2}$$

```
[Out] -1/4*B*g*(b*x+a)^2/(-a*d+b*c)/i^3/(d*x+c)^2+1/2*g*(b*x+a)^2*(A+B*ln(e*(b*x+
a)/(d*x+c)))/(-a*d+b*c)/i^3/(d*x+c)^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2562, 2341}

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = \frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2i^3(c+dx)^2(bc-ad)} - \frac{Bg(a+bx)^2}{4i^3(c+dx)^2(bc-ad)}$$

```
[In] Int[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x)^3,x]
```

```
[Out] -1/4*(B*g*(a + b*x)^2)/((b*c - a*d)*i^3*(c + d*x)^2) + (g*(a + b*x)^2*(A +
B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*c - a*d)*i^3*(c + d*x)^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{g\text{Subst}\left(\int x(A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^3}$$
$$= -\frac{Bg(a+bx)^2}{4(bc-ad)i^3(c+dx)^2} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)i^3(c+dx)^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 207 vs. 2(85) = 170.

Time = 0.10 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.44

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$$
$$= \frac{g\left(\frac{(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2(c+dx)^2} - \frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2(c+dx)} + \frac{bB\left(\frac{1}{c+dx}+\frac{b\log(a+bx)}{bc-ad}-\frac{b\log(c+dx)}{bc-ad}\right)}{d^2} - \frac{B\left(\frac{bc-ad}{(c+dx)^2}+\frac{2b}{c+dx}+\frac{2b^2\log(a+bx)}{bc-ad}-\frac{2b^2\log(c+dx)}{bc-ad}\right)}{4d^2}\right)}{i^3}$$

```
[In] Integrate[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x
)^3,x]
```

```
[Out] (g*(((b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d^2*(c + d*x)^2)
- (b*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d^2*(c + d*x)) + (b*B*((c + d*x
)^(-1) + (b*Log[a + b*x])/(b*c - a*d) - (b*Log[c + d*x])/(b*c - a*d)))/d^2
- (B*((b*c - a*d)/(c + d*x)^2 + (2*b)/(c + d*x) + (2*b^2*Log[a + b*x])/(b*c
 - a*d) - (2*b^2*Log[c + d*x])/(b*c - a*d)))/(4*d^2)))/i^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(81) = 162.

Time = 0.81 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.05

method	result
norman	$\frac{-\frac{2Aadg+2Abcg-Badg-Bbcg}{4i\,d^2}-\frac{(2Abg-Bbg)x}{2id}-\frac{B\,a^2 g\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2i(ad-cb)}-\frac{B\,b^2 g\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2(ad-cb)i}-\frac{Babgx\ln\left(\frac{e(bx+a)}{dx+c}\right)}{i(ad-cb)}}{i^2(dx+c)^2}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{g\,d^2 A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^2e^3i^3}+\frac{g\,d^2 B\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2}-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{4}\right)}{(ad-cb)^2e^3i^3}\right)}{d^2}$
default	$-\frac{e(ad-cb)\left(\frac{g\,d^2 A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^2e^3i^3}+\frac{g\,d^2 B\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2}-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{4}\right)}{(ad-cb)^2e^3i^3}\right)}{d^2}$
parallelrisch	$-\frac{-B\,a^2 b\,d^4 g+B\,b^3c^2d^2g+4Axa\,b^2d^4g-4Ax\,b^3c\,d^3g-2Bxa\,b^2d^4g+2Bx\,b^3c\,d^3g+2B\ln\left(\frac{e(bx+a)}{dx+c}\right)a^2b\,d^4g+2B\,x^2\ln\left(\frac{e}{}\right.}{4i^3(dx+c)^2(ad-cb)b\,d^4}$
risch	$-\frac{Bg(2bdx+ad+cb)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2d^2i^3(dx+c)^2}-\frac{g\left(-2B\ln(-dx-c)b^2d^2x^2+2B\ln(bx+a)b^2d^2x^2-4B\ln(-dx-c)b^2cdx+4B\ln(bx+\right.}{}$
parts	$\frac{gA\left(-\frac{b}{d^2(dx+c)}-\frac{ad-cb}{2d^2(dx+c)^2}\right)}{i^3}-\frac{gBd\left(a\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2}-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{4}\right)-\frac{cb\left(\left(\frac{be}{d}+\frac{(ad-c}{d(dx+}\right.\right.}{}\right.}{i^3(ad-cb)^2e^2}$

```
[In] int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x,method=_RETURNV
ERBOSE)
```

```
[Out] (-1/4*(2*A*a*d*g+2*A*b*c*g-B*a*d*g-B*b*c*g)/i/d^2-1/2*(2*A*b*g-B*b*g)/i/d*x
-1/2*B*a^2*g/i/(a*d-b*c)*ln(e*(b*x+a)/(d*x+c))-1/2*B*b^2*g/(a*d-b*c)/i*x^2*
ln(e*(b*x+a)/(d*x+c))-B*a*b*g/i/(a*d-b*c)*x*ln(e*(b*x+a)/(d*x+c)))/i^2/(d*x
+c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(81) = 162.

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.18

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = -\frac{2\left(((2A-B)b^2cd-(2A-B)abd^2)gx+((2A-B)b^2c^2-(2A-B)a^2d^2)g-2\left(Bb^2d^2gx^2+2Babd^2gx\right.\right.}{4\left((bcd^4-ad^5)i^3x^2+2\left(bc^2d^3-acd^4\right)i^3x+\left(bc^3d^2-ac^2d^3\right)i^3\right)}$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algorit
hm="fricas")
```

```
[Out] -1/4*(2*((2*A - B)*b^2*c*d - (2*A - B)*a*b*d^2)*g*x + ((2*A - B)*b^2*c^2 -
(2*A - B)*a^2*d^2)*g - 2*(B*b^2*d^2*g*x^2 + 2*B*a*b*d^2*g*x + B*a^2*d^2*g)*
log((b*e*x + a*e)/(d*x + c)))/((b*c*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a
*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. 2(71) = 142.

Time = 2.32 (sec) , antiderivative size = 382, normalized size of antiderivative = 4.49

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = \frac{Bb^2g\log\left(x+\frac{-\frac{Ba^2b^2d^2g}{ad-bc}+\frac{2Bab^3cdg}{ad-bc}+Bab^2dg-\frac{Bb^4c^2g}{ad-bc}+Bb^3cg}{2Bb^3dg}\right)}{2d^2i^3\,(ad-bc)} - \frac{Bb^2g\log\left(x+\frac{\frac{Ba^2b^2d^2g}{ad-bc}-\frac{2Bab^3cdg}{ad-bc}+Bab^2dg+\frac{Bb^4c^2g}{ad-bc}+Bb^3cg}{2Bb^3dg}\right)}{2d^2i^3\,(ad-bc)} + \frac{-2Aadg-2Abcg+Badg+Bbcg+x(-4Abdg+2Bbdg)}{4c^2d^2i^3+8cd^3i^3x+4d^4i^3x^2} + \frac{(-Badg-Bbcg-2Bbdgx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2c^2d^2i^3+4cd^3i^3x+2d^4i^3x^2}$$

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**3,x)
```

```
[Out] B*b**2*g*log(x + (-B*a**2*b**2*d**2*g/(a*d - b*c) + 2*B*a*b**3*c*d*g/(a*d -
 b*c) + B*a*b**2*d*g - B*b**4*c**2*g/(a*d - b*c) + B*b**3*c*g)/(2*B*b**3*d*
g))/(2*d**2*i**3*(a*d - b*c)) - B*b**2*g*log(x + (B*a**2*b**2*d**2*g/(a*d -
```

```
 b*c) - 2*B*a*b**3*c*d*g/(a*d - b*c) + B*a*b**2*d*g + B*b**4*c**2*g/(a*d -
b*c) + B*b**3*c*g)/(2*B*b**3*d*g))/(2*d**2*i**3*(a*d - b*c)) + (-2*A*a*d*g
- 2*A*b*c*g + B*a*d*g + B*b*c*g + x*(-4*A*b*d*g + 2*B*b*d*g))/(4*c**2*d**2*
i**3 + 8*c*d**3*i**3*x + 4*d**4*i**3*x**2) + (-B*a*d*g - B*b*c*g - 2*B*b*d*
g*x)*log(e*(a + b*x)/(c + d*x))/(2*c**2*d**2*i**3 + 4*c*d**3*i**3*x + 2*d**
4*i**3*x**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(81) = 162$.

Time = 0.22 (sec) , antiderivative size = 567, normalized size of antiderivative = 6.67

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = -\frac{1}{4}\,Bbg\left(\frac{2\,(2\,dx+c)\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{d^4i^3x^2+2\,cd^3i^3x+c^2d^2i^3}-\frac{bc^2-3\,acd+2\,(bcd-2\,ad^2)x}{(bcd^4-ad^5)i^3x^2+2\,(bc^2d^3-acd^4)i^3x+(bc^3d^2-ac^2d^3)i^3}-\right.$$
$$+\frac{1}{4}\,Bag\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)i^3x^2+2\,(bc^2d^2-acd^3)i^3x+(bc^3d-ac^2d^2)i^3}-\frac{2\,\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{d^3i^3x^2+2\,cd^2i^3x+c^2di^3}+\frac{}{(b^2c^2}\right.$$
$$-\frac{(2\,dx+c)Abg}{2\,(d^4i^3x^2+2\,cd^3i^3x+c^2d^2i^3)}-\frac{Aag}{2\,(d^3i^3x^2+2\,cd^2i^3x+c^2di^3)}$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algorit
hm="maxima")
```

```
[Out] -1/4*B*b*g*(2*(2*d*x + c)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^4*i^3*x^2
 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - (b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*x)
/((b*c*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 -
a*c^2*d^3)*i^3) - 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*b*c*
d^3 + a^2*d^4)*i^3) + 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*
b*c*d^3 + a^2*d^4)*i^3)) + 1/4*B*a*g*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a
*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3)
- 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2
*d*i^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*
b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) - 1/2*(2*d*x +
c)*A*b*g/(d^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - 1/2*A*a*g/(d^3*i^3*x
^2 + 2*c*d^2*i^3*x + c^2*d*i^3)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.55

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)\left(\frac{2\,(bex+ae)^2Bg\log\left(\frac{bex+ae}{dx+c}\right)}{(dx+c)^2ei^3}+\frac{(bex+ae)^2(2\,Ag-B$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algorit
hm="giac")
```

```
[Out] 1/4*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
*(2*(b*e*x + a*e)^2*B*g*log((b*e*x + a*e)/(d*x + c))/((d*x + c)^2*e*i^3) +
(b*e*x + a*e)^2*(2*A*g - B*g)/((d*x + c)^2*e*i^3))
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.33

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$$
$$=-\frac{x\,(2\,A\,b\,d\,g-B\,b\,d\,g)+A\,a\,d\,g+A\,b\,c\,g-\frac{B\,a\,d\,g}{2}-\frac{B\,b\,c\,g}{2}}{2\,c^2\,d^2\,i^3+4\,c\,d^3\,i^3\,x+2\,d^4\,i^3\,x^2}$$
$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{B\,a\,g}{2\,d^2\,i^3}+\frac{B\,b\,c\,g}{2\,d^3\,i^3}+\frac{B\,b\,g\,x}{d^2\,i^3}\right)}{2\,c\,x+d\,x^2+\frac{c^2}{d}}+\frac{B\,b^2\,g\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,1\mathrm{i}}{d^2\,i^3\,\left(a\,d-b\,c\right)}$$

```
[In] int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^3,x)
```

```
[Out] (B*b^2*g*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*1i)/(d^2*i^3*(a*d - b*c
)) - (log((e*(a + b*x))/(c + d*x))*((B*a*g)/(2*d^2*i^3) + (B*b*c*g)/(2*d^3*
i^3) + (B*b*g*x)/(d^2*i^3)))/(2*c*x + d*x^2 + c^2/d) - (x*(2*A*b*d*g - B*b*
d*g) + A*a*d*g + A*b*c*g - (B*a*d*g)/2 - (B*b*c*g)/2)/(2*c^2*d^2*i^3 + 2*d^
4*i^3*x^2 + 4*c*d^3*i^3*x)
```

3.50 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 144

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx = \frac{B}{4di^3(c+dx)^2}+\frac{bB}{2d(bc-ad)i^3(c+dx)}+\frac{b^2B\log(a+bx)}{2d(bc-ad)^2i^3} - \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2di^3(c+dx)^2}-\frac{b^2B\log(c+dx)}{2d(bc-ad)^2i^3}$$

```
[Out] 1/4*B/d/i^3/(d*x+c)^2+1/2*b*B/d/(-a*d+b*c)/i^3/(d*x+c)+1/2*b^2*B*ln(b*x+a)/
d/(-a*d+b*c)^2/i^3+1/2*(-A-B*ln(e*(b*x+a)/(d*x+c)))/d/i^3/(d*x+c)^2-1/2*b^2
*B*ln(d*x+c)/d/(-a*d+b*c)^2/i^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2548, 21, 46}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx = -\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2di^3(c+dx)^2}+\frac{b^2B\log(a+bx)}{2di^3(bc-ad)^2}-\frac{b^2B\log(c+dx)}{2di^3(bc-ad)^2} + \frac{bB}{2di^3(c+dx)(bc-ad)}+\frac{B}{4di^3(c+dx)^2}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(c*i + d*i*x)^3,x]
```

```
[Out] B/(4*d*i^3*(c + d*x)^2) + (b*B)/(2*d*(b*c - a*d)*i^3*(c + d*x)) + (b^2*B*Lo
g[a + b*x])/(2*d*(b*c - a*d)^2*i^3) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/
(2*d*i^3*(c + d*x)^2) - (b^2*B*Log[c + d*x])/(2*d*(b*c - a*d)^2*i^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2548

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Dist[B*n*((b*c
- a*d)/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c -
 a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2di^3(c+dx)^2}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)(c+dx)(ci+dix)^2}\,dx}{2di}\\
&= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2di^3(c+dx)^2}+\frac{(B(bc-ad))\int\frac{1}{(a+bx)(c+dx)^3}\,dx}{2di^3}\\
&= -\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2di^3(c+dx)^2}\\
&\quad+\frac{(B(bc-ad))\int\left(\frac{b^3}{(bc-ad)^3(a+bx)}-\frac{d}{(bc-ad)(c+dx)^3}-\frac{bd}{(bc-ad)^2(c+dx)^2}-\frac{b^2d}{(bc-ad)^3(c+dx)}\right)dx}{2di^3}\\
&= \frac{B}{4di^3(c+dx)^2}+\frac{bB}{2d(bc-ad)i^3(c+dx)}+\frac{b^2B\log(a+bx)}{2d(bc-ad)^2i^3}\\
&\quad-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2di^3(c+dx)^2}-\frac{b^2B\log(c+dx)}{2d(bc-ad)^2i^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.77

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx = \frac{-2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{B\left((bc-ad)(3bc-ad+2bdx)+2b^2(c+dx)^2\log(a+bx)-2b^2(c+dx)^2\log(c+dx)\right)}{(bc-ad)^2}}{4di^3(c+dx)^2}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(c*i + d*i*x)^3,x]
```

```
[Out] (-2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*((b*c - a*d)*(3*b*c - a*d + 2
*b*d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c + d*x]))
/(b*c - a*d)^2)/(4*d*i^3*(c + d*x)^2)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.53

method	result
parts	$-\frac{A}{2i^3(dx+c)^2 d}-\frac{Bd\Bigg(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2}-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{4}-\frac{be\bigg(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{d}}{i^3(ad-cb)^2e^2}$
norman	$\frac{\frac{Bb^2cx\ln\left(\frac{e(bx+a)}{dx+c}\right)}{\left(a^2d^2-2abcd+b^2c^2\right)i}-\frac{2Aad^2-2Abcd-Bad^2+3Bbcd}{4id^2(ad-cb)}-\frac{Bbx}{2i(ad-cb)}-\frac{Ba(ad-2cb)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2i\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{b^2Bdx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2\left(a^2d^2-2abcd+b^2c^2\right)i}}{i^2(dx+c)^2}$
parallelrisch	$-\frac{-4Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3cd^4-4B\ln\left(\frac{e(bx+a)}{dx+c}\right)ab^2cd^4-2Bx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3d^5+2Bxab^2d^5-2Bxb^3cd^4+2B\ln\bigg(\frac{e(bx+c}{dx+c}}{4i^3(dx+c)^2(a^2d^2-2abcd+b^2c^2)bd^4}$
risch	$-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2di^3(dx+c)^2}-\frac{2B\ln(dx+c)b^2d^2x^2-2B\ln(-bx-a)b^2d^2x^2+4B\ln(dx+c)b^2cdx-4B\ln(-bx-a)b^2cdx+2B\ln(dx-}{4(a^2d^2-2a}$
derivativedivides	$-\frac{e(ad-cb)\Bigg(-\frac{d^2Ab\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{(ad-cb)^3e^2i^3}+\frac{d^3A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^3e^3i^3}-\frac{d^2Bb\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad-cb)e}{d(dx+c)}-\frac{be}{d}\right)}{(ad-cb)^3e^2i^3}}{d^2}$
default	$-\frac{e(ad-cb)\Bigg(-\frac{d^2Ab\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{(ad-cb)^3e^2i^3}+\frac{d^3A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^3e^3i^3}-\frac{d^2Bb\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad-cb)e}{d(dx+c)}-\frac{be}{d}\right)}{(ad-cb)^3e^2i^3}}{d^2}$

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*A/i^3/(d*x+c)^2/d-B/i^3*d/(a*d-b*c)^2/e^2*(1/2*(b*e/d+(a*d-b*c)*e/d/(d
*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^
2-b*e/d*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d
-b*c)*e/d/(d*x+c)-b*e/d))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.53

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx = -\frac{(2A-3B)b^2c^2-4(A-B)abcd+(2A-B)a^2d^2-2(Bb^2cd-Babd^2)x-2(Bb^2d^2x^2+2Bb^2cdx+2\ldots}{4((b^2c^2d^3-2abcd^4+a^2d^5)i^3x^2+2(b^2c^3d^2-2abc^2d^3+a^2cd^4)i^3x+(b^2c^4d-2abc^3d^2\ldots}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((2*A - 3*B)*b^2*c^2 - 4*(A - B)*a*b*c*d + (2*A - B)*a^2*d^2 - 2*(B*b^
2*c*d - B*a*b*d^2)*x - 2*(B*b^2*d^2*x^2 + 2*B*b^2*c*d*x + 2*B*a*b*c*d - B*a
^2*d^2)*log((b*e*x + a*e)/(d*x + c)))/((b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5
)*i^3*x^2 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*i^3*x + (b^2*c^4*d
- 2*a*b*c^3*d^2 + a^2*c^2*d^3)*i^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(124) = 248.

Time = 1.08 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.93

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx = -\frac{Bb^2\log\left(x+\frac{-\frac{Ba^3b^2d^3}{(ad-bc)^2}+\frac{3Ba^2b^3cd^2}{(ad-bc)^2}-\frac{3Bab^4c^2d}{(ad-bc)^2}+Bab^2d+\frac{Bb^5c^3}{(ad-bc)^2}+Bb^3c}{2Bb^3d}\right)}{2di^3(ad-bc)^2} + \frac{Bb^2\log\left(x+\frac{\frac{Ba^3b^2d^3}{(ad-bc)^2}-\frac{3Ba^2b^3cd^2}{(ad-bc)^2}+\frac{3Bab^4c^2d}{(ad-bc)^2}+Bab^2d-\frac{Bb^5c^3}{(ad-bc)^2}+Bb^3c}{2Bb^3d}\right)}{2di^3(ad-bc)^2} - \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2c^2di^3+4cd^2i^3x+2d^3i^3x^2} + \frac{-2Aad+2Abc+Bad-3Bbc-2Bbdx}{4ac^2d^2i^3-4bc^3di^3+x^2\cdot(4ad^4i^3-4bcd^3i^3)+x(8acd^3i^3-8bc^2d^2i^3)}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**3,x)
```

```
[Out] -B*b**2*log(x + (-B*a**3*b**2*d**3/(a*d - b*c)**2 + 3*B*a**2*b**3*c*d**2/(a
*d - b*c)**2 - 3*B*a*b**4*c**2*d/(a*d - b*c)**2 + B*a*b**2*d + B*b**5*c**3/
(a*d - b*c)**2 + B*b**3*c)/(2*B*b**3*d))/(2*d*i**3*(a*d - b*c)**2) + B*b**2
*log(x + (B*a**3*b**2*d**3/(a*d - b*c)**2 - 3*B*a**2*b**3*c*d**2/(a*d - b*c
)**2 + 3*B*a*b**4*c**2*d/(a*d - b*c)**2 + B*a*b**2*d - B*b**5*c**3/(a*d - b
*c)**2 + B*b**3*c)/(2*B*b**3*d))/(2*d*i**3*(a*d - b*c)**2) - B*log(e*(a + b
*x)/(c + d*x))/(2*c**2*d*i**3 + 4*c*d**2*i**3*x + 2*d**3*i**3*x**2) + (-2*A
*a*d + 2*A*b*c + B*a*d - 3*B*b*c - 2*B*b*d*x)/(4*a*c**2*d**2*i**3 - 4*b*c**
3*d*i**3 + x**2*(4*a*d**4*i**3 - 4*b*c*d**3*i**3) + x*(8*a*c*d**3*i**3 - 8*
b*c**2*d**2*i**3))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.77

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx$$
$$=\frac{1}{4}B\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)i^3x^2+2\,(bc^2d^2-acd^3)i^3x+(bc^3d-ac^2d^2)i^3}-\frac{2\,\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{d^3i^3x^2+2\,cd^2i^3x+c^2di^3}+\frac{2\,b^2\,\mathrm{l}}{(b^2c^2d-2}\right.$$
$$-\frac{A}{2\,(d^3i^3x^2+2\,cd^2i^3x+c^2di^3)}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algorithm="maxima")
```

```
[Out] 1/4*B*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 -
a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) - 2*log(b*e*x/(d*x + c) + a*e/(
d*x + c))/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) + 2*b^2*log(b*x + a)/((
b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d -
2*a*b*c*d^2 + a^2*d^3)*i^3)) - 1/2*A/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i
^3)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.64

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(2\left(\frac{2\,(bex+ae)Bb}{(bci^3-adi^3)(dx+c)}-\frac{(bex+ae)^2Bd}{(bcei^3-adei^3)(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)-\frac{(bex+ae)^2(2\,Ad-Bd)}{(bcei^3-adei^3)(dx+c)^2}+\right.$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*(2*(b*e*x + a*e)*B*b/((b*c*i^3 - a*d*i^3)*(d*x + c)) - (b*e*x + a*e)
^2*B*d/((b*c*e*i^3 - a*d*e*i^3)*(d*x + c)^2))*log((b*e*x + a*e)/(d*x + c))
- (b*e*x + a*e)^2*(2*A*d - B*d)/((b*c*e*i^3 - a*d*e*i^3)*(d*x + c)^2) + 4*(
b*e*x + a*e)*(A*b - B*b)/((b*c*i^3 - a*d*i^3)*(d*x + c)))*(b*c/((b*c*e - a*
d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.44

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx=\frac{B\,b^2\,\mathrm{atanh}\left(\frac{2\,a^2\,d^3\,i^3-2\,b^2\,c^2\,d\,i^3}{2\,d\,i^3\,{\left(a\,d-b\,c\right)}^2}+\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{d\,i^3\,{\left(a\,d-b\,c\right)}^2}-\frac{B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{2\,d^2\,i^3\,\left(2\,c\,x+d\,x^2+\frac{c^2}{d}\right)}-\frac{\frac{2\,A\,a\,d-2\,A\,b\,c-B\,a\,d+3\,B\,b\,c}{2\,\left(a\,d-b\,c\right)}+\frac{B\,b\,d\,x}{a\,d-b\,c}}{2\,c^2\,d\,i^3+4\,c\,d^2\,i^3\,x+2\,d^3\,i^3\,x^2}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/(c*i + d*i*x)^3,x)
```

```
[Out] (B*b^2*atanh((2*a^2*d^3*i^3 - 2*b^2*c^2*d*i^3)/(2*d*i^3*(a*d - b*c)^2) + (2
*b*d*x)/(a*d - b*c)))/(d*i^3*(a*d - b*c)^2) - (B*log((e*(a + b*x))/(c + d*x
)))/(2*d^2*i^3*(2*c*x + d*x^2 + c^2/d)) - ((2*A*a*d - 2*A*b*c - B*a*d + 3*B
*b*c)/(2*(a*d - b*c)) + (B*b*d*x)/(a*d - b*c))/(2*c^2*d*i^3 + 2*d^3*i^3*x^2
 + 4*c*d^2*i^3*x)
```

3.51 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 243

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx &= -\frac{B\left(4b-\frac{d(a+bx)}{c+dx}\right)^2}{4(bc-ad)^3gi^3}-\frac{b^2B\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^3gi^3}\\ &\quad+\frac{d^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3gi^3(c+dx)^2}\\ &\quad-\frac{2bd(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3gi^3(c+dx)}\\ &\quad+\frac{b^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3gi^3}\end{aligned}$$

```
[Out] -1/4*B*(4*b-d*(b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g/i^3-1/2*b^2*B*ln((b*x+a)/(d
*x+c))^2/(-a*d+b*c)^3/g/i^3+1/2*d^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(
-a*d+b*c)^3/g/i^3/(d*x+c)^2-2*b*d*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d
+b*c)^3/g/i^3/(d*x+c)+b^2*ln((b*x+a)/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/(
-a*d+b*c)^3/g/i^3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2562, 45, 2372, 2338}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx = \frac{b^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{gi^3(bc-ad)^3} + \frac{d^2(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2gi^3(c+dx)^2(bc-ad)^3} - \frac{2bd(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{gi^3(c+dx)(bc-ad)^3} - \frac{b^2B\log^2\left(\frac{a+bx}{c+dx}\right)}{2gi^3(bc-ad)^3} - \frac{B\left(4b-\frac{d(a+bx)}{c+dx}\right)^2}{4gi^3(bc-ad)^3}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)*(c*i + d*i*x)^3),x]
```

```
[Out] -1/4*(B*(4*b - (d*(a + b*x))/(c + d*x))^2)/((b*c - a*d)^3*g*i^3) - (b^2*B*L
og[(a + b*x)/(c + d*x)]^2)/(2*(b*c - a*d)^3*g*i^3) + (d^2*(a + b*x)^2*(A +
B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*c - a*d)^3*g*i^3*(c + d*x)^2) - (2*b
*d*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^3*g*i^3*(c
+ d*x)) + (b^2*Log[(a + b*x)/(c + d*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/((b*c - a*d)^3*g*i^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex))}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3}$$

$$= \frac{d^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3gi^3(c+dx)^2} - \frac{2bd(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3gi^3(c+dx)} + \frac{b^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3gi^3} - \frac{B\text{Subst}\left(\int \left(\frac{1}{2}d(-4b+dx)+\frac{b^2\log(x)}{x}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3}$$

$$= -\frac{B\left(4b-\frac{d(a+bx)}{c+dx}\right)^2}{4(bc-ad)^3gi^3} + \frac{d^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3gi^3(c+dx)^2} - \frac{2bd(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3gi^3(c+dx)} + \frac{b^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3gi^3} - \frac{(b^2B)\,\text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3}$$

$$= -\frac{B\left(4b-\frac{d(a+bx)}{c+dx}\right)^2}{4(bc-ad)^3gi^3} - \frac{b^2B\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^3gi^3} + \frac{d^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3gi^3(c+dx)^2} - \frac{2bd(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3gi^3(c+dx)} + \frac{b^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3gi^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.72

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx$$
$$= \frac{2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+4b(bc-ad)(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+4b^2(c+dx)^2\log(a+bx}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)*(c*i + d*i*x)
^3),x]
```

```
[Out] (2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*b*(b*c - a*d)*(c
+ d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*b^2*(c + d*x)^2*Log[a + b*x
]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 4*b^2*(c + d*x)^2*(A + B*Log[(e*(a
 + b*x))/(c + d*x)])*Log[c + d*x] - 4*b*B*(c + d*x)*(b*c - a*d + b*(c + d*x
)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]) - B*((b*c - a*d)^2 + 2*b*(b*c -
a*d)*(c + d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c +
 d*x]) - 2*b^2*B*(c + d*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*
x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*b^2*B*(
c + d*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*
x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(4*(b*c - a*d)^3*g*i^3*(c +
d*x)^2)
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.51

method	result
parts	$\frac{A\left(-\frac{1}{2(ad-cb)(dx+c)^2}+\frac{b^2\ln(dx+c)}{(ad-cb)^3}+\frac{b}{(ad-cb)^2(dx+c)}-\frac{b^2\ln(bx+a)}{(ad-cb)^3}\right)}{g\,i^3}-\frac{Bd\left(\frac{d\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2}-\cdots\right)}{ad-cb}\cdots\right)}{\cdots}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{d^2Ab^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^3(ad-cb)^4g}-\frac{2d^3Ab\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^3(ad-cb)^4g}+\frac{d^4A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2e^3i^3(ad-cb)^4g}+\frac{d^2Bb^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2e\,i^3(ad-cb)^4g}-\cdots\right)}{\cdots}$
default	$-\frac{e(ad-cb)\left(\frac{d^2Ab^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^3(ad-cb)^4g}-\frac{2d^3Ab\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^3(ad-cb)^4g}+\frac{d^4A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2e^3i^3(ad-cb)^4g}+\frac{d^2Bb^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2e\,i^3(ad-cb)^4g}-\cdots\right)}{\cdots}$
parallelrisch	$-\frac{-4Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^3bc^4d^2-8Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^2b^2c^5d+12Axa^3bc^4d^2-8Axa^2b^2c^5d-10Bxa^3bc^4d^2+8Bxa^2b^2c^5d\cdots}{\cdots}$
norman	$\frac{-\frac{2Aad^3-6Abcd^2-Bad^3+7Bbcd^2}{4gi(ad-cb)^2d^2}-\frac{\left(2Ab^2c^2+Ba^2d^2-4Babcd\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2gi\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}+\frac{\left(2Abd^2-3Bbd^2\right)x}{2ig\left(a^2d^2-2abcd+b^2c^2\right)d}-\frac{Bb^2c^2\ln\left(\cdots\right.}{2gi\left(a^3d^3-3a^2bcd^2\cdots\right.}}{\cdots}$
risch	$-\frac{A}{2g\,i^3(ad-cb)(dx+c)^2}+\frac{Ab^2\ln(dx+c)}{g\,i^3(ad-cb)^3}+\frac{Ab}{g\,i^3(ad-cb)^2(dx+c)}-\frac{Ab^2\ln(bx+a)}{g\,i^3(ad-cb)^3}+\frac{3B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)b^2}{2g\,i^3(ad-cb)^3}+\cdots$

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^3,x,method=_RETURNV
ERBOSE)
```

```
[Out] A/g/i^3*(-1/2/(a*d-b*c)/(d*x+c)^2+b^2/(a*d-b*c)^3*ln(d*x+c)+b/(a*d-b*c)^2/(
d*x+c)-b^2/(a*d-b*c)^3*ln(b*x+a))-B/g/i^3*d/(a*d-b*c)^2/e^2*(d/(a*d-b*c)*(1
/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4*(b*e
/d+(a*d-b*c)*e/d/(d*x+c))^2)-2*b*e/(a*d-b*c)*((b*e/d+(a*d-b*c)*e/d/(d*x+c))
*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)+1/2/d/(a*d-b*
c)*e^2*b^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.46

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx = \frac{(6A-7B)b^2c^2-8(A-B)abcd+(2A-B)a^2d^2+2\left(Bb^2d^2x^2+2Bb^2cdx+Bb^2c^2\right)\log\left(\frac{bex+ae}{dx+c}\right)^2+2((2}{4\left((b^3c^3d^2-3ab^2c^2d^3+3a^2bcd^4-a^3d^5)gi^3x^2+2(b^3c^4d\right.}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^3,x, algorit
hm="fricas")
```

```
[Out] 1/4*((6*A - 7*B)*b^2*c^2 - 8*(A - B)*a*b*c*d + (2*A - B)*a^2*d^2 + 2*(B*b^2
*d^2*x^2 + 2*B*b^2*c*d*x + B*b^2*c^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*((
2*A - 3*B)*b^2*c*d - (2*A - 3*B)*a*b*d^2)*x + 2*((2*A - 3*B)*b^2*d^2*x^2 +
2*A*b^2*c^2 - 4*B*a*b*c*d + B*a^2*d^2 + 2*(2*(A - B)*b^2*c*d - B*a*b*d^2)*x
)*log((b*e*x + a*e)/(d*x + c)))/((b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c
*d^4 - a^3*d^5)*g*i^3*x^2 + 2*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^
3 - a^3*c*d^4)*g*i^3*x + (b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c
^2*d^3)*g*i^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(207) = 414$.

Time = 2.51 (sec) , antiderivative size = 889, normalized size of antiderivative = 3.66

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx &= -\frac{Bb^2\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2a^3d^3gi^3-6a^2bcd^2gi^3+6ab^2c^2dgi^3-2b^3c^3gi^3}\\ &+\frac{b^2\cdot(2A-3B)\log\left(x+\frac{2Aab^2d+2Ab^3c-3Bab^2d-3Bb^3c-\frac{a^4b^2d^4\cdot(2A-3B)}{(ad-bc)^3}+\frac{4a^3b^3cd^3\cdot(2A-3B)}{(ad-bc)^3}-\frac{6a^2b^4c^2d^2\cdot(2A-3B)}{(ad-bc)^3}+\frac{4ab^5c^3d(2A-3B}{(ad-bc)^3}}{4Ab^3d-6Bb^3d}\right.}{2gi^3(ad-bc)^3}\\ &-\frac{b^2\cdot(2A-3B)\log\left(x+\frac{2Aab^2d+2Ab^3c-3Bab^2d-3Bb^3c+\frac{a^4b^2d^4\cdot(2A-3B)}{(ad-bc)^3}-\frac{4a^3b^3cd^3\cdot(2A-3B)}{(ad-bc)^3}+\frac{6a^2b^4c^2d^2\cdot(2A-3B)}{(ad-bc)^3}-\frac{4ab^5c^3d(2A-3B}{(ad-bc)^3}}{4Ab^3d-6Bb^3d}\right.}{2gi^3(ad-bc)^3}\\ &+\frac{(-Bad+3Bbc+2Bbdx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2a^2c^2d^2gi^3+4a^2cd^3gi^3x+2a^2d^4gi^3x^2-4abc^3dgi^3-8abc^2d^2gi^3x-4abcd^3gi^3x^2+2b^2c^4gi^3+4b^2c^3dgi^3x}\\ &+\frac{-2Aad+6Abc+Bad-7Bbc+x(4Abd-6Bbd)}{4a^2c^2d^2gi^3-8abc^3dgi^3+4b^2c^4gi^3+x^2\cdot(4a^2d^4gi^3-8abcd^3gi^3+4b^2c^2d^2gi^3)+x(8a^2cd^3gi^3-16abc^2d^2}\end{aligned}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)**3,x)
```

```
[Out] -B*b**2*log(e*(a + b*x)/(c + d*x))**2/(2*a**3*d**3*g*i**3 - 6*a**2*b*c*d**2
*g*i**3 + 6*a*b**2*c**2*d*g*i**3 - 2*b**3*c**3*g*i**3) + b**2*(2*A - 3*B)*l
og(x + (2*A*a*b**2*d + 2*A*b**3*c - 3*B*a*b**2*d - 3*B*b**3*c - a**4*b**2*d
**4*(2*A - 3*B)/(a*d - b*c)**3 + 4*a**3*b**3*c*d**3*(2*A - 3*B)/(a*d - b*c)
**3 - 6*a**2*b**4*c**2*d**2*(2*A - 3*B)/(a*d - b*c)**3 + 4*a*b**5*c**3*d*(2
*A - 3*B)/(a*d - b*c)**3 - b**6*c**4*(2*A - 3*B)/(a*d - b*c)**3)/(4*A*b**3*
d - 6*B*b**3*d))/(2*g*i**3*(a*d - b*c)**3) - b**2*(2*A - 3*B)*log(x + (2*A*
a*b**2*d + 2*A*b**3*c - 3*B*a*b**2*d - 3*B*b**3*c + a**4*b**2*d**4*(2*A - 3
*B)/(a*d - b*c)**3 - 4*a**3*b**3*c*d**3*(2*A - 3*B)/(a*d - b*c)**3 + 6*a**2
*b**4*c**2*d**2*(2*A - 3*B)/(a*d - b*c)**3 - 4*a*b**5*c**3*d*(2*A - 3*B)/(a
*d - b*c)**3 + b**6*c**4*(2*A - 3*B)/(a*d - b*c)**3)/(4*A*b**3*d - 6*B*b**3
*d))/(2*g*i**3*(a*d - b*c)**3) + (-B*a*d + 3*B*b*c + 2*B*b*d*x)*log(e*(a +
b*x)/(c + d*x))/(2*a**2*c**2*d**2*g*i**3 + 4*a**2*c*d**3*g*i**3*x + 2*a**2*
d**4*g*i**3*x**2 - 4*a*b*c**3*d*g*i**3 - 8*a*b*c**2*d**2*g*i**3*x - 4*a*b*c
*d**3*g*i**3*x**2 + 2*b**2*c**4*g*i**3 + 4*b**2*c**3*d*g*i**3*x + 2*b**2*c*
*2*d**2*g*i**3*x**2) + (-2*A*a*d + 6*A*b*c + B*a*d - 7*B*b*c + x*(4*A*b*d -
 6*B*b*d))/(4*a**2*c**2*d**2*g*i**3 - 8*a*b*c**3*d*g*i**3 + 4*b**2*c**4*g*i
**3 + x**2*(4*a**2*d**4*g*i**3 - 8*a*b*c*d**3*g*i**3 + 4*b**2*c**2*d**2*g*i
**3) + x*(8*a**2*c*d**3*g*i**3 - 16*a*b*c**2*d**2*g*i**3 + 8*b**2*c**3*d*g*
i**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 885 vs. $2(237) = 474$.

Time = 0.26 (sec) , antiderivative size = 885, normalized size of antiderivative = 3.64

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx$$
$$= \frac{1}{2}B\left(\frac{2\,bdx+3\,bc-ad}{(b^2c^2d^2-2\,abcd^3+a^2d^4)gi^3x^2+2\,(b^2c^3d-2\,abc^2d^2+a^2cd^3)gi^3x+(b^2c^4-2\,abc^3d+a^2c^2d^2)gi^3}+\cdots+\frac{ae}{dx+c}\right)$$
$$+\frac{1}{2}A\left(\frac{2\,bdx+3\,bc-ad}{(b^2c^2d^2-2\,abcd^3+a^2d^4)gi^3x^2+2\,(b^2c^3d-2\,abc^2d^2+a^2cd^3)gi^3x+(b^2c^4-2\,abc^3d+a^2c^2d^2)gi^3}\cdots\right.$$
$$-\frac{\left(7\,b^2c^2-8\,abcd+a^2d^2+2\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\log\left(bx+a\right)^2+2\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\log\left(dx\cdots\right.\right.}{4\,(b^3c^5gi^3-3\,ab^2c^4dgi^3+3\,a^2bc^3d^2gi^3-a^3c^2d^3gi^3+(\cdots}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^3,x, algorit
hm="maxima")
```

```
[Out] 1/2*B*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*g*i^3
*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3*x + (b^2*c^4 - 2*a*b
```

```
*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2*d
 + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c)/((b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3))*log(b*e*x/(d*x + c) + a*e/(d*x +
c)) + 1/2*A*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)
*g*i^3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3*x + (b^2*c^4 -
 2*a*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c)/((b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3)) - 1/4*(7*b^2*c^2 - 8*a*b*c*
d + a^2*d^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a)^2 + 2*(b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x + c)^2 + 6*(b^2*c*d - a*b*d^2)*
x + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a) - 2*(3*b^2*d^2*x^2
 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*
x + a))*log(d*x + c))*B/(b^3*c^5*g*i^3 - 3*a*b^2*c^4*d*g*i^3 + 3*a^2*b*c^3*
d^2*g*i^3 - a^3*c^2*d^3*g*i^3 + (b^3*c^3*d^2*g*i^3 - 3*a*b^2*c^2*d^3*g*i^3
+ 3*a^2*b*c*d^4*g*i^3 - a^3*d^5*g*i^3)*x^2 + 2*(b^3*c^4*d*g*i^3 - 3*a*b^2*c
^3*d^2*g*i^3 + 3*a^2*b*c^2*d^3*g*i^3 - a^3*c*d^4*g*i^3)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.82

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(\frac{2\,Bb^2e\log\left(\frac{bex+ae}{dx+c}\right)^2}{b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3}+\frac{4\,Ab^2e\log\left(\frac{bex+ae}{dx+c}\right)}{b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3}-2\left(\frac{4\,(bex+ae)Bbd}{(b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3)(dx\right.\right.$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^3,x, algorit
hm="giac")
```

```
[Out] 1/4*(2*B*b^2*e*log((b*e*x + a*e)/(d*x + c))^2/(b^2*c^2*g*i^3 - 2*a*b*c*d*g*
i^3 + a^2*d^2*g*i^3) + 4*A*b^2*e*log((b*e*x + a*e)/(d*x + c))/(b^2*c^2*g*i^
3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3) - 2*(4*(b*e*x + a*e)*B*b*d/((b^2*c^2*g
*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)) - (b*e*x + a*e)^2*B*d^2/
((b^2*c^2*e*g*i^3 - 2*a*b*c*d*e*g*i^3 + a^2*d^2*e*g*i^3)*(d*x + c)^2))*log(
(b*e*x + a*e)/(d*x + c)) + (2*A*d^2 - B*d^2)*(b*e*x + a*e)^2/((b^2*c^2*e*g*
i^3 - 2*a*b*c*d*e*g*i^3 + a^2*d^2*e*g*i^3)*(d*x + c)^2) - 8*(A*b*d - B*b*d)
*(b*e*x + a*e)/((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)
))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 545, normalized size of antiderivative = 2.24

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx = \frac{3\,A\,b\,c}{2\,g\,i^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^2}-\frac{A\,a\,d}{2\,g\,i^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^2}-\frac{B\,b^2\,{\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}^2}{2\,g\,i^3\,{\left(a\,d-b\,c\right)}^3}+\frac{B\,a\,d}{4\,g\,i^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^2}-\frac{7\,B\,b\,c}{4\,g\,i^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^2}-\frac{B\,a^2\,d^2\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{2\,g\,i^3\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^2}-\frac{3\,B\,b^2\,c^2\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{2\,g\,i^3\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^2}+\frac{A\,b\,d\,x}{g\,i^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^2}-\frac{3\,B\,b\,d\,x}{2\,g\,i^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^2}+\frac{B\,a\,b\,d^2\,x\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{g\,i^3\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^2}-\frac{B\,b^2\,c\,d\,x\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{g\,i^3\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^2}+\frac{2\,B\,a\,b\,c\,d\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{g\,i^3\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^2}+\frac{A\,b^2\,\mathrm{atan}\left(\frac{a\,d\,1{}\mathrm{i}+b\,c\,1{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}\right)\,2{}\mathrm{i}}{g\,i^3\,{\left(a\,d-b\,c\right)}^3}-\frac{B\,b^2\,\mathrm{atan}\left(\frac{a\,d\,1{}\mathrm{i}+b\,c\,1{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}\right)\,3{}\mathrm{i}}{g\,i^3\,{\left(a\,d-b\,c\right)}^3}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)*(c*i + d*i*x)^3),x)
```

```
[Out] (A*b^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(g*i^3*(a*d - b*c
)^3) - (B*b^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*3i)/(g*i^3*(a*
d - b*c)^3) - (B*b^2*log((e*(a + b*x))/(c + d*x))^2)/(2*g*i^3*(a*d - b*c)^3
) - (A*a*d)/(2*g*i^3*(a*d - b*c)^2*(c + d*x)^2) + (3*A*b*c)/(2*g*i^3*(a*d -
 b*c)^2*(c + d*x)^2) + (B*a*d)/(4*g*i^3*(a*d - b*c)^2*(c + d*x)^2) - (7*B*b
*c)/(4*g*i^3*(a*d - b*c)^2*(c + d*x)^2) - (B*a^2*d^2*log((e*(a + b*x))/(c +
 d*x)))/(2*g*i^3*(a*d - b*c)^3*(c + d*x)^2) - (3*B*b^2*c^2*log((e*(a + b*x)
)/(c + d*x)))/(2*g*i^3*(a*d - b*c)^3*(c + d*x)^2) + (A*b*d*x)/(g*i^3*(a*d -
 b*c)^2*(c + d*x)^2) - (3*B*b*d*x)/(2*g*i^3*(a*d - b*c)^2*(c + d*x)^2) + (B
*a*b*d^2*x*log((e*(a + b*x))/(c + d*x)))/(g*i^3*(a*d - b*c)^3*(c + d*x)^2)
- (B*b^2*c*d*x*log((e*(a + b*x))/(c + d*x)))/(g*i^3*(a*d - b*c)^3*(c + d*x)
^2) + (2*B*a*b*c*d*log((e*(a + b*x))/(c + d*x)))/(g*i^3*(a*d - b*c)^3*(c +
d*x)^2)
```

3.52 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 365

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx &= \frac{Bd^3(a+bx)^2}{4(bc-ad)^4g^2i^3(c+dx)^2}-\frac{3bBd^2(a+bx)}{(bc-ad)^4g^2i^3(c+dx)} \\ &\quad -\frac{b^3B(c+dx)}{(bc-ad)^4g^2i^3(a+bx)}+\frac{3b^2Bd\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^2i^3} \\ &\quad -\frac{d^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^2i^3(c+dx)^2} \\ &\quad +\frac{3bd^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3(c+dx)} \\ &\quad -\frac{b^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3(a+bx)} \\ &\quad -\frac{3b^2d\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3}\end{aligned}$$

```
[Out] 1/4*B*d^3*(b*x+a)^2/(-a*d+b*c)^4/g^2/i^3/(d*x+c)^2-3*b*B*d^2*(b*x+a)/(-a*d+
b*c)^4/g^2/i^3/(d*x+c)-b^3*B*(d*x+c)/(-a*d+b*c)^4/g^2/i^3/(b*x+a)+3/2*b^2*B
*d*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^4/g^2/i^3-1/2*d^3*(b*x+a)^2*(A+B*ln(e*(
b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^2/i^3/(d*x+c)^2+3*b*d^2*(b*x+a)*(A+B*ln(e*(
b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^2/i^3/(d*x+c)-b^3*(d*x+c)*(A+B*ln(e*(b*x+a)
/(d*x+c)))/(-a*d+b*c)^4/g^2/i^3/(b*x+a)-3*b^2*d*ln((b*x+a)/(d*x+c))*(A+B*ln
(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^2/i^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2562, 45, 2372, 2338}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = -\frac{b^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2i^3(a+bx)(bc-ad)^4} - \frac{3b^2d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2i^3(bc-ad)^4} - \frac{d^3(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^2i^3(c+dx)^2(bc-ad)^4} + \frac{3bd^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2i^3(c+dx)(bc-ad)^4} - \frac{b^3B(c+dx)}{g^2i^3(a+bx)(bc-ad)^4} + \frac{3b^2Bd\log^2\left(\frac{a+bx}{c+dx}\right)}{2g^2i^3(bc-ad)^4} + \frac{Bd^3(a+bx)^2}{4g^2i^3(c+dx)^2(bc-ad)^4} - \frac{3bBd^2(a+bx)}{g^2i^3(c+dx)(bc-ad)^4}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^2*(c*i + d*i*x)^3),
x]
```

```
[Out] (B*d^3*(a + b*x)^2)/(4*(b*c - a*d)^4*g^2*i^3*(c + d*x)^2) - (3*b*B*d^2*(a +
 b*x))/((b*c - a*d)^4*g^2*i^3*(c + d*x)) - (b^3*B*(c + d*x))/((b*c - a*d)^4
*g^2*i^3*(a + b*x)) + (3*b^2*B*d*Log[(a + b*x)/(c + d*x)]^2)/(2*(b*c - a*d)
^4*g^2*i^3) - (d^3*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*
c - a*d)^4*g^2*i^3*(c + d*x)^2) + (3*b*d^2*(a + b*x)*(A + B*Log[(e*(a + b*x
))/(c + d*x)]))/((b*c - a*d)^4*g^2*i^3*(c + d*x)) - (b^3*(c + d*x)*(A + B*L
og[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^4*g^2*i^3*(a + b*x)) - (3*b^2*d*
Log[(a + b*x)/(c + d*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)
^4*g^2*i^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)^3(A+B\log(ex))}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}\\
&= -\frac{d^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^2i^3(c+dx)^2} + \frac{3bd^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3(c+dx)}\\
&\quad - \frac{b^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3(a+bx)} - \frac{3b^2d\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3}\\
&\quad - \frac{B\text{Subst}\left(\int \left(3bd^2-\frac{b^3}{x^2}-\frac{d^3x}{2}-\frac{3b^2d\log(x)}{x}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}\\
&= \frac{Bd^3(a+bx)^2}{4(bc-ad)^4g^2i^3(c+dx)^2} - \frac{3bBd^2(a+bx)}{(bc-ad)^4g^2i^3(c+dx)}\\
&\quad - \frac{b^3B(c+dx)}{(bc-ad)^4g^2i^3(a+bx)} - \frac{d^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^2i^3(c+dx)^2}\\
&\quad + \frac{3bd^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3(c+dx)} - \frac{b^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3(a+bx)}\\
&\quad - \frac{3b^2d\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3} + \frac{(3b^2Bd)\,\text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}
\end{aligned}$$

$$= \frac{Bd^3(a+bx)^2}{4(bc-ad)^4g^2i^3(c+dx)^2} - \frac{3bBd^2(a+bx)}{(bc-ad)^4g^2i^3(c+dx)} - \frac{b^3B(c+dx)}{(bc-ad)^4g^2i^3(a+bx)} + \frac{3b^2Bd\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^2i^3} - \frac{d^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^2i^3(c+dx)^2} + \frac{3bd^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3(c+dx)} - \frac{b^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3(a+bx)} - \frac{3b^2d\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.24

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = \frac{-\frac{4b^3Bc}{a+bx} + \frac{4ab^2Bd}{a+bx} + \frac{Bd(bc-ad)^2}{(c+dx)^2} + \frac{8b^2Bcd}{c+dx} - \frac{8abBd^2}{c+dx} + \frac{2bBd(bc-ad)}{c+dx} + 6b^2Bd\log(a+bx) - \frac{4b^2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{a+bx}}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^2*(c*i + d*i*
x)^3),x]
```

```
[Out] ((-4*b^3*B*c)/(a + b*x) + (4*a*b^2*B*d)/(a + b*x) + (B*d*(b*c - a*d)^2)/(c
+ d*x)^2 + (8*b^2*B*c*d)/(c + d*x) - (8*a*b*B*d^2)/(c + d*x) + (2*b*B*d*(b*
c - a*d))/(c + d*x) + 6*b^2*B*d*Log[a + b*x] - (4*b^2*(b*c - a*d)*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]))/(a + b*x) - (2*d*(b*c - a*d)^2*(A + B*Log[(e*(
a + b*x))/(c + d*x)]))/(c + d*x)^2 - (8*b*d*(b*c - a*d)*(A + B*Log[(e*(a +
b*x))/(c + d*x)]))/(c + d*x) - 12*b^2*d*Log[a + b*x]*(A + B*Log[(e*(a + b*x
))/(c + d*x)]) - 6*b^2*B*d*Log[c + d*x] + 12*b^2*d*(A + B*Log[(e*(a + b*x))
/(c + d*x)])*Log[c + d*x] + 6*b^2*B*d*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(
b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) -
6*b^2*B*d*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x
] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(4*(b*c - a*d)^4*g^2*i^3)
```

Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.39

method	result
parts	$\frac{A\left(-\frac{d}{2(ad-cb)^2(dx+c)^2}+\frac{3d\,b^2\ln(dx+c)}{(ad-cb)^4}+\frac{2db}{(ad-cb)^3(dx+c)}+\frac{b^2}{(ad-cb)^3(bx+a)}-\frac{3d\,b^2\ln(bx+a)}{(ad-cb)^4}\right)}{g^2 i^3}-\frac{Bd\left(d^2\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{\cdots}\right.\right.}{\cdots}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{d^2 A\,b^3}{i^3(ad-cb)^5 g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{3d^3 A\,b^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^3(ad-cb)^5 g^2}-\frac{3d^4 Ab\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2 i^3(ad-cb)^5 g^2}+\frac{d^5 A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2e^3 i^3(ad-cb)^5 g^2}-\cdots\right.}{\cdots}$
default	$-\frac{e(ad-cb)\left(\frac{d^2 A\,b^3}{i^3(ad-cb)^5 g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{3d^3 A\,b^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^3(ad-cb)^5 g^2}-\frac{3d^4 Ab\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2 i^3(ad-cb)^5 g^2}+\frac{d^5 A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2e^3 i^3(ad-cb)^5 g^2}-\cdots\right.}{\cdots}$
risch	$-\frac{Ad}{2g^2 i^3(ad-cb)^2(dx+c)^2}+\frac{3Ad\,b^2\ln(dx+c)}{g^2 i^3(ad-cb)^4}+\frac{2Adb}{g^2 i^3(ad-cb)^3(dx+c)}+\frac{A\,b^2}{g^2 i^3(ad-cb)^3(bx+a)}-\frac{3Ad\,b^2\ln(bx+a)}{g^2 i^3(ad-cb)^4}-\cdots$
parallelrisch	$-\frac{6B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2 a\,b^5 d^7+12B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2 b^6 c\,d^6+12A\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^5 d^7+24A\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^6 c\,d^6-18B\cdots}{\cdots}$
norman	Expression too large to display

[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x,method=_RETURNVERBOSE)

[Out] A/g^2/i^3*(-1/2*d/(a*d-b*c)^2/(d*x+c)^2+3*d/(a*d-b*c)^4*b^2*ln(d*x+c)+2*d/(a*d-b*c)^3*b/(d*x+c)+b^2/(a*d-b*c)^3/(b*x+a)-3*d/(a*d-b*c)^4*b^2*ln(b*x+a))-B/g^2/i^3*d/(a*d-b*c)^2/e^2*(d^2/(a*d-b*c)^2*(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-3*d/(a*d-b*c)^2*b*e*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)+3/2/(a*d-b*c)^2*b^2*e^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/d/(a*d-b*c)^2*b^3*e^3*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.84

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = -\frac{4\,(A+B)b^3c^3+3\,(2\,A-5\,B)ab^2c^2d-12\,(A-B)a^2bcd^2+(2\,A-B)a^3d^3+6\,((2\,A-B)b^3cd^2-(2\,A}{4\,((b^5c^4d^2-4\,a}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x, algor
ithm="fricas")
```

```
[Out] -1/4*(4*(A + B)*b^3*c^3 + 3*(2*A - 5*B)*a*b^2*c^2*d - 12*(A - B)*a^2*b*c*d^
2 + (2*A - B)*a^3*d^3 + 6*((2*A - B)*b^3*c*d^2 - (2*A - B)*a*b^2*d^3)*x^2 +
 6*(B*b^3*d^3*x^3 + B*a*b^2*c^2*d + (2*B*b^3*c*d^2 + B*a*b^2*d^3)*x^2 + (B*
b^3*c^2*d + 2*B*a*b^2*c*d^2)*x)*log((b*e*x + a*e)/(d*x + c))^2 + 3*((6*A -
B)*b^3*c^2*d - 2*(2*A + B)*a*b^2*c*d^2 - (2*A - 3*B)*a^2*b*d^3)*x + 2*(3*(2
*A - B)*b^3*d^3*x^3 + 2*B*b^3*c^3 + 6*A*a*b^2*c^2*d - 6*B*a^2*b*c*d^2 + B*a
^3*d^3 + 3*(4*A*b^3*c*d^2 + (2*A - 3*B)*a*b^2*d^3)*x^2 + 3*(2*(A + B)*b^3*c
^2*d + 4*(A - B)*a*b^2*c*d^2 - B*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c))
)/((b^5*c^4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a
^4*b*d^6)*g^2*i^3*x^3 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3
- 2*a^3*b^2*c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*g^2*i^3*x^2 + (b^5*c^6 - 2*a
*b^4*c^5*d - 2*a^2*b^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^
5*c*d^5)*g^2*i^3*x + (a*b^4*c^6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a
^4*b*c^3*d^3 + a^5*c^2*d^4)*g^2*i^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2/(d*i*x+c*i)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1721 vs. $2(359) = 718$.

Time = 0.33 (sec) , antiderivative size = 1721, normalized size of antiderivative = 4.72

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x, algor
ithm="maxima")
```

```
[Out] -1/2*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a
*b*d^2)*x)/((b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*g
^2*i^3*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d
^4 - a^4*d^5)*g^2*i^3*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*
a^3*b*c^2*d^3 - 2*a^4*c*d^4)*g^2*i^3*x + (a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a
^3*b*c^3*d^2 - a^4*c^2*d^3)*g^2*i^3) + 6*b^2*d*log(b*x + a)/((b^4*c^4 - 4*a
*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3) - 6*b^2*
d*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^
3 + a^4*d^4)*g^2*i^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 1/2*A*((6*b^2
*d^2*x^2 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a*b*d^2)*x)/((b
^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*g^2*i^3*x^3 + (
2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*
g^2*i^3*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3
- 2*a^4*c*d^4)*g^2*i^3*x + (a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 -
 a^4*c^2*d^3)*g^2*i^3) + 6*b^2*d*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6
*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3) - 6*b^2*d*log(d*x + c)
/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g
^2*i^3)) - 1/4*(4*b^3*c^3 - 15*a*b^2*c^2*d + 12*a^2*b*c*d^2 - a^3*d^3 - 6*(
b^3*c*d^2 - a*b^2*d^3)*x^2 - 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 +
a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(b*x + a)^2 - 6*(b^3*d^3
*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c
*d^2)*x)*log(d*x + c)^2 - 3*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x - 6
*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d +
2*a*b^2*c*d^2)*x)*log(b*x + a) + 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^
2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x + 2*(b^3*d^3*x^3 + a*b^2
*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log
(b*x + a))*log(d*x + c))*B/(a*b^4*c^6*g^2*i^3 - 4*a^2*b^3*c^5*d*g^2*i^3 + 6
*a^3*b^2*c^4*d^2*g^2*i^3 - 4*a^4*b*c^3*d^3*g^2*i^3 + a^5*c^2*d^4*g^2*i^3 +
(b^5*c^4*d^2*g^2*i^3 - 4*a*b^4*c^3*d^3*g^2*i^3 + 6*a^2*b^3*c^2*d^4*g^2*i^3
- 4*a^3*b^2*c*d^5*g^2*i^3 + a^4*b*d^6*g^2*i^3)*x^3 + (2*b^5*c^5*d*g^2*i^3 -
 7*a*b^4*c^4*d^2*g^2*i^3 + 8*a^2*b^3*c^3*d^3*g^2*i^3 - 2*a^3*b^2*c^2*d^4*g^
2*i^3 - 2*a^4*b*c*d^5*g^2*i^3 + a^5*d^6*g^2*i^3)*x^2 + (b^5*c^6*g^2*i^3 - 2
*a*b^4*c^5*d*g^2*i^3 - 2*a^2*b^3*c^4*d^2*g^2*i^3 + 8*a^3*b^2*c^3*d^3*g^2*i^
3 - 7*a^4*b*c^2*d^4*g^2*i^3 + 2*a^5*c*d^5*g^2*i^3)*x)
```

Giac [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{(bgx+ag)^2(dix+ci)^3}\,dx$$

[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x, algorithm="giac")

[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)/((b*g*x + a*g)^2*(d*i*x + c*i)^3), x)

Mupad [B] (verification not implemented)

Time = 5.71 (sec) , antiderivative size = 983, normalized size of antiderivative = 2.69

$$
\begin{aligned}
\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = {} & \frac{A\,b^2\,c^2}{g^2\,i^3\,{\left(a\,d-b\,c\right)}^3\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \\
& - \frac{A\,a^2\,d^2}{2\,g^2\,i^3\,{\left(a\,d-b\,c\right)}^3\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \\
& - \frac{3\,B\,b^2\,d\,{\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}^2}{2\,g^2\,i^3\,{\left(a\,d-b\,c\right)}^4} \\
& + \frac{B\,a^2\,d^2}{4\,g^2\,i^3\,{\left(a\,d-b\,c\right)}^3\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \\
& + \frac{B\,b^2\,c^2}{g^2\,i^3\,{\left(a\,d-b\,c\right)}^3\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \\
& - \frac{B\,a\,d\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{2\,g^2\,i^3\,{\left(a\,d-b\,c\right)}^2\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \\
& - \frac{B\,b\,c\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{g^2\,i^3\,{\left(a\,d-b\,c\right)}^2\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \\
& + \frac{3\,A\,b^2\,d^2\,x^2}{g^2\,i^3\,{\left(a\,d-b\,c\right)}^3\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \\
& - \frac{3\,B\,b^2\,d^2\,x^2}{2\,g^2\,i^3\,{\left(a\,d-b\,c\right)}^3\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \\
& + \frac{5\,A\,a\,b\,c\,d}{2\,g^2\,i^3\,{\left(a\,d-b\,c\right)}^3\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \\
& - \frac{11\,B\,a\,b\,c\,d}{4\,g^2\,i^3\,{\left(a\,d-b\,c\right)}^3\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \\
& - \frac{3\,B\,b\,d\,x\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{2\,g^2\,i^3\,{\left(a\,d-b\,c\right)}^2\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \\
& + \frac{3\,B\,b^2\,d^2\,x^2\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{g^2\,i^3\,{\left(a\,d-b\,c\right)}^3\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \\
& + \frac{3\,A\,a\,b\,d^2\,x}{2\,g^2\,i^3\,{\left(a\,d-b\,c\right)}^3\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \\
& - \frac{9\,B\,a\,b\,d^2\,x}{4\,g^2\,i^3\,{\left(a\,d-b\,c\right)}^3\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \\
& + \frac{9\,A\,b^2\,c\,d\,x}{2\,g^2\,i^3\,{\left(a\,d-b\,c\right)}^3\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \\
& - \frac{3\,B\,b^2\,c\,d\,x}{4\,g^2\,i^3\,{\left(a\,d-b\,c\right)}^3\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \\
& + \frac{3\,B\,a\,b\,c\,d\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{g^2\,i^3\,{\left(a\,d-b\,c\right)}^3\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2} \\
& 3\,B\,a\,b\,d^2\,x\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)
\end{aligned}
$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^2*(c*i + d*i*x)^3),
x)

[Out] (A*b^2*d*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*6i)/(g^2*i^3*(a*d -
 b*c)^4) - (3*B*b^2*d*log((e*(a + b*x))/(c + d*x))^2)/(2*g^2*i^3*(a*d - b*c
)^4) - (B*b^2*d*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*3i)/(g^2*i^3
*(a*d - b*c)^4) - (A*a^2*d^2)/(2*g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^
2) + (A*b^2*c^2)/(g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) + (B*a^2*d^2
)/(4*g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) + (B*b^2*c^2)/(g^2*i^3*(a
*d - b*c)^3*(a + b*x)*(c + d*x)^2) - (B*a*d*log((e*(a + b*x))/(c + d*x)))/(
2*g^2*i^3*(a*d - b*c)^2*(a + b*x)*(c + d*x)^2) - (B*b*c*log((e*(a + b*x))/(
c + d*x)))/(g^2*i^3*(a*d - b*c)^2*(a + b*x)*(c + d*x)^2) + (3*A*b^2*d^2*x^2
)/(g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) - (3*B*b^2*d^2*x^2)/(2*g^2*
i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) + (5*A*a*b*c*d)/(2*g^2*i^3*(a*d -
b*c)^3*(a + b*x)*(c + d*x)^2) - (11*B*a*b*c*d)/(4*g^2*i^3*(a*d - b*c)^3*(a
+ b*x)*(c + d*x)^2) - (3*B*b*d*x*log((e*(a + b*x))/(c + d*x)))/(2*g^2*i^3*(
a*d - b*c)^2*(a + b*x)*(c + d*x)^2) + (3*B*b^2*d^2*x^2*log((e*(a + b*x))/(c
 + d*x)))/(g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) + (3*A*a*b*d^2*x)/(
2*g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) - (9*B*a*b*d^2*x)/(4*g^2*i^3
*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) + (9*A*b^2*c*d*x)/(2*g^2*i^3*(a*d - b
*c)^3*(a + b*x)*(c + d*x)^2) - (3*B*b^2*c*d*x)/(4*g^2*i^3*(a*d - b*c)^3*(a
+ b*x)*(c + d*x)^2) + (3*B*a*b*c*d*log((e*(a + b*x))/(c + d*x)))/(g^2*i^3*(
a*d - b*c)^3*(a + b*x)*(c + d*x)^2) + (3*B*a*b*d^2*x*log((e*(a + b*x))/(c +
 d*x)))/(g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) + (3*B*b^2*c*d*x*log(
(e*(a + b*x))/(c + d*x)))/(g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2)
```

3.53 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 463

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx = &-\frac{Bd^4(a+bx)^2}{4(bc-ad)^5g^3i^3(c+dx)^2}+\frac{4bBd^3(a+bx)}{(bc-ad)^5g^3i^3(c+dx)}\\&+\frac{4b^3Bd(c+dx)}{(bc-ad)^5g^3i^3(a+bx)}-\frac{b^4B(c+dx)^2}{4(bc-ad)^5g^3i^3(a+bx)^2}\\&-\frac{3b^2Bd^2\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}+\frac{d^4(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^5g^3i^3(c+dx)^2}\\&-\frac{4bd^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^3i^3(c+dx)}\\&+\frac{4b^3d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^3i^3(a+bx)}\\&-\frac{b^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^5g^3i^3(a+bx)^2}\\&+\frac{6b^2d^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^3i^3}\end{aligned}$$

```
[Out] -1/4*B*d^4*(b*x+a)^2/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2+4*b*B*d^3*(b*x+a)/(-a*d
+b*c)^5/g^3/i^3/(d*x+c)+4*b^3*B*d*(d*x+c)/(-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/4*
b^4*B*(d*x+c)^2/(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2-3*b^2*B*d^2*ln((b*x+a)/(d*x+
c))^2/(-a*d+b*c)^5/g^3/i^3+1/2*d^4*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-
a*d+b*c)^5/g^3/i^3/(d*x+c)^2-4*b*d^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-
a*d+b*c)^5/g^3/i^3/(d*x+c)+4*b^3*d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*
```

```
d+b*c)^5/g^3/i^3/(b*x+a)-1/2*b^4*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*
d+b*c)^5/g^3/i^3/(b*x+a)^2+6*b^2*d^2*ln((b*x+a)/(d*x+c))*(A+B*ln(e*(b*x+a)/
(d*x+c)))/(-a*d+b*c)^5/g^3/i^3
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2562, 45, 2372, 2338}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx = -\frac{b^4(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^3i^3(a+bx)^2(bc-ad)^5} + \frac{4b^3d(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^3i^3(a+bx)(bc-ad)^5} + \frac{6b^2d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^3i^3(bc-ad)^5} + \frac{d^4(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^3i^3(c+dx)^2(bc-ad)^5} - \frac{4bd^3(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^3i^3(c+dx)(bc-ad)^5} - \frac{b^4B(c+dx)^2}{4g^3i^3(a+bx)^2(bc-ad)^5} + \frac{4b^3Bd(c+dx)}{g^3i^3(a+bx)(bc-ad)^5} - \frac{3b^2Bd^2\log^2\left(\frac{a+bx}{c+dx}\right)}{g^3i^3(bc-ad)^5} - \frac{Bd^4(a+bx)^2}{4g^3i^3(c+dx)^2(bc-ad)^5} + \frac{4bBd^3(a+bx)}{g^3i^3(c+dx)(bc-ad)^5}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^3*(c*i + d*i*x)^3),
x]
```

```
[Out] -1/4*(B*d^4*(a + b*x)^2)/((b*c - a*d)^5*g^3*i^3*(c + d*x)^2) + (4*b*B*d^3*(
a + b*x))/((b*c - a*d)^5*g^3*i^3*(c + d*x)) + (4*b^3*B*d*(c + d*x))/((b*c -
 a*d)^5*g^3*i^3*(a + b*x)) - (b^4*B*(c + d*x)^2)/(4*(b*c - a*d)^5*g^3*i^3*(
a + b*x)^2) - (3*b^2*B*d^2*Log[(a + b*x)/(c + d*x)]^2)/((b*c - a*d)^5*g^3*i
^3) + (d^4*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*c - a*d)
^5*g^3*i^3*(c + d*x)^2) - (4*b*d^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c +
d*x)]))/((b*c - a*d)^5*g^3*i^3*(c + d*x)) + (4*b^3*d*(c + d*x)*(A + B*Log[(
e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^5*g^3*i^3*(a + b*x)) - (b^4*(c + d*x
)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*c - a*d)^5*g^3*i^3*(a + b*x
)^2) + (6*b^2*d^2*Log[(a + b*x)/(c + d*x)]*(A + B*Log[(e*(a + b*x))/(c + d*
x)]))/((b*c - a*d)^5*g^3*i^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)^4(A+B\log(ex))}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}\\
&= \frac{d^4(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^5g^3i^3(c+dx)^2}-\frac{4bd^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^3i^3(c+dx)}\\
&\quad+\frac{4b^3d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^3i^3(a+bx)}-\frac{b^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^5g^3i^3(a+bx)^2}\\
&\quad+\frac{6b^2d^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^3i^3}\\
&\quad-\frac{B\text{Subst}\left(\int \left(\frac{1}{2}\left(-8bd^3-\frac{b^4}{x^3}+\frac{8b^3d}{x^2}+d^4x\right)+\frac{6b^2d^2\log(x)}{x}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}
\end{aligned}
$$

$$= \frac{d^4(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^5g^3i^3(c+dx)^2} - \frac{4bd^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^3i^3(c+dx)}$$
$$+ \frac{4b^3d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^5g^3i^3(a+bx)^2}$$
$$+ \frac{6b^2d^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^3i^3}$$
$$- \frac{B\text{Subst}\left(\int\left(-8bd^3-\frac{b^4}{x^3}+\frac{8b^3d}{x^2}+d^4x\right)dx, x, \frac{a+bx}{c+dx}\right)}{2(bc-ad)^5g^3i^3}$$
$$- \frac{(6b^2Bd^2)\,\text{Subst}\left(\int\frac{\log(x)}{x}dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}$$
$$= -\frac{Bd^4(a+bx)^2}{4(bc-ad)^5g^3i^3(c+dx)^2} + \frac{4bBd^3(a+bx)}{(bc-ad)^5g^3i^3(c+dx)} + \frac{4b^3Bd(c+dx)}{(bc-ad)^5g^3i^3(a+bx)}$$
$$- \frac{b^4B(c+dx)^2}{4(bc-ad)^5g^3i^3(a+bx)^2} - \frac{3b^2Bd^2\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3} + \frac{d^4(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^5g^3i^3(c+dx)^2}$$
$$- \frac{4bd^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^3i^3(c+dx)} + \frac{4b^3d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^3i^3(a+bx)}$$
$$- \frac{b^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^5g^3i^3(a+bx)^2} + \frac{6b^2d^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^3i^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.63 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.15

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx =$$

$$-\frac{\frac{b^2B(bc-ad)^2}{(a+bx)^2} - \frac{12b^3Bcd}{a+bx} + \frac{12ab^2Bd^2}{a+bx} - \frac{2b^2Bd(bc-ad)}{a+bx} + \frac{Bd^2(bc-ad)^2}{(c+dx)^2} + \frac{12b^2Bcd^2}{c+dx} - \frac{12abBd^3}{c+dx} + \frac{2bBd^2(bc-ad)}{c+dx} + \frac{2b^2(bc-ad}{}}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^3*(c*i + d*i*
x)^3),x]
```

```
[Out] -1/4*((b^2*B*(b*c - a*d)^2)/(a + b*x)^2 - (12*b^3*B*c*d)/(a + b*x) + (12*a*
b^2*B*d^2)/(a + b*x) - (2*b^2*B*d*(b*c - a*d))/(a + b*x) + (B*d^2*(b*c - a*
d)^2)/(c + d*x)^2 + (12*b^2*B*c*d^2)/(c + d*x) - (12*a*b*B*d^3)/(c + d*x) +
 (2*b*B*d^2*(b*c - a*d))/(c + d*x) + (2*b^2*(b*c - a*d)^2*(A + B*Log[(e*(a
```

```
+ b*x))/(c + d*x)]))/(a + b*x)^2 - (12*b^2*d*(b*c - a*d)*(A + B*Log[(e*(a +
 b*x))/(c + d*x)]))/(a + b*x) - (2*d^2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x
))/(c + d*x)]))/(c + d*x)^2 - (12*b*d^2*(b*c - a*d)*(A + B*Log[(e*(a + b*x)
)/(c + d*x)]))/(c + d*x) - 24*b^2*d^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x))
/(c + d*x)]) + 24*b^2*d^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x]
 + 12*b^2*B*d^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*
d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 12*b^2*B*d^2*((2*Log[(
d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b
*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)^5*g^3*i^3)
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 649, normalized size of antiderivative = 1.40

method	result
parts	$\frac{A\left(-\frac{d^2}{2(ad-cb)^3(dx+c)^2}+\frac{6d^2b^2\ln(dx+c)}{(ad-cb)^5}+\frac{3d^2b}{(ad-cb)^4(dx+c)}+\frac{b^2}{2(ad-cb)^3(bx+a)^2}-\frac{6d^2b^2\ln(bx+a)}{(ad-cb)^5}+\frac{3b^2d}{(ad-cb)^4(bx+a)}\right)}{g^3i^3}-\frac{Bd\Big(}{}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^2eAb^4}{2i^3(ad-cb)^6g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{4d^3Ab^3}{i^3(ad-cb)^6g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{6d^4Ab^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^3(ad-cb)^6g^3}-\frac{4d^5Ab\left(\frac{be}{d}+\frac{(a}{d(}\right.}{e^2i^3(ad-cb}\right.}{}$
default	$-\frac{e(ad-cb)\left(-\frac{d^2eAb^4}{2i^3(ad-cb)^6g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{4d^3Ab^3}{i^3(ad-cb)^6g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{6d^4Ab^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^3(ad-cb)^6g^3}-\frac{4d^5Ab\left(\frac{be}{d}+\frac{(a}{d(}\right.}{e^2i^3(ad-cb}\right.}{}$
risch	Expression too large to display
parallelrisch	Expression too large to display
norman	Expression too large to display

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x,method=_RETUR
NVERBOSE)

[Out] A/g^3/i^3*(-1/2*d^2/(a*d-b*c)^3/(d*x+c)^2+6*d^2/(a*d-b*c)^5*b^2*ln(d*x+c)+3
*d^2/(a*d-b*c)^4*b/(d*x+c)+1/2*b^2/(a*d-b*c)^3/(b*x+a)^2-6*d^2/(a*d-b*c)^5*
b^2*ln(b*x+a)+3*b^2/(a*d-b*c)^4*d/(b*x+a))-B/g^3/i^3*d/(a*d-b*c)^2/e^2*(d^3
/(a*d-b*c)^3*(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d
*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-4*d^2/(a*d-b*c)^3*b*e*((b*e/d+(
a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)
```

```
-b*e/d)+3*d/(a*d-b*c)^3*b^2*e^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-4/(a*d-b*
c)^3*b^3*e^3*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+
c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+1/d/(a*d-b*c)^3*b^4*e^4*(-1/2/(b*e/d+(
a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c
)*e/d/(d*x+c))^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1011 vs. 2(455) = 910.

Time = 0.33 (sec) , antiderivative size = 1011, normalized size of antiderivative = 2.18

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx = -\frac{(2A+B)b^4c^4-16(A+B)ab^3c^3d+30Ba^2b^2c^2d^2+16(A-B)a^3bcd^3-(2A-B)a^4d^4-24(Ab^4cd^3}{}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x, algor
ithm="fricas")
```

```
[Out] -1/4*((2*A + B)*b^4*c^4 - 16*(A + B)*a*b^3*c^3*d + 30*B*a^2*b^2*c^2*d^2 + 1
6*(A - B)*a^3*b*c*d^3 - (2*A - B)*a^4*d^4 - 24*(A*b^4*c*d^3 - A*a*b^3*d^4)*
x^3 - 12*((3*A + B)*b^4*c^2*d^2 - 2*B*a*b^3*c*d^3 - (3*A - B)*a^2*b^2*d^4)*
x^2 - 12*(B*b^4*d^4*x^4 + B*a^2*b^2*c^2*d^2 + 2*(B*b^4*c*d^3 + B*a*b^3*d^4)
*x^3 + (B*b^4*c^2*d^2 + 4*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*x^2 + 2*(B*a*b^3*c
^2*d^2 + B*a^2*b^2*c*d^3)*x)*log((b*e*x + a*e)/(d*x + c))^2 - 4*((2*A + 3*B
)*b^4*c^3*d + 3*(4*A - B)*a*b^3*c^2*d^2 - 3*(4*A + B)*a^2*b^2*c*d^3 - (2*A
- 3*B)*a^3*b*d^4)*x - 2*(12*A*b^4*d^4*x^4 - B*b^4*c^4 + 8*B*a*b^3*c^3*d + 1
2*A*a^2*b^2*c^2*d^2 - 8*B*a^3*b*c*d^3 + B*a^4*d^4 + 12*((2*A + B)*b^4*c*d^3
 + (2*A - B)*a*b^3*d^4)*x^3 + 6*((2*A + 3*B)*b^4*c^2*d^2 + 8*A*a*b^3*c*d^3
+ (2*A - 3*B)*a^2*b^2*d^4)*x^2 + 4*(B*b^4*c^3*d + 6*(A + B)*a*b^3*c^2*d^2 +
 6*(A - B)*a^2*b^2*c*d^3 - B*a^3*b*d^4)*x)*log((b*e*x + a*e)/(d*x + c)))/((
b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^2*d^5 + 5
*a^4*b^3*c*d^6 - a^5*b^2*d^7)*g^3*i^3*x^4 + 2*(b^7*c^6*d - 4*a*b^6*c^5*d^2
+ 5*a^2*b^5*c^4*d^3 - 5*a^4*b^3*c^2*d^5 + 4*a^5*b^2*c*d^6 - a^6*b*d^7)*g^3*
i^3*x^3 + (b^7*c^7 - a*b^6*c^6*d - 9*a^2*b^5*c^5*d^2 + 25*a^3*b^4*c^4*d^3 -
 25*a^4*b^3*c^3*d^4 + 9*a^5*b^2*c^2*d^5 + a^6*b*c*d^6 - a^7*d^7)*g^3*i^3*x^
2 + 2*(a*b^6*c^7 - 4*a^2*b^5*c^6*d + 5*a^3*b^4*c^5*d^2 - 5*a^5*b^2*c^3*d^4
+ 4*a^6*b*c^2*d^5 - a^7*c*d^6)*g^3*i^3*x + (a^2*b^5*c^7 - 5*a^3*b^4*c^6*d +
 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5)*g
^3*i^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2106 vs. 2(430) = 860.

Time = 138.12 (sec) , antiderivative size = 2106, normalized size of antiderivative = 4.55

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3/(d*i*x+c*i)**3,x)
```

```
[Out] 6*A*b**2*d**2*log(x + (-6*A*a**6*b**2*d**8/(a*d - b*c)**5 + 36*A*a**5*b**3*
c*d**7/(a*d - b*c)**5 - 90*A*a**4*b**4*c**2*d**6/(a*d - b*c)**5 + 120*A*a**
3*b**5*c**3*d**5/(a*d - b*c)**5 - 90*A*a**2*b**6*c**4*d**4/(a*d - b*c)**5 +
 36*A*a*b**7*c**5*d**3/(a*d - b*c)**5 + 6*A*a*b**2*d**3 - 6*A*b**8*c**6*d**
2/(a*d - b*c)**5 + 6*A*b**3*c*d**2)/(12*A*b**3*d**3))/(g**3*i**3*(a*d - b*c
)**5) - 6*A*b**2*d**2*log(x + (6*A*a**6*b**2*d**8/(a*d - b*c)**5 - 36*A*a**
5*b**3*c*d**7/(a*d - b*c)**5 + 90*A*a**4*b**4*c**2*d**6/(a*d - b*c)**5 - 12
0*A*a**3*b**5*c**3*d**5/(a*d - b*c)**5 + 90*A*a**2*b**6*c**4*d**4/(a*d - b*
c)**5 - 36*A*a*b**7*c**5*d**3/(a*d - b*c)**5 + 6*A*a*b**2*d**3 + 6*A*b**8*c
**6*d**2/(a*d - b*c)**5 + 6*A*b**3*c*d**2)/(12*A*b**3*d**3))/(g**3*i**3*(a*
d - b*c)**5) - 3*B*b**2*d**2*log(e*(a + b*x)/(c + d*x))**2/(a**5*d**5*g**3*
i**3 - 5*a**4*b*c*d**4*g**3*i**3 + 10*a**3*b**2*c**2*d**3*g**3*i**3 - 10*a*
*2*b**3*c**3*d**2*g**3*i**3 + 5*a*b**4*c**4*d*g**3*i**3 - b**5*c**5*g**3*i*
*3) + (-B*a**3*d**3 + 7*B*a**2*b*c*d**2 + 4*B*a**2*b*d**3*x + 7*B*a*b**2*c*
*2*d + 28*B*a*b**2*c*d**2*x + 18*B*a*b**2*d**3*x**2 - B*b**3*c**3 + 4*B*b**
3*c**2*d*x + 18*B*b**3*c*d**2*x**2 + 12*B*b**3*d**3*x**3)*log(e*(a + b*x)/(
c + d*x))/(2*a**6*c**2*d**4*g**3*i**3 + 4*a**6*c*d**5*g**3*i**3*x + 2*a**6*
d**6*g**3*i**3*x**2 - 8*a**5*b*c**3*d**3*g**3*i**3 - 12*a**5*b*c**2*d**4*g*
*3*i**3*x + 4*a**5*b*d**6*g**3*i**3*x**3 + 12*a**4*b**2*c**4*d**2*g**3*i**3
 + 8*a**4*b**2*c**3*d**3*g**3*i**3*x - 18*a**4*b**2*c**2*d**4*g**3*i**3*x**
2 - 12*a**4*b**2*c*d**5*g**3*i**3*x**3 + 2*a**4*b**2*d**6*g**3*i**3*x**4 -
8*a**3*b**3*c**5*d*g**3*i**3 + 8*a**3*b**3*c**4*d**2*g**3*i**3*x + 32*a**3*
b**3*c**3*d**3*g**3*i**3*x**2 + 8*a**3*b**3*c**2*d**4*g**3*i**3*x**3 - 8*a*
*3*b**3*c*d**5*g**3*i**3*x**4 + 2*a**2*b**4*c**6*g**3*i**3 - 12*a**2*b**4*c
**5*d*g**3*i**3*x - 18*a**2*b**4*c**4*d**2*g**3*i**3*x**2 + 8*a**2*b**4*c**
3*d**3*g**3*i**3*x**3 + 12*a**2*b**4*c**2*d**4*g**3*i**3*x**4 + 4*a*b**5*c*
*6*g**3*i**3*x - 12*a*b**5*c**4*d**2*g**3*i**3*x**3 - 8*a*b**5*c**3*d**3*g*
*3*i**3*x**4 + 2*b**6*c**6*g**3*i**3*x**2 + 4*b**6*c**5*d*g**3*i**3*x**3 +
2*b**6*c**4*d**2*g**3*i**3*x**4) + (-2*A*a**3*d**3 + 14*A*a**2*b*c*d**2 + 1
4*A*a*b**2*c**2*d - 2*A*b**3*c**3 + 24*A*b**3*d**3*x**3 + B*a**3*d**3 - 15*
B*a**2*b*c*d**2 + 15*B*a*b**2*c**2*d - B*b**3*c**3 + x**2*(36*A*a*b**2*d**3
 + 36*A*b**3*c*d**2 - 12*B*a*b**2*d**3 + 12*B*b**3*c*d**2) + x*(8*A*a**2*b*
d**3 + 56*A*a*b**2*c*d**2 + 8*A*b**3*c**2*d - 12*B*a**2*b*d**3 + 12*B*b**3*
c**2*d))/(4*a**6*c**2*d**4*g**3*i**3 - 16*a**5*b*c**3*d**3*g**3*i**3 + 24*a
```

```
**4*b**2*c**4*d**2*g**3*i**3 - 16*a**3*b**3*c**5*d*g**3*i**3 + 4*a**2*b**4*
c**6*g**3*i**3 + x**4*(4*a**4*b**2*d**6*g**3*i**3 - 16*a**3*b**3*c*d**5*g**
3*i**3 + 24*a**2*b**4*c**2*d**4*g**3*i**3 - 16*a*b**5*c**3*d**3*g**3*i**3 +
 4*b**6*c**4*d**2*g**3*i**3) + x**3*(8*a**5*b*d**6*g**3*i**3 - 24*a**4*b**2
*c*d**5*g**3*i**3 + 16*a**3*b**3*c**2*d**4*g**3*i**3 + 16*a**2*b**4*c**3*d*
*3*g**3*i**3 - 24*a*b**5*c**4*d**2*g**3*i**3 + 8*b**6*c**5*d*g**3*i**3) + x
**2*(4*a**6*d**6*g**3*i**3 - 36*a**4*b**2*c**2*d**4*g**3*i**3 + 64*a**3*b**
3*c**3*d**3*g**3*i**3 - 36*a**2*b**4*c**4*d**2*g**3*i**3 + 4*b**6*c**6*g**3
*i**3) + x*(8*a**6*c*d**5*g**3*i**3 - 24*a**5*b*c**2*d**4*g**3*i**3 + 16*a*
*4*b**2*c**3*d**3*g**3*i**3 + 16*a**3*b**3*c**4*d**2*g**3*i**3 - 24*a**2*b*
*4*c**5*d*g**3*i**3 + 8*a*b**5*c**6*g**3*i**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2380 vs. $2(455) = 910$.

Time = 0.35 (sec) , antiderivative size = 2380, normalized size of antiderivative = 5.14

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x, algor
ithm="maxima")
```

```
[Out] 1/2*B*((12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d^3
+ 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 + a^2*b*d^3
)*x)/((b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5
+ a^4*b^2*d^6)*g^3*i^3*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4*c^3
*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*g^3*i^3*x^3 + (b^6*
c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d^6)
*g^3*i^3*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b
^2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*g^3*i^3*x + (a^2*b^4*c^6 - 4*a^3*
b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4)*g^3*i^3) + 1
2*b^2*d^2*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*
a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3) - 12*b^2*d^2*log(d*x +
c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*
a^4*b*c*d^4 - a^5*d^5)*g^3*i^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 1/2
*A*((12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d^3 + 1
8*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 + a^2*b*d^3)*x
)/((b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5 + a
^4*b^2*d^6)*g^3*i^3*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4*c^3*d^
3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*g^3*i^3*x^3 + (b^6*c^6
 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d^6)*g^
3*i^3*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b^2*
c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*g^3*i^3*x + (a^2*b^4*c^6 - 4*a^3*b^3
```

```
*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4)*g^3*i^3) + 12*b
^2*d^2*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3
*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3) - 12*b^2*d^2*log(d*x + c)/
((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4
*b*c*d^4 - a^5*d^5)*g^3*i^3)) - 1/4*(b^4*c^4 - 16*a*b^3*c^3*d + 30*a^2*b^2*
c^2*d^2 - 16*a^3*b*c*d^3 + a^4*d^4 - 12*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*
b^2*d^4)*x^2 + 12*(b^4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4
)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2
+ a^2*b^2*c*d^3)*x)*log(b*x + a)^2 - 24*(b^4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*
(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x
^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a)*log(d*x + c) + 12*(b
^4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2
 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*
log(d*x + c)^2 - 12*(b^4*c^3*d - a*b^3*c^2*d^2 - a^2*b^2*c*d^3 + a^3*b*d^4)
*x)*B/(a^2*b^5*c^7*g^3*i^3 - 5*a^3*b^4*c^6*d*g^3*i^3 + 10*a^4*b^3*c^5*d^2*g
^3*i^3 - 10*a^5*b^2*c^4*d^3*g^3*i^3 + 5*a^6*b*c^3*d^4*g^3*i^3 - a^7*c^2*d^5
*g^3*i^3 + (b^7*c^5*d^2*g^3*i^3 - 5*a*b^6*c^4*d^3*g^3*i^3 + 10*a^2*b^5*c^3*
d^4*g^3*i^3 - 10*a^3*b^4*c^2*d^5*g^3*i^3 + 5*a^4*b^3*c*d^6*g^3*i^3 - a^5*b^
2*d^7*g^3*i^3)*x^4 + 2*(b^7*c^6*d*g^3*i^3 - 4*a*b^6*c^5*d^2*g^3*i^3 + 5*a^2
*b^5*c^4*d^3*g^3*i^3 - 5*a^4*b^3*c^2*d^5*g^3*i^3 + 4*a^5*b^2*c*d^6*g^3*i^3
- a^6*b*d^7*g^3*i^3)*x^3 + (b^7*c^7*g^3*i^3 - a*b^6*c^6*d*g^3*i^3 - 9*a^2*b
^5*c^5*d^2*g^3*i^3 + 25*a^3*b^4*c^4*d^3*g^3*i^3 - 25*a^4*b^3*c^3*d^4*g^3*i^
3 + 9*a^5*b^2*c^2*d^5*g^3*i^3 + a^6*b*c*d^6*g^3*i^3 - a^7*d^7*g^3*i^3)*x^2
+ 2*(a*b^6*c^7*g^3*i^3 - 4*a^2*b^5*c^6*d*g^3*i^3 + 5*a^3*b^4*c^5*d^2*g^3*i^
3 - 5*a^5*b^2*c^3*d^4*g^3*i^3 + 4*a^6*b*c^2*d^5*g^3*i^3 - a^7*c*d^6*g^3*i^3
)*x)
```

Giac [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{(bgx+ag)^3(dix+ci)^3}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x, algor
ithm="giac")
```

```
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)/((b*g*x + a*g)^3*(d*i*x + c*i)
^3), x)
```

Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 1443, normalized size of antiderivative = 3.12

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^3*(c*i + d*i*x)^3),
x)
[Out] (A*b^2*d^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*12i)/(g^3*i^3*(a*
d - b*c)^5) - (3*B*b^2*d^2*log((e*(a + b*x))/(c + d*x))^2)/(g^3*i^3*(a*d -
b*c)^5) - (A*a^3*d^3)/(2*g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) - (
A*b^3*c^3)/(2*g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) + (B*a^3*d^3)/
(4*g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) - (B*b^3*c^3)/(4*g^3*i^3*
(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) - (B*a*d*log((e*(a + b*x))/(c + d*x)
))/(2*g^3*i^3*(a*d - b*c)^2*(a + b*x)^2*(c + d*x)^2) - (B*b*c*log((e*(a + b
*x))/(c + d*x)))/(2*g^3*i^3*(a*d - b*c)^2*(a + b*x)^2*(c + d*x)^2) + (6*A*b
^3*d^3*x^3)/(g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) + (7*A*a*b^2*c^
2*d)/(2*g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) + (7*A*a^2*b*c*d^2)/
(2*g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) + (15*B*a*b^2*c^2*d)/(4*g
^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) - (15*B*a^2*b*c*d^2)/(4*g^3*i
^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) + (2*A*a^2*b*d^3*x)/(g^3*i^3*(a*d
 - b*c)^4*(a + b*x)^2*(c + d*x)^2) - (3*B*a^2*b*d^3*x)/(g^3*i^3*(a*d - b*c)
^4*(a + b*x)^2*(c + d*x)^2) + (2*A*b^3*c^2*d*x)/(g^3*i^3*(a*d - b*c)^4*(a +
 b*x)^2*(c + d*x)^2) + (3*B*b^3*c^2*d*x)/(g^3*i^3*(a*d - b*c)^4*(a + b*x)^2
*(c + d*x)^2) + (9*A*a*b^2*d^3*x^2)/(g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c +
 d*x)^2) - (3*B*a*b^2*d^3*x^2)/(g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)
^2) + (9*A*b^3*c*d^2*x^2)/(g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) +
 (3*B*b^3*c*d^2*x^2)/(g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) - (B*b
*d*x*log((e*(a + b*x))/(c + d*x)))/(g^3*i^3*(a*d - b*c)^2*(a + b*x)^2*(c +
d*x)^2) + (6*B*b^3*d^3*x^3*log((e*(a + b*x))/(c + d*x)))/(g^3*i^3*(a*d - b*
c)^4*(a + b*x)^2*(c + d*x)^2) + (9*B*a*b^2*d^3*x^2*log((e*(a + b*x))/(c + d
*x)))/(g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) + (9*B*b^3*c*d^2*x^2*
log((e*(a + b*x))/(c + d*x)))/(g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^
2) + (14*A*a*b^2*c*d^2*x)/(g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) +
 (3*B*a*b^2*c^2*d*log((e*(a + b*x))/(c + d*x)))/(g^3*i^3*(a*d - b*c)^4*(a +
 b*x)^2*(c + d*x)^2) + (3*B*a^2*b*c*d^2*log((e*(a + b*x))/(c + d*x)))/(g^3*
i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) + (3*B*a^2*b*d^3*x*log((e*(a + b
*x))/(c + d*x)))/(g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) + (3*B*b^3
*c^2*d*x*log((e*(a + b*x))/(c + d*x)))/(g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(
c + d*x)^2) + (12*B*a*b^2*c*d^2*x*log((e*(a + b*x))/(c + d*x)))/(g^3*i^3*(a
*d - b*c)^4*(a + b*x)^2*(c + d*x)^2)
```

3.54 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 563

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx &= \frac{Bd^5(a+bx)^2}{4(bc-ad)^6g^4i^3(c+dx)^2}-\frac{5bBd^4(a+bx)}{(bc-ad)^6g^4i^3(c+dx)} \\ &-\frac{10b^3Bd^2(c+dx)}{(bc-ad)^6g^4i^3(a+bx)}+\frac{5b^4Bd(c+dx)^2}{4(bc-ad)^6g^4i^3(a+bx)^2} \\ &-\frac{b^5B(c+dx)^3}{9(bc-ad)^6g^4i^3(a+bx)^3}+\frac{5b^2Bd^3\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3} \\ &-\frac{d^5(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2} \\ &+\frac{5bd^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(c+dx)} \\ &-\frac{10b^3d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(a+bx)} \\ &+\frac{5b^4d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2} \\ &-\frac{b^5(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^6g^4i^3(a+bx)^3} \\ &-\frac{10b^2d^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3}\end{aligned}$$

```
[Out] 1/4*B*d^5*(b*x+a)^2/(-a*d+b*c)^6/g^4/i^3/(d*x+c)^2-5*b*B*d^4*(b*x+a)/(-a*d+
b*c)^6/g^4/i^3/(d*x+c)-10*b^3*B*d^2*(d*x+c)/(-a*d+b*c)^6/g^4/i^3/(b*x+a)+5/
```

```
4*b^4*B*d*(d*x+c)^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-1/9*b^5*B*(d*x+c)^3/(-a*
d+b*c)^6/g^4/i^3/(b*x+a)^3+5*b^2*B*d^3*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^6/g
^4/i^3-1/2*d^5*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^6/g^4/i^3/(
d*x+c)^2+5*b*d^4*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^6/g^4/i^3/(
d*x+c)-10*b^3*d^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^6/g^4/i^3/
(b*x+a)+5/2*b^4*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^6/g^4/i^
3/(b*x+a)^2-1/3*b^5*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^6/g^4/
i^3/(b*x+a)^3-10*b^2*d^3*ln((b*x+a)/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-
a*d+b*c)^6/g^4/i^3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2562, 45, 2372, 12, 14, 2338}

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = -\frac{b^5(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3g^4i^3(a+bx)^3(bc-ad)^6} + \frac{5b^4d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^4i^3(a+bx)^2(bc-ad)^6} - \frac{10b^3d^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^4i^3(a+bx)(bc-ad)^6} - \frac{10b^2d^3\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^4i^3(bc-ad)^6} - \frac{d^5(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^4i^3(c+dx)^2(bc-ad)^6} + \frac{5bd^4(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^4i^3(c+dx)(bc-ad)^6} - \frac{b^5B(c+dx)^3}{9g^4i^3(a+bx)^3(bc-ad)^6} + \frac{5b^4Bd(c+dx)^2}{4g^4i^3(a+bx)^2(bc-ad)^6} - \frac{10b^3Bd^2(c+dx)}{g^4i^3(a+bx)(bc-ad)^6} + \frac{5b^2Bd^3\log^2\left(\frac{a+bx}{c+dx}\right)}{g^4i^3(bc-ad)^6} + \frac{Bd^5(a+bx)^2}{4g^4i^3(c+dx)^2(bc-ad)^6} - \frac{5bBd^4(a+bx)}{g^4i^3(c+dx)(bc-ad)^6}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^4*(c*i + d*i*x)^3),
x]

[Out] (B*d^5*(a + b*x)^2)/(4*(b*c - a*d)^6*g^4*i^3*(c + d*x)^2) - (5*b*B*d^4*(a +
 b*x))/((b*c - a*d)^6*g^4*i^3*(c + d*x)) - (10*b^3*B*d^2*(c + d*x))/((b*c -
```

```
 a*d)^6*g^4*i^3*(a + b*x)) + (5*b^4*B*d*(c + d*x)^2)/(4*(b*c - a*d)^6*g^4*i
^3*(a + b*x)^2) - (b^5*B*(c + d*x)^3)/(9*(b*c - a*d)^6*g^4*i^3*(a + b*x)^3)
 + (5*b^2*B*d^3*Log[(a + b*x)/(c + d*x)]^2)/((b*c - a*d)^6*g^4*i^3) - (d^5*
(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*c - a*d)^6*g^4*i^3*
(c + d*x)^2) + (5*b*d^4*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b
*c - a*d)^6*g^4*i^3*(c + d*x)) - (10*b^3*d^2*(c + d*x)*(A + B*Log[(e*(a + b
*x))/(c + d*x)]))/((b*c - a*d)^6*g^4*i^3*(a + b*x)) + (5*b^4*d*(c + d*x)^2*
(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*c - a*d)^6*g^4*i^3*(a + b*x)^2)
 - (b^5*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*(b*c - a*d)^6*
g^4*i^3*(a + b*x)^3) - (10*b^2*d^3*Log[(a + b*x)/(c + d*x)]*(A + B*Log[(e*(
a + b*x))/(c + d*x)]))/((b*c - a*d)^6*g^4*i^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
```

```
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)^5(A+B\log(ex))}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3} \\
&= -\frac{d^5(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2} + \frac{5bd^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(c+dx)} \\
&\quad - \frac{10b^3d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2} \\
&\quad - \frac{b^5(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^6g^4i^3(a+bx)^3} - \frac{10b^2d^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3} \\
&\quad - \frac{B\text{Subst}\left(\int \frac{-2b^5+15b^4dx-60b^3d^2x^2+30bd^4x^4-3d^5x^5-60b^2d^3x^3\log(x)}{6x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3} \\
&= -\frac{d^5(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2} + \frac{5bd^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(c+dx)} \\
&\quad - \frac{10b^3d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2} \\
&\quad - \frac{b^5(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^6g^4i^3(a+bx)^3} - \frac{10b^2d^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3} \\
&\quad - \frac{B\text{Subst}\left(\int \frac{-2b^5+15b^4dx-60b^3d^2x^2+30bd^4x^4-3d^5x^5-60b^2d^3x^3\log(x)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{6(bc-ad)^6g^4i^3} \\
&= -\frac{d^5(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2} + \frac{5bd^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(c+dx)} \\
&\quad - \frac{10b^3d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2} \\
&\quad - \frac{b^5(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^6g^4i^3(a+bx)^3} - \frac{10b^2d^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3} \\
&\quad - \frac{B\text{Subst}\left(\int \left(\frac{-2b^5+15b^4dx-60b^3d^2x^2+30bd^4x^4-3d^5x^5}{x^4} - \frac{60b^2d^3\log(x)}{x}\right)dx, x, \frac{a+bx}{c+dx}\right)}{6(bc-ad)^6g^4i^3}
\end{aligned}$$

$$= -\frac{d^5(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2}+\frac{5bd^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(c+dx)}$$
$$-\frac{10b^3d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(a+bx)}+\frac{5b^4d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2}$$
$$-\frac{b^5(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^6g^4i^3(a+bx)^3}-\frac{10b^2d^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3}$$
$$-\frac{B\text{Subst}\left(\int\frac{-2b^5+15b^4dx-60b^3d^2x^2+30bd^4x^4-3d^5x^5}{x^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{6(bc-ad)^6g^4i^3}$$
$$+\frac{(10b^2Bd^3)\,\text{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}$$

$$=\frac{5b^2Bd^3\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}-\frac{d^5(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2}$$
$$+\frac{5bd^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(c+dx)}-\frac{10b^3d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(a+bx)}$$
$$+\frac{5b^4d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2}-\frac{b^5(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^6g^4i^3(a+bx)^3}$$
$$-\frac{10b^2d^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3}$$
$$-\frac{B\text{Subst}\left(\int\left(30bd^4-\frac{2b^5}{x^4}+\frac{15b^4d}{x^3}-\frac{60b^3d^2}{x^2}-3d^5x\right)dx,x,\frac{a+bx}{c+dx}\right)}{6(bc-ad)^6g^4i^3}$$

$$=\frac{Bd^5(a+bx)^2}{4(bc-ad)^6g^4i^3(c+dx)^2}-\frac{5bBd^4(a+bx)}{(bc-ad)^6g^4i^3(c+dx)}-\frac{10b^3Bd^2(c+dx)}{(bc-ad)^6g^4i^3(a+bx)}$$
$$+\frac{5b^4Bd(c+dx)^2}{4(bc-ad)^6g^4i^3(a+bx)^2}-\frac{b^5B(c+dx)^3}{9(bc-ad)^6g^4i^3(a+bx)^3}+\frac{5b^2Bd^3\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}$$
$$-\frac{d^5(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2}+\frac{5bd^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(c+dx)}$$
$$-\frac{10b^3d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(a+bx)}+\frac{5b^4d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2}$$
$$-\frac{b^5(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^6g^4i^3(a+bx)^3}-\frac{10b^2d^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.89 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.13

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx =$$

$$-\frac{\frac{4b^2B(bc-ad)^3}{(a+bx)^3} - \frac{33b^2Bd(bc-ad)^2}{(a+bx)^2} + \frac{216b^3Bcd^2}{a+bx} - \frac{216ab^2Bd^3}{a+bx} + \frac{66b^2Bd^2(bc-ad)}{a+bx} - \frac{9Bd^3(bc-ad)^2}{(c+dx)^2} - \frac{144b^2Bcd^3}{c+dx} + \frac{144abBd^4}{c+dx} - \cdots}{\cdots}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^4*(c*i + d*i*
x)^3),x]

[Out] -1/36*((4*b^2*B*(b*c - a*d)^3)/(a + b*x)^3 - (33*b^2*B*d*(b*c - a*d)^2)/(a
+ b*x)^2 + (216*b^3*B*c*d^2)/(a + b*x) - (216*a*b^2*B*d^3)/(a + b*x) + (66*
b^2*B*d^2*(b*c - a*d))/(a + b*x) - (9*B*d^3*(b*c - a*d)^2)/(c + d*x)^2 - (1
44*b^2*B*c*d^3)/(c + d*x) + (144*a*b*B*d^4)/(c + d*x) - (18*b*B*d^3*(b*c -
a*d))/(c + d*x) + 120*b^2*B*d^3*Log[a + b*x] + (12*b^2*(b*c - a*d)^3*(A + B
*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x)^3 - (54*b^2*d*(b*c - a*d)^2*(A +
B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x)^2 + (216*b^2*d^2*(b*c - a*d)*(A
+ B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) + (18*d^3*(b*c - a*d)^2*(A + B
*Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x)^2 + (144*b*d^3*(b*c - a*d)*(A + B
*Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x) + 360*b^2*d^3*Log[a + b*x]*(A + B
*Log[(e*(a + b*x))/(c + d*x)]) - 120*b^2*B*d^3*Log[c + d*x] - 360*b^2*d^3*(
A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 180*b^2*B*d^3*(Log[a + b
*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a
+ b*x))/(-(b*c) + a*d)]) + 180*b^2*B*d^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*
d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
)/((b*c - a*d)^6*g^4*i^3)
```

Maple [A] (verified)

Time = 7.99 (sec) , antiderivative size = 787, normalized size of antiderivative = 1.40

method	result
parts	$\frac{A\left(-\frac{d^3}{2(ad-cb)^4(dx+c)^2}+\frac{10d^3b^2\ln(dx+c)}{(ad-cb)^6}+\frac{4d^3b}{(ad-cb)^5(dx+c)}+\frac{b^2}{3(ad-cb)^3(bx+a)^3}-\frac{10d^3b^2\ln(bx+a)}{(ad-cb)^6}+\frac{6b^2d^2}{(ad-cb)^5(bx+a)}+\frac{3\cdots}{2(ad-cb\cdots}\right.}{g^4i^3}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{d^2e^2Ab^5}{3i^3(ad-cb)^7g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{5d^3eAb^4}{2i^3(ad-cb)^7g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{10d^4Ab^3}{i^3(ad-cb)^7g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{10d^5Ab^2\ln\cdots}{e\,i^3(\cdots}\right.}{\cdots}$
default	$-\frac{e(ad-cb)\left(\frac{d^2e^2Ab^5}{3i^3(ad-cb)^7g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{5d^3eAb^4}{2i^3(ad-cb)^7g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{10d^4Ab^3}{i^3(ad-cb)^7g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{10d^5Ab^2\ln\cdots}{e\,i^3(\cdots}\right.}{\cdots}$
risch	Expression too large to display
parallelrisch	Expression too large to display
norman	Expression too large to display

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x,method=_RETUR
NVERBOSE)
```

```
[Out] A/g^4/i^3*(-1/2*d^3/(a*d-b*c)^4/(d*x+c)^2+10*d^3/(a*d-b*c)^6*b^2*ln(d*x+c)+
4*d^3/(a*d-b*c)^5*b/(d*x+c)+1/3*b^2/(a*d-b*c)^3/(b*x+a)^3-10*d^3/(a*d-b*c)^
6*b^2*ln(b*x+a)+6*b^2/(a*d-b*c)^5*d^2/(b*x+a)+3/2*b^2/(a*d-b*c)^4*d/(b*x+a)
^2)-B/g^4/i^3*d/(a*d-b*c)^2/e^2*(d^4/(a*d-b*c)^4*(1/2*(b*e/d+(a*d-b*c)*e/d/
(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^2)-5*d^3/(a*d-b*c)^4*b*e*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c
)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)+5*d^2/(a*d-b*c)^4*b^2*e^2*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))^2-10*d/(a*d-b*c)^4*b^3*e^3*(-1/(b*e/d+(a*d-b*c)*
e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)
))+5/(a*d-b*c)^4*b^4*e^4*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*
d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-1/d/(a*d-b*c)^4*b^
5*e^5*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))
-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1509 vs. 2(551) = 1102.

Time = 0.35 (sec) , antiderivative size = 1509, normalized size of antiderivative = 2.68

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x, algor
ithm="fricas")
```

```
[Out] -1/36*(4*(3*A + B)*b^5*c^5 - 45*(2*A + B)*a*b^4*c^4*d + 360*(A + B)*a^2*b^3
*c^3*d^2 - 10*(12*A + 49*B)*a^3*b^2*c^2*d^3 - 180*(A - B)*a^4*b*c*d^4 + 9*(
2*A - B)*a^5*d^5 + 120*((3*A + B)*b^5*c*d^4 - (3*A + B)*a*b^4*d^5)*x^4 + 60
*(3*(3*A + 2*B)*b^5*c^2*d^3 + 2*(3*A - 2*B)*a*b^4*c*d^4 - (15*A + 2*B)*a^2*
b^3*d^5)*x^3 + 20*((6*A + 11*B)*b^5*c^3*d^2 + 21*(3*A + B)*a*b^4*c^2*d^3 -
3*(12*A + 13*B)*a^2*b^3*c*d^4 - (33*A - 7*B)*a^3*b^2*d^5)*x^2 + 180*(B*b^5*
d^5*x^5 + B*a^3*b^2*c^2*d^3 + (2*B*b^5*c*d^4 + 3*B*a*b^4*d^5)*x^4 + (B*b^5*
c^2*d^3 + 6*B*a*b^4*c*d^4 + 3*B*a^2*b^3*d^5)*x^3 + (3*B*a*b^4*c^2*d^3 + 6*B
*a^2*b^3*c*d^4 + B*a^3*b^2*d^5)*x^2 + (3*B*a^2*b^3*c^2*d^3 + 2*B*a^3*b^2*c*
d^4)*x)*log((b*e*x + a*e)/(d*x + c))^2 - 5*((6*A + 5*B)*b^5*c^4*d - 36*(2*A
 + 3*B)*a*b^4*c^3*d^2 - 6*(24*A - 13*B)*a^2*b^3*c^2*d^3 + 4*(48*A + 13*B)*a
^3*b^2*c*d^4 + 9*(2*A - 3*B)*a^4*b*d^5)*x + 6*(20*(3*A + B)*b^5*d^5*x^5 + 2
*B*b^5*c^5 - 15*B*a*b^4*c^4*d + 60*B*a^2*b^3*c^3*d^2 + 60*A*a^3*b^2*c^2*d^3
 - 30*B*a^4*b*c*d^4 + 3*B*a^5*d^5 + 20*((6*A + 5*B)*b^5*c*d^4 + 9*A*a*b^4*d
^5)*x^4 + 10*((6*A + 11*B)*b^5*c^2*d^3 + 18*(2*A + B)*a*b^4*c*d^4 + 9*(2*A
- B)*a^2*b^3*d^5)*x^3 + 10*(2*B*b^5*c^3*d^2 + 9*(2*A + 3*B)*a*b^4*c^2*d^3 +
 36*A*a^2*b^3*c*d^4 + 3*(2*A - 3*B)*a^3*b^2*d^5)*x^2 - 5*(B*b^5*c^4*d - 12*
B*a*b^4*c^3*d^2 - 36*(A + B)*a^2*b^3*c^2*d^3 - 24*(A - B)*a^3*b^2*c*d^4 + 3
*B*a^4*b*d^5)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^9*c^6*d^2 - 6*a*b^8*c^5*
d^3 + 15*a^2*b^7*c^4*d^4 - 20*a^3*b^6*c^3*d^5 + 15*a^4*b^5*c^2*d^6 - 6*a^5*
b^4*c*d^7 + a^6*b^3*d^8)*g^4*i^3*x^5 + (2*b^9*c^7*d - 9*a*b^8*c^6*d^2 + 12*
a^2*b^7*c^5*d^3 + 5*a^3*b^6*c^4*d^4 - 30*a^4*b^5*c^3*d^5 + 33*a^5*b^4*c^2*d
^6 - 16*a^6*b^3*c*d^7 + 3*a^7*b^2*d^8)*g^4*i^3*x^4 + (b^9*c^8 - 18*a^2*b^7*
c^6*d^2 + 52*a^3*b^6*c^5*d^3 - 60*a^4*b^5*c^4*d^4 + 24*a^5*b^4*c^3*d^5 + 10
*a^6*b^3*c^2*d^6 - 12*a^7*b^2*c*d^7 + 3*a^8*b*d^8)*g^4*i^3*x^3 + (3*a*b^8*c
^8 - 12*a^2*b^7*c^7*d + 10*a^3*b^6*c^6*d^2 + 24*a^4*b^5*c^5*d^3 - 60*a^5*b^
4*c^4*d^4 + 52*a^6*b^3*c^3*d^5 - 18*a^7*b^2*c^2*d^6 + a^9*d^8)*g^4*i^3*x^2
+ (3*a^2*b^7*c^8 - 16*a^3*b^6*c^7*d + 33*a^4*b^5*c^6*d^2 - 30*a^5*b^4*c^5*d
^3 + 5*a^6*b^3*c^4*d^4 + 12*a^7*b^2*c^3*d^5 - 9*a^8*b*c^2*d^6 + 2*a^9*c*d^7
)*g^4*i^3*x + (a^3*b^6*c^8 - 6*a^4*b^5*c^7*d + 15*a^5*b^4*c^6*d^2 - 20*a^6*
b^3*c^5*d^3 + 15*a^7*b^2*c^4*d^4 - 6*a^8*b*c^3*d^5 + a^9*c^2*d^6)*g^4*i^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4/(d*i*x+c*i)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3816 vs. $2(551) = 1102$.

Time = 0.57 (sec) , antiderivative size = 3816, normalized size of antiderivative = 6.78

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x, algor
ithm="maxima")
```

```
[Out] -1/6*B*((60*b^4*d^4*x^4 + 2*b^4*c^4 - 13*a*b^3*c^3*d + 47*a^2*b^2*c^2*d^2 +
 27*a^3*b*c*d^3 - 3*a^4*d^4 + 30*(3*b^4*c*d^3 + 5*a*b^3*d^4)*x^3 + 10*(2*b^
4*c^2*d^2 + 23*a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2 - 5*(b^4*c^3*d - 11*a*b^3*
c^2*d^2 - 35*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x)/((b^8*c^5*d^2 - 5*a*b^7*c^4*d^
3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*b^3*d^7
)*g^4*i^3*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 + 10*a^3
*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7)*g^4*i
^3*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c^4*d^3 -
 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d^7)*g^4*
i^3*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a^4*b^4*c^4
*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a^8*d^7)*g^4
*i^3*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5*d^2 - 10*a^5*
b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*d^6)*g^4*i^3*x
+ (a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*a^6*b^2*c^4*d^3
+ 5*a^7*b*c^3*d^4 - a^8*c^2*d^5)*g^4*i^3) + 60*b^2*d^3*log(b*x + a)/((b^6*c
^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c
^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3) - 60*b^2*d^3*log(d*x + c)/((b^6*
c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*
c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3))*log(b*e*x/(d*x + c) + a*e/(d*x
 + c)) - 1/6*A*((60*b^4*d^4*x^4 + 2*b^4*c^4 - 13*a*b^3*c^3*d + 47*a^2*b^2*c
^2*d^2 + 27*a^3*b*c*d^3 - 3*a^4*d^4 + 30*(3*b^4*c*d^3 + 5*a*b^3*d^4)*x^3 +
10*(2*b^4*c^2*d^2 + 23*a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2 - 5*(b^4*c^3*d - 1
1*a*b^3*c^2*d^2 - 35*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x)/((b^8*c^5*d^2 - 5*a*b^
```

7*c^4*d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5
*b^3*d^7)*g^4*i^3*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3
+ 10*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^
7)*g^4*i^3*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c
^4*d^3 - 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d
^7)*g^4*i^3*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a^4
*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a^8*
d^7)*g^4*i^3*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5*d^2 -
 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*d^6)*g^
4*i^3*x + (a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*a^6*b^2*
c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5)*g^4*i^3) + 60*b^2*d^3*log(b*x + a)
/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a
^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3) - 60*b^2*d^3*log(d*x + c
)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*
a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3)) - 1/36*(4*b^5*c^5 - 45
*a*b^4*c^4*d + 360*a^2*b^3*c^3*d^2 - 490*a^3*b^2*c^2*d^3 + 180*a^4*b*c*d^4
- 9*a^5*d^5 + 120*(b^5*c*d^4 - a*b^4*d^5)*x^4 + 120*(3*b^5*c^2*d^3 - 2*a*b^
4*c*d^4 - a^2*b^3*d^5)*x^3 + 20*(11*b^5*c^3*d^2 + 21*a*b^4*c^2*d^3 - 39*a^2
*b^3*c*d^4 + 7*a^3*b^2*d^5)*x^2 - 180*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b
^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)
*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c
^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a)^2 - 180*(b^5*d^5*x^5 + a^3*b^2*c^
2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*
a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 +
(3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(d*x + c)^2 - 5*(5*b^5*c^4*d -
108*a*b^4*c^3*d^2 + 78*a^2*b^3*c^2*d^3 + 52*a^3*b^2*c*d^4 - 27*a^4*b*d^5)*x
 + 120*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (
b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2
*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*lo
g(b*x + a) - 120*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^
5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d
^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*
d^4)*x - 3*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4
 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6
*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x
)*log(b*x + a))*log(d*x + c))*B/(a^3*b^6*c^8*g^4*i^3 - 6*a^4*b^5*c^7*d*g^4*
i^3 + 15*a^5*b^4*c^6*d^2*g^4*i^3 - 20*a^6*b^3*c^5*d^3*g^4*i^3 + 15*a^7*b^2*
c^4*d^4*g^4*i^3 - 6*a^8*b*c^3*d^5*g^4*i^3 + a^9*c^2*d^6*g^4*i^3 + (b^9*c^6*
d^2*g^4*i^3 - 6*a*b^8*c^5*d^3*g^4*i^3 + 15*a^2*b^7*c^4*d^4*g^4*i^3 - 20*a^3
*b^6*c^3*d^5*g^4*i^3 + 15*a^4*b^5*c^2*d^6*g^4*i^3 - 6*a^5*b^4*c*d^7*g^4*i^3
 + a^6*b^3*d^8*g^4*i^3)*x^5 + (2*b^9*c^7*d*g^4*i^3 - 9*a*b^8*c^6*d^2*g^4*i^
3 + 12*a^2*b^7*c^5*d^3*g^4*i^3 + 5*a^3*b^6*c^4*d^4*g^4*i^3 - 30*a^4*b^5*c^3
*d^5*g^4*i^3 + 33*a^5*b^4*c^2*d^6*g^4*i^3 - 16*a^6*b^3*c*d^7*g^4*i^3 + 3*a^
7*b^2*d^8*g^4*i^3)*x^4 + (b^9*c^8*g^4*i^3 - 18*a^2*b^7*c^6*d^2*g^4*i^3 + 52
*a^3*b^6*c^5*d^3*g^4*i^3 - 60*a^4*b^5*c^4*d^4*g^4*i^3 + 24*a^5*b^4*c^3*d^5*

```
g^4*i^3 + 10*a^6*b^3*c^2*d^6*g^4*i^3 - 12*a^7*b^2*c*d^7*g^4*i^3 + 3*a^8*b*d
^8*g^4*i^3)*x^3 + (3*a*b^8*c^8*g^4*i^3 - 12*a^2*b^7*c^7*d*g^4*i^3 + 10*a^3*
b^6*c^6*d^2*g^4*i^3 + 24*a^4*b^5*c^5*d^3*g^4*i^3 - 60*a^5*b^4*c^4*d^4*g^4*i
^3 + 52*a^6*b^3*c^3*d^5*g^4*i^3 - 18*a^7*b^2*c^2*d^6*g^4*i^3 + a^9*d^8*g^4*
i^3)*x^2 + (3*a^2*b^7*c^8*g^4*i^3 - 16*a^3*b^6*c^7*d*g^4*i^3 + 33*a^4*b^5*c
^6*d^2*g^4*i^3 - 30*a^5*b^4*c^5*d^3*g^4*i^3 + 5*a^6*b^3*c^4*d^4*g^4*i^3 + 1
2*a^7*b^2*c^3*d^5*g^4*i^3 - 9*a^8*b*c^2*d^6*g^4*i^3 + 2*a^9*c*d^7*g^4*i^3)*
x)
```

Giac [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{(bgx+ag)^4(dix+ci)^3}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x, algor
ithm="giac")
```

```
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)/((b*g*x + a*g)^4*(d*i*x + c*i)
^3), x)
```

Mupad [B] (verification not implemented)

Time = 13.33 (sec) , antiderivative size = 2291, normalized size of antiderivative = 4.07

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx=\text{Too large to display}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^4*(c*i + d*i*x)^3),
x)
```

```
[Out] ((12*A*b^4*c^4 - 18*A*a^4*d^4 + 9*B*a^4*d^4 + 4*B*b^4*c^4 + 282*A*a^2*b^2*c
^2*d^2 + 319*B*a^2*b^2*c^2*d^2 - 78*A*a*b^3*c^3*d + 162*A*a^3*b*c*d^3 - 41*
B*a*b^3*c^3*d - 171*B*a^3*b*c*d^3)/(6*(a*d - b*c)) + (10*x^2*(33*A*a^2*b^2*
d^4 - 7*B*a^2*b^2*d^4 + 6*A*b^4*c^2*d^2 + 11*B*b^4*c^2*d^2 + 69*A*a*b^3*c*d
^3 + 32*B*a*b^3*c*d^3))/(3*(a*d - b*c)) + (5*x*(18*A*a^3*b*d^4 - 27*B*a^3*b
*d^4 - 6*A*b^4*c^3*d - 5*B*b^4*c^3*d + 66*A*a*b^3*c^2*d^2 + 210*A*a^2*b^2*c
*d^3 + 103*B*a*b^3*c^2*d^2 + 25*B*a^2*b^2*c*d^3))/(6*(a*d - b*c)) + (10*x^3
*(15*A*a*b^3*d^4 + 2*B*a*b^3*d^4 + 9*A*b^4*c*d^3 + 6*B*b^4*c*d^3))/(a*d - b
*c) + (20*x^4*(3*A*b^4*d^4 + B*b^4*d^4))/(a*d - b*c))/(x^5*(6*a^4*b^3*d^6*g
^4*i^3 + 6*b^7*c^4*d^2*g^4*i^3 - 24*a*b^6*c^3*d^3*g^4*i^3 - 24*a^3*b^4*c*d^
5*g^4*i^3 + 36*a^2*b^5*c^2*d^4*g^4*i^3) + x*(18*a^2*b^5*c^6*g^4*i^3 + 12*a^
7*c*d^5*g^4*i^3 - 60*a^3*b^4*c^5*d*g^4*i^3 - 30*a^6*b*c^2*d^4*g^4*i^3 + 60*
```

a^4*b^3*c^4*d^2*g^4*i^3) + x^2*(6*a^7*d^6*g^4*i^3 + 18*a*b^6*c^6*g^4*i^3 +
12*a^6*b*c*d^5*g^4*i^3 - 36*a^2*b^5*c^5*d*g^4*i^3 - 30*a^3*b^4*c^4*d^2*g^4*
i^3 + 120*a^4*b^3*c^3*d^3*g^4*i^3 - 90*a^5*b^2*c^2*d^4*g^4*i^3) + x^3*(6*b^
7*c^6*g^4*i^3 + 18*a^6*b*d^6*g^4*i^3 + 12*a*b^6*c^5*d*g^4*i^3 - 36*a^5*b^2*
c*d^5*g^4*i^3 - 90*a^2*b^5*c^4*d^2*g^4*i^3 + 120*a^3*b^4*c^3*d^3*g^4*i^3 -
30*a^4*b^3*c^2*d^4*g^4*i^3) + x^4*(18*a^5*b^2*d^6*g^4*i^3 + 12*b^7*c^5*d*g^
4*i^3 - 30*a*b^6*c^4*d^2*g^4*i^3 - 60*a^4*b^3*c*d^5*g^4*i^3 + 60*a^3*b^4*c^
2*d^4*g^4*i^3) + 6*a^3*b^4*c^6*g^4*i^3 + 6*a^7*c^2*d^4*g^4*i^3 - 24*a^4*b^3
*c^5*d*g^4*i^3 - 24*a^6*b*c^3*d^3*g^4*i^3 + 36*a^5*b^2*c^4*d^2*g^4*i^3) + (
log((e*(a + b*x))/(c + d*x))*(x^2*((5*B*b*d*(a*d + b*c))/(g^4*i^3*(a^2*d^2
+ b^2*c^2 - 2*a*b*c*d)^2) + (5*B*b*d*(2*a*d + b*c))/(3*g^4*i^3*(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d)^2) + (10*B*b^2*d^3*((2*a*c*(a*d - b*c))/d + ((a*d + b*c
)^2*(a*d - b*c))/(b*d^2)))/(g^4*i^3*(a*d - b*c)^4*(a^2*d^2 + b^2*c^2 - 2*a*
b*c*d))) + x^3*((5*B*b^2*d^2)/(g^4*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) +
(20*B*b^2*d^2*(a*d + b*c))/(g^4*i^3*(a*d - b*c)^3*(a^2*d^2 + b^2*c^2 - 2*a
*b*c*d))) + x*((5*B*(a*d + b*c)*(2*a*d + b*c))/(3*g^4*i^3*(a^2*d^2 + b^2*c^
2 - 2*a*b*c*d)^2) - (5*B)/(6*g^4*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (5*
B*a*b*c*d)/(g^4*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) + (20*B*a*b*c*d*(a*d
+ b*c))/(g^4*i^3*(a*d - b*c)^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - (B*(3*a
*d + 2*b*c))/(6*g^4*i^3*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (5*B*a*c
*(2*a*d + b*c))/(3*g^4*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) + (10*B*b^3*d
^3*x^4)/(g^4*i^3*(a*d - b*c)^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (10*B*a^2
*b*c^2*d)/(g^4*i^3*(a*d - b*c)^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))))/(b^2*d*
x^5 + (x^4*(3*a*b^2*d^2 + 2*b^3*c*d))/(b*d) + (a^3*c^2)/(b*d) + (x^2*(a^3*d
^2 + 3*a*b^2*c^2 + 6*a^2*b*c*d))/(b*d) + (x^3*(b^3*c^2 + 3*a^2*b*d^2 + 6*a*
b^2*c*d))/(b*d) + (x*(3*a^2*b*c^2 + 2*a^3*c*d))/(b*d)) + (b^2*d^3*atan((b^2
d^3(3*A + B)*((a^6*d^6*g^4*i^3 - b^6*c^6*g^4*i^3 + 4*a*b^5*c^5*d*g^4*i^3
- 4*a^5*b*c*d^5*g^4*i^3 - 5*a^2*b^4*c^4*d^2*g^4*i^3 + 5*a^4*b^2*c^2*d^4*g^4
*i^3)/(a^5*d^5*g^4*i^3 - b^5*c^5*g^4*i^3 + 5*a*b^4*c^4*d*g^4*i^3 - 5*a^4*b*
c*d^4*g^4*i^3 - 10*a^2*b^3*c^3*d^2*g^4*i^3 + 10*a^3*b^2*c^2*d^3*g^4*i^3) +
2*b*d*x)*(a^5*d^5*g^4*i^3 - b^5*c^5*g^4*i^3 + 5*a*b^4*c^4*d*g^4*i^3 - 5*a^4
*b*c*d^4*g^4*i^3 - 10*a^2*b^3*c^3*d^2*g^4*i^3 + 10*a^3*b^2*c^2*d^3*g^4*i^3)
*10i)/(g^4*i^3*(a*d - b*c)^6*(30*A*b^2*d^3 + 10*B*b^2*d^3)))*(3*A + B)*20i)
/(3*g^4*i^3*(a*d - b*c)^6) - (5*B*b^2*d^3*log((e*(a + b*x))/(c + d*x))^2)/(
g^4*i^3*(a*d - b*c)^4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))

3.55 $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

Optimal result

Integrand size = 40, antiderivative size = 539

$$\begin{aligned}
&\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx\\
&=\frac{3B^2(bc-ad)^4g^3ix}{10bd^3}-\frac{3B^2(bc-ad)^3g^3i(c+dx)^2}{20d^4}+\frac{bB^2(bc-ad)^2g^3i(c+dx)^3}{30d^4}\\
&\quad-\frac{B(bc-ad)^2g^3i(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^2d}\\
&\quad-\frac{B(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^2}\\
&\quad+\frac{(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^2}\\
&\quad+\frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b}\\
&\quad+\frac{B(bc-ad)^3g^3i(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^2d^2}\\
&\quad-\frac{B(bc-ad)^4g^3i(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^2d^3}\\
&\quad-\frac{B(bc-ad)^5g^3i\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^2d^4}\\
&\quad-\frac{B^2(bc-ad)^5g^3i\log(c+dx)}{10b^2d^4}-\frac{B^2(bc-ad)^5g^3i\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{10b^2d^4}
\end{aligned}$$

```
[Out] 3/10*B^2*(-a*d+b*c)^4*g^3*i*x/b/d^3-3/20*B^2*(-a*d+b*c)^3*g^3*i*(d*x+c)^2/d
^4+1/30*b*B^2*(-a*d+b*c)^2*g^3*i*(d*x+c)^3/d^4-1/30*B*(-a*d+b*c)^2*g^3*i*(b
*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/d-1/10*B*(-a*d+b*c)*g^3*i*(b*x+a)^4
*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2+1/20*(-a*d+b*c)*g^3*i*(b*x+a)^4*(A+B*ln(e*
(b*x+a)/(d*x+c)))^2/b^2+1/5*g^3*i*(b*x+a)^4*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+
c)))^2/b+1/60*B*(-a*d+b*c)^3*g^3*i*(b*x+a)^2*(3*A+B+3*B*ln(e*(b*x+a)/(d*x+c
)))/b^2/d^2-1/60*B*(-a*d+b*c)^4*g^3*i*(b*x+a)*(6*A+5*B+6*B*ln(e*(b*x+a)/(d*
x+c)))/b^2/d^3-1/60*B*(-a*d+b*c)^5*g^3*i*ln((-a*d+b*c)/b/(d*x+c))*(6*A+11*B
+6*B*ln(e*(b*x+a)/(d*x+c)))/b^2/d^4-1/10*B^2*(-a*d+b*c)^5*g^3*i*ln(d*x+c)/b
^2/d^4-1/10*B^2*(-a*d+b*c)^5*g^3*i*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/d^4
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2562, 2383, 2381, 2384, 2354, 2438, 2373, 45}

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$$
$$=-\frac{Bg^3i(bc-ad)^5\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A+11B\right)}{60b^2d^4}$$
$$-\frac{Bg^3i(a+bx)(bc-ad)^4\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A+5B\right)}{60b^2d^3}$$
$$+\frac{Bg^3i(a+bx)^2(bc-ad)^3\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{60b^2d^2}$$
$$-\frac{Bg^3i(a+bx)^3(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{30b^2d}$$
$$+\frac{g^3i(a+bx)^4(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{20b^2}$$
$$-\frac{Bg^3i(a+bx)^4(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{10b^2}$$
$$+\frac{g^3i(a+bx)^4(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b}-\frac{B^2g^3i(bc-ad)^5\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{10b^2d^4}$$
$$-\frac{B^2g^3i(bc-ad)^5\log(c+dx)}{10b^2d^4}-\frac{3B^2g^3i(c+dx)^2(bc-ad)^3}{20d^4}$$
$$+\frac{bB^2g^3i(c+dx)^3(bc-ad)^2}{30d^4}+\frac{3B^2g^3ix(bc-ad)^4}{10bd^3}$$

```
[In] Int[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

[Out] (3*B^2*(b*c - a*d)^4*g^3*i*x)/(10*b*d^3) - (3*B^2*(b*c - a*d)^3*g^3*i*(c + d*x)^2)/(20*d^4) + (b*B^2*(b*c - a*d)^2*g^3*i*(c + d*x)^3)/(30*d^4) - (B*(b*c - a*d)^2*g^3*i*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(30*b^2*d) - (B*(b*c - a*d)*g^3*i*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(10*b^2) + ((b*c - a*d)*g^3*i*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(20*b^2) + (g^3*i*(a + b*x)^4*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(5*b) + (B*(b*c - a*d)^3*g^3*i*(a + b*x)^2*(3*A + B + 3*B*Log[(e*(a + b*x))/(c + d*x)]))/(60*b^2*d^2) - (B*(b*c - a*d)^4*g^3*i*(a + b*x)*(6*A + 5*B + 6*B*Log[(e*(a + b*x))/(c + d*x)]))/(60*b^2*d^3) - (B*(b*c - a*d)^5*g^3*i*Log[(b*c - a*d)/(b*(c + d*x))]*(6*A + 11*B + 6*B*Log[(e*(a + b*x))/(c + d*x)]))/(60*b^2*d^4) - (B^2*(b*c - a*d)^5*g^3*i*Log[c + d*x])/(10*b^2*d^4) - (B^2*(b*c - a*d)^5*g^3*i*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(10*b^2*d^4)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e), Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2383

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
```

```
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Dist[(m + q + 2)/(d*(q + 1)), Int[(
f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Dist[b*n*(p/(d*(q +
 1))), Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x]) /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ
[q, -1] && GtQ[m, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \left((bc-ad)^5 g^3 i\right) \text{Subst}\left(\int \frac{x^3 (A+B\log(ex))^2}{(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{g^3 i (a+bx)^4 (c+dx) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} + \frac{\left((bc-ad)^5 g^3 i\right) \text{Subst}\left(\int \frac{x^3 (A+B\log(ex))^2}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b} - \frac{\left(2B(bc-ad)^5 g^3 i\right) \text{Subst}\left(\int \frac{x^3 (A+B\log(ex))}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b}$$

$$= -\frac{B(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^2} + \frac{(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^2} + \frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} - \frac{(B(bc-ad)^5g^3i)\,\text{Subst}\left(\int\frac{x^3(A+B\log(ex))}{(b-dx)^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{10b^2} + \frac{(B^2(bc-ad)^5g^3i)\,\text{Subst}\left(\int\frac{x^3}{(b-dx)^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{10b^2}$$

$$= -\frac{B(bc-ad)^2g^3i(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^2d} - \frac{B(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^2} + \frac{(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^2} + \frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} + \frac{(B^2(bc-ad)^5g^3i)\,\text{Subst}\left(\int\left(\frac{b^3}{d^3(b-dx)^4}-\frac{3b^2}{d^3(b-dx)^3}+\frac{3b}{d^3(b-dx)^2}-\frac{1}{d^3(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{10b^2} + \frac{(B(bc-ad)^5g^3i)\,\text{Subst}\left(\int\frac{x^2(3A+B+3B\log(ex))}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{30b^2d}$$

$$= \frac{3B^2(bc-ad)^4g^3ix}{10bd^3} - \frac{3B^2(bc-ad)^3g^3i(c+dx)^2}{20d^4} + \frac{bB^2(bc-ad)^2g^3i(c+dx)^3}{30d^4} - \frac{B(bc-ad)^2g^3i(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^2d} - \frac{B(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^2} + \frac{(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^2} + \frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} + \frac{B(bc-ad)^3g^3i(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^2d^2} - \frac{B^2(bc-ad)^5g^3i\log(c+dx)}{10b^2d^4} - \frac{(B(bc-ad)^5g^3i)\operatorname{Subst}\left(\int \frac{x(3B+2(3A+B)+6B\log(ex))}{(b-dx)^2}dx, x, \frac{a+bx}{c+dx}\right)}{60b^2d^2}$$

$$= \frac{3B^2(bc-ad)^4g^3ix}{10bd^3} - \frac{3B^2(bc-ad)^3g^3i(c+dx)^2}{20d^4} + \frac{bB^2(bc-ad)^2g^3i(c+dx)^3}{30d^4} - \frac{B(bc-ad)^2g^3i(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^2d} - \frac{B(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^2} + \frac{(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^2} + \frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} + \frac{B(bc-ad)^3g^3i(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^2d^2} - \frac{B(bc-ad)^4g^3i(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^2d^3} - \frac{B^2(bc-ad)^5g^3i\log(c+dx)}{10b^2d^4} + \frac{(B(bc-ad)^5g^3i)\operatorname{Subst}\left(\int \frac{9B+2(3A+B)+6B\log(ex)}{b-dx}dx, x, \frac{a+bx}{c+dx}\right)}{60b^2d^3}$$

$$
\begin{aligned}
&= \frac{3B^2(bc-ad)^4g^3ix}{10bd^3} - \frac{3B^2(bc-ad)^3g^3i(c+dx)^2}{20d^4} + \frac{bB^2(bc-ad)^2g^3i(c+dx)^3}{30d^4} \\
&\quad - \frac{B(bc-ad)^2g^3i(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^2d} \\
&\quad - \frac{B(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^2} \\
&\quad + \frac{(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^2} \\
&\quad + \frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} \\
&\quad + \frac{B(bc-ad)^3g^3i(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^2d^2} \\
&\quad - \frac{B(bc-ad)^4g^3i(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^2d^3} \\
&\quad - \frac{B(bc-ad)^5g^3i\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^2d^4} \\
&\quad - \frac{B^2(bc-ad)^5g^3i\log(c+dx)}{10b^2d^4} + \frac{(B^2(bc-ad)^5g^3i)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{10b^2d^4}
\end{aligned}
$$

$$= \frac{3B^2(bc-ad)^4g^3ix}{10bd^3} - \frac{3B^2(bc-ad)^3g^3i(c+dx)^2}{20d^4} + \frac{bB^2(bc-ad)^2g^3i(c+dx)^3}{30d^4}$$
$$- \frac{B(bc-ad)^2g^3i(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^2d}$$
$$- \frac{B(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^2}$$
$$+ \frac{(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^2}$$
$$+ \frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b}$$
$$+ \frac{B(bc-ad)^3g^3i(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^2d^2}$$
$$- \frac{B(bc-ad)^4g^3i(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^2d^3}$$
$$- \frac{B(bc-ad)^5g^3i\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^2d^4}$$
$$- \frac{B^2(bc-ad)^5g^3i\log(c+dx)}{10b^2d^4} - \frac{B^2(bc-ad)^5g^3i\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{10b^2d^4}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 905, normalized size of antiderivative = 1.68

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{g^3i\left(5(bc-ad)(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + 4d(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - \frac{5B(bc-ad)^2\left(6Abd(\right.}{}\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)^2,x]
```

```
[Out] (g^3*i*(5*(b*c - a*d)*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 +
4*d*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (5*B*(b*c - a*d)^2
*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))
/(c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c
+ d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(b*
c - a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*x
)])*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2
*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (-(b*c) + a*d)*Lo
```

```
g[c + d*x]) + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log
[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*d^4
) + (B*(b*c - a*d)*(24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d)^3*(a + b*
x)*Log[(e*(a + b*x))/(c + d*x)] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*L
og[(e*(a + b*x))/(c + d*x)]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*
(a + b*x))/(c + d*x)]) - 6*d^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*
x)]) - 24*B*(b*c - a*d)^4*Log[c + d*x] - 24*(b*c - a*d)^4*(A + B*Log[(e*(a
+ b*x))/(c + d*x)])*Log[c + d*x] + 4*B*(b*c - a*d)^2*(2*b*d*(b*c - a*d)*x -
 d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*(6*b*d*(b*
c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*
c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^3*(b*d*x + (-(b*c) + a*d)*Log[c
 + d*x]) + 12*B*(b*c - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c
 + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*d^4))
)/(20*b^2)
```

Maple [F]

$$\int (bgx+ag)^3 (dix+ci) \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
[Out] int((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)^3(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algor
ithm="fricas")
[Out] integral(A^2*b^3*d*g^3*i*x^4 + A^2*a^3*c*g^3*i + (A^2*b^3*c + 3*A^2*a*b^2*d
)*g^3*i*x^3 + 3*(A^2*a*b^2*c + A^2*a^2*b*d)*g^3*i*x^2 + (3*A^2*a^2*b*c + A^
2*a^3*d)*g^3*i*x + (B^2*b^3*d*g^3*i*x^4 + B^2*a^3*c*g^3*i + (B^2*b^3*c + 3*
B^2*a*b^2*d)*g^3*i*x^3 + 3*(B^2*a*b^2*c + B^2*a^2*b*d)*g^3*i*x^2 + (3*B^2*a
^2*b*c + B^2*a^3*d)*g^3*i*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^3*d*
g^3*i*x^4 + A*B*a^3*c*g^3*i + (A*B*b^3*c + 3*A*B*a*b^2*d)*g^3*i*x^3 + 3*(A*
B*a*b^2*c + A*B*a^2*b*d)*g^3*i*x^2 + (3*A*B*a^2*b*c + A*B*a^3*d)*g^3*i*x)*l
og((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**3*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3186 vs. $2(514) = 1028$.

Time = 0.33 (sec) , antiderivative size = 3186, normalized size of antiderivative = 5.91

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algor
ithm="maxima")
[Out] 1/5*A^2*b^3*d*g^3*i*x^5 + 1/4*A^2*b^3*c*g^3*i*x^4 + 3/4*A^2*a*b^2*d*g^3*i*x
^4 + A^2*a*b^2*c*g^3*i*x^3 + A^2*a^2*b*d*g^3*i*x^3 + 3/2*A^2*a^2*b*c*g^3*i*
x^2 + 1/2*A^2*a^3*d*g^3*i*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) +
 a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a^3*c*g^3*i + 3*(x^2*log(b*e*x/(d
*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b
*c - a*d)*x/(b*d))*A*B*a^2*b*c*g^3*i + (2*x^3*log(b*e*x/(d*x + c) + a*e/(d*
x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b
*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a*b^2*c*g^3*i + 1/12*(6
*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*
log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^
3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b^3*c*g^3*i + (x^2*log(b*e
*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2
 - (b*c - a*d)*x/(b*d))*A*B*a^3*d*g^3*i + (2*x^3*log(b*e*x/(d*x + c) + a*e/
(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d -
a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b*d*g^3*i + 1/4*
(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^
4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*
d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*a*b^2*d*g^3*i + 1/30*(12
*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^
5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*
b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b
^4*d^4))*A*B*b^3*d*g^3*i + A^2*a^3*c*g^3*i*x - 1/60*(6*a^4*c*d^4*g^3*i - (6
*g^3*i*log(e) + 5*g^3*i)*b^4*c^5 + (30*g^3*i*log(e) + 19*g^3*i)*a*b^3*c^4*d
```

- (60*g^3*i*log(e) + 23*g^3*i)*a^2*b^2*c^3*d^2 + 3*(20*g^3*i*log(e) + g^3*i)*a^3*b*c^2*d^3)*B^2*log(d*x + c)/(b*d^4) + 1/10*(b^5*c^5*g^3*i - 5*a*b^4*c^4*d*g^3*i + 10*a^2*b^3*c^3*d^2*g^3*i - 10*a^3*b^2*c^2*d^3*g^3*i + 5*a^4*b*c*d^4*g^3*i - a^5*d^5*g^3*i)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^4) + 1/60*(12*B^2*b^5*d^5*g^3*i*x^5*log(e)^2 + 3*((5*g^3*i*log(e)^2 - 2*g^3*i*log(e))*b^5*c*d^4 + (15*g^3*i*log(e)^2 + 2*g^3*i*log(e))*a*b^4*d^5)*B^2*x^4 - 2*((g^3*i*log(e) - g^3*i)*b^5*c^2*d^3 - 2*(15*g^3*i*log(e)^2 - 5*g^3*i*log(e) - g^3*i)*a*b^4*c*d^4 - (30*g^3*i*log(e)^2 + 11*g^3*i*log(e) + g^3*i)*a^2*b^3*d^5)*B^2*x^3 + ((3*g^3*i*log(e) - 2*g^3*i)*b^5*c^3*d^2 - 3*(5*g^3*i*log(e) - 4*g^3*i)*a*b^4*c^2*d^3 + 3*(30*g^3*i*log(e)^2 - 5*g^3*i*log(e) - 6*g^3*i)*a^2*b^3*c*d^4 + (30*g^3*i*log(e)^2 + 27*g^3*i*log(e) + 8*g^3*i)*a^3*b^2*d^5)*B^2*x^2 - ((6*g^3*i*log(e) - g^3*i)*b^5*c^4*d - 2*(15*g^3*i*log(e) - 4*g^3*i)*a*b^4*c^3*d^2 + 12*(5*g^3*i*log(e) - 2*g^3*i)*a^2*b^3*c^2*d^3 - 2*(30*g^3*i*log(e)^2 + 15*g^3*i*log(e) - 14*g^3*i)*a^3*b^2*c*d^4 - (6*g^3*i*log(e) + 11*g^3*i)*a^4*b*d^5)*B^2*x + 3*(4*B^2*b^5*d^5*g^3*i*x^5 + 20*B^2*a^3*b^2*c*d^4*g^3*i*x + 5*(b^5*c*d^4*g^3*i + 3*a*b^4*d^5*g^3*i)*B^2*x^4 + 20*(a*b^4*c*d^4*g^3*i + a^2*b^3*d^5*g^3*i)*B^2*x^3 + 10*(3*a^2*b^3*c*d^4*g^3*i + a^3*b^2*d^5*g^3*i)*B^2*x^2 + (5*a^4*b*c*d^4*g^3*i - a^5*d^5*g^3*i)*B^2)*log(b*x + a)^2 + 3*(4*B^2*b^5*d^5*g^3*i*x^5 + 20*B^2*a^3*b^2*c*d^4*g^3*i*x + 5*(b^5*c*d^4*g^3*i + 3*a*b^4*d^5*g^3*i)*B^2*x^4 + 20*(a*b^4*c*d^4*g^3*i + a^2*b^3*d^5*g^3*i)*B^2*x^3 + 10*(3*a^2*b^3*c*d^4*g^3*i + a^3*b^2*d^5*g^3*i)*B^2*x^2 - (b^5*c^5*g^3*i - 5*a*b^4*c^4*d*g^3*i + 10*a^2*b^3*c^3*d^2*g^3*i - 10*a^3*b^2*c^2*d^3*g^3*i)*B^2)*log(d*x + c)^2 + (24*B^2*b^5*d^5*g^3*i*x^5*log(e) + 6*((5*g^3*i*log(e) - g^3*i)*b^5*c*d^4 + (15*g^3*i*log(e) + g^3*i)*a*b^4*d^5)*B^2*x^4 - 2*(b^5*c^2*d^3*g^3*i - 10*(6*g^3*i*log(e) - g^3*i)*a*b^4*c*d^4 - (60*g^3*i*log(e) + 11*g^3*i)*a^2*b^3*d^5)*B^2*x^3 + 3*(b^5*c^3*d^2*g^3*i - 5*a*b^4*c^2*d^3*g^3*i + 5*(12*g^3*i*log(e) - g^3*i)*a^2*b^3*c*d^4 + (20*g^3*i*log(e) + 9*g^3*i)*a^3*b^2*d^5)*B^2*x^2 - 6*(b^5*c^4*d*g^3*i - 5*a*b^4*c^3*d^2*g^3*i + 10*a^2*b^3*c^2*d^3*g^3*i - a^4*b*d^5*g^3*i - 5*(4*g^3*i*log(e) + g^3*i)*a^3*b^2*c*d^4)*B^2*x - (6*a*b^4*c^4*d*g^3*i - 27*a^2*b^3*c^3*d^2*g^3*i + 47*a^3*b^2*c^2*d^3*g^3*i - (30*g^3*i*log(e) + 31*g^3*i)*a^4*b*c*d^4 + (6*g^3*i*log(e) + 5*g^3*i)*a^5*d^5)*B^2)*log(b*x + a) - (24*B^2*b^5*d^5*g^3*i*x^5*log(e) + 6*((5*g^3*i*log(e) - g^3*i)*b^5*c*d^4 + (15*g^3*i*log(e) + g^3*i)*a*b^4*d^5)*B^2*x^4 - 2*(b^5*c^2*d^3*g^3*i - 10*(6*g^3*i*log(e) - g^3*i)*a*b^4*c*d^4 - (60*g^3*i*log(e) + 11*g^3*i)*a^2*b^3*d^5)*B^2*x^3 + 3*(b^5*c^3*d^2*g^3*i - 5*a*b^4*c^2*d^3*g^3*i + 5*(12*g^3*i*log(e) - g^3*i)*a^2*b^3*c*d^4 + (20*g^3*i*log(e) + 9*g^3*i)*a^3*b^2*d^5)*B^2*x^2 - 6*(b^5*c^4*d*g^3*i - 5*a*b^4*c^3*d^2*g^3*i + 10*a^2*b^3*c^2*d^3*g^3*i - a^4*b*d^5*g^3*i - 5*(4*g^3*i*log(e) + g^3*i)*a^3*b^2*c*d^4)*B^2*x + 6*(4*B^2*b^5*d^5*g^3*i*x^5 + 20*B^2*a^3*b^2*c*d^4*g^3*i*x + 5*(b^5*c*d^4*g^3*i + 3*a*b^4*d^5*g^3*i)*B^2*x^4 + 20*(a*b^4*c*d^4*g^3*i + a^2*b^3*d^5*g^3*i)*B^2*x^3 + 10*(3*a^2*b^3*c*d^4*g^3*i + a^3*b^2*d^5*g^3*i)*B^2*x^2 + (5*a^4*b*c*d^4*g^3*i - a^5*d^5*g^3*i)*B^2)*log(b*x + a))*log(d*x + c))/(b^2*d^4)

Giac [F]

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$=\int (bgx+ag)^3(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algor
ithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^3*(d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)^
2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$=\int (a\,g+b\,g\,x)^3\,(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

```
[In] int((a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

```
[Out] int((a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x
)
```

3.56 $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 40, antiderivative size = 450

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= -\frac{B^2(bc-ad)^3g^2ix}{3bd^2}+\frac{B^2(bc-ad)^2g^2i(c+dx)^2}{12d^3}$$
$$-\frac{B(bc-ad)^2g^2i(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2d}$$
$$-\frac{B(bc-ad)g^2i(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^2}$$
$$+\frac{(bc-ad)g^2i(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{12b^2}$$
$$+\frac{g^2i(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b}$$
$$+\frac{B(bc-ad)^3g^2i(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2d^2}$$
$$+\frac{B(bc-ad)^4g^2i\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2d^3}$$
$$+\frac{B^2(bc-ad)^4g^2i\log(c+dx)}{6b^2d^3}+\frac{B^2(bc-ad)^4g^2i\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{6b^2d^3}$$

```
[Out] -1/3*B^2*(-a*d+b*c)^3*g^2*i*x/b/d^2+1/12*B^2*(-a*d+b*c)^2*g^2*i*(d*x+c)^2/d
^3-1/12*B*(-a*d+b*c)^2*g^2*i*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/d-1/
6*B*(-a*d+b*c)*g^2*i*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2+1/12*(-a*d+b
```

```
*c)*g^2*i*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^2+1/4*g^2*i*(b*x+a)^3*(
d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+1/12*B*(-a*d+b*c)^3*g^2*i*(b*x+a)*(2
*A+B+2*B*ln(e*(b*x+a)/(d*x+c)))/b^2/d^2+1/12*B*(-a*d+b*c)^4*g^2*i*ln((-a*d+
b*c)/b/(d*x+c))*(2*A+3*B+2*B*ln(e*(b*x+a)/(d*x+c)))/b^2/d^3+1/6*B^2*(-a*d+b
*c)^4*g^2*i*ln(d*x+c)/b^2/d^3+1/6*B^2*(-a*d+b*c)^4*g^2*i*polylog(2,d*(b*x+a
)/b/(d*x+c))/b^2/d^3
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2562, 2383, 2381, 2384, 2354, 2438, 2373, 45}

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$$
$$=\frac{Bg^2i(bc-ad)^4\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+3B\right)}{12b^2d^3}+\frac{Bg^2i(a+bx)(bc-ad)^3\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{12b^2d^2}-\frac{Bg^2i(a+bx)^2(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{12b^2d}+\frac{g^2i(a+bx)^3(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{12b^2}-\frac{Bg^2i(a+bx)^3(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6b^2}+\frac{g^2i(a+bx)^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b}+\frac{B^2g^2i(bc-ad)^4\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{6b^2d^3}+\frac{B^2g^2i(bc-ad)^4\log(c+dx)}{6b^2d^3}+\frac{B^2g^2i(c+dx)^2(bc-ad)^2}{12d^3}-\frac{B^2g^2ix(bc-ad)^3}{3bd^2}$$

```
[In] Int[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] -1/3*(B^2*(b*c - a*d)^3*g^2*i*x)/(b*d^2) + (B^2*(b*c - a*d)^2*g^2*i*(c + d*
x)^2)/(12*d^3) - (B*(b*c - a*d)^2*g^2*i*(a + b*x)^2*(A + B*Log[(e*(a + b*x)
)/(c + d*x)]))/(12*b^2*d) - (B*(b*c - a*d)*g^2*i*(a + b*x)^3*(A + B*Log[(e*
(a + b*x))/(c + d*x)]))/(6*b^2) + ((b*c - a*d)*g^2*i*(a + b*x)^3*(A + B*Log
[(e*(a + b*x))/(c + d*x)])^2)/(12*b^2) + (g^2*i*(a + b*x)^3*(c + d*x)*(A +
B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*b) + (B*(b*c - a*d)^3*g^2*i*(a + b*x)
*(2*A + B + 2*B*Log[(e*(a + b*x))/(c + d*x)]))/(12*b^2*d^2) + (B*(b*c - a*d
)^4*g^2*i*Log[(b*c - a*d)/(b*(c + d*x))]*(2*A + 3*B + 2*B*Log[(e*(a + b*x))
/(c + d*x)]))/(12*b^2*d^3) + (B^2*(b*c - a*d)^4*g^2*i*Log[c + d*x])/(6*b^2*
```

```
d^3) + (B^2*(b*c - a*d)^4*g^2*i*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(6
*b^2*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2383

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Dist[(m + q + 2)/(d*(q + 1)), Int[(
f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Dist[b*n*(p/(d*(q +
 1))), Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x]) /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ
[q, -1] && GtQ[m, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
```

```
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \left((bc-ad)^4 g^2 i\right) \text{Subst}\left(\int \frac{x^2 (A+B\log(ex))^2}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{g^2 i (a+bx)^3 (c+dx) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} + \frac{\left((bc-ad)^4 g^2 i\right) \text{Subst}\left(\int \frac{x^2(A+B\log(ex))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{4b} - \frac{\left(B(bc-ad)^4 g^2 i\right) \text{Subst}\left(\int \frac{x^2(A+B\log(ex))}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b}$$

$$= -\frac{B(bc-ad) g^2 i (a+bx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^2} + \frac{(bc-ad) g^2 i (a+bx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{12b^2} + \frac{g^2 i (a+bx)^3 (c+dx) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)^4 g^2 i\right) \text{Subst}\left(\int \frac{x^2(A+B\log(ex))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{6b^2} + \frac{\left(B^2(bc-ad)^4 g^2 i\right) \text{Subst}\left(\int \frac{x^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{6b^2}$$

$$= -\frac{B(bc-ad)^2g^2i(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2d} - \frac{B(bc-ad)g^2i(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^2} + \frac{(bc-ad)g^2i(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{12b^2} + \frac{g^2i(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} + \frac{(B^2(bc-ad)^4g^2i)\operatorname{Subst}\left(\int\left(\frac{b^2}{d^2(b-dx)^3}-\frac{2b}{d^2(b-dx)^2}+\frac{1}{d^2(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{6b^2} + \frac{(B(bc-ad)^4g^2i)\operatorname{Subst}\left(\int\frac{x(2A+B+2B\log(ex))}{(b-dx)^2}dx,x,\frac{a+bx}{c+dx}\right)}{12b^2d}$$

$$= -\frac{B^2(bc-ad)^3g^2ix}{3bd^2} + \frac{B^2(bc-ad)^2g^2i(c+dx)^2}{12d^3} - \frac{B(bc-ad)^2g^2i(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2d} - \frac{B(bc-ad)g^2i(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^2} + \frac{(bc-ad)g^2i(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{12b^2} + \frac{g^2i(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} + \frac{B(bc-ad)^3g^2i(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2d^2} + \frac{B^2(bc-ad)^4g^2i\log(c+dx)}{6b^2d^3} - \frac{(B(bc-ad)^4g^2i)\operatorname{Subst}\left(\int\frac{2A+3B+2B\log(ex)}{b-dx}dx,x,\frac{a+bx}{c+dx}\right)}{12b^2d^2}$$

$$= -\frac{B^2(bc-ad)^3g^2ix}{3bd^2} + \frac{B^2(bc-ad)^2g^2i(c+dx)^2}{12d^3} - \frac{B(bc-ad)^2g^2i(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2d} - \frac{B(bc-ad)g^2i(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^2} + \frac{(bc-ad)g^2i(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{12b^2} + \frac{g^2i(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} + \frac{B(bc-ad)^3g^2i(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2d^2} + \frac{B(bc-ad)^4g^2i\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2d^3} + \frac{B^2(bc-ad)^4g^2i\log(c+dx)}{6b^2d^3} - \frac{(B^2(bc-ad)^4g^2i)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{6b^2d^3}$$

$$= -\frac{B^2(bc-ad)^3g^2ix}{3bd^2} + \frac{B^2(bc-ad)^2g^2i(c+dx)^2}{12d^3} - \frac{B(bc-ad)^2g^2i(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2d} - \frac{B(bc-ad)g^2i(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^2} + \frac{(bc-ad)g^2i(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{12b^2} + \frac{g^2i(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} + \frac{B(bc-ad)^3g^2i(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2d^2} + \frac{B(bc-ad)^4g^2i\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2d^3} + \frac{B^2(bc-ad)^4g^2i\log(c+dx)}{6b^2d^3} + \frac{B^2(bc-ad)^4g^2i\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{6b^2d^3}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.51

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\frac{g^2i\left(4(bc-ad)(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+3d(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{4B(bc-ad)^2\left(2Abd(bc\right.}{}\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)^2,x]
```

```
[Out] (g^2*i*(4*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 +
3*d*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (4*B*(b*c - a*d)^2
*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x))/(c
+ d*x)] - d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*(b*c -
 a*d)^2*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
*Log[c + d*x] + B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + B*(b*
c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*
x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3 - (B*(b*c - a*d)*(6*A*b
*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(c + d
*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(b*c - a*d
)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log
[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c -
 a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (-(b*c) + a*d)*Log[c + d
*x]) + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*
x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3))/(12*b^2
)
```

Maple [F]

$$\int (bgx+ag)^2\,(dix+ci)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2dx$$

```
[In] int((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \int (bgx+ag)^2(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algor
ithm="fricas")
```

```
[Out] integral(A^2*b^2*d*g^2*i*x^3 + A^2*a^2*c*g^2*i + (A^2*b^2*c + 2*A^2*a*b*d)*
g^2*i*x^2 + (2*A^2*a*b*c + A^2*a^2*d)*g^2*i*x + (B^2*b^2*d*g^2*i*x^3 + B^2*
a^2*c*g^2*i + (B^2*b^2*c + 2*B^2*a*b*d)*g^2*i*x^2 + (2*B^2*a*b*c + B^2*a^2*
d)*g^2*i*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^2*d*g^2*i*x^3 + A*B*a
^2*c*g^2*i + (A*B*b^2*c + 2*A*B*a*b*d)*g^2*i*x^2 + (2*A*B*a*b*c + A*B*a^2*d
)*g^2*i*x)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**2*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2243 vs. $2(429) = 858$.

Time = 0.31 (sec) , antiderivative size = 2243, normalized size of antiderivative = 4.98

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algor
ithm="maxima")
```

```
[Out] 1/4*A^2*b^2*d*g^2*i*x^4 + 1/3*A^2*b^2*c*g^2*i*x^3 + 2/3*A^2*a*b*d*g^2*i*x^3
 + A^2*a*b*c*g^2*i*x^2 + 1/2*A^2*a^2*d*g^2*i*x^2 + 2*(x*log(b*e*x/(d*x + c)
 + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a^2*c*g^2*i +
2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*lo
```

g(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a*b*c*g^2*i + 1/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*c*g^2*i + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^2*d*g^2*i + 2/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a*b*d*g^2*i + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b^2*d*g^2*i + A^2*a^2*c*g^2*i*x - 1/12*(2*a^3*c*d^3*g^2*i + (2*g^2*i*log(e) + g^2*i)*b^3*c^4 - 2*(4*g^2*i*log(e) + g^2*i)*a*b^2*c^3*d + (12*g^2*i*log(e) - g^2*i)*a^2*b*c^2*d^2)*B^2*log(d*x + c)/(b*d^3) - 1/6*(b^4*c^4*g^2*i - 4*a*b^3*c^3*d*g^2*i + 6*a^2*b^2*c^2*d^2*g^2*i - 4*a^3*b*c*d^3*g^2*i + a^4*d^4*g^2*i)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^3) + 1/12*(3*B^2*b^4*d^4*g^2*i*x^4*log(e)^2 + 2*((2*g^2*i*log(e)^2 - g^2*i*log(e))*b^4*c*d^3 + (4*g^2*i*log(e)^2 + g^2*i*log(e))*a*b^3*d^4)*B^2*x^3 - ((g^2*i*log(e) - g^2*i)*b^4*c^2*d^2 - 2*(6*g^2*i*log(e)^2 - 2*g^2*i*log(e) - g^2*i)*a*b^3*c*d^3 - (6*g^2*i*log(e)^2 + 5*g^2*i*log(e) + g^2*i)*a^2*b^2*d^4)*B^2*x^2 + ((2*g^2*i*log(e) - g^2*i)*b^4*c^3*d - (8*g^2*i*log(e) - 5*g^2*i)*a*b^3*c^2*d^2 + (12*g^2*i*log(e)^2 + 4*g^2*i*log(e) - 7*g^2*i)*a^2*b^2*c*d^3 + (2*g^2*i*log(e) + 3*g^2*i)*a^3*b*d^4)*B^2*x + (3*B^2*b^4*d^4*g^2*i*x^4 + 12*B^2*a^2*b^2*c*d^3*g^2*i*x + 4*(b^4*c*d^3*g^2*i + 2*a*b^3*d^4*g^2*i)*B^2*x^3 + 6*(2*a*b^3*c*d^3*g^2*i + a^2*b^2*d^4*g^2*i)*B^2*x^2 + (4*a^3*b*c*d^3*g^2*i - a^4*d^4*g^2*i)*B^2)*log(b*x + a)^2 + (3*B^2*b^4*d^4*g^2*i*x^4 + 12*B^2*a^2*b^2*c*d^3*g^2*i*x + 4*(b^4*c*d^3*g^2*i + 2*a*b^3*d^4*g^2*i)*B^2*x^3 + 6*(2*a*b^3*c*d^3*g^2*i + a^2*b^2*d^4*g^2*i)*B^2*x^2 + (b^4*c^4*g^2*i - 4*a*b^3*c^3*d*g^2*i + 6*a^2*b^2*c^2*d^2*g^2*i)*B^2)*log(d*x + c)^2 + (6*B^2*b^4*d^4*g^2*i*x^4*log(e) + 2*((4*g^2*i*log(e) - g^2*i)*b^4*c*d^3 + (8*g^2*i*log(e) + g^2*i)*a*b^3*d^4)*B^2*x^3 - (b^4*c^2*d^2*g^2*i - 4*(6*g^2*i*log(e) - g^2*i)*a*b^3*c*d^3 - (12*g^2*i*log(e) + 5*g^2*i)*a^2*b^2*d^4)*B^2*x^2 + 2*(b^4*c^3*d*g^2*i - 4*a*b^3*c^2*d^2*g^2*i + a^3*b*d^4*g^2*i + 2*(6*g^2*i*log(e) + g^2*i)*a^2*b^2*c*d^3)*B^2*x + (2*a*b^3*c^3*d*g^2*i - 7*a^2*b^2*c^2*d^2*g^2*i + 2*(4*g^2*i*log(e) + 3*g^2*i)*a^3*b*c*d^3 - (2*g^2*i*log(e) + g^2*i)*a^4*d^4)*B^2)*log(b*x + a) - (6*B^2*b^4*d^4*g^2*i*x^4*log(e) + 2*((4*g^2*i*log(e) - g^2*i)*b^4*c*d^3 + (8*g^2*i*log(e) + g^2*i)*a*b^3*d^4)*B^2*x^3 - (b^4*c^2*d^2*g^2*i - 4*(6*g^2*i*log(e) - g^2*i)*a*b^3*c*d^3 - (12*g^2*i*log(e) + 5*g^2*i)*a^2*b^2*d^4)*B^2*x^2 + 2*(b^4*c^3*d*g^2*i - 4*a*b^3*c^2*d^2*g^2*i + a^3*b*d^4*g^2*i + 2*(6*g^2*i*log(e) + g^2*i)*a^2*b^2*c*d^3)*B^2*x + 2*(3*B^2*b^4*d^4*g^2*i*x^4 + 12*B^2*a^2*b^2*c*d^3*g^2*i*x + 4*(b^4*c*d^3*g^2*i + 2*a*b^3*d^4*g^2*i)*B^2*x^3 + 6*(2*a*b^3*c*d^3*g^2*i + a^2*b^2*d^4*g^2*i)*B^2*x^2 + (4*a^3*b*c*d^3*g^2*i - a^4*d^4*g^2*i)*B^2)*log(b*x + a))*log(d*x + c))/(b^2*d^3)

Giac [F]

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)^2(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")

[Out] integrate((b*g*x + a*g)^2*(d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (a\,g+b\,g\,x)^2\,(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

[In] int((a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)

[Out] int((a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)

3.57 $\int (ag+bgx)(ci+dix) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 38, antiderivative size = 343

$$
\begin{aligned}
&\int (ag+bgx)(ci+dix) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx \\
&= \frac{B^2(bc-ad)^2 gix}{3bd} - \frac{B(bc-ad)^2 gi(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2d} \\
&\quad - \frac{B(bc-ad)gi(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2} \\
&\quad + \frac{(bc-ad)gi(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b^2} \\
&\quad + \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} \\
&\quad - \frac{B(bc-ad)^3 gi\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2d^2} \\
&\quad - \frac{B^2(bc-ad)^3 gi\log(c+dx)}{3b^2d^2} - \frac{B^2(bc-ad)^3 gi\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^2d^2}
\end{aligned}
$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*g*i*x/b/d-1/3*B*(-a*d+b*c)^2*g*i*(b*x+a)*(A+B*ln(e*(b*
x+a)/(d*x+c)))/b^2/d-1/3*B*(-a*d+b*c)*g*i*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+
c)))/b^2+1/6*(-a*d+b*c)*g*i*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^2+1/3
*g*i*(b*x+a)^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b-1/3*B*(-a*d+b*c)^3*g
*i*ln((-a*d+b*c)/b/(d*x+c))*(A+B+B*ln(e*(b*x+a)/(d*x+c)))/b^2/d^2-1/3*B^2*(
-a*d+b*c)^3*g*i*ln(d*x+c)/b^2/d^2-1/3*B^2*(-a*d+b*c)^3*g*i*polylog(2,d*(b*x
+a)/b/(d*x+c))/b^2/d^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2562, 2383, 2381, 2384, 2354, 2438, 2373, 45}

$$\begin{aligned}
&\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx\\
&=-\frac{Bgi(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A+B\right)}{3b^2d^2}\\
&-\frac{Bgi(a+bx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b^2d}\\
&+\frac{gi(a+bx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{6b^2}\\
&-\frac{Bgi(a+bx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b^2}\\
&+\frac{gi(a+bx)^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b}-\frac{B^2gi(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^2d^2}\\
&-\frac{B^2gi(bc-ad)^3\log(c+dx)}{3b^2d^2}+\frac{B^2gix(bc-ad)^2}{3bd}
\end{aligned}$$

```
[In] Int[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^2*g*i*x)/(3*b*d) - (B*(b*c - a*d)^2*g*i*(a + b*x)*(A + B*L
og[(e*(a + b*x))/(c + d*x)]))/(3*b^2*d) - (B*(b*c - a*d)*g*i*(a + b*x)^2*(A
 + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b^2) + ((b*c - a*d)*g*i*(a + b*x)^2*
(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(6*b^2) + (g*i*(a + b*x)^2*(c + d*x
)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*b) - (B*(b*c - a*d)^3*g*i*Log[
(b*c - a*d)/(b*(c + d*x))]*(A + B + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b^2
*d^2) - (B^2*(b*c - a*d)^3*g*i*Log[c + d*x])/(3*b^2*d^2) - (B^2*(b*c - a*d)
^3*g*i*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*b^2*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
```

```
Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2383

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Dist[(m + q + 2)/(d*(q + 1)), Int[(
f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Dist[b*n*(p/(d*(q +
 1))), Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x]) /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ
[q, -1] && GtQ[m, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
```

```
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \left((bc-ad)^3 gi\right) \text{Subst}\left(\int \frac{x(A+B\log(ex))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} + \frac{\left((bc-ad)^3 gi\right)\text{Subst}\left(\int \frac{x(A+B\log(ex))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b} - \frac{\left(2B(bc-ad)^3 gi\right)\text{Subst}\left(\int \frac{x(A+B\log(ex))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b}$$

$$= -\frac{B(bc-ad)gi(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2} + \frac{(bc-ad)gi(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b^2} + \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} - \frac{\left(B(bc-ad)^3 gi\right)\text{Subst}\left(\int \frac{x(A+B\log(ex))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b^2} + \frac{\left(B^2(bc-ad)^3 gi\right)\text{Subst}\left(\int \frac{x}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b^2}$$

$$= -\frac{B(bc-ad)^2 gi(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2d} - \frac{B(bc-ad)gi(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2} + \frac{(bc-ad)gi(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b^2} + \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} + \frac{(B^2(bc-ad)^3 gi)\operatorname{Subst}\left(\int\left(\frac{b}{d(-b+dx)^2}+\frac{1}{d(-b+dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{3b^2} + \frac{(B(bc-ad)^3 gi)\operatorname{Subst}\left(\int\frac{A+B+B\log(ex)}{b-dx}dx, x, \frac{a+bx}{c+dx}\right)}{3b^2d}$$

$$= \frac{B^2(bc-ad)^2 gix}{3bd} - \frac{B(bc-ad)^2 gi(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2d} - \frac{B(bc-ad)gi(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2} + \frac{(bc-ad)gi(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b^2} + \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} - \frac{B(bc-ad)^3 gi\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2d^2} - \frac{B^2(bc-ad)^3 gi\log(c+dx)}{3b^2d^2} + \frac{(B^2(bc-ad)^3 gi)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}dx, x, \frac{a+bx}{c+dx}\right)}{3b^2d^2}$$

$$= \frac{B^2(bc-ad)^2 gix}{3bd} - \frac{B(bc-ad)^2 gi(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2d} - \frac{B(bc-ad)gi(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2} + \frac{(bc-ad)gi(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b^2} + \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} - \frac{B(bc-ad)^3 gi\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2d^2} - \frac{B^2(bc-ad)^3 gi\log(c+dx)}{3b^2d^2} - \frac{B^2(bc-ad)^3 gi\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{3b^2d^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 869 vs. $2(343) = 686$.

Time = 0.41 (sec) , antiderivative size = 869, normalized size of antiderivative = 2.53

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$

$$= \frac{gi\left(-6Ab^2Bcd(bc-ad)x + 6aAbBd^2(-bc+ad)x + 4AbBd(bc-ad)(bc+ad)x - 6bB^2cd(bc-ad)(a+b\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^
2,x]
```

```
[Out] (g*i*(-6*A*b^2*B*c*d*(b*c - a*d)*x + 6*a*A*b*B*d^2*(-(b*c) + a*d)*x + 4*A*b
*B*d*(b*c - a*d)*(b*c + a*d)*x - 6*b*B^2*c*d*(b*c - a*d)*(a + b*x)*Log[(e*(
a + b*x))/(c + d*x)] + 6*a*B^2*d^2*(-(b*c) + a*d)*(a + b*x)*Log[(e*(a + b*x
))/(c + d*x)] + 4*B^2*d*(b*c - a*d)*(b*c + a*d)*(a + b*x)*Log[(e*(a + b*x))
/(c + d*x)] - 2*b^2*B*d^2*(b*c - a*d)*x^2*(A + B*Log[(e*(a + b*x))/(c + d*x
)]) + 6*a^2*b*B*c*d^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2
*a^3*B*d^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*a*b^2*c*d^
2*x*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 3*b^2*d^2*(b*c + a*d)*x^2*(A +
 B*Log[(e*(a + b*x))/(c + d*x)])^2 + 2*b^3*d^3*x^3*(A + B*Log[(e*(a + b*x))
/(c + d*x)])^2 + 6*b*B^2*c*(b*c - a*d)^2*Log[c + d*x] + 6*a*B^2*d*(b*c - a*
d)^2*Log[c + d*x] - 4*B^2*(b*c - a*d)^2*(b*c + a*d)*Log[c + d*x] + 2*b^3*B*
c^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 6*a*b^2*B*c^2*d*(A
+ B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 2*B^2*(b*c - a*d)*(a^2*d^2
*Log[a + b*x] - b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - 3*a^2*b*B^2*
```

```
c*d^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*P
olyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + a^3*B^2*d^3*(Log[a + b*x]*(Log[a
 + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-
(b*c) + a*d)]) - b^3*B^2*c^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c
+ d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 3*a*b^2*B
^2*c^2*d*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x]
 + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(6*b^2*d^2)
```

Maple [F]

$$\int (bgx + ag)(dix + ci)\left(A + B\ln\left(\frac{e(bx + a)}{dx + c}\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag + bgx)(ci + dix)\left(A + B\log\left(\frac{e(a + bx)}{c + dx}\right)\right)^2 dx$$
$$= \int (bgx + ag)(dix + ci)\left(B\log\left(\frac{(bx + a)e}{dx + c}\right) + A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorit
hm="fricas")
```

```
[Out] integral(A^2*b*d*g*i*x^2 + A^2*a*c*g*i + (A^2*b*c + A^2*a*d)*g*i*x + (B^2*b
*d*g*i*x^2 + B^2*a*c*g*i + (B^2*b*c + B^2*a*d)*g*i*x)*log((b*e*x + a*e)/(d*
x + c))^2 + 2*(A*B*b*d*g*i*x^2 + A*B*a*c*g*i + (A*B*b*c + A*B*a*d)*g*i*x)*l
og((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag + bgx)(ci + dix)\left(A + B\log\left(\frac{e(a + bx)}{c + dx}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1252 vs. 2(326) = 652.

Time = 0.30 (sec) , antiderivative size = 1252, normalized size of antiderivative = 3.65

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorit
hm="maxima")
```

```
[Out] 1/3*A^2*b*d*g*i*x^3 + 1/2*A^2*b*c*g*i*x^2 + 1/2*A^2*a*d*g*i*x^2 + 2*(x*log(
b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B
*a*c*g*i + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2
 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*b*c*g*i + (x^2*log(b*e*x
/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 -
 (b*c - a*d)*x/(b*d))*A*B*a*d*g*i + 1/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d
*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*
b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b*d*g*i + A^2*a*c*g*i*
x + 1/3*(b^2*c^3*g*i*log(e) - a^2*c*d^2*g*i - (3*g*i*log(e) - g*i)*a*b*c^2*
d)*B^2*log(d*x + c)/(b*d^2) + 1/3*(b^3*c^3*g*i - 3*a*b^2*c^2*d*g*i + 3*a^2*
b*c*d^2*g*i - a^3*d^3*g*i)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1)
 + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^2) + 1/6*(2*B^2*b^3*d^3*g*
i*x^3*log(e)^2 + ((3*g*i*log(e)^2 - 2*g*i*log(e))*b^3*c*d^2 + (3*g*i*log(e)
^2 + 2*g*i*log(e))*a*b^2*d^3)*B^2*x^2 - 2*((g*i*log(e) - g*i)*b^3*c^2*d - (
3*g*i*log(e)^2 - 2*g*i)*a*b^2*c*d^2 - (g*i*log(e) + g*i)*a^2*b*d^3)*B^2*x +
 (2*B^2*b^3*d^3*g*i*x^3 + 6*B^2*a*b^2*c*d^2*g*i*x + 3*(b^3*c*d^2*g*i + a*b^
2*d^3*g*i)*B^2*x^2 + (3*a^2*b*c*d^2*g*i - a^3*d^3*g*i)*B^2)*log(b*x + a)^2
+ (2*B^2*b^3*d^3*g*i*x^3 + 6*B^2*a*b^2*c*d^2*g*i*x + 3*(b^3*c*d^2*g*i + a*b
^2*d^3*g*i)*B^2*x^2 - (b^3*c^3*g*i - 3*a*b^2*c^2*d*g*i)*B^2)*log(d*x + c)^2
 + 2*(2*B^2*b^3*d^3*g*i*x^3*log(e) + ((3*g*i*log(e) - g*i)*b^3*c*d^2 + (3*g
*i*log(e) + g*i)*a*b^2*d^3)*B^2*x^2 + (6*a*b^2*c*d^2*g*i*log(e) - b^3*c^2*d
*g*i + a^2*b*d^3*g*i)*B^2*x - (a^3*d^3*g*i*log(e) + a*b^2*c^2*d*g*i - (3*g*
i*log(e) + g*i)*a^2*b*c*d^2)*B^2)*log(b*x + a) - 2*(2*B^2*b^3*d^3*g*i*x^3*l
og(e) + ((3*g*i*log(e) - g*i)*b^3*c*d^2 + (3*g*i*log(e) + g*i)*a*b^2*d^3)*B
^2*x^2 + (6*a*b^2*c*d^2*g*i*log(e) - b^3*c^2*d*g*i + a^2*b*d^3*g*i)*B^2*x +
 (2*B^2*b^3*d^3*g*i*x^3 + 6*B^2*a*b^2*c*d^2*g*i*x + 3*(b^3*c*d^2*g*i + a*b^
2*d^3*g*i)*B^2*x^2 + (3*a^2*b*c*d^2*g*i - a^3*d^3*g*i)*B^2)*log(b*x + a))*l
og(d*x + c))/(b^2*d^2)
```

Giac [F]

$$\int (ag + bgx)(ci + dix) \left(A + B \log \left(\frac{e(a + bx)}{c + dx}\right)\right)^2 dx$$
$$= \int (bgx + ag)(dix + ci)\left(B \log \left(\frac{(bx + a)e}{dx + c}\right) + A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorit
hm="giac")
```

```
[Out] integrate((b*g*x + a*g)*(d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)^2,
 x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag + bgx)(ci + dix) \left(A + B \log \left(\frac{e(a + bx)}{c + dx}\right)\right)^2 dx$$
$$= \int (a\,g + b\,g\,x)\,(c\,i + d\,i\,x)\,\left(A + B\,\ln\left(\frac{e\,(a + b\,x)}{c + d\,x}\right)\right)^2 dx$$

```
[In] int((a*g + b*g*x)*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

```
[Out] int((a*g + b*g*x)*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.58 $\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

Optimal result

Integrand size = 30, antiderivative size = 203

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = -\frac{B(bc-ad)i(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2}+\frac{i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d}+\frac{B^2(bc-ad)^2i\log(c+dx)}{b^2d}+\frac{B(bc-ad)^2i\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d}-\frac{B^2(bc-ad)^2i\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d}$$

```
[Out] -B*(-a*d+b*c)*i*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2+1/2*i*(d*x+c)^2*(A+
B*ln(e*(b*x+a)/(d*x+c)))^2/d+B^2*(-a*d+b*c)^2*i*ln(d*x+c)/b^2/d+B*(-a*d+b*c
)^2*i*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^2/d-B^2*(-a*d
+b*c)^2*i*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used

$= \{2552, 2356, 2389, 2379, 2438, 2351, 31\}$

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{Bi(bc-ad)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^2d} - \frac{Bi(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^2} + \frac{i(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d} - \frac{B^2i(bc-ad)^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d} + \frac{B^2i(bc-ad)^2\log(c+dx)}{b^2d}$$

```
[In] Int[(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] -((B*(b*c - a*d)*i*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/b^2) + (
i*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*d) + (B^2*(b*c - a
*d)^2*i*Log[c + d*x])/(b^2*d) + (B*(b*c - a*d)^2*i*(A + B*Log[(e*(a + b*x))
/(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^2*d) - (B^2*(b*c - a*
d)^2*i*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b^2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
```

```
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left((bc-ad)^2 i\right)\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)\\
&= \frac{i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d} - \frac{\left(B(bc-ad)^2 i\right)\text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{d}\\
&= \frac{i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d}\\
&\quad - \frac{\left(B(bc-ad)^2 i\right)\text{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b}\\
&\quad - \frac{\left(B(bc-ad)^2 i\right)\text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bd}\end{aligned}$$

$$= -\frac{B(bc-ad)i(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2} + \frac{i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d} + \frac{B(bc-ad)^2i\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d} + \frac{(B^2(bc-ad)^2i)\,\text{Subst}\left(\int\frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^2} - \frac{(B^2(bc-ad)^2i)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^2d}$$

$$= -\frac{B(bc-ad)i(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2} + \frac{i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d} + \frac{B^2(bc-ad)^2i\log(c+dx)}{b^2d} + \frac{B(bc-ad)^2i\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d} - \frac{B^2(bc-ad)^2i\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.01

$$\int(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \frac{i\left((c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - \frac{B(bc-ad)\left((-bBc+aBd)\log^2(a+bx)+2\left(Abdx+Bd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+(-bBc+aBd)\log(\right.\right.}{}\right.}{2d}$$

```
[In] Integrate[(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (i*((c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(b*c - a*d)*((-
(b*B*c) + a*B*d)*Log[a + b*x]^2 + 2*(A*b*d*x + B*d*(a + b*x)*Log[(e*(a + b*
x))/(c + d*x)] + (-(b*B*c) + a*B*d)*Log[c + d*x]) + 2*(b*c - a*d)*Log[a + b
*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)] + B*Log[(b*(c + d*x))/(b*c - a*d)])
 + 2*B*(b*c - a*d)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/b^2))/(2*d)
```

Maple [F]

$$\int (dix+ci)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

```
[In] int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
[Out] int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
[Out] integral(A^2*d*i*x + A^2*c*i + (B^2*d*i*x + B^2*c*i)*log((b*e*x + a*e)/(d*x
 + c))^2 + 2*(A*B*d*i*x + A*B*c*i)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 633 vs. 2(200) = 400.

Time = 0.28 (sec) , antiderivative size = 633, normalized size of antiderivative = 3.12

$$\begin{aligned}
&\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx \\
&= \frac{1}{2}A^2dix^2 + 2\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)ABci \\
&\quad + \left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)ABdi \\
&\quad + A^2cix - \frac{((i\log(e)-i)bc^2+acdi)B^2\log(dx+c)}{bd} \\
&\quad - \frac{(b^2c^2i-2\,abcdi+a^2d^2i)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{b^2d} \\
&\quad + \frac{B^2b^2d^2ix^2\log(e)^2+2\left(abd^2i\log(e)+\left(i\log(e)^2-i\log(e)\right)b^2cd\right)B^2x+\left(B^2b^2d^2ix^2+2\,B^2b^2cdix+(2\,a\ldots\right.}{}
\end{aligned}$$

[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")

[Out] 1/2*A^2*d*i*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*c*i + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*d*i + A^2*c*i*x - ((i*log(e) - i)*b*c^2 + a*c*d*i)*B^2*log(d*x + c)/(b*d) - (b^2*c^2*i - 2*a*b*c*d*i + a^2*d^2*i)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d) + 1/2*(B^2*b^2*d^2*i*x^2*log(e)^2 + 2*(a*b*d^2*i*log(e) + (i*log(e)^2 - i*log(e))*b^2*c*d)*B^2*x + (B^2*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i*x + (2*a*b*c*d*i - a^2*d^2*i)*B^2)*log(b*x + a)^2 + (B^2*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i*x + B^2*b^2*c^2*i)*log(d*x + c)^2 + 2*(B^2*b^2*d^2*i*x^2*log(e) + ((2*i*log(e) - i)*b^2*c*d + a*b*d^2*i)*B^2*x + ((2*i*log(e) - i)*a*b*c*d - (i*log(e) - i)*a^2*d^2)*B^2)*log(b*x + a) - 2*(B^2*b^2*d^2*i*x^2*log(e) + ((2*i*log(e) - i)*b^2*c*d + a*b*d^2*i)*B^2*x + (B^2*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i*x + (2*a*b*c*d*i - a^2*d^2*i)*B^2)*log(b*x + a))*log(d*x + c))/(b^2*d)

Giac [F]

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\int (dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")

[Out] integrate((d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)

Mupad [F(-1)]

Timed out.

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\int (c\,i+d\,i\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

[In] int((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)

[Out] int((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)

3.59 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 286

$$\begin{aligned}
\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx
&= \frac{2B(bc-ad)i\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g}+\frac{di(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2g}\\
&-\frac{(bc-ad)i\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g}\\
&+\frac{2B^2(bc-ad)i\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2g}\\
&+\frac{2B(bc-ad)i\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g}\\
&+\frac{2B^2(bc-ad)i\,\mathrm{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g}
\end{aligned}$$

```
[Out] 2*B*(-a*d+b*c)*i*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/g
+d*i*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^2/g-(-a*d+b*c)*i*(A+B*ln(e*(b*
x+a)/(d*x+c)))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^2/g+2*B^2*(-a*d+b*c)*i*polylog
(2,d*(b*x+a)/b/(d*x+c))/b^2/g+2*B*(-a*d+b*c)*i*(A+B*ln(e*(b*x+a)/(d*x+c)))*
polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/g+2*B^2*(-a*d+b*c)*i*polylog(3,b*(d*x+c)
/d/(b*x+a))/b^2/g
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2562, 2389, 2379, 2421, 6724, 2355, 2354, 2438}

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$$
$$=\frac{2Bi(bc-ad)\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^2g}$$
$$+\frac{2Bi(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^2g}$$
$$+\frac{di(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^2g}$$
$$-\frac{i(bc-ad)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^2g}$$
$$+\frac{2B^2i(bc-ad)\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2g}+\frac{2B^2i(bc-ad)\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g}$$

```
[In] Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x),x]
```

```
[Out] (2*B*(b*c - a*d)*i*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/
(c + d*x)]))/(b^2*g) + (d*i*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^
2)/(b^2*g) - ((b*c - a*d)*i*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1 -
(b*(c + d*x))/(d*(a + b*x))])/(b^2*g) + (2*B^2*(b*c - a*d)*i*PolyLog[2, (d*
(a + b*x))/(b*(c + d*x))])/(b^2*g) + (2*B*(b*c - a*d)*i*(A + B*Log[(e*(a +
b*x))/(c + d*x)])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b^2*g) + (2*B^2
*(b*c - a*d)*i*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))])/(b^2*g)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
```

```
, p}, x] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)i)\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{g}$$

$$= \frac{((bc-ad)i)\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg} + \frac{(d(bc-ad)i)\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg}$$

$$= \frac{di(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2g} - \frac{(bc-ad)i\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g} + \frac{(2B(bc-ad)i)\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)(A+B\log(ex))}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g} - \frac{(2Bd(bc-ad)i)\text{Subst}\left(\int \frac{A+B\log(ex)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g}$$

$$= \frac{2B(bc-ad)i\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g} + \frac{di(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2g} - \frac{(bc-ad)i\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g} + \frac{2B(bc-ad)i\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g} - \frac{(2B^2(bc-ad)i)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g} - \frac{(2B^2(bc-ad)i)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g}$$

$$= \frac{2B(bc-ad)i\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g} + \frac{di(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2g}$$
$$- \frac{(bc-ad)i\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g} + \frac{2B^2(bc-ad)i\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^2g}$$
$$+ \frac{2B(bc-ad)i\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g} + \frac{2B^2(bc-ad)i\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1214 vs. $2(286) = 572$.

Time = 0.82 (sec) , antiderivative size = 1214, normalized size of antiderivative = 4.24

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{i\left(3A^2bdx+3A^2(bc-ad)\log(a+bx)-3AB\left(ad\log^2\left(\frac{a}{b}+x\right)-2ad\log\left(\frac{a}{b}+x\right)(1+\log(a+bx))+2\left(-\right.\right.\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g
*x),x]
```

```
[Out] (i*(3*A^2*b*d*x + 3*A^2*(b*c - a*d)*Log[a + b*x] - 3*A*B*(a*d*Log[a/b + x]^
2 - 2*a*d*Log[a/b + x]*(1 + Log[a + b*x]) + 2*(-(b*c) + a*d + Log[c/d + x]*
(b*c + a*d*Log[a + b*x] - a*d*Log[(d*(a + b*x))/(-(b*c) + a*d)]) + (-(b*d*x
) + a*d*Log[a + b*x])*Log[(e*(a + b*x))/(c + d*x)]) - 2*a*d*PolyLog[2, (b*(
c + d*x))/(b*c - a*d)]) + 3*A*b*B*c*(Log[a/b + x]^2 - 2*Log[a + b*x]*(Log[a
/b + x] - Log[c/d + x] - Log[(e*(a + b*x))/(c + d*x)]) - 2*(Log[c/d + x]*Lo
g[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) -
 B^2*(a*d*Log[a/b + x]^3 - 3*d*(2*b*x - 2*(a + b*x)*Log[a/b + x] + (a + b*x
)*Log[a/b + x]^2) - 3*b*(2*d*x - 2*(c + d*x)*Log[c/d + x] + (c + d*x)*Log[c
/d + x]^2) - 3*d*(b*x - a*Log[a + b*x])*(-Log[a/b + x] + Log[c/d + x] + Log
[(e*(a + b*x))/(c + d*x)])^2 + 6*(a*d + 2*b*d*x - b*d*x*Log[c/d + x] - b*c*
Log[c + d*x] + Log[a/b + x]*(-(d*(a + b*x)) + d*(a + b*x)*Log[c/d + x] + (b
*c - a*d)*Log[(b*(c + d*x))/(b*c - a*d)]) + (b*c - a*d)*PolyLog[2, (d*(a +
b*x))/(-(b*c) + a*d)]) - 3*(Log[a/b + x] - Log[c/d + x] - Log[(e*(a + b*x))
/(c + d*x)])*(-2*b*c + 2*a*d - 2*d*(a + b*x)*Log[a/b + x] + a*d*Log[a/b + x
]^2 + 2*Log[c/d + x]*(b*(c + d*x) - a*d*Log[(d*(a + b*x))/(-(b*c) + a*d)])
- 2*a*d*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 3*a*d*(Log[a/b + x]^2*(Log
[c/d + x] - Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Log[a/b + x]*PolyLog[2, (d*
(a + b*x))/(-(b*c) + a*d)] + 2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)]) +
3*a*d*(Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[c/d + x]*Po
```

```
lyLog[2, (b*(c + d*x))/(b*c - a*d)] - 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d
)])) + b*B^2*c*(Log[a/b + x]^3 + 3*Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c)
 + a*d)] + 3*Log[a + b*x]*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))
/(c + d*x)])^2 + 3*Log[a/b + x]^2*(-Log[c/d + x] + Log[(b*(c + d*x))/(b*c -
 a*d)]) + 6*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 6*Log[c
/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 3*(Log[a/b + x] - Log[c/d +
 x] - Log[(e*(a + b*x))/(c + d*x)])*(Log[a/b + x]^2 - 2*(Log[c/d + x]*Log[(
d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - 6*
PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*PolyLog[3, (b*(c + d*x))/(b*c
- a*d)])))/(3*b^2*g)
```

Maple [F]

$$\int \frac{(dix+ci)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{bgx+ag}dx$$

```
[In] int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x)
[Out] int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algorit
hm="fricas")
[Out] integral((A^2*d*i*x + A^2*c*i + (B^2*d*i*x + B^2*c*i)*log((b*e*x + a*e)/(d*
x + c))^2 + 2*(A*B*d*i*x + A*B*c*i)*log((b*e*x + a*e)/(d*x + c)))/(b*g*x +
a*g), x)
```

Sympy [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \frac{i\left(\int \frac{A^2c}{a+bx}\,dx+\int \frac{A^2dx}{a+bx}\,dx+\int \frac{B^2c\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a+bx}\,dx+\int \frac{2ABc\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx+\int \frac{B^2dx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a+bx}\,dx+\right.}{g}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g),x)
```

```
[Out] i*(Integral(A**2*c/(a + b*x), x) + Integral(A**2*d*x/(a + b*x), x) + Integr
al(B**2*c*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a + b*x), x) + Integral(
2*A*B*c*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) + Integral(B**2*
d*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a + b*x), x) + Integral(2*A*B*
d*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algorit
hm="maxima")
```

```
[Out] A^2*d*i*(x/(b*g) - a*log(b*x + a)/(b^2*g)) + A^2*c*i*log(b*g*x + a*g)/(b*g)
 + (B^2*b*d*i*x + (b*c*i - a*d*i)*B^2*log(b*x + a))*log(d*x + c)^2/(b^2*g)
- integrate(-(B^2*b^2*c^2*i*log(e)^2 + 2*A*B*b^2*c^2*i*log(e) + (B^2*b^2*d^
2*i*log(e)^2 + 2*A*B*b^2*d^2*i*log(e))*x^2 + (B^2*b^2*d^2*i*x^2 + 2*B^2*b^2
*c*d*i*x + B^2*b^2*c^2*i)*log(b*x + a)^2 + 2*(B^2*b^2*c*d*i*log(e)^2 + 2*A*
B*b^2*c*d*i*log(e))*x + 2*(B^2*b^2*c^2*i*log(e) + A*B*b^2*c^2*i + (B^2*b^2*
d^2*i*log(e) + A*B*b^2*d^2*i)*x^2 + 2*(B^2*b^2*c*d*i*log(e) + A*B*b^2*c*d*i
)*x)*log(b*x + a) - 2*(B^2*b^2*c^2*i*log(e) + A*B*b^2*c^2*i + ((i*log(e) +
i)*B^2*b^2*d^2 + A*B*b^2*d^2*i)*x^2 + (2*A*B*b^2*c*d*i + (2*b^2*c*d*i*log(e
) + a*b*d^2*i)*B^2)*x + (B^2*b^2*d^2*i*x^2 + (3*b^2*c*d*i - a*b*d^2*i)*B^2*
x + (b^2*c^2*i + a*b*c*d*i - a^2*d^2*i)*B^2)*log(b*x + a))*log(d*x + c))/(b
^3*d*g*x^2 + a*b^2*c*g + (b^3*c*g + a*b^2*d*g)*x), x)
```

Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algorit
hm="giac")
```

```
[Out] integrate((d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*g),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

```
[In] int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x),x)
[Out] int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x), x
)
```

3.60 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 241

$$\begin{aligned}
&\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx \\
&= -\frac{2B^2i(c+dx)}{bg^2(a+bx)} - \frac{2Bi(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)} \\
&\quad - \frac{i(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{bg^2(a+bx)} - \frac{di\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} \\
&\quad + \frac{2Bdi\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} + \frac{2B^2di\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}
\end{aligned}$$

```
[Out] -2*B^2*i*(d*x+c)/b/g^2/(b*x+a)-2*B*i*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/
g^2/(b*x+a)-i*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b/g^2/(b*x+a)-d*i*(A+B*
ln(e*(b*x+a)/(d*x+c)))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^2/g^2+2*B*d*i*(A+B*ln(
e*(b*x+a)/(d*x+c)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/g^2+2*B^2*d*i*polylo
g(3,b*(d*x+c)/d/(b*x+a))/b^2/g^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {2562, 2380, 2342, 2341, 2379, 2421, 6724}

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}dx = \frac{2Bdi\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^2g^2} - \frac{di\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^2g^2} - \frac{2Bi(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg^2(a+bx)} - \frac{i(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{bg^2(a+bx)} + \frac{2B^2di\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} - \frac{2B^2i(c+dx)}{bg^2(a+bx)}$$

```
[In] Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)^2,
x]
```

```
[Out] (-2*B^2*i*(c + d*x))/(b*g^2*(a + b*x)) - (2*B*i*(c + d*x)*(A + B*Log[(e*(a
+ b*x))/(c + d*x)]))/(b*g^2*(a + b*x)) - (i*(c + d*x)*(A + B*Log[(e*(a + b*
x))/(c + d*x)])^2)/(b*g^2*(a + b*x)) - (d*i*(A + B*Log[(e*(a + b*x))/(c + d
*x)])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^2*g^2) + (2*B*d*i*(A + B*L
og[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b^2*
g^2) + (2*B^2*d*i*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))])/(b^2*g^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{i\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^2(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^2}\\ &= \frac{i\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^2} + \frac{(di)\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^2}\end{aligned}$$

$$= -\frac{i(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{bg^2(a+bx)} - \frac{di\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}$$
$$+ \frac{(2Bi)\text{Subst}\left(\int \frac{A+B\log(ex)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^2}$$
$$+ \frac{(2Bdi)\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)(A+B\log(ex))}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2}$$

$$= -\frac{2B^2i(c+dx)}{bg^2(a+bx)} - \frac{2Bi(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)}$$
$$- \frac{i(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{bg^2(a+bx)} - \frac{di\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}$$
$$+ \frac{2Bdi\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}$$
$$- \frac{(2B^2di)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2}$$

$$= -\frac{2B^2i(c+dx)}{bg^2(a+bx)} - \frac{2Bi(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)}$$
$$- \frac{i(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{bg^2(a+bx)} - \frac{di\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}$$
$$+ \frac{2Bdi\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} + \frac{2B^2di\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1407 vs. 2(241) = 482.

Time = 1.21 (sec) , antiderivative size = 1407, normalized size of antiderivative = 5.84

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$$
$$= \frac{i\left(\frac{3A^2(-bc+ad)}{a+bx} + 3A^2d\log(a+bx) - \frac{6AbBc\left(-d(a+bx)\log\left(\frac{c}{d}+x\right)+d(a+bx)\log\left(\frac{d(a+bx)}{-bc+ad}\right)+(bc-ad)\left(1+\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\right)}{(bc-ad)(a+bx)} + \frac{3bB^2c\left(\right.}{}\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g
*x)^2,x]
```

```
[Out] (i*((3*A^2*(-(b*c) + a*d))/(a + b*x) + 3*A^2*d*Log[a + b*x] - (6*A*b*B*c*(-
(d*(a + b*x)*Log[c/d + x]) + d*(a + b*x)*Log[(d*(a + b*x))/(-(b*c) + a*d)]
+ (b*c - a*d)*(1 + Log[(e*(a + b*x))/(c + d*x)])))/((b*c - a*d)*(a + b*x))
+ (3*b*B^2*c*(-2*b*c + 2*a*d - 2*d*(a + b*x)*Log[a + b*x] - 2*(b*c - a*d)*L
og[(e*(a + b*x))/(c + d*x)] - 2*d*(a + b*x)*Log[a + b*x]*Log[(e*(a + b*x))/
(c + d*x)] - (b*c - a*d)*Log[(e*(a + b*x))/(c + d*x)]^2 + 2*d*(a + b*x)*Log
[c + d*x] - 2*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)]*Log[(b*c - a*d)/(b*c
 + b*d*x)] + d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/
(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + d*(a + b*x)*(
Log[(b*c - a*d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(
b*c - a*d)/(b*c + b*d*x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/((b
*c - a*d)*(a + b*x)) + 3*A*B*d*(Log[a/b + x]^2 - 2*Log[a/b + x]*Log[a + b*x
] - 2*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[a + b*x]*((a*d
)/(b*c - a*d) + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)]) + 2*a*((a + b*
x)^(-1) + Log[(e*(a + b*x))/(c + d*x)]/(a + b*x) + (d*Log[c + d*x])/(-(b*c)
 + a*d)) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + (B^2*d*((b*c - a*d)*(
a + b*x)*Log[a/b + x]^3 + 3*a*(b*c - a*d)*(2 + 2*Log[a/b + x] + Log[a/b + x
]^2) + 3*(b*c - a*d)*(a + (a + b*x)*Log[a + b*x])*(-Log[a/b + x] + Log[c/d
+ x] + Log[(e*(a + b*x))/(c + d*x)])^2 + 3*a*(d*(a + b*x)*Log[a/b + x]^2 +
2*((-(b*c) + a*d)*Log[c/d + x] + d*(a + b*x)*(Log[a + b*x] - Log[c + d*x]))
 - 2*Log[a/b + x]*((b*c - a*d)*Log[c/d + x] + d*(a + b*x)*Log[(b*(c + d*x))
/(b*c - a*d)]) - 2*d*(a + b*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) +
3*a*(Log[c/d + x]*(b*(c + d*x)*Log[c/d + x] - 2*d*(a + b*x)*Log[(d*(a + b*x
))/(-(b*c) + a*d)]) - 2*d*(a + b*x)*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
- 3*(Log[a/b + x] - Log[c/d + x] - Log[(e*(a + b*x))/(c + d*x)])*((b*c - a*
d)*(a + b*x)*Log[a/b + x]^2 + 2*a*(b*c - a*d)*(1 + Log[a/b + x]) + 2*a*(-(b
*c) + a*d)*Log[c/d + x] + 2*a*d*(a + b*x)*(Log[a + b*x] - Log[c + d*x]) - 2
*(b*c - a*d)*(a + b*x)*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Po
lyLog[2, (b*(c + d*x))/(b*c - a*d)])) - 3*(b*c - a*d)*(a + b*x)*(Log[a/b +
x]^2*(Log[c/d + x] - Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Log[a/b + x]*PolyL
og[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2*PolyLog[3, (d*(a + b*x))/(-(b*c) +
a*d)]) + 3*(b*c - a*d)*(a + b*x)*(Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c)
+ a*d)] + 2*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 2*PolyLog[
3, (b*(c + d*x))/(b*c - a*d)])))/((b*c - a*d)*(a + b*x))))/(3*b^2*g^2)
```

Maple [F]

$$\int \frac{(dix+ci)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(bgx+ag)^2}dx$$

```
[In] int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x)

[Out] int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x)
```

Fricas [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algor
ithm="fricas")
```

```
[Out] integral((A^2*d*i*x + A^2*c*i + (B^2*d*i*x + B^2*c*i)*log((b*e*x + a*e)/(d*
x + c))^2 + 2*(A*B*d*i*x + A*B*c*i)*log((b*e*x + a*e)/(d*x + c)))/(b^2*g^2*
x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx=\text{Timed out}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algor
ithm="maxima")
```

```
[Out] A^2*d*i*(a/(b^3*g^2*x + a*b^2*g^2) + log(b*x + a)/(b^2*g^2)) - 2*A*B*c*i*(l
og(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2) + 1/(b^2*g^2*x +
a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/((b^2*c -
a*b*d)*g^2)) - A^2*c*i/(b^2*g^2*x + a*b*g^2) - ((b*c*i - a*d*i)*B^2 - (B^2*
b*d*i*x + B^2*a*d*i)*log(b*x + a))*log(d*x + c)^2/(b^3*g^2*x + a*b^2*g^2) -
 integrate(-(B^2*b^2*c^2*i*log(e)^2 + (B^2*b^2*d^2*i*log(e)^2 + 2*A*B*b^2*d
^2*i*log(e))*x^2 + (B^2*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i*x + B^2*b^2*c^2*i)*
log(b*x + a)^2 + 2*(B^2*b^2*c*d*i*log(e)^2 + A*B*b^2*c*d*i*log(e))*x + 2*(B
^2*b^2*c^2*i*log(e) + (B^2*b^2*d^2*i*log(e) + A*B*b^2*d^2*i)*x^2 + (2*B^2*b
^2*c*d*i*log(e) + A*B*b^2*c*d*i)*x)*log(b*x + a) - 2*((b^2*c^2*i*log(e) - a
```

```
*b*c*d*i + a^2*d^2*i)*B^2 + (B^2*b^2*d^2*i*log(e) + A*B*b^2*d^2*i)*x^2 + (A
*B*b^2*c*d*i + ((2*i*log(e) - i)*b^2*c*d + a*b*d^2*i)*B^2)*x + (2*B^2*b^2*d
^2*i*x^2 + 2*(b^2*c*d*i + a*b*d^2*i)*B^2*x + (b^2*c^2*i + a^2*d^2*i)*B^2)*l
og(b*x + a))*log(d*x + c))/(b^4*d*g^2*x^3 + a^2*b^2*c*g^2 + (b^4*c*g^2 + 2*
a*b^3*d*g^2)*x^2 + (2*a*b^3*c*g^2 + a^2*b^2*d*g^2)*x), x)
```

Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algor
ithm="giac")
```

```
[Out] integrate((d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*g)^
2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{{(a\,g+b\,g\,x)}^2}\,dx$$

```
[In] int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^2,
x)
```

```
[Out] int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^2,
 x)
```

3.61 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 141

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{B^2i(c+dx)^2}{4(bc-ad)g^3(a+bx)^2} - \frac{Bi(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)g^3(a+bx)^2} - \frac{i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)g^3(a+bx)^2}$$

```
[Out] -1/4*B^2*i*(d*x+c)^2/(-a*d+b*c)/g^3/(b*x+a)^2-1/2*B*i*(d*x+c)^2*(A+B*ln(e*(
b*x+a)/(d*x+c)))/(-a*d+b*c)/g^3/(b*x+a)^2-1/2*i*(d*x+c)^2*(A+B*ln(e*(b*x+a)
/(d*x+c)))^2/(-a*d+b*c)/g^3/(b*x+a)^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2562, 2342, 2341}

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{i(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g^3(a+bx)^2(bc-ad)} - \frac{Bi(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^3(a+bx)^2(bc-ad)} - \frac{B^2i(c+dx)^2}{4g^3(a+bx)^2(bc-ad)}$$

```
[In] Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)^3,
x]
```

```
[Out] -1/4*(B^2*i*(c + d*x)^2)/((b*c - a*d)*g^3*(a + b*x)^2) - (B*i*(c + d*x)^2*(
A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*c - a*d)*g^3*(a + b*x)^2) - (i*(
c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(b*c - a*d)*g^3*(a +
b*x)^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^3}\\
&= -\frac{i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)g^3(a+bx)^2} + \frac{(Bi)\text{Subst}\left(\int \frac{A+B\log(ex)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^3}\\
&= -\frac{B^2i(c+dx)^2}{4(bc-ad)g^3(a+bx)^2} - \frac{Bi(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)g^3(a+bx)^2}\\
&\quad - \frac{i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)g^3(a+bx)^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.53 (sec) , antiderivative size = 765, normalized size of antiderivative = 5.43

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{i\left(2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-4d(-bc+ad)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+4Bd(a+bx)\left(\right.\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g
*x)^3,x]
```

```
[Out] -1/4*(i*(2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - 4*d*(-(b*
c) + a*d)*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 4*B*d*(a + b*x
)*(2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*d*(a + b*x)*Log[a
 + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*d*(a + b*x)*(A + B*Log[(e*
(a + b*x))/(c + d*x)])*Log[c + d*x] + 2*B*(b*c - a*d + d*(a + b*x)*Log[a +
b*x] - d*(a + b*x)*Log[c + d*x]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x
] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c)
+ a*d)]) + B*d*(a + b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*
x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + B*(2*(b*c -
a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*d*(-(b*c) + a*d)*(a + b*x)*
(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 4*d^2*(a + b*x)^2*Log[a + b*x]*(A +
B*Log[(e*(a + b*x))/(c + d*x)]) + 4*d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x)
)/(c + d*x)])*Log[c + d*x] - 4*B*d*(a + b*x)*(b*c - a*d + d*(a + b*x)*Log[a
 + b*x] - d*(a + b*x)*Log[c + d*x]) + B*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)
*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x
]) + 2*B*d^2*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/
(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2*B*d^2*(a +
b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] +
 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))))/(b^2*(b*c - a*d)*g^3*(a + b*x)
^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(135) = 270.

Time = 0.80 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.34

method	result
parts	$\frac{i\,A^2\left(-\frac{-ad+cb}{2b^2(bx+a)^2}-\frac{d}{b^2(bx+a)}\right)}{g^3}-\frac{i\,B^2e^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}\right)}{g^3(ad-cb)}-\frac{2iBA}{}$
norman	$\frac{\frac{B^2cdix\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g(ad-cb)}+\frac{ciBd(2A+B)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(ad-cb)}-\frac{2A^2aid+2A^2bci+2adiBA+2bciBA+adi\,B^2+B^2bci}{4g\,b^2}-\frac{\left(2A^2id+2diBA+B^2\right.}{2gb}}{(bx+a)^2g^2}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{i\,d^2e\,A^2}{2(ad-cb)^2g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{2i\,d^2eAB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}\right)}{(ad-cb)^2g^3}+\frac{i\,d^2e\,B^2\left(-\frac{\ln\left(\frac{be}{d}\right.}{2\left(\frac{be}{d}\right.}\right.}{}\right.}{d^2}$
default	$-\frac{e(ad-cb)\left(-\frac{i\,d^2e\,A^2}{2(ad-cb)^2g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{2i\,d^2eAB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}\right)}{(ad-cb)^2g^3}+\frac{i\,d^2e\,B^2\left(-\frac{\ln\left(\frac{be}{d}\right.}{2\left(\frac{be}{d}\right.}\right.}{}\right.}{d^2}$
risch	$\frac{i\,A^2ad}{2g^3b^2(bx+a)^2}-\frac{i\,A^2c}{2g^3b(bx+a)^2}-\frac{i\,A^2d}{g^3b^2(bx+a)}+\frac{i\,B^2e^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2g^3(ad-cb)\left(\frac{be}{d}+\frac{ea}{dx+c}-\frac{ecb}{d(dx+c)}\right)^2}+\frac{i\,B^2e^2\ln\left(\frac{be}{d}+\frac{(ad-c}{d(dx+}\right.}{2g^3(ad-cb)\left(\frac{be}{d}+\frac{ea}{dx+c}-\right.}$
parallelrisch	$-\frac{2A^2a^2b^2d^3i+B^2a^2b^2d^3i-B^2b^4c^2di-2A^2c^2i\,b^4d+2AB\,a^2b^2d^3i-2AB\,b^4c^2di-8ABx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^4c\,d^2i-4AB\,x^2\ln}{}$

```
[In] int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x,method=_RETUR
NVERBOSE)
```

```
[Out] i*A^2/g^3*(-1/2*(-a*d+b*c)/b^2/(b*x+a)^2-d/b^2/(b*x+a))-i*B^2/g^3/(a*d-b*c)
*e^2*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^
2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(
b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-2*i*B*A/g^3/(a*d-b*c)*e^2*(-1/2/(b*e/d+(a*d
-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e
/d/(d*x+c))^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(135) = 270$.

Time = 0.31 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.05

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$-\frac{2\left((2A^2+2AB+B^2)b^2cd-(2A^2+2AB+B^2)abd^2\right)ix+2\left(B^2b^2d^2ix^2+2B^2b^2cdix+B^2b^2c^2i\right)\log\left(\right.}{4\left((b^5c-\right.}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algor
ithm="fricas")
```

```
[Out] -1/4*(2*((2*A^2 + 2*A*B + B^2)*b^2*c*d - (2*A^2 + 2*A*B + B^2)*a*b*d^2)*i*x
 + 2*(B^2*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i*x + B^2*b^2*c^2*i)*log((b*e*x + a
*e)/(d*x + c))^2 + ((2*A^2 + 2*A*B + B^2)*b^2*c^2 - (2*A^2 + 2*A*B + B^2)*a
^2*d^2)*i + 2*((2*A*B + B^2)*b^2*d^2*i*x^2 + 2*(2*A*B + B^2)*b^2*c*d*i*x +
(2*A*B + B^2)*b^2*c^2*i)*log((b*e*x + a*e)/(d*x + c)))/((b^5*c - a*b^4*d)*g
^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 714 vs. $2(122) = 244$.

Time = 5.02 (sec) , antiderivative size = 714, normalized size of antiderivative = 5.06

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{Bd^2i(2A+B)\log\left(x+\frac{2ABad^3i+2ABbcd^2i+B^2ad^3i+B^2bcd^2i-\frac{Ba^2d^4i(2A+B)}{ad-bc}+\frac{2Babcd^3i(2A+B)}{ad-bc}-\frac{Bb^2c^2d^2i(2A+B)}{ad-bc}}{4ABbd^3i+2B^2bd^3i}\right)}{2b^2g^3(ad-bc)} + \frac{Bd^2i(2A+B)\log\left(x+\frac{2ABad^3i+2ABbcd^2i+B^2ad^3i+B^2bcd^2i+\frac{Ba^2d^4i(2A+B)}{ad-bc}-\frac{2Babcd^3i(2A+B)}{ad-bc}+\frac{Bb^2c^2d^2i(2A+B)}{ad-bc}}{4ABbd^3i+2B^2bd^3i}\right)}{2b^2g^3(ad-bc)} + \frac{(B^2c^2i+2B^2cdix+B^2d^2ix^2)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2a^3dg^3-2a^2bcg^3+4a^2bdg^3x-4ab^2cg^3x+2ab^2dg^3x^2-2b^3cg^3x^2} + \frac{-2A^2adi-2A^2bci-2ABadi-2ABbci-B^2adi-B^2bci+x(-4A^2bdi-4ABbdi-2B^2bdi)}{4a^2b^2g^3+8ab^3g^3x+4b^4g^3x^2} + \frac{(-2ABadi-2ABbci-4ABbdix-B^2adi-B^2bci-2B^2bdix)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2a^2b^2g^3+4ab^3g^3x+2b^4g^3x^2}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3,x)
```

```
[Out] -B*d**2*i*(2*A + B)*log(x + (2*A*B*a*d**3*i + 2*A*B*b*c*d**2*i + B**2*a*d**
3*i + B**2*b*c*d**2*i - B*a**2*d**4*i*(2*A + B)/(a*d - b*c) + 2*B*a*b*c*d**
3*i*(2*A + B)/(a*d - b*c) - B*b**2*c**2*d**2*i*(2*A + B)/(a*d - b*c))/(4*A*
B*b*d**3*i + 2*B**2*b*d**3*i))/(2*b**2*g**3*(a*d - b*c)) + B*d**2*i*(2*A +
B)*log(x + (2*A*B*a*d**3*i + 2*A*B*b*c*d**2*i + B**2*a*d**3*i + B**2*b*c*d*
*2*i + B*a**2*d**4*i*(2*A + B)/(a*d - b*c) - 2*B*a*b*c*d**3*i*(2*A + B)/(a*
d - b*c) + B*b**2*c**2*d**2*i*(2*A + B)/(a*d - b*c))/(4*A*B*b*d**3*i + 2*B*
*2*b*d**3*i))/(2*b**2*g**3*(a*d - b*c)) + (B**2*c**2*i + 2*B**2*c*d*i*x + B
**2*d**2*i*x**2)*log(e*(a + b*x)/(c + d*x))**2/(2*a**3*d*g**3 - 2*a**2*b*c*
g**3 + 4*a**2*b*d*g**3*x - 4*a*b**2*c*g**3*x + 2*a*b**2*d*g**3*x**2 - 2*b**
```

```
3*c*g**3*x**2) + (-2*A**2*a*d*i - 2*A**2*b*c*i - 2*A*B*a*d*i - 2*A*B*b*c*i
- B**2*a*d*i - B**2*b*c*i + x*(-4*A**2*b*d*i - 4*A*B*b*d*i - 2*B**2*b*d*i))
/(4*a**2*b**2*g**3 + 8*a*b**3*g**3*x + 4*b**4*g**3*x**2) + (-2*A*B*a*d*i -
2*A*B*b*c*i - 4*A*B*b*d*i*x - B**2*a*d*i - B**2*b*c*i - 2*B**2*b*d*i*x)*log
(e*(a + b*x)/(c + d*x))/(2*a**2*b**2*g**3 + 4*a*b**3*g**3*x + 2*b**4*g**3*x
**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1987 vs. $2(135) = 270$.

Time = 0.30 (sec) , antiderivative size = 1987, normalized size of antiderivative = 14.09

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algor
ithm="maxima")
```

```
[Out] -1/2*(2*b*x + a)*B^2*d*i*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^4*g^3*x^
2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) + 1/4*(2*((2*b*d*x - b*c + 3*a*d)/((b^4*c
- a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*
g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2
*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3))*log(b*e*x/(d*x + c
) + a*e/(d*x + c)) - (b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*
a*b*d^2*x + a^2*d^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^
2)*log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x
+ a^2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b
^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))*log(d*x + c))/(a^2*b^3*c^
2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*d^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3
+ a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*c^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2
*g^3)*x))*B^2*c*i - 1/4*(2*((3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*x)/((b^5
*c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^
2*d)*g^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*c*d + a^2*
b^2*d^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a*b^3*c*d +
a^2*b^2*d^2)*g^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + (7*a*b^2*c^2 - 8*
a^2*b*c*d + a^3*d^2 - 2*(2*a^2*b*c*d - a^3*d^2 + (2*b^3*c*d - a*b^2*d^2)*x^
2 + 2*(2*a*b^2*c*d - a^2*b*d^2)*x)*log(b*x + a)^2 - 2*(2*a^2*b*c*d - a^3*d^
2 + (2*b^3*c*d - a*b^2*d^2)*x^2 + 2*(2*a*b^2*c*d - a^2*b*d^2)*x)*log(d*x +
c)^2 + 2*(4*b^3*c^2 - 5*a*b^2*c*d + a^2*b*d^2)*x + 2*(4*a^2*b*c*d - a^3*d^2
 + (4*b^3*c*d - a*b^2*d^2)*x^2 + 2*(4*a*b^2*c*d - a^2*b*d^2)*x)*log(b*x + a
) - 2*(4*a^2*b*c*d - a^3*d^2 + (4*b^3*c*d - a*b^2*d^2)*x^2 + 2*(4*a*b^2*c*d
 - a^2*b*d^2)*x - 2*(2*a^2*b*c*d - a^3*d^2 + (2*b^3*c*d - a*b^2*d^2)*x^2 +
2*(2*a*b^2*c*d - a^2*b*d^2)*x)*log(b*x + a))*log(d*x + c))/(a^2*b^4*c^2*g^3
 - 2*a^3*b^3*c*d*g^3 + a^4*b^2*d^2*g^3 + (b^6*c^2*g^3 - 2*a*b^5*c*d*g^3 + a
```

```
^2*b^4*d^2*g^3)*x^2 + 2*(a*b^5*c^2*g^3 - 2*a^2*b^4*c*d*g^3 + a^3*b^3*d^2*g^
3)*x))*B^2*d*i - 1/2*A*B*d*i*(2*(2*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x
+ c))/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) + (3*a*b*c - a^2*d + 2*(2
*b^2*c - a*b*d)*x)/((b^5*c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3
*x + (a^2*b^3*c - a^3*b^2*d)*g^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*
c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((
b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3)) + 1/2*A*B*c*i*((2*b*d*x - b*c +
3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^
2*c - a^3*b*d)*g^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^3*g^3*x^2 +
 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d +
a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g
^3)) - 1/2*B^2*c*i*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^3*g^3*x^2 + 2*
a*b^2*g^3*x + a^2*b*g^3) - 1/2*(2*b*x + a)*A^2*d*i/(b^4*g^3*x^2 + 2*a*b^3*g
^3*x + a^2*b^2*g^3) - 1/2*A^2*c*i/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.46

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{1}{4}\left(\frac{2\,(dx+c)^2B^2e^3i\log\left(\frac{bex+ae}{dx+c}\right)^2}{(bex+ae)^2g^3}+\frac{2\,(2\,ABe^3i+B^2e^3i)(dx+c)^2\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^2g^3}+\frac{(2\,A^2e^3i+2\,ABe^3i+}{(bex+ae)}\right.$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algor
ithm="giac")
```

```
[Out] -1/4*(2*(d*x + c)^2*B^2*e^3*i*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a*e)
^2*g^3) + 2*(2*A*B*e^3*i + B^2*e^3*i)*(d*x + c)^2*log((b*e*x + a*e)/(d*x +
c))/((b*e*x + a*e)^2*g^3) + (2*A^2*e^3*i + 2*A*B*e^3*i + B^2*e^3*i)*(d*x +
c)^2/((b*e*x + a*e)^2*g^3))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*
e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 2.81 (sec) , antiderivative size = 469, normalized size of antiderivative = 3.33

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{x\,(2\,b\,d\,i\,A^2+2\,b\,d\,i\,A\,B+b\,d\,i\,B^2)+A^2\,a\,d\,i+A^2\,b\,c\,i+\frac{B^2\,a\,d\,i}{2}+\frac{B^2\,b\,c\,i}{2}+A\,B\,a\,d\,i+A\,B\,b\,c\,i}{2\,a^2\,b^2\,g^3+4\,a\,b^3\,g^3\,x+2\,b^4\,g^3\,x^2} - \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{\frac{B^2\,c\,i}{2\,b^2\,g^3}+\frac{B^2\,a\,d\,i}{2\,b^3\,g^3}+\frac{B^2\,d\,i\,x}{b^2\,g^3}}{2\,a\,x+b\,x^2+\frac{a^2}{b}}-\frac{B^2\,d^2\,i}{2\,b^2\,g^3\,(a\,d-b\,c)}\right) - \frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(x\left(\frac{B^2\,i}{b^2\,g^3}+\frac{2\,A\,B\,i}{b^2\,g^3}\right)+\frac{A\,B\,a\,i}{b^3\,g^3}+\frac{B\,i\,(A\,b\,c-B\,a\,d+B\,b\,c)}{b^3\,d\,g^3}+\frac{B^2\,d^2\,i\left(\frac{2\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2}{2\,b\,d^3}+\frac{a\,(a\,d-b\,c)}{2\,b\,d^2}\right)}{b^2\,g^3\,(a\,d-b\,c)}\right)}{\frac{b\,x^2}{d}+\frac{a^2}{b\,d}+\frac{2\,a\,x}{d}} - \frac{B\,d^2\,i\,\mathrm{atan}\left(\frac{\left(\frac{2\,c\,b^3\,g^3+2\,a\,d\,b^2\,g^3}{2\,b^2\,g^3}+2\,b\,d\,x\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(2\,A+B)\,1\mathrm{i}}{b^2\,g^3\,(a\,d-b\,c)}$$

```
[In] int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^3,
x)
```

```
[Out] - (x*(2*A^2*b*d*i + B^2*b*d*i + 2*A*B*b*d*i) + A^2*a*d*i + A^2*b*c*i + (B^2
*a*d*i)/2 + (B^2*b*c*i)/2 + A*B*a*d*i + A*B*b*c*i)/(2*a^2*b^2*g^3 + 2*b^4*g
^3*x^2 + 4*a*b^3*g^3*x) - log((e*(a + b*x))/(c + d*x))^2*(((B^2*c*i)/(2*b^2
*g^3) + (B^2*a*d*i)/(2*b^3*g^3) + (B^2*d*i*x)/(b^2*g^3))/(2*a*x + b*x^2 + a
^2/b) - (B^2*d^2*i)/(2*b^2*g^3*(a*d - b*c))) - (log((e*(a + b*x))/(c + d*x)
)*(x*((B^2*i)/(b^2*g^3) + (2*A*B*i)/(b^2*g^3)) + (A*B*a*i)/(b^3*g^3) + (B*i
*(A*b*c - B*a*d + B*b*c))/(b^3*d*g^3) + (B^2*d^2*i*((2*a^2*d^2 + b^2*c^2 -
3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)))/(b^2*g^3*(a*d - b*c))))/
((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - (B*d^2*i*atan((((2*b^3*c*g^3 + 2*a*b^
2*d*g^3)/(2*b^2*g^3) + 2*b*d*x)*1i)/(a*d - b*c))*(2*A + B)*1i)/(b^2*g^3*(a*
d - b*c))
```

3.62 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 287

$$\begin{aligned}\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx &= \frac{B^2di(c+dx)^2}{4(bc-ad)^2g^4(a+bx)^2} \\ &- \frac{2bB^2i(c+dx)^3}{27(bc-ad)^2g^4(a+bx)^3} \\ &+ \frac{Bdi(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^2g^4(a+bx)^2} \\ &- \frac{2bBi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^2g^4(a+bx)^3} \\ &+ \frac{di(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^2g^4(a+bx)^2} \\ &- \frac{bi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^2g^4(a+bx)^3}\end{aligned}$$

```
[Out] 1/4*B^2*d*i*(d*x+c)^2/(-a*d+b*c)^2/g^4/(b*x+a)^2-2/27*b*B^2*i*(d*x+c)^3/(-a
*d+b*c)^2/g^4/(b*x+a)^3+1/2*B*d*i*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a
*d+b*c)^2/g^4/(b*x+a)^2-2/9*b*B*i*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a
*d+b*c)^2/g^4/(b*x+a)^3+1/2*d*i*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a
*d+b*c)^2/g^4/(b*x+a)^2-1/3*b*i*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a
*d+b*c)^2/g^4/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2562, 2395, 2342, 2341}

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{bi(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g^4(a+bx)^3(bc-ad)^2} - \frac{2bBi(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{9g^4(a+bx)^3(bc-ad)^2} + \frac{di(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g^4(a+bx)^2(bc-ad)^2} + \frac{Bdi(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^4(a+bx)^2(bc-ad)^2} - \frac{2bB^2i(c+dx)^3}{27g^4(a+bx)^3(bc-ad)^2} + \frac{B^2di(c+dx)^2}{4g^4(a+bx)^2(bc-ad)^2}$$

```
[In] Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)^4,
x]
```

```
[Out] (B^2*d*i*(c + d*x)^2)/(4*(b*c - a*d)^2*g^4*(a + b*x)^2) - (2*b*B^2*i*(c + d
*x)^3)/(27*(b*c - a*d)^2*g^4*(a + b*x)^3) + (B*d*i*(c + d*x)^2*(A + B*Log[(
e*(a + b*x))/(c + d*x)]))/(2*(b*c - a*d)^2*g^4*(a + b*x)^2) - (2*b*B*i*(c +
 d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(9*(b*c - a*d)^2*g^4*(a + b*x
)^3) + (d*i*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(b*c - a
*d)^2*g^4*(a + b*x)^2) - (b*i*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x
)])^2)/(3*(b*c - a*d)^2*g^4*(a + b*x)^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^4} \\
&= \frac{i\text{Subst}\left(\int \left(\frac{b(A+B\log(ex))^2}{x^4}-\frac{d(A+B\log(ex))^2}{x^3}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^4} \\
&= \frac{(bi)\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^4}-\frac{(di)\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^4} \\
&= \frac{di(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^2g^4(a+bx)^2}-\frac{bi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^2g^4(a+bx)^3} \\
&\quad+\frac{(2bBi)\text{Subst}\left(\int \frac{A+B\log(ex)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bc-ad)^2g^4}-\frac{(Bdi)\text{Subst}\left(\int \frac{A+B\log(ex)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^4} \\
&= \frac{B^2di(c+dx)^2}{4(bc-ad)^2g^4(a+bx)^2}-\frac{2bB^2i(c+dx)^3}{27(bc-ad)^2g^4(a+bx)^3} \\
&\quad+\frac{Bdi(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^2g^4(a+bx)^2}-\frac{2bBi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^2g^4(a+bx)^3} \\
&\quad+\frac{di(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^2g^4(a+bx)^2}-\frac{bi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^2g^4(a+bx)^3}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.60 (sec) , antiderivative size = 1032, normalized size of antiderivative = 3.60

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{i\left(36(bc-ad)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+54d(bc-ad)^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+2B\left(12A(bc\right.\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g
*x)^4,x]
```

```
[Out] -1/108*(i*(36*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 54*d*(
b*c - a*d)^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 2*B*(12*A*(
b*c - a*d)^3 + 4*B*(b*c - a*d)^3 - 18*A*d*(b*c - a*d)^2*(a + b*x) - 15*B*d*
(b*c - a*d)^2*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(b*c
- a*d)*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x] + 66*B*d^3*(a + b*x)
^3*Log[a + b*x] - 18*B*d^3*(a + b*x)^3*Log[a + b*x]^2 + 12*B*(b*c - a*d)^3*
Log[(e*(a + b*x))/(c + d*x)] - 18*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b
*x))/(c + d*x)] + 36*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[(e*(a + b*x))/(c + d
*x)] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] - 36*
A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3*(a + b*x)^3*Log[c + d*x] + 36*B*d
^3*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 36*B*d^3*(a
 + b*x)^3*Log[(e*(a + b*x))/(c + d*x)]*Log[c + d*x] - 18*B*d^3*(a + b*x)^3*
Log[c + d*x]^2 + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c -
 a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 36
*B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 27*B*d*(a + b*x
)*(2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*d*(-(b*c) + a*d
)*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 4*d^2*(a + b*x)^2*Log[a
+ b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*d^2*(a + b*x)^2*(A + B*Log[
(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 4*B*d*(a + b*x)*(b*c - a*d + d*(a
+ b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) + B*((b*c - a*d)^2 + 2*d*(-
(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2
*Log[c + d*x]) + 2*B*d^2*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b
*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2
*B*d^2*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Lo
g[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))))/(b^2*(b*c - a*d)^2
*g^4*(a + b*x)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(275) = 550$.

Time = 0.99 (sec) , antiderivative size = 643, normalized size of antiderivative = 2.24

method	result
parts	$\frac{i A^{2}\left(-\frac{-ad+cb}{3b^{2}(bx+a)^{3}}-\frac{d}{2b^{2}(bx+a)^{2}}\right)}{g^{4}}-\frac{i B^{2}(ad-cb)^{2}e^{2}\left(\frac{d^{4}\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}}-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}}\right)}{(ad-cb)^{4}}\cdots\right)}{g^{4}d^{3}\cdots}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{i d^{2}e^{2}A^{2}b}{3(ad-cb)^{3}g^{4}\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{3}}-\frac{i d^{3}e A^{2}}{2(ad-cb)^{3}g^{4}\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}}-\frac{2i d^{2}e^{2}ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{3}}-\frac{1}{9\left(\frac{be}{d}+\frac{(ad\cdots}{d(d\cdots}\right.}\cdots\right)}{(ad-cb)^{3}g^{4}}\cdots\right)}{\cdots}$
default	$-\frac{e(ad-cb)\left(\frac{i d^{2}e^{2}A^{2}b}{3(ad-cb)^{3}g^{4}\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{3}}-\frac{i d^{3}e A^{2}}{2(ad-cb)^{3}g^{4}\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}}-\frac{2i d^{2}e^{2}ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{3}}-\frac{1}{9\left(\frac{be}{d}+\frac{(ad\cdots}{d(d\cdots}\right.}\cdots\right)}{(ad-cb)^{3}g^{4}}\cdots\right)}{\cdots}$
norman	$-\frac{\frac{18A^{2}a^{2}bd^{2}i+18A^{2}ab^{2}cdi-36A^{2}b^{3}c^{2}i+30ABa^{2}bd^{2}i+30ABab^{2}cdi-24ABb^{3}c^{2}i+19B^{2}a^{2}bd^{2}i+19B^{2}ab^{2}cdi-8B^{2}b^{3}c^{2}i}{108gb^{3}(ad-cb)}-\frac{(18A^{2}a\cdots}{\cdots}}{\cdots}$
parallelrisch	$-\frac{-216ABx\ln\left(\frac{e(bx+a)}{dx+c}\right)ab^{5}cd^{3}i-27B^{2}ab^{5}c^{2}d^{2}i-54ABab^{5}c^{2}d^{2}i-18B^{2}x^{3}\ln\left(\frac{e(bx+a)}{dx+c}\right)^{2}b^{6}d^{4}i-30B^{2}x^{3}\ln\left(\frac{e(bx+a)}{dx+c}\right)b\cdots}{\cdots}$
risch	Expression too large to display

```
[In] int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x,method=_RETUR
NVERBOSE)
```

```
[Out] i*A^2/g^4*(-1/3*(-a*d+b*c)/b^2/(b*x+a)^3-1/2*d/b^2/(b*x+a)^2)-i*B^2/g^4/d^3
*(a*d-b*c)^2*e^2*(d^4/(a*d-b*c)^4*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(
b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d
+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-d^3/(a*d-b*c)^
4*b*e*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))
^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/27
/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))-2*i*B*A/g^4/d^3*(a*d-b*c)^2*e^2*(d^4/(a*
d-b*c)^4*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+
c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-d^3/(a*d-b*c)^4*b*e*(-1/3/(b*e/d+(
a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c
)*e/d/(d*x+c))^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(275) = 550$.

Time = 0.31 (sec) , antiderivative size = 601, normalized size of antiderivative = 2.09

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^4}\,dx$$
$$= \frac{6\left(\left(6\,AB + 5\,B^2\right)b^3cd^2 - \left(6\,AB + 5\,B^2\right)ab^2d^3\right)ix^2 - 3\left(\left(18\,A^2 + 6\,AB - B^2\right)b^3c^2d - 18\left(2\,A^2 + 2\,AB + B^2\right)\right.}{}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algor
ithm="fricas")
```

```
[Out] 1/108*(6*((6*A*B + 5*B^2)*b^3*c*d^2 - (6*A*B + 5*B^2)*a*b^2*d^3)*i*x^2 - 3*
((18*A^2 + 6*A*B - B^2)*b^3*c^2*d - 18*(2*A^2 + 2*A*B + B^2)*a*b^2*c*d^2 +
(18*A^2 + 30*A*B + 19*B^2)*a^2*b*d^3)*i*x + 18*(B^2*b^3*d^3*i*x^3 + 3*B^2*a
*b^2*d^3*i*x^2 - 3*(B^2*b^3*c^2*d - 2*B^2*a*b^2*c*d^2)*i*x - (2*B^2*b^3*c^3
 - 3*B^2*a*b^2*c^2*d)*i)*log((b*e*x + a*e)/(d*x + c))^2 - (4*(9*A^2 + 6*A*B
 + 2*B^2)*b^3*c^3 - 27*(2*A^2 + 2*A*B + B^2)*a*b^2*c^2*d + (18*A^2 + 30*A*B
 + 19*B^2)*a^3*d^3)*i + 6*((6*A*B + 5*B^2)*b^3*d^3*i*x^3 + 3*(2*B^2*b^3*c*d
^2 + 3*(2*A*B + B^2)*a*b^2*d^3)*i*x^2 - 3*((6*A*B + B^2)*b^3*c^2*d - 6*(2*A
*B + B^2)*a*b^2*c*d^2)*i*x - (4*(3*A*B + B^2)*b^3*c^3 - 9*(2*A*B + B^2)*a*b
^2*c^2*d)*i)*log((b*e*x + a*e)/(d*x + c)))/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^
5*d^2)*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a
^2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*
c*d + a^5*b^2*d^2)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1387 vs. $2(267) = 534$.

Time = 10.34 (sec) , antiderivative size = 1387, normalized size of antiderivative = 4.83

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{Bd^3i(6A+5B)\log\left(x+\frac{6ABad^4i+6ABbcd^3i+5B^2ad^4i+5B^2bcd^3i-\frac{Ba^3d^6i(6A+5B)}{(ad-bc)^2}+\frac{3Ba^2bcd^5i(6A+5B)}{(ad-bc)^2}-\frac{3Bab^2c^2d^4i(6A+5B)}{(ad-bc)^2}+\frac{Bb}{}}{12ABbd^4i+10B^2bd^4i}\right.}{18b^2g^4(ad-bc)^2}$$

$$+\frac{Bd^3i(6A+5B)\log\left(x+\frac{6ABad^4i+6ABbcd^3i+5B^2ad^4i+5B^2bcd^3i+\frac{Ba^3d^6i(6A+5B)}{(ad-bc)^2}-\frac{3Ba^2bcd^5i(6A+5B)}{(ad-bc)^2}+\frac{3Bab^2c^2d^4i(6A+5B)}{(ad-bc)^2}-\frac{Bb^3}{}}{12ABbd^4i+10B^2bd^4i}\right.}{18b^2g^4(ad-bc)^2}$$

$$+\frac{(3B^2ac^2di+6B^2acd^2ix+3B^2ad^3ix^2-2B^2bc^3i-3B^2bc^2dix+}{6a^5d^2g^4-12a^4bcdg^4+18a^4bd^2g^4x+6a^3b^2c^2g^4-36a^3b^2cdg^4x+18a^3b^2d^2g^4x^2+18a^2b^3c^2g^4x-36a^2b^3c}$$

$$+\frac{(-6ABa^2d^2i-6ABabcdi-18ABabd^2ix+12ABb^2c^2i+18ABb^2cdix-5B^2a^2d^2i-5B^2abcdi-15B^2a}{18a^4b^2dg^4-18a^3b^3cg^4+54a^3b^3dg^4x-54a^2b^4cg^4x+54a^2b^4dg^4x^2-54ab^5c}$$

$$+\frac{-18A^2a^2d^2i-18A^2abcdi+36A^2b^2c^2i-30ABa^2d^2i-30ABabcdi+24ABb^2c^2i-19B^2a^2d^2i-19B^2ab}{108a^4b^2dg^4-108a^3b^3cg^4+x^3\cdot(108ab^5dg^4-}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**4,x)
```

```
[Out] -B*d**3*i*(6*A + 5*B)*log(x + (6*A*B*a*d**4*i + 6*A*B*b*c*d**3*i + 5*B**2*a
*d**4*i + 5*B**2*b*c*d**3*i - B*a**3*d**6*i*(6*A + 5*B)/(a*d - b*c)**2 + 3*
B*a**2*b*c*d**5*i*(6*A + 5*B)/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**4*i*(6*A
+ 5*B)/(a*d - b*c)**2 + B*b**3*c**3*d**3*i*(6*A + 5*B)/(a*d - b*c)**2)/(12*
A*B*b*d**4*i + 10*B**2*b*d**4*i))/(18*b**2*g**4*(a*d - b*c)**2) + B*d**3*i*
(6*A + 5*B)*log(x + (6*A*B*a*d**4*i + 6*A*B*b*c*d**3*i + 5*B**2*a*d**4*i +
5*B**2*b*c*d**3*i + B*a**3*d**6*i*(6*A + 5*B)/(a*d - b*c)**2 - 3*B*a**2*b*c
*d**5*i*(6*A + 5*B)/(a*d - b*c)**2 + 3*B*a*b**2*c**2*d**4*i*(6*A + 5*B)/(a*
d - b*c)**2 - B*b**3*c**3*d**3*i*(6*A + 5*B)/(a*d - b*c)**2)/(12*A*B*b*d**4
*i + 10*B**2*b*d**4*i))/(18*b**2*g**4*(a*d - b*c)**2) + (3*B**2*a*c**2*d*i
+ 6*B**2*a*c*d**2*i*x + 3*B**2*a*d**3*i*x**2 - 2*B**2*b*c**3*i - 3*B**2*b*c
**2*d*i*x + B**2*b*d**3*i*x**3)*log(e*(a + b*x)/(c + d*x))**2/(6*a**5*d**2*
g**4 - 12*a**4*b*c*d*g**4 + 18*a**4*b*d**2*g**4*x + 6*a**3*b**2*c**2*g**4 -
 36*a**3*b**2*c*d*g**4*x + 18*a**3*b**2*d**2*g**4*x**2 + 18*a**2*b**3*c**2*
g**4*x - 36*a**2*b**3*c*d*g**4*x**2 + 6*a**2*b**3*d**2*g**4*x**3 + 18*a*b**
4*c**2*g**4*x**2 - 12*a*b**4*c*d*g**4*x**3 + 6*b**5*c**2*g**4*x**3) + (-6*A
*B*a**2*d**2*i - 6*A*B*a*b*c*d*i - 18*A*B*a*b*d**2*i*x + 12*A*B*b**2*c**2*i
 + 18*A*B*b**2*c*d*i*x - 5*B**2*a**2*d**2*i - 5*B**2*a*b*c*d*i - 15*B**2*a*
b*d**2*i*x + 4*B**2*b**2*c**2*i + 3*B**2*b**2*c*d*i*x - 6*B**2*b**2*d**2*i*
x**2)*log(e*(a + b*x)/(c + d*x))/(18*a**4*b**2*d*g**4 - 18*a**3*b**3*c*g**4
 + 54*a**3*b**3*d*g**4*x - 54*a**2*b**4*c*g**4*x + 54*a**2*b**4*d*g**4*x**2
```

```
- 54*a*b**5*c*g**4*x**2 + 18*a*b**5*d*g**4*x**3 - 18*b**6*c*g**4*x**3) + (
-18*A**2*a**2*d**2*i - 18*A**2*a*b*c*d*i + 36*A**2*b**2*c**2*i - 30*A*B*a**
2*d**2*i - 30*A*B*a*b*c*d*i + 24*A*B*b**2*c**2*i - 19*B**2*a**2*d**2*i - 19
*B**2*a*b*c*d*i + 8*B**2*b**2*c**2*i + x**2*(-36*A*B*b**2*d**2*i - 30*B**2*
b**2*d**2*i) + x*(-54*A**2*a*b*d**2*i + 54*A**2*b**2*c*d*i - 90*A*B*a*b*d**
2*i + 18*A*B*b**2*c*d*i - 57*B**2*a*b*d**2*i - 3*B**2*b**2*c*d*i))/(108*a**
4*b**2*d*g**4 - 108*a**3*b**3*c*g**4 + x**3*(108*a*b**5*d*g**4 - 108*b**6*c
*g**4) + x**2*(324*a**2*b**4*d*g**4 - 324*a*b**5*c*g**4) + x*(324*a**3*b**3
*d*g**4 - 324*a**2*b**4*c*g**4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3282 vs. $2(275) = 550$.

Time = 0.42 (sec) , antiderivative size = 3282, normalized size of antiderivative = 11.44

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algor
ithm="maxima")
```

```
[Out] -1/6*(3*b*x + a)*B^2*d*i*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^5*g^4*x^
3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - 1/54*(6*((6*b^2*d^2*
x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6
*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d +
a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x
+ (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4
*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x +
c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4))*log(b*e*x
/(d*x + c) + a*e/(d*x + c)) + (4*b^3*c^3 - 27*a*b^2*c^2*d + 108*a^2*b*c*d^2
 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*
d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18*(b^3*d^3*x^3 + 3*a*b
^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 - 3*(5*b^3*c^2*d - 54*
a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b
*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*
a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x
 + a^3*d^3)*log(b*x + a))*log(d*x + c))/(a^3*b^4*c^3*g^4 - 3*a^4*b^3*c^2*d*
g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^4 + (b^7*c^3*g^4 - 3*a*b^6*c^2*d*g^
4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g^4)*x^3 + 3*(a*b^6*c^3*g^4 - 3*a^2*b
^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 - a^4*b^3*d^3*g^4)*x^2 + 3*(a^2*b^5*c^3*
g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^3*c*d^2*g^4 - a^5*b^2*d^3*g^4)*x))*B^2*
c*i - 1/108*(6*((5*a*b^2*c^2 - 22*a^2*b*c*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*
b^2*d^2)*x^2 + 3*(3*b^3*c^2 - 16*a*b^2*c*d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*
a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d
```

^2)*g^4*x^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^
4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4) - 6*(3*b*c*d^2 - a*d^3)*log(b*x +
a)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4) + 6*(3*
b*c*d^2 - a*d^3)*log(d*x + c)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 -
a^3*b^2*d^3)*g^4))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + (19*a*b^3*c^3 -
189*a^2*b^2*c^2*d + 189*a^3*b*c*d^2 - 19*a^4*d^3 - 6*(27*b^4*c^2*d - 32*a*b
^3*c*d^2 + 5*a^2*b^2*d^3)*x^2 + 18*(3*a^3*b*c*d^2 - a^4*d^3 + (3*b^4*c*d^2
- a*b^3*d^3)*x^3 + 3*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^2 + 3*(3*a^2*b^2*c*d^2
- a^3*b*d^3)*x)*log(b*x + a)^2 + 18*(3*a^3*b*c*d^2 - a^4*d^3 + (3*b^4*c*d^
2 - a*b^3*d^3)*x^3 + 3*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^2 + 3*(3*a^2*b^2*c*d
^2 - a^3*b*d^3)*x)*log(d*x + c)^2 + 3*(9*b^4*c^3 - 125*a*b^3*c^2*d + 135*a^
2*b^2*c*d^2 - 19*a^3*b*d^3)*x - 6*(27*a^3*b*c*d^2 - 5*a^4*d^3 + (27*b^4*c*d
^2 - 5*a*b^3*d^3)*x^3 + 3*(27*a*b^3*c*d^2 - 5*a^2*b^2*d^3)*x^2 + 3*(27*a^2*
b^2*c*d^2 - 5*a^3*b*d^3)*x)*log(b*x + a) + 6*(27*a^3*b*c*d^2 - 5*a^4*d^3 +
(27*b^4*c*d^2 - 5*a*b^3*d^3)*x^3 + 3*(27*a*b^3*c*d^2 - 5*a^2*b^2*d^3)*x^2 +
3*(27*a^2*b^2*c*d^2 - 5*a^3*b*d^3)*x - 6*(3*a^3*b*c*d^2 - a^4*d^3 + (3*b^4
*c*d^2 - a*b^3*d^3)*x^3 + 3*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^2 + 3*(3*a^2*b^
2*c*d^2 - a^3*b*d^3)*x)*log(b*x + a))*log(d*x + c))/(a^3*b^5*c^3*g^4 - 3*a^
4*b^4*c^2*d*g^4 + 3*a^5*b^3*c*d^2*g^4 - a^6*b^2*d^3*g^4 + (b^8*c^3*g^4 - 3*
a*b^7*c^2*d*g^4 + 3*a^2*b^6*c*d^2*g^4 - a^3*b^5*d^3*g^4)*x^3 + 3*(a*b^7*c^3
*g^4 - 3*a^2*b^6*c^2*d*g^4 + 3*a^3*b^5*c*d^2*g^4 - a^4*b^4*d^3*g^4)*x^2 + 3
*(a^2*b^6*c^3*g^4 - 3*a^3*b^5*c^2*d*g^4 + 3*a^4*b^4*c*d^2*g^4 - a^5*b^3*d^3
*g^4)*x))*B^2*d*i - 1/18*A*B*d*i*(6*(3*b*x + a)*log(b*e*x/(d*x + c) + a*e/(
d*x + c))/(b^5*g^4*x^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) +
(5*a*b^2*c^2 - 22*a^2*b*c*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 +
3*(3*b^3*c^2 - 16*a*b^2*c*d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2
*b^5*d^2)*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3
*(a^2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b
^3*c*d + a^5*b^2*d^2)*g^4) - 6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 -
3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4) + 6*(3*b*c*d^2 - a*d^3
)*log(d*x + c)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g
^4)) - 1/9*A*B*c*i*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3
*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 +
3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^
3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*
g^4) + 6*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^
2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c
^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3
*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*B^2*c*i*log(b*e*x/(
d*x + c) + a*e/(d*x + c))^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*
x + a^3*b*g^4) - 1/6*(3*b*x + a)*A^2*d*i/(b^5*g^4*x^3 + 3*a*b^4*g^4*x^2 + 3
*a^2*b^3*g^4*x + a^3*b^2*g^4) - 1/3*A^2*c*i/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2
+ 3*a^2*b^2*g^4*x + a^3*b*g^4)

Giac [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.59

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{1}{108}\left(\frac{18\left(2\,B^2be^4i-\frac{3\,(bex+ae)B^2de^3i}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^3bcg^4}{(dx+c)^3}-\frac{(bex+ae)^3adg^4}{(dx+c)^3}}+\frac{6\left(12\,ABbe^4i+4\,B^2be^4i-\frac{18\,(bex+ae)ABde^3i}{dx+c}-\frac{9\,(be}{}\right.}{\frac{(bex+ae)^3bcg^4}{(dx+c)^3}-\frac{(bex+ae)^3adg^4}{(dx+c)^3}}\right.$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algor
ithm="giac")
```

```
[Out] -1/108*(18*(2*B^2*b*e^4*i - 3*(b*e*x + a*e)*B^2*d*e^3*i/(d*x + c))*log((b*e
*x + a*e)/(d*x + c))^2/((b*e*x + a*e)^3*b*c*g^4/(d*x + c)^3 - (b*e*x + a*e)
^3*a*d*g^4/(d*x + c)^3) + 6*(12*A*B*b*e^4*i + 4*B^2*b*e^4*i - 18*(b*e*x + a
*e)*A*B*d*e^3*i/(d*x + c) - 9*(b*e*x + a*e)*B^2*d*e^3*i/(d*x + c))*log((b*e
*x + a*e)/(d*x + c))/((b*e*x + a*e)^3*b*c*g^4/(d*x + c)^3 - (b*e*x + a*e)^3
*a*d*g^4/(d*x + c)^3) + (36*A^2*b*e^4*i + 24*A*B*b*e^4*i + 8*B^2*b*e^4*i -
54*(b*e*x + a*e)*A^2*d*e^3*i/(d*x + c) - 54*(b*e*x + a*e)*A*B*d*e^3*i/(d*x
+ c) - 27*(b*e*x + a*e)*B^2*d*e^3*i/(d*x + c))/((b*e*x + a*e)^3*b*c*g^4/(d*
x + c)^3 - (b*e*x + a*e)^3*a*d*g^4/(d*x + c)^3))*(b*c/((b*c*e - a*d*e)*(b*c
 - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 955, normalized size of antiderivative = 3.33

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$$

$$= -\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{\frac{B^2\,c\,i}{3\,b^2\,g^4}+\frac{B^2\,a\,d\,i}{6\,b^3\,g^4}+\frac{B^2\,d\,i\,x}{2\,b^2\,g^4}}{3\,a^2\,x+\frac{a^3}{b}+b^2\,x^3+3\,a\,b\,x^2}-\frac{B^2\,d^3\,i}{6\,b^2\,g^4\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)$$

$$-\frac{\frac{18\,i\,A^2\,a^2\,d^2+18\,i\,A^2\,a\,b\,c\,d-36\,i\,A^2\,b^2\,c^2+30\,i\,A\,B\,a^2\,d^2+30\,i\,A\,B\,a\,b\,c\,d-24\,i\,A\,B\,b^2\,c^2+19\,i\,B^2\,a^2\,d^2+19\,i\,B^2\,a\,b\,c\,d-8\,i\,B^2\,b^2\,c^2}{6\,(a\,d-b\,c)}+\frac{x^2\,(5\,i\,}{}}{18\,a^3\,b^2\,g^4+54\,a^2\,b^3\,g^4\,x+54\,a\,b^4\,g}$$

$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(x\left(\frac{A\,B\,i}{b^2\,g^4}+\frac{B^2\,d^3\,i\left(b\left(\frac{3\,a^2\,d^2-4\,a\,b\,c\,d+b^2\,c^2}{6\,b\,d^3}+\frac{a\,(a\,d-b\,c)}{3\,b\,d^2}\right)+\frac{3\,a^2\,d^2-4\,a\,b\,c\,d+b^2\,c^2}{3\,d^3}+\frac{2\,a\,(a\,d-b\,c)}{3\,d^2}\right)}{3\,b^2\,g^4\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)+\frac{A\,B\,a\,i}{3\,b^3\,g^4}+\frac{B\,i\,(}{}\right.}{\frac{3\,a^2\,x}{d}+\frac{a}{b}}$$

$$-\frac{B\,d^3\,i\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x-\frac{18\,b^4\,c^2\,g^4-18\,a^2\,b^2\,d^2\,g^4}{18\,b^2\,g^4\,(a\,d-b\,c)}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(6\,A+5\,B)\,1\mathrm{i}}{9\,b^2\,g^4\,(a\,d-b\,c)^2}$$

```
[In] int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^4,
x)
```

```
[Out] - log((e*(a + b*x))/(c + d*x))^2*(((B^2*c*i)/(3*b^2*g^4) + (B^2*a*d*i)/(6*b
^3*g^4) + (B^2*d*i*x)/(2*b^2*g^4))/(3*a^2*x + a^3/b + b^2*x^3 + 3*a*b*x^2)
- (B^2*d^3*i)/(6*b^2*g^4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((18*A^2*a^2*d
^2*i - 36*A^2*b^2*c^2*i + 19*B^2*a^2*d^2*i - 8*B^2*b^2*c^2*i + 30*A*B*a^2*d
^2*i - 24*A*B*b^2*c^2*i + 18*A^2*a*b*c*d*i + 19*B^2*a*b*c*d*i + 30*A*B*a*b*
c*d*i)/(6*(a*d - b*c)) + (x^2*(5*B^2*b^2*d^2*i + 6*A*B*b^2*d^2*i))/(a*d - b
*c) + (x*(18*A^2*a*b*d^2*i + 19*B^2*a*b*d^2*i - 18*A^2*b^2*c*d*i + B^2*b^2*
c*d*i + 30*A*B*a*b*d^2*i - 6*A*B*b^2*c*d*i))/(2*(a*d - b*c)))/(18*a^3*b^2*g
^4 + 18*b^5*g^4*x^3 + 54*a^2*b^3*g^4*x + 54*a*b^4*g^4*x^2) - (log((e*(a + b
*x))/(c + d*x))*(x*((A*B*i)/(b^2*g^4) + (B^2*d^3*i*(b*((3*a^2*d^2 + b^2*c^2
 - 4*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(3*b*d^2)) + (3*a^2*d^2 + b^2*c^2
 - 4*a*b*c*d)/(3*d^3) + (2*a*(a*d - b*c))/(3*d^2)))/(3*b^2*g^4*(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d))) + (A*B*a*i)/(3*b^3*g^4) + (B*i*(2*A*b*c - B*a*d + B*b
*c))/(3*b^3*d*g^4) + (B^2*d^3*i*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*
d^3) + (a*(a*d - b*c))/(3*b*d^2)) + (3*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d -
6*a^2*b*c*d^2)/(3*b*d^4)))/(3*b^2*g^4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (B
^2*d^3*i*x^2*((b^2*c - a*b*d)/(3*d^2) - (2*b*(a*d - b*c))/(3*d^2)))/(3*b^2*
g^4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))))/((3*a^2*x)/d + a^3/(b*d) + (b^2*x^3)
/d + (3*a*b*x^2)/d) - (B*d^3*i*atan(((2*b*d*x - (18*b^4*c^2*g^4 - 18*a^2*b^
2*d^2*g^4)/(18*b^2*g^4*(a*d - b*c)))*1i)/(a*d - b*c))*(6*A + 5*B)*1i)/(9*b^
2*g^4*(a*d - b*c)^2)
```

3.63 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 445

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{B^2d^2i(c+dx)^2}{4(bc-ad)^3g^5(a+bx)^2} + \frac{4bB^2di(c+dx)^3}{27(bc-ad)^3g^5(a+bx)^3} - \frac{b^2B^2i(c+dx)^4}{32(bc-ad)^3g^5(a+bx)^4} - \frac{Bd^2i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^5(a+bx)^2} + \frac{4bBdi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^3g^5(a+bx)^3} - \frac{b^2Bi(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)^3g^5(a+bx)^4} - \frac{d^2i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^3g^5(a+bx)^2} + \frac{2bdi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^3g^5(a+bx)^3} - \frac{b^2i(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^3g^5(a+bx)^4}$$

```
[Out] -1/4*B^2*d^2*i*(d*x+c)^2/(-a*d+b*c)^3/g^5/(b*x+a)^2+4/27*b*B^2*d*i*(d*x+c)^
3/(-a*d+b*c)^3/g^5/(b*x+a)^3-1/32*b^2*B^2*i*(d*x+c)^4/(-a*d+b*c)^3/g^5/(b*x
+a)^4-1/2*B*d^2*i*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^5/(b
*x+a)^2+4/9*b*B*d*i*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^5/
(b*x+a)^3-1/8*b^2*B*i*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^
5/(b*x+a)^4-1/2*d^2*i*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/
g^5/(b*x+a)^2+2/3*b*d*i*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^
3/g^5/(b*x+a)^3-1/4*b^2*i*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c
)^3/g^5/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2562, 2395, 2342, 2341}

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}dx = -\frac{b^2i(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4g^5(a+bx)^4(bc-ad)^3} - \frac{b^2Bi(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{8g^5(a+bx)^4(bc-ad)^3} - \frac{d^2i(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g^5(a+bx)^2(bc-ad)^3} - \frac{Bd^2i(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^5(a+bx)^2(bc-ad)^3} + \frac{2bdi(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g^5(a+bx)^3(bc-ad)^3} + \frac{4bBdi(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{9g^5(a+bx)^3(bc-ad)^3} - \frac{b^2B^2i(c+dx)^4}{32g^5(a+bx)^4(bc-ad)^3} - \frac{B^2d^2i(c+dx)^2}{4g^5(a+bx)^2(bc-ad)^3} + \frac{4bB^2di(c+dx)^3}{27g^5(a+bx)^3(bc-ad)^3}$$

```
[In] Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)^5,
x]
```

```
[Out] -1/4*(B^2*d^2*i*(c + d*x)^2)/((b*c - a*d)^3*g^5*(a + b*x)^2) + (4*b*B^2*d*i
*(c + d*x)^3)/(27*(b*c - a*d)^3*g^5*(a + b*x)^3) - (b^2*B^2*i*(c + d*x)^4)/
```

```
(32*(b*c - a*d)^3*g^5*(a + b*x)^4) - (B*d^2*i*(c + d*x)^2*(A + B*Log[(e*(a
+ b*x))/(c + d*x)]))/(2*(b*c - a*d)^3*g^5*(a + b*x)^2) + (4*b*B*d*i*(c + d*
x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(9*(b*c - a*d)^3*g^5*(a + b*x)^3
) - (b^2*B*i*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(8*(b*c - a*
d)^3*g^5*(a + b*x)^4) - (d^2*i*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)])^2)/(2*(b*c - a*d)^3*g^5*(a + b*x)^2) + (2*b*d*i*(c + d*x)^3*(A + B*Log
[(e*(a + b*x))/(c + d*x)])^2)/(3*(b*c - a*d)^3*g^5*(a + b*x)^3) - (b^2*i*(c
 + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*(b*c - a*d)^3*g^5*(a +
 b*x)^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{i\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex))^2}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^5}$$

$$= \frac{i\text{Subst}\left(\int \left(\frac{b^2(A+B\log(ex))^2}{x^5} - \frac{2bd(A+B\log(ex))^2}{x^4} + \frac{d^2(A+B\log(ex))^2}{x^3}\right) dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^5}$$

$$= \frac{(b^2i)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^5} dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^5} - \frac{(2bdi)\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^4} dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^5} + \frac{(d^2i)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^3} dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^5}$$

$$= -\frac{d^2i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^3g^5(a+bx)^2} + \frac{2bdi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^3g^5(a+bx)^3} - \frac{b^2i(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^3g^5(a+bx)^4} + \frac{(b^2Bi)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^5} dx, x, \frac{a+bx}{c+dx}\right)}{2(bc-ad)^3g^5} - \frac{(4bBdi)\text{Subst}\left(\int \frac{A+B\log(ex)}{x^4} dx, x, \frac{a+bx}{c+dx}\right)}{3(bc-ad)^3g^5} + \frac{(Bd^2i)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^3} dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^5}$$

$$= -\frac{B^2d^2i(c+dx)^2}{4(bc-ad)^3g^5(a+bx)^2} + \frac{4bB^2di(c+dx)^3}{27(bc-ad)^3g^5(a+bx)^3} - \frac{b^2B^2i(c+dx)^4}{32(bc-ad)^3g^5(a+bx)^4} - \frac{Bd^2i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^5(a+bx)^2} + \frac{4bBdi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^3g^5(a+bx)^3} - \frac{b^2Bi(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)^3g^5(a+bx)^4} - \frac{d^2i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^3g^5(a+bx)^2} + \frac{2bdi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^3g^5(a+bx)^3} - \frac{b^2i(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^3g^5(a+bx)^4}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.69 (sec) , antiderivative size = 1255, normalized size of antiderivative = 2.82

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5} dx =$$

$$-\frac{i\left(216(bc-ad)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - 288d(-bc+ad)^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + 16Bd(a\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g
*x)^5,x]
```

```
[Out] -1/864*(i*(216*(b*c - a*d)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - 288*d
*(-(b*c) + a*d)^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 16*B*d
*(a + b*x)*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3 - 18*A*d*(b*c - a*d)^2*(
a + b*x) - 15*B*d*(b*c - a*d)^2*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x)^
2 + 66*B*d^2*(b*c - a*d)*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x] +
66*B*d^3*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*(a + b*x)^3*Log[a + b*x]^2 + 1
2*B*(b*c - a*d)^3*Log[(e*(a + b*x))/(c + d*x)] - 18*B*d*(b*c - a*d)^2*(a +
b*x)*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[(e
*(a + b*x))/(c + d*x)] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(e*(a + b*x)
)/(c + d*x)] - 36*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3*(a + b*x)^3*Log
[c + d*x] + 36*B*d^3*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c +
d*x] - 36*B*d^3*(a + b*x)^3*Log[(e*(a + b*x))/(c + d*x)]*Log[c + d*x] - 18*
B*d^3*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(b
*(c + d*x))/(b*c - a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-
(b*c) + a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
 + 3*B*(36*A*(b*c - a*d)^4 + 9*B*(b*c - a*d)^4 + 48*A*d*(-(b*c) + a*d)^3*(a
 + b*x) + 28*B*d*(-(b*c) + a*d)^3*(a + b*x) + 72*A*d^2*(b*c - a*d)^2*(a + b
*x)^2 + 78*B*d^2*(b*c - a*d)^2*(a + b*x)^2 + 144*A*d^3*(-(b*c) + a*d)*(a +
b*x)^3 + 300*B*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 144*A*d^4*(a + b*x)^4*Log[a
 + b*x] - 300*B*d^4*(a + b*x)^4*Log[a + b*x] + 72*B*d^4*(a + b*x)^4*Log[a +
 b*x]^2 + 36*B*(b*c - a*d)^4*Log[(e*(a + b*x))/(c + d*x)] + 48*B*d*(-(b*c)
+ a*d)^3*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 72*B*d^2*(b*c - a*d)^2*(a
 + b*x)^2*Log[(e*(a + b*x))/(c + d*x)] + 144*B*d^3*(-(b*c) + a*d)*(a + b*x)
^3*Log[(e*(a + b*x))/(c + d*x)] - 144*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[(e
*(a + b*x))/(c + d*x)] + 144*A*d^4*(a + b*x)^4*Log[c + d*x] + 300*B*d^4*(a
+ b*x)^4*Log[c + d*x] - 144*B*d^4*(a + b*x)^4*Log[(d*(a + b*x))/(-(b*c) + a
*d)]*Log[c + d*x] + 144*B*d^4*(a + b*x)^4*Log[(e*(a + b*x))/(c + d*x)]*Log[
c + d*x] + 72*B*d^4*(a + b*x)^4*Log[c + d*x]^2 - 144*B*d^4*(a + b*x)^4*Log[
a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 144*B*d^4*(a + b*x)^4*PolyLog[2,
(d*(a + b*x))/(-(b*c) + a*d)] - 144*B*d^4*(a + b*x)^4*PolyLog[2, (b*(c + d*
x))/(b*c - a*d)])))/(b^2*(b*c - a*d)^3*g^5*(a + b*x)^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(427) = 854$.

Time = 1.61 (sec) , antiderivative size = 926, normalized size of antiderivative = 2.08

method	result
parts	$\frac{i\,A^{2}\left(-\frac{d}{3b^{2}(bx+a)^{3}}-\frac{-ad+cb}{4b^{2}(bx+a)^{4}}\right)}{g^{5}}-\frac{i\,B^{2}(ad-cb)^{2}e^{2}\left(\frac{d^{5}\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}}-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}}\right.}{(ad-cb)^{5}}\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display
norman	Expression too large to display
parallelrisch	Expression too large to display
risch	Expression too large to display

```
[In] int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x,method=_RETUR
NVERBOSE)
```

```
[Out] i*A^2/g^5*(-1/3*d/b^2/(b*x+a)^3-1/4*(-a*d+b*c)/b^2/(b*x+a)^4)-i*B^2/g^5/d^3
*(a*d-b*c)^2*e^2*(d^5/(a*d-b*c)^5*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(
b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d
+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-2*d^4/(a*d-b*c
)^5*b*e*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c
))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/
27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)+d^3/(a*d-b*c)^5*e^2*b^2*(-1/4/(b*e/d+(a
*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/8/(b*e/d+(a*d-b*
c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/32/(b*e/d+(a*d-b*c)*e/d
/(d*x+c))^4))-2*i*B*A/g^5/d^3*(a*d-b*c)^2*e^2*(d^5/(a*d-b*c)^5*(-1/2/(b*e/d
+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b
*c)*e/d/(d*x+c))^2)-2*d^4/(a*d-b*c)^5*b*e*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c
))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)+d
^3/(a*d-b*c)^5*e^2*b^2*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-
b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 985 vs. $2(427) = 854$.

Time = 0.31 (sec) , antiderivative size = 985, normalized size of antiderivative = 2.21

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^{2}}{(ag+bgx)^{5}}\,dx =$$

$$-\frac{12\left(\left(12\,AB+13\,B^{2}\right)b^{4}cd^{3}-\left(12\,AB+13\,B^{2}\right)ab^{3}d^{4}\right)ix^{3}-6\left(\left(12\,AB+B^{2}\right)b^{4}c^{2}d^{2}-16\left(6\,AB+5\,B^{2}\right)ab^{3}\right.}{}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algor
ithm="fricas")
```

```
[Out] -1/864*(12*((12*A*B + 13*B^2)*b^4*c*d^3 - (12*A*B + 13*B^2)*a*b^3*d^4)*i*x^
3 - 6*((12*A*B + B^2)*b^4*c^2*d^2 - 16*(6*A*B + 5*B^2)*a*b^3*c*d^3 + (84*A*
B + 79*B^2)*a^2*b^2*d^4)*i*x^2 + 4*((72*A^2 + 12*A*B - 5*B^2)*b^4*c^3*d - 1
2*(18*A^2 + 6*A*B - B^2)*a*b^3*c^2*d^2 + 108*(2*A^2 + 2*A*B + B^2)*a^2*b^2*
c*d^3 - (72*A^2 + 156*A*B + 115*B^2)*a^3*b*d^4)*i*x + 72*(B^2*b^4*d^4*i*x^4
 + 4*B^2*a*b^3*d^4*i*x^3 + 6*B^2*a^2*b^2*d^4*i*x^2 + 4*(B^2*b^4*c^3*d - 3*B
^2*a*b^3*c^2*d^2 + 3*B^2*a^2*b^2*c*d^3)*i*x + (3*B^2*b^4*c^4 - 8*B^2*a*b^3*
c^3*d + 6*B^2*a^2*b^2*c^2*d^2)*i)*log((b*e*x + a*e)/(d*x + c))^2 + (27*(8*A
^2 + 4*A*B + B^2)*b^4*c^4 - 64*(9*A^2 + 6*A*B + 2*B^2)*a*b^3*c^3*d + 216*(2
*A^2 + 2*A*B + B^2)*a^2*b^2*c^2*d^2 - (72*A^2 + 156*A*B + 115*B^2)*a^4*d^4)
*i + 12*((12*A*B + 13*B^2)*b^4*d^4*i*x^4 + 4*(3*B^2*b^4*c*d^3 + 2*(6*A*B +
5*B^2)*a*b^3*d^4)*i*x^3 - 6*(B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 - 6*(2*A*B
 + B^2)*a^2*b^2*d^4)*i*x^2 + 4*((12*A*B + B^2)*b^4*c^3*d - 6*(6*A*B + B^2)*
a*b^3*c^2*d^2 + 18*(2*A*B + B^2)*a^2*b^2*c*d^3)*i*x + (9*(4*A*B + B^2)*b^4*
c^4 - 32*(3*A*B + B^2)*a*b^3*c^3*d + 36*(2*A*B + B^2)*a^2*b^2*c^2*d^2)*i)*l
og((b*e*x + a*e)/(d*x + c)))/((b^9*c^3 - 3*a*b^8*c^2*d + 3*a^2*b^7*c*d^2 -
a^3*b^6*d^3)*g^5*x^4 + 4*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*a^3*b^6*c*d^2 - a
^4*b^5*d^3)*g^5*x^3 + 6*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*a^4*b^5*c*d^2 -
a^5*b^4*d^3)*g^5*x^2 + 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2 -
 a^6*b^3*d^3)*g^5*x + (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d^2 - a^
7*b^2*d^3)*g^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx=\text{Timed out}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**5,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4808 vs. $2(427) = 854$.

Time = 0.57 (sec) , antiderivative size = 4808, normalized size of antiderivative = 10.80

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx=\text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algor
ithm="maxima")
```

```
[Out] -1/12*(4*b*x + a)*B^2*d*i*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^6*g^5*x
^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g^5) +
 1/288*(12*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 +
 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^
2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*
d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d
^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*
d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2
*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)
*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 -
 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^
4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log(b*e*x/
(d*x + c) + a*e/(d*x + c)) - (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2*
d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(
13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 +
 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a
)^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x
 + a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 324*a^2*b^
2*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2
*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*b^4*d^4*x^4 + 100
*a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x + 25*a^4*d^4 - 12*(b
^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)
*log(b*x + a))*log(d*x + c))/(a^4*b^5*c^4*g^5 - 4*a^5*b^4*c^3*d*g^5 + 6*a^6
*b^3*c^2*d^2*g^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d^4*g^5 + (b^9*c^4*g^5 - 4*a
*b^8*c^3*d*g^5 + 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b^6*c*d^3*g^5 + a^4*b^5*d^4*
g^5)*x^4 + 4*(a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^5 + 6*a^3*b^6*c^2*d^2*g^5 -
 4*a^4*b^5*c*d^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*(a^2*b^7*c^4*g^5 - 4*a^3*b^
6*c^3*d*g^5 + 6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*c*d^3*g^5 + a^6*b^3*d^4*g^5
)*x^2 + 4*(a^3*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5 + 6*a^5*b^4*c^2*d^2*g^5 -
4*a^6*b^3*c*d^3*g^5 + a^7*b^2*d^4*g^5)*x))*B^2*c*i - 1/864*(12*((7*a*b^3*c^
3 - 33*a^2*b^2*c^2*d + 75*a^3*b*c*d^2 - 13*a^4*d^3 + 12*(4*b^4*c*d^2 - a*b^
3*d^3)*x^3 - 6*(4*b^4*c^2*d - 29*a*b^3*c*d^2 + 7*a^2*b^2*d^3)*x^2 + 4*(4*b^
4*c^3 - 21*a*b^3*c^2*d + 57*a^2*b^2*c*d^2 - 13*a^3*b*d^3)*x)/((b^9*c^3 - 3*
a*b^8*c^2*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*g^5*x^4 + 4*(a*b^8*c^3 - 3*a^2
*b^7*c^2*d + 3*a^3*b^6*c*d^2 - a^4*b^5*d^3)*g^5*x^3 + 6*(a^2*b^7*c^3 - 3*a^
3*b^6*c^2*d + 3*a^4*b^5*c*d^2 - a^5*b^4*d^3)*g^5*x^2 + 4*(a^3*b^6*c^3 - 3*a
^4*b^5*c^2*d + 3*a^5*b^4*c*d^2 - a^6*b^3*d^3)*g^5*x + (a^4*b^5*c^3 - 3*a^5*
b^4*c^2*d + 3*a^6*b^3*c*d^2 - a^7*b^2*d^3)*g^5) + 12*(4*b*c*d^3 - a*d^4)*lo
g(b*x + a)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3
+ a^4*b^2*d^4)*g^5) - 12*(4*b*c*d^3 - a*d^4)*log(d*x + c)/((b^6*c^4 - 4*a*b
^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5))*log(b*e
*x/(d*x + c) + a*e/(d*x + c)) + (37*a*b^4*c^4 - 304*a^2*b^3*c^3*d + 1512*a^
3*b^2*c^2*d^2 - 1360*a^4*b*c*d^3 + 115*a^5*d^4 + 12*(88*b^5*c^2*d^2 - 101*a
*b^4*c*d^3 + 13*a^2*b^3*d^4)*x^3 - 6*(40*b^5*c^3*d - 609*a*b^4*c^2*d^2 + 64
8*a^2*b^3*c*d^3 - 79*a^3*b^2*d^4)*x^2 - 72*(4*a^4*b*c*d^3 - a^5*d^4 + (4*b^
```

5*c*d^3 - a*b^4*d^4)*x^4 + 4*(4*a*b^4*c*d^3 - a^2*b^3*d^4)*x^3 + 6*(4*a^2*b
^3*c*d^3 - a^3*b^2*d^4)*x^2 + 4*(4*a^3*b^2*c*d^3 - a^4*b*d^4)*x)*log(b*x +
a)^2 - 72*(4*a^4*b*c*d^3 - a^5*d^4 + (4*b^5*c*d^3 - a*b^4*d^4)*x^4 + 4*(4*a
*b^4*c*d^3 - a^2*b^3*d^4)*x^3 + 6*(4*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^2 + 4*(
4*a^3*b^2*c*d^3 - a^4*b*d^4)*x)*log(d*x + c)^2 + 4*(16*b^5*c^4 - 163*a*b^4*
c^3*d + 1068*a^2*b^3*c^2*d^2 - 1036*a^3*b^2*c*d^3 + 115*a^4*b*d^4)*x + 12*(
88*a^4*b*c*d^3 - 13*a^5*d^4 + (88*b^5*c*d^3 - 13*a*b^4*d^4)*x^4 + 4*(88*a*b
^4*c*d^3 - 13*a^2*b^3*d^4)*x^3 + 6*(88*a^2*b^3*c*d^3 - 13*a^3*b^2*d^4)*x^2
+ 4*(88*a^3*b^2*c*d^3 - 13*a^4*b*d^4)*x)*log(b*x + a) - 12*(88*a^4*b*c*d^3
- 13*a^5*d^4 + (88*b^5*c*d^3 - 13*a*b^4*d^4)*x^4 + 4*(88*a*b^4*c*d^3 - 13*a
^2*b^3*d^4)*x^3 + 6*(88*a^2*b^3*c*d^3 - 13*a^3*b^2*d^4)*x^2 + 4*(88*a^3*b^2
*c*d^3 - 13*a^4*b*d^4)*x - 12*(4*a^4*b*c*d^3 - a^5*d^4 + (4*b^5*c*d^3 - a*b
^4*d^4)*x^4 + 4*(4*a*b^4*c*d^3 - a^2*b^3*d^4)*x^3 + 6*(4*a^2*b^3*c*d^3 - a^
3*b^2*d^4)*x^2 + 4*(4*a^3*b^2*c*d^3 - a^4*b*d^4)*x)*log(b*x + a))*log(d*x +
c))/(a^4*b^6*c^4*g^5 - 4*a^5*b^5*c^3*d*g^5 + 6*a^6*b^4*c^2*d^2*g^5 - 4*a^7
*b^3*c*d^3*g^5 + a^8*b^2*d^4*g^5 + (b^10*c^4*g^5 - 4*a*b^9*c^3*d*g^5 + 6*a^
2*b^8*c^2*d^2*g^5 - 4*a^3*b^7*c*d^3*g^5 + a^4*b^6*d^4*g^5)*x^4 + 4*(a*b^9*c
^4*g^5 - 4*a^2*b^8*c^3*d*g^5 + 6*a^3*b^7*c^2*d^2*g^5 - 4*a^4*b^6*c*d^3*g^5
+ a^5*b^5*d^4*g^5)*x^3 + 6*(a^2*b^8*c^4*g^5 - 4*a^3*b^7*c^3*d*g^5 + 6*a^4*b
^6*c^2*d^2*g^5 - 4*a^5*b^5*c*d^3*g^5 + a^6*b^4*d^4*g^5)*x^2 + 4*(a^3*b^7*c^
4*g^5 - 4*a^4*b^6*c^3*d*g^5 + 6*a^5*b^5*c^2*d^2*g^5 - 4*a^6*b^4*c*d^3*g^5 +
a^7*b^3*d^4*g^5)*x))*B^2*d*i - 1/72*A*B*d*i*(12*(4*b*x + a)*log(b*e*x/(d*x
+ c) + a*e/(d*x + c))/(b^6*g^5*x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 +
4*a^3*b^3*g^5*x + a^4*b^2*g^5) + (7*a*b^3*c^3 - 33*a^2*b^2*c^2*d + 75*a^3*
b*c*d^2 - 13*a^4*d^3 + 12*(4*b^4*c*d^2 - a*b^3*d^3)*x^3 - 6*(4*b^4*c^2*d -
29*a*b^3*c*d^2 + 7*a^2*b^2*d^3)*x^2 + 4*(4*b^4*c^3 - 21*a*b^3*c^2*d + 57*a^
2*b^2*c*d^2 - 13*a^3*b*d^3)*x)/((b^9*c^3 - 3*a*b^8*c^2*d + 3*a^2*b^7*c*d^2
- a^3*b^6*d^3)*g^5*x^4 + 4*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*a^3*b^6*c*d^2 -
a^4*b^5*d^3)*g^5*x^3 + 6*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*a^4*b^5*c*d^2
- a^5*b^4*d^3)*g^5*x^2 + 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2
- a^6*b^3*d^3)*g^5*x + (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d^2 -
a^7*b^2*d^3)*g^5) + 12*(4*b*c*d^3 - a*d^4)*log(b*x + a)/((b^6*c^4 - 4*a*b^5
*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5) - 12*(4*b*
c*d^3 - a*d^4)*log(d*x + c)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 -
4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5)) + 1/24*A*B*c*i*((12*b^3*d^3*x^3 - 3*b
^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*
b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 -
3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a
^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*
a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3
*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^
5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) - 12*log(b*e*x/(d*x + c) +
a*e/(d*x + c))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b
^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a
^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((

```
b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*
g^5)) - 1/4*B^2*c*i*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^5*g^5*x^4 + 4
*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/12*(4
*b*x + a)*A^2*d*i/(b^6*g^5*x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^
3*b^3*g^5*x + a^4*b^2*g^5) - 1/4*A^2*c*i/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6
*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

Giac [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 744, normalized size of antiderivative = 1.67

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{1}{864}\left(\frac{72\left(3B^2b^2e^5i-\frac{8(bex+ae)B^2bde^4i}{dx+c}+\frac{6(bex+ae)^2B^2d^2e^3i}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^4b^2c^2g^5}{(dx+c)^4}-\frac{2(bex+ae)^4abcdg^5}{(dx+c)^4}+\frac{(bex+ae)^4a^2d^2g^5}{(dx+c)^4}}+\frac{12\left(36ABb^2e^5i+9B^2b^2e^5i-\right.}{}\right.$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algor
ithm="giac")
```

```
[Out] -1/864*(72*(3*B^2*b^2*e^5*i - 8*(b*e*x + a*e)*B^2*b*d*e^4*i/(d*x + c) + 6*(
b*e*x + a*e)^2*B^2*d^2*e^3*i/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))^2/((
b*e*x + a*e)^4*b^2*c^2*g^5/(d*x + c)^4 - 2*(b*e*x + a*e)^4*a*b*c*d*g^5/(d*x
 + c)^4 + (b*e*x + a*e)^4*a^2*d^2*g^5/(d*x + c)^4) + 12*(36*A*B*b^2*e^5*i +
 9*B^2*b^2*e^5*i - 96*(b*e*x + a*e)*A*B*b*d*e^4*i/(d*x + c) - 32*(b*e*x + a
*e)*B^2*b*d*e^4*i/(d*x + c) + 72*(b*e*x + a*e)^2*A*B*d^2*e^3*i/(d*x + c)^2
+ 36*(b*e*x + a*e)^2*B^2*d^2*e^3*i/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c)
)/((b*e*x + a*e)^4*b^2*c^2*g^5/(d*x + c)^4 - 2*(b*e*x + a*e)^4*a*b*c*d*g^5/
(d*x + c)^4 + (b*e*x + a*e)^4*a^2*d^2*g^5/(d*x + c)^4) + (216*A^2*b^2*e^5*i
 + 108*A*B*b^2*e^5*i + 27*B^2*b^2*e^5*i - 576*(b*e*x + a*e)*A^2*b*d*e^4*i/(
d*x + c) - 384*(b*e*x + a*e)*A*B*b*d*e^4*i/(d*x + c) - 128*(b*e*x + a*e)*B^
2*b*d*e^4*i/(d*x + c) + 432*(b*e*x + a*e)^2*A^2*d^2*e^3*i/(d*x + c)^2 + 432
*(b*e*x + a*e)^2*A*B*d^2*e^3*i/(d*x + c)^2 + 216*(b*e*x + a*e)^2*B^2*d^2*e^
3*i/(d*x + c)^2)/((b*e*x + a*e)^4*b^2*c^2*g^5/(d*x + c)^4 - 2*(b*e*x + a*e)
^4*a*b*c*d*g^5/(d*x + c)^4 + (b*e*x + a*e)^4*a^2*d^2*g^5/(d*x + c)^4))*(b*c
/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 7.44 (sec) , antiderivative size = 1870, normalized size of antiderivative = 4.20

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

```
[In] int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^5,
x)
```

```
[Out] ((72*A^2*a^3*d^3*i + 216*A^2*b^3*c^3*i + 115*B^2*a^3*d^3*i + 27*B^2*b^3*c^3
*i + 156*A*B*a^3*d^3*i + 108*A*B*b^3*c^3*i - 360*A^2*a*b^2*c^2*d*i + 72*A^2
*a^2*b*c*d^2*i - 101*B^2*a*b^2*c^2*d*i + 115*B^2*a^2*b*c*d^2*i - 276*A*B*a*
b^2*c^2*d*i + 156*A*B*a^2*b*c*d^2*i)/(12*(a*d - b*c)) + (x^2*(79*B^2*a*b^2*
d^3*i - B^2*b^3*c*d^2*i + 84*A*B*a*b^2*d^3*i - 12*A*B*b^3*c*d^2*i))/(2*(a*d
 - b*c)) + (x*(72*A^2*a^2*b*d^3*i + 115*B^2*a^2*b*d^3*i + 72*A^2*b^3*c^2*d*
i - 5*B^2*b^3*c^2*d*i + 156*A*B*a^2*b*d^3*i + 12*A*B*b^3*c^2*d*i - 144*A^2*
a*b^2*c*d^2*i + 7*B^2*a*b^2*c*d^2*i - 60*A*B*a*b^2*c*d^2*i))/(3*(a*d - b*c)
) + (d*x^3*(13*B^2*b^3*d^2*i + 12*A*B*b^3*d^2*i))/(a*d - b*c))/(x*(288*a^3*
b^4*c*g^5 - 288*a^4*b^3*d*g^5) - x^3*(288*a^2*b^5*d*g^5 - 288*a*b^6*c*g^5)
+ x^4*(72*b^7*c*g^5 - 72*a*b^6*d*g^5) + x^2*(432*a^2*b^5*c*g^5 - 432*a^3*b^
4*d*g^5) + 72*a^4*b^3*c*g^5 - 72*a^5*b^2*d*g^5) - log((e*(a + b*x))/(c + d*
x))^2*(((B^2*c*i)/(4*b^2*g^5) + (B^2*a*d*i)/(12*b^3*g^5) + (B^2*d*i*x)/(3*b
^2*g^5))/(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3) - (B^2*d^4
*i)/(12*b^2*g^5*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (lo
g((e*(a + b*x))/(c + d*x))*(x*((2*A*B*i)/(3*b^2*g^5) + (B^2*d^4*i*(b*(a*((4
*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (
6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(12*b*d^4)) + a*(b*((
4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) +
(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*d^3) + (a*(a*d - b*c))/(2*d^2)) + (6*a
^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(4*d^4)))/(6*b^2*g^5*(a^
3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) + (A*B*a*i)/(6*b^3*g^5)
+ (B*i*(3*A*b*c - B*a*d + B*b*c))/(6*b^3*d*g^5) + (B^2*d^4*i*(a*(a*((4*a^2*
d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (6*a^3
*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(12*b*d^4)) + (4*a^4*d^4 +
 b^4*c^4 + 10*a^2*b^2*c^2*d^2 - 5*a*b^3*c^3*d - 10*a^3*b*c*d^3)/(4*b*d^5)))
/(6*b^2*g^5*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (B^2*d^4
*i*x^2*(b*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c)
)/(4*b*d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*d^3) + (a*(a*d - b*c))/
(2*d^2)) - a*((b^2*c - a*b*d)/(4*d^2) - (b*(a*d - b*c))/(2*d^2)) + (b^3*c^2
 + 4*a^2*b*d^2 - 5*a*b^2*c*d)/(4*d^3)))/(6*b^2*g^5*(a^3*d^3 - b^3*c^3 + 3*a
*b^2*c^2*d - 3*a^2*b*c*d^2)) - (B^2*d^4*i*x^3*(b*((b^2*c - a*b*d)/(4*d^2) -
 (b*(a*d - b*c))/(2*d^2)) + (b^3*c - a*b^2*d)/(4*d^2)))/(6*b^2*g^5*(a^3*d^3
 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))))/((4*a^3*x)/d + a^4/(b*d) + (
b^3*x^4)/d + (6*a^2*b*x^2)/d + (4*a*b^2*x^3)/d) - (B*d^4*i*atan((B*d^4*i*(1
```

```
2*A + 13*B)*(72*b^5*c^3*g^5 + 72*a^3*b^2*d^3*g^5 - 72*a*b^4*c^2*d*g^5 - 72*
a^2*b^3*c*d^2*g^5)*1i)/(72*b^2*g^5*(13*B^2*d^4*i + 12*A*B*d^4*i)*(a*d - b*c
)^3) + (B*d^5*i*x*(12*A + 13*B)*(b^4*c^2*g^5 + a^2*b^2*d^2*g^5 - 2*a*b^3*c*
d*g^5)*2i)/(b*g^5*(13*B^2*d^4*i + 12*A*B*d^4*i)*(a*d - b*c)^3))*(12*A + 13*
B)*1i)/(36*b^2*g^5*(a*d - b*c)^3)
```

3.64 $\int (ag+bgx)^3(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 42, antiderivative size = 711

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$

$$= \frac{3B^2(bc-ad)^5g^3i^2x}{20b^2d^3}+\frac{B^2(bc-ad)^2g^3i^2(a+bx)^4}{60b^3}-\frac{3B^2(bc-ad)^4g^3i^2(c+dx)^2}{40bd^4}$$
$$+\frac{B^2(bc-ad)^3g^3i^2(c+dx)^3}{60d^4}-\frac{B(bc-ad)^3g^3i^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{90b^3d}$$
$$-\frac{B(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^3}$$
$$-\frac{B(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15b^2}$$
$$+\frac{(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{60b^3}$$
$$+\frac{(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{15b^2}$$
$$+\frac{g^3i^2(a+bx)^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b}$$
$$+\frac{B(bc-ad)^4g^3i^2(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{180b^3d^2}$$
$$-\frac{B(bc-ad)^5g^3i^2(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{180b^3d^3}$$
$$-\frac{B(bc-ad)^6g^3i^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{180b^3d^4}$$
$$-\frac{B^2(bc-ad)^6g^3i^2\log(c+dx)}{20b^3d^4}-\frac{B^2(bc-ad)^6g^3i^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{30b^3d^4}$$

```
[Out] 3/20*B^2*(-a*d+b*c)^5*g^3*i^2*x/b^2/d^3+1/60*B^2*(-a*d+b*c)^2*g^3*i^2*(b*x+
a)^4/b^3-3/40*B^2*(-a*d+b*c)^4*g^3*i^2*(d*x+c)^2/b/d^4+1/60*B^2*(-a*d+b*c)^
3*g^3*i^2*(d*x+c)^3/d^4-1/90*B*(-a*d+b*c)^3*g^3*i^2*(b*x+a)^3*(A+B*ln(e*(b*
x+a)/(d*x+c)))/b^3/d-1/20*B*(-a*d+b*c)^2*g^3*i^2*(b*x+a)^4*(A+B*ln(e*(b*x+a
)/(d*x+c)))/b^3-1/15*B*(-a*d+b*c)*g^3*i^2*(b*x+a)^4*(d*x+c)*(A+B*ln(e*(b*x+
a)/(d*x+c)))/b^2+1/60*(-a*d+b*c)^2*g^3*i^2*(b*x+a)^4*(A+B*ln(e*(b*x+a)/(d*x
+c)))^2/b^3+1/15*(-a*d+b*c)*g^3*i^2*(b*x+a)^4*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*
x+c)))^2/b^2+1/6*g^3*i^2*(b*x+a)^4*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/
b+1/180*B*(-a*d+b*c)^4*g^3*i^2*(b*x+a)^2*(3*A+B+3*B*ln(e*(b*x+a)/(d*x+c)))/
b^3/d^2-1/180*B*(-a*d+b*c)^5*g^3*i^2*(b*x+a)*(6*A+5*B+6*B*ln(e*(b*x+a)/(d*x
```

```
+c)))/b^3/d^3-1/180*B*(-a*d+b*c)^6*g^3*i^2*ln((-a*d+b*c)/b/(d*x+c))*(6*A+11
*B+6*B*ln(e*(b*x+a)/(d*x+c)))/b^3/d^4-1/20*B^2*(-a*d+b*c)^6*g^3*i^2*ln(d*x+
c)/b^3/d^4-1/30*B^2*(-a*d+b*c)^6*g^3*i^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3
/d^4
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 711, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2562, 2383, 2381, 2384, 2354, 2438, 2373, 45, 47, 37, 2382, 12, 79}

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$$

$$= -\frac{Bg^3i^2(bc-ad)^6\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A+11B\right)}{180b^3d^4} - \frac{Bg^3i^2(a+bx)(bc-ad)^5\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A+5B\right)}{180b^3d^3} + \frac{Bg^3i^2(a+bx)^2(bc-ad)^4\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{180b^3d^2} - \frac{Bg^3i^2(a+bx)^3(bc-ad)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{90b^3d} + \frac{g^3i^2(a+bx)^4(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{60b^3} - \frac{Bg^3i^2(a+bx)^4(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{20b^3} + \frac{g^3i^2(a+bx)^4(c+dx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{15b^2} - \frac{Bg^3i^2(a+bx)^4(c+dx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{15b^2} + \frac{g^3i^2(a+bx)^4(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{6b} - \frac{B^2g^3i^2(bc-ad)^6\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{30b^3d^4} - \frac{B^2g^3i^2(bc-ad)^6\log(c+dx)}{20b^3d^4} + \frac{B^2g^3i^2(a+bx)^4(bc-ad)^2}{60b^3} + \frac{3B^2g^3i^2x(bc-ad)^5}{20b^2d^3} - \frac{3B^2g^3i^2(c+dx)^2(bc-ad)^4}{40bd^4} + \frac{B^2g^3i^2(c+dx)^3(bc-ad)^3}{60d^4}$$

```
[In] Int[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,
x]
```

```
[Out] (3*B^2*(b*c - a*d)^5*g^3*i^2*x)/(20*b^2*d^3) + (B^2*(b*c - a*d)^2*g^3*i^2*(
a + b*x)^4)/(60*b^3) - (3*B^2*(b*c - a*d)^4*g^3*i^2*(c + d*x)^2)/(40*b*d^4)
 + (B^2*(b*c - a*d)^3*g^3*i^2*(c + d*x)^3)/(60*d^4) - (B*(b*c - a*d)^3*g^3*
i^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(90*b^3*d) - (B*(b*c
- a*d)^2*g^3*i^2*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(20*b^3)
 - (B*(b*c - a*d)*g^3*i^2*(a + b*x)^4*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c
 + d*x)]))/(15*b^2) + ((b*c - a*d)^2*g^3*i^2*(a + b*x)^4*(A + B*Log[(e*(a +
 b*x))/(c + d*x)])^2)/(60*b^3) + ((b*c - a*d)*g^3*i^2*(a + b*x)^4*(c + d*x)
*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(15*b^2) + (g^3*i^2*(a + b*x)^4*(c
 + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(6*b) + (B*(b*c - a*d)^4*
g^3*i^2*(a + b*x)^2*(3*A + B + 3*B*Log[(e*(a + b*x))/(c + d*x)]))/(180*b^3*
d^2) - (B*(b*c - a*d)^5*g^3*i^2*(a + b*x)*(6*A + 5*B + 6*B*Log[(e*(a + b*x)
)/(c + d*x)]))/(180*b^3*d^3) - (B*(b*c - a*d)^6*g^3*i^2*Log[(b*c - a*d)/(b*
(c + d*x))]*(6*A + 11*B + 6*B*Log[(e*(a + b*x))/(c + d*x)]))/(180*b^3*d^4)
- (B^2*(b*c - a*d)^6*g^3*i^2*Log[c + d*x])/(20*b^3*d^4) - (B^2*(b*c - a*d)^
6*g^3*i^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(30*b^3*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2383

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
```

```
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Dist[(m + q + 2)/(d*(q + 1)), Int[(
f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Dist[b*n*(p/(d*(q +
 1))), Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x]) /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ
[q, -1] && GtQ[m, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \left((bc-ad)^6 g^3 i^2\right) \text{Subst}\left(\int \frac{x^3(A+B\log(ex))^2}{(b-dx)^7}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{g^3 i^2 (a+bx)^4 (c+dx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b} + \frac{\left((bc-ad)^6 g^3 i^2\right) \text{Subst}\left(\int \frac{x^3(A+B\log(ex))^2}{(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b} - \frac{\left(B(bc-ad)^6 g^3 i^2\right) \text{Subst}\left(\int \frac{x^3(A+B\log(ex))}{(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b}$$

$$= -\frac{B(bc-ad)^2 g^3 i^2 (a+bx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^3} - \frac{B(bc-ad) g^3 i^2 (a+bx)^4 (c+dx) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15b^2} + \frac{(bc-ad) g^3 i^2 (a+bx)^4 (c+dx) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{15b^2} + \frac{g^3 i^2 (a+bx)^4 (c+dx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b} + \frac{((bc-ad)^6 g^3 i^2) \operatorname{Subst}\left(\int \frac{x^3 (A+B\log(ex))^2}{(b-dx)^5} \, dx, x, \frac{a+bx}{c+dx}\right)}{15b^2} - \frac{(2B(bc-ad)^6 g^3 i^2) \operatorname{Subst}\left(\int \frac{x^3 (A+B\log(ex))}{(b-dx)^5} \, dx, x, \frac{a+bx}{c+dx}\right)}{15b^2} + \frac{(B^2 (bc-ad)^6 g^3 i^2) \operatorname{Subst}\left(\int \frac{x^3 (5b-dx)}{20b^2 (b-dx)^5} \, dx, x, \frac{a+bx}{c+dx}\right)}{3b}$$

$$= -\frac{B(bc-ad)^2 g^3 i^2 (a+bx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^3} - \frac{B(bc-ad) g^3 i^2 (a+bx)^4 (c+dx) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15b^2} + \frac{(bc-ad)^2 g^3 i^2 (a+bx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{60b^3} + \frac{(bc-ad) g^3 i^2 (a+bx)^4 (c+dx) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{15b^2} + \frac{g^3 i^2 (a+bx)^4 (c+dx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b} - \frac{(B(bc-ad)^6 g^3 i^2) \operatorname{Subst}\left(\int \frac{x^3 (A+B\log(ex))}{(b-dx)^4} \, dx, x, \frac{a+bx}{c+dx}\right)}{30b^3} + \frac{(B^2 (bc-ad)^6 g^3 i^2) \operatorname{Subst}\left(\int \frac{x^3 (5b-dx)}{(b-dx)^5} \, dx, x, \frac{a+bx}{c+dx}\right)}{60b^3} + \frac{(B^2 (bc-ad)^6 g^3 i^2) \operatorname{Subst}\left(\int \frac{x^3}{(b-dx)^4} \, dx, x, \frac{a+bx}{c+dx}\right)}{30b^3}$$

$$= \frac{B^2(bc-ad)^2g^3i^2(a+bx)^4}{60b^3} - \frac{B(bc-ad)^3g^3i^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{90b^3d}$$
$$- \frac{B(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^3}$$
$$- \frac{B(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15b^2}$$
$$+ \frac{(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{60b^3}$$
$$+ \frac{(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{15b^2}$$
$$+ \frac{g^3i^2(a+bx)^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b}$$
$$+ \frac{(B^2(bc-ad)^6g^3i^2)\operatorname{Subst}\left(\int \frac{x^3}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{60b^3}$$
$$+ \frac{(B^2(bc-ad)^6g^3i^2)\operatorname{Subst}\left(\int \left(\frac{b^3}{d^3(b-dx)^4} - \frac{3b^2}{d^3(b-dx)^3} + \frac{3b}{d^3(b-dx)^2} - \frac{1}{d^3(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{30b^3}$$
$$+ \frac{(B(bc-ad)^6g^3i^2)\operatorname{Subst}\left(\int \frac{x^2(3A+B+3B\log(ex))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{90b^3d}$$

$$= \frac{B^2(bc-ad)^5g^3i^2x}{10b^2d^3} + \frac{B^2(bc-ad)^2g^3i^2(a+bx)^4}{60b^3} - \frac{B^2(bc-ad)^4g^3i^2(c+dx)^2}{20bd^4}$$
$$+ \frac{B^2(bc-ad)^3g^3i^2(c+dx)^3}{90d^4} - \frac{B(bc-ad)^3g^3i^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{90b^3d}$$
$$- \frac{B(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^3}$$
$$- \frac{B(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15b^2}$$
$$+ \frac{(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{60b^3}$$
$$+ \frac{(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{15b^2}$$
$$+ \frac{g^3i^2(a+bx)^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b}$$
$$+ \frac{B(bc-ad)^4g^3i^2(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{180b^3d^2}$$
$$- \frac{B^2(bc-ad)^6g^3i^2\log(c+dx)}{30b^3d^4}$$
$$+ \frac{(B^2(bc-ad)^6g^3i^2)\operatorname{Subst}\left(\int\left(\frac{b^3}{d^3(b-dx)^4} - \frac{3b^2}{d^3(b-dx)^3} + \frac{3b}{d^3(b-dx)^2} - \frac{1}{d^3(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{60b^3}$$
$$- \frac{(B(bc-ad)^6g^3i^2)\operatorname{Subst}\left(\int\frac{x(3B+2(3A+B)+6B\log(ex))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{180b^3d^2}$$

$$= \frac{3B^2(bc-ad)^5g^3i^2x}{20b^2d^3} + \frac{B^2(bc-ad)^2g^3i^2(a+bx)^4}{60b^3} - \frac{3B^2(bc-ad)^4g^3i^2(c+dx)^2}{40bd^4}$$

$$+ \frac{B^2(bc-ad)^3g^3i^2(c+dx)^3}{60d^4} - \frac{B(bc-ad)^3g^3i^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{90b^3d}$$

$$- \frac{B(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^3}$$

$$- \frac{B(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15b^2}$$

$$+ \frac{(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{60b^3}$$

$$+ \frac{(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{15b^2}$$

$$+ \frac{g^3i^2(a+bx)^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b}$$

$$+ \frac{B(bc-ad)^4g^3i^2(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{180b^3d^2}$$

$$- \frac{B(bc-ad)^5g^3i^2(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{180b^3d^3}$$

$$- \frac{B^2(bc-ad)^6g^3i^2\log(c+dx)}{20b^3d^4}$$

$$+ \frac{(B(bc-ad)^6g^3i^2)\,\text{Subst}\left(\int \frac{9B+2(3A+B)+6B\log(ex)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{180b^3d^3}$$

$$= \frac{3B^2(bc-ad)^5g^3i^2x}{20b^2d^3} + \frac{B^2(bc-ad)^2g^3i^2(a+bx)^4}{60b^3} - \frac{3B^2(bc-ad)^4g^3i^2(c+dx)^2}{40bd^4}$$
$$+ \frac{B^2(bc-ad)^3g^3i^2(c+dx)^3}{60d^4} - \frac{B(bc-ad)^3g^3i^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{90b^3d}$$
$$- \frac{B(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^3}$$
$$- \frac{B(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15b^2}$$
$$+ \frac{(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{60b^3}$$
$$+ \frac{(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{15b^2}$$
$$+ \frac{g^3i^2(a+bx)^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b}$$
$$+ \frac{B(bc-ad)^4g^3i^2(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{180b^3d^2}$$
$$- \frac{B(bc-ad)^5g^3i^2(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{180b^3d^3}$$
$$- \frac{B(bc-ad)^6g^3i^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{180b^3d^4}$$
$$- \frac{B^2(bc-ad)^6g^3i^2\log(c+dx)}{20b^3d^4}$$
$$+ \frac{(B^2(bc-ad)^6g^3i^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{30b^3d^4}$$

$$= \frac{3B^2(bc-ad)^5g^3i^2x}{20b^2d^3} + \frac{B^2(bc-ad)^2g^3i^2(a+bx)^4}{60b^3} - \frac{3B^2(bc-ad)^4g^3i^2(c+dx)^2}{40bd^4} + \frac{B^2(bc-ad)^3g^3i^2(c+dx)^3}{60d^4} - \frac{B(bc-ad)^3g^3i^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{90b^3d} - \frac{B(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^3} - \frac{B(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15b^2} + \frac{(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{60b^3} + \frac{(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{15b^2} + \frac{g^3i^2(a+bx)^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b} + \frac{B(bc-ad)^4g^3i^2(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{180b^3d^2} - \frac{B(bc-ad)^5g^3i^2(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{180b^3d^3} - \frac{B(bc-ad)^6g^3i^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{180b^3d^4} - \frac{B^2(bc-ad)^6g^3i^2\log(c+dx)}{20b^3d^4} - \frac{B^2(bc-ad)^6g^3i^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{30b^3d^4}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1559 vs. 2(711) = 1422.

Time = 0.79 (sec) , antiderivative size = 1559, normalized size of antiderivative = 2.19

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$

$$= \frac{g^3i^2\left(15(bc-ad)^2(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + 24d(bc-ad)(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + 10d\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x
)])^2,x]
```

```
[Out] (g^3*i^2*(15*(b*c - a*d)^2*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])
^2 + 24*d*(b*c - a*d)*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 +
```

```
10*d^2*(a + b*x)^6*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (5*B*(b*c - a*d
)^3*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*
x))/(c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/
(c + d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*
(b*c - a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c +
d*x)])*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2
- 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (-(b*c) + a*d)
*Log[c + d*x]) + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] -
Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^4
 + (2*B*(b*c - a*d)^2*(24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d)^3*(a +
 b*x)*Log[(e*(a + b*x))/(c + d*x)] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A +
B*Log[(e*(a + b*x))/(c + d*x)]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[
(e*(a + b*x))/(c + d*x)]) - 6*d^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c +
 d*x)]) - 24*B*(b*c - a*d)^4*Log[c + d*x] - 24*(b*c - a*d)^4*(A + B*Log[(e*
(a + b*x))/(c + d*x)])*Log[c + d*x] + 4*B*(b*c - a*d)^2*(2*b*d*(b*c - a*d)*
x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*(6*b*d*
(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*
(b*c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^3*(b*d*x + (-(b*c) + a*d)*Lo
g[c + d*x]) + 12*B*(b*c - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Lo
g[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^4 -
 (B*(b*c - a*d)*(24*b^2*B*c*d*(b*c - a*d)^3*x + 120*A*b*d*(b*c - a*d)^4*x +
 130*b*B*d*(b*c - a*d)^4*x + 24*a*b*B*d^2*(-(b*c) + a*d)^3*x - 12*b*B*c*d^2
*(b*c - a*d)^2*(a + b*x)^2 + 12*a*B*d^3*(b*c - a*d)^2*(a + b*x)^2 + 35*B*d^
2*(-(b*c) + a*d)^3*(a + b*x)^2 + 8*b*B*c*d^3*(b*c - a*d)*(a + b*x)^3 + 10*B
*d^3*(b*c - a*d)^2*(a + b*x)^3 + 8*a*B*d^4*(-(b*c) + a*d)*(a + b*x)^3 - 6*b
*B*c*d^4*(a + b*x)^4 + 6*a*B*d^5*(a + b*x)^4 + 120*B*d*(b*c - a*d)^4*(a + b
*x)*Log[(e*(a + b*x))/(c + d*x)] + 60*d^2*(-(b*c) + a*d)^3*(a + b*x)^2*(A +
 B*Log[(e*(a + b*x))/(c + d*x)]) + 40*d^3*(b*c - a*d)^2*(a + b*x)^3*(A + B*
Log[(e*(a + b*x))/(c + d*x)]) + 30*d^4*(-(b*c) + a*d)*(a + b*x)^4*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]) + 24*d^5*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(
c + d*x)]) - 24*b*B*c*(b*c - a*d)^4*Log[c + d*x] + 24*a*B*d*(b*c - a*d)^4*L
og[c + d*x] - 250*B*(b*c - a*d)^5*Log[c + d*x] - 120*(b*c - a*d)^5*(A + B*L
og[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 60*B*(b*c - a*d)^5*((2*Log[(d*(
a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c
 + d*x))/(b*c - a*d)])))/(6*d^4)))/(60*b^3)
```

Maple [F]

$$\int (bgx+ag)^3 (dix+ci)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\int (bgx+ag)^3(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, alg
orithm="fricas")
```

```
[Out] integral(A^2*b^3*d^2*g^3*i^2*x^5 + A^2*a^3*c^2*g^3*i^2 + (2*A^2*b^3*c*d + 3
*A^2*a*b^2*d^2)*g^3*i^2*x^4 + (A^2*b^3*c^2 + 6*A^2*a*b^2*c*d + 3*A^2*a^2*b*
d^2)*g^3*i^2*x^3 + (3*A^2*a*b^2*c^2 + 6*A^2*a^2*b*c*d + A^2*a^3*d^2)*g^3*i^
2*x^2 + (3*A^2*a^2*b*c^2 + 2*A^2*a^3*c*d)*g^3*i^2*x + (B^2*b^3*d^2*g^3*i^2*
x^5 + B^2*a^3*c^2*g^3*i^2 + (2*B^2*b^3*c*d + 3*B^2*a*b^2*d^2)*g^3*i^2*x^4 +
 (B^2*b^3*c^2 + 6*B^2*a*b^2*c*d + 3*B^2*a^2*b*d^2)*g^3*i^2*x^3 + (3*B^2*a*b
^2*c^2 + 6*B^2*a^2*b*c*d + B^2*a^3*d^2)*g^3*i^2*x^2 + (3*B^2*a^2*b*c^2 + 2*
B^2*a^3*c*d)*g^3*i^2*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^3*d^2*g^3
*i^2*x^5 + A*B*a^3*c^2*g^3*i^2 + (2*A*B*b^3*c*d + 3*A*B*a*b^2*d^2)*g^3*i^2*
x^4 + (A*B*b^3*c^2 + 6*A*B*a*b^2*c*d + 3*A*B*a^2*b*d^2)*g^3*i^2*x^3 + (3*A*
B*a*b^2*c^2 + 6*A*B*a^2*b*c*d + A*B*a^3*d^2)*g^3*i^2*x^2 + (3*A*B*a^2*b*c^2
 + 2*A*B*a^3*c*d)*g^3*i^2*x)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**3*(d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5178 vs. $2(680) = 1360$.

Time = 0.37 (sec) , antiderivative size = 5178, normalized size of antiderivative = 7.28

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, alg
orithm="maxima")
```

```
[Out] 1/6*A^2*b^3*d^2*g^3*i^2*x^6 + 2/5*A^2*b^3*c*d*g^3*i^2*x^5 + 3/5*A^2*a*b^2*d
^2*g^3*i^2*x^5 + 1/4*A^2*b^3*c^2*g^3*i^2*x^4 + 3/2*A^2*a*b^2*c*d*g^3*i^2*x^
4 + 3/4*A^2*a^2*b*d^2*g^3*i^2*x^4 + A^2*a*b^2*c^2*g^3*i^2*x^3 + 2*A^2*a^2*b
*c*d*g^3*i^2*x^3 + 1/3*A^2*a^3*d^2*g^3*i^2*x^3 + 3/2*A^2*a^2*b*c^2*g^3*i^2*
x^2 + A^2*a^3*c*d*g^3*i^2*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) +
 a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a^3*c^2*g^3*i^2 + 3*(x^2*log(b*e*
x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2
- (b*c - a*d)*x/(b*d))*A*B*a^2*b*c^2*g^3*i^2 + (2*x^3*log(b*e*x/(d*x + c) +
 a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c
*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a*b^2*c^2*g^3*i
^2 + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/
b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*
d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b^3*c^2*g^3*i^
2 + 2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^
2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^3*c*d*g^3*i^2 + 2*(2*x^3*lo
g(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x
 + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*
A*B*a^2*b*c*d*g^3*i^2 + 1/2*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6
*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)
*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*
A*B*a*b^2*c*d*g^3*i^2 + 1/15*(12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) +
 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*
d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^
2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*b^3*c*d*g^3*i^2 + 1/3*(2*x^3*l
og(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*
x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))
*A*B*a^3*d^2*g^3*i^2 + 1/4*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*
a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*
x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A
*B*a^2*b*d^2*g^3*i^2 + 1/10*(12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) +
12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d
^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2
 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*a*b^2*d^2*g^3*i^2 + 1/180*(60*x
^6*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 60*a^6*log(b*x + a)/b^6 + 60*c^6*
log(d*x + c)/d^6 - (12*(b^5*c*d^4 - a*b^4*d^5)*x^5 - 15*(b^5*c^2*d^3 - a^2*
b^3*d^5)*x^4 + 20*(b^5*c^3*d^2 - a^3*b^2*d^5)*x^3 - 30*(b^5*c^4*d - a^4*b*d
^5)*x^2 + 60*(b^5*c^5 - a^5*d^5)*x)/(b^5*d^5))*A*B*b^3*d^2*g^3*i^2 + A^2*a^
3*c^2*g^3*i^2*x - 1/180*(33*a^4*b*c^2*d^4*g^3*i^2 - 6*a^5*c*d^5*g^3*i^2 - 2
*(3*g^3*i^2*log(e) + g^3*i^2)*b^5*c^6 + 6*(6*g^3*i^2*log(e) + g^3*i^2)*a*b^
4*c^5*d - 3*(30*g^3*i^2*log(e) - g^3*i^2)*a^2*b^3*c^4*d^2 + 2*(60*g^3*i^2*l
og(e) - 17*g^3*i^2)*a^3*b^2*c^3*d^3)*B^2*log(d*x + c)/(b^2*d^4) + 1/30*(b^6
*c^6*g^3*i^2 - 6*a*b^5*c^5*d*g^3*i^2 + 15*a^2*b^4*c^4*d^2*g^3*i^2 - 20*a^3*
b^3*c^3*d^3*g^3*i^2 + 15*a^4*b^2*c^2*d^4*g^3*i^2 - 6*a^5*b*c*d^5*g^3*i^2 +
a^6*d^6*g^3*i^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-
(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d^4) + 1/360*(60*B^2*b^6*d^6*g^3*i^2*x
```

```
^6*log(e)^2 + 24*((6*g^3*i^2*log(e)^2 - g^3*i^2*log(e))*b^6*c*d^5 + (9*g^3*
i^2*log(e)^2 + g^3*i^2*log(e))*a*b^5*d^6)*B^2*x^5 + 6*((15*g^3*i^2*log(e)^2
 - 7*g^3*i^2*log(e) + g^3*i^2)*b^6*c^2*d^4 + 2*(45*g^3*i^2*log(e)^2 - 3*g^3
*i^2*log(e) - g^3*i^2)*a*b^5*c*d^5 + (45*g^3*i^2*log(e)^2 + 13*g^3*i^2*log(
e) + g^3*i^2)*a^2*b^4*d^6)*B^2*x^4 - 2*((2*g^3*i^2*log(e) - 3*g^3*i^2)*b^6*
c^3*d^3 - 3*(60*g^3*i^2*log(e)^2 - 26*g^3*i^2*log(e) + g^3*i^2)*a*b^5*c^2*d
^4 - 3*(120*g^3*i^2*log(e)^2 + 14*g^3*i^2*log(e) - 5*g^3*i^2)*a^2*b^4*c*d^5
 - (60*g^3*i^2*log(e)^2 + 38*g^3*i^2*log(e) + 9*g^3*i^2)*a^3*b^3*d^6)*B^2*x
^3 + ((6*g^3*i^2*log(e) - 7*g^3*i^2)*b^6*c^4*d^2 - 2*(18*g^3*i^2*log(e) - 2
3*g^3*i^2)*a*b^5*c^3*d^3 + 60*(9*g^3*i^2*log(e)^2 - 3*g^3*i^2*log(e) - g^3*
i^2)*a^2*b^4*c^2*d^4 + 2*(180*g^3*i^2*log(e)^2 + 102*g^3*i^2*log(e) + 5*g^3
*i^2)*a^3*b^3*c*d^5 + (6*g^3*i^2*log(e) + 11*g^3*i^2)*a^4*b^2*d^6)*B^2*x^2
 - 2*(2*(3*g^3*i^2*log(e) - 2*g^3*i^2)*b^6*c^5*d - 9*(4*g^3*i^2*log(e) - 3*g
^3*i^2)*a*b^5*c^4*d^2 + (90*g^3*i^2*log(e) - 77*g^3*i^2)*a^2*b^4*c^3*d^3 -
(180*g^3*i^2*log(e)^2 + 30*g^3*i^2*log(e) - 97*g^3*i^2)*a^3*b^3*c^2*d^4 - 3
*(12*g^3*i^2*log(e) + 17*g^3*i^2)*a^4*b^2*c*d^5 + 2*(3*g^3*i^2*log(e) + 4*g
^3*i^2)*a^5*b*d^6)*B^2*x + 6*(10*B^2*b^6*d^6*g^3*i^2*x^6 + 60*B^2*a^3*b^3*c
^2*d^4*g^3*i^2*x + 12*(2*b^6*c*d^5*g^3*i^2 + 3*a*b^5*d^6*g^3*i^2)*B^2*x^5 +
 15*(b^6*c^2*d^4*g^3*i^2 + 6*a*b^5*c*d^5*g^3*i^2 + 3*a^2*b^4*d^6*g^3*i^2)*B
^2*x^4 + 20*(3*a*b^5*c^2*d^4*g^3*i^2 + 6*a^2*b^4*c*d^5*g^3*i^2 + a^3*b^3*d^
6*g^3*i^2)*B^2*x^3 + 30*(3*a^2*b^4*c^2*d^4*g^3*i^2 + 2*a^3*b^3*c*d^5*g^3*i^
2)*B^2*x^2 + (15*a^4*b^2*c^2*d^4*g^3*i^2 - 6*a^5*b*c*d^5*g^3*i^2 + a^6*d^6*
g^3*i^2)*B^2)*log(b*x + a)^2 + 6*(10*B^2*b^6*d^6*g^3*i^2*x^6 + 60*B^2*a^3*b
^3*c^2*d^4*g^3*i^2*x + 12*(2*b^6*c*d^5*g^3*i^2 + 3*a*b^5*d^6*g^3*i^2)*B^2*x
^5 + 15*(b^6*c^2*d^4*g^3*i^2 + 6*a*b^5*c*d^5*g^3*i^2 + 3*a^2*b^4*d^6*g^3*i^
2)*B^2*x^4 + 20*(3*a*b^5*c^2*d^4*g^3*i^2 + 6*a^2*b^4*c*d^5*g^3*i^2 + a^3*b^
3*d^6*g^3*i^2)*B^2*x^3 + 30*(3*a^2*b^4*c^2*d^4*g^3*i^2 + 2*a^3*b^3*c*d^5*g^
3*i^2)*B^2*x^2 - (b^6*c^6*g^3*i^2 - 6*a*b^5*c^5*d*g^3*i^2 + 15*a^2*b^4*c^4*
d^2*g^3*i^2 - 20*a^3*b^3*c^3*d^3*g^3*i^2)*B^2)*log(d*x + c)^2 + 2*(60*B^2*b
^6*d^6*g^3*i^2*x^6*log(e) + 12*((12*g^3*i^2*log(e) - g^3*i^2)*b^6*c*d^5 + (
18*g^3*i^2*log(e) + g^3*i^2)*a*b^5*d^6)*B^2*x^5 + 3*((30*g^3*i^2*log(e) - 7
*g^3*i^2)*b^6*c^2*d^4 + 6*(30*g^3*i^2*log(e) - g^3*i^2)*a*b^5*c*d^5 + (90*g
^3*i^2*log(e) + 13*g^3*i^2)*a^2*b^4*d^6)*B^2*x^4 - 2*(b^6*c^3*d^3*g^3*i^2 -
 3*(60*g^3*i^2*log(e) - 13*g^3*i^2)*a*b^5*c^2*d^4 - 3*(120*g^3*i^2*log(e) +
 7*g^3*i^2)*a^2*b^4*c*d^5 - (60*g^3*i^2*log(e) + 19*g^3*i^2)*a^3*b^3*d^6)*B
^2*x^3 + 3*(b^6*c^4*d^2*g^3*i^2 - 6*a*b^5*c^3*d^3*g^3*i^2 + a^4*b^2*d^6*g^3
*i^2 + 30*(6*g^3*i^2*log(e) - g^3*i^2)*a^2*b^4*c^2*d^4 + 2*(60*g^3*i^2*log(
e) + 17*g^3*i^2)*a^3*b^3*c*d^5)*B^2*x^2 - 6*(b^6*c^5*d*g^3*i^2 - 6*a*b^5*c^
4*d^2*g^3*i^2 + 15*a^2*b^4*c^3*d^3*g^3*i^2 - 6*a^4*b^2*c*d^5*g^3*i^2 + a^5*
b*d^6*g^3*i^2 - 5*(12*g^3*i^2*log(e) + g^3*i^2)*a^3*b^3*c^2*d^4)*B^2*x - (6
*a*b^5*c^5*d*g^3*i^2 - 33*a^2*b^4*c^4*d^2*g^3*i^2 + 74*a^3*b^3*c^3*d^3*g^3*
i^2 - 9*(10*g^3*i^2*log(e) + 7*g^3*i^2)*a^4*b^2*c^2*d^4 + 18*(2*g^3*i^2*log
(e) + g^3*i^2)*a^5*b*c*d^5 - 2*(3*g^3*i^2*log(e) + g^3*i^2)*a^6*d^6)*B^2)*l
og(b*x + a) - 2*(60*B^2*b^6*d^6*g^3*i^2*x^6*log(e) + 12*((12*g^3*i^2*log(e)
 - g^3*i^2)*b^6*c*d^5 + (18*g^3*i^2*log(e) + g^3*i^2)*a*b^5*d^6)*B^2*x^5 +
```

```
3*((30*g^3*i^2*log(e) - 7*g^3*i^2)*b^6*c^2*d^4 + 6*(30*g^3*i^2*log(e) - g^3
*i^2)*a*b^5*c*d^5 + (90*g^3*i^2*log(e) + 13*g^3*i^2)*a^2*b^4*d^6)*B^2*x^4 -
 2*(b^6*c^3*d^3*g^3*i^2 - 3*(60*g^3*i^2*log(e) - 13*g^3*i^2)*a*b^5*c^2*d^4
- 3*(120*g^3*i^2*log(e) + 7*g^3*i^2)*a^2*b^4*c*d^5 - (60*g^3*i^2*log(e) + 1
9*g^3*i^2)*a^3*b^3*d^6)*B^2*x^3 + 3*(b^6*c^4*d^2*g^3*i^2 - 6*a*b^5*c^3*d^3*
g^3*i^2 + a^4*b^2*d^6*g^3*i^2 + 30*(6*g^3*i^2*log(e) - g^3*i^2)*a^2*b^4*c^2
*d^4 + 2*(60*g^3*i^2*log(e) + 17*g^3*i^2)*a^3*b^3*c*d^5)*B^2*x^2 - 6*(b^6*c
^5*d*g^3*i^2 - 6*a*b^5*c^4*d^2*g^3*i^2 + 15*a^2*b^4*c^3*d^3*g^3*i^2 - 6*a^4
*b^2*c*d^5*g^3*i^2 + a^5*b*d^6*g^3*i^2 - 5*(12*g^3*i^2*log(e) + g^3*i^2)*a^
3*b^3*c^2*d^4)*B^2*x + 6*(10*B^2*b^6*d^6*g^3*i^2*x^6 + 60*B^2*a^3*b^3*c^2*d
^4*g^3*i^2*x + 12*(2*b^6*c*d^5*g^3*i^2 + 3*a*b^5*d^6*g^3*i^2)*B^2*x^5 + 15*
(b^6*c^2*d^4*g^3*i^2 + 6*a*b^5*c*d^5*g^3*i^2 + 3*a^2*b^4*d^6*g^3*i^2)*B^2*x
^4 + 20*(3*a*b^5*c^2*d^4*g^3*i^2 + 6*a^2*b^4*c*d^5*g^3*i^2 + a^3*b^3*d^6*g^
3*i^2)*B^2*x^3 + 30*(3*a^2*b^4*c^2*d^4*g^3*i^2 + 2*a^3*b^3*c*d^5*g^3*i^2)*B
^2*x^2 + (15*a^4*b^2*c^2*d^4*g^3*i^2 - 6*a^5*b*c*d^5*g^3*i^2 + a^6*d^6*g^3*
i^2)*B^2)*log(b*x + a))*log(d*x + c))/(b^3*d^4)
```

Giac [F]

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \int (bgx+ag)^3(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, alg
orithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^3*(d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A
)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \int {\left(a\,g+b\,g\,x\right)}^3\,{\left(c\,i+d\,i\,x\right)}^2\,{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2\,dx$$

```
[In] int((a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2,
x)
```

```
[Out] int((a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2,
 x)
```

3.65 $\int (ag+bgx)^2(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 42, antiderivative size = 761

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$= -\frac{B^2(bc-ad)^4g^2i^2x}{10b^2d^2} - \frac{B^2(bc-ad)^3g^2i^2(c+dx)^2}{20bd^3} + \frac{B^2(bc-ad)^2g^2i^2(c+dx)^3}{30d^3}$$
$$+ \frac{B^2(bc-ad)^5g^2i^2\log\left(\frac{a+bx}{c+dx}\right)}{30b^3d^3} - \frac{B(bc-ad)^3g^2i^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^3d}$$
$$- \frac{B(bc-ad)^2g^2i^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15b^3}$$
$$- \frac{B(bc-ad)^3g^2i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5bd^3}$$
$$+ \frac{4B(bc-ad)^2g^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15d^3}$$
$$- \frac{bB(bc-ad)g^2i^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10d^3}$$
$$+ \frac{(bc-ad)^2g^2i^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{30b^3}$$
$$+ \frac{(bc-ad)g^2i^2(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^2}$$
$$+ \frac{g^2i^2(a+bx)^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b}$$
$$+ \frac{B(bc-ad)^4g^2i^2(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^3d^2}$$
$$+ \frac{B(bc-ad)^5g^2i^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^3d^3}$$
$$+ \frac{B^2(bc-ad)^5g^2i^2\log(c+dx)}{10b^3d^3} + \frac{B^2(bc-ad)^5g^2i^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{15b^3d^3}$$

```
[Out] -1/10*B^2*(-a*d+b*c)^4*g^2*i^2*x/b^2/d^2-1/20*B^2*(-a*d+b*c)^3*g^2*i^2*(d*x
+c)^2/b/d^3+1/30*B^2*(-a*d+b*c)^2*g^2*i^2*(d*x+c)^3/d^3+1/30*B^2*(-a*d+b*c)
^5*g^2*i^2*ln((b*x+a)/(d*x+c))/b^3/d^3-1/30*B*(-a*d+b*c)^3*g^2*i^2*(b*x+a)^
2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/d-1/15*B*(-a*d+b*c)^2*g^2*i^2*(b*x+a)^3*(
A+B*ln(e*(b*x+a)/(d*x+c)))/b^3-1/5*B*(-a*d+b*c)^3*g^2*i^2*(d*x+c)^2*(A+B*ln
(e*(b*x+a)/(d*x+c)))/b/d^3+4/15*B*(-a*d+b*c)^2*g^2*i^2*(d*x+c)^3*(A+B*ln(e*
(b*x+a)/(d*x+c)))/d^3-1/10*b*B*(-a*d+b*c)*g^2*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+
```

```
a)/(d*x+c)))/d^3+1/30*(-a*d+b*c)^2*g^2*i^2*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x
+c)))^2/b^3+1/10*(-a*d+b*c)*g^2*i^2*(b*x+a)^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*
x+c)))^2/b^2+1/5*g^2*i^2*(b*x+a)^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/
b+1/30*B*(-a*d+b*c)^4*g^2*i^2*(b*x+a)*(2*A+B+2*B*ln(e*(b*x+a)/(d*x+c)))/b^3
/d^2+1/30*B*(-a*d+b*c)^5*g^2*i^2*ln((-a*d+b*c)/b/(d*x+c))*(2*A+3*B+2*B*ln(e
*(b*x+a)/(d*x+c)))/b^3/d^3+1/10*B^2*(-a*d+b*c)^5*g^2*i^2*ln(d*x+c)/b^3/d^3+
1/15*B^2*(-a*d+b*c)^5*g^2*i^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^3
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 761, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.262$, Rules

used = {2562, 2383, 2381, 2384, 2354, 2438, 2373, 45, 2382, 12, 907}

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{Bg^2i^2(bc-ad)^5\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+3B\right)}{30b^3d^3} + \frac{Bg^2i^2(a+bx)(bc-ad)^4\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{30b^3d^2} - \frac{Bg^2i^2(a+bx)^2(bc-ad)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{30b^3d} + \frac{g^2i^2(a+bx)^3(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{30b^3} - \frac{Bg^2i^2(a+bx)^3(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{15b^3} + \frac{g^2i^2(a+bx)^3(c+dx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{10b^2} - \frac{Bg^2i^2(c+dx)^2(bc-ad)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5bd^3} + \frac{4Bg^2i^2(c+dx)^3(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{15d^3} - \frac{bBg^2i^2(c+dx)^4(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{10d^3} + \frac{g^2i^2(a+bx)^3(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b} + \frac{B^2g^2i^2(bc-ad)^5\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{15b^3d^3} + \frac{B^2g^2i^2(bc-ad)^5\log\left(\frac{a+bx}{c+dx}\right)}{30b^3d^3} + \frac{B^2g^2i^2(bc-ad)^5\log(c+dx)}{10b^3d^3} - \frac{B^2g^2i^2x(bc-ad)^4}{10b^2d^2} - \frac{B^2g^2i^2(c+dx)^2(bc-ad)^3}{20bd^3} + \frac{B^2g^2i^2(c+dx)^3(bc-ad)^2}{30d^3}$$

```
[In] Int[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,
x]
```

```
[Out] -1/10*(B^2*(b*c - a*d)^4*g^2*i^2*x)/(b^2*d^2) - (B^2*(b*c - a*d)^3*g^2*i^2*
(c + d*x)^2)/(20*b*d^3) + (B^2*(b*c - a*d)^2*g^2*i^2*(c + d*x)^3)/(30*d^3)
+ (B^2*(b*c - a*d)^5*g^2*i^2*Log[(a + b*x)/(c + d*x)])/(30*b^3*d^3) - (B*(b
*c - a*d)^3*g^2*i^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(30*b
```

```
^3*d) - (B*(b*c - a*d)^2*g^2*i^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c +
d*x)]))/(15*b^3) - (B*(b*c - a*d)^3*g^2*i^2*(c + d*x)^2*(A + B*Log[(e*(a +
b*x))/(c + d*x)]))/(5*b*d^3) + (4*B*(b*c - a*d)^2*g^2*i^2*(c + d*x)^3*(A +
B*Log[(e*(a + b*x))/(c + d*x)]))/(15*d^3) - (b*B*(b*c - a*d)*g^2*i^2*(c + d
*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(10*d^3) + ((b*c - a*d)^2*g^2*i
^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(30*b^3) + ((b*c - a
*d)*g^2*i^2*(a + b*x)^3*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(
10*b^2) + (g^2*i^2*(a + b*x)^3*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)])^2)/(5*b) + (B*(b*c - a*d)^4*g^2*i^2*(a + b*x)*(2*A + B + 2*B*Log[(e*(a
 + b*x))/(c + d*x)]))/(30*b^3*d^2) + (B*(b*c - a*d)^5*g^2*i^2*Log[(b*c - a*
d)/(b*(c + d*x))]*(2*A + 3*B + 2*B*Log[(e*(a + b*x))/(c + d*x)]))/(30*b^3*d
^3) + (B^2*(b*c - a*d)^5*g^2*i^2*Log[c + d*x])/(10*b^3*d^3) + (B^2*(b*c - a
*d)^5*g^2*i^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(15*b^3*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
```

```
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2383

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Dist[(m + q + 2)/(d*(q + 1)), Int[(
f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Dist[b*n*(p/(d*(q +
 1))), Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x]) /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ
[q, -1] && GtQ[m, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
```

```
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((bc-ad)^5g^2i^2\right)\text{Subst}\left(\int \frac{x^2(A+B\log(ex))^2}{(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)\\
&= \frac{g^2i^2(a+bx)^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b}\\
&\quad+\frac{\left(2(bc-ad)^5g^2i^2\right)\text{Subst}\left(\int \frac{x^2(A+B\log(ex))^2}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b}\\
&\quad-\frac{\left(2B(bc-ad)^5g^2i^2\right)\text{Subst}\left(\int \frac{x^2(A+B\log(ex))}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b}\\
&= -\frac{B(bc-ad)^3g^2i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5bd^3}\\
&\quad+\frac{4B(bc-ad)^2g^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15d^3}\\
&\quad-\frac{bB(bc-ad)g^2i^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10d^3}\\
&\quad+\frac{(bc-ad)g^2i^2(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^2}\\
&\quad+\frac{g^2i^2(a+bx)^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b}\\
&\quad+\frac{\left((bc-ad)^5g^2i^2\right)\text{Subst}\left(\int \frac{x^2(A+B\log(ex))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{10b^2}\\
&\quad-\frac{\left(B(bc-ad)^5g^2i^2\right)\text{Subst}\left(\int \frac{x^2(A+B\log(ex))}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b^2}\\
&\quad+\frac{\left(2B^2(bc-ad)^5g^2i^2\right)\text{Subst}\left(\int \frac{b^2-4bdx+6d^2x^2}{12d^3x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b}
\end{aligned}$$

$$= -\frac{B(bc-ad)^2g^2i^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15b^3} - \frac{B(bc-ad)^3g^2i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5bd^3} + \frac{4B(bc-ad)^2g^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15d^3} - \frac{bB(bc-ad)g^2i^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10d^3} + \frac{(bc-ad)^2g^2i^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{30b^3} + \frac{(bc-ad)g^2i^2(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^2} + \frac{g^2i^2(a+bx)^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} - \frac{\left(B(bc-ad)^5g^2i^2\right)\text{Subst}\left(\int\frac{x^2(A+B\log(ex))}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{15b^3} + \frac{\left(B^2(bc-ad)^5g^2i^2\right)\text{Subst}\left(\int\frac{x^2}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{15b^3} + \frac{\left(B^2(bc-ad)^5g^2i^2\right)\text{Subst}\left(\int\frac{b^2-4bdx+6d^2x^2}{x(b-dx)^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{30bd^3}$$

$$= -\frac{B(bc-ad)^3 g^2 i^2 (a+bx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^3 d}$$
$$-\frac{B(bc-ad)^2 g^2 i^2 (a+bx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15b^3}$$
$$-\frac{B(bc-ad)^3 g^2 i^2 (c+dx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5bd^3}$$
$$+\frac{4B(bc-ad)^2 g^2 i^2 (c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15d^3}$$
$$-\frac{bB(bc-ad) g^2 i^2 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10d^3}$$
$$+\frac{(bc-ad)^2 g^2 i^2 (a+bx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{30b^3}$$
$$+\frac{(bc-ad) g^2 i^2 (a+bx)^3 (c+dx) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^2}$$
$$+\frac{g^2 i^2 (a+bx)^3 (c+dx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b}$$
$$+\frac{(B^2(bc-ad)^5 g^2 i^2)\operatorname{Subst}\left(\int \left(\frac{b^2}{d^2(b-dx)^3}-\frac{2b}{d^2(b-dx)^2}+\frac{1}{d^2(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{15b^3}$$
$$+\frac{(B^2(bc-ad)^5 g^2 i^2)\operatorname{Subst}\left(\int \left(\frac{1}{b^2x}+\frac{3bd}{(b-dx)^4}-\frac{5d}{(b-dx)^3}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{30bd^3}$$
$$+\frac{(B(bc-ad)^5 g^2 i^2)\operatorname{Subst}\left(\int \frac{x(2A+B+2B\log(ex))}{(b-dx)^2}dx, x, \frac{a+bx}{c+dx}\right)}{30b^3 d}$$

$$= -\frac{B^2(bc-ad)^4g^2i^2x}{10b^2d^2} - \frac{B^2(bc-ad)^3g^2i^2(c+dx)^2}{20bd^3} + \frac{B^2(bc-ad)^2g^2i^2(c+dx)^3}{30d^3}$$
$$+ \frac{B^2(bc-ad)^5g^2i^2\log\left(\frac{a+bx}{c+dx}\right)}{30b^3d^3} - \frac{B(bc-ad)^3g^2i^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^3d}$$
$$- \frac{B(bc-ad)^2g^2i^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15b^3}$$
$$- \frac{B(bc-ad)^3g^2i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5bd^3}$$
$$+ \frac{4B(bc-ad)^2g^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15d^3}$$
$$- \frac{bB(bc-ad)g^2i^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10d^3}$$
$$+ \frac{(bc-ad)^2g^2i^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{30b^3}$$
$$+ \frac{(bc-ad)g^2i^2(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^2}$$
$$+ \frac{g^2i^2(a+bx)^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b}$$
$$+ \frac{B(bc-ad)^4g^2i^2(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^3d^2}$$
$$+ \frac{B^2(bc-ad)^5g^2i^2\log(c+dx)}{10b^3d^3}$$
$$- \frac{\left(B(bc-ad)^5g^2i^2\right)\text{Subst}\left(\int \frac{2A+3B+2B\log(ex)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{30b^3d^2}$$

$$= -\frac{B^2(bc-ad)^4 g^2 i^2 x}{10b^2 d^2} - \frac{B^2(bc-ad)^3 g^2 i^2 (c+dx)^2}{20bd^3} + \frac{B^2(bc-ad)^2 g^2 i^2 (c+dx)^3}{30d^3} + \frac{B^2(bc-ad)^5 g^2 i^2 \log\left(\frac{a+bx}{c+dx}\right)}{30b^3 d^3} - \frac{B(bc-ad)^3 g^2 i^2 (a+bx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^3 d} - \frac{B(bc-ad)^2 g^2 i^2 (a+bx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15b^3} - \frac{B(bc-ad)^3 g^2 i^2 (c+dx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5bd^3} + \frac{4B(bc-ad)^2 g^2 i^2 (c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15d^3} - \frac{bB(bc-ad) g^2 i^2 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10d^3} + \frac{(bc-ad)^2 g^2 i^2 (a+bx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{30b^3} + \frac{(bc-ad) g^2 i^2 (a+bx)^3 (c+dx) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^2} + \frac{g^2 i^2 (a+bx)^3 (c+dx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} + \frac{B(bc-ad)^4 g^2 i^2 (a+bx) \left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^3 d^2} + \frac{B(bc-ad)^5 g^2 i^2 \log\left(\frac{bc-ad}{b(c+dx)}\right) \left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^3 d^3} + \frac{B^2(bc-ad)^5 g^2 i^2 \log(c+dx)}{10b^3 d^3} - \frac{(B^2(bc-ad)^5 g^2 i^2)\,\mathrm{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{15b^3 d^3}$$

$$= -\frac{B^2(bc-ad)^4g^2i^2x}{10b^2d^2} - \frac{B^2(bc-ad)^3g^2i^2(c+dx)^2}{20bd^3} + \frac{B^2(bc-ad)^2g^2i^2(c+dx)^3}{30d^3}$$
$$+ \frac{B^2(bc-ad)^5g^2i^2\log\left(\frac{a+bx}{c+dx}\right)}{30b^3d^3} - \frac{B(bc-ad)^3g^2i^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^3d}$$
$$- \frac{B(bc-ad)^2g^2i^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15b^3}$$
$$- \frac{B(bc-ad)^3g^2i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5bd^3}$$
$$+ \frac{4B(bc-ad)^2g^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15d^3}$$
$$- \frac{bB(bc-ad)g^2i^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10d^3}$$
$$+ \frac{(bc-ad)^2g^2i^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{30b^3}$$
$$+ \frac{(bc-ad)g^2i^2(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^2}$$
$$+ \frac{g^2i^2(a+bx)^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b}$$
$$+ \frac{B(bc-ad)^4g^2i^2(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^3d^2}$$
$$+ \frac{B(bc-ad)^5g^2i^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^3d^3}$$
$$+ \frac{B^2(bc-ad)^5g^2i^2\log(c+dx)}{10b^3d^3} + \frac{B^2(bc-ad)^5g^2i^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{15b^3d^3}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 1194, normalized size of antiderivative = 1.57

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{g^2i^2\left(20d^3(bc-ad)^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + 30d^4(bc-ad)(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x
)])^2,x]
```

```
[Out] (g^2*i^2*(20*d^3*(b*c - a*d)^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*
x)])^2 + 30*d^4*(b*c - a*d)*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)^2 + 12*d^5*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 20*B*(b*c
 - a*d)^3*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a +
b*x))/(c + d*x)] - d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2
*B*(b*c - a*d)^2*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c
 + d*x)])*Log[c + d*x] + B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]
) + B*(b*c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*L
og[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - 10*B*(b*c - a*d)^
2*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x)
)/(c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c
 + d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(b
*c - a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*
x)])*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 -
2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (-(b*c) + a*d)*L
og[c + d*x]) + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Lo
g[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + B*(b
*c - a*d)*(24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[(e
*(a + b*x))/(c + d*x)] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(a
+ b*x))/(c + d*x)]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(a + b*x)
)/(c + d*x)]) - 6*d^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 24
*B*(b*c - a*d)^4*Log[c + d*x] - 24*(b*c - a*d)^4*(A + B*Log[(e*(a + b*x))/(
c + d*x)])*Log[c + d*x] + 4*B*(b*c - a*d)^2*(2*b*d*(b*c - a*d)*x - d^2*(a +
 b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*(6*b*d*(b*c - a*d)^
2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^
3*Log[c + d*x]) + 12*B*(b*c - a*d)^3*(b*d*x + (-(b*c) + a*d)*Log[c + d*x])
+ 12*B*(b*c - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*
Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))))/(60*b^3*d^3)
```

Maple [F]

$$\int (bgx + ag)^2 (dix + ci)^2 \left(A + B \ln \left(\frac{e(bx + a)}{dx + c}\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \int (bgx+ag)^2(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, alg
orithm="fricas")
```

```
[Out] integral(A^2*b^2*d^2*g^2*i^2*x^4 + A^2*a^2*c^2*g^2*i^2 + 2*(A^2*b^2*c*d + A
^2*a*b*d^2)*g^2*i^2*x^3 + (A^2*b^2*c^2 + 4*A^2*a*b*c*d + A^2*a^2*d^2)*g^2*i
^2*x^2 + 2*(A^2*a*b*c^2 + A^2*a^2*c*d)*g^2*i^2*x + (B^2*b^2*d^2*g^2*i^2*x^4
 + B^2*a^2*c^2*g^2*i^2 + 2*(B^2*b^2*c*d + B^2*a*b*d^2)*g^2*i^2*x^3 + (B^2*b
^2*c^2 + 4*B^2*a*b*c*d + B^2*a^2*d^2)*g^2*i^2*x^2 + 2*(B^2*a*b*c^2 + B^2*a^
2*c*d)*g^2*i^2*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^2*d^2*g^2*i^2*x
^4 + A*B*a^2*c^2*g^2*i^2 + 2*(A*B*b^2*c*d + A*B*a*b*d^2)*g^2*i^2*x^3 + (A*B
*b^2*c^2 + 4*A*B*a*b*c*d + A*B*a^2*d^2)*g^2*i^2*x^2 + 2*(A*B*a*b*c^2 + A*B*
a^2*c*d)*g^2*i^2*x)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**2*(d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3656 vs. $2(728) = 1456$.

Time = 0.34 (sec) , antiderivative size = 3656, normalized size of antiderivative = 4.80

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, alg
orithm="maxima")
```

```
[Out] 1/5*A^2*b^2*d^2*g^2*i^2*x^5 + 1/2*A^2*b^2*c*d*g^2*i^2*x^4 + 1/2*A^2*a*b*d^2
*g^2*i^2*x^4 + 1/3*A^2*b^2*c^2*g^2*i^2*x^3 + 4/3*A^2*a*b*c*d*g^2*i^2*x^3 +
```

1/3*A^2*a^2*d^2*g^2*i^2*x^3 + A^2*a*b*c^2*g^2*i^2*x^2 + A^2*a^2*c*d*g^2*i^2
x^2 + 2(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log
(d*x + c)/d)*A*B*a^2*c^2*g^2*i^2 + 2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x +
c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*
B*a*b*c^2*g^2*i^2 + 1/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3
*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(
b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*c^2*g^2*i^2 + 2*(x^2*log(b*e*x/(d*
x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*
c - a*d)*x/(b*d))*A*B*a^2*c*d*g^2*i^2 + 4/3*(2*x^3*log(b*e*x/(d*x + c) + a*
e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d
- a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a*b*c*d*g^2*i^2 +
1/6*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 +
6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^
2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b^2*c*d*g^2*i^2 + 1/
3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*
c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/
(b^2*d^2))*A*B*a^2*d^2*g^2*i^2 + 1/6*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x
+ c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a
*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b
^3*d^3))*A*B*a*b*d^2*g^2*i^2 + 1/30*(12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x
+ c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 -
a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*
d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*b^2*d^2*g^2*i^2 + A^2*a
^2*c^2*g^2*i^2*x - 1/30*(2*b^4*c^5*g^2*i^2*log(e) + 9*a^3*b*c^2*d^3*g^2*i^2
- 2*a^4*c*d^4*g^2*i^2 - 2*(5*g^2*i^2*log(e) - g^2*i^2)*a*b^3*c^4*d + (20*g
^2*i^2*log(e) - 9*g^2*i^2)*a^2*b^2*c^3*d^2)*B^2*log(d*x + c)/(b^2*d^3) - 1/
15*(b^5*c^5*g^2*i^2 - 5*a*b^4*c^4*d*g^2*i^2 + 10*a^2*b^3*c^3*d^2*g^2*i^2 -
10*a^3*b^2*c^2*d^3*g^2*i^2 + 5*a^4*b*c*d^4*g^2*i^2 - a^5*d^5*g^2*i^2)*(log(
b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a
*d)))*B^2/(b^3*d^3) + 1/60*(12*B^2*b^5*d^5*g^2*i^2*x^5*log(e)^2 + 6*((5*g^2
*i^2*log(e)^2 - g^2*i^2*log(e))*b^5*c*d^4 + (5*g^2*i^2*log(e)^2 + g^2*i^2*l
og(e))*a*b^4*d^5)*B^2*x^4 + 2*((10*g^2*i^2*log(e)^2 - 6*g^2*i^2*log(e) + g^
2*i^2)*b^5*c^2*d^3 + 2*(20*g^2*i^2*log(e)^2 - g^2*i^2)*a*b^4*c*d^4 + (10*g^
2*i^2*log(e)^2 + 6*g^2*i^2*log(e) + g^2*i^2)*a^2*b^3*d^5)*B^2*x^3 - ((2*g^2
*i^2*log(e) - 3*g^2*i^2)*b^5*c^3*d^2 - 3*(20*g^2*i^2*log(e)^2 - 10*g^2*i^2*
log(e) - g^2*i^2)*a*b^4*c^2*d^3 - 3*(20*g^2*i^2*log(e)^2 + 10*g^2*i^2*log(e
) - g^2*i^2)*a^2*b^3*c*d^4 - (2*g^2*i^2*log(e) + 3*g^2*i^2)*a^3*b^2*d^5)*B^
2*x^2 + 2*(2*(g^2*i^2*log(e) - g^2*i^2)*b^5*c^4*d - (10*g^2*i^2*log(e) - 11
*g^2*i^2)*a*b^4*c^3*d^2 + 6*(5*g^2*i^2*log(e)^2 - 3*g^2*i^2)*a^2*b^3*c^2*d^
3 + (10*g^2*i^2*log(e) + 11*g^2*i^2)*a^3*b^2*c*d^4 - 2*(g^2*i^2*log(e) + g^
2*i^2)*a^4*b*d^5)*B^2*x + 2*(6*B^2*b^5*d^5*g^2*i^2*x^5 + 30*B^2*a^2*b^3*c^2
*d^3*g^2*i^2*x + 15*(b^5*c*d^4*g^2*i^2 + a*b^4*d^5*g^2*i^2)*B^2*x^4 + 10*(b
^5*c^2*d^3*g^2*i^2 + 4*a*b^4*c*d^4*g^2*i^2 + a^2*b^3*d^5*g^2*i^2)*B^2*x^3 +
30*(a*b^4*c^2*d^3*g^2*i^2 + a^2*b^3*c*d^4*g^2*i^2)*B^2*x^2 + (10*a^3*b^2*c
^2*d^3*g^2*i^2 - 5*a^4*b*c*d^4*g^2*i^2 + a^5*d^5*g^2*i^2)*B^2)*log(b*x + a)

```
^2 + 2*(6*B^2*b^5*d^5*g^2*i^2*x^5 + 30*B^2*a^2*b^3*c^2*d^3*g^2*i^2*x + 15*(
b^5*c*d^4*g^2*i^2 + a*b^4*d^5*g^2*i^2)*B^2*x^4 + 10*(b^5*c^2*d^3*g^2*i^2 +
4*a*b^4*c*d^4*g^2*i^2 + a^2*b^3*d^5*g^2*i^2)*B^2*x^3 + 30*(a*b^4*c^2*d^3*g^
2*i^2 + a^2*b^3*c*d^4*g^2*i^2)*B^2*x^2 + (b^5*c^5*g^2*i^2 - 5*a*b^4*c^4*d*g
^2*i^2 + 10*a^2*b^3*c^3*d^2*g^2*i^2)*B^2)*log(d*x + c)^2 + 2*(12*B^2*b^5*d^
5*g^2*i^2*x^5*log(e) + 3*((10*g^2*i^2*log(e) - g^2*i^2)*b^5*c*d^4 + (10*g^2
*i^2*log(e) + g^2*i^2)*a*b^4*d^5)*B^2*x^4 + 2*(40*a*b^4*c*d^4*g^2*i^2*log(e
) + (10*g^2*i^2*log(e) - 3*g^2*i^2)*b^5*c^2*d^3 + (10*g^2*i^2*log(e) + 3*g^
2*i^2)*a^2*b^3*d^5)*B^2*x^3 - (b^5*c^3*d^2*g^2*i^2 - a^3*b^2*d^5*g^2*i^2 -
15*(4*g^2*i^2*log(e) - g^2*i^2)*a*b^4*c^2*d^3 - 15*(4*g^2*i^2*log(e) + g^2*
i^2)*a^2*b^3*c*d^4)*B^2*x^2 + 2*(30*a^2*b^3*c^2*d^3*g^2*i^2*log(e) + b^5*c^
4*d*g^2*i^2 - 5*a*b^4*c^3*d^2*g^2*i^2 + 5*a^3*b^2*c*d^4*g^2*i^2 - a^4*b*d^5
*g^2*i^2)*B^2*x + (2*a^5*d^5*g^2*i^2*log(e) + 2*a*b^4*c^4*d*g^2*i^2 - 9*a^2
*b^3*c^3*d^2*g^2*i^2 + (20*g^2*i^2*log(e) + 9*g^2*i^2)*a^3*b^2*c^2*d^3 - 2*
(5*g^2*i^2*log(e) + g^2*i^2)*a^4*b*c*d^4)*B^2)*log(b*x + a) - 2*(12*B^2*b^5
*d^5*g^2*i^2*x^5*log(e) + 3*((10*g^2*i^2*log(e) - g^2*i^2)*b^5*c*d^4 + (10*
g^2*i^2*log(e) + g^2*i^2)*a*b^4*d^5)*B^2*x^4 + 2*(40*a*b^4*c*d^4*g^2*i^2*lo
g(e) + (10*g^2*i^2*log(e) - 3*g^2*i^2)*b^5*c^2*d^3 + (10*g^2*i^2*log(e) + 3
*g^2*i^2)*a^2*b^3*d^5)*B^2*x^3 - (b^5*c^3*d^2*g^2*i^2 - a^3*b^2*d^5*g^2*i^2
 - 15*(4*g^2*i^2*log(e) - g^2*i^2)*a*b^4*c^2*d^3 - 15*(4*g^2*i^2*log(e) + g
^2*i^2)*a^2*b^3*c*d^4)*B^2*x^2 + 2*(30*a^2*b^3*c^2*d^3*g^2*i^2*log(e) + b^5
*c^4*d*g^2*i^2 - 5*a*b^4*c^3*d^2*g^2*i^2 + 5*a^3*b^2*c*d^4*g^2*i^2 - a^4*b*
d^5*g^2*i^2)*B^2*x + 2*(6*B^2*b^5*d^5*g^2*i^2*x^5 + 30*B^2*a^2*b^3*c^2*d^3*
g^2*i^2*x + 15*(b^5*c*d^4*g^2*i^2 + a*b^4*d^5*g^2*i^2)*B^2*x^4 + 10*(b^5*c^
2*d^3*g^2*i^2 + 4*a*b^4*c*d^4*g^2*i^2 + a^2*b^3*d^5*g^2*i^2)*B^2*x^3 + 30*(
a*b^4*c^2*d^3*g^2*i^2 + a^2*b^3*c*d^4*g^2*i^2)*B^2*x^2 + (10*a^3*b^2*c^2*d^
3*g^2*i^2 - 5*a^4*b*c*d^4*g^2*i^2 + a^5*d^5*g^2*i^2)*B^2)*log(b*x + a))*log
(d*x + c))/(b^3*d^3)
```

Giac [F]

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \int (bgx+ag)^2(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, alg
orithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^2*(d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A
)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^2\,(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

```
[In] int((a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2,
x)
[Out] int((a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2,
 x)
```

3.66 $\int (ag+bgx)(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 40, antiderivative size = 589

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{B^2(bc-ad)^3gi^2x}{12b^2d}+\frac{B^2(bc-ad)^2gi^2(c+dx)^2}{12bd^2}-\frac{B^2(bc-ad)^4gi^2\log\left(\frac{a+bx}{c+dx}\right)}{12b^3d^2}$$
$$-\frac{B(bc-ad)^3gi^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^3d}$$
$$-\frac{B(bc-ad)^2gi^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^3}$$
$$+\frac{B(bc-ad)^2gi^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4bd^2}$$
$$-\frac{B(bc-ad)gi^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^2}$$
$$+\frac{(bc-ad)^2gi^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{12b^3}$$
$$+\frac{(bc-ad)gi^2(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b^2}$$
$$+\frac{gi^2(a+bx)^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b}$$
$$-\frac{B(bc-ad)^4gi^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^3d^2}$$
$$-\frac{B^2(bc-ad)^4gi^2\log(c+dx)}{4b^3d^2}-\frac{B^2(bc-ad)^4gi^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{6b^3d^2}$$

```
[Out] 1/12*B^2*(-a*d+b*c)^3*g*i^2*x/b^2/d+1/12*B^2*(-a*d+b*c)^2*g*i^2*(d*x+c)^2/b
/d^2-1/12*B^2*(-a*d+b*c)^4*g*i^2*ln((b*x+a)/(d*x+c))/b^3/d^2-1/6*B*(-a*d+b*
c)^3*g*i^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/d-1/6*B*(-a*d+b*c)^2*g*i
^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3+1/4*B*(-a*d+b*c)^2*g*i^2*(d*x+
c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d^2-1/6*B*(-a*d+b*c)*g*i^2*(d*x+c)^3*(A+
B*ln(e*(b*x+a)/(d*x+c)))/d^2+1/12*(-a*d+b*c)^2*g*i^2*(b*x+a)^2*(A+B*ln(e*(b
*x+a)/(d*x+c)))^2/b^3+1/6*(-a*d+b*c)*g*i^2*(b*x+a)^2*(d*x+c)*(A+B*ln(e*(b*x
+a)/(d*x+c)))^2/b^2+1/4*g*i^2*(b*x+a)^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)
))^2/b-1/6*B*(-a*d+b*c)^4*g*i^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B+B*ln(e*(b*x+a
)/(d*x+c)))/b^3/d^2-1/4*B^2*(-a*d+b*c)^4*g*i^2*ln(d*x+c)/b^3/d^2-1/6*B^2*(-
a*d+b*c)^4*g*i^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^2
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {2562, 2383, 2381, 2384, 2354, 2438, 2373, 45, 2382, 12, 78}

$$\int (ag+bgx)(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= -\frac{Bgi^2(bc-ad)^4\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A+B\right)}{6b^3d^2} - \frac{Bgi^2(a+bx)(bc-ad)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6b^3d} + \frac{gi^2(a+bx)^2(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{12b^3} - \frac{Bgi^2(a+bx)^2(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6b^3} + \frac{gi^2(a+bx)^2(c+dx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{6b^2} + \frac{Bgi^2(c+dx)^2(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4bd^2} - \frac{Bgi^2(c+dx)^3(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6d^2} + \frac{gi^2(a+bx)^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b} - \frac{B^2gi^2(bc-ad)^4\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{6b^3d^2} - \frac{B^2gi^2(bc-ad)^4\log\left(\frac{a+bx}{c+dx}\right)}{12b^3d^2} - \frac{B^2gi^2(bc-ad)^4\log(c+dx)}{4b^3d^2} + \frac{B^2gi^2x(bc-ad)^3}{12b^2d} + \frac{B^2gi^2(c+dx)^2(bc-ad)^2}{12bd^2}$$

```
[In] Int[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^3*g*i^2*x)/(12*b^2*d) + (B^2*(b*c - a*d)^2*g*i^2*(c + d*x)
^2)/(12*b*d^2) - (B^2*(b*c - a*d)^4*g*i^2*Log[(a + b*x)/(c + d*x)])/(12*b^3
*d^2) - (B*(b*c - a*d)^3*g*i^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)
]))/(6*b^3*d) - (B*(b*c - a*d)^2*g*i^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))
/(c + d*x)]))/(6*b^3) + (B*(b*c - a*d)^2*g*i^2*(c + d*x)^2*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]))/(4*b*d^2) - (B*(b*c - a*d)*g*i^2*(c + d*x)^3*(A + B*L
og[(e*(a + b*x))/(c + d*x)]))/(6*d^2) + ((b*c - a*d)^2*g*i^2*(a + b*x)^2*(A
 + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(12*b^3) + ((b*c - a*d)*g*i^2*(a + b*
```

```
x)^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(6*b^2) + (g*i^2*(a
+ b*x)^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*b) - (B*(b*
c - a*d)^4*g*i^2*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B + B*Log[(e*(a + b*x)
)/(c + d*x)]))/(6*b^3*d^2) - (B^2*(b*c - a*d)^4*g*i^2*Log[c + d*x])/(4*b^3*
d^2) - (B^2*(b*c - a*d)^4*g*i^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(6
*b^3*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
```

```
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2383

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Dist[(m + q + 2)/(d*(q + 1)), Int[(
f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Dist[b*n*(p/(d*(q +
 1))), Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x]) /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ
[q, -1] && GtQ[m, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \left((bc-ad)^4gi^2\right)\text{Subst}\left(\int \frac{x(A+B\log(ex))^2}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{gi^2(a+bx)^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} + \frac{\left((bc-ad)^4gi^2\right)\text{Subst}\left(\int \frac{x(A+B\log(ex))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b} - \frac{\left(B(bc-ad)^4gi^2\right)\text{Subst}\left(\int \frac{x(A+B\log(ex))}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b}$$

$$= \frac{B(bc-ad)^2gi^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4bd^2} - \frac{B(bc-ad)gi^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^2} + \frac{(bc-ad)gi^2(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b^2} + \frac{gi^2(a+bx)^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} + \frac{\left((bc-ad)^4gi^2\right)\text{Subst}\left(\int \frac{x(A+B\log(ex))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{6b^2} - \frac{\left(B(bc-ad)^4gi^2\right)\text{Subst}\left(\int \frac{x(A+B\log(ex))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b^2} + \frac{\left(B^2(bc-ad)^4gi^2\right)\text{Subst}\left(\int \frac{-b+3dx}{6d^2x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b}$$

$$= -\frac{B(bc-ad)^2gi^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^3} + \frac{B(bc-ad)^2gi^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4bd^2} - \frac{B(bc-ad)gi^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^2} + \frac{(bc-ad)^2gi^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{12b^3} + \frac{(bc-ad)gi^2(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b^2} + \frac{gi^2(a+bx)^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} - \frac{\left(B(bc-ad)^4gi^2\right)\text{Subst}\left(\int\frac{x(A+B\log(ex))}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{6b^3} + \frac{\left(B^2(bc-ad)^4gi^2\right)\text{Subst}\left(\int\frac{x}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{6b^3} + \frac{\left(B^2(bc-ad)^4gi^2\right)\text{Subst}\left(\int\frac{-b+3dx}{x(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{12bd^2}$$

$$= -\frac{B(bc-ad)^3 gi^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^3d} - \frac{B(bc-ad)^2 gi^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^3} + \frac{B(bc-ad)^2 gi^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4bd^2} - \frac{B(bc-ad)gi^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^2} + \frac{(bc-ad)^2 gi^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{12b^3} + \frac{(bc-ad)gi^2(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b^2} + \frac{gi^2(a+bx)^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} + \frac{(B^2(bc-ad)^4 gi^2)\,\text{Subst}\left(\int\left(\frac{b}{d(-b+dx)^2}+\frac{1}{d(-b+dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{6b^3} + \frac{(B^2(bc-ad)^4 gi^2)\,\text{Subst}\left(\int\left(-\frac{1}{b^2x}+\frac{2d}{(b-dx)^3}-\frac{d}{b(b-dx)^2}-\frac{d}{b^2(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{12bd^2} + \frac{(B(bc-ad)^4 gi^2)\,\text{Subst}\left(\int\frac{A+B+B\log(ex)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{6b^3d}$$

$$=\frac{B^2(bc-ad)^3gi^2x}{12b^2d}+\frac{B^2(bc-ad)^2gi^2(c+dx)^2}{12bd^2}-\frac{B^2(bc-ad)^4gi^2\log\left(\frac{a+bx}{c+dx}\right)}{12b^3d^2}$$
$$-\frac{B(bc-ad)^3gi^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^3d}$$
$$-\frac{B(bc-ad)^2gi^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^3}$$
$$+\frac{B(bc-ad)^2gi^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4bd^2}$$
$$-\frac{B(bc-ad)gi^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^2}$$
$$+\frac{(bc-ad)^2gi^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{12b^3}$$
$$+\frac{(bc-ad)gi^2(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b^2}$$
$$+\frac{gi^2(a+bx)^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b}$$
$$-\frac{B(bc-ad)^4gi^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^3d^2}$$
$$-\frac{B^2(bc-ad)^4gi^2\log(c+dx)}{4b^3d^2}+\frac{(B^2(bc-ad)^4gi^2)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{6b^3d^2}$$

$$= \frac{B^2(bc-ad)^3 gi^2 x}{12b^2 d} + \frac{B^2(bc-ad)^2 gi^2 (c+dx)^2}{12bd^2} - \frac{B^2(bc-ad)^4 gi^2 \log\left(\frac{a+bx}{c+dx}\right)}{12b^3 d^2}$$
$$- \frac{B(bc-ad)^3 gi^2 (a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^3 d}$$
$$- \frac{B(bc-ad)^2 gi^2 (a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^3}$$
$$+ \frac{B(bc-ad)^2 gi^2 (c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4bd^2}$$
$$- \frac{B(bc-ad) gi^2 (c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^2}$$
$$+ \frac{(bc-ad)^2 gi^2 (a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{12b^3}$$
$$+ \frac{(bc-ad) gi^2 (a+bx)^2 (c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b^2}$$
$$+ \frac{gi^2 (a+bx)^2 (c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b}$$
$$- \frac{B(bc-ad)^4 gi^2 \log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^3 d^2}$$
$$- \frac{B^2(bc-ad)^4 gi^2 \log(c+dx)}{4b^3 d^2} - \frac{B^2(bc-ad)^4 gi^2 \operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{6b^3 d^2}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 677, normalized size of antiderivative = 1.15

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{gi^2\left(-4(bc-ad)(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + 3b(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + \frac{4B(bc-ad)^2\left(2Abd(\right.}{}\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)^2,x]
```

```
[Out] (g*i^2*(-4*(b*c - a*d)*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 +
 3*b*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (4*B*(b*c - a*d)^
2*(2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*(b*d*x + (b*c - a*d)*Log[a + b*x])
 + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + b^2*(c + d*x)
^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*(b*c - a*d)^2*Log[a + b*x]*(A +
```

```
 B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*(b*c - a*d)^2*Log[c + d*x] - B*(b*c
- a*d)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) -
2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/b^3 - (B*(b*c - a*d)*(6*A*b*d
*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*(b*d*x + (b*c - a*d)*Log[a + b*x]) - B
*(b*c - a*d)*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c - a*d)^2*Log[a
 + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 3*b
^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*b^3*(c
+ d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*(b*c - a*d)^3*Log[a + b*x
]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(b*c - a*d)^3*Log[c + d*x] - 3
*B*(b*c - a*d)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a
*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/b^3))/(12*d^2)
```

Maple [F]

$$\int (bgx + ag)\,(dix + ci)^2 \left(A + B\ln\left(\frac{e(bx + a)}{dx + c}\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ag + bgx)(ci + dix)^2 \left(A + B\log\left(\frac{e(a + bx)}{c + dx}\right)\right)^2 dx$$
$$= \int (bgx + ag)(dix + ci)^2 \left(B\log\left(\frac{(bx + a)e}{dx + c}\right) + A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algor
ithm="fricas")
```

```
[Out] integral(A^2*b*d^2*g*i^2*x^3 + A^2*a*c^2*g*i^2 + (2*A^2*b*c*d + A^2*a*d^2)*
g*i^2*x^2 + (A^2*b*c^2 + 2*A^2*a*c*d)*g*i^2*x + (B^2*b*d^2*g*i^2*x^3 + B^2*
a*c^2*g*i^2 + (2*B^2*b*c*d + B^2*a*d^2)*g*i^2*x^2 + (B^2*b*c^2 + 2*B^2*a*c*
d)*g*i^2*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b*d^2*g*i^2*x^3 + A*B*a
*c^2*g*i^2 + (2*A*B*b*c*d + A*B*a*d^2)*g*i^2*x^2 + (A*B*b*c^2 + 2*A*B*a*c*d
)*g*i^2*x)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2259 vs. $2(562) = 1124$.

Time = 0.32 (sec) , antiderivative size = 2259, normalized size of antiderivative = 3.84

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algor
ithm="maxima")
```

```
[Out] 1/4*A^2*b*d^2*g*i^2*x^4 + 2/3*A^2*b*c*d*g*i^2*x^3 + 1/3*A^2*a*d^2*g*i^2*x^3
 + 1/2*A^2*b*c^2*g*i^2*x^2 + A^2*a*c*d*g*i^2*x^2 + 2*(x*log(b*e*x/(d*x + c)
 + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a*c^2*g*i^2 +
(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(
d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*b*c^2*g*i^2 + 2*(x^2*log(b*e*x/(d*x
 + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c
 - a*d)*x/(b*d))*A*B*a*c*d*g*i^2 + 2/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*
x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b
*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b*c*d*g*i^2 + 1/3*(2*x^
3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log
(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^
2))*A*B*a*d^2*g*i^2 + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*
a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*
x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A
*B*b*d^2*g*i^2 + A^2*a*c^2*g*i^2*x - 1/12*(7*a^2*b*c^2*d^2*g*i^2 - 2*a^3*c*
d^3*g*i^2 - (2*g*i^2*log(e) - g*i^2)*b^3*c^4 + 2*(4*g*i^2*log(e) - 3*g*i^2)
*a*b^2*c^3*d)*B^2*log(d*x + c)/(b^2*d^2) + 1/6*(b^4*c^4*g*i^2 - 4*a*b^3*c^3
*d*g*i^2 + 6*a^2*b^2*c^2*d^2*g*i^2 - 4*a^3*b*c*d^3*g*i^2 + a^4*d^4*g*i^2)*(
log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c
 - a*d)))*B^2/(b^3*d^2) + 1/12*(3*B^2*b^4*d^4*g*i^2*x^4*log(e)^2 + 2*((4*g*
i^2*log(e)^2 - g*i^2*log(e))*b^4*c*d^3 + (2*g*i^2*log(e)^2 + g*i^2*log(e))*
a*b^3*d^4)*B^2*x^3 + ((6*g*i^2*log(e)^2 - 5*g*i^2*log(e) + g*i^2)*b^4*c^2*d
^2 + 2*(6*g*i^2*log(e)^2 + 2*g*i^2*log(e) - g*i^2)*a*b^3*c*d^3 + (g*i^2*log
```

```
(e) + g*i^2)*a^2*b^2*d^4)*B^2*x^2 - ((2*g*i^2*log(e) - 3*g*i^2)*b^4*c^3*d -
 (12*g*i^2*log(e)^2 - 4*g*i^2*log(e) - 7*g*i^2)*a*b^3*c^2*d^2 - (8*g*i^2*lo
g(e) + 5*g*i^2)*a^2*b^2*c*d^3 + (2*g*i^2*log(e) + g*i^2)*a^3*b*d^4)*B^2*x +
 (3*B^2*b^4*d^4*g*i^2*x^4 + 12*B^2*a*b^3*c^2*d^2*g*i^2*x + 4*(2*b^4*c*d^3*g
*i^2 + a*b^3*d^4*g*i^2)*B^2*x^3 + 6*(b^4*c^2*d^2*g*i^2 + 2*a*b^3*c*d^3*g*i^
2)*B^2*x^2 + (6*a^2*b^2*c^2*d^2*g*i^2 - 4*a^3*b*c*d^3*g*i^2 + a^4*d^4*g*i^2
)*B^2)*log(b*x + a)^2 + (3*B^2*b^4*d^4*g*i^2*x^4 + 12*B^2*a*b^3*c^2*d^2*g*i
^2*x + 4*(2*b^4*c*d^3*g*i^2 + a*b^3*d^4*g*i^2)*B^2*x^3 + 6*(b^4*c^2*d^2*g*i
^2 + 2*a*b^3*c*d^3*g*i^2)*B^2*x^2 - (b^4*c^4*g*i^2 - 4*a*b^3*c^3*d*g*i^2)*B
^2)*log(d*x + c)^2 + (6*B^2*b^4*d^4*g*i^2*x^4*log(e) + 2*((8*g*i^2*log(e) -
 g*i^2)*b^4*c*d^3 + (4*g*i^2*log(e) + g*i^2)*a*b^3*d^4)*B^2*x^3 + (a^2*b^2*
d^4*g*i^2 + (12*g*i^2*log(e) - 5*g*i^2)*b^4*c^2*d^2 + 4*(6*g*i^2*log(e) + g
*i^2)*a*b^3*c*d^3)*B^2*x^2 - 2*(b^4*c^3*d*g*i^2 - 4*a^2*b^2*c*d^3*g*i^2 + a
^3*b*d^4*g*i^2 - 2*(6*g*i^2*log(e) - g*i^2)*a*b^3*c^2*d^2)*B^2*x - (2*a*b^3
*c^3*d*g*i^2 - (12*g*i^2*log(e) + g*i^2)*a^2*b^2*c^2*d^2 + 2*(4*g*i^2*log(e
) - g*i^2)*a^3*b*c*d^3 - (2*g*i^2*log(e) - g*i^2)*a^4*d^4)*B^2)*log(b*x + a
) - (6*B^2*b^4*d^4*g*i^2*x^4*log(e) + 2*((8*g*i^2*log(e) - g*i^2)*b^4*c*d^3
 + (4*g*i^2*log(e) + g*i^2)*a*b^3*d^4)*B^2*x^3 + (a^2*b^2*d^4*g*i^2 + (12*g
*i^2*log(e) - 5*g*i^2)*b^4*c^2*d^2 + 4*(6*g*i^2*log(e) + g*i^2)*a*b^3*c*d^3
)*B^2*x^2 - 2*(b^4*c^3*d*g*i^2 - 4*a^2*b^2*c*d^3*g*i^2 + a^3*b*d^4*g*i^2 -
2*(6*g*i^2*log(e) - g*i^2)*a*b^3*c^2*d^2)*B^2*x + 2*(3*B^2*b^4*d^4*g*i^2*x^
4 + 12*B^2*a*b^3*c^2*d^2*g*i^2*x + 4*(2*b^4*c*d^3*g*i^2 + a*b^3*d^4*g*i^2)*
B^2*x^3 + 6*(b^4*c^2*d^2*g*i^2 + 2*a*b^3*c*d^3*g*i^2)*B^2*x^2 + (6*a^2*b^2*
c^2*d^2*g*i^2 - 4*a^3*b*c*d^3*g*i^2 + a^4*d^4*g*i^2)*B^2)*log(b*x + a))*log
(d*x + c))/(b^3*d^2)
```

Giac [F]

$$\int (ag+bgx)(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)(dix+ci)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algor
ithm="giac")
```

```
[Out] integrate((b*g*x + a*g)*(d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^
2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)\,\left(c\,i+d\,i\,x\right)^2\,\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)^2\,dx$$

```
[In] int((a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
[Out] int((a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x
)
```

3.67 $\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 32, antiderivative size = 334

$$\begin{aligned}
&\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx \\
&= \frac{B^2(bc-ad)^2 i^2 x}{3b^2} + \frac{B^2(bc-ad)^3 i^2 \log\left(\frac{a+bx}{c+dx}\right)}{3b^3 d} \\
&\quad - \frac{2B(bc-ad)^2 i^2 (a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^3} \\
&\quad - \frac{B(bc-ad) i^2 (c+dx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd} + \frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d} \\
&\quad + \frac{B^2(bc-ad)^3 i^2 \log(c+dx)}{b^3 d} + \frac{2B(bc-ad)^3 i^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3 d} \\
&\quad - \frac{2B^2(bc-ad)^3 i^2 \operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3 d}
\end{aligned}$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*i^2*x/b^2+1/3*B^2*(-a*d+b*c)^3*i^2*ln((b*x+a)/(d*x+c))
/b^3/d-2/3*B*(-a*d+b*c)^2*i^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3-1/3*B
*(-a*d+b*c)*i^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/3*i^2*(d*x+c)^3
*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d+B^2*(-a*d+b*c)^3*i^2*ln(d*x+c)/b^3/d+2/3*B
*(-a*d+b*c)^3*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3
/d-2/3*B^2*(-a*d+b*c)^3*i^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2552, 2356, 2389, 2379, 2438, 2351, 31, 46}

$$\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$$
$$=\frac{2Bi^2(bc-ad)^3\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b^3d}-\frac{2Bi^2(a+bx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b^3}-\frac{Bi^2(c+dx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3bd}+\frac{i^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d}-\frac{2B^2i^2(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d}+\frac{B^2i^2(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d}+\frac{B^2i^2(bc-ad)^3\log(c+dx)}{b^3d}+\frac{B^2i^2x(bc-ad)^2}{3b^2}$$

```
[In] Int[(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^2*i^2*x)/(3*b^2) + (B^2*(b*c - a*d)^3*i^2*Log[(a + b*x)/(c
 + d*x)])/(3*b^3*d) - (2*B*(b*c - a*d)^2*i^2*(a + b*x)*(A + B*Log[(e*(a + b
*x))/(c + d*x)]))/(3*b^3) - (B*(b*c - a*d)*i^2*(c + d*x)^2*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]))/(3*b*d) + (i^2*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(
c + d*x)])^2)/(3*d) + (B^2*(b*c - a*d)^3*i^2*Log[c + d*x])/(b^3*d) + (2*B*(
b*c - a*d)^3*i^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x))
/(d*(a + b*x))])/(3*b^3*d) - (2*B^2*(b*c - a*d)^3*i^2*PolyLog[2, (b*(c + d*
x))/(d*(a + b*x))])/(3*b^3*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \left((bc-ad)^3 i^2\right) \text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$=\frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d}-\frac{(2B(bc-ad)^3i^2)\operatorname{Subst}\left(\int\frac{A+B\log(ex)}{x(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{3d}$$

$$=\frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d}-\frac{(2B(bc-ad)^3i^2)\operatorname{Subst}\left(\int\frac{A+B\log(ex)}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b}-\frac{(2B(bc-ad)^3i^2)\operatorname{Subst}\left(\int\frac{A+B\log(ex)}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{3bd}$$

$$=-\frac{B(bc-ad)i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd}+\frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d}-\frac{(2B(bc-ad)^3i^2)\operatorname{Subst}\left(\int\frac{A+B\log(ex)}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^2}-\frac{(2B(bc-ad)^3i^2)\operatorname{Subst}\left(\int\frac{A+B\log(ex)}{x(b-dx)}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^2d}+\frac{(B^2(bc-ad)^3i^2)\operatorname{Subst}\left(\int\frac{1}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{3bd}$$

$$=-\frac{2B(bc-ad)^2i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^3}-\frac{B(bc-ad)i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd}+\frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d}+\frac{2B(bc-ad)^3i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d}+\frac{(2B^2(bc-ad)^3i^2)\operatorname{Subst}\left(\int\frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^3}-\frac{(2B^2(bc-ad)^3i^2)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^3d}+\frac{(B^2(bc-ad)^3i^2)\operatorname{Subst}\left(\int\left(\frac{1}{b^2x}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{3bd}$$

$$= \frac{B^2(bc-ad)^2 i^2 x}{3b^2} + \frac{B^2(bc-ad)^3 i^2 \log\left(\frac{a+bx}{c+dx}\right)}{3b^3 d} - \frac{2B(bc-ad)^2 i^2 (a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^3} - \frac{B(bc-ad) i^2 (c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd} + \frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d} + \frac{B^2(bc-ad)^3 i^2 \log(c+dx)}{b^3 d} + \frac{2B(bc-ad)^3 i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3 d} - \frac{2B^2(bc-ad)^3 i^2 \mathrm{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3 d}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.86

$$\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$

$$= \frac{i^2\left((c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - \frac{B(bc-ad)\left(2Abd(bc-ad)x-B(bc-ad)(bdx+(bc-ad)\log(a+bx))+2Bd(bc-ad)(a+bx)\log(\right.}{}\right.}{}$$

```
[In] Integrate[(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (i^2*((c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(b*c - a*d)*(
2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*(b*d*x + (b*c - a*d)*Log[a + b*x]) +
2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + b^2*(c + d*x)^2*
(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*(b*c - a*d)^2*Log[a + b*x]*(A + B*
Log[(e*(a + b*x))/(c + d*x)]) - 2*B*(b*c - a*d)^2*Log[c + d*x] - B*(b*c - a
*d)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*P
olyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/b^3))/(3*d)
```

Maple [F]

$$\int (dix + ci)^2 \left(A + B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ci + dix)^2 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (dix + ci)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2 dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x^2
 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*d
^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log((b*e*x + a*e)/(d*x + c)), x
)
```

Sympy [F(-1)]

Timed out.

$$\int (ci + dix)^2 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1202 vs. $2(319) = 638$.

Time = 0.28 (sec) , antiderivative size = 1202, normalized size of antiderivative = 3.60

$$\int (ci + dix)^2 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*A^2*d^2*i^2*x^3 + A^2*c*d*i^2*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x
 + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*c^2*i^2 + 2*(x^2*log(b*e*
x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2
- (b*c - a*d)*x/(b*d))*A*B*c*d*i^2 + 1/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(
d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a
*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*d^2*i^2 + A^2*c^2*i^2
*x - 1/3*(5*a*b*c^2*d*i^2 - 2*a^2*c*d^2*i^2 + (2*i^2*log(e) - 3*i^2)*b^2*c^
3)*B^2*log(d*x + c)/(b^2*d) - 2/3*(b^3*c^3*i^2 - 3*a*b^2*c^2*d*i^2 + 3*a^2*
b*c*d^2*i^2 - a^3*d^3*i^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1)
 + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d) + 1/3*(B^2*b^3*d^3*i^2*x^
3*log(e)^2 + (a*b^2*d^3*i^2*log(e) + (3*i^2*log(e)^2 - i^2*log(e))*b^3*c*d^
2)*B^2*x^2 + ((3*i^2*log(e)^2 - 4*i^2*log(e) + i^2)*b^3*c^2*d + 2*(3*i^2*lo
g(e) - i^2)*a*b^2*c*d^2 - (2*i^2*log(e) - i^2)*a^2*b*d^3)*B^2*x + (B^2*b^3*
d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i^2*x + (3*a*b^2*c^
2*d*i^2 - 3*a^2*b*c*d^2*i^2 + a^3*d^3*i^2)*B^2)*log(b*x + a)^2 + (B^2*b^3*d
^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i^2*x + B^2*b^3*c^3*
i^2)*log(d*x + c)^2 + (2*B^2*b^3*d^3*i^2*x^3*log(e) + (a*b^2*d^3*i^2 + (6*i
^2*log(e) - i^2)*b^3*c*d^2)*B^2*x^2 + 2*(3*a*b^2*c*d^2*i^2 - a^2*b*d^3*i^2
+ (3*i^2*log(e) - 2*i^2)*b^3*c^2*d)*B^2*x + (2*(3*i^2*log(e) - 2*i^2)*a*b^2
*c^2*d - (6*i^2*log(e) - 7*i^2)*a^2*b*c*d^2 + (2*i^2*log(e) - 3*i^2)*a^3*d^
3)*B^2)*log(b*x + a) - (2*B^2*b^3*d^3*i^2*x^3*log(e) + (a*b^2*d^3*i^2 + (6*
i^2*log(e) - i^2)*b^3*c*d^2)*B^2*x^2 + 2*(3*a*b^2*c*d^2*i^2 - a^2*b*d^3*i^2
 + (3*i^2*log(e) - 2*i^2)*b^3*c^2*d)*B^2*x + 2*(B^2*b^3*d^3*i^2*x^3 + 3*B^2
*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i^2*x + (3*a*b^2*c^2*d*i^2 - 3*a^2*b*c
*d^2*i^2 + a^3*d^3*i^2)*B^2)*log(b*x + a))*log(d*x + c))/(b^3*d)
```

Giac [F]

$$\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (dix+ci)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate((d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (c\,i+d\,i\,x)^2 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

```
[In] int((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
[Out] int((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.68 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 535

$$
\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx \\
&= -\frac{Bd(bc-ad)i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g} \\
&\quad+\frac{2B(bc-ad)^2i^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g} \\
&\quad+\frac{d(bc-ad)i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g}+\frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg} \\
&\quad+\frac{B^2(bc-ad)^2i^2\log(c+dx)}{b^3g}+\frac{B(bc-ad)^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad-\frac{(bc-ad)^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad+\frac{2B^2(bc-ad)^2i^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^3g}-\frac{B^2(bc-ad)^2i^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad+\frac{2B(bc-ad)^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad+\frac{2B^2(bc-ad)^2i^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}
\end{aligned}
$$

```
[Out] -B*d*(-a*d+b*c)*i^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/g+2*B*(-a*d+b*c
)^2*i^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/g+d*(-a*d+
b*c)*i^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^3/g+1/2*i^2*(d*x+c)^2*(A+B
*ln(e*(b*x+a)/(d*x+c)))^2/b/g+B^2*(-a*d+b*c)^2*i^2*ln(d*x+c)/b^3/g+B*(-a*d+
b*c)^2*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g-(-a*
d+b*c)^2*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g+
2*B^2*(-a*d+b*c)^2*i^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/g-B^2*(-a*d+b*c)^
2*i^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g+2*B*(-a*d+b*c)^2*i^2*(A+B*ln(e*(
b*x+a)/(d*x+c)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g+2*B^2*(-a*d+b*c)^2*i^
2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^3/g
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.262, Rules used = {2562, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31}

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{2Bi^2(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3g}$$
$$+\frac{di^2(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^3g}$$
$$-\frac{Bdi^2(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3g}$$
$$+\frac{2Bi^2(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3g}$$
$$-\frac{i^2(bc-ad)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^3g}$$
$$+\frac{Bi^2(bc-ad)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3g}$$
$$+\frac{i^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2bg}+\frac{2B^2i^2(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^3g}$$
$$-\frac{B^2i^2(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}$$
$$+\frac{2B^2i^2(bc-ad)^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}+\frac{B^2i^2(bc-ad)^2\log(c+dx)}{b^3g}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x),
x]
```

```
[Out] -((B*d*(b*c - a*d)*i^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^3
*g)) + (2*B*(b*c - a*d)^2*i^2*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*
(a + b*x))/(c + d*x)]))/(b^3*g) + (d*(b*c - a*d)*i^2*(a + b*x)*(A + B*Log[(
e*(a + b*x))/(c + d*x)])^2)/(b^3*g) + (i^2*(c + d*x)^2*(A + B*Log[(e*(a + b
*x))/(c + d*x)])^2)/(2*b*g) + (B^2*(b*c - a*d)^2*i^2*Log[c + d*x])/(b^3*g)
+ (B*(b*c - a*d)^2*i^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c +
 d*x))/(d*(a + b*x))])/(b^3*g) - ((b*c - a*d)^2*i^2*(A + B*Log[(e*(a + b*x)
)/(c + d*x)])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^3*g) + (2*B^2*(b*c
 - a*d)^2*i^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^3*g) - (B^2*(b*c
- a*d)^2*i^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b^3*g) + (2*B*(b*c -
 a*d)^2*i^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (b*(c + d*x))/(
d*(a + b*x))])/(b^3*g) + (2*B^2*(b*c - a*d)^2*i^2*PolyLog[3, (b*(c + d*x))/
(d*(a + b*x))])/(b^3*g)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
```

```
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)^2 i^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{g}$$

$$= \frac{((bc-ad)^2 i^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg} + \frac{(d(bc-ad)^2 i^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg}$$

$$= \frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg} + \frac{((bc-ad)^2 i^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2 g} - \frac{(B(bc-ad)^2 i^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg} + \frac{(d(bc-ad)^2 i^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2 g}$$

$$= \frac{d(bc-ad)i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3 g} + \frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg} - \frac{(bc-ad)^2 i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3 g} + \frac{(2B(bc-ad)^2 i^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)(A+B\log(ex))}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3 g} - \frac{(B(bc-ad)^2 i^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2 g} - \frac{(2Bd(bc-ad)^2 i^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3 g} - \frac{(Bd(bc-ad)^2 i^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2 g}$$

$$= -\frac{Bd(bc-ad)i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g}$$
$$+\frac{2B(bc-ad)^2i^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g}$$
$$+\frac{d(bc-ad)i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g}$$
$$+\frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg}$$
$$+\frac{B(bc-ad)^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}$$
$$-\frac{(bc-ad)^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}$$
$$+\frac{2B(bc-ad)^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}$$
$$-\frac{(B^2(bc-ad)^2i^2)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3g}$$
$$-\frac{(2B^2(bc-ad)^2i^2)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3g}$$
$$-\frac{(2B^2(bc-ad)^2i^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3g}$$
$$+\frac{(B^2d(bc-ad)^2i^2)\operatorname{Subst}\left(\int\frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3g}$$

$$
\begin{aligned}
&= -\frac{Bd(bc-ad)i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g} \\
&+ \frac{2B(bc-ad)^2i^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g} \\
&+ \frac{d(bc-ad)i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g} \\
&+ \frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg} + \frac{B^2(bc-ad)^2i^2\log(c+dx)}{b^3g} \\
&+ \frac{B(bc-ad)^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&- \frac{(bc-ad)^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&+ \frac{2B^2(bc-ad)^2i^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^3g} - \frac{B^2(bc-ad)^2i^2\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&+ \frac{2B(bc-ad)^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&+ \frac{2B^2(bc-ad)^2i^2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2615 vs. $2(535) = 1070$.

Time = 2.44 (sec) , antiderivative size = 2615, normalized size of antiderivative = 4.89

$$
\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b
*g*x),x]
```

```
[Out] (i^2*(12*A^2*b*d*(2*b*c - a*d)*x + 6*A^2*b^2*d^2*x^2 + 12*A^2*(b*c - a*d)^2
*Log[a + b*x] - 24*A*b*B*c*(a*d*Log[a/b + x]^2 - 2*a*d*Log[a/b + x]*(1 + Lo
g[a + b*x]) + 2*(-(b*c) + a*d + Log[c/d + x]*(b*c + a*d*Log[a + b*x] - a*d*
Log[(d*(a + b*x))/(-(b*c) + a*d)]) + (-(b*d*x) + a*d*Log[a + b*x])*Log[(e*(
a + b*x))/(c + d*x)]) - 2*a*d*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 12*A
*b^2*B*c^2*(Log[a/b + x]^2 - 2*Log[a + b*x]*(Log[a/b + x] - Log[c/d + x] -
Log[(e*(a + b*x))/(c + d*x)]) - 2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) +
```

```
 a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + 6*A*B*(-4*a*d^2*(a + b*x
)*(-1 + Log[a/b + x]) + 2*a^2*d^2*Log[a/b + x]^2 + 4*a*b*d*(c + d*x)*(-1 +
Log[c/d + x]) + d^2*(b*x*(2*a - b*x) + 2*b^2*x^2*Log[a/b + x] - 2*a^2*Log[a
 + b*x]) - 2*d^2*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x])*(Log[a/b + x] - Lo
g[c/d + x] - Log[(e*(a + b*x))/(c + d*x)]) + b^2*(d*x*(-2*c + d*x) - 2*d^2*
x^2*Log[c/d + x] + 2*c^2*Log[c + d*x]) - 4*a^2*d^2*(Log[c/d + x]*Log[(d*(a
+ b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - 8*b*B^2
*c*(a*d*Log[a/b + x]^3 - 3*d*(2*b*x - 2*(a + b*x)*Log[a/b + x] + (a + b*x)*
Log[a/b + x]^2) - 3*b*(2*d*x - 2*(c + d*x)*Log[c/d + x] + (c + d*x)*Log[c/d
 + x]^2) - 3*d*(b*x - a*Log[a + b*x])*(-Log[a/b + x] + Log[c/d + x] + Log[(
e*(a + b*x))/(c + d*x)])^2 + 6*(a*d + 2*b*d*x - b*d*x*Log[c/d + x] - b*c*Lo
g[c + d*x] + Log[a/b + x]*(-(d*(a + b*x)) + d*(a + b*x)*Log[c/d + x] + (b*c
 - a*d)*Log[(b*(c + d*x))/(b*c - a*d)]) + (b*c - a*d)*PolyLog[2, (d*(a + b*
x))/(-(b*c) + a*d)]) - 3*(Log[a/b + x] - Log[c/d + x] - Log[(e*(a + b*x))/(
c + d*x)])*(-2*b*c + 2*a*d - 2*d*(a + b*x)*Log[a/b + x] + a*d*Log[a/b + x]^
2 + 2*Log[c/d + x]*(b*(c + d*x) - a*d*Log[(d*(a + b*x))/(-(b*c) + a*d)]) -
2*a*d*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 3*a*d*(Log[a/b + x]^2*(Log[c
/d + x] - Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Log[a/b + x]*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)] + 2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)]) + 3*
a*d*(Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[c/d + x]*Poly
Log[2, (b*(c + d*x))/(b*c - a*d)] - 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]
)) + B^2*(4*a^2*d^2*Log[a/b + x]^3 - 12*a*d^2*(2*b*x - 2*(a + b*x)*Log[a/b
+ x] + (a + b*x)*Log[a/b + x]^2) - 3*d^2*(b*x*(6*a - b*x) + (-6*a^2 - 4*a*b
*x + 2*b^2*x^2)*Log[a/b + x] + 2*(a^2 - b^2*x^2)*Log[a/b + x]^2) - 12*a*b*d
*(2*d*x - 2*(c + d*x)*Log[c/d + x] + (c + d*x)*Log[c/d + x]^2) - 3*b^2*(d*x
*(6*c - d*x) + (-6*c^2 - 4*c*d*x + 2*d^2*x^2)*Log[c/d + x] + 2*(c^2 - d^2*x
^2)*Log[c/d + x]^2) + 6*d^2*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x])*(-Log[a
/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])^2 - 6*(Log[a/b + x]
- Log[c/d + x] - Log[(e*(a + b*x))/(c + d*x)])*(-4*a*d^2*(a + b*x)*(-1 + Lo
g[a/b + x]) + 2*a^2*d^2*Log[a/b + x]^2 + 4*a*b*d*(c + d*x)*(-1 + Log[c/d +
x]) + d^2*(b*x*(2*a - b*x) + 2*b^2*x^2*Log[a/b + x] - 2*a^2*Log[a + b*x]) +
 b^2*(d*x*(-2*c + d*x) - 2*d^2*x^2*Log[c/d + x] + 2*c^2*Log[c + d*x]) - 4*a
^2*d^2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c +
 d*x))/(b*c - a*d)])) + 6*(2*a*b*c*d + 3*b^2*c*d*x + 3*a*b*d^2*x - b^2*d^2*
x^2 - 2*a*b*d^2*x*Log[c/d + x] + b^2*d^2*x^2*Log[c/d + x] - a^2*d^2*Log[a +
 b*x] - b^2*c^2*Log[c + d*x] - 2*a*b*c*d*Log[c + d*x] - Log[a/b + x]*(b*d*(
2*a*c + b*x*(2*c - d*x)) - 2*d^2*(a^2 - b^2*x^2)*Log[c/d + x] + (-2*b^2*c^2
 + 2*a^2*d^2)*Log[(b*(c + d*x))/(b*c - a*d)]) + 2*(b^2*c^2 - a^2*d^2)*PolyL
og[2, (d*(a + b*x))/(-(b*c) + a*d)] + 4*a*d*(a*d + 2*b*d*x - b*d*x*Log[c/d
+ x] - b*c*Log[c + d*x] + Log[a/b + x]*(-(d*(a + b*x)) + d*(a + b*x)*Log[c/
d + x] + (b*c - a*d)*Log[(b*(c + d*x))/(b*c - a*d)]) + (b*c - a*d)*PolyLog[
2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2*a^2*d^2*(Log[a/b + x]^2*(Log[c/d + x]
 - Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Log[a/b + x]*PolyLog[2, (d*(a + b*x)
)/(-(b*c) + a*d)] + 2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)])) + 12*a^2*d
^2*(Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[c/d + x]*PolyL
```

```
og[2, (b*(c + d*x))/(b*c - a*d)] - 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])
) + 4*b^2*B^2*c^2*(Log[a/b + x]^3 + 3*Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b
*c) + a*d)] + 3*Log[a + b*x]*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a + b*
x))/(c + d*x)])^2 + 3*Log[a/b + x]^2*(-Log[c/d + x] + Log[(b*(c + d*x))/(b*
c - a*d)]) + 6*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 6*Lo
g[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 3*(Log[a/b + x] - Log[c/
d + x] - Log[(e*(a + b*x))/(c + d*x)])*(Log[a/b + x]^2 - 2*(Log[c/d + x]*Lo
g[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) -
 6*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*PolyLog[3, (b*(c + d*x))/(b
*c - a*d)])))/(12*b^3*g)
```

Maple [F]

$$\int \frac{(dix+ci)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{bgx+ag}dx$$

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x)
```

```
[Out] int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algor
ithm="fricas")
```

```
[Out] integral((A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x^
2 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*
d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log((b*e*x + a*e)/(d*x + c)))/
(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$$

$$= \frac{i^2\left(\int \frac{A^2c^2}{a+bx}\,dx + \int \frac{A^2d^2x^2}{a+bx}\,dx + \int \frac{B^2c^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a+bx}\,dx + \int \frac{2ABc^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx + \int \frac{2A^2cdx}{a+bx}\,dx + \int \frac{B^2d^2x}{}\right.}{}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g),x)
```

```
[Out] i**2*(Integral(A**2*c**2/(a + b*x), x) + Integral(A**2*d**2*x**2/(a + b*x),
 x) + Integral(B**2*c**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a + b*x),
 x) + Integral(2*A*B*c**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x
) + Integral(2*A**2*c*d*x/(a + b*x), x) + Integral(B**2*d**2*x**2*log(a*e/(
c + d*x) + b*e*x/(c + d*x))**2/(a + b*x), x) + Integral(2*A*B*d**2*x**2*log
(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) + Integral(2*B**2*c*d*x*log
(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a + b*x), x) + Integral(4*A*B*c*d*x*l
og(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algor
ithm="maxima")
```

```
[Out] 2*A^2*c*d*i^2*(x/(b*g) - a*log(b*x + a)/(b^2*g)) + 1/2*A^2*d^2*i^2*(2*a^2*l
og(b*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A^2*c^2*i^2*log(b*g*x + a*
g)/(b*g) + 1/2*(B^2*b^2*d^2*i^2*x^2 + 2*(2*b^2*c*d*i^2 - a*b*d^2*i^2)*B^2*x
 + 2*(b^2*c^2*i^2 - 2*a*b*c*d*i^2 + a^2*d^2*i^2)*B^2*log(b*x + a))*log(d*x
+ c)^2/(b^3*g) - integrate(-(B^2*b^3*c^3*i^2*log(e)^2 + 2*A*B*b^3*c^3*i^2*l
og(e) + (B^2*b^3*d^3*i^2*log(e)^2 + 2*A*B*b^3*d^3*i^2*log(e))*x^3 + 3*(B^2*
b^3*c*d^2*i^2*log(e)^2 + 2*A*B*b^3*c*d^2*i^2*log(e))*x^2 + (B^2*b^3*d^3*i^2
*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i^2*x + B^2*b^3*c^3*i^2)*l
og(b*x + a)^2 + 3*(B^2*b^3*c^2*d*i^2*log(e)^2 + 2*A*B*b^3*c^2*d*i^2*log(e))
*x + 2*(B^2*b^3*c^3*i^2*log(e) + A*B*b^3*c^3*i^2 + (B^2*b^3*d^3*i^2*log(e)
+ A*B*b^3*d^3*i^2)*x^3 + 3*(B^2*b^3*c*d^2*i^2*log(e) + A*B*b^3*c*d^2*i^2)*x
^2 + 3*(B^2*b^3*c^2*d*i^2*log(e) + A*B*b^3*c^2*d*i^2)*x)*log(b*x + a) - (2*
B^2*b^3*c^3*i^2*log(e) + 2*A*B*b^3*c^3*i^2 + (2*A*B*b^3*d^3*i^2 + (2*i^2*lo
g(e) + i^2)*B^2*b^3*d^3)*x^3 + (6*A*B*b^3*c*d^2*i^2 - (a*b^2*d^3*i^2 - 2*(3
*i^2*log(e) + 2*i^2)*b^3*c*d^2)*B^2)*x^2 + 2*(3*A*B*b^3*c^2*d*i^2 + (3*b^3*
c^2*d*i^2*log(e) + 2*a*b^2*c*d^2*i^2 - a^2*b*d^3*i^2)*B^2)*x + 2*(B^2*b^3*d
^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + (4*b^3*c^2*d*i^2 - 2*a*b^2*c*d^2*i^2
 + a^2*b*d^3*i^2)*B^2*x + (b^3*c^3*i^2 + a*b^2*c^2*d*i^2 - 2*a^2*b*c*d^2*i^
2 + a^3*d^3*i^2)*B^2)*log(b*x + a))*log(d*x + c))/(b^4*d*g*x^2 + a*b^3*c*g
+ (b^4*c*g + a*b^3*d*g)*x), x)
```

Giac [F]

$$\int \frac{(ci + dix)^2 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag + bgx}\, dx = \int \frac{(dix + ci)^2 \left(B \log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2}{bgx + ag}\, dx$$

[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algorithm="giac")

[Out] integrate((d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*g), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci + dix)^2 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag + bgx}\, dx = \int \frac{(c\,i + d\,i\,x)^2 \left(A + B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{a\,g + b\,g\,x}\, dx$$

[In] int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x), x)

[Out] int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x), x)

3.69 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 442

$$\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx \\
&= -\frac{2B^2(bc-ad)i^2(c+dx)}{b^2g^2(a+bx)} - \frac{2B(bc-ad)i^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g^2(a+bx)} \\
&\quad + \frac{2Bd(bc-ad)i^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^2} \\
&\quad + \frac{d^2i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g^2} - \frac{(bc-ad)i^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2g^2(a+bx)} \\
&\quad - \frac{2d(bc-ad)i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} \\
&\quad + \frac{2B^2d(bc-ad)i^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^3g^2} \\
&\quad + \frac{4Bd(bc-ad)i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} \\
&\quad + \frac{4B^2d(bc-ad)i^2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}
\end{aligned}$$

```
[Out] -2*B^2*(-a*d+b*c)*i^2*(d*x+c)/b^2/g^2/(b*x+a)-2*B*(-a*d+b*c)*i^2*(d*x+c)*(A
+B*ln(e*(b*x+a)/(d*x+c)))/b^2/g^2/(b*x+a)+2*B*d*(-a*d+b*c)*i^2*ln((-a*d+b*c
)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/g^2+d^2*i^2*(b*x+a)*(A+B*ln(e*
```

```
(b*x+a)/(d*x+c)))^2/b^3/g^2-(-a*d+b*c)*i^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c
)))^2/b^2/g^2/(b*x+a)-2*d*(-a*d+b*c)*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1
-b*(d*x+c)/d/(b*x+a))/b^3/g^2+2*B^2*d*(-a*d+b*c)*i^2*polylog(2,d*(b*x+a)/b/
(d*x+c))/b^3/g^2+4*B*d*(-a*d+b*c)*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2
,b*(d*x+c)/d/(b*x+a))/b^3/g^2+4*B^2*d*(-a*d+b*c)*i^2*polylog(3,b*(d*x+c)/d/
(b*x+a))/b^3/g^2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2562, 2395, 2342, 2341, 2355, 2354, 2438, 2379, 2421, 6724}

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}dx$$
$$=\frac{d^2i^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^3g^2}+\frac{4Bdi^2(bc-ad)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3g^2}+\frac{2Bdi^2(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3g^2}-\frac{2di^2(bc-ad)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^3g^2}-\frac{i^2(c+dx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^2g^2(a+bx)}-\frac{2Bi^2(c+dx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^2g^2(a+bx)}+\frac{2B^2di^2(bc-ad)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^3g^2}+\frac{4B^2di^2(bc-ad)\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}-\frac{2B^2i^2(c+dx)(bc-ad)}{b^2g^2(a+bx)}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)^
2,x]
```

```
[Out] (-2*B^2*(b*c - a*d)*i^2*(c + d*x))/(b^2*g^2*(a + b*x)) - (2*B*(b*c - a*d)*i
^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^2*g^2*(a + b*x)) + (2
*B*d*(b*c - a*d)*i^2*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)
)/(c + d*x)]))/(b^3*g^2) + (d^2*i^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c +
 d*x)])^2)/(b^3*g^2) - ((b*c - a*d)*i^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/
```

```
(c + d*x)])^2)/(b^2*g^2*(a + b*x)) - (2*d*(b*c - a*d)*i^2*(A + B*Log[(e*(a
+ b*x))/(c + d*x)])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^3*g^2) + (2*
B^2*d*(b*c - a*d)*i^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^3*g^2) +
(4*B*d*(b*c - a*d)*i^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (b*(
c + d*x))/(d*(a + b*x))])/(b^3*g^2) + (4*B^2*d*(b*c - a*d)*i^2*PolyLog[3, (
b*(c + d*x))/(d*(a + b*x))])/(b^3*g^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
```

```
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\left((bc-ad)i^2\right)\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^2(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^2}$$

$$= \frac{\left((bc-ad)i^2\right)\text{Subst}\left(\int \left(\frac{(A+B\log(ex))^2}{b^2x^2}+\frac{d^2(A+B\log(ex))^2}{b^2(b-dx)^2}+\frac{2d(A+B\log(ex))^2}{b^2x(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{g^2}$$

$$= \frac{((bc-ad)i^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2} + \frac{(2d(bc-ad)i^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2} + \frac{(d^2(bc-ad)i^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2}$$

$$= \frac{d^2i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g^2} - \frac{(bc-ad)i^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2g^2(a+bx)} - \frac{2d(bc-ad)i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} + \frac{(2B(bc-ad)i^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2} + \frac{(4Bd(bc-ad)i^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)(A+B\log(ex))}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^2} - \frac{(2Bd^2(bc-ad)i^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^2}$$

$$
\begin{aligned}
&= -\frac{2B^2(bc-ad)i^2(c+dx)}{b^2g^2(a+bx)} - \frac{2B(bc-ad)i^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g^2(a+bx)} \\
&+ \frac{2Bd(bc-ad)i^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^2} \\
&+ \frac{d^2i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g^2} \\
&- \frac{(bc-ad)i^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2g^2(a+bx)} \\
&- \frac{2d(bc-ad)i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} \\
&+ \frac{4Bd(bc-ad)i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\mathrm{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} \\
&- \frac{(2B^2d(bc-ad)i^2)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^2} \\
&- \frac{(4B^2d(bc-ad)i^2)\,\mathrm{Subst}\left(\int\frac{\mathrm{Li}_2\left(\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2B^2(bc-ad)i^2(c+dx)}{b^2g^2(a+bx)} - \frac{2B(bc-ad)i^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g^2(a+bx)} \\
&+ \frac{2Bd(bc-ad)i^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^2} \\
&+ \frac{d^2i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g^2} \\
&- \frac{(bc-ad)i^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2g^2(a+bx)} \\
&- \frac{2d(bc-ad)i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} \\
&+ \frac{2B^2d(bc-ad)i^2\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^3g^2} \\
&+ \frac{4Bd(bc-ad)i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} \\
&+ \frac{4B^2d(bc-ad)i^2\operatorname{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2649 vs. 2(442) = 884.

Time = 2.54 (sec) , antiderivative size = 2649, normalized size of antiderivative = 5.99

$$
\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b
*g*x)^2,x]
```

```
[Out] (i^2*(3*A^2*b*d^2*x - (3*A^2*(b*c - a*d)^2)/(a + b*x) + 6*A^2*d*(b*c - a*d)
*Log[a + b*x] - (6*A*b^2*B*c^2*(-(d*(a + b*x)*Log[c/d + x]) + d*(a + b*x)*L
og[(d*(a + b*x))/(-(b*c) + a*d)] + (b*c - a*d)*(1 + Log[(e*(a + b*x))/(c +
d*x)])))/((b*c - a*d)*(a + b*x)) + (3*b^2*B^2*c^2*(-2*b*c + 2*a*d - 2*d*(a
+ b*x)*Log[a + b*x] - 2*(b*c - a*d)*Log[(e*(a + b*x))/(c + d*x)] - 2*d*(a +
 b*x)*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] - (b*c - a*d)*Log[(e*(a + b
*x))/(c + d*x)]^2 + 2*d*(a + b*x)*Log[c + d*x] - 2*d*(a + b*x)*Log[(e*(a +
b*x))/(c + d*x)]*Log[(b*c - a*d)/(b*c + b*d*x)] + d*(a + b*x)*(Log[a + b*x]
*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b
*x))/(-(b*c) + a*d)]) + d*(a + b*x)*(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*Log[
```

(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c - a*d)/(b*c + b*d*x)]) - 2*PolyLog
[2, (b*(c + d*x))/(b*c - a*d)])))/((b*c - a*d)*(a + b*x)) + 6*A*b*B*c*d*(Lo
g[a/b + x]^2 - 2*Log[a/b + x]*Log[a + b*x] - 2*Log[c/d + x]*Log[(d*(a + b*x
))/(-(b*c) + a*d)] + 2*Log[a + b*x]*((a*d)/(b*c - a*d) + Log[c/d + x] + Log
[(e*(a + b*x))/(c + d*x)]) + 2*a*((a + b*x)^(-1) + Log[(e*(a + b*x))/(c + d
*x)]/(a + b*x) + (d*Log[c + d*x])/(-(b*c) + a*d)) - 2*PolyLog[2, (b*(c + d*
x))/(b*c - a*d)]) + 6*A*B*d^2*((a + b*x)*(-1 + Log[a/b + x]) - a*Log[a/b +
x]^2 - (a^2*(1 + Log[a/b + x]))/(a + b*x) - b*(c/d + x)*(-1 + Log[c/d + x])
 + (a^2*Log[c/d + x])/(a + b*x) + (b*x - a^2/(a + b*x) - 2*a*Log[a + b*x])*
(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)]) + (a^2*d*(Log
[a + b*x] - Log[c + d*x]))/(-(b*c) + a*d) + 2*a*(Log[c/d + x]*Log[(d*(a + b
*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + B^2*d^2*(6
*b*x - 6*(a + b*x)*Log[a/b + x] + 3*(a + b*x)*Log[a/b + x]^2 - 2*a*Log[a/b
+ x]^3 - (3*a^2*(2 + 2*Log[a/b + x] + Log[a/b + x]^2))/(a + b*x) + (3*b*(2*
d*x - 2*(c + d*x)*Log[c/d + x] + (c + d*x)*Log[c/d + x]^2))/d + 3*(b*x - a^
2/(a + b*x) - 2*a*Log[a + b*x])*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a +
 b*x))/(c + d*x)])^2 - (6*(a*d + 2*b*d*x - b*d*x*Log[c/d + x] - b*c*Log[c +
 d*x] + Log[a/b + x]*(-(d*(a + b*x)) + d*(a + b*x)*Log[c/d + x] + (b*c - a*
d)*Log[(b*(c + d*x))/(b*c - a*d)]) + (b*c - a*d)*PolyLog[2, (d*(a + b*x))/(
-(b*c) + a*d)]))/d + (3*a^2*(d*(a + b*x)*Log[a/b + x]^2 + 2*((-(b*c) + a*d)
Log[c/d + x] + d(a + b*x)*(Log[a + b*x] - Log[c + d*x])) - 2*Log[a/b + x]
*((b*c - a*d)*Log[c/d + x] + d*(a + b*x)*Log[(b*(c + d*x))/(b*c - a*d)]) -
2*d*(a + b*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/((-(b*c) + a*d)*(a
 + b*x)) + (3*a^2*(-(b*(c + d*x)*Log[c/d + x]^2) + 2*d*(a + b*x)*Log[c/d +
x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*d*(a + b*x)*PolyLog[2, (b*(c + d*x
))/(b*c - a*d)]))/((b*c - a*d)*(a + b*x)) + 6*(-Log[a/b + x] + Log[c/d + x]
 + Log[(e*(a + b*x))/(c + d*x)])*((a + b*x)*(-1 + Log[a/b + x]) - a*Log[a/b
 + x]^2 - (a^2*(1 + Log[a/b + x]))/(a + b*x) - b*(c/d + x)*(-1 + Log[c/d +
x]) + (a^2*Log[c/d + x])/(a + b*x) + (a^2*d*(Log[a + b*x] - Log[c + d*x]))/
(-(b*c) + a*d) + 2*a*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Poly
Log[2, (b*(c + d*x))/(b*c - a*d)])) + 6*a*(Log[a/b + x]^2*(Log[c/d + x] - L
og[(b*(c + d*x))/(b*c - a*d)]) - 2*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-
(b*c) + a*d)] + 2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)]) - 6*a*(Log[c/d
+ x]^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[c/d + x]*PolyLog[2, (b*(c
+ d*x))/(b*c - a*d)] - 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])) + (2*b*B^2
*c*d*((b*c - a*d)*(a + b*x)*Log[a/b + x]^3 + 3*a*(b*c - a*d)*(2 + 2*Log[a/b
 + x] + Log[a/b + x]^2) + 3*(b*c - a*d)*(a + (a + b*x)*Log[a + b*x])*(-Log[
a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])^2 + 3*a*(d*(a + b*x
)*Log[a/b + x]^2 + 2*((-(b*c) + a*d)*Log[c/d + x] + d*(a + b*x)*(Log[a + b*
x] - Log[c + d*x])) - 2*Log[a/b + x]*((b*c - a*d)*Log[c/d + x] + d*(a + b*x
)*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*d*(a + b*x)*PolyLog[2, (d*(a + b*x))/
(-(b*c) + a*d)]) + 3*a*(Log[c/d + x]*(b*(c + d*x)*Log[c/d + x] - 2*d*(a + b
*x)*Log[(d*(a + b*x))/(-(b*c) + a*d)]) - 2*d*(a + b*x)*PolyLog[2, (b*(c + d
*x))/(b*c - a*d)]) - 3*(Log[a/b + x] - Log[c/d + x] - Log[(e*(a + b*x))/(c
+ d*x)])*((b*c - a*d)*(a + b*x)*Log[a/b + x]^2 + 2*a*(b*c - a*d)*(1 + Log[a

```
/b + x]) + 2*a*(-(b*c) + a*d)*Log[c/d + x] + 2*a*d*(a + b*x)*(Log[a + b*x]
- Log[c + d*x]) - 2*(b*c - a*d)*(a + b*x)*(Log[c/d + x]*Log[(d*(a + b*x))/(
-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - 3*(b*c - a*d)*(a
 + b*x)*(Log[a/b + x]^2*(Log[c/d + x] - Log[(b*(c + d*x))/(b*c - a*d)]) - 2
*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2*PolyLog[3, (d*(a
 + b*x))/(-(b*c) + a*d)]) + 3*(b*c - a*d)*(a + b*x)*(Log[c/d + x]^2*Log[(d*
(a + b*x))/(-(b*c) + a*d)] + 2*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c -
 a*d)] - 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])))/((b*c - a*d)*(a + b*x))
))/(3*b^3*g^2)
```

Maple [F]

$$\int \frac{(dix+ci)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(bgx+ag)^2}\,dx$$

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x)
```

```
[Out] int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, alg
orithm="fricas")
```

```
[Out] integral((A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x^
2 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*
d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log((b*e*x + a*e)/(d*x + c)))/
(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, alg
orithm="maxima")
```

```
[Out] -A^2*(a^2/(b^4*g^2*x + a*b^3*g^2) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*g^2
))*d^2*i^2 + 2*A^2*c*d*i^2*(a/(b^3*g^2*x + a*b^2*g^2) + log(b*x + a)/(b^2*g
^2)) - 2*A*B*c^2*i^2*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b
*g^2) + 1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*
log(d*x + c)/((b^2*c - a*b*d)*g^2)) - A^2*c^2*i^2/(b^2*g^2*x + a*b*g^2) + (
B^2*b^2*d^2*i^2*x^2 + B^2*a*b*d^2*i^2*x - (b^2*c^2*i^2 - 2*a*b*c*d*i^2 + a^
2*d^2*i^2)*B^2 + 2*((b^2*c*d*i^2 - a*b*d^2*i^2)*B^2*x + (a*b*c*d*i^2 - a^2*
d^2*i^2)*B^2)*log(b*x + a))*log(d*x + c)^2/(b^4*g^2*x + a*b^3*g^2) - integr
ate(-(B^2*b^3*c^3*i^2*log(e)^2 + (B^2*b^3*d^3*i^2*log(e)^2 + 2*A*B*b^3*d^3*
i^2*log(e))*x^3 + 3*(B^2*b^3*c*d^2*i^2*log(e)^2 + 2*A*B*b^3*c*d^2*i^2*log(e
))*x^2 + (B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i
^2*x + B^2*b^3*c^3*i^2)*log(b*x + a)^2 + (3*B^2*b^3*c^2*d*i^2*log(e)^2 + 4*
A*B*b^3*c^2*d*i^2*log(e))*x + 2*(B^2*b^3*c^3*i^2*log(e) + (B^2*b^3*d^3*i^2*
log(e) + A*B*b^3*d^3*i^2)*x^3 + 3*(B^2*b^3*c*d^2*i^2*log(e) + A*B*b^3*c*d^2
*i^2)*x^2 + (3*B^2*b^3*c^2*d*i^2*log(e) + 2*A*B*b^3*c^2*d*i^2)*x)*log(b*x +
 a) - 2*((A*B*b^3*d^3*i^2 + (i^2*log(e) + i^2)*B^2*b^3*d^3)*x^3 + (b^3*c^3*
i^2*log(e) - a*b^2*c^2*d*i^2 + 2*a^2*b*c*d^2*i^2 - a^3*d^3*i^2)*B^2 + (3*A*
B*b^3*c*d^2*i^2 + (3*b^3*c*d^2*i^2*log(e) + 2*a*b^2*d^3*i^2)*B^2)*x^2 + (2*
A*B*b^3*c^2*d*i^2 + (2*a*b^2*c*d^2*i^2 + (3*i^2*log(e) - i^2)*b^3*c^2*d)*B^
2)*x + (B^2*b^3*d^3*i^2*x^3 + (5*b^3*c*d^2*i^2 - 2*a*b^2*d^3*i^2)*B^2*x^2 +
 (3*b^3*c^2*d*i^2 + 4*a*b^2*c*d^2*i^2 - 4*a^2*b*d^3*i^2)*B^2*x + (b^3*c^3*i
^2 + 2*a^2*b*c*d^2*i^2 - 2*a^3*d^3*i^2)*B^2)*log(b*x + a))*log(d*x + c))/(b
^5*d*g^2*x^3 + a^2*b^3*c*g^2 + (b^5*c*g^2 + 2*a*b^4*d*g^2)*x^2 + (2*a*b^4*c
*g^2 + a^2*b^3*d*g^2)*x), x)
```

Giac [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, alg
orithm="giac")
```

```
[Out] integrate((d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*g
)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(c\,i+d\,i\,x)^2 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(a\,g+b\,g\,x)^2}\,dx$$

```
[In] int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^
2,x)
```

```
[Out] int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^
2, x)
```

3.70 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 387

$$
\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx \\
&= -\frac{2B^2di^2(c+dx)}{b^2g^3(a+bx)}-\frac{B^2i^2(c+dx)^2}{4bg^3(a+bx)^2}-\frac{2Bdi^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g^3(a+bx)} \\
&\quad -\frac{Bi^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a+bx)^2}-\frac{di^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2g^3(a+bx)} \\
&\quad -\frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg^3(a+bx)^2}-\frac{d^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} \\
&\quad +\frac{2Bd^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3}+\frac{2B^2d^2i^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3}
\end{aligned}
$$

```
[Out] -2*B^2*d*i^2*(d*x+c)/b^2/g^3/(b*x+a)-1/4*B^2*i^2*(d*x+c)^2/b/g^3/(b*x+a)^2-
2*B*d*i^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/g^3/(b*x+a)-1/2*B*i^2*(d*
x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g^3/(b*x+a)^2-d*i^2*(d*x+c)*(A+B*ln(e*
(b*x+a)/(d*x+c)))^2/b^2/g^3/(b*x+a)-1/2*i^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*
x+c)))^2/b/g^3/(b*x+a)^2-d^2*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1-b*(d*x+
c)/d/(b*x+a))/b^3/g^3+2*B*d^2*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,b*(
d*x+c)/d/(b*x+a))/b^3/g^3+2*B^2*d^2*i^2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^3/
g^3
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2562, 2380, 2342, 2341, 2379, 2421, 6724}

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$=\frac{2Bd^2i^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3g^3}$$
$$-\frac{d^2i^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^3g^3}$$
$$-\frac{di^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^2g^3(a+bx)}-\frac{2Bdi^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^2g^3(a+bx)}$$
$$-\frac{i^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2bg^3(a+bx)^2}-\frac{Bi^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2bg^3(a+bx)^2}$$
$$+\frac{2B^2d^2i^2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3}-\frac{2B^2di^2(c+dx)}{b^2g^3(a+bx)}-\frac{B^2i^2(c+dx)^2}{4bg^3(a+bx)^2}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)^
3,x]
```

```
[Out] (-2*B^2*d*i^2*(c + d*x))/(b^2*g^3*(a + b*x)) - (B^2*i^2*(c + d*x)^2)/(4*b*g
^3*(a + b*x)^2) - (2*B*d*i^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])
)/(b^2*g^3*(a + b*x)) - (B*i^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)]))/(2*b*g^3*(a + b*x)^2) - (d*i^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c
+ d*x)])^2)/(b^2*g^3*(a + b*x)) - (i^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))
/(c + d*x)])^2)/(2*b*g^3*(a + b*x)^2) - (d^2*i^2*(A + B*Log[(e*(a + b*x))/(
c + d*x)])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^3*g^3) + (2*B*d^2*i^2
*(A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x)
)])/(b^3*g^3) + (2*B^2*d^2*i^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))])/(b^
3*g^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{i^2\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^3(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^3}$$

$$= \frac{i^2\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^3} + \frac{(di^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^2(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^3}$$

$$= -\frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(Bi^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^3} + \frac{(di^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^3} + \frac{(d^2i^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^3}$$

$$= -\frac{B^2i^2(c+dx)^2}{4bg^3(a+bx)^2} - \frac{Bi^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a+bx)^2} - \frac{di^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2g^3(a+bx)} - \frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg^3(a+bx)^2} - \frac{d^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} + \frac{(2Bdi^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^3} + \frac{(2Bd^2i^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)(A+B\log(ex))}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^3}$$

$$= -\frac{2B^2di^2(c+dx)}{b^2g^3(a+bx)} - \frac{B^2i^2(c+dx)^2}{4bg^3(a+bx)^2} - \frac{2Bdi^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g^3(a+bx)} - \frac{Bi^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a+bx)^2} - \frac{di^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2g^3(a+bx)} - \frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg^3(a+bx)^2} - \frac{d^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} + \frac{2Bd^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} - \frac{(2B^2d^2i^2)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^3}$$

$$= -\frac{2B^2di^2(c+dx)}{b^2g^3(a+bx)} - \frac{B^2i^2(c+dx)^2}{4bg^3(a+bx)^2} - \frac{2Bdi^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g^3(a+bx)}$$
$$-\frac{Bi^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a+bx)^2}$$
$$-\frac{di^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2g^3(a+bx)} - \frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg^3(a+bx)^2}$$
$$-\frac{d^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3}$$
$$+\frac{2Bd^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} + \frac{2B^2d^2i^2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3426 vs. $2(387) = 774$.

Time = 4.11 (sec) , antiderivative size = 3426, normalized size of antiderivative = 8.85

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b
*g*x)^3,x]
```

```
[Out] (i^2*((-6*A^2*(b*c - a*d)^2)/(a + b*x)^2 + (24*A^2*d*(-(b*c) + a*d))/(a + b
*x) + 12*A^2*d^2*Log[a + b*x] - (6*A*b^2*B*c^2*(b^2*c^2 - 4*a*b*c*d + a^2*d
^2 - 2*b^2*c*d*x - 2*a*b*d^2*x - 2*b^2*d^2*x^2 + 2*d^2*(a + b*x)^2*Log[c/d
+ x] - 2*d^2*(a + b*x)^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*b^2*c^2*Log[
(e*(a + b*x))/(c + d*x)] - 4*a*b*c*d*Log[(e*(a + b*x))/(c + d*x)] + 2*a^2*d
^2*Log[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^2*(a + b*x)^2) - (12*A*b*B*c
*d*(3*a*b^2*c^2 - 4*a^2*b*c*d + a^3*d^2 + 4*b^3*c^2*x - 6*a*b^2*c*d*x + 2*a
^2*b*d^2*x - 2*d*(-2*b*c + a*d)*(a + b*x)^2*Log[a + b*x] + 2*(b*c - a*d)^2*
(a + 2*b*x)*Log[(e*(a + b*x))/(c + d*x)] - 4*a^2*b*c*d*Log[c + d*x] + 2*a^3
*d^2*Log[c + d*x] - 8*a*b^2*c*d*x*Log[c + d*x] + 4*a^2*b*d^2*x*Log[c + d*x]
 - 4*b^3*c*d*x^2*Log[c + d*x] + 2*a*b^2*d^2*x^2*Log[c + d*x]))/((b*c - a*d)
^2*(a + b*x)^2) + (3*b^2*B^2*c^2*(-(b*c - a*d)^2 + 6*d*(b*c - a*d)*(a + b*x
) + 6*d^2*(a + b*x)^2*Log[a + b*x] - 2*d^2*(a + b*x)^2*Log[a + b*x]^2 - 2*(
b*c - a*d)^2*Log[(e*(a + b*x))/(c + d*x)] + 4*d*(b*c - a*d)*(a + b*x)*Log[(
e*(a + b*x))/(c + d*x)] + 4*d^2*(a + b*x)^2*Log[a + b*x]*Log[(e*(a + b*x))/
(c + d*x)] - 2*(b*c - a*d)^2*Log[(e*(a + b*x))/(c + d*x)]^2 - 6*d^2*(a + b*
x)^2*Log[c + d*x] + 4*d^2*(a + b*x)^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c -
 a*d)] - 4*d^2*(a + b*x)^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[(b*c - a*d
```

)/(b*c + b*d*x)] + 4*d^2*(a + b*x)^2*Log[(e*(a + b*x))/(c + d*x)]*Log[(b*c
- a*d)/(b*c + b*d*x)] - 2*d^2*(a + b*x)^2*Log[(b*c - a*d)/(b*c + b*d*x)]^2
+ 4*d^2*(a + b*x)^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 4*d^2*(a + b
*x)^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)^2*(a + b*x)^2) +
 6*A*B*d^2*(2*Log[a/b + x]^2 + (8*a*(1 + Log[a/b + x]))/(a + b*x) - (a^2*(1
 + 2*Log[a/b + x]))/(a + b*x)^2 + 2*((a*(3*a + 4*b*x))/(a + b*x)^2 + 2*Log[
a + b*x])*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)]) + (
8*a*((-(b*c) + a*d)*Log[c/d + x] + d*(a + b*x)*(Log[a + b*x] - Log[c + d*x]
)))/((b*c - a*d)*(a + b*x)) + (2*a^2*(Log[c/d + x] + (d*(a + b*x)*(b*c - a*
d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]))/(b*c - a*d)^2))/(
a + b*x)^2 - 4*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2,
 (b*(c + d*x))/(b*c - a*d)])) + (6*b*B^2*c*d*(-4*(b*c - a*d)^2*(a + b*x)*(2
 + 2*Log[a/b + x] + Log[a/b + x]^2) + a*(b*c - a*d)^2*(1 + 2*Log[a/b + x] +
 2*Log[a/b + x]^2) - 2*(b*c - a*d)^2*(a + 2*b*x)*(-Log[a/b + x] + Log[c/d +
 x] + Log[(e*(a + b*x))/(c + d*x)])^2 + 2*(Log[a/b + x] - Log[c/d + x] - Lo
g[(e*(a + b*x))/(c + d*x)])*(4*(b*c - a*d)^2*(a + b*x)*(1 + Log[a/b + x]) -
 a*(b*c - a*d)^2*(1 + 2*Log[a/b + x]) - 4*(b*c - a*d)*(a + b*x)*((b*c - a*d
)*Log[c/d + x] - d*(a + b*x)*(Log[a + b*x] - Log[c + d*x])) + 2*a*((b*c - a
*d)^2*Log[c/d + x] + d*(a + b*x)*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*
(a + b*x)*Log[c + d*x]))) - 4*(b*c - a*d)*(a + b*x)*(d*(a + b*x)*Log[a/b +
x]^2 + 2*((-(b*c) + a*d)*Log[c/d + x] + d*(a + b*x)*(Log[a + b*x] - Log[c +
 d*x])) - 2*Log[a/b + x]*((b*c - a*d)*Log[c/d + x] + d*(a + b*x)*Log[(b*(c
+ d*x))/(b*c - a*d)]) - 2*d*(a + b*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*
d)]) + 2*a*(d*(-(b*c) + a*d)*(a + b*x) - (b*c - a*d)^2*(1 + 2*Log[a/b + x])
Log[c/d + x] - d^2(a + b*x)^2*Log[a + b*x] + d^2*(a + b*x)^2*Log[c + d*x]
 - d*(a + b*x)*(d*(a + b*x)*Log[a/b + x]^2 + 2*(b*c - a*d)*(1 + Log[a/b + x
]) - 2*d*(a + b*x)*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2
, (d*(a + b*x))/(-(b*c) + a*d)]))) - 4*(b*c - a*d)*(a + b*x)*(Log[c/d + x]*
(b*(c + d*x)*Log[c/d + x] - 2*d*(a + b*x)*Log[(d*(a + b*x))/(-(b*c) + a*d)]
) - 2*d*(a + b*x)*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 2*a*((b*c - a*d)
^2*Log[c/d + x]^2 - d*(a + b*x)*(d*(a + b*x)*Log[c/d + x]^2 - 2*Log[c/d + x
]*(b*c - a*d + d*(a + b*x)*Log[(d*(a + b*x))/(-(b*c) + a*d)]) + 2*d*(a + b*
x)*(Log[a + b*x] - Log[c + d*x]) - 2*d*(a + b*x)*PolyLog[2, (b*(c + d*x))/(
b*c - a*d)]))))/((b*c - a*d)^2*(a + b*x)^2) + B^2*d^2*(4*Log[a/b + x]^3 + (
24*a*(2 + 2*Log[a/b + x] + Log[a/b + x]^2))/(a + b*x) - (3*a^2*(1 + 2*Log[a
/b + x] + 2*Log[a/b + x]^2))/(a + b*x)^2 + (6*(a*(3*a + 4*b*x) + 2*(a + b*x
)^2*Log[a + b*x])*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*
x)])^2)/(a + b*x)^2 + (24*a*(-(b*(c + d*x)*Log[c/d + x]^2) + 2*d*(a + b*x)*
Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*d*(a + b*x)*PolyLog[2, (
b*(c + d*x))/(b*c - a*d)]))/((-(b*c) + a*d)*(a + b*x)) + 6*(-Log[a/b + x] +
 Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])*(2*Log[a/b + x]^2 + (8*a*(1 +
 Log[a/b + x]))/(a + b*x) - (a^2*(1 + 2*Log[a/b + x]))/(a + b*x)^2 + (2*a^2
*Log[c/d + x])/(a + b*x)^2 - (8*a*Log[c/d + x])/(a + b*x) + (8*a*d*(-Log[a
+ b*x] + Log[c + d*x]))/(-(b*c) + a*d) + (2*a^2*d*(b*c - a*d + d*(a + b*x)*
Log[a + b*x] - d*(a + b*x)*Log[c + d*x]))/((b*c - a*d)^2*(a + b*x)) - 4*(Lo

```
g[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*
c - a*d)])) + (6*a^2*(-Log[c/d + x]^2 + (d*(a + b*x)*(d*(a + b*x)*Log[c/d +
 x]^2 - 2*Log[c/d + x]*(b*c - a*d + d*(a + b*x)*Log[(d*(a + b*x))/(-(b*c) +
 a*d)]) + 2*d*(a + b*x)*(Log[a + b*x] - Log[c + d*x]) - 2*d*(a + b*x)*PolyL
og[2, (b*(c + d*x))/(b*c - a*d)]))/(b*c - a*d)^2))/(a + b*x)^2 + 6*(2*Log[a
/b + x]^2*(-Log[c/d + x] + Log[(b*(c + d*x))/(b*c - a*d)]) + 4*Log[a/b + x]
*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + (4*a*(d*(a + b*x)*Log[a/b + x]^
2 + 2*((-(b*c) + a*d)*Log[c/d + x] + d*(a + b*x)*(Log[a + b*x] - Log[c + d*
x])) - 2*Log[a/b + x]*((b*c - a*d)*Log[c/d + x] + d*(a + b*x)*Log[(b*(c + d
*x))/(b*c - a*d)]) - 2*d*(a + b*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]
))/((b*c - a*d)*(a + b*x)) + (a^2*(-(d*(-(b*c) + a*d)*(a + b*x)) + (b*c - a
*d)^2*(1 + 2*Log[a/b + x])*Log[c/d + x] + d^2*(a + b*x)^2*Log[a + b*x] - d^
2*(a + b*x)^2*Log[c + d*x] + d*(a + b*x)*(d*(a + b*x)*Log[a/b + x]^2 + 2*(b
*c - a*d)*(1 + Log[a/b + x]) - 2*d*(a + b*x)*(Log[a/b + x]*Log[(b*(c + d*x)
)/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))))/((b*c - a*d)^
2*(a + b*x)^2) - 4*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)]) + 12*(Log[c/d
+ x]^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[c/d + x]*PolyLog[2, (b*(c
+ d*x))/(b*c - a*d)] - 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]))))/(12*b^3*
g^3)
```

Maple [F]

$$\int \frac{(dix+ci)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(bgx+ag)^3}\,dx$$

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x)
```

```
[Out] int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, alg
orithm="fricas")
```

```
[Out] integral((A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x^
2 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*
d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log((b*e*x + a*e)/(d*x + c)))/
(b^3*g^3*x^3 + 3*a*b^2*g^3*x^2 + 3*a^2*b*g^3*x + a^3*g^3), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= \frac{i^2\left(\int \frac{A^2c^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{A^2d^2x^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{B^2c^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{2ABc^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\right.}{}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3,x)
```

```
[Out] i**2*(Integral(A**2*c**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x
) + Integral(A**2*d**2*x**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)
, x) + Integral(B**2*c**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a**3 + 3
*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*c**2*log(a*e/(c
 + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),
 x) + Integral(2*A**2*c*d*x/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)
, x) + Integral(B**2*d**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a**
3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*d**2*x**2*
log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b
**3*x**3), x) + Integral(2*B**2*c*d*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))*
*2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(4*A*B*c*d
*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2
+ b**3*x**3), x))/g**3
```

Maxima [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, alg
orithm="maxima")
```

```
[Out] -A*B*c*d*i^2*(2*(2*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^4*g^3*x
^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) + (3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*
x)/((b^5*c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c
- a^3*b^2*d)*g^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*c*
d + a^2*b^2*d^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a*b^
3*c*d + a^2*b^2*d^2)*g^3)) + 1/2*A^2*d^2*i^2*((4*a*b*x + 3*a^2)/(b^5*g^3*x^
2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(b^3*g^3)) + 1/2*A*B*c^2*
i^2*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*
b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*
```

```
x + c))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^
3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*
b^2*c*d + a^2*b*d^2)*g^3)) - (2*b*x + a)*A^2*c*d*i^2/(b^4*g^3*x^2 + 2*a*b^3
*g^3*x + a^2*b^2*g^3) - 1/2*A^2*c^2*i^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*
b*g^3) - 1/2*(4*(b^2*c*d*i^2 - a*b*d^2*i^2)*B^2*x + (b^2*c^2*i^2 + 2*a*b*c*
d*i^2 - 3*a^2*d^2*i^2)*B^2 - 2*(B^2*b^2*d^2*i^2*x^2 + 2*B^2*a*b*d^2*i^2*x +
 B^2*a^2*d^2*i^2)*log(b*x + a))*log(d*x + c)^2/(b^5*g^3*x^2 + 2*a*b^4*g^3*x
 + a^2*b^3*g^3) - integrate(-(3*B^2*b^3*c^2*d*i^2*x*log(e)^2 + B^2*b^3*c^3*
i^2*log(e)^2 + (B^2*b^3*d^3*i^2*log(e)^2 + 2*A*B*b^3*d^3*i^2*log(e))*x^3 +
(3*B^2*b^3*c*d^2*i^2*log(e)^2 + 2*A*B*b^3*c*d^2*i^2*log(e))*x^2 + (B^2*b^3*
d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i^2*x + B^2*b^3*c^3
*i^2)*log(b*x + a)^2 + 2*(3*B^2*b^3*c^2*d*i^2*x*log(e) + B^2*b^3*c^3*i^2*lo
g(e) + (B^2*b^3*d^3*i^2*log(e) + A*B*b^3*d^3*i^2)*x^3 + (3*B^2*b^3*c*d^2*i^
2*log(e) + A*B*b^3*c*d^2*i^2)*x^2)*log(b*x + a) + ((6*a*b^2*c*d^2*i^2 - 7*a
^2*b*d^3*i^2 - (6*i^2*log(e) - i^2)*b^3*c^2*d)*B^2*x - 2*(B^2*b^3*d^3*i^2*l
og(e) + A*B*b^3*d^3*i^2)*x^3 - (2*b^3*c^3*i^2*log(e) - a*b^2*c^2*d*i^2 - 2*
a^2*b*c*d^2*i^2 + 3*a^3*d^3*i^2)*B^2 - 2*(A*B*b^3*c*d^2*i^2 + (2*a*b^2*d^3*
i^2 + (3*i^2*log(e) - 2*i^2)*b^3*c*d^2)*B^2)*x^2 - 2*(2*B^2*b^3*d^3*i^2*x^3
 + 3*(b^3*c*d^2*i^2 + a*b^2*d^3*i^2)*B^2*x^2 + 3*(b^3*c^2*d*i^2 + a^2*b*d^3
*i^2)*B^2*x + (b^3*c^3*i^2 + a^3*d^3*i^2)*B^2)*log(b*x + a))*log(d*x + c))/
(b^6*d*g^3*x^4 + a^3*b^3*c*g^3 + (b^6*c*g^3 + 3*a*b^5*d*g^3)*x^3 + 3*(a*b^5
*c*g^3 + a^2*b^4*d*g^3)*x^2 + (3*a^2*b^4*c*g^3 + a^3*b^3*d*g^3)*x), x)
```

Giac [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, alg
orithm="giac")
```

```
[Out] integrate((d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*g
)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx=\int \frac{(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(a\,g+b\,g\,x)^3}\,dx$$

```
[In] int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^
3,x)
```

```
[Out] int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^
3, x)
```

3.71 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 147

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{2B^2i^2(c+dx)^3}{27(bc-ad)g^4(a+bx)^3} - \frac{2Bi^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)g^4(a+bx)^3} - \frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)g^4(a+bx)^3}$$

```
[Out] -2/27*B^2*i^2*(d*x+c)^3/(-a*d+b*c)/g^4/(b*x+a)^3-2/9*B*i^2*(d*x+c)^3*(A+B*l
n(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)/g^4/(b*x+a)^3-1/3*i^2*(d*x+c)^3*(A+B*ln(e*
(b*x+a)/(d*x+c)))^2/(-a*d+b*c)/g^4/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2562, 2342, 2341}

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{i^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g^4(a+bx)^3(bc-ad)} - \frac{2Bi^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{9g^4(a+bx)^3(bc-ad)} - \frac{2B^2i^2(c+dx)^3}{27g^4(a+bx)^3(bc-ad)}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)^
4,x]
```

```
[Out] (-2*B^2*i^2*(c + d*x)^3)/(27*(b*c - a*d)*g^4*(a + b*x)^3) - (2*B*i^2*(c + d
*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(9*(b*c - a*d)*g^4*(a + b*x)^3)
 - (i^2*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*(b*c - a*d)*
g^4*(a + b*x)^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{i^2\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^4}$$

$$= -\frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)g^4(a+bx)^3} + \frac{(2Bi^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bc-ad)g^4}$$

$$= -\frac{2B^2i^2(c+dx)^3}{27(bc-ad)g^4(a+bx)^3} - \frac{2Bi^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)g^4(a+bx)^3} - \frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)g^4(a+bx)^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.08 (sec) , antiderivative size = 1352, normalized size of antiderivative = 9.20

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{i^2\left(18(bc-ad)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+54d(bc-ad)^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-54d^2(-bc\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b
*g*x)^4,x]
```

```
[Out] -1/54*(i^2*(18*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 54*d*
(b*c - a*d)^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - 54*d^2*(-(
b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + B*(12*A*(b
*c - a*d)^3 + 4*B*(b*c - a*d)^3 - 18*A*d*(b*c - a*d)^2*(a + b*x) - 15*B*d*(
b*c - a*d)^2*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(b*c -
 a*d)*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x] + 66*B*d^3*(a + b*x)^
3*Log[a + b*x] - 18*B*d^3*(a + b*x)^3*Log[a + b*x]^2 + 12*B*(b*c - a*d)^3*L
og[(e*(a + b*x))/(c + d*x)] - 18*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*
x))/(c + d*x)] + 36*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[(e*(a + b*x))/(c + d*
x)] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] - 36*A
*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3*(a + b*x)^3*Log[c + d*x] + 36*B*d^
3*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 36*B*d^3*(a
+ b*x)^3*Log[(e*(a + b*x))/(c + d*x)]*Log[c + d*x] - 18*B*d^3*(a + b*x)^3*L
og[c + d*x]^2 + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c -
a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 36*
B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 54*B*d^2*(a + b*
x)^2*(2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*d*(a + b*x)*Lo
g[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*d*(a + b*x)*(A + B*Log[
(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 2*B*(b*c - a*d + d*(a + b*x)*Log[a
 + b*x] - d*(a + b*x)*Log[c + d*x]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a +
b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*
c) + a*d)]) + B*d*(a + b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c +
 d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + 27*B*d*(a
 + b*x)*(2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*d*(-(b*c)
 + a*d)*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 4*d^2*(a + b*x)^2*
Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*d^2*(a + b*x)^2*(A +
B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 4*B*d*(a + b*x)*(b*c - a*d +
 d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) + B*((b*c - a*d)^2 +
2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a +
b*x)^2*Log[c + d*x]) + 2*B*d^2*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*
```

```
Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)
]) - 2*B*d^2*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*
x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b^3*(b*c -
a*d)*g^4*(a + b*x)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(141) = 282$.

Time = 1.05 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.50

method	result
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{i^2d^2e^2A^2}{3(ad-cb)^2g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{2i^2d^2e^2AB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}\right)}{(ad-cb)^2g^4}+\frac{i^2d^2e^2B^2\left(-\frac{\ln(\ldots}{3(\ldots}\ldots\right.}{\ldots}\ldots\right)}{d^2}$
default	$-\frac{e(ad-cb)\left(-\frac{i^2d^2e^2A^2}{3(ad-cb)^2g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{2i^2d^2e^2AB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}\right)}{(ad-cb)^2g^4}+\frac{i^2d^2e^2B^2\left(-\frac{\ln(\ldots}{3(\ldots}\ldots\right.}{\ldots}\ldots\right)}{d^2}$
parts	$\frac{i^2A^2\left(-\frac{a^2d^2-2abcd+b^2c^2}{3b^3(bx+a)^3}+\frac{d(ad-cb)}{b^3(bx+a)^2}-\frac{d^2}{b^3(bx+a)}\right)}{g^4}-\frac{i^2B^2e^3\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{9\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{2}{27\left(\frac{be}{d}+\frac{(ad-}{d(dx}\ldots\right.}\ldots\right.}{g^4(ad-cb)}$
risch	$-\frac{i^2A^2a^2d^2}{3g^4b^3(bx+a)^3}+\frac{2i^2A^2acd}{3g^4b^2(bx+a)^3}-\frac{i^2A^2c^2}{3g^4b(bx+a)^3}+\frac{i^2A^2d^2a}{g^4b^3(bx+a)^2}-\frac{i^2A^2dc}{g^4b^2(bx+a)^2}-\frac{i^2A^2d^2}{g^4b^3(bx+a)}+\frac{i^2B^2e^3\ldots}{3g^4(ad-cb)\ldots}$
norman	$\frac{B^2cd^2i^2x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g(ad-cb)}+\frac{B^2c^2di^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g(ad-cb)}-\frac{9A^2a^2d^2i^2+9A^2abcd\,i^2+9A^2b^2c^2i^2+6AB\,a^2d^2i^2+6ABabcd\,i^2+6AB\,b^2c^2\ldots}{27g\,b^3}$
parallelrisch	$-\frac{-54ABx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^5cd^3i^2-54ABx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^5c^2d^2i^2-18ABx^3\ln\left(\frac{e(bx+a)}{dx+c}\right)b^5d^4i^2-27B^2x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2b\ldots}{\ldots}$

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x,method=_RET
URNVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(-1/3*i^2*d^2*e^2/(a*d-b*c)^2/g^4*A^2/(b*e/d+(a*d-b*c)*e
/d/(d*x+c))^3+2*i^2*d^2*e^2/(a*d-b*c)^2/g^4*A*B*(-1/3/(b*e/d+(a*d-b*c)*e/d/
(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^3)+i^2*d^2*e^2/(a*d-b*c)^2/g^4*B^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*
ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(141) = 282$.

Time = 0.31 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.02

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{3\left((9A^2+6AB+2B^2)b^3cd^2-(9A^2+6AB+2B^2)ab^2d^3\right)i^2x^2+3\left((9A^2+6AB+2B^2)b^3c^2d-(9A\right.}{}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, alg
orithm="fricas")
```

```
[Out] -1/27*(3*((9*A^2 + 6*A*B + 2*B^2)*b^3*c*d^2 - (9*A^2 + 6*A*B + 2*B^2)*a*b^2
*d^3)*i^2*x^2 + 3*((9*A^2 + 6*A*B + 2*B^2)*b^3*c^2*d - (9*A^2 + 6*A*B + 2*B
^2)*a^2*b*d^3)*i^2*x + ((9*A^2 + 6*A*B + 2*B^2)*b^3*c^3 - (9*A^2 + 6*A*B +
2*B^2)*a^3*d^3)*i^2 + 9*(B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*
B^2*b^3*c^2*d*i^2*x + B^2*b^3*c^3*i^2)*log((b*e*x + a*e)/(d*x + c))^2 + 6*(
(3*A*B + B^2)*b^3*d^3*i^2*x^3 + 3*(3*A*B + B^2)*b^3*c*d^2*i^2*x^2 + 3*(3*A*
B + B^2)*b^3*c^2*d*i^2*x + (3*A*B + B^2)*b^3*c^3*i^2)*log((b*e*x + a*e)/(d*
x + c)))/((b^7*c - a*b^6*d)*g^4*x^3 + 3*(a*b^6*c - a^2*b^5*d)*g^4*x^2 + 3*(
a^2*b^5*c - a^3*b^4*d)*g^4*x + (a^3*b^4*c - a^4*b^3*d)*g^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1182 vs. $2(131) = 262$.

Time = 28.41 (sec) , antiderivative size = 1182, normalized size of antiderivative = 8.04

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{2Bd^3i^2\cdot(3A+B)\log\left(x+\frac{6ABad^4i^2+6ABbcd^3i^2+2B^2ad^4i^2+2B^2bcd^3i^2-\frac{2Ba^2d^5i^2\cdot(3A+B)}{ad-bc}+\frac{4Babcd^4i^2\cdot(3A+B)}{ad-bc}-\frac{2Bb^2c^2d^3i^2\cdot(3}{ad-bc}}{12ABbd^4i^2+4B^2bd^4i^2}\right.}{9b^3g^4(ad-bc)} + \frac{2Bd^3i^2\cdot(3A+B)\log\left(x+\frac{6ABad^4i^2+6ABbcd^3i^2+2B^2ad^4i^2+2B^2bcd^3i^2+\frac{2Ba^2d^5i^2\cdot(3A+B)}{ad-bc}-\frac{4Babcd^4i^2\cdot(3A+B)}{ad-bc}+\frac{2Bb^2c^2d^3i^2\cdot(3}{ad-bc}}{12ABbd^4i^2+4B^2bd^4i^2}\right.}{9b^3g^4(ad-bc)} + \frac{(B^2c^3i^2+3B^2c^2di^2x+3B^2cd^2i^2x^2+B^2d^3i^2x^3)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{3a^4dg^4-3a^3bcg^4+9a^3bdg^4x-9a^2b^2cg^4x+9a^2b^2dg^4x^2-9ab^3cg^4x^2+3ab^3dg^4x^3-3b^4cg^4x^3} + \frac{-9A^2a^2d^2i^2-9A^2abcdi^2-9A^2b^2c^2i^2-6ABa^2d^2i^2-6ABabcdi^2-6ABb^2c^2i^2-2B^2a^2d^2i^2-2B^2abc}{} + \frac{(-6ABa^2d^2i^2-6ABabcdi^2-18ABabd^2i^2x-6ABb^2c^2i^2-18ABb^2cdi^2x-18ABb^2d^2i^2x^2-2B^2a^2d^2}{9a^3b^3g^4+27a^2b^4g^4x+27ab^5g}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**4,x)
```

```
[Out] -2*B*d**3*i**2*(3*A + B)*log(x + (6*A*B*a*d**4*i**2 + 6*A*B*b*c*d**3*i**2 +
 2*B**2*a*d**4*i**2 + 2*B**2*b*c*d**3*i**2 - 2*B*a**2*d**5*i**2*(3*A + B)/(
a*d - b*c) + 4*B*a*b*c*d**4*i**2*(3*A + B)/(a*d - b*c) - 2*B*b**2*c**2*d**3
*i**2*(3*A + B)/(a*d - b*c))/(12*A*B*b*d**4*i**2 + 4*B**2*b*d**4*i**2))/(9*
b**3*g**4*(a*d - b*c)) + 2*B*d**3*i**2*(3*A + B)*log(x + (6*A*B*a*d**4*i**2
 + 6*A*B*b*c*d**3*i**2 + 2*B**2*a*d**4*i**2 + 2*B**2*b*c*d**3*i**2 + 2*B*a*
*2*d**5*i**2*(3*A + B)/(a*d - b*c) - 4*B*a*b*c*d**4*i**2*(3*A + B)/(a*d - b
*c) + 2*B*b**2*c**2*d**3*i**2*(3*A + B)/(a*d - b*c))/(12*A*B*b*d**4*i**2 +
4*B**2*b*d**4*i**2))/(9*b**3*g**4*(a*d - b*c)) + (B**2*c**3*i**2 + 3*B**2*c
**2*d*i**2*x + 3*B**2*c*d**2*i**2*x**2 + B**2*d**3*i**2*x**3)*log(e*(a + b*
x)/(c + d*x))**2/(3*a**4*d*g**4 - 3*a**3*b*c*g**4 + 9*a**3*b*d*g**4*x - 9*a
**2*b**2*c*g**4*x + 9*a**2*b**2*d*g**4*x**2 - 9*a*b**3*c*g**4*x**2 + 3*a*b*
*3*d*g**4*x**3 - 3*b**4*c*g**4*x**3) + (-9*A**2*a**2*d**2*i**2 - 9*A**2*a*b
*c*d*i**2 - 9*A**2*b**2*c**2*i**2 - 6*A*B*a**2*d**2*i**2 - 6*A*B*a*b*c*d*i*
*2 - 6*A*B*b**2*c**2*i**2 - 2*B**2*a**2*d**2*i**2 - 2*B**2*a*b*c*d*i**2 - 2
*B**2*b**2*c**2*i**2 + x**2*(-27*A**2*b**2*d**2*i**2 - 18*A*B*b**2*d**2*i**
2 - 6*B**2*b**2*d**2*i**2) + x*(-27*A**2*a*b*d**2*i**2 - 27*A**2*b**2*c*d*i
**2 - 18*A*B*a*b*d**2*i**2 - 18*A*B*b**2*c*d*i**2 - 6*B**2*a*b*d**2*i**2 -
6*B**2*b**2*c*d*i**2))/(27*a**3*b**3*g**4 + 81*a**2*b**4*g**4*x + 81*a*b**5
*g**4*x**2 + 27*b**6*g**4*x**3) + (-6*A*B*a**2*d**2*i**2 - 6*A*B*a*b*c*d*i*
*2 - 18*A*B*a*b*d**2*i**2*x - 6*A*B*b**2*c**2*i**2 - 18*A*B*b**2*c*d*i**2*x
 - 18*A*B*b**2*d**2*i**2*x**2 - 2*B**2*a**2*d**2*i**2 - 2*B**2*a*b*c*d*i**2
 - 6*B**2*a*b*d**2*i**2*x - 2*B**2*b**2*c**2*i**2 - 6*B**2*b**2*c*d*i**2*x
- 6*B**2*b**2*d**2*i**2*x**2)*log(e*(a + b*x)/(c + d*x))/(9*a**3*b**3*g**4
+ 27*a**2*b**4*g**4*x + 27*a*b**5*g**4*x**2 + 9*b**6*g**4*x**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5532 vs. $2(141) = 282$.

Time = 0.58 (sec) , antiderivative size = 5532, normalized size of antiderivative = 37.63

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, alg
orithm="maxima")
```

```
[Out] -1/3*(3*b*x + a)*B^2*c*d*i^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^5*g^
4*x^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - 1/3*(3*b^2*x^2 +
 3*a*b*x + a^2)*B^2*d^2*i^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^6*g^4
*x^3 + 3*a*b^5*g^4*x^2 + 3*a^2*b^4*g^4*x + a^3*b^3*g^4) - 1/54*(6*((6*b^2*d
^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((
```

b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + (4*b^3*c^3 - 27*a*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/(a^3*b^4*c^3*g^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^4 + (b^7*c^3*g^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g^4)*x^3 + 3*(a*b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 - a^4*b^3*d^3*g^4)*x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^3*c*d^2*g^4 - a^5*b^2*d^3*g^4)*x))*B^2*c^2*i^2 - 1/54*(6*((5*a*b^2*c^2 - 22*a^2*b*c*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c^2 - 16*a*b^2*c*d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4) - 6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4) + 6*(3*b*c*d^2 - a*d^3)*log(d*x + c)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + (19*a*b^3*c^3 - 189*a^2*b^2*c^2*d + 189*a^3*b*c*d^2 - 19*a^4*d^3 - 6*(27*b^4*c^2*d - 32*a*b^3*c*d^2 + 5*a^2*b^2*d^3)*x^2 + 18*(3*a^3*b*c*d^2 - a^4*d^3 + (3*b^4*c*d^2 - a*b^3*d^3)*x^3 + 3*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^2 + 3*(3*a^2*b^2*c*d^2 - a^3*b*d^3)*x)*log(b*x + a)^2 + 18*(3*a^3*b*c*d^2 - a^4*d^3 + (3*b^4*c*d^2 - a*b^3*d^3)*x^3 + 3*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^2 + 3*(3*a^2*b^2*c*d^2 - a^3*b*d^3)*x)*log(d*x + c)^2 + 3*(9*b^4*c^3 - 125*a*b^3*c^2*d + 135*a^2*b^2*c*d^2 - 19*a^3*b*d^3)*x - 6*(27*a^3*b*c*d^2 - 5*a^4*d^3 + (27*b^4*c*d^2 - 5*a*b^3*d^3)*x^3 + 3*(27*a*b^3*c*d^2 - 5*a^2*b^2*d^3)*x^2 + 3*(27*a^2*b^2*c*d^2 - 5*a^3*b*d^3)*x)*log(b*x + a) + 6*(27*a^3*b*c*d^2 - 5*a^4*d^3 + (27*b^4*c*d^2 - 5*a*b^3*d^3)*x^3 + 3*(27*a*b^3*c*d^2 - 5*a^2*b^2*d^3)*x^2 + 3*(27*a^2*b^2*c*d^2 - 5*a^3*b*d^3)*x - 6*(3*a^3*b*c*d^2 - a^4*d^3 + (3*b^4*c*d^2 - a*b^3*d^3)*x^3 + 3*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^2 + 3*(3*a^2*b^2*c*d^2 - a^3*b*d^3)*x)*log(b*x + a))*log(d*x + c))/(a^3*b^5*c^3*g^4 - 3*a^4*b^4*c^2*d*g^4 + 3*a^5*b^3*c*d^2*g^4 - a^6*b^2*d^3*g^4 + (b^8*c^3*g^4 - 3*a*b^7*c^2*d*g^4 + 3*a^2*b^6*c*d^2*g^4 - a^3*b^5*d^3*g^4)*x^3 + 3*(a*b^7*c^3*g^4 - 3*a^2*b^6*c^2*d*g^4 + 3*a^3*b^5*c*d^2*g^4 - a^4*b^4*d^3*g^4)*x^2 + 3*(a^2*b^6*c^3*g^4 - 3*a^3*b^5*c^2*d*g^4 + 3*a^4*b^4*c*d^2*g^4 - a^5*b^3*d^3*g^4)*x))*B^2*c*d*i^2 - 1/54*(6*((11*a^2*b^2*c^2 - 7*a^3*b*c*d + 2*a^4*d^2 + 6*(3*b^4*c^2 - 3*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 3*(9*a*b^3*c^2 - 7*a^2*b^2*c*d + 2*a^3*b*d^2)*x)/((b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*g^4*x^

3 + 3*(a*b^7*c^2 - 2*a^2*b^6*c*d + a^3*b^5*d^2)*g^4*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d + a^4*b^4*d^2)*g^4*x + (a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2)*g^4) + 6*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4) - 6*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(d*x + c)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + (85*a^2*b^3*c^3 - 108*a^3*b^2*c^2*d + 27*a^4*b*c*d^2 - 4*a^5*d^3 + 6*(18*b^5*c^3 - 27*a*b^4*c^2*d + 11*a^2*b^3*c*d^2 - 2*a^3*b^2*d^3)*x^2 - 18*(3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2 + a^5*d^3 + (3*b^5*c^2*d - 3*a*b^4*c*d^2 + a^2*b^3*d^3)*x^3 + 3*(3*a*b^4*c^2*d - 3*a^2*b^3*c*d^2 + a^3*b^2*d^3)*x^2 + 3*(3*a^2*b^3*c^2*d - 3*a^3*b^2*c*d^2 + a^4*b*d^3)*x)*log(b*x + a)^2 - 18*(3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2 + a^5*d^3 + (3*b^5*c^2*d - 3*a*b^4*c*d^2 + a^2*b^3*d^3)*x^3 + 3*(3*a*b^4*c^2*d - 3*a^2*b^3*c*d^2 + a^3*b^2*d^3)*x^2 + 3*(3*a^2*b^3*c^2*d - 3*a^3*b^2*c*d^2 + a^4*b*d^3)*x)*log(d*x + c)^2 + 3*(63*a*b^4*c^3 - 86*a^2*b^3*c^2*d + 27*a^3*b^2*c*d^2 - 4*a^4*b*d^3)*x + 6*(18*a^3*b^2*c^2*d - 9*a^4*b*c*d^2 + 2*a^5*d^3 + (18*b^5*c^2*d - 9*a*b^4*c*d^2 + 2*a^2*b^3*d^3)*x^3 + 3*(18*a*b^4*c^2*d - 9*a^2*b^3*c*d^2 + 2*a^3*b^2*d^3)*x^2 + 3*(18*a^2*b^3*c^2*d - 9*a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*log(b*x + a) - 6*(18*a^3*b^2*c^2*d - 9*a^4*b*c*d^2 + 2*a^5*d^3 + (18*b^5*c^2*d - 9*a*b^4*c*d^2 + 2*a^2*b^3*d^3)*x^3 + 3*(18*a*b^4*c^2*d - 9*a^2*b^3*c*d^2 + 2*a^3*b^2*d^3)*x^2 + 3*(18*a^2*b^3*c^2*d - 9*a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x - 6*(3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2 + a^5*d^3 + (3*b^5*c^2*d - 3*a*b^4*c*d^2 + a^2*b^3*d^3)*x^3 + 3*(3*a*b^4*c^2*d - 3*a^2*b^3*c*d^2 + a^3*b^2*d^3)*x^2 + 3*(3*a^2*b^3*c^2*d - 3*a^3*b^2*c*d^2 + a^4*b*d^3)*x)*log(b*x + a))*log(d*x + c))/(a^3*b^6*c^3*g^4 - 3*a^4*b^5*c^2*d*g^4 + 3*a^5*b^4*c*d^2*g^4 - a^6*b^3*d^3*g^4 + (b^9*c^3*g^4 - 3*a*b^8*c^2*d*g^4 + 3*a^2*b^7*c*d^2*g^4 - a^3*b^6*d^3*g^4)*x^3 + 3*(a*b^8*c^3*g^4 - 3*a^2*b^7*c^2*d*g^4 + 3*a^3*b^6*c*d^2*g^4 - a^4*b^5*d^3*g^4)*x^2 + 3*(a^2*b^7*c^3*g^4 - 3*a^3*b^6*c^2*d*g^4 + 3*a^4*b^5*c*d^2*g^4 - a^5*b^4*d^3*g^4)*x))*B^2*d^2*i^2 - 1/9*A*B*d^2*i^2*(6*(3*b^2*x^2 + 3*a*b*x + a^2)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^6*g^4*x^3 + 3*a*b^5*g^4*x^2 + 3*a^2*b^4*g^4*x + a^3*b^3*g^4) + (11*a^2*b^2*c^2 - 7*a^3*b*c*d + 2*a^4*d^2 + 6*(3*b^4*c^2 - 3*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 3*(9*a*b^3*c^2 - 7*a^2*b^2*c*d + 2*a^3*b*d^2)*x)/((b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*g^4*x^3 + 3*(a*b^7*c^2 - 2*a^2*b^6*c*d + a^3*b^5*d^2)*g^4*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d + a^4*b^4*d^2)*g^4*x + (a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2)*g^4) + 6*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4) - 6*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(d*x + c)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4)) - 1/9*A*B*c*d*i^2*(6*(3*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^5*g^4*x^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) + (5*a*b^2*c^2 - 22*a^2*b*c*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c^2 - 16*a*b^2*c*d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4) - 6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c

```
^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4) + 6*(3*b*c*d^2 - a
*d^3)*log(d*x + c)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^
3)*g^4)) - 1/9*A*B*c^2*i^2*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2
*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^
4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^
2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5
*b*d^2)*g^4) + 6*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^4*g^4*x^3 + 3*a*b^
3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3
*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4
*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*B^2*c^2*i^2
*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*
a^2*b^2*g^4*x + a^3*b*g^4) - 1/3*(3*b*x + a)*A^2*c*d*i^2/(b^5*g^4*x^3 + 3*a
*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - 1/3*(3*b^2*x^2 + 3*a*b*x +
a^2)*A^2*d^2*i^2/(b^6*g^4*x^3 + 3*a*b^5*g^4*x^2 + 3*a^2*b^4*g^4*x + a^3*b^3
*g^4) - 1/3*A^2*c^2*i^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x +
a^3*b*g^4)
```

Giac [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.49

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{1}{27}\left(\frac{9\,(dx+c)^3B^2e^4i^2\log\left(\frac{bex+ae}{dx+c}\right)^2}{(bex+ae)^3g^4} + \frac{6\,(3\,ABe^4i^2+B^2e^4i^2)(dx+c)^3\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^3g^4} + \frac{(9\,A^2e^4i^2+6\,AB}{(b}\right.$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, alg
orithm="giac")
```

```
[Out] -1/27*(9*(d*x + c)^3*B^2*e^4*i^2*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a
*e)^3*g^4) + 6*(3*A*B*e^4*i^2 + B^2*e^4*i^2)*(d*x + c)^3*log((b*e*x + a*e)/
(d*x + c))/((b*e*x + a*e)^3*g^4) + (9*A^2*e^4*i^2 + 6*A*B*e^4*i^2 + 2*B^2*e
^4*i^2)*(d*x + c)^3/((b*e*x + a*e)^3*g^4))*(b*c/((b*c*e - a*d*e)*(b*c - a*d
)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 3.99 (sec) , antiderivative size = 1153, normalized size of antiderivative = 7.84

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{x^2\,(9\,A^2\,b^2\,d^2\,i^2+6\,A\,B\,b^2\,d^2\,i^2+2\,B^2\,b^2\,d^2\,i^2)+x\,(9\,c\,A^2\,b^2\,d\,i^2+9\,a\,A^2\,b\,d^2\,i^2+6\,c\,A\,B\,b^2\,d\,i^2+6\,a\,A\,B}{}$$

$$-\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{x\left(b\left(\frac{B^2\,c\,d\,i^2}{3\,b^3\,g^4}+\frac{B^2\,a\,d^2\,i^2}{3\,b^4\,g^4}\right)+\frac{2\,B^2\,c\,d\,i^2}{3\,b^2\,g^4}+\frac{2\,B^2\,a\,d^2\,i^2}{3\,b^3\,g^4}\right)+a\left(\frac{B^2\,c\,d\,i^2}{3\,b^3\,g^4}+\frac{B^2\,a\,d^2\,i^2}{3\,b^4\,g^4}\right)+\frac{B^2\,c^2\,i^2}{3\,b^2\,g^4}+}{3\,a^2\,x+\frac{a^3}{b}+b^2\,x^3+3\,a\,b\,x^2}\right.$$

$$\left.-\frac{B^2\,d^3\,i^2}{3\,b^3\,g^4\,(a\,d-b\,c)}\right)$$

$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(x\left(b\left(\frac{B\,i^2\,(2\,A\,b\,c-B\,a\,d+B\,b\,c)}{3\,b^4\,g^4}+\frac{2\,A\,B\,a\,d\,i^2}{3\,b^4\,g^4}\right)+\frac{2\,B\,i^2\,(2\,A\,b\,c-B\,a\,d+B\,b\,c)}{3\,b^3\,g^4}+\frac{2\,B^2\,d^3\,i^2\left(b\left(\frac{3\,a^2\,d^2-4\,a\,b\,c\,d+b^2}{6\,b\,d^3}\right.\right.}{}\right.\right.}{}$$

$$-\frac{B\,d^3\,i^2\,\mathrm{atan}\left(\frac{\left(\frac{9\,c\,b^4\,g^4+9\,a\,d\,b^3\,g^4}{9\,b^3\,g^4}+2\,b\,d\,x\right)1\mathrm{i}}{a\,d-b\,c}\right)(3\,A+B)\,4\mathrm{i}}{9\,b^3\,g^4\,(a\,d-b\,c)}$$

```
[In] int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^
4,x)
```

```
[Out] - (x^2*(9*A^2*b^2*d^2*i^2 + 2*B^2*b^2*d^2*i^2 + 6*A*B*b^2*d^2*i^2) + x*(9*A
^2*a*b*d^2*i^2 + 2*B^2*a*b*d^2*i^2 + 9*A^2*b^2*c*d*i^2 + 2*B^2*b^2*c*d*i^2
+ 6*A*B*a*b*d^2*i^2 + 6*A*B*b^2*c*d*i^2) + 3*A^2*a^2*d^2*i^2 + 3*A^2*b^2*c^
2*i^2 + (2*B^2*a^2*d^2*i^2)/3 + (2*B^2*b^2*c^2*i^2)/3 + 2*A*B*a^2*d^2*i^2 +
 2*A*B*b^2*c^2*i^2 + 3*A^2*a*b*c*d*i^2 + (2*B^2*a*b*c*d*i^2)/3 + 2*A*B*a*b*
c*d*i^2)/(9*a^3*b^3*g^4 + 9*b^6*g^4*x^3 + 27*a^2*b^4*g^4*x + 27*a*b^5*g^4*x
^2) - log((e*(a + b*x))/(c + d*x))^2*((x*(b*((B^2*c*d*i^2)/(3*b^3*g^4) + (B
^2*a*d^2*i^2)/(3*b^4*g^4)) + (2*B^2*c*d*i^2)/(3*b^2*g^4) + (2*B^2*a*d^2*i^2
)/(3*b^3*g^4)) + a*((B^2*c*d*i^2)/(3*b^3*g^4) + (B^2*a*d^2*i^2)/(3*b^4*g^4)
) + (B^2*c^2*i^2)/(3*b^2*g^4) + (B^2*d^2*i^2*x^2)/(b^2*g^4))/(3*a^2*x + a^3
/b + b^2*x^3 + 3*a*b*x^2) - (B^2*d^3*i^2)/(3*b^3*g^4*(a*d - b*c))) - (log((
e*(a + b*x))/(c + d*x))*(x*(b*((B*i^2*(2*A*b*c - B*a*d + B*b*c))/(3*b^4*g^4
) + (2*A*B*a*d*i^2)/(3*b^4*g^4)) + (2*B*i^2*(2*A*b*c - B*a*d + B*b*c))/(3*b
^3*g^4) + (2*B^2*d^3*i^2*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) +
(a*(a*d - b*c))/(3*b*d^2)) + (3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(3*d^3) + (2
*a*(a*d - b*c))/(3*d^2)))/(3*b^3*g^4*(a*d - b*c)) + (4*A*B*a*d*i^2)/(3*b^3*
g^4)) + x^2*((2*A*B*d*i^2)/(b^2*g^4) - (2*B^2*d^3*i^2*((b^2*c - a*b*d)/(3*d
^2) - (2*b*(a*d - b*c))/(3*d^2)))/(3*b^3*g^4*(a*d - b*c))) + a*((B*i^2*(2*A
```

```
*b*c - B*a*d + B*b*c))/(3*b^4*g^4) + (2*A*B*a*d*i^2)/(3*b^4*g^4)) + (2*B*i^
2*(A*b^2*c^2 - B*a^2*d^2 + B*a*b*c*d))/(3*b^4*d*g^4) + (2*B^2*d^3*i^2*(a*((
3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(3*b*d^2)) + (
3*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d^2)/(3*b*d^4)))/(3*b^3*g^4
*(a*d - b*c))))/((3*a^2*x)/d + a^3/(b*d) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (
B*d^3*i^2*atan((((9*b^4*c*g^4 + 9*a*b^3*d*g^4)/(9*b^3*g^4) + 2*b*d*x)*1i)/(
a*d - b*c))*(3*A + B)*4i)/(9*b^3*g^4*(a*d - b*c))
```

3.72 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 299

$$\begin{aligned}\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx &= \frac{2B^2di^2(c+dx)^3}{27(bc-ad)^2g^5(a+bx)^3} \\ &\quad - \frac{bB^2i^2(c+dx)^4}{32(bc-ad)^2g^5(a+bx)^4} \\ &\quad + \frac{2Bdi^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^2g^5(a+bx)^3} \\ &\quad - \frac{bBi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)^2g^5(a+bx)^4} \\ &\quad + \frac{di^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^2g^5(a+bx)^3} \\ &\quad - \frac{bi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^2g^5(a+bx)^4}\end{aligned}$$

```
[Out] 2/27*B^2*d*i^2*(d*x+c)^3/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/32*b*B^2*i^2*(d*x+c)^
4/(-a*d+b*c)^2/g^5/(b*x+a)^4+2/9*B*d*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c
)))/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/8*b*B*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x
+c)))/(-a*d+b*c)^2/g^5/(b*x+a)^4+1/3*d*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x
+c)))^2/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/4*b*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d
*x+c)))^2/(-a*d+b*c)^2/g^5/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2562, 2395, 2342, 2341}

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}dx = -\frac{bi^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4g^5(a+bx)^4(bc-ad)^2} - \frac{bBi^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{8g^5(a+bx)^4(bc-ad)^2} + \frac{di^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g^5(a+bx)^3(bc-ad)^2} + \frac{2Bdi^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{9g^5(a+bx)^3(bc-ad)^2} - \frac{bB^2i^2(c+dx)^4}{32g^5(a+bx)^4(bc-ad)^2} + \frac{2B^2di^2(c+dx)^3}{27g^5(a+bx)^3(bc-ad)^2}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)^
5,x]
```

```
[Out] (2*B^2*d*i^2*(c + d*x)^3)/(27*(b*c - a*d)^2*g^5*(a + b*x)^3) - (b*B^2*i^2*(
c + d*x)^4)/(32*(b*c - a*d)^2*g^5*(a + b*x)^4) + (2*B*d*i^2*(c + d*x)^3*(A
+ B*Log[(e*(a + b*x))/(c + d*x)]))/(9*(b*c - a*d)^2*g^5*(a + b*x)^3) - (b*B
*i^2*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(8*(b*c - a*d)^2*g^5
*(a + b*x)^4) + (d*i^2*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/
(3*(b*c - a*d)^2*g^5*(a + b*x)^3) - (b*i^2*(c + d*x)^4*(A + B*Log[(e*(a + b
*x))/(c + d*x)])^2)/(4*(b*c - a*d)^2*g^5*(a + b*x)^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i^2\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex))^2}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^5}\\
&= \frac{i^2\text{Subst}\left(\int \left(\frac{b(A+B\log(ex))^2}{x^5}-\frac{d(A+B\log(ex))^2}{x^4}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^5}\\
&= \frac{(bi^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^5}-\frac{(di^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^5}\\
&= \frac{di^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^2g^5(a+bx)^3}-\frac{bi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^2g^5(a+bx)^4}\\
&\quad+\frac{(bBi^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{2(bc-ad)^2g^5}-\frac{(2Bdi^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bc-ad)^2g^5}\\
&= \frac{2B^2di^2(c+dx)^3}{27(bc-ad)^2g^5(a+bx)^3}-\frac{bB^2i^2(c+dx)^4}{32(bc-ad)^2g^5(a+bx)^4}\\
&\quad+\frac{2Bdi^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^2g^5(a+bx)^3}-\frac{bBi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)^2g^5(a+bx)^4}\\
&\quad+\frac{di^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^2g^5(a+bx)^3}-\frac{bi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^2g^5(a+bx)^4}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.46 (sec) , antiderivative size = 1703, normalized size of antiderivative = 5.70

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{i^2\left(216(bc-ad)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-576d(-bc+ad)^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+432d^2\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b
*g*x)^5,x]
```

```
[Out] -1/864*(i^2*(216*(b*c - a*d)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - 576
*d*(-(b*c) + a*d)^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 432*
d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 32*B
*d*(a + b*x)*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3 - 18*A*d*(b*c - a*d)^2
*(a + b*x) - 15*B*d*(b*c - a*d)^2*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x
)^2 + 66*B*d^2*(b*c - a*d)*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x]
+ 66*B*d^3*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*(a + b*x)^3*Log[a + b*x]^2 +
 12*B*(b*c - a*d)^3*Log[(e*(a + b*x))/(c + d*x)] - 18*B*d*(b*c - a*d)^2*(a
+ b*x)*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[
(e*(a + b*x))/(c + d*x)] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(e*(a + b*
x))/(c + d*x)] - 36*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3*(a + b*x)^3*L
og[c + d*x] + 36*B*d^3*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c
+ d*x] - 36*B*d^3*(a + b*x)^3*Log[(e*(a + b*x))/(c + d*x)]*Log[c + d*x] - 1
8*B*d^3*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[
(b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/
(-(b*c) + a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)
]) + 3*B*(36*A*(b*c - a*d)^4 + 9*B*(b*c - a*d)^4 + 48*A*d*(-(b*c) + a*d)^3*
(a + b*x) + 28*B*d*(-(b*c) + a*d)^3*(a + b*x) + 72*A*d^2*(b*c - a*d)^2*(a +
 b*x)^2 + 78*B*d^2*(b*c - a*d)^2*(a + b*x)^2 + 144*A*d^3*(-(b*c) + a*d)*(a
+ b*x)^3 + 300*B*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 144*A*d^4*(a + b*x)^4*Log
[a + b*x] - 300*B*d^4*(a + b*x)^4*Log[a + b*x] + 72*B*d^4*(a + b*x)^4*Log[a
 + b*x]^2 + 36*B*(b*c - a*d)^4*Log[(e*(a + b*x))/(c + d*x)] + 48*B*d*(-(b*c
) + a*d)^3*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 72*B*d^2*(b*c - a*d)^2*
(a + b*x)^2*Log[(e*(a + b*x))/(c + d*x)] + 144*B*d^3*(-(b*c) + a*d)*(a + b*
x)^3*Log[(e*(a + b*x))/(c + d*x)] - 144*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[
(e*(a + b*x))/(c + d*x)] + 144*A*d^4*(a + b*x)^4*Log[c + d*x] + 300*B*d^4*(
a + b*x)^4*Log[c + d*x] - 144*B*d^4*(a + b*x)^4*Log[(d*(a + b*x))/(-(b*c) +
 a*d)]*Log[c + d*x] + 144*B*d^4*(a + b*x)^4*Log[(e*(a + b*x))/(c + d*x)]*Lo
g[c + d*x] + 72*B*d^4*(a + b*x)^4*Log[c + d*x]^2 - 144*B*d^4*(a + b*x)^4*Lo
g[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 144*B*d^4*(a + b*x)^4*PolyLog[2
```

```
, (d*(a + b*x))/(-(b*c) + a*d)] - 144*B*d^4*(a + b*x)^4*PolyLog[2, (b*(c +
d*x))/(b*c - a*d)]) + 216*B*d^2*(a + b*x)^2*(2*(b*c - a*d)^2*(A + B*Log[(e*
(a + b*x))/(c + d*x)]) + 4*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[(e*(a + b*
x))/(c + d*x)]) - 4*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(
c + d*x)]) + 4*d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c +
 d*x] - 4*B*d*(a + b*x)*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)
*Log[c + d*x]) + B*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a
 + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) + 2*B*d^2*(a + b*x
)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Pol
yLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2*B*d^2*(a + b*x)^2*((2*Log[(d*(a
+ b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c +
 d*x))/(b*c - a*d)]))))/(b^3*(b*c - a*d)^2*g^5*(a + b*x)^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(287) = 574$.

Time = 1.64 (sec) , antiderivative size = 685, normalized size of antiderivative = 2.29

method	result
parts	$\frac{i^2A^2\left(\frac{2d(ad-cb)}{3b^3(bx+a)^3}-\frac{a^2d^2-2abcd+b^2c^2}{4b^3(bx+a)^4}-\frac{d^2}{2b^3(bx+a)^2}\right)}{g^5}-\frac{i^2B^2(ad-cb)^3e^3\left(\frac{d^5\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{9\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}\cdots\right)}{(ad-cb)^5}\cdots\right)}{\cdots}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{i^2d^2e^3A^2b}{4(ad-cb)^3g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{i^2d^3e^2A^2}{3(ad-cb)^3g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{2i^2d^2e^3ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{\cdots}{16\left(\frac{be}{d}+\cdots\right)}\cdots\right)}{(ad-cb)^3g^5}\cdots\right)}{\cdots}$
default	$-\frac{e(ad-cb)\left(\frac{i^2d^2e^3A^2b}{4(ad-cb)^3g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{i^2d^3e^2A^2}{3(ad-cb)^3g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{2i^2d^2e^3ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{\cdots}{16\left(\frac{be}{d}+\cdots\right)}\cdots\right)}{(ad-cb)^3g^5}\cdots\right)}{\cdots}$
norman	Expression too large to display
parallelrisch	Expression too large to display
risch	Expression too large to display

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x,method=_RET
URNVERBOSE)
```

```
[Out] i^2*A^2/g^5*(2/3*d*(a*d-b*c)/b^3/(b*x+a)^3-1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/
b^3/(b*x+a)^4-1/2*d^2/b^3/(b*x+a)^2)-i^2*B^2/g^5/d^4*(a*d-b*c)^3*e^3*(d^5/(
```

```
a*d-b*c)^5*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*
x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))
-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-d^4/(a*d-b*c)^5*b*e*(-1/4/(b*e/d+(a*
d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/8/(b*e/d+(a*d-b*c
)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/32/(b*e/d+(a*d-b*c)*e/d/
(d*x+c))^4))-2*i^2*B*A/g^5/d^4*(a*d-b*c)^3*e^3*(d^5/(a*d-b*c)^5*(-1/3/(b*e/
d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-
b*c)*e/d/(d*x+c))^3)-d^4/(a*d-b*c)^5*b*e*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 837 vs. $2(287) = 574$.

Time = 0.30 (sec) , antiderivative size = 837, normalized size of antiderivative = 2.80

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$$
$$= \frac{12\left((12\,AB+7\,B^2)b^4cd^3-(12\,AB+7\,B^2)ab^3d^4\right)i^2x^3-6\left((72\,A^2+12\,AB-5\,B^2)b^4c^2d^2-16\,(9\,A^2+6\,A\right.}{}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, alg
orithm="fricas")
```

```
[Out] 1/864*(12*((12*A*B + 7*B^2)*b^4*c*d^3 - (12*A*B + 7*B^2)*a*b^3*d^4)*i^2*x^3
 - 6*((72*A^2 + 12*A*B - 5*B^2)*b^4*c^2*d^2 - 16*(9*A^2 + 6*A*B + 2*B^2)*a*
b^3*c*d^3 + (72*A^2 + 84*A*B + 37*B^2)*a^2*b^2*d^4)*i^2*x^2 - 4*((144*A^2 +
 60*A*B + 11*B^2)*b^4*c^3*d - 24*(9*A^2 + 6*A*B + 2*B^2)*a*b^3*c^2*d^2 + (7
2*A^2 + 84*A*B + 37*B^2)*a^3*b*d^4)*i^2*x - (27*(8*A^2 + 4*A*B + B^2)*b^4*c
^4 - 32*(9*A^2 + 6*A*B + 2*B^2)*a*b^3*c^3*d + (72*A^2 + 84*A*B + 37*B^2)*a^
4*d^4)*i^2 + 72*(B^2*b^4*d^4*i^2*x^4 + 4*B^2*a*b^3*d^4*i^2*x^3 - 6*(B^2*b^4
*c^2*d^2 - 2*B^2*a*b^3*c*d^3)*i^2*x^2 - 4*(2*B^2*b^4*c^3*d - 3*B^2*a*b^3*c^
2*d^2)*i^2*x - (3*B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d)*i^2)*log((b*e*x + a*e)/(
d*x + c))^2 + 12*((12*A*B + 7*B^2)*b^4*d^4*i^2*x^4 + 4*(3*B^2*b^4*c*d^3 + 4
*(3*A*B + B^2)*a*b^3*d^4)*i^2*x^3 - 6*((12*A*B + B^2)*b^4*c^2*d^2 - 8*(3*A*
B + B^2)*a*b^3*c*d^3)*i^2*x^2 - 4*((24*A*B + 5*B^2)*b^4*c^3*d - 12*(3*A*B +
 B^2)*a*b^3*c^2*d^2)*i^2*x - (9*(4*A*B + B^2)*b^4*c^4 - 16*(3*A*B + B^2)*a*
b^3*c^3*d)*i^2)*log((b*e*x + a*e)/(d*x + c)))/((b^9*c^2 - 2*a*b^8*c*d + a^2
*b^7*d^2)*g^5*x^4 + 4*(a*b^8*c^2 - 2*a^2*b^7*c*d + a^3*b^6*d^2)*g^5*x^3 + 6
*(a^2*b^7*c^2 - 2*a^3*b^6*c*d + a^4*b^5*d^2)*g^5*x^2 + 4*(a^3*b^6*c^2 - 2*a
^4*b^5*c*d + a^5*b^4*d^2)*g^5*x + (a^4*b^5*c^2 - 2*a^5*b^4*c*d + a^6*b^3*d^
2)*g^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2055 vs. $2(277) = 554$.

Time = 69.34 (sec) , antiderivative size = 2055, normalized size of antiderivative = 6.87

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**5,x)
```

```
[Out] -B*d**4*i**2*(12*A + 7*B)*log(x + (12*A*B*a*d**5*i**2 + 12*A*B*b*c*d**4*i**
2 + 7*B**2*a*d**5*i**2 + 7*B**2*b*c*d**4*i**2 - B*a**3*d**7*i**2*(12*A + 7*
B)/(a*d - b*c)**2 + 3*B*a**2*b*c*d**6*i**2*(12*A + 7*B)/(a*d - b*c)**2 - 3*
B*a*b**2*c**2*d**5*i**2*(12*A + 7*B)/(a*d - b*c)**2 + B*b**3*c**3*d**4*i**2
*(12*A + 7*B)/(a*d - b*c)**2)/(24*A*B*b*d**5*i**2 + 14*B**2*b*d**5*i**2))/(
72*b**3*g**5*(a*d - b*c)**2) + B*d**4*i**2*(12*A + 7*B)*log(x + (12*A*B*a*d
**5*i**2 + 12*A*B*b*c*d**4*i**2 + 7*B**2*a*d**5*i**2 + 7*B**2*b*c*d**4*i**2
 + B*a**3*d**7*i**2*(12*A + 7*B)/(a*d - b*c)**2 - 3*B*a**2*b*c*d**6*i**2*(1
2*A + 7*B)/(a*d - b*c)**2 + 3*B*a*b**2*c**2*d**5*i**2*(12*A + 7*B)/(a*d - b
*c)**2 - B*b**3*c**3*d**4*i**2*(12*A + 7*B)/(a*d - b*c)**2)/(24*A*B*b*d**5*
i**2 + 14*B**2*b*d**5*i**2))/(72*b**3*g**5*(a*d - b*c)**2) + (4*B**2*a*c**3
*d*i**2 + 12*B**2*a*c**2*d**2*i**2*x + 12*B**2*a*c*d**3*i**2*x**2 + 4*B**2*
a*d**4*i**2*x**3 - 3*B**2*b*c**4*i**2 - 8*B**2*b*c**3*d*i**2*x - 6*B**2*b*c
**2*d**2*i**2*x**2 + B**2*b*d**4*i**2*x**4)*log(e*(a + b*x)/(c + d*x))**2/(
12*a**6*d**2*g**5 - 24*a**5*b*c*d*g**5 + 48*a**5*b*d**2*g**5*x + 12*a**4*b*
*2*c**2*g**5 - 96*a**4*b**2*c*d*g**5*x + 72*a**4*b**2*d**2*g**5*x**2 + 48*a
**3*b**3*c**2*g**5*x - 144*a**3*b**3*c*d*g**5*x**2 + 48*a**3*b**3*d**2*g**5
*x**3 + 72*a**2*b**4*c**2*g**5*x**2 - 96*a**2*b**4*c*d*g**5*x**3 + 12*a**2*
b**4*d**2*g**5*x**4 + 48*a*b**5*c**2*g**5*x**3 - 24*a*b**5*c*d*g**5*x**4 +
12*b**6*c**2*g**5*x**4) + (-12*A*B*a**3*d**3*i**2 - 12*A*B*a**2*b*c*d**2*i*
*2 - 48*A*B*a**2*b*d**3*i**2*x - 12*A*B*a*b**2*c**2*d*i**2 - 48*A*B*a*b**2*
c*d**2*i**2*x - 72*A*B*a*b**2*d**3*i**2*x**2 + 36*A*B*b**3*c**3*i**2 + 96*A
*B*b**3*c**2*d*i**2*x + 72*A*B*b**3*c*d**2*i**2*x**2 - 7*B**2*a**3*d**3*i**
2 - 7*B**2*a**2*b*c*d**2*i**2 - 28*B**2*a**2*b*d**3*i**2*x - 7*B**2*a*b**2*
c**2*d*i**2 - 28*B**2*a*b**2*c*d**2*i**2*x - 42*B**2*a*b**2*d**3*i**2*x**2
+ 9*B**2*b**3*c**3*i**2 + 20*B**2*b**3*c**2*d*i**2*x + 6*B**2*b**3*c*d**2*i
**2*x**2 - 12*B**2*b**3*d**3*i**2*x**3)*log(e*(a + b*x)/(c + d*x))/(72*a**5
*b**3*d*g**5 - 72*a**4*b**4*c*g**5 + 288*a**4*b**4*d*g**5*x - 288*a**3*b**5
*c*g**5*x + 432*a**3*b**5*d*g**5*x**2 - 432*a**2*b**6*c*g**5*x**2 + 288*a**
2*b**6*d*g**5*x**3 - 288*a*b**7*c*g**5*x**3 + 72*a*b**7*d*g**5*x**4 - 72*b*
*8*c*g**5*x**4) + (-72*A**2*a**3*d**3*i**2 - 72*A**2*a**2*b*c*d**2*i**2 - 7
2*A**2*a*b**2*c**2*d*i**2 + 216*A**2*b**3*c**3*i**2 - 84*A*B*a**3*d**3*i**2
 - 84*A*B*a**2*b*c*d**2*i**2 - 84*A*B*a*b**2*c**2*d*i**2 + 108*A*B*b**3*c**
3*i**2 - 37*B**2*a**3*d**3*i**2 - 37*B**2*a**2*b*c*d**2*i**2 - 37*B**2*a*b*
```

```
*2*c**2*d*i**2 + 27*B**2*b**3*c**3*i**2 + x**3*(-144*A*B*b**3*d**3*i**2 - 8
4*B**2*b**3*d**3*i**2) + x**2*(-432*A**2*a*b**2*d**3*i**2 + 432*A**2*b**3*c
*d**2*i**2 - 504*A*B*a*b**2*d**3*i**2 + 72*A*B*b**3*c*d**2*i**2 - 222*B**2*
a*b**2*d**3*i**2 - 30*B**2*b**3*c*d**2*i**2) + x*(-288*A**2*a**2*b*d**3*i**
2 - 288*A**2*a*b**2*c*d**2*i**2 + 576*A**2*b**3*c**2*d*i**2 - 336*A*B*a**2*
b*d**3*i**2 - 336*A*B*a*b**2*c*d**2*i**2 + 240*A*B*b**3*c**2*d*i**2 - 148*B
**2*a**2*b*d**3*i**2 - 148*B**2*a*b**2*c*d**2*i**2 + 44*B**2*b**3*c**2*d*i*
*2))/(864*a**5*b**3*d*g**5 - 864*a**4*b**4*c*g**5 + x**4*(864*a*b**7*d*g**5
 - 864*b**8*c*g**5) + x**3*(3456*a**2*b**6*d*g**5 - 3456*a*b**7*c*g**5) + x
**2*(5184*a**3*b**5*d*g**5 - 5184*a**2*b**6*c*g**5) + x*(3456*a**4*b**4*d*g
**5 - 3456*a**3*b**5*c*g**5))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8031 vs. 2(287) = 574.

Time = 0.79 (sec) , antiderivative size = 8031, normalized size of antiderivative = 26.86

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, alg
orithm="maxima")
```

```
[Out] -1/6*(4*b*x + a)*B^2*c*d*i^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^6*g^
5*x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g^5
) - 1/12*(6*b^2*x^2 + 4*a*b*x + a^2)*B^2*d^2*i^2*log(b*e*x/(d*x + c) + a*e/
(d*x + c))^2/(b^7*g^5*x^4 + 4*a*b^6*g^5*x^3 + 6*a^2*b^5*g^5*x^2 + 4*a^3*b^4
*g^5*x + a^4*b^3*g^5) + 1/288*(12*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c
^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b
^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a
^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^
3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a
^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*
a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6
*b^2*c*d^2 - a^7*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*
d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x
+ c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*
b*d^4)*g^5))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - (9*b^4*c^4 - 64*a*b^3*c
^3*d + 216*a^2*b^2*c^2*d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3
 - a*b^3*d^4)*x^3 + 6*(13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*
x^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x
 + a^4*d^4)*log(b*x + a)^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*
d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b
^3*c^2*d^2 + 324*a^2*b^2*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*
```

b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*b^4*d^4*x^4 + 100*a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x + 25*a^4*d^4 - 12*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a))*log(d*x + c))/(a^4*b^5*c^4*g^5 - 4*a^5*b^4*c^3*d*g^5 + 6*a^6*b^3*c^2*d^2*g^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d^4*g^5 + (b^9*c^4*g^5 - 4*a*b^8*c^3*d*g^5 + 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b^6*c*d^3*g^5 + a^4*b^5*d^4*g^5)*x^4 + 4*(a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^5 + 6*a^3*b^6*c^2*d^2*g^5 - 4*a^4*b^5*c*d^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*(a^2*b^7*c^4*g^5 - 4*a^3*b^6*c^3*d*g^5 + 6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*c*d^3*g^5 + a^6*b^3*d^4*g^5)*x^2 + 4*(a^3*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5 + 6*a^5*b^4*c^2*d^2*g^5 - 4*a^6*b^3*c*d^3*g^5 + a^7*b^2*d^4*g^5)*x))*B^2*c^2*i^2 - 1/432*(12*((7*a*b^3*c^3 - 33*a^2*b^2*c^2*d + 75*a^3*b*c*d^2 - 13*a^4*d^3 + 12*(4*b^4*c*d^2 - a*b^3*d^3)*x^3 - 6*(4*b^4*c^2*d - 29*a*b^3*c*d^2 + 7*a^2*b^2*d^3)*x^2 + 4*(4*b^4*c^3 - 21*a*b^3*c^2*d + 57*a^2*b^2*c*d^2 - 13*a^3*b*d^3)*x)/((b^9*c^3 - 3*a*b^8*c^2*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*g^5*x^4 + 4*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*a^3*b^6*c*d^2 - a^4*b^5*d^3)*g^5*x^3 + 6*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*a^4*b^5*c*d^2 - a^5*b^4*d^3)*g^5*x^2 + 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2 - a^6*b^3*d^3)*g^5*x + (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d^2 - a^7*b^2*d^3)*g^5) + 12*(4*b*c*d^3 - a*d^4)*log(b*x + a)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5) - 12*(4*b*c*d^3 - a*d^4)*log(d*x + c)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + (37*a*b^4*c^4 - 304*a^2*b^3*c^3*d + 1512*a^3*b^2*c^2*d^2 - 1360*a^4*b*c*d^3 + 115*a^5*d^4 + 12*(88*b^5*c^2*d^2 - 101*a*b^4*c*d^3 + 13*a^2*b^3*d^4)*x^3 - 6*(40*b^5*c^3*d - 609*a*b^4*c^2*d^2 + 648*a^2*b^3*c*d^3 - 79*a^3*b^2*d^4)*x^2 - 72*(4*a^4*b*c*d^3 - a^5*d^4 + (4*b^5*c*d^3 - a*b^4*d^4)*x^4 + 4*(4*a*b^4*c*d^3 - a^2*b^3*d^4)*x^3 + 6*(4*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^2 + 4*(4*a^3*b^2*c*d^3 - a^4*b*d^4)*x)*log(b*x + a)^2 - 72*(4*a^4*b*c*d^3 - a^5*d^4 + (4*b^5*c*d^3 - a*b^4*d^4)*x^4 + 4*(4*a*b^4*c*d^3 - a^2*b^3*d^4)*x^3 + 6*(4*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^2 + 4*(4*a^3*b^2*c*d^3 - a^4*b*d^4)*x)*log(d*x + c)^2 + 4*(16*b^5*c^4 - 163*a*b^4*c^3*d + 1068*a^2*b^3*c^2*d^2 - 1036*a^3*b^2*c*d^3 + 115*a^4*b*d^4)*x + 12*(88*a^4*b*c*d^3 - 13*a^5*d^4 + (88*b^5*c*d^3 - 13*a*b^4*d^4)*x^4 + 4*(88*a*b^4*c*d^3 - 13*a^2*b^3*d^4)*x^3 + 6*(88*a^2*b^3*c*d^3 - 13*a^3*b^2*d^4)*x^2 + 4*(88*a^3*b^2*c*d^3 - 13*a^4*b*d^4)*x)*log(b*x + a) - 12*(88*a^4*b*c*d^3 - 13*a^5*d^4 + (88*b^5*c*d^3 - 13*a*b^4*d^4)*x^4 + 4*(88*a*b^4*c*d^3 - 13*a^2*b^3*d^4)*x^3 + 6*(88*a^2*b^3*c*d^3 - 13*a^3*b^2*d^4)*x^2 + 4*(88*a^3*b^2*c*d^3 - 13*a^4*b*d^4)*x - 12*(4*a^4*b*c*d^3 - a^5*d^4 + (4*b^5*c*d^3 - a*b^4*d^4)*x^4 + 4*(4*a*b^4*c*d^3 - a^2*b^3*d^4)*x^3 + 6*(4*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^2 + 4*(4*a^3*b^2*c*d^3 - a^4*b*d^4)*x)*log(b*x + a))*log(d*x + c))/(a^4*b^6*c^4*g^5 - 4*a^5*b^5*c^3*d*g^5 + 6*a^6*b^4*c^2*d^2*g^5 - 4*a^7*b^3*c*d^3*g^5 + a^8*b^2*d^4*g^5 + (b^10*c^4*g^5 - 4*a*b^9*c^3*d*g^5 + 6*a^2*b^8*c^2*d^2*g^5 - 4*a^3*b^7*c*d^3*g^5 + a^4*b^6*d^4*g^5)*x^4 + 4*(a*b^9*c^4*g^5 - 4*a^2*b^8*c^3*d*g^5 + 6*a^3*b^7*c^2*d^2*g^5 - 4*a^4*b^6*c*d^3*g^5 + a^5*b^5*d^4*g^5)*x^3 + 6*(a^2*b^8*c^4*g^5 - 4*

a^3*b^7*c^3*d*g^5 + 6*a^4*b^6*c^2*d^2*g^5 - 4*a^5*b^5*c*d^3*g^5 + a^6*b^4*d
^4*g^5)*x^2 + 4*(a^3*b^7*c^4*g^5 - 4*a^4*b^6*c^3*d*g^5 + 6*a^5*b^5*c^2*d^2*
g^5 - 4*a^6*b^4*c*d^3*g^5 + a^7*b^3*d^4*g^5)*x))*B^2*c*d*i^2 - 1/864*(12*((
13*a^2*b^3*c^3 - 75*a^3*b^2*c^2*d + 33*a^4*b*c*d^2 - 7*a^5*d^3 - 12*(6*b^5*
c^2*d - 4*a*b^4*c*d^2 + a^2*b^3*d^3)*x^3 + 6*(6*b^5*c^3 - 46*a*b^4*c^2*d +
29*a^2*b^3*c*d^2 - 7*a^3*b^2*d^3)*x^2 + 4*(10*a*b^4*c^3 - 63*a^2*b^3*c^2*d
+ 33*a^3*b^2*c*d^2 - 7*a^4*b*d^3)*x)/((b^10*c^3 - 3*a*b^9*c^2*d + 3*a^2*b^8
*c*d^2 - a^3*b^7*d^3)*g^5*x^4 + 4*(a*b^9*c^3 - 3*a^2*b^8*c^2*d + 3*a^3*b^7*
c*d^2 - a^4*b^6*d^3)*g^5*x^3 + 6*(a^2*b^8*c^3 - 3*a^3*b^7*c^2*d + 3*a^4*b^6
*c*d^2 - a^5*b^5*d^3)*g^5*x^2 + 4*(a^3*b^7*c^3 - 3*a^4*b^6*c^2*d + 3*a^5*b^
5*c*d^2 - a^6*b^4*d^3)*g^5*x + (a^4*b^6*c^3 - 3*a^5*b^5*c^2*d + 3*a^6*b^4*c
*d^2 - a^7*b^3*d^3)*g^5) - 12*(6*b^2*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*log(b
*x + a)/((b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a
^4*b^3*d^4)*g^5) + 12*(6*b^2*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*log(d*x + c)/
((b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d
^4)*g^5))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + (115*a^2*b^4*c^4 - 1360*a^
3*b^3*c^3*d + 1512*a^4*b^2*c^2*d^2 - 304*a^5*b*c*d^3 + 37*a^6*d^4 - 12*(108
*b^6*c^3*d - 148*a*b^5*c^2*d^2 + 47*a^2*b^4*c*d^3 - 7*a^3*b^3*d^4)*x^3 + 6*
(36*b^6*c^4 - 712*a*b^5*c^3*d + 903*a^2*b^4*c^2*d^2 - 264*a^3*b^3*c*d^3 + 3
7*a^4*b^2*d^4)*x^2 + 72*(6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4 + (6*b
^6*c^2*d^2 - 4*a*b^5*c*d^3 + a^2*b^4*d^4)*x^4 + 4*(6*a*b^5*c^2*d^2 - 4*a^2*
b^4*c*d^3 + a^3*b^3*d^4)*x^3 + 6*(6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4
*b^2*d^4)*x^2 + 4*(6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c*d^3 + a^5*b*d^4)*x)*log(
b*x + a)^2 + 72*(6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4 + (6*b^6*c^2*d
^2 - 4*a*b^5*c*d^3 + a^2*b^4*d^4)*x^4 + 4*(6*a*b^5*c^2*d^2 - 4*a^2*b^4*c*d^
3 + a^3*b^3*d^4)*x^3 + 6*(6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4
)*x^2 + 4*(6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c*d^3 + a^5*b*d^4)*x)*log(d*x + c)
^2 + 4*(76*a*b^5*c^4 - 1057*a^2*b^4*c^3*d + 1248*a^3*b^3*c^2*d^2 - 304*a^4*
b^2*c*d^3 + 37*a^5*b*d^4)*x - 12*(108*a^4*b^2*c^2*d^2 - 40*a^5*b*c*d^3 + 7*
a^6*d^4 + (108*b^6*c^2*d^2 - 40*a*b^5*c*d^3 + 7*a^2*b^4*d^4)*x^4 + 4*(108*a
*b^5*c^2*d^2 - 40*a^2*b^4*c*d^3 + 7*a^3*b^3*d^4)*x^3 + 6*(108*a^2*b^4*c^2*d
^2 - 40*a^3*b^3*c*d^3 + 7*a^4*b^2*d^4)*x^2 + 4*(108*a^3*b^3*c^2*d^2 - 40*a^
4*b^2*c*d^3 + 7*a^5*b*d^4)*x)*log(b*x + a) + 12*(108*a^4*b^2*c^2*d^2 - 40*a
^5*b*c*d^3 + 7*a^6*d^4 + (108*b^6*c^2*d^2 - 40*a*b^5*c*d^3 + 7*a^2*b^4*d^4)
x^4 + 4(108*a*b^5*c^2*d^2 - 40*a^2*b^4*c*d^3 + 7*a^3*b^3*d^4)*x^3 + 6*(10
8*a^2*b^4*c^2*d^2 - 40*a^3*b^3*c*d^3 + 7*a^4*b^2*d^4)*x^2 + 4*(108*a^3*b^3*
c^2*d^2 - 40*a^4*b^2*c*d^3 + 7*a^5*b*d^4)*x - 12*(6*a^4*b^2*c^2*d^2 - 4*a^5
*b*c*d^3 + a^6*d^4 + (6*b^6*c^2*d^2 - 4*a*b^5*c*d^3 + a^2*b^4*d^4)*x^4 + 4*
(6*a*b^5*c^2*d^2 - 4*a^2*b^4*c*d^3 + a^3*b^3*d^4)*x^3 + 6*(6*a^2*b^4*c^2*d^
2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*x^2 + 4*(6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c
*d^3 + a^5*b*d^4)*x)*log(b*x + a))*log(d*x + c))/(a^4*b^7*c^4*g^5 - 4*a^5*b
^6*c^3*d*g^5 + 6*a^6*b^5*c^2*d^2*g^5 - 4*a^7*b^4*c*d^3*g^5 + a^8*b^3*d^4*g^
5 + (b^11*c^4*g^5 - 4*a*b^10*c^3*d*g^5 + 6*a^2*b^9*c^2*d^2*g^5 - 4*a^3*b^8*
c*d^3*g^5 + a^4*b^7*d^4*g^5)*x^4 + 4*(a*b^10*c^4*g^5 - 4*a^2*b^9*c^3*d*g^5
+ 6*a^3*b^8*c^2*d^2*g^5 - 4*a^4*b^7*c*d^3*g^5 + a^5*b^6*d^4*g^5)*x^3 + 6*(a

^2*b^9*c^4*g^5 - 4*a^3*b^8*c^3*d*g^5 + 6*a^4*b^7*c^2*d^2*g^5 - 4*a^5*b^6*c*
d^3*g^5 + a^6*b^5*d^4*g^5)*x^2 + 4*(a^3*b^8*c^4*g^5 - 4*a^4*b^7*c^3*d*g^5 +
6*a^5*b^6*c^2*d^2*g^5 - 4*a^6*b^5*c*d^3*g^5 + a^7*b^4*d^4*g^5)*x))*B^2*d^2
*i^2 - 1/72*A*B*d^2*i^2*(12*(6*b^2*x^2 + 4*a*b*x + a^2)*log(b*e*x/(d*x + c)
+ a*e/(d*x + c))/(b^7*g^5*x^4 + 4*a*b^6*g^5*x^3 + 6*a^2*b^5*g^5*x^2 + 4*a^
3*b^4*g^5*x + a^4*b^3*g^5) + (13*a^2*b^3*c^3 - 75*a^3*b^2*c^2*d + 33*a^4*b*
c*d^2 - 7*a^5*d^3 - 12*(6*b^5*c^2*d - 4*a*b^4*c*d^2 + a^2*b^3*d^3)*x^3 + 6*
(6*b^5*c^3 - 46*a*b^4*c^2*d + 29*a^2*b^3*c*d^2 - 7*a^3*b^2*d^3)*x^2 + 4*(10
*a*b^4*c^3 - 63*a^2*b^3*c^2*d + 33*a^3*b^2*c*d^2 - 7*a^4*b*d^3)*x)/((b^10*c
^3 - 3*a*b^9*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7*d^3)*g^5*x^4 + 4*(a*b^9*c^3
- 3*a^2*b^8*c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*d^3)*g^5*x^3 + 6*(a^2*b^8*c^3
- 3*a^3*b^7*c^2*d + 3*a^4*b^6*c*d^2 - a^5*b^5*d^3)*g^5*x^2 + 4*(a^3*b^7*c^
3 - 3*a^4*b^6*c^2*d + 3*a^5*b^5*c*d^2 - a^6*b^4*d^3)*g^5*x + (a^4*b^6*c^3 -
3*a^5*b^5*c^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d^3)*g^5) - 12*(6*b^2*c^2*d^2
- 4*a*b*c*d^3 + a^2*d^4)*log(b*x + a)/((b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5
*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4)*g^5) + 12*(6*b^2*c^2*d^2 - 4*a*b*
c*d^3 + a^2*d^4)*log(d*x + c)/((b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2
- 4*a^3*b^4*c*d^3 + a^4*b^3*d^4)*g^5)) - 1/36*A*B*c*d*i^2*(12*(4*b*x + a)*
log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^6*g^5*x^4 + 4*a*b^5*g^5*x^3 + 6*a^2
*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g^5) + (7*a*b^3*c^3 - 33*a^2*b^2*c
^2*d + 75*a^3*b*c*d^2 - 13*a^4*d^3 + 12*(4*b^4*c*d^2 - a*b^3*d^3)*x^3 - 6*(
4*b^4*c^2*d - 29*a*b^3*c*d^2 + 7*a^2*b^2*d^3)*x^2 + 4*(4*b^4*c^3 - 21*a*b^3
*c^2*d + 57*a^2*b^2*c*d^2 - 13*a^3*b*d^3)*x)/((b^9*c^3 - 3*a*b^8*c^2*d + 3*
a^2*b^7*c*d^2 - a^3*b^6*d^3)*g^5*x^4 + 4*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*a
^3*b^6*c*d^2 - a^4*b^5*d^3)*g^5*x^3 + 6*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*
a^4*b^5*c*d^2 - a^5*b^4*d^3)*g^5*x^2 + 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3
*a^5*b^4*c*d^2 - a^6*b^3*d^3)*g^5*x + (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^
6*b^3*c*d^2 - a^7*b^2*d^3)*g^5) + 12*(4*b*c*d^3 - a*d^4)*log(b*x + a)/((b^6
*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g
^5) - 12*(4*b*c*d^3 - a*d^4)*log(d*x + c)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2
*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5)) + 1/24*A*B*c^2*i^2*((12
*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6
*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^
3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 +
4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6
*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 +
4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (
a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) - 12*log(
b*e*x/(d*x + c) + a*e/(d*x + c))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3
*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4
*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d
^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c
*d^3 + a^4*b*d^4)*g^5)) - 1/4*B^2*c^2*i^2*log(b*e*x/(d*x + c) + a*e/(d*x +
c))^2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x
+ a^4*b*g^5) - 1/6*(4*b*x + a)*A^2*c*d*i^2/(b^6*g^5*x^4 + 4*a*b^5*g^5*x^3 +

```
6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g^5) - 1/12*(6*b^2*x^2 + 4*a
*b*x + a^2)*A^2*d^2*i^2/(b^7*g^5*x^4 + 4*a*b^6*g^5*x^3 + 6*a^2*b^5*g^5*x^2
+ 4*a^3*b^4*g^5*x + a^4*b^3*g^5) - 1/4*A^2*c^2*i^2/(b^5*g^5*x^4 + 4*a*b^4*g
^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

Giac [A] (verification not implemented)

none

Time = 0.58 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.60

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{1}{864}\left(\frac{72\left(3\,B^2be^5i^2-\frac{4\,(bex+ae)B^2de^4i^2}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^4bcg^5}{(dx+c)^4}-\frac{(bex+ae)^4adg^5}{(dx+c)^4}}+\frac{12\left(36\,ABbe^5i^2+9\,B^2be^5i^2-\frac{48\,(bex+ae)ABde^4i^2}{dx+c}\right.}{\frac{(bex+ae)^4bcg^5}{(dx+c)^4}-\frac{(bex+ae)}{(dx+}}\right.$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, alg
orithm="giac")
```

```
[Out] -1/864*(72*(3*B^2*b*e^5*i^2 - 4*(b*e*x + a*e)*B^2*d*e^4*i^2/(d*x + c))*log(
(b*e*x + a*e)/(d*x + c))^2/((b*e*x + a*e)^4*b*c*g^5/(d*x + c)^4 - (b*e*x +
a*e)^4*a*d*g^5/(d*x + c)^4) + 12*(36*A*B*b*e^5*i^2 + 9*B^2*b*e^5*i^2 - 48*(
b*e*x + a*e)*A*B*d*e^4*i^2/(d*x + c) - 16*(b*e*x + a*e)*B^2*d*e^4*i^2/(d*x
+ c))*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^4*b*c*g^5/(d*x + c)^4 - (
b*e*x + a*e)^4*a*d*g^5/(d*x + c)^4) + (216*A^2*b*e^5*i^2 + 108*A*B*b*e^5*i^
2 + 27*B^2*b*e^5*i^2 - 288*(b*e*x + a*e)*A^2*d*e^4*i^2/(d*x + c) - 192*(b*e
*x + a*e)*A*B*d*e^4*i^2/(d*x + c) - 64*(b*e*x + a*e)*B^2*d*e^4*i^2/(d*x + c
))/((b*e*x + a*e)^4*b*c*g^5/(d*x + c)^4 - (b*e*x + a*e)^4*a*d*g^5/(d*x + c)
^4))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d))
)
```

Mupad [B] (verification not implemented)

Time = 6.38 (sec) , antiderivative size = 1940, normalized size of antiderivative = 6.49

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

```
[In] int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^
5,x)
```

```
[Out] - log((e*(a + b*x))/(c + d*x))^2*((x*(b*((B^2*c*d*i^2)/(6*b^3*g^5) + (B^2*a
*d^2*i^2)/(12*b^4*g^5)) + (B^2*c*d*i^2)/(2*b^2*g^5) + (B^2*a*d^2*i^2)/(4*b^
```

3*g^5)) + a*((B^2*c*d*i^2)/(6*b^3*g^5) + (B^2*a*d^2*i^2)/(12*b^4*g^5)) + (B ^2*c^2*i^2)/(4*b^2*g^5) + (B^2*d^2*i^2*x^2)/(2*b^2*g^5))/(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3) - (B^2*d^4*i^2)/(12*b^3*g^5*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((72*A^2*a^3*d^3*i^2 - 216*A^2*b^3*c^3*i^2 + 37* B^2*a^3*d^3*i^2 - 27*B^2*b^3*c^3*i^2 + 84*A*B*a^3*d^3*i^2 - 108*A*B*b^3*c^3 *i^2 + 72*A^2*a*b^2*c^2*d*i^2 + 72*A^2*a^2*b*c*d^2*i^2 + 37*B^2*a*b^2*c^2*d *i^2 + 37*B^2*a^2*b*c*d^2*i^2 + 84*A*B*a*b^2*c^2*d*i^2 + 84*A*B*a^2*b*c*d^2 *i^2)/(12*(a*d - b*c)) + (x^3*(7*B^2*b^3*d^3*i^2 + 12*A*B*b^3*d^3*i^2))/(a* d - b*c) + (x*(72*A^2*a^2*b*d^3*i^2 + 37*B^2*a^2*b*d^3*i^2 - 144*A^2*b^3*c^ 2*d*i^2 - 11*B^2*b^3*c^2*d*i^2 + 72*A^2*a*b^2*c*d^2*i^2 + 37*B^2*a*b^2*c*d^ 2*i^2 + 84*A*B*a^2*b*d^3*i^2 - 60*A*B*b^3*c^2*d*i^2 + 84*A*B*a*b^2*c*d^2*i^ 2))/(3*(a*d - b*c)) + (x^2*(72*A^2*a*b^2*d^3*i^2 + 37*B^2*a*b^2*d^3*i^2 - 7 2*A^2*b^3*c*d^2*i^2 + 5*B^2*b^3*c*d^2*i^2 + 84*A*B*a*b^2*d^3*i^2 - 12*A*B*b ^3*c*d^2*i^2))/(2*(a*d - b*c)))/(72*a^4*b^3*g^5 + 72*b^7*g^5*x^4 + 288*a^3* b^4*g^5*x + 288*a*b^6*g^5*x^3 + 432*a^2*b^5*g^5*x^2) - (log((e*(a + b*x))/(c + d*x))*(x^2*((A*B*d*i^2)/(b^2*g^5) + (B^2*d^4*i^2*(b*(b*((4*a^2*d^2 + b^ 2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (4*a^2*d^2 + b ^2*c^2 - 5*a*b*c*d)/(6*d^3) + (a*(a*d - b*c))/(2*d^2)) - a*((b^2*c - a*b*d) /(4*d^2) - (b*(a*d - b*c))/(2*d^2)) + (b^3*c^2 + 4*a^2*b*d^2 - 5*a*b^2*c*d) /(4*d^3)))/(6*b^3*g^5*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) + a*((B*i^2*(4*A*b* c - B*a*d + B*b*c))/(12*b^4*g^5) + (A*B*a*d*i^2)/(6*b^4*g^5)) + x*(b*((B*i^ 2*(4*A*b*c - B*a*d + B*b*c))/(12*b^4*g^5) + (A*B*a*d*i^2)/(6*b^4*g^5)) + (B *i^2*(4*A*b*c - B*a*d + B*b*c))/(4*b^3*g^5) + (B^2*d^4*i^2*(b*(a*((4*a^2*d^ 2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (6*a^3*d ^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(12*b*d^4)) + a*(b*((4*a^2*d ^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (4*a^2* d^2 + b^2*c^2 - 5*a*b*c*d)/(6*d^3) + (a*(a*d - b*c))/(2*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(4*d^4)))/(6*b^3*g^5*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (A*B*a*d*i^2)/(2*b^3*g^5)) + (B*i^2*(6*A*b^2*c^2 - 2*B*a^2*d^2 + B*b^2*c^2 + B*a*b*c*d))/(12*b^4*d*g^5) + (B^2*d^4*i^2*(a*(a* ((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(12*b*d^4)) + (4*a ^4*d^4 + b^4*c^4 + 10*a^2*b^2*c^2*d^2 - 5*a*b^3*c^3*d - 10*a^3*b*c*d^3)/(4* b*d^5)))/(6*b^3*g^5*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (B^2*d^4*i^2*x^3*(b* ((b^2*c - a*b*d)/(4*d^2) - (b*(a*d - b*c))/(2*d^2)) + (b^3*c - a*b^2*d)/(4* d^2)))/(6*b^3*g^5*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))))/((4*a^3*x)/d + a^4/(b* d) + (b^3*x^4)/d + (6*a^2*b*x^2)/d + (4*a*b^2*x^3)/d) - (B*d^4*i^2*atan(((2 *b*d*x - (72*b^5*c^2*g^5 - 72*a^2*b^3*d^2*g^5)/(72*b^3*g^5*(a*d - b*c)))*1i)/(a*d - b*c))*(12*A + 7*B)*1i)/(36*b^3*g^5*(a*d - b*c)^2)

3.73 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 463

$$\begin{aligned}\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = &-\frac{2B^2d^2i^2(c+dx)^3}{27(bc-ad)^3g^6(a+bx)^3} \\ &+\frac{bB^2di^2(c+dx)^4}{16(bc-ad)^3g^6(a+bx)^4} \\ &-\frac{2b^2B^2i^2(c+dx)^5}{125(bc-ad)^3g^6(a+bx)^5} \\ &-\frac{2Bd^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^3g^6(a+bx)^3} \\ &+\frac{bBdi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^3g^6(a+bx)^4} \\ &-\frac{2b^2Bi^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{25(bc-ad)^3g^6(a+bx)^5} \\ &-\frac{d^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^3g^6(a+bx)^3} \\ &+\frac{bdi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^3g^6(a+bx)^4} \\ &-\frac{b^2i^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5(bc-ad)^3g^6(a+bx)^5}\end{aligned}$$

```
[Out] -2/27*B^2*d^2*i^2*(d*x+c)^3/(-a*d+b*c)^3/g^6/(b*x+a)^3+1/16*b*B^2*d*i^2*(d*
x+c)^4/(-a*d+b*c)^3/g^6/(b*x+a)^4-2/125*b^2*B^2*i^2*(d*x+c)^5/(-a*d+b*c)^3/
g^6/(b*x+a)^5-2/9*B*d^2*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c
)^3/g^6/(b*x+a)^3+1/4*b*B*d*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d
+b*c)^3/g^6/(b*x+a)^4-2/25*b^2*B*i^2*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/
(-a*d+b*c)^3/g^6/(b*x+a)^5-1/3*d^2*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c))
)^2/(-a*d+b*c)^3/g^6/(b*x+a)^3+1/2*b*d*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x
+c)))^2/(-a*d+b*c)^3/g^6/(b*x+a)^4-1/5*b^2*i^2*(d*x+c)^5*(A+B*ln(e*(b*x+a)/
(d*x+c)))^2/(-a*d+b*c)^3/g^6/(b*x+a)^5
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2562, 2395, 2342, 2341}

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}dx = -\frac{b^2i^2(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5g^6(a+bx)^5(bc-ad)^3} - \frac{2b^2Bi^2(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{25g^6(a+bx)^5(bc-ad)^3} - \frac{d^2i^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g^6(a+bx)^3(bc-ad)^3} - \frac{2Bd^2i^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{9g^6(a+bx)^3(bc-ad)^3} + \frac{bdi^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g^6(a+bx)^4(bc-ad)^3} + \frac{bBdi^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4g^6(a+bx)^4(bc-ad)^3} - \frac{2b^2B^2i^2(c+dx)^5}{125g^6(a+bx)^5(bc-ad)^3} - \frac{2B^2d^2i^2(c+dx)^3}{27g^6(a+bx)^3(bc-ad)^3} + \frac{bB^2di^2(c+dx)^4}{16g^6(a+bx)^4(bc-ad)^3}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)^
6,x]
```

```
[Out] (-2*B^2*d^2*i^2*(c + d*x)^3)/(27*(b*c - a*d)^3*g^6*(a + b*x)^3) + (b*B^2*d*
i^2*(c + d*x)^4)/(16*(b*c - a*d)^3*g^6*(a + b*x)^4) - (2*b^2*B^2*i^2*(c + d
```

```
*x)^5)/(125*(b*c - a*d)^3*g^6*(a + b*x)^5) - (2*B*d^2*i^2*(c + d*x)^3*(A +
B*Log[(e*(a + b*x))/(c + d*x)]))/(9*(b*c - a*d)^3*g^6*(a + b*x)^3) + (b*B*d
*i^2*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*(b*c - a*d)^3*g^6
*(a + b*x)^4) - (2*b^2*B*i^2*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)
]))/(25*(b*c - a*d)^3*g^6*(a + b*x)^5) - (d^2*i^2*(c + d*x)^3*(A + B*Log[(e
*(a + b*x))/(c + d*x)])^2)/(3*(b*c - a*d)^3*g^6*(a + b*x)^3) + (b*d*i^2*(c
+ d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(b*c - a*d)^3*g^6*(a +
b*x)^4) - (b^2*i^2*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(5*(
b*c - a*d)^3*g^6*(a + b*x)^5)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{i^2 \text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex))^2}{x^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3 g^6}$$

$$= \frac{i^2 \text{Subst}\left(\int \left(\frac{b^2(A+B\log(ex))^2}{x^6} - \frac{2bd(A+B\log(ex))^2}{x^5} + \frac{d^2(A+B\log(ex))^2}{x^4}\right) dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3 g^6}$$

$$= \frac{(b^2 i^2) \text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^6} dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3 g^6} - \frac{(2bdi^2) \text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^5} dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3 g^6} + \frac{(d^2 i^2) \text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^4} dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3 g^6}$$

$$= -\frac{d^2 i^2 (c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^3 g^6 (a+bx)^3} + \frac{bdi^2 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^3 g^6 (a+bx)^4} - \frac{b^2 i^2 (c+dx)^5 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5(bc-ad)^3 g^6 (a+bx)^5} + \frac{(2b^2 B i^2) \text{Subst}\left(\int \frac{A+B\log(ex)}{x^6} dx, x, \frac{a+bx}{c+dx}\right)}{5(bc-ad)^3 g^6} - \frac{(bBdi^2) \text{Subst}\left(\int \frac{A+B\log(ex)}{x^5} dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3 g^6} + \frac{(2Bd^2 i^2) \text{Subst}\left(\int \frac{A+B\log(ex)}{x^4} dx, x, \frac{a+bx}{c+dx}\right)}{3(bc-ad)^3 g^6}$$

$$= -\frac{2B^2 d^2 i^2 (c+dx)^3}{27(bc-ad)^3 g^6 (a+bx)^3} + \frac{bB^2 di^2 (c+dx)^4}{16(bc-ad)^3 g^6 (a+bx)^4} - \frac{2b^2 B^2 i^2 (c+dx)^5}{125(bc-ad)^3 g^6 (a+bx)^5} - \frac{2Bd^2 i^2 (c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^3 g^6 (a+bx)^3} + \frac{bBdi^2 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^3 g^6 (a+bx)^4} - \frac{2b^2 B i^2 (c+dx)^5 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{25(bc-ad)^3 g^6 (a+bx)^5} - \frac{d^2 i^2 (c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^3 g^6 (a+bx)^3} + \frac{bdi^2 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^3 g^6 (a+bx)^4} - \frac{b^2 i^2 (c+dx)^5 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5(bc-ad)^3 g^6 (a+bx)^5}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.00 (sec) , antiderivative size = 2010, normalized size of antiderivative = 4.34

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6} dx = \text{Result too large to show}$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b
*g*x)^6,x]
```

```
[Out] -1/54000*(i^2*(10800*(b*c - a*d)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 +
 27000*d*(b*c - a*d)^4*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - 1
8000*d^2*(-(b*c) + a*d)^3*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^
```

2 + 1000*B*d^2*(a + b*x)^2*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3 - 18*A*d
*(b*c - a*d)^2*(a + b*x) - 15*B*d*(b*c - a*d)^2*(a + b*x) + 36*A*d^2*(b*c -
a*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a*d)*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3
*Log[a + b*x] + 66*B*d^3*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*(a + b*x)^3*Lo
g[a + b*x]^2 + 12*B*(b*c - a*d)^3*Log[(e*(a + b*x))/(c + d*x)] - 18*B*d*(b*
c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^2*(b*c - a*d)*(a
+ b*x)^2*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*
Log[(e*(a + b*x))/(c + d*x)] - 36*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3
*(a + b*x)^3*Log[c + d*x] + 36*B*d^3*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c)
+ a*d)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[(e*(a + b*x))/(c + d*x)]*Lo
g[c + d*x] - 18*B*d^3*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*(a + b*x)^3*Log
[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2,
(d*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*x
))/(b*c - a*d)]) + 375*B*d*(a + b*x)*(36*A*(b*c - a*d)^4 + 9*B*(b*c - a*d)^
4 + 48*A*d*(-(b*c) + a*d)^3*(a + b*x) + 28*B*d*(-(b*c) + a*d)^3*(a + b*x) +
72*A*d^2*(b*c - a*d)^2*(a + b*x)^2 + 78*B*d^2*(b*c - a*d)^2*(a + b*x)^2 +
144*A*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 300*B*d^3*(-(b*c) + a*d)*(a + b*x)^3
- 144*A*d^4*(a + b*x)^4*Log[a + b*x] - 300*B*d^4*(a + b*x)^4*Log[a + b*x]
+ 72*B*d^4*(a + b*x)^4*Log[a + b*x]^2 + 36*B*(b*c - a*d)^4*Log[(e*(a + b*x)
)/(c + d*x)] + 48*B*d*(-(b*c) + a*d)^3*(a + b*x)*Log[(e*(a + b*x))/(c + d*x
)] + 72*B*d^2*(b*c - a*d)^2*(a + b*x)^2*Log[(e*(a + b*x))/(c + d*x)] + 144*
B*d^3*(-(b*c) + a*d)*(a + b*x)^3*Log[(e*(a + b*x))/(c + d*x)] - 144*B*d^4*(
a + b*x)^4*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] + 144*A*d^4*(a + b*x)^
4*Log[c + d*x] + 300*B*d^4*(a + b*x)^4*Log[c + d*x] - 144*B*d^4*(a + b*x)^4
Log[(d(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] + 144*B*d^4*(a + b*x)^4*Log
[(e*(a + b*x))/(c + d*x)]*Log[c + d*x] + 72*B*d^4*(a + b*x)^4*Log[c + d*x]^
2 - 144*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 144
*B*d^4*(a + b*x)^4*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 144*B*d^4*(a
+ b*x)^4*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 6*B*(720*A*(b*c - a*d)^5
+ 144*B*(b*c - a*d)^5 - 900*A*d*(b*c - a*d)^4*(a + b*x) - 405*B*d*(b*c - a*
d)^4*(a + b*x) + 1200*A*d^2*(b*c - a*d)^3*(a + b*x)^2 + 940*B*d^2*(b*c - a*
d)^3*(a + b*x)^2 - 1800*A*d^3*(b*c - a*d)^2*(a + b*x)^3 - 2310*B*d^3*(b*c -
a*d)^2*(a + b*x)^3 + 3600*A*d^4*(b*c - a*d)*(a + b*x)^4 + 8220*B*d^4*(b*c
- a*d)*(a + b*x)^4 + 3600*A*d^5*(a + b*x)^5*Log[a + b*x] + 8220*B*d^5*(a +
b*x)^5*Log[a + b*x] - 1800*B*d^5*(a + b*x)^5*Log[a + b*x]^2 + 720*B*(b*c -
a*d)^5*Log[(e*(a + b*x))/(c + d*x)] - 900*B*d*(b*c - a*d)^4*(a + b*x)*Log[(
e*(a + b*x))/(c + d*x)] + 1200*B*d^2*(b*c - a*d)^3*(a + b*x)^2*Log[(e*(a +
b*x))/(c + d*x)] - 1800*B*d^3*(b*c - a*d)^2*(a + b*x)^3*Log[(e*(a + b*x))/(
c + d*x)] + 3600*B*d^4*(b*c - a*d)*(a + b*x)^4*Log[(e*(a + b*x))/(c + d*x)]
+ 3600*B*d^5*(a + b*x)^5*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] - 3600*
A*d^5*(a + b*x)^5*Log[c + d*x] - 8220*B*d^5*(a + b*x)^5*Log[c + d*x] + 3600
*B*d^5*(a + b*x)^5*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 3600*B*
d^5*(a + b*x)^5*Log[(e*(a + b*x))/(c + d*x)]*Log[c + d*x] - 1800*B*d^5*(a +
b*x)^5*Log[c + d*x]^2 + 3600*B*d^5*(a + b*x)^5*Log[a + b*x]*Log[(b*(c + d*
x))/(b*c - a*d)] + 3600*B*d^5*(a + b*x)^5*PolyLog[2, (d*(a + b*x))/(-(b*c)

```
+ a*d)] + 3600*B*d^5*(a + b*x)^5*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(
b^3*(b*c - a*d)^3*g^6*(a + b*x)^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 967 vs. 2(445) = 890.

Time = 1.81 (sec) , antiderivative size = 968, normalized size of antiderivative = 2.09

method	result	size
parts	Expression too large to display	968
derivativedivides	Expression too large to display	1082
default	Expression too large to display	1082
norman	Expression too large to display	1977
parallelrisch	Expression too large to display	2225
risch	Expression too large to display	5135

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x,method=_RET
URNVERBOSE)
```

```
[Out] i^2*A^2/g^6*(-1/3*d^2/b^3/(b*x+a)^3+1/2*d*(a*d-b*c)/b^3/(b*x+a)^4-1/5*(a^2*
d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^5)-i^2*B^2/g^6/d^4*(a*d-b*c)^3*e^3*(d^6/
(a*d-b*c)^6*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d
*x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-2*d^5/(a*d-b*c)^6*b*e*(-1/4/(b*e/d+
(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/8/(b*e/d+(a*d-
b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/32/(b*e/d+(a*d-b*c)*e
/d/(d*x+c))^4)+d^4/(a*d-b*c)^6*e^2*b^2*(-1/5/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^
5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/25/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5*ln
(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/125/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5))-2*i^2
*B*A/g^6/d^4*(a*d-b*c)^3*e^3*(d^6/(a*d-b*c)^6*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d
*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^
3)-2*d^5/(a*d-b*c)^6*b*e*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*
d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4)+d^4/(a*d-b*c)^6*e
^2*b^2*(-1/5/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)-1/25/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1323 vs. 2(445) = 890.

Time = 0.34 (sec) , antiderivative size = 1323, normalized size of antiderivative = 2.86

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x, alg
orithm="fricas")
```

```
[Out] -1/54000*(60*((60*A*B + 47*B^2)*b^5*c*d^4 - (60*A*B + 47*B^2)*a*b^4*d^5)*i^
2*x^4 - 30*((60*A*B - 13*B^2)*b^5*c^2*d^3 - 50*(12*A*B + 7*B^2)*a*b^4*c*d^4
 + 3*(180*A*B + 121*B^2)*a^2*b^3*d^5)*i^2*x^3 + 10*(2*(900*A^2 + 60*A*B - 4
3*B^2)*b^5*c^3*d^2 - 75*(72*A^2 + 12*A*B - 5*B^2)*a*b^4*c^2*d^3 + 600*(9*A^
2 + 6*A*B + 2*B^2)*a^2*b^3*c*d^4 - (1800*A^2 + 2820*A*B + 1489*B^2)*a^3*b^2
*d^5)*i^2*x^2 + 5*(27*(200*A^2 + 60*A*B + 7*B^2)*b^5*c^4*d - 100*(144*A^2 +
 60*A*B + 11*B^2)*a*b^4*c^3*d^2 + 1200*(9*A^2 + 6*A*B + 2*B^2)*a^2*b^3*c^2*
d^3 - (1800*A^2 + 2820*A*B + 1489*B^2)*a^4*b*d^5)*i^2*x + (432*(25*A^2 + 10
*A*B + 2*B^2)*b^5*c^5 - 3375*(8*A^2 + 4*A*B + B^2)*a*b^4*c^4*d + 2000*(9*A^
2 + 6*A*B + 2*B^2)*a^2*b^3*c^3*d^2 - (1800*A^2 + 2820*A*B + 1489*B^2)*a^5*d
^5)*i^2 + 1800*(B^2*b^5*d^5*i^2*x^5 + 5*B^2*a*b^4*d^5*i^2*x^4 + 10*B^2*a^2*
b^3*d^5*i^2*x^3 + 10*(B^2*b^5*c^3*d^2 - 3*B^2*a*b^4*c^2*d^3 + 3*B^2*a^2*b^3
*c*d^4)*i^2*x^2 + 5*(3*B^2*b^5*c^4*d - 8*B^2*a*b^4*c^3*d^2 + 6*B^2*a^2*b^3*
c^2*d^3)*i^2*x + (6*B^2*b^5*c^5 - 15*B^2*a*b^4*c^4*d + 10*B^2*a^2*b^3*c^3*d
^2)*i^2)*log((b*e*x + a*e)/(d*x + c))^2 + 60*((60*A*B + 47*B^2)*b^5*d^5*i^2
*x^5 + 5*(12*B^2*b^5*c*d^4 + 5*(12*A*B + 7*B^2)*a*b^4*d^5)*i^2*x^4 - 10*(3*
B^2*b^5*c^2*d^3 - 30*B^2*a*b^4*c*d^4 - 20*(3*A*B + B^2)*a^2*b^3*d^5)*i^2*x^
3 + 10*(2*(30*A*B + B^2)*b^5*c^3*d^2 - 15*(12*A*B + B^2)*a*b^4*c^2*d^3 + 60
*(3*A*B + B^2)*a^2*b^3*c*d^4)*i^2*x^2 + 5*(9*(20*A*B + 3*B^2)*b^5*c^4*d - 2
0*(24*A*B + 5*B^2)*a*b^4*c^3*d^2 + 120*(3*A*B + B^2)*a^2*b^3*c^2*d^3)*i^2*x
 + (72*(5*A*B + B^2)*b^5*c^5 - 225*(4*A*B + B^2)*a*b^4*c^4*d + 200*(3*A*B +
 B^2)*a^2*b^3*c^3*d^2)*i^2)*log((b*e*x + a*e)/(d*x + c)))/((b^11*c^3 - 3*a*
b^10*c^2*d + 3*a^2*b^9*c*d^2 - a^3*b^8*d^3)*g^6*x^5 + 5*(a*b^10*c^3 - 3*a^2
*b^9*c^2*d + 3*a^3*b^8*c*d^2 - a^4*b^7*d^3)*g^6*x^4 + 10*(a^2*b^9*c^3 - 3*a
^3*b^8*c^2*d + 3*a^4*b^7*c*d^2 - a^5*b^6*d^3)*g^6*x^3 + 10*(a^3*b^8*c^3 - 3
*a^4*b^7*c^2*d + 3*a^5*b^6*c*d^2 - a^6*b^5*d^3)*g^6*x^2 + 5*(a^4*b^7*c^3 -
3*a^5*b^6*c^2*d + 3*a^6*b^5*c*d^2 - a^7*b^4*d^3)*g^6*x + (a^5*b^6*c^3 - 3*a
^6*b^5*c^2*d + 3*a^7*b^4*c*d^2 - a^8*b^3*d^3)*g^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**6,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10880 vs. $2(445) = 890$.

Time = 1.13 (sec) , antiderivative size = 10880, normalized size of antiderivative = 23.50

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x, alg
orithm="maxima")
[Out] -1/10*(5*b*x + a)*B^2*c*d*i^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^7*g
^6*x^5 + 5*a*b^6*g^6*x^4 + 10*a^2*b^5*g^6*x^3 + 10*a^3*b^4*g^6*x^2 + 5*a^4*
b^3*g^6*x + a^5*b^2*g^6) - 1/30*(10*b^2*x^2 + 5*a*b*x + a^2)*B^2*d^2*i^2*lo
g(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^8*g^6*x^5 + 5*a*b^7*g^6*x^4 + 10*a^
2*b^6*g^6*x^3 + 10*a^3*b^5*g^6*x^2 + 5*a^4*b^4*g^6*x + a^5*b^3*g^6) - 1/900
0*(60*((60*b^4*d^4*x^4 + 12*b^4*c^4 - 63*a*b^3*c^3*d + 137*a^2*b^2*c^2*d^2
- 163*a^3*b*c*d^3 + 137*a^4*d^4 - 30*(b^4*c*d^3 - 9*a*b^3*d^4)*x^3 + 10*(2*
b^4*c^2*d^2 - 13*a*b^3*c*d^3 + 47*a^2*b^2*d^4)*x^2 - 5*(3*b^4*c^3*d - 17*a*
b^3*c^2*d^2 + 43*a^2*b^2*c*d^3 - 77*a^3*b*d^4)*x)/((b^10*c^4 - 4*a*b^9*c^3*
d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*g^6*x^5 + 5*(a*b^9*c
^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 + a^5*b^5*d^4)*g
^6*x^4 + 10*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2 - 4*a^5*b^5*
c*d^3 + a^6*b^4*d^4)*g^6*x^3 + 10*(a^3*b^7*c^4 - 4*a^4*b^6*c^3*d + 6*a^5*b^
5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*g^6*x^2 + 5*(a^4*b^6*c^4 - 4*a^5
*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d^4)*g^6*x + (a^
5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4*a^8*b^2*c*d^3 + a^9*b*d
^4)*g^6) + 60*d^5*log(b*x + a)/((b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d
^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5)*g^6) - 60*d^5*log(d*
x + c)/((b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2 - 10*a^3*b^3*c^2*d^3
+ 5*a^4*b^2*c*d^4 - a^5*b*d^5)*g^6))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) +
 (144*b^5*c^5 - 1125*a*b^4*c^4*d + 4000*a^2*b^3*c^3*d^2 - 9000*a^3*b^2*c^2*
d^3 + 18000*a^4*b*c*d^4 - 12019*a^5*d^5 + 8220*(b^5*c*d^4 - a*b^4*d^5)*x^4
```

- 30*(77*b^5*c^2*d^3 - 1250*a*b^4*c*d^4 + 1173*a^2*b^3*d^5)*x^3 + 10*(94*b^
5*c^3*d^2 - 975*a*b^4*c^2*d^3 + 6600*a^2*b^3*c*d^4 - 5719*a^3*b^2*d^5)*x^2
- 1800*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b^2*d^5
*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*log(b*x + a)^2 - 1800*(b^5*d^5*x^5 + 5*a*b^
4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d
^5)*log(d*x + c)^2 - 5*(81*b^5*c^4*d - 700*a*b^4*c^3*d^2 + 3000*a^2*b^3*c^2
*d^3 - 10800*a^3*b^2*c*d^4 + 8419*a^4*b*d^5)*x + 8220*(b^5*d^5*x^5 + 5*a*b^
4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d
^5)*log(b*x + a) - 60*(137*b^5*d^5*x^5 + 685*a*b^4*d^5*x^4 + 1370*a^2*b^3*d
^5*x^3 + 1370*a^3*b^2*d^5*x^2 + 685*a^4*b*d^5*x + 137*a^5*d^5 - 60*(b^5*d^5
*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*
d^5*x + a^5*d^5)*log(b*x + a))*log(d*x + c))/(a^5*b^6*c^5*g^6 - 5*a^6*b^5*c
^4*d*g^6 + 10*a^7*b^4*c^3*d^2*g^6 - 10*a^8*b^3*c^2*d^3*g^6 + 5*a^9*b^2*c*d^
4*g^6 - a^10*b*d^5*g^6 + (b^11*c^5*g^6 - 5*a*b^10*c^4*d*g^6 + 10*a^2*b^9*c^
3*d^2*g^6 - 10*a^3*b^8*c^2*d^3*g^6 + 5*a^4*b^7*c*d^4*g^6 - a^5*b^6*d^5*g^6)
x^5 + 5(a*b^10*c^5*g^6 - 5*a^2*b^9*c^4*d*g^6 + 10*a^3*b^8*c^3*d^2*g^6 - 1
0*a^4*b^7*c^2*d^3*g^6 + 5*a^5*b^6*c*d^4*g^6 - a^6*b^5*d^5*g^6)*x^4 + 10*(a^
2*b^9*c^5*g^6 - 5*a^3*b^8*c^4*d*g^6 + 10*a^4*b^7*c^3*d^2*g^6 - 10*a^5*b^6*c
^2*d^3*g^6 + 5*a^6*b^5*c*d^4*g^6 - a^7*b^4*d^5*g^6)*x^3 + 10*(a^3*b^8*c^5*g
^6 - 5*a^4*b^7*c^4*d*g^6 + 10*a^5*b^6*c^3*d^2*g^6 - 10*a^6*b^5*c^2*d^3*g^6
+ 5*a^7*b^4*c*d^4*g^6 - a^8*b^3*d^5*g^6)*x^2 + 5*(a^4*b^7*c^5*g^6 - 5*a^5*b
^6*c^4*d*g^6 + 10*a^6*b^5*c^3*d^2*g^6 - 10*a^7*b^4*c^2*d^3*g^6 + 5*a^8*b^3*
c*d^4*g^6 - a^9*b^2*d^5*g^6)*x))*B^2*c^2*i^2 - 1/18000*(60*((27*a*b^4*c^4 -
148*a^2*b^3*c^3*d + 352*a^3*b^2*c^2*d^2 - 548*a^4*b*c*d^3 + 77*a^5*d^4 - 6
0*(5*b^5*c*d^3 - a*b^4*d^4)*x^4 + 30*(5*b^5*c^2*d^2 - 46*a*b^4*c*d^3 + 9*a^
2*b^3*d^4)*x^3 - 10*(10*b^5*c^3*d - 67*a*b^4*c^2*d^2 + 248*a^2*b^3*c*d^3 -
47*a^3*b^2*d^4)*x^2 + 5*(15*b^5*c^4 - 88*a*b^4*c^3*d + 232*a^2*b^3*c^2*d^2
- 428*a^3*b^2*c*d^3 + 77*a^4*b*d^4)*x)/((b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*
b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4)*g^6*x^5 + 5*(a*b^10*c^4 - 4*a^
2*b^9*c^3*d + 6*a^3*b^8*c^2*d^2 - 4*a^4*b^7*c*d^3 + a^5*b^6*d^4)*g^6*x^4 +
10*(a^2*b^9*c^4 - 4*a^3*b^8*c^3*d + 6*a^4*b^7*c^2*d^2 - 4*a^5*b^6*c*d^3 + a
^6*b^5*d^4)*g^6*x^3 + 10*(a^3*b^8*c^4 - 4*a^4*b^7*c^3*d + 6*a^5*b^6*c^2*d^2
- 4*a^6*b^5*c*d^3 + a^7*b^4*d^4)*g^6*x^2 + 5*(a^4*b^7*c^4 - 4*a^5*b^6*c^3*
d + 6*a^6*b^5*c^2*d^2 - 4*a^7*b^4*c*d^3 + a^8*b^3*d^4)*g^6*x + (a^5*b^6*c^4
- 4*a^6*b^5*c^3*d + 6*a^7*b^4*c^2*d^2 - 4*a^8*b^3*c*d^3 + a^9*b^2*d^4)*g^6
) - 60*(5*b*c*d^4 - a*d^5)*log(b*x + a)/((b^7*c^5 - 5*a*b^6*c^4*d + 10*a^2*
b^5*c^3*d^2 - 10*a^3*b^4*c^2*d^3 + 5*a^4*b^3*c*d^4 - a^5*b^2*d^5)*g^6) + 60
*(5*b*c*d^4 - a*d^5)*log(d*x + c)/((b^7*c^5 - 5*a*b^6*c^4*d + 10*a^2*b^5*c^
3*d^2 - 10*a^3*b^4*c^2*d^3 + 5*a^4*b^3*c*d^4 - a^5*b^2*d^5)*g^6))*log(b*e*x
/(d*x + c) + a*e/(d*x + c)) + (549*a*b^5*c^5 - 4625*a^2*b^4*c^4*d + 19000*a
^3*b^3*c^3*d^2 - 63000*a^4*b^2*c^2*d^3 + 51875*a^5*b*c*d^4 - 3799*a^6*d^5 -
60*(625*b^6*c^2*d^3 - 702*a*b^5*c*d^4 + 77*a^2*b^4*d^5)*x^4 + 30*(325*b^6*
c^3*d^2 - 5667*a*b^5*c^2*d^3 + 5975*a^2*b^4*c*d^4 - 633*a^3*b^3*d^5)*x^3 -
10*(350*b^6*c^4*d - 3949*a*b^5*c^3*d^2 + 29475*a^2*b^4*c^2*d^3 - 28775*a^3*
b^3*c*d^4 + 2899*a^4*b^2*d^5)*x^2 + 1800*(5*a^5*b*c*d^4 - a^6*d^5 + (5*b^6*

c*d^4 - a*b^5*d^5)*x^5 + 5*(5*a*b^5*c*d^4 - a^2*b^4*d^5)*x^4 + 10*(5*a^2*b^
4*c*d^4 - a^3*b^3*d^5)*x^3 + 10*(5*a^3*b^3*c*d^4 - a^4*b^2*d^5)*x^2 + 5*(5*
a^4*b^2*c*d^4 - a^5*b*d^5)*x)*log(b*x + a)^2 + 1800*(5*a^5*b*c*d^4 - a^6*d^
5 + (5*b^6*c*d^4 - a*b^5*d^5)*x^5 + 5*(5*a*b^5*c*d^4 - a^2*b^4*d^5)*x^4 + 1
0*(5*a^2*b^4*c*d^4 - a^3*b^3*d^5)*x^3 + 10*(5*a^3*b^3*c*d^4 - a^4*b^2*d^5)*
x^2 + 5*(5*a^4*b^2*c*d^4 - a^5*b*d^5)*x)*log(d*x + c)^2 + 5*(225*b^6*c^5 -
2201*a*b^5*c^4*d + 10900*a^2*b^4*c^3*d^2 - 46200*a^3*b^3*c^2*d^3 + 41075*a^
4*b^2*c*d^4 - 3799*a^5*b*d^5)*x - 60*(625*a^5*b*c*d^4 - 77*a^6*d^5 + (625*b
^6*c*d^4 - 77*a*b^5*d^5)*x^5 + 5*(625*a*b^5*c*d^4 - 77*a^2*b^4*d^5)*x^4 + 1
0*(625*a^2*b^4*c*d^4 - 77*a^3*b^3*d^5)*x^3 + 10*(625*a^3*b^3*c*d^4 - 77*a^4
*b^2*d^5)*x^2 + 5*(625*a^4*b^2*c*d^4 - 77*a^5*b*d^5)*x)*log(b*x + a) + 60*(
625*a^5*b*c*d^4 - 77*a^6*d^5 + (625*b^6*c*d^4 - 77*a*b^5*d^5)*x^5 + 5*(625*
a*b^5*c*d^4 - 77*a^2*b^4*d^5)*x^4 + 10*(625*a^2*b^4*c*d^4 - 77*a^3*b^3*d^5)
x^3 + 10(625*a^3*b^3*c*d^4 - 77*a^4*b^2*d^5)*x^2 + 5*(625*a^4*b^2*c*d^4 -
77*a^5*b*d^5)*x - 60*(5*a^5*b*c*d^4 - a^6*d^5 + (5*b^6*c*d^4 - a*b^5*d^5)*
x^5 + 5*(5*a*b^5*c*d^4 - a^2*b^4*d^5)*x^4 + 10*(5*a^2*b^4*c*d^4 - a^3*b^3*d
^5)*x^3 + 10*(5*a^3*b^3*c*d^4 - a^4*b^2*d^5)*x^2 + 5*(5*a^4*b^2*c*d^4 - a^5
*b*d^5)*x)*log(b*x + a))*log(d*x + c))/(a^5*b^7*c^5*g^6 - 5*a^6*b^6*c^4*d*g
^6 + 10*a^7*b^5*c^3*d^2*g^6 - 10*a^8*b^4*c^2*d^3*g^6 + 5*a^9*b^3*c*d^4*g^6
- a^10*b^2*d^5*g^6 + (b^12*c^5*g^6 - 5*a*b^11*c^4*d*g^6 + 10*a^2*b^10*c^3*d
^2*g^6 - 10*a^3*b^9*c^2*d^3*g^6 + 5*a^4*b^8*c*d^4*g^6 - a^5*b^7*d^5*g^6)*x^
5 + 5*(a*b^11*c^5*g^6 - 5*a^2*b^10*c^4*d*g^6 + 10*a^3*b^9*c^3*d^2*g^6 - 10*
a^4*b^8*c^2*d^3*g^6 + 5*a^5*b^7*c*d^4*g^6 - a^6*b^6*d^5*g^6)*x^4 + 10*(a^2*
b^10*c^5*g^6 - 5*a^3*b^9*c^4*d*g^6 + 10*a^4*b^8*c^3*d^2*g^6 - 10*a^5*b^7*c^
2*d^3*g^6 + 5*a^6*b^6*c*d^4*g^6 - a^7*b^5*d^5*g^6)*x^3 + 10*(a^3*b^9*c^5*g^
6 - 5*a^4*b^8*c^4*d*g^6 + 10*a^5*b^7*c^3*d^2*g^6 - 10*a^6*b^6*c^2*d^3*g^6 +
5*a^7*b^5*c*d^4*g^6 - a^8*b^4*d^5*g^6)*x^2 + 5*(a^4*b^8*c^5*g^6 - 5*a^5*b^
7*c^4*d*g^6 + 10*a^6*b^6*c^3*d^2*g^6 - 10*a^7*b^5*c^2*d^3*g^6 + 5*a^8*b^4*c
*d^4*g^6 - a^9*b^3*d^5*g^6)*x))*B^2*c*d*i^2 - 1/54000*(60*((47*a^2*b^4*c^4
- 278*a^3*b^3*c^3*d + 822*a^4*b^2*c^2*d^2 - 278*a^5*b*c*d^3 + 47*a^6*d^4 +
60*(10*b^6*c^2*d^2 - 5*a*b^5*c*d^3 + a^2*b^4*d^4)*x^4 - 30*(10*b^6*c^3*d -
95*a*b^5*c^2*d^2 + 46*a^2*b^4*c*d^3 - 9*a^3*b^3*d^4)*x^3 + 10*(20*b^6*c^4 -
140*a*b^5*c^3*d + 537*a^2*b^4*c^2*d^2 - 248*a^3*b^3*c*d^3 + 47*a^4*b^2*d^4
)*x^2 + 5*(35*a*b^5*c^4 - 218*a^2*b^4*c^3*d + 702*a^3*b^3*c^2*d^2 - 278*a^4
*b^2*c*d^3 + 47*a^5*b*d^4)*x)/((b^12*c^4 - 4*a*b^11*c^3*d + 6*a^2*b^10*c^2*
d^2 - 4*a^3*b^9*c*d^3 + a^4*b^8*d^4)*g^6*x^5 + 5*(a*b^11*c^4 - 4*a^2*b^10*c
^3*d + 6*a^3*b^9*c^2*d^2 - 4*a^4*b^8*c*d^3 + a^5*b^7*d^4)*g^6*x^4 + 10*(a^2
*b^10*c^4 - 4*a^3*b^9*c^3*d + 6*a^4*b^8*c^2*d^2 - 4*a^5*b^7*c*d^3 + a^6*b^6
*d^4)*g^6*x^3 + 10*(a^3*b^9*c^4 - 4*a^4*b^8*c^3*d + 6*a^5*b^7*c^2*d^2 - 4*a
^6*b^6*c*d^3 + a^7*b^5*d^4)*g^6*x^2 + 5*(a^4*b^8*c^4 - 4*a^5*b^7*c^3*d + 6*
a^6*b^6*c^2*d^2 - 4*a^7*b^5*c*d^3 + a^8*b^4*d^4)*g^6*x + (a^5*b^7*c^4 - 4*a
^6*b^6*c^3*d + 6*a^7*b^5*c^2*d^2 - 4*a^8*b^4*c*d^3 + a^9*b^3*d^4)*g^6) + 60
*(10*b^2*c^2*d^3 - 5*a*b*c*d^4 + a^2*d^5)*log(b*x + a)/((b^8*c^5 - 5*a*b^7*
c^4*d + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 + 5*a^4*b^4*c*d^4 - a^5*b^3
*d^5)*g^6) - 60*(10*b^2*c^2*d^3 - 5*a*b*c*d^4 + a^2*d^5)*log(d*x + c)/((b^8

*c^5 - 5*a*b^7*c^4*d + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 + 5*a^4*b^4*
c*d^4 - a^5*b^3*d^5)*g^6))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + (1489*a^2
*b^5*c^5 - 14375*a^3*b^4*c^4*d + 85000*a^4*b^3*c^3*d^2 - 85000*a^5*b^2*c^2*
d^3 + 14375*a^6*b*c*d^4 - 1489*a^7*d^5 + 60*(1100*b^7*c^3*d^2 - 1425*a*b^6*
c^2*d^3 + 372*a^2*b^5*c*d^4 - 47*a^3*b^4*d^5)*x^4 - 30*(500*b^7*c^4*d - 982
5*a*b^6*c^3*d^2 + 11937*a^2*b^5*c^2*d^3 - 2975*a^3*b^4*c*d^4 + 363*a^4*b^3*
d^5)*x^3 + 10*(400*b^7*c^5 - 5450*a*b^6*c^4*d + 49189*a^2*b^5*c^3*d^2 - 555
25*a^3*b^4*c^2*d^3 + 12875*a^4*b^3*c*d^4 - 1489*a^5*b^2*d^5)*x^2 - 1800*(10
*a^5*b^2*c^2*d^3 - 5*a^6*b*c*d^4 + a^7*d^5 + (10*b^7*c^2*d^3 - 5*a*b^6*c*d^
4 + a^2*b^5*d^5)*x^5 + 5*(10*a*b^6*c^2*d^3 - 5*a^2*b^5*c*d^4 + a^3*b^4*d^5)
x^4 + 10(10*a^2*b^5*c^2*d^3 - 5*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^3 + 10*(10
*a^3*b^4*c^2*d^3 - 5*a^4*b^3*c*d^4 + a^5*b^2*d^5)*x^2 + 5*(10*a^4*b^3*c^2*d
^3 - 5*a^5*b^2*c*d^4 + a^6*b*d^5)*x)*log(b*x + a)^2 - 1800*(10*a^5*b^2*c^2*
d^3 - 5*a^6*b*c*d^4 + a^7*d^5 + (10*b^7*c^2*d^3 - 5*a*b^6*c*d^4 + a^2*b^5*d
^5)*x^5 + 5*(10*a*b^6*c^2*d^3 - 5*a^2*b^5*c*d^4 + a^3*b^4*d^5)*x^4 + 10*(10
*a^2*b^5*c^2*d^3 - 5*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^3 + 10*(10*a^3*b^4*c^2*
d^3 - 5*a^4*b^3*c*d^4 + a^5*b^2*d^5)*x^2 + 5*(10*a^4*b^3*c^2*d^3 - 5*a^5*b^
2*c*d^4 + a^6*b*d^5)*x)*log(d*x + c)^2 + 5*(925*a*b^6*c^5 - 9911*a^2*b^5*c^
4*d + 67900*a^3*b^4*c^3*d^2 - 71800*a^4*b^3*c^2*d^3 + 14375*a^5*b^2*c*d^4 -
 1489*a^6*b*d^5)*x + 60*(1100*a^5*b^2*c^2*d^3 - 325*a^6*b*c*d^4 + 47*a^7*d^
5 + (1100*b^7*c^2*d^3 - 325*a*b^6*c*d^4 + 47*a^2*b^5*d^5)*x^5 + 5*(1100*a*b
^6*c^2*d^3 - 325*a^2*b^5*c*d^4 + 47*a^3*b^4*d^5)*x^4 + 10*(1100*a^2*b^5*c^2
*d^3 - 325*a^3*b^4*c*d^4 + 47*a^4*b^3*d^5)*x^3 + 10*(1100*a^3*b^4*c^2*d^3 -
 325*a^4*b^3*c*d^4 + 47*a^5*b^2*d^5)*x^2 + 5*(1100*a^4*b^3*c^2*d^3 - 325*a^
5*b^2*c*d^4 + 47*a^6*b*d^5)*x)*log(b*x + a) - 60*(1100*a^5*b^2*c^2*d^3 - 32
5*a^6*b*c*d^4 + 47*a^7*d^5 + (1100*b^7*c^2*d^3 - 325*a*b^6*c*d^4 + 47*a^2*b
^5*d^5)*x^5 + 5*(1100*a*b^6*c^2*d^3 - 325*a^2*b^5*c*d^4 + 47*a^3*b^4*d^5)*x
^4 + 10*(1100*a^2*b^5*c^2*d^3 - 325*a^3*b^4*c*d^4 + 47*a^4*b^3*d^5)*x^3 + 1
0*(1100*a^3*b^4*c^2*d^3 - 325*a^4*b^3*c*d^4 + 47*a^5*b^2*d^5)*x^2 + 5*(1100
*a^4*b^3*c^2*d^3 - 325*a^5*b^2*c*d^4 + 47*a^6*b*d^5)*x - 60*(10*a^5*b^2*c^2
*d^3 - 5*a^6*b*c*d^4 + a^7*d^5 + (10*b^7*c^2*d^3 - 5*a*b^6*c*d^4 + a^2*b^5*
d^5)*x^5 + 5*(10*a*b^6*c^2*d^3 - 5*a^2*b^5*c*d^4 + a^3*b^4*d^5)*x^4 + 10*(1
0*a^2*b^5*c^2*d^3 - 5*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^3 + 10*(10*a^3*b^4*c^2
*d^3 - 5*a^4*b^3*c*d^4 + a^5*b^2*d^5)*x^2 + 5*(10*a^4*b^3*c^2*d^3 - 5*a^5*b
^2*c*d^4 + a^6*b*d^5)*x)*log(b*x + a))*log(d*x + c))/(a^5*b^8*c^5*g^6 - 5*a
^6*b^7*c^4*d*g^6 + 10*a^7*b^6*c^3*d^2*g^6 - 10*a^8*b^5*c^2*d^3*g^6 + 5*a^9*
b^4*c*d^4*g^6 - a^10*b^3*d^5*g^6 + (b^13*c^5*g^6 - 5*a*b^12*c^4*d*g^6 + 10*
a^2*b^11*c^3*d^2*g^6 - 10*a^3*b^10*c^2*d^3*g^6 + 5*a^4*b^9*c*d^4*g^6 - a^5*
b^8*d^5*g^6)*x^5 + 5*(a*b^12*c^5*g^6 - 5*a^2*b^11*c^4*d*g^6 + 10*a^3*b^10*c
^3*d^2*g^6 - 10*a^4*b^9*c^2*d^3*g^6 + 5*a^5*b^8*c*d^4*g^6 - a^6*b^7*d^5*g^6
)*x^4 + 10*(a^2*b^11*c^5*g^6 - 5*a^3*b^10*c^4*d*g^6 + 10*a^4*b^9*c^3*d^2*g^
6 - 10*a^5*b^8*c^2*d^3*g^6 + 5*a^6*b^7*c*d^4*g^6 - a^7*b^6*d^5*g^6)*x^3 + 1
0*(a^3*b^10*c^5*g^6 - 5*a^4*b^9*c^4*d*g^6 + 10*a^5*b^8*c^3*d^2*g^6 - 10*a^6
*b^7*c^2*d^3*g^6 + 5*a^7*b^6*c*d^4*g^6 - a^8*b^5*d^5*g^6)*x^2 + 5*(a^4*b^9*
c^5*g^6 - 5*a^5*b^8*c^4*d*g^6 + 10*a^6*b^7*c^3*d^2*g^6 - 10*a^7*b^6*c^2*d^3

*g^6 + 5*a^8*b^5*c*d^4*g^6 - a^9*b^4*d^5*g^6)*x))*B^2*d^2*i^2 - 1/900*A*B*d ^2*i^2*(60*(10*b^2*x^2 + 5*a*b*x + a^2)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^8*g^6*x^5 + 5*a*b^7*g^6*x^4 + 10*a^2*b^6*g^6*x^3 + 10*a^3*b^5*g^6*x^2 + 5*a^4*b^4*g^6*x + a^5*b^3*g^6) + (47*a^2*b^4*c^4 - 278*a^3*b^3*c^3*d + 82 2*a^4*b^2*c^2*d^2 - 278*a^5*b*c*d^3 + 47*a^6*d^4 + 60*(10*b^6*c^2*d^2 - 5*a *b^5*c*d^3 + a^2*b^4*d^4)*x^4 - 30*(10*b^6*c^3*d - 95*a*b^5*c^2*d^2 + 46*a^ 2*b^4*c*d^3 - 9*a^3*b^3*d^4)*x^3 + 10*(20*b^6*c^4 - 140*a*b^5*c^3*d + 537*a ^2*b^4*c^2*d^2 - 248*a^3*b^3*c*d^3 + 47*a^4*b^2*d^4)*x^2 + 5*(35*a*b^5*c^4 - 218*a^2*b^4*c^3*d + 702*a^3*b^3*c^2*d^2 - 278*a^4*b^2*c*d^3 + 47*a^5*b*d^ 4)*x)/((b^12*c^4 - 4*a*b^11*c^3*d + 6*a^2*b^10*c^2*d^2 - 4*a^3*b^9*c*d^3 + a^4*b^8*d^4)*g^6*x^5 + 5*(a*b^11*c^4 - 4*a^2*b^10*c^3*d + 6*a^3*b^9*c^2*d^2 - 4*a^4*b^8*c*d^3 + a^5*b^7*d^4)*g^6*x^4 + 10*(a^2*b^10*c^4 - 4*a^3*b^9*c^ 3*d + 6*a^4*b^8*c^2*d^2 - 4*a^5*b^7*c*d^3 + a^6*b^6*d^4)*g^6*x^3 + 10*(a^3* b^9*c^4 - 4*a^4*b^8*c^3*d + 6*a^5*b^7*c^2*d^2 - 4*a^6*b^6*c*d^3 + a^7*b^5*d ^4)*g^6*x^2 + 5*(a^4*b^8*c^4 - 4*a^5*b^7*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^7* b^5*c*d^3 + a^8*b^4*d^4)*g^6*x + (a^5*b^7*c^4 - 4*a^6*b^6*c^3*d + 6*a^7*b^5 *c^2*d^2 - 4*a^8*b^4*c*d^3 + a^9*b^3*d^4)*g^6) + 60*(10*b^2*c^2*d^3 - 5*a*b *c*d^4 + a^2*d^5)*log(b*x + a)/((b^8*c^5 - 5*a*b^7*c^4*d + 10*a^2*b^6*c^3*d ^2 - 10*a^3*b^5*c^2*d^3 + 5*a^4*b^4*c*d^4 - a^5*b^3*d^5)*g^6) - 60*(10*b^2* c^2*d^3 - 5*a*b*c*d^4 + a^2*d^5)*log(d*x + c)/((b^8*c^5 - 5*a*b^7*c^4*d + 1 0*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 + 5*a^4*b^4*c*d^4 - a^5*b^3*d^5)*g^6)) - 1/300*A*B*c*d*i^2*(60*(5*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c)) /(b^7*g^6*x^5 + 5*a*b^6*g^6*x^4 + 10*a^2*b^5*g^6*x^3 + 10*a^3*b^4*g^6*x^2 + 5*a^4*b^3*g^6*x + a^5*b^2*g^6) + (27*a*b^4*c^4 - 148*a^2*b^3*c^3*d + 352*a ^3*b^2*c^2*d^2 - 548*a^4*b*c*d^3 + 77*a^5*d^4 - 60*(5*b^5*c*d^3 - a*b^4*d^4)*x^4 + 30*(5*b^5*c^2*d^2 - 46*a*b^4*c*d^3 + 9*a^2*b^3*d^4)*x^3 - 10*(10*b^ 5*c^3*d - 67*a*b^4*c^2*d^2 + 248*a^2*b^3*c*d^3 - 47*a^3*b^2*d^4)*x^2 + 5*(1 5*b^5*c^4 - 88*a*b^4*c^3*d + 232*a^2*b^3*c^2*d^2 - 428*a^3*b^2*c*d^3 + 77*a ^4*b*d^4)*x)/((b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c* d^3 + a^4*b^7*d^4)*g^6*x^5 + 5*(a*b^10*c^4 - 4*a^2*b^9*c^3*d + 6*a^3*b^8*c^ 2*d^2 - 4*a^4*b^7*c*d^3 + a^5*b^6*d^4)*g^6*x^4 + 10*(a^2*b^9*c^4 - 4*a^3*b^ 8*c^3*d + 6*a^4*b^7*c^2*d^2 - 4*a^5*b^6*c*d^3 + a^6*b^5*d^4)*g^6*x^3 + 10*(a^3*b^8*c^4 - 4*a^4*b^7*c^3*d + 6*a^5*b^6*c^2*d^2 - 4*a^6*b^5*c*d^3 + a^7*b ^4*d^4)*g^6*x^2 + 5*(a^4*b^7*c^4 - 4*a^5*b^6*c^3*d + 6*a^6*b^5*c^2*d^2 - 4* a^7*b^4*c*d^3 + a^8*b^3*d^4)*g^6*x + (a^5*b^6*c^4 - 4*a^6*b^5*c^3*d + 6*a^7 *b^4*c^2*d^2 - 4*a^8*b^3*c*d^3 + a^9*b^2*d^4)*g^6) - 60*(5*b*c*d^4 - a*d^5) *log(b*x + a)/((b^7*c^5 - 5*a*b^6*c^4*d + 10*a^2*b^5*c^3*d^2 - 10*a^3*b^4*c ^2*d^3 + 5*a^4*b^3*c*d^4 - a^5*b^2*d^5)*g^6) + 60*(5*b*c*d^4 - a*d^5)*log(d *x + c)/((b^7*c^5 - 5*a*b^6*c^4*d + 10*a^2*b^5*c^3*d^2 - 10*a^3*b^4*c^2*d^3 + 5*a^4*b^3*c*d^4 - a^5*b^2*d^5)*g^6)) - 1/150*A*B*c^2*i^2*((60*b^4*d^4*x^ 4 + 12*b^4*c^4 - 63*a*b^3*c^3*d + 137*a^2*b^2*c^2*d^2 - 163*a^3*b*c*d^3 + 1 37*a^4*d^4 - 30*(b^4*c*d^3 - 9*a*b^3*d^4)*x^3 + 10*(2*b^4*c^2*d^2 - 13*a*b^ 3*c*d^3 + 47*a^2*b^2*d^4)*x^2 - 5*(3*b^4*c^3*d - 17*a*b^3*c^2*d^2 + 43*a^2* b^2*c*d^3 - 77*a^3*b*d^4)*x)/((b^10*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*g^6*x^5 + 5*(a*b^9*c^4 - 4*a^2*b^8*c^3*d

```
+ 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 + a^5*b^5*d^4)*g^6*x^4 + 10*(a^2*b^8*
c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2 - 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*
g^6*x^3 + 10*(a^3*b^7*c^4 - 4*a^4*b^6*c^3*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*b^4
*c*d^3 + a^7*b^3*d^4)*g^6*x^2 + 5*(a^4*b^6*c^4 - 4*a^5*b^5*c^3*d + 6*a^6*b^
4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d^4)*g^6*x + (a^5*b^5*c^4 - 4*a^6*b^4
*c^3*d + 6*a^7*b^3*c^2*d^2 - 4*a^8*b^2*c*d^3 + a^9*b*d^4)*g^6) + 60*log(b*e
*x/(d*x + c) + a*e/(d*x + c))/(b^6*g^6*x^5 + 5*a*b^5*g^6*x^4 + 10*a^2*b^4*g
^6*x^3 + 10*a^3*b^3*g^6*x^2 + 5*a^4*b^2*g^6*x + a^5*b*g^6) + 60*d^5*log(b*x
 + a)/((b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2 - 10*a^3*b^3*c^2*d^3 +
 5*a^4*b^2*c*d^4 - a^5*b*d^5)*g^6) - 60*d^5*log(d*x + c)/((b^6*c^5 - 5*a*b^
5*c^4*d + 10*a^2*b^4*c^3*d^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b
*d^5)*g^6)) - 1/5*B^2*c^2*i^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^6*g
^6*x^5 + 5*a*b^5*g^6*x^4 + 10*a^2*b^4*g^6*x^3 + 10*a^3*b^3*g^6*x^2 + 5*a^4*
b^2*g^6*x + a^5*b*g^6) - 1/10*(5*b*x + a)*A^2*c*d*i^2/(b^7*g^6*x^5 + 5*a*b^
6*g^6*x^4 + 10*a^2*b^5*g^6*x^3 + 10*a^3*b^4*g^6*x^2 + 5*a^4*b^3*g^6*x + a^5
*b^2*g^6) - 1/30*(10*b^2*x^2 + 5*a*b*x + a^2)*A^2*d^2*i^2/(b^8*g^6*x^5 + 5*
a*b^7*g^6*x^4 + 10*a^2*b^6*g^6*x^3 + 10*a^3*b^5*g^6*x^2 + 5*a^4*b^4*g^6*x +
 a^5*b^3*g^6) - 1/5*A^2*c^2*i^2/(b^6*g^6*x^5 + 5*a*b^5*g^6*x^4 + 10*a^2*b^4
*g^6*x^3 + 10*a^3*b^3*g^6*x^2 + 5*a^4*b^2*g^6*x + a^5*b*g^6)
```

Giac [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 780, normalized size of antiderivative = 1.68

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = -\frac{1}{54000}\left(\frac{1800\left(6\,B^2b^2e^6i^2-\frac{15\,(bex+ae)B^2bde^5i^2}{dx+c}+\frac{10\,(bex+ae)^2B^2d^2e^4i^2}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^5b^2c^2g^6}{(dx+c)^5}-\frac{2\,(bex+ae)^5abcdg^6}{(dx+c)^5}+\frac{(bex+ae)^5a^2d^2g^6}{(dx+c)^5}}+\frac{60\left(360\,ABb^2e^6i^2+\right.}{}\right.$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x, alg
orithm="giac")
```

```
[Out] -1/54000*(1800*(6*B^2*b^2*e^6*i^2 - 15*(b*e*x + a*e)*B^2*b*d*e^5*i^2/(d*x +
 c) + 10*(b*e*x + a*e)^2*B^2*d^2*e^4*i^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*
x + c))^2/((b*e*x + a*e)^5*b^2*c^2*g^6/(d*x + c)^5 - 2*(b*e*x + a*e)^5*a*b*
c*d*g^6/(d*x + c)^5 + (b*e*x + a*e)^5*a^2*d^2*g^6/(d*x + c)^5) + 60*(360*A*
B*b^2*e^6*i^2 + 72*B^2*b^2*e^6*i^2 - 900*(b*e*x + a*e)*A*B*b*d*e^5*i^2/(d*x
 + c) - 225*(b*e*x + a*e)*B^2*b*d*e^5*i^2/(d*x + c) + 600*(b*e*x + a*e)^2*A
*B*d^2*e^4*i^2/(d*x + c)^2 + 200*(b*e*x + a*e)^2*B^2*d^2*e^4*i^2/(d*x + c)^
2)*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^5*b^2*c^2*g^6/(d*x + c)^5 -
2*(b*e*x + a*e)^5*a*b*c*d*g^6/(d*x + c)^5 + (b*e*x + a*e)^5*a^2*d^2*g^6/(d*
```

```
x + c)^5) + (10800*A^2*b^2*e^6*i^2 + 4320*A*B*b^2*e^6*i^2 + 864*B^2*b^2*e^6
*i^2 - 27000*(b*e*x + a*e)*A^2*b*d*e^5*i^2/(d*x + c) - 13500*(b*e*x + a*e)*
A*B*b*d*e^5*i^2/(d*x + c) - 3375*(b*e*x + a*e)*B^2*b*d*e^5*i^2/(d*x + c) +
18000*(b*e*x + a*e)^2*A^2*d^2*e^4*i^2/(d*x + c)^2 + 12000*(b*e*x + a*e)^2*A
*B*d^2*e^4*i^2/(d*x + c)^2 + 4000*(b*e*x + a*e)^2*B^2*d^2*e^4*i^2/(d*x + c)
^2)/((b*e*x + a*e)^5*b^2*c^2*g^6/(d*x + c)^5 - 2*(b*e*x + a*e)^5*a*b*c*d*g^
6/(d*x + c)^5 + (b*e*x + a*e)^5*a^2*d^2*g^6/(d*x + c)^5))*(b*c/((b*c*e - a*
d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 3434, normalized size of antiderivative = 7.42

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

```
[In] int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^
6,x)
```

```
[Out] ((1800*A^2*a^4*d^4*i^2 + 10800*A^2*b^4*c^4*i^2 + 1489*B^2*a^4*d^4*i^2 + 864
*B^2*b^4*c^4*i^2 + 2820*A*B*a^4*d^4*i^2 + 4320*A*B*b^4*c^4*i^2 - 16200*A^2*
a*b^3*c^3*d*i^2 + 1800*A^2*a^3*b*c*d^3*i^2 - 2511*B^2*a*b^3*c^3*d*i^2 + 148
9*B^2*a^3*b*c*d^3*i^2 + 1800*A^2*a^2*b^2*c^2*d^2*i^2 + 1489*B^2*a^2*b^2*c^2
*d^2*i^2 + 2820*A*B*a^2*b^2*c^2*d^2*i^2 - 9180*A*B*a*b^3*c^3*d*i^2 + 2820*A
*B*a^3*b*c*d^3*i^2)/(60*(a*d - b*c)) + (x^3*(363*B^2*a*b^3*d^4*i^2 + 13*B^2
*b^4*c*d^3*i^2 + 540*A*B*a*b^3*d^4*i^2 - 60*A*B*b^4*c*d^3*i^2))/(2*(a*d - b
*c)) + (x*(1800*A^2*a^3*b*d^4*i^2 + 1489*B^2*a^3*b*d^4*i^2 + 5400*A^2*b^4*c
^3*d*i^2 + 189*B^2*b^4*c^3*d*i^2 - 9000*A^2*a*b^3*c^2*d^2*i^2 + 1800*A^2*a^
2*b^2*c*d^3*i^2 - 911*B^2*a*b^3*c^2*d^2*i^2 + 1489*B^2*a^2*b^2*c*d^3*i^2 +
2820*A*B*a^3*b*d^4*i^2 + 1620*A*B*b^4*c^3*d*i^2 - 4380*A*B*a*b^3*c^2*d^2*i^
2 + 2820*A*B*a^2*b^2*c*d^3*i^2))/(12*(a*d - b*c)) + (x^2*(1800*A^2*a^2*b^2*
d^4*i^2 + 1489*B^2*a^2*b^2*d^4*i^2 + 1800*A^2*b^4*c^2*d^2*i^2 - 86*B^2*b^4*
c^2*d^2*i^2 - 3600*A^2*a*b^3*c*d^3*i^2 + 289*B^2*a*b^3*c*d^3*i^2 + 2820*A*B
*a^2*b^2*d^4*i^2 + 120*A*B*b^4*c^2*d^2*i^2 - 780*A*B*a*b^3*c*d^3*i^2))/(6*(
a*d - b*c)) + (d*x^4*(47*B^2*b^4*d^3*i^2 + 60*A*B*b^4*d^3*i^2))/(a*d - b*c)
)/(x*(4500*a^4*b^5*c*g^6 - 4500*a^5*b^4*d*g^6) - x^4*(4500*a^2*b^7*d*g^6 -
4500*a*b^8*c*g^6) + x^5*(900*b^9*c*g^6 - 900*a*b^8*d*g^6) + x^2*(9000*a^3*b
^6*c*g^6 - 9000*a^4*b^5*d*g^6) + x^3*(9000*a^2*b^7*c*g^6 - 9000*a^3*b^6*d*g
^6) + 900*a^5*b^4*c*g^6 - 900*a^6*b^3*d*g^6) - log((e*(a + b*x))/(c + d*x))
^2*((x*(b*((B^2*c*d*i^2)/(10*b^3*g^6) + (B^2*a*d^2*i^2)/(30*b^4*g^6)) + (2*
B^2*c*d*i^2)/(5*b^2*g^6) + (2*B^2*a*d^2*i^2)/(15*b^3*g^6)) + a*((B^2*c*d*i^
2)/(10*b^3*g^6) + (B^2*a*d^2*i^2)/(30*b^4*g^6)) + (B^2*c^2*i^2)/(5*b^2*g^6)
 + (B^2*d^2*i^2*x^2)/(3*b^2*g^6))/(5*a^4*x + a^5/b + b^4*x^5 + 10*a^3*b*x^2
 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3) - (B^2*d^5*i^2)/(30*b^3*g^6*(a^3*d^3 - b^3
```

*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (log((e*(a + b*x))/(c + d*x))*(a*
((B*i^2*(6*A*b*c - B*a*d + B*b*c))/(30*b^4*g^6) + (A*B*a*d*i^2)/(15*b^4*g^6
)) + x*(b*((B*i^2*(6*A*b*c - B*a*d + B*b*c))/(30*b^4*g^6) + (A*B*a*d*i^2)/(
15*b^4*g^6)) + (2*B*i^2*(6*A*b*c - B*a*d + B*b*c))/(15*b^3*g^6) + (B^2*d^5*
i^2*((10*a^4*d^4 + b^4*c^4 + 15*a^2*b^2*c^2*d^2 - 6*a*b^3*c^3*d - 20*a^3*b*
c*d^3)/(5*d^5) + b*(a*(a*((5*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)/(20*b*d^3) + (a
*(a*d - b*c))/(5*b*d^2)) + (10*a^3*d^3 - b^3*c^3 + 6*a*b^2*c^2*d - 15*a^2*b
*c*d^2)/(30*b*d^4)) + (10*a^4*d^4 + b^4*c^4 + 15*a^2*b^2*c^2*d^2 - 6*a*b^3*
c^3*d - 20*a^3*b*c*d^3)/(20*b*d^5)) + a*(b*(a*((5*a^2*d^2 + b^2*c^2 - 6*a*b
*c*d)/(20*b*d^3) + (a*(a*d - b*c))/(5*b*d^2)) + (10*a^3*d^3 - b^3*c^3 + 6*a
*b^2*c^2*d - 15*a^2*b*c*d^2)/(30*b*d^4)) + a*(b*((5*a^2*d^2 + b^2*c^2 - 6*a
*b*c*d)/(20*b*d^3) + (a*(a*d - b*c))/(5*b*d^2)) + (5*a^2*d^2 + b^2*c^2 - 6*
a*b*c*d)/(10*d^3) + (2*a*(a*d - b*c))/(5*d^2)) + (10*a^3*d^3 - b^3*c^3 + 6*
a*b^2*c^2*d - 15*a^2*b*c*d^2)/(10*d^4))))/(15*b^3*g^6*(a^3*d^3 - b^3*c^3 +
3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (4*A*B*a*d*i^2)/(15*b^3*g^6)) + x^2*((2*A
*B*d*i^2)/(3*b^2*g^6) + (B^2*d^5*i^2*(a*(b*(b*((5*a^2*d^2 + b^2*c^2 - 6*a*b
*c*d)/(20*b*d^3) + (a*(a*d - b*c))/(5*b*d^2)) + (5*a^2*d^2 + b^2*c^2 - 6*a*
b*c*d)/(10*d^3) + (2*a*(a*d - b*c))/(5*d^2)) - a*((b^2*c - a*b*d)/(5*d^2) -
 (2*b*(a*d - b*c))/(5*d^2)) + (3*(b^3*c^2 + 5*a^2*b*d^2 - 6*a*b^2*c*d))/(20
*d^3)) - (b^4*c^3 - 10*a^3*b*d^3 + 15*a^2*b^2*c*d^2 - 6*a*b^3*c^2*d)/(5*d^4
) + b*(b*(a*((5*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)/(20*b*d^3) + (a*(a*d - b*c))
/(5*b*d^2)) + (10*a^3*d^3 - b^3*c^3 + 6*a*b^2*c^2*d - 15*a^2*b*c*d^2)/(30*b
d^4)) + a(b*((5*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)/(20*b*d^3) + (a*(a*d - b*c
))/(5*b*d^2)) + (5*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)/(10*d^3) + (2*a*(a*d - b*
c))/(5*d^2)) + (10*a^3*d^3 - b^3*c^3 + 6*a*b^2*c^2*d - 15*a^2*b*c*d^2)/(10*
d^4))))/(15*b^3*g^6*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) +
 (B*i^2*(6*A*b^2*c^2 - B*a^2*d^2 + B*b^2*c^2))/(15*b^4*d*g^6) + (B^2*d^5*i^
2*(a*(a*(a*((5*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)/(20*b*d^3) + (a*(a*d - b*c))/
(5*b*d^2)) + (10*a^3*d^3 - b^3*c^3 + 6*a*b^2*c^2*d - 15*a^2*b*c*d^2)/(30*b*
d^4)) + (10*a^4*d^4 + b^4*c^4 + 15*a^2*b^2*c^2*d^2 - 6*a*b^3*c^3*d - 20*a^3
*b*c*d^3)/(20*b*d^5)) + (5*a^5*d^5 - b^5*c^5 - 15*a^2*b^3*c^3*d^2 + 20*a^3*
b^2*c^2*d^3 + 6*a*b^4*c^4*d - 15*a^4*b*c*d^4)/(5*b*d^6)))/(15*b^3*g^6*(a^3*
d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (B^2*d^5*i^2*x^3*((b^4*c^
2 + 5*a^2*b^2*d^2 - 6*a*b^3*c*d)/(5*d^3) + b*(b*(b*((5*a^2*d^2 + b^2*c^2 -
6*a*b*c*d)/(20*b*d^3) + (a*(a*d - b*c))/(5*b*d^2)) + (5*a^2*d^2 + b^2*c^2 -
 6*a*b*c*d)/(10*d^3) + (2*a*(a*d - b*c))/(5*d^2)) - a*((b^2*c - a*b*d)/(5*d
^2) - (2*b*(a*d - b*c))/(5*d^2)) + (3*(b^3*c^2 + 5*a^2*b*d^2 - 6*a*b^2*c*d)
)/(20*d^3)) - a*(b*((b^2*c - a*b*d)/(5*d^2) - (2*b*(a*d - b*c))/(5*d^2)) +
(b^3*c - a*b^2*d)/(5*d^2))))/(15*b^3*g^6*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
 - 3*a^2*b*c*d^2)) - (B^2*d^5*i^2*x^4*(b*(b*((b^2*c - a*b*d)/(5*d^2) - (2*b
*(a*d - b*c))/(5*d^2)) + (b^3*c - a*b^2*d)/(5*d^2)) + (b^4*c - a*b^3*d)/(5*
d^2)))/(15*b^3*g^6*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))))/(
(5*a^4*x)/d + a^5/(b*d) + (b^4*x^5)/d + (10*a^3*b*x^2)/d + (5*a*b^3*x^4)/d
+ (10*a^2*b^2*x^3)/d) - (B*d^5*i^2*atan((B*d^5*i^2*(60*A + 47*B)*(900*b^6*c
^3*g^6 + 900*a^3*b^3*d^3*g^6 - 900*a*b^5*c^2*d*g^6 - 900*a^2*b^4*c*d^2*g^6)

```
*1i)/(900*b^3*g^6*(47*B^2*d^5*i^2 + 60*A*B*d^5*i^2)*(a*d - b*c)^3) + (B*d^6
*i^2*x*(60*A + 47*B)*(b^5*c^2*g^6 + a^2*b^3*d^2*g^6 - 2*a*b^4*c*d*g^6)*2i)/
(b^2*g^6*(47*B^2*d^5*i^2 + 60*A*B*d^5*i^2)*(a*d - b*c)^3))*(60*A + 47*B)*1i
)/(450*b^3*g^6*(a*d - b*c)^3)
```

3.74 $\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 42, antiderivative size = 1089

$$
\begin{aligned}
&\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx \\
&= \frac{5B^2(bc-ad)^6g^3i^3x}{84b^3d^3}+\frac{B^2(bc-ad)^3g^3i^3(a+bx)^4}{140b^4}-\frac{29B^2(bc-ad)^5g^3i^3(c+dx)^2}{840b^2d^4} \\
&\quad+\frac{47B^2(bc-ad)^4g^3i^3(c+dx)^3}{1260bd^4}-\frac{13B^2(bc-ad)^3g^3i^3(c+dx)^4}{420d^4} \\
&\quad+\frac{bB^2(bc-ad)^2g^3i^3(c+dx)^5}{105d^4}-\frac{B^2(bc-ad)^7g^3i^3\log\left(\frac{a+bx}{c+dx}\right)}{210b^4d^4} \\
&\quad-\frac{B(bc-ad)^4g^3i^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{210b^4d} \\
&\quad-\frac{3B(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{140b^4} \\
&\quad-\frac{B(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{35b^3} \\
&\quad+\frac{2B(bc-ad)^4g^3i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{21bd^4} \\
&\quad-\frac{3B(bc-ad)^3g^3i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{14d^4} \\
&\quad+\frac{6bB(bc-ad)^2g^3i^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{35d^4} \\
&\quad-\frac{b^2B(bc-ad)g^3i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{21d^4} \\
&\quad+\frac{(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{140b^4} \\
&\quad+\frac{(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{35b^3} \\
&\quad+\frac{(bc-ad)g^3i^3(a+bx)^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{14b^2} \\
&\quad+\frac{g^3i^3(a+bx)^4(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{7b} \\
&\quad+\frac{B(bc-ad)^5g^3i^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{420b^4d^2} \\
&\quad-\frac{B(bc-ad)^6g^3i^3(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{420b^4d^3} \\
&\quad-\frac{B(bc-ad)^7g^3i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{420b^4d^4} \\
&\quad 11B^2(bc-ad)^7g^3i^3\log(c+dx) \qquad B^2(bc-ad)^7g^3i^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)
\end{aligned}
$$

```
[Out] 5/84*B^2*(-a*d+b*c)^6*g^3*i^3*x/b^3/d^3+1/140*B^2*(-a*d+b*c)^3*g^3*i^3*(b*x
+a)^4/b^4-29/840*B^2*(-a*d+b*c)^5*g^3*i^3*(d*x+c)^2/b^2/d^4+47/1260*B^2*(-a
*d+b*c)^4*g^3*i^3*(d*x+c)^3/b/d^4-13/420*B^2*(-a*d+b*c)^3*g^3*i^3*(d*x+c)^4
/d^4+1/105*b*B^2*(-a*d+b*c)^2*g^3*i^3*(d*x+c)^5/d^4-1/210*B^2*(-a*d+b*c)^7*
g^3*i^3*ln((b*x+a)/(d*x+c))/b^4/d^4-1/210*B*(-a*d+b*c)^4*g^3*i^3*(b*x+a)^3*
(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/d-3/140*B*(-a*d+b*c)^3*g^3*i^3*(b*x+a)^4*(A
+B*ln(e*(b*x+a)/(d*x+c)))/b^4-1/35*B*(-a*d+b*c)^2*g^3*i^3*(b*x+a)^4*(d*x+c)
*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3+2/21*B*(-a*d+b*c)^4*g^3*i^3*(d*x+c)^3*(A+B
*ln(e*(b*x+a)/(d*x+c)))/b/d^4-3/14*B*(-a*d+b*c)^3*g^3*i^3*(d*x+c)^4*(A+B*ln
(e*(b*x+a)/(d*x+c)))/d^4+6/35*b*B*(-a*d+b*c)^2*g^3*i^3*(d*x+c)^5*(A+B*ln(e*
(b*x+a)/(d*x+c)))/d^4-1/21*b^2*B*(-a*d+b*c)*g^3*i^3*(d*x+c)^6*(A+B*ln(e*(b*
x+a)/(d*x+c)))/d^4+1/140*(-a*d+b*c)^3*g^3*i^3*(b*x+a)^4*(A+B*ln(e*(b*x+a)/(
d*x+c)))^2/b^4+1/35*(-a*d+b*c)^2*g^3*i^3*(b*x+a)^4*(d*x+c)*(A+B*ln(e*(b*x+a
)/(d*x+c)))^2/b^3+1/14*(-a*d+b*c)*g^3*i^3*(b*x+a)^4*(d*x+c)^2*(A+B*ln(e*(b*
x+a)/(d*x+c)))^2/b^2+1/7*g^3*i^3*(b*x+a)^4*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x
+c)))^2/b+1/420*B*(-a*d+b*c)^5*g^3*i^3*(b*x+a)^2*(3*A+B+3*B*ln(e*(b*x+a)/(d
*x+c)))/b^4/d^2-1/420*B*(-a*d+b*c)^6*g^3*i^3*(b*x+a)*(6*A+5*B+6*B*ln(e*(b*x
+a)/(d*x+c)))/b^4/d^3-1/420*B*(-a*d+b*c)^7*g^3*i^3*ln((-a*d+b*c)/b/(d*x+c))
*(6*A+11*B+6*B*ln(e*(b*x+a)/(d*x+c)))/b^4/d^4-11/420*B^2*(-a*d+b*c)^7*g^3*i
^3*ln(d*x+c)/b^4/d^4-1/70*B^2*(-a*d+b*c)^7*g^3*i^3*polylog(2,d*(b*x+a)/b/(d
*x+c))/b^4/d^4
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 1089, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules

used = {2562, 2383, 2381, 2384, 2354, 2438, 2373, 45, 47, 37, 2382, 12, 79, 1634}

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$

$$\begin{aligned}
&= -\frac{B^2g^3i^3\log\left(\frac{a+bx}{c+dx}\right)(bc-ad)^7}{210b^4d^4} \\
&- \frac{Bg^3i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(bc-ad)^7}{420b^4d^4} \\
&- \frac{11B^2g^3i^3\log(c+dx)(bc-ad)^7}{420b^4d^4} - \frac{B^2g^3i^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)(bc-ad)^7}{70b^4d^4} \\
&+ \frac{5B^2g^3i^3x(bc-ad)^6}{84b^3d^3} - \frac{Bg^3i^3(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(bc-ad)^6}{420b^4d^3} \\
&- \frac{29B^2g^3i^3(c+dx)^2(bc-ad)^5}{840b^2d^4} \\
&+ \frac{Bg^3i^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(bc-ad)^5}{420b^4d^2} \\
&+ \frac{47B^2g^3i^3(c+dx)^3(bc-ad)^4}{1260bd^4} - \frac{Bg^3i^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(bc-ad)^4}{210b^4d} \\
&+ \frac{2Bg^3i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(bc-ad)^4}{21bd^4} + \frac{B^2g^3i^3(a+bx)^4(bc-ad)^3}{140b^4} \\
&- \frac{13B^2g^3i^3(c+dx)^4(bc-ad)^3}{420d^4} + \frac{g^3i^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(bc-ad)^3}{140b^4} \\
&- \frac{3Bg^3i^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(bc-ad)^3}{140b^4} \\
&- \frac{3Bg^3i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(bc-ad)^3}{14d^4} + \frac{bB^2g^3i^3(c+dx)^5(bc-ad)^2}{105d^4} \\
&+ \frac{g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(bc-ad)^2}{35b^3} \\
&+ \frac{6bBg^3i^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(bc-ad)^2}{35d^4} \\
&- \frac{Bg^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(bc-ad)^2}{35b^3} \\
&+ \frac{g^3i^3(a+bx)^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(bc-ad)}{14b^2} \\
&- \frac{b^2Bg^3i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(bc-ad)}{21d^4} \\
&+ \frac{g^3i^3(a+bx)^4(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{7b}
\end{aligned}$$

```
[In] Int[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,
x]
```

```
[Out] (5*B^2*(b*c - a*d)^6*g^3*i^3*x)/(84*b^3*d^3) + (B^2*(b*c - a*d)^3*g^3*i^3*(
a + b*x)^4)/(140*b^4) - (29*B^2*(b*c - a*d)^5*g^3*i^3*(c + d*x)^2)/(840*b^2
*d^4) + (47*B^2*(b*c - a*d)^4*g^3*i^3*(c + d*x)^3)/(1260*b*d^4) - (13*B^2*(
b*c - a*d)^3*g^3*i^3*(c + d*x)^4)/(420*d^4) + (b*B^2*(b*c - a*d)^2*g^3*i^3*
(c + d*x)^5)/(105*d^4) - (B^2*(b*c - a*d)^7*g^3*i^3*Log[(a + b*x)/(c + d*x)
])/(210*b^4*d^4) - (B*(b*c - a*d)^4*g^3*i^3*(a + b*x)^3*(A + B*Log[(e*(a +
b*x))/(c + d*x)]))/(210*b^4*d) - (3*B*(b*c - a*d)^3*g^3*i^3*(a + b*x)^4*(A
+ B*Log[(e*(a + b*x))/(c + d*x)]))/(140*b^4) - (B*(b*c - a*d)^2*g^3*i^3*(a
+ b*x)^4*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(35*b^3) + (2*B*(b
*c - a*d)^4*g^3*i^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(21*b
*d^4) - (3*B*(b*c - a*d)^3*g^3*i^3*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c
+ d*x)]))/(14*d^4) + (6*b*B*(b*c - a*d)^2*g^3*i^3*(c + d*x)^5*(A + B*Log[(e
*(a + b*x))/(c + d*x)]))/(35*d^4) - (b^2*B*(b*c - a*d)*g^3*i^3*(c + d*x)^6*
(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(21*d^4) + ((b*c - a*d)^3*g^3*i^3*(a
+ b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(140*b^4) + ((b*c - a*d)^2
*g^3*i^3*(a + b*x)^4*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(35*
b^3) + ((b*c - a*d)*g^3*i^3*(a + b*x)^4*(c + d*x)^2*(A + B*Log[(e*(a + b*x)
)/(c + d*x)])^2)/(14*b^2) + (g^3*i^3*(a + b*x)^4*(c + d*x)^3*(A + B*Log[(e*
(a + b*x))/(c + d*x)])^2)/(7*b) + (B*(b*c - a*d)^5*g^3*i^3*(a + b*x)^2*(3*A
 + B + 3*B*Log[(e*(a + b*x))/(c + d*x)]))/(420*b^4*d^2) - (B*(b*c - a*d)^6*
g^3*i^3*(a + b*x)*(6*A + 5*B + 6*B*Log[(e*(a + b*x))/(c + d*x)]))/(420*b^4*
d^3) - (B*(b*c - a*d)^7*g^3*i^3*Log[(b*c - a*d)/(b*(c + d*x))]*(6*A + 11*B
+ 6*B*Log[(e*(a + b*x))/(c + d*x)]))/(420*b^4*d^4) - (11*B^2*(b*c - a*d)^7*
g^3*i^3*Log[c + d*x])/(420*b^4*d^4) - (B^2*(b*c - a*d)^7*g^3*i^3*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))])/(70*b^4*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
```

```
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2383

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Dist[(m + q + 2)/(d*(q + 1)), Int[(
f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Dist[b*n*(p/(d*(q +
 1))), Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x]) /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ
[q, -1] && GtQ[m, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \left((bc-ad)^7 g^3 i^3\right) \text{Subst}\left(\int \frac{x^3 (A+B\log(ex))^2}{(b-dx)^8}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{g^3 i^3 (a+bx)^4 (c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{7b} + \frac{\left(3(bc-ad)^7 g^3 i^3\right) \text{Subst}\left(\int \frac{x^3 (A+B\log(ex))^2}{(b-dx)^7}\,dx, x, \frac{a+bx}{c+dx}\right)}{7b} - \frac{\left(2B(bc-ad)^7 g^3 i^3\right) \text{Subst}\left(\int \frac{x^3 (A+B\log(ex))}{(b-dx)^7}\,dx, x, \frac{a+bx}{c+dx}\right)}{7b}$$

$$= \frac{2B(bc-ad)^4 g^3 i^3 (c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{21bd^4} - \frac{3B(bc-ad)^3 g^3 i^3 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{14d^4} + \frac{6bB(bc-ad)^2 g^3 i^3 (c+dx)^5 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{35d^4} - \frac{b^2 B(bc-ad) g^3 i^3 (c+dx)^6 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{21d^4} + \frac{(bc-ad) g^3 i^3 (a+bx)^4 (c+dx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{14b^2} + \frac{g^3 i^3 (a+bx)^4 (c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{7b} + \frac{\left((bc-ad)^7 g^3 i^3\right) \text{Subst}\left(\int \frac{x^3 (A+B\log(ex))^2}{(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{7b^2} - \frac{\left(B(bc-ad)^7 g^3 i^3\right) \text{Subst}\left(\int \frac{x^3 (A+B\log(ex))}{(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{7b^2} + \frac{\left(2B^2 (bc-ad)^7 g^3 i^3\right) \text{Subst}\left(\int \frac{-b^3+6b^2 dx-15bd^2x^2+20d^3x^3}{60d^4 x (b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{7b}$$

$$
\begin{aligned}
&= -\frac{B(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{140b^4} \\
&- \frac{B(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{35b^3} \\
&+ \frac{2B(bc-ad)^4g^3i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{21bd^4} \\
&- \frac{3B(bc-ad)^3g^3i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{14d^4} \\
&+ \frac{6bB(bc-ad)^2g^3i^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{35d^4} \\
&- \frac{b^2B(bc-ad)g^3i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{21d^4} \\
&+ \frac{(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{35b^3} \\
&+ \frac{(bc-ad)g^3i^3(a+bx)^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{14b^2} \\
&+ \frac{g^3i^3(a+bx)^4(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{7b} \\
&+ \frac{((bc-ad)^7g^3i^3)\,\text{Subst}\left(\int\frac{x^3(A+B\log(ex))^2}{(b-dx)^5}\,dx,x,\frac{a+bx}{c+dx}\right)}{35b^3} \\
&- \frac{(2B(bc-ad)^7g^3i^3)\,\text{Subst}\left(\int\frac{x^3(A+B\log(ex))}{(b-dx)^5}\,dx,x,\frac{a+bx}{c+dx}\right)}{35b^3} \\
&+ \frac{(B^2(bc-ad)^7g^3i^3)\,\text{Subst}\left(\int\frac{x^3(5b-dx)}{20b^2(b-dx)^5}\,dx,x,\frac{a+bx}{c+dx}\right)}{7b^2} \\
&+ \frac{(B^2(bc-ad)^7g^3i^3)\,\text{Subst}\left(\int\frac{-b^3+6b^2dx-15bd^2x^2+20d^3x^3}{x(b-dx)^6}\,dx,x,\frac{a+bx}{c+dx}\right)}{210bd^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3B(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{140b^4} \\
&- \frac{B(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{35b^3} \\
&+ \frac{2B(bc-ad)^4g^3i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{21bd^4} \\
&- \frac{3B(bc-ad)^3g^3i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{14d^4} \\
&+ \frac{6bB(bc-ad)^2g^3i^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{35d^4} \\
&- \frac{b^2B(bc-ad)g^3i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{21d^4} \\
&+ \frac{(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{140b^4} \\
&+ \frac{(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{35b^3} \\
&+ \frac{(bc-ad)g^3i^3(a+bx)^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{14b^2} \\
&+ \frac{g^3i^3(a+bx)^4(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{7b} \\
&- \frac{(B(bc-ad)^7g^3i^3)\,\text{Subst}\left(\int\frac{x^3(A+B\log(ex))}{(b-dx)^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{70b^4} \\
&+ \frac{(B^2(bc-ad)^7g^3i^3)\,\text{Subst}\left(\int\frac{x^3(5b-dx)}{(b-dx)^5}\,dx,x,\frac{a+bx}{c+dx}\right)}{140b^4} \\
&+ \frac{(B^2(bc-ad)^7g^3i^3)\,\text{Subst}\left(\int\frac{x^3}{(b-dx)^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{70b^4} \\
&+ \frac{(B^2(bc-ad)^7g^3i^3)\,\text{Subst}\left(\int\left(-\frac{1}{b^3x}+\frac{10b^2d}{(b-dx)^6}-\frac{26bd}{(b-dx)^5}+\frac{19d}{(b-dx)^4}-\frac{d}{b(b-dx)^3}-\frac{d}{b^2(b-dx)^2}-\frac{d}{b^3(b-dx)}\right)\right.}{210bd^4}
\end{aligned}
$$

$$= -\frac{B^2(bc-ad)^6g^3i^3x}{210b^3d^3} + \frac{B^2(bc-ad)^3g^3i^3(a+bx)^4}{140b^4} - \frac{B^2(bc-ad)^5g^3i^3(c+dx)^2}{420b^2d^4}$$
$$+ \frac{19B^2(bc-ad)^4g^3i^3(c+dx)^3}{630bd^4} - \frac{13B^2(bc-ad)^3g^3i^3(c+dx)^4}{420d^4}$$
$$+ \frac{bB^2(bc-ad)^2g^3i^3(c+dx)^5}{105d^4} - \frac{B^2(bc-ad)^7g^3i^3\log\left(\frac{a+bx}{c+dx}\right)}{210b^4d^4}$$
$$- \frac{B(bc-ad)^4g^3i^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{210b^4d}$$
$$- \frac{3B(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{140b^4}$$
$$- \frac{B(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{35b^3}$$
$$+ \frac{2B(bc-ad)^4g^3i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{21bd^4}$$
$$- \frac{3B(bc-ad)^3g^3i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{14d^4}$$
$$+ \frac{6bB(bc-ad)^2g^3i^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{35d^4}$$
$$- \frac{b^2B(bc-ad)g^3i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{21d^4}$$
$$+ \frac{(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{140b^4}$$
$$+ \frac{(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{35b^3}$$
$$+ \frac{(bc-ad)g^3i^3(a+bx)^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{14b^2}$$
$$+ \frac{g^3i^3(a+bx)^4(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{7b} - \frac{B^2(bc-ad)^7g^3i^3\log(c+dx)}{210b^4d^4}$$
$$+ \frac{\left(B^2(bc-ad)^7g^3i^3\right)\text{Subst}\left(\int \frac{x^3}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{140b^4}$$
$$+ \frac{\left(B^2(bc-ad)^7g^3i^3\right)\text{Subst}\left(\int \left(\frac{b^3}{d^3(b-dx)^4} - \frac{3b^2}{d^3(b-dx)^3} + \frac{3b}{d^3(b-dx)^2} - \frac{1}{d^3(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{70b^4}$$
$$+ \frac{\left(B(bc-ad)^7g^3i^3\right)\text{Subst}\left(\int \frac{x^2(3A+B+3B\log(ex))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{210b^4d}$$

$$= \frac{4B^2(bc-ad)^6g^3i^3x}{105b^3d^3} + \frac{B^2(bc-ad)^3g^3i^3(a+bx)^4}{140b^4} - \frac{B^2(bc-ad)^5g^3i^3(c+dx)^2}{42b^2d^4}$$
$$+ \frac{11B^2(bc-ad)^4g^3i^3(c+dx)^3}{315bd^4} - \frac{13B^2(bc-ad)^3g^3i^3(c+dx)^4}{420d^4}$$
$$+ \frac{bB^2(bc-ad)^2g^3i^3(c+dx)^5}{105d^4} - \frac{B^2(bc-ad)^7g^3i^3\log\left(\frac{a+bx}{c+dx}\right)}{210b^4d^4}$$
$$- \frac{B(bc-ad)^4g^3i^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{210b^4d}$$
$$- \frac{3B(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{140b^4}$$
$$- \frac{B(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{35b^3}$$
$$+ \frac{2B(bc-ad)^4g^3i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{21bd^4}$$
$$- \frac{3B(bc-ad)^3g^3i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{14d^4}$$
$$+ \frac{6bB(bc-ad)^2g^3i^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{35d^4}$$
$$- \frac{b^2B(bc-ad)g^3i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{21d^4}$$
$$+ \frac{(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{140b^4}$$
$$+ \frac{(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{35b^3}$$
$$+ \frac{(bc-ad)g^3i^3(a+bx)^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{14b^2}$$
$$+ \frac{g^3i^3(a+bx)^4(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{7b}$$
$$+ \frac{B(bc-ad)^5g^3i^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{420b^4d^2}$$
$$- \frac{2B^2(bc-ad)^7g^3i^3\log(c+dx)}{105b^4d^4}$$
$$+ \frac{(B^2(bc-ad)^7g^3i^3)\,\text{Subst}\left(\int\left(\frac{b^3}{d^3(b-dx)^4} - \frac{3b^2}{d^3(b-dx)^3} + \frac{3b}{d^3(b-dx)^2} - \frac{1}{d^3(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{140b^4}$$
$$- \frac{(B(bc-ad)^7g^3i^3)\,\text{Subst}\left(\int\frac{x(3B+2(3A+B)+6B\log(ex))}{(b-dx)^2}dx, x, \frac{a+bx}{c+dx}\right)}{420b^4d^2}$$

$$
\begin{aligned}
&= \frac{5B^2(bc-ad)^6g^3i^3x}{84b^3d^3} + \frac{B^2(bc-ad)^3g^3i^3(a+bx)^4}{140b^4} - \frac{29B^2(bc-ad)^5g^3i^3(c+dx)^2}{840b^2d^4} \\
&+ \frac{47B^2(bc-ad)^4g^3i^3(c+dx)^3}{1260bd^4} - \frac{13B^2(bc-ad)^3g^3i^3(c+dx)^4}{420d^4} \\
&+ \frac{bB^2(bc-ad)^2g^3i^3(c+dx)^5}{105d^4} - \frac{B^2(bc-ad)^7g^3i^3\log\left(\frac{a+bx}{c+dx}\right)}{210b^4d^4} \\
&- \frac{B(bc-ad)^4g^3i^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{210b^4d} \\
&- \frac{3B(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{140b^4} \\
&- \frac{B(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{35b^3} \\
&+ \frac{2B(bc-ad)^4g^3i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{21bd^4} \\
&- \frac{3B(bc-ad)^3g^3i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{14d^4} \\
&+ \frac{6bB(bc-ad)^2g^3i^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{35d^4} \\
&- \frac{b^2B(bc-ad)g^3i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{21d^4} \\
&+ \frac{(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{140b^4} \\
&+ \frac{(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{35b^3} \\
&+ \frac{(bc-ad)g^3i^3(a+bx)^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{14b^2} \\
&+ \frac{g^3i^3(a+bx)^4(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{7b} \\
&+ \frac{B(bc-ad)^5g^3i^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{420b^4d^2} \\
&- \frac{B(bc-ad)^6g^3i^3(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{420b^4d^3} \\
&- \frac{11B^2(bc-ad)^7g^3i^3\log(c+dx)}{420b^4d^4} \\
&+ \frac{(B(bc-ad)^7g^3i^3)\,\text{Subst}\left(\int\frac{9B+2(3A+B)+6B\log(ex)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{420b^4d^3}
\end{aligned}
$$

$$= \frac{5B^2(bc-ad)^6g^3i^3x}{84b^3d^3} + \frac{B^2(bc-ad)^3g^3i^3(a+bx)^4}{140b^4} - \frac{29B^2(bc-ad)^5g^3i^3(c+dx)^2}{840b^2d^4} + \frac{47B^2(bc-ad)^4g^3i^3(c+dx)^3}{1260bd^4} - \frac{13B^2(bc-ad)^3g^3i^3(c+dx)^4}{420d^4} + \frac{bB^2(bc-ad)^2g^3i^3(c+dx)^5}{105d^4} - \frac{B^2(bc-ad)^7g^3i^3\log\left(\frac{a+bx}{c+dx}\right)}{210b^4d^4} - \frac{B(bc-ad)^4g^3i^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{210b^4d} - \frac{3B(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{140b^4} - \frac{B(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{35b^3} + \frac{2B(bc-ad)^4g^3i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{21bd^4} - \frac{3B(bc-ad)^3g^3i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{14d^4} + \frac{6bB(bc-ad)^2g^3i^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{35d^4} - \frac{b^2B(bc-ad)g^3i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{21d^4} + \frac{(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{140b^4} + \frac{(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{35b^3} + \frac{(bc-ad)g^3i^3(a+bx)^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{14b^2} + \frac{g^3i^3(a+bx)^4(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{7b} + \frac{B(bc-ad)^5g^3i^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{420b^4d^2} - \frac{B(bc-ad)^6g^3i^3(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{420b^4d^3} - \frac{B(bc-ad)^7g^3i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{420b^4d^4} - \frac{11B^2(bc-ad)^7g^3i^3\log(c+dx)}{420b^4d^4} + \frac{(B^2(bc-ad)^7g^3i^3)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{70b^4d^4}$$

$$
\begin{aligned}
&= \frac{5B^2(bc-ad)^6g^3i^3x}{84b^3d^3} + \frac{B^2(bc-ad)^3g^3i^3(a+bx)^4}{140b^4} - \frac{29B^2(bc-ad)^5g^3i^3(c+dx)^2}{840b^2d^4} \\
&+ \frac{47B^2(bc-ad)^4g^3i^3(c+dx)^3}{1260bd^4} - \frac{13B^2(bc-ad)^3g^3i^3(c+dx)^4}{420d^4} \\
&+ \frac{bB^2(bc-ad)^2g^3i^3(c+dx)^5}{105d^4} - \frac{B^2(bc-ad)^7g^3i^3\log\left(\frac{a+bx}{c+dx}\right)}{210b^4d^4} \\
&- \frac{B(bc-ad)^4g^3i^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{210b^4d} \\
&- \frac{3B(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{140b^4} \\
&- \frac{B(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{35b^3} \\
&+ \frac{2B(bc-ad)^4g^3i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{21bd^4} \\
&- \frac{3B(bc-ad)^3g^3i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{14d^4} \\
&+ \frac{6bB(bc-ad)^2g^3i^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{35d^4} \\
&- \frac{b^2B(bc-ad)g^3i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{21d^4} \\
&+ \frac{(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{140b^4} \\
&+ \frac{(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{35b^3} \\
&+ \frac{(bc-ad)g^3i^3(a+bx)^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{14b^2} \\
&+ \frac{g^3i^3(a+bx)^4(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{7b} \\
&+ \frac{B(bc-ad)^5g^3i^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{420b^4d^2} \\
&- \frac{B(bc-ad)^6g^3i^3(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{420b^4d^3} \\
&- \frac{B(bc-ad)^7g^3i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{420b^4d^4} \\
&- \frac{11B^2(bc-ad)^7g^3i^3\log(c+dx)}{420b^4d^4} - \frac{B^2(bc-ad)^7g^3i^3\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{70b^4d^4}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2330 vs. $2(1089) = 2178$.

Time = 1.73 (sec) , antiderivative size = 2330, normalized size of antiderivative = 2.14

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x
)])^2,x]
```

```
[Out] (g^3*i^3*(35*(b*c - a*d)^3*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])
^2 + 84*d*(b*c - a*d)^2*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
+ 70*d^2*(b*c - a*d)*(a + b*x)^6*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 2
0*d^3*(a + b*x)^7*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (35*B*(b*c - a*d
)^4*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*
x))/(c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/
(c + d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*
(b*c - a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c +
d*x)])*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2
- 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (-(b*c) + a*d)
*Log[c + d*x]) + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] -
Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*
d^4) + (7*B*(b*c - a*d)^3*(24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d)^3*
(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(
A + B*Log[(e*(a + b*x))/(c + d*x)]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*
Log[(e*(a + b*x))/(c + d*x)]) - 6*d^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/
(c + d*x)]) - 24*B*(b*c - a*d)^4*Log[c + d*x] - 24*(b*c - a*d)^4*(A + B*Log
[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 4*B*(b*c - a*d)^2*(2*b*d*(b*c - a
*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*(6*
b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3
- 6*(b*c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^3*(b*d*x + (-(b*c) + a*d
)*Log[c + d*x]) + 12*B*(b*c - a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)]
- Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d
^4 - (7*B*(b*c - a*d)^2*(24*b^2*B*c*d*(b*c - a*d)^3*x + 120*A*b*d*(b*c - a*
d)^4*x + 130*b*B*d*(b*c - a*d)^4*x + 24*a*b*B*d^2*(-(b*c) + a*d)^3*x - 12*b
*B*c*d^2*(b*c - a*d)^2*(a + b*x)^2 + 12*a*B*d^3*(b*c - a*d)^2*(a + b*x)^2 +
 35*B*d^2*(-(b*c) + a*d)^3*(a + b*x)^2 + 8*b*B*c*d^3*(b*c - a*d)*(a + b*x)^
3 + 10*B*d^3*(b*c - a*d)^2*(a + b*x)^3 + 8*a*B*d^4*(-(b*c) + a*d)*(a + b*x)
^3 - 6*b*B*c*d^4*(a + b*x)^4 + 6*a*B*d^5*(a + b*x)^4 + 120*B*d*(b*c - a*d)^
4*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 60*d^2*(-(b*c) + a*d)^3*(a + b*x
)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 40*d^3*(b*c - a*d)^2*(a + b*x)^3
*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 30*d^4*(-(b*c) + a*d)*(a + b*x)^4*(
A + B*Log[(e*(a + b*x))/(c + d*x)]) + 24*d^5*(a + b*x)^5*(A + B*Log[(e*(a +
 b*x))/(c + d*x)]) - 24*b*B*c*(b*c - a*d)^4*Log[c + d*x] + 24*a*B*d*(b*c -
```

a*d)^4*Log[c + d*x] - 250*B*(b*c - a*d)^5*Log[c + d*x] - 120*(b*c - a*d)^5*
(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 60*B*(b*c - a*d)^5*((2*
Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[
2, (b*(c + d*x))/(b*c - a*d)])))/(6*d^4) + (B*(b*c - a*d)*(60*b^2*B*c*d*(b*
c - a*d)^4*x - 60*a*b*B*d^2*(b*c - a*d)^4*x + 360*A*b*d*(b*c - a*d)^5*x + 4
62*b*B*d*(b*c - a*d)^5*x - 30*b*B*c*d^2*(b*c - a*d)^3*(a + b*x)^2 + 30*a*B*
d^3*(b*c - a*d)^3*(a + b*x)^2 - 141*B*d^2*(b*c - a*d)^4*(a + b*x)^2 + 20*b*
B*c*d^3*(b*c - a*d)^2*(a + b*x)^3 - 20*a*B*d^4*(b*c - a*d)^2*(a + b*x)^3 +
54*B*d^3*(b*c - a*d)^3*(a + b*x)^3 - 15*b*B*c*d^4*(b*c - a*d)*(a + b*x)^4 +
15*a*B*d^5*(b*c - a*d)*(a + b*x)^4 - 18*B*d^4*(b*c - a*d)^2*(a + b*x)^4 +
12*b*B*c*d^5*(a + b*x)^5 - 12*a*B*d^6*(a + b*x)^5 + 360*B*d*(b*c - a*d)^5*(
a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 180*d^2*(b*c - a*d)^4*(a + b*x)^2*(
A + B*Log[(e*(a + b*x))/(c + d*x)]) + 120*d^3*(b*c - a*d)^3*(a + b*x)^3*(A
+ B*Log[(e*(a + b*x))/(c + d*x)]) - 90*d^4*(b*c - a*d)^2*(a + b*x)^4*(A + B
Log[(e(a + b*x))/(c + d*x)]) + 72*d^5*(b*c - a*d)*(a + b*x)^5*(A + B*Log[
(e*(a + b*x))/(c + d*x)]) - 60*d^6*(a + b*x)^6*(A + B*Log[(e*(a + b*x))/(c
+ d*x)]) - 60*b*B*c*(b*c - a*d)^5*Log[c + d*x] + 60*a*B*d*(b*c - a*d)^5*Log
[c + d*x] - 822*B*(b*c - a*d)^6*Log[c + d*x] - 360*(b*c - a*d)^6*(A + B*Log
[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 180*B*(b*c - a*d)^6*((2*Log[(d*(a
+ b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c
+ d*x))/(b*c - a*d)])))/(9*d^4)))/(140*b^4)

Maple [F]

$$\int (bgx+ag)^3 (dix+ci)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

[In] int((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)

[Out] int((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)

Fricas [F]

$$\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)^3(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, alg
orithm="fricas")

[Out] integral(A^2*b^3*d^3*g^3*i^3*x^6 + A^2*a^3*c^3*g^3*i^3 + 3*(A^2*b^3*c*d^2 +
A^2*a*b^2*d^3)*g^3*i^3*x^5 + 3*(A^2*b^3*c^2*d + 3*A^2*a*b^2*c*d^2 + A^2*a^
2*b*d^3)*g^3*i^3*x^4 + (A^2*b^3*c^3 + 9*A^2*a*b^2*c^2*d + 9*A^2*a^2*b*c*d^2

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 + A^2*a^3*d^3)*g^3*i^3*x^3 + 3*(A^2*a*b^2*c^3 + 3*A^2*a^2*b*c^2*d + A^2*a^
3*c*d^2)*g^3*i^3*x^2 + 3*(A^2*a^2*b*c^3 + A^2*a^3*c^2*d)*g^3*i^3*x + (B^2*b
^3*d^3*g^3*i^3*x^6 + B^2*a^3*c^3*g^3*i^3 + 3*(B^2*b^3*c*d^2 + B^2*a*b^2*d^3
)*g^3*i^3*x^5 + 3*(B^2*b^3*c^2*d + 3*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3)*g^3*i
^3*x^4 + (B^2*b^3*c^3 + 9*B^2*a*b^2*c^2*d + 9*B^2*a^2*b*c*d^2 + B^2*a^3*d^3
)*g^3*i^3*x^3 + 3*(B^2*a*b^2*c^3 + 3*B^2*a^2*b*c^2*d + B^2*a^3*c*d^2)*g^3*i
^3*x^2 + 3*(B^2*a^2*b*c^3 + B^2*a^3*c^2*d)*g^3*i^3*x)*log((b*e*x + a*e)/(d*
x + c))^2 + 2*(A*B*b^3*d^3*g^3*i^3*x^6 + A*B*a^3*c^3*g^3*i^3 + 3*(A*B*b^3*c
*d^2 + A*B*a*b^2*d^3)*g^3*i^3*x^5 + 3*(A*B*b^3*c^2*d + 3*A*B*a*b^2*c*d^2 +
A*B*a^2*b*d^3)*g^3*i^3*x^4 + (A*B*b^3*c^3 + 9*A*B*a*b^2*c^2*d + 9*A*B*a^2*b
*c*d^2 + A*B*a^3*d^3)*g^3*i^3*x^3 + 3*(A*B*a*b^2*c^3 + 3*A*B*a^2*b*c^2*d +
A*B*a^3*c*d^2)*g^3*i^3*x^2 + 3*(A*B*a^2*b*c^3 + A*B*a^3*c^2*d)*g^3*i^3*x)*l
og((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**3*(d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6921 vs. $2(1042) = 2084$.

Time = 0.40 (sec) , antiderivative size = 6921, normalized size of antiderivative = 6.36

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, alg
orithm="maxima")
[Out] 1/7*A^2*b^3*d^3*g^3*i^3*x^7 + 1/2*A^2*b^3*c*d^2*g^3*i^3*x^6 + 1/2*A^2*a*b^2
*d^3*g^3*i^3*x^6 + 3/5*A^2*b^3*c^2*d*g^3*i^3*x^5 + 9/5*A^2*a*b^2*c*d^2*g^3*
i^3*x^5 + 3/5*A^2*a^2*b*d^3*g^3*i^3*x^5 + 1/4*A^2*b^3*c^3*g^3*i^3*x^4 + 9/4
*A^2*a*b^2*c^2*d*g^3*i^3*x^4 + 9/4*A^2*a^2*b*c*d^2*g^3*i^3*x^4 + 1/4*A^2*a^
3*d^3*g^3*i^3*x^4 + A^2*a*b^2*c^3*g^3*i^3*x^3 + 3*A^2*a^2*b*c^2*d*g^3*i^3*x
^3 + A^2*a^3*c*d^2*g^3*i^3*x^3 + 3/2*A^2*a^2*b*c^3*g^3*i^3*x^2 + 3/2*A^2*a^
3*c^2*d*g^3*i^3*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x
 + a)/b - c*log(d*x + c)/d)*A*B*a^3*c^3*g^3*i^3 + 3*(x^2*log(b*e*x/(d*x + c
```

```
) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a
*d)*x/(b*d))*A*B*a^2*b*c^3*g^3*i^3 + (2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x
+ c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d
^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a*b^2*c^3*g^3*i^3 + 1/12*
(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^
4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*
d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b^3*c^3*g^3*i^3 + 3*(x^2
*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x
+ c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^3*c^2*d*g^3*i^3 + 3*(2*x^3*log(b*e*x/
(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^
3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*
b*c^2*d*g^3*i^3 + 3/4*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*l
og(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 -
 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*a*
b^2*c^2*d*g^3*i^3 + 1/10*(12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*
a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)
*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 -
12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*b^3*c^2*d*g^3*i^3 + (2*x^3*log(b*e
*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)
/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a
^3*c*d^2*g^3*i^3 + 3/4*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*
log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3
- 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*a
^2*b*c*d^2*g^3*i^3 + 3/10*(12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 12
*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4
)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 -
 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*a*b^2*c*d^2*g^3*i^3 + 1/60*(60*x^
6*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 60*a^6*log(b*x + a)/b^6 + 60*c^6*l
og(d*x + c)/d^6 - (12*(b^5*c*d^4 - a*b^4*d^5)*x^5 - 15*(b^5*c^2*d^3 - a^2*b
^3*d^5)*x^4 + 20*(b^5*c^3*d^2 - a^3*b^2*d^5)*x^3 - 30*(b^5*c^4*d - a^4*b*d^
5)*x^2 + 60*(b^5*c^5 - a^5*d^5)*x)/(b^5*d^5))*A*B*b^3*c*d^2*g^3*i^3 + 1/12*
(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^
4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*
d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*a^3*d^3*g^3*i^3 + 1/10*(
12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*
c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^
2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/
(b^4*d^4))*A*B*a^2*b*d^3*g^3*i^3 + 1/60*(60*x^6*log(b*e*x/(d*x + c) + a*e/(
d*x + c)) - 60*a^6*log(b*x + a)/b^6 + 60*c^6*log(d*x + c)/d^6 - (12*(b^5*c*
d^4 - a*b^4*d^5)*x^5 - 15*(b^5*c^2*d^3 - a^2*b^3*d^5)*x^4 + 20*(b^5*c^3*d^2
 - a^3*b^2*d^5)*x^3 - 30*(b^5*c^4*d - a^4*b*d^5)*x^2 + 60*(b^5*c^5 - a^5*d^
5)*x)/(b^5*d^5))*A*B*a*b^2*d^3*g^3*i^3 + 1/210*(60*x^7*log(b*e*x/(d*x + c)
+ a*e/(d*x + c)) + 60*a^7*log(b*x + a)/b^7 - 60*c^7*log(d*x + c)/d^7 - (10*
(b^6*c*d^5 - a*b^5*d^6)*x^6 - 12*(b^6*c^2*d^4 - a^2*b^4*d^6)*x^5 + 15*(b^6*
c^3*d^3 - a^3*b^3*d^6)*x^4 - 20*(b^6*c^4*d^2 - a^4*b^2*d^6)*x^3 + 30*(b^6*c
```

^5*d - a^5*b*d^6)*x^2 - 60*(b^6*c^6 - a^6*d^6)*x)/(b^6*d^6))*A*B*b^3*d^3*g^3*i^3 + A^2*a^3*c^3*g^3*i^3*x + 1/420*(6*b^6*c^7*g^3*i^3*log(e) - 107*a^4*b^2*c^3*d^4*g^3*i^3 + 39*a^5*b*c^2*d^5*g^3*i^3 - 6*a^6*c*d^6*g^3*i^3 - 6*(7*g^3*i^3*log(e) - g^3*i^3)*a*b^5*c^6*d + 3*(42*g^3*i^3*log(e) - 13*g^3*i^3)*a^2*b^4*c^5*d^2 - (210*g^3*i^3*log(e) - 107*g^3*i^3)*a^3*b^3*c^4*d^3)*B^2*log(d*x + c)/(b^3*d^4) + 1/70*(b^7*c^7*g^3*i^3 - 7*a*b^6*c^6*d*g^3*i^3 + 21*a^2*b^5*c^5*d^2*g^3*i^3 - 35*a^3*b^4*c^4*d^3*g^3*i^3 + 35*a^4*b^3*c^3*d^4*g^3*i^3 - 21*a^5*b^2*c^2*d^5*g^3*i^3 + 7*a^6*b*c*d^6*g^3*i^3 - a^7*d^7*g^3*i^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^4*d^4) + 1/2520*(360*B^2*b^7*d^7*g^3*i^3*x^7*log(e)^2 + 60*((21*g^3*i^3*log(e)^2 - 2*g^3*i^3*log(e))*b^7*c*d^6 + (21*g^3*i^3*log(e)^2 + 2*g^3*i^3*log(e))*a*b^6*d^7)*B^2*x^6 + 24*((63*g^3*i^3*log(e)^2 - 15*g^3*i^3*log(e) + g^3*i^3)*b^7*c^2*d^5 + (189*g^3*i^3*log(e)^2 - 2*g^3*i^3)*a*b^6*c*d^6 + (63*g^3*i^3*log(e)^2 + 15*g^3*i^3*log(e) + g^3*i^3)*a^2*b^5*d^7)*B^2*x^5 + 6*((105*g^3*i^3*log(e)^2 - 51*g^3*i^3*log(e) + 10*g^3*i^3)*b^7*c^3*d^4 + (945*g^3*i^3*log(e)^2 - 147*g^3*i^3*log(e) - 10*g^3*i^3)*a*b^6*c^2*d^5 + (945*g^3*i^3*log(e)^2 + 147*g^3*i^3*log(e) - 10*g^3*i^3)*a^2*b^5*c*d^6 + (105*g^3*i^3*log(e)^2 + 51*g^3*i^3*log(e) + 10*g^3*i^3)*a^3*b^4*d^7)*B^2*x^4 - 2*((6*g^3*i^3*log(e) - 11*g^3*i^3)*b^7*c^4*d^3 - 4*(315*g^3*i^3*log(e)^2 - 147*g^3*i^3*log(e) + 19*g^3*i^3)*a*b^6*c^3*d^4 - 6*(630*g^3*i^3*log(e)^2 - 29*g^3*i^3)*a^2*b^5*c^2*d^5 - 4*(315*g^3*i^3*log(e)^2 + 147*g^3*i^3*log(e) + 19*g^3*i^3)*a^3*b^4*c*d^6 - (6*g^3*i^3*log(e) + 11*g^3*i^3)*a^4*b^3*d^7)*B^2*x^3 + 3*(3*(2*g^3*i^3*log(e) - 3*g^3*i^3)*b^7*c^5*d^2 - (42*g^3*i^3*log(e) - 67*g^3*i^3)*a*b^6*c^4*d^3 + 2*(630*g^3*i^3*log(e)^2 - 252*g^3*i^3*log(e) - 29*g^3*i^3)*a^2*b^5*c^3*d^4 + 2*(630*g^3*i^3*log(e)^2 + 252*g^3*i^3*log(e) - 29*g^3*i^3)*a^3*b^4*c^2*d^5 + (42*g^3*i^3*log(e) + 67*g^3*i^3)*a^4*b^3*c*d^6 - 3*(2*g^3*i^3*log(e) + 3*g^3*i^3)*a^5*b^2*d^7)*B^2*x^2 - 6*(6*(g^3*i^3*log(e) - g^3*i^3)*b^7*c^6*d - 3*(14*g^3*i^3*log(e) - 15*g^3*i^3)*a*b^6*c^5*d^2 + 2*(63*g^3*i^3*log(e) - 73*g^3*i^3)*a^2*b^5*c^4*d^3 - 2*(210*g^3*i^3*log(e)^2 - 107*g^3*i^3)*a^3*b^4*c^3*d^4 - 2*(63*g^3*i^3*log(e) + 73*g^3*i^3)*a^4*b^3*c^2*d^5 + 3*(14*g^3*i^3*log(e) + 15*g^3*i^3)*a^5*b^2*c*d^6 - 6*(g^3*i^3*log(e) + g^3*i^3)*a^6*b*d^7)*B^2*x + 18*(20*B^2*b^7*d^7*g^3*i^3*x^7 + 140*B^2*a^3*b^4*c^3*d^4*g^3*i^3*x + 70*(b^7*c*d^6*g^3*i^3 + a*b^6*d^7*g^3*i^3)*B^2*x^6 + 84*(b^7*c^2*d^5*g^3*i^3 + 3*a*b^6*c*d^6*g^3*i^3 + a^2*b^5*d^7*g^3*i^3)*B^2*x^5 + 35*(b^7*c^3*d^4*g^3*i^3 + 9*a*b^6*c^2*d^5*g^3*i^3 + 9*a^2*b^5*c*d^6*g^3*i^3 + a^3*b^4*d^7*g^3*i^3)*B^2*x^4 + 140*(a*b^6*c^3*d^4*g^3*i^3 + 3*a^2*b^5*c^2*d^5*g^3*i^3 + a^3*b^4*c*d^6*g^3*i^3)*B^2*x^3 + 210*(a^2*b^5*c^3*d^4*g^3*i^3 + a^3*b^4*c^2*d^5*g^3*i^3)*B^2*x^2 + (35*a^4*b^3*c^3*d^4*g^3*i^3 - 21*a^5*b^2*c^2*d^5*g^3*i^3 + 7*a^6*b*c*d^6*g^3*i^3 - a^7*d^7*g^3*i^3)*B^2)*log(b*x + a)^2 + 18*(20*B^2*b^7*d^7*g^3*i^3*x^7 + 140*B^2*a^3*b^4*c^3*d^4*g^3*i^3*x + 70*(b^7*c*d^6*g^3*i^3 + a*b^6*d^7*g^3*i^3)*B^2*x^6 + 84*(b^7*c^2*d^5*g^3*i^3 + 3*a*b^6*c*d^6*g^3*i^3 + a^2*b^5*d^7*g^3*i^3)*B^2*x^5 + 35*(b^7*c^3*d^4*g^3*i^3 + 9*a*b^6*c^2*d^5*g^3*i^3 + 9*a^2*b^5*c*d^6*g^3*i^3 + a^3*b^4*d^7*g^3*i^3)*B^2*x^4 + 140*(a*b^6*c^3*d^4*g^3*i^3 + 3*a^2*b^5*c^2*d^5*g^3*i^3 + a^3*b^4*c*d^6*g^3*i^3)*B^2*

x^3 + 210*(a^2*b^5*c^3*d^4*g^3*i^3 + a^3*b^4*c^2*d^5*g^3*i^3)*B^2*x^2 - (b^
7*c^7*g^3*i^3 - 7*a*b^6*c^6*d*g^3*i^3 + 21*a^2*b^5*c^5*d^2*g^3*i^3 - 35*a^3
*b^4*c^4*d^3*g^3*i^3)*B^2)*log(d*x + c)^2 + 6*(120*B^2*b^7*d^7*g^3*i^3*x^7*
log(e) + 20*((21*g^3*i^3*log(e) - g^3*i^3)*b^7*c*d^6 + (21*g^3*i^3*log(e) +
g^3*i^3)*a*b^6*d^7)*B^2*x^6 + 12*(126*a*b^6*c*d^6*g^3*i^3*log(e) + (42*g^3
*i^3*log(e) - 5*g^3*i^3)*b^7*c^2*d^5 + (42*g^3*i^3*log(e) + 5*g^3*i^3)*a^2*
b^5*d^7)*B^2*x^5 + 3*((70*g^3*i^3*log(e) - 17*g^3*i^3)*b^7*c^3*d^4 + 7*(90*
g^3*i^3*log(e) - 7*g^3*i^3)*a*b^6*c^2*d^5 + 7*(90*g^3*i^3*log(e) + 7*g^3*i^
3)*a^2*b^5*c*d^6 + (70*g^3*i^3*log(e) + 17*g^3*i^3)*a^3*b^4*d^7)*B^2*x^4 +
2*(1260*a^2*b^5*c^2*d^5*g^3*i^3*log(e) - b^7*c^4*d^3*g^3*i^3 + a^4*b^3*d^7*
g^3*i^3 + 14*(30*g^3*i^3*log(e) - 7*g^3*i^3)*a*b^6*c^3*d^4 + 14*(30*g^3*i^3
*log(e) + 7*g^3*i^3)*a^3*b^4*c*d^6)*B^2*x^3 + 3*(b^7*c^5*d^2*g^3*i^3 - 7*a*
b^6*c^4*d^3*g^3*i^3 + 7*a^4*b^3*c*d^6*g^3*i^3 - a^5*b^2*d^7*g^3*i^3 + 84*(5
*g^3*i^3*log(e) - g^3*i^3)*a^2*b^5*c^3*d^4 + 84*(5*g^3*i^3*log(e) + g^3*i^3
)*a^3*b^4*c^2*d^5)*B^2*x^2 + 6*(140*a^3*b^4*c^3*d^4*g^3*i^3*log(e) - b^7*c^
6*d*g^3*i^3 + 7*a*b^6*c^5*d^2*g^3*i^3 - 21*a^2*b^5*c^4*d^3*g^3*i^3 + 21*a^4
*b^3*c^2*d^5*g^3*i^3 - 7*a^5*b^2*c*d^6*g^3*i^3 + a^6*b*d^7*g^3*i^3)*B^2*x -
(6*a^7*d^7*g^3*i^3*log(e) + 6*a*b^6*c^6*d*g^3*i^3 - 39*a^2*b^5*c^5*d^2*g^3
*i^3 + 107*a^3*b^4*c^4*d^3*g^3*i^3 - (210*g^3*i^3*log(e) + 107*g^3*i^3)*a^4
*b^3*c^3*d^4 + 3*(42*g^3*i^3*log(e) + 13*g^3*i^3)*a^5*b^2*c^2*d^5 - 6*(7*g^
3*i^3*log(e) + g^3*i^3)*a^6*b*c*d^6)*B^2)*log(b*x + a) - 6*(120*B^2*b^7*d^7
*g^3*i^3*x^7*log(e) + 20*((21*g^3*i^3*log(e) - g^3*i^3)*b^7*c*d^6 + (21*g^3
*i^3*log(e) + g^3*i^3)*a*b^6*d^7)*B^2*x^6 + 12*(126*a*b^6*c*d^6*g^3*i^3*log
(e) + (42*g^3*i^3*log(e) - 5*g^3*i^3)*b^7*c^2*d^5 + (42*g^3*i^3*log(e) + 5*
g^3*i^3)*a^2*b^5*d^7)*B^2*x^5 + 3*((70*g^3*i^3*log(e) - 17*g^3*i^3)*b^7*c^3
d^4 + 7(90*g^3*i^3*log(e) - 7*g^3*i^3)*a*b^6*c^2*d^5 + 7*(90*g^3*i^3*log(
e) + 7*g^3*i^3)*a^2*b^5*c*d^6 + (70*g^3*i^3*log(e) + 17*g^3*i^3)*a^3*b^4*d^
7)*B^2*x^4 + 2*(1260*a^2*b^5*c^2*d^5*g^3*i^3*log(e) - b^7*c^4*d^3*g^3*i^3 +
a^4*b^3*d^7*g^3*i^3 + 14*(30*g^3*i^3*log(e) - 7*g^3*i^3)*a*b^6*c^3*d^4 + 1
4*(30*g^3*i^3*log(e) + 7*g^3*i^3)*a^3*b^4*c*d^6)*B^2*x^3 + 3*(b^7*c^5*d^2*g
^3*i^3 - 7*a*b^6*c^4*d^3*g^3*i^3 + 7*a^4*b^3*c*d^6*g^3*i^3 - a^5*b^2*d^7*g^
3*i^3 + 84*(5*g^3*i^3*log(e) - g^3*i^3)*a^2*b^5*c^3*d^4 + 84*(5*g^3*i^3*log
(e) + g^3*i^3)*a^3*b^4*c^2*d^5)*B^2*x^2 + 6*(140*a^3*b^4*c^3*d^4*g^3*i^3*lo
g(e) - b^7*c^6*d*g^3*i^3 + 7*a*b^6*c^5*d^2*g^3*i^3 - 21*a^2*b^5*c^4*d^3*g^3
*i^3 + 21*a^4*b^3*c^2*d^5*g^3*i^3 - 7*a^5*b^2*c*d^6*g^3*i^3 + a^6*b*d^7*g^3
*i^3)*B^2*x + 6*(20*B^2*b^7*d^7*g^3*i^3*x^7 + 140*B^2*a^3*b^4*c^3*d^4*g^3*i
^3*x + 70*(b^7*c*d^6*g^3*i^3 + a*b^6*d^7*g^3*i^3)*B^2*x^6 + 84*(b^7*c^2*d^5
*g^3*i^3 + 3*a*b^6*c*d^6*g^3*i^3 + a^2*b^5*d^7*g^3*i^3)*B^2*x^5 + 35*(b^7*c
^3*d^4*g^3*i^3 + 9*a*b^6*c^2*d^5*g^3*i^3 + 9*a^2*b^5*c*d^6*g^3*i^3 + a^3*b^
4*d^7*g^3*i^3)*B^2*x^4 + 140*(a*b^6*c^3*d^4*g^3*i^3 + 3*a^2*b^5*c^2*d^5*g^3
*i^3 + a^3*b^4*c*d^6*g^3*i^3)*B^2*x^3 + 210*(a^2*b^5*c^3*d^4*g^3*i^3 + a^3*
b^4*c^2*d^5*g^3*i^3)*B^2*x^2 + (35*a^4*b^3*c^3*d^4*g^3*i^3 - 21*a^5*b^2*c^2
*d^5*g^3*i^3 + 7*a^6*b*c*d^6*g^3*i^3 - a^7*d^7*g^3*i^3)*B^2)*log(b*x + a))*
log(d*x + c))/(b^4*d^4)

Giac [F]

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \int (bgx+ag)^3(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, alg
orithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^3*(d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A
)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \int (a\,g+b\,g\,x)^3\,(c\,i+d\,i\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

```
[In] int((a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2,
x)
```

```
[Out] int((a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2,
 x)
```

3.75 $\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 42, antiderivative size = 908

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$= -\frac{7B^2(bc-ad)^5g^2i^3x}{180b^3d^2}-\frac{7B^2(bc-ad)^4g^2i^3(c+dx)^2}{360b^2d^3}-\frac{B^2(bc-ad)^3g^2i^3(c+dx)^3}{60bd^3}+\frac{B^2(bc-ad)^2g^2i^3(c+dx)^4}{60d^3}+\frac{B^2(bc-ad)^6g^2i^3\log\left(\frac{a+bx}{c+dx}\right)}{36b^4d^3}-\frac{B(bc-ad)^4g^2i^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^4d}-\frac{B(bc-ad)^3g^2i^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^4}-\frac{B(bc-ad)^4g^2i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^2d^3}+\frac{B(bc-ad)^3g^2i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{45bd^3}+\frac{7B(bc-ad)^2g^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60d^3}-\frac{bB(bc-ad)g^2i^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15d^3}+\frac{(bc-ad)^3g^2i^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{60b^4}+\frac{(bc-ad)^2g^2i^3(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^3}+\frac{(bc-ad)g^2i^3(a+bx)^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^2}+\frac{g^2i^3(a+bx)^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b}+\frac{B(bc-ad)^5g^2i^3(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^4d^2}+\frac{B(bc-ad)^6g^2i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^4d^3}+\frac{11B^2(bc-ad)^6g^2i^3\log(c+dx)}{180b^4d^3}+\frac{B^2(bc-ad)^6g^2i^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{30b^4d^3}$$

```
[Out] -7/180*B^2*(-a*d+b*c)^5*g^2*i^3*x/b^3/d^2-7/360*B^2*(-a*d+b*c)^4*g^2*i^3*(d
*x+c)^2/b^2/d^3-1/60*B^2*(-a*d+b*c)^3*g^2*i^3*(d*x+c)^3/b/d^3+1/60*B^2*(-a*
d+b*c)^2*g^2*i^3*(d*x+c)^4/d^3+1/36*B^2*(-a*d+b*c)^6*g^2*i^3*ln((b*x+a)/(d*
x+c))/b^4/d^3-1/60*B*(-a*d+b*c)^4*g^2*i^3*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+
c)))/b^4/d-1/30*B*(-a*d+b*c)^3*g^2*i^3*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c))
)/b^4-1/10*B*(-a*d+b*c)^4*g^2*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2
/d^3+1/45*B*(-a*d+b*c)^3*g^2*i^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d^
3+7/60*B*(-a*d+b*c)^2*g^2*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3-1/1
5*b*B*(-a*d+b*c)*g^2*i^3*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3+1/60*(-a
*d+b*c)^3*g^2*i^3*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^4+1/20*(-a*d+b*
c)^2*g^2*i^3*(b*x+a)^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^3+1/10*(-a*d
+b*c)*g^2*i^3*(b*x+a)^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^2+1/6*g^2
*i^3*(b*x+a)^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+1/60*B*(-a*d+b*c)^
5*g^2*i^3*(b*x+a)*(2*A+B+2*B*ln(e*(b*x+a)/(d*x+c)))/b^4/d^2+1/60*B*(-a*d+b*
c)^6*g^2*i^3*ln((-a*d+b*c)/b/(d*x+c))*(2*A+3*B+2*B*ln(e*(b*x+a)/(d*x+c)))/b
^4/d^3+11/180*B^2*(-a*d+b*c)^6*g^2*i^3*ln(d*x+c)/b^4/d^3+1/30*B^2*(-a*d+b*c
)^6*g^2*i^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/d^3
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 908, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.262$, Rules

used = {2562, 2383, 2381, 2384, 2354, 2438, 2373, 45, 2382, 12, 907}

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{B^2g^2i^3\log\left(\frac{a+bx}{c+dx}\right)(bc-ad)^6}{36b^4d^3}$$
$$+\frac{Bg^2i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(bc-ad)^6}{60b^4d^3}$$
$$+\frac{11B^2g^2i^3\log(c+dx)(bc-ad)^6}{180b^4d^3}+\frac{B^2g^2i^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)(bc-ad)^6}{30b^4d^3}$$
$$-\frac{7B^2g^2i^3x(bc-ad)^5}{180b^3d^2}+\frac{Bg^2i^3(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(bc-ad)^5}{60b^4d^2}$$
$$-\frac{7B^2g^2i^3(c+dx)^2(bc-ad)^4}{360b^2d^3}-\frac{Bg^2i^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(bc-ad)^4}{60b^4d}$$
$$-\frac{Bg^2i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(bc-ad)^4}{10b^2d^3}$$
$$-\frac{B^2g^2i^3(c+dx)^3(bc-ad)^3}{60bd^3}+\frac{g^2i^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(bc-ad)^3}{60b^4}$$
$$-\frac{Bg^2i^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(bc-ad)^3}{30b^4}$$
$$+\frac{Bg^2i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(bc-ad)^3}{45bd^3}+\frac{B^2g^2i^3(c+dx)^4(bc-ad)^2}{60d^3}$$
$$+\frac{g^2i^3(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(bc-ad)^2}{20b^3}$$
$$+\frac{7Bg^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(bc-ad)^2}{60d^3}$$
$$+\frac{g^2i^3(a+bx)^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(bc-ad)}{10b^2}$$
$$-\frac{bBg^2i^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(bc-ad)}{15d^3}$$
$$+\frac{g^2i^3(a+bx)^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b}$$

```
[In] Int[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,
x]
```

```
[Out] (-7*B^2*(b*c - a*d)^5*g^2*i^3*x)/(180*b^3*d^2) - (7*B^2*(b*c - a*d)^4*g^2*i
^3*(c + d*x)^2)/(360*b^2*d^3) - (B^2*(b*c - a*d)^3*g^2*i^3*(c + d*x)^3)/(60
*b*d^3) + (B^2*(b*c - a*d)^2*g^2*i^3*(c + d*x)^4)/(60*d^3) + (B^2*(b*c - a*
d)^6*g^2*i^3*Log[(a + b*x)/(c + d*x)])/(36*b^4*d^3) - (B*(b*c - a*d)^4*g^2*
i^3*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(60*b^4*d) - (B*(b*c
- a*d)^3*g^2*i^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(30*b^4)
 - (B*(b*c - a*d)^4*g^2*i^3*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/(10*b^2*d^3) + (B*(b*c - a*d)^3*g^2*i^3*(c + d*x)^3*(A + B*Log[(e*(a + b
*x))/(c + d*x)]))/(45*b*d^3) + (7*B*(b*c - a*d)^2*g^2*i^3*(c + d*x)^4*(A +
B*Log[(e*(a + b*x))/(c + d*x)]))/(60*d^3) - (b*B*(b*c - a*d)*g^2*i^3*(c + d
*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(15*d^3) + ((b*c - a*d)^3*g^2*i
^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(60*b^4) + ((b*c - a
*d)^2*g^2*i^3*(a + b*x)^3*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)
/(20*b^3) + ((b*c - a*d)*g^2*i^3*(a + b*x)^3*(c + d*x)^2*(A + B*Log[(e*(a +
 b*x))/(c + d*x)])^2)/(10*b^2) + (g^2*i^3*(a + b*x)^3*(c + d*x)^3*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)])^2)/(6*b) + (B*(b*c - a*d)^5*g^2*i^3*(a + b*x)*(
2*A + B + 2*B*Log[(e*(a + b*x))/(c + d*x)]))/(60*b^4*d^2) + (B*(b*c - a*d)^
6*g^2*i^3*Log[(b*c - a*d)/(b*(c + d*x))]*(2*A + 3*B + 2*B*Log[(e*(a + b*x))
/(c + d*x)]))/(60*b^4*d^3) + (11*B^2*(b*c - a*d)^6*g^2*i^3*Log[c + d*x])/(1
80*b^4*d^3) + (B^2*(b*c - a*d)^6*g^2*i^3*PolyLog[2, (d*(a + b*x))/(b*(c + d
*x))])/(30*b^4*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
```

```
Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2383

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Dist[(m + q + 2)/(d*(q + 1)), Int[(
f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Dist[b*n*(p/(d*(q +
 1))), Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x]) /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ
[q, -1] && GtQ[m, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left((bc-ad)^6g^2i^3\right)\text{Subst}\left(\int\frac{x^2(A+B\log(ex))^2}{(b-dx)^7}\,dx,x,\frac{a+bx}{c+dx}\right)\\ &= \frac{g^2i^3(a+bx)^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b}\\ &\quad+\frac{\left((bc-ad)^6g^2i^3\right)\text{Subst}\left(\int\frac{x^2(A+B\log(ex))^2}{(b-dx)^6}\,dx,x,\frac{a+bx}{c+dx}\right)}{2b}\\ &\quad-\frac{\left(B(bc-ad)^6g^2i^3\right)\text{Subst}\left(\int\frac{x^2(A+B\log(ex))}{(b-dx)^6}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b}\end{aligned}$$

$$= -\frac{B(bc-ad)^3 g^2 i^3 (c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9bd^3} + \frac{B(bc-ad)^2 g^2 i^3 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^3} - \frac{bB(bc-ad) g^2 i^3 (c+dx)^5 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15d^3} + \frac{(bc-ad) g^2 i^3 (a+bx)^3 (c+dx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^2} + \frac{g^2 i^3 (a+bx)^3 (c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b} + \frac{((bc-ad)^6 g^2 i^3) \operatorname{Subst}\left(\int \frac{x^2(A+B\log(ex))^2}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b^2} - \frac{(B(bc-ad)^6 g^2 i^3) \operatorname{Subst}\left(\int \frac{x^2(A+B\log(ex))}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b^2} + \frac{(B^2(bc-ad)^6 g^2 i^3) \operatorname{Subst}\left(\int \frac{b^2-5bdx+10d^2x^2}{30d^3x(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b}$$

$$= -\frac{B(bc-ad)^4 g^2 i^3 (c+dx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^2d^3}$$
$$+ \frac{B(bc-ad)^3 g^2 i^3 (c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{45bd^3}$$
$$+ \frac{7B(bc-ad)^2 g^2 i^3 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60d^3}$$
$$- \frac{bB(bc-ad) g^2 i^3 (c+dx)^5 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15d^3}$$
$$+ \frac{(bc-ad)^2 g^2 i^3 (a+bx)^3 (c+dx) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^3}$$
$$+ \frac{(bc-ad) g^2 i^3 (a+bx)^3 (c+dx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^2}$$
$$+ \frac{g^2 i^3 (a+bx)^3 (c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b}$$
$$+ \frac{((bc-ad)^6 g^2 i^3)\,\text{Subst}\left(\int \frac{x^2(A+B\log(ex))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{20b^3}$$
$$- \frac{(B(bc-ad)^6 g^2 i^3)\,\text{Subst}\left(\int \frac{x^2(A+B\log(ex))}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{10b^3}$$
$$+ \frac{(B^2(bc-ad)^6 g^2 i^3)\,\text{Subst}\left(\int \frac{b^2-4bdx+6d^2x^2}{12d^3x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b^2}$$
$$+ \frac{(B^2(bc-ad)^6 g^2 i^3)\,\text{Subst}\left(\int \frac{b^2-5bdx+10d^2x^2}{x(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{90bd^3}$$

$$= -\frac{B(bc-ad)^3 g^2 i^3 (a+bx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^4} - \frac{B(bc-ad)^4 g^2 i^3 (c+dx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^2d^3} + \frac{B(bc-ad)^3 g^2 i^3 (c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{45bd^3} + \frac{7B(bc-ad)^2 g^2 i^3 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60d^3} - \frac{bB(bc-ad) g^2 i^3 (c+dx)^5 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15d^3} + \frac{(bc-ad)^3 g^2 i^3 (a+bx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{60b^4} + \frac{(bc-ad)^2 g^2 i^3 (a+bx)^3 (c+dx) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^3} + \frac{(bc-ad) g^2 i^3 (a+bx)^3 (c+dx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^2} + \frac{g^2 i^3 (a+bx)^3 (c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b} - \frac{\left(B(bc-ad)^6 g^2 i^3\right) \text{Subst}\left(\int \frac{x^2(A+B\log(ex))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{30b^4} + \frac{\left(B^2(bc-ad)^6 g^2 i^3\right) \text{Subst}\left(\int \frac{x^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{30b^4} + \frac{\left(B^2(bc-ad)^6 g^2 i^3\right) \text{Subst}\left(\int \frac{b^2-4bdx+6d^2x^2}{x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{60b^2d^3} + \frac{\left(B^2(bc-ad)^6 g^2 i^3\right) \text{Subst}\left(\int \left(\frac{1}{b^3x} + \frac{6bd}{(b-dx)^5} - \frac{9d}{(b-dx)^4} + \frac{d}{b(b-dx)^3} + \frac{d}{b^2(b-dx)^2} + \frac{d}{b^3(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{90bd^3}$$

$$
\begin{aligned}
&= \frac{B^2(bc-ad)^5 g^2 i^3 x}{90 b^3 d^2} + \frac{B^2(bc-ad)^4 g^2 i^3 (c+dx)^2}{180 b^2 d^3} - \frac{B^2(bc-ad)^3 g^2 i^3 (c+dx)^3}{30 b d^3} \\
&+ \frac{B^2(bc-ad)^2 g^2 i^3 (c+dx)^4}{60 d^3} + \frac{B^2(bc-ad)^6 g^2 i^3 \log\left(\frac{a+bx}{c+dx}\right)}{90 b^4 d^3} \\
&- \frac{B(bc-ad)^4 g^2 i^3 (a+bx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60 b^4 d} \\
&- \frac{B(bc-ad)^3 g^2 i^3 (a+bx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30 b^4} \\
&- \frac{B(bc-ad)^4 g^2 i^3 (c+dx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10 b^2 d^3} \\
&+ \frac{B(bc-ad)^3 g^2 i^3 (c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{45 b d^3} \\
&+ \frac{7B(bc-ad)^2 g^2 i^3 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60 d^3} \\
&- \frac{bB(bc-ad) g^2 i^3 (c+dx)^5 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15 d^3} \\
&+ \frac{(bc-ad)^3 g^2 i^3 (a+bx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{60 b^4} \\
&+ \frac{(bc-ad)^2 g^2 i^3 (a+bx)^3 (c+dx) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20 b^3} \\
&+ \frac{(bc-ad) g^2 i^3 (a+bx)^3 (c+dx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10 b^2} \\
&+ \frac{g^2 i^3 (a+bx)^3 (c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b} + \frac{B^2(bc-ad)^6 g^2 i^3 \log(c+dx)}{90 b^4 d^3} \\
&+ \frac{(B^2(bc-ad)^6 g^2 i^3)\,\mathrm{Subst}\left(\int \left(\frac{b^2}{d^2(b-dx)^3} - \frac{2b}{d^2(b-dx)^2} + \frac{1}{d^2(b-dx)}\right) dx, x, \frac{a+bx}{c+dx}\right)}{30 b^4} \\
&+ \frac{(B^2(bc-ad)^6 g^2 i^3)\,\mathrm{Subst}\left(\int \left(\frac{1}{b^2 x} + \frac{3bd}{(b-dx)^4} - \frac{5d}{(b-dx)^3} + \frac{d}{b(b-dx)^2} + \frac{d}{b^2(b-dx)}\right) dx, x, \frac{a+bx}{c+dx}\right)}{60 b^2 d^3} \\
&+ \frac{(B(bc-ad)^6 g^2 i^3)\,\mathrm{Subst}\left(\int \frac{x(2A+B+2B\log(ex))}{(b-dx)^2} dx, x, \frac{a+bx}{c+dx}\right)}{60 b^4 d}
\end{aligned}
$$

$$= -\frac{7B^2(bc-ad)^5g^2i^3x}{180b^3d^2} - \frac{7B^2(bc-ad)^4g^2i^3(c+dx)^2}{360b^2d^3} - \frac{B^2(bc-ad)^3g^2i^3(c+dx)^3}{60bd^3} + \frac{B^2(bc-ad)^2g^2i^3(c+dx)^4}{60d^3} + \frac{B^2(bc-ad)^6g^2i^3\log\left(\frac{a+bx}{c+dx}\right)}{36b^4d^3} - \frac{B(bc-ad)^4g^2i^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^4d} - \frac{B(bc-ad)^3g^2i^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^4} - \frac{B(bc-ad)^4g^2i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^2d^3} + \frac{B(bc-ad)^3g^2i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{45bd^3} + \frac{7B(bc-ad)^2g^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60d^3} - \frac{bB(bc-ad)g^2i^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15d^3} + \frac{(bc-ad)^3g^2i^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{60b^4} + \frac{(bc-ad)^2g^2i^3(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^3} + \frac{(bc-ad)g^2i^3(a+bx)^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^2} + \frac{g^2i^3(a+bx)^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b} + \frac{B(bc-ad)^5g^2i^3(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^4d^2} + \frac{11B^2(bc-ad)^6g^2i^3\log(c+dx)}{180b^4d^3} - \frac{(B(bc-ad)^6g^2i^3)\,\text{Subst}\left(\int\frac{2A+3B+2B\log(ex)}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{60b^4d^2}$$

$$
\begin{aligned}
&= -\frac{7B^2(bc-ad)^5g^2i^3x}{180b^3d^2} - \frac{7B^2(bc-ad)^4g^2i^3(c+dx)^2}{360b^2d^3} \\
&- \frac{B^2(bc-ad)^3g^2i^3(c+dx)^3}{60bd^3} + \frac{B^2(bc-ad)^2g^2i^3(c+dx)^4}{60d^3} \\
&+ \frac{B^2(bc-ad)^6g^2i^3\log\left(\frac{a+bx}{c+dx}\right)}{36b^4d^3} - \frac{B(bc-ad)^4g^2i^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^4d} \\
&- \frac{B(bc-ad)^3g^2i^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^4} \\
&- \frac{B(bc-ad)^4g^2i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^2d^3} \\
&+ \frac{B(bc-ad)^3g^2i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{45bd^3} \\
&+ \frac{7B(bc-ad)^2g^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60d^3} \\
&- \frac{bB(bc-ad)g^2i^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15d^3} \\
&+ \frac{(bc-ad)^3g^2i^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{60b^4} \\
&+ \frac{(bc-ad)^2g^2i^3(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^3} \\
&+ \frac{(bc-ad)g^2i^3(a+bx)^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^2} \\
&+ \frac{g^2i^3(a+bx)^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b} \\
&+ \frac{B(bc-ad)^5g^2i^3(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^4d^2} \\
&+ \frac{B(bc-ad)^6g^2i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^4d^3} \\
&+ \frac{11B^2(bc-ad)^6g^2i^3\log(c+dx)}{180b^4d^3} \\
&- \frac{(B^2(bc-ad)^6g^2i^3)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{30b^4d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{7B^2(bc-ad)^5g^2i^3x}{180b^3d^2} - \frac{7B^2(bc-ad)^4g^2i^3(c+dx)^2}{360b^2d^3} \\
&\quad - \frac{B^2(bc-ad)^3g^2i^3(c+dx)^3}{60bd^3} + \frac{B^2(bc-ad)^2g^2i^3(c+dx)^4}{60d^3} \\
&\quad + \frac{B^2(bc-ad)^6g^2i^3\log\left(\frac{a+bx}{c+dx}\right)}{36b^4d^3} - \frac{B(bc-ad)^4g^2i^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^4d} \\
&\quad - \frac{B(bc-ad)^3g^2i^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^4} \\
&\quad - \frac{B(bc-ad)^4g^2i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^2d^3} \\
&\quad + \frac{B(bc-ad)^3g^2i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{45bd^3} \\
&\quad + \frac{7B(bc-ad)^2g^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60d^3} \\
&\quad - \frac{bB(bc-ad)g^2i^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15d^3} \\
&\quad + \frac{(bc-ad)^3g^2i^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{60b^4} \\
&\quad + \frac{(bc-ad)^2g^2i^3(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^3} \\
&\quad + \frac{(bc-ad)g^2i^3(a+bx)^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^2} \\
&\quad + \frac{g^2i^3(a+bx)^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b} \\
&\quad + \frac{B(bc-ad)^5g^2i^3(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^4d^2} \\
&\quad + \frac{B(bc-ad)^6g^2i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^4d^3} \\
&\quad + \frac{11B^2(bc-ad)^6g^2i^3\log(c+dx)}{180b^4d^3} + \frac{B^2(bc-ad)^6g^2i^3\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{30b^4d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 1555, normalized size of antiderivative = 1.71

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$

$$=\frac{g^2i^3\left(15(bc-ad)^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-24b(bc-ad)(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+1\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x
)])^2,x]
```

```
[Out] (g^2*i^3*(15*(b*c - a*d)^2*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])
^2 - 24*b*(b*c - a*d)*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 +
10*b^2*(c + d*x)^6*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (5*B*(b*c - a*d
)^3*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*(b*d*x + (b*c - a*d)*Log[a
 + b*x]) - B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c -
a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(c +
 d*x)] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
 + 2*b^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*(b*c - a*d)^3
*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(b*c - a*d)^3*Log[
c + d*x] - 3*B*(b*c - a*d)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*
x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/b^4 + (2*
B*(b*c - a*d)^2*(24*A*b*d*(b*c - a*d)^3*x - 12*B*(b*c - a*d)^3*(b*d*x + (b*
c - a*d)*Log[a + b*x]) - 4*B*(b*c - a*d)^2*(2*b*d*(b*c - a*d)*x + b^2*(c +
d*x)^2 + 2*(b*c - a*d)^2*Log[a + b*x]) - B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2
*x + 3*b^2*(b*c - a*d)*(c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Lo
g[a + b*x]) + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] +
 12*b^2*(b*c - a*d)^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 8*
b^3*(b*c - a*d)*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*b^4*(c
 + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 24*(b*c - a*d)^4*Log[a + b
*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 24*B*(b*c - a*d)^4*Log[c + d*x]
- 12*B*(b*c - a*d)^4*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c
 - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/b^4 - (B*(b*c - a
*d)*(120*A*b*d*(b*c - a*d)^4*x - 60*B*(b*c - a*d)^4*(b*d*x + (b*c - a*d)*Lo
g[a + b*x]) - 20*B*(b*c - a*d)^3*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2
*(b*c - a*d)^2*Log[a + b*x]) - 5*B*(b*c - a*d)^2*(6*b*d*(b*c - a*d)^2*x + 3
*b^2*(b*c - a*d)*(c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a +
b*x]) - 2*B*(b*c - a*d)*(12*b*d*(b*c - a*d)^3*x + 6*b^2*(b*c - a*d)^2*(c +
d*x)^2 + 4*b^3*(b*c - a*d)*(c + d*x)^3 + 3*b^4*(c + d*x)^4 + 12*(b*c - a*d)
^4*Log[a + b*x]) + 120*B*d*(b*c - a*d)^4*(a + b*x)*Log[(e*(a + b*x))/(c + d
*x)] + 60*b^2*(b*c - a*d)^3*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
) + 40*b^3*(b*c - a*d)^2*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) +
 30*b^4*(b*c - a*d)*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 24*b
```

```
^5*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 120*(b*c - a*d)^5*Log
[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 120*B*(b*c - a*d)^5*Log[c
+ d*x] - 60*B*(b*c - a*d)^5*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x
))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(6*b^4)))/
(60*d^3)
```

Maple [F]

$$\int (bgx+ag)^2 (dix+ci)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\begin{aligned}&\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx\\&=\int (bgx+ag)^2(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx\end{aligned}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, alg
orithm="fricas")
```

```
[Out] integral(A^2*b^2*d^3*g^2*i^3*x^5 + A^2*a^2*c^3*g^2*i^3 + (3*A^2*b^2*c*d^2 +
 2*A^2*a*b*d^3)*g^2*i^3*x^4 + (3*A^2*b^2*c^2*d + 6*A^2*a*b*c*d^2 + A^2*a^2*
d^3)*g^2*i^3*x^3 + (A^2*b^2*c^3 + 6*A^2*a*b*c^2*d + 3*A^2*a^2*c*d^2)*g^2*i^
3*x^2 + (2*A^2*a*b*c^3 + 3*A^2*a^2*c^2*d)*g^2*i^3*x + (B^2*b^2*d^3*g^2*i^3*
x^5 + B^2*a^2*c^3*g^2*i^3 + (3*B^2*b^2*c*d^2 + 2*B^2*a*b*d^3)*g^2*i^3*x^4 +
 (3*B^2*b^2*c^2*d + 6*B^2*a*b*c*d^2 + B^2*a^2*d^3)*g^2*i^3*x^3 + (B^2*b^2*c
^3 + 6*B^2*a*b*c^2*d + 3*B^2*a^2*c*d^2)*g^2*i^3*x^2 + (2*B^2*a*b*c^3 + 3*B^
2*a^2*c^2*d)*g^2*i^3*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^2*d^3*g^2
*i^3*x^5 + A*B*a^2*c^3*g^2*i^3 + (3*A*B*b^2*c*d^2 + 2*A*B*a*b*d^3)*g^2*i^3*
x^4 + (3*A*B*b^2*c^2*d + 6*A*B*a*b*c*d^2 + A*B*a^2*d^3)*g^2*i^3*x^3 + (A*B*
b^2*c^3 + 6*A*B*a*b*c^2*d + 3*A*B*a^2*c*d^2)*g^2*i^3*x^2 + (2*A*B*a*b*c^3 +
 3*A*B*a^2*c^2*d)*g^2*i^3*x)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**2*(d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5196 vs. $2(869) = 1738$.

Time = 0.36 (sec) , antiderivative size = 5196, normalized size of antiderivative = 5.72

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, alg
orithm="maxima")
[Out] 1/6*A^2*b^2*d^3*g^2*i^3*x^6 + 3/5*A^2*b^2*c*d^2*g^2*i^3*x^5 + 2/5*A^2*a*b*d
^3*g^2*i^3*x^5 + 3/4*A^2*b^2*c^2*d*g^2*i^3*x^4 + 3/2*A^2*a*b*c*d^2*g^2*i^3*
x^4 + 1/4*A^2*a^2*d^3*g^2*i^3*x^4 + 1/3*A^2*b^2*c^3*g^2*i^3*x^3 + 2*A^2*a*b
*c^2*d*g^2*i^3*x^3 + A^2*a^2*c*d^2*g^2*i^3*x^3 + A^2*a*b*c^3*g^2*i^3*x^2 +
3/2*A^2*a^2*c^2*d*g^2*i^3*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) +
 a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a^2*c^3*g^2*i^3 + 2*(x^2*log(b*e*
x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2
- (b*c - a*d)*x/(b*d))*A*B*a*b*c^3*g^2*i^3 + 1/3*(2*x^3*log(b*e*x/(d*x + c)
 + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2
*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*c^3*g^2*i
^3 + 3*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c
^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^2*c^2*d*g^2*i^3 + 2*(2*x^3
*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(
d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2
))*A*B*a*b*c^2*d*g^2*i^3 + 1/4*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c))
- 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d
^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3
))*A*B*b^2*c^2*d*g^2*i^3 + (2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*
a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 -
2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*c*d^2*g^2*i^3 + 1/2*(6*x^4*log(
b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x +
 c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 +
6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*a*b*c*d^2*g^2*i^3 + 1/10*(12*x^5*lo
```

g(b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d
x + c)/d^5 - (3(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4
)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)
)*A*B*b^2*c*d^2*g^2*i^3 + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c))
- 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d
^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3
))*A*B*a^2*d^3*g^2*i^3 + 1/15*(12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c))
+ 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3
*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x
^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*a*b*d^3*g^2*i^3 + 1/180*(60*x
^6*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 60*a^6*log(b*x + a)/b^6 + 60*c^6*
log(d*x + c)/d^6 - (12*(b^5*c*d^4 - a*b^4*d^5)*x^5 - 15*(b^5*c^2*d^3 - a^2*
b^3*d^5)*x^4 + 20*(b^5*c^3*d^2 - a^3*b^2*d^5)*x^3 - 30*(b^5*c^4*d - a^4*b*d
^5)*x^2 + 60*(b^5*c^5 - a^5*d^5)*x)/(b^5*d^5))*A*B*b^2*d^3*g^2*i^3 + A^2*a^
2*c^3*g^2*i^3*x - 1/180*(74*a^3*b^2*c^3*d^3*g^2*i^3 - 33*a^4*b*c^2*d^4*g^2*
i^3 + 6*a^5*c*d^5*g^2*i^3 + 2*(3*g^2*i^3*log(e) - g^2*i^3)*b^5*c^6 - 18*(2*
g^2*i^3*log(e) - g^2*i^3)*a*b^4*c^5*d + 9*(10*g^2*i^3*log(e) - 7*g^2*i^3)*a
^2*b^3*c^4*d^2)*B^2*log(d*x + c)/(b^3*d^3) - 1/30*(b^6*c^6*g^2*i^3 - 6*a*b^
5*c^5*d*g^2*i^3 + 15*a^2*b^4*c^4*d^2*g^2*i^3 - 20*a^3*b^3*c^3*d^3*g^2*i^3 +
15*a^4*b^2*c^2*d^4*g^2*i^3 - 6*a^5*b*c*d^5*g^2*i^3 + a^6*d^6*g^2*i^3)*(log
(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c -
a*d)))*B^2/(b^4*d^3) + 1/360*(60*B^2*b^6*d^6*g^2*i^3*x^6*log(e)^2 + 24*((9*
g^2*i^3*log(e)^2 - g^2*i^3*log(e))*b^6*c*d^5 + (6*g^2*i^3*log(e)^2 + g^2*i^
3*log(e))*a*b^5*d^6)*B^2*x^5 + 6*((45*g^2*i^3*log(e)^2 - 13*g^2*i^3*log(e)
+ g^2*i^3)*b^6*c^2*d^4 + 2*(45*g^2*i^3*log(e)^2 + 3*g^2*i^3*log(e) - g^2*i^
3)*a*b^5*c*d^5 + (15*g^2*i^3*log(e)^2 + 7*g^2*i^3*log(e) + g^2*i^3)*a^2*b^4
*d^6)*B^2*x^4 + 2*((60*g^2*i^3*log(e)^2 - 38*g^2*i^3*log(e) + 9*g^2*i^3)*b^
6*c^3*d^3 + 3*(120*g^2*i^3*log(e)^2 - 14*g^2*i^3*log(e) - 5*g^2*i^3)*a*b^5*
c^2*d^4 + 3*(60*g^2*i^3*log(e)^2 + 26*g^2*i^3*log(e) + g^2*i^3)*a^2*b^4*c*d
^5 + (2*g^2*i^3*log(e) + 3*g^2*i^3)*a^3*b^3*d^6)*B^2*x^3 - ((6*g^2*i^3*log(
e) - 11*g^2*i^3)*b^6*c^4*d^2 - 2*(180*g^2*i^3*log(e)^2 - 102*g^2*i^3*log(e)
+ 5*g^2*i^3)*a*b^5*c^3*d^3 - 60*(9*g^2*i^3*log(e)^2 + 3*g^2*i^3*log(e) - g
^2*i^3)*a^2*b^4*c^2*d^4 - 2*(18*g^2*i^3*log(e) + 23*g^2*i^3)*a^3*b^3*c*d^5
+ (6*g^2*i^3*log(e) + 7*g^2*i^3)*a^4*b^2*d^6)*B^2*x^2 + 2*(2*(3*g^2*i^3*log
(e) - 4*g^2*i^3)*b^6*c^5*d - 3*(12*g^2*i^3*log(e) - 17*g^2*i^3)*a*b^5*c^4*d
^2 + (180*g^2*i^3*log(e)^2 - 30*g^2*i^3*log(e) - 97*g^2*i^3)*a^2*b^4*c^3*d^
3 + (90*g^2*i^3*log(e) + 77*g^2*i^3)*a^3*b^3*c^2*d^4 - 9*(4*g^2*i^3*log(e)
+ 3*g^2*i^3)*a^4*b^2*c*d^5 + 2*(3*g^2*i^3*log(e) + 2*g^2*i^3)*a^5*b*d^6)*B^
2*x + 6*(10*B^2*b^6*d^6*g^2*i^3*x^6 + 60*B^2*a^2*b^4*c^3*d^3*g^2*i^3*x + 12
*(3*b^6*c*d^5*g^2*i^3 + 2*a*b^5*d^6*g^2*i^3)*B^2*x^5 + 15*(3*b^6*c^2*d^4*g^
2*i^3 + 6*a*b^5*c*d^5*g^2*i^3 + a^2*b^4*d^6*g^2*i^3)*B^2*x^4 + 20*(b^6*c^3*
d^3*g^2*i^3 + 6*a*b^5*c^2*d^4*g^2*i^3 + 3*a^2*b^4*c*d^5*g^2*i^3)*B^2*x^3 +
30*(2*a*b^5*c^3*d^3*g^2*i^3 + 3*a^2*b^4*c^2*d^4*g^2*i^3)*B^2*x^2 + (20*a^3*
b^3*c^3*d^3*g^2*i^3 - 15*a^4*b^2*c^2*d^4*g^2*i^3 + 6*a^5*b*c*d^5*g^2*i^3 -
a^6*d^6*g^2*i^3)*B^2)*log(b*x + a)^2 + 6*(10*B^2*b^6*d^6*g^2*i^3*x^6 + 60*B

^2*a^2*b^4*c^3*d^3*g^2*i^3*x + 12*(3*b^6*c*d^5*g^2*i^3 + 2*a*b^5*d^6*g^2*i^
3)*B^2*x^5 + 15*(3*b^6*c^2*d^4*g^2*i^3 + 6*a*b^5*c*d^5*g^2*i^3 + a^2*b^4*d^
6*g^2*i^3)*B^2*x^4 + 20*(b^6*c^3*d^3*g^2*i^3 + 6*a*b^5*c^2*d^4*g^2*i^3 + 3*
a^2*b^4*c*d^5*g^2*i^3)*B^2*x^3 + 30*(2*a*b^5*c^3*d^3*g^2*i^3 + 3*a^2*b^4*c^
2*d^4*g^2*i^3)*B^2*x^2 + (b^6*c^6*g^2*i^3 - 6*a*b^5*c^5*d*g^2*i^3 + 15*a^2*
b^4*c^4*d^2*g^2*i^3)*B^2)*log(d*x + c)^2 + 2*(60*B^2*b^6*d^6*g^2*i^3*x^6*lo
g(e) + 12*((18*g^2*i^3*log(e) - g^2*i^3)*b^6*c*d^5 + (12*g^2*i^3*log(e) + g
^2*i^3)*a*b^5*d^6)*B^2*x^5 + 3*((90*g^2*i^3*log(e) - 13*g^2*i^3)*b^6*c^2*d^
4 + 6*(30*g^2*i^3*log(e) + g^2*i^3)*a*b^5*c*d^5 + (30*g^2*i^3*log(e) + 7*g^
2*i^3)*a^2*b^4*d^6)*B^2*x^4 + 2*(a^3*b^3*d^6*g^2*i^3 + (60*g^2*i^3*log(e) -
19*g^2*i^3)*b^6*c^3*d^3 + 3*(120*g^2*i^3*log(e) - 7*g^2*i^3)*a*b^5*c^2*d^4
+ 3*(60*g^2*i^3*log(e) + 13*g^2*i^3)*a^2*b^4*c*d^5)*B^2*x^3 - 3*(b^6*c^4*d
^2*g^2*i^3 - 6*a^3*b^3*c*d^5*g^2*i^3 + a^4*b^2*d^6*g^2*i^3 - 2*(60*g^2*i^3*
log(e) - 17*g^2*i^3)*a*b^5*c^3*d^3 - 30*(6*g^2*i^3*log(e) + g^2*i^3)*a^2*b^
4*c^2*d^4)*B^2*x^2 + 6*(b^6*c^5*d*g^2*i^3 - 6*a*b^5*c^4*d^2*g^2*i^3 + 15*a^
3*b^3*c^2*d^4*g^2*i^3 - 6*a^4*b^2*c*d^5*g^2*i^3 + a^5*b*d^6*g^2*i^3 + 5*(12
*g^2*i^3*log(e) - g^2*i^3)*a^2*b^4*c^3*d^3)*B^2*x + (6*a*b^5*c^5*d*g^2*i^3
- 33*a^2*b^4*c^4*d^2*g^2*i^3 + 2*(60*g^2*i^3*log(e) + 17*g^2*i^3)*a^3*b^3*c
^3*d^3 - 3*(30*g^2*i^3*log(e) + g^2*i^3)*a^4*b^2*c^2*d^4 + 6*(6*g^2*i^3*log
(e) - g^2*i^3)*a^5*b*c*d^5 - 2*(3*g^2*i^3*log(e) - g^2*i^3)*a^6*d^6)*B^2)*l
og(b*x + a) - 2*(60*B^2*b^6*d^6*g^2*i^3*x^6*log(e) + 12*((18*g^2*i^3*log(e)
- g^2*i^3)*b^6*c*d^5 + (12*g^2*i^3*log(e) + g^2*i^3)*a*b^5*d^6)*B^2*x^5 +
3*((90*g^2*i^3*log(e) - 13*g^2*i^3)*b^6*c^2*d^4 + 6*(30*g^2*i^3*log(e) + g^
2*i^3)*a*b^5*c*d^5 + (30*g^2*i^3*log(e) + 7*g^2*i^3)*a^2*b^4*d^6)*B^2*x^4 +
2*(a^3*b^3*d^6*g^2*i^3 + (60*g^2*i^3*log(e) - 19*g^2*i^3)*b^6*c^3*d^3 + 3*
(120*g^2*i^3*log(e) - 7*g^2*i^3)*a*b^5*c^2*d^4 + 3*(60*g^2*i^3*log(e) + 13*
g^2*i^3)*a^2*b^4*c*d^5)*B^2*x^3 - 3*(b^6*c^4*d^2*g^2*i^3 - 6*a^3*b^3*c*d^5*
g^2*i^3 + a^4*b^2*d^6*g^2*i^3 - 2*(60*g^2*i^3*log(e) - 17*g^2*i^3)*a*b^5*c^
3*d^3 - 30*(6*g^2*i^3*log(e) + g^2*i^3)*a^2*b^4*c^2*d^4)*B^2*x^2 + 6*(b^6*c
^5*d*g^2*i^3 - 6*a*b^5*c^4*d^2*g^2*i^3 + 15*a^3*b^3*c^2*d^4*g^2*i^3 - 6*a^4
*b^2*c*d^5*g^2*i^3 + a^5*b*d^6*g^2*i^3 + 5*(12*g^2*i^3*log(e) - g^2*i^3)*a^
2*b^4*c^3*d^3)*B^2*x + 6*(10*B^2*b^6*d^6*g^2*i^3*x^6 + 60*B^2*a^2*b^4*c^3*d
^3*g^2*i^3*x + 12*(3*b^6*c*d^5*g^2*i^3 + 2*a*b^5*d^6*g^2*i^3)*B^2*x^5 + 15*
(3*b^6*c^2*d^4*g^2*i^3 + 6*a*b^5*c*d^5*g^2*i^3 + a^2*b^4*d^6*g^2*i^3)*B^2*x
^4 + 20*(b^6*c^3*d^3*g^2*i^3 + 6*a*b^5*c^2*d^4*g^2*i^3 + 3*a^2*b^4*c*d^5*g^
2*i^3)*B^2*x^3 + 30*(2*a*b^5*c^3*d^3*g^2*i^3 + 3*a^2*b^4*c^2*d^4*g^2*i^3)*B
^2*x^2 + (20*a^3*b^3*c^3*d^3*g^2*i^3 - 15*a^4*b^2*c^2*d^4*g^2*i^3 + 6*a^5*b
*c*d^5*g^2*i^3 - a^6*d^6*g^2*i^3)*B^2)*log(b*x + a))*log(d*x + c))/(b^4*d^3
)

Giac [F]

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\int (bgx+ag)^2(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, alg
orithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^2*(d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A
)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^2\,(c\,i+d\,i\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

```
[In] int((a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2,
x)
```

```
[Out] int((a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2,
 x)
```

3.76 $\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 40, antiderivative size = 730

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\frac{B^2(bc-ad)^4gi^3x}{60b^3d}+\frac{B^2(bc-ad)^3gi^3(c+dx)^2}{30b^2d^2}+\frac{B^2(bc-ad)^2gi^3(c+dx)^3}{30bd^2}$$
$$-\frac{B^2(bc-ad)^5gi^3\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d^2}-\frac{B(bc-ad)^4gi^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^4d}$$
$$-\frac{B(bc-ad)^3gi^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^4}$$
$$+\frac{3B(bc-ad)^3gi^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^2d^2}$$
$$+\frac{B(bc-ad)^2gi^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^2}$$
$$-\frac{B(bc-ad)gi^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10d^2}$$
$$+\frac{(bc-ad)^3gi^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^4}$$
$$+\frac{(bc-ad)^2gi^3(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^3}$$
$$+\frac{3(bc-ad)gi^3(a+bx)^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^2}$$
$$+\frac{gi^3(a+bx)^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b}$$
$$-\frac{B(bc-ad)^5gi^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^4d^2}$$
$$-\frac{11B^2(bc-ad)^5gi^3\log(c+dx)}{60b^4d^2}-\frac{B^2(bc-ad)^5gi^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{10b^4d^2}$$

```
[Out] 1/60*B^2*(-a*d+b*c)^4*g*i^3*x/b^3/d+1/30*B^2*(-a*d+b*c)^3*g*i^3*(d*x+c)^2/b
^2/d^2+1/30*B^2*(-a*d+b*c)^2*g*i^3*(d*x+c)^3/b/d^2-1/12*B^2*(-a*d+b*c)^5*g*
i^3*ln((b*x+a)/(d*x+c))/b^4/d^2-1/10*B*(-a*d+b*c)^4*g*i^3*(b*x+a)*(A+B*ln(e
*(b*x+a)/(d*x+c)))/b^4/d-1/10*B*(-a*d+b*c)^3*g*i^3*(b*x+a)^2*(A+B*ln(e*(b*x
+a)/(d*x+c)))/b^4+3/20*B*(-a*d+b*c)^3*g*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*
x+c)))/b^2/d^2+1/30*B*(-a*d+b*c)^2*g*i^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c
)))/b/d^2-1/10*B*(-a*d+b*c)*g*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^2
```

```
+1/20*(-a*d+b*c)^3*g*i^3*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^4+1/10*(
-a*d+b*c)^2*g*i^3*(b*x+a)^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^3+3/20*
(-a*d+b*c)*g*i^3*(b*x+a)^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^2+1/5*
g*i^3*(b*x+a)^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b-1/10*B*(-a*d+b*c)
^5*g*i^3*ln((-a*d+b*c)/b/(d*x+c))*(A+B+B*ln(e*(b*x+a)/(d*x+c)))/b^4/d^2-11/
60*B^2*(-a*d+b*c)^5*g*i^3*ln(d*x+c)/b^4/d^2-1/10*B^2*(-a*d+b*c)^5*g*i^3*pol
ylog(2,d*(b*x+a)/b/(d*x+c))/b^4/d^2
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 730, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules

used = {2562, 2383, 2381, 2384, 2354, 2438, 2373, 45, 2382, 12, 78}

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= -\frac{Bgi^3(bc-ad)^5\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A+B\right)}{10b^4d^2} - \frac{Bgi^3(a+bx)(bc-ad)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{10b^4d} + \frac{gi^3(a+bx)^2(bc-ad)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{20b^4} - \frac{Bgi^3(a+bx)^2(bc-ad)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{10b^4} + \frac{gi^3(a+bx)^2(c+dx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{10b^3} + \frac{3Bgi^3(c+dx)^2(bc-ad)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{20b^2d^2} + \frac{3gi^3(a+bx)^2(c+dx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{20b^2} + \frac{Bgi^3(c+dx)^3(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{30bd^2} - \frac{Bgi^3(c+dx)^4(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{10d^2} + \frac{gi^3(a+bx)^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b} - \frac{B^2gi^3(bc-ad)^5\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{10b^4d^2} - \frac{B^2gi^3(bc-ad)^5\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d^2} - \frac{11B^2gi^3(bc-ad)^5\log(c+dx)}{60b^4d^2} + \frac{B^2gi^3x(bc-ad)^4}{60b^3d} + \frac{B^2gi^3(c+dx)^2(bc-ad)^3}{30b^2d^2} + \frac{B^2gi^3(c+dx)^3(bc-ad)^2}{30bd^2}$$

```
[In] Int[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^4*g*i^3*x)/(60*b^3*d) + (B^2*(b*c - a*d)^3*g*i^3*(c + d*x)
^2)/(30*b^2*d^2) + (B^2*(b*c - a*d)^2*g*i^3*(c + d*x)^3)/(30*b*d^2) - (B^2*
(b*c - a*d)^5*g*i^3*Log[(a + b*x)/(c + d*x)])/(12*b^4*d^2) - (B*(b*c - a*d)
^4*g*i^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(10*b^4*d) - (B*(b
*c - a*d)^3*g*i^3*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(10*b^4
) + (3*B*(b*c - a*d)^3*g*i^3*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)
]))/(20*b^2*d^2) + (B*(b*c - a*d)^2*g*i^3*(c + d*x)^3*(A + B*Log[(e*(a + b*
```

```
x))/(c + d*x)]))/(30*b*d^2) - (B*(b*c - a*d)*g*i^3*(c + d*x)^4*(A + B*Log[(
e*(a + b*x))/(c + d*x)]))/(10*d^2) + ((b*c - a*d)^3*g*i^3*(a + b*x)^2*(A +
B*Log[(e*(a + b*x))/(c + d*x)])^2)/(20*b^4) + ((b*c - a*d)^2*g*i^3*(a + b*x
)^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(10*b^3) + (3*(b*c -
a*d)*g*i^3*(a + b*x)^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/
(20*b^2) + (g*i^3*(a + b*x)^2*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x
)])^2)/(5*b) - (B*(b*c - a*d)^5*g*i^3*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B
 + B*Log[(e*(a + b*x))/(c + d*x)]))/(10*b^4*d^2) - (11*B^2*(b*c - a*d)^5*g*
i^3*Log[c + d*x])/(60*b^4*d^2) - (B^2*(b*c - a*d)^5*g*i^3*PolyLog[2, (d*(a
+ b*x))/(b*(c + d*x))])/(10*b^4*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2383

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Dist[(m + q + 2)/(d*(q + 1)), Int[(
f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Dist[b*n*(p/(d*(q +
 1))), Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x]) /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ
[q, -1] && GtQ[m, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
```

`tegersQ[m, q]`

Rubi steps

$$\text{integral} = \left((bc-ad)^5 gi^3\right) \text{Subst}\left(\int \frac{x(A+B\log(ex))^2}{(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{gi^3(a+bx)^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} + \frac{\left(3(bc-ad)^5gi^3\right)\text{Subst}\left(\int\frac{x(A+B\log(ex))^2}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b} - \frac{\left(2B(bc-ad)^5gi^3\right)\text{Subst}\left(\int\frac{x(A+B\log(ex))}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b}$$

$$= \frac{2B(bc-ad)^2gi^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15bd^2} - \frac{B(bc-ad)gi^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10d^2} + \frac{3(bc-ad)gi^3(a+bx)^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^2} + \frac{gi^3(a+bx)^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} + \frac{\left(3(bc-ad)^5gi^3\right)\text{Subst}\left(\int\frac{x(A+B\log(ex))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{10b^2} - \frac{\left(3B(bc-ad)^5gi^3\right)\text{Subst}\left(\int\frac{x(A+B\log(ex))}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{10b^2} + \frac{\left(2B^2(bc-ad)^5gi^3\right)\text{Subst}\left(\int\frac{-b+4dx}{12d^2x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b}$$

$$= \frac{3B(bc-ad)^3gi^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^2d^2} + \frac{B(bc-ad)^2gi^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^2} - \frac{B(bc-ad)gi^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10d^2} + \frac{(bc-ad)^2gi^3(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^3} + \frac{3(bc-ad)gi^3(a+bx)^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^2} + \frac{gi^3(a+bx)^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} + \frac{((bc-ad)^5gi^3)\,\mathrm{Subst}\left(\int\frac{x(A+B\log(ex))^2}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{10b^3} - \frac{(B(bc-ad)^5gi^3)\,\mathrm{Subst}\left(\int\frac{x(A+B\log(ex))}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{5b^3} + \frac{(3B^2(bc-ad)^5gi^3)\,\mathrm{Subst}\left(\int\frac{-b+3dx}{6d^2x(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{10b^2} + \frac{(B^2(bc-ad)^5gi^3)\,\mathrm{Subst}\left(\int\frac{-b+4dx}{x(b-dx)^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{30bd^2}$$

$$= -\frac{B(bc-ad)^3gi^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^4} + \frac{3B(bc-ad)^3gi^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^2d^2} + \frac{B(bc-ad)^2gi^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^2} - \frac{B(bc-ad)gi^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10d^2} + \frac{(bc-ad)^3gi^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^4} + \frac{(bc-ad)^2gi^3(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^3} + \frac{3(bc-ad)gi^3(a+bx)^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^2} + \frac{gi^3(a+bx)^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} - \frac{(B(bc-ad)^5gi^3)\,\text{Subst}\left(\int\frac{x(A+B\log(ex))}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{10b^4} + \frac{(B^2(bc-ad)^5gi^3)\,\text{Subst}\left(\int\frac{x}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{10b^4} + \frac{(B^2(bc-ad)^5gi^3)\,\text{Subst}\left(\int\frac{-b+3dx}{x(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{20b^2d^2} + \frac{(B^2(bc-ad)^5gi^3)\,\text{Subst}\left(\int\left(-\frac{1}{b^3x}+\frac{3d}{(b-dx)^4}-\frac{d}{b(b-dx)^3}-\frac{d}{b^2(b-dx)^2}-\frac{d}{b^3(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{30bd^2}$$

$$= -\frac{B^2(bc-ad)^4gi^3x}{30b^3d} - \frac{B^2(bc-ad)^3gi^3(c+dx)^2}{60b^2d^2} + \frac{B^2(bc-ad)^2gi^3(c+dx)^3}{30bd^2} - \frac{B^2(bc-ad)^5gi^3\log\left(\frac{a+bx}{c+dx}\right)}{30b^4d^2} - \frac{B(bc-ad)^4gi^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^4d} - \frac{B(bc-ad)^3gi^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^4} + \frac{3B(bc-ad)^3gi^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^2d^2} + \frac{B(bc-ad)^2gi^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^2} - \frac{B(bc-ad)gi^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10d^2} + \frac{(bc-ad)^3gi^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^4} + \frac{(bc-ad)^2gi^3(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^3} + \frac{3(bc-ad)gi^3(a+bx)^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^2} + \frac{gi^3(a+bx)^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} - \frac{B^2(bc-ad)^5gi^3\log(c+dx)}{30b^4d^2} + \frac{(B^2(bc-ad)^5gi^3)\operatorname{Subst}\left(\int\left(\frac{b}{d(-b+dx)^2}+\frac{1}{d(-b+dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{10b^4} + \frac{(B^2(bc-ad)^5gi^3)\operatorname{Subst}\left(\int\left(-\frac{1}{b^2x}+\frac{2d}{(b-dx)^3}-\frac{d}{b(b-dx)^2}-\frac{d}{b^2(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{20b^2d^2} + \frac{(B(bc-ad)^5gi^3)\operatorname{Subst}\left(\int\frac{A+B+B\log(ex)}{b-dx}dx,x,\frac{a+bx}{c+dx}\right)}{10b^4d}$$

$$= \frac{B^2(bc-ad)^4 gi^3 x}{60b^3 d} + \frac{B^2(bc-ad)^3 gi^3 (c+dx)^2}{30b^2 d^2} + \frac{B^2(bc-ad)^2 gi^3 (c+dx)^3}{30bd^2} - \frac{B^2(bc-ad)^5 gi^3 \log\left(\frac{a+bx}{c+dx}\right)}{12b^4 d^2} - \frac{B(bc-ad)^4 gi^3 (a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^4 d} - \frac{B(bc-ad)^3 gi^3 (a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^4} + \frac{3B(bc-ad)^3 gi^3 (c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^2 d^2} + \frac{B(bc-ad)^2 gi^3 (c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^2} - \frac{B(bc-ad) gi^3 (c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10d^2} + \frac{(bc-ad)^3 gi^3 (a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^4} + \frac{(bc-ad)^2 gi^3 (a+bx)^2 (c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^3} + \frac{3(bc-ad) gi^3 (a+bx)^2 (c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^2} + \frac{gi^3 (a+bx)^2 (c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} - \frac{B(bc-ad)^5 gi^3 \log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^4 d^2} - \frac{11B^2(bc-ad)^5 gi^3 \log(c+dx)}{60b^4 d^2} + \frac{(B^2(bc-ad)^5 gi^3)\,\mathrm{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{10b^4 d^2}$$

$$= \frac{B^2(bc-ad)^4gi^3x}{60b^3d} + \frac{B^2(bc-ad)^3gi^3(c+dx)^2}{30b^2d^2} + \frac{B^2(bc-ad)^2gi^3(c+dx)^3}{30bd^2} - \frac{B^2(bc-ad)^5gi^3\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d^2} - \frac{B(bc-ad)^4gi^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^4d} - \frac{B(bc-ad)^3gi^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^4} + \frac{3B(bc-ad)^3gi^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^2d^2} + \frac{B(bc-ad)^2gi^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^2} - \frac{B(bc-ad)gi^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10d^2} + \frac{(bc-ad)^3gi^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^4} + \frac{(bc-ad)^2gi^3(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^3} + \frac{3(bc-ad)gi^3(a+bx)^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^2} + \frac{gi^3(a+bx)^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} - \frac{B(bc-ad)^5gi^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^4d^2} - \frac{11B^2(bc-ad)^5gi^3\log(c+dx)}{60b^4d^2} - \frac{B^2(bc-ad)^5gi^3\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{10b^4d^2}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 901, normalized size of antiderivative = 1.23

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \frac{gi^3\left(-5(bc-ad)(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+4b(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{5B(bc-ad)^2\left(6Abd(\right.}{}\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)^2,x]
```

[Out] (g*i^3*(-5*(b*c - a*d)*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 4*b*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (5*B*(b*c - a*d)^2*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*(b*d*x + (b*c - a*d)*Log[a + b*x]) - B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*b^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*(b*c - a*d)^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(b*c - a*d)^3*Log[c + d*x] - 3*B*(b*c - a*d)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(3*b^4) - (B*(b*c - a*d)*(24*A*b*d*(b*c - a*d)^3*x - 12*B*(b*c - a*d)^3*(b*d*x + (b*c - a*d)*Log[a + b*x]) - 4*B*(b*c - a*d)^2*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c - a*d)^2*Log[a + b*x]) - B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]) + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 12*b^2*(b*c - a*d)^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 8*b^3*(b*c - a*d)*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*b^4*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 24*(b*c - a*d)^4*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 24*B*(b*c - a*d)^4*Log[c + d*x] - 12*B*(b*c - a*d)^4*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(3*b^4)))/(20*d^2)

Maple [F]

$$\int (bgx+ag)\,(dix+ci)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

[In] int((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)

[Out] int((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)

Fricas [F]

$$\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")

[Out] integral(A^2*b*d^3*g*i^3*x^4 + A^2*a*c^3*g*i^3 + (3*A^2*b*c*d^2 + A^2*a*d^3)*g*i^3*x^3 + 3*(A^2*b*c^2*d + A^2*a*c*d^2)*g*i^3*x^2 + (A^2*b*c^3 + 3*A^2*a*c^2*d)*g*i^3*x + (B^2*b*d^3*g*i^3*x^4 + B^2*a*c^3*g*i^3 + (3*B^2*b*c*d^2

```
+ B^2*a*d^3)*g*i^3*x^3 + 3*(B^2*b*c^2*d + B^2*a*c*d^2)*g*i^3*x^2 + (B^2*b*c
^3 + 3*B^2*a*c^2*d)*g*i^3*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b*d^3*
g*i^3*x^4 + A*B*a*c^3*g*i^3 + (3*A*B*b*c*d^2 + A*B*a*d^3)*g*i^3*x^3 + 3*(A*
B*b*c^2*d + A*B*a*c*d^2)*g*i^3*x^2 + (A*B*b*c^3 + 3*A*B*a*c^2*d)*g*i^3*x)*l
og((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3218 vs. $2(697) = 1394$.

Time = 0.32 (sec) , antiderivative size = 3218, normalized size of antiderivative = 4.41

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algor
ithm="maxima")
```

```
[Out] 1/5*A^2*b*d^3*g*i^3*x^5 + 3/4*A^2*b*c*d^2*g*i^3*x^4 + 1/4*A^2*a*d^3*g*i^3*x
^4 + A^2*b*c^2*d*g*i^3*x^3 + A^2*a*c*d^2*g*i^3*x^3 + 1/2*A^2*b*c^3*g*i^3*x^
2 + 3/2*A^2*a*c^2*d*g*i^3*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) +
 a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a*c^3*g*i^3 + (x^2*log(b*e*x/(d*x
 + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c
 - a*d)*x/(b*d))*A*B*b*c^3*g*i^3 + 3*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x +
c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*
B*a*c^2*d*g*i^3 + (2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b
*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^
2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b*c^2*d*g*i^3 + (2*x^3*log(b*e*x/(d*x + c) +
 a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c
*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a*c*d^2*g*i^3 +
 1/4*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 +
 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a
^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b*c*d^2*g*i^3 + 1/1
2*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*
```

c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*
b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*a*d^3*g*i^3 + 1/30*(12
*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^
5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*
b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b
^4*d^4))*A*B*b*d^3*g*i^3 + A^2*a*c^3*g*i^3*x - 1/60*(47*a^2*b^2*c^3*d^2*g*i
^3 - 27*a^3*b*c^2*d^3*g*i^3 + 6*a^4*c*d^4*g*i^3 - (6*g*i^3*log(e) - 5*g*i^3
)*b^4*c^5 + (30*g*i^3*log(e) - 31*g*i^3)*a*b^3*c^4*d)*B^2*log(d*x + c)/(b^3
d^2) + 1/10(b^5*c^5*g*i^3 - 5*a*b^4*c^4*d*g*i^3 + 10*a^2*b^3*c^3*d^2*g*i^
3 - 10*a^3*b^2*c^2*d^3*g*i^3 + 5*a^4*b*c*d^4*g*i^3 - a^5*d^5*g*i^3)*(log(b*
x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d
)))*B^2/(b^4*d^2) + 1/60*(12*B^2*b^5*d^5*g*i^3*x^5*log(e)^2 + 3*((15*g*i^3*
log(e)^2 - 2*g*i^3*log(e))*b^5*c*d^4 + (5*g*i^3*log(e)^2 + 2*g*i^3*log(e))*
a*b^4*d^5)*B^2*x^4 + 2*((30*g*i^3*log(e)^2 - 11*g*i^3*log(e) + g*i^3)*b^5*c
^2*d^3 + 2*(15*g*i^3*log(e)^2 + 5*g*i^3*log(e) - g*i^3)*a*b^4*c*d^4 + (g*i^
3*log(e) + g*i^3)*a^2*b^3*d^5)*B^2*x^3 + ((30*g*i^3*log(e)^2 - 27*g*i^3*log
(e) + 8*g*i^3)*b^5*c^3*d^2 + 3*(30*g*i^3*log(e)^2 + 5*g*i^3*log(e) - 6*g*i^
3)*a*b^4*c^2*d^3 + 3*(5*g*i^3*log(e) + 4*g*i^3)*a^2*b^3*c*d^4 - (3*g*i^3*lo
g(e) + 2*g*i^3)*a^3*b^2*d^5)*B^2*x^2 - ((6*g*i^3*log(e) - 11*g*i^3)*b^5*c^4
d - 2(30*g*i^3*log(e)^2 - 15*g*i^3*log(e) - 14*g*i^3)*a*b^4*c^3*d^2 - 12*
(5*g*i^3*log(e) + 2*g*i^3)*a^2*b^3*c^2*d^3 + 2*(15*g*i^3*log(e) + 4*g*i^3)*
a^3*b^2*c*d^4 - (6*g*i^3*log(e) + g*i^3)*a^4*b*d^5)*B^2*x + 3*(4*B^2*b^5*d^
5*g*i^3*x^5 + 20*B^2*a*b^4*c^3*d^2*g*i^3*x + 5*(3*b^5*c*d^4*g*i^3 + a*b^4*d
^5*g*i^3)*B^2*x^4 + 20*(b^5*c^2*d^3*g*i^3 + a*b^4*c*d^4*g*i^3)*B^2*x^3 + 10
*(b^5*c^3*d^2*g*i^3 + 3*a*b^4*c^2*d^3*g*i^3)*B^2*x^2 + (10*a^2*b^3*c^3*d^2*
g*i^3 - 10*a^3*b^2*c^2*d^3*g*i^3 + 5*a^4*b*c*d^4*g*i^3 - a^5*d^5*g*i^3)*B^2
)*log(b*x + a)^2 + 3*(4*B^2*b^5*d^5*g*i^3*x^5 + 20*B^2*a*b^4*c^3*d^2*g*i^3*
x + 5*(3*b^5*c*d^4*g*i^3 + a*b^4*d^5*g*i^3)*B^2*x^4 + 20*(b^5*c^2*d^3*g*i^3
 + a*b^4*c*d^4*g*i^3)*B^2*x^3 + 10*(b^5*c^3*d^2*g*i^3 + 3*a*b^4*c^2*d^3*g*i
^3)*B^2*x^2 - (b^5*c^5*g*i^3 - 5*a*b^4*c^4*d*g*i^3)*B^2)*log(d*x + c)^2 + (
24*B^2*b^5*d^5*g*i^3*x^5*log(e) + 6*((15*g*i^3*log(e) - g*i^3)*b^5*c*d^4 +
(5*g*i^3*log(e) + g*i^3)*a*b^4*d^5)*B^2*x^4 + 2*(a^2*b^3*d^5*g*i^3 + (60*g*
i^3*log(e) - 11*g*i^3)*b^5*c^2*d^3 + 10*(6*g*i^3*log(e) + g*i^3)*a*b^4*c*d^
4)*B^2*x^3 + 3*(5*a^2*b^3*c*d^4*g*i^3 - a^3*b^2*d^5*g*i^3 + (20*g*i^3*log(e
) - 9*g*i^3)*b^5*c^3*d^2 + 5*(12*g*i^3*log(e) + g*i^3)*a*b^4*c^2*d^3)*B^2*x
^2 - 6*(b^5*c^4*d*g*i^3 - 10*a^2*b^3*c^2*d^3*g*i^3 + 5*a^3*b^2*c*d^4*g*i^3
- a^4*b*d^5*g*i^3 - 5*(4*g*i^3*log(e) - g*i^3)*a*b^4*c^3*d^2)*B^2*x - (6*a*
b^4*c^4*d*g*i^3 - 3*(20*g*i^3*log(e) - g*i^3)*a^2*b^3*c^3*d^2 + (60*g*i^3*l
og(e) - 23*g*i^3)*a^3*b^2*c^2*d^3 - (30*g*i^3*log(e) - 19*g*i^3)*a^4*b*c*d^
4 + (6*g*i^3*log(e) - 5*g*i^3)*a^5*d^5)*B^2)*log(b*x + a) - (24*B^2*b^5*d^5
*g*i^3*x^5*log(e) + 6*((15*g*i^3*log(e) - g*i^3)*b^5*c*d^4 + (5*g*i^3*log(e
) + g*i^3)*a*b^4*d^5)*B^2*x^4 + 2*(a^2*b^3*d^5*g*i^3 + (60*g*i^3*log(e) - 1
1*g*i^3)*b^5*c^2*d^3 + 10*(6*g*i^3*log(e) + g*i^3)*a*b^4*c*d^4)*B^2*x^3 + 3
*(5*a^2*b^3*c*d^4*g*i^3 - a^3*b^2*d^5*g*i^3 + (20*g*i^3*log(e) - 9*g*i^3)*b
^5*c^3*d^2 + 5*(12*g*i^3*log(e) + g*i^3)*a*b^4*c^2*d^3)*B^2*x^2 - 6*(b^5*c^

```
4*d*g*i^3 - 10*a^2*b^3*c^2*d^3*g*i^3 + 5*a^3*b^2*c*d^4*g*i^3 - a^4*b*d^5*g*
i^3 - 5*(4*g*i^3*log(e) - g*i^3)*a*b^4*c^3*d^2)*B^2*x + 6*(4*B^2*b^5*d^5*g*
i^3*x^5 + 20*B^2*a*b^4*c^3*d^2*g*i^3*x + 5*(3*b^5*c*d^4*g*i^3 + a*b^4*d^5*g
*i^3)*B^2*x^4 + 20*(b^5*c^2*d^3*g*i^3 + a*b^4*c*d^4*g*i^3)*B^2*x^3 + 10*(b^
5*c^3*d^2*g*i^3 + 3*a*b^4*c^2*d^3*g*i^3)*B^2*x^2 + (10*a^2*b^3*c^3*d^2*g*i^
3 - 10*a^3*b^2*c^2*d^3*g*i^3 + 5*a^4*b*c*d^4*g*i^3 - a^5*d^5*g*i^3)*B^2)*lo
g(b*x + a))*log(d*x + c))/(b^4*d^2)
```

Giac [F]

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\int (bgx+ag)(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algor
ithm="giac")
```

```
[Out] integrate((b*g*x + a*g)*(d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^
2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)\,\left(c\,i+d\,i\,x\right)^3\,\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)^2\,dx$$

```
[In] int((a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

```
[Out] int((a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x
)
```

3.77 $\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

Optimal result

Integrand size = 32, antiderivative size = 420

$$\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \frac{5B^2(bc-ad)^3i^3x}{12b^3}+\frac{B^2(bc-ad)^2i^3(c+dx)^2}{12b^2d}+\frac{5B^2(bc-ad)^4i^3\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d} - \frac{B(bc-ad)^3i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^4} - \frac{B(bc-ad)^2i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b^2d} - \frac{B(bc-ad)i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6bd}+\frac{i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4d} + \frac{11B^2(bc-ad)^4i^3\log(c+dx)}{12b^4d}+\frac{B(bc-ad)^4i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d} - \frac{B^2(bc-ad)^4i^3\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d}$$

```
[Out] 5/12*B^2*(-a*d+b*c)^3*i^3*x/b^3+1/12*B^2*(-a*d+b*c)^2*i^3*(d*x+c)^2/b^2/d+5
/12*B^2*(-a*d+b*c)^4*i^3*ln((b*x+a)/(d*x+c))/b^4/d-1/2*B*(-a*d+b*c)^3*i^3*(
b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4-1/4*B*(-a*d+b*c)^2*i^3*(d*x+c)^2*(A+
B*ln(e*(b*x+a)/(d*x+c)))/b^2/d-1/6*B*(-a*d+b*c)*i^3*(d*x+c)^3*(A+B*ln(e*(b*
x+a)/(d*x+c)))/b/d+1/4*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d+11/12*
B^2*(-a*d+b*c)^4*i^3*ln(d*x+c)/b^4/d+1/2*B*(-a*d+b*c)^4*i^3*(A+B*ln(e*(b*x+
a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/d-1/2*B^2*(-a*d+b*c)^4*i^3*polyl
og(2,b*(d*x+c)/d/(b*x+a))/b^4/d
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2552, 2356, 2389, 2379, 2438, 2351, 31, 46}

$$\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{Bi^3(bc-ad)^4 \log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b^4d} - \frac{Bi^3(a+bx)(bc-ad)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b^4} - \frac{Bi^3(c+dx)^2(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b^2d} - \frac{Bi^3(c+dx)^3(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6bd} + \frac{i^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d} - \frac{B^2i^3(bc-ad)^4\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d} + \frac{5B^2i^3(bc-ad)^4\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d} + \frac{11B^2i^3(bc-ad)^4\log(c+dx)}{12b^4d} + \frac{5B^2i^3x(bc-ad)^3}{12b^3} + \frac{B^2i^3(c+dx)^2(bc-ad)^2}{12b^2d}$$

```
[In] Int[(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (5*B^2*(b*c - a*d)^3*i^3*x)/(12*b^3) + (B^2*(b*c - a*d)^2*i^3*(c + d*x)^2)/
(12*b^2*d) + (5*B^2*(b*c - a*d)^4*i^3*Log[(a + b*x)/(c + d*x)])/(12*b^4*d)
- (B*(b*c - a*d)^3*i^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*b
^4) - (B*(b*c - a*d)^2*i^3*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
)/(4*b^2*d) - (B*(b*c - a*d)*i^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c +
d*x)]))/(6*b*d) + (i^3*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/
(4*d) + (11*B^2*(b*c - a*d)^4*i^3*Log[c + d*x])/(12*b^4*d) + (B*(b*c - a*d)
^4*i^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a + b
*x))])/(2*b^4*d) - (B^2*(b*c - a*d)^4*i^3*PolyLog[2, (b*(c + d*x))/(d*(a +
b*x))])/(2*b^4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] && !IGtQ[q, 0]) || (EqQ[p, 2] &&
NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\text{integral} = \left((bc-ad)^4 i^3\right) \text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4d} - \frac{\left(B(bc-ad)^4 i^3\right)\text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{2d}$$

$$= \frac{i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4d} - \frac{\left(B(bc-ad)^4 i^3\right)\text{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b} - \frac{\left(B(bc-ad)^4 i^3\right)\text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{2bd}$$

$$= -\frac{B(bc-ad)i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6bd} + \frac{i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4d} - \frac{\left(B(bc-ad)^4 i^3\right)\text{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b^2} - \frac{\left(B(bc-ad)^4 i^3\right)\text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b^2d} + \frac{\left(B^2(bc-ad)^4 i^3\right)\text{Subst}\left(\int \frac{1}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{6bd}$$

$$
\begin{aligned}
&= -\frac{B(bc-ad)^2 i^3 (c+dx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b^2 d} \\
&\quad - \frac{B(bc-ad) i^3 (c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6bd} \\
&\quad + \frac{i^3 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4d} \\
&\quad - \frac{(B(bc-ad)^4 i^3)\,\mathrm{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b^3} \\
&\quad - \frac{(B(bc-ad)^4 i^3)\,\mathrm{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b^3 d} \\
&\quad + \frac{(B^2(bc-ad)^4 i^3)\,\mathrm{Subst}\left(\int \frac{1}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{4b^2 d} \\
&\quad + \frac{(B^2(bc-ad)^4 i^3)\,\mathrm{Subst}\left(\int \left(\frac{1}{b^3 x} + \frac{d}{b(b-dx)^3} + \frac{d}{b^2(b-dx)^2} + \frac{d}{b^3(b-dx)}\right) dx, x, \frac{a+bx}{c+dx}\right)}{6bd} \\
&= \frac{B^2(bc-ad)^3 i^3 x}{6b^3} + \frac{B^2(bc-ad)^2 i^3 (c+dx)^2}{12b^2 d} + \frac{B^2(bc-ad)^4 i^3 \log\left(\frac{a+bx}{c+dx}\right)}{6b^4 d} \\
&\quad - \frac{B(bc-ad)^3 i^3 (a+bx) \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^4} \\
&\quad - \frac{B(bc-ad)^2 i^3 (c+dx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b^2 d} \\
&\quad - \frac{B(bc-ad) i^3 (c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6bd} \\
&\quad + \frac{i^3 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4d} + \frac{B^2(bc-ad)^4 i^3 \log(c+dx)}{6b^4 d} \\
&\quad + \frac{B(bc-ad)^4 i^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4 d} \\
&\quad + \frac{(B^2(bc-ad)^4 i^3)\,\mathrm{Subst}\left(\int \frac{1}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b^4} \\
&\quad - \frac{(B^2(bc-ad)^4 i^3)\,\mathrm{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b^4 d} \\
&\quad + \frac{(B^2(bc-ad)^4 i^3)\,\mathrm{Subst}\left(\int \left(\frac{1}{b^2 x} + \frac{d}{b(b-dx)^2} + \frac{d}{b^2(b-dx)}\right) dx, x, \frac{a+bx}{c+dx}\right)}{4b^2 d}
\end{aligned}
$$

$$= \frac{5B^2(bc-ad)^3i^3x}{12b^3} + \frac{B^2(bc-ad)^2i^3(c+dx)^2}{12b^2d} + \frac{5B^2(bc-ad)^4i^3\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d} - \frac{B(bc-ad)^3i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^4} - \frac{B(bc-ad)^2i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b^2d} - \frac{B(bc-ad)i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6bd} + \frac{i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4d} + \frac{11B^2(bc-ad)^4i^3\log(c+dx)}{12b^4d} + \frac{B(bc-ad)^4i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d} - \frac{B^2(bc-ad)^4i^3\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.93

$$\int (ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \frac{i^3\left((c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - \frac{B(bc-ad)\left(6Abd(bc-ad)^2x-3B(bc-ad)^2(bdx+(bc-ad)\log(a+bx))-B(bc-ad)\left(2bd(bc-ad)\right.\right.}{}\right.}{}$$

```
[In] Integrate[(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (i^3*((c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(b*c - a*d)*(
6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*(b*d*x + (b*c - a*d)*Log[a + b*
x]) - B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c - a*d)^
2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)
] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*
b^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*(b*c - a*d)^3*Log[
a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(b*c - a*d)^3*Log[c + d
*x] - 3*B*(b*c - a*d)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(
b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(3*b^4)))/(4*d
)
```

Maple [F]

$$\int (dix + ci)^3 \left(A + B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

```
[In] int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

Fricas [F]

$$\int (ci + dix)^3 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (dix + ci)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2 dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas
")
```

```
[Out] integral(A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*c^
3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2*c^
3*i^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*d^2*i^
3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log((b*e*x + a*e)/(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ci + dix)^3 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1789 vs. $2(399) = 798$.

Time = 0.29 (sec) , antiderivative size = 1789, normalized size of antiderivative = 4.26

$$\int (ci + dix)^3 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima
")
```

```
[Out] 1/4*A^2*d^3*i^3*x^4 + A^2*c*d^2*i^3*x^3 + 3/2*A^2*c^2*d*i^3*x^2 + 2*(x*log(
b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B
*c^3*i^3 + 3*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b
^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*c^2*d*i^3 + (2*x^3*log
(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x
+ c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A
*B*c*d^2*i^3 + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log
(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*d^3*
i^3 + A^2*c^3*i^3*x - 1/12*(26*a*b^2*c^3*d*i^3 - 21*a^2*b*c^2*d^2*i^3 + 6*a
^3*c*d^3*i^3 + (6*i^3*log(e) - 11*i^3)*b^3*c^4)*B^2*log(d*x + c)/(b^3*d) -
1/2*(b^4*c^4*i^3 - 4*a*b^3*c^3*d*i^3 + 6*a^2*b^2*c^2*d^2*i^3 - 4*a^3*b*c*d^
3*i^3 + a^4*d^4*i^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dil
og(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^4*d) + 1/12*(3*B^2*b^4*d^4*i^3*x^4*l
og(e)^2 + 2*(a*b^3*d^4*i^3*log(e) + (6*i^3*log(e)^2 - i^3*log(e))*b^4*c*d^3
)*B^2*x^3 + ((18*i^3*log(e)^2 - 9*i^3*log(e) + i^3)*b^4*c^2*d^2 + 2*(6*i^3*
log(e) - i^3)*a*b^3*c*d^3 - (3*i^3*log(e) - i^3)*a^2*b^2*d^4)*B^2*x^2 + ((1
2*i^3*log(e)^2 - 18*i^3*log(e) + 7*i^3)*b^4*c^3*d + (36*i^3*log(e) - 19*i^3
)*a*b^3*c^2*d^2 - (24*i^3*log(e) - 17*i^3)*a^2*b^2*c*d^3 + (6*i^3*log(e) -
5*i^3)*a^3*b*d^4)*B^2*x + 3*(B^2*b^4*d^4*i^3*x^4 + 4*B^2*b^4*c*d^3*i^3*x^3
+ 6*B^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2*b^4*c^3*d*i^3*x + (4*a*b^3*c^3*d*i^3 -
6*a^2*b^2*c^2*d^2*i^3 + 4*a^3*b*c*d^3*i^3 - a^4*d^4*i^3)*B^2)*log(b*x + a)^
2 + 3*(B^2*b^4*d^4*i^3*x^4 + 4*B^2*b^4*c*d^3*i^3*x^3 + 6*B^2*b^4*c^2*d^2*i^
3*x^2 + 4*B^2*b^4*c^3*d*i^3*x + B^2*b^4*c^4*i^3)*log(d*x + c)^2 + (6*B^2*b^
4*d^4*i^3*x^4*log(e) + 2*(a*b^3*d^4*i^3 + (12*i^3*log(e) - i^3)*b^4*c*d^3)*
B^2*x^3 + 3*(4*a*b^3*c*d^3*i^3 - a^2*b^2*d^4*i^3 + 3*(4*i^3*log(e) - i^3)*b
^4*c^2*d^2)*B^2*x^2 + 6*(6*a*b^3*c^2*d^2*i^3 - 4*a^2*b^2*c*d^3*i^3 + a^3*b*
d^4*i^3 + (4*i^3*log(e) - 3*i^3)*b^4*c^3*d)*B^2*x + (6*(4*i^3*log(e) - 3*i^
3)*a*b^3*c^3*d - 9*(4*i^3*log(e) - 5*i^3)*a^2*b^2*c^2*d^2 + 2*(12*i^3*log(e
) - 19*i^3)*a^3*b*c*d^3 - (6*i^3*log(e) - 11*i^3)*a^4*d^4)*B^2)*log(b*x + a
) - (6*B^2*b^4*d^4*i^3*x^4*log(e) + 2*(a*b^3*d^4*i^3 + (12*i^3*log(e) - i^3
)*b^4*c*d^3)*B^2*x^3 + 3*(4*a*b^3*c*d^3*i^3 - a^2*b^2*d^4*i^3 + 3*(4*i^3*lo
g(e) - i^3)*b^4*c^2*d^2)*B^2*x^2 + 6*(6*a*b^3*c^2*d^2*i^3 - 4*a^2*b^2*c*d^3
*i^3 + a^3*b*d^4*i^3 + (4*i^3*log(e) - 3*i^3)*b^4*c^3*d)*B^2*x + 6*(B^2*b^4
*d^4*i^3*x^4 + 4*B^2*b^4*c*d^3*i^3*x^3 + 6*B^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2*
b^4*c^3*d*i^3*x + (4*a*b^3*c^3*d*i^3 - 6*a^2*b^2*c^2*d^2*i^3 + 4*a^3*b*c*d^
3*i^3 - a^4*d^4*i^3)*B^2)*log(b*x + a))*log(d*x + c))/(b^4*d)
```

Giac [F]

$$\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (dix+ci)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
[Out] integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (c\,i+d\,i\,x)^3 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

```
[In] int((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
[Out] int((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

3.78 $\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 712

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{B^2d(bc-ad)^2i^3x}{3b^3g} + \frac{B^2(bc-ad)^3i^3\log\left(\frac{a+bx}{c+dx}\right)}{3b^4g}$$
$$- \frac{5Bd(bc-ad)^2i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^4g}$$
$$- \frac{B(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2g}$$
$$+ \frac{2B(bc-ad)^3i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g}$$
$$+ \frac{d(bc-ad)^2i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^4g}$$
$$+ \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g} + \frac{i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3bg}$$
$$+ \frac{2B^2(bc-ad)^3i^3\log(c+dx)}{b^4g} + \frac{5B(bc-ad)^3i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^4g}$$
$$- \frac{(bc-ad)^3i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}$$
$$+ \frac{2B^2(bc-ad)^3i^3\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g} - \frac{5B^2(bc-ad)^3i^3\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3b^4g}$$
$$+ \frac{2B(bc-ad)^3i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}$$
$$+ \frac{2B^2(bc-ad)^3i^3\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}$$

```
[Out] 1/3*B^2*d*(-a*d+b*c)^2*i^3*x/b^3/g+1/3*B^2*(-a*d+b*c)^3*i^3*ln((b*x+a)/(d*x
+c))/b^4/g-5/3*B*d*(-a*d+b*c)^2*i^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4
/g-1/3*B*(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/g+2*B*(-a
*d+b*c)^3*i^3*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/g+d*
(-a*d+b*c)^2*i^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^4/g+1/2*(-a*d+b*c)
*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^2/g+1/3*i^3*(d*x+c)^3*(A+B*l
n(e*(b*x+a)/(d*x+c)))^2/b/g+2*B^2*(-a*d+b*c)^3*i^3*ln(d*x+c)/b^4/g+5/3*B*(-
```

```
a*d+b*c)^3*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g-
(-a*d+b*c)^3*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^
4/g+2*B^2*(-a*d+b*c)^3*i^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/g-5/3*B^2*(-a
*d+b*c)^3*i^3*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g+2*B*(-a*d+b*c)^3*i^3*(A+
B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g+2*B^2*(-a*d+b
*c)^3*i^3*polylog(3,b*(d*x+c)/d/(b*x+a))/b^4/g
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 712, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules

used = {2562, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31, 46}

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{2Bi^3(bc-ad)^3 \operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4g}$$
$$+ \frac{di^3(a+bx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^4g}$$
$$- \frac{5Bdi^3(a+bx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b^4g}$$
$$+ \frac{2Bi^3(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4g}$$
$$- \frac{i^3(bc-ad)^3\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^4g}$$
$$+ \frac{5Bi^3(bc-ad)^3\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b^4g}$$
$$+ \frac{i^3(c+dx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b^2g}$$
$$- \frac{Bi^3(c+dx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b^2g} + \frac{i^3(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3bg}$$
$$+ \frac{2B^2i^3(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g} - \frac{5B^2i^3(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3b^4g}$$
$$+ \frac{2B^2i^3(bc-ad)^3\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g} + \frac{B^2i^3(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{3b^4g}$$
$$+ \frac{2B^2i^3(bc-ad)^3\log(c+dx)}{b^4g} + \frac{B^2di^3x(bc-ad)^2}{3b^3g}$$

```
[In] Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x),
x]
```

```
[Out] (B^2*d*(b*c - a*d)^2*i^3*x)/(3*b^3*g) + (B^2*(b*c - a*d)^3*i^3*Log[(a + b*x
)/(c + d*x)])/(3*b^4*g) - (5*B*d*(b*c - a*d)^2*i^3*(a + b*x)*(A + B*Log[(e*
(a + b*x))/(c + d*x)]))/(3*b^4*g) - (B*(b*c - a*d)*i^3*(c + d*x)^2*(A + B*L
og[(e*(a + b*x))/(c + d*x)]))/(3*b^2*g) + (2*B*(b*c - a*d)^3*i^3*Log[(b*c -
 a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^4*g) + (d*(b*
c - a*d)^2*i^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(b^4*g) +
```

```
((b*c - a*d)*i^3*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*b^2
*g) + (i^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*b*g) + (2
*B^2*(b*c - a*d)^3*i^3*Log[c + d*x])/(b^4*g) + (5*B*(b*c - a*d)^3*i^3*(A +
B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(3*b^
4*g) - ((b*c - a*d)^3*i^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1 - (b
*(c + d*x))/(d*(a + b*x))])/(b^4*g) + (2*B^2*(b*c - a*d)^3*i^3*PolyLog[2, (
d*(a + b*x))/(b*(c + d*x))])/(b^4*g) - (5*B^2*(b*c - a*d)^3*i^3*PolyLog[2,
(b*(c + d*x))/(d*(a + b*x))])/(3*b^4*g) + (2*B*(b*c - a*d)^3*i^3*(A + B*Log
[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b^4*g)
 + (2*B^2*(b*c - a*d)^3*i^3*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))])/(b^4*g
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\left((bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x(b-dx)^4} \, dx, x, \frac{a+bx}{c+dx}\right)}{g}$$

$$= \frac{\left((bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x(b-dx)^3} \, dx, x, \frac{a+bx}{c+dx}\right)}{bg} + \frac{\left(d(bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(b-dx)^4} \, dx, x, \frac{a+bx}{c+dx}\right)}{bg}$$

$$= \frac{i^3(c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3bg} + \frac{\left((bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x(b-dx)^2} \, dx, x, \frac{a+bx}{c+dx}\right)}{b^2 g} - \frac{\left(2B(bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)^3} \, dx, x, \frac{a+bx}{c+dx}\right)}{3bg} + \frac{\left(d(bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(b-dx)^3} \, dx, x, \frac{a+bx}{c+dx}\right)}{b^2 g}$$

$$= \frac{(bc-ad) i^3 (c+dx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2 g} + \frac{i^3(c+dx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3bg} + \frac{\left((bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x(b-dx)} \, dx, x, \frac{a+bx}{c+dx}\right)}{b^3 g} - \frac{\left(2B(bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)^2} \, dx, x, \frac{a+bx}{c+dx}\right)}{3b^2 g} - \frac{\left(B(bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)^2} \, dx, x, \frac{a+bx}{c+dx}\right)}{b^2 g} + \frac{\left(d(bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(b-dx)^2} \, dx, x, \frac{a+bx}{c+dx}\right)}{b^3 g} - \frac{\left(2Bd(bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^3} \, dx, x, \frac{a+bx}{c+dx}\right)}{3b^2 g}$$

$$= -\frac{B(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2g}$$
$$+\frac{d(bc-ad)^2i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^4g}$$
$$+\frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g}$$
$$+\frac{i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3bg}$$
$$-\frac{(bc-ad)^3i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}$$
$$+\frac{(2B(bc-ad)^3i^3)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)(A+B\log(ex))}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^4g}$$
$$-\frac{(2B(bc-ad)^3i^3)\operatorname{Subst}\left(\int\frac{A+B\log(ex)}{x(b-dx)}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^3g}$$
$$-\frac{(B(bc-ad)^3i^3)\operatorname{Subst}\left(\int\frac{A+B\log(ex)}{x(b-dx)}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3g}$$
$$+\frac{(B^2(bc-ad)^3i^3)\operatorname{Subst}\left(\int\frac{1}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^2g}$$
$$-\frac{(2Bd(bc-ad)^3i^3)\operatorname{Subst}\left(\int\frac{A+B\log(ex)}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^4g}$$
$$-\frac{(2Bd(bc-ad)^3i^3)\operatorname{Subst}\left(\int\frac{A+B\log(ex)}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^3g}$$
$$-\frac{(Bd(bc-ad)^3i^3)\operatorname{Subst}\left(\int\frac{A+B\log(ex)}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3g}$$

$$= -\frac{5Bd(bc-ad)^2 i^3 (a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^4 g} - \frac{B(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2 g} + \frac{2B(bc-ad)^3 i^3 \log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4 g} + \frac{d(bc-ad)^2 i^3 (a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^4 g} + \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2 g} + \frac{i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3bg} + \frac{5B(bc-ad)^3 i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^4 g} - \frac{(bc-ad)^3 i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4 g} + \frac{2B(bc-ad)^3 i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\mathrm{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4 g} - \frac{(2B^2(bc-ad)^3 i^3)\,\mathrm{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b^4 g} - \frac{(B^2(bc-ad)^3 i^3)\,\mathrm{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4 g} - \frac{(2B^2(bc-ad)^3 i^3)\,\mathrm{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4 g} - \frac{(2B^2(bc-ad)^3 i^3)\,\mathrm{Subst}\left(\int \frac{\mathrm{Li}_2\left(\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4 g} + \frac{(B^2(bc-ad)^3 i^3)\,\mathrm{Subst}\left(\int \left(\frac{1}{b^2 x}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{3b^2 g} + \frac{(2B^2 d(bc-ad)^3 i^3)\,\mathrm{Subst}\left(\int \frac{1}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b^4 g} + \frac{(B^2 d(bc-ad)^3 i^3)\,\mathrm{Subst}\left(\int \frac{1}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4 g}$$

$$
\begin{aligned}
&= \frac{B^2d(bc-ad)^2i^3x}{3b^3g} + \frac{B^2(bc-ad)^3i^3\log\left(\frac{a+bx}{c+dx}\right)}{3b^4g} \\
&- \frac{5Bd(bc-ad)^2i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^4g} \\
&- \frac{B(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2g} \\
&+ \frac{2B(bc-ad)^3i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g} \\
&+ \frac{d(bc-ad)^2i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^4g} \\
&+ \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g} \\
&+ \frac{i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3bg} + \frac{2B^2(bc-ad)^3i^3\log(c+dx)}{b^4g} \\
&+ \frac{5B(bc-ad)^3i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^4g} \\
&- \frac{(bc-ad)^3i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g} \\
&+ \frac{2B^2(bc-ad)^3i^3\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g} - \frac{5B^2(bc-ad)^3i^3\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{3b^4g} \\
&+ \frac{2B(bc-ad)^3i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g} \\
&+ \frac{2B^2(bc-ad)^3i^3\operatorname{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5044 vs. $2(712) = 1424$.

Time = 4.05 (sec) , antiderivative size = 5044, normalized size of antiderivative = 7.08

$$
\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b
*g*x),x]
[Out] Result too large to show
```

Maple [F]

$$\int \frac{(dix+ci)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{bgx+ag}dx$$

[In] int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x)

[Out] int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x)

Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algorithm="fricas")

[Out] integral((A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*c^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2*c^3*i^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*d^2*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log((b*e*x + a*e)/(d*x + c)))/(b*g*x + a*g), x)

Sympy [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \frac{i^3\left(\int \frac{A^2c^3}{a+bx}\,dx + \int \frac{A^2d^3x^3}{a+bx}\,dx + \int \frac{B^2c^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a+bx}\,dx + \int \frac{2ABc^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx + \int \frac{3A^2cd^2x^2}{a+bx}\,dx + \int \cdots\right.}{\cdots}$$

[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g),x)

[Out] i**3*(Integral(A**2*c**3/(a + b*x), x) + Integral(A**2*d**3*x**3/(a + b*x), x) + Integral(B**2*c**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a + b*x), x) + Integral(2*A*B*c**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) + Integral(3*A**2*c*d**2*x**2/(a + b*x), x) + Integral(3*A**2*c**2*d*x/(a + b*x), x) + Integral(B**2*d**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a + b*x), x) + Integral(2*A*B*d**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) + Integral(3*B**2*c*d**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a + b*x), x) + Integral(3*B**2*c**2*d*x*log(a*e/(c + d*x) +

```
 b*e*x/(c + d*x))**2/(a + b*x), x) + Integral(6*A*B*c*d**2*x**2*log(a*e/(c
+ d*x) + b*e*x/(c + d*x))/(a + b*x), x) + Integral(6*A*B*c**2*d*x*log(a*e/(
c + d*x) + b*e*x/(c + d*x))/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{(dix+ci)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algor
ithm="maxima")
```

```
[Out] 3*A^2*c^2*d*i^3*(x/(b*g) - a*log(b*x + a)/(b^2*g)) - 1/6*A^2*d^3*i^3*(6*a^3
*log(b*x + a)/(b^4*g) - (2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/(b^3*g)) + 3/2*A^
2*c*d^2*i^3*(2*a^2*log(b*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A^2*c^
3*i^3*log(b*g*x + a*g)/(b*g) + 1/6*(2*B^2*b^3*d^3*i^3*x^3 + 3*(3*b^3*c*d^2*
i^3 - a*b^2*d^3*i^3)*B^2*x^2 + 6*(3*b^3*c^2*d*i^3 - 3*a*b^2*c*d^2*i^3 + a^2
*b*d^3*i^3)*B^2*x + 6*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^2*b*c*d^2*i^3
- a^3*d^3*i^3)*B^2*log(b*x + a))*log(d*x + c)^2/(b^4*g) - integrate(-1/3*(3
*B^2*b^4*c^4*i^3*log(e)^2 + 6*A*B*b^4*c^4*i^3*log(e) + 3*(B^2*b^4*d^4*i^3*l
og(e)^2 + 2*A*B*b^4*d^4*i^3*log(e))*x^4 + 12*(B^2*b^4*c*d^3*i^3*log(e)^2 +
2*A*B*b^4*c*d^3*i^3*log(e))*x^3 + 18*(B^2*b^4*c^2*d^2*i^3*log(e)^2 + 2*A*B*
b^4*c^2*d^2*i^3*log(e))*x^2 + 3*(B^2*b^4*d^4*i^3*x^4 + 4*B^2*b^4*c*d^3*i^3*
x^3 + 6*B^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2*b^4*c^3*d*i^3*x + B^2*b^4*c^4*i^3)*
log(b*x + a)^2 + 12*(B^2*b^4*c^3*d*i^3*log(e)^2 + 2*A*B*b^4*c^3*d*i^3*log(e
))*x + 6*(B^2*b^4*c^4*i^3*log(e) + A*B*b^4*c^4*i^3 + (B^2*b^4*d^4*i^3*log(e
) + A*B*b^4*d^4*i^3)*x^4 + 4*(B^2*b^4*c*d^3*i^3*log(e) + A*B*b^4*c*d^3*i^3)
*x^3 + 6*(B^2*b^4*c^2*d^2*i^3*log(e) + A*B*b^4*c^2*d^2*i^3)*x^2 + 4*(B^2*b^
4*c^3*d*i^3*log(e) + A*B*b^4*c^3*d*i^3)*x)*log(b*x + a) - (6*B^2*b^4*c^4*i^
3*log(e) + 6*A*B*b^4*c^4*i^3 + 2*(3*A*B*b^4*d^4*i^3 + (3*i^3*log(e) + i^3)*
B^2*b^4*d^4)*x^4 + (24*A*B*b^4*c*d^3*i^3 - (a*b^3*d^4*i^3 - 3*(8*i^3*log(e)
 + 3*i^3)*b^4*c*d^3)*B^2)*x^3 + 3*(12*A*B*b^4*c^2*d^2*i^3 - (3*a*b^3*c*d^3*
i^3 - a^2*b^2*d^4*i^3 - 6*(2*i^3*log(e) + i^3)*b^4*c^2*d^2)*B^2)*x^2 + 6*(4
*A*B*b^4*c^3*d*i^3 + (4*b^4*c^3*d*i^3*log(e) + 3*a*b^3*c^2*d^2*i^3 - 3*a^2*
b^2*c*d^3*i^3 + a^3*b*d^4*i^3)*B^2)*x + 6*(B^2*b^4*d^4*i^3*x^4 + 4*B^2*b^4*
c*d^3*i^3*x^3 + 6*B^2*b^4*c^2*d^2*i^3*x^2 + (5*b^4*c^3*d*i^3 - 3*a*b^3*c^2*
d^2*i^3 + 3*a^2*b^2*c*d^3*i^3 - a^3*b*d^4*i^3)*B^2*x + (b^4*c^4*i^3 + a*b^3
*c^3*d*i^3 - 3*a^2*b^2*c^2*d^2*i^3 + 3*a^3*b*c*d^3*i^3 - a^4*d^4*i^3)*B^2)*
log(b*x + a))*log(d*x + c))/(b^5*d*g*x^2 + a*b^4*c*g + (b^5*c*g + a*b^4*d*g
)*x), x)
```

Giac [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algor
ithm="giac")
```

```
[Out] integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*g
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \int \frac{(c\,i+d\,i\,x)^3 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

```
[In] int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x),
x)
```

```
[Out] int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x),
 x)
```

3.79 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 692

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$$

$$= -\frac{2B^2(bc-ad)^2i^3(c+dx)}{b^3g^2(a+bx)} - \frac{Bd^2(bc-ad)i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^2} - \frac{2B(bc-ad)^2i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^2(a+bx)} + \frac{4Bd(bc-ad)^2i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^2} + \frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^4g^2} - \frac{(bc-ad)^2i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g^2(a+bx)} + \frac{di^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g^2} + \frac{B^2d(bc-ad)^2i^3\log(c+dx)}{b^4g^2} + \frac{Bd(bc-ad)^2i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} - \frac{3d(bc-ad)^2i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} + \frac{4B^2d(bc-ad)^2i^3\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g^2} - \frac{B^2d(bc-ad)^2i^3\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} + \frac{6Bd(bc-ad)^2i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} + \frac{6B^2d(bc-ad)^2i^3\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$

```
[Out] -2*B^2*(-a*d+b*c)^2*i^3*(d*x+c)/b^3/g^2/(b*x+a)-B*d^2*(-a*d+b*c)*i^3*(b*x+a
)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/g^2-2*B*(-a*d+b*c)^2*i^3*(d*x+c)*(A+B*ln(
e*(b*x+a)/(d*x+c)))/b^3/g^2/(b*x+a)+4*B*d*(-a*d+b*c)^2*i^3*ln((-a*d+b*c)/b/
(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/g^2+2*d^2*(-a*d+b*c)*i^3*(b*x+a)*(
A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^4/g^2-(-a*d+b*c)^2*i^3*(d*x+c)*(A+B*ln(e*(b*
x+a)/(d*x+c)))^2/b^3/g^2/(b*x+a)+1/2*d*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x
+c)))^2/b^2/g^2+B^2*d*(-a*d+b*c)^2*i^3*ln(d*x+c)/b^4/g^2+B*d*(-a*d+b*c)^2*i
^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^2-3*d*(-a*d+
b*c)^2*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^2+
```

```
4*B^2*d*(-a*d+b*c)^2*i^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/g^2-B^2*d*(-a*d
+b*c)^2*i^3*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^2+6*B*d*(-a*d+b*c)^2*i^3*(
A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^2+6*B^2*d*(
-a*d+b*c)^2*i^3*polylog(3,b*(d*x+c)/d/(b*x+a))/b^4/g^2
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2562, 2395, 2342, 2341, 2356, 2389, 2379, 2438, 2351, 31, 2355, 2354, 2421, 6724}

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$$
$$= \frac{2d^2i^3(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^4g^2} - \frac{Bd^2i^3(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4g^2} + \frac{6Bdi^3(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4g^2} + \frac{4Bdi^3(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4g^2} - \frac{3di^3(bc-ad)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^4g^2} + \frac{Bdi^3(bc-ad)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4g^2} - \frac{i^3(c+dx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^3g^2(a+bx)} - \frac{2Bi^3(c+dx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3g^2(a+bx)} + \frac{di^3(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b^2g^2} + \frac{4B^2di^3(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g^2} - \frac{B^2di^3(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} + \frac{6B^2di^3(bc-ad)^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} + \frac{B^2di^3(bc-ad)^2\log(c+dx)}{b^4g^2} - \frac{2B^2i^3(c+dx)(bc-ad)^2}{b^3g^2(a+bx)}$$

```
[In] Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)^
2,x]
```

```
[Out] (-2*B^2*(b*c - a*d)^2*i^3*(c + d*x))/(b^3*g^2*(a + b*x)) - (B*d^2*(b*c - a*
d)*i^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^4*g^2) - (2*B*(b*
c - a*d)^2*i^3*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^3*g^2*(a
+ b*x)) + (4*B*d*(b*c - a*d)^2*i^3*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]))/(b^4*g^2) + (2*d^2*(b*c - a*d)*i^3*(a + b*x)*(
A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(b^4*g^2) - ((b*c - a*d)^2*i^3*(c +
d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(b^3*g^2*(a + b*x)) + (d*i^3*(
c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*b^2*g^2) + (B^2*d*(b*
c - a*d)^2*i^3*Log[c + d*x])/(b^4*g^2) + (B*d*(b*c - a*d)^2*i^3*(A + B*Log[
(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^4*g^2) -
 (3*d*(b*c - a*d)^2*i^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1 - (b*(
c + d*x))/(d*(a + b*x))])/(b^4*g^2) + (4*B^2*d*(b*c - a*d)^2*i^3*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))])/(b^4*g^2) - (B^2*d*(b*c - a*d)^2*i^3*PolyLog
[2, (b*(c + d*x))/(d*(a + b*x))])/(b^4*g^2) + (6*B*d*(b*c - a*d)^2*i^3*(A +
 B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(
b^4*g^2) + (6*B^2*d*(b*c - a*d)^2*i^3*PolyLog[3, (b*(c + d*x))/(d*(a + b*x)
)])/(b^4*g^2)
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 2341</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

<u>Rule 2342</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

<u>Rule 2351</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

<u>Rule 2354</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
```

```
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)^2 i^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^2(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^2}$$

$$= \frac{((bc-ad)^2 i^3)\,\text{Subst}\left(\int \left(\frac{(A+B\log(ex))^2}{b^3x^2} + \frac{d^2(A+B\log(ex))^2}{b^2(b-dx)^3} + \frac{2d^2(A+B\log(ex))^2}{b^3(b-dx)^2} + \frac{3d(A+B\log(ex))^2}{b^3x(b-dx)}\right) dx, x, \frac{a+bx}{c+dx}\right)}{g^2}$$

$$= \frac{((bc-ad)^2 i^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^2} + \frac{(3d(bc-ad)^2 i^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^2} + \frac{(2d^2(bc-ad)^2 i^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^2} + \frac{(d^2(bc-ad)^2 i^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2}$$

$$= \frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^4g^2} - \frac{(bc-ad)^2i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g^2(a+bx)} + \frac{di^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g^2} - \frac{3d(bc-ad)^2i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} + \frac{(2B(bc-ad)^2i^3)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{x^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3g^2} + \frac{(6Bd(bc-ad)^2i^3)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)(A+B\log(ex))}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^4g^2} - \frac{(Bd(bc-ad)^2i^3)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^2g^2} - \frac{(4Bd^2(bc-ad)^2i^3)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^4g^2}$$

$$= -\frac{2B^2(bc-ad)^2i^3(c+dx)}{b^3g^2(a+bx)} - \frac{2B(bc-ad)^2i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^2(a+bx)}$$
$$+ \frac{4Bd(bc-ad)^2i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^2}$$
$$+ \frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^4g^2}$$
$$- \frac{(bc-ad)^2i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g^2(a+bx)}$$
$$+ \frac{di^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g^2}$$
$$- \frac{3d(bc-ad)^2i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$+ \frac{6Bd(bc-ad)^2i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$- \frac{(Bd(bc-ad)^2i^3)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^2}$$
$$- \frac{(4B^2d(bc-ad)^2i^3)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4g^2}$$
$$- \frac{(6B^2d(bc-ad)^2i^3)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4g^2}$$
$$- \frac{(Bd^2(bc-ad)^2i^3)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^2}$$

$$= -\frac{2B^2(bc-ad)^2i^3(c+dx)}{b^3g^2(a+bx)} - \frac{Bd^2(bc-ad)i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^2}$$
$$-\frac{2B(bc-ad)^2i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^2(a+bx)}$$
$$+\frac{4Bd(bc-ad)^2i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^2}$$
$$+\frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^4g^2}$$
$$-\frac{(bc-ad)^2i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g^2(a+bx)}$$
$$+\frac{di^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g^2}$$
$$+\frac{Bd(bc-ad)^2i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$-\frac{3d(bc-ad)^2i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$+\frac{4B^2d(bc-ad)^2i^3\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g^2}$$
$$+\frac{6Bd(bc-ad)^2i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$+\frac{6B^2d(bc-ad)^2i^3\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$-\frac{(B^2d(bc-ad)^2i^3)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^4g^2}$$
$$+\frac{(B^2d^2(bc-ad)^2i^3)\,\text{Subst}\left(\int\frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^4g^2}$$

$$= -\frac{2B^2(bc-ad)^2i^3(c+dx)}{b^3g^2(a+bx)} - \frac{Bd^2(bc-ad)i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^2}$$
$$- \frac{2B(bc-ad)^2i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^2(a+bx)}$$
$$+ \frac{4Bd(bc-ad)^2i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^2}$$
$$+ \frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^4g^2}$$
$$- \frac{(bc-ad)^2i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g^2(a+bx)}$$
$$+ \frac{di^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g^2} + \frac{B^2d(bc-ad)^2i^3\log(c+dx)}{b^4g^2}$$
$$+ \frac{Bd(bc-ad)^2i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$- \frac{3d(bc-ad)^2i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$+ \frac{4B^2d(bc-ad)^2i^3\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g^2} - \frac{B^2d(bc-ad)^2i^3\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$+ \frac{6Bd(bc-ad)^2i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$+ \frac{6B^2d(bc-ad)^2i^3\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4506 vs. $2(692) = 1384$.

Time = 5.81 (sec) , antiderivative size = 4506, normalized size of antiderivative = 6.51

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b
*g*x)^2,x]
```

```
[Out] (i^3*(4*A^2*b*d^2*(3*b*c - 2*a*d)*x + 2*A^2*b^2*d^3*x^2 - (4*A^2*(b*c - a*d
)^3)/(a + b*x) + 12*A^2*d*(b*c - a*d)^2*Log[a + b*x] - (8*A*b^3*B*c^3*(-(d*
```

(a + b*x)*Log[c/d + x]) + d*(a + b*x)*Log[(d*(a + b*x))/(-(b*c) + a*d)] + (
b*c - a*d)*(1 + Log[(e*(a + b*x))/(c + d*x)])))/((b*c - a*d)*(a + b*x)) + (
4*b^3*B^2*c^3*(-2*b*c + 2*a*d - 2*d*(a + b*x)*Log[a + b*x] - 2*(b*c - a*d)*
Log[(e*(a + b*x))/(c + d*x)] - 2*d*(a + b*x)*Log[a + b*x]*Log[(e*(a + b*x))
/(c + d*x)] - (b*c - a*d)*Log[(e*(a + b*x))/(c + d*x)]^2 + 2*d*(a + b*x)*Lo
g[c + d*x] - 2*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)]*Log[(b*c - a*d)/(b*
c + b*d*x)] + d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))
/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + d*(a + b*x)*
(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[
(b*c - a*d)/(b*c + b*d*x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/((
b*c - a*d)*(a + b*x)) + 12*A*b^2*B*c^2*d*(Log[a/b + x]^2 - 2*Log[a/b + x]*L
og[a + b*x] - 2*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[a +
b*x]*((a*d)/(b*c - a*d) + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)]) + 2*
a*((a + b*x)^(-1) + Log[(e*(a + b*x))/(c + d*x)]/(a + b*x) + (d*Log[c + d*x
])/(-(b*c) + a*d)) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 4*A*B*d^3*(
4*a^2 - (4*a*b*c)/d + a*b*x - (b^2*c*x)/d + (2*a^3)/(a + b*x) + 3*a^2*Log[a
/b + x]^2 + (4*a*b*c*Log[c/d + x])/d - a^2*Log[a + b*x] + (2*a^3*d*Log[a +
b*x])/(b*c - a*d) + 6*a^2*Log[c/d + x]*Log[a + b*x] - 2*a^2*Log[a/b + x]*(2
 + 3*Log[a + b*x]) - 6*a^2*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] -
 4*a*b*x*Log[(e*(a + b*x))/(c + d*x)] + b^2*x^2*Log[(e*(a + b*x))/(c + d*x)
] + (2*a^3*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x) + 6*a^2*Log[a + b*x]*Log
[(e*(a + b*x))/(c + d*x)] + (b^2*c^2*Log[c + d*x])/d^2 + (2*a^3*d*Log[c + d
*x])/(-(b*c) + a*d) - 6*a^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 24*A*b
*B*c*d^2*((a + b*x)*(-1 + Log[a/b + x]) - a*Log[a/b + x]^2 - (a^2*(1 + Log[
a/b + x]))/(a + b*x) - b*(c/d + x)*(-1 + Log[c/d + x]) + (a^2*Log[c/d + x])
/(a + b*x) + (b*x - a^2/(a + b*x) - 2*a*Log[a + b*x])*(-Log[a/b + x] + Log[
c/d + x] + Log[(e*(a + b*x))/(c + d*x)]) + (a^2*d*(Log[a + b*x] - Log[c + d
*x]))/(-(b*c) + a*d) + 2*a*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]
+ PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + 4*b*B^2*c*d^2*(6*b*x - 6*(a + b
*x)*Log[a/b + x] + 3*(a + b*x)*Log[a/b + x]^2 - 2*a*Log[a/b + x]^3 - (3*a^2
*(2 + 2*Log[a/b + x] + Log[a/b + x]^2))/(a + b*x) + (3*b*(2*d*x - 2*(c + d*
x)*Log[c/d + x] + (c + d*x)*Log[c/d + x]^2))/d + 3*(b*x - a^2/(a + b*x) - 2
*a*Log[a + b*x])*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x
)])^2 - (6*(a*d + 2*b*d*x - b*d*x*Log[c/d + x] - b*c*Log[c + d*x] + Log[a/b
 + x]*(-(d*(a + b*x)) + d*(a + b*x)*Log[c/d + x] + (b*c - a*d)*Log[(b*(c +
d*x))/(b*c - a*d)]) + (b*c - a*d)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])
)/d + (3*a^2*(d*(a + b*x)*Log[a/b + x]^2 + 2*((-(b*c) + a*d)*Log[c/d + x] +
 d*(a + b*x)*(Log[a + b*x] - Log[c + d*x])) - 2*Log[a/b + x]*((b*c - a*d)*L
og[c/d + x] + d*(a + b*x)*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*d*(a + b*x)*P
olyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/((-(b*c) + a*d)*(a + b*x)) + (3*a
^2*(-(b*(c + d*x)*Log[c/d + x]^2) + 2*d*(a + b*x)*Log[c/d + x]*Log[(d*(a +
b*x))/(-(b*c) + a*d)] + 2*d*(a + b*x)*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]
))/((b*c - a*d)*(a + b*x)) + 6*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a +
b*x))/(c + d*x)])*((a + b*x)*(-1 + Log[a/b + x]) - a*Log[a/b + x]^2 - (a^2*
(1 + Log[a/b + x]))/(a + b*x) - b*(c/d + x)*(-1 + Log[c/d + x]) + (a^2*Log[

```
c/d + x])/(a + b*x) + (a^2*d*(Log[a + b*x] - Log[c + d*x]))/(-(b*c) + a*d)
+ 2*a*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c +
d*x))/(b*c - a*d)])) + 6*a*(Log[a/b + x]^2*(Log[c/d + x] - Log[(b*(c + d*x)
)/(b*c - a*d)]) - 2*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] +
 2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)]) - 6*a*(Log[c/d + x]^2*Log[(d*(
a + b*x))/(-(b*c) + a*d)] + 2*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c -
a*d)] - 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])) + B^2*d^3*(b*x*(-6*a + b*
x) + (6*a^2 + 4*a*b*x - 2*b^2*x^2)*Log[a/b + x] - 2*(a^2 - b^2*x^2)*Log[a/b
 + x]^2 + 4*a^2*Log[a/b + x]^3 + (4*a^3*(2 + 2*Log[a/b + x] + Log[a/b + x]^
2))/(a + b*x) - 8*a*(2*b*x - 2*(a + b*x)*Log[a/b + x] + (a + b*x)*Log[a/b +
 x]^2) - (8*a*b*(2*d*x - 2*(c + d*x)*Log[c/d + x] + (c + d*x)*Log[c/d + x]^
2))/d + (b^2*(d*x*(-6*c + d*x) + (6*c^2 + 4*c*d*x - 2*d^2*x^2)*Log[c/d + x]
 - 2*(c^2 - d^2*x^2)*Log[c/d + x]^2))/d^2 + 2*(-4*a*b*x + b^2*x^2 + (2*a^3)
/(a + b*x) + 6*a^2*Log[a + b*x])*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a
+ b*x))/(c + d*x)])^2 + 4*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))
/(c + d*x)])*(4*a^2 - (4*a*b*c)/d + a*b*x - (b^2*c*x)/d + (2*a^3)/(a + b*x)
 + (-4*a^2 - 4*a*b*x + b^2*x^2 + (2*a^3)/(a + b*x))*Log[a/b + x] + 3*a^2*Lo
g[a/b + x]^2 + (4*a*b*c*Log[c/d + x])/d + 4*a*b*x*Log[c/d + x] - b^2*x^2*Lo
g[c/d + x] - (2*a^3*Log[c/d + x])/(a + b*x) - a^2*Log[a + b*x] + (2*a^3*d*L
og[a + b*x])/(b*c - a*d) - 6*a^2*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a
*d)] + (b^2*c^2*Log[c + d*x])/d^2 + (2*a^3*d*Log[c + d*x])/(-(b*c) + a*d) -
 6*a^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + (4*a^3*(-(b*(c + d*x)*Log[c
/d + x]^2) + 2*d*(a + b*x)*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] +
 2*d*(a + b*x)*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((-(b*c) + a*d)*(a +
 b*x)) + 2*((8*a*(a*d + 2*b*d*x - b*d*x*Log[c/d + x] - b*c*Log[c + d*x] + L
og[a/b + x]*(-(d*(a + b*x)) + d*(a + b*x)*Log[c/d + x] + (b*c - a*d)*Log[(b
*(c + d*x))/(b*c - a*d)]) + (b*c - a*d)*PolyLog[2, (d*(a + b*x))/(-(b*c) +
a*d)]))/d - (-2*a*b*c*d - 3*b^2*c*d*x - 3*a*b*d^2*x + b^2*d^2*x^2 + 2*a*b*d
^2*x*Log[c/d + x] - b^2*d^2*x^2*Log[c/d + x] + a^2*d^2*Log[a + b*x] + b^2*c
^2*Log[c + d*x] + 2*a*b*c*d*Log[c + d*x] + Log[a/b + x]*(b*d*(2*a*c + b*x*(
2*c - d*x)) - 2*d^2*(a^2 - b^2*x^2)*Log[c/d + x] + (-2*b^2*c^2 + 2*a^2*d^2)
*Log[(b*(c + d*x))/(b*c - a*d)]) + (-2*b^2*c^2 + 2*a^2*d^2)*PolyLog[2, (d*(
a + b*x))/(-(b*c) + a*d)])/d^2 + (2*a^3*(d*(a + b*x)*Log[a/b + x]^2 + 2*((-
(b*c) + a*d)*Log[c/d + x] + d*(a + b*x)*(Log[a + b*x] - Log[c + d*x])) - 2*
Log[a/b + x]*((b*c - a*d)*Log[c/d + x] + d*(a + b*x)*Log[(b*(c + d*x))/(b*c
 - a*d)]) - 2*d*(a + b*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/((b*c
- a*d)*(a + b*x)) - 6*a^2*(Log[a/b + x]^2*(Log[c/d + x] - Log[(b*(c + d*x))
/(b*c - a*d)]) - 2*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] +
2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)]) + 12*a^2*(Log[c/d + x]^2*Log[(
d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c
 - a*d)] - 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])) + (4*b^2*B^2*c^2*d*((b
*c - a*d)*(a + b*x)*Log[a/b + x]^3 + 3*a*(b*c - a*d)*(2 + 2*Log[a/b + x] +
Log[a/b + x]^2) + 3*(b*c - a*d)*(a + (a + b*x)*Log[a + b*x])*(-Log[a/b + x]
 + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])^2 + 3*a*(d*(a + b*x)*Log[a/
b + x]^2 + 2*((-(b*c) + a*d)*Log[c/d + x] + d*(a + b*x)*(Log[a + b*x] - Log
```

```
[c + d*x])) - 2*Log[a/b + x]*((b*c - a*d)*Log[c/d + x] + d*(a + b*x)*Log[(b
*(c + d*x))/(b*c - a*d)]) - 2*d*(a + b*x)*PolyLog[2, (d*(a + b*x))/(-(b*c)
+ a*d)]) + 3*a*(Log[c/d + x]*(b*(c + d*x)*Log[c/d + x] - 2*d*(a + b*x)*Log[
(d*(a + b*x))/(-(b*c) + a*d)]) - 2*d*(a + b*x)*PolyLog[2, (b*(c + d*x))/(b*
c - a*d)]) - 3*(Log[a/b + x] - Log[c/d + x] - Log[(e*(a + b*x))/(c + d*x)])
*((b*c - a*d)*(a + b*x)*Log[a/b + x]^2 + 2*a*(b*c - a*d)*(1 + Log[a/b + x])
 + 2*a*(-(b*c) + a*d)*Log[c/d + x] + 2*a*d*(a + b*x)*(Log[a + b*x] - Log[c
+ d*x]) - 2*(b*c - a*d)*(a + b*x)*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) +
 a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - 3*(b*c - a*d)*(a + b*x)*
(Log[a/b + x]^2*(Log[c/d + x] - Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Log[a/b
 + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2*PolyLog[3, (d*(a + b*x))
/(-(b*c) + a*d)]) + 3*(b*c - a*d)*(a + b*x)*(Log[c/d + x]^2*Log[(d*(a + b*x
))/(-(b*c) + a*d)] + 2*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] -
 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])))/((b*c - a*d)*(a + b*x))))/(4*b^
4*g^2)
```

Maple [F]

$$\int \frac{(dix+ci)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(bgx+ag)^2}\,dx$$

```
[In] int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x)
[Out] int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, alg
orithm="fricas")
[Out] integral((A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*c
^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2*c
^3*i^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*d^2*i
^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log((b*e*x + a*e)/(d*x + c)))/(b^
2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, alg
orithm="maxima")
[Out] -3*A^2*(a^2/(b^4*g^2*x + a*b^3*g^2) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*g
^2))*c*d^2*i^3 + 1/2*(2*a^3/(b^5*g^2*x + a*b^4*g^2) + 6*a^2*log(b*x + a)/(b
^4*g^2) + (b*x^2 - 4*a*x)/(b^3*g^2))*A^2*d^3*i^3 + 3*A^2*c^2*d*i^3*(a/(b^3*
g^2*x + a*b^2*g^2) + log(b*x + a)/(b^2*g^2)) - 2*A*B*c^3*i^3*(log(b*e*x/(d*
x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2) + 1/(b^2*g^2*x + a*b*g^2) + d
*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2))
 - A^2*c^3*i^3/(b^2*g^2*x + a*b*g^2) + 1/2*(B^2*b^3*d^3*i^3*x^3 + 3*(2*b^3*
c*d^2*i^3 - a*b^2*d^3*i^3)*B^2*x^2 + 2*(3*a*b^2*c*d^2*i^3 - 2*a^2*b*d^3*i^3
)*B^2*x - 2*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^2*b*c*d^2*i^3 - a^3*d^3*
i^3)*B^2 + 6*((b^3*c^2*d*i^3 - 2*a*b^2*c*d^2*i^3 + a^2*b*d^3*i^3)*B^2*x + (
a*b^2*c^2*d*i^3 - 2*a^2*b*c*d^2*i^3 + a^3*d^3*i^3)*B^2)*log(b*x + a))*log(d
*x + c)^2/(b^5*g^2*x + a*b^4*g^2) - integrate(-(B^2*b^4*c^4*i^3*log(e)^2 +
(B^2*b^4*d^4*i^3*log(e)^2 + 2*A*B*b^4*d^4*i^3*log(e))*x^4 + 4*(B^2*b^4*c*d^
3*i^3*log(e)^2 + 2*A*B*b^4*c*d^3*i^3*log(e))*x^3 + 6*(B^2*b^4*c^2*d^2*i^3*l
og(e)^2 + 2*A*B*b^4*c^2*d^2*i^3*log(e))*x^2 + (B^2*b^4*d^4*i^3*x^4 + 4*B^2*
b^4*c*d^3*i^3*x^3 + 6*B^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2*b^4*c^3*d*i^3*x + B^2
*b^4*c^4*i^3)*log(b*x + a)^2 + 2*(2*B^2*b^4*c^3*d*i^3*log(e)^2 + 3*A*B*b^4*
c^3*d*i^3*log(e))*x + 2*(B^2*b^4*c^4*i^3*log(e) + (B^2*b^4*d^4*i^3*log(e) +
 A*B*b^4*d^4*i^3)*x^4 + 4*(B^2*b^4*c*d^3*i^3*log(e) + A*B*b^4*c*d^3*i^3)*x^
3 + 6*(B^2*b^4*c^2*d^2*i^3*log(e) + A*B*b^4*c^2*d^2*i^3)*x^2 + (4*B^2*b^4*c
^3*d*i^3*log(e) + 3*A*B*b^4*c^3*d*i^3)*x)*log(b*x + a) - ((2*A*B*b^4*d^4*i^
3 + (2*i^3*log(e) + i^3)*B^2*b^4*d^4)*x^4 + 2*(4*A*B*b^4*c*d^3*i^3 - (a*b^3
*d^4*i^3 - (4*i^3*log(e) + 3*i^3)*b^4*c*d^3)*B^2)*x^3 + 2*(b^4*c^4*i^3*log(
e) - a*b^3*c^3*d*i^3 + 3*a^2*b^2*c^2*d^2*i^3 - 3*a^3*b*c*d^3*i^3 + a^4*d^4*
```

```
i^3)*B^2 + (12*A*B*b^4*c^2*d^2*i^3 + (12*b^4*c^2*d^2*i^3*log(e) + 12*a*b^3*
c*d^3*i^3 - 7*a^2*b^2*d^4*i^3)*B^2)*x^2 + 2*(3*A*B*b^4*c^3*d*i^3 + (3*a*b^3
*c^2*d^2*i^3 - a^3*b*d^4*i^3 + (4*i^3*log(e) - i^3)*b^4*c^3*d)*B^2)*x + 2*(
B^2*b^4*d^4*i^3*x^4 + 4*B^2*b^4*c*d^3*i^3*x^3 + 3*(3*b^4*c^2*d^2*i^3 - 2*a*
b^3*c*d^3*i^3 + a^2*b^2*d^4*i^3)*B^2*x^2 + 2*(2*b^4*c^3*d*i^3 + 3*a*b^3*c^2
*d^2*i^3 - 6*a^2*b^2*c*d^3*i^3 + 3*a^3*b*d^4*i^3)*B^2*x + (b^4*c^4*i^3 + 3*
a^2*b^2*c^2*d^2*i^3 - 6*a^3*b*c*d^3*i^3 + 3*a^4*d^4*i^3)*B^2)*log(b*x + a))
*log(d*x + c))/(b^6*d*g^2*x^3 + a^2*b^4*c*g^2 + (b^6*c*g^2 + 2*a*b^5*d*g^2)
*x^2 + (2*a*b^5*c*g^2 + a^2*b^4*d*g^2)*x), x)
```

Giac [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, alg
orithm="giac")
```

```
[Out] integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*g
)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(c\,i+d\,i\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(a\,g+b\,g\,x)^2}\,dx$$

```
[In] int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^
2,x)
```

```
[Out] int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^
2, x)
```

3.80 $\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 604

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= -\frac{4B^2d(bc-ad)i^3(c+dx)}{b^3g^3(a+bx)} - \frac{B^2(bc-ad)i^3(c+dx)^2}{4b^2g^3(a+bx)^2}$$
$$- \frac{4Bd(bc-ad)i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^3(a+bx)}$$
$$- \frac{B(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g^3(a+bx)^2}$$
$$+ \frac{2Bd^2(bc-ad)i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^3}$$
$$+ \frac{d^3i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^4g^3} - \frac{2d(bc-ad)i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g^3(a+bx)}$$
$$- \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g^3(a+bx)^2}$$
$$- \frac{3d^2(bc-ad)i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}$$
$$+ \frac{2B^2d^2(bc-ad)i^3\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g^3}$$
$$+ \frac{6Bd^2(bc-ad)i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}$$
$$+ \frac{6B^2d^2(bc-ad)i^3\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}$$

```
[Out] -4*B^2*d*(-a*d+b*c)*i^3*(d*x+c)/b^3/g^3/(b*x+a)-1/4*B^2*(-a*d+b*c)*i^3*(d*x
+c)^2/b^2/g^3/(b*x+a)^2-4*B*d*(-a*d+b*c)*i^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x
+c)))/b^3/g^3/(b*x+a)-1/2*B*(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x
+c)))/b^2/g^3/(b*x+a)^2+2*B*d^2*(-a*d+b*c)*i^3*ln((-a*d+b*c)/b/(d*x+c))*(A+
B*ln(e*(b*x+a)/(d*x+c)))/b^4/g^3+d^3*i^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c))
)^2/b^4/g^3-2*d*(-a*d+b*c)*i^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^3/g^
3/(b*x+a)-1/2*(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^2/g^
3/(b*x+a)^2-3*d^2*(-a*d+b*c)*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1-b*(d*x+
c)/d/(b*x+a))/b^4/g^3+2*B^2*d^2*(-a*d+b*c)*i^3*polylog(2,d*(b*x+a)/b/(d*x+c
))/b^4/g^3+6*B*d^2*(-a*d+b*c)*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,b*(
```

```
d*x+c)/d/(b*x+a))/b^4/g^3+6*B^2*d^2*(-a*d+b*c)*i^3*polylog(3,b*(d*x+c)/d/(b
*x+a))/b^4/g^3
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2562, 2395, 2342, 2341, 2355, 2354, 2438, 2379, 2421, 6724}

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}dx$$
$$= \frac{d^3i^3(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^4g^3} + \frac{6Bd^2i^3(bc-ad)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4g^3} + \frac{2Bd^2i^3(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4g^3} - \frac{3d^2i^3(bc-ad)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^4g^3} - \frac{2di^3(c+dx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^3g^3(a+bx)} - \frac{4Bdi^3(c+dx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3g^3(a+bx)} - \frac{i^3(c+dx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b^2g^3(a+bx)^2} - \frac{Bi^3(c+dx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b^2g^3(a+bx)^2} + \frac{2B^2d^2i^3(bc-ad)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g^3} + \frac{6B^2d^2i^3(bc-ad)\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} - \frac{4B^2di^3(c+dx)(bc-ad)}{b^3g^3(a+bx)} - \frac{B^2i^3(c+dx)^2(bc-ad)}{4b^2g^3(a+bx)^2}$$

```
[In] Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)^
3,x]
```

```
[Out] (-4*B^2*d*(b*c - a*d)*i^3*(c + d*x))/(b^3*g^3*(a + b*x)) - (B^2*(b*c - a*d)
*i^3*(c + d*x)^2)/(4*b^2*g^3*(a + b*x)^2) - (4*B*d*(b*c - a*d)*i^3*(c + d*x
```

```
)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^3*g^3*(a + b*x)) - (B*(b*c - a*d
)*i^3*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*b^2*g^3*(a + b*x
)^2) + (2*B*d^2*(b*c - a*d)*i^3*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(
e*(a + b*x))/(c + d*x)]))/(b^4*g^3) + (d^3*i^3*(a + b*x)*(A + B*Log[(e*(a +
 b*x))/(c + d*x)])^2)/(b^4*g^3) - (2*d*(b*c - a*d)*i^3*(c + d*x)*(A + B*Log
[(e*(a + b*x))/(c + d*x)])^2)/(b^3*g^3*(a + b*x)) - ((b*c - a*d)*i^3*(c + d
*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*b^2*g^3*(a + b*x)^2) - (3*
d^2*(b*c - a*d)*i^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1 - (b*(c +
d*x))/(d*(a + b*x))])/(b^4*g^3) + (2*B^2*d^2*(b*c - a*d)*i^3*PolyLog[2, (d*
(a + b*x))/(b*(c + d*x))])/(b^4*g^3) + (6*B*d^2*(b*c - a*d)*i^3*(A + B*Log[
(e*(a + b*x))/(c + d*x)])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b^4*g^3
) + (6*B^2*d^2*(b*c - a*d)*i^3*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))])/(b^
4*g^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
```

```
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)i^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^3(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^3}$$

$$= \frac{((bc-ad)i^3)\,\text{Subst}\left(\int \left(\frac{(A+B\log(ex))^2}{b^2x^3} + \frac{2d(A+B\log(ex))^2}{b^3x^2} + \frac{d^3(A+B\log(ex))^2}{b^3(b-dx)^2} + \frac{3d^2(A+B\log(ex))^2}{b^3x(b-dx)}\right)\,dx, x, \frac{a+b}{c+d}\right.}{g^3}$$

$$= \frac{((bc-ad)i^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^3} + \frac{(2d(bc-ad)i^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^3} + \frac{(3d^2(bc-ad)i^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^3} + \frac{(d^3(bc-ad)i^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^3}$$

$$= \frac{d^3i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^4g^3} - \frac{2d(bc-ad)i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g^3(a+bx)} - \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g^3(a+bx)^2} - \frac{3d^2(bc-ad)i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} + \frac{(B(bc-ad)i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^3} + \frac{(4Bd(bc-ad)i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^3} + \frac{(6Bd^2(bc-ad)i^3)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)(A+B\log(ex))}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4g^3} - \frac{(2Bd^3(bc-ad)i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4g^3}$$

$$
\begin{aligned}
&= -\frac{4B^2d(bc-ad)i^3(c+dx)}{b^3g^3(a+bx)} - \frac{B^2(bc-ad)i^3(c+dx)^2}{4b^2g^3(a+bx)^2} \\
&- \frac{4Bd(bc-ad)i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^3(a+bx)} \\
&- \frac{B(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g^3(a+bx)^2} \\
&+ \frac{2Bd^2(bc-ad)i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^3} \\
&+ \frac{d^3i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^4g^3} \\
&- \frac{2d(bc-ad)i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g^3(a+bx)} \\
&- \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g^3(a+bx)^2} \\
&- \frac{3d^2(bc-ad)i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} \\
&+ \frac{6Bd^2(bc-ad)i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\mathrm{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} \\
&- \frac{\left(2B^2d^2(bc-ad)i^3\right)\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^4g^3} \\
&- \frac{\left(6B^2d^2(bc-ad)i^3\right)\mathrm{Subst}\left(\int\frac{\mathrm{Li}_2\left(\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^4g^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4B^2d(bc-ad)i^3(c+dx)}{b^3g^3(a+bx)} - \frac{B^2(bc-ad)i^3(c+dx)^2}{4b^2g^3(a+bx)^2} \\
&- \frac{4Bd(bc-ad)i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^3(a+bx)} \\
&- \frac{B(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g^3(a+bx)^2} \\
&+ \frac{2Bd^2(bc-ad)i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^3} \\
&+ \frac{d^3i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^4g^3} \\
&- \frac{2d(bc-ad)i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g^3(a+bx)} \\
&- \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g^3(a+bx)^2} \\
&- \frac{3d^2(bc-ad)i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} \\
&+ \frac{2B^2d^2(bc-ad)i^3\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g^3} \\
&+ \frac{6Bd^2(bc-ad)i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} \\
&+ \frac{6B^2d^2(bc-ad)i^3\operatorname{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5989 vs. $2(604) = 1208$.

Time = 6.89 (sec) , antiderivative size = 5989, normalized size of antiderivative = 9.92

$$
\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b
*g*x)^3,x]
```

```
[Out] Result too large to show
```

Maple [F]

$$\int \frac{(dix+ci)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(bgx+ag)^3}dx$$

[In] int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x)

[Out] int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x)

Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, alg orithm="fricas")

[Out] integral((A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*c ^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2*c ^3*i^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*d^2*i ^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log((b*e*x + a*e)/(d*x + c)))/(b^ 3*g^3*x^3 + 3*a*b^2*g^3*x^2 + 3*a^2*b*g^3*x + a^3*g^3), x)

Sympy [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= \frac{i^3\left(\int \frac{A^2c^3}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{A^2d^3x^3}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{B^2c^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{2ABc^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\right.}{}$$

[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3,x)

[Out] i**3*(Integral(A**2*c**3/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(A**2*d**3*x**3/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3) , x) + Integral(B**2*c**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a**3 + 3 *a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*c**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(3*A**2*c*d**2*x**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3 *x**3), x) + Integral(3*A**2*c**2*d*x/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 +

```
b**3*x**3), x) + Integral(B**2*d**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x
))**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*
d**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**
2*x**2 + b**3*x**3), x) + Integral(3*B**2*c*d**2*x**2*log(a*e/(c + d*x) + b
*e*x/(c + d*x))**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + In
tegral(3*B**2*c**2*d*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a**3 + 3*a*
*2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(6*A*B*c*d**2*x**2*log(a*
e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x*
*3), x) + Integral(6*A*B*c**2*d*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**
3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x))/g**3
```

Maxima [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, alg
orithm="maxima")
```

```
[Out] -3/2*A*B*c^2*d*i^3*(2*(2*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^4
*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) + (3*a*b*c - a^2*d + 2*(2*b^2*c - a
*b*d)*x)/((b^5*c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*
b^3*c - a^3*b^2*d)*g^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*
b^3*c*d + a^2*b^2*d^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 -
2*a*b^3*c*d + a^2*b^2*d^2)*g^3)) - 1/2*A^2*d^3*i^3*((6*a^2*b*x + 5*a^3)/(b^
6*g^3*x^2 + 2*a*b^5*g^3*x + a^2*b^4*g^3) - 2*x/(b^3*g^3) + 6*a*log(b*x + a)
/(b^4*g^3)) + 3/2*A^2*c*d^2*i^3*((4*a*b*x + 3*a^2)/(b^5*g^3*x^2 + 2*a*b^4*g
^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(b^3*g^3)) + 1/2*A*B*c^3*i^3*((2*b*d*x
 - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x
+ (a^2*b^2*c - a^3*b*d)*g^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^3*
g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b
^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2)*g^3)) - 3/2*(2*b*x + a)*A^2*c^2*d*i^3/(b^4*g^3*x^2 + 2*a*b^3*g^3*x
+ a^2*b^2*g^3) - 1/2*A^2*c^3*i^3/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)
+ 1/2*(2*B^2*b^3*d^3*i^3*x^3 + 4*B^2*a*b^2*d^3*i^3*x^2 - 2*(3*b^3*c^2*d*i^3
 - 6*a*b^2*c*d^2*i^3 + 2*a^2*b*d^3*i^3)*B^2*x - (b^3*c^3*i^3 + 3*a*b^2*c^2*
d*i^3 - 9*a^2*b*c*d^2*i^3 + 5*a^3*d^3*i^3)*B^2 + 6*((b^3*c*d^2*i^3 - a*b^2*
d^3*i^3)*B^2*x^2 + 2*(a*b^2*c*d^2*i^3 - a^2*b*d^3*i^3)*B^2*x + (a^2*b*c*d^2
*i^3 - a^3*d^3*i^3)*B^2)*log(b*x + a))*log(d*x + c)^2/(b^6*g^3*x^2 + 2*a*b^
5*g^3*x + a^2*b^4*g^3) - integrate(-(4*B^2*b^4*c^3*d*i^3*x*log(e)^2 + B^2*b
^4*c^4*i^3*log(e)^2 + (B^2*b^4*d^4*i^3*log(e)^2 + 2*A*B*b^4*d^4*i^3*log(e))
*x^4 + 4*(B^2*b^4*c*d^3*i^3*log(e)^2 + 2*A*B*b^4*c*d^3*i^3*log(e))*x^3 + 6*
(B^2*b^4*c^2*d^2*i^3*log(e)^2 + A*B*b^4*c^2*d^2*i^3*log(e))*x^2 + (B^2*b^4*
```

```
d^4*i^3*x^4 + 4*B^2*b^4*c*d^3*i^3*x^3 + 6*B^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2*b
^4*c^3*d*i^3*x + B^2*b^4*c^4*i^3)*log(b*x + a)^2 + 2*(4*B^2*b^4*c^3*d*i^3*x
*log(e) + B^2*b^4*c^4*i^3*log(e) + (B^2*b^4*d^4*i^3*log(e) + A*B*b^4*d^4*i^
3)*x^4 + 4*(B^2*b^4*c*d^3*i^3*log(e) + A*B*b^4*c*d^3*i^3)*x^3 + 3*(2*B^2*b^
4*c^2*d^2*i^3*log(e) + A*B*b^4*c^2*d^2*i^3)*x^2)*log(b*x + a) - (2*(A*B*b^4
*d^4*i^3 + (i^3*log(e) + i^3)*B^2*b^4*d^4)*x^4 - (9*a*b^3*c^2*d^2*i^3 - 21*
a^2*b^2*c*d^3*i^3 + 9*a^3*b*d^4*i^3 - (8*i^3*log(e) - i^3)*b^4*c^3*d)*B^2*x
 + 2*(4*A*B*b^4*c*d^3*i^3 + (4*b^4*c*d^3*i^3*log(e) + 3*a*b^3*d^4*i^3)*B^2)
*x^3 + (2*b^4*c^4*i^3*log(e) - a*b^3*c^3*d*i^3 - 3*a^2*b^2*c^2*d^2*i^3 + 9*
a^3*b*c*d^3*i^3 - 5*a^4*d^4*i^3)*B^2 + 6*(A*B*b^4*c^2*d^2*i^3 + (2*a*b^3*c*
d^3*i^3 + (2*i^3*log(e) - i^3)*b^4*c^2*d^2)*B^2)*x^2 + 2*(B^2*b^4*d^4*i^3*x
^4 + (7*b^4*c*d^3*i^3 - 3*a*b^3*d^4*i^3)*B^2*x^3 + 3*(2*b^4*c^2*d^2*i^3 + 3
*a*b^3*c*d^3*i^3 - 3*a^2*b^2*d^4*i^3)*B^2*x^2 + (4*b^4*c^3*d*i^3 + 9*a^2*b^
2*c*d^3*i^3 - 9*a^3*b*d^4*i^3)*B^2*x + (b^4*c^4*i^3 + 3*a^3*b*c*d^3*i^3 - 3
*a^4*d^4*i^3)*B^2)*log(b*x + a))*log(d*x + c))/(b^7*d*g^3*x^4 + a^3*b^4*c*g
^3 + (b^7*c*g^3 + 3*a*b^6*d*g^3)*x^3 + 3*(a*b^6*c*g^3 + a^2*b^5*d*g^3)*x^2
+ (3*a^2*b^5*c*g^3 + a^3*b^4*d*g^3)*x), x)
```

Giac [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, alg
orithm="giac")
```

```
[Out] integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*g
)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(c\,i+d\,i\,x)^3 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(a\,g+b\,g\,x)^3}\,dx$$

```
[In] int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^
3,x)
```

```
[Out] int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^
3, x)
```

3.81 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 147

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{B^2i^3(c+dx)^4}{32(bc-ad)g^5(a+bx)^4} - \frac{Bi^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)g^5(a+bx)^4} - \frac{i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)g^5(a+bx)^4}$$

```
[Out] -1/32*B^2*i^3*(d*x+c)^4/(-a*d+b*c)/g^5/(b*x+a)^4-1/8*B*i^3*(d*x+c)^4*(A+B*l
n(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)/g^5/(b*x+a)^4-1/4*i^3*(d*x+c)^4*(A+B*ln(e*
(b*x+a)/(d*x+c)))^2/(-a*d+b*c)/g^5/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2562, 2342, 2341}

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{i^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4g^5(a+bx)^4(bc-ad)} - \frac{Bi^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{8g^5(a+bx)^4(bc-ad)} - \frac{B^2i^3(c+dx)^4}{32g^5(a+bx)^4(bc-ad)}$$

```
[In] Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)^
5,x]
```

```
[Out] -1/32*(B^2*i^3*(c + d*x)^4)/((b*c - a*d)*g^5*(a + b*x)^4) - (B*i^3*(c + d*x
)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(8*(b*c - a*d)*g^5*(a + b*x)^4) -
 (i^3*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*(b*c - a*d)*g^
5*(a + b*x)^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i^3 \text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^5} \\
&= -\frac{i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)g^5(a+bx)^4} + \frac{(Bi^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{2(bc-ad)g^5} \\
&= -\frac{B^2 i^3(c+dx)^4}{32(bc-ad)g^5(a+bx)^4} - \frac{Bi^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)g^5(a+bx)^4} \\
&\quad - \frac{i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)g^5(a+bx)^4}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.71 (sec) , antiderivative size = 2401, normalized size of antiderivative = 16.33

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b
*g*x)^5,x]
```

```
[Out] -1/4*((b*c - a*d)^3*i^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(b^4*g^5*(a
 + b*x)^4) - (d*(b*c - a*d)^2*i^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(
b^4*g^5*(a + b*x)^3) - (3*d^2*(b*c - a*d)*i^3*(A + B*Log[(e*(a + b*x))/(c +
 d*x)])^2)/(2*b^4*g^5*(a + b*x)^2) - (d^3*i^3*(A + B*Log[(e*(a + b*x))/(c +
 d*x)])^2)/(b^4*g^5*(a + b*x)) + (B*(b*c - a*d)^4*i^3*(-1/4*A/((b*c - a*d)*
(a + b*x)^4) - B/(16*(b*c - a*d)*(a + b*x)^4) + (A*d)/(3*(b*c - a*d)^2*(a +
 b*x)^3) + (7*B*d)/(36*(b*c - a*d)^2*(a + b*x)^3) - (A*d^2)/(2*(b*c - a*d)^
3*(a + b*x)^2) - (13*B*d^2)/(24*(b*c - a*d)^3*(a + b*x)^2) + (A*d^3)/((b*c
- a*d)^4*(a + b*x)) + (25*B*d^3)/(12*(b*c - a*d)^4*(a + b*x)) + (A*d^4*Log[
a + b*x])/(b*c - a*d)^5 + (25*B*d^4*Log[a + b*x])/(12*(b*c - a*d)^5) - (B*d
^4*Log[a + b*x]^2)/(2*(b*c - a*d)^5) - (B*Log[(e*(a + b*x))/(c + d*x)])/(4*
(b*c - a*d)*(a + b*x)^4) + (B*d*Log[(e*(a + b*x))/(c + d*x)])/(3*(b*c - a*d
)^2*(a + b*x)^3) - (B*d^2*Log[(e*(a + b*x))/(c + d*x)])/(2*(b*c - a*d)^3*(a
 + b*x)^2) + (B*d^3*Log[(e*(a + b*x))/(c + d*x)])/((b*c - a*d)^4*(a + b*x))
 + (B*d^4*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)])/(b*c - a*d)^5 - (A*d^4
*Log[c + d*x])/(b*c - a*d)^5 - (25*B*d^4*Log[c + d*x])/(12*(b*c - a*d)^5) +
 (B*d^4*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*c - a*d)^5 - (B*
d^4*Log[(e*(a + b*x))/(c + d*x)]*Log[c + d*x])/(b*c - a*d)^5 - (B*d^4*Log[c
 + d*x]^2)/(2*(b*c - a*d)^5) + (B*d^4*Log[a + b*x]*Log[(b*(c + d*x))/(b*c -
 a*d)])/(b*c - a*d)^5 + (B*d^4*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b
*c - a*d)^5 + (B*d^4*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*c - a*d)^5))
/(2*b^4*g^5) + (2*B*d*(b*c - a*d)^3*i^3*(-1/3*A/((b*c - a*d)*(a + b*x)^3) -
 B/(9*(b*c - a*d)*(a + b*x)^3) + (A*d)/(2*(b*c - a*d)^2*(a + b*x)^2) + (5*B
*d)/(12*(b*c - a*d)^2*(a + b*x)^2) - (A*d^2)/((b*c - a*d)^3*(a + b*x)) - (1
1*B*d^2)/(6*(b*c - a*d)^3*(a + b*x)) - (A*d^3*Log[a + b*x])/(b*c - a*d)^4 -
 (11*B*d^3*Log[a + b*x])/(6*(b*c - a*d)^4) + (B*d^3*Log[a + b*x]^2)/(2*(b*c
 - a*d)^4) - (B*Log[(e*(a + b*x))/(c + d*x)])/(3*(b*c - a*d)*(a + b*x)^3) +
 (B*d*Log[(e*(a + b*x))/(c + d*x)])/(2*(b*c - a*d)^2*(a + b*x)^2) - (B*d^2*
Log[(e*(a + b*x))/(c + d*x)])/((b*c - a*d)^3*(a + b*x)) - (B*d^3*Log[a + b*
x]*Log[(e*(a + b*x))/(c + d*x)])/(b*c - a*d)^4 + (A*d^3*Log[c + d*x])/(b*c
- a*d)^4 + (11*B*d^3*Log[c + d*x])/(6*(b*c - a*d)^4) - (B*d^3*Log[-((d*(a +
 b*x))/(b*c - a*d))]*Log[c + d*x])/(b*c - a*d)^4 + (B*d^3*Log[(e*(a + b*x))
/(c + d*x)]*Log[c + d*x])/(b*c - a*d)^4 + (B*d^3*Log[c + d*x]^2)/(2*(b*c -
```

```
a*d)^4) - (B*d^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b*c - a*d)^4
 - (B*d^3*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*c - a*d)^4 - (B*d^3*
PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*c - a*d)^4))/(b^4*g^5) + (3*B*d^2
*(b*c - a*d)^2*i^3*(-1/2*(A + B*Log[(e*(a + b*x))/(c + d*x)])/((b*c - a*d)*
(a + b*x)^2) + (d*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^2*(a +
 b*x)) + (d^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b*c - a*d
)^3 - (d^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x])/(b*c - a*d)^3
 - (B*(1/((b*c - a*d)*(a + b*x)^2) - (2*d)/((b*c - a*d)^2*(a + b*x)) - (2*d
^2*Log[a + b*x])/(b*c - a*d)^3 + (2*d^2*Log[c + d*x])/(b*c - a*d)^3))/4 + (
B*d*(1/((b*c - a*d)*(a + b*x)) + (d*Log[a + b*x])/(b*c - a*d)^2 - (d*Log[c
+ d*x])/(b*c - a*d)^2))/(b*c - a*d) - (B*d^2*(Log[a + b*x]^2 - 2*Log[a + b*
x]*Log[(b*(c + d*x))/(b*c - a*d)] - 2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d
))]))/(2*(b*c - a*d)^3) + (B*d^2*(2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c
 + d*x] - Log[c + d*x]^2 + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(2*(b*
c - a*d)^3)))/(b^4*g^5) + (2*B*d^3*(b*c - a*d)*i^3*(-((A + B*Log[(e*(a + b*
x))/(c + d*x)])/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x]*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]))/(b*c - a*d)^2 + (d*(A + B*Log[(e*(a + b*x))/(c + d*x)
])*Log[c + d*x])/(b*c - a*d)^2 - B*(1/((b*c - a*d)*(a + b*x)) + (d*Log[a +
b*x])/(b*c - a*d)^2 - (d*Log[c + d*x])/(b*c - a*d)^2) + (B*d*(Log[a + b*x]^
2 - 2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 2*PolyLog[2, -((d*(a +
b*x))/(b*c - a*d))]))/(2*(b*c - a*d)^2) - (B*d*(2*Log[-((d*(a + b*x))/(b*c
- a*d))]*Log[c + d*x] - Log[c + d*x]^2 + 2*PolyLog[2, (b*(c + d*x))/(b*c -
a*d)]))/(2*(b*c - a*d)^2)))/(b^4*g^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(141) = 282$.

Time = 1.65 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.50

method	result
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{i^3d^2e^3A^2}{4(ad-cb)^2g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}+\frac{2i^3d^2e^3AB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{1}{16\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}\right)}{(ad-cb)^2g^5}+\frac{i^3d^2e^3B^2\left(-\frac{\ln}{4}\right.}{}\right.}{d^2}$
default	$-\frac{e(ad-cb)\left(-\frac{i^3d^2e^3A^2}{4(ad-cb)^2g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}+\frac{2i^3d^2e^3AB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{1}{16\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}\right)}{(ad-cb)^2g^5}+\frac{i^3d^2e^3B^2\left(-\frac{\ln}{4}\right.}{}\right.}{d^2}$
parts	$\frac{i^3A^2\left(-\frac{d\left(a^2d^2-2abcd+b^2c^2\right)}{b^4(bx+a)^3}+\frac{3d^2(ad-cb)}{2b^4(bx+a)^2}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{4b^4(bx+a)^4}-\frac{d^3}{b^4(bx+a)}\right)}{g^5}-\frac{i^3B^2e^4\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)}{d(dx+c)}\right.}{4\left(\frac{be}{d}+\frac{(ad-cb)}{d(dx+c)}\right.}\right.}{}$
risch	$-\frac{i^3A^2d^3a^2}{g^5b^4(bx+a)^3}+\frac{2i^3A^2d^2ac}{g^5b^3(bx+a)^3}-\frac{i^3A^2d\,c^2}{g^5b^2(bx+a)^3}+\frac{3i^3A^2d^3a}{2g^5b^4(bx+a)^2}-\frac{3i^3A^2d^2c}{2g^5b^3(bx+a)^2}+\frac{i^3A^2a^3d^3}{4g^5b^4(bx+a)^4}-\frac{3i^3A^2}{4g^5b^3(}$
norman	$\frac{B^2c\,d^3i^3x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{(ad-cb)g}+\frac{B^2c^3d\,i^3x\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g(ad-cb)}+\frac{\left(8i^3A^2c^3+4i^3c^3BA+i^3c^3B^2\right)x}{8ga}+\frac{3\left(8A^2a\,c^2d\,i^3+8A^2b\,c^3i^3+4ABa\,c^2d\right.}{16}$
parallelrisch	$\frac{B^2x^4a^6c\,d^4i^3+64AB\,x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6c^2d^3i^3+96AB\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6c^3d^2i^3+64ABx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6c^4d\,i^3+16AB\,x}{}$

```
[In] int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x,method=_RET
URNVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(-1/4*i^3*d^2*e^3/(a*d-b*c)^2/g^5*A^2/(b*e/d+(a*d-b*c)*e
/d/(d*x+c))^4+2*i^3*d^2*e^3/(a*d-b*c)^2/g^5*A*B*(-1/4/(b*e/d+(a*d-b*c)*e/d/
(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c
))^4)+i^3*d^2*e^3/(a*d-b*c)^2/g^5*B^2*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4
*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/8/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b
*e/d+(a*d-b*c)*e/d/(d*x+c))-1/32/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(141) = 282$.

Time = 0.31 (sec) , antiderivative size = 559, normalized size of antiderivative = 3.80

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{4\left((8A^2+4AB+B^2)b^4cd^3-(8A^2+4AB+B^2)ab^3d^4\right)i^3x^3+6\left((8A^2+4AB+B^2)b^4c^2d^2-(8A^2+\right.}{}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, alg
orithm="fricas")
```

```
[Out] -1/32*(4*((8*A^2 + 4*A*B + B^2)*b^4*c*d^3 - (8*A^2 + 4*A*B + B^2)*a*b^3*d^4
)*i^3*x^3 + 6*((8*A^2 + 4*A*B + B^2)*b^4*c^2*d^2 - (8*A^2 + 4*A*B + B^2)*a^
2*b^2*d^4)*i^3*x^2 + 4*((8*A^2 + 4*A*B + B^2)*b^4*c^3*d - (8*A^2 + 4*A*B +
B^2)*a^3*b*d^4)*i^3*x + ((8*A^2 + 4*A*B + B^2)*b^4*c^4 - (8*A^2 + 4*A*B + B
^2)*a^4*d^4)*i^3 + 8*(B^2*b^4*d^4*i^3*x^4 + 4*B^2*b^4*c*d^3*i^3*x^3 + 6*B^2
*b^4*c^2*d^2*i^3*x^2 + 4*B^2*b^4*c^3*d*i^3*x + B^2*b^4*c^4*i^3)*log((b*e*x
+ a*e)/(d*x + c))^2 + 4*((4*A*B + B^2)*b^4*d^4*i^3*x^4 + 4*(4*A*B + B^2)*b^
4*c*d^3*i^3*x^3 + 6*(4*A*B + B^2)*b^4*c^2*d^2*i^3*x^2 + 4*(4*A*B + B^2)*b^4
*c^3*d*i^3*x + (4*A*B + B^2)*b^4*c^4*i^3)*log((b*e*x + a*e)/(d*x + c)))/((b
^9*c - a*b^8*d)*g^5*x^4 + 4*(a*b^8*c - a^2*b^7*d)*g^5*x^3 + 6*(a^2*b^7*c -
a^3*b^6*d)*g^5*x^2 + 4*(a^3*b^6*c - a^4*b^5*d)*g^5*x + (a^4*b^5*c - a^5*b^4
*d)*g^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**5,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11688 vs. $2(141) = 282$.

Time = 1.03 (sec) , antiderivative size = 11688, normalized size of antiderivative = 79.51

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, alg
orithm="maxima")
```

```
[Out] -1/4*(4*b*x + a)*B^2*c^2*d*i^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^6*
g^5*x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g
^5) - 1/4*(6*b^2*x^2 + 4*a*b*x + a^2)*B^2*c*d^2*i^3*log(b*e*x/(d*x + c) + a
*e/(d*x + c))^2/(b^7*g^5*x^4 + 4*a*b^6*g^5*x^3 + 6*a^2*b^5*g^5*x^2 + 4*a^3*
b^4*g^5*x + a^4*b^3*g^5) - 1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)*
B^2*d^3*i^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^8*g^5*x^4 + 4*a*b^7*g
```

^5*x^3 + 6*a^2*b^6*g^5*x^2 + 4*a^3*b^5*g^5*x + a^4*b^4*g^5) + 1/288*(12*((1
2*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 -
6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d
^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 +
4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 +
6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 +
4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x +
(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) + 12*d^4
*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d
^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^
2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log(b*e*x/(d*x + c) + a*
e/(d*x + c)) - (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2*d^2 - 576*a^3*
b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(13*b^4*c^2*d^2
- 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x
^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a)^2 + 72*(b^4*
d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*lo
g(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 324*a^2*b^2*c*d^3 - 271*
a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a
^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*b^4*d^4*x^4 + 100*a*b^3*d^4*x^3
+ 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x + 25*a^4*d^4 - 12*(b^4*d^4*x^4 + 4
*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a))
*log(d*x + c))/(a^4*b^5*c^4*g^5 - 4*a^5*b^4*c^3*d*g^5 + 6*a^6*b^3*c^2*d^2*g
^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d^4*g^5 + (b^9*c^4*g^5 - 4*a*b^8*c^3*d*g^5
+ 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b^6*c*d^3*g^5 + a^4*b^5*d^4*g^5)*x^4 + 4*(
a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^5 + 6*a^3*b^6*c^2*d^2*g^5 - 4*a^4*b^5*c*d
^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*(a^2*b^7*c^4*g^5 - 4*a^3*b^6*c^3*d*g^5 +
6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*c*d^3*g^5 + a^6*b^3*d^4*g^5)*x^2 + 4*(a^3
*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5 + 6*a^5*b^4*c^2*d^2*g^5 - 4*a^6*b^3*c*d^
3*g^5 + a^7*b^2*d^4*g^5)*x))*B^2*c^3*i^3 - 1/288*(12*((7*a*b^3*c^3 - 33*a^2
*b^2*c^2*d + 75*a^3*b*c*d^2 - 13*a^4*d^3 + 12*(4*b^4*c*d^2 - a*b^3*d^3)*x^3
- 6*(4*b^4*c^2*d - 29*a*b^3*c*d^2 + 7*a^2*b^2*d^3)*x^2 + 4*(4*b^4*c^3 - 21
*a*b^3*c^2*d + 57*a^2*b^2*c*d^2 - 13*a^3*b*d^3)*x)/((b^9*c^3 - 3*a*b^8*c^2*
d + 3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*g^5*x^4 + 4*(a*b^8*c^3 - 3*a^2*b^7*c^2*d
+ 3*a^3*b^6*c*d^2 - a^4*b^5*d^3)*g^5*x^3 + 6*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*
d + 3*a^4*b^5*c*d^2 - a^5*b^4*d^3)*g^5*x^2 + 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2
*d + 3*a^5*b^4*c*d^2 - a^6*b^3*d^3)*g^5*x + (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d
+ 3*a^6*b^3*c*d^2 - a^7*b^2*d^3)*g^5) + 12*(4*b*c*d^3 - a*d^4)*log(b*x + a)
/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*
d^4)*g^5) - 12*(4*b*c*d^3 - a*d^4)*log(d*x + c)/((b^6*c^4 - 4*a*b^5*c^3*d +
6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5))*log(b*e*x/(d*x +
c) + a*e/(d*x + c)) + (37*a*b^4*c^4 - 304*a^2*b^3*c^3*d + 1512*a^3*b^2*c^2*
d^2 - 1360*a^4*b*c*d^3 + 115*a^5*d^4 + 12*(88*b^5*c^2*d^2 - 101*a*b^4*c*d^3
+ 13*a^2*b^3*d^4)*x^3 - 6*(40*b^5*c^3*d - 609*a*b^4*c^2*d^2 + 648*a^2*b^3*
c*d^3 - 79*a^3*b^2*d^4)*x^2 - 72*(4*a^4*b*c*d^3 - a^5*d^4 + (4*b^5*c*d^3 -
a*b^4*d^4)*x^4 + 4*(4*a*b^4*c*d^3 - a^2*b^3*d^4)*x^3 + 6*(4*a^2*b^3*c*d^3 -

a^3*b^2*d^4)*x^2 + 4*(4*a^3*b^2*c*d^3 - a^4*b*d^4)*x)*log(b*x + a)^2 - 72*
(4*a^4*b*c*d^3 - a^5*d^4 + (4*b^5*c*d^3 - a*b^4*d^4)*x^4 + 4*(4*a*b^4*c*d^3
 - a^2*b^3*d^4)*x^3 + 6*(4*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^2 + 4*(4*a^3*b^2*
c*d^3 - a^4*b*d^4)*x)*log(d*x + c)^2 + 4*(16*b^5*c^4 - 163*a*b^4*c^3*d + 10
68*a^2*b^3*c^2*d^2 - 1036*a^3*b^2*c*d^3 + 115*a^4*b*d^4)*x + 12*(88*a^4*b*c
*d^3 - 13*a^5*d^4 + (88*b^5*c*d^3 - 13*a*b^4*d^4)*x^4 + 4*(88*a*b^4*c*d^3 -
 13*a^2*b^3*d^4)*x^3 + 6*(88*a^2*b^3*c*d^3 - 13*a^3*b^2*d^4)*x^2 + 4*(88*a^
3*b^2*c*d^3 - 13*a^4*b*d^4)*x)*log(b*x + a) - 12*(88*a^4*b*c*d^3 - 13*a^5*d
^4 + (88*b^5*c*d^3 - 13*a*b^4*d^4)*x^4 + 4*(88*a*b^4*c*d^3 - 13*a^2*b^3*d^4
)*x^3 + 6*(88*a^2*b^3*c*d^3 - 13*a^3*b^2*d^4)*x^2 + 4*(88*a^3*b^2*c*d^3 - 1
3*a^4*b*d^4)*x - 12*(4*a^4*b*c*d^3 - a^5*d^4 + (4*b^5*c*d^3 - a*b^4*d^4)*x^
4 + 4*(4*a*b^4*c*d^3 - a^2*b^3*d^4)*x^3 + 6*(4*a^2*b^3*c*d^3 - a^3*b^2*d^4)
x^2 + 4(4*a^3*b^2*c*d^3 - a^4*b*d^4)*x)*log(b*x + a))*log(d*x + c))/(a^4*
b^6*c^4*g^5 - 4*a^5*b^5*c^3*d*g^5 + 6*a^6*b^4*c^2*d^2*g^5 - 4*a^7*b^3*c*d^3
*g^5 + a^8*b^2*d^4*g^5 + (b^10*c^4*g^5 - 4*a*b^9*c^3*d*g^5 + 6*a^2*b^8*c^2*
d^2*g^5 - 4*a^3*b^7*c*d^3*g^5 + a^4*b^6*d^4*g^5)*x^4 + 4*(a*b^9*c^4*g^5 - 4
*a^2*b^8*c^3*d*g^5 + 6*a^3*b^7*c^2*d^2*g^5 - 4*a^4*b^6*c*d^3*g^5 + a^5*b^5*
d^4*g^5)*x^3 + 6*(a^2*b^8*c^4*g^5 - 4*a^3*b^7*c^3*d*g^5 + 6*a^4*b^6*c^2*d^2
*g^5 - 4*a^5*b^5*c*d^3*g^5 + a^6*b^4*d^4*g^5)*x^2 + 4*(a^3*b^7*c^4*g^5 - 4*
a^4*b^6*c^3*d*g^5 + 6*a^5*b^5*c^2*d^2*g^5 - 4*a^6*b^4*c*d^3*g^5 + a^7*b^3*d
^4*g^5)*x))*B^2*c^2*d*i^3 - 1/288*(12*((13*a^2*b^3*c^3 - 75*a^3*b^2*c^2*d +
 33*a^4*b*c*d^2 - 7*a^5*d^3 - 12*(6*b^5*c^2*d - 4*a*b^4*c*d^2 + a^2*b^3*d^3
)*x^3 + 6*(6*b^5*c^3 - 46*a*b^4*c^2*d + 29*a^2*b^3*c*d^2 - 7*a^3*b^2*d^3)*x
^2 + 4*(10*a*b^4*c^3 - 63*a^2*b^3*c^2*d + 33*a^3*b^2*c*d^2 - 7*a^4*b*d^3)*x
)/((b^10*c^3 - 3*a*b^9*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7*d^3)*g^5*x^4 + 4*(
a*b^9*c^3 - 3*a^2*b^8*c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*d^3)*g^5*x^3 + 6*(a
^2*b^8*c^3 - 3*a^3*b^7*c^2*d + 3*a^4*b^6*c*d^2 - a^5*b^5*d^3)*g^5*x^2 + 4*(
a^3*b^7*c^3 - 3*a^4*b^6*c^2*d + 3*a^5*b^5*c*d^2 - a^6*b^4*d^3)*g^5*x + (a^4
*b^6*c^3 - 3*a^5*b^5*c^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d^3)*g^5) - 12*(6*b^
2*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*log(b*x + a)/((b^7*c^4 - 4*a*b^6*c^3*d +
 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4)*g^5) + 12*(6*b^2*c^2*d^
2 - 4*a*b*c*d^3 + a^2*d^4)*log(d*x + c)/((b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b
^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4)*g^5))*log(b*e*x/(d*x + c) + a*e
/(d*x + c)) + (115*a^2*b^4*c^4 - 1360*a^3*b^3*c^3*d + 1512*a^4*b^2*c^2*d^2
- 304*a^5*b*c*d^3 + 37*a^6*d^4 - 12*(108*b^6*c^3*d - 148*a*b^5*c^2*d^2 + 47
*a^2*b^4*c*d^3 - 7*a^3*b^3*d^4)*x^3 + 6*(36*b^6*c^4 - 712*a*b^5*c^3*d + 903
*a^2*b^4*c^2*d^2 - 264*a^3*b^3*c*d^3 + 37*a^4*b^2*d^4)*x^2 + 72*(6*a^4*b^2*
c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4 + (6*b^6*c^2*d^2 - 4*a*b^5*c*d^3 + a^2*b^
4*d^4)*x^4 + 4*(6*a*b^5*c^2*d^2 - 4*a^2*b^4*c*d^3 + a^3*b^3*d^4)*x^3 + 6*(6
*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*x^2 + 4*(6*a^3*b^3*c^2*d^
2 - 4*a^4*b^2*c*d^3 + a^5*b*d^4)*x)*log(b*x + a)^2 + 72*(6*a^4*b^2*c^2*d^2
- 4*a^5*b*c*d^3 + a^6*d^4 + (6*b^6*c^2*d^2 - 4*a*b^5*c*d^3 + a^2*b^4*d^4)*x
^4 + 4*(6*a*b^5*c^2*d^2 - 4*a^2*b^4*c*d^3 + a^3*b^3*d^4)*x^3 + 6*(6*a^2*b^4
*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*x^2 + 4*(6*a^3*b^3*c^2*d^2 - 4*a^
4*b^2*c*d^3 + a^5*b*d^4)*x)*log(d*x + c)^2 + 4*(76*a*b^5*c^4 - 1057*a^2*b^4

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*c^3*d + 1248*a^3*b^3*c^2*d^2 - 304*a^4*b^2*c*d^3 + 37*a^5*b*d^4)*x - 12*(1
08*a^4*b^2*c^2*d^2 - 40*a^5*b*c*d^3 + 7*a^6*d^4 + (108*b^6*c^2*d^2 - 40*a*b
^5*c*d^3 + 7*a^2*b^4*d^4)*x^4 + 4*(108*a*b^5*c^2*d^2 - 40*a^2*b^4*c*d^3 + 7
*a^3*b^3*d^4)*x^3 + 6*(108*a^2*b^4*c^2*d^2 - 40*a^3*b^3*c*d^3 + 7*a^4*b^2*d
^4)*x^2 + 4*(108*a^3*b^3*c^2*d^2 - 40*a^4*b^2*c*d^3 + 7*a^5*b*d^4)*x)*log(b
*x + a) + 12*(108*a^4*b^2*c^2*d^2 - 40*a^5*b*c*d^3 + 7*a^6*d^4 + (108*b^6*c
^2*d^2 - 40*a*b^5*c*d^3 + 7*a^2*b^4*d^4)*x^4 + 4*(108*a*b^5*c^2*d^2 - 40*a^
2*b^4*c*d^3 + 7*a^3*b^3*d^4)*x^3 + 6*(108*a^2*b^4*c^2*d^2 - 40*a^3*b^3*c*d^
3 + 7*a^4*b^2*d^4)*x^2 + 4*(108*a^3*b^3*c^2*d^2 - 40*a^4*b^2*c*d^3 + 7*a^5*
b*d^4)*x - 12*(6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4 + (6*b^6*c^2*d^2
 - 4*a*b^5*c*d^3 + a^2*b^4*d^4)*x^4 + 4*(6*a*b^5*c^2*d^2 - 4*a^2*b^4*c*d^3
+ a^3*b^3*d^4)*x^3 + 6*(6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*
x^2 + 4*(6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c*d^3 + a^5*b*d^4)*x)*log(b*x + a))*
log(d*x + c))/(a^4*b^7*c^4*g^5 - 4*a^5*b^6*c^3*d*g^5 + 6*a^6*b^5*c^2*d^2*g^
5 - 4*a^7*b^4*c*d^3*g^5 + a^8*b^3*d^4*g^5 + (b^11*c^4*g^5 - 4*a*b^10*c^3*d*
g^5 + 6*a^2*b^9*c^2*d^2*g^5 - 4*a^3*b^8*c*d^3*g^5 + a^4*b^7*d^4*g^5)*x^4 +
4*(a*b^10*c^4*g^5 - 4*a^2*b^9*c^3*d*g^5 + 6*a^3*b^8*c^2*d^2*g^5 - 4*a^4*b^7
*c*d^3*g^5 + a^5*b^6*d^4*g^5)*x^3 + 6*(a^2*b^9*c^4*g^5 - 4*a^3*b^8*c^3*d*g^
5 + 6*a^4*b^7*c^2*d^2*g^5 - 4*a^5*b^6*c*d^3*g^5 + a^6*b^5*d^4*g^5)*x^2 + 4*
(a^3*b^8*c^4*g^5 - 4*a^4*b^7*c^3*d*g^5 + 6*a^5*b^6*c^2*d^2*g^5 - 4*a^6*b^5*
c*d^3*g^5 + a^7*b^4*d^4*g^5)*x))*B^2*c*d^2*i^3 - 1/288*(12*((25*a^3*b^3*c^3
 - 23*a^4*b^2*c^2*d + 13*a^5*b*c*d^2 - 3*a^6*d^3 + 12*(4*b^6*c^3 - 6*a*b^5*
c^2*d + 4*a^2*b^4*c*d^2 - a^3*b^3*d^3)*x^3 + 6*(18*a*b^5*c^3 - 22*a^2*b^4*c
^2*d + 13*a^3*b^3*c*d^2 - 3*a^4*b^2*d^3)*x^2 + 4*(22*a^2*b^4*c^3 - 23*a^3*b
^3*c^2*d + 13*a^4*b^2*c*d^2 - 3*a^5*b*d^3)*x)/((b^11*c^3 - 3*a*b^10*c^2*d +
 3*a^2*b^9*c*d^2 - a^3*b^8*d^3)*g^5*x^4 + 4*(a*b^10*c^3 - 3*a^2*b^9*c^2*d +
 3*a^3*b^8*c*d^2 - a^4*b^7*d^3)*g^5*x^3 + 6*(a^2*b^9*c^3 - 3*a^3*b^8*c^2*d
+ 3*a^4*b^7*c*d^2 - a^5*b^6*d^3)*g^5*x^2 + 4*(a^3*b^8*c^3 - 3*a^4*b^7*c^2*d
 + 3*a^5*b^6*c*d^2 - a^6*b^5*d^3)*g^5*x + (a^4*b^7*c^3 - 3*a^5*b^6*c^2*d +
3*a^6*b^5*c*d^2 - a^7*b^4*d^3)*g^5) + 12*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 + 4
*a^2*b*c*d^3 - a^3*d^4)*log(b*x + a)/((b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*
c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*g^5) - 12*(4*b^3*c^3*d - 6*a*b^2*c
^2*d^2 + 4*a^2*b*c*d^3 - a^3*d^4)*log(d*x + c)/((b^8*c^4 - 4*a*b^7*c^3*d +
6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*g^5))*log(b*e*x/(d*x + c
) + a*e/(d*x + c)) + (415*a^3*b^4*c^4 - 576*a^4*b^3*c^3*d + 216*a^5*b^2*c^2
*d^2 - 64*a^6*b*c*d^3 + 9*a^7*d^4 + 12*(48*b^7*c^4 - 84*a*b^6*c^3*d + 52*a^
2*b^5*c^2*d^2 - 19*a^3*b^4*c*d^3 + 3*a^4*b^3*d^4)*x^3 + 6*(252*a*b^6*c^4 -
400*a^2*b^5*c^3*d + 203*a^3*b^4*c^2*d^2 - 64*a^4*b^3*c*d^3 + 9*a^5*b^2*d^4)
*x^2 - 72*(4*a^4*b^3*c^3*d - 6*a^5*b^2*c^2*d^2 + 4*a^6*b*c*d^3 - a^7*d^4 +
(4*b^7*c^3*d - 6*a*b^6*c^2*d^2 + 4*a^2*b^5*c*d^3 - a^3*b^4*d^4)*x^4 + 4*(4*
a*b^6*c^3*d - 6*a^2*b^5*c^2*d^2 + 4*a^3*b^4*c*d^3 - a^4*b^3*d^4)*x^3 + 6*(4
*a^2*b^5*c^3*d - 6*a^3*b^4*c^2*d^2 + 4*a^4*b^3*c*d^3 - a^5*b^2*d^4)*x^2 + 4
*(4*a^3*b^4*c^3*d - 6*a^4*b^3*c^2*d^2 + 4*a^5*b^2*c*d^3 - a^6*b*d^4)*x)*log
(b*x + a)^2 - 72*(4*a^4*b^3*c^3*d - 6*a^5*b^2*c^2*d^2 + 4*a^6*b*c*d^3 - a^7
*d^4 + (4*b^7*c^3*d - 6*a*b^6*c^2*d^2 + 4*a^2*b^5*c*d^3 - a^3*b^4*d^4)*x^4
```

+ 4*(4*a*b^6*c^3*d - 6*a^2*b^5*c^2*d^2 + 4*a^3*b^4*c*d^3 - a^4*b^3*d^4)*x^3
 + 6*(4*a^2*b^5*c^3*d - 6*a^3*b^4*c^2*d^2 + 4*a^4*b^3*c*d^3 - a^5*b^2*d^4)*
x^2 + 4*(4*a^3*b^4*c^3*d - 6*a^4*b^3*c^2*d^2 + 4*a^5*b^2*c*d^3 - a^6*b*d^4)
*x)*log(d*x + c)^2 + 4*(340*a^2*b^5*c^4 - 501*a^3*b^4*c^3*d + 216*a^4*b^3*c
^2*d^2 - 64*a^5*b^2*c*d^3 + 9*a^6*b*d^4)*x + 12*(48*a^4*b^3*c^3*d - 36*a^5*
b^2*c^2*d^2 + 16*a^6*b*c*d^3 - 3*a^7*d^4 + (48*b^7*c^3*d - 36*a*b^6*c^2*d^2
 + 16*a^2*b^5*c*d^3 - 3*a^3*b^4*d^4)*x^4 + 4*(48*a*b^6*c^3*d - 36*a^2*b^5*c
^2*d^2 + 16*a^3*b^4*c*d^3 - 3*a^4*b^3*d^4)*x^3 + 6*(48*a^2*b^5*c^3*d - 36*a
^3*b^4*c^2*d^2 + 16*a^4*b^3*c*d^3 - 3*a^5*b^2*d^4)*x^2 + 4*(48*a^3*b^4*c^3*
d - 36*a^4*b^3*c^2*d^2 + 16*a^5*b^2*c*d^3 - 3*a^6*b*d^4)*x)*log(b*x + a) -
12*(48*a^4*b^3*c^3*d - 36*a^5*b^2*c^2*d^2 + 16*a^6*b*c*d^3 - 3*a^7*d^4 + (4
8*b^7*c^3*d - 36*a*b^6*c^2*d^2 + 16*a^2*b^5*c*d^3 - 3*a^3*b^4*d^4)*x^4 + 4*
(48*a*b^6*c^3*d - 36*a^2*b^5*c^2*d^2 + 16*a^3*b^4*c*d^3 - 3*a^4*b^3*d^4)*x^
3 + 6*(48*a^2*b^5*c^3*d - 36*a^3*b^4*c^2*d^2 + 16*a^4*b^3*c*d^3 - 3*a^5*b^2
*d^4)*x^2 + 4*(48*a^3*b^4*c^3*d - 36*a^4*b^3*c^2*d^2 + 16*a^5*b^2*c*d^3 - 3
*a^6*b*d^4)*x - 12*(4*a^4*b^3*c^3*d - 6*a^5*b^2*c^2*d^2 + 4*a^6*b*c*d^3 - a
^7*d^4 + (4*b^7*c^3*d - 6*a*b^6*c^2*d^2 + 4*a^2*b^5*c*d^3 - a^3*b^4*d^4)*x^
4 + 4*(4*a*b^6*c^3*d - 6*a^2*b^5*c^2*d^2 + 4*a^3*b^4*c*d^3 - a^4*b^3*d^4)*x
^3 + 6*(4*a^2*b^5*c^3*d - 6*a^3*b^4*c^2*d^2 + 4*a^4*b^3*c*d^3 - a^5*b^2*d^4
)*x^2 + 4*(4*a^3*b^4*c^3*d - 6*a^4*b^3*c^2*d^2 + 4*a^5*b^2*c*d^3 - a^6*b*d^
4)*x)*log(b*x + a))*log(d*x + c))/(a^4*b^8*c^4*g^5 - 4*a^5*b^7*c^3*d*g^5 +
6*a^6*b^6*c^2*d^2*g^5 - 4*a^7*b^5*c*d^3*g^5 + a^8*b^4*d^4*g^5 + (b^12*c^4*g
^5 - 4*a*b^11*c^3*d*g^5 + 6*a^2*b^10*c^2*d^2*g^5 - 4*a^3*b^9*c*d^3*g^5 + a^
4*b^8*d^4*g^5)*x^4 + 4*(a*b^11*c^4*g^5 - 4*a^2*b^10*c^3*d*g^5 + 6*a^3*b^9*c
^2*d^2*g^5 - 4*a^4*b^8*c*d^3*g^5 + a^5*b^7*d^4*g^5)*x^3 + 6*(a^2*b^10*c^4*g
^5 - 4*a^3*b^9*c^3*d*g^5 + 6*a^4*b^8*c^2*d^2*g^5 - 4*a^5*b^7*c*d^3*g^5 + a^
6*b^6*d^4*g^5)*x^2 + 4*(a^3*b^9*c^4*g^5 - 4*a^4*b^8*c^3*d*g^5 + 6*a^5*b^7*c
^2*d^2*g^5 - 4*a^6*b^6*c*d^3*g^5 + a^7*b^5*d^4*g^5)*x))*B^2*d^3*i^3 - 1/24*
A*B*d^3*i^3*(12*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)*log(b*e*x/(d*x
+ c) + a*e/(d*x + c))/(b^8*g^5*x^4 + 4*a*b^7*g^5*x^3 + 6*a^2*b^6*g^5*x^2 +
4*a^3*b^5*g^5*x + a^4*b^4*g^5) + (25*a^3*b^3*c^3 - 23*a^4*b^2*c^2*d + 13*a^
5*b*c*d^2 - 3*a^6*d^3 + 12*(4*b^6*c^3 - 6*a*b^5*c^2*d + 4*a^2*b^4*c*d^2 - a
^3*b^3*d^3)*x^3 + 6*(18*a*b^5*c^3 - 22*a^2*b^4*c^2*d + 13*a^3*b^3*c*d^2 - 3
*a^4*b^2*d^3)*x^2 + 4*(22*a^2*b^4*c^3 - 23*a^3*b^3*c^2*d + 13*a^4*b^2*c*d^2
 - 3*a^5*b*d^3)*x)/((b^11*c^3 - 3*a*b^10*c^2*d + 3*a^2*b^9*c*d^2 - a^3*b^8*
d^3)*g^5*x^4 + 4*(a*b^10*c^3 - 3*a^2*b^9*c^2*d + 3*a^3*b^8*c*d^2 - a^4*b^7*
d^3)*g^5*x^3 + 6*(a^2*b^9*c^3 - 3*a^3*b^8*c^2*d + 3*a^4*b^7*c*d^2 - a^5*b^6
*d^3)*g^5*x^2 + 4*(a^3*b^8*c^3 - 3*a^4*b^7*c^2*d + 3*a^5*b^6*c*d^2 - a^6*b^
5*d^3)*g^5*x + (a^4*b^7*c^3 - 3*a^5*b^6*c^2*d + 3*a^6*b^5*c*d^2 - a^7*b^4*d
^3)*g^5) + 12*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3 - a^3*d^4)*log
(b*x + a)/((b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 +
 a^4*b^4*d^4)*g^5) - 12*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3 - a^
3*d^4)*log(d*x + c)/((b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b
^5*c*d^3 + a^4*b^4*d^4)*g^5)) - 1/24*A*B*c*d^2*i^3*(12*(6*b^2*x^2 + 4*a*b*x
 + a^2)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^7*g^5*x^4 + 4*a*b^6*g^5*x^3

+ 6*a^2*b^5*g^5*x^2 + 4*a^3*b^4*g^5*x + a^4*b^3*g^5) + (13*a^2*b^3*c^3 - 7
5*a^3*b^2*c^2*d + 33*a^4*b*c*d^2 - 7*a^5*d^3 - 12*(6*b^5*c^2*d - 4*a*b^4*c*
d^2 + a^2*b^3*d^3)*x^3 + 6*(6*b^5*c^3 - 46*a*b^4*c^2*d + 29*a^2*b^3*c*d^2 -
7*a^3*b^2*d^3)*x^2 + 4*(10*a*b^4*c^3 - 63*a^2*b^3*c^2*d + 33*a^3*b^2*c*d^2
- 7*a^4*b*d^3)*x)/((b^10*c^3 - 3*a*b^9*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7*d
^3)*g^5*x^4 + 4*(a*b^9*c^3 - 3*a^2*b^8*c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*d^
3)*g^5*x^3 + 6*(a^2*b^8*c^3 - 3*a^3*b^7*c^2*d + 3*a^4*b^6*c*d^2 - a^5*b^5*d
^3)*g^5*x^2 + 4*(a^3*b^7*c^3 - 3*a^4*b^6*c^2*d + 3*a^5*b^5*c*d^2 - a^6*b^4*
d^3)*g^5*x + (a^4*b^6*c^3 - 3*a^5*b^5*c^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d^3
)*g^5) - 12*(6*b^2*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*log(b*x + a)/((b^7*c^4
- 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4)*g^5) +
12*(6*b^2*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*log(d*x + c)/((b^7*c^4 - 4*a*b^
6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4)*g^5)) - 1/24*A
*B*c^2*d*i^3*(12*(4*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^6*g^5*
x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g^5)
+ (7*a*b^3*c^3 - 33*a^2*b^2*c^2*d + 75*a^3*b*c*d^2 - 13*a^4*d^3 + 12*(4*b^4
*c*d^2 - a*b^3*d^3)*x^3 - 6*(4*b^4*c^2*d - 29*a*b^3*c*d^2 + 7*a^2*b^2*d^3)*
x^2 + 4*(4*b^4*c^3 - 21*a*b^3*c^2*d + 57*a^2*b^2*c*d^2 - 13*a^3*b*d^3)*x)/(
(b^9*c^3 - 3*a*b^8*c^2*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*g^5*x^4 + 4*(a*b^
8*c^3 - 3*a^2*b^7*c^2*d + 3*a^3*b^6*c*d^2 - a^4*b^5*d^3)*g^5*x^3 + 6*(a^2*b
^7*c^3 - 3*a^3*b^6*c^2*d + 3*a^4*b^5*c*d^2 - a^5*b^4*d^3)*g^5*x^2 + 4*(a^3*
b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2 - a^6*b^3*d^3)*g^5*x + (a^4*b^5
*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d^2 - a^7*b^2*d^3)*g^5) + 12*(4*b*c*d^
3 - a*d^4)*log(b*x + a)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a
^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5) - 12*(4*b*c*d^3 - a*d^4)*log(d*x + c)/((b^
6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*
g^5)) + 1/24*A*B*c^3*i^3*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23
*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d
- 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*
d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d
^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*
d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c
*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^
2 - a^7*b*d^3)*g^5) - 12*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^5*g^5*x^4
+ 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d
^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c
*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*
a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*B^2*c^3*i^3*log(
b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b
^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*(4*b*x + a)*A^2*c^2*d*i^3/(
b^6*g^5*x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b
^2*g^5) - 1/4*(6*b^2*x^2 + 4*a*b*x + a^2)*A^2*c*d^2*i^3/(b^7*g^5*x^4 + 4*a*
b^6*g^5*x^3 + 6*a^2*b^5*g^5*x^2 + 4*a^3*b^4*g^5*x + a^4*b^3*g^5) - 1/4*(4*b
^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)*A^2*d^3*i^3/(b^8*g^5*x^4 + 4*a*b^7*
g^5*x^3 + 6*a^2*b^6*g^5*x^2 + 4*a^3*b^5*g^5*x + a^4*b^4*g^5) - 1/4*A^2*c^3*

```
i^3/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x +
a^4*b*g^5)
```

Giac [A] (verification not implemented)

none

Time = 0.64 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.48

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{1}{32}\left(\frac{8\,(dx+c)^4B^2e^5i^3\log\left(\frac{bex+ae}{dx+c}\right)^2}{(bex+ae)^4g^5}+\frac{4\,(4\,ABe^5i^3+B^2e^5i^3)(dx+c)^4\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^4g^5}+\frac{(8\,A^2e^5i^3+4\,ABe\right.$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, alg
orithm="giac")
```

```
[Out] -1/32*(8*(d*x + c)^4*B^2*e^5*i^3*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a
*e)^4*g^5) + 4*(4*A*B*e^5*i^3 + B^2*e^5*i^3)*(d*x + c)^4*log((b*e*x + a*e)/
(d*x + c))/((b*e*x + a*e)^4*g^5) + (8*A^2*e^5*i^3 + 4*A*B*e^5*i^3 + B^2*e^5
*i^3)*(d*x + c)^4/((b*e*x + a*e)^4*g^5))*(b*c/((b*c*e - a*d*e)*(b*c - a*d))
 - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 1565, normalized size of antiderivative = 10.65

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

```
[In] int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^
5,x)
```

```
[Out] -(24*A^2*a^4*d^4*i^3 - 24*A^2*b^4*c^4*i^3 + 3*B^2*a^4*d^4*i^3 - 3*B^2*b^4*c
^4*i^3 + 12*A*B*a^4*d^4*i^3 - 12*A*B*b^4*c^4*i^3 - 24*B^2*b^4*c^4*i^3*log((
e*(a + b*x))/(c + d*x))^2 + B^2*a^4*d^4*i^3*atan((a*d*1i + b*c*1i + b*d*x*2
i)/(a*d - b*c))*24i + 12*B^2*a^4*d^4*i^3*log((e*(a + b*x))/(c + d*x)) - 12*
B^2*b^4*c^4*i^3*log((e*(a + b*x))/(c + d*x)) - 24*B^2*b^4*d^4*i^3*x^4*log((
e*(a + b*x))/(c + d*x))^2 + 96*A^2*a^3*b*d^4*i^3*x + 12*B^2*a^3*b*d^4*i^3*x
 - 96*A^2*b^4*c^3*d*i^3*x - 12*B^2*b^4*c^3*d*i^3*x + B^2*b^4*d^4*i^3*x^4*at
an((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*24i + A*B*a^4*d^4*i^3*atan((a*
d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*96i + 96*A^2*a*b^3*d^4*i^3*x^3 + 12*
B^2*a*b^3*d^4*i^3*x^3 - 96*A^2*b^4*c*d^3*i^3*x^3 - 12*B^2*b^4*c*d^3*i^3*x^3
```

```
 + 48*A*B*a^4*d^4*i^3*log((e*(a + b*x))/(c + d*x)) - 48*A*B*b^4*c^4*i^3*log
((e*(a + b*x))/(c + d*x)) + 144*A^2*a^2*b^2*d^4*i^3*x^2 + 18*B^2*a^2*b^2*d^
4*i^3*x^2 - 144*A^2*b^4*c^2*d^2*i^3*x^2 - 18*B^2*b^4*c^2*d^2*i^3*x^2 + 48*A
*B*a^3*b*d^4*i^3*x - 48*A*B*b^4*c^3*d*i^3*x + 48*B^2*a*b^3*d^4*i^3*x^3*log(
(e*(a + b*x))/(c + d*x)) - 96*B^2*b^4*c^3*d*i^3*x*log((e*(a + b*x))/(c + d*
x))^2 - 48*B^2*b^4*c*d^3*i^3*x^3*log((e*(a + b*x))/(c + d*x)) + A*B*b^4*d^4
*i^3*x^4*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*96i + B^2*a^3*b*d^4
*i^3*x*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*96i + 48*A*B*a*b^3*d^
4*i^3*x^3 - 48*A*B*b^4*c*d^3*i^3*x^3 + 72*B^2*a^2*b^2*d^4*i^3*x^2*log((e*(a
 + b*x))/(c + d*x)) - 72*B^2*b^4*c^2*d^2*i^3*x^2*log((e*(a + b*x))/(c + d*x
)) - 96*B^2*b^4*c*d^3*i^3*x^3*log((e*(a + b*x))/(c + d*x))^2 + B^2*a*b^3*d^
4*i^3*x^3*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*96i + 48*B^2*a^3*b
*d^4*i^3*x*log((e*(a + b*x))/(c + d*x)) - 48*B^2*b^4*c^3*d*i^3*x*log((e*(a
+ b*x))/(c + d*x)) + 72*A*B*a^2*b^2*d^4*i^3*x^2 - 72*A*B*b^4*c^2*d^2*i^3*x^
2 - 144*B^2*b^4*c^2*d^2*i^3*x^2*log((e*(a + b*x))/(c + d*x))^2 + B^2*a^2*b^
2*d^4*i^3*x^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*144i + A*B*a^3
*b*d^4*i^3*x*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*384i + 288*A*B*
a^2*b^2*d^4*i^3*x^2*log((e*(a + b*x))/(c + d*x)) - 288*A*B*b^4*c^2*d^2*i^3*
x^2*log((e*(a + b*x))/(c + d*x)) + A*B*a*b^3*d^4*i^3*x^3*atan((a*d*1i + b*c
*1i + b*d*x*2i)/(a*d - b*c))*384i + 192*A*B*a^3*b*d^4*i^3*x*log((e*(a + b*x
))/(c + d*x)) - 192*A*B*b^4*c^3*d*i^3*x*log((e*(a + b*x))/(c + d*x)) + A*B*
a^2*b^2*d^4*i^3*x^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*576i + 1
92*A*B*a*b^3*d^4*i^3*x^3*log((e*(a + b*x))/(c + d*x)) - 192*A*B*b^4*c*d^3*i
^3*x^3*log((e*(a + b*x))/(c + d*x)))/(96*b^4*g^5*(a*d - b*c)*(a + b*x)^4)
```

3.82 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 299

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \frac{B^2di^3(c+dx)^4}{32(bc-ad)^2g^6(a+bx)^4} - \frac{2bB^2i^3(c+dx)^5}{125(bc-ad)^2g^6(a+bx)^5} + \frac{Bdi^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)^2g^6(a+bx)^4} - \frac{2bBi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{25(bc-ad)^2g^6(a+bx)^5} + \frac{di^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^2g^6(a+bx)^4} - \frac{bi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5(bc-ad)^2g^6(a+bx)^5}$$

```
[Out] 1/32*B^2*d*i^3*(d*x+c)^4/(-a*d+b*c)^2/g^6/(b*x+a)^4-2/125*b*B^2*i^3*(d*x+c)
^5/(-a*d+b*c)^2/g^6/(b*x+a)^5+1/8*B*d*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+
c)))/(-a*d+b*c)^2/g^6/(b*x+a)^4-2/25*b*B*i^3*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d
*x+c)))/(-a*d+b*c)^2/g^6/(b*x+a)^5+1/4*d*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d
*x+c)))^2/(-a*d+b*c)^2/g^6/(b*x+a)^4-1/5*b*i^3*(d*x+c)^5*(A+B*ln(e*(b*x+a)/
(d*x+c)))^2/(-a*d+b*c)^2/g^6/(b*x+a)^5
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2562, 2395, 2342, 2341}

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}dx = -\frac{bi^3(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5g^6(a+bx)^5(bc-ad)^2} - \frac{2bBi^3(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{25g^6(a+bx)^5(bc-ad)^2} + \frac{di^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4g^6(a+bx)^4(bc-ad)^2} + \frac{Bdi^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{8g^6(a+bx)^4(bc-ad)^2} - \frac{2bB^2i^3(c+dx)^5}{125g^6(a+bx)^5(bc-ad)^2} + \frac{B^2di^3(c+dx)^4}{32g^6(a+bx)^4(bc-ad)^2}$$

```
[In] Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)^
6,x]
```

```
[Out] (B^2*d*i^3*(c + d*x)^4)/(32*(b*c - a*d)^2*g^6*(a + b*x)^4) - (2*b*B^2*i^3*(
c + d*x)^5)/(125*(b*c - a*d)^2*g^6*(a + b*x)^5) + (B*d*i^3*(c + d*x)^4*(A +
 B*Log[(e*(a + b*x))/(c + d*x)]))/(8*(b*c - a*d)^2*g^6*(a + b*x)^4) - (2*b*
B*i^3*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(25*(b*c - a*d)^2*g
^6*(a + b*x)^5) + (d*i^3*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
)/(4*(b*c - a*d)^2*g^6*(a + b*x)^4) - (b*i^3*(c + d*x)^5*(A + B*Log[(e*(a +
 b*x))/(c + d*x)])^2)/(5*(b*c - a*d)^2*g^6*(a + b*x)^5)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i^3\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex))^2}{x^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^6}\\
&= \frac{i^3\text{Subst}\left(\int \left(\frac{b(A+B\log(ex))^2}{x^6}-\frac{d(A+B\log(ex))^2}{x^5}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^6}\\
&= \frac{(bi^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^6}-\frac{(di^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^6}\\
&= \frac{di^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^2g^6(a+bx)^4}-\frac{bi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5(bc-ad)^2g^6(a+bx)^5}\\
&\quad+\frac{(2bBi^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{5(bc-ad)^2g^6}-\frac{(Bdi^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{2(bc-ad)^2g^6}\\
&= \frac{B^2di^3(c+dx)^4}{32(bc-ad)^2g^6(a+bx)^4}-\frac{2bB^2i^3(c+dx)^5}{125(bc-ad)^2g^6(a+bx)^5}\\
&\quad+\frac{Bdi^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)^2g^6(a+bx)^4}-\frac{2bBi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{25(bc-ad)^2g^6(a+bx)^5}\\
&\quad+\frac{di^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^2g^6(a+bx)^4}-\frac{bi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5(bc-ad)^2g^6(a+bx)^5}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.53 (sec) , antiderivative size = 2456, normalized size of antiderivative = 8.21

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b
*g*x)^6,x]
```

```
[Out] -1/36000*(i^3*(7200*(b*c - a*d)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 +
27000*d*(b*c - a*d)^4*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - 36
000*d^2*(-(b*c) + a*d)^3*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
 + 18000*d^3*(b*c - a*d)^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])
^2 + 2000*B*d^2*(a + b*x)^2*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3 - 18*A*
d*(b*c - a*d)^2*(a + b*x) - 15*B*d*(b*c - a*d)^2*(a + b*x) + 36*A*d^2*(b*c
- a*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a*d)*(a + b*x)^2 + 36*A*d^3*(a + b*x)^
3*Log[a + b*x] + 66*B*d^3*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*(a + b*x)^3*L
og[a + b*x]^2 + 12*B*(b*c - a*d)^3*Log[(e*(a + b*x))/(c + d*x)] - 18*B*d*(b
*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^2*(b*c - a*d)*(
a + b*x)^2*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]
*Log[(e*(a + b*x))/(c + d*x)] - 36*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^
3*(a + b*x)^3*Log[c + d*x] + 36*B*d^3*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c)
 + a*d)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[(e*(a + b*x))/(c + d*x)]*L
og[c + d*x] - 18*B*d^3*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*(a + b*x)^3*Lo
g[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2,
 (d*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*
x))/(b*c - a*d)]) + 375*B*d*(a + b*x)*(36*A*(b*c - a*d)^4 + 9*B*(b*c - a*d)
^4 + 48*A*d*(-(b*c) + a*d)^3*(a + b*x) + 28*B*d*(-(b*c) + a*d)^3*(a + b*x)
+ 72*A*d^2*(b*c - a*d)^2*(a + b*x)^2 + 78*B*d^2*(b*c - a*d)^2*(a + b*x)^2 +
 144*A*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 300*B*d^3*(-(b*c) + a*d)*(a + b*x)^
3 - 144*A*d^4*(a + b*x)^4*Log[a + b*x] - 300*B*d^4*(a + b*x)^4*Log[a + b*x]
 + 72*B*d^4*(a + b*x)^4*Log[a + b*x]^2 + 36*B*(b*c - a*d)^4*Log[(e*(a + b*x
))/(c + d*x)] + 48*B*d*(-(b*c) + a*d)^3*(a + b*x)*Log[(e*(a + b*x))/(c + d*
x)] + 72*B*d^2*(b*c - a*d)^2*(a + b*x)^2*Log[(e*(a + b*x))/(c + d*x)] + 144
*B*d^3*(-(b*c) + a*d)*(a + b*x)^3*Log[(e*(a + b*x))/(c + d*x)] - 144*B*d^4*
(a + b*x)^4*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] + 144*A*d^4*(a + b*x)
^4*Log[c + d*x] + 300*B*d^4*(a + b*x)^4*Log[c + d*x] - 144*B*d^4*(a + b*x)^
4*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] + 144*B*d^4*(a + b*x)^4*Lo
g[(e*(a + b*x))/(c + d*x)]*Log[c + d*x] + 72*B*d^4*(a + b*x)^4*Log[c + d*x]
^2 - 144*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 14
4*B*d^4*(a + b*x)^4*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 144*B*d^4*(a
 + b*x)^4*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 4*B*(720*A*(b*c - a*d)^5
```

```
 + 144*B*(b*c - a*d)^5 - 900*A*d*(b*c - a*d)^4*(a + b*x) - 405*B*d*(b*c - a
*d)^4*(a + b*x) + 1200*A*d^2*(b*c - a*d)^3*(a + b*x)^2 + 940*B*d^2*(b*c - a
*d)^3*(a + b*x)^2 - 1800*A*d^3*(b*c - a*d)^2*(a + b*x)^3 - 2310*B*d^3*(b*c
- a*d)^2*(a + b*x)^3 + 3600*A*d^4*(b*c - a*d)*(a + b*x)^4 + 8220*B*d^4*(b*c
 - a*d)*(a + b*x)^4 + 3600*A*d^5*(a + b*x)^5*Log[a + b*x] + 8220*B*d^5*(a +
 b*x)^5*Log[a + b*x] - 1800*B*d^5*(a + b*x)^5*Log[a + b*x]^2 + 720*B*(b*c -
 a*d)^5*Log[(e*(a + b*x))/(c + d*x)] - 900*B*d*(b*c - a*d)^4*(a + b*x)*Log[
(e*(a + b*x))/(c + d*x)] + 1200*B*d^2*(b*c - a*d)^3*(a + b*x)^2*Log[(e*(a +
 b*x))/(c + d*x)] - 1800*B*d^3*(b*c - a*d)^2*(a + b*x)^3*Log[(e*(a + b*x))/
(c + d*x)] + 3600*B*d^4*(b*c - a*d)*(a + b*x)^4*Log[(e*(a + b*x))/(c + d*x)
] + 3600*B*d^5*(a + b*x)^5*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] - 3600
*A*d^5*(a + b*x)^5*Log[c + d*x] - 8220*B*d^5*(a + b*x)^5*Log[c + d*x] + 360
0*B*d^5*(a + b*x)^5*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 3600*B
*d^5*(a + b*x)^5*Log[(e*(a + b*x))/(c + d*x)]*Log[c + d*x] - 1800*B*d^5*(a
+ b*x)^5*Log[c + d*x]^2 + 3600*B*d^5*(a + b*x)^5*Log[a + b*x]*Log[(b*(c + d
*x))/(b*c - a*d)] + 3600*B*d^5*(a + b*x)^5*PolyLog[2, (d*(a + b*x))/(-(b*c)
 + a*d)] + 3600*B*d^5*(a + b*x)^5*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) +
9000*B*d^3*(a + b*x)^3*(2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
) + 4*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 4*d
^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*d^2*(a
 + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 4*B*d*(a + b*
x)*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) + B*((
b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x
] + 2*d^2*(a + b*x)^2*Log[c + d*x]) + 2*B*d^2*(a + b*x)^2*(Log[a + b*x]*(Lo
g[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))
/(-(b*c) + a*d)]) - 2*B*d^2*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d
)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))
))/(b^4*(b*c - a*d)^2*g^6*(a + b*x)^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(287) = 574$.

Time = 1.86 (sec) , antiderivative size = 722, normalized size of antiderivative = 2.41

method	result
derivativedivides	$-\frac{e(ad-cb)\left(\frac{i^3 d^2 e^4 A^2 b}{5(ad-cb)^3 g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}-\frac{i^3 d^3 e^3 A^2}{4(ad-cb)^3 g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{2i^3 d^2 e^4 ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}-\frac{}{25\left(\frac{be}{d}+\right.}\right.}{(ad-cb)^3 g^6}\right.}{}$
default	$-\frac{e(ad-cb)\left(\frac{i^3 d^2 e^4 A^2 b}{5(ad-cb)^3 g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}-\frac{i^3 d^3 e^3 A^2}{4(ad-cb)^3 g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{2i^3 d^2 e^4 ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}-\frac{}{25\left(\frac{be}{d}+\right.}\right.}{(ad-cb)^3 g^6}\right.}{}$
parts	$\frac{i^3 A^2\left(-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{5b^4(bx+a)^5}+\frac{d^2(ad-cb)}{b^4(bx+a)^3}-\frac{d^3}{2b^4(bx+a)^2}-\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)}{4b^4(bx+a)^4}\right)}{g^6}-\frac{i^3B^2(ad-cb)^4e^4\left(\frac{d^6\left(-\right.}{}\right.}{}$
norman	Expression too large to display
parallelrisch	Expression too large to display
risch	Expression too large to display

```
[In] int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x,method=_RET
URNVERBOSE)
```

```
[Out] -1/d^2*e*(a*d-b*c)*(1/5*i^3*d^2*e^4/(a*d-b*c)^3/g^6*A^2*b/(b*e/d+(a*d-b*c)*
e/d/(d*x+c))^5-1/4*i^3*d^3*e^3/(a*d-b*c)^3/g^6*A^2/(b*e/d+(a*d-b*c)*e/d/(d*
x+c))^4-2*i^3*d^2*e^4/(a*d-b*c)^3/g^6*A*B*b*(-1/5/(b*e/d+(a*d-b*c)*e/d/(d*x
+c))^5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/25/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5
)+2*i^3*d^3*e^3/(a*d-b*c)^3/g^6*A*B*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4)-i^3*d^
2*e^4/(a*d-b*c)^3/g^6*B^2*b*(-1/5/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5*ln(b*e/d+
(a*d-b*c)*e/d/(d*x+c))^2-2/25/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5*ln(b*e/d+(a*d
-b*c)*e/d/(d*x+c))-2/125/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5)+i^3*d^3*e^3/(a*d-
b*c)^3/g^6*B^2*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d
/(d*x+c))^2-1/8/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x
+c))-1/32/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1045 vs. 2(287) = 574.

Time = 0.34 (sec) , antiderivative size = 1045, normalized size of antiderivative = 3.49

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx$$
$$= \frac{20\left(\left(20\,AB+9\,B^2\right)b^5cd^4-\left(20\,AB+9\,B^2\right)ab^4d^5\right)i^3x^4-10\left(\left(200\,A^2+20\,AB-11\,B^2\right)b^5c^2d^3-50\left(8\,A^2+\right.\right.}{}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x, alg
orithm="fricas")
```

```
[Out] 1/4000*(20*((20*A*B + 9*B^2)*b^5*c*d^4 - (20*A*B + 9*B^2)*a*b^4*d^5)*i^3*x^
4 - 10*((200*A^2 + 20*A*B - 11*B^2)*b^5*c^2*d^3 - 50*(8*A^2 + 4*A*B + B^2)*
a*b^4*c*d^4 + (200*A^2 + 180*A*B + 61*B^2)*a^2*b^3*d^5)*i^3*x^3 - 10*(2*(20
0*A^2 + 60*A*B + 7*B^2)*b^5*c^3*d^2 - 75*(8*A^2 + 4*A*B + B^2)*a*b^4*c^2*d^
3 + (200*A^2 + 180*A*B + 61*B^2)*a^3*b^2*d^5)*i^3*x^2 - 5*((600*A^2 + 220*A
*B + 39*B^2)*b^5*c^4*d - 100*(8*A^2 + 4*A*B + B^2)*a*b^4*c^3*d^2 + (200*A^2
 + 180*A*B + 61*B^2)*a^4*b*d^5)*i^3*x - (32*(25*A^2 + 10*A*B + 2*B^2)*b^5*c
^5 - 125*(8*A^2 + 4*A*B + B^2)*a*b^4*c^4*d + (200*A^2 + 180*A*B + 61*B^2)*a
^5*d^5)*i^3 + 200*(B^2*b^5*d^5*i^3*x^5 + 5*B^2*a*b^4*d^5*i^3*x^4 - 10*(B^2*
b^5*c^2*d^3 - 2*B^2*a*b^4*c*d^4)*i^3*x^3 - 10*(2*B^2*b^5*c^3*d^2 - 3*B^2*a*
b^4*c^2*d^3)*i^3*x^2 - 5*(3*B^2*b^5*c^4*d - 4*B^2*a*b^4*c^3*d^2)*i^3*x - (4
*B^2*b^5*c^5 - 5*B^2*a*b^4*c^4*d)*i^3)*log((b*e*x + a*e)/(d*x + c))^2 + 20*
((20*A*B + 9*B^2)*b^5*d^5*i^3*x^5 + 5*(4*B^2*b^5*c*d^4 + 5*(4*A*B + B^2)*a*
b^4*d^5)*i^3*x^4 - 10*((20*A*B + B^2)*b^5*c^2*d^3 - 10*(4*A*B + B^2)*a*b^4*
c*d^4)*i^3*x^3 - 10*(2*(20*A*B + 3*B^2)*b^5*c^3*d^2 - 15*(4*A*B + B^2)*a*b^
4*c^2*d^3)*i^3*x^2 - 5*((60*A*B + 11*B^2)*b^5*c^4*d - 20*(4*A*B + B^2)*a*b^
4*c^3*d^2)*i^3*x - (16*(5*A*B + B^2)*b^5*c^5 - 25*(4*A*B + B^2)*a*b^4*c^4*d
)*i^3)*log((b*e*x + a*e)/(d*x + c)))/((b^11*c^2 - 2*a*b^10*c*d + a^2*b^9*d^
2)*g^6*x^5 + 5*(a*b^10*c^2 - 2*a^2*b^9*c*d + a^3*b^8*d^2)*g^6*x^4 + 10*(a^2
*b^9*c^2 - 2*a^3*b^8*c*d + a^4*b^7*d^2)*g^6*x^3 + 10*(a^3*b^8*c^2 - 2*a^4*b
^7*c*d + a^5*b^6*d^2)*g^6*x^2 + 5*(a^4*b^7*c^2 - 2*a^5*b^6*c*d + a^6*b^5*d^
2)*g^6*x + (a^5*b^6*c^2 - 2*a^6*b^5*c*d + a^7*b^4*d^2)*g^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**6,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15765 vs. $2(287) = 574$.

Time = 1.50 (sec) , antiderivative size = 15765, normalized size of antiderivative = 52.73

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x, alg
orithm="maxima")
[Out] -3/20*(5*b*x + a)*B^2*c^2*d*i^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^7
*g^6*x^5 + 5*a*b^6*g^6*x^4 + 10*a^2*b^5*g^6*x^3 + 10*a^3*b^4*g^6*x^2 + 5*a^
4*b^3*g^6*x + a^5*b^2*g^6) - 1/10*(10*b^2*x^2 + 5*a*b*x + a^2)*B^2*c*d^2*i^
3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^8*g^6*x^5 + 5*a*b^7*g^6*x^4 + 1
0*a^2*b^6*g^6*x^3 + 10*a^3*b^5*g^6*x^2 + 5*a^4*b^4*g^6*x + a^5*b^3*g^6) - 1
/20*(10*b^3*x^3 + 10*a*b^2*x^2 + 5*a^2*b*x + a^3)*B^2*d^3*i^3*log(b*e*x/(d*
x + c) + a*e/(d*x + c))^2/(b^9*g^6*x^5 + 5*a*b^8*g^6*x^4 + 10*a^2*b^7*g^6*x
^3 + 10*a^3*b^6*g^6*x^2 + 5*a^4*b^5*g^6*x + a^5*b^4*g^6) - 1/9000*(60*((60*
b^4*d^4*x^4 + 12*b^4*c^4 - 63*a*b^3*c^3*d + 137*a^2*b^2*c^2*d^2 - 163*a^3*b
*c*d^3 + 137*a^4*d^4 - 30*(b^4*c*d^3 - 9*a*b^3*d^4)*x^3 + 10*(2*b^4*c^2*d^2
 - 13*a*b^3*c*d^3 + 47*a^2*b^2*d^4)*x^2 - 5*(3*b^4*c^3*d - 17*a*b^3*c^2*d^2
 + 43*a^2*b^2*c*d^3 - 77*a^3*b*d^4)*x)/((b^10*c^4 - 4*a*b^9*c^3*d + 6*a^2*b
^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*g^6*x^5 + 5*(a*b^9*c^4 - 4*a^2*
b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 + a^5*b^5*d^4)*g^6*x^4 + 10
*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2 - 4*a^5*b^5*c*d^3 + a^6
*b^4*d^4)*g^6*x^3 + 10*(a^3*b^7*c^4 - 4*a^4*b^6*c^3*d + 6*a^5*b^5*c^2*d^2 -
 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*g^6*x^2 + 5*(a^4*b^6*c^4 - 4*a^5*b^5*c^3*d
+ 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d^4)*g^6*x + (a^5*b^5*c^4 -
 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4*a^8*b^2*c*d^3 + a^9*b*d^4)*g^6) +
60*d^5*log(b*x + a)/((b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2 - 10*a^3
*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5)*g^6) - 60*d^5*log(d*x + c)/((b^
6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2
```

*c*d^4 - a^5*b*d^5)*g^6))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + (144*b^5*c
^5 - 1125*a*b^4*c^4*d + 4000*a^2*b^3*c^3*d^2 - 9000*a^3*b^2*c^2*d^3 + 18000
*a^4*b*c*d^4 - 12019*a^5*d^5 + 8220*(b^5*c*d^4 - a*b^4*d^5)*x^4 - 30*(77*b^
5*c^2*d^3 - 1250*a*b^4*c*d^4 + 1173*a^2*b^3*d^5)*x^3 + 10*(94*b^5*c^3*d^2 -
975*a*b^4*c^2*d^3 + 6600*a^2*b^3*c*d^4 - 5719*a^3*b^2*d^5)*x^2 - 1800*(b^5
*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b^2*d^5*x^2 + 5*a^
4*b*d^5*x + a^5*d^5)*log(b*x + a)^2 - 1800*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 +
10*a^2*b^3*d^5*x^3 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*log(d*x
+ c)^2 - 5*(81*b^5*c^4*d - 700*a*b^4*c^3*d^2 + 3000*a^2*b^3*c^2*d^3 - 1080
0*a^3*b^2*c*d^4 + 8419*a^4*b*d^5)*x + 8220*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 +
10*a^2*b^3*d^5*x^3 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*log(b*x
+ a) - 60*(137*b^5*d^5*x^5 + 685*a*b^4*d^5*x^4 + 1370*a^2*b^3*d^5*x^3 + 13
70*a^3*b^2*d^5*x^2 + 685*a^4*b*d^5*x + 137*a^5*d^5 - 60*(b^5*d^5*x^5 + 5*a*
b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5
*d^5)*log(b*x + a))*log(d*x + c))/(a^5*b^6*c^5*g^6 - 5*a^6*b^5*c^4*d*g^6 +
10*a^7*b^4*c^3*d^2*g^6 - 10*a^8*b^3*c^2*d^3*g^6 + 5*a^9*b^2*c*d^4*g^6 - a^1
0*b*d^5*g^6 + (b^11*c^5*g^6 - 5*a*b^10*c^4*d*g^6 + 10*a^2*b^9*c^3*d^2*g^6 -
10*a^3*b^8*c^2*d^3*g^6 + 5*a^4*b^7*c*d^4*g^6 - a^5*b^6*d^5*g^6)*x^5 + 5*(a
*b^10*c^5*g^6 - 5*a^2*b^9*c^4*d*g^6 + 10*a^3*b^8*c^3*d^2*g^6 - 10*a^4*b^7*c
^2*d^3*g^6 + 5*a^5*b^6*c*d^4*g^6 - a^6*b^5*d^5*g^6)*x^4 + 10*(a^2*b^9*c^5*g
^6 - 5*a^3*b^8*c^4*d*g^6 + 10*a^4*b^7*c^3*d^2*g^6 - 10*a^5*b^6*c^2*d^3*g^6
+ 5*a^6*b^5*c*d^4*g^6 - a^7*b^4*d^5*g^6)*x^3 + 10*(a^3*b^8*c^5*g^6 - 5*a^4*
b^7*c^4*d*g^6 + 10*a^5*b^6*c^3*d^2*g^6 - 10*a^6*b^5*c^2*d^3*g^6 + 5*a^7*b^4
*c*d^4*g^6 - a^8*b^3*d^5*g^6)*x^2 + 5*(a^4*b^7*c^5*g^6 - 5*a^5*b^6*c^4*d*g^
6 + 10*a^6*b^5*c^3*d^2*g^6 - 10*a^7*b^4*c^2*d^3*g^6 + 5*a^8*b^3*c*d^4*g^6 -
a^9*b^2*d^5*g^6)*x))*B^2*c^3*i^3 - 1/12000*(60*((27*a*b^4*c^4 - 148*a^2*b^
3*c^3*d + 352*a^3*b^2*c^2*d^2 - 548*a^4*b*c*d^3 + 77*a^5*d^4 - 60*(5*b^5*c*
d^3 - a*b^4*d^4)*x^4 + 30*(5*b^5*c^2*d^2 - 46*a*b^4*c*d^3 + 9*a^2*b^3*d^4)*
x^3 - 10*(10*b^5*c^3*d - 67*a*b^4*c^2*d^2 + 248*a^2*b^3*c*d^3 - 47*a^3*b^2*
d^4)*x^2 + 5*(15*b^5*c^4 - 88*a*b^4*c^3*d + 232*a^2*b^3*c^2*d^2 - 428*a^3*b
^2*c*d^3 + 77*a^4*b*d^4)*x)/((b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2
- 4*a^3*b^8*c*d^3 + a^4*b^7*d^4)*g^6*x^5 + 5*(a*b^10*c^4 - 4*a^2*b^9*c^3*d
+ 6*a^3*b^8*c^2*d^2 - 4*a^4*b^7*c*d^3 + a^5*b^6*d^4)*g^6*x^4 + 10*(a^2*b^9
*c^4 - 4*a^3*b^8*c^3*d + 6*a^4*b^7*c^2*d^2 - 4*a^5*b^6*c*d^3 + a^6*b^5*d^4)
*g^6*x^3 + 10*(a^3*b^8*c^4 - 4*a^4*b^7*c^3*d + 6*a^5*b^6*c^2*d^2 - 4*a^6*b^
5*c*d^3 + a^7*b^4*d^4)*g^6*x^2 + 5*(a^4*b^7*c^4 - 4*a^5*b^6*c^3*d + 6*a^6*b
^5*c^2*d^2 - 4*a^7*b^4*c*d^3 + a^8*b^3*d^4)*g^6*x + (a^5*b^6*c^4 - 4*a^6*b^
5*c^3*d + 6*a^7*b^4*c^2*d^2 - 4*a^8*b^3*c*d^3 + a^9*b^2*d^4)*g^6) - 60*(5*b
*c*d^4 - a*d^5)*log(b*x + a)/((b^7*c^5 - 5*a*b^6*c^4*d + 10*a^2*b^5*c^3*d^2
- 10*a^3*b^4*c^2*d^3 + 5*a^4*b^3*c*d^4 - a^5*b^2*d^5)*g^6) + 60*(5*b*c*d^4
- a*d^5)*log(d*x + c)/((b^7*c^5 - 5*a*b^6*c^4*d + 10*a^2*b^5*c^3*d^2 - 10*
a^3*b^4*c^2*d^3 + 5*a^4*b^3*c*d^4 - a^5*b^2*d^5)*g^6))*log(b*e*x/(d*x + c)
+ a*e/(d*x + c)) + (549*a*b^5*c^5 - 4625*a^2*b^4*c^4*d + 19000*a^3*b^3*c^3*
d^2 - 63000*a^4*b^2*c^2*d^3 + 51875*a^5*b*c*d^4 - 3799*a^6*d^5 - 60*(625*b^
6*c^2*d^3 - 702*a*b^5*c*d^4 + 77*a^2*b^4*d^5)*x^4 + 30*(325*b^6*c^3*d^2 - 5

667*a*b^5*c^2*d^3 + 5975*a^2*b^4*c*d^4 - 633*a^3*b^3*d^5)*x^3 - 10*(350*b^6
*c^4*d - 3949*a*b^5*c^3*d^2 + 29475*a^2*b^4*c^2*d^3 - 28775*a^3*b^3*c*d^4 +
2899*a^4*b^2*d^5)*x^2 + 1800*(5*a^5*b*c*d^4 - a^6*d^5 + (5*b^6*c*d^4 - a*b
^5*d^5)*x^5 + 5*(5*a*b^5*c*d^4 - a^2*b^4*d^5)*x^4 + 10*(5*a^2*b^4*c*d^4 - a
^3*b^3*d^5)*x^3 + 10*(5*a^3*b^3*c*d^4 - a^4*b^2*d^5)*x^2 + 5*(5*a^4*b^2*c*d
^4 - a^5*b*d^5)*x)*log(b*x + a)^2 + 1800*(5*a^5*b*c*d^4 - a^6*d^5 + (5*b^6*
c*d^4 - a*b^5*d^5)*x^5 + 5*(5*a*b^5*c*d^4 - a^2*b^4*d^5)*x^4 + 10*(5*a^2*b^
4*c*d^4 - a^3*b^3*d^5)*x^3 + 10*(5*a^3*b^3*c*d^4 - a^4*b^2*d^5)*x^2 + 5*(5*
a^4*b^2*c*d^4 - a^5*b*d^5)*x)*log(d*x + c)^2 + 5*(225*b^6*c^5 - 2201*a*b^5*
c^4*d + 10900*a^2*b^4*c^3*d^2 - 46200*a^3*b^3*c^2*d^3 + 41075*a^4*b^2*c*d^4
- 3799*a^5*b*d^5)*x - 60*(625*a^5*b*c*d^4 - 77*a^6*d^5 + (625*b^6*c*d^4 -
77*a*b^5*d^5)*x^5 + 5*(625*a*b^5*c*d^4 - 77*a^2*b^4*d^5)*x^4 + 10*(625*a^2*
b^4*c*d^4 - 77*a^3*b^3*d^5)*x^3 + 10*(625*a^3*b^3*c*d^4 - 77*a^4*b^2*d^5)*x
^2 + 5*(625*a^4*b^2*c*d^4 - 77*a^5*b*d^5)*x)*log(b*x + a) + 60*(625*a^5*b*c
*d^4 - 77*a^6*d^5 + (625*b^6*c*d^4 - 77*a*b^5*d^5)*x^5 + 5*(625*a*b^5*c*d^4
- 77*a^2*b^4*d^5)*x^4 + 10*(625*a^2*b^4*c*d^4 - 77*a^3*b^3*d^5)*x^3 + 10*(
625*a^3*b^3*c*d^4 - 77*a^4*b^2*d^5)*x^2 + 5*(625*a^4*b^2*c*d^4 - 77*a^5*b*d
^5)*x - 60*(5*a^5*b*c*d^4 - a^6*d^5 + (5*b^6*c*d^4 - a*b^5*d^5)*x^5 + 5*(5*
a*b^5*c*d^4 - a^2*b^4*d^5)*x^4 + 10*(5*a^2*b^4*c*d^4 - a^3*b^3*d^5)*x^3 + 1
0*(5*a^3*b^3*c*d^4 - a^4*b^2*d^5)*x^2 + 5*(5*a^4*b^2*c*d^4 - a^5*b*d^5)*x)*
log(b*x + a))*log(d*x + c))/(a^5*b^7*c^5*g^6 - 5*a^6*b^6*c^4*d*g^6 + 10*a^7
*b^5*c^3*d^2*g^6 - 10*a^8*b^4*c^2*d^3*g^6 + 5*a^9*b^3*c*d^4*g^6 - a^10*b^2*
d^5*g^6 + (b^12*c^5*g^6 - 5*a*b^11*c^4*d*g^6 + 10*a^2*b^10*c^3*d^2*g^6 - 10
*a^3*b^9*c^2*d^3*g^6 + 5*a^4*b^8*c*d^4*g^6 - a^5*b^7*d^5*g^6)*x^5 + 5*(a*b^
11*c^5*g^6 - 5*a^2*b^10*c^4*d*g^6 + 10*a^3*b^9*c^3*d^2*g^6 - 10*a^4*b^8*c^2
*d^3*g^6 + 5*a^5*b^7*c*d^4*g^6 - a^6*b^6*d^5*g^6)*x^4 + 10*(a^2*b^10*c^5*g^
6 - 5*a^3*b^9*c^4*d*g^6 + 10*a^4*b^8*c^3*d^2*g^6 - 10*a^5*b^7*c^2*d^3*g^6 +
5*a^6*b^6*c*d^4*g^6 - a^7*b^5*d^5*g^6)*x^3 + 10*(a^3*b^9*c^5*g^6 - 5*a^4*b
^8*c^4*d*g^6 + 10*a^5*b^7*c^3*d^2*g^6 - 10*a^6*b^6*c^2*d^3*g^6 + 5*a^7*b^5*
c*d^4*g^6 - a^8*b^4*d^5*g^6)*x^2 + 5*(a^4*b^8*c^5*g^6 - 5*a^5*b^7*c^4*d*g^6
+ 10*a^6*b^6*c^3*d^2*g^6 - 10*a^7*b^5*c^2*d^3*g^6 + 5*a^8*b^4*c*d^4*g^6 -
a^9*b^3*d^5*g^6)*x))*B^2*c^2*d*i^3 - 1/18000*(60*((47*a^2*b^4*c^4 - 278*a^3
*b^3*c^3*d + 822*a^4*b^2*c^2*d^2 - 278*a^5*b*c*d^3 + 47*a^6*d^4 + 60*(10*b^
6*c^2*d^2 - 5*a*b^5*c*d^3 + a^2*b^4*d^4)*x^4 - 30*(10*b^6*c^3*d - 95*a*b^5*
c^2*d^2 + 46*a^2*b^4*c*d^3 - 9*a^3*b^3*d^4)*x^3 + 10*(20*b^6*c^4 - 140*a*b^
5*c^3*d + 537*a^2*b^4*c^2*d^2 - 248*a^3*b^3*c*d^3 + 47*a^4*b^2*d^4)*x^2 + 5
*(35*a*b^5*c^4 - 218*a^2*b^4*c^3*d + 702*a^3*b^3*c^2*d^2 - 278*a^4*b^2*c*d^
3 + 47*a^5*b*d^4)*x)/((b^12*c^4 - 4*a*b^11*c^3*d + 6*a^2*b^10*c^2*d^2 - 4*a
^3*b^9*c*d^3 + a^4*b^8*d^4)*g^6*x^5 + 5*(a*b^11*c^4 - 4*a^2*b^10*c^3*d + 6*
a^3*b^9*c^2*d^2 - 4*a^4*b^8*c*d^3 + a^5*b^7*d^4)*g^6*x^4 + 10*(a^2*b^10*c^4
- 4*a^3*b^9*c^3*d + 6*a^4*b^8*c^2*d^2 - 4*a^5*b^7*c*d^3 + a^6*b^6*d^4)*g^6
x^3 + 10(a^3*b^9*c^4 - 4*a^4*b^8*c^3*d + 6*a^5*b^7*c^2*d^2 - 4*a^6*b^6*c*
d^3 + a^7*b^5*d^4)*g^6*x^2 + 5*(a^4*b^8*c^4 - 4*a^5*b^7*c^3*d + 6*a^6*b^6*c
^2*d^2 - 4*a^7*b^5*c*d^3 + a^8*b^4*d^4)*g^6*x + (a^5*b^7*c^4 - 4*a^6*b^6*c^
3*d + 6*a^7*b^5*c^2*d^2 - 4*a^8*b^4*c*d^3 + a^9*b^3*d^4)*g^6) + 60*(10*b^2*

c^2*d^3 - 5*a*b*c*d^4 + a^2*d^5)*log(b*x + a)/((b^8*c^5 - 5*a*b^7*c^4*d + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 + 5*a^4*b^4*c*d^4 - a^5*b^3*d^5)*g^6) - 60*(10*b^2*c^2*d^3 - 5*a*b*c*d^4 + a^2*d^5)*log(d*x + c)/((b^8*c^5 - 5*a*b^7*c^4*d + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 + 5*a^4*b^4*c*d^4 - a^5*b^3*d^5)*g^6))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + (1489*a^2*b^5*c^5 - 14375*a^3*b^4*c^4*d + 85000*a^4*b^3*c^3*d^2 - 85000*a^5*b^2*c^2*d^3 + 14375*a^6*b*c*d^4 - 1489*a^7*d^5 + 60*(1100*b^7*c^3*d^2 - 1425*a*b^6*c^2*d^3 + 372*a^2*b^5*c*d^4 - 47*a^3*b^4*d^5)*x^4 - 30*(500*b^7*c^4*d - 9825*a*b^6*c^3*d^2 + 11937*a^2*b^5*c^2*d^3 - 2975*a^3*b^4*c*d^4 + 363*a^4*b^3*d^5)*x^3 + 10*(400*b^7*c^5 - 5450*a*b^6*c^4*d + 49189*a^2*b^5*c^3*d^2 - 55525*a^3*b^4*c^2*d^3 + 12875*a^4*b^3*c*d^4 - 1489*a^5*b^2*d^5)*x^2 - 1800*(10*a^5*b^2*c^2*d^3 - 5*a^6*b*c*d^4 + a^7*d^5 + (10*b^7*c^2*d^3 - 5*a*b^6*c*d^4 + a^2*b^5*d^5)*x^5 + 5*(10*a*b^6*c^2*d^3 - 5*a^2*b^5*c*d^4 + a^3*b^4*d^5)*x^4 + 10*(10*a^2*b^5*c^2*d^3 - 5*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^3 + 10*(10*a^3*b^4*c^2*d^3 - 5*a^4*b^3*c*d^4 + a^5*b^2*d^5)*x^2 + 5*(10*a^4*b^3*c^2*d^3 - 5*a^5*b^2*c*d^4 + a^6*b*d^5)*x)*log(b*x + a)^2 - 1800*(10*a^5*b^2*c^2*d^3 - 5*a^6*b*c*d^4 + a^7*d^5 + (10*b^7*c^2*d^3 - 5*a*b^6*c*d^4 + a^2*b^5*d^5)*x^5 + 5*(10*a*b^6*c^2*d^3 - 5*a^2*b^5*c*d^4 + a^3*b^4*d^5)*x^4 + 10*(10*a^2*b^5*c^2*d^3 - 5*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^3 + 10*(10*a^3*b^4*c^2*d^3 - 5*a^4*b^3*c*d^4 + a^5*b^2*d^5)*x^2 + 5*(10*a^4*b^3*c^2*d^3 - 5*a^5*b^2*c*d^4 + a^6*b*d^5)*x)*log(d*x + c)^2 + 5*(925*a*b^6*c^5 - 9911*a^2*b^5*c^4*d + 67900*a^3*b^4*c^3*d^2 - 71800*a^4*b^3*c^2*d^3 + 14375*a^5*b^2*c*d^4 - 1489*a^6*b*d^5)*x + 60*(1100*a^5*b^2*c^2*d^3 - 325*a^6*b*c*d^4 + 47*a^7*d^5 + (1100*b^7*c^2*d^3 - 325*a*b^6*c*d^4 + 47*a^2*b^5*d^5)*x^5 + 5*(1100*a*b^6*c^2*d^3 - 325*a^2*b^5*c*d^4 + 47*a^3*b^4*d^5)*x^4 + 10*(1100*a^2*b^5*c^2*d^3 - 325*a^3*b^4*c*d^4 + 47*a^4*b^3*d^5)*x^3 + 10*(1100*a^3*b^4*c^2*d^3 - 325*a^4*b^3*c*d^4 + 47*a^5*b^2*d^5)*x^2 + 5*(1100*a^4*b^3*c^2*d^3 - 325*a^5*b^2*c*d^4 + 47*a^6*b*d^5)*x)*log(b*x + a) - 60*(1100*a^5*b^2*c^2*d^3 - 325*a^6*b*c*d^4 + 47*a^7*d^5 + (1100*b^7*c^2*d^3 - 325*a*b^6*c*d^4 + 47*a^2*b^5*d^5)*x^5 + 5*(1100*a*b^6*c^2*d^3 - 325*a^2*b^5*c*d^4 + 47*a^3*b^4*d^5)*x^4 + 10*(1100*a^2*b^5*c^2*d^3 - 325*a^3*b^4*c*d^4 + 47*a^4*b^3*d^5)*x^3 + 10*(1100*a^3*b^4*c^2*d^3 - 325*a^4*b^3*c*d^4 + 47*a^5*b^2*d^5)*x^2 + 5*(1100*a^4*b^3*c^2*d^3 - 325*a^5*b^2*c*d^4 + 47*a^6*b*d^5)*x - 60*(10*a^5*b^2*c^2*d^3 - 5*a^6*b*c*d^4 + a^7*d^5 + (10*b^7*c^2*d^3 - 5*a*b^6*c*d^4 + a^2*b^5*d^5)*x^5 + 5*(10*a*b^6*c^2*d^3 - 5*a^2*b^5*c*d^4 + a^3*b^4*d^5)*x^4 + 10*(10*a^2*b^5*c^2*d^3 - 5*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^3 + 10*(10*a^3*b^4*c^2*d^3 - 5*a^4*b^3*c*d^4 + a^5*b^2*d^5)*x^2 + 5*(10*a^4*b^3*c^2*d^3 - 5*a^5*b^2*c*d^4 + a^6*b*d^5)*x)*log(b*x + a))*log(d*x + c))/(a^5*b^8*c^5*g^6 - 5*a^6*b^7*c^4*d*g^6 + 10*a^7*b^6*c^3*d^2*g^6 - 10*a^8*b^5*c^2*d^3*g^6 + 5*a^9*b^4*c*d^4*g^6 - a^10*b^3*d^5*g^6 + (b^13*c^5*g^6 - 5*a*b^12*c^4*d*g^6 + 10*a^2*b^11*c^3*d^2*g^6 - 10*a^3*b^10*c^2*d^3*g^6 + 5*a^4*b^9*c*d^4*g^6 - a^5*b^8*d^5*g^6)*x^5 + 5*(a*b^12*c^5*g^6 - 5*a^2*b^11*c^4*d*g^6 + 10*a^3*b^10*c^3*d^2*g^6 - 10*a^4*b^9*c^2*d^3*g^6 + 5*a^5*b^8*c*d^4*g^6 - a^6*b^7*d^5*g^6)*x^4 + 10*(a^2*b^11*c^5*g^6 - 5*a^3*b^10*c^4*d*g^6 + 10*a^4*b^9*c^3*d^2*g^6 - 10*a^5*b^8*c^2*d^3*g^6 + 5*a^6*b^7*c*d^4*g^6 - a^7*b^6*d^5*g^6)*x^3 + 10*(a^3*b^

10*c^5*g^6 - 5*a^4*b^9*c^4*d*g^6 + 10*a^5*b^8*c^3*d^2*g^6 - 10*a^6*b^7*c^2*
d^3*g^6 + 5*a^7*b^6*c*d^4*g^6 - a^8*b^5*d^5*g^6)*x^2 + 5*(a^4*b^9*c^5*g^6 -
 5*a^5*b^8*c^4*d*g^6 + 10*a^6*b^7*c^3*d^2*g^6 - 10*a^7*b^6*c^2*d^3*g^6 + 5*
a^8*b^5*c*d^4*g^6 - a^9*b^4*d^5*g^6)*x))*B^2*c*d^2*i^3 - 1/36000*(60*((77*a
^3*b^4*c^4 - 548*a^4*b^3*c^3*d + 352*a^5*b^2*c^2*d^2 - 148*a^6*b*c*d^3 + 27
*a^7*d^4 - 60*(10*b^7*c^3*d - 10*a*b^6*c^2*d^2 + 5*a^2*b^5*c*d^3 - a^3*b^4*
d^4)*x^4 + 30*(10*b^7*c^4 - 100*a*b^6*c^3*d + 95*a^2*b^5*c^2*d^2 - 46*a^3*b
^4*c*d^3 + 9*a^4*b^3*d^4)*x^3 + 10*(50*a*b^6*c^4 - 410*a^2*b^5*c^3*d + 337*
a^3*b^4*c^2*d^2 - 148*a^4*b^3*c*d^3 + 27*a^5*b^2*d^4)*x^2 + 5*(65*a^2*b^5*c
^4 - 488*a^3*b^4*c^3*d + 352*a^4*b^3*c^2*d^2 - 148*a^5*b^2*c*d^3 + 27*a^6*b
*d^4)*x)/((b^13*c^4 - 4*a*b^12*c^3*d + 6*a^2*b^11*c^2*d^2 - 4*a^3*b^10*c*d^
3 + a^4*b^9*d^4)*g^6*x^5 + 5*(a*b^12*c^4 - 4*a^2*b^11*c^3*d + 6*a^3*b^10*c^
2*d^2 - 4*a^4*b^9*c*d^3 + a^5*b^8*d^4)*g^6*x^4 + 10*(a^2*b^11*c^4 - 4*a^3*b
^10*c^3*d + 6*a^4*b^9*c^2*d^2 - 4*a^5*b^8*c*d^3 + a^6*b^7*d^4)*g^6*x^3 + 10
*(a^3*b^10*c^4 - 4*a^4*b^9*c^3*d + 6*a^5*b^8*c^2*d^2 - 4*a^6*b^7*c*d^3 + a^
7*b^6*d^4)*g^6*x^2 + 5*(a^4*b^9*c^4 - 4*a^5*b^8*c^3*d + 6*a^6*b^7*c^2*d^2 -
 4*a^7*b^6*c*d^3 + a^8*b^5*d^4)*g^6*x + (a^5*b^8*c^4 - 4*a^6*b^7*c^3*d + 6*
a^7*b^6*c^2*d^2 - 4*a^8*b^5*c*d^3 + a^9*b^4*d^4)*g^6) - 60*(10*b^3*c^3*d^2
- 10*a*b^2*c^2*d^3 + 5*a^2*b*c*d^4 - a^3*d^5)*log(b*x + a)/((b^9*c^5 - 5*a*
b^8*c^4*d + 10*a^2*b^7*c^3*d^2 - 10*a^3*b^6*c^2*d^3 + 5*a^4*b^5*c*d^4 - a^5
*b^4*d^5)*g^6) + 60*(10*b^3*c^3*d^2 - 10*a*b^2*c^2*d^3 + 5*a^2*b*c*d^4 - a^
3*d^5)*log(d*x + c)/((b^9*c^5 - 5*a*b^8*c^4*d + 10*a^2*b^7*c^3*d^2 - 10*a^3
*b^6*c^2*d^3 + 5*a^4*b^5*c*d^4 - a^5*b^4*d^5)*g^6))*log(b*e*x/(d*x + c) + a
*e/(d*x + c)) + (3799*a^3*b^5*c^5 - 51875*a^4*b^4*c^4*d + 63000*a^5*b^3*c^3
*d^2 - 19000*a^6*b^2*c^2*d^3 + 4625*a^7*b*c*d^4 - 549*a^8*d^5 - 60*(900*b^8
*c^4*d - 1400*a*b^7*c^3*d^2 + 675*a^2*b^6*c^2*d^3 - 202*a^3*b^5*c*d^4 + 27*
a^4*b^4*d^5)*x^4 + 30*(300*b^8*c^5 - 7700*a*b^7*c^4*d + 11175*a^2*b^6*c^3*d
^2 - 5017*a^3*b^5*c^2*d^3 + 1425*a^4*b^4*c*d^4 - 183*a^5*b^3*d^5)*x^3 + 10*
(1900*a*b^7*c^5 - 33950*a^2*b^6*c^4*d + 45999*a^3*b^5*c^3*d^2 - 18025*a^4*b
^4*c^2*d^3 + 4625*a^5*b^3*c*d^4 - 549*a^6*b^2*d^5)*x^2 + 1800*(10*a^5*b^3*c
^3*d^2 - 10*a^6*b^2*c^2*d^3 + 5*a^7*b*c*d^4 - a^8*d^5 + (10*b^8*c^3*d^2 - 1
0*a*b^7*c^2*d^3 + 5*a^2*b^6*c*d^4 - a^3*b^5*d^5)*x^5 + 5*(10*a*b^7*c^3*d^2
- 10*a^2*b^6*c^2*d^3 + 5*a^3*b^5*c*d^4 - a^4*b^4*d^5)*x^4 + 10*(10*a^2*b^6*
c^3*d^2 - 10*a^3*b^5*c^2*d^3 + 5*a^4*b^4*c*d^4 - a^5*b^3*d^5)*x^3 + 10*(10*
a^3*b^5*c^3*d^2 - 10*a^4*b^4*c^2*d^3 + 5*a^5*b^3*c*d^4 - a^6*b^2*d^5)*x^2 +
 5*(10*a^4*b^4*c^3*d^2 - 10*a^5*b^3*c^2*d^3 + 5*a^6*b^2*c*d^4 - a^7*b*d^5)*
x)*log(b*x + a)^2 + 1800*(10*a^5*b^3*c^3*d^2 - 10*a^6*b^2*c^2*d^3 + 5*a^7*b
*c*d^4 - a^8*d^5 + (10*b^8*c^3*d^2 - 10*a*b^7*c^2*d^3 + 5*a^2*b^6*c*d^4 - a
^3*b^5*d^5)*x^5 + 5*(10*a*b^7*c^3*d^2 - 10*a^2*b^6*c^2*d^3 + 5*a^3*b^5*c*d^
4 - a^4*b^4*d^5)*x^4 + 10*(10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 + 5*a^4*
b^4*c*d^4 - a^5*b^3*d^5)*x^3 + 10*(10*a^3*b^5*c^3*d^2 - 10*a^4*b^4*c^2*d^3
+ 5*a^5*b^3*c*d^4 - a^6*b^2*d^5)*x^2 + 5*(10*a^4*b^4*c^3*d^2 - 10*a^5*b^3*c
^2*d^3 + 5*a^6*b^2*c*d^4 - a^7*b*d^5)*x)*log(d*x + c)^2 + 5*(2875*a^2*b^6*c
^5 - 43451*a^3*b^5*c^4*d + 55500*a^4*b^4*c^3*d^2 - 19000*a^5*b^3*c^2*d^3 +
4625*a^6*b^2*c*d^4 - 549*a^7*b*d^5)*x - 60*(900*a^5*b^3*c^3*d^2 - 500*a^6*b

```
^2*c^2*d^3 + 175*a^7*b*c*d^4 - 27*a^8*d^5 + (900*b^8*c^3*d^2 - 500*a*b^7*c^
2*d^3 + 175*a^2*b^6*c*d^4 - 27*a^3*b^5*d^5)*x^5 + 5*(900*a*b^7*c^3*d^2 - 50
0*a^2*b^6*c^2*d^3 + 175*a^3*b^5*c*d^4 - 27*a^4*b^4*d^5)*x^4 + 10*(900*a^2*b
^6*c^3*d^2 - 500*a^3*b^5*c^2*d^3 + 175*a^4*b^4*c*d^4 - 27*a^5*b^3*d^5)*x^3
+ 10*(900*a^3*b^5*c^3*d^2 - 500*a^4*b^4*c^2*d^3 + 175*a^5*b^3*c*d^4 - 27*a^
6*b^2*d^5)*x^2 + 5*(900*a^4*b^4*c^3*d^2 - 500*a^5*b^3*c^2*d^3 + 175*a^6*b^2
*c*d^4 - 27*a^7*b*d^5)*x)*log(b*x + a) + 60*(900*a^5*b^3*c^3*d^2 - 500*a^6*
b^2*c^2*d^3 + 175*a^7*b*c*d^4 - 27*a^8*d^5 + (900*b^8*c^3*d^2 - 500*a*b^7*c
^2*d^3 + 175*a^2*b^6*c*d^4 - 27*a^3*b^5*d^5)*x^5 + 5*(900*a*b^7*c^3*d^2 - 5
00*a^2*b^6*c^2*d^3 + 175*a^3*b^5*c*d^4 - 27*a^4*b^4*d^5)*x^4 + 10*(900*a^2*
b^6*c^3*d^2 - 500*a^3*b^5*c^2*d^3 + 175*a^4*b^4*c*d^4 - 27*a^5*b^3*d^5)*x^3
 + 10*(900*a^3*b^5*c^3*d^2 - 500*a^4*b^4*c^2*d^3 + 175*a^5*b^3*c*d^4 - 27*a
^6*b^2*d^5)*x^2 + 5*(900*a^4*b^4*c^3*d^2 - 500*a^5*b^3*c^2*d^3 + 175*a^6*b^
2*c*d^4 - 27*a^7*b*d^5)*x - 60*(10*a^5*b^3*c^3*d^2 - 10*a^6*b^2*c^2*d^3 + 5
*a^7*b*c*d^4 - a^8*d^5 + (10*b^8*c^3*d^2 - 10*a*b^7*c^2*d^3 + 5*a^2*b^6*c*d
^4 - a^3*b^5*d^5)*x^5 + 5*(10*a*b^7*c^3*d^2 - 10*a^2*b^6*c^2*d^3 + 5*a^3*b^
5*c*d^4 - a^4*b^4*d^5)*x^4 + 10*(10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 +
5*a^4*b^4*c*d^4 - a^5*b^3*d^5)*x^3 + 10*(10*a^3*b^5*c^3*d^2 - 10*a^4*b^4*c^
2*d^3 + 5*a^5*b^3*c*d^4 - a^6*b^2*d^5)*x^2 + 5*(10*a^4*b^4*c^3*d^2 - 10*a^5
*b^3*c^2*d^3 + 5*a^6*b^2*c*d^4 - a^7*b*d^5)*x)*log(b*x + a))*log(d*x + c))/
(a^5*b^9*c^5*g^6 - 5*a^6*b^8*c^4*d*g^6 + 10*a^7*b^7*c^3*d^2*g^6 - 10*a^8*b^
6*c^2*d^3*g^6 + 5*a^9*b^5*c*d^4*g^6 - a^10*b^4*d^5*g^6 + (b^14*c^5*g^6 - 5*
a*b^13*c^4*d*g^6 + 10*a^2*b^12*c^3*d^2*g^6 - 10*a^3*b^11*c^2*d^3*g^6 + 5*a^
4*b^10*c*d^4*g^6 - a^5*b^9*d^5*g^6)*x^5 + 5*(a*b^13*c^5*g^6 - 5*a^2*b^12*c^
4*d*g^6 + 10*a^3*b^11*c^3*d^2*g^6 - 10*a^4*b^10*c^2*d^3*g^6 + 5*a^5*b^9*c*d
^4*g^6 - a^6*b^8*d^5*g^6)*x^4 + 10*(a^2*b^12*c^5*g^6 - 5*a^3*b^11*c^4*d*g^6
 + 10*a^4*b^10*c^3*d^2*g^6 - 10*a^5*b^9*c^2*d^3*g^6 + 5*a^6*b^8*c*d^4*g^6 -
 a^7*b^7*d^5*g^6)*x^3 + 10*(a^3*b^11*c^5*g^6 - 5*a^4*b^10*c^4*d*g^6 + 10*a^
5*b^9*c^3*d^2*g^6 - 10*a^6*b^8*c^2*d^3*g^6 + 5*a^7*b^7*c*d^4*g^6 - a^8*b^6*
d^5*g^6)*x^2 + 5*(a^4*b^10*c^5*g^6 - 5*a^5*b^9*c^4*d*g^6 + 10*a^6*b^8*c^3*d
^2*g^6 - 10*a^7*b^7*c^2*d^3*g^6 + 5*a^8*b^6*c*d^4*g^6 - a^9*b^5*d^5*g^6)*x)
)*B^2*d^3*i^3 - 1/600*A*B*d^3*i^3*(60*(10*b^3*x^3 + 10*a*b^2*x^2 + 5*a^2*b*
x + a^3)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^9*g^6*x^5 + 5*a*b^8*g^6*x^
4 + 10*a^2*b^7*g^6*x^3 + 10*a^3*b^6*g^6*x^2 + 5*a^4*b^5*g^6*x + a^5*b^4*g^6
) + (77*a^3*b^4*c^4 - 548*a^4*b^3*c^3*d + 352*a^5*b^2*c^2*d^2 - 148*a^6*b*c
*d^3 + 27*a^7*d^4 - 60*(10*b^7*c^3*d - 10*a*b^6*c^2*d^2 + 5*a^2*b^5*c*d^3 -
 a^3*b^4*d^4)*x^4 + 30*(10*b^7*c^4 - 100*a*b^6*c^3*d + 95*a^2*b^5*c^2*d^2 -
 46*a^3*b^4*c*d^3 + 9*a^4*b^3*d^4)*x^3 + 10*(50*a*b^6*c^4 - 410*a^2*b^5*c^3
*d + 337*a^3*b^4*c^2*d^2 - 148*a^4*b^3*c*d^3 + 27*a^5*b^2*d^4)*x^2 + 5*(65*
a^2*b^5*c^4 - 488*a^3*b^4*c^3*d + 352*a^4*b^3*c^2*d^2 - 148*a^5*b^2*c*d^3 +
 27*a^6*b*d^4)*x)/((b^13*c^4 - 4*a*b^12*c^3*d + 6*a^2*b^11*c^2*d^2 - 4*a^3*
b^10*c*d^3 + a^4*b^9*d^4)*g^6*x^5 + 5*(a*b^12*c^4 - 4*a^2*b^11*c^3*d + 6*a^
3*b^10*c^2*d^2 - 4*a^4*b^9*c*d^3 + a^5*b^8*d^4)*g^6*x^4 + 10*(a^2*b^11*c^4
- 4*a^3*b^10*c^3*d + 6*a^4*b^9*c^2*d^2 - 4*a^5*b^8*c*d^3 + a^6*b^7*d^4)*g^6
*x^3 + 10*(a^3*b^10*c^4 - 4*a^4*b^9*c^3*d + 6*a^5*b^8*c^2*d^2 - 4*a^6*b^7*c
```

*d^3 + a^7*b^6*d^4)*g^6*x^2 + 5*(a^4*b^9*c^4 - 4*a^5*b^8*c^3*d + 6*a^6*b^7*
c^2*d^2 - 4*a^7*b^6*c*d^3 + a^8*b^5*d^4)*g^6*x + (a^5*b^8*c^4 - 4*a^6*b^7*c
^3*d + 6*a^7*b^6*c^2*d^2 - 4*a^8*b^5*c*d^3 + a^9*b^4*d^4)*g^6) - 60*(10*b^3
*c^3*d^2 - 10*a*b^2*c^2*d^3 + 5*a^2*b*c*d^4 - a^3*d^5)*log(b*x + a)/((b^9*c
^5 - 5*a*b^8*c^4*d + 10*a^2*b^7*c^3*d^2 - 10*a^3*b^6*c^2*d^3 + 5*a^4*b^5*c*
d^4 - a^5*b^4*d^5)*g^6) + 60*(10*b^3*c^3*d^2 - 10*a*b^2*c^2*d^3 + 5*a^2*b*c
*d^4 - a^3*d^5)*log(d*x + c)/((b^9*c^5 - 5*a*b^8*c^4*d + 10*a^2*b^7*c^3*d^2
 - 10*a^3*b^6*c^2*d^3 + 5*a^4*b^5*c*d^4 - a^5*b^4*d^5)*g^6)) - 1/300*A*B*c*
d^2*i^3*(60*(10*b^2*x^2 + 5*a*b*x + a^2)*log(b*e*x/(d*x + c) + a*e/(d*x + c
))/(b^8*g^6*x^5 + 5*a*b^7*g^6*x^4 + 10*a^2*b^6*g^6*x^3 + 10*a^3*b^5*g^6*x^2
 + 5*a^4*b^4*g^6*x + a^5*b^3*g^6) + (47*a^2*b^4*c^4 - 278*a^3*b^3*c^3*d + 8
22*a^4*b^2*c^2*d^2 - 278*a^5*b*c*d^3 + 47*a^6*d^4 + 60*(10*b^6*c^2*d^2 - 5*
a*b^5*c*d^3 + a^2*b^4*d^4)*x^4 - 30*(10*b^6*c^3*d - 95*a*b^5*c^2*d^2 + 46*a
^2*b^4*c*d^3 - 9*a^3*b^3*d^4)*x^3 + 10*(20*b^6*c^4 - 140*a*b^5*c^3*d + 537*
a^2*b^4*c^2*d^2 - 248*a^3*b^3*c*d^3 + 47*a^4*b^2*d^4)*x^2 + 5*(35*a*b^5*c^4
 - 218*a^2*b^4*c^3*d + 702*a^3*b^3*c^2*d^2 - 278*a^4*b^2*c*d^3 + 47*a^5*b*d
^4)*x)/((b^12*c^4 - 4*a*b^11*c^3*d + 6*a^2*b^10*c^2*d^2 - 4*a^3*b^9*c*d^3 +
 a^4*b^8*d^4)*g^6*x^5 + 5*(a*b^11*c^4 - 4*a^2*b^10*c^3*d + 6*a^3*b^9*c^2*d^
2 - 4*a^4*b^8*c*d^3 + a^5*b^7*d^4)*g^6*x^4 + 10*(a^2*b^10*c^4 - 4*a^3*b^9*c
^3*d + 6*a^4*b^8*c^2*d^2 - 4*a^5*b^7*c*d^3 + a^6*b^6*d^4)*g^6*x^3 + 10*(a^3
*b^9*c^4 - 4*a^4*b^8*c^3*d + 6*a^5*b^7*c^2*d^2 - 4*a^6*b^6*c*d^3 + a^7*b^5*
d^4)*g^6*x^2 + 5*(a^4*b^8*c^4 - 4*a^5*b^7*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^7
*b^5*c*d^3 + a^8*b^4*d^4)*g^6*x + (a^5*b^7*c^4 - 4*a^6*b^6*c^3*d + 6*a^7*b^
5*c^2*d^2 - 4*a^8*b^4*c*d^3 + a^9*b^3*d^4)*g^6) + 60*(10*b^2*c^2*d^3 - 5*a*
b*c*d^4 + a^2*d^5)*log(b*x + a)/((b^8*c^5 - 5*a*b^7*c^4*d + 10*a^2*b^6*c^3*
d^2 - 10*a^3*b^5*c^2*d^3 + 5*a^4*b^4*c*d^4 - a^5*b^3*d^5)*g^6) - 60*(10*b^2
*c^2*d^3 - 5*a*b*c*d^4 + a^2*d^5)*log(d*x + c)/((b^8*c^5 - 5*a*b^7*c^4*d +
10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 + 5*a^4*b^4*c*d^4 - a^5*b^3*d^5)*g^
6)) - 1/200*A*B*c^2*d*i^3*(60*(5*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x +
c))/(b^7*g^6*x^5 + 5*a*b^6*g^6*x^4 + 10*a^2*b^5*g^6*x^3 + 10*a^3*b^4*g^6*x^
2 + 5*a^4*b^3*g^6*x + a^5*b^2*g^6) + (27*a*b^4*c^4 - 148*a^2*b^3*c^3*d + 35
2*a^3*b^2*c^2*d^2 - 548*a^4*b*c*d^3 + 77*a^5*d^4 - 60*(5*b^5*c*d^3 - a*b^4*
d^4)*x^4 + 30*(5*b^5*c^2*d^2 - 46*a*b^4*c*d^3 + 9*a^2*b^3*d^4)*x^3 - 10*(10
*b^5*c^3*d - 67*a*b^4*c^2*d^2 + 248*a^2*b^3*c*d^3 - 47*a^3*b^2*d^4)*x^2 + 5
*(15*b^5*c^4 - 88*a*b^4*c^3*d + 232*a^2*b^3*c^2*d^2 - 428*a^3*b^2*c*d^3 + 7
7*a^4*b*d^4)*x)/((b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8
*c*d^3 + a^4*b^7*d^4)*g^6*x^5 + 5*(a*b^10*c^4 - 4*a^2*b^9*c^3*d + 6*a^3*b^8
*c^2*d^2 - 4*a^4*b^7*c*d^3 + a^5*b^6*d^4)*g^6*x^4 + 10*(a^2*b^9*c^4 - 4*a^3
*b^8*c^3*d + 6*a^4*b^7*c^2*d^2 - 4*a^5*b^6*c*d^3 + a^6*b^5*d^4)*g^6*x^3 + 1
0*(a^3*b^8*c^4 - 4*a^4*b^7*c^3*d + 6*a^5*b^6*c^2*d^2 - 4*a^6*b^5*c*d^3 + a^
7*b^4*d^4)*g^6*x^2 + 5*(a^4*b^7*c^4 - 4*a^5*b^6*c^3*d + 6*a^6*b^5*c^2*d^2 -
 4*a^7*b^4*c*d^3 + a^8*b^3*d^4)*g^6*x + (a^5*b^6*c^4 - 4*a^6*b^5*c^3*d + 6*
a^7*b^4*c^2*d^2 - 4*a^8*b^3*c*d^3 + a^9*b^2*d^4)*g^6) - 60*(5*b*c*d^4 - a*d
^5)*log(b*x + a)/((b^7*c^5 - 5*a*b^6*c^4*d + 10*a^2*b^5*c^3*d^2 - 10*a^3*b^
4*c^2*d^3 + 5*a^4*b^3*c*d^4 - a^5*b^2*d^5)*g^6) + 60*(5*b*c*d^4 - a*d^5)*lo

```
g(d*x + c)/((b^7*c^5 - 5*a*b^6*c^4*d + 10*a^2*b^5*c^3*d^2 - 10*a^3*b^4*c^2*
d^3 + 5*a^4*b^3*c*d^4 - a^5*b^2*d^5)*g^6)) - 1/150*A*B*c^3*i^3*((60*b^4*d^4
*x^4 + 12*b^4*c^4 - 63*a*b^3*c^3*d + 137*a^2*b^2*c^2*d^2 - 163*a^3*b*c*d^3
+ 137*a^4*d^4 - 30*(b^4*c*d^3 - 9*a*b^3*d^4)*x^3 + 10*(2*b^4*c^2*d^2 - 13*a
*b^3*c*d^3 + 47*a^2*b^2*d^4)*x^2 - 5*(3*b^4*c^3*d - 17*a*b^3*c^2*d^2 + 43*a
^2*b^2*c*d^3 - 77*a^3*b*d^4)*x)/((b^10*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*
d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*g^6*x^5 + 5*(a*b^9*c^4 - 4*a^2*b^8*c^3
*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 + a^5*b^5*d^4)*g^6*x^4 + 10*(a^2*b
^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2 - 4*a^5*b^5*c*d^3 + a^6*b^4*d^
4)*g^6*x^3 + 10*(a^3*b^7*c^4 - 4*a^4*b^6*c^3*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*
b^4*c*d^3 + a^7*b^3*d^4)*g^6*x^2 + 5*(a^4*b^6*c^4 - 4*a^5*b^5*c^3*d + 6*a^6
*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d^4)*g^6*x + (a^5*b^5*c^4 - 4*a^6*
b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4*a^8*b^2*c*d^3 + a^9*b*d^4)*g^6) + 60*log(
b*e*x/(d*x + c) + a*e/(d*x + c))/(b^6*g^6*x^5 + 5*a*b^5*g^6*x^4 + 10*a^2*b^
4*g^6*x^3 + 10*a^3*b^3*g^6*x^2 + 5*a^4*b^2*g^6*x + a^5*b*g^6) + 60*d^5*log(
b*x + a)/((b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2 - 10*a^3*b^3*c^2*d^
3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5)*g^6) - 60*d^5*log(d*x + c)/((b^6*c^5 - 5*a
*b^5*c^4*d + 10*a^2*b^4*c^3*d^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^
5*b*d^5)*g^6)) - 1/5*B^2*c^3*i^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^
6*g^6*x^5 + 5*a*b^5*g^6*x^4 + 10*a^2*b^4*g^6*x^3 + 10*a^3*b^3*g^6*x^2 + 5*a
^4*b^2*g^6*x + a^5*b*g^6) - 3/20*(5*b*x + a)*A^2*c^2*d*i^3/(b^7*g^6*x^5 + 5
*a*b^6*g^6*x^4 + 10*a^2*b^5*g^6*x^3 + 10*a^3*b^4*g^6*x^2 + 5*a^4*b^3*g^6*x
+ a^5*b^2*g^6) - 1/10*(10*b^2*x^2 + 5*a*b*x + a^2)*A^2*c*d^2*i^3/(b^8*g^6*x
^5 + 5*a*b^7*g^6*x^4 + 10*a^2*b^6*g^6*x^3 + 10*a^3*b^5*g^6*x^2 + 5*a^4*b^4*
g^6*x + a^5*b^3*g^6) - 1/20*(10*b^3*x^3 + 10*a*b^2*x^2 + 5*a^2*b*x + a^3)*A
^2*d^3*i^3/(b^9*g^6*x^5 + 5*a*b^8*g^6*x^4 + 10*a^2*b^7*g^6*x^3 + 10*a^3*b^6
*g^6*x^2 + 5*a^4*b^5*g^6*x + a^5*b^4*g^6) - 1/5*A^2*c^3*i^3/(b^6*g^6*x^5 +
5*a*b^5*g^6*x^4 + 10*a^2*b^4*g^6*x^3 + 10*a^3*b^3*g^6*x^2 + 5*a^4*b^2*g^6*x
 + a^5*b*g^6)
```

Giac [A] (verification not implemented)

none

Time = 0.64 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.60

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = -\frac{1}{4000}\left(\frac{200\left(4B^2be^6i^3-\frac{5(bex+ae)B^2de^5i^3}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^5bcg^6}{(dx+c)^5}-\frac{(bex+ae)^5adg^6}{(dx+c)^5}}+\frac{20\left(80ABbe^6i^3+16B^2be^6i^3-\frac{100(bex+ae)ABde^5}{dx+c}\right.}{\frac{(bex+ae)^5bcg^6}{(dx+c)^5}-\frac{(bex+}{(dx}}\right.$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x, alg
orithm="giac")
```

```
[Out] -1/4000*(200*(4*B^2*b*e^6*i^3 - 5*(b*e*x + a*e)*B^2*d*e^5*i^3/(d*x + c))*lo
g((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a*e)^5*b*c*g^6/(d*x + c)^5 - (b*e*x
+ a*e)^5*a*d*g^6/(d*x + c)^5) + 20*(80*A*B*b*e^6*i^3 + 16*B^2*b*e^6*i^3 - 1
00*(b*e*x + a*e)*A*B*d*e^5*i^3/(d*x + c) - 25*(b*e*x + a*e)*B^2*d*e^5*i^3/(
d*x + c))*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^5*b*c*g^6/(d*x + c)^5
 - (b*e*x + a*e)^5*a*d*g^6/(d*x + c)^5) + (800*A^2*b*e^6*i^3 + 320*A*B*b*e^
6*i^3 + 64*B^2*b*e^6*i^3 - 1000*(b*e*x + a*e)*A^2*d*e^5*i^3/(d*x + c) - 500
*(b*e*x + a*e)*A*B*d*e^5*i^3/(d*x + c) - 125*(b*e*x + a*e)*B^2*d*e^5*i^3/(d
*x + c))/((b*e*x + a*e)^5*b*c*g^6/(d*x + c)^5 - (b*e*x + a*e)^5*a*d*g^6/(d*
x + c)^5))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c -
 a*d)))
```

Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 3720, normalized size of antiderivative = 12.44

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx=\text{Too large to display}$$

```
[In] int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^
6,x)
```

```
[Out] - log((e*(a + b*x))/(c + d*x))^2*((x*(a*(b*((B^2*a*d^3*i^3)/(20*b^5*g^6) +
(B^2*c*d^2*i^3)/(10*b^4*g^6)) + (3*B^2*a*d^3*i^3)/(20*b^4*g^6) + (3*B^2*c*d
^2*i^3)/(10*b^3*g^6)) + b*(a*((B^2*a*d^3*i^3)/(20*b^5*g^6) + (B^2*c*d^2*i^3
)/(10*b^4*g^6)) + (3*B^2*c^2*d*i^3)/(20*b^3*g^6)) + (3*B^2*c^2*d*i^3)/(5*b^
2*g^6)) + x^2*(b*(b*((B^2*a*d^3*i^3)/(20*b^5*g^6) + (B^2*c*d^2*i^3)/(10*b^4
*g^6)) + (3*B^2*a*d^3*i^3)/(20*b^4*g^6) + (3*B^2*c*d^2*i^3)/(10*b^3*g^6)) +
 (3*B^2*a*d^3*i^3)/(10*b^3*g^6) + (3*B^2*c*d^2*i^3)/(5*b^2*g^6)) + a*(a*((B
^2*a*d^3*i^3)/(20*b^5*g^6) + (B^2*c*d^2*i^3)/(10*b^4*g^6)) + (3*B^2*c^2*d*i
^3)/(20*b^3*g^6)) + (B^2*c^3*i^3)/(5*b^2*g^6) + (B^2*d^3*i^3*x^3)/(2*b^2*g^
6))/(5*a^4*x + a^5/b + b^4*x^5 + 10*a^3*b*x^2 + 5*a*b^3*x^4 + 10*a^2*b^2*x^
3) - (B^2*d^5*i^3)/(20*b^4*g^6*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((200*A^
2*a^4*d^4*i^3 - 800*A^2*b^4*c^4*i^3 + 61*B^2*a^4*d^4*i^3 - 64*B^2*b^4*c^4*i
^3 + 180*A*B*a^4*d^4*i^3 - 320*A*B*b^4*c^4*i^3 + 200*A^2*a*b^3*c^3*d*i^3 +
200*A^2*a^3*b*c*d^3*i^3 + 61*B^2*a*b^3*c^3*d*i^3 + 61*B^2*a^3*b*c*d^3*i^3 +
 200*A^2*a^2*b^2*c^2*d^2*i^3 + 61*B^2*a^2*b^2*c^2*d^2*i^3 + 180*A*B*a^2*b^2
*c^2*d^2*i^3 + 180*A*B*a*b^3*c^3*d*i^3 + 180*A*B*a^3*b*c*d^3*i^3)/(20*(a*d
- b*c)) + (x^4*(9*B^2*b^4*d^4*i^3 + 20*A*B*b^4*d^4*i^3))/(a*d - b*c) + (x^3
*(200*A^2*a*b^3*d^4*i^3 + 61*B^2*a*b^3*d^4*i^3 - 200*A^2*b^4*c*d^3*i^3 + 11
*B^2*b^4*c*d^3*i^3 + 180*A*B*a*b^3*d^4*i^3 - 20*A*B*b^4*c*d^3*i^3))/(2*(a*d
 - b*c)) + (x*(200*A^2*a^3*b*d^4*i^3 + 61*B^2*a^3*b*d^4*i^3 - 600*A^2*b^4*c
^3*d*i^3 - 39*B^2*b^4*c^3*d*i^3 + 200*A^2*a*b^3*c^2*d^2*i^3 + 200*A^2*a^2*b
^2*c*d^3*i^3 + 61*B^2*a*b^3*c^2*d^2*i^3 + 61*B^2*a^2*b^2*c*d^3*i^3 + 180*A*
```

B*a^3*b*d^4*i^3 - 220*A*B*b^4*c^3*d*i^3 + 180*A*B*a*b^3*c^2*d^2*i^3 + 180*A
*B*a^2*b^2*c*d^3*i^3))/(4*(a*d - b*c)) + (x^2*(200*A^2*a^2*b^2*d^4*i^3 + 61
*B^2*a^2*b^2*d^4*i^3 - 400*A^2*b^4*c^2*d^2*i^3 - 14*B^2*b^4*c^2*d^2*i^3 + 2
00*A^2*a*b^3*c*d^3*i^3 + 61*B^2*a*b^3*c*d^3*i^3 + 180*A*B*a^2*b^2*d^4*i^3 -
120*A*B*b^4*c^2*d^2*i^3 + 180*A*B*a*b^3*c*d^3*i^3))/(2*(a*d - b*c)))/(200*
a^5*b^4*g^6 + 200*b^9*g^6*x^5 + 1000*a^4*b^5*g^6*x + 1000*a*b^8*g^6*x^4 + 2
000*a^3*b^6*g^6*x^2 + 2000*a^2*b^7*g^6*x^3) - (log((e*(a + b*x))/(c + d*x))
(x^3((A*B*d^2*i^3)/(b^2*g^6) + (B^2*d^5*i^3*((b^4*c^2 + 5*a^2*b^2*d^2 - 6
*a*b^3*c*d)/(5*d^3) + b*(b*(b*((5*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)/(20*b*d^3)
+ (a*(a*d - b*c))/(5*b*d^2)) + (5*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)/(10*d^3)
+ (2*a*(a*d - b*c))/(5*d^2)) - a*((b^2*c - a*b*d)/(5*d^2) - (2*b*(a*d - b*c
))/(5*d^2)) + (3*(b^3*c^2 + 5*a^2*b*d^2 - 6*a*b^2*c*d))/(20*d^3)) - a*(b*((
b^2*c - a*b*d)/(5*d^2) - (2*b*(a*d - b*c))/(5*d^2)) + (b^3*c - a*b^2*d)/(5*
d^2))))/(10*b^4*g^6*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) + a*(a*((B*d*i^3*(6*A
*b*c - B*a*d + B*b*c))/(30*b^5*g^6) + (A*B*a*d^2*i^3)/(10*b^5*g^6)) + (B*i^
3*(6*A*b^2*c^2 - B*a^2*d^2 + B*b^2*c^2))/(20*b^5*g^6)) + x*(b*(a*((B*d*i^3*
(6*A*b*c - B*a*d + B*b*c))/(30*b^5*g^6) + (A*B*a*d^2*i^3)/(10*b^5*g^6)) + (
B*i^3*(6*A*b^2*c^2 - B*a^2*d^2 + B*b^2*c^2))/(20*b^5*g^6)) + a*(b*((B*d*i^3
*(6*A*b*c - B*a*d + B*b*c))/(30*b^5*g^6) + (A*B*a*d^2*i^3)/(10*b^5*g^6)) +
(B*d*i^3*(6*A*b*c - B*a*d + B*b*c))/(10*b^4*g^6) + (3*A*B*a*d^2*i^3)/(10*b^
4*g^6)) + (B*i^3*(6*A*b^2*c^2 - B*a^2*d^2 + B*b^2*c^2))/(5*b^4*g^6) + (B^2*
d^5*i^3*((10*a^4*d^4 + b^4*c^4 + 15*a^2*b^2*c^2*d^2 - 6*a*b^3*c^3*d - 20*a^
3*b*c*d^3)/(5*d^5) + b*(a*(a*((5*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)/(20*b*d^3)
+ (a*(a*d - b*c))/(5*b*d^2)) + (10*a^3*d^3 - b^3*c^3 + 6*a*b^2*c^2*d - 15*a
^2*b*c*d^2)/(30*b*d^4)) + (10*a^4*d^4 + b^4*c^4 + 15*a^2*b^2*c^2*d^2 - 6*a*
b^3*c^3*d - 20*a^3*b*c*d^3)/(20*b*d^5)) + a*(b*(a*((5*a^2*d^2 + b^2*c^2 - 6
*a*b*c*d)/(20*b*d^3) + (a*(a*d - b*c))/(5*b*d^2)) + (10*a^3*d^3 - b^3*c^3 +
6*a*b^2*c^2*d - 15*a^2*b*c*d^2)/(30*b*d^4)) + a*(b*((5*a^2*d^2 + b^2*c^2 -
6*a*b*c*d)/(20*b*d^3) + (a*(a*d - b*c))/(5*b*d^2)) + (5*a^2*d^2 + b^2*c^2
- 6*a*b*c*d)/(10*d^3) + (2*a*(a*d - b*c))/(5*d^2)) + (10*a^3*d^3 - b^3*c^3
+ 6*a*b^2*c^2*d - 15*a^2*b*c*d^2)/(10*d^4))))/(10*b^4*g^6*(a^2*d^2 + b^2*c^
2 - 2*a*b*c*d))) + x^2*(b*(b*((B*d*i^3*(6*A*b*c - B*a*d + B*b*c))/(30*b^5*g
^6) + (A*B*a*d^2*i^3)/(10*b^5*g^6)) + (B*d*i^3*(6*A*b*c - B*a*d + B*b*c))/(
10*b^4*g^6) + (3*A*B*a*d^2*i^3)/(10*b^4*g^6)) + (B*d*i^3*(6*A*b*c - B*a*d +
B*b*c))/(5*b^3*g^6) + (3*A*B*a*d^2*i^3)/(5*b^3*g^6) + (B^2*d^5*i^3*(a*(b*(
b*((5*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)/(20*b*d^3) + (a*(a*d - b*c))/(5*b*d^2)
) + (5*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)/(10*d^3) + (2*a*(a*d - b*c))/(5*d^2))
- a*((b^2*c - a*b*d)/(5*d^2) - (2*b*(a*d - b*c))/(5*d^2)) + (3*(b^3*c^2 +
5*a^2*b*d^2 - 6*a*b^2*c*d))/(20*d^3)) - (b^4*c^3 - 10*a^3*b*d^3 + 15*a^2*b^
2*c*d^2 - 6*a*b^3*c^2*d)/(5*d^4) + b*(b*(a*((5*a^2*d^2 + b^2*c^2 - 6*a*b*c*
d)/(20*b*d^3) + (a*(a*d - b*c))/(5*b*d^2)) + (10*a^3*d^3 - b^3*c^3 + 6*a*b^
2*c^2*d - 15*a^2*b*c*d^2)/(30*b*d^4)) + a*(b*((5*a^2*d^2 + b^2*c^2 - 6*a*b*
c*d)/(20*b*d^3) + (a*(a*d - b*c))/(5*b*d^2)) + (5*a^2*d^2 + b^2*c^2 - 6*a*b
*c*d)/(10*d^3) + (2*a*(a*d - b*c))/(5*d^2)) + (10*a^3*d^3 - b^3*c^3 + 6*a*b
^2*c^2*d - 15*a^2*b*c*d^2)/(10*d^4))))/(10*b^4*g^6*(a^2*d^2 + b^2*c^2 - 2*a

```
*b*c*d))) + (B*i^3*(4*A*b^3*c^3 - B*a^3*d^3 + B*b^3*c^3 - B*a*b^2*c^2*d + B
*a^2*b*c*d^2))/(10*b^5*d*g^6) + (B^2*d^5*i^3*(a*(a*(a*((5*a^2*d^2 + b^2*c^2
 - 6*a*b*c*d)/(20*b*d^3) + (a*(a*d - b*c))/(5*b*d^2)) + (10*a^3*d^3 - b^3*c
^3 + 6*a*b^2*c^2*d - 15*a^2*b*c*d^2)/(30*b*d^4)) + (10*a^4*d^4 + b^4*c^4 +
15*a^2*b^2*c^2*d^2 - 6*a*b^3*c^3*d - 20*a^3*b*c*d^3)/(20*b*d^5)) + (5*a^5*d
^5 - b^5*c^5 - 15*a^2*b^3*c^3*d^2 + 20*a^3*b^2*c^2*d^3 + 6*a*b^4*c^4*d - 15
*a^4*b*c*d^4)/(5*b*d^6)))/(10*b^4*g^6*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (B
^2*d^5*i^3*x^4*(b*(b*((b^2*c - a*b*d)/(5*d^2) - (2*b*(a*d - b*c))/(5*d^2))
+ (b^3*c - a*b^2*d)/(5*d^2)) + (b^4*c - a*b^3*d)/(5*d^2)))/(10*b^4*g^6*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d))))/((5*a^4*x)/d + a^5/(b*d) + (b^4*x^5)/d + (10
*a^3*b*x^2)/d + (5*a*b^3*x^4)/d + (10*a^2*b^2*x^3)/d) - (B*d^5*i^3*atan(((2
*b*d*x - (200*b^6*c^2*g^6 - 200*a^2*b^4*d^2*g^6)/(200*b^4*g^6*(a*d - b*c)))
*1i)/(a*d - b*c))*(20*A + 9*B)*1i)/(100*b^4*g^6*(a*d - b*c)^2)
```

3.83 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 463

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx = -\frac{B^2d^2i^3(c+dx)^4}{32(bc-ad)^3g^7(a+bx)^4} + \frac{4bB^2di^3(c+dx)^5}{125(bc-ad)^3g^7(a+bx)^5} - \frac{b^2B^2i^3(c+dx)^6}{108(bc-ad)^3g^7(a+bx)^6} - \frac{Bd^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)^3g^7(a+bx)^4} + \frac{4bBdi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{25(bc-ad)^3g^7(a+bx)^5} - \frac{b^2Bi^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{18(bc-ad)^3g^7(a+bx)^6} - \frac{d^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^3g^7(a+bx)^4} + \frac{2bdi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5(bc-ad)^3g^7(a+bx)^5} - \frac{b^2i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6(bc-ad)^3g^7(a+bx)^6}$$

```
[Out] -1/32*B^2*d^2*i^3*(d*x+c)^4/(-a*d+b*c)^3/g^7/(b*x+a)^4+4/125*b*B^2*d*i^3*(d
*x+c)^5/(-a*d+b*c)^3/g^7/(b*x+a)^5-1/108*b^2*B^2*i^3*(d*x+c)^6/(-a*d+b*c)^3
/g^7/(b*x+a)^6-1/8*B*d^2*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*
c)^3/g^7/(b*x+a)^4+4/25*b*B*d*i^3*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a
*d+b*c)^3/g^7/(b*x+a)^5-1/18*b^2*B*i^3*(d*x+c)^6*(A+B*ln(e*(b*x+a)/(d*x+c))
)/(-a*d+b*c)^3/g^7/(b*x+a)^6-1/4*d^2*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c
)))^2/(-a*d+b*c)^3/g^7/(b*x+a)^4+2/5*b*d*i^3*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d
*x+c)))^2/(-a*d+b*c)^3/g^7/(b*x+a)^5-1/6*b^2*i^3*(d*x+c)^6*(A+B*ln(e*(b*x+a
)/(d*x+c)))^2/(-a*d+b*c)^3/g^7/(b*x+a)^6
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2562, 2395, 2342, 2341}

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}dx=-\frac{b^2i^3(c+dx)^6\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{6g^7(a+bx)^6(bc-ad)^3}-\frac{b^2Bi^3(c+dx)^6\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{18g^7(a+bx)^6(bc-ad)^3}-\frac{d^2i^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4g^7(a+bx)^4(bc-ad)^3}-\frac{Bd^2i^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{8g^7(a+bx)^4(bc-ad)^3}+\frac{2bdi^3(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5g^7(a+bx)^5(bc-ad)^3}+\frac{4bBdi^3(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{25g^7(a+bx)^5(bc-ad)^3}-\frac{b^2B^2i^3(c+dx)^6}{108g^7(a+bx)^6(bc-ad)^3}-\frac{B^2d^2i^3(c+dx)^4}{32g^7(a+bx)^4(bc-ad)^3}+\frac{4bB^2di^3(c+dx)^5}{125g^7(a+bx)^5(bc-ad)^3}$$

```
[In] Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)^
7,x]
```

```
[Out] -1/32*(B^2*d^2*i^3*(c + d*x)^4)/((b*c - a*d)^3*g^7*(a + b*x)^4) + (4*b*B^2*
d*i^3*(c + d*x)^5)/(125*(b*c - a*d)^3*g^7*(a + b*x)^5) - (b^2*B^2*i^3*(c +
```

```
d*x)^6)/(108*(b*c - a*d)^3*g^7*(a + b*x)^6) - (B*d^2*i^3*(c + d*x)^4*(A + B
*Log[(e*(a + b*x))/(c + d*x)]))/(8*(b*c - a*d)^3*g^7*(a + b*x)^4) + (4*b*B*
d*i^3*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(25*(b*c - a*d)^3*g
^7*(a + b*x)^5) - (b^2*B*i^3*(c + d*x)^6*(A + B*Log[(e*(a + b*x))/(c + d*x)
]))/(18*(b*c - a*d)^3*g^7*(a + b*x)^6) - (d^2*i^3*(c + d*x)^4*(A + B*Log[(e
*(a + b*x))/(c + d*x)])^2)/(4*(b*c - a*d)^3*g^7*(a + b*x)^4) + (2*b*d*i^3*(
c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(5*(b*c - a*d)^3*g^7*(a
+ b*x)^5) - (b^2*i^3*(c + d*x)^6*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(6
*(b*c - a*d)^3*g^7*(a + b*x)^6)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{i^3 \text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex))^2}{x^7}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3 g^7}$$

$$=\frac{i^3\text{Subst}\left(\int\left(\frac{b^2(A+B\log(ex))^2}{x^7}-\frac{2bd(A+B\log(ex))^2}{x^6}+\frac{d^2(A+B\log(ex))^2}{x^5}\right)dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^7}$$

$$=\frac{(b^2i^3)\,\text{Subst}\left(\int\frac{(A+B\log(ex))^2}{x^7}dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^7}-\frac{(2bdi^3)\,\text{Subst}\left(\int\frac{(A+B\log(ex))^2}{x^6}dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^7}+\frac{(d^2i^3)\,\text{Subst}\left(\int\frac{(A+B\log(ex))^2}{x^5}dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^7}$$

$$=-\frac{d^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^3g^7(a+bx)^4}+\frac{2bdi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5(bc-ad)^3g^7(a+bx)^5}-\frac{b^2i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6(bc-ad)^3g^7(a+bx)^6}+\frac{(b^2Bi^3)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{x^7}dx,x,\frac{a+bx}{c+dx}\right)}{3(bc-ad)^3g^7}-\frac{(4bBdi^3)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{x^6}dx,x,\frac{a+bx}{c+dx}\right)}{5(bc-ad)^3g^7}+\frac{(Bd^2i^3)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{x^5}dx,x,\frac{a+bx}{c+dx}\right)}{2(bc-ad)^3g^7}$$

$$=-\frac{B^2d^2i^3(c+dx)^4}{32(bc-ad)^3g^7(a+bx)^4}+\frac{4bB^2di^3(c+dx)^5}{125(bc-ad)^3g^7(a+bx)^5}-\frac{b^2B^2i^3(c+dx)^6}{108(bc-ad)^3g^7(a+bx)^6}-\frac{Bd^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)^3g^7(a+bx)^4}+\frac{4bBdi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{25(bc-ad)^3g^7(a+bx)^5}-\frac{b^2Bi^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{18(bc-ad)^3g^7(a+bx)^6}-\frac{d^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^3g^7(a+bx)^4}+\frac{2bdi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5(bc-ad)^3g^7(a+bx)^5}-\frac{b^2i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6(bc-ad)^3g^7(a+bx)^6}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.69 (sec) , antiderivative size = 2606, normalized size of antiderivative = 5.63

$$\int\frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx=\text{Result too large to show}$$

```
[In] Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b
*g*x)^7,x]
```

```
[Out] (i^3*(-6000*A*B*(b*c - a*d)^6 - 1000*B^2*(b*c - a*d)^6 + 25920*a*A*B*d*(-(b
*c) + a*d)^5 + 5184*a*B^2*d*(-(b*c) + a*d)^5 - 25920*A*b*B*d*(b*c - a*d)^5*
x - 5184*b*B^2*d*(b*c - a*d)^5*x + 32400*a*A*B*d^2*(b*c - a*d)^4*(a + b*x)
```

+ 14580*a*B^2*d^2*(b*c - a*d)^4*(a + b*x) + 7200*A*B*d*(b*c - a*d)^5*(a + b*x) + 2640*B^2*d*(b*c - a*d)^5*(a + b*x) + 32400*A*b*B*d^2*(b*c - a*d)^4*x*(a + b*x) + 14580*b*B^2*d^2*(b*c - a*d)^4*x*(a + b*x) - 49500*A*B*d^2*(b*c - a*d)^4*(a + b*x)^2 - 15675*B^2*d^2*(b*c - a*d)^4*(a + b*x)^2 + 43200*a*A*B*d^3*(-(b*c) + a*d)^3*(a + b*x)^2 + 33840*a*B^2*d^3*(-(b*c) + a*d)^3*(a + b*x)^2 - 43200*A*b*B*d^3*(b*c - a*d)^3*x*(a + b*x)^2 - 33840*b*B^2*d^3*(b*c - a*d)^3*x*(a + b*x)^2 + 64800*a*A*B*d^4*(b*c - a*d)^2*(a + b*x)^3 + 83160*a*B^2*d^4*(b*c - a*d)^2*(a + b*x)^3 + 42000*A*B*d^3*(b*c - a*d)^3*(a + b*x)^3 + 34900*B^2*d^3*(b*c - a*d)^3*(a + b*x)^3 + 64800*A*b*B*d^4*(b*c - a*d)^2*x*(a + b*x)^3 + 83160*b*B^2*d^4*(b*c - a*d)^2*x*(a + b*x)^3 - 63000*A*B*d^4*(b*c - a*d)^2*(a + b*x)^4 - 83850*B^2*d^4*(b*c - a*d)^2*(a + b*x)^4 + 129600*a*A*B*d^5*(-(b*c) + a*d)*(a + b*x)^4 + 295920*a*B^2*d^5*(-(b*c) + a*d)*(a + b*x)^4 - 129600*A*b*B*d^5*(b*c - a*d)*x*(a + b*x)^4 - 295920*b*B^2*d^5*(b*c - a*d)*x*(a + b*x)^4 + 126000*A*B*d^5*(b*c - a*d)*(a + b*x)^5 + 293700*B^2*d^5*(b*c - a*d)*(a + b*x)^5 - 129600*a*A*B*d^6*(a + b*x)^5*Log[a + b*x] - 295920*a*B^2*d^6*(a + b*x)^5*Log[a + b*x] - 129600*A*b*B*d^6*x*(a + b*x)^5*Log[a + b*x] - 295920*b*B^2*d^6*x*(a + b*x)^5*Log[a + b*x] + 126000*A*B*d^6*(a + b*x)^6*Log[a + b*x] + 293700*B^2*d^6*(a + b*x)^6*Log[a + b*x] + 64800*a*B^2*d^6*(a + b*x)^5*Log[a + b*x]^2 + 64800*b*B^2*d^6*x*(a + b*x)^5*Log[a + b*x]^2 - 63000*B^2*d^6*(a + b*x)^6*Log[a + b*x]^2 - 6000*B^2*(b*c - a*d)^6*Log[(e*(a + b*x))/(c + d*x)] + 25920*a*B^2*d*(-(b*c) + a*d)^5*Log[(e*(a + b*x))/(c + d*x)] - 25920*b*B^2*d*(b*c - a*d)^5*x*Log[(e*(a + b*x))/(c + d*x)] + 32400*a*B^2*d^2*(b*c - a*d)^4*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 7200*B^2*d*(b*c - a*d)^5*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 32400*b*B^2*d^2*(b*c - a*d)^4*x*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 49500*B^2*d^2*(b*c - a*d)^4*(a + b*x)^2*Log[(e*(a + b*x))/(c + d*x)] + 43200*a*B^2*d^3*(-(b*c) + a*d)^3*(a + b*x)^2*Log[(e*(a + b*x))/(c + d*x)] - 43200*b*B^2*d^3*(b*c - a*d)^3*x*(a + b*x)^2*Log[(e*(a + b*x))/(c + d*x)] + 64800*a*B^2*d^4*(b*c - a*d)^2*(a + b*x)^3*Log[(e*(a + b*x))/(c + d*x)] + 42000*B^2*d^3*(b*c - a*d)^3*(a + b*x)^3*Log[(e*(a + b*x))/(c + d*x)] + 64800*b*B^2*d^4*(b*c - a*d)^2*x*(a + b*x)^3*Log[(e*(a + b*x))/(c + d*x)] - 63000*B^2*d^4*(b*c - a*d)^2*(a + b*x)^4*Log[(e*(a + b*x))/(c + d*x)] + 129600*a*B^2*d^5*(-(b*c) + a*d)*(a + b*x)^4*Log[(e*(a + b*x))/(c + d*x)] - 129600*b*B^2*d^5*(b*c - a*d)*x*(a + b*x)^4*Log[(e*(a + b*x))/(c + d*x)] + 126000*B^2*d^5*(b*c - a*d)*(a + b*x)^5*Log[(e*(a + b*x))/(c + d*x)] - 129600*a*B^2*d^6*(a + b*x)^5*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] - 129600*b*B^2*d^6*x*(a + b*x)^5*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] + 126000*B^2*d^6*(a + b*x)^6*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] - 18000*(b*c - a*d)^6*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 64800*d*(-(b*c) + a*d)^5*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - 81000*d^2*(b*c - a*d)^4*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 36000*d^3*(-(b*c) + a*d)^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 129600*a*A*B*d^6*(a + b*x)^5*Log[c + d*x] + 295920*a*B^2*d^6*(a + b*x)^5*Log[c + d*x] + 129600*A*b*B*d^6*x*(a + b*x)^5*Log[c + d*x] + 295920*b*B^2*d^6*x*(a + b*x)^5*Log[c + d*x] - 126000*A*B*d^6*(a + b*x)^6*Log[c + d*x] - 293700*B^2*d^6*(a + b*x)^6*Log[c +

```
d*x] - 129600*a*B^2*d^6*(a + b*x)^5*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c
 + d*x] - 129600*b*B^2*d^6*x*(a + b*x)^5*Log[(d*(a + b*x))/(-(b*c) + a*d)]*
Log[c + d*x] + 126000*B^2*d^6*(a + b*x)^6*Log[(d*(a + b*x))/(-(b*c) + a*d)]
*Log[c + d*x] + 129600*a*B^2*d^6*(a + b*x)^5*Log[(e*(a + b*x))/(c + d*x)]*L
og[c + d*x] + 129600*b*B^2*d^6*x*(a + b*x)^5*Log[(e*(a + b*x))/(c + d*x)]*L
og[c + d*x] - 126000*B^2*d^6*(a + b*x)^6*Log[(e*(a + b*x))/(c + d*x)]*Log[c
 + d*x] + 64800*a*B^2*d^6*(a + b*x)^5*Log[c + d*x]^2 + 64800*b*B^2*d^6*x*(a
 + b*x)^5*Log[c + d*x]^2 - 63000*B^2*d^6*(a + b*x)^6*Log[c + d*x]^2 - 12960
0*a*B^2*d^6*(a + b*x)^5*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 12960
0*b*B^2*d^6*x*(a + b*x)^5*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 126
000*B^2*d^6*(a + b*x)^6*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 12960
0*a*B^2*d^6*(a + b*x)^5*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 129600*b
*B^2*d^6*x*(a + b*x)^5*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 126000*B^
2*d^6*(a + b*x)^6*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 129600*a*B^2*d
^6*(a + b*x)^5*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 129600*b*B^2*d^6*x*(
a + b*x)^5*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 126000*B^2*d^6*(a + b*x)
^6*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(108000*b^4*(b*c - a*d)^3*g^7*(a
 + b*x)^6)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1018 vs. $2(445) = 890$.

Time = 2.52 (sec) , antiderivative size = 1019, normalized size of antiderivative = 2.20

method	result	size
parts	Expression too large to display	1019
derivativedivides	Expression too large to display	1082
default	Expression too large to display	1082
norman	Expression too large to display	2555
parallelrisch	Expression too large to display	2803
risch	Expression too large to display	6548

```
[In] int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^7,x,method=_RET
URNVERBOSE)
```

```
[Out] i^3*A^2/g^7*(-3/5*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/(b*x+a)^5-1/3*d^3/b^4/(
b*x+a)^3+3/4*d^2*(a*d-b*c)/b^4/(b*x+a)^4-1/6*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^
2*c^2*d+b^3*c^3)/b^4/(b*x+a)^6)-i^3*B^2/g^7/d^5*(a*d-b*c)^4*e^4*(d^7/(a*d-b
*c)^7*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))
^2-1/8/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/32
/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4)-2*d^6/(a*d-b*c)^7*b*e*(-1/5/(b*e/d+(a*d-b
*c)*e/d/(d*x+c))^5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/25/(b*e/d+(a*d-b*c)*
e/d/(d*x+c))^5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/125/(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^5)+d^5/(a*d-b*c)^7*e^2*b^2*(-1/6/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^6*ln
```

```
(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/18/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^6*ln(b*e
/d+(a*d-b*c)*e/d/(d*x+c))-1/108/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^6))-2*i^3*B*A
/g^7/d^5*(a*d-b*c)^4*e^4*(d^7/(a*d-b*c)^7*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c
))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4)-
2*d^6/(a*d-b*c)^7*b*e*(-1/5/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5*ln(b*e/d+(a*d-b
*c)*e/d/(d*x+c))-1/25/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5)+d^5/(a*d-b*c)^7*e^2*
b^2*(-1/6/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^6*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1
/36/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^6))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1610 vs. $2(445) = 890$.

Time = 0.37 (sec) , antiderivative size = 1610, normalized size of antiderivative = 3.48

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^7,x, alg
orithm="fricas")
```

```
[Out] -1/108000*(60*((60*A*B + 37*B^2)*b^6*c*d^5 - (60*A*B + 37*B^2)*a*b^5*d^6)*i
^3*x^5 - 30*((60*A*B - 23*B^2)*b^6*c^2*d^4 - 36*(20*A*B + 9*B^2)*a*b^5*c*d^
5 + (660*A*B + 347*B^2)*a^2*b^4*d^6)*i^3*x^4 + 20*((1800*A^2 + 60*A*B - 53*
B^2)*b^6*c^3*d^3 - 27*(200*A^2 + 20*A*B - 11*B^2)*a*b^5*c^2*d^4 + 675*(8*A^
2 + 4*A*B + B^2)*a^2*b^4*c*d^5 - (1800*A^2 + 2220*A*B + 919*B^2)*a^3*b^3*d^
6)*i^3*x^3 + 15*((5400*A^2 + 1140*A*B + 73*B^2)*b^6*c^4*d^2 - 72*(200*A^2 +
 60*A*B + 7*B^2)*a*b^5*c^3*d^3 + 1350*(8*A^2 + 4*A*B + B^2)*a^2*b^4*c^2*d^4
 - (1800*A^2 + 2220*A*B + 919*B^2)*a^4*b^2*d^6)*i^3*x^2 + 6*(8*(1350*A^2 +
390*A*B + 53*B^2)*b^6*c^5*d - 45*(600*A^2 + 220*A*B + 39*B^2)*a*b^5*c^4*d^2
 + 2250*(8*A^2 + 4*A*B + B^2)*a^2*b^4*c^3*d^3 - (1800*A^2 + 2220*A*B + 919*
B^2)*a^5*b*d^6)*i^3*x + (1000*(18*A^2 + 6*A*B + B^2)*b^6*c^6 - 1728*(25*A^2
 + 10*A*B + 2*B^2)*a*b^5*c^5*d + 3375*(8*A^2 + 4*A*B + B^2)*a^2*b^4*c^4*d^2
 - (1800*A^2 + 2220*A*B + 919*B^2)*a^6*d^6)*i^3 + 1800*(B^2*b^6*d^6*i^3*x^6
 + 6*B^2*a*b^5*d^6*i^3*x^5 + 15*B^2*a^2*b^4*d^6*i^3*x^4 + 20*(B^2*b^6*c^3*d
^3 - 3*B^2*a*b^5*c^2*d^4 + 3*B^2*a^2*b^4*c*d^5)*i^3*x^3 + 15*(3*B^2*b^6*c^4
*d^2 - 8*B^2*a*b^5*c^3*d^3 + 6*B^2*a^2*b^4*c^2*d^4)*i^3*x^2 + 6*(6*B^2*b^6*
c^5*d - 15*B^2*a*b^5*c^4*d^2 + 10*B^2*a^2*b^4*c^3*d^3)*i^3*x + (10*B^2*b^6*
c^6 - 24*B^2*a*b^5*c^5*d + 15*B^2*a^2*b^4*c^4*d^2)*i^3)*log((b*e*x + a*e)/(
d*x + c))^2 + 60*((60*A*B + 37*B^2)*b^6*d^6*i^3*x^6 + 6*(10*B^2*b^6*c*d^5 +
 3*(20*A*B + 9*B^2)*a*b^5*d^6)*i^3*x^5 - 15*(2*B^2*b^6*c^2*d^4 - 24*B^2*a*b
^5*c*d^5 - 15*(4*A*B + B^2)*a^2*b^4*d^6)*i^3*x^4 + 20*((60*A*B + B^2)*b^6*c
^3*d^3 - 9*(20*A*B + B^2)*a*b^5*c^2*d^4 + 45*(4*A*B + B^2)*a^2*b^4*c*d^5)*i
^3*x^3 + 15*((180*A*B + 19*B^2)*b^6*c^4*d^2 - 24*(20*A*B + 3*B^2)*a*b^5*c^3
*d^3 + 90*(4*A*B + B^2)*a^2*b^4*c^2*d^4)*i^3*x^2 + 6*(4*(90*A*B + 13*B^2)*b
```

```
^6*c^5*d - 15*(60*A*B + 11*B^2)*a*b^5*c^4*d^2 + 150*(4*A*B + B^2)*a^2*b^4*c
^3*d^3)*i^3*x + (100*(6*A*B + B^2)*b^6*c^6 - 288*(5*A*B + B^2)*a*b^5*c^5*d
+ 225*(4*A*B + B^2)*a^2*b^4*c^4*d^2)*i^3)*log((b*e*x + a*e)/(d*x + c)))/((b
^13*c^3 - 3*a*b^12*c^2*d + 3*a^2*b^11*c*d^2 - a^3*b^10*d^3)*g^7*x^6 + 6*(a*
b^12*c^3 - 3*a^2*b^11*c^2*d + 3*a^3*b^10*c*d^2 - a^4*b^9*d^3)*g^7*x^5 + 15*
(a^2*b^11*c^3 - 3*a^3*b^10*c^2*d + 3*a^4*b^9*c*d^2 - a^5*b^8*d^3)*g^7*x^4 +
 20*(a^3*b^10*c^3 - 3*a^4*b^9*c^2*d + 3*a^5*b^8*c*d^2 - a^6*b^7*d^3)*g^7*x^
3 + 15*(a^4*b^9*c^3 - 3*a^5*b^8*c^2*d + 3*a^6*b^7*c*d^2 - a^7*b^6*d^3)*g^7*
x^2 + 6*(a^5*b^8*c^3 - 3*a^6*b^7*c^2*d + 3*a^7*b^6*c*d^2 - a^8*b^5*d^3)*g^7
*x + (a^6*b^7*c^3 - 3*a^7*b^6*c^2*d + 3*a^8*b^5*c*d^2 - a^9*b^4*d^3)*g^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx=\text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**7,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20330 vs. $2(445) = 890$.

Time = 2.08 (sec) , antiderivative size = 20330, normalized size of antiderivative = 43.91

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx=\text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^7,x, alg
orithm="maxima")
[Out] -1/10*(6*b*x + a)*B^2*c^2*d*i^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^8
*g^7*x^6 + 6*a*b^7*g^7*x^5 + 15*a^2*b^6*g^7*x^4 + 20*a^3*b^5*g^7*x^3 + 15*a
^4*b^4*g^7*x^2 + 6*a^5*b^3*g^7*x + a^6*b^2*g^7) - 1/20*(15*b^2*x^2 + 6*a*b*
x + a^2)*B^2*c*d^2*i^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^9*g^7*x^6
+ 6*a*b^8*g^7*x^5 + 15*a^2*b^7*g^7*x^4 + 20*a^3*b^6*g^7*x^3 + 15*a^4*b^5*g^
7*x^2 + 6*a^5*b^4*g^7*x + a^6*b^3*g^7) - 1/60*(20*b^3*x^3 + 15*a*b^2*x^2 +
6*a^2*b*x + a^3)*B^2*d^3*i^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^10*g
^7*x^6 + 6*a*b^9*g^7*x^5 + 15*a^2*b^8*g^7*x^4 + 20*a^3*b^7*g^7*x^3 + 15*a^4
*b^6*g^7*x^2 + 6*a^5*b^5*g^7*x + a^6*b^4*g^7) + 1/10800*(60*((60*b^5*d^5*x^
5 - 10*b^5*c^5 + 62*a*b^4*c^4*d - 163*a^2*b^3*c^3*d^2 + 237*a^3*b^2*c^2*d^3
 - 213*a^4*b*c*d^4 + 147*a^5*d^5 - 30*(b^5*c*d^4 - 11*a*b^4*d^5)*x^4 + 20*(
```

b^5*c^2*d^3 - 8*a*b^4*c*d^4 + 37*a^2*b^3*d^5)*x^3 - 15*(b^5*c^3*d^2 - 7*a*b ^4*c^2*d^3 + 23*a^2*b^3*c*d^4 - 57*a^3*b^2*d^5)*x^2 + 6*(2*b^5*c^4*d - 13*a *b^4*c^3*d^2 + 37*a^2*b^3*c^2*d^3 - 63*a^3*b^2*c*d^4 + 87*a^4*b*d^5)*x)/((b ^12*c^5 - 5*a*b^11*c^4*d + 10*a^2*b^10*c^3*d^2 - 10*a^3*b^9*c^2*d^3 + 5*a^4 *b^8*c*d^4 - a^5*b^7*d^5)*g^7*x^6 + 6*(a*b^11*c^5 - 5*a^2*b^10*c^4*d + 10*a ^3*b^9*c^3*d^2 - 10*a^4*b^8*c^2*d^3 + 5*a^5*b^7*c*d^4 - a^6*b^6*d^5)*g^7*x^ 5 + 15*(a^2*b^10*c^5 - 5*a^3*b^9*c^4*d + 10*a^4*b^8*c^3*d^2 - 10*a^5*b^7*c^ 2*d^3 + 5*a^6*b^6*c*d^4 - a^7*b^5*d^5)*g^7*x^4 + 20*(a^3*b^9*c^5 - 5*a^4*b^ 8*c^4*d + 10*a^5*b^7*c^3*d^2 - 10*a^6*b^6*c^2*d^3 + 5*a^7*b^5*c*d^4 - a^8*b ^4*d^5)*g^7*x^3 + 15*(a^4*b^8*c^5 - 5*a^5*b^7*c^4*d + 10*a^6*b^6*c^3*d^2 - 10*a^7*b^5*c^2*d^3 + 5*a^8*b^4*c*d^4 - a^9*b^3*d^5)*g^7*x^2 + 6*(a^5*b^7*c^ 5 - 5*a^6*b^6*c^4*d + 10*a^7*b^5*c^3*d^2 - 10*a^8*b^4*c^2*d^3 + 5*a^9*b^3*c *d^4 - a^10*b^2*d^5)*g^7*x + (a^6*b^6*c^5 - 5*a^7*b^5*c^4*d + 10*a^8*b^4*c^ 3*d^2 - 10*a^9*b^3*c^2*d^3 + 5*a^10*b^2*c*d^4 - a^11*b*d^5)*g^7) + 60*d^6*l og(b*x + a)/((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3 *d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*g^7) - 60*d^6*log(d*x + c)/((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^ 3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*g^7))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - (100*b^6*c^6 - 864*a*b^5*c^5*d + 3375*a^2*b^4*c^4*d ^2 - 8000*a^3*b^3*c^3*d^3 + 13500*a^4*b^2*c^2*d^4 - 21600*a^5*b*c*d^5 + 134 89*a^6*d^6 - 8820*(b^6*c*d^5 - a*b^5*d^6)*x^5 + 90*(29*b^6*c^2*d^4 - 548*a* b^5*c*d^5 + 519*a^2*b^4*d^6)*x^4 - 60*(19*b^6*c^3*d^3 - 231*a*b^5*c^2*d^4 + 1875*a^2*b^4*c*d^5 - 1663*a^3*b^3*d^6)*x^3 + 15*(37*b^6*c^4*d^2 - 376*a*b^ 5*c^3*d^3 + 1950*a^2*b^4*c^2*d^4 - 8800*a^3*b^3*c*d^5 + 7189*a^4*b^2*d^6)*x ^2 + 1800*(b^6*d^6*x^6 + 6*a*b^5*d^6*x^5 + 15*a^2*b^4*d^6*x^4 + 20*a^3*b^3* d^6*x^3 + 15*a^4*b^2*d^6*x^2 + 6*a^5*b*d^6*x + a^6*d^6)*log(b*x + a)^2 + 18 00*(b^6*d^6*x^6 + 6*a*b^5*d^6*x^5 + 15*a^2*b^4*d^6*x^4 + 20*a^3*b^3*d^6*x^3 + 15*a^4*b^2*d^6*x^2 + 6*a^5*b*d^6*x + a^6*d^6)*log(d*x + c)^2 - 6*(44*b^6 *c^5*d - 405*a*b^5*c^4*d^2 + 1750*a^2*b^4*c^3*d^3 - 5000*a^3*b^3*c^2*d^4 + 13500*a^4*b^2*c*d^5 - 9889*a^5*b*d^6)*x - 8820*(b^6*d^6*x^6 + 6*a*b^5*d^6*x ^5 + 15*a^2*b^4*d^6*x^4 + 20*a^3*b^3*d^6*x^3 + 15*a^4*b^2*d^6*x^2 + 6*a^5*b *d^6*x + a^6*d^6)*log(b*x + a) + 180*(49*b^6*d^6*x^6 + 294*a*b^5*d^6*x^5 + 735*a^2*b^4*d^6*x^4 + 980*a^3*b^3*d^6*x^3 + 735*a^4*b^2*d^6*x^2 + 294*a^5*b *d^6*x + 49*a^6*d^6 - 20*(b^6*d^6*x^6 + 6*a*b^5*d^6*x^5 + 15*a^2*b^4*d^6*x^ 4 + 20*a^3*b^3*d^6*x^3 + 15*a^4*b^2*d^6*x^2 + 6*a^5*b*d^6*x + a^6*d^6)*log(b*x + a))*log(d*x + c))/(a^6*b^7*c^6*g^7 - 6*a^7*b^6*c^5*d*g^7 + 15*a^8*b^5 *c^4*d^2*g^7 - 20*a^9*b^4*c^3*d^3*g^7 + 15*a^10*b^3*c^2*d^4*g^7 - 6*a^11*b^ 2*c*d^5*g^7 + a^12*b*d^6*g^7 + (b^13*c^6*g^7 - 6*a*b^12*c^5*d*g^7 + 15*a^2* b^11*c^4*d^2*g^7 - 20*a^3*b^10*c^3*d^3*g^7 + 15*a^4*b^9*c^2*d^4*g^7 - 6*a^5 *b^8*c*d^5*g^7 + a^6*b^7*d^6*g^7)*x^6 + 6*(a*b^12*c^6*g^7 - 6*a^2*b^11*c^5* d*g^7 + 15*a^3*b^10*c^4*d^2*g^7 - 20*a^4*b^9*c^3*d^3*g^7 + 15*a^5*b^8*c^2*d ^4*g^7 - 6*a^6*b^7*c*d^5*g^7 + a^7*b^6*d^6*g^7)*x^5 + 15*(a^2*b^11*c^6*g^7 - 6*a^3*b^10*c^5*d*g^7 + 15*a^4*b^9*c^4*d^2*g^7 - 20*a^5*b^8*c^3*d^3*g^7 + 15*a^6*b^7*c^2*d^4*g^7 - 6*a^7*b^6*c*d^5*g^7 + a^8*b^5*d^6*g^7)*x^4 + 20*(a ^3*b^10*c^6*g^7 - 6*a^4*b^9*c^5*d*g^7 + 15*a^5*b^8*c^4*d^2*g^7 - 20*a^6*b^7 *c^3*d^3*g^7 + 15*a^7*b^6*c^2*d^4*g^7 - 6*a^8*b^5*c*d^5*g^7 + a^9*b^4*d^6*g^7)*x^3 + 15*(a^4*b^9*c^6*g^7 - 6*a^5*b^8*c^5*d*g^7 + 15*a^6*b^7*c^4*d^2*g^7 - 20*a^7*b^6*c^3*d^3*g^7 + 15*a^8*b^5*c^2*d^4*g^7 - 6*a^9*b^4*c*d^5*g^7 + a^10*b^3*d^6*g^7)*x^2 + 6*(a^5*b^8*c^6*g^7 - 6*a^6*b^7*c^5*d*g^7 + 15*a^7*b^6*c^4*d^2*g^7 - 20*a^8*b^5*c^3*d^3*g^7 + 15*a^9*b^4*c^2*d^4*g^7 - 6*a^10*b^3*c*d^5*g^7 + a^11*b^2*d^6*g^7)*x))*B^2*c^3*i^3 - 1/18000*(60*((22*a*b^5*c^5 - 140*a^2*b^4*c^4*d + 385*a^3*b^3*c^3*d^2 - 615*a^4*b^2*c^2*d^3 + 735*a^5*b*c*d^4 - 87*a^6*d^5 + 60*(6*b^6*c*d^4 - a*b^5*d^5)*x^5 - 30*(6*b^6*c^2*d^3 - 67*a*b^5*c*d^4 + 11*a^2*b^4*d^5)*x^4 + 20*(6*b^6*c^3*d^2 - 49*a*b^5*c^2*d^3 + 230*a^2*b^4*c*d^4 - 37*a^3*b^3*d^5)*x^3 - 15*(6*b^6*c^4*d - 43*a*b^5*c^3*d^2 + 145*a^2*b^4*c^2*d^3 - 365*a^3*b^3*c*d^4 + 57*a^4*b^2*d^5)*x^2 + 6*(12*b^6*c^5 - 80*a*b^5*c^4*d + 235*a^2*b^4*c^3*d^2 - 415*a^3*b^3*c^2*d^3 + 585*a^4*b^2*c*d^4 - 87*a^5*b*d^5)*x)/((b^13*c^5 - 5*a*b^12*c^4*d + 10*a^2*b^11*c^3*d^2 - 10*a^3*b^10*c^2*d^3 + 5*a^4*b^9*c*d^4 - a^5*b^8*d^5)*g^7*x^6 + 6*(a*b^12*c^5 - 5*a^2*b^11*c^4*d + 10*a^3*b^10*c^3*d^2 - 10*a^4*b^9*c^2*d^3 + 5*a^5*b^8*c*d^4 - a^6*b^7*d^5)*g^7*x^5 + 15*(a^2*b^11*c^5 - 5*a^3*b^10*c^4*d + 10*a^4*b^9*c^3*d^2 - 10*a^5*b^8*c^2*d^3 + 5*a^6*b^7*c*d^4 - a^7*b^6*d^5)*g^7*x^4 + 20*(a^3*b^10*c^5 - 5*a^4*b^9*c^4*d + 10*a^5*b^8*c^3*d^2 - 10*a^6*b^7*c^2*d^3 + 5*a^7*b^6*c*d^4 - a^8*b^5*d^5)*g^7*x^3 + 15*(a^4*b^9*c^5 - 5*a^5*b^8*c^4*d + 10*a^6*b^7*c^3*d^2 - 10*a^7*b^6*c^2*d^3 + 5*a^8*b^5*c*d^4 - a^9*b^4*d^5)*g^7*x^2 + 6*(a^5*b^8*c^5 - 5*a^6*b^7*c^4*d + 10*a^7*b^6*c^3*d^2 - 10*a^8*b^5*c^2*d^3 + 5*a^9*b^4*c*d^4 - a^10*b^3*d^5)*g^7*x + (a^6*b^7*c^5 - 5*a^7*b^6*c^4*d + 10*a^8*b^5*c^3*d^2 - 10*a^9*b^4*c^2*d^3 + 5*a^10*b^3*c*d^4 - a^11*b^2*d^5)*g^7) + 60*(6*b*c*d^5 - a*d^6)*log(b*x + a)/((b^8*c^6 - 6*a*b^7*c^5*d + 15*a^2*b^6*c^4*d^2 - 20*a^3*b^5*c^3*d^3 + 15*a^4*b^4*c^2*d^4 - 6*a^5*b^3*c*d^5 + a^6*b^2*d^6)*g^7) - 60*(6*b*c*d^5 - a*d^6)*log(d*x + c)/((b^8*c^6 - 6*a*b^7*c^5*d + 15*a^2*b^6*c^4*d^2 - 20*a^3*b^5*c^3*d^3 + 15*a^4*b^4*c^2*d^4 - 6*a^5*b^3*c*d^5 + a^6*b^2*d^6)*g^7))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + (364*a*b^6*c^6 - 3294*a^2*b^5*c^5*d + 13875*a^3*b^4*c^4*d^2 - 38000*a^4*b^3*c^3*d^3 + 94500*a^5*b^2*c^2*d^4 - 72114*a^6*b*c*d^5 + 4669*a^7*d^6 + 180*(274*b^7*c^2*d^4 - 303*a*b^6*c*d^5 + 29*a^2*b^5*d^6)*x^5 - 90*(154*b^7*c^3*d^3 - 3057*a*b^6*c^2*d^4 + 3202*a^2*b^5*c*d^5 - 299*a^3*b^4*d^6)*x^4 + 60*(94*b^7*c^4*d^2 - 1205*a*b^6*c^3*d^3 + 10401*a^2*b^5*c^2*d^4 - 10213*a^3*b^4*c*d^5 + 923*a^4*b^3*d^6)*x^3 - 15*(162*b^7*c^5*d - 1753*a*b^6*c^4*d^2 + 9796*a^2*b^5*c^3*d^3 - 48150*a^3*b^4*c^2*d^4 + 43714*a^4*b^3*c*d^5 - 3769*a^5*b^2*d^6)*x^2 - 1800*(6*a^6*b*c*d^5 - a^7*d^6 + (6*b^7*c*d^5 - a*b^6*d^6)*x^6 + 6*(6*a*b^6*c*d^5 - a^2*b^5*d^6)*x^5 + 15*(6*a^2*b^5*c*d^5 - a^3*b^4*d^6)*x^4 + 20*(6*a^3*b^4*c*d^5 - a^4*b^3*d^6)*x^3 + 15*(6*a^4*b^3*c*d^5 - a^5*b^2*d^6)*x^2 + 6*(6*a^5*b^2*c*d^5 - a^6*b*d^6)*x)*log(b*x + a)^2 - 1800*(6*a^6*b*c*d^5 - a^7*d^6 + (6*b^7*c*d^5 - a*b^6*d^6)*x^6 + 6*(6*a*b^6*c*d^5 - a^2*b^5*d^6)*x^5 + 15*(6*a^2*b^5*c*d^5 - a^3*b^4*d^6)*x^4 + 20*(6*a^3*b^4*c*d^5 - a^4*b^3*d^6)*x^3 + 15*(6*a^4*b^3*c*d^5 - a^5*b^2*d^6)*x^2 + 6*(6*a^5*b^2*c*d^5 - a^6*b*d^6)*x)*log(d*x + c)^2 + 6*(144*b^7*c^6 - 1454*a*b^6*c^5*d + 7005*a^2*b^5*c^4*d^2 - 22750*a^3*b^4*c^3*d^3 + 71000*a^4*b^3*c^2*d^4 - 58614*a^5*b^2*c*d^5 + 4669*a^6*b*d^6)*x

+ 180*(274*a^6*b*c*d^5 - 29*a^7*d^6 + (274*b^7*c*d^5 - 29*a*b^6*d^6)*x^6 + 6*(274*a*b^6*c*d^5 - 29*a^2*b^5*d^6)*x^5 + 15*(274*a^2*b^5*c*d^5 - 29*a^3*b^4*d^6)*x^4 + 20*(274*a^3*b^4*c*d^5 - 29*a^4*b^3*d^6)*x^3 + 15*(274*a^4*b^3*c*d^5 - 29*a^5*b^2*d^6)*x^2 + 6*(274*a^5*b^2*c*d^5 - 29*a^6*b*d^6)*x)*log(b*x + a) - 180*(274*a^6*b*c*d^5 - 29*a^7*d^6 + (274*b^7*c*d^5 - 29*a*b^6*d^6)*x^6 + 6*(274*a*b^6*c*d^5 - 29*a^2*b^5*d^6)*x^5 + 15*(274*a^2*b^5*c*d^5 - 29*a^3*b^4*d^6)*x^4 + 20*(274*a^3*b^4*c*d^5 - 29*a^4*b^3*d^6)*x^3 + 15*(274*a^4*b^3*c*d^5 - 29*a^5*b^2*d^6)*x^2 + 6*(274*a^5*b^2*c*d^5 - 29*a^6*b*d^6)*x - 20*(6*a^6*b*c*d^5 - a^7*d^6 + (6*b^7*c*d^5 - a*b^6*d^6)*x^6 + 6*(6*a*b^6*c*d^5 - a^2*b^5*d^6)*x^5 + 15*(6*a^2*b^5*c*d^5 - a^3*b^4*d^6)*x^4 + 20*(6*a^3*b^4*c*d^5 - a^4*b^3*d^6)*x^3 + 15*(6*a^4*b^3*c*d^5 - a^5*b^2*d^6)*x^2 + 6*(6*a^5*b^2*c*d^5 - a^6*b*d^6)*x)*log(b*x + a))*log(d*x + c))/(a^6*b^8*c^6*g^7 - 6*a^7*b^7*c^5*d*g^7 + 15*a^8*b^6*c^4*d^2*g^7 - 20*a^9*b^5*c^3*d^3*g^7 + 15*a^10*b^4*c^2*d^4*g^7 - 6*a^11*b^3*c*d^5*g^7 + a^12*b^2*d^6*g^7 + (b^14*c^6*g^7 - 6*a*b^13*c^5*d*g^7 + 15*a^2*b^12*c^4*d^2*g^7 - 20*a^3*b^11*c^3*d^3*g^7 + 15*a^4*b^10*c^2*d^4*g^7 - 6*a^5*b^9*c*d^5*g^7 + a^6*b^8*d^6*g^7)*x^6 + 6*(a*b^13*c^6*g^7 - 6*a^2*b^12*c^5*d*g^7 + 15*a^3*b^11*c^4*d^2*g^7 - 20*a^4*b^10*c^3*d^3*g^7 + 15*a^5*b^9*c^2*d^4*g^7 - 6*a^6*b^8*c*d^5*g^7 + a^7*b^7*d^6*g^7)*x^5 + 15*(a^2*b^12*c^6*g^7 - 6*a^3*b^11*c^5*d*g^7 + 15*a^4*b^10*c^4*d^2*g^7 - 20*a^5*b^9*c^3*d^3*g^7 + 15*a^6*b^8*c^2*d^4*g^7 - 6*a^7*b^7*c*d^5*g^7 + a^8*b^6*d^6*g^7)*x^4 + 20*(a^3*b^11*c^6*g^7 - 6*a^4*b^10*c^5*d*g^7 + 15*a^5*b^9*c^4*d^2*g^7 - 20*a^6*b^8*c^3*d^3*g^7 + 15*a^7*b^7*c^2*d^4*g^7 - 6*a^8*b^6*c*d^5*g^7 + a^9*b^5*d^6*g^7)*x^3 + 15*(a^4*b^10*c^6*g^7 - 6*a^5*b^9*c^5*d*g^7 + 15*a^6*b^8*c^4*d^2*g^7 - 20*a^7*b^7*c^3*d^3*g^7 + 15*a^8*b^6*c^2*d^4*g^7 - 6*a^9*b^5*c*d^5*g^7 + a^10*b^4*d^6*g^7)*x^2 + 6*(a^5*b^9*c^6*g^7 - 6*a^6*b^8*c^5*d*g^7 + 15*a^7*b^7*c^4*d^2*g^7 - 20*a^8*b^6*c^3*d^3*g^7 + 15*a^9*b^5*c^2*d^4*g^7 - 6*a^10*b^4*c*d^5*g^7 + a^11*b^3*d^6*g^7)*x))*B^2*c^2*d*i^3 - 1/36000*(60*((37*a^2*b^5*c^5 - 245*a^3*b^4*c^4*d + 730*a^4*b^3*c^3*d^2 - 1470*a^5*b^2*c^2*d^3 + 405*a^6*b*c*d^4 - 57*a^7*d^5 - 60*(15*b^7*c^2*d^3 - 6*a*b^6*c*d^4 + a^2*b^5*d^5)*x^5 + 30*(15*b^7*c^3*d^2 - 171*a*b^6*c^2*d^3 + 67*a^2*b^5*c*d^4 - 11*a^3*b^4*d^5)*x^4 - 20*(15*b^7*c^4*d - 126*a*b^6*c^3*d^2 + 604*a^2*b^5*c^2*d^3 - 230*a^3*b^4*c*d^4 + 37*a^4*b^3*d^5)*x^3 + 15*(15*b^7*c^5 - 111*a*b^6*c^4*d + 388*a^2*b^5*c^3*d^2 - 1000*a^3*b^4*c^2*d^3 + 365*a^4*b^3*c*d^4 - 57*a^5*b^2*d^5)*x^2 + 6*(27*a*b^6*c^5 - 185*a^2*b^5*c^4*d + 580*a^3*b^4*c^3*d^2 - 1270*a^4*b^3*c^2*d^3 + 405*a^5*b^2*c*d^4 - 57*a^6*b*d^5)*x)/((b^14*c^5 - 5*a*b^13*c^4*d + 10*a^2*b^12*c^3*d^2 - 10*a^3*b^11*c^2*d^3 + 5*a^4*b^10*c*d^4 - a^5*b^9*d^5)*g^7*x^6 + 6*(a*b^13*c^5 - 5*a^2*b^12*c^4*d + 10*a^3*b^11*c^3*d^2 - 10*a^4*b^10*c^2*d^3 + 5*a^5*b^9*c*d^4 - a^6*b^8*d^5)*g^7*x^5 + 15*(a^2*b^12*c^5 - 5*a^3*b^11*c^4*d + 10*a^4*b^10*c^3*d^2 - 10*a^5*b^9*c^2*d^3 + 5*a^6*b^8*c*d^4 - a^7*b^7*d^5)*g^7*x^4 + 20*(a^3*b^11*c^5 - 5*a^4*b^10*c^4*d + 10*a^5*b^9*c^3*d^2 - 10*a^6*b^8*c^2*d^3 + 5*a^7*b^7*c*d^4 - a^8*b^6*d^5)*g^7*x^3 + 15*(a^4*b^10*c^5 - 5*a^5*b^9*c^4*d + 10*a^6*b^8*c^3*d^2 - 10*a^7*b^7*c^2*d^3 + 5*a^8*b^6*c*d^4 - a^9*b^5*d^5)*g^7*x^2 + 6*(a^5*b^9*c^5 - 5*a^6*b^8*c^4*d + 10*a^7*b^7*c^3*d^2 - 10*a^8*b^6*c^2*d^3 + 5*a^9*b^5*c*d^4 - a^10*b^4*d^5)*g^

7*x + (a^6*b^8*c^5 - 5*a^7*b^7*c^4*d + 10*a^8*b^6*c^3*d^2 - 10*a^9*b^5*c^2*
d^3 + 5*a^10*b^4*c*d^4 - a^11*b^3*d^5)*g^7) - 60*(15*b^2*c^2*d^4 - 6*a*b*c*
d^5 + a^2*d^6)*log(b*x + a)/((b^9*c^6 - 6*a*b^8*c^5*d + 15*a^2*b^7*c^4*d^2
- 20*a^3*b^6*c^3*d^3 + 15*a^4*b^5*c^2*d^4 - 6*a^5*b^4*c*d^5 + a^6*b^3*d^6)*
g^7) + 60*(15*b^2*c^2*d^4 - 6*a*b*c*d^5 + a^2*d^6)*log(d*x + c)/((b^9*c^6 -
6*a*b^8*c^5*d + 15*a^2*b^7*c^4*d^2 - 20*a^3*b^6*c^3*d^3 + 15*a^4*b^5*c^2*d
^4 - 6*a^5*b^4*c*d^5 + a^6*b^3*d^6)*g^7))*log(b*e*x/(d*x + c) + a*e/(d*x +
c)) + (919*a^2*b^6*c^6 - 8934*a^3*b^5*c^5*d + 43125*a^4*b^4*c^4*d^2 - 17000
0*a^5*b^3*c^3*d^3 + 155625*a^6*b^2*c^2*d^4 - 22794*a^7*b*c*d^5 + 2059*a^8*d
^6 - 180*(625*b^8*c^3*d^3 - 779*a*b^7*c^2*d^4 + 173*a^2*b^6*c*d^5 - 19*a^3*
b^5*d^6)*x^5 + 90*(325*b^8*c^4*d^2 - 6934*a*b^7*c^3*d^3 + 8182*a^2*b^6*c^2*
d^4 - 1762*a^3*b^5*c*d^5 + 189*a^4*b^4*d^6)*x^4 - 60*(175*b^8*c^5*d - 2449*
a*b^7*c^4*d^2 + 23290*a^2*b^6*c^3*d^3 - 25786*a^3*b^5*c^2*d^4 + 5323*a^4*b^
4*c*d^5 - 553*a^5*b^3*d^6)*x^3 + 15*(225*b^8*c^6 - 2802*a*b^7*c^5*d + 18193
*a^2*b^6*c^4*d^2 - 104356*a^3*b^5*c^3*d^3 + 107475*a^4*b^4*c^2*d^4 - 20794*
a^5*b^3*c*d^5 + 2059*a^6*b^2*d^6)*x^2 + 1800*(15*a^6*b^2*c^2*d^4 - 6*a^7*b*
c*d^5 + a^8*d^6 + (15*b^8*c^2*d^4 - 6*a*b^7*c*d^5 + a^2*b^6*d^6)*x^6 + 6*(1
5*a*b^7*c^2*d^4 - 6*a^2*b^6*c*d^5 + a^3*b^5*d^6)*x^5 + 15*(15*a^2*b^6*c^2*d
^4 - 6*a^3*b^5*c*d^5 + a^4*b^4*d^6)*x^4 + 20*(15*a^3*b^5*c^2*d^4 - 6*a^4*b^
4*c*d^5 + a^5*b^3*d^6)*x^3 + 15*(15*a^4*b^4*c^2*d^4 - 6*a^5*b^3*c*d^5 + a^6
*b^2*d^6)*x^2 + 6*(15*a^5*b^3*c^2*d^4 - 6*a^6*b^2*c*d^5 + a^7*b*d^6)*x)*log
(b*x + a)^2 + 1800*(15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5 + a^8*d^6 + (15*b^8*
c^2*d^4 - 6*a*b^7*c*d^5 + a^2*b^6*d^6)*x^6 + 6*(15*a*b^7*c^2*d^4 - 6*a^2*b^
6*c*d^5 + a^3*b^5*d^6)*x^5 + 15*(15*a^2*b^6*c^2*d^4 - 6*a^3*b^5*c*d^5 + a^4
*b^4*d^6)*x^4 + 20*(15*a^3*b^5*c^2*d^4 - 6*a^4*b^4*c*d^5 + a^5*b^3*d^6)*x^3
+ 15*(15*a^4*b^4*c^2*d^4 - 6*a^5*b^3*c*d^5 + a^6*b^2*d^6)*x^2 + 6*(15*a^5*
b^3*c^2*d^4 - 6*a^6*b^2*c*d^5 + a^7*b*d^6)*x)*log(d*x + c)^2 + 6*(549*a*b^7
*c^6 - 5744*a^2*b^6*c^5*d + 30555*a^3*b^5*c^4*d^2 - 138250*a^4*b^4*c^3*d^3
+ 133625*a^5*b^3*c^2*d^4 - 22794*a^6*b^2*c*d^5 + 2059*a^7*b*d^6)*x - 180*(6
25*a^6*b^2*c^2*d^4 - 154*a^7*b*c*d^5 + 19*a^8*d^6 + (625*b^8*c^2*d^4 - 154*
a*b^7*c*d^5 + 19*a^2*b^6*d^6)*x^6 + 6*(625*a*b^7*c^2*d^4 - 154*a^2*b^6*c*d^
5 + 19*a^3*b^5*d^6)*x^5 + 15*(625*a^2*b^6*c^2*d^4 - 154*a^3*b^5*c*d^5 + 19*
a^4*b^4*d^6)*x^4 + 20*(625*a^3*b^5*c^2*d^4 - 154*a^4*b^4*c*d^5 + 19*a^5*b^3
*d^6)*x^3 + 15*(625*a^4*b^4*c^2*d^4 - 154*a^5*b^3*c*d^5 + 19*a^6*b^2*d^6)*x
^2 + 6*(625*a^5*b^3*c^2*d^4 - 154*a^6*b^2*c*d^5 + 19*a^7*b*d^6)*x)*log(b*x
+ a) + 180*(625*a^6*b^2*c^2*d^4 - 154*a^7*b*c*d^5 + 19*a^8*d^6 + (625*b^8*c
^2*d^4 - 154*a*b^7*c*d^5 + 19*a^2*b^6*d^6)*x^6 + 6*(625*a*b^7*c^2*d^4 - 154
*a^2*b^6*c*d^5 + 19*a^3*b^5*d^6)*x^5 + 15*(625*a^2*b^6*c^2*d^4 - 154*a^3*b^
5*c*d^5 + 19*a^4*b^4*d^6)*x^4 + 20*(625*a^3*b^5*c^2*d^4 - 154*a^4*b^4*c*d^5
+ 19*a^5*b^3*d^6)*x^3 + 15*(625*a^4*b^4*c^2*d^4 - 154*a^5*b^3*c*d^5 + 19*a
^6*b^2*d^6)*x^2 + 6*(625*a^5*b^3*c^2*d^4 - 154*a^6*b^2*c*d^5 + 19*a^7*b*d^6
)*x - 20*(15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5 + a^8*d^6 + (15*b^8*c^2*d^4 -
6*a*b^7*c*d^5 + a^2*b^6*d^6)*x^6 + 6*(15*a*b^7*c^2*d^4 - 6*a^2*b^6*c*d^5 +
a^3*b^5*d^6)*x^5 + 15*(15*a^2*b^6*c^2*d^4 - 6*a^3*b^5*c*d^5 + a^4*b^4*d^6)*
x^4 + 20*(15*a^3*b^5*c^2*d^4 - 6*a^4*b^4*c*d^5 + a^5*b^3*d^6)*x^3 + 15*(15*

a^4*b^4*c^2*d^4 - 6*a^5*b^3*c*d^5 + a^6*b^2*d^6)*x^2 + 6*(15*a^5*b^3*c^2*d^
4 - 6*a^6*b^2*c*d^5 + a^7*b*d^6)*x)*log(b*x + a))*log(d*x + c))/(a^6*b^9*c^
6*g^7 - 6*a^7*b^8*c^5*d*g^7 + 15*a^8*b^7*c^4*d^2*g^7 - 20*a^9*b^6*c^3*d^3*g
^7 + 15*a^10*b^5*c^2*d^4*g^7 - 6*a^11*b^4*c*d^5*g^7 + a^12*b^3*d^6*g^7 + (b
^15*c^6*g^7 - 6*a*b^14*c^5*d*g^7 + 15*a^2*b^13*c^4*d^2*g^7 - 20*a^3*b^12*c^
3*d^3*g^7 + 15*a^4*b^11*c^2*d^4*g^7 - 6*a^5*b^10*c*d^5*g^7 + a^6*b^9*d^6*g^
7)*x^6 + 6*(a*b^14*c^6*g^7 - 6*a^2*b^13*c^5*d*g^7 + 15*a^3*b^12*c^4*d^2*g^7
 - 20*a^4*b^11*c^3*d^3*g^7 + 15*a^5*b^10*c^2*d^4*g^7 - 6*a^6*b^9*c*d^5*g^7
+ a^7*b^8*d^6*g^7)*x^5 + 15*(a^2*b^13*c^6*g^7 - 6*a^3*b^12*c^5*d*g^7 + 15*a
^4*b^11*c^4*d^2*g^7 - 20*a^5*b^10*c^3*d^3*g^7 + 15*a^6*b^9*c^2*d^4*g^7 - 6*
a^7*b^8*c*d^5*g^7 + a^8*b^7*d^6*g^7)*x^4 + 20*(a^3*b^12*c^6*g^7 - 6*a^4*b^1
1*c^5*d*g^7 + 15*a^5*b^10*c^4*d^2*g^7 - 20*a^6*b^9*c^3*d^3*g^7 + 15*a^7*b^8
*c^2*d^4*g^7 - 6*a^8*b^7*c*d^5*g^7 + a^9*b^6*d^6*g^7)*x^3 + 15*(a^4*b^11*c^
6*g^7 - 6*a^5*b^10*c^5*d*g^7 + 15*a^6*b^9*c^4*d^2*g^7 - 20*a^7*b^8*c^3*d^3*
g^7 + 15*a^8*b^7*c^2*d^4*g^7 - 6*a^9*b^6*c*d^5*g^7 + a^10*b^5*d^6*g^7)*x^2
+ 6*(a^5*b^10*c^6*g^7 - 6*a^6*b^9*c^5*d*g^7 + 15*a^7*b^8*c^4*d^2*g^7 - 20*a
^8*b^7*c^3*d^3*g^7 + 15*a^9*b^6*c^2*d^4*g^7 - 6*a^10*b^5*c*d^5*g^7 + a^11*b
^4*d^6*g^7)*x))*B^2*c*d^2*i^3 - 1/108000*(60*((57*a^3*b^5*c^5 - 405*a^4*b^4
*c^4*d + 1470*a^5*b^3*c^3*d^2 - 730*a^6*b^2*c^2*d^3 + 245*a^7*b*c*d^4 - 37*
a^8*d^5 + 60*(20*b^8*c^3*d^2 - 15*a*b^7*c^2*d^3 + 6*a^2*b^6*c*d^4 - a^3*b^5
*d^5)*x^5 - 30*(20*b^8*c^4*d - 235*a*b^7*c^3*d^2 + 171*a^2*b^6*c^2*d^3 - 67
*a^3*b^5*c*d^4 + 11*a^4*b^4*d^5)*x^4 + 20*(20*b^8*c^5 - 175*a*b^7*c^4*d + 8
66*a^2*b^6*c^3*d^2 - 604*a^3*b^5*c^2*d^3 + 230*a^4*b^4*c*d^4 - 37*a^5*b^3*d
^5)*x^3 + 15*(35*a*b^7*c^5 - 271*a^2*b^6*c^4*d + 1128*a^3*b^5*c^3*d^2 - 700
*a^4*b^4*c^2*d^3 + 245*a^5*b^3*c*d^4 - 37*a^6*b^2*d^5)*x^2 + 6*(47*a^2*b^6*
c^5 - 345*a^3*b^5*c^4*d + 1320*a^4*b^4*c^3*d^2 - 730*a^5*b^3*c^2*d^3 + 245*
a^6*b^2*c*d^4 - 37*a^7*b*d^5)*x)/((b^15*c^5 - 5*a*b^14*c^4*d + 10*a^2*b^13*
c^3*d^2 - 10*a^3*b^12*c^2*d^3 + 5*a^4*b^11*c*d^4 - a^5*b^10*d^5)*g^7*x^6 +
6*(a*b^14*c^5 - 5*a^2*b^13*c^4*d + 10*a^3*b^12*c^3*d^2 - 10*a^4*b^11*c^2*d^
3 + 5*a^5*b^10*c*d^4 - a^6*b^9*d^5)*g^7*x^5 + 15*(a^2*b^13*c^5 - 5*a^3*b^12
*c^4*d + 10*a^4*b^11*c^3*d^2 - 10*a^5*b^10*c^2*d^3 + 5*a^6*b^9*c*d^4 - a^7*
b^8*d^5)*g^7*x^4 + 20*(a^3*b^12*c^5 - 5*a^4*b^11*c^4*d + 10*a^5*b^10*c^3*d^
2 - 10*a^6*b^9*c^2*d^3 + 5*a^7*b^8*c*d^4 - a^8*b^7*d^5)*g^7*x^3 + 15*(a^4*b
^11*c^5 - 5*a^5*b^10*c^4*d + 10*a^6*b^9*c^3*d^2 - 10*a^7*b^8*c^2*d^3 + 5*a^
8*b^7*c*d^4 - a^9*b^6*d^5)*g^7*x^2 + 6*(a^5*b^10*c^5 - 5*a^6*b^9*c^4*d + 10
*a^7*b^8*c^3*d^2 - 10*a^8*b^7*c^2*d^3 + 5*a^9*b^6*c*d^4 - a^10*b^5*d^5)*g^7
*x + (a^6*b^9*c^5 - 5*a^7*b^8*c^4*d + 10*a^8*b^7*c^3*d^2 - 10*a^9*b^6*c^2*d
^3 + 5*a^10*b^5*c*d^4 - a^11*b^4*d^5)*g^7) + 60*(20*b^3*c^3*d^3 - 15*a*b^2*
c^2*d^4 + 6*a^2*b*c*d^5 - a^3*d^6)*log(b*x + a)/((b^10*c^6 - 6*a*b^9*c^5*d
+ 15*a^2*b^8*c^4*d^2 - 20*a^3*b^7*c^3*d^3 + 15*a^4*b^6*c^2*d^4 - 6*a^5*b^5*
c*d^5 + a^6*b^4*d^6)*g^7) - 60*(20*b^3*c^3*d^3 - 15*a*b^2*c^2*d^4 + 6*a^2*b
*c*d^5 - a^3*d^6)*log(d*x + c)/((b^10*c^6 - 6*a*b^9*c^5*d + 15*a^2*b^8*c^4*
d^2 - 20*a^3*b^7*c^3*d^3 + 15*a^4*b^6*c^2*d^4 - 6*a^5*b^5*c*d^5 + a^6*b^4*d
^6)*g^7))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + (2059*a^3*b^6*c^6 - 22794*
a^4*b^5*c^5*d + 155625*a^5*b^4*c^4*d^2 - 170000*a^6*b^3*c^3*d^3 + 43125*a^7

*b^2*c^2*d^4 - 8934*a^8*b*c*d^5 + 919*a^9*d^6 + 60*(2200*b^9*c^4*d^2 - 3175
*a*b^8*c^3*d^3 + 1257*a^2*b^7*c^2*d^4 - 319*a^3*b^6*c*d^5 + 37*a^4*b^5*d^6)
x^5 - 30(1000*b^9*c^5*d - 24075*a*b^8*c^4*d^2 + 32822*a^2*b^7*c^3*d^3 - 1
2466*a^3*b^6*c^2*d^4 + 3066*a^4*b^5*c*d^5 - 347*a^5*b^4*d^6)*x^4 + 20*(400*
b^9*c^6 - 6825*a*b^8*c^5*d + 78267*a^2*b^7*c^4*d^2 - 100130*a^3*b^6*c^3*d^3
 + 35778*a^4*b^5*c^2*d^4 - 8409*a^5*b^4*c*d^5 + 919*a^6*b^3*d^6)*x^3 + 15*(
925*a*b^8*c^6 - 12222*a^2*b^7*c^5*d + 106533*a^3*b^6*c^4*d^2 - 128396*a^4*b
^5*c^3*d^3 + 41175*a^5*b^4*c^2*d^4 - 8934*a^6*b^3*c*d^5 + 919*a^7*b^2*d^6)*
x^2 - 1800*(20*a^6*b^3*c^3*d^3 - 15*a^7*b^2*c^2*d^4 + 6*a^8*b*c*d^5 - a^9*d
^6 + (20*b^9*c^3*d^3 - 15*a*b^8*c^2*d^4 + 6*a^2*b^7*c*d^5 - a^3*b^6*d^6)*x^
6 + 6*(20*a*b^8*c^3*d^3 - 15*a^2*b^7*c^2*d^4 + 6*a^3*b^6*c*d^5 - a^4*b^5*d^
6)*x^5 + 15*(20*a^2*b^7*c^3*d^3 - 15*a^3*b^6*c^2*d^4 + 6*a^4*b^5*c*d^5 - a^
5*b^4*d^6)*x^4 + 20*(20*a^3*b^6*c^3*d^3 - 15*a^4*b^5*c^2*d^4 + 6*a^5*b^4*c*
d^5 - a^6*b^3*d^6)*x^3 + 15*(20*a^4*b^5*c^3*d^3 - 15*a^5*b^4*c^2*d^4 + 6*a^
6*b^3*c*d^5 - a^7*b^2*d^6)*x^2 + 6*(20*a^5*b^4*c^3*d^3 - 15*a^6*b^3*c^2*d^4
 + 6*a^7*b^2*c*d^5 - a^8*b*d^6)*x)*log(b*x + a)^2 - 1800*(20*a^6*b^3*c^3*d^
3 - 15*a^7*b^2*c^2*d^4 + 6*a^8*b*c*d^5 - a^9*d^6 + (20*b^9*c^3*d^3 - 15*a*b
^8*c^2*d^4 + 6*a^2*b^7*c*d^5 - a^3*b^6*d^6)*x^6 + 6*(20*a*b^8*c^3*d^3 - 15*
a^2*b^7*c^2*d^4 + 6*a^3*b^6*c*d^5 - a^4*b^5*d^6)*x^5 + 15*(20*a^2*b^7*c^3*d
^3 - 15*a^3*b^6*c^2*d^4 + 6*a^4*b^5*c*d^5 - a^5*b^4*d^6)*x^4 + 20*(20*a^3*b
^6*c^3*d^3 - 15*a^4*b^5*c^2*d^4 + 6*a^5*b^4*c*d^5 - a^6*b^3*d^6)*x^3 + 15*(
20*a^4*b^5*c^3*d^3 - 15*a^5*b^4*c^2*d^4 + 6*a^6*b^3*c*d^5 - a^7*b^2*d^6)*x^
2 + 6*(20*a^5*b^4*c^3*d^3 - 15*a^6*b^3*c^2*d^4 + 6*a^7*b^2*c*d^5 - a^8*b*d^
6)*x)*log(d*x + c)^2 + 6*(1489*a^2*b^7*c^6 - 17604*a^3*b^6*c^5*d + 132255*a
^4*b^5*c^4*d^2 - 151250*a^5*b^4*c^3*d^3 + 43125*a^6*b^3*c^2*d^4 - 8934*a^7*
b^2*c*d^5 + 919*a^8*b*d^6)*x + 60*(2200*a^6*b^3*c^3*d^3 - 975*a^7*b^2*c^2*d
^4 + 282*a^8*b*c*d^5 - 37*a^9*d^6 + (2200*b^9*c^3*d^3 - 975*a*b^8*c^2*d^4 +
 282*a^2*b^7*c*d^5 - 37*a^3*b^6*d^6)*x^6 + 6*(2200*a*b^8*c^3*d^3 - 975*a^2*
b^7*c^2*d^4 + 282*a^3*b^6*c*d^5 - 37*a^4*b^5*d^6)*x^5 + 15*(2200*a^2*b^7*c^
3*d^3 - 975*a^3*b^6*c^2*d^4 + 282*a^4*b^5*c*d^5 - 37*a^5*b^4*d^6)*x^4 + 20*
(2200*a^3*b^6*c^3*d^3 - 975*a^4*b^5*c^2*d^4 + 282*a^5*b^4*c*d^5 - 37*a^6*b^
3*d^6)*x^3 + 15*(2200*a^4*b^5*c^3*d^3 - 975*a^5*b^4*c^2*d^4 + 282*a^6*b^3*c
*d^5 - 37*a^7*b^2*d^6)*x^2 + 6*(2200*a^5*b^4*c^3*d^3 - 975*a^6*b^3*c^2*d^4
+ 282*a^7*b^2*c*d^5 - 37*a^8*b*d^6)*x)*log(b*x + a) - 60*(2200*a^6*b^3*c^3*
d^3 - 975*a^7*b^2*c^2*d^4 + 282*a^8*b*c*d^5 - 37*a^9*d^6 + (2200*b^9*c^3*d^
3 - 975*a*b^8*c^2*d^4 + 282*a^2*b^7*c*d^5 - 37*a^3*b^6*d^6)*x^6 + 6*(2200*a
*b^8*c^3*d^3 - 975*a^2*b^7*c^2*d^4 + 282*a^3*b^6*c*d^5 - 37*a^4*b^5*d^6)*x^
5 + 15*(2200*a^2*b^7*c^3*d^3 - 975*a^3*b^6*c^2*d^4 + 282*a^4*b^5*c*d^5 - 37
*a^5*b^4*d^6)*x^4 + 20*(2200*a^3*b^6*c^3*d^3 - 975*a^4*b^5*c^2*d^4 + 282*a^
5*b^4*c*d^5 - 37*a^6*b^3*d^6)*x^3 + 15*(2200*a^4*b^5*c^3*d^3 - 975*a^5*b^4*
c^2*d^4 + 282*a^6*b^3*c*d^5 - 37*a^7*b^2*d^6)*x^2 + 6*(2200*a^5*b^4*c^3*d^3
 - 975*a^6*b^3*c^2*d^4 + 282*a^7*b^2*c*d^5 - 37*a^8*b*d^6)*x - 60*(20*a^6*b
^3*c^3*d^3 - 15*a^7*b^2*c^2*d^4 + 6*a^8*b*c*d^5 - a^9*d^6 + (20*b^9*c^3*d^3
 - 15*a*b^8*c^2*d^4 + 6*a^2*b^7*c*d^5 - a^3*b^6*d^6)*x^6 + 6*(20*a*b^8*c^3*
d^3 - 15*a^2*b^7*c^2*d^4 + 6*a^3*b^6*c*d^5 - a^4*b^5*d^6)*x^5 + 15*(20*a^2*

b^7*c^3*d^3 - 15*a^3*b^6*c^2*d^4 + 6*a^4*b^5*c*d^5 - a^5*b^4*d^6)*x^4 + 20*
(20*a^3*b^6*c^3*d^3 - 15*a^4*b^5*c^2*d^4 + 6*a^5*b^4*c*d^5 - a^6*b^3*d^6)*x
^3 + 15*(20*a^4*b^5*c^3*d^3 - 15*a^5*b^4*c^2*d^4 + 6*a^6*b^3*c*d^5 - a^7*b^
2*d^6)*x^2 + 6*(20*a^5*b^4*c^3*d^3 - 15*a^6*b^3*c^2*d^4 + 6*a^7*b^2*c*d^5 -
a^8*b*d^6)*x)*log(b*x + a))*log(d*x + c))/(a^6*b^10*c^6*g^7 - 6*a^7*b^9*c^
5*d*g^7 + 15*a^8*b^8*c^4*d^2*g^7 - 20*a^9*b^7*c^3*d^3*g^7 + 15*a^10*b^6*c^2
*d^4*g^7 - 6*a^11*b^5*c*d^5*g^7 + a^12*b^4*d^6*g^7 + (b^16*c^6*g^7 - 6*a*b^
15*c^5*d*g^7 + 15*a^2*b^14*c^4*d^2*g^7 - 20*a^3*b^13*c^3*d^3*g^7 + 15*a^4*b
^12*c^2*d^4*g^7 - 6*a^5*b^11*c*d^5*g^7 + a^6*b^10*d^6*g^7)*x^6 + 6*(a*b^15*
c^6*g^7 - 6*a^2*b^14*c^5*d*g^7 + 15*a^3*b^13*c^4*d^2*g^7 - 20*a^4*b^12*c^3*
d^3*g^7 + 15*a^5*b^11*c^2*d^4*g^7 - 6*a^6*b^10*c*d^5*g^7 + a^7*b^9*d^6*g^7)
x^5 + 15(a^2*b^14*c^6*g^7 - 6*a^3*b^13*c^5*d*g^7 + 15*a^4*b^12*c^4*d^2*g^
7 - 20*a^5*b^11*c^3*d^3*g^7 + 15*a^6*b^10*c^2*d^4*g^7 - 6*a^7*b^9*c*d^5*g^7
+ a^8*b^8*d^6*g^7)*x^4 + 20*(a^3*b^13*c^6*g^7 - 6*a^4*b^12*c^5*d*g^7 + 15*
a^5*b^11*c^4*d^2*g^7 - 20*a^6*b^10*c^3*d^3*g^7 + 15*a^7*b^9*c^2*d^4*g^7 - 6
*a^8*b^8*c*d^5*g^7 + a^9*b^7*d^6*g^7)*x^3 + 15*(a^4*b^12*c^6*g^7 - 6*a^5*b^
11*c^5*d*g^7 + 15*a^6*b^10*c^4*d^2*g^7 - 20*a^7*b^9*c^3*d^3*g^7 + 15*a^8*b^
8*c^2*d^4*g^7 - 6*a^9*b^7*c*d^5*g^7 + a^10*b^6*d^6*g^7)*x^2 + 6*(a^5*b^11*c
^6*g^7 - 6*a^6*b^10*c^5*d*g^7 + 15*a^7*b^9*c^4*d^2*g^7 - 20*a^8*b^8*c^3*d^3
*g^7 + 15*a^9*b^7*c^2*d^4*g^7 - 6*a^10*b^6*c*d^5*g^7 + a^11*b^5*d^6*g^7)*x)
)*B^2*d^3*i^3 - 1/1800*A*B*d^3*i^3*(60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b
*x + a^3)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^10*g^7*x^6 + 6*a*b^9*g^7*
x^5 + 15*a^2*b^8*g^7*x^4 + 20*a^3*b^7*g^7*x^3 + 15*a^4*b^6*g^7*x^2 + 6*a^5*
b^5*g^7*x + a^6*b^4*g^7) + (57*a^3*b^5*c^5 - 405*a^4*b^4*c^4*d + 1470*a^5*b
^3*c^3*d^2 - 730*a^6*b^2*c^2*d^3 + 245*a^7*b*c*d^4 - 37*a^8*d^5 + 60*(20*b^
8*c^3*d^2 - 15*a*b^7*c^2*d^3 + 6*a^2*b^6*c*d^4 - a^3*b^5*d^5)*x^5 - 30*(20*
b^8*c^4*d - 235*a*b^7*c^3*d^2 + 171*a^2*b^6*c^2*d^3 - 67*a^3*b^5*c*d^4 + 11
*a^4*b^4*d^5)*x^4 + 20*(20*b^8*c^5 - 175*a*b^7*c^4*d + 866*a^2*b^6*c^3*d^2
- 604*a^3*b^5*c^2*d^3 + 230*a^4*b^4*c*d^4 - 37*a^5*b^3*d^5)*x^3 + 15*(35*a*
b^7*c^5 - 271*a^2*b^6*c^4*d + 1128*a^3*b^5*c^3*d^2 - 700*a^4*b^4*c^2*d^3 +
245*a^5*b^3*c*d^4 - 37*a^6*b^2*d^5)*x^2 + 6*(47*a^2*b^6*c^5 - 345*a^3*b^5*c
^4*d + 1320*a^4*b^4*c^3*d^2 - 730*a^5*b^3*c^2*d^3 + 245*a^6*b^2*c*d^4 - 37*
a^7*b*d^5)*x)/((b^15*c^5 - 5*a*b^14*c^4*d + 10*a^2*b^13*c^3*d^2 - 10*a^3*b^
12*c^2*d^3 + 5*a^4*b^11*c*d^4 - a^5*b^10*d^5)*g^7*x^6 + 6*(a*b^14*c^5 - 5*a
^2*b^13*c^4*d + 10*a^3*b^12*c^3*d^2 - 10*a^4*b^11*c^2*d^3 + 5*a^5*b^10*c*d^
4 - a^6*b^9*d^5)*g^7*x^5 + 15*(a^2*b^13*c^5 - 5*a^3*b^12*c^4*d + 10*a^4*b^1
1*c^3*d^2 - 10*a^5*b^10*c^2*d^3 + 5*a^6*b^9*c*d^4 - a^7*b^8*d^5)*g^7*x^4 +
20*(a^3*b^12*c^5 - 5*a^4*b^11*c^4*d + 10*a^5*b^10*c^3*d^2 - 10*a^6*b^9*c^2*
d^3 + 5*a^7*b^8*c*d^4 - a^8*b^7*d^5)*g^7*x^3 + 15*(a^4*b^11*c^5 - 5*a^5*b^1
0*c^4*d + 10*a^6*b^9*c^3*d^2 - 10*a^7*b^8*c^2*d^3 + 5*a^8*b^7*c*d^4 - a^9*b
^6*d^5)*g^7*x^2 + 6*(a^5*b^10*c^5 - 5*a^6*b^9*c^4*d + 10*a^7*b^8*c^3*d^2 -
10*a^8*b^7*c^2*d^3 + 5*a^9*b^6*c*d^4 - a^10*b^5*d^5)*g^7*x + (a^6*b^9*c^5 -
5*a^7*b^8*c^4*d + 10*a^8*b^7*c^3*d^2 - 10*a^9*b^6*c^2*d^3 + 5*a^10*b^5*c*d
^4 - a^11*b^4*d^5)*g^7) + 60*(20*b^3*c^3*d^3 - 15*a*b^2*c^2*d^4 + 6*a^2*b*c
*d^5 - a^3*d^6)*log(b*x + a)/((b^10*c^6 - 6*a*b^9*c^5*d + 15*a^2*b^8*c^4*d^

2 - 20*a^3*b^7*c^3*d^3 + 15*a^4*b^6*c^2*d^4 - 6*a^5*b^5*c*d^5 + a^6*b^4*d^6)*g^7) - 60*(20*b^3*c^3*d^3 - 15*a*b^2*c^2*d^4 + 6*a^2*b*c*d^5 - a^3*d^6)*l og(d*x + c)/((b^10*c^6 - 6*a*b^9*c^5*d + 15*a^2*b^8*c^4*d^2 - 20*a^3*b^7*c^ 3*d^3 + 15*a^4*b^6*c^2*d^4 - 6*a^5*b^5*c*d^5 + a^6*b^4*d^6)*g^7)) - 1/600*A *B*c*d^2*i^3*(60*(15*b^2*x^2 + 6*a*b*x + a^2)*log(b*e*x/(d*x + c) + a*e/(d* x + c))/(b^9*g^7*x^6 + 6*a*b^8*g^7*x^5 + 15*a^2*b^7*g^7*x^4 + 20*a^3*b^6*g^ 7*x^3 + 15*a^4*b^5*g^7*x^2 + 6*a^5*b^4*g^7*x + a^6*b^3*g^7) + (37*a^2*b^5*c ^5 - 245*a^3*b^4*c^4*d + 730*a^4*b^3*c^3*d^2 - 1470*a^5*b^2*c^2*d^3 + 405*a ^6*b*c*d^4 - 57*a^7*d^5 - 60*(15*b^7*c^2*d^3 - 6*a*b^6*c*d^4 + a^2*b^5*d^5) *x^5 + 30*(15*b^7*c^3*d^2 - 171*a*b^6*c^2*d^3 + 67*a^2*b^5*c*d^4 - 11*a^3*b ^4*d^5)*x^4 - 20*(15*b^7*c^4*d - 126*a*b^6*c^3*d^2 + 604*a^2*b^5*c^2*d^3 - 230*a^3*b^4*c*d^4 + 37*a^4*b^3*d^5)*x^3 + 15*(15*b^7*c^5 - 111*a*b^6*c^4*d + 388*a^2*b^5*c^3*d^2 - 1000*a^3*b^4*c^2*d^3 + 365*a^4*b^3*c*d^4 - 57*a^5*b ^2*d^5)*x^2 + 6*(27*a*b^6*c^5 - 185*a^2*b^5*c^4*d + 580*a^3*b^4*c^3*d^2 - 1 270*a^4*b^3*c^2*d^3 + 405*a^5*b^2*c*d^4 - 57*a^6*b*d^5)*x)/((b^14*c^5 - 5*a *b^13*c^4*d + 10*a^2*b^12*c^3*d^2 - 10*a^3*b^11*c^2*d^3 + 5*a^4*b^10*c*d^4 - a^5*b^9*d^5)*g^7*x^6 + 6*(a*b^13*c^5 - 5*a^2*b^12*c^4*d + 10*a^3*b^11*c^3 *d^2 - 10*a^4*b^10*c^2*d^3 + 5*a^5*b^9*c*d^4 - a^6*b^8*d^5)*g^7*x^5 + 15*(a ^2*b^12*c^5 - 5*a^3*b^11*c^4*d + 10*a^4*b^10*c^3*d^2 - 10*a^5*b^9*c^2*d^3 + 5*a^6*b^8*c*d^4 - a^7*b^7*d^5)*g^7*x^4 + 20*(a^3*b^11*c^5 - 5*a^4*b^10*c^4 *d + 10*a^5*b^9*c^3*d^2 - 10*a^6*b^8*c^2*d^3 + 5*a^7*b^7*c*d^4 - a^8*b^6*d^ 5)*g^7*x^3 + 15*(a^4*b^10*c^5 - 5*a^5*b^9*c^4*d + 10*a^6*b^8*c^3*d^2 - 10*a ^7*b^7*c^2*d^3 + 5*a^8*b^6*c*d^4 - a^9*b^5*d^5)*g^7*x^2 + 6*(a^5*b^9*c^5 - 5*a^6*b^8*c^4*d + 10*a^7*b^7*c^3*d^2 - 10*a^8*b^6*c^2*d^3 + 5*a^9*b^5*c*d^4 - a^10*b^4*d^5)*g^7*x + (a^6*b^8*c^5 - 5*a^7*b^7*c^4*d + 10*a^8*b^6*c^3*d^ 2 - 10*a^9*b^5*c^2*d^3 + 5*a^10*b^4*c*d^4 - a^11*b^3*d^5)*g^7) - 60*(15*b^2 *c^2*d^4 - 6*a*b*c*d^5 + a^2*d^6)*log(b*x + a)/((b^9*c^6 - 6*a*b^8*c^5*d + 15*a^2*b^7*c^4*d^2 - 20*a^3*b^6*c^3*d^3 + 15*a^4*b^5*c^2*d^4 - 6*a^5*b^4*c* d^5 + a^6*b^3*d^6)*g^7) + 60*(15*b^2*c^2*d^4 - 6*a*b*c*d^5 + a^2*d^6)*log(d *x + c)/((b^9*c^6 - 6*a*b^8*c^5*d + 15*a^2*b^7*c^4*d^2 - 20*a^3*b^6*c^3*d^3 + 15*a^4*b^5*c^2*d^4 - 6*a^5*b^4*c*d^5 + a^6*b^3*d^6)*g^7)) - 1/300*A*B*c^ 2*d*i^3*(60*(6*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^8*g^7*x^6 + 6*a*b^7*g^7*x^5 + 15*a^2*b^6*g^7*x^4 + 20*a^3*b^5*g^7*x^3 + 15*a^4*b^4*g^7 *x^2 + 6*a^5*b^3*g^7*x + a^6*b^2*g^7) + (22*a*b^5*c^5 - 140*a^2*b^4*c^4*d + 385*a^3*b^3*c^3*d^2 - 615*a^4*b^2*c^2*d^3 + 735*a^5*b*c*d^4 - 87*a^6*d^5 + 60*(6*b^6*c*d^4 - a*b^5*d^5)*x^5 - 30*(6*b^6*c^2*d^3 - 67*a*b^5*c*d^4 + 11 *a^2*b^4*d^5)*x^4 + 20*(6*b^6*c^3*d^2 - 49*a*b^5*c^2*d^3 + 230*a^2*b^4*c*d^ 4 - 37*a^3*b^3*d^5)*x^3 - 15*(6*b^6*c^4*d - 43*a*b^5*c^3*d^2 + 145*a^2*b^4* c^2*d^3 - 365*a^3*b^3*c*d^4 + 57*a^4*b^2*d^5)*x^2 + 6*(12*b^6*c^5 - 80*a*b^ 5*c^4*d + 235*a^2*b^4*c^3*d^2 - 415*a^3*b^3*c^2*d^3 + 585*a^4*b^2*c*d^4 - 8 7*a^5*b*d^5)*x)/((b^13*c^5 - 5*a*b^12*c^4*d + 10*a^2*b^11*c^3*d^2 - 10*a^3* b^10*c^2*d^3 + 5*a^4*b^9*c*d^4 - a^5*b^8*d^5)*g^7*x^6 + 6*(a*b^12*c^5 - 5*a ^2*b^11*c^4*d + 10*a^3*b^10*c^3*d^2 - 10*a^4*b^9*c^2*d^3 + 5*a^5*b^8*c*d^4 - a^6*b^7*d^5)*g^7*x^5 + 15*(a^2*b^11*c^5 - 5*a^3*b^10*c^4*d + 10*a^4*b^9*c ^3*d^2 - 10*a^5*b^8*c^2*d^3 + 5*a^6*b^7*c*d^4 - a^7*b^6*d^5)*g^7*x^4 + 20*(

a^3*b^10*c^5 - 5*a^4*b^9*c^4*d + 10*a^5*b^8*c^3*d^2 - 10*a^6*b^7*c^2*d^3 + 5*a^7*b^6*c*d^4 - a^8*b^5*d^5)*g^7*x^3 + 15*(a^4*b^9*c^5 - 5*a^5*b^8*c^4*d + 10*a^6*b^7*c^3*d^2 - 10*a^7*b^6*c^2*d^3 + 5*a^8*b^5*c*d^4 - a^9*b^4*d^5)*g^7*x^2 + 6*(a^5*b^8*c^5 - 5*a^6*b^7*c^4*d + 10*a^7*b^6*c^3*d^2 - 10*a^8*b^5*c^2*d^3 + 5*a^9*b^4*c*d^4 - a^10*b^3*d^5)*g^7*x + (a^6*b^7*c^5 - 5*a^7*b^6*c^4*d + 10*a^8*b^5*c^3*d^2 - 10*a^9*b^4*c^2*d^3 + 5*a^10*b^3*c*d^4 - a^11*b^2*d^5)*g^7) + 60*(6*b*c*d^5 - a*d^6)*log(b*x + a)/((b^8*c^6 - 6*a*b^7*c^5*d + 15*a^2*b^6*c^4*d^2 - 20*a^3*b^5*c^3*d^3 + 15*a^4*b^4*c^2*d^4 - 6*a^5*b^3*c*d^5 + a^6*b^2*d^6)*g^7) - 60*(6*b*c*d^5 - a*d^6)*log(d*x + c)/((b^8*c^6 - 6*a*b^7*c^5*d + 15*a^2*b^6*c^4*d^2 - 20*a^3*b^5*c^3*d^3 + 15*a^4*b^4*c^2*d^4 - 6*a^5*b^3*c*d^5 + a^6*b^2*d^6)*g^7)) + 1/180*A*B*c^3*i^3*((60*b^5*d^5*x^5 - 10*b^5*c^5 + 62*a*b^4*c^4*d - 163*a^2*b^3*c^3*d^2 + 237*a^3*b^2*c^2*d^3 - 213*a^4*b*c*d^4 + 147*a^5*d^5 - 30*(b^5*c*d^4 - 11*a*b^4*d^5)*x^4 + 20*(b^5*c^2*d^3 - 8*a*b^4*c*d^4 + 37*a^2*b^3*d^5)*x^3 - 15*(b^5*c^3*d^2 - 7*a*b^4*c^2*d^3 + 23*a^2*b^3*c*d^4 - 57*a^3*b^2*d^5)*x^2 + 6*(2*b^5*c^4*d - 13*a*b^4*c^3*d^2 + 37*a^2*b^3*c^2*d^3 - 63*a^3*b^2*c*d^4 + 87*a^4*b*d^5)*x)/((b^12*c^5 - 5*a*b^11*c^4*d + 10*a^2*b^10*c^3*d^2 - 10*a^3*b^9*c^2*d^3 + 5*a^4*b^8*c*d^4 - a^5*b^7*d^5)*g^7*x^6 + 6*(a*b^11*c^5 - 5*a^2*b^10*c^4*d + 10*a^3*b^9*c^3*d^2 - 10*a^4*b^8*c^2*d^3 + 5*a^5*b^7*c*d^4 - a^6*b^6*d^5)*g^7*x^5 + 15*(a^2*b^10*c^5 - 5*a^3*b^9*c^4*d + 10*a^4*b^8*c^3*d^2 - 10*a^5*b^7*c^2*d^3 + 5*a^6*b^6*c*d^4 - a^7*b^5*d^5)*g^7*x^4 + 20*(a^3*b^9*c^5 - 5*a^4*b^8*c^4*d + 10*a^5*b^7*c^3*d^2 - 10*a^6*b^6*c^2*d^3 + 5*a^7*b^5*c*d^4 - a^8*b^4*d^5)*g^7*x^3 + 15*(a^4*b^8*c^5 - 5*a^5*b^7*c^4*d + 10*a^6*b^6*c^3*d^2 - 10*a^7*b^5*c^2*d^3 + 5*a^8*b^4*c*d^4 - a^9*b^3*d^5)*g^7*x^2 + 6*(a^5*b^7*c^5 - 5*a^6*b^6*c^4*d + 10*a^7*b^5*c^3*d^2 - 10*a^8*b^4*c^2*d^3 + 5*a^9*b^3*c*d^4 - a^10*b^2*d^5)*g^7*x + (a^6*b^6*c^5 - 5*a^7*b^5*c^4*d + 10*a^8*b^4*c^3*d^2 - 10*a^9*b^3*c^2*d^3 + 5*a^10*b^2*c*d^4 - a^11*b*d^5)*g^7) - 60*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^7*g^7*x^6 + 6*a*b^6*g^7*x^5 + 15*a^2*b^5*g^7*x^4 + 20*a^3*b^4*g^7*x^3 + 15*a^4*b^3*g^7*x^2 + 6*a^5*b^2*g^7*x + a^6*b*g^7) + 60*d^6*log(b*x + a)/((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*g^7) - 60*d^6*log(d*x + c)/((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*g^7)) - 1/6*B^2*c^3*i^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^7*g^7*x^6 + 6*a*b^6*g^7*x^5 + 15*a^2*b^5*g^7*x^4 + 20*a^3*b^4*g^7*x^3 + 15*a^4*b^3*g^7*x^2 + 6*a^5*b^2*g^7*x + a^6*b*g^7) - 1/10*(6*b*x + a)*A^2*c^2*d*i^3/(b^8*g^7*x^6 + 6*a*b^7*g^7*x^5 + 15*a^2*b^6*g^7*x^4 + 20*a^3*b^5*g^7*x^3 + 15*a^4*b^4*g^7*x^2 + 6*a^5*b^3*g^7*x + a^6*b^2*g^7) - 1/20*(15*b^2*x^2 + 6*a*b*x + a^2)*A^2*c*d^2*i^3/(b^9*g^7*x^6 + 6*a*b^8*g^7*x^5 + 15*a^2*b^7*g^7*x^4 + 20*a^3*b^6*g^7*x^3 + 15*a^4*b^5*g^7*x^2 + 6*a^5*b^4*g^7*x + a^6*b^3*g^7) - 1/60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)*A^2*d^3*i^3/(b^10*g^7*x^6 + 6*a*b^9*g^7*x^5 + 15*a^2*b^8*g^7*x^4 + 20*a^3*b^7*g^7*x^3 + 15*a^4*b^6*g^7*x^2 + 6*a^5*b^5*g^7*x + a^6*b^4*g^7) - 1/6*A^2*c^3*i^3/(b^7*g^7*x^6 + 6*a*b^6*g^7*x^5 + 15*a^2*b^5*g^7*x^4 + 20*a^3*b^4*g^7*x^3 + 15*a^4*b^3*g^7*x^2 + 6*a^5*b^2*g^7*x + a^6*b*g^7)

Giac [A] (verification not implemented)

none

Time = 0.66 (sec) , antiderivative size = 780, normalized size of antiderivative = 1.68

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx = -\frac{1}{108000}\left(\frac{1800\left(10\,B^2b^2e^7i^3-\frac{24\,(bex+ae)B^2bde^6i^3}{dx+c}+\frac{15\,(bex+ae)^2B^2d^2e^5i^3}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^6b^2c^2g^7}{(dx+c)^6}-\frac{2\,(bex+ae)^6abcdg^7}{(dx+c)^6}+\frac{(bex+ae)^6a^2d^2g^7}{(dx+c)^6}}+\frac{60\left(600\,ABb^2e^7i^3\right.}{}\right.$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^7,x, alg
orithm="giac")
```

```
[Out] -1/108000*(1800*(10*B^2*b^2*e^7*i^3 - 24*(b*e*x + a*e)*B^2*b*d*e^6*i^3/(d*x
 + c) + 15*(b*e*x + a*e)^2*B^2*d^2*e^5*i^3/(d*x + c)^2)*log((b*e*x + a*e)/(
d*x + c))^2/((b*e*x + a*e)^6*b^2*c^2*g^7/(d*x + c)^6 - 2*(b*e*x + a*e)^6*a*
b*c*d*g^7/(d*x + c)^6 + (b*e*x + a*e)^6*a^2*d^2*g^7/(d*x + c)^6) + 60*(600*
A*B*b^2*e^7*i^3 + 100*B^2*b^2*e^7*i^3 - 1440*(b*e*x + a*e)*A*B*b*d*e^6*i^3/
(d*x + c) - 288*(b*e*x + a*e)*B^2*b*d*e^6*i^3/(d*x + c) + 900*(b*e*x + a*e)
^2*A*B*d^2*e^5*i^3/(d*x + c)^2 + 225*(b*e*x + a*e)^2*B^2*d^2*e^5*i^3/(d*x +
 c)^2)*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^6*b^2*c^2*g^7/(d*x + c)^
6 - 2*(b*e*x + a*e)^6*a*b*c*d*g^7/(d*x + c)^6 + (b*e*x + a*e)^6*a^2*d^2*g^7
/(d*x + c)^6) + (18000*A^2*b^2*e^7*i^3 + 6000*A*B*b^2*e^7*i^3 + 1000*B^2*b^
2*e^7*i^3 - 43200*(b*e*x + a*e)*A^2*b*d*e^6*i^3/(d*x + c) - 17280*(b*e*x +
a*e)*A*B*b*d*e^6*i^3/(d*x + c) - 3456*(b*e*x + a*e)*B^2*b*d*e^6*i^3/(d*x +
c) + 27000*(b*e*x + a*e)^2*A^2*d^2*e^5*i^3/(d*x + c)^2 + 13500*(b*e*x + a*e
)^2*A*B*d^2*e^5*i^3/(d*x + c)^2 + 3375*(b*e*x + a*e)^2*B^2*d^2*e^5*i^3/(d*x
 + c)^2)/((b*e*x + a*e)^6*b^2*c^2*g^7/(d*x + c)^6 - 2*(b*e*x + a*e)^6*a*b*c
*d*g^7/(d*x + c)^6 + (b*e*x + a*e)^6*a^2*d^2*g^7/(d*x + c)^6))*(b*c/((b*c*e
 - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 10.60 (sec) , antiderivative size = 6275, normalized size of antiderivative = 13.55

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx = \text{Too large to display}$$

```
[In] int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^
7,x)
```

```
[Out] ((1800*A^2*a^5*d^5*i^3 + 18000*A^2*b^5*c^5*i^3 + 919*B^2*a^5*d^5*i^3 + 1000
*B^2*b^5*c^5*i^3 + 2220*A*B*a^5*d^5*i^3 + 6000*A*B*b^5*c^5*i^3 - 25200*A^2*
```

```
a*b^4*c^4*d*i^3 + 1800*A^2*a^4*b*c*d^4*i^3 - 2456*B^2*a*b^4*c^4*d*i^3 + 919
*B^2*a^4*b*c*d^4*i^3 + 1800*A^2*a^2*b^3*c^3*d^2*i^3 + 1800*A^2*a^3*b^2*c^2*
d^3*i^3 + 919*B^2*a^2*b^3*c^3*d^2*i^3 + 919*B^2*a^3*b^2*c^2*d^3*i^3 + 2220*
A*B*a^2*b^3*c^3*d^2*i^3 + 2220*A*B*a^3*b^2*c^2*d^3*i^3 - 11280*A*B*a*b^4*c^
4*d*i^3 + 2220*A*B*a^4*b*c*d^4*i^3)/(60*(a*d - b*c)) + (x^4*(347*B^2*a*b^4*
d^5*i^3 + 23*B^2*b^5*c*d^4*i^3 + 660*A*B*a*b^4*d^5*i^3 - 60*A*B*b^5*c*d^4*i
^3))/(2*(a*d - b*c)) + (x^2*(1800*A^2*a^3*b^2*d^5*i^3 + 919*B^2*a^3*b^2*d^5
*i^3 + 5400*A^2*b^5*c^3*d^2*i^3 + 73*B^2*b^5*c^3*d^2*i^3 - 9000*A^2*a*b^4*c
^2*d^3*i^3 + 1800*A^2*a^2*b^3*c*d^4*i^3 - 431*B^2*a*b^4*c^2*d^3*i^3 + 919*B
^2*a^2*b^3*c*d^4*i^3 + 2220*A*B*a^3*b^2*d^5*i^3 + 1140*A*B*b^5*c^3*d^2*i^3
- 3180*A*B*a*b^4*c^2*d^3*i^3 + 2220*A*B*a^2*b^3*c*d^4*i^3))/(4*(a*d - b*c))
 + (x^3*(1800*A^2*a^2*b^3*d^5*i^3 + 919*B^2*a^2*b^3*d^5*i^3 + 1800*A^2*b^5*
c^2*d^3*i^3 - 53*B^2*b^5*c^2*d^3*i^3 - 3600*A^2*a*b^4*c*d^4*i^3 + 244*B^2*a
*b^4*c*d^4*i^3 + 2220*A*B*a^2*b^3*d^5*i^3 + 60*A*B*b^5*c^2*d^3*i^3 - 480*A*
B*a*b^4*c*d^4*i^3))/(3*(a*d - b*c)) + (x*(1800*A^2*a^4*b*d^5*i^3 + 919*B^2*
a^4*b*d^5*i^3 + 10800*A^2*b^5*c^4*d*i^3 + 424*B^2*b^5*c^4*d*i^3 - 16200*A^2
*a*b^4*c^3*d^2*i^3 + 1800*A^2*a^3*b^2*c*d^4*i^3 - 1331*B^2*a*b^4*c^3*d^2*i^
3 + 919*B^2*a^3*b^2*c*d^4*i^3 + 2220*A*B*a^4*b*d^5*i^3 + 3120*A*B*b^5*c^4*d
*i^3 + 1800*A^2*a^2*b^3*c^2*d^3*i^3 + 919*B^2*a^2*b^3*c^2*d^3*i^3 - 6780*A*
B*a*b^4*c^3*d^2*i^3 + 2220*A*B*a^3*b^2*c*d^4*i^3 + 2220*A*B*a^2*b^3*c^2*d^3
*i^3))/(10*(a*d - b*c)) + (d*x^5*(37*B^2*b^5*d^4*i^3 + 60*A*B*b^5*d^4*i^3))
/(a*d - b*c))/(x*(10800*a^5*b^6*c*g^7 - 10800*a^6*b^5*d*g^7) - x^5*(10800*a
^2*b^9*d*g^7 - 10800*a*b^10*c*g^7) + x^6*(1800*b^11*c*g^7 - 1800*a*b^10*d*g
^7) + x^2*(27000*a^4*b^7*c*g^7 - 27000*a^5*b^6*d*g^7) + x^4*(27000*a^2*b^9*
c*g^7 - 27000*a^3*b^8*d*g^7) + x^3*(36000*a^3*b^8*c*g^7 - 36000*a^4*b^7*d*g
^7) + 1800*a^6*b^5*c*g^7 - 1800*a^7*b^4*d*g^7) - log((e*(a + b*x))/(c + d*x
))^2*((x*(a*(b*((B^2*a*d^3*i^3)/(60*b^5*g^7) + (B^2*c*d^2*i^3)/(20*b^4*g^7)
) + (B^2*a*d^3*i^3)/(15*b^4*g^7) + (B^2*c*d^2*i^3)/(5*b^3*g^7)) + b*(a*((B^
2*a*d^3*i^3)/(60*b^5*g^7) + (B^2*c*d^2*i^3)/(20*b^4*g^7)) + (B^2*c^2*d*i^3)
/(10*b^3*g^7)) + (B^2*c^2*d*i^3)/(2*b^2*g^7)) + x^2*(b*(b*((B^2*a*d^3*i^3)/
(60*b^5*g^7) + (B^2*c*d^2*i^3)/(20*b^4*g^7)) + (B^2*a*d^3*i^3)/(15*b^4*g^7)
 + (B^2*c*d^2*i^3)/(5*b^3*g^7)) + (B^2*a*d^3*i^3)/(6*b^3*g^7) + (B^2*c*d^2*
i^3)/(2*b^2*g^7)) + a*(a*((B^2*a*d^3*i^3)/(60*b^5*g^7) + (B^2*c*d^2*i^3)/(2
0*b^4*g^7)) + (B^2*c^2*d*i^3)/(10*b^3*g^7)) + (B^2*c^3*i^3)/(6*b^2*g^7) + (
B^2*d^3*i^3*x^3)/(3*b^2*g^7))/(6*a^5*x + a^6/b + b^5*x^6 + 15*a^4*b*x^2 + 6
*a*b^4*x^5 + 20*a^3*b^2*x^3 + 15*a^2*b^3*x^4) - (B^2*d^6*i^3)/(60*b^4*g^7*(
a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (log((e*(a + b*x))/(
c + d*x))*(a*(a*((B*d*i^3*(9*A*b*c - B*a*d + B*b*c))/(90*b^5*g^7) + (A*B*a*
d^2*i^3)/(30*b^5*g^7)) + (B*i^3*(36*A*b^2*c^2 - 3*B*a^2*d^2 + 5*B*b^2*c^2 -
 2*B*a*b*c*d))/(180*b^5*g^7)) + x^2*(b*(b*((B*d*i^3*(9*A*b*c - B*a*d + B*b*
c))/(90*b^5*g^7) + (A*B*a*d^2*i^3)/(30*b^5*g^7)) + (2*B*d*i^3*(9*A*b*c - B*
a*d + B*b*c))/(45*b^4*g^7) + (2*A*B*a*d^2*i^3)/(15*b^4*g^7)) + (B*d*i^3*(9*
A*b*c - B*a*d + B*b*c))/(9*b^3*g^7) + (A*B*a*d^2*i^3)/(3*b^3*g^7) + (B^2*d^
6*i^3*(b*((20*a^4*d^4 + b^4*c^4 + 21*a^2*b^2*c^2*d^2 - 7*a*b^3*c^3*d - 35*a
^3*b*c*d^3)/(15*d^5) + b*(a*(a*((6*a^2*d^2 + b^2*c^2 - 7*a*b*c*d)/(30*b*d^3
```

) + (a*(a*d - b*c))/(6*b*d^2)) + (15*a^3*d^3 - b^3*c^3 + 7*a*b^2*c^2*d - 21
*a^2*b*c*d^2)/(60*b*d^4)) + (20*a^4*d^4 + b^4*c^4 + 21*a^2*b^2*c^2*d^2 - 7*
a*b^3*c^3*d - 35*a^3*b*c*d^3)/(60*b*d^5)) + a*(b*(a*((6*a^2*d^2 + b^2*c^2 -
 7*a*b*c*d)/(30*b*d^3) + (a*(a*d - b*c))/(6*b*d^2)) + (15*a^3*d^3 - b^3*c^3
 + 7*a*b^2*c^2*d - 21*a^2*b*c*d^2)/(60*b*d^4)) + a*(b*((6*a^2*d^2 + b^2*c^2
 - 7*a*b*c*d)/(30*b*d^3) + (a*(a*d - b*c))/(6*b*d^2)) + (6*a^2*d^2 + b^2*c^
2 - 7*a*b*c*d)/(15*d^3) + (a*(a*d - b*c))/(3*d^2)) + (15*a^3*d^3 - b^3*c^3
+ 7*a*b^2*c^2*d - 21*a^2*b*c*d^2)/(20*d^4))) + a*(a*(b*(b*((6*a^2*d^2 + b^2
*c^2 - 7*a*b*c*d)/(30*b*d^3) + (a*(a*d - b*c))/(6*b*d^2)) + (6*a^2*d^2 + b^
2*c^2 - 7*a*b*c*d)/(15*d^3) + (a*(a*d - b*c))/(3*d^2)) - a*((b^2*c - a*b*d)
/(6*d^2) - (b*(a*d - b*c))/(3*d^2)) + (b^3*c^2 + 6*a^2*b*d^2 - 7*a*b^2*c*d)
/(10*d^3)) - (b^4*c^3 - 15*a^3*b*d^3 + 21*a^2*b^2*c*d^2 - 7*a*b^3*c^2*d)/(1
0*d^4) + b*(b*(a*((6*a^2*d^2 + b^2*c^2 - 7*a*b*c*d)/(30*b*d^3) + (a*(a*d -
b*c))/(6*b*d^2)) + (15*a^3*d^3 - b^3*c^3 + 7*a*b^2*c^2*d - 21*a^2*b*c*d^2)/
(60*b*d^4)) + a*(b*((6*a^2*d^2 + b^2*c^2 - 7*a*b*c*d)/(30*b*d^3) + (a*(a*d
- b*c))/(6*b*d^2)) + (6*a^2*d^2 + b^2*c^2 - 7*a*b*c*d)/(15*d^3) + (a*(a*d -
 b*c))/(3*d^2)) + (15*a^3*d^3 - b^3*c^3 + 7*a*b^2*c^2*d - 21*a^2*b*c*d^2)/(
20*d^4))) + (b^5*c^4 + 20*a^4*b*d^4 - 35*a^3*b^2*c*d^3 + 21*a^2*b^3*c^2*d^2
 - 7*a*b^4*c^3*d)/(6*d^5)))/(30*b^4*g^7*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
- 3*a^2*b*c*d^2))) + x*(b*(a*((B*d*i^3*(9*A*b*c - B*a*d + B*b*c))/(90*b^5*g
^7) + (A*B*a*d^2*i^3)/(30*b^5*g^7)) + (B*i^3*(36*A*b^2*c^2 - 3*B*a^2*d^2 +
5*B*b^2*c^2 - 2*B*a*b*c*d))/(180*b^5*g^7)) + a*(b*((B*d*i^3*(9*A*b*c - B*a*
d + B*b*c))/(90*b^5*g^7) + (A*B*a*d^2*i^3)/(30*b^5*g^7)) + (2*B*d*i^3*(9*A*
b*c - B*a*d + B*b*c))/(45*b^4*g^7) + (2*A*B*a*d^2*i^3)/(15*b^4*g^7)) + (B*i
^3*(36*A*b^2*c^2 - 3*B*a^2*d^2 + 5*B*b^2*c^2 - 2*B*a*b*c*d))/(36*b^4*g^7) +
 (B^2*d^6*i^3*(a*((20*a^4*d^4 + b^4*c^4 + 21*a^2*b^2*c^2*d^2 - 7*a*b^3*c^3*
d - 35*a^3*b*c*d^3)/(15*d^5) + b*(a*(a*((6*a^2*d^2 + b^2*c^2 - 7*a*b*c*d)/(
30*b*d^3) + (a*(a*d - b*c))/(6*b*d^2)) + (15*a^3*d^3 - b^3*c^3 + 7*a*b^2*c^
2*d - 21*a^2*b*c*d^2)/(60*b*d^4)) + (20*a^4*d^4 + b^4*c^4 + 21*a^2*b^2*c^2*
d^2 - 7*a*b^3*c^3*d - 35*a^3*b*c*d^3)/(60*b*d^5)) + a*(b*(a*((6*a^2*d^2 + b
^2*c^2 - 7*a*b*c*d)/(30*b*d^3) + (a*(a*d - b*c))/(6*b*d^2)) + (15*a^3*d^3 -
 b^3*c^3 + 7*a*b^2*c^2*d - 21*a^2*b*c*d^2)/(60*b*d^4)) + a*(b*((6*a^2*d^2 +
 b^2*c^2 - 7*a*b*c*d)/(30*b*d^3) + (a*(a*d - b*c))/(6*b*d^2)) + (6*a^2*d^2
+ b^2*c^2 - 7*a*b*c*d)/(15*d^3) + (a*(a*d - b*c))/(3*d^2)) + (15*a^3*d^3 -
b^3*c^3 + 7*a*b^2*c^2*d - 21*a^2*b*c*d^2)/(20*d^4))) + (15*a^5*d^5 - b^5*c^
5 - 21*a^2*b^3*c^3*d^2 + 35*a^3*b^2*c^2*d^3 + 7*a*b^4*c^4*d - 35*a^4*b*c*d^
4)/(6*d^6) + b*(a*(a*(a*((6*a^2*d^2 + b^2*c^2 - 7*a*b*c*d)/(30*b*d^3) + (a*
(a*d - b*c))/(6*b*d^2)) + (15*a^3*d^3 - b^3*c^3 + 7*a*b^2*c^2*d - 21*a^2*b*
c*d^2)/(60*b*d^4)) + (20*a^4*d^4 + b^4*c^4 + 21*a^2*b^2*c^2*d^2 - 7*a*b^3*c
^3*d - 35*a^3*b*c*d^3)/(60*b*d^5)) + (15*a^5*d^5 - b^5*c^5 - 21*a^2*b^3*c^3
*d^2 + 35*a^3*b^2*c^2*d^3 + 7*a*b^4*c^4*d - 35*a^4*b*c*d^4)/(30*b*d^6))))/(
30*b^4*g^7*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) + x^3*((2*
A*B*d^2*i^3)/(3*b^2*g^7) + (B^2*d^6*i^3*(b*(a*(b*(b*((6*a^2*d^2 + b^2*c^2 -
 7*a*b*c*d)/(30*b*d^3) + (a*(a*d - b*c))/(6*b*d^2)) + (6*a^2*d^2 + b^2*c^2
- 7*a*b*c*d)/(15*d^3) + (a*(a*d - b*c))/(3*d^2)) - a*((b^2*c - a*b*d)/(6*d^

2) - (b*(a*d - b*c))/(3*d^2)) + (b^3*c^2 + 6*a^2*b*d^2 - 7*a*b^2*c*d)/(10*d
^3)) - (b^4*c^3 - 15*a^3*b*d^3 + 21*a^2*b^2*c*d^2 - 7*a*b^3*c^2*d)/(10*d^4)
 + b*(b*(a*((6*a^2*d^2 + b^2*c^2 - 7*a*b*c*d)/(30*b*d^3) + (a*(a*d - b*c))/
(6*b*d^2)) + (15*a^3*d^3 - b^3*c^3 + 7*a*b^2*c^2*d - 21*a^2*b*c*d^2)/(60*b*
d^4)) + a*(b*((6*a^2*d^2 + b^2*c^2 - 7*a*b*c*d)/(30*b*d^3) + (a*(a*d - b*c)
)/(6*b*d^2)) + (6*a^2*d^2 + b^2*c^2 - 7*a*b*c*d)/(15*d^3) + (a*(a*d - b*c))
/(3*d^2)) + (15*a^3*d^3 - b^3*c^3 + 7*a*b^2*c^2*d - 21*a^2*b*c*d^2)/(20*d^4
))) - (b^5*c^3 - 15*a^3*b^2*d^3 + 21*a^2*b^3*c*d^2 - 7*a*b^4*c^2*d)/(6*d^4)
 + a*((2*(b^4*c^2 + 6*a^2*b^2*d^2 - 7*a*b^3*c*d))/(15*d^3) + b*(b*(b*((6*a^
2*d^2 + b^2*c^2 - 7*a*b*c*d)/(30*b*d^3) + (a*(a*d - b*c))/(6*b*d^2)) + (6*a
^2*d^2 + b^2*c^2 - 7*a*b*c*d)/(15*d^3) + (a*(a*d - b*c))/(3*d^2)) - a*((b^2
*c - a*b*d)/(6*d^2) - (b*(a*d - b*c))/(3*d^2)) + (b^3*c^2 + 6*a^2*b*d^2 - 7
*a*b^2*c*d)/(10*d^3)) - a*(b*((b^2*c - a*b*d)/(6*d^2) - (b*(a*d - b*c))/(3*
d^2)) + (b^3*c - a*b^2*d)/(6*d^2)))))/(30*b^4*g^7*(a^3*d^3 - b^3*c^3 + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2))) + (B*i^3*(60*A*b^3*c^3 - 6*B*a^3*d^3 + 11*B*b^
3*c^3 - 8*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2))/(180*b^5*d*g^7) + (B^2*d^6*i^3*
(a*(a*(a*(a*((6*a^2*d^2 + b^2*c^2 - 7*a*b*c*d)/(30*b*d^3) + (a*(a*d - b*c))
/(6*b*d^2)) + (15*a^3*d^3 - b^3*c^3 + 7*a*b^2*c^2*d - 21*a^2*b*c*d^2)/(60*b
*d^4)) + (20*a^4*d^4 + b^4*c^4 + 21*a^2*b^2*c^2*d^2 - 7*a*b^3*c^3*d - 35*a^
3*b*c*d^3)/(60*b*d^5)) + (15*a^5*d^5 - b^5*c^5 - 21*a^2*b^3*c^3*d^2 + 35*a^
3*b^2*c^2*d^3 + 7*a*b^4*c^4*d - 35*a^4*b*c*d^4)/(30*b*d^6)) + (6*a^6*d^6 +
b^6*c^6 + 21*a^2*b^4*c^4*d^2 - 35*a^3*b^3*c^3*d^3 + 35*a^4*b^2*c^2*d^4 - 7*
a*b^5*c^5*d - 21*a^5*b*c*d^5)/(6*b*d^7)))/(30*b^4*g^7*(a^3*d^3 - b^3*c^3 +
3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (B^2*d^6*i^3*x^5*((b^5*c - a*b^4*d)/(6*d^
2) + b*(b*(b*((b^2*c - a*b*d)/(6*d^2) - (b*(a*d - b*c))/(3*d^2)) + (b^3*c -
 a*b^2*d)/(6*d^2)) + (b^4*c - a*b^3*d)/(6*d^2))))/(30*b^4*g^7*(a^3*d^3 - b^
3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (B^2*d^6*i^3*x^4*((b^5*c^2 + 6*a^
2*b^3*d^2 - 7*a*b^4*c*d)/(6*d^3) + b*((2*(b^4*c^2 + 6*a^2*b^2*d^2 - 7*a*b^3
*c*d))/(15*d^3) + b*(b*(b*((6*a^2*d^2 + b^2*c^2 - 7*a*b*c*d)/(30*b*d^3) + (
a*(a*d - b*c))/(6*b*d^2)) + (6*a^2*d^2 + b^2*c^2 - 7*a*b*c*d)/(15*d^3) + (a
*(a*d - b*c))/(3*d^2)) - a*((b^2*c - a*b*d)/(6*d^2) - (b*(a*d - b*c))/(3*d^
2)) + (b^3*c^2 + 6*a^2*b*d^2 - 7*a*b^2*c*d)/(10*d^3)) - a*(b*((b^2*c - a*b*
d)/(6*d^2) - (b*(a*d - b*c))/(3*d^2)) + (b^3*c - a*b^2*d)/(6*d^2))) - a*(b*
(b*((b^2*c - a*b*d)/(6*d^2) - (b*(a*d - b*c))/(3*d^2)) + (b^3*c - a*b^2*d)/
(6*d^2)) + (b^4*c - a*b^3*d)/(6*d^2)))))/(30*b^4*g^7*(a^3*d^3 - b^3*c^3 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2))))/((6*a^5*x)/d + a^6/(b*d) + (b^5*x^6)/d + (1
5*a^4*b*x^2)/d + (6*a*b^4*x^5)/d + (20*a^3*b^2*x^3)/d + (15*a^2*b^3*x^4)/d)
 - (B*d^6*i^3*atan((B*d^6*i^3*(60*A + 37*B)*(1800*b^7*c^3*g^7 + 1800*a^3*b^
4*d^3*g^7 - 1800*a*b^6*c^2*d*g^7 - 1800*a^2*b^5*c*d^2*g^7)*1i)/(1800*b^4*g^
7*(37*B^2*d^6*i^3 + 60*A*B*d^6*i^3)*(a*d - b*c)^3) + (B*d^7*i^3*x*(60*A + 3
7*B)*(b^6*c^2*g^7 + a^2*b^4*d^2*g^7 - 2*a*b^5*c*d*g^7)*2i)/(b^3*g^7*(37*B^2
*d^6*i^3 + 60*A*B*d^6*i^3)*(a*d - b*c)^3))*(60*A + 37*B)*1i)/(900*b^4*g^7*(
a*d - b*c)^3)

3.84 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 718

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$$

$$= \frac{bB^2(bc-ad)^2g^3x}{3d^3i} + \frac{B^2(bc-ad)^3g^3\log\left(\frac{a+bx}{c+dx}\right)}{3d^4i} + \frac{7B(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^3i} - \frac{b^2B(bc-ad)g^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^4i} + \frac{6B(bc-ad)^3g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4i} + \frac{3(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i} - \frac{3b^2(bc-ad)g^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^4i} + \frac{b^3g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d^4i} + \frac{(bc-ad)^3g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^4i} - \frac{2B^2(bc-ad)^3g^3\log(c+dx)}{d^4i} - \frac{7B(bc-ad)^3g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3d^4i} + \frac{6B^2(bc-ad)^3g^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i} + \frac{2B(bc-ad)^3g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i} + \frac{7B^2(bc-ad)^3g^3\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3d^4i} - \frac{2B^2(bc-ad)^3g^3\,\mathrm{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}$$

```
[Out] 1/3*b*B^2*(-a*d+b*c)^2*g^3*x/d^3/i+1/3*B^2*(-a*d+b*c)^3*g^3*ln((b*x+a)/(d*x
+c))/d^4/i+7/3*B*(-a*d+b*c)^2*g^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3/i
-1/3*b^2*B*(-a*d+b*c)*g^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4/i+6*B*(
-a*d+b*c)^3*g^3*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4/i+
3*(-a*d+b*c)^2*g^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i-3/2*b^2*(-a*
d+b*c)*g^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^4/i+1/3*b^3*g^3*(d*x+c
```

```
)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^4/i+(-a*d+b*c)^3*g^3*ln((-a*d+b*c)/b/(d
*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^4/i-2*B^2*(-a*d+b*c)^3*g^3*ln(d*x+c)
/d^4/i-7/3*B*(-a*d+b*c)^3*g^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/
(b*x+a))/d^4/i+6*B^2*(-a*d+b*c)^3*g^3*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i+
2*B*(-a*d+b*c)^3*g^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x
+c))/d^4/i+7/3*B^2*(-a*d+b*c)^3*g^3*polylog(2,b*(d*x+c)/d/(b*x+a))/d^4/i-2*
B^2*(-a*d+b*c)^3*g^3*polylog(3,d*(b*x+a)/b/(d*x+c))/d^4/i
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 718, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2562, 2395, 2356, 2389, 2379, 2438, 2351, 31, 46, 2355, 2354, 2421, 6724}

$$\begin{aligned}
&\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx \\
&= \frac{b^3g^3(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d^4i}-\frac{3b^2g^3(c+dx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d^4i} \\
&\quad-\frac{b^2Bg^3(c+dx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^4i} \\
&\quad+\frac{2Bg^3(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^4i} \\
&\quad+\frac{g^3(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^4i} \\
&\quad+\frac{6Bg^3(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^4i} \\
&\quad-\frac{7Bg^3(bc-ad)^3\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^4i} \\
&\quad+\frac{3g^3(a+bx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^3i} \\
&\quad+\frac{7Bg^3(a+bx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^3i} \\
&\quad+\frac{6B^2g^3(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}+\frac{7B^2g^3(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3d^4i} \\
&\quad-\frac{2B^2g^3(bc-ad)^3\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}+\frac{B^2g^3(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{3d^4i} \\
&\quad-\frac{2B^2g^3(bc-ad)^3\log(c+dx)}{d^4i}+\frac{bB^2g^3x(bc-ad)^2}{3d^3i}
\end{aligned}$$

```
[In] Int[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x),
x]
```

```
[Out] (b*B^2*(b*c - a*d)^2*g^3*x)/(3*d^3*i) + (B^2*(b*c - a*d)^3*g^3*Log[(a + b*x
)/(c + d*x)])/(3*d^4*i) + (7*B*(b*c - a*d)^2*g^3*(a + b*x)*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]))/(3*d^3*i) - (b^2*B*(b*c - a*d)*g^3*(c + d*x)^2*(A + B
*Log[(e*(a + b*x))/(c + d*x)]))/(3*d^4*i) + (6*B*(b*c - a*d)^3*g^3*Log[(b*c
 - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d^4*i) + (3*(
b*c - a*d)^2*g^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d^3*i)
- (3*b^2*(b*c - a*d)*g^3*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
)/(2*d^4*i) + (b^3*g^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/
(3*d^4*i) + ((b*c - a*d)^3*g^3*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e
*(a + b*x))/(c + d*x)])^2)/(d^4*i) - (2*B^2*(b*c - a*d)^3*g^3*Log[c + d*x])
/(d^4*i) - (7*B*(b*c - a*d)^3*g^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[
1 - (b*(c + d*x))/(d*(a + b*x))])/(3*d^4*i) + (6*B^2*(b*c - a*d)^3*g^3*Poly
Log[2, (d*(a + b*x))/(b*(c + d*x))])/(d^4*i) + (2*B*(b*c - a*d)^3*g^3*(A +
B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d
^4*i) + (7*B^2*(b*c - a*d)^3*g^3*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(
3*d^4*i) - (2*B^2*(b*c - a*d)^3*g^3*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))]
)/(d^4*i)
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 46</u>

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

<u>Rule 2351</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

<u>Rule 2354</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)^3g^3)\,\text{Subst}\left(\int \frac{x^3(A+B\log(ex))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{i}$$

$$= \frac{((bc-ad)^3g^3)\,\text{Subst}\left(\int \left(\frac{b^3(A+B\log(ex))^2}{d^3(b-dx)^4} - \frac{3b^2(A+B\log(ex))^2}{d^3(b-dx)^3} + \frac{3b(A+B\log(ex))^2}{d^3(b-dx)^2} - \frac{(A+B\log(ex))^2}{d^3(b-dx)}\right) dx, x, \frac{a+bx}{c+dx}\right)}{i}$$

$$= -\frac{((bc-ad)^3g^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i} + \frac{(3b(bc-ad)^3g^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i} - \frac{(3b^2(bc-ad)^3g^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i} + \frac{(b^3(bc-ad)^3g^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i}$$

$$= \frac{3(bc-ad)^2 g^3 (a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3 i} - \frac{3b^2(bc-ad)g^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^4 i} + \frac{b^3 g^3 (c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d^4 i} + \frac{(bc-ad)^3 g^3 \log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^4 i} - \frac{(2B(bc-ad)^3 g^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex))\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4 i} + \frac{(3b^2 B(bc-ad)^3 g^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4 i} - \frac{(2b^3 B(bc-ad)^3 g^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3d^4 i} - \frac{(6B(bc-ad)^3 g^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3 i}$$

$$
\begin{aligned}
= & \frac{6B(bc-ad)^3 g^3 \log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4 i} \\
& + \frac{3(bc-ad)^2 g^3 (a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3 i} \\
& - \frac{3b^2(bc-ad) g^3 (c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^4 i} \\
& + \frac{b^3 g^3 (c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d^4 i} \\
& + \frac{(bc-ad)^3 g^3 \log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^4 i} \\
& + \frac{2B(bc-ad)^3 g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4 i} \\
& + \frac{(3bB(bc-ad)^3 g^3)\,\mathrm{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4 i} \\
& - \frac{(2b^2 B(bc-ad)^3 g^3)\,\mathrm{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{3d^4 i} \\
& - \frac{(2B^2(bc-ad)^3 g^3)\,\mathrm{Subst}\left(\int \frac{\mathrm{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4 i} \\
& - \frac{(6B^2(bc-ad)^3 g^3)\,\mathrm{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4 i} \\
& + \frac{(3bB(bc-ad)^3 g^3)\,\mathrm{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3 i} \\
& - \frac{(2b^2 B(bc-ad)^3 g^3)\,\mathrm{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3d^3 i}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3B(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i} \\
&\quad - \frac{b^2B(bc-ad)g^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^4i} \\
&\quad + \frac{6B(bc-ad)^3g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4i} \\
&\quad + \frac{3(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i} \\
&\quad - \frac{3b^2(bc-ad)g^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^4i} \\
&\quad + \frac{b^3g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d^4i} \\
&\quad + \frac{(bc-ad)^3g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^4i} \\
&\quad - \frac{3B(bc-ad)^3g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{d^4i} \\
&\quad + \frac{6B^2(bc-ad)^3g^3\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i} \\
&\quad + \frac{2B(bc-ad)^3g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i} \\
&\quad - \frac{2B^2(bc-ad)^3g^3\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i} \\
&\quad - \frac{(2bB(bc-ad)^3g^3)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{x(b-dx)}\,dx,x,\frac{a+bx}{c+dx}\right)}{3d^4i} \\
&\quad + \frac{(3B^2(bc-ad)^3g^3)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^4i} \\
&\quad + \frac{(b^2B^2(bc-ad)^3g^3)\,\text{Subst}\left(\int\frac{1}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{3d^4i} \\
&\quad - \frac{(2bB(bc-ad)^3g^3)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{3d^3i} \\
&\quad - \frac{(3B^2(bc-ad)^3g^3)\,\text{Subst}\left(\int\frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i}
\end{aligned}
$$

$$= \frac{7B(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^3i} - \frac{b^2B(bc-ad)g^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^4i} + \frac{6B(bc-ad)^3g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4i} + \frac{3(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i} - \frac{3b^2(bc-ad)g^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^4i} + \frac{b^3g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d^4i} + \frac{(bc-ad)^3g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^4i} - \frac{3B^2(bc-ad)^3g^3\log(c+dx)}{d^4i} - \frac{7B(bc-ad)^3g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3d^4i} + \frac{6B^2(bc-ad)^3g^3\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i} + \frac{2B(bc-ad)^3g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i} + \frac{3B^2(bc-ad)^3g^3\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{d^4i} - \frac{2B^2(bc-ad)^3g^3\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i} - \frac{(2B^2(bc-ad)^3g^3)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{3d^4i} + \frac{(b^2B^2(bc-ad)^3g^3)\,\text{Subst}\left(\int\left(\frac{1}{b^2x}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{3d^4i} + \frac{(2B^2(bc-ad)^3g^3)\,\text{Subst}\left(\int\frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{3d^3i}$$

$$= \frac{bB^2(bc-ad)^2g^3x}{3d^3i} + \frac{B^2(bc-ad)^3g^3\log\left(\frac{a+bx}{c+dx}\right)}{3d^4i} + \frac{7B(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^3i} - \frac{b^2B(bc-ad)g^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^4i} + \frac{6B(bc-ad)^3g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4i} + \frac{3(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i} - \frac{3b^2(bc-ad)g^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^4i} + \frac{b^3g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d^4i} + \frac{(bc-ad)^3g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^4i} - \frac{2B^2(bc-ad)^3g^3\log(c+dx)}{d^4i} - \frac{7B(bc-ad)^3g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3d^4i} + \frac{6B^2(bc-ad)^3g^3\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i} + \frac{2B(bc-ad)^3g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i} + \frac{7B^2(bc-ad)^3g^3\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{3d^4i} - \frac{2B^2(bc-ad)^3g^3\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 1020, normalized size of antiderivative = 1.42

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$$

$$= \frac{g^3\left(6bd(bc-ad)^2x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+3d^2(-bc+ad)(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+2d^3(a+b\right.}{}$$

```
[In] Integrate[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d
*i*x),x]
```

```
[Out] (g^3*(6*b*d*(b*c - a*d)^2*x*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 3*d^2*
(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 2*d^3*(
a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - 6*A^2*(b*c - a*d)^3*Log
[c + d*x] + 12*A*B*(b*c - a*d)^3*Log[(e*(a + b*x))/(c + d*x)]*Log[(b*c - a*
d)/(b*c + b*d*x)] + 6*B^2*(b*c - a*d)^3*Log[(e*(a + b*x))/(c + d*x)]^2*Log[
(b*c - a*d)/(b*c + b*d*x)] - 6*A*B*(b*c - a*d)^3*(Log[(b*c - a*d)/(b*c + b*
d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c - a*d)/(b*c + b*d*x)]
) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 6*B*(b*c - a*d)^2*(2*a*d*Log
[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b*c*(A + B*Log[(e*(a + b
*x))/(c + d*x)])*Log[c + d*x] - a*B*d*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(
b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) +
b*B*c*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] +
2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + 3*B*(b*c - a*d)^2*(2*A*b*d*x +
2*B*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 2*B*(b*c - a*d)*Log[c + d*x]
 - 2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + B*(b*c
 - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x]
+ 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + 2*B*(b*c - a*d)*(2*A*b*d*(b*c
 - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - d^2*
(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*(b*c - a*d)^2*Log[c
+ d*x] - 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x]
+ B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + B*(b*c - a*d)^2*((2
*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog
[2, (b*(c + d*x))/(b*c - a*d)])) + 12*B^2*(b*c - a*d)^3*(Log[(e*(a + b*x))/
(c + d*x)]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - PolyLog[3, (d*(a + b*x
))/(b*(c + d*x))])))/(6*d^4*i)
```

Maple [F]

$$\int \frac{(bgx + ag)^3 \left(A + B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{dix + ci}dx$$

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x)
```

```
[Out] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algor
ithm="fricas")
```

```
[Out] integral((A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a
^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a
^3*g^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g
^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log((b*e*x + a*e)/(d*x + c)))/(d*
i*x + c*i), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$$
$$=\frac{g^3\left(\int \frac{A^2a^3}{c+dx}\,dx+\int \frac{A^2b^3x^3}{c+dx}\,dx+\int \frac{B^2a^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{c+dx}\,dx+\int \frac{2ABa^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx+\int \frac{3A^2ab^2x^2}{c+dx}\,dx+\int \frac{3A}{c}\right.}{}$$

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i),x)
```

```
[Out] g**3*(Integral(A**2*a**3/(c + d*x), x) + Integral(A**2*b**3*x**3/(c + d*x),
 x) + Integral(B**2*a**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c + d*x),
 x) + Integral(2*A*B*a**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x
) + Integral(3*A**2*a*b**2*x**2/(c + d*x), x) + Integral(3*A**2*a**2*b*x/(c
 + d*x), x) + Integral(B**2*b**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))*
*2/(c + d*x), x) + Integral(2*A*B*b**3*x**3*log(a*e/(c + d*x) + b*e*x/(c +
d*x))/(c + d*x), x) + Integral(3*B**2*a*b**2*x**2*log(a*e/(c + d*x) + b*e*x
/(c + d*x))**2/(c + d*x), x) + Integral(3*B**2*a**2*b*x*log(a*e/(c + d*x) +
 b*e*x/(c + d*x))**2/(c + d*x), x) + Integral(6*A*B*a*b**2*x**2*log(a*e/(c
+ d*x) + b*e*x/(c + d*x))/(c + d*x), x) + Integral(6*A*B*a**2*b*x*log(a*e/(
c + d*x) + b*e*x/(c + d*x))/(c + d*x), x))/i
```

Maxima [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algor
ithm="maxima")
```

```
[Out] 3*A^2*a^2*b*g^3*(x/(d*i) - c*log(d*x + c)/(d^2*i)) - 1/6*A^2*b^3*g^3*(6*c^3
*log(d*x + c)/(d^4*i) - (2*d^2*x^3 - 3*c*d*x^2 + 6*c^2*x)/(d^3*i)) + 3/2*A^
2*a*b^2*g^3*(2*c^2*log(d*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A^2*a^
3*g^3*log(d*i*x + c*i)/(d*i) - 1/6*(2*(b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*
a^2*b*c*d^2*g^3 - a^3*d^3*g^3)*B^2*log(d*x + c)^3 - (2*B^2*b^3*d^3*g^3*x^3
- 3*(b^3*c*d^2*g^3 - 3*a*b^2*d^3*g^3)*B^2*x^2 + 6*(b^3*c^2*d*g^3 - 3*a*b^2*
c*d^2*g^3 + 3*a^2*b*d^3*g^3)*B^2*x)*log(d*x + c)^2)/(d^4*i) - integrate(-1/
3*(3*B^2*a^3*d^2*g^3*log(e)^2 + 6*A*B*a^3*d^2*g^3*log(e) + 3*(B^2*b^3*d^2*g
^3*log(e)^2 + 2*A*B*b^3*d^2*g^3*log(e))*x^3 + 9*(B^2*a*b^2*d^2*g^3*log(e)^2
 + 2*A*B*a*b^2*d^2*g^3*log(e))*x^2 + 3*(B^2*b^3*d^2*g^3*x^3 + 3*B^2*a*b^2*d
^2*g^3*x^2 + 3*B^2*a^2*b*d^2*g^3*x + B^2*a^3*d^2*g^3)*log(b*x + a)^2 + 9*(B
^2*a^2*b*d^2*g^3*log(e)^2 + 2*A*B*a^2*b*d^2*g^3*log(e))*x + 6*(B^2*a^3*d^2*
g^3*log(e) + A*B*a^3*d^2*g^3 + (B^2*b^3*d^2*g^3*log(e) + A*B*b^3*d^2*g^3)*x
^3 + 3*(B^2*a*b^2*d^2*g^3*log(e) + A*B*a*b^2*d^2*g^3)*x^2 + 3*(B^2*a^2*b*d^
2*g^3*log(e) + A*B*a^2*b*d^2*g^3)*x)*log(b*x + a) - (6*B^2*a^3*d^2*g^3*log(
e) + 6*A*B*a^3*d^2*g^3 + 2*(3*A*B*b^3*d^2*g^3 + (3*g^3*log(e) + g^3)*B^2*b^
3*d^2)*x^3 + 3*(6*A*B*a*b^2*d^2*g^3 - (b^3*c*d*g^3 - 3*(2*g^3*log(e) + g^3)
*a*b^2*d^2)*B^2)*x^2 + 6*(3*A*B*a^2*b*d^2*g^3 + (b^3*c^2*g^3 - 3*a*b^2*c*d*
g^3 + 3*(g^3*log(e) + g^3)*a^2*b*d^2)*B^2)*x + 6*(B^2*b^3*d^2*g^3*x^3 + 3*B
^2*a*b^2*d^2*g^3*x^2 + 3*B^2*a^2*b*d^2*g^3*x + B^2*a^3*d^2*g^3)*log(b*x + a
))*log(d*x + c))/(d^3*i*x + c*d^2*i), x)
```

Giac [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algor
ithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*i
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{(a\,g+b\,g\,x)^3 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

```
[In] int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x),
x)
```

```
[Out] int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x),
 x)
```

3.85 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 536

$$\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx\\
&=-\frac{B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2i}\\
&-\frac{4B(bc-ad)^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i}\\
&-\frac{2(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i}+\frac{b^2g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^3i}\\
&-\frac{(bc-ad)^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i}+\frac{B^2(bc-ad)^2g^2\log(c+dx)}{d^3i}\\
&+\frac{B(bc-ad)^2g^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{d^3i}\\
&-\frac{4B^2(bc-ad)^2g^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}\\
&-\frac{2B(bc-ad)^2g^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}\\
&-\frac{B^2(bc-ad)^2g^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{d^3i}+\frac{2B^2(bc-ad)^2g^2\,\mathrm{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}
\end{aligned}$$

```
[Out] -B*(-a*d+b*c)*g^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^2/i-4*B*(-a*d+b*c)^
2*g^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3/i-2*(-a*d+b*
```

```
c)*g^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^2/i+1/2*b^2*g^2*(d*x+c)^2*(A
+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i-(-a*d+b*c)^2*g^2*ln((-a*d+b*c)/b/(d*x+c))
*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i+B^2*(-a*d+b*c)^2*g^2*ln(d*x+c)/d^3/i+B
*(-a*d+b*c)^2*g^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/d^3
/i-4*B^2*(-a*d+b*c)^2*g^2*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i-2*B*(-a*d+b*
c)^2*g^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i-B
^2*(-a*d+b*c)^2*g^2*polylog(2,b*(d*x+c)/d/(b*x+a))/d^3/i+2*B^2*(-a*d+b*c)^2
*g^2*polylog(3,d*(b*x+a)/b/(d*x+c))/d^3/i
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2562, 2395, 2356, 2389, 2379, 2438, 2351, 31, 2355, 2354, 2421, 6724}

$$\begin{aligned}
&\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx\\
&=\frac{b^2g^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d^3i}\\
&-\frac{2Bg^2(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^3i}\\
&-\frac{g^2(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^3i}\\
&-\frac{4Bg^2(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^3i}\\
&+\frac{Bg^2(bc-ad)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^3i}\\
&-\frac{2g^2(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^2i}\\
&-\frac{Bg^2(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^2i}\\
&-\frac{4B^2g^2(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}-\frac{B^2g^2(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{d^3i}\\
&+\frac{2B^2g^2(bc-ad)^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}+\frac{B^2g^2(bc-ad)^2\log(c+dx)}{d^3i}
\end{aligned}$$

```
[In] Int[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x),
x]
```

```
[Out] -((B*(b*c - a*d)*g^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d^2*i
)) - (4*B*(b*c - a*d)^2*g^2*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]))/(d^3*i) - (2*(b*c - a*d)*g^2*(a + b*x)*(A + B*Log[(e*
(a + b*x))/(c + d*x)])^2)/(d^2*i) + (b^2*g^2*(c + d*x)^2*(A + B*Log[(e*(a +
 b*x))/(c + d*x)])^2)/(2*d^3*i) - ((b*c - a*d)^2*g^2*Log[(b*c - a*d)/(b*(c
+ d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d^3*i) + (B^2*(b*c - a*d)
^2*g^2*Log[c + d*x])/(d^3*i) + (B*(b*c - a*d)^2*g^2*(A + B*Log[(e*(a + b*x)
)/(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(d^3*i) - (4*B^2*(b*c -
 a*d)^2*g^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^3*i) - (2*B*(b*c -
a*d)^2*g^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (d*(a + b*x))/(b
*(c + d*x))])/(d^3*i) - (B^2*(b*c - a*d)^2*g^2*PolyLog[2, (b*(c + d*x))/(d*
(a + b*x))])/(d^3*i) + (2*B^2*(b*c - a*d)^2*g^2*PolyLog[3, (d*(a + b*x))/(b
*(c + d*x))])/(d^3*i)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
```

```
NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{((bc-ad)^2g^2)\,\text{Subst}\left(\int \frac{x^2(A+B\log(ex))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{i} \\
&= \frac{((bc-ad)^2g^2)\,\text{Subst}\left(\int \left(\frac{b^2(A+B\log(ex))^2}{d^2(b-dx)^3} - \frac{2b(A+B\log(ex))^2}{d^2(b-dx)^2} + \frac{(A+B\log(ex))^2}{d^2(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{i} \\
&= \frac{((bc-ad)^2g^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i} \\
&\quad - \frac{(2b(bc-ad)^2g^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i} \\
&\quad + \frac{(b^2(bc-ad)^2g^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i} \\
&= -\frac{2(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i} \\
&\quad + \frac{b^2g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^3i} \\
&\quad - \frac{(bc-ad)^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i} \\
&\quad + \frac{(2B(bc-ad)^2g^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i} \\
&\quad - \frac{(b^2B(bc-ad)^2g^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i} \\
&\quad + \frac{(4B(bc-ad)^2g^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4B(bc-ad)^2 g^2 \log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3 i} \\
&\quad - \frac{2(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2 i} \\
&\quad + \frac{b^2 g^2 (c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^3 i} \\
&\quad - \frac{(bc-ad)^2 g^2 \log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3 i} \\
&\quad - \frac{2B(bc-ad)^2 g^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3 i} \\
&\quad - \frac{\left(bB(bc-ad)^2 g^2\right)\mathrm{Subst}\left(\int \frac{A+B\log(ex)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3 i} \\
&\quad + \frac{\left(2B^2(bc-ad)^2 g^2\right)\mathrm{Subst}\left(\int \frac{\mathrm{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3 i} \\
&\quad + \frac{\left(4B^2(bc-ad)^2 g^2\right)\mathrm{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3 i} \\
&\quad - \frac{\left(bB(bc-ad)^2 g^2\right)\mathrm{Subst}\left(\int \frac{A+B\log(ex)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2 i}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2i} \\
&- \frac{4B(bc-ad)^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i} \\
&- \frac{2(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i} \\
&+ \frac{b^2g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^3i} \\
&- \frac{(bc-ad)^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i} \\
&+ \frac{B(bc-ad)^2g^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{d^3i} \\
&- \frac{4B^2(bc-ad)^2g^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i} \\
&- \frac{2B(bc-ad)^2g^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i} \\
&+ \frac{2B^2(bc-ad)^2g^2\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i} - \frac{(B^2(bc-ad)^2g^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i} \\
&+ \frac{(B^2(bc-ad)^2g^2)\,\text{Subst}\left(\int \frac{1}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2i}\\
&- \frac{4B(bc-ad)^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i}\\
&- \frac{2(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i}\\
&+ \frac{b^2g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^3i}\\
&- \frac{(bc-ad)^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i} + \frac{B^2(bc-ad)^2g^2\log(c+dx)}{d^3i}\\
&+ \frac{B(bc-ad)^2g^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{d^3i}\\
&- \frac{4B^2(bc-ad)^2g^2\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}\\
&- \frac{2B(bc-ad)^2g^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}\\
&- \frac{B^2(bc-ad)^2g^2\mathrm{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{d^3i} + \frac{2B^2(bc-ad)^2g^2\mathrm{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.36

$$
\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx
$$

$$
= \frac{g^2\left(-2bd(bc-ad)x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + d^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + 2A^2(bc-ad)^2\log(c\right.}{}
$$

```
[In] Integrate[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d
*i*x),x]
```

```
[Out] (g^2*(-2*b*d*(b*c - a*d)*x*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + d^2*(a
+ b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 2*A^2*(b*c - a*d)^2*Log[c
 + d*x] - 4*A*B*(b*c - a*d)^2*Log[(e*(a + b*x))/(c + d*x)]*Log[(b*c - a*d)/
(b*c + b*d*x)] - 2*B^2*(b*c - a*d)^2*Log[(e*(a + b*x))/(c + d*x)]^2*Log[(b*
c - a*d)/(b*c + b*d*x)] + 2*A*B*(b*c - a*d)^2*(Log[(b*c - a*d)/(b*c + b*d*x
)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c - a*d)/(b*c + b*d*x)]) -
 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 2*B*(b*c - a*d)*(2*a*d*Log[a +
```

```
b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b*c*(A + B*Log[(e*(a + b*x))/
(c + d*x)])*Log[c + d*x] - a*B*d*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c
+ d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*c
*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*Pol
yLog[2, (b*(c + d*x))/(b*c - a*d)])) - B*(b*c - a*d)*(2*A*b*d*x + 2*B*d*(a
+ b*x)*Log[(e*(a + b*x))/(c + d*x)] - 2*B*(b*c - a*d)*Log[c + d*x] - 2*(b*c
 - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + B*(b*c - a*d)*(
(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyL
og[2, (b*(c + d*x))/(b*c - a*d)])) - 4*B^2*(b*c - a*d)^2*(Log[(e*(a + b*x))
/(c + d*x)]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - PolyLog[3, (d*(a + b*
x))/(b*(c + d*x))])))/(2*d^3*i)
```

Maple [F]

$$\int \frac{(bgx+ag)^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{dix+ci}dx$$

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x)
```

```
[Out] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algor
ithm="fricas")
```

```
[Out] integral((A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^
2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*
b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log((b*e*x + a*e)/(d*x + c)))/
(d*i*x + c*i), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$$
$$= \frac{g^2\left(\int \frac{A^2a^2}{c+dx}\,dx + \int \frac{A^2b^2x^2}{c+dx}\,dx + \int \frac{B^2a^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{c+dx}\,dx + \int \frac{2ABa^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx + \int \frac{2A^2abx}{c+dx}\,dx + \int \frac{B^2b^2}{}\right.}{}$$

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i),x)
```

```
[Out] g**2*(Integral(A**2*a**2/(c + d*x), x) + Integral(A**2*b**2*x**2/(c + d*x),
 x) + Integral(B**2*a**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c + d*x),
 x) + Integral(2*A*B*a**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x
) + Integral(2*A**2*a*b*x/(c + d*x), x) + Integral(B**2*b**2*x**2*log(a*e/(
c + d*x) + b*e*x/(c + d*x))**2/(c + d*x), x) + Integral(2*A*B*b**2*x**2*log
(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x) + Integral(2*B**2*a*b*x*log
(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c + d*x), x) + Integral(4*A*B*a*b*x*l
og(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x))/i
```

Maxima [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algor
ithm="maxima")
```

```
[Out] 2*A^2*a*b*g^2*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + 1/2*A^2*b^2*g^2*(2*c^2*l
og(d*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A^2*a^2*g^2*log(d*i*x + c*
i)/(d*i) + 1/6*(2*(b^2*c^2*g^2 - 2*a*b*c*d*g^2 + a^2*d^2*g^2)*B^2*log(d*x +
 c)^3 + 3*(B^2*b^2*d^2*g^2*x^2 - 2*(b^2*c*d*g^2 - 2*a*b*d^2*g^2)*B^2*x)*log
(d*x + c)^2)/(d^3*i) - integrate(-(B^2*a^2*d*g^2*log(e)^2 + 2*A*B*a^2*d*g^2
*log(e) + (B^2*b^2*d*g^2*log(e)^2 + 2*A*B*b^2*d*g^2*log(e))*x^2 + (B^2*b^2*
d*g^2*x^2 + 2*B^2*a*b*d*g^2*x + B^2*a^2*d*g^2)*log(b*x + a)^2 + 2*(B^2*a*b*
d*g^2*log(e)^2 + 2*A*B*a*b*d*g^2*log(e))*x + 2*(B^2*a^2*d*g^2*log(e) + A*B*
a^2*d*g^2 + (B^2*b^2*d*g^2*log(e) + A*B*b^2*d*g^2)*x^2 + 2*(B^2*a*b*d*g^2*l
og(e) + A*B*a*b*d*g^2)*x)*log(b*x + a) - (2*B^2*a^2*d*g^2*log(e) + 2*A*B*a^
2*d*g^2 + (2*A*B*b^2*d*g^2 + (2*g^2*log(e) + g^2)*B^2*b^2*d)*x^2 + 2*(2*A*B
*a*b*d*g^2 - (b^2*c*g^2 - 2*(g^2*log(e) + g^2)*a*b*d)*B^2)*x + 2*(B^2*b^2*d
*g^2*x^2 + 2*B^2*a*b*d*g^2*x + B^2*a^2*d*g^2)*log(b*x + a))*log(d*x + c))/(
d^2*i*x + c*d*i), x)
```

Giac [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algor
ithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*i
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

```
[In] int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x),
x)
```

```
[Out] int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x),
 x)
```

3.86 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 283

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \frac{2B(bc-ad)g\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2i} + \frac{g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{di} + \frac{(bc-ad)g\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i} + \frac{2B^2(bc-ad)g\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i} + \frac{2B(bc-ad)g\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i} - \frac{2B^2(bc-ad)g\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}$$

```
[Out] 2*B*(-a*d+b*c)*g*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^2/i
+g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d/i+(-a*d+b*c)*g*ln((-a*d+b*c)/b/(
d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^2/i+2*B^2*(-a*d+b*c)*g*polylog(2,d*
(b*x+a)/b/(d*x+c))/d^2/i+2*B*(-a*d+b*c)*g*(A+B*ln(e*(b*x+a)/(d*x+c)))*polyl
og(2,d*(b*x+a)/b/(d*x+c))/d^2/i-2*B^2*(-a*d+b*c)*g*polylog(3,d*(b*x+a)/b/(d
*x+c))/d^2/i
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {2562, 2395, 2355, 2354, 2438, 2421, 6724}

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$$
$$=\frac{2Bg(bc-ad)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^2i}$$
$$+\frac{2Bg(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^2i}$$
$$+\frac{g(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^2i}+\frac{g(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{di}$$
$$+\frac{2B^2g(bc-ad)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}-\frac{2B^2g(bc-ad)\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}$$

```
[In] Int[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x),x]
```

```
[Out] (2*B*(b*c - a*d)*g*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/
(c + d*x)]))/(d^2*i) + (g*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)
/(d*i) + ((b*c - a*d)*g*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b
*x))/(c + d*x)])^2)/(d^2*i) + (2*B^2*(b*c - a*d)*g*PolyLog[2, (d*(a + b*x))
/(b*(c + d*x))])/(d^2*i) + (2*B*(b*c - a*d)*g*(A + B*Log[(e*(a + b*x))/(c +
 d*x)])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^2*i) - (2*B^2*(b*c - a*
d)*g*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(d^2*i)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)g)\text{Subst}\left(\int \frac{x(A+B\log(ex))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{i}$$

$$= \frac{((bc-ad)g)\text{Subst}\left(\int \left(\frac{b(A+B\log(ex))^2}{d(-b+dx)^2} + \frac{(A+B\log(ex))^2}{d(-b+dx)}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{i}$$

$$= \frac{((bc-ad)g)\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{-b+dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{di} + \frac{(b(bc-ad)g)\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(-b+dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{di}$$

$$= \frac{g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{di} + \frac{(bc-ad)g\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i} - \frac{(2B(bc-ad)g)\text{Subst}\left(\int \frac{(A+B\log(ex))\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i} + \frac{(2B(bc-ad)g)\text{Subst}\left(\int \frac{A+B\log(ex)}{-b+dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{di}$$

$$= \frac{2B(bc-ad)g\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2i} + \frac{g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{di} + \frac{(bc-ad)g\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i} + \frac{2B(bc-ad)g\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i} - \frac{(2B^2(bc-ad)g)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i} - \frac{(2B^2(bc-ad)g)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i}$$

$$= \frac{2B(bc-ad)g\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2i} + \frac{g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{di} + \frac{(bc-ad)g\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i} + \frac{2B^2(bc-ad)g\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i} + \frac{2B(bc-ad)g\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i} - \frac{2B^2(bc-ad)g\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1227 vs. 2(283) = 566.

Time = 0.66 (sec) , antiderivative size = 1227, normalized size of antiderivative = 4.34

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$$
$$=\frac{g\left(3A^2bdx-3A^2(bc-ad)\log(c+dx)-3aABd\left(\log^2\left(\frac{c}{d}+x\right)+2\left(\log\left(\frac{a}{b}+x\right)-\log\left(\frac{c}{d}+x\right)-\log\left(\frac{e(a+}{c+d}\right.\right.\right.\right.}{}$$

```
[In] Integrate[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i
*x),x]
```

```
[Out] (g*(3*A^2*b*d*x - 3*A^2*(b*c - a*d)*Log[c + d*x] - 3*a*A*B*d*(Log[c/d + x]^
2 + 2*(Log[a/b + x] - Log[c/d + x] - Log[(e*(a + b*x))/(c + d*x)])*Log[c +
d*x] - 2*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a +
b*x))/(-(b*c) + a*d)])) - 3*A*B*(-2*d*(a + b*x)*(-1 + Log[a/b + x]) + 2*b*(
c + d*x)*(-1 + Log[c/d + x]) - b*c*Log[c/d + x]^2 + 2*b*(Log[a/b + x] - Log
[c/d + x] - Log[(e*(a + b*x))/(c + d*x)])*(d*x - c*Log[c + d*x]) + 2*b*c*(L
og[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*
c) + a*d)])) + a*B^2*d*(Log[c/d + x]^3 + 3*Log[c/d + x]^2*(-Log[a/b + x] +
Log[(d*(a + b*x))/(-(b*c) + a*d)]) + 3*(-Log[a/b + x] + Log[c/d + x] + Log[
(e*(a + b*x))/(c + d*x)])^2*Log[c + d*x] + 3*Log[a/b + x]^2*Log[(b*(c + d*x
))/(b*c - a*d)] + 6*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] +
 3*(Log[a/b + x] - Log[c/d + x] - Log[(e*(a + b*x))/(c + d*x)])*(Log[c/d +
x]^2 - 2*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a +
b*x))/(-(b*c) + a*d)])) + 6*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*
d)] - 6*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*PolyLog[3, (b*(c + d*x
))/(b*c - a*d)]) + B^2*(3*d*(2*b*x - 2*(a + b*x)*Log[a/b + x] + (a + b*x)*L
og[a/b + x]^2) - b*c*Log[c/d + x]^3 + 3*b*(2*d*x - 2*(c + d*x)*Log[c/d + x]
 + (c + d*x)*Log[c/d + x]^2) + 3*b*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(
a + b*x))/(c + d*x)])^2*(d*x - c*Log[c + d*x]) - 6*(a*d + 2*b*d*x - b*d*x*L
og[c/d + x] - b*c*Log[c + d*x] + Log[a/b + x]*(-(d*(a + b*x)) + d*(a + b*x)
*Log[c/d + x] + (b*c - a*d)*Log[(b*(c + d*x))/(b*c - a*d)]) + (b*c - a*d)*P
olyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 3*(Log[a/b + x] - Log[c/d + x] -
 Log[(e*(a + b*x))/(c + d*x)])*(-2*d*(a + b*x)*(-1 + Log[a/b + x]) + 2*b*(c
 + d*x)*(-1 + Log[c/d + x]) - b*c*Log[c/d + x]^2 + 2*b*c*(Log[a/b + x]*Log[
(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])) - 3
*b*c*(Log[a/b + x]^2*Log[(b*(c + d*x))/(b*c - a*d)] + 2*Log[a/b + x]*PolyLo
g[2, (d*(a + b*x))/(-(b*c) + a*d)] - 2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a
*d)]) + 3*b*c*(Log[c/d + x]^2*(Log[a/b + x] - Log[(d*(a + b*x))/(-(b*c) + a
*d)]) - 2*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 2*PolyLog[3,
 (b*(c + d*x))/(b*c - a*d)]))))/(3*d^2*i)
```

Maple [F]

$$\int \frac{(bgx+ag)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{dix+ci}dx$$

[In] int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x)

[Out] int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x)

Fricas [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algorithm="fricas")

[Out] integral((A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((b*e*x + a*e)/(d*x + c)))/(d*i*x + c*i), x)

Sympy [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \frac{g\left(\int \frac{A^2a}{c+dx}\,dx + \int \frac{A^2bx}{c+dx}\,dx + \int \frac{B^2a\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{c+dx}\,dx + \int \frac{2ABa\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx + \int \frac{B^2bx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{c+dx}\,dx + \dots}{i}$$

[In] integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i),x)

[Out] g*(Integral(A**2*a/(c + d*x), x) + Integral(A**2*b*x/(c + d*x), x) + Integral(B**2*a*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c + d*x), x) + Integral(2*A*B*a*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x) + Integral(B**2*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c + d*x), x) + Integral(2*A*B*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x))/i

Maxima [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx=\int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algorit
hm="maxima")
```

```
[Out] A^2*b*g*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + A^2*a*g*log(d*i*x + c*i)/(d*i)
 + 1/3*(3*B^2*b*d*g*x*log(d*x + c)^2 - (b*c*g - a*d*g)*B^2*log(d*x + c)^3)/
(d^2*i) - integrate(-(B^2*a*g*log(e)^2 + 2*A*B*a*g*log(e) + (B^2*b*g*x + B^
2*a*g)*log(b*x + a)^2 + (B^2*b*g*log(e)^2 + 2*A*B*b*g*log(e))*x + 2*(B^2*a*
g*log(e) + A*B*a*g + (B^2*b*g*log(e) + A*B*b*g)*x)*log(b*x + a) - 2*(B^2*a*
g*log(e) + A*B*a*g + ((g*log(e) + g)*B^2*b + A*B*b*g)*x + (B^2*b*g*x + B^2*
a*g)*log(b*x + a))*log(d*x + c))/(d*i*x + c*i), x)
```

Giac [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx=\int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algorit
hm="giac")
```

```
[Out] integrate((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*i),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx=\int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

```
[In] int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x),x)
```

```
[Out] int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x), x
)
```

3.87 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 127

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{di} - \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{di} + \frac{2B^2\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{di}$$

```
[Out] -ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d/i-2*B*(A+B*ln(e*(
b*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))/d/i+2*B^2*polylog(3,d*(b*x+
a)/b/(d*x+c))/d/i
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2552, 2354, 2421, 6724}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = -\frac{2B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{di} - \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{di} + \frac{2B^2\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{di}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(c*i + d*i*x),x]
```

```
[Out] -((Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(
d*i)) - (2*B*(A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (d*(a + b*x))/
(b*(c + d*x))])/(d*i) + (2*B^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(d*
i)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{i}\\ &= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{di}+\frac{(2B)\text{Subst}\left(\int \frac{(A+B\log(ex))\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{di}\end{aligned}$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{di} - \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{di} + \frac{(2B^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{di}$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{di} - \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{di} + \frac{2B^2\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{di}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.98

$$\int\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$$

$$= \frac{A^2\log(c+dx) + 2AB\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log\left(\frac{bc-ad}{bc+bdx}\right) - 2AB\log\left(\frac{e(a+bx)}{c+dx}\right)\log\left(\frac{bc-ad}{bc+bdx}\right) - B^2\log^2\left(\frac{e(a+bx)}{c+dx}\right)\log\left(\frac{b}{b}\right.}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(c*i + d*i*x),x]
```

```
[Out] (A^2*Log[c + d*x] + 2*A*B*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[(b*c - a*d)
/(b*c + b*d*x)] - 2*A*B*Log[(e*(a + b*x))/(c + d*x)]*Log[(b*c - a*d)/(b*c +
 b*d*x)] - B^2*Log[(e*(a + b*x))/(c + d*x)]^2*Log[(b*c - a*d)/(b*c + b*d*x)
] + A*B*Log[(b*c - a*d)/(b*c + b*d*x)]^2 - 2*B^2*Log[(e*(a + b*x))/(c + d*x
)]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 2*A*B*PolyLog[2, (b*(c + d*x))
/(b*c - a*d)] + 2*B^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(d*i)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(127) = 254.

Time = 1.41 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.66

method	result
parts	$\frac{A^2\ln(dx+c)}{id} - \frac{B^2\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)+2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\mathrm{Li}_2\left(\frac{d\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)-2}{id}$
risch	$\frac{A^2\ln(dx+c)}{id} - \frac{B^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)}{di} - \frac{2B^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\mathrm{Li}_2\left(\frac{d\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)}{di}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{dA^2\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{ie(ad-cb)}+\frac{dB^2\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)+2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\mathrm{Li}}{ie(ad-cb)}\right.}{}$
default	$-\frac{e(ad-cb)\left(\frac{dA^2\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{ie(ad-cb)}+\frac{dB^2\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)+2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\mathrm{Li}}{ie(ad-cb)}\right.}{}$

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x,method=_RETURNVERBOSE)
```

```
[Out] A^2/i*ln(d*x+c)/d-B^2/i/d*(ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(1-1/b/e*d*(
b*e/d+(a*d-b*c)*e/d/(d*x+c)))+2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*polylog(2,1
/b/e*d*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-2*polylog(3,1/b/e*d*(b*e/d+(a*d-b*c)*
e/d/(d*x+c))))+2*B*A/i*(-dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/
d-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)
/b/e)/d)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algorithm="fricas")

[Out] integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x + c)) + A^2)/(d*i*x + c*i), x)

Sympy [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx=\frac{\int \frac{A^2}{c+dx}\,dx+\int \frac{B^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{c+dx}\,dx+\int \frac{2AB\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx}{i}$$

[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i),x)

[Out] (Integral(A**2/(c + d*x), x) + Integral(B**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c + d*x), x) + Integral(2*A*B*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x))/i

Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algorithm="maxima")

[Out] 1/3*B^2*log(d*x + c)^3/(d*i) + A^2*log(d*i*x + c*i)/(d*i) - integrate(-(B^2*log(b*x + a)^2 + B^2*log(e)^2 + 2*A*B*log(e) + 2*(B^2*log(e) + A*B)*log(b*x + a) - 2*(B^2*log(b*x + a) + B^2*log(e) + A*B)*log(d*x + c))/(d*i*x + c*i), x)

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algorithm="giac")
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*i), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(c*i + d*i*x),x)
[Out] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(c*i + d*i*x), x)
```

3.88 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 44

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx=\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)gi}$$

```
[Out] 1/3*(A+B*ln(e*(b*x+a)/(d*x+c)))^3/B/(-a*d+b*c)/g/i
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2562, 2339, 30}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx=\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^3}{3Bgi(bc-ad)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)*(c*i + d*i*x)),x]
```

```
[Out] (A + B*Log[(e*(a + b*x))/(c + d*x)])^3/(3*B*(b*c - a*d)*g*i)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
```

```
x]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)gi}$$

$$= \frac{\text{Subst}\left(\int x^2\,dx, x, A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B(bc-ad)gi}$$

$$= \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)gi}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.80

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = \frac{3A^2\log\left(\frac{e(a+bx)}{c+dx}\right)+3AB\log^2\left(\frac{e(a+bx)}{c+dx}\right)+B^2\log^3\left(\frac{e(a+bx)}{c+dx}\right)}{3bcgi-3adgi}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)*(c*i + d*i*
x)),x]
```

```
[Out] (3*A^2*Log[(e*(a + b*x))/(c + d*x)] + 3*A*B*Log[(e*(a + b*x))/(c + d*x)]^2
+ B^2*Log[(e*(a + b*x))/(c + d*x)]^3)/(3*b*c*g*i - 3*a*d*g*i)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(42) = 84$.

Time = 0.66 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.41

method	result	size
parallelrisch	$-\frac{B^2 \ln\left(\frac{e(bx+a)}{dx+c}\right)^3 b^2 d^2+3AB \ln\left(\frac{e(bx+a)}{dx+c}\right)^2 b^2 d^2+3A^2 \ln\left(\frac{e(bx+a)}{dx+c}\right) b^2 d^2}{3b^2 d^2 gi(ad-cb)}$	106
norman	$-\frac{B^2 \ln\left(\frac{e(bx+a)}{dx+c}\right)^3}{3gi(ad-cb)}-\frac{A^2 \ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi(ad-cb)}-\frac{BA \ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{gi(ad-cb)}$	113
parts	$\frac{A^2\left(\frac{\ln(dx+c)}{ad-cb}-\frac{\ln(bx+a)}{ad-cb}\right)}{gi}-\frac{B^2 \ln\left(\frac{e(bx+a)}{dx+c}\right)^3}{3gi(ad-cb)}-\frac{BA \ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{gi(ad-cb)}$	123
risch	$\frac{A^2 \ln(dx+c)}{gi(ad-cb)}-\frac{A^2 \ln(bx+a)}{gi(ad-cb)}-\frac{B^2 \ln\left(\frac{e(bx+a)}{dx+c}\right)^3}{3gi(ad-cb)}-\frac{BA \ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{gi(ad-cb)}$	130
derivativedivides	$-\frac{e(ad-cb)\left(\frac{d^2 A^2 \ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei(ad-cb)^2 g}+\frac{d^2 AB \ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{ei(ad-cb)^2 g}+\frac{d^2 B^2 \ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}{3ei(ad-cb)^2 g}\right)}{d^2}$	182
default	$-\frac{e(ad-cb)\left(\frac{d^2 A^2 \ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei(ad-cb)^2 g}+\frac{d^2 AB \ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{ei(ad-cb)^2 g}+\frac{d^2 B^2 \ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}{3ei(ad-cb)^2 g}\right)}{d^2}$	182

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i),x,method=_RETURNV
ERBOSE)
```

```
[Out] -1/3*(B^2*ln(e*(b*x+a)/(d*x+c))^3*b^2*d^2+3*A*B*ln(e*(b*x+a)/(d*x+c))^2*b^2
*d^2+3*A^2*ln(e*(b*x+a)/(d*x+c))*b^2*d^2)/b^2/d^2/g/i/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(42) = 84$.

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.98

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx=\frac{B^2\log\left(\frac{bex+ae}{dx+c}\right)^3+3\,AB\log\left(\frac{bex+ae}{dx+c}\right)^2+3\,A^2\log\left(\frac{bex+ae}{dx+c}\right)}{3\,(bc-ad)gi}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i),x, algorit
hm="fricas")
```

```
[Out] 1/3*(B^2*log((b*e*x + a*e)/(d*x + c))^3 + 3*A*B*log((b*e*x + a*e)/(d*x + c)
)^2 + 3*A^2*log((b*e*x + a*e)/(d*x + c)))/((b*c - a*d)*g*i)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(31) = 62$.

Time = 0.38 (sec) , antiderivative size = 206, normalized size of antiderivative = 4.68

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = A^2\left(\frac{\log\left(x+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{gi\,(ad-bc)} - \frac{\log\left(x+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{gi\,(ad-bc)}\right) - \frac{AB\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{adgi-bcgi} - \frac{B^2\log\left(\frac{e(a+bx)}{c+dx}\right)^3}{3adgi-3bcgi}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)/(d*i*x+c*i),x)
```

```
[Out] A**2*(log(x + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*
c**2/(a*d - b*c) + b*c)/(2*b*d))/(g*i*(a*d - b*c)) - log(x + (a**2*d**2/(a*
d - b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*
d))/(g*i*(a*d - b*c))) - A*B*log(e*(a + b*x)/(c + d*x))**2/(a*d*g*i - b*c*g
*i) - B**2*log(e*(a + b*x)/(c + d*x))**3/(3*a*d*g*i - 3*b*c*g*i)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(42) = 84$.

Time = 0.22 (sec) , antiderivative size = 397, normalized size of antiderivative = 9.02

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = B^2\left(\frac{\log(bx+a)}{(bc-ad)gi} - \frac{\log(dx+c)}{(bc-ad)gi}\right)\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)^2 + 2\,AB\left(\frac{\log(bx+a)}{(bc-ad)gi} - \frac{\log(dx+c)}{(bc-ad)gi}\right)\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right) - \frac{1}{3}B^2\left(\frac{3\left(\log(bx+a)^2-2\log(bx+a)\log(dx+c)+\log(dx+c)^2\right)\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{bcgi-adgi} - \frac{\log(bx+a)^3 \cdots}{\cdots}\right) + A^2\left(\frac{\log(bx+a)}{(bc-ad)gi} - \frac{\log(dx+c)}{(bc-ad)gi}\right) - \frac{\left(\log(bx+a)^2-2\log(bx+a)\log(dx+c)+\log(dx+c)^2\right)AB}{bcgi-adgi}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i),x, algorit
hm="maxima")
```

```
[Out] B^2*(log(b*x + a)/((b*c - a*d)*g*i) - log(d*x + c)/((b*c - a*d)*g*i))*log(b
*e*x/(d*x + c) + a*e/(d*x + c))^2 + 2*A*B*(log(b*x + a)/((b*c - a*d)*g*i) -
 log(d*x + c)/((b*c - a*d)*g*i))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 1/3
*B^2*(3*(log(b*x + a)^2 - 2*log(b*x + a)*log(d*x + c) + log(d*x + c)^2)*log
(b*e*x/(d*x + c) + a*e/(d*x + c))/(b*c*g*i - a*d*g*i) - (log(b*x + a)^3 - 3
*log(b*x + a)^2*log(d*x + c) + 3*log(b*x + a)*log(d*x + c)^2 - log(d*x + c)
^3)/(b*c*g*i - a*d*g*i)) + A^2*(log(b*x + a)/((b*c - a*d)*g*i) - log(d*x +
c)/((b*c - a*d)*g*i)) - (log(b*x + a)^2 - 2*log(b*x + a)*log(d*x + c) + log
(d*x + c)^2)*A*B/(b*c*g*i - a*d*g*i)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(42) = 84$.

Time = 0.47 (sec) , antiderivative size = 132, normalized size of antiderivative = 3.00

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = \frac{\left(B^2e\log\left(\frac{bex+ae}{dx+c}\right)^3+3\,ABe\log\left(\frac{bex+ae}{dx+c}\right)^2+3\,A^2e\log\left(\frac{bex+ae}{dx+c}\right)\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)}{3\,gi}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i),x, algorit
hm="giac")
```

```
[Out] 1/3*(B^2*e*log((b*e*x + a*e)/(d*x + c))^3 + 3*A*B*e*log((b*e*x + a*e)/(d*x
+ c))^2 + 3*A^2*e*log((b*e*x + a*e)/(d*x + c)))*(b*c/((b*c*e - a*d*e)*(b*c
- a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/(g*i)
```

Mupad [B] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.18

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = -\frac{-6\mathrm{i}\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,A^2+3\,A\,B\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2+B^2\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^3}{3\,g\,i\,(a\,d-b\,c)}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)*(c*i + d*i*x)),x)
```

```
[Out] -(B^2*log((e*(a + b*x))/(c + d*x))^3 - A^2*atan((a*d*1i + b*c*1i + b*d*x*2i
)/(a*d - b*c))*6i + 3*A*B*log((e*(a + b*x))/(c + d*x))^2)/(3*g*i*(a*d - b*c
))
```

3.89 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 183

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{2bB^2(c+dx)}{(bc-ad)^2g^2i(a+bx)} - \frac{2bB(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^2i(a+bx)} - \frac{b(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^2g^2i(a+bx)} - \frac{d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^2g^2i}$$

```
[Out] -2*b*B^2*(d*x+c)/(-a*d+b*c)^2/g^2/i/(b*x+a)-2*b*B*(d*x+c)*(A+B*ln(e*(b*x+a)
/(d*x+c)))/(-a*d+b*c)^2/g^2/i/(b*x+a)-b*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))
^2/(-a*d+b*c)^2/g^2/i/(b*x+a)-1/3*d*(A+B*ln(e*(b*x+a)/(d*x+c)))^3/B/(-a*d+b
*c)^2/g^2/i
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2562, 2395, 2342, 2341, 2339, 30}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{d\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^3}{3Bg^2i(bc-ad)^2} - \frac{b(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^2i(a+bx)(bc-ad)^2} - \frac{2bB(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2i(a+bx)(bc-ad)^2} - \frac{2bB^2(c+dx)}{g^2i(a+bx)(bc-ad)^2}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^2*(c*i + d*i*x)),
x]
```

```
[Out] (-2*b*B^2*(c + d*x))/((b*c - a*d)^2*g^2*i*(a + b*x)) - (2*b*B*(c + d*x)*(A
+ B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^2*g^2*i*(a + b*x)) - (b*(c
+ d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*c - a*d)^2*g^2*i*(a + b*
x)) - (d*(A + B*Log[(e*(a + b*x))/(c + d*x)])^3)/(3*B*(b*c - a*d)^2*g^2*i)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
```

```
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^2i} \\
&= \frac{\text{Subst}\left(\int \left(\frac{b(A+B\log(ex))^2}{x^2}-\frac{d(A+B\log(ex))^2}{x}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^2i} \\
&= \frac{b\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^2i}-\frac{d\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^2i} \\
&= -\frac{b(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^2g^2i(a+bx)}+\frac{(2bB)\text{Subst}\left(\int \frac{A+B\log(ex)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^2i} \\
&\quad -\frac{d\text{Subst}\left(\int x^2\,dx, x, A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B(bc-ad)^2g^2i} \\
&= -\frac{2bB^2(c+dx)}{(bc-ad)^2g^2i(a+bx)}-\frac{2bB(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^2i(a+bx)} \\
&\quad -\frac{b(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^2g^2i(a+bx)}-\frac{d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^2g^2i}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.02

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{3(A^2+2AB+2B^2)\,d(a+bx)\log(a+bx)+6B(A+B)(bc-ad)\log\left(\frac{e(a+bx)}{c+dx}\right)+3B(aAd+Abdx+bB}{3(bc-a}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^2*(c*i + d*
i*x)),x]
```

```
[Out] -1/3*(3*(A^2 + 2*A*B + 2*B^2)*d*(a + b*x)*Log[a + b*x] + 6*B*(A + B)*(b*c -
 a*d)*Log[(e*(a + b*x))/(c + d*x)] + 3*B*(a*A*d + A*b*d*x + b*B*(c + d*x))*
Log[(e*(a + b*x))/(c + d*x)]^2 + B^2*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x
)]^3 + 3*(A^2 + 2*A*B + 2*B^2)*(b*c - a*d - d*(a + b*x)*Log[c + d*x]))/((b*
c - a*d)^2*g^2*i*(a + b*x))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(181) = 362.

Time = 0.98 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.19

method	result
norman	$-\frac{-\frac{\left(A^2ad+2ABbc+2B^2bc\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{B(Aad+Bbc)\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{gi\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{B^2ad\ln\left(\frac{e(bx+a)}{dx+c}\right)^3}{3gi\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{b\left(A^2d+2ABd+2B^2d\right)x\ln\left(\frac{e(}{}\right.}{gi\left(a^2d^2-2abcd+b^2c^2\right.}}{g(bx+a)}$
parts	$\frac{A^2\left(\frac{d\ln(dx+c)}{(ad-cb)^2}+\frac{1}{(bx+a)(ad-cb)}-\frac{d\ln(bx+a)}{(ad-cb)^2}\right)}{g^2i}-\frac{B^2\left(\frac{d^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}{3(ad-cb)^2}-\frac{dbe\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{2\ln\left(\frac{be}{d}+\frac{(ad-}{d(dx+}\right.}{\frac{be}{d}+\frac{(ad-cb}{d(dx+c}}\right.}{(ad-cb)^2}\right.}{g^2id}$
parallelrisch	$-\frac{3ABx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2a^3bc^2d+6ABx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^3bc^2d+B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^3a^4c^2d+3B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2a^3bc^3-3A^2xa}{}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{d^2A^2b}{i(ad-cb)^3g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^3A^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei(ad-cb)^3g^2}-\frac{2d^2ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{i(ad-cb)^3g^2}+\frac{d^3}{}\right.}{}$
default	$-\frac{e(ad-cb)\left(\frac{d^2A^2b}{i(ad-cb)^3g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^3A^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei(ad-cb)^3g^2}-\frac{2d^2ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{i(ad-cb)^3g^2}+\frac{d^3}{}\right.}{}$
risch	$\frac{A^2d\ln(dx+c)}{g^2i(ad-cb)^2}+\frac{A^2}{g^2i(bx+a)(ad-cb)}-\frac{A^2d\ln(bx+a)}{g^2i(ad-cb)^2}-\frac{B^2d\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}{3g^2i(ad-cb)^2}-\frac{B^2be\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{g^2i(ad-cb)^2\left(\frac{be}{d}+\frac{ea}{dx+c}-\frac{ec}{d(dx}\right.}$

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x,method=_RETUR
NVERBOSE)
```

```
[Out] (-(A^2*a*d+2*A*B*b*c+2*B^2*b*c)/g/i/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x+a
)/(d*x+c))-B*(A*a*d+B*b*c)/g/i/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x+a)/(d*
x+c))^2-1/3*B^2*a*d/g/i/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x+a)/(d*x+c))^3
-1/g/i*b*(A^2*d+2*A*B*d+2*B^2*d)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)
/(d*x+c))-(A^2+2*A*B+2*B^2)*b/g/i/a/(a*d-b*c)*x-1/3*b*B^2*d/g/i/(a^2*d^2-2*
a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)/(d*x+c))^3-B*b*d*(A+B)/g/i/(a^2*d^2-2*a*b*c
*d+b^2*c^2)*x*ln(e*(b*x+a)/(d*x+c))^2)/g/(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.26

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{(B^2bdx+B^2ad)\log\left(\frac{bex+ae}{dx+c}\right)^3+3\,(A^2+2\,AB+2\,B^2)bc-3\,(A^2+2\,AB+2\,B^2)ad+3\,(B^2bc+ABad+\ldots}{3\,((b^3c^2-2\,ab^2cd+a^2bd^2)g^2ix+\ldots}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x, algor
ithm="fricas")
```

```
[Out] -1/3*((B^2*b*d*x + B^2*a*d)*log((b*e*x + a*e)/(d*x + c))^3 + 3*(A^2 + 2*A*B
 + 2*B^2)*b*c - 3*(A^2 + 2*A*B + 2*B^2)*a*d + 3*(B^2*b*c + A*B*a*d + (A*B +
 B^2)*b*d*x)*log((b*e*x + a*e)/(d*x + c))^2 + 3*(A^2*a*d + (A^2 + 2*A*B + 2
*B^2)*b*d*x + 2*(A*B + B^2)*b*c)*log((b*e*x + a*e)/(d*x + c)))/((b^3*c^2 -
2*a*b^2*c*d + a^2*b*d^2)*g^2*i*x + (a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*g^2*
i)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(158) = 316$.

Time = 0.74 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.96

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$$
$$= -\frac{B^2d\log\left(\frac{e(a+bx)}{c+dx}\right)^3}{3a^2d^2g^2i-6abcdg^2i+3b^2c^2g^2i}+\frac{(2AB+2B^2)\log\left(\frac{e(a+bx)}{c+dx}\right)}{a^2dg^2i-abcg^2i+abdg^2ix-b^2cg^2ix}$$
$$+\left(A^2+2AB+2B^2\right)\left(\frac{d\log\left(x+\frac{-\frac{a^3d^4}{(ad-bc)^2}+\frac{3a^2bcd^3}{(ad-bc)^2}-\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2+\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{g^2i\,(ad-bc)^2}\right.$$
$$-\frac{d\log\left(x+\frac{\frac{a^3d^4}{(ad-bc)^2}-\frac{3a^2bcd^3}{(ad-bc)^2}+\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2-\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{g^2i\,(ad-bc)^2}$$
$$\left.+\frac{1}{a^2dg^2i-abcg^2i+x\,(abdg^2i-b^2cg^2i)}\right)$$
$$+\frac{(-ABad-ABbdx-B^2bc-B^2bdx)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{a^3d^2g^2i-2a^2bcdg^2i+a^2bd^2g^2ix+ab^2c^2g^2i-2ab^2cdg^2ix+b^3c^2g^2ix}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2/(d*i*x+c*i),x)
```

```
[Out] -B**2*d*log(e*(a + b*x)/(c + d*x))**3/(3*a**2*d**2*g**2*i - 6*a*b*c*d*g**2*
i + 3*b**2*c**2*g**2*i) + (2*A*B + 2*B**2)*log(e*(a + b*x)/(c + d*x))/(a**2
*d*g**2*i - a*b*c*g**2*i + a*b*d*g**2*i*x - b**2*c*g**2*i*x) + (A**2 + 2*A*
B + 2*B**2)*(d*log(x + (-a**3*d**4/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*d -
b*c)**2 - 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 + b**3*c**3*d/(a*d - b
*c)**2 + b*c*d)/(2*b*d**2))/(g**2*i*(a*d - b*c)**2) - d*log(x + (a**3*d**4/
(a*d - b*c)**2 - 3*a**2*b*c*d**3/(a*d - b*c)**2 + 3*a*b**2*c**2*d**2/(a*d -
 b*c)**2 + a*d**2 - b**3*c**3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(g**2*i
*(a*d - b*c)**2) + 1/(a**2*d*g**2*i - a*b*c*g**2*i + x*(a*b*d*g**2*i - b**2
*c*g**2*i))) + (-A*B*a*d - A*B*b*d*x - B**2*b*c - B**2*b*d*x)*log(e*(a + b*
x)/(c + d*x))**2/(a**3*d**2*g**2*i - 2*a**2*b*c*d*g**2*i + a**2*b*d**2*g**2
*i*x + a*b**2*c**2*g**2*i - 2*a*b**2*c*d*g**2*i*x + b**3*c**2*g**2*i*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1008 vs. $2(181) = 362$.

Time = 0.26 (sec) , antiderivative size = 1008, normalized size of antiderivative = 5.51

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx =$$

$$-B^2\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i}+\frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}-\frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)\log\Bigg(\frac{\cdots}{dx}\cdots+\frac{ae}{dx+c}\Bigg)^2$$

$$-2\,AB\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i}+\frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}-\frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)\log\Bigg(\cdots+\frac{ae}{dx+c}\Bigg)$$

$$+\frac{1}{3}B^2\left(\frac{3\left((bdx+ad)\log(bx+a)^2+(bdx+ad)\log(dx+c)^2-2\,bc+2\,ad-2\,(bdx+ad)\log(bx+a)+\cdots\right.}{ab^2c^2g^2i-2\,a^2bcdg^2i+a^3d^2g^2i+(b^3c^2g^2i-2\,a\cdots}\right.$$

$$-A^2\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i}+\frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}-\frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)$$

$$+\frac{\left((bdx+ad)\log(bx+a)^2+(bdx+ad)\log(dx+c)^2-2\,bc+2\,ad-2\,(bdx+ad)\log(bx+a)+2\,(bdx+\cdots\right.}{ab^2c^2g^2i-2\,a^2bcdg^2i+a^3d^2g^2i+(b^3c^2g^2i-2\,ab^2cdg^2i+a^2bd^2\cdots}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x, algor
ithm="maxima")
```

```
[Out] -B^2*(1/((b^2*c - a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/
((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*g^2*i))*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2 - 2*A*B*(1/((
b^2*c - a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/((b^2*c^2
- 2*a*b*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)*g^2*i))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 1/3*B^2*(3*((b*d*x + a*
d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d - 2*(b*d*x
 + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a))*log(d*x
 + c))*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(a*b^2*c^2*g^2*i - 2*a^2*b*c*d*
g^2*i + a^3*d^2*g^2*i + (b^3*c^2*g^2*i - 2*a*b^2*c*d*g^2*i + a^2*b*d^2*g^2*
i)*x) - ((b*d*x + a*d)*log(b*x + a)^3 - (b*d*x + a*d)*log(d*x + c)^3 - 3*(b
*d*x + a*d)*log(b*x + a)^2 - 3*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a))*l
og(d*x + c)^2 + 6*b*c - 6*a*d + 6*(b*d*x + a*d)*log(b*x + a) - 3*(2*b*d*x +
 (b*d*x + a*d)*log(b*x + a)^2 + 2*a*d - 2*(b*d*x + a*d)*log(b*x + a))*log(d
*x + c))/(a*b^2*c^2*g^2*i - 2*a^2*b*c*d*g^2*i + a^3*d^2*g^2*i + (b^3*c^2*g^
2*i - 2*a*b^2*c*d*g^2*i + a^2*b*d^2*g^2*i)*x)) - A^2*(1/((b^2*c - a*b*d)*g^
2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/((b^2*c^2 - 2*a*b*c*d + a^2
```

```
*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g^2*i)) + ((
b*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d
- 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a
))*log(d*x + c))*A*B/(a*b^2*c^2*g^2*i - 2*a^2*b*c*d*g^2*i + a^3*d^2*g^2*i +
 (b^3*c^2*g^2*i - 2*a*b^2*c*d*g^2*i + a^2*b*d^2*g^2*i)*x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2(dix+ci)}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x, algor
ithm="giac")
```

```
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/((b*g*x + a*g)^2*(d*i*x + c*
i)), x)
```

Mupad [B] (verification not implemented)

Time = 2.97 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.29

$$\begin{aligned}
&\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx\\
&=\frac{A^2+2\,A\,B+2\,B^2}{(a\,d-b\,c)\,\left(a\,g^2\,i+b\,g^2\,i\,x\right)}-\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{B\,d\,(A+B)}{g^2\,i\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right.\\
&\qquad\left.-\frac{B^2\,(a\,d-b\,c)}{b\,d\,g^2\,i\,\left(\frac{x}{d}+\frac{a}{b\,d}\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)\\
&\quad-\frac{B^2\,d\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^3}{3\,g^2\,i\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{2\,B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,(a\,d-b\,c)\,(A+B)}{b\,d\,g^2\,i\,\left(\frac{x}{d}+\frac{a}{b\,d}\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\\
&\quad+\frac{d\,\mathrm{atan}\left(\frac{d\left(2\,b\,d\,x+\frac{a^2\,d^2\,g^2\,i-b^2\,c^2\,g^2\,i}{g^2\,i\,(a\,d-b\,c)}\right)\,\left(A^2+2\,A\,B+2\,B^2\right)\,1\mathrm{i}}{(a\,d-b\,c)\,\left(d\,A^2+2\,d\,A\,B+2\,d\,B^2\right)}\right)\,\left(A^2+2\,A\,B+2\,B^2\right)\,2\mathrm{i}}{g^2\,i\,(a\,d-b\,c)^2}
\end{aligned}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^2*(c*i + d*i*x)),
x)
```

```
[Out] (A^2 + 2*B^2 + 2*A*B)/((a*d - b*c)*(a*g^2*i + b*g^2*i*x)) - log((e*(a + b*x
))/(c + d*x))^2*((B*d*(A + B))/(g^2*i*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (B
^2*(a*d - b*c))/(b*d*g^2*i*(x/d + a/(b*d))*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
```

```
) - (B^2*d*log((e*(a + b*x))/(c + d*x))^3)/(3*g^2*i*(a^2*d^2 + b^2*c^2 - 2*
a*b*c*d)) + (d*atan((d*(2*b*d*x + (a^2*d^2*g^2*i - b^2*c^2*g^2*i)/(g^2*i*(a
*d - b*c)))*(A^2 + 2*B^2 + 2*A*B)*1i)/((a*d - b*c)*(A^2*d + 2*B^2*d + 2*A*B
*d)))*(A^2 + 2*B^2 + 2*A*B)*2i)/(g^2*i*(a*d - b*c)^2) + (2*B*log((e*(a + b*
x))/(c + d*x))*(a*d - b*c)*(A + B))/(b*d*g^2*i*(x/d + a/(b*d))*(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d))
```

3.90 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 343

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = \frac{4bB^2d(c+dx)}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2B^2(c+dx)^2}{4(bc-ad)^3g^3i(a+bx)^2} + \frac{4bBd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2B(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2} + \frac{2bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^3g^3i(a+bx)^2} + \frac{d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^3g^3i}$$

```
[Out] 4*b*B^2*d*(d*x+c)/(-a*d+b*c)^3/g^3/i/(b*x+a)-1/4*b^2*B^2*(d*x+c)^2/(-a*d+b*
c)^3/g^3/i/(b*x+a)^2+4*b*B*d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)
^3/g^3/i/(b*x+a)-1/2*b^2*B*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)
^3/g^3/i/(b*x+a)^2+2*b*d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3
/g^3/i/(b*x+a)-1/2*b^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3
/g^3/i/(b*x+a)^2+1/3*d^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^3/B/(-a*d+b*c)^3/g^3/i
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2562, 2395, 2342, 2341, 2339, 30}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = -\frac{b^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g^3i(a+bx)^2(bc-ad)^3} - \frac{b^2B(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^3i(a+bx)^2(bc-ad)^3} + \frac{d^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^3}{3Bg^3i(bc-ad)^3} + \frac{2bd(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^3i(a+bx)(bc-ad)^3} + \frac{4bBd(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^3i(a+bx)(bc-ad)^3} - \frac{b^2B^2(c+dx)^2}{4g^3i(a+bx)^2(bc-ad)^3} + \frac{4bB^2d(c+dx)}{g^3i(a+bx)(bc-ad)^3}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^3*(c*i + d*i*x)),
x]
```

```
[Out] (4*b*B^2*d*(c + d*x))/((b*c - a*d)^3*g^3*i*(a + b*x)) - (b^2*B^2*(c + d*x)^
2)/(4*(b*c - a*d)^3*g^3*i*(a + b*x)^2) + (4*b*B*d*(c + d*x)*(A + B*Log[(e*(
a + b*x))/(c + d*x)]))/((b*c - a*d)^3*g^3*i*(a + b*x)) - (b^2*B*(c + d*x)^2
*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*c - a*d)^3*g^3*i*(a + b*x)^2)
+ (2*b*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*c - a*d)^3*g
^3*i*(a + b*x)) - (b^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/
(2*(b*c - a*d)^3*g^3*i*(a + b*x)^2) + (d^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)])^3)/(3*B*(b*c - a*d)^3*g^3*i)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i}\\
&= \frac{\text{Subst}\left(\int \left(\frac{b^2(A+B\log(ex))^2}{x^3} - \frac{2bd(A+B\log(ex))^2}{x^2} + \frac{d^2(A+B\log(ex))^2}{x}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i}\\
&= \frac{b^2\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i} - \frac{(2bd)\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i}\\
&\quad + \frac{d^2\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i}
\end{aligned}
$$

$$= \frac{2bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^3g^3i(a+bx)^2} + \frac{(b^2B)\operatorname{Subst}\left(\int\frac{A+B\log(ex)}{x^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i} - \frac{(4bBd)\operatorname{Subst}\left(\int\frac{A+B\log(ex)}{x^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i} + \frac{d^2\operatorname{Subst}\left(\int x^2\,dx,x,A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B(bc-ad)^3g^3i}$$

$$= \frac{4bB^2d(c+dx)}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2B^2(c+dx)^2}{4(bc-ad)^3g^3i(a+bx)^2} + \frac{4bBd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2B(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2} + \frac{2bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^3g^3i(a+bx)^2} + \frac{d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^3g^3i}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.93

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = \frac{-3(2A^2+2AB+B^2)(bc-ad)^2+6(2A^2+6AB+7B^2)d(bc-ad)(a+bx)+6(2A^2+6AB+7B^2)d^2(a}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^3*(c*i + d*
i*x)),x]
```

```
[Out] (-3*(2*A^2 + 2*A*B + B^2)*(b*c - a*d)^2 + 6*(2*A^2 + 6*A*B + 7*B^2)*d*(b*c
- a*d)*(a + b*x) + 6*(2*A^2 + 6*A*B + 7*B^2)*d^2*(a + b*x)^2*Log[a + b*x] -
 6*B*(b*c - a*d)*(-6*a*A*d - 7*a*B*d + b*B*(c - 6*d*x) + 2*A*b*(c - 2*d*x))
*Log[(e*(a + b*x))/(c + d*x)] - 6*B*(-2*a^2*A*d^2 - 4*a*b*d*(A*d*x + B*(c +
 d*x)) + b^2*(-2*A*d^2*x^2 + B*(c^2 - 2*c*d*x - 3*d^2*x^2)))*Log[(e*(a + b*
x))/(c + d*x)]^2 + 4*B^2*d^2*(a + b*x)^2*Log[(e*(a + b*x))/(c + d*x)]^3 - 6
*(2*A^2 + 6*A*B + 7*B^2)*d^2*(a + b*x)^2*Log[c + d*x])/(12*(b*c - a*d)^3*g^
3*i*(a + b*x)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 766 vs. $2(335) = 670$.

Time = 1.31 (sec) , antiderivative size = 767, normalized size of antiderivative = 2.24

method	result
parts	$\frac{A^2\left(\frac{d^2\ln(dx+c)}{(ad-cb)^3}+\frac{1}{2(ad-cb)(bx+a)^2}+\frac{d}{(ad-cb)^2(bx+a)}-\frac{d^2\ln(bx+a)}{(ad-cb)^3}\right)}{g^3 i}-\frac{B^2\left(\frac{d^3\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}{3(ad-cb)^3}-\frac{2d^2be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad}{d(}\right.}{\frac{be}{d}+\frac{(ad}{d(}}\right.}{}\right.}{}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^2eA^2b^2}{2i(ad-cb)^4g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{2d^3A^2b}{i(ad-cb)^4g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^4A^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei(ad-cb)^4g^3}+\frac{2d^2eABb^2\left(-\frac{\ln(}{2(}\right.}{}\right.}{}$
default	$-\frac{e(ad-cb)\left(-\frac{d^2eA^2b^2}{2i(ad-cb)^4g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{2d^3A^2b}{i(ad-cb)^4g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^4A^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei(ad-cb)^4g^3}+\frac{2d^2eABb^2\left(-\frac{\ln(}{2(}\right.}{}\right.}{}$
norman	$\frac{6A^2ab^2d-2A^2b^3c+14ABab^2d-2ABb^3c+15B^2ab^2d-B^2b^3c}{4gib^2\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{\left(2A^2a^2d^2+8ABabcd-2ABb^2c^2+8B^2abcd-B^2b^2c^2\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2ig\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}$
risch	$\frac{A^2d^2\ln(dx+c)}{g^3i(ad-cb)^3}+\frac{A^2}{2g^3i(ad-cb)(bx+a)^2}+\frac{A^2d}{g^3i(ad-cb)^2(bx+a)}-\frac{A^2d^2\ln(bx+a)}{g^3i(ad-cb)^3}-\frac{B^2d^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}{3g^3i(ad-cb)^3}-$
parallelrisch	Expression too large to display

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x,method=_RETUR
NVERBOSE)
```

```
[Out] A^2/g^3/i*(d^2/(a*d-b*c)^3*ln(d*x+c)+1/2/(a*d-b*c)/(b*x+a)^2+d/(a*d-b*c)^2/
(b*x+a)-d^2/(a*d-b*c)^3*ln(b*x+a))-B^2/g^3/i/d*(1/3*d^3/(a*d-b*c)^3*ln(b*e/
d+(a*d-b*c)*e/d/(d*x+c))^3-2*d^2/(a*d-b*c)^3*b*e*(-1/(b*e/d+(a*d-b*c)*e/d/(
d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+d/(a*d-b*c)
^3*e^2*b^2*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*
x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))
-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))-2*B*A/g^3/i/d*(1/2*d^3/(a*d-b*c)^3*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2*d^2/(a*d-b*c)^3*b*e*(-1/(b*e/d+(a*d-b*c)
*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c
)))+d/(a*d-b*c)^3*e^2*b^2*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a
*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.57

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = -\frac{3\,(2\,A^2+2\,AB+B^2)b^2c^2-24\,(A^2+2\,AB+2\,B^2)abcd+3\,(6\,A^2+14\,AB+15\,B^2)a^2d^2-4\,(B^2b^2d^2x^2}{}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x, algor
ithm="fricas")
```

```
[Out] -1/12*(3*(2*A^2 + 2*A*B + B^2)*b^2*c^2 - 24*(A^2 + 2*A*B + 2*B^2)*a*b*c*d +
 3*(6*A^2 + 14*A*B + 15*B^2)*a^2*d^2 - 4*(B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x
 + B^2*a^2*d^2)*log((b*e*x + a*e)/(d*x + c))^3 - 6*((2*A*B + 3*B^2)*b^2*d^2
*x^2 - B^2*b^2*c^2 + 4*B^2*a*b*c*d + 2*A*B*a^2*d^2 + 2*(B^2*b^2*c*d + 2*(A*
B + B^2)*a*b*d^2)*x)*log((b*e*x + a*e)/(d*x + c))^2 - 6*((2*A^2 + 6*A*B + 7
*B^2)*b^2*c*d - (2*A^2 + 6*A*B + 7*B^2)*a*b*d^2)*x - 6*((2*A^2 + 6*A*B + 7*
B^2)*b^2*d^2*x^2 + 2*A^2*a^2*d^2 - (2*A*B + B^2)*b^2*c^2 + 8*(A*B + B^2)*a*
b*c*d + 2*((2*A*B + 3*B^2)*b^2*c*d + 2*(A^2 + 2*A*B + 2*B^2)*a*b*d^2)*x)*lo
g((b*e*x + a*e)/(d*x + c)))/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a
^3*b^2*d^3)*g^3*i*x^2 + 2*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 -
a^4*b*d^3)*g^3*i*x + (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d
^3)*g^3*i)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1488 vs. 2(303) = 606.

Time = 4.24 (sec) , antiderivative size = 1488, normalized size of antiderivative = 4.34

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3/(d*i*x+c*i),x)
```

```
[Out] -B**2*d**2*log(e*(a + b*x)/(c + d*x))**3/(3*a**3*d**3*g**3*i - 9*a**2*b*c*d
**2*g**3*i + 9*a*b**2*c**2*d*g**3*i - 3*b**3*c**3*g**3*i) + d**2*(2*A**2 +
6*A*B + 7*B**2)*log(x + (2*A**2*a*d**3 + 2*A**2*b*c*d**2 + 6*A*B*a*d**3 + 6
*A*B*b*c*d**2 + 7*B**2*a*d**3 + 7*B**2*b*c*d**2 - a**4*d**6*(2*A**2 + 6*A*B
 + 7*B**2)/(a*d - b*c)**3 + 4*a**3*b*c*d**5*(2*A**2 + 6*A*B + 7*B**2)/(a*d
- b*c)**3 - 6*a**2*b**2*c**2*d**4*(2*A**2 + 6*A*B + 7*B**2)/(a*d - b*c)**3
```

```
+ 4*a*b**3*c**3*d**3*(2*A**2 + 6*A*B + 7*B**2)/(a*d - b*c)**3 - b**4*c**4*d
**2*(2*A**2 + 6*A*B + 7*B**2)/(a*d - b*c)**3)/(4*A**2*b*d**3 + 12*A*B*b*d**
3 + 14*B**2*b*d**3))/(2*g**3*i*(a*d - b*c)**3) - d**2*(2*A**2 + 6*A*B + 7*B
**2)*log(x + (2*A**2*a*d**3 + 2*A**2*b*c*d**2 + 6*A*B*a*d**3 + 6*A*B*b*c*d*
*2 + 7*B**2*a*d**3 + 7*B**2*b*c*d**2 + a**4*d**6*(2*A**2 + 6*A*B + 7*B**2)/
(a*d - b*c)**3 - 4*a**3*b*c*d**5*(2*A**2 + 6*A*B + 7*B**2)/(a*d - b*c)**3 +
 6*a**2*b**2*c**2*d**4*(2*A**2 + 6*A*B + 7*B**2)/(a*d - b*c)**3 - 4*a*b**3*
c**3*d**3*(2*A**2 + 6*A*B + 7*B**2)/(a*d - b*c)**3 + b**4*c**4*d**2*(2*A**2
 + 6*A*B + 7*B**2)/(a*d - b*c)**3)/(4*A**2*b*d**3 + 12*A*B*b*d**3 + 14*B**2
*b*d**3))/(2*g**3*i*(a*d - b*c)**3) + (6*A*B*a*d - 2*A*B*b*c + 4*A*B*b*d*x
+ 7*B**2*a*d - B**2*b*c + 6*B**2*b*d*x)*log(e*(a + b*x)/(c + d*x))/(2*a**4*
d**2*g**3*i - 4*a**3*b*c*d*g**3*i + 4*a**3*b*d**2*g**3*i*x + 2*a**2*b**2*c*
*2*g**3*i - 8*a**2*b**2*c*d*g**3*i*x + 2*a**2*b**2*d**2*g**3*i*x**2 + 4*a*b
**3*c**2*g**3*i*x - 4*a*b**3*c*d*g**3*i*x**2 + 2*b**4*c**2*g**3*i*x**2) + (
-2*A*B*a**2*d**2 - 4*A*B*a*b*d**2*x - 2*A*B*b**2*d**2*x**2 - 4*B**2*a*b*c*d
 - 4*B**2*a*b*d**2*x + B**2*b**2*c**2 - 2*B**2*b**2*c*d*x - 3*B**2*b**2*d**
2*x**2)*log(e*(a + b*x)/(c + d*x))**2/(2*a**5*d**3*g**3*i - 6*a**4*b*c*d**2
*g**3*i + 4*a**4*b*d**3*g**3*i*x + 6*a**3*b**2*c**2*d*g**3*i - 12*a**3*b**2
*c*d**2*g**3*i*x + 2*a**3*b**2*d**3*g**3*i*x**2 - 2*a**2*b**3*c**3*g**3*i +
 12*a**2*b**3*c**2*d*g**3*i*x - 6*a**2*b**3*c*d**2*g**3*i*x**2 - 4*a*b**4*c
**3*g**3*i*x + 6*a*b**4*c**2*d*g**3*i*x**2 - 2*b**5*c**3*g**3*i*x**2) + (6*
A**2*a*d - 2*A**2*b*c + 14*A*B*a*d - 2*A*B*b*c + 15*B**2*a*d - B**2*b*c + x
*(4*A**2*b*d + 12*A*B*b*d + 14*B**2*b*d))/(4*a**4*d**2*g**3*i - 8*a**3*b*c*
d*g**3*i + 4*a**2*b**2*c**2*g**3*i + x**2*(4*a**2*b**2*d**2*g**3*i - 8*a*b*
*3*c*d*g**3*i + 4*b**4*c**2*g**3*i) + x*(8*a**3*b*d**2*g**3*i - 16*a**2*b**
2*c*d*g**3*i + 8*a*b**3*c**2*g**3*i))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2115 vs. $2(335) = 670$.

Time = 0.35 (sec) , antiderivative size = 2115, normalized size of antiderivative = 6.17

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^3(ci + dix)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x, algor
ithm="maxima")
```

```
[Out] 1/2*B^2*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3
*i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i*x + (a^2*b^2*c^2 -
 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c)/((b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i))*log(b*e*x/(d*x + c) + a*e/(d*x
+ c))^2 + A*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^
```

```
2)*g^3*i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i*x + (a^2*b^2
*c^2 - 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c)/((b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i))*log(b*e*x/(d*x + c) + a*e
/(d*x + c)) - 1/12*B^2*(6*(b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2
 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a
^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d
^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 -
 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))*log(d*x + c))*log(b*
e*x/(d*x + c) + a*e/(d*x + c))/(a^2*b^3*c^3*g^3*i - 3*a^3*b^2*c^2*d*g^3*i +
 3*a^4*b*c*d^2*g^3*i - a^5*d^3*g^3*i + (b^5*c^3*g^3*i - 3*a*b^4*c^2*d*g^3*i
 + 3*a^2*b^3*c*d^2*g^3*i - a^3*b^2*d^3*g^3*i)*x^2 + 2*(a*b^4*c^3*g^3*i - 3*
a^2*b^3*c^2*d*g^3*i + 3*a^3*b^2*c*d^2*g^3*i - a^4*b*d^3*g^3*i)*x) + (3*b^2*
c^2 - 48*a*b*c*d + 45*a^2*d^2 - 4*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log
(b*x + a)^3 + 4*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^3 + 18*(
b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^2 + 6*(3*b^2*d^2*x^2 + 6*
a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a
))*log(d*x + c)^2 - 42*(b^2*c*d - a*b*d^2)*x - 42*(b^2*d^2*x^2 + 2*a*b*d^2*
x + a^2*d^2)*log(b*x + a) + 6*(7*b^2*d^2*x^2 + 14*a*b*d^2*x + 7*a^2*d^2 + 2
*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^2 - 6*(b^2*d^2*x^2 + 2*
a*b*d^2*x + a^2*d^2)*log(b*x + a))*log(d*x + c))/(a^2*b^3*c^3*g^3*i - 3*a^3
*b^2*c^2*d*g^3*i + 3*a^4*b*c*d^2*g^3*i - a^5*d^3*g^3*i + (b^5*c^3*g^3*i - 3
*a*b^4*c^2*d*g^3*i + 3*a^2*b^3*c*d^2*g^3*i - a^3*b^2*d^3*g^3*i)*x^2 + 2*(a*
b^4*c^3*g^3*i - 3*a^2*b^3*c^2*d*g^3*i + 3*a^3*b^2*c*d^2*g^3*i - a^4*b*d^3*g
^3*i)*x)) + 1/2*A^2*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*
b^2*d^2)*g^3*i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i*x + (a
^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c)/((b^3
*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i)) - 1/2*(b^2*c^2 - 8*
a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^
2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a
*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2
*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2
)*log(b*x + a))*log(d*x + c))*A*B/(a^2*b^3*c^3*g^3*i - 3*a^3*b^2*c^2*d*g^3*
i + 3*a^4*b*c*d^2*g^3*i - a^5*d^3*g^3*i + (b^5*c^3*g^3*i - 3*a*b^4*c^2*d*g^
3*i + 3*a^2*b^3*c*d^2*g^3*i - a^3*b^2*d^3*g^3*i)*x^2 + 2*(a*b^4*c^3*g^3*i -
 3*a^2*b^3*c^2*d*g^3*i + 3*a^3*b^2*c*d^2*g^3*i - a^4*b*d^3*g^3*i)*x)
```

Giac [A] (verification not implemented)

none

Time = 58.16 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.62

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = -\frac{1}{4}\left(\frac{2\,(dx+c)^2B^2e^3\log\left(\frac{bex+ae}{dx+c}\right)^2}{(bex+ae)^2g^3i}+\frac{2\,(2\,ABe^3+B^2e^3)(dx+c)^2\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^2g^3i}+\frac{(2\,A^2e^3+2\,ABe^3+B^2}{(bex+ae)^2g}\right.$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x, algor
ithm="giac")
```

```
[Out] -1/4*(2*(d*x + c)^2*B^2*e^3*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a*e)^2
*g^3*i) + 2*(2*A*B*e^3 + B^2*e^3)*(d*x + c)^2*log((b*e*x + a*e)/(d*x + c))/
((b*e*x + a*e)^2*g^3*i) + (2*A^2*e^3 + 2*A*B*e^3 + B^2*e^3)*(d*x + c)^2/((b
*e*x + a*e)^2*g^3*i))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*
d*e)*(b*c - a*d)))^2
```

Mupad [B] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 981, normalized size of antiderivative = 2.86

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx$$
$$= \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{\frac{B^2\,d^2\,\left(\frac{2\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2}{2\,b\,d^3}+\frac{a\,(a\,d-b\,c)}{2\,b\,d^2}\right)}{g^3\,i\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}+\frac{B^2\,x\,(a\,d-b\,c)}{g^3\,i\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}}{\frac{b\,x^2}{d}+\frac{a^2}{b\,d}+\frac{2\,a\,x}{d}} - \frac{B\,d^2\,(2\,A+3\,B)}{2\,g^3\,i\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}\right)$$
$$- \frac{\frac{6\,A^2\,a\,d-2\,A^2\,b\,c+15\,B^2\,a\,d-B^2\,b\,c+14\,A\,B\,a\,d-2\,A\,B\,b\,c}{2\,(a\,d-b\,c)}+\frac{x\,(2\,b\,d\,A^2+6\,b\,d\,A\,B+7\,b\,d\,B^2)}{a\,d-b\,c}}{x^2\,\left(2\,b^3\,c\,g^3\,i-2\,a\,b^2\,d\,g^3\,i\right)+x\,\left(4\,a\,b^2\,c\,g^3\,i-4\,a^2\,b\,d\,g^3\,i\right)-2\,a^3\,d\,g^3\,i+2\,a^2\,b\,c\,g^3\,i}$$
$$+ \frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(\frac{B\,d^2\,\left(\frac{2\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2}{2\,b\,d^3}+\frac{a\,(a\,d-b\,c)}{2\,b\,d^2}\right)(2\,A+3\,B)}{g^3\,i\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}-\frac{B^2}{b\,d\,g^3\,i\,(a\,d-b\,c)}+\frac{B\,x\,(2\,A+3\,B)\,(a\,d-b\,c)}{g^3\,i\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}\right)}{\frac{b\,x^2}{d}+\frac{a^2}{b\,d}+\frac{2\,a\,x}{d}}$$
$$- \frac{B^2\,d^2\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^3}{3\,g^3\,i\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}$$
$$+ \frac{d^2\,\mathrm{atan}\left(\frac{d^2\,\left(A^2+3\,A\,B+\frac{7\,B^2}{2}\right)\left(2\,i\,a^3\,d^3\,g^3-2\,i\,a^2\,b\,c\,d^2\,g^3-2\,i\,a\,b^2\,c^2\,d\,g^3+2\,i\,b^3\,c^3\,g^3\right)\,1\mathrm{i}}{g^3\,i\,(a\,d-b\,c)^3\,(2\,A^2\,d^2+6\,A\,B\,d^2+7\,B^2\,d^2)}+\frac{b\,d^3\,x\,\left(i\,a^2\,d^2\,g^3-2\,i\,a\,b\,c\,d\,g^3+i\,b^2\,c^2\,g^3\right)\left(A^2\right.}{g^3\,i\,(a\,d-b\,c)^3\,(2\,A^2\,d^2+6\,A\,B\,d^2+7}\right.}{g^3\,i\,(a\,d-b\,c)^3}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^3*(c*i + d*i*x)),
x)
```

```
[Out] log((e*(a + b*x))/(c + d*x))^2*(((B^2*d^2*((2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d
)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)))/(g^3*i*(a^3*d^3 - b^3*c^3 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2)) + (B^2*x*(a*d - b*c))/(g^3*i*(a^3*d^3 - b^3*c^3
+ 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - (B
*d^2*(2*A + 3*B))/(2*g^3*i*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d
^2))) - ((6*A^2*a*d - 2*A^2*b*c + 15*B^2*a*d - B^2*b*c + 14*A*B*a*d - 2*A*B
*b*c)/(2*(a*d - b*c)) + (x*(2*A^2*b*d + 7*B^2*b*d + 6*A*B*b*d))/(a*d - b*c)
)/(x^2*(2*b^3*c*g^3*i - 2*a*b^2*d*g^3*i) + x*(4*a*b^2*c*g^3*i - 4*a^2*b*d*g
^3*i) - 2*a^3*d*g^3*i + 2*a^2*b*c*g^3*i) + (log((e*(a + b*x))/(c + d*x))*((
B*d^2*((2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d
^2))*(2*A + 3*B))/(g^3*i*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
)) - B^2/(b*d*g^3*i*(a*d - b*c)) + (B*x*(2*A + 3*B)*(a*d - b*c))/(g^3*i*(a^
3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))))/((b*x^2)/d + a^2/(b*d)
+ (2*a*x)/d) - (B^2*d^2*log((e*(a + b*x))/(c + d*x))^3)/(3*g^3*i*(a^3*d^3 -
```

```
 b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d^2*atan((d^2*(A^2 + (7*B^2)/
2 + 3*A*B)*(2*a^3*d^3*g^3*i + 2*b^3*c^3*g^3*i - 2*a*b^2*c^2*d*g^3*i - 2*a^2
*b*c*d^2*g^3*i)*1i)/(g^3*i*(a*d - b*c)^3*(2*A^2*d^2 + 7*B^2*d^2 + 6*A*B*d^2
)) + (b*d^3*x*(a^2*d^2*g^3*i + b^2*c^2*g^3*i - 2*a*b*c*d*g^3*i)*(A^2 + (7*B
^2)/2 + 3*A*B)*4i)/(g^3*i*(a*d - b*c)^3*(2*A^2*d^2 + 7*B^2*d^2 + 6*A*B*d^2)
))*(A^2 + (7*B^2)/2 + 3*A*B)*2i)/(g^3*i*(a*d - b*c)^3)
```

3.91 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 507

$$
\begin{aligned}
\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = &-\frac{6bB^2d^2(c+dx)}{(bc-ad)^4g^4i(a+bx)}+\frac{3b^2B^2d(c+dx)^2}{4(bc-ad)^4g^4i(a+bx)^2} \\
&-\frac{2b^3B^2(c+dx)^3}{27(bc-ad)^4g^4i(a+bx)^3} \\
&-\frac{6bBd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^4i(a+bx)} \\
&+\frac{3b^2Bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2} \\
&-\frac{2b^3B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^4g^4i(a+bx)^3} \\
&-\frac{3bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^4i(a+bx)} \\
&+\frac{3b^2d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^4g^4i(a+bx)^2} \\
&-\frac{b^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^4g^4i(a+bx)^3} \\
&-\frac{d^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^4g^4i}
\end{aligned}
$$

```
[Out] -6*b*B^2*d^2*(d*x+c)/(-a*d+b*c)^4/g^4/i/(b*x+a)+3/4*b^2*B^2*d*(d*x+c)^2/(-a
*d+b*c)^4/g^4/i/(b*x+a)^2-2/27*b^3*B^2*(d*x+c)^3/(-a*d+b*c)^4/g^4/i/(b*x+a)
^3-6*b*B*d^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^4/i/(b*x+a)
+3/2*b^2*B*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^4/i/(b*x+
a)^2-2/9*b^3*B*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^4/i/(b*
x+a)^3-3*b*d^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^4/i/(b*
x+a)+3/2*b^2*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^4/i/(
b*x+a)^2-1/3*b^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^4/i
/(b*x+a)^3-1/3*d^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^3/B/(-a*d+b*c)^4/g^4/i
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2562, 2395, 2342, 2341, 2339, 30}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = -\frac{b^3(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g^4i(a+bx)^3(bc-ad)^4} - \frac{2b^3B(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{9g^4i(a+bx)^3(bc-ad)^4} + \frac{3b^2d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g^4i(a+bx)^2(bc-ad)^4} + \frac{3b^2Bd(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^4i(a+bx)^2(bc-ad)^4} - \frac{d^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^3}{3Bg^4i(bc-ad)^4} - \frac{3bd^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^4i(a+bx)(bc-ad)^4} - \frac{6bBd^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^4i(a+bx)(bc-ad)^4} - \frac{2b^3B^2(c+dx)^3}{27g^4i(a+bx)^3(bc-ad)^4} + \frac{3b^2B^2d(c+dx)^2}{4g^4i(a+bx)^2(bc-ad)^4} - \frac{6bB^2d^2(c+dx)}{g^4i(a+bx)(bc-ad)^4}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^4*(c*i + d*i*x)),
x]
```

```
[Out] (-6*b*B^2*d^2*(c + d*x))/((b*c - a*d)^4*g^4*i*(a + b*x)) + (3*b^2*B^2*d*(c
+ d*x)^2)/(4*(b*c - a*d)^4*g^4*i*(a + b*x)^2) - (2*b^3*B^2*(c + d*x)^3)/(27
*(b*c - a*d)^4*g^4*i*(a + b*x)^3) - (6*b*B*d^2*(c + d*x)*(A + B*Log[(e*(a +
 b*x))/(c + d*x)]))/((b*c - a*d)^4*g^4*i*(a + b*x)) + (3*b^2*B*d*(c + d*x)^
2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*c - a*d)^4*g^4*i*(a + b*x)^2)
 - (2*b^3*B*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(9*(b*c - a*d
)^4*g^4*i*(a + b*x)^3) - (3*b*d^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d
*x)])^2)/((b*c - a*d)^4*g^4*i*(a + b*x)) + (3*b^2*d*(c + d*x)^2*(A + B*Log[
(e*(a + b*x))/(c + d*x)])^2)/(2*(b*c - a*d)^4*g^4*i*(a + b*x)^2) - (b^3*(c
+ d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*(b*c - a*d)^4*g^4*i*(a
+ b*x)^3) - (d^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^3)/(3*B*(b*c - a*d)^4
*g^4*i)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)^3(A+B\log(ex))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^3(A+B\log(ex))^2}{x^4} - \frac{3b^2d(A+B\log(ex))^2}{x^3} + \frac{3bd^2(A+B\log(ex))^2}{x^2} - \frac{d^3(A+B\log(ex))^2}{x}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i}$$

$$= \frac{b^3\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i} - \frac{(3b^2d)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i} + \frac{(3bd^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i} - \frac{d^3\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i}$$

$$= -\frac{3bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^4i(a+bx)} + \frac{3b^2d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^4g^4i(a+bx)^2} - \frac{b^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^4g^4i(a+bx)^3} + \frac{(2b^3B)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bc-ad)^4g^4i} - \frac{(3b^2Bd)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i} + \frac{(6bBd^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i} - \frac{d^3\text{Subst}\left(\int x^2\,dx, x, A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B(bc-ad)^4g^4i}$$

$$= -\frac{6bB^2d^2(c+dx)}{(bc-ad)^4g^4i(a+bx)} + \frac{3b^2B^2d(c+dx)^2}{4(bc-ad)^4g^4i(a+bx)^2} - \frac{2b^3B^2(c+dx)^3}{27(bc-ad)^4g^4i(a+bx)^3} - \frac{6bBd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^4i(a+bx)} + \frac{3b^2Bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2} - \frac{2b^3B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^4g^4i(a+bx)^3} - \frac{3bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^4i(a+bx)} + \frac{3b^2d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^4g^4i(a+bx)^2} - \frac{b^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^4g^4i(a+bx)^3} - \frac{d^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^4g^4i}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 442, normalized size of antiderivative = 0.87

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = -\frac{4(9A^2+6AB+2B^2)(bc-ad)^3 - 3(18A^2+30AB+19B^2)d(bc-ad)^2(a+bx) - 6(18A^2+66AB+8\cdots}{\cdots}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^4*(c*i + d*
i*x)),x]
```

```
[Out] -1/108*(4*(9*A^2 + 6*A*B + 2*B^2)*(b*c - a*d)^3 - 3*(18*A^2 + 30*A*B + 19*B
^2)*d*(b*c - a*d)^2*(a + b*x) - 6*(18*A^2 + 66*A*B + 85*B^2)*d^2*(-(b*c) +
a*d)*(a + b*x)^2 + 6*(18*A^2 + 66*A*B + 85*B^2)*d^3*(a + b*x)^3*Log[a + b*x
] + 6*B*(b*c - a*d)*(4*(3*A + B)*(b*c - a*d)^2 + 3*(6*A + 5*B)*d*(-(b*c) +
a*d)*(a + b*x) + 6*(6*A + 11*B)*d^2*(a + b*x)^2)*Log[(e*(a + b*x))/(c + d*x
)] + 18*B*(6*a^3*A*d^3 + 18*a^2*b*d^2*(A*d*x + B*(c + d*x)) + 9*a*b^2*d*(2*
A*d^2*x^2 + B*(-c^2 + 2*c*d*x + 3*d^2*x^2)) + b^3*(6*A*d^3*x^3 + B*(2*c^3 -
 3*c^2*d*x + 6*c*d^2*x^2 + 11*d^3*x^3)))*Log[(e*(a + b*x))/(c + d*x)]^2 + 3
6*B^2*d^3*(a + b*x)^3*Log[(e*(a + b*x))/(c + d*x)]^3 - 6*(18*A^2 + 66*A*B +
 85*B^2)*d^3*(a + b*x)^3*Log[c + d*x])/((b*c - a*d)^4*g^4*i*(a + b*x)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1075 vs. 2(493) = 986.

Time = 1.84 (sec) , antiderivative size = 1076, normalized size of antiderivative = 2.12

method	result	size
parts	Expression too large to display	1076
derivativedivides	Expression too large to display	1244
default	Expression too large to display	1244
risch	Expression too large to display	1422
norman	Expression too large to display	1693
parallelrisch	Expression too large to display	1967

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x,method=_RETUR
NVERBOSE)
```

```
[Out] A^2/g^4/i*(d^3/(a*d-b*c)^4*ln(d*x+c)+1/3/(a*d-b*c)/(b*x+a)^3+1/2*d/(a*d-b*c
)^2/(b*x+a)^2+d^2/(a*d-b*c)^3/(b*x+a)-d^3/(a*d-b*c)^4*ln(b*x+a))-B^2/g^4/i/
d*(1/3*d^4/(a*d-b*c)^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3-3*d^3/(a*d-b*c)^4*
b*e*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(
b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-
b*c)*e/d/(d*x+c)))+3*d^2/(a*d-b*c)^4*b^2*e^2*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*
x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))
^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-d/(
a*d-b*c)^4*b^3*e^3*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)
*e/d/(d*x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/
(d*x+c))-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))-2*B*A/g^4/i/d*(1/2*d^4/(a*d
-b*c)^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-3*d^3/(a*d-b*c)^4*b*e*(-1/(b*e/d+
(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e
/d/(d*x+c)))+3*d^2/(a*d-b*c)^4*b^2*e^2*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^
2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-d/(a
*d-b*c)^4*b^3*e^3*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*
e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 940, normalized size of antiderivative = 1.85

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = -\frac{4\,(9\,A^2+6\,AB+2\,B^2)b^3c^3-81\,(2\,A^2+2\,AB+B^2)ab^2c^2d+324\,(A^2+2\,AB+2\,B^2)a^2bcd^2-(198\,A^2}{}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x, algor
ithm="fricas")
```

```
[Out] -1/108*(4*(9*A^2 + 6*A*B + 2*B^2)*b^3*c^3 - 81*(2*A^2 + 2*A*B + B^2)*a*b^2*
c^2*d + 324*(A^2 + 2*A*B + 2*B^2)*a^2*b*c*d^2 - (198*A^2 + 510*A*B + 575*B^
2)*a^3*d^3 + 36*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x
+ B^2*a^3*d^3)*log((b*e*x + a*e)/(d*x + c))^3 + 6*((18*A^2 + 66*A*B + 85*B^
2)*b^3*c*d^2 - (18*A^2 + 66*A*B + 85*B^2)*a*b^2*d^3)*x^2 + 18*((6*A*B + 11*
B^2)*b^3*d^3*x^3 + 2*B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 18*B^2*a^2*b*c*d^2 +
 6*A*B*a^3*d^3 + 3*(2*B^2*b^3*c*d^2 + 3*(2*A*B + 3*B^2)*a*b^2*d^3)*x^2 - 3*
(B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 - 6*(A*B + B^2)*a^2*b*d^3)*x)*log((b*e*x
 + a*e)/(d*x + c))^2 - 3*((18*A^2 + 30*A*B + 19*B^2)*b^3*c^2*d - 54*(2*A^2
+ 6*A*B + 7*B^2)*a*b^2*c*d^2 + (90*A^2 + 294*A*B + 359*B^2)*a^2*b*d^3)*x +
6*((18*A^2 + 66*A*B + 85*B^2)*b^3*d^3*x^3 + 18*A^2*a^3*d^3 + 4*(3*A*B + B^2
)*b^3*c^3 - 27*(2*A*B + B^2)*a*b^2*c^2*d + 108*(A*B + B^2)*a^2*b*c*d^2 + 3*
(2*(6*A*B + 11*B^2)*b^3*c*d^2 + 9*(2*A^2 + 6*A*B + 7*B^2)*a*b^2*d^3)*x^2 -
3*((6*A*B + 5*B^2)*b^3*c^2*d - 18*(2*A*B + 3*B^2)*a*b^2*c*d^2 - 18*(A^2 + 2
*A*B + 2*B^2)*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^7*c^4 - 4*a*b
^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4)*g^4*i*x^3 + 3
*(a*b^6*c^4 - 4*a^2*b^5*c^3*d + 6*a^3*b^4*c^2*d^2 - 4*a^4*b^3*c*d^3 + a^5*b
^2*d^4)*g^4*i*x^2 + 3*(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 6*a^4*b^3*c^2*d^2 -
4*a^5*b^2*c*d^3 + a^6*b*d^4)*g^4*i*x + (a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a
^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)*g^4*i)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2388 vs. $2(459) = 918$.

Time = 29.82 (sec) , antiderivative size = 2388, normalized size of antiderivative = 4.71

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**4/(d*i*x+c*i),x)
```

```
[Out] -B**2*d**3*log(e*(a + b*x)/(c + d*x))**3/(3*a**4*d**4*g**4*i - 12*a**3*b*c*
d**3*g**4*i + 18*a**2*b**2*c**2*d**2*g**4*i - 12*a*b**3*c**3*d*g**4*i + 3*b
**4*c**4*g**4*i) + d**3*(18*A**2 + 66*A*B + 85*B**2)*log(x + (18*A**2*a*d**
4 + 18*A**2*b*c*d**3 + 66*A*B*a*d**4 + 66*A*B*b*c*d**3 + 85*B**2*a*d**4 + 8
5*B**2*b*c*d**3 - a**5*d**8*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**4 + 5
*a**4*b*c*d**7*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**4 - 10*a**3*b**2*c
**2*d**6*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**4 + 10*a**2*b**3*c**3*d*
*5*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**4 - 5*a*b**4*c**4*d**4*(18*A**
2 + 66*A*B + 85*B**2)/(a*d - b*c)**4 + b**5*c**5*d**3*(18*A**2 + 66*A*B + 8
5*B**2)/(a*d - b*c)**4)/(36*A**2*b*d**4 + 132*A*B*b*d**4 + 170*B**2*b*d**4)
```

```
)/(18*g**4*i*(a*d - b*c)**4) - d**3*(18*A**2 + 66*A*B + 85*B**2)*log(x + (1
8*A**2*a*d**4 + 18*A**2*b*c*d**3 + 66*A*B*a*d**4 + 66*A*B*b*c*d**3 + 85*B**
2*a*d**4 + 85*B**2*b*c*d**3 + a**5*d**8*(18*A**2 + 66*A*B + 85*B**2)/(a*d -
 b*c)**4 - 5*a**4*b*c*d**7*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**4 + 10
*a**3*b**2*c**2*d**6*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**4 - 10*a**2*
b**3*c**3*d**5*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**4 + 5*a*b**4*c**4*
d**4*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**4 - b**5*c**5*d**3*(18*A**2
+ 66*A*B + 85*B**2)/(a*d - b*c)**4)/(36*A**2*b*d**4 + 132*A*B*b*d**4 + 170*
B**2*b*d**4))/(18*g**4*i*(a*d - b*c)**4) + (66*A*B*a**2*d**2 - 42*A*B*a*b*c
*d + 90*A*B*a*b*d**2*x + 12*A*B*b**2*c**2 - 18*A*B*b**2*c*d*x + 36*A*B*b**2
*d**2*x**2 + 85*B**2*a**2*d**2 - 23*B**2*a*b*c*d + 147*B**2*a*b*d**2*x + 4*
B**2*b**2*c**2 - 15*B**2*b**2*c*d*x + 66*B**2*b**2*d**2*x**2)*log(e*(a + b*
x)/(c + d*x))/(18*a**6*d**3*g**4*i - 54*a**5*b*c*d**2*g**4*i + 54*a**5*b*d*
*3*g**4*i*x + 54*a**4*b**2*c**2*d*g**4*i - 162*a**4*b**2*c*d**2*g**4*i*x +
54*a**4*b**2*d**3*g**4*i*x**2 - 18*a**3*b**3*c**3*g**4*i + 162*a**3*b**3*c*
*2*d*g**4*i*x - 162*a**3*b**3*c*d**2*g**4*i*x**2 + 18*a**3*b**3*d**3*g**4*i
*x**3 - 54*a**2*b**4*c**3*g**4*i*x + 162*a**2*b**4*c**2*d*g**4*i*x**2 - 54*
a**2*b**4*c*d**2*g**4*i*x**3 - 54*a*b**5*c**3*g**4*i*x**2 + 54*a*b**5*c**2*
d*g**4*i*x**3 - 18*b**6*c**3*g**4*i*x**3) + (-6*A*B*a**3*d**3 - 18*A*B*a**2
*b*d**3*x - 18*A*B*a*b**2*d**3*x**2 - 6*A*B*b**3*d**3*x**3 - 18*B**2*a**2*b
*c*d**2 - 18*B**2*a**2*b*d**3*x + 9*B**2*a*b**2*c**2*d - 18*B**2*a*b**2*c*d
**2*x - 27*B**2*a*b**2*d**3*x**2 - 2*B**2*b**3*c**3 + 3*B**2*b**3*c**2*d*x
- 6*B**2*b**3*c*d**2*x**2 - 11*B**2*b**3*d**3*x**3)*log(e*(a + b*x)/(c + d*
x))**2/(6*a**7*d**4*g**4*i - 24*a**6*b*c*d**3*g**4*i + 18*a**6*b*d**4*g**4*
i*x + 36*a**5*b**2*c**2*d**2*g**4*i - 72*a**5*b**2*c*d**3*g**4*i*x + 18*a**
5*b**2*d**4*g**4*i*x**2 - 24*a**4*b**3*c**3*d*g**4*i + 108*a**4*b**3*c**2*d
**2*g**4*i*x - 72*a**4*b**3*c*d**3*g**4*i*x**2 + 6*a**4*b**3*d**4*g**4*i*x*
*3 + 6*a**3*b**4*c**4*g**4*i - 72*a**3*b**4*c**3*d*g**4*i*x + 108*a**3*b**4
*c**2*d**2*g**4*i*x**2 - 24*a**3*b**4*c*d**3*g**4*i*x**3 + 18*a**2*b**5*c**
4*g**4*i*x - 72*a**2*b**5*c**3*d*g**4*i*x**2 + 36*a**2*b**5*c**2*d**2*g**4*
i*x**3 + 18*a*b**6*c**4*g**4*i*x**2 - 24*a*b**6*c**3*d*g**4*i*x**3 + 6*b**7
*c**4*g**4*i*x**3) + (198*A**2*a**2*d**2 - 126*A**2*a*b*c*d + 36*A**2*b**2*
c**2 + 510*A*B*a**2*d**2 - 138*A*B*a*b*c*d + 24*A*B*b**2*c**2 + 575*B**2*a*
*2*d**2 - 73*B**2*a*b*c*d + 8*B**2*b**2*c**2 + x**2*(108*A**2*b**2*d**2 + 3
96*A*B*b**2*d**2 + 510*B**2*b**2*d**2) + x*(270*A**2*a*b*d**2 - 54*A**2*b**
2*c*d + 882*A*B*a*b*d**2 - 90*A*B*b**2*c*d + 1077*B**2*a*b*d**2 - 57*B**2*b
**2*c*d))/(108*a**6*d**3*g**4*i - 324*a**5*b*c*d**2*g**4*i + 324*a**4*b**2*
c**2*d*g**4*i - 108*a**3*b**3*c**3*g**4*i + x**3*(108*a**3*b**3*d**3*g**4*i
 - 324*a**2*b**4*c*d**2*g**4*i + 324*a*b**5*c**2*d*g**4*i - 108*b**6*c**3*g
**4*i) + x**2*(324*a**4*b**2*d**3*g**4*i - 972*a**3*b**3*c*d**2*g**4*i + 97
2*a**2*b**4*c**2*d*g**4*i - 324*a*b**5*c**3*g**4*i) + x*(324*a**5*b*d**3*g*
*4*i - 972*a**4*b**2*c*d**2*g**4*i + 972*a**3*b**3*c**2*d*g**4*i - 324*a**2
*b**4*c**3*g**4*i))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3434 vs. $2(493) = 986$.

Time = 0.48 (sec) , antiderivative size = 3434, normalized size of antiderivative = 6.77

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x, algor
ithm="maxima")
```

```
[Out] -1/6*B^2*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
- 5*a*b*d^2)*x)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*
g^4*i*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)
*g^4*i*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3
)*g^4*i*x + (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*g^4*i
) + 6*d^3*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^
3*b*c*d^3 + a^4*d^4)*g^4*i) - 6*d^3*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i))*log(b*e*x/(d*x + c)
+ a*e/(d*x + c))^2 - 1/3*A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a
^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c
*d^2 - a^3*b^3*d^3)*g^4*i*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*
c*d^2 - a^4*b^2*d^3)*g^4*i*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b
^2*c*d^2 - a^5*b*d^3)*g^4*i*x + (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*
d^2 - a^6*d^3)*g^4*i) + 6*d^3*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i) - 6*d^3*log(d*x + c)/((b^4*
c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i))*
log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 1/108*B^2*(6*(4*b^3*c^3 - 27*a*b^2*c
^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(
b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 1
8*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2
- 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a
*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 +
33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d
^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))*log(b*e*x/(d*
x + c) + a*e/(d*x + c))/(a^3*b^4*c^4*g^4*i - 4*a^4*b^3*c^3*d*g^4*i + 6*a^5*
b^2*c^2*d^2*g^4*i - 4*a^6*b*c*d^3*g^4*i + a^7*d^4*g^4*i + (b^7*c^4*g^4*i -
4*a*b^6*c^3*d*g^4*i + 6*a^2*b^5*c^2*d^2*g^4*i - 4*a^3*b^4*c*d^3*g^4*i + a^4
*b^3*d^4*g^4*i)*x^3 + 3*(a*b^6*c^4*g^4*i - 4*a^2*b^5*c^3*d*g^4*i + 6*a^3*b^
4*c^2*d^2*g^4*i - 4*a^4*b^3*c*d^3*g^4*i + a^5*b^2*d^4*g^4*i)*x^2 + 3*(a^2*b
^5*c^4*g^4*i - 4*a^3*b^4*c^3*d*g^4*i + 6*a^4*b^3*c^2*d^2*g^4*i - 4*a^5*b^2*
c*d^3*g^4*i + a^6*b*d^4*g^4*i)*x) + (8*b^3*c^3 - 81*a*b^2*c^2*d + 648*a^2*b
*c*d^2 - 575*a^3*d^3 + 36*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x +
a^3*d^3)*log(b*x + a)^3 - 36*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x
```

```
 + a^3*d^3)*log(d*x + c)^3 + 510*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 198*(b^3*d^3
*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18*(11*b
^3*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^
3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c)^2
 - 3*(19*b^3*c^2*d - 378*a*b^2*c*d^2 + 359*a^2*b*d^3)*x + 510*(b^3*d^3*x^3
+ 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(85*b^3*d^3*x
^3 + 255*a*b^2*d^3*x^2 + 255*a^2*b*d^3*x + 85*a^3*d^3 + 18*(b^3*d^3*x^3 + 3
*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 66*(b^3*d^3*x^3
+ 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/(a
^3*b^4*c^4*g^4*i - 4*a^4*b^3*c^3*d*g^4*i + 6*a^5*b^2*c^2*d^2*g^4*i - 4*a^6*
b*c*d^3*g^4*i + a^7*d^4*g^4*i + (b^7*c^4*g^4*i - 4*a*b^6*c^3*d*g^4*i + 6*a^
2*b^5*c^2*d^2*g^4*i - 4*a^3*b^4*c*d^3*g^4*i + a^4*b^3*d^4*g^4*i)*x^3 + 3*(a
*b^6*c^4*g^4*i - 4*a^2*b^5*c^3*d*g^4*i + 6*a^3*b^4*c^2*d^2*g^4*i - 4*a^4*b^
3*c*d^3*g^4*i + a^5*b^2*d^4*g^4*i)*x^2 + 3*(a^2*b^5*c^4*g^4*i - 4*a^3*b^4*c
^3*d*g^4*i + 6*a^4*b^3*c^2*d^2*g^4*i - 4*a^5*b^2*c*d^3*g^4*i + a^6*b*d^4*g^
4*i)*x)) - 1/6*A^2*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3
*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3
*b^3*d^3)*g^4*i*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^
4*b^2*d^3)*g^4*i*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 -
 a^5*b*d^3)*g^4*i*x + (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*
d^3)*g^4*i) + 6*d^3*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*
d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i) - 6*d^3*log(d*x + c)/((b^4*c^4 - 4*a*
b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i)) - 1/18*(4*
b^3*c^3 - 27*a*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a
*b^2*d^3)*x^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3
)*log(b*x + a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*
d^3)*log(d*x + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 6
6*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) -
6*(11*b^3*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3
*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x
 + c))*A*B/(a^3*b^4*c^4*g^4*i - 4*a^4*b^3*c^3*d*g^4*i + 6*a^5*b^2*c^2*d^2*g
^4*i - 4*a^6*b*c*d^3*g^4*i + a^7*d^4*g^4*i + (b^7*c^4*g^4*i - 4*a*b^6*c^3*d
*g^4*i + 6*a^2*b^5*c^2*d^2*g^4*i - 4*a^3*b^4*c*d^3*g^4*i + a^4*b^3*d^4*g^4*
i)*x^3 + 3*(a*b^6*c^4*g^4*i - 4*a^2*b^5*c^3*d*g^4*i + 6*a^3*b^4*c^2*d^2*g^4
*i - 4*a^4*b^3*c*d^3*g^4*i + a^5*b^2*d^4*g^4*i)*x^2 + 3*(a^2*b^5*c^4*g^4*i
- 4*a^3*b^4*c^3*d*g^4*i + 6*a^4*b^3*c^2*d^2*g^4*i - 4*a^5*b^2*c*d^3*g^4*i +
 a^6*b*d^4*g^4*i)*x)
```

Giac [A] (verification not implemented)

none

Time = 71.59 (sec) , antiderivative size = 451, normalized size of antiderivative = 0.89

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = -\frac{1}{108}\left(\frac{18\left(2\,B^2be^4-\frac{3\,(bex+ae)B^2de^3}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^3bcg^4i}{(dx+c)^3}-\frac{(bex+ae)^3adg^4i}{(dx+c)^3}}+\frac{6\left(12\,ABbe^4+4\,B^2be^4-\frac{18\,(bex+ae)ABde^3}{dx+c}-\frac{9\,(bex+ae)\cdot}{dx+c}\right.}{\frac{(bex+ae)^3bcg^4i}{(dx+c)^3}-\frac{(bex+ae)^3adg^4i}{(dx+c)^3}}\right.$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x, algor
ithm="giac")
```

```
[Out] -1/108*(18*(2*B^2*b*e^4 - 3*(b*e*x + a*e)*B^2*d*e^3/(d*x + c))*log((b*e*x +
 a*e)/(d*x + c))^2/((b*e*x + a*e)^3*b*c*g^4*i/(d*x + c)^3 - (b*e*x + a*e)^3
*a*d*g^4*i/(d*x + c)^3) + 6*(12*A*B*b*e^4 + 4*B^2*b*e^4 - 18*(b*e*x + a*e)*
A*B*d*e^3/(d*x + c) - 9*(b*e*x + a*e)*B^2*d*e^3/(d*x + c))*log((b*e*x + a*e
)/(d*x + c))/((b*e*x + a*e)^3*b*c*g^4*i/(d*x + c)^3 - (b*e*x + a*e)^3*a*d*g
^4*i/(d*x + c)^3) + (36*A^2*b*e^4 + 24*A*B*b*e^4 + 8*B^2*b*e^4 - 54*(b*e*x
+ a*e)*A^2*d*e^3/(d*x + c) - 54*(b*e*x + a*e)*A*B*d*e^3/(d*x + c) - 27*(b*e
*x + a*e)*B^2*d*e^3/(d*x + c))/((b*e*x + a*e)^3*b*c*g^4*i/(d*x + c)^3 - (b*
e*x + a*e)^3*a*d*g^4*i/(d*x + c)^3))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a
*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

Mupad [B] (verification not implemented)

Time = 8.27 (sec) , antiderivative size = 1882, normalized size of antiderivative = 3.71

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^4*(c*i + d*i*x)),
x)
```

```
[Out] log((e*(a + b*x))/(c + d*x))^2*(((B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*
c*d)/(6*b*d^3) + (a*(a*d - b*c))/(3*b*d^2)) + (3*a^3*d^3 - b^3*c^3 + 4*a*b^
2*c^2*d - 6*a^2*b*c*d^2)/(3*b*d^4)))/(g^4*i*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*
c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (B^2*d^3*x^2*((b^2*c - a*b*d)/(
3*d^2) - (2*b*(a*d - b*c))/(3*d^2)))/(g^4*i*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*
c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^3*x*(b*((3*a^2*d^2 + b^2
*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(3*b*d^2)) + (3*a^2*d^2 + b^2
*c^2 - 4*a*b*c*d)/(3*d^3) + (2*a*(a*d - b*c))/(3*d^2)))/(g^4*i*(a^4*d^4 + b
```

```
^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))/((3*a^2*x)/d
+ a^3/(b*d) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (d^3*(11*B^2 + 6*A*B))/(6*g^4*
i*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))
 + ((198*A^2*a^2*d^2 + 36*A^2*b^2*c^2 + 575*B^2*a^2*d^2 + 8*B^2*b^2*c^2 + 5
10*A*B*a^2*d^2 + 24*A*B*b^2*c^2 - 126*A^2*a*b*c*d - 73*B^2*a*b*c*d - 138*A*
B*a*b*c*d)/(6*(a*d - b*c)) + (x*(90*A^2*a*b*d^2 + 359*B^2*a*b*d^2 - 18*A^2*
b^2*c*d - 19*B^2*b^2*c*d + 294*A*B*a*b*d^2 - 30*A*B*b^2*c*d))/(2*(a*d - b*c
)) + (d*x^2*(18*A^2*b^2*d + 85*B^2*b^2*d + 66*A*B*b^2*d))/(a*d - b*c))/(x*(
54*a^4*b*d^2*g^4*i + 54*a^2*b^3*c^2*g^4*i - 108*a^3*b^2*c*d*g^4*i) + x^2*(5
4*a*b^4*c^2*g^4*i + 54*a^3*b^2*d^2*g^4*i - 108*a^2*b^3*c*d*g^4*i) + x^3*(18
*b^5*c^2*g^4*i + 18*a^2*b^3*d^2*g^4*i - 36*a*b^4*c*d*g^4*i) + 18*a^5*d^2*g^
4*i + 18*a^3*b^2*c^2*g^4*i - 36*a^4*b*c*d*g^4*i) - (log((e*(a + b*x))/(c +
d*x))*(x*(B^2/(g^4*i*(a*d - b*c)^2) - (d^3*(11*B^2 + 6*A*B)*(b*((3*a^2*d^2
+ b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(3*b*d^2)) + (3*a^2*d^2
+ b^2*c^2 - 4*a*b*c*d)/(3*d^3) + (2*a*(a*d - b*c))/(3*d^2)))/(3*g^4*i*(a^4*
d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))) + (5*B
^2*a*d - 3*B^2*b*c)/(3*b*d*g^4*i*(a*d - b*c)^2) + (B^2*a)/(3*b*g^4*i*(a*d -
 b*c)^2) - (d^3*(11*B^2 + 6*A*B)*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d)/(6*b
*d^3) + (a*(a*d - b*c))/(3*b*d^2)) + (3*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d -
 6*a^2*b*c*d^2)/(3*b*d^4)))/(3*g^4*i*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2
 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (d^3*x^2*((b^2*c - a*b*d)/(3*d^2) - (2
*b*(a*d - b*c))/(3*d^2))*(11*B^2 + 6*A*B))/(3*g^4*i*(a^4*d^4 + b^4*c^4 + 6*
a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))))/((3*a^2*x)/d + a^3/(b*d
) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (B^2*d^3*log((e*(a + b*x))/(c + d*x))^3)
/(3*g^4*i*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*
c*d^3)) + (d^3*atan((d^3*(A^2 + (85*B^2)/18 + (11*A*B)/3)*(18*a^4*d^4*g^4*i
 - 18*b^4*c^4*g^4*i + 36*a*b^3*c^3*d*g^4*i - 36*a^3*b*c*d^3*g^4*i)*1i)/(g^4
*i*(a*d - b*c)^4*(18*A^2*d^3 + 85*B^2*d^3 + 66*A*B*d^3)) + (b*d^4*x*(A^2 +
(85*B^2)/18 + (11*A*B)/3)*(a^3*d^3*g^4*i - b^3*c^3*g^4*i + 3*a*b^2*c^2*d*g^
4*i - 3*a^2*b*c*d^2*g^4*i)*36i)/(g^4*i*(a*d - b*c)^4*(18*A^2*d^3 + 85*B^2*d
^3 + 66*A*B*d^3)))*(A^2 + (85*B^2)/18 + (11*A*B)/3)*2i)/(g^4*i*(a*d - b*c)^
4)
```

3.92 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 722

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$$
$$= \frac{2AB(bc-ad)^2g^3(a+bx)}{d^3i^2(c+dx)} - \frac{2B^2(bc-ad)^2g^3(a+bx)}{d^3i^2(c+dx)}$$
$$+ \frac{2B^2(bc-ad)^2g^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^3i^2(c+dx)}$$
$$- \frac{bB(bc-ad)g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i^2}$$
$$- \frac{6bB(bc-ad)^2g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4i^2}$$
$$- \frac{3b(bc-ad)g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^2}$$
$$- \frac{(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^2(c+dx)} + \frac{b^3g^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^4i^2}$$
$$- \frac{3b(bc-ad)^2g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^4i^2} + \frac{bB^2(bc-ad)^2g^3\log(c+dx)}{d^4i^2}$$
$$+ \frac{bB(bc-ad)^2g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{d^4i^2}$$
$$- \frac{6bB^2(bc-ad)^2g^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}$$
$$- \frac{6bB(bc-ad)^2g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}$$
$$- \frac{bB^2(bc-ad)^2g^3\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{d^4i^2} + \frac{6bB^2(bc-ad)^2g^3\,\mathrm{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}$$

```
[Out] 2*A*B*(-a*d+b*c)^2*g^3*(b*x+a)/d^3/i^2/(d*x+c)-2*B^2*(-a*d+b*c)^2*g^3*(b*x+
a)/d^3/i^2/(d*x+c)+2*B^2*(-a*d+b*c)^2*g^3*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/d^3
/i^2/(d*x+c)-b*B*(-a*d+b*c)*g^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3/i^2
-6*b*B*(-a*d+b*c)^2*g^3*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c))
)/d^4/i^2-3*b*(-a*d+b*c)*g^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i^2-
(-a*d+b*c)^2*g^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i^2/(d*x+c)+1/2*
b^3*g^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^4/i^2-3*b*(-a*d+b*c)^2*g^
3*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^4/i^2+b*B^2*(-a*
```

```
d+b*c)^2*g^3*ln(d*x+c)/d^4/i^2+b*B*(-a*d+b*c)^2*g^3*(A+B*ln(e*(b*x+a)/(d*x+
c)))*ln(1-b*(d*x+c)/d/(b*x+a))/d^4/i^2-6*b*B^2*(-a*d+b*c)^2*g^3*polylog(2,d
*(b*x+a)/b/(d*x+c))/d^4/i^2-6*b*B*(-a*d+b*c)^2*g^3*(A+B*ln(e*(b*x+a)/(d*x+c
)))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i^2-b*B^2*(-a*d+b*c)^2*g^3*polylog(2
,b*(d*x+c)/d/(b*x+a))/d^4/i^2+6*b*B^2*(-a*d+b*c)^2*g^3*polylog(3,d*(b*x+a)/
b/(d*x+c))/d^4/i^2
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 722, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules

used = {2562, 2395, 2333, 2332, 2356, 2389, 2379, 2438, 2351, 31, 2355, 2354, 2421, 6724}

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$$
$$= \frac{b^3g^3(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d^4i^2} - \frac{6bBg^3(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^4i^2} - \frac{3bg^3(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^4i^2} - \frac{6bBg^3(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^4i^2} + \frac{bBg^3(bc-ad)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^4i^2} - \frac{3bg^3(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^3i^2} - \frac{g^3(a+bx)(bc-ad)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^3i^2(c+dx)} - \frac{bBg^3(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^3i^2} + \frac{2ABg^3(a+bx)(bc-ad)^2}{d^3i^2(c+dx)} - \frac{6bB^2g^3(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2} - \frac{bB^2g^3(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{d^4i^2} + \frac{6bB^2g^3(bc-ad)^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2} + \frac{bB^2g^3(bc-ad)^2\log(c+dx)}{d^4i^2} + \frac{2B^2g^3(a+bx)(bc-ad)^2\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^3i^2(c+dx)} - \frac{2B^2g^3(a+bx)(bc-ad)^2}{d^3i^2(c+dx)}$$

```
[In] Int[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x)^
2,x]
```

```
[Out] (2*A*B*(b*c - a*d)^2*g^3*(a + b*x))/(d^3*i^2*(c + d*x)) - (2*B^2*(b*c - a*d
)^2*g^3*(a + b*x))/(d^3*i^2*(c + d*x)) + (2*B^2*(b*c - a*d)^2*g^3*(a + b*x)
*Log[(e*(a + b*x))/(c + d*x)])/(d^3*i^2*(c + d*x)) - (b*B*(b*c - a*d)*g^3*(
a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d^3*i^2) - (6*b*B*(b*c - a*
d)^2*g^3*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/(d^4*i^2) - (3*b*(b*c - a*d)*g^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c +
 d*x)])^2)/(d^3*i^2) - ((b*c - a*d)^2*g^3*(a + b*x)*(A + B*Log[(e*(a + b*x)
```

```
)/(c + d*x)])^2)/(d^3*i^2*(c + d*x)) + (b^3*g^3*(c + d*x)^2*(A + B*Log[(e*(
a + b*x))/(c + d*x)])^2)/(2*d^4*i^2) - (3*b*(b*c - a*d)^2*g^3*Log[(b*c - a*
d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d^4*i^2) + (b*B^
2*(b*c - a*d)^2*g^3*Log[c + d*x])/(d^4*i^2) + (b*B*(b*c - a*d)^2*g^3*(A + B
*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(d^4*i
^2) - (6*b*B^2*(b*c - a*d)^2*g^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(
d^4*i^2) - (6*b*B*(b*c - a*d)^2*g^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Po
lyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^4*i^2) - (b*B^2*(b*c - a*d)^2*g^3
*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(d^4*i^2) + (6*b*B^2*(b*c - a*d)^
2*g^3*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(d^4*i^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left((bc-ad)^2g^3\right)\text{Subst}\left(\int \frac{x^3(A+B\log(ex))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{i^2} \\
&= \frac{\left((bc-ad)^2g^3\right)\text{Subst}\left(\int \left(-\frac{(A+B\log(ex))^2}{d^3}+\frac{b^3(A+B\log(ex))^2}{d^3(b-dx)^3}-\frac{3b^2(A+B\log(ex))^2}{d^3(b-dx)^2}+\frac{3b(A+B\log(ex))^2}{d^3(b-dx)}\right)dx, x,\right.}{i^2} \\
&= -\frac{\left((bc-ad)^2g^3\right)\text{Subst}\left(\int (A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^2} \\
&\quad + \frac{\left(3b(bc-ad)^2g^3\right)\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^2} \\
&\quad - \frac{\left(3b^2(bc-ad)^2g^3\right)\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^2} \\
&\quad + \frac{\left(b^3(bc-ad)^2g^3\right)\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^2}
\end{aligned}$$

$$= -\frac{3b(bc-ad)g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^2} - \frac{(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^2(c+dx)} + \frac{b^3g^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^4i^2} - \frac{3b(bc-ad)^2g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^4i^2} + \frac{(6bB(bc-ad)^2g^3)\,\text{Subst}\left(\int\frac{(A+B\log(ex))\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^4i^2} - \frac{(b^3B(bc-ad)^2g^3)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^4i^2} + \frac{(2B(bc-ad)^2g^3)\,\text{Subst}\left(\int(A+B\log(ex))\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i^2} + \frac{(6bB(bc-ad)^2g^3)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i^2}$$

$$= \frac{2AB(bc-ad)^2g^3(a+bx)}{d^3i^2(c+dx)} - \frac{6bB(bc-ad)^2g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4i^2}$$
$$- \frac{3b(bc-ad)g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^2}$$
$$- \frac{(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^2(c+dx)}$$
$$+ \frac{b^3g^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^4i^2}$$
$$- \frac{3b(bc-ad)^2g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^4i^2}$$
$$- \frac{6bB(bc-ad)^2g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}$$
$$- \frac{(b^2B(bc-ad)^2g^3)\,\mathrm{Subst}\left(\int\frac{A+B\log(ex)}{x(b-dx)}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^4i^2}$$
$$+ \frac{(6bB^2(bc-ad)^2g^3)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^4i^2}$$
$$+ \frac{(6bB^2(bc-ad)^2g^3)\,\mathrm{Subst}\left(\int\frac{\mathrm{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^4i^2}$$
$$- \frac{(b^2B(bc-ad)^2g^3)\,\mathrm{Subst}\left(\int\frac{A+B\log(ex)}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i^2}$$
$$+ \frac{(2B^2(bc-ad)^2g^3)\,\mathrm{Subst}\left(\int\log(ex)\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i^2}$$

$$
\begin{aligned}
&= \frac{2AB(bc-ad)^2g^3(a+bx)}{d^3i^2(c+dx)} - \frac{2B^2(bc-ad)^2g^3(a+bx)}{d^3i^2(c+dx)} \\
&+ \frac{2B^2(bc-ad)^2g^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^3i^2(c+dx)} \\
&- \frac{bB(bc-ad)g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i^2} \\
&- \frac{6bB(bc-ad)^2g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4i^2} \\
&- \frac{3b(bc-ad)g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^2} \\
&- \frac{(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^2(c+dx)} \\
&+ \frac{b^3g^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^4i^2} \\
&- \frac{3b(bc-ad)^2g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^4i^2} \\
&+ \frac{bB(bc-ad)^2g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{d^4i^2} \\
&- \frac{6bB^2(bc-ad)^2g^3\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2} \\
&- \frac{6bB(bc-ad)^2g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2} \\
&+ \frac{6bB^2(bc-ad)^2g^3\operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2} \\
&- \frac{(bB^2(bc-ad)^2g^3)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4i^2} \\
&+ \frac{(bB^2(bc-ad)^2g^3)\operatorname{Subst}\left(\int\frac{1}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2AB(bc-ad)^2g^3(a+bx)}{d^3i^2(c+dx)} - \frac{2B^2(bc-ad)^2g^3(a+bx)}{d^3i^2(c+dx)} \\
&+ \frac{2B^2(bc-ad)^2g^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^3i^2(c+dx)} \\
&- \frac{bB(bc-ad)g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i^2} \\
&- \frac{6bB(bc-ad)^2g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4i^2} \\
&- \frac{3b(bc-ad)g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^2} \\
&- \frac{(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^2(c+dx)} \\
&+ \frac{b^3g^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^4i^2} \\
&- \frac{3b(bc-ad)^2g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^4i^2} \\
&+ \frac{bB^2(bc-ad)^2g^3\log(c+dx)}{d^4i^2} \\
&+ \frac{bB(bc-ad)^2g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{d^4i^2} \\
&- \frac{6bB^2(bc-ad)^2g^3\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2} \\
&- \frac{6bB(bc-ad)^2g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2} \\
&- \frac{bB^2(bc-ad)^2g^3\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{d^4i^2} + \frac{6bB^2(bc-ad)^2g^3\operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4443 vs. 2(722) = 1444.

Time = 6.29 (sec) , antiderivative size = 4443, normalized size of antiderivative = 6.15

$$
\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d
*i*x)^2,x]
```

```
[Out] (g^3*(-4*A^2*b^2*d*(2*b*c - 3*a*d)*x + 2*A^2*b^3*d^2*x^2 + (4*A^2*(b*c - a*
d)^3)/(c + d*x) + 12*A^2*b*(b*c - a*d)^2*Log[c + d*x] + (8*a^3*A*B*d^3*(b*c
 - a*d + b*(c + d*x)*Log[a/b + x] + (-(b*c) + a*d)*Log[(e*(a + b*x))/(c + d
*x)] - b*c*Log[(b*(c + d*x))/(b*c - a*d)] - b*d*x*Log[(b*(c + d*x))/(b*c -
a*d)]))/((b*c - a*d)*(c + d*x)) + 12*a^2*A*b*B*d^2*(-Log[c/d + x]^2 + 2*Log
[c/d + x]*Log[c + d*x] + 2*(-(c/(c + d*x)) + (b*c*Log[a + b*x])/(-(b*c) + a
*d) + (b*c*Log[c + d*x])/(b*c - a*d) - Log[a/b + x]*Log[c + d*x] + Log[(e*(
a + b*x))/(c + d*x)]*(c/(c + d*x) + Log[c + d*x]) + Log[a/b + x]*Log[(b*(c
+ d*x))/(b*c - a*d)]) + 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 4*A*b
^3*B*(-4*c^2 + (4*a*c*d)/b - c*d*x + (a*d^2*x)/b - (2*c^3)/(c + d*x) + 4*c^
2*Log[c/d + x] - 3*c^2*Log[c/d + x]^2 - (a^2*d^2*Log[a + b*x])/b^2 + (2*b*c
^3*Log[a + b*x])/(-(b*c) + a*d) - 4*c*d*x*Log[(e*(a + b*x))/(c + d*x)] + d^
2*x^2*Log[(e*(a + b*x))/(c + d*x)] + (2*c^3*Log[(e*(a + b*x))/(c + d*x)])/(
c + d*x) + c^2*Log[c + d*x] + (2*b*c^3*Log[c + d*x])/(b*c - a*d) + 6*c^2*Lo
g[c/d + x]*Log[c + d*x] + 6*c^2*Log[(e*(a + b*x))/(c + d*x)]*Log[c + d*x] -
 (2*c*Log[a/b + x]*(2*a*d + 3*b*c*Log[c + d*x] - 3*b*c*Log[(b*(c + d*x))/(b
*c - a*d)]))/b + 6*c^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 24*a*A*b
^2*B*d*(d*(a/b + x)*(-1 + Log[a/b + x]) - (c^2*Log[a/b + x])/(c + d*x) - (c
 + d*x)*(-1 + Log[c/d + x]) + c*Log[c/d + x]^2 + (c^2*(1 + Log[c/d + x]))/(
c + d*x) + (b*c^2*(Log[a + b*x] - Log[c + d*x]))/(b*c - a*d) + (-Log[a/b +
x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])*(d*x - c^2/(c + d*x) - 2*
c*Log[c + d*x]) - 2*c*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLo
g[2, (d*(a + b*x))/(-(b*c) + a*d)])) - (4*a^3*B^2*d^3*(2*b*c - 2*a*d + 2*b*
(c + d*x)*Log[a + b*x] - 2*(b*c - a*d)*Log[(e*(a + b*x))/(c + d*x)] - 2*b*(
c + d*x)*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] + (b*c - a*d)*Log[(e*(a
+ b*x))/(c + d*x)]^2 - 2*b*(c + d*x)*Log[c + d*x] - 2*b*(c + d*x)*Log[(e*(a
 + b*x))/(c + d*x)]*Log[(b*c - a*d)/(b*c + b*d*x)] + b*(c + d*x)*(Log[a + b
*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a
+ b*x))/(-(b*c) + a*d)]) + b*(c + d*x)*(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*L
og[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c - a*d)/(b*c + b*d*x)]) - 2*Poly
Log[2, (b*(c + d*x))/(b*c - a*d)])))/((b*c - a*d)*(c + d*x)) + 4*a*b^2*B^2*
d*(6*d*x + (3*d*(2*b*x - 2*(a + b*x)*Log[a/b + x] + (a + b*x)*Log[a/b + x]^
2))/b - 6*(c + d*x)*Log[c/d + x] + 3*(c + d*x)*Log[c/d + x]^2 - 2*c*Log[c/d
 + x]^3 - (3*c^2*(2 + 2*Log[c/d + x] + Log[c/d + x]^2))/(c + d*x) + 3*(-Log
[a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])^2*(d*x - c^2/(c +
d*x) - 2*c*Log[c + d*x]) - (6*(a*d + 2*b*d*x - b*d*x*Log[c/d + x] - b*c*Log
[c + d*x] + Log[a/b + x]*(-(d*(a + b*x)) + d*(a + b*x)*Log[c/d + x] + (b*c
- a*d)*Log[(b*(c + d*x))/(b*c - a*d)]) + (b*c - a*d)*PolyLog[2, (d*(a + b*x
))/(-(b*c) + a*d)]))/b + (3*c^2*(-(d*(a + b*x)*Log[a/b + x]^2) + 2*b*(c + d
*x)*Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + 2*b*(c + d*x)*PolyLog[2,
(d*(a + b*x))/(-(b*c) + a*d)]))/((-(b*c) + a*d)*(c + d*x)) + 6*(-Log[a/b +
x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])*(d*(a/b + x)*(-1 + Log[a/
b + x]) - (c^2*Log[a/b + x])/(c + d*x) - (c + d*x)*(-1 + Log[c/d + x]) + c*
Log[c/d + x]^2 + (c^2*(1 + Log[c/d + x]))/(c + d*x) + (b*c^2*(Log[a + b*x]
- Log[c + d*x]))/(b*c - a*d) - 2*c*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a
```

```
*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])) + 3*c^2*((2*Log[a/b + x]*
(1 + Log[c/d + x]))/(c + d*x) + (b*(Log[c/d + x]^2 - 2*Log[a + b*x] - 2*Log
[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[c + d*x]))/(b*c - a*d)
- (2*b*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*c - a*d)) - 6*c*(Log[a/b +
 x]^2*Log[(b*(c + d*x))/(b*c - a*d)] + 2*Log[a/b + x]*PolyLog[2, (d*(a + b*
x))/(-(b*c) + a*d)] - 2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)]) + 6*c*(Lo
g[c/d + x]^2*(Log[a/b + x] - Log[(d*(a + b*x))/(-(b*c) + a*d)]) - 2*Log[c/d
 + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 2*PolyLog[3, (b*(c + d*x))/(b
*c - a*d)])) + b^3*B^2*(d*x*(-6*c + d*x) - (8*c*d*(2*b*x - 2*(a + b*x)*Log[
a/b + x] + (a + b*x)*Log[a/b + x]^2))/b + (d^2*(b*x*(-6*a + b*x) + (6*a^2 +
 4*a*b*x - 2*b^2*x^2)*Log[a/b + x] - 2*(a^2 - b^2*x^2)*Log[a/b + x]^2))/b^2
 + (6*c^2 + 4*c*d*x - 2*d^2*x^2)*Log[c/d + x] - 2*(c^2 - d^2*x^2)*Log[c/d +
 x]^2 + 4*c^2*Log[c/d + x]^3 + (4*c^3*(2 + 2*Log[c/d + x] + Log[c/d + x]^2)
)/(c + d*x) - 8*c*(2*d*x - 2*(c + d*x)*Log[c/d + x] + (c + d*x)*Log[c/d + x
]^2) + 2*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])^2*(-
4*c*d*x + d^2*x^2 + (2*c^3)/(c + d*x) + 6*c^2*Log[c + d*x]) + 4*(-Log[a/b +
 x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])*(-4*c^2 + (4*a*c*d)/b -
c*d*x + (a*d^2*x)/b - (2*c^3)/(c + d*x) + 4*c^2*Log[c/d + x] + 4*c*d*x*Log[
c/d + x] - d^2*x^2*Log[c/d + x] - (2*c^3*Log[c/d + x])/(c + d*x) - 3*c^2*Lo
g[c/d + x]^2 - (a^2*d^2*Log[a + b*x])/b^2 + (2*b*c^3*Log[a + b*x])/(-(b*c)
+ a*d) + c^2*Log[c + d*x] + (2*b*c^3*Log[c + d*x])/(b*c - a*d) + Log[a/b +
x]*((-4*a*c*d)/b - 4*c*d*x + d^2*x^2 + (2*c^3)/(c + d*x) + 6*c^2*Log[(b*(c
+ d*x))/(b*c - a*d)]) + 6*c^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + (
4*c^3*(-(d*(a + b*x)*Log[a/b + x]^2) + 2*b*(c + d*x)*Log[a/b + x]*Log[(b*(c
 + d*x))/(b*c - a*d)] + 2*b*(c + d*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*
d)]))/((b*c - a*d)*(c + d*x)) + 12*c^2*(Log[a/b + x]^2*Log[(b*(c + d*x))/(b
*c - a*d)] + 2*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 2*Po
lyLog[3, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*((8*c*(a*d + 2*b*d*x - b*d*x*Lo
g[c/d + x] - b*c*Log[c + d*x] + Log[a/b + x]*(-(d*(a + b*x)) + d*(a + b*x)*
Log[c/d + x] + (b*c - a*d)*Log[(b*(c + d*x))/(b*c - a*d)]) + (b*c - a*d)*Po
lyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/b - (-2*a*b*c*d - 3*b^2*c*d*x - 3*
a*b*d^2*x + b^2*d^2*x^2 + 2*a*b*d^2*x*Log[c/d + x] - b^2*d^2*x^2*Log[c/d +
x] + a^2*d^2*Log[a + b*x] + b^2*c^2*Log[c + d*x] + 2*a*b*c*d*Log[c + d*x] +
 Log[a/b + x]*(b*d*(2*a*c + b*x*(2*c - d*x)) - 2*d^2*(a^2 - b^2*x^2)*Log[c/
d + x] + (-2*b^2*c^2 + 2*a^2*d^2)*Log[(b*(c + d*x))/(b*c - a*d)]) + (-2*b^2
*c^2 + 2*a^2*d^2)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/b^2 + (2*c^3*(2
*(b*c - a*d)*Log[a/b + x]*(1 + Log[c/d + x]) + b*(c + d*x)*(Log[c/d + x]^2
- 2*Log[a + b*x] - 2*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log
[c + d*x]) - 2*b*(c + d*x)*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((-(b*c)
 + a*d)*(c + d*x)) - 6*c^2*(Log[c/d + x]^2*(Log[a/b + x] - Log[(d*(a + b*x)
)/(-(b*c) + a*d)]) - 2*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] +
 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]))) + (4*a^2*b*B^2*d^2*((b*c - a*d)
*(c + d*x)*Log[c/d + x]^3 + 3*c*(b*c - a*d)*(2 + 2*Log[c/d + x] + Log[c/d +
 x]^2) + 3*(b*c - a*d)*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c
 + d*x)])^2*(c + (c + d*x)*Log[c + d*x]) + 3*c*Log[a/b + x]*(-(d*(a + b*x)*
```

```
Log[a/b + x]) + 2*b*(c + d*x)*Log[(b*(c + d*x))/(b*c - a*d)]) + 6*b*c*(c +
d*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 3*(Log[a/b + x] - Log[c/d +
 x] - Log[(e*(a + b*x))/(c + d*x)])*(-2*c*(b*c - a*d)*Log[a/b + x] + (b*c -
 a*d)*(c + d*x)*Log[c/d + x]^2 + 2*c*(b*c - a*d)*(1 + Log[c/d + x]) + 2*b*c
*(c + d*x)*(Log[a + b*x] - Log[c + d*x]) - 2*(b*c - a*d)*(c + d*x)*(Log[a/b
 + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a
*d)])) + 3*(b*c - a*d)*(c + d*x)*(Log[a/b + x]^2*Log[(b*(c + d*x))/(b*c - a
*d)] + 2*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 2*PolyLog[
3, (d*(a + b*x))/(-(b*c) + a*d)]) - 3*(c*(2*(b*c - a*d)*Log[a/b + x]*(1 + L
og[c/d + x]) + b*(c + d*x)*(Log[c/d + x]^2 - 2*Log[a + b*x] - 2*Log[c/d + x
]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[c + d*x]) - 2*b*(c + d*x)*PolyL
og[2, (b*(c + d*x))/(b*c - a*d)]) + (b*c - a*d)*(c + d*x)*(Log[c/d + x]^2*(
Log[a/b + x] - Log[(d*(a + b*x))/(-(b*c) + a*d)]) - 2*Log[c/d + x]*PolyLog[
2, (b*(c + d*x))/(b*c - a*d)] + 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])))
/((b*c - a*d)*(c + d*x))))/(4*d^4*i^2)
```

Maple [F]

$$\int \frac{(bgx+ag)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(dix+ci)^2}dx$$

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x)
```

```
[Out] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, alg
orithm="fricas")
```

```
[Out] integral((A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a
^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a
^3*g^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g
^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log((b*e*x + a*e)/(d*x + c)))/(d^
2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, alg
orithm="maxima")
[Out] 1/2*(2*c^3/(d^5*i^2*x + c*d^4*i^2) + 6*c^2*log(d*x + c)/(d^4*i^2) + (d*x^2
- 4*c*x)/(d^3*i^2))*A^2*b^3*g^3 - 3*A^2*a*b^2*(c^2/(d^4*i^2*x + c*d^3*i^2)
- x/(d^2*i^2) + 2*c*log(d*x + c)/(d^3*i^2))*g^3 + 3*A^2*a^2*b*g^3*(c/(d^3*i
^2*x + c*d^2*i^2) + log(d*x + c)/(d^2*i^2)) - 2*A*B*a^3*g^3*(log(b*e*x/(d*x
 + c) + a*e/(d*x + c))/(d^2*i^2*x + c*d*i^2) - 1/(d^2*i^2*x + c*d*i^2) - b*
log(b*x + a)/((b*c*d - a*d^2)*i^2) + b*log(d*x + c)/((b*c*d - a*d^2)*i^2))
- A^2*a^3*g^3/(d^2*i^2*x + c*d*i^2) + 1/2*(2*((b^3*c^2*d*g^3 - 2*a*b^2*c*d^
2*g^3 + a^2*b*d^3*g^3)*B^2*x + (b^3*c^3*g^3 - 2*a*b^2*c^2*d*g^3 + a^2*b*c*d
^2*g^3)*B^2)*log(d*x + c)^3 + (B^2*b^3*d^3*g^3*x^3 - 3*(b^3*c*d^2*g^3 - 2*a
*b^2*d^3*g^3)*B^2*x^2 - 2*(2*b^3*c^2*d*g^3 - 3*a*b^2*c*d^2*g^3)*B^2*x + 2*(
b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 - a^3*d^3*g^3)*B^2)*log
(d*x + c)^2)/(d^5*i^2*x + c*d^4*i^2) - integrate(-(B^2*a^3*d^3*g^3*log(e)^2
 + (B^2*b^3*d^3*g^3*log(e)^2 + 2*A*B*b^3*d^3*g^3*log(e))*x^3 + 3*(B^2*a*b^2
*d^3*g^3*log(e)^2 + 2*A*B*a*b^2*d^3*g^3*log(e))*x^2 + (B^2*b^3*d^3*g^3*x^3
+ 3*B^2*a*b^2*d^3*g^3*x^2 + 3*B^2*a^2*b*d^3*g^3*x + B^2*a^3*d^3*g^3)*log(b*
x + a)^2 + 3*(B^2*a^2*b*d^3*g^3*log(e)^2 + 2*A*B*a^2*b*d^3*g^3*log(e))*x +
2*(B^2*a^3*d^3*g^3*log(e) + (B^2*b^3*d^3*g^3*log(e) + A*B*b^3*d^3*g^3)*x^3
+ 3*(B^2*a*b^2*d^3*g^3*log(e) + A*B*a*b^2*d^3*g^3)*x^2 + 3*(B^2*a^2*b*d^3*g
^3*log(e) + A*B*a^2*b*d^3*g^3)*x)*log(b*x + a) - ((2*A*B*b^3*d^3*g^3 + (2*g
^3*log(e) + g^3)*B^2*b^3*d^3)*x^3 + 2*(b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*
a^2*b*c*d^2*g^3 + (g^3*log(e) - g^3)*a^3*d^3)*B^2 + 3*(2*A*B*a*b^2*d^3*g^3
- (b^3*c*d^2*g^3 - 2*(g^3*log(e) + g^3)*a*b^2*d^3)*B^2)*x^2 + 2*(3*A*B*a^2*
b*d^3*g^3 + (3*a^2*b*d^3*g^3*log(e) - 2*b^3*c^2*d*g^3 + 3*a*b^2*c*d^2*g^3)*
B^2)*x + 2*(B^2*b^3*d^3*g^3*x^3 + 3*B^2*a*b^2*d^3*g^3*x^2 + 3*B^2*a^2*b*d^3
*g^3*x + B^2*a^3*d^3*g^3)*log(b*x + a))*log(d*x + c))/(d^5*i^2*x^2 + 2*c*d^
4*i^2*x + c^2*d^3*i^2), x)
```

Giac [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, alg
orithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*i
)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(a\,g+b\,g\,x)^3 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(c\,i+d\,i\,x)^2}\,dx$$

```
[In] int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)^
2,x)
```

```
[Out] int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)^
2, x)
```

3.93 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 469

$$\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx \\
&= -\frac{2AB(bc-ad)g^2(a+bx)}{d^2i^2(c+dx)}+\frac{2B^2(bc-ad)g^2(a+bx)}{d^2i^2(c+dx)} \\
&\quad-\frac{2B^2(bc-ad)g^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^2i^2(c+dx)} \\
&\quad+\frac{2bB(bc-ad)g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i^2} \\
&\quad+\frac{bg^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^2}+\frac{(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^2(c+dx)} \\
&\quad+\frac{2b(bc-ad)g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^2} \\
&\quad+\frac{2bB^2(bc-ad)g^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2} \\
&\quad+\frac{4bB(bc-ad)g^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2} \\
&\quad-\frac{4bB^2(bc-ad)g^2\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2}
\end{aligned}$$

```
[Out] -2*A*B*(-a*d+b*c)*g^2*(b*x+a)/d^2/i^2/(d*x+c)+2*B^2*(-a*d+b*c)*g^2*(b*x+a)/
d^2/i^2/(d*x+c)-2*B^2*(-a*d+b*c)*g^2*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/d^2/i^2/
```

```
(d*x+c)+2*b*B*(-a*d+b*c)*g^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*
x+c)))/d^3/i^2+b*g^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^2/i^2+(-a*d+b*
c)*g^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^2/i^2/(d*x+c)+2*b*(-a*d+b*c)
*g^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i^2+2*b*B^2
*(-a*d+b*c)*g^2*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i^2+4*b*B*(-a*d+b*c)*g^2
*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i^2-4*b*B^2
*(-a*d+b*c)*g^2*polylog(3,d*(b*x+a)/b/(d*x+c))/d^3/i^2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2562, 2395, 2333, 2332, 2355, 2354, 2438, 2421, 6724}

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2} \, dx$$
$$= \frac{4bBg^2(bc-ad)\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^3i^2} + \frac{2bg^2(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^3i^2} + \frac{2bBg^2(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^3i^2} + \frac{bg^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^2i^2} + \frac{g^2(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^2i^2(c+dx)} - \frac{2ABg^2(a+bx)(bc-ad)}{d^2i^2(c+dx)} + \frac{2bB^2g^2(bc-ad)\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2} - \frac{4bB^2g^2(bc-ad)\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2} - \frac{2B^2g^2(a+bx)(bc-ad)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^2i^2(c+dx)} + \frac{2B^2g^2(a+bx)(bc-ad)}{d^2i^2(c+dx)}$$

```
[In] Int[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x)^
2,x]
```

```
[Out] (-2*A*B*(b*c - a*d)*g^2*(a + b*x))/(d^2*i^2*(c + d*x)) + (2*B^2*(b*c - a*d)
*g^2*(a + b*x))/(d^2*i^2*(c + d*x)) - (2*B^2*(b*c - a*d)*g^2*(a + b*x)*Log[
(e*(a + b*x))/(c + d*x)])/(d^2*i^2*(c + d*x)) + (2*b*B*(b*c - a*d)*g^2*Log[
(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d^3*i^2)
+ (b*g^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d^2*i^2) + ((b*
```

c - a*d)*g^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d^2*i^2*(c + d*x)) + (2*b*(b*c - a*d)*g^2*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d^3*i^2) + (2*b*B^2*(b*c - a*d)*g^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^3*i^2) + (4*b*B*(b*c - a*d)*g^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^3*i^2) - (4*b*B^2*(b*c - a*d)*g^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(d^3*i^2)

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
```

```
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{((bc-ad)g^2)\,\text{Subst}\left(\int \frac{x^2(A+B\log(ex))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{i^2}\\
&= \frac{((bc-ad)g^2)\,\text{Subst}\left(\int \left(\frac{(A+B\log(ex))^2}{d^2}+\frac{b^2(A+B\log(ex))^2}{d^2(-b+dx)^2}+\frac{2b(A+B\log(ex))^2}{d^2(-b+dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{i^2}\\
&= \frac{((bc-ad)g^2)\,\text{Subst}\left(\int (A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^2}\\
&\quad+\frac{(2b(bc-ad)g^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{-b+dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^2}\\
&\quad+\frac{(b^2(bc-ad)g^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(-b+dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^2}\end{aligned}$$

$$= \frac{bg^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^2} + \frac{(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^2(c+dx)}$$
$$+ \frac{2b(bc-ad)g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^2}$$
$$- \frac{(4bB(bc-ad)g^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^2}$$
$$- \frac{(2B(bc-ad)g^2)\,\text{Subst}\left(\int (A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^2}$$
$$+ \frac{(2bB(bc-ad)g^2)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{-b+dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^2}$$

$$= -\frac{2AB(bc-ad)g^2(a+bx)}{d^2i^2(c+dx)} + \frac{2bB(bc-ad)g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i^2}$$
$$+ \frac{bg^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^2}$$
$$+ \frac{(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^2(c+dx)}$$
$$+ \frac{2b(bc-ad)g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^2}$$
$$+ \frac{4bB(bc-ad)g^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2}$$
$$- \frac{(2bB^2(bc-ad)g^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^2}$$
$$- \frac{(4bB^2(bc-ad)g^2)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^2}$$
$$- \frac{(2B^2(bc-ad)g^2)\,\text{Subst}\left(\int \log(ex)\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^2}$$

$$
\begin{aligned}
&= -\frac{2AB(bc-ad)g^2(a+bx)}{d^2i^2(c+dx)} + \frac{2B^2(bc-ad)g^2(a+bx)}{d^2i^2(c+dx)} \\
&- \frac{2B^2(bc-ad)g^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^2i^2(c+dx)} \\
&+ \frac{2bB(bc-ad)g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i^2} \\
&+ \frac{bg^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^2} \\
&+ \frac{(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^2(c+dx)} \\
&+ \frac{2b(bc-ad)g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^2} \\
&+ \frac{2bB^2(bc-ad)g^2\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2} \\
&+ \frac{4bB(bc-ad)g^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2} \\
&- \frac{4bB^2(bc-ad)g^2\operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2622 vs. $2(469) = 938$.

Time = 3.22 (sec) , antiderivative size = 2622, normalized size of antiderivative = 5.59

$$
\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d
*i*x)^2,x]
```

```
[Out] (g^2*(3*A^2*b^2*d*x - (3*A^2*(b*c - a*d)^2)/(c + d*x) + 6*A^2*b*(-(b*c) + a
*d)*Log[c + d*x] + (6*a^2*A*B*d^2*(b*c - a*d + b*(c + d*x)*Log[a/b + x] + (
-(b*c) + a*d)*Log[(e*(a + b*x))/(c + d*x)] - b*c*Log[(b*(c + d*x))/(b*c - a
*d)] - b*d*x*Log[(b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)*(c + d*x)) + 6*a
*A*b*B*d*(-Log[c/d + x]^2 + 2*Log[c/d + x]*Log[c + d*x] + 2*(-(c/(c + d*x))
 + (b*c*Log[a + b*x])/(-(b*c) + a*d) + (b*c*Log[c + d*x])/(b*c - a*d) - Log
[a/b + x]*Log[c + d*x] + Log[(e*(a + b*x))/(c + d*x)]*(c/(c + d*x) + Log[c
+ d*x]) + Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)]) + 2*PolyLog[2, (d*(a
```

+ b*x))/(-(b*c) + a*d)]) + 6*A*b^2*B*(d*(a/b + x)*(-1 + Log[a/b + x]) - (c
^2*Log[a/b + x])/(c + d*x) - (c + d*x)*(-1 + Log[c/d + x]) + c*Log[c/d + x]
^2 + (c^2*(1 + Log[c/d + x]))/(c + d*x) + (b*c^2*(Log[a + b*x] - Log[c + d*
x]))/(b*c - a*d) + (-Log[a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d
x)])(d*x - c^2/(c + d*x) - 2*c*Log[c + d*x]) - 2*c*(Log[a/b + x]*Log[(b*(
c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])) - (3*a^
2*B^2*d^2*(2*b*c - 2*a*d + 2*b*(c + d*x)*Log[a + b*x] - 2*(b*c - a*d)*Log[(
e*(a + b*x))/(c + d*x)] - 2*b*(c + d*x)*Log[a + b*x]*Log[(e*(a + b*x))/(c +
d*x)] + (b*c - a*d)*Log[(e*(a + b*x))/(c + d*x)]^2 - 2*b*(c + d*x)*Log[c +
d*x] - 2*b*(c + d*x)*Log[(e*(a + b*x))/(c + d*x)]*Log[(b*c - a*d)/(b*c + b
*d*x)] + b*(c + d*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c
- a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*(c + d*x)*(Log[
(b*c - a*d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c
- a*d)/(b*c + b*d*x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/((b*c -
a*d)*(c + d*x)) + b^2*B^2*(6*d*x + (3*d*(2*b*x - 2*(a + b*x)*Log[a/b + x]
+ (a + b*x)*Log[a/b + x]^2))/b - 6*(c + d*x)*Log[c/d + x] + 3*(c + d*x)*Log
[c/d + x]^2 - 2*c*Log[c/d + x]^3 - (3*c^2*(2 + 2*Log[c/d + x] + Log[c/d + x
]^2))/(c + d*x) + 3*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c +
d*x)])^2*(d*x - c^2/(c + d*x) - 2*c*Log[c + d*x]) - (6*(a*d + 2*b*d*x - b*d
*x*Log[c/d + x] - b*c*Log[c + d*x] + Log[a/b + x]*(-(d*(a + b*x)) + d*(a +
b*x)*Log[c/d + x] + (b*c - a*d)*Log[(b*(c + d*x))/(b*c - a*d)]) + (b*c - a*
d)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/b + (3*c^2*(-(d*(a + b*x)*Log
[a/b + x]^2) + 2*b*(c + d*x)*Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] +
2*b*(c + d*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/((-(b*c) + a*d)*(c
+ d*x)) + 6*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])*
(d*(a/b + x)*(-1 + Log[a/b + x]) - (c^2*Log[a/b + x])/(c + d*x) - (c + d*x)
*(-1 + Log[c/d + x]) + c*Log[c/d + x]^2 + (c^2*(1 + Log[c/d + x]))/(c + d*x
) + (b*c^2*(Log[a + b*x] - Log[c + d*x]))/(b*c - a*d) - 2*c*(Log[a/b + x]*L
og[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))
+ 3*c^2*((2*Log[a/b + x]*(1 + Log[c/d + x]))/(c + d*x) + (b*(Log[c/d + x]^2
- 2*Log[a + b*x] - 2*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Lo
g[c + d*x]))/(b*c - a*d) - (2*b*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*c
- a*d)) - 6*c*(Log[a/b + x]^2*Log[(b*(c + d*x))/(b*c - a*d)] + 2*Log[a/b +
x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 2*PolyLog[3, (d*(a + b*x))/(
-(b*c) + a*d)]) + 6*c*(Log[c/d + x]^2*(Log[a/b + x] - Log[(d*(a + b*x))/(-(
b*c) + a*d)]) - 2*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 2*Po
lyLog[3, (b*(c + d*x))/(b*c - a*d)])) + (2*a*b*B^2*d*((b*c - a*d)*(c + d*x)
*Log[c/d + x]^3 + 3*c*(b*c - a*d)*(2 + 2*Log[c/d + x] + Log[c/d + x]^2) + 3
*(b*c - a*d)*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])^
2*(c + (c + d*x)*Log[c + d*x]) + 3*c*Log[a/b + x]*(-(d*(a + b*x)*Log[a/b +
x]) + 2*b*(c + d*x)*Log[(b*(c + d*x))/(b*c - a*d)]) + 6*b*c*(c + d*x)*PolyL
og[2, (d*(a + b*x))/(-(b*c) + a*d)] + 3*(Log[a/b + x] - Log[c/d + x] - Log[
(e*(a + b*x))/(c + d*x)])*(-2*c*(b*c - a*d)*Log[a/b + x] + (b*c - a*d)*(c +
d*x)*Log[c/d + x]^2 + 2*c*(b*c - a*d)*(1 + Log[c/d + x]) + 2*b*c*(c + d*x)
*(Log[a + b*x] - Log[c + d*x]) - 2*(b*c - a*d)*(c + d*x)*(Log[a/b + x]*Log[

```
(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])) + 3
*(b*c - a*d)*(c + d*x)*(Log[a/b + x]^2*Log[(b*(c + d*x))/(b*c - a*d)] + 2*L
og[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 2*PolyLog[3, (d*(a +
 b*x))/(-(b*c) + a*d)]) - 3*(c*(2*(b*c - a*d)*Log[a/b + x]*(1 + Log[c/d + x
]) + b*(c + d*x)*(Log[c/d + x]^2 - 2*Log[a + b*x] - 2*Log[c/d + x]*Log[(d*(
a + b*x))/(-(b*c) + a*d)] + 2*Log[c + d*x]) - 2*b*(c + d*x)*PolyLog[2, (b*(
c + d*x))/(b*c - a*d)]) + (b*c - a*d)*(c + d*x)*(Log[c/d + x]^2*(Log[a/b +
x] - Log[(d*(a + b*x))/(-(b*c) + a*d)]) - 2*Log[c/d + x]*PolyLog[2, (b*(c +
 d*x))/(b*c - a*d)] + 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]))))/((b*c - a
*d)*(c + d*x))))/(3*d^3*i^2)
```

Maple [F]

$$\int \frac{(bgx+ag)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(dix+ci)^2}dx$$

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, alg
orithm="fricas")
```

```
[Out] integral((A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^
2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*
b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log((b*e*x + a*e)/(d*x + c)))/
(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, alg
orithm="maxima")
```

```
[Out] -A^2*b^2*(c^2/(d^4*i^2*x + c*d^3*i^2) - x/(d^2*i^2) + 2*c*log(d*x + c)/(d^3
*i^2))*g^2 + 2*A^2*a*b*g^2*(c/(d^3*i^2*x + c*d^2*i^2) + log(d*x + c)/(d^2*i
^2)) - 2*A*B*a^2*g^2*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^2*i^2*x + c*d
*i^2) - 1/(d^2*i^2*x + c*d*i^2) - b*log(b*x + a)/((b*c*d - a*d^2)*i^2) + b*
log(d*x + c)/((b*c*d - a*d^2)*i^2)) - A^2*a^2*g^2/(d^2*i^2*x + c*d*i^2) - 1
/3*(2*((b^2*c*d*g^2 - a*b*d^2*g^2)*B^2*x + (b^2*c^2*g^2 - a*b*c*d*g^2)*B^2)
*log(d*x + c)^3 - 3*(B^2*b^2*d^2*g^2*x^2 + B^2*b^2*c*d*g^2*x - (b^2*c^2*g^2
 - 2*a*b*c*d*g^2 + a^2*d^2*g^2)*B^2)*log(d*x + c)^2)/(d^4*i^2*x + c*d^3*i^2
) - integrate(-(B^2*a^2*d^2*g^2*log(e)^2 + (B^2*b^2*d^2*g^2*log(e)^2 + 2*A*
B*b^2*d^2*g^2*log(e))*x^2 + (B^2*b^2*d^2*g^2*x^2 + 2*B^2*a*b*d^2*g^2*x + B^
2*a^2*d^2*g^2)*log(b*x + a)^2 + 2*(B^2*a*b*d^2*g^2*log(e)^2 + 2*A*B*a*b*d^2
*g^2*log(e))*x + 2*(B^2*a^2*d^2*g^2*log(e) + (B^2*b^2*d^2*g^2*log(e) + A*B*
b^2*d^2*g^2)*x^2 + 2*(B^2*a*b*d^2*g^2*log(e) + A*B*a*b*d^2*g^2)*x)*log(b*x
+ a) + 2*((b^2*c^2*g^2 - 2*a*b*c*d*g^2 - (g^2*log(e) - g^2)*a^2*d^2)*B^2 -
(A*B*b^2*d^2*g^2 + (g^2*log(e) + g^2)*B^2*b^2*d^2)*x^2 - (2*A*B*a*b*d^2*g^2
 + (2*a*b*d^2*g^2*log(e) + b^2*c*d*g^2)*B^2)*x - (B^2*b^2*d^2*g^2*x^2 + 2*B
^2*a*b*d^2*g^2*x + B^2*a^2*d^2*g^2)*log(b*x + a))*log(d*x + c))/(d^4*i^2*x^
2 + 2*c*d^3*i^2*x + c^2*d^2*i^2), x)
```

Giac [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, alg
orithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*i
)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(c\,i+d\,i\,x)^2}\,dx$$

```
[In] int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)^
2,x)
[Out] int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)^
2, x)
```

3.94 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2} dx$

Optimal result

Integrand size = 40, antiderivative size = 261

$$\begin{aligned}
&\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2} dx \\
&= \frac{2ABg(a+bx)}{di^2(c+dx)} - \frac{2B^2g(a+bx)}{di^2(c+dx)} + \frac{2B^2g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{di^2(c+dx)} \\
&\quad - \frac{g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{di^2(c+dx)} - \frac{bg\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^2} \\
&\quad - \frac{2bBg\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2} + \frac{2bB^2g\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2}
\end{aligned}$$

```
[Out] 2*A*B*g*(b*x+a)/d/i^2/(d*x+c)-2*B^2*g*(b*x+a)/d/i^2/(d*x+c)+2*B^2*g*(b*x+a)
*ln(e*(b*x+a)/(d*x+c))/d/i^2/(d*x+c)-g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^
2/d/i^2/(d*x+c)-b*g*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/
d^2/i^2-2*b*B*g*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))/
d^2/i^2+2*b*B^2*g*polylog(3,d*(b*x+a)/b/(d*x+c))/d^2/i^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {2562, 2395, 2333, 2332, 2354, 2421, 6724}

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$$
$$=-\frac{2bBg\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^2i^2}$$
$$-\frac{bg\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^2i^2}-\frac{g(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{di^2(c+dx)}$$
$$+\frac{2ABg(a+bx)}{di^2(c+dx)}+\frac{2bB^2g\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2}$$
$$+\frac{2B^2g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{di^2(c+dx)}-\frac{2B^2g(a+bx)}{di^2(c+dx)}$$

```
[In] Int[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x)^2,
x]
```

```
[Out] (2*A*B*g*(a + b*x))/(d*i^2*(c + d*x)) - (2*B^2*g*(a + b*x))/(d*i^2*(c + d*x
)) + (2*B^2*g*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(d*i^2*(c + d*x)) - (
g*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d*i^2*(c + d*x)) - (b*
g*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d
^2*i^2) - (2*b*B*g*(A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (d*(a +
b*x))/(b*(c + d*x))])/(d^2*i^2) + (2*b*B^2*g*PolyLog[3, (d*(a + b*x))/(b*(c
 + d*x))])/(d^2*i^2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
```

```
Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g\text{Subst}\left(\int \frac{x(A+B\log(ex))^2}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{i^2}\\
&= \frac{g\text{Subst}\left(\int \left(-\frac{(A+B\log(ex))^2}{d}-\frac{b(A+B\log(ex))^2}{d(-b+dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{i^2}\\
&= -\frac{g\text{Subst}\left(\int (A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{di^2}-\frac{(bg)\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{-b+dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{di^2}
\end{aligned}$$

$$= -\frac{g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{di^2(c+dx)} - \frac{bg\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^2} + \frac{(2bBg)\text{Subst}\left(\int \frac{(A+B\log(ex))\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^2} + \frac{(2Bg)\text{Subst}\left(\int(A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{di^2}$$

$$= \frac{2ABg(a+bx)}{di^2(c+dx)} - \frac{g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{di^2(c+dx)} - \frac{bg\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^2} - \frac{2bBg\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2} + \frac{(2bB^2g)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^2} + \frac{(2B^2g)\,\text{Subst}\left(\int\log(ex)\,dx, x, \frac{a+bx}{c+dx}\right)}{di^2}$$

$$= \frac{2ABg(a+bx)}{di^2(c+dx)} - \frac{2B^2g(a+bx)}{di^2(c+dx)} + \frac{2B^2g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{di^2(c+dx)} - \frac{g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{di^2(c+dx)} - \frac{bg\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^2} - \frac{2bBg\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2} + \frac{2bB^2g\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1412 vs. $2(261) = 522$.

Time = 1.02 (sec) , antiderivative size = 1412, normalized size of antiderivative = 5.41

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$$

$$= \frac{g\left(\frac{3A^2(bc-ad)}{c+dx} + 3A^2b\log(c+dx) + \frac{6aABd\left(bc-ad+b(c+dx)\log\left(\frac{a}{b}+x\right)+(-bc+ad)\log\left(\frac{e(a+bx)}{c+dx}\right)-bc\log\left(\frac{b(c+dx)}{bc-ad}\right)-bdx\log\left(\frac{b(c+dx}{bc-ac}\right.\right.}{(bc-ad)(c+dx)}\right.}{}$$

```
[In] Integrate[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i
*x)^2,x]
```

[Out] (g*((3*A^2*(b*c - a*d))/(c + d*x) + 3*A^2*b*Log[c + d*x] + (6*a*A*B*d*(b*c - a*d + b*(c + d*x)*Log[a/b + x] + (-(b*c) + a*d)*Log[(e*(a + b*x))/(c + d*x)] - b*c*Log[(b*(c + d*x))/(b*c - a*d)] - b*d*x*Log[(b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)*(c + d*x)) + 3*A*b*B*(-Log[c/d + x]^2 + 2*Log[c/d + x]*Log[c + d*x] + 2*(-(c/(c + d*x)) + (b*c*Log[a + b*x])/(-(b*c) + a*d) + (b*c*Log[c + d*x])/(b*c - a*d) - Log[a/b + x]*Log[c + d*x] + Log[(e*(a + b*x))/(c + d*x)]*(c/(c + d*x) + Log[c + d*x]) + Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)]) + 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - (3*a*B^2*d*(2*b*c - 2*a*d + 2*b*(c + d*x)*Log[a + b*x] - 2*(b*c - a*d)*Log[(e*(a + b*x))/(c + d*x)] - 2*b*(c + d*x)*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] + (b*c - a*d)*Log[(e*(a + b*x))/(c + d*x)]^2 - 2*b*(c + d*x)*Log[c + d*x] - 2*b*(c + d*x)*Log[(e*(a + b*x))/(c + d*x)]*Log[(b*c - a*d)/(b*c + b*d*x)] + b*(c + d*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*(c + d*x)*(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c - a*d)/(b*c + b*d*x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/((b*c - a*d)*(c + d*x)) + (b*B^2*((b*c - a*d)*(c + d*x)*Log[c/d + x]^3 + 3*c*(b*c - a*d)*(2 + 2*Log[c/d + x] + Log[c/d + x]^2) + 3*(b*c - a*d)*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])^2*(c + (c + d*x)*Log[c + d*x]) + 3*c*Log[a/b + x]*(-(d*(a + b*x)*Log[a/b + x]) + 2*b*(c + d*x)*Log[(b*(c + d*x))/(b*c - a*d)]) + 6*b*c*(c + d*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 3*(Log[a/b + x] - Log[c/d + x] - Log[(e*(a + b*x))/(c + d*x)])*(-2*c*(b*c - a*d)*Log[a/b + x] + (b*c - a*d)*(c + d*x)*Log[c/d + x]^2 + 2*c*(b*c - a*d)*(1 + Log[c/d + x]) + 2*b*c*(c + d*x)*(Log[a + b*x] - Log[c + d*x]) - 2*(b*c - a*d)*(c + d*x)*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])) + 3*(b*c - a*d)*(c + d*x)*(Log[a/b + x]^2*Log[(b*(c + d*x))/(b*c - a*d)] + 2*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)]) - 3*(c*(2*(b*c - a*d)*Log[a/b + x]*(1 + Log[c/d + x]) + b*(c + d*x)*(Log[c/d + x]^2 - 2*Log[a + b*x] - 2*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[c + d*x]) - 2*b*(c + d*x)*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + (b*c - a*d)*(c + d*x)*(Log[c/d + x]^2*(Log[a/b + x] - Log[(d*(a + b*x))/(-(b*c) + a*d)]) - 2*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]))))/((b*c - a*d)*(c + d*x))))/(3*d^2*i^2)

Maple [F]

$$\int \frac{(bgx + ag)\left(A + B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(dix + ci)^2}dx$$

[In] int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x)

[Out] int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x)

Fricas [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, algor
ithm="fricas")
```

```
[Out] integral((A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b*e*x + a*e)/(d*
x + c))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((b*e*x + a*e)/(d*x + c)))/(d^2*i^2*
x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, algor
ithm="maxima")
```

```
[Out] A^2*b*g*(c/(d^3*i^2*x + c*d^2*i^2) + log(d*x + c)/(d^2*i^2)) - 2*A*B*a*g*(l
og(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^2*i^2*x + c*d*i^2) - 1/(d^2*i^2*x +
c*d*i^2) - b*log(b*x + a)/((b*c*d - a*d^2)*i^2) + b*log(d*x + c)/((b*c*d -
a*d^2)*i^2)) - A^2*a*g/(d^2*i^2*x + c*d*i^2) + 1/3*(3*(b*c*g - a*d*g)*B^2*l
og(d*x + c)^2 + (B^2*b*d*g*x + B^2*b*c*g)*log(d*x + c)^3)/(d^3*i^2*x + c*d^
2*i^2) - integrate(-(B^2*a*d*g*log(e)^2 + (B^2*b*d*g*x + B^2*a*d*g)*log(b*x
 + a)^2 + (B^2*b*d*g*log(e)^2 + 2*A*B*b*d*g*log(e))*x + 2*(B^2*a*d*g*log(e)
 + (B^2*b*d*g*log(e) + A*B*b*d*g)*x)*log(b*x + a) - 2*(((g*log(e) - g)*a*d
+ b*c*g)*B^2 + (B^2*b*d*g*log(e) + A*B*b*d*g)*x + (B^2*b*d*g*x + B^2*a*d*g)
*log(b*x + a))*log(d*x + c))/(d^3*i^2*x^2 + 2*c*d^2*i^2*x + c^2*d*i^2), x)
```

Giac [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, algor
ithm="giac")
```

```
[Out] integrate((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*i)^
2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx=\int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{{(c\,i+d\,i\,x)}^2}\,dx$$

```
[In] int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)^2,
x)
```

```
[Out] int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)^2,
 x)
```

3.95 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 152

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = -\frac{2AB(a+bx)}{(bc-ad)i^2(c+dx)}+\frac{2B^2(a+bx)}{(bc-ad)i^2(c+dx)} - \frac{2B^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)i^2(c+dx)} + \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)i^2(c+dx)}$$

```
[Out] -2*A*B*(b*x+a)/(-a*d+b*c)/i^2/(d*x+c)+2*B^2*(b*x+a)/(-a*d+b*c)/i^2/(d*x+c)-
2*B^2*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(-a*d+b*c)/i^2/(d*x+c)+(b*x+a)*(A+B*ln(
e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)/i^2/(d*x+c)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2552, 2333, 2332}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{i^2(c+dx)(bc-ad)} - \frac{2AB(a+bx)}{i^2(c+dx)(bc-ad)} - \frac{2B^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{i^2(c+dx)(bc-ad)} + \frac{2B^2(a+bx)}{i^2(c+dx)(bc-ad)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(c*i + d*i*x)^2,x]
```

```
[Out] (-2*A*B*(a + b*x))/((b*c - a*d)*i^2*(c + d*x)) + (2*B^2*(a + b*x))/((b*c -
a*d)*i^2*(c + d*x)) - (2*B^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/((b*c
- a*d)*i^2*(c + d*x)) + ((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/
((b*c - a*d)*i^2*(c + d*x))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}\\
&= \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)i^2(c+dx)} - \frac{(2B)\text{Subst}\left(\int (A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}\\
&= -\frac{2AB(a+bx)}{(bc-ad)i^2(c+dx)} + \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)i^2(c+dx)}\\
&\quad - \frac{(2B^2)\,\text{Subst}\left(\int \log(ex)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}\\
&= -\frac{2AB(a+bx)}{(bc-ad)i^2(c+dx)} + \frac{2B^2(a+bx)}{(bc-ad)i^2(c+dx)}\\
&\quad - \frac{2B^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)i^2(c+dx)} + \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)i^2(c+dx)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.07

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$$
$$= \frac{-\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + \frac{B\left(2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+2b(c+dx)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2b(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\cdots}{\cdots}}{\cdots}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(c*i + d*i*x)^2,x]
```

```
[Out] (-(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(2*(b*c - a*d)*(A + B*Log[(e*
(a + b*x))/(c + d*x)]) + 2*b*(c + d*x)*Log[a + b*x]*(A + B*Log[(e*(a + b*x)
)/(c + d*x)]) - 2*b*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c +
d*x] - 2*B*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]
) - b*B*(c + d*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c -
a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*(c + d*x)*((2*Lo
g[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2,
 (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(d*i^2*(c + d*x))
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.20

method	result
norman	$\frac{\frac{(A^2-2BA+2B^2)x}{ic}-\frac{B^2 a \ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{i(ad-cb)}-\frac{B^2 bx \ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{(ad-cb)i}-\frac{2aB(-B+A)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{i(ad-cb)}-\frac{2Bb(-B+A)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{i(ad-cb)}}{i(dx+c)}$
parts	$-\frac{A^2}{i^2(dx+c)d}-\frac{B^2\left(\frac{e(bx+a)\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{dx+c}-\frac{2e(bx+a)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{dx+c}+\frac{2e(bx+a)}{dx+c}\right)}{i^2e(ad-cb)}-\frac{2BA\left(\frac{e(bx+a)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{dx+c}-\frac{e(bx+}{dx+}\right.}{i^2e(ad-cb)}$
parallelrisch	$-\frac{-2ABab\,d^3+2AB\,b^2c\,d^2+B^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)^2b^2d^3-2B^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)b^2d^3+B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2ab\,d^3-2B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{i^2(dx+c)b\,d^3(ad-cb}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{d^2A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{(ad-cb)^2e^2i^2}+\frac{2d^2AB\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad-cb)e}{d(dx+c)}-\frac{be}{d}\right)}{(ad-cb)^2e^2i^2}+\frac{d^2B^2\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\right.}{}\right.}{d^2}$
default	$-\frac{e(ad-cb)\left(\frac{d^2A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{(ad-cb)^2e^2i^2}+\frac{2d^2AB\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad-cb)e}{d(dx+c)}-\frac{be}{d}\right)}{(ad-cb)^2e^2i^2}+\frac{d^2B^2\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\right.}{}\right.}{d^2}$
risch	$-\frac{A^2}{i^2(dx+c)d}-\frac{B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2bx}{i^2(ad-cb)(dx+c)}-\frac{B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2a}{i^2(ad-cb)(dx+c)}+\frac{2B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)bx}{i^2(ad-cb)(dx+c)}+\frac{2B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a}{i^2(ad-cb)(dx+c)}-\frac{2B}{i^2(ad-c}$

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((A^2-2*A*B+2*B^2)/i/c*x-B^2*a/i/(a*d-b*c)*ln(e*(b*x+a)/(d*x+c))^2-B^2*b/(a
*d-b*c)/i*x*ln(e*(b*x+a)/(d*x+c))^2-2*a*B*(-B+A)/i/(a*d-b*c)*ln(e*(b*x+a)/(
d*x+c))-2*B*b*(-B+A)/i/(a*d-b*c)*x*ln(e*(b*x+a)/(d*x+c)))/i/(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.02

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx =$$

$$-\frac{(A^2-2\,AB+2\,B^2)bc-(A^2-2\,AB+2\,B^2)ad-(B^2bdx+B^2ad)\log\left(\frac{bex+ae}{dx+c}\right)^2-2\left((AB-B^2)bdx+(\right.}{(bcd^2-ad^3)i^2x+(bc^2d-acd^2)i^2}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, algorithm="fricas
")
```

```
[Out] -((A^2 - 2*A*B + 2*B^2)*b*c - (A^2 - 2*A*B + 2*B^2)*a*d - (B^2*b*d*x + B^2*
a*d)*log((b*e*x + a*e)/(d*x + c))^2 - 2*((A*B - B^2)*b*d*x + (A*B - B^2)*a*
d)*log((b*e*x + a*e)/(d*x + c)))/((b*c*d^2 - a*d^3)*i^2*x + (b*c^2*d - a*c*
d^2)*i^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(128) = 256$.

Time = 1.14 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.84

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \frac{2Bb(A-B)\log\left(x+\frac{2ABabd+2ABb^2c-2B^2abd-2B^2b^2c-\frac{2Ba^2bd^2(A-B)}{ad-bc}+\frac{4Bab^2cd(A-B)}{ad-bc}-\frac{2Bb^3c^2(A-B)}{ad-bc}}{4ABb^2d-4B^2b^2d}\right)}{di^2\left(ad-bc\right)} - \frac{2Bb(A-B)\log\left(x+\frac{2ABabd+2ABb^2c-2B^2abd-2B^2b^2c+\frac{2Ba^2bd^2(A-B)}{ad-bc}-\frac{4Bab^2cd(A-B)}{ad-bc}+\frac{2Bb^3c^2(A-B)}{ad-bc}}{4ABb^2d-4B^2b^2d}\right)}{di^2\left(ad-bc\right)} + \frac{(-2AB+2B^2)\log\left(\frac{e(a+bx)}{c+dx}\right)}{cdi^2+d^2i^2x} + \frac{(-B^2a-B^2bx)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{acdi^2+ad^2i^2x-bc^2i^2-bcdi^2x} + \frac{-A^2+2AB-2B^2}{cdi^2+d^2i^2x}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**2,x)
```

```
[Out] 2*B*b*(A - B)*log(x + (2*A*B*a*b*d + 2*A*B*b**2*c - 2*B**2*a*b*d - 2*B**2*b
**2*c - 2*B*a**2*b*d**2*(A - B)/(a*d - b*c) + 4*B*a*b**2*c*d*(A - B)/(a*d -
 b*c) - 2*B*b**3*c**2*(A - B)/(a*d - b*c))/(4*A*B*b**2*d - 4*B**2*b**2*d))/
(d*i**2*(a*d - b*c)) - 2*B*b*(A - B)*log(x + (2*A*B*a*b*d + 2*A*B*b**2*c -
2*B**2*a*b*d - 2*B**2*b**2*c + 2*B*a**2*b*d**2*(A - B)/(a*d - b*c) - 4*B*a*
b**2*c*d*(A - B)/(a*d - b*c) + 2*B*b**3*c**2*(A - B)/(a*d - b*c))/(4*A*B*b*
*2*d - 4*B**2*b**2*d))/(d*i**2*(a*d - b*c)) + (-2*A*B + 2*B**2)*log(e*(a +
b*x)/(c + d*x))/(c*d*i**2 + d**2*i**2*x) + (-B**2*a - B**2*b*x)*log(e*(a +
b*x)/(c + d*x))**2/(a*c*d*i**2 + a*d**2*i**2*x - b*c**2*i**2 - b*c*d*i**2*x
) + (-A**2 + 2*A*B - 2*B**2)/(c*d*i**2 + d**2*i**2*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(152) = 304$.

Time = 0.22 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.74

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \left(2\left(\frac{1}{d^2i^2x+cdi^2}+\frac{b\log(bx+a)}{(bcd-ad^2)i^2}-\frac{b\log(dx+c)}{(bcd-ad^2)i^2}\right)\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{(bdx+bc)\log(bx+a)^2 -}{} \right. - 2\,AB\left(\frac{\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{d^2i^2x+cdi^2}-\frac{1}{d^2i^2x+cdi^2}-\frac{b\log(bx+a)}{(bcd-ad^2)i^2}+\frac{b\log(dx+c)}{(bcd-ad^2)i^2}\right) - \frac{B^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)^2}{d^2i^2x+cdi^2}-\frac{A^2}{d^2i^2x+cdi^2}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, algorithm="maxima
")
```

```
[Out] (2*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*i^2) - b*log(
d*x + c)/((b*c*d - a*d^2)*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - ((b*
d*x + b*c)*log(b*x + a)^2 + (b*d*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*a*d +
2*(b*d*x + b*c)*log(b*x + a) - 2*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a))
*log(d*x + c))/(b*c^2*d*i^2 - a*c*d^2*i^2 + (b*c*d^2*i^2 - a*d^3*i^2)*x))*B
^2 - 2*A*B*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^2*i^2*x + c*d*i^2) - 1/
(d^2*i^2*x + c*d*i^2) - b*log(b*x + a)/((b*c*d - a*d^2)*i^2) + b*log(d*x +
c)/((b*c*d - a*d^2)*i^2)) - B^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(d^2
*i^2*x + c*d*i^2) - A^2/(d^2*i^2*x + c*d*i^2)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.14

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$$
$$=\left(\frac{(bex+ae)B^2\log\left(\frac{bex+ae}{dx+c}\right)^2}{(dx+c)i^2}+\frac{2\,(bex+ae)(AB-B^2)\log\left(\frac{bex+ae}{dx+c}\right)}{(dx+c)i^2}+\frac{(bex+ae)(A^2-2\,AB+2\,B^2)}{(dx+c)i^2}\right)\Bigg(\frac{}{(}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, algorithm="giac")
```

```
[Out] ((b*e*x + a*e)*B^2*log((b*e*x + a*e)/(d*x + c))^2/((d*x + c)*i^2) + 2*(b*e*
x + a*e)*(A*B - B^2)*log((b*e*x + a*e)/(d*x + c))/((d*x + c)*i^2) + (b*e*x
+ a*e)*(A^2 - 2*A*B + 2*B^2)/((d*x + c)*i^2))*(b*c/((b*c*e - a*d*e)*(b*c -
a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.46

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx=\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{2\,B^2}{b\,d^2\,i^2}-\frac{2\,A\,B}{b\,d^2\,i^2}\right)}{\frac{x}{b}+\frac{c}{b\,d}}$$
$$-\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{B^2}{d^2\,i^2\,\left(x+\frac{c}{d}\right)}+\frac{B^2\,b}{d\,i^2\,\left(a\,d-b\,c\right)}\right)$$
$$-\frac{A^2-2\,A\,B+2\,B^2}{x\,d^2\,i^2+c\,d\,i^2}$$
$$+\frac{B\,b\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{a\,d^2\,i^2+b\,c\,d\,i^2}{d\,i^2}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,\left(A-B\right)\,4\mathrm{i}}{d\,i^2\,\left(a\,d-b\,c\right)}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(c*i + d*i*x)^2,x)
```

```
[Out] (log((e*(a + b*x))/(c + d*x))*((2*B^2)/(b*d^2*i^2) - (2*A*B)/(b*d^2*i^2)))/
(x/b + c/(b*d)) - log((e*(a + b*x))/(c + d*x))^2*(B^2/(d^2*i^2*(x + c/d)) +
 (B^2*b)/(d*i^2*(a*d - b*c))) - (A^2 + 2*B^2 - 2*A*B)/(d^2*i^2*x + c*d*i^2)
 + (B*b*atan(((2*b*d*x + (a*d^2*i^2 + b*c*d*i^2)/(d*i^2))*1i)/(a*d - b*c))*
(A - B)*4i)/(d*i^2*(a*d - b*c))
```

3.96 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 214

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx = \frac{2ABd(a+bx)}{(bc-ad)^2gi^2(c+dx)} - \frac{2B^2d(a+bx)}{(bc-ad)^2gi^2(c+dx)} + \frac{2B^2d(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^2gi^2(c+dx)} - \frac{d(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^2gi^2(c+dx)} + \frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^2gi^2}$$

```
[Out] 2*A*B*d*(b*x+a)/(-a*d+b*c)^2/g/i^2/(d*x+c)-2*B^2*d*(b*x+a)/(-a*d+b*c)^2/g/i
^2/(d*x+c)+2*B^2*d*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(-a*d+b*c)^2/g/i^2/(d*x+c)
-d*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^2/g/i^2/(d*x+c)+1/3*b*(
A+B*ln(e*(b*x+a)/(d*x+c)))^3/B/(-a*d+b*c)^2/g/i^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2562, 2388, 2339, 30, 2333, 2332}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx = \frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^3}{3Bgi^2(bc-ad)^2} - \frac{d(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{gi^2(c+dx)(bc-ad)^2} + \frac{2ABd(a+bx)}{gi^2(c+dx)(bc-ad)^2} + \frac{2B^2d(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{gi^2(c+dx)(bc-ad)^2} - \frac{2B^2d(a+bx)}{gi^2(c+dx)(bc-ad)^2}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)*(c*i + d*i*x)^2),
x]
```

```
[Out] (2*A*B*d*(a + b*x))/((b*c - a*d)^2*g*i^2*(c + d*x)) - (2*B^2*d*(a + b*x))/(
(b*c - a*d)^2*g*i^2*(c + d*x)) + (2*B^2*d*(a + b*x)*Log[(e*(a + b*x))/(c +
d*x)])/((b*c - a*d)^2*g*i^2*(c + d*x)) - (d*(a + b*x)*(A + B*Log[(e*(a + b*
x))/(c + d*x)])^2)/((b*c - a*d)^2*g*i^2*(c + d*x)) + (b*(A + B*Log[(e*(a +
b*x))/(c + d*x)])^3)/(3*B*(b*c - a*d)^2*g*i^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
```

```
x]
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2gi^2} \\
&= \frac{b\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2gi^2} - \frac{d\text{Subst}\left(\int (A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2gi^2} \\
&= -\frac{d(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^2gi^2(c+dx)} + \frac{b\text{Subst}\left(\int x^2\,dx, x, A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B(bc-ad)^2gi^2} \\
&\quad + \frac{(2Bd)\text{Subst}\left(\int (A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2gi^2} \\
&= \frac{2ABd(a+bx)}{(bc-ad)^2gi^2(c+dx)} - \frac{d(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^2gi^2(c+dx)} \\
&\quad + \frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^2gi^2} + \frac{(2B^2d)\,\text{Subst}\left(\int \log(ex)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2gi^2} \\
&= \frac{2ABd(a+bx)}{(bc-ad)^2gi^2(c+dx)} - \frac{2B^2d(a+bx)}{(bc-ad)^2gi^2(c+dx)} + \frac{2B^2d(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^2gi^2(c+dx)} \\
&\quad - \frac{d(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^2gi^2(c+dx)} + \frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^2gi^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.87

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \frac{3b(A^2-2AB+2B^2)\,(c+dx)\log(a+bx)+6(A-B)B(bc-ad)\log\left(\frac{e(a+bx)}{c+dx}\right)+3B(-Bd(a+bx)+Ab}{3(bc-a}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)*(c*i + d*i*
x)^2),x]
```

```
[Out] (3*b*(A^2 - 2*A*B + 2*B^2)*(c + d*x)*Log[a + b*x] + 6*(A - B)*B*(b*c - a*d)
*Log[(e*(a + b*x))/(c + d*x)] + 3*B*(-(B*d*(a + b*x)) + A*b*(c + d*x))*Log[
(e*(a + b*x))/(c + d*x)]^2 + b*B^2*(c + d*x)*Log[(e*(a + b*x))/(c + d*x)]^3
 - 3*(A^2 - 2*A*B + 2*B^2)*(-(b*c) + a*d + b*(c + d*x)*Log[c + d*x]))/(3*(b
*c - a*d)^2*g*i^2*(c + d*x))
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.86

method	result
norman	$\frac{\frac{\left(A^2bc-2ABad+2B^2ad\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{B(Abc-Bad)\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{gi\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{d\left(A^2b-2ABb+2B^2b\right)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{\left(A^2-2BA+2B^2\right)}{gic(ad-cb)}}{i(dx+c)}$
parallelrisch	$\frac{-3A^2ab^2d^4+3A^2b^3cd^3+6ABab^2d^4-6ABb^3cd^3-6B^2ab^2d^4+6B^2b^3cd^3+B^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)^3b^3d^4-3B^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{}$
parts	$\frac{A^2\left(-\frac{1}{(ad-cb)(dx+c)}-\frac{b\ln(dx+c)}{(ad-cb)^2}+\frac{b\ln(bx+a)}{(ad-cb)^2}\right)}{g\,i^2}-\frac{B^2\left(\frac{d\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2-2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\right.\right.}{ad-cb}\right.}{g\,i^2(ad-cb)e}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^2A^2b\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^2(ad-cb)^3g}+\frac{d^3A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^2(ad-cb)^3g}-\frac{d^2ABb\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{e\,i^2(ad-cb)^3g}+\frac{2d^3AB\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\right.}{e^2i^2}\right.}{}$
default	$-\frac{e(ad-cb)\left(-\frac{d^2A^2b\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^2(ad-cb)^3g}+\frac{d^3A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^2(ad-cb)^3g}-\frac{d^2ABb\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{e\,i^2(ad-cb)^3g}+\frac{2d^3AB\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\right.}{e^2i^2}\right.}{}$
risch	$-\frac{A^2}{g\,i^2(ad-cb)(dx+c)}-\frac{A^2b\ln(dx+c)}{g\,i^2(ad-cb)^2}+\frac{A^2b\ln(bx+a)}{g\,i^2(ad-cb)^2}-\frac{B^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2b}{g\,i^2(ad-cb)^2}-\frac{B^2d\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2a}{g\,i^2(ad-cb)^2(dx+c)}+$

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x,method=_RETUR
```

```
NVERBOSE)
```

```
[Out] ((A^2*b*c-2*A*B*a*d+2*B^2*a*d)/g/i/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x+a)
/(d*x+c))+B*(A*b*c-B*a*d)/g/i/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x+a)/(d*x
+c))^2+1/g/i*d*(A^2*b-2*A*B*b+2*B^2*b)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(
b*x+a)/(d*x+c))+(A^2-2*A*B+2*B^2)*d/g/i/c/(a*d-b*c)*x+b*d*B*(-B+A)/g/i/(a^2
*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)/(d*x+c))^2+1/3*B^2*b*c/g/i/(a^2*d^2-
2*a*b*c*d+b^2*c^2)*ln(e*(b*x+a)/(d*x+c))^3+1/3*b*B^2*d/g/i/(a^2*d^2-2*a*b*c
*d+b^2*c^2)*x*ln(e*(b*x+a)/(d*x+c))^3)/i/(d*x+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.10

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx = \frac{(B^2bdx+B^2bc)\log\left(\frac{bex+ae}{dx+c}\right)^3+3\,(A^2-2\,AB+2\,B^2)bc-3\,(A^2-2\,AB+2\,B^2)ad+3\,(ABbc-B^2ad+(A}{3\,((b^2c^2d-2\,abcd^2+a^2d^3)gi^2x+(b^2}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x, algor
ithm="fricas")
```

```
[Out] 1/3*((B^2*b*d*x + B^2*b*c)*log((b*e*x + a*e)/(d*x + c))^3 + 3*(A^2 - 2*A*B
+ 2*B^2)*b*c - 3*(A^2 - 2*A*B + 2*B^2)*a*d + 3*(A*B*b*c - B^2*a*d + (A*B -
B^2)*b*d*x)*log((b*e*x + a*e)/(d*x + c))^2 + 3*(A^2*b*c + (A^2 - 2*A*B + 2*
B^2)*b*d*x - 2*(A*B - B^2)*a*d)*log((b*e*x + a*e)/(d*x + c)))/((b^2*c^2*d -
 2*a*b*c*d^2 + a^2*d^3)*g*i^2*x + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*g*i^2
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(185) = 370$.

Time = 0.73 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.52

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \frac{B^2b\log\left(\frac{e(a+bx)}{c+dx}\right)^3}{3a^2d^2gi^2 - 6abcdgi^2 + 3b^2c^2gi^2} + \frac{(-2AB+2B^2)\log\left(\frac{e(a+bx)}{c+dx}\right)}{acdgi^2 + ad^2gi^2x - bc^2gi^2 - bcdgi^2x} + \left(A^2 - 2AB + 2B^2\right)\left(-\frac{b\log\left(x + \frac{-\frac{a^3bd^3}{(ad-bc)^2} + \frac{3a^2b^2cd^2}{(ad-bc)^2} - \frac{3ab^3c^2d}{(ad-bc)^2} + abd + \frac{b^4c^3}{(ad-bc)^2} + b^2c}{2b^2d}\right)}{gi^2(ad-bc)^2} + \frac{b\log\left(x + \frac{\frac{a^3bd^3}{(ad-bc)^2} - \frac{3a^2b^2cd^2}{(ad-bc)^2} + \frac{3ab^3c^2d}{(ad-bc)^2} + abd - \frac{b^4c^3}{(ad-bc)^2} + b^2c}{2b^2d}\right)}{gi^2(ad-bc)^2} - \frac{1}{acdgi^2 - bc^2gi^2 + x(ad^2gi^2 - bcdgi^2)}\right) + \frac{(ABbc + ABbdx - B^2ad - B^2bdx)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{a^2cd^2gi^2 + a^2d^3gi^2x - 2abc^2dgi^2 - 2abcd^2gi^2x + b^2c^3gi^2 + b^2c^2dgi^2x}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)/(d*i*x+c*i)**2,x)
```

```
[Out] B**2*b*log(e*(a + b*x)/(c + d*x))**3/(3*a**2*d**2*g*i**2 - 6*a*b*c*d*g*i**2
 + 3*b**2*c**2*g*i**2) + (-2*A*B + 2*B**2)*log(e*(a + b*x)/(c + d*x))/(a*c*
d*g*i**2 + a*d**2*g*i**2*x - b*c**2*g*i**2 - b*c*d*g*i**2*x) + (A**2 - 2*A*
B + 2*B**2)*(-b*log(x + (-a**3*b*d**3/(a*d - b*c)**2 + 3*a**2*b**2*c*d**2/(
a*d - b*c)**2 - 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d + b**4*c**3/(a*d - b
*c)**2 + b**2*c)/(2*b**2*d))/(g*i**2*(a*d - b*c)**2) + b*log(x + (a**3*b*d*
*3/(a*d - b*c)**2 - 3*a**2*b**2*c*d**2/(a*d - b*c)**2 + 3*a*b**3*c**2*d/(a*
d - b*c)**2 + a*b*d - b**4*c**3/(a*d - b*c)**2 + b**2*c)/(2*b**2*d))/(g*i**
2*(a*d - b*c)**2) - 1/(a*c*d*g*i**2 - b*c**2*g*i**2 + x*(a*d**2*g*i**2 - b*
c*d*g*i**2))) + (A*B*b*c + A*B*b*d*x - B**2*a*d - B**2*b*d*x)*log(e*(a + b*
x)/(c + d*x))**2/(a**2*c*d**2*g*i**2 + a**2*d**3*g*i**2*x - 2*a*b*c**2*d*g*
i**2 - 2*a*b*c*d**2*g*i**2*x + b**2*c**3*g*i**2 + b**2*c**2*d*g*i**2*x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1004 vs. $2(212) = 424$.

Time = 0.26 (sec) , antiderivative size = 1004, normalized size of antiderivative = 4.69

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= B^2\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2} + \frac{b\log(bx+a)}{(b^2c^2-2abcd+a^2d^2)gi^2} - \frac{b\log(dx+c)}{(b^2c^2-2abcd+a^2d^2)gi^2}\right)\log\left(\frac{be\ldots}{dx+\ldots} + \frac{ae}{dx+c}\right)^2$$
$$+2AB\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2} + \frac{b\log(bx+a)}{(b^2c^2-2abcd+a^2d^2)gi^2} - \frac{b\log(dx+c)}{(b^2c^2-2abcd+a^2d^2)gi^2}\right)\log\left(\ldots + \frac{ae}{dx+c}\right)$$
$$-\frac{1}{3}B^2\left(\frac{3\left((bdx+bc)\log(bx+a)^2+(bdx+bc)\log(dx+c)^2+2bc-2ad+2(bdx+bc)\log(bx+a)-2\ldots\right.}{b^2c^3gi^2-2abc^2dgi^2+a^2cd^2gi^2+(b^2c^2dgi^2-2ab\ldots}\right.$$
$$+A^2\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2} + \frac{b\log(bx+a)}{(b^2c^2-2abcd+a^2d^2)gi^2} - \frac{b\log(dx+c)}{(b^2c^2-2abcd+a^2d^2)gi^2}\right)$$
$$-\frac{\left((bdx+bc)\log(bx+a)^2+(bdx+bc)\log(dx+c)^2+2bc-2ad+2(bdx+bc)\log(bx+a)-2(bdx+b\ldots\right.}{b^2c^3gi^2-2abc^2dgi^2+a^2cd^2gi^2+(b^2c^2dgi^2-2abcd^2gi^2+a^2d^3g\ldots}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x, algor
ithm="maxima")
```

```
[Out] B^2*(1/((b*c*d - a*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/(
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*g*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2 + 2*A*B*(1/((b
*c*d - a*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/((b^2*c^2 -
 2*a*b*c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*g*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 1/3*B^2*(3*((b*d*x + b*c
)*log(b*x + a)^2 + (b*d*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*a*d + 2*(b*d*x
+ b*c)*log(b*x + a) - 2*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a))*log(d*x
+ c))*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*c^3*g*i^2 - 2*a*b*c^2*d*g*i
^2 + a^2*c*d^2*g*i^2 + (b^2*c^2*d*g*i^2 - 2*a*b*c*d^2*g*i^2 + a^2*d^3*g*i^2
)*x) - ((b*d*x + b*c)*log(b*x + a)^3 - (b*d*x + b*c)*log(d*x + c)^3 + 3*(b*
d*x + b*c)*log(b*x + a)^2 + 3*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a))*lo
g(d*x + c)^2 + 6*b*c - 6*a*d + 6*(b*d*x + b*c)*log(b*x + a) - 3*(2*b*d*x +
(b*d*x + b*c)*log(b*x + a)^2 + 2*b*c + 2*(b*d*x + b*c)*log(b*x + a))*log(d*
x + c))/(b^2*c^3*g*i^2 - 2*a*b*c^2*d*g*i^2 + a^2*c*d^2*g*i^2 + (b^2*c^2*d*g
*i^2 - 2*a*b*c*d^2*g*i^2 + a^2*d^3*g*i^2)*x)) + A^2*(1/((b*c*d - a*d^2)*g*i
^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/((b^2*c^2 - 2*a*b*c*d + a^2*
```

```
d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g*i^2)) - ((b
*d*x + b*c)*log(b*x + a)^2 + (b*d*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*a*d +
 2*(b*d*x + b*c)*log(b*x + a) - 2*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a)
)*log(d*x + c))*A*B/(b^2*c^3*g*i^2 - 2*a*b*c^2*d*g*i^2 + a^2*c*d^2*g*i^2 +
(b^2*c^2*d*g*i^2 - 2*a*b*c*d^2*g*i^2 + a^2*d^3*g*i^2)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.58

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$
$$=\frac{1}{3}\left(\frac{B^2be\log\left(\frac{bex+ae}{dx+c}\right)^3}{bcgi^2-adgi^2}+\frac{3\,A^2be\log\left(\frac{bex+ae}{dx+c}\right)}{bcgi^2-adgi^2}+3\left(\frac{ABbe}{bcgi^2-adgi^2}-\frac{(bex+ae)B^2d}{(bcgi^2-adgi^2)(dx+c)}\right)\log\left(\frac{bex+}{dx+}\right.\right.$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x, algor
ithm="giac")
```

```
[Out] 1/3*(B^2*b*e*log((b*e*x + a*e)/(d*x + c))^3/(b*c*g*i^2 - a*d*g*i^2) + 3*A^2
*b*e*log((b*e*x + a*e)/(d*x + c))/(b*c*g*i^2 - a*d*g*i^2) + 3*(A*B*b*e/(b*c
*g*i^2 - a*d*g*i^2) - (b*e*x + a*e)*B^2*d/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c
)))*log((b*e*x + a*e)/(d*x + c))^2 - 6*(A*B*d - B^2*d)*(b*e*x + a*e)*log((b
*e*x + a*e)/(d*x + c))/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)) - 3*(A^2*d - 2*A
*B*d + 2*B^2*d)*(b*e*x + a*e)/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)))*(b*c/((b
*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.98

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$
$$=\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{B\,b\,(A-B)}{g\,i^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}-\frac{B^2\,(a\,d-b\,c)}{b\,d\,g\,i^2\,\left(\frac{x}{b}+\frac{c}{b\,d}\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)-\frac{A^2-2\,A\,B+2\,B^2}{(a\,d-b\,c)\,\left(c\,g\,i^2+d\,g\,i^2\,x\right)}+\frac{B^2\,b\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^3}{3\,g\,i^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}-\frac{2\,B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,(A-B)\,(a\,d-b\,c)}{b\,d\,g\,i^2\,\left(\frac{x}{b}+\frac{c}{b\,d}\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}-\frac{b\,\mathrm{atan}\left(\frac{b\,\left(2\,b\,d\,x+\frac{a^2\,d^2\,g\,i^2-b^2\,c^2\,g\,i^2}{g\,i^2\,(a\,d-b\,c)}\right)\,\left(A^2-2\,A\,B+2\,B^2\right)\,1\mathrm{i}}{(a\,d-b\,c)\,\left(b\,A^2-2\,b\,A\,B+2\,b\,B^2\right)}\right)\,\left(A^2-2\,A\,B+2\,B^2\right)\,2\mathrm{i}}{g\,i^2\,{(a\,d-b\,c)}^2}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)*(c*i + d*i*x)^2),
x)
```

```
[Out] log((e*(a + b*x))/(c + d*x))^2*((B*b*(A - B))/(g*i^2*(a^2*d^2 + b^2*c^2 - 2
*a*b*c*d)) - (B^2*(a*d - b*c))/(b*d*g*i^2*(x/b + c/(b*d))*(a^2*d^2 + b^2*c^
2 - 2*a*b*c*d))) - (A^2 + 2*B^2 - 2*A*B)/((a*d - b*c)*(c*g*i^2 + d*g*i^2*x)
) + (B^2*b*log((e*(a + b*x))/(c + d*x))^3)/(3*g*i^2*(a^2*d^2 + b^2*c^2 - 2*
a*b*c*d)) - (b*atan((b*(2*b*d*x + (a^2*d^2*g*i^2 - b^2*c^2*g*i^2)/(g*i^2*(a
*d - b*c)))*(A^2 + 2*B^2 - 2*A*B)*1i)/((a*d - b*c)*(A^2*b + 2*B^2*b - 2*A*B
*b)))*(A^2 + 2*B^2 - 2*A*B)*2i)/(g*i^2*(a*d - b*c)^2) - (2*B*log((e*(a + b*
x))/(c + d*x))*(A - B)*(a*d - b*c))/(b*d*g*i^2*(x/b + c/(b*d))*(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d))
```

3.97 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 365

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = -\frac{2ABd^2(a+bx)}{(bc-ad)^3g^2i^2(c+dx)}+\frac{2B^2d^2(a+bx)}{(bc-ad)^3g^2i^2(c+dx)} - \frac{2b^2B^2(c+dx)}{(bc-ad)^3g^2i^2(a+bx)}-\frac{2B^2d^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^3g^2i^2(c+dx)} - \frac{2b^2B(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^2i^2(a+bx)} + \frac{d^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3g^2i^2(c+dx)} - \frac{b^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3g^2i^2(a+bx)} - \frac{2bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^3g^2i^2}$$

```
[Out] -2*A*B*d^2*(b*x+a)/(-a*d+b*c)^3/g^2/i^2/(d*x+c)+2*B^2*d^2*(b*x+a)/(-a*d+b*c
)^3/g^2/i^2/(d*x+c)-2*b^2*B^2*(d*x+c)/(-a*d+b*c)^3/g^2/i^2/(b*x+a)-2*B^2*d^
2*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(-a*d+b*c)^3/g^2/i^2/(d*x+c)-2*b^2*B*(d*x+c
)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^2/i^2/(b*x+a)+d^2*(b*x+a)*(A+B
*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^2/i^2/(d*x+c)-b^2*(d*x+c)*(A+B*ln(
e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^2/i^2/(b*x+a)-2/3*b*d*(A+B*ln(e*(b*x+a
)/(d*x+c)))^3/B/(-a*d+b*c)^3/g^2/i^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2562, 2395, 2333, 2332, 2342, 2341, 2339, 30}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = -\frac{b^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^2i^2(a+bx)(bc-ad)^3} - \frac{2b^2B(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2i^2(a+bx)(bc-ad)^3} + \frac{d^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^2i^2(c+dx)(bc-ad)^3} - \frac{2ABd^2(a+bx)}{g^2i^2(c+dx)(bc-ad)^3} - \frac{2bd\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^3}{3Bg^2i^2(bc-ad)^3} - \frac{2b^2B^2(c+dx)}{g^2i^2(a+bx)(bc-ad)^3} - \frac{2B^2d^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{g^2i^2(c+dx)(bc-ad)^3} + \frac{2B^2d^2(a+bx)}{g^2i^2(c+dx)(bc-ad)^3}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^2*(c*i + d*i*x)^2
),x]
```

```
[Out] (-2*A*B*d^2*(a + b*x))/((b*c - a*d)^3*g^2*i^2*(c + d*x)) + (2*B^2*d^2*(a +
b*x))/((b*c - a*d)^3*g^2*i^2*(c + d*x)) - (2*b^2*B^2*(c + d*x))/((b*c - a*d
)^3*g^2*i^2*(a + b*x)) - (2*B^2*d^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])
/((b*c - a*d)^3*g^2*i^2*(c + d*x)) - (2*b^2*B*(c + d*x)*(A + B*Log[(e*(a +
b*x))/(c + d*x)]))/((b*c - a*d)^3*g^2*i^2*(a + b*x)) + (d^2*(a + b*x)*(A +
B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*c - a*d)^3*g^2*i^2*(c + d*x)) - (b^2
*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*c - a*d)^3*g^2*i^2*(
a + b*x)) - (2*b*d*(A + B*Log[(e*(a + b*x))/(c + d*x)])^3)/(3*B*(b*c - a*d)
^3*g^2*i^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}$$

$$= \frac{\text{Subst}\left(\int \left(d^2(A+B\log(ex))^2 + \frac{b^2(A+B\log(ex))^2}{x^2} - \frac{2bd(A+B\log(ex))^2}{x}\right) dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3 g^2 i^2}$$

$$= \frac{b^2\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^2} dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3 g^2 i^2} - \frac{(2bd)\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x} dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3 g^2 i^2} + \frac{d^2\text{Subst}\left(\int (A+B\log(ex))^2 dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3 g^2 i^2}$$

$$= \frac{d^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3 g^2 i^2 (c+dx)} - \frac{b^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3 g^2 i^2 (a+bx)} + \frac{(2b^2B)\text{Subst}\left(\int \frac{A+B\log(ex)}{x^2} dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3 g^2 i^2} - \frac{(2bd)\text{Subst}\left(\int x^2 dx, x, A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B(bc-ad)^3 g^2 i^2} - \frac{(2Bd^2)\text{Subst}\left(\int (A+B\log(ex)) dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3 g^2 i^2}$$

$$= -\frac{2ABd^2(a+bx)}{(bc-ad)^3 g^2 i^2 (c+dx)} - \frac{2b^2B^2(c+dx)}{(bc-ad)^3 g^2 i^2 (a+bx)} - \frac{2b^2B(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3 g^2 i^2 (a+bx)} + \frac{d^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3 g^2 i^2 (c+dx)} - \frac{b^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3 g^2 i^2 (a+bx)} - \frac{2bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^3 g^2 i^2} - \frac{(2B^2d^2)\text{Subst}\left(\int \log(ex) dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3 g^2 i^2}$$

$$= -\frac{2ABd^2(a+bx)}{(bc-ad)^3 g^2 i^2 (c+dx)} + \frac{2B^2d^2(a+bx)}{(bc-ad)^3 g^2 i^2 (c+dx)} - \frac{2b^2B^2(c+dx)}{(bc-ad)^3 g^2 i^2 (a+bx)} - \frac{2B^2d^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^3 g^2 i^2 (c+dx)} - \frac{2b^2B(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3 g^2 i^2 (a+bx)} + \frac{d^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3 g^2 i^2 (c+dx)} - \frac{b^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3 g^2 i^2 (a+bx)} - \frac{2bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^3 g^2 i^2}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.84

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx =$$
$$-\frac{-3(A^2-2AB+2B^2)\,d(-bc+ad)(a+bx)+3b(A^2+2AB+2B^2)\,(bc-ad)(c+dx)+6b(A^2+2B^2)}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^2*(c*i + d*
i*x)^2),x]
```

```
[Out] -1/3*(-3*(A^2 - 2*A*B + 2*B^2)*d*(-(b*c) + a*d)*(a + b*x) + 3*b*(A^2 + 2*A*
B + 2*B^2)*(b*c - a*d)*(c + d*x) + 6*b*(A^2 + 2*B^2)*d*(a + b*x)*(c + d*x)*
Log[a + b*x] + 6*B*(b*c - a*d)*(A*b*c + b*B*c + a*A*d - a*B*d + 2*A*b*d*x)*
Log[(e*(a + b*x))/(c + d*x)] + 3*B*(-(a^2*B*d^2) + 2*a*b*d*(-(B*d*x) + A*(c
 + d*x)) + b^2*(2*A*d*x*(c + d*x) + B*c*(c + 2*d*x)))*Log[(e*(a + b*x))/(c
+ d*x)]^2 + 2*b*B^2*d*(a + b*x)*(c + d*x)*Log[(e*(a + b*x))/(c + d*x)]^3 -
6*b*(A^2 + 2*B^2)*d*(a + b*x)*(c + d*x)*Log[c + d*x])/((b*c - a*d)^3*g^2*i^
2*(a + b*x)*(c + d*x))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. 2(363) = 726.

Time = 1.22 (sec) , antiderivative size = 766, normalized size of antiderivative = 2.10

method	result
parts	$\frac{A^2\left(-\frac{d}{(ad-cb)^2(dx+c)}-\frac{2db\ln(dx+c)}{(ad-cb)^3}-\frac{b}{(ad-cb)^2(bx+a)}+\frac{2db\ln(bx+a)}{(ad-cb)^3}\right)}{g^2i^2} - \frac{B^2\left(\frac{d^2\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2-2\right.}{}\right.}{}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^2A^2b^2}{i^2(ad-cb)^4g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{2d^3A^2b\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^2(ad-cb)^4g^2}+\frac{d^4A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^2(ad-cb)^4g^2}+\frac{2d^2ABb^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-}{d(dx}\right.}{\frac{be}{d}+\frac{(ad-}{d(dx}}\right.}{i^2(ad-}\right.}{}$
default	$-\frac{e(ad-cb)\left(-\frac{d^2A^2b^2}{i^2(ad-cb)^4g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{2d^3A^2b\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^2(ad-cb)^4g^2}+\frac{d^4A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^2(ad-cb)^4g^2}+\frac{2d^2ABb^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-}{d(dx}\right.}{\frac{be}{d}+\frac{(ad-}{d(dx}}\right.}{i^2(ad-}\right.}{}$
norman	$\frac{\frac{\left(2A^2abcd-2ABa^2d^2+2ABb^2c^2+2B^2a^2d^2+2B^2b^2c^2\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^3d^3-3a^2bc\,d^2+3ab^2c^2d-b^3c^3\right)}+\frac{B\left(2Aabcd-Ba^2d^2+Bb^2c^2\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{gi\left(a^3d^3-3a^2bc\,d^2+3ab^2c^2d-b^3c^3\right)}+\frac{\left(2A^2abd^2+2\right.}{}}{}$
parallelrisch	Expression too large to display
risch	Expression too large to display

[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x,method=_RETURNVERBOSE)

[Out] 1/g^2*A^2/i^2*(-d/(a*d-b*c)^2/(d*x+c)-2*d/(a*d-b*c)^3*b*ln(d*x+c)-b/(a*d-b*c)^2/(b*x+a)+2*d/(a*d-b*c)^3*b*ln(b*x+a))-B^2/g^2/i^2/(a*d-b*c)/e*(d^2/(a*d-b*c)^2*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+2*(a*d-b*c)*e/d/(d*x+c)+2*b*e/d)-2/3/(a*d-b*c)^2*b*d*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3+1/(a*d-b*c)^2*e^2*b^2*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))-2*B*A/g^2/i^2/(a*d-b*c)/e*(d^2/(a*d-b*c)^2*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)-1/(a*d-b*c)^2*b*d*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+1/(a*d-b*c)^2*e^2*b^2*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.41

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = \frac{12\,ABabcd-3\,(A^2+2\,AB+2\,B^2)b^2c^2+3\,(A^2-2\,AB+2\,B^2)a^2d^2-2\,(B^2b^2d^2x^2+B^2abcd+(B^2b^2cd}{}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, alg
orithm="fricas")
```

```
[Out] 1/3*(12*A*B*a*b*c*d - 3*(A^2 + 2*A*B + 2*B^2)*b^2*c^2 + 3*(A^2 - 2*A*B + 2*
B^2)*a^2*d^2 - 2*(B^2*b^2*d^2*x^2 + B^2*a*b*c*d + (B^2*b^2*c*d + B^2*a*b*d^
2)*x)*log((b*e*x + a*e)/(d*x + c))^3 - 3*(2*A*B*b^2*d^2*x^2 + B^2*b^2*c^2 +
 2*A*B*a*b*c*d - B^2*a^2*d^2 + 2*((A*B + B^2)*b^2*c*d + (A*B - B^2)*a*b*d^2
)*x)*log((b*e*x + a*e)/(d*x + c))^2 - 6*((A^2 + 2*B^2)*b^2*c*d - (A^2 + 2*B
^2)*a*b*d^2)*x - 6*((A^2 + 2*B^2)*b^2*d^2*x^2 + A^2*a*b*c*d + (A*B + B^2)*b
^2*c^2 - (A*B - B^2)*a^2*d^2 + ((A^2 + 2*A*B + 2*B^2)*b^2*c*d + (A^2 - 2*A*
B + 2*B^2)*a*b*d^2)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^4*c^3*d - 3*a*b^3*
c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*g^2*i^2*x^2 + (b^4*c^4 - 2*a*b^3*c^3
*d + 2*a^3*b*c*d^3 - a^4*d^4)*g^2*i^2*x + (a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*
a^3*b*c^2*d^2 - a^4*c*d^3)*g^2*i^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1404 vs. 2(335) = 670.

Time = 3.08 (sec) , antiderivative size = 1404, normalized size of antiderivative = 3.85

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2/(d*i*x+c*i)**2,x)
```

```
[Out] 2*B**2*b*d*log(e*(a + b*x)/(c + d*x))**3/(3*a**3*d**3*g**2*i**2 - 9*a**2*b*
c*d**2*g**2*i**2 + 9*a*b**2*c**2*d*g**2*i**2 - 3*b**3*c**3*g**2*i**2) - 2*b
*d*(A**2 + 2*B**2)*log(x + (2*A**2*a*b*d**2 + 2*A**2*b**2*c*d + 4*B**2*a*b*
d**2 + 4*B**2*b**2*c*d - 2*a**4*b*d**5*(A**2 + 2*B**2)/(a*d - b*c)**3 + 8*a
**3*b**2*c*d**4*(A**2 + 2*B**2)/(a*d - b*c)**3 - 12*a**2*b**3*c**2*d**3*(A*
*2 + 2*B**2)/(a*d - b*c)**3 + 8*a*b**4*c**3*d**2*(A**2 + 2*B**2)/(a*d - b*c
)**3 - 2*b**5*c**4*d*(A**2 + 2*B**2)/(a*d - b*c)**3)/(4*A**2*b**2*d**2 + 8*
```

```
B**2*b**2*d**2))/(g**2*i**2*(a*d - b*c)**3) + 2*b*d*(A**2 + 2*B**2)*log(x +
 (2*A**2*a*b*d**2 + 2*A**2*b**2*c*d + 4*B**2*a*b*d**2 + 4*B**2*b**2*c*d + 2
*a**4*b*d**5*(A**2 + 2*B**2)/(a*d - b*c)**3 - 8*a**3*b**2*c*d**4*(A**2 + 2*
B**2)/(a*d - b*c)**3 + 12*a**2*b**3*c**2*d**3*(A**2 + 2*B**2)/(a*d - b*c)**
3 - 8*a*b**4*c**3*d**2*(A**2 + 2*B**2)/(a*d - b*c)**3 + 2*b**5*c**4*d*(A**2
 + 2*B**2)/(a*d - b*c)**3)/(4*A**2*b**2*d**2 + 8*B**2*b**2*d**2))/(g**2*i**
2*(a*d - b*c)**3) + (-2*A*B*a*d - 2*A*B*b*c - 4*A*B*b*d*x + 2*B**2*a*d - 2*
B**2*b*c)*log(e*(a + b*x)/(c + d*x))/(a**3*c*d**2*g**2*i**2 + a**3*d**3*g**
2*i**2*x - 2*a**2*b*c**2*d*g**2*i**2 - a**2*b*c*d**2*g**2*i**2*x + a**2*b*d
**3*g**2*i**2*x**2 + a*b**2*c**3*g**2*i**2 - a*b**2*c**2*d*g**2*i**2*x - 2*
a*b**2*c*d**2*g**2*i**2*x**2 + b**3*c**3*g**2*i**2*x + b**3*c**2*d*g**2*i**
2*x**2) + (2*A*B*a*b*c*d + 2*A*B*a*b*d**2*x + 2*A*B*b**2*c*d*x + 2*A*B*b**2
*d**2*x**2 - B**2*a**2*d**2 - 2*B**2*a*b*d**2*x + B**2*b**2*c**2 + 2*B**2*b
**2*c*d*x)*log(e*(a + b*x)/(c + d*x))**2/(a**4*c*d**3*g**2*i**2 + a**4*d**4
*g**2*i**2*x - 3*a**3*b*c**2*d**2*g**2*i**2 - 2*a**3*b*c*d**3*g**2*i**2*x +
 a**3*b*d**4*g**2*i**2*x**2 + 3*a**2*b**2*c**3*d*g**2*i**2 - 3*a**2*b**2*c*
d**3*g**2*i**2*x**2 - a*b**3*c**4*g**2*i**2 + 2*a*b**3*c**3*d*g**2*i**2*x +
 3*a*b**3*c**2*d**2*g**2*i**2*x**2 - b**4*c**4*g**2*i**2*x - b**4*c**3*d*g*
*2*i**2*x**2) - (A**2*a*d + A**2*b*c - 2*A*B*a*d + 2*A*B*b*c + 2*B**2*a*d +
 2*B**2*b*c + x*(2*A**2*b*d + 4*B**2*b*d))/(a**3*c*d**2*g**2*i**2 - 2*a**2*
b*c**2*d*g**2*i**2 + a*b**2*c**3*g**2*i**2 + x**2*(a**2*b*d**3*g**2*i**2 -
2*a*b**2*c*d**2*g**2*i**2 + b**3*c**2*d*g**2*i**2) + x*(a**3*d**3*g**2*i**2
 - a**2*b*c*d**2*g**2*i**2 - a*b**2*c**2*d*g**2*i**2 + b**3*c**3*g**2*i**2)
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1995 vs. $2(363) = 726$.

Time = 0.31 (sec) , antiderivative size = 1995, normalized size of antiderivative = 5.47

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, alg
orithm="maxima")
```

```
[Out] -B^2*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*g^2*i^
2*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*g^2*i^2*x + (a*b^2*
c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2) + 2*b*d*log(b*x + a)/((b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2) - 2*b*d*log(d*x + c)/((b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2))*log(b*e*x/(d*x +
 c) + a*e/(d*x + c))^2 - 2*A*B*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2*a*b^2
*c*d^2 + a^2*b*d^3)*g^2*i^2*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^
3*d^3)*g^2*i^2*x + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2) + 2*b*d
```

```
*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2)
 - 2*b*d*log(d*x + c)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
g^2*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 2/3*B^2*(3*(b^2*c^2 - 2*a*
b*c*d + a^2*d^2 - (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(b*x +
 a)^2 + 2*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(b*x + a)*log(
d*x + c) - (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(d*x + c)^2)*
log(b*e*x/(d*x + c) + a*e/(d*x + c))/(a*b^3*c^4*g^2*i^2 - 3*a^2*b^2*c^3*d*g
^2*i^2 + 3*a^3*b*c^2*d^2*g^2*i^2 - a^4*c*d^3*g^2*i^2 + (b^4*c^3*d*g^2*i^2 -
 3*a*b^3*c^2*d^2*g^2*i^2 + 3*a^2*b^2*c*d^3*g^2*i^2 - a^3*b*d^4*g^2*i^2)*x^2
 + (b^4*c^4*g^2*i^2 - 2*a*b^3*c^3*d*g^2*i^2 + 2*a^3*b*c*d^3*g^2*i^2 - a^4*d
^4*g^2*i^2)*x) + (3*b^2*c^2 - 3*a^2*d^2 + (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d
 + a*b*d^2)*x)*log(b*x + a)^3 + 3*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d
^2)*x)*log(b*x + a)*log(d*x + c)^2 - (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*
b*d^2)*x)*log(d*x + c)^3 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + a*b*c
*d + (b^2*c*d + a*b*d^2)*x)*log(b*x + a) - 3*(2*b^2*d^2*x^2 + 2*a*b*c*d + (
b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(b*x + a)^2 + 2*(b^2*c*d
+ a*b*d^2)*x)*log(d*x + c))/(a*b^3*c^4*g^2*i^2 - 3*a^2*b^2*c^3*d*g^2*i^2 +
3*a^3*b*c^2*d^2*g^2*i^2 - a^4*c*d^3*g^2*i^2 + (b^4*c^3*d*g^2*i^2 - 3*a*b^3*
c^2*d^2*g^2*i^2 + 3*a^2*b^2*c*d^3*g^2*i^2 - a^3*b*d^4*g^2*i^2)*x^2 + (b^4*c
^4*g^2*i^2 - 2*a*b^3*c^3*d*g^2*i^2 + 2*a^3*b*c*d^3*g^2*i^2 - a^4*d^4*g^2*i^
2)*x)) - A^2*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3
)*g^2*i^2*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*g^2*i^2*x +
 (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2) + 2*b*d*log(b*x + a)/((b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2) - 2*b*d*log(d*x +
 c)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2)) - 2*(b^2
*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x
)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(b*
x + a)*log(d*x + c) - (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(d
*x + c)^2)*A*B/(a*b^3*c^4*g^2*i^2 - 3*a^2*b^2*c^3*d*g^2*i^2 + 3*a^3*b*c^2*d
^2*g^2*i^2 - a^4*c*d^3*g^2*i^2 + (b^4*c^3*d*g^2*i^2 - 3*a*b^3*c^2*d^2*g^2*i
^2 + 3*a^2*b^2*c*d^3*g^2*i^2 - a^3*b*d^4*g^2*i^2)*x^2 + (b^4*c^4*g^2*i^2 -
2*a*b^3*c^3*d*g^2*i^2 + 2*a^3*b*c*d^3*g^2*i^2 - a^4*d^4*g^2*i^2)*x)
```

Giac [A] (verification not implemented)

none

Time = 57.24 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.56

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = -\left(\frac{(dx+c)B^2e^2\log\left(\frac{bex+ae}{dx+c}\right)^2}{(bex+ae)g^2i^2} + \frac{2\left(ABe^2+B^2e^2\right)(dx+c)\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)g^2i^2} + \frac{(A^2e^2+2\,ABe^2+2\,B^2e^2)(dx}{(bex+ae)g^2i^2}\right.$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, alg
orithm="giac")
```

```
[Out] -((d*x + c)*B^2*e^2*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a*e)*g^2*i^2)
+ 2*(A*B*e^2 + B^2*e^2)*(d*x + c)*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*
e)*g^2*i^2) + (A^2*e^2 + 2*A*B*e^2 + 2*B^2*e^2)*(d*x + c)/((b*e*x + a*e)*g^
2*i^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*
d)))^2
```

Mupad [B] (verification not implemented)

Time = 3.76 (sec) , antiderivative size = 731, normalized size of antiderivative = 2.00

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = \frac{2\,B^2\,b\,d\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^3}{3\,g^2\,i^2\,\left(a\,d-b\,c\right)^3}$$
$$-\frac{\frac{A^2\,a\,d+A^2\,b\,c+2\,B^2\,a\,d+2\,B^2\,b\,c-2\,A\,B\,a\,d+2\,A\,B\,b\,c}{a\,d-b\,c}+\frac{2\,x\,\left(b\,d\,A^2+2\,b\,d\,B^2\right)}{a\,d-b\,c}}{x\,\left(a^2\,d^2\,g^2\,i^2-b^2\,c^2\,g^2\,i^2\right)+x^2\,\left(a\,b\,d^2\,g^2\,i^2-b^2\,c\,d\,g^2\,i^2\right)-a\,b\,c^2\,g^2\,i^2+a^2\,c\,d\,g^2\,i^2}$$
$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{2\,\left(B^2\,b\,c-B^2\,a\,d+A\,B\,a\,d+A\,B\,b\,c\right)}{g^2\,i^2\,\left(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d\right)}+\frac{4\,A\,B\,x}{g^2\,i^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)}{x^2+\frac{x\,(a\,d+b\,c)}{b\,d}+\frac{a\,c}{b\,d}}$$
$$-\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\,\left(\frac{\frac{B^2\,(a\,d+b\,c)}{g^2\,i^2\,\left(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d\right)}+\frac{2\,B^2\,x}{g^2\,i^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}}{x^2+\frac{x\,(a\,d+b\,c)}{b\,d}+\frac{a\,c}{b\,d}}\right.$$
$$\left.-\frac{2\,A\,B\,b\,d}{g^2\,i^2\,\left(a\,d-b\,c\right)^3}\right)$$
$$-\frac{b\,d\,\mathrm{atan}\left(\frac{b\,d\,\left(A^2+2\,B^2\right)\,\left(\frac{a^3\,d^3\,g^2\,i^2-a^2\,b\,c\,d^2\,g^2\,i^2-a\,b^2\,c^2\,d\,g^2\,i^2+b^3\,c^3\,g^2\,i^2}{a^2\,d^2\,g^2\,i^2-2\,a\,b\,c\,d\,g^2\,i^2+b^2\,c^2\,g^2\,i^2}+2\,b\,d\,x\right)\,\left(a^2\,d^2\,g^2\,i^2-2\,a\,b\,c\,d\,g^2\,i^2+b^2\,c^2\,g^2\,i^2\right)\,2\mathrm{i}}{g^2\,i^2\,\left(a\,d-b\,c\right)^3\,\left(2\,b\,d\,A^2+4\,b\,d\,B^2\right)}\right)\,\left(A^2+2\right.}{g^2\,i^2\,\left(a\,d-b\,c\right)^3}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^2*(c*i + d*i*x)^2
),x)
```

```
[Out] (2*B^2*b*d*log((e*(a + b*x))/(c + d*x))^3)/(3*g^2*i^2*(a*d - b*c)^3) - ((A^
2*a*d + A^2*b*c + 2*B^2*a*d + 2*B^2*b*c - 2*A*B*a*d + 2*A*B*b*c)/(a*d - b*c
) + (2*x*(A^2*b*d + 2*B^2*b*d))/(a*d - b*c))/(x*(a^2*d^2*g^2*i^2 - b^2*c^2*
g^2*i^2) + x^2*(a*b*d^2*g^2*i^2 - b^2*c*d*g^2*i^2) - a*b*c^2*g^2*i^2 + a^2*
c*d*g^2*i^2) - (log((e*(a + b*x))/(c + d*x))*((2*(B^2*b*c - B^2*a*d + A*B*a
*d + A*B*b*c))/(g^2*i^2*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (4*A*B*x
)/(g^2*i^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))))/(x^2 + (x*(a*d + b*c))/(b*d)
+ (a*c)/(b*d)) - (b*d*atan((b*d*(A^2 + 2*B^2)*((a^3*d^3*g^2*i^2 + b^3*c^3*g
^2*i^2 - a*b^2*c^2*d*g^2*i^2 - a^2*b*c*d^2*g^2*i^2)/(a^2*d^2*g^2*i^2 + b^2*
c^2*g^2*i^2 - 2*a*b*c*d*g^2*i^2) + 2*b*d*x)*(a^2*d^2*g^2*i^2 + b^2*c^2*g^2*
```

```
i^2 - 2*a*b*c*d*g^2*i^2)*2i)/(g^2*i^2*(a*d - b*c)^3*(2*A^2*b*d + 4*B^2*b*d)
))*(A^2 + 2*B^2)*4i)/(g^2*i^2*(a*d - b*c)^3) - log((e*(a + b*x))/(c + d*x))
^2*(((B^2*(a*d + b*c))/(g^2*i^2*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) +
(2*B^2*x)/(g^2*i^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(x^2 + (x*(a*d + b*c))
/(b*d) + (a*c)/(b*d)) - (2*A*B*b*d)/(g^2*i^2*(a*d - b*c)^3))
```

3.98 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 523

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \frac{2ABd^3(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} - \frac{2B^2d^3(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} + \frac{6b^2B^2d(c+dx)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3B^2(c+dx)^2}{4(bc-ad)^4g^3i^2(a+bx)^2} + \frac{2B^2d^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^4g^3i^2(c+dx)} + \frac{6b^2Bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3B(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} - \frac{d^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^3i^2(c+dx)} + \frac{3b^2d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^4g^3i^2(a+bx)^2} + \frac{bd^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{B(bc-ad)^4g^3i^2}$$

```
[Out] 2*A*B*d^3*(b*x+a)/(-a*d+b*c)^4/g^3/i^2/(d*x+c)-2*B^2*d^3*(b*x+a)/(-a*d+b*c)
^4/g^3/i^2/(d*x+c)+6*b^2*B^2*d*(d*x+c)/(-a*d+b*c)^4/g^3/i^2/(b*x+a)-1/4*b^3
*B^2*(d*x+c)^2/(-a*d+b*c)^4/g^3/i^2/(b*x+a)^2+2*B^2*d^3*(b*x+a)*ln(e*(b*x+a
)/(d*x+c))/(-a*d+b*c)^4/g^3/i^2/(d*x+c)+6*b^2*B*d*(d*x+c)*(A+B*ln(e*(b*x+a)
/(d*x+c)))/(-a*d+b*c)^4/g^3/i^2/(b*x+a)-1/2*b^3*B*(d*x+c)^2*(A+B*ln(e*(b*x+
a)/(d*x+c)))/(-a*d+b*c)^4/g^3/i^2/(b*x+a)^2-d^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(
d*x+c)))^2/(-a*d+b*c)^4/g^3/i^2/(d*x+c)+3*b^2*d*(d*x+c)*(A+B*ln(e*(b*x+a)/(
d*x+c)))^2/(-a*d+b*c)^4/g^3/i^2/(b*x+a)-1/2*b^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)
/(d*x+c)))^2/(-a*d+b*c)^4/g^3/i^2/(b*x+a)^2+b*d^2*(A+B*ln(e*(b*x+a)/(d*x+c)
))^3/B/(-a*d+b*c)^4/g^3/i^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2562, 2395, 2333, 2332, 2342, 2341, 2339, 30}

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = & -\frac{b^3(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g^3i^2(a+bx)^2(bc-ad)^4} \\ & -\frac{b^3B(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^3i^2(a+bx)^2(bc-ad)^4} \\ & +\frac{3b^2d(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^3i^2(a+bx)(bc-ad)^4} \\ & +\frac{6b^2Bd(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^3i^2(a+bx)(bc-ad)^4} \\ & -\frac{d^3(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^3i^2(c+dx)(bc-ad)^4} \\ & +\frac{2ABd^3(a+bx)}{g^3i^2(c+dx)(bc-ad)^4}+\frac{bd^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^3}{Bg^3i^2(bc-ad)^4} \\ & -\frac{b^3B^2(c+dx)^2}{4g^3i^2(a+bx)^2(bc-ad)^4}+\frac{6b^2B^2d(c+dx)}{g^3i^2(a+bx)(bc-ad)^4} \\ & +\frac{2B^2d^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{g^3i^2(c+dx)(bc-ad)^4}-\frac{2B^2d^3(a+bx)}{g^3i^2(c+dx)(bc-ad)^4}\end{aligned}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^3*(c*i + d*i*x)^2
),x]
```

```
[Out] (2*A*B*d^3*(a + b*x))/((b*c - a*d)^4*g^3*i^2*(c + d*x)) - (2*B^2*d^3*(a + b
*x))/((b*c - a*d)^4*g^3*i^2*(c + d*x)) + (6*b^2*B^2*d*(c + d*x))/((b*c - a*
```

```
d)^4*g^3*i^2*(a + b*x)) - (b^3*B^2*(c + d*x)^2)/(4*(b*c - a*d)^4*g^3*i^2*(a
 + b*x)^2) + (2*B^2*d^3*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/((b*c - a*d
)^4*g^3*i^2*(c + d*x)) + (6*b^2*B*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c +
 d*x)]))/((b*c - a*d)^4*g^3*i^2*(a + b*x)) - (b^3*B*(c + d*x)^2*(A + B*Log[
(e*(a + b*x))/(c + d*x)]))/(2*(b*c - a*d)^4*g^3*i^2*(a + b*x)^2) - (d^3*(a
+ b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*c - a*d)^4*g^3*i^2*(c +
d*x)) + (3*b^2*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*c -
a*d)^4*g^3*i^2*(a + b*x)) - (b^3*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c +
d*x)])^2)/(2*(b*c - a*d)^4*g^3*i^2*(a + b*x)^2) + (b*d^2*(A + B*Log[(e*(a +
 b*x))/(c + d*x)])^3)/(B*(b*c - a*d)^4*g^3*i^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)^3(A+B\log(ex))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2}$$

$$= \frac{\text{Subst}\left(\int \left(-d^3(A+B\log(ex))^2 + \frac{b^3(A+B\log(ex))^2}{x^3} - \frac{3b^2d(A+B\log(ex))^2}{x^2} + \frac{3bd^2(A+B\log(ex))^2}{x}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2}$$

$$= \frac{b^3\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2} - \frac{(3b^2d)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2} + \frac{(3bd^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2} - \frac{d^3\text{Subst}\left(\int (A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2}$$

$$= -\frac{d^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^3i^2(c+dx)} + \frac{3b^2d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^4g^3i^2(a+bx)^2} + \frac{(b^3B)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2} - \frac{(6b^2Bd)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2} + \frac{(3bd^2)\,\text{Subst}\left(\int x^2\,dx, x, A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B(bc-ad)^4g^3i^2} + \frac{(2Bd^3)\,\text{Subst}\left(\int (A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2}$$

$$= \frac{2ABd^3(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} + \frac{6b^2B^2d(c+dx)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3B^2(c+dx)^2}{4(bc-ad)^4g^3i^2(a+bx)^2} + \frac{6b^2Bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3B(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} - \frac{d^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^3i^2(c+dx)} + \frac{3b^2d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^4g^3i^2(a+bx)^2} + \frac{bd^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{B(bc-ad)^4g^3i^2} + \frac{(2B^2d^3)\operatorname{Subst}\left(\int\log(ex)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2}$$

$$= \frac{2ABd^3(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} - \frac{2B^2d^3(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} + \frac{6b^2B^2d(c+dx)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3B^2(c+dx)^2}{4(bc-ad)^4g^3i^2(a+bx)^2} + \frac{2B^2d^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^4g^3i^2(c+dx)} + \frac{6b^2Bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3B(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} - \frac{d^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^3i^2(c+dx)} + \frac{3b^2d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^4g^3i^2(a+bx)^2} + \frac{bd^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{B(bc-ad)^4g^3i^2}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 466, normalized size of antiderivative = 0.89

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx$$

$$= \frac{4(A^2-2AB+2B^2)\,d^2(bc-ad)(a+bx)^2 - b(2A^2+2AB+B^2)\,(bc-ad)^2(c+dx) + 2b(4A^2+10AB+1}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^3*(c*i + d*
i*x)^2),x]
```

```
[Out] (4*(A^2 - 2*A*B + 2*B^2)*d^2*(b*c - a*d)*(a + b*x)^2 - b*(2*A^2 + 2*A*B + B
^2)*(b*c - a*d)^2*(c + d*x) + 2*b*(4*A^2 + 10*A*B + 11*B^2)*d*(b*c - a*d)*(
a + b*x)*(c + d*x) + 6*b*(2*A^2 + 2*A*B + 5*B^2)*d^2*(a + b*x)^2*(c + d*x)*
```

```
Log[a + b*x] + 2*B*(b*c - a*d)*(4*(A - B)*d^2*(a + b*x)^2 - b*(2*A + B)*(b*
c - a*d)*(c + d*x) + 2*b*(4*A + 5*B)*d*(a + b*x)*(c + d*x))*Log[(e*(a + b*x
))/(c + d*x)] - 2*B*(2*a^3*B*d^3 - 6*a^2*b*d^2*(-(B*d*x) + A*(c + d*x)) - 6
*a*b^2*d*(2*A*d*x*(c + d*x) + B*c*(c + 2*d*x)) + b^3*(-6*A*d^2*x^2*(c + d*x
) + B*(c^3 - 3*c^2*d*x - 9*c*d^2*x^2 - 3*d^3*x^3)))*Log[(e*(a + b*x))/(c +
d*x)]^2 + 4*b*B^2*d^2*(a + b*x)^2*(c + d*x)*Log[(e*(a + b*x))/(c + d*x)]^3
- 6*b*(2*A^2 + 2*A*B + 5*B^2)*d^2*(a + b*x)^2*(c + d*x)*Log[c + d*x])/(4*(b
*c - a*d)^4*g^3*i^2*(a + b*x)^2*(c + d*x))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. $2(517) = 1034$.

Time = 2.71 (sec) , antiderivative size = 1080, normalized size of antiderivative = 2.07

method	result	size
parts	Expression too large to display	1080
derivativedivides	Expression too large to display	1228
default	Expression too large to display	1228
risch	Expression too large to display	1736
parallelrisch	Expression too large to display	1763
norman	Expression too large to display	1849

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x,method=_RET
URNVERBOSE)
```

```
[Out] A^2/g^3/i^2*(-d^2/(a*d-b*c)^3/(d*x+c)-3*d^2/(a*d-b*c)^4*b*ln(d*x+c)-1/2*b/(
a*d-b*c)^2/(b*x+a)^2+3*d^2/(a*d-b*c)^4*b*ln(b*x+a)-2*b/(a*d-b*c)^3*d/(b*x+a
))-B^2/g^3/i^2/(a*d-b*c)/e*(d^3/(a*d-b*c)^3*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*
ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+
(a*d-b*c)*e/d/(d*x+c))+2*(a*d-b*c)*e/d/(d*x+c)+2*b*e/d)-1/(a*d-b*c)^3*b*d^2
*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3+3/(a*d-b*c)^3*b^2*d*e^2*(-1/(b*e/d+(a*
d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-b*c)*e/
d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))
-1/(a*d-b*c)^3*b^3*e^3*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-
b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*
e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))-2*B*A/g^3/i^2/(a*d-b*c)/
e*(d^3/(a*d-b*c)^3*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d
*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)-3/2/(a*d-b*c)^3*b*d^2*e*ln(b*e/d+(a*d-b
*c)*e/d/(d*x+c))^2+3/(a*d-b*c)^3*b^2*d*e^2*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-1/(a*d-b
*c)^3*b^3*e^3*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/
(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 1005, normalized size of antiderivative = 1.92

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx =$$
$$-\frac{(2\,A^2 + 2\,AB + B^2)b^3c^3 - 12\,(A^2 + 2\,AB + 2\,B^2)ab^2c^2d + 3\,(2\,A^2 + 10\,AB + 5\,B^2)a^2bcd^2 + 4\,(A^2 - 2\,A}{}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, alg
orithm="fricas")
```

```
[Out] -1/4*((2*A^2 + 2*A*B + B^2)*b^3*c^3 - 12*(A^2 + 2*A*B + 2*B^2)*a*b^2*c^2*d
+ 3*(2*A^2 + 10*A*B + 5*B^2)*a^2*b*c*d^2 + 4*(A^2 - 2*A*B + 2*B^2)*a^3*d^3
- 4*(B^2*b^3*d^3*x^3 + B^2*a^2*b*c*d^2 + (B^2*b^3*c*d^2 + 2*B^2*a*b^2*d^3)*
x^2 + (2*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c))^3
 - 6*((2*A^2 + 2*A*B + 5*B^2)*b^3*c*d^2 - (2*A^2 + 2*A*B + 5*B^2)*a*b^2*d^3
)*x^2 - 2*(3*(2*A*B + B^2)*b^3*d^3*x^3 - B^2*b^3*c^3 + 6*B^2*a*b^2*c^2*d +
6*A*B*a^2*b*c*d^2 - 2*B^2*a^3*d^3 + 3*(4*A*B*a*b^2*d^3 + (2*A*B + 3*B^2)*b^
3*c*d^2)*x^2 + 3*(B^2*b^3*c^2*d + 4*(A*B + B^2)*a*b^2*c*d^2 + 2*(A*B - B^2)
*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c))^2 - 3*((2*A^2 + 6*A*B + 7*B^2)*
b^3*c^2*d + 2*(2*A^2 - 2*A*B + 3*B^2)*a*b^2*c*d^2 - (6*A^2 + 2*A*B + 13*B^2
)*a^2*b*d^3)*x - 2*(3*(2*A^2 + 2*A*B + 5*B^2)*b^3*d^3*x^3 + 6*A^2*a^2*b*c*d
^2 - (2*A*B + B^2)*b^3*c^3 + 12*(A*B + B^2)*a*b^2*c^2*d - 4*(A*B - B^2)*a^3
*d^3 + 3*((2*A^2 + 6*A*B + 7*B^2)*b^3*c*d^2 + 4*(A^2 + 2*B^2)*a*b^2*d^3)*x^
2 + 3*((2*A*B + 3*B^2)*b^3*c^2*d + 4*(A^2 + 2*A*B + 2*B^2)*a*b^2*c*d^2 + 2*
(A^2 - 2*A*B + 2*B^2)*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^6*c^4
*d - 4*a*b^5*c^3*d^2 + 6*a^2*b^4*c^2*d^3 - 4*a^3*b^3*c*d^4 + a^4*b^2*d^5)*g
^3*i^2*x^3 + (b^6*c^5 - 2*a*b^5*c^4*d - 2*a^2*b^4*c^3*d^2 + 8*a^3*b^3*c^2*d
^3 - 7*a^4*b^2*c*d^4 + 2*a^5*b*d^5)*g^3*i^2*x^2 + (2*a*b^5*c^5 - 7*a^2*b^4*
c^4*d + 8*a^3*b^3*c^3*d^2 - 2*a^4*b^2*c^2*d^3 - 2*a^5*b*c*d^4 + a^6*d^5)*g^
3*i^2*x + (a^2*b^4*c^5 - 4*a^3*b^3*c^4*d + 6*a^4*b^2*c^3*d^2 - 4*a^5*b*c^2*
d^3 + a^6*c*d^4)*g^3*i^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3/(d*i*x+c*i)**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4187 vs. $2(517) = 1034$.

Time = 0.51 (sec) , antiderivative size = 4187, normalized size of antiderivative = 8.01

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, alg
orithm="maxima")
[Out] 1/2*B^2*((6*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*
a*b*d^2)*x)/((b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*
g^3*i^2*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3
- 2*a^4*b*d^4)*g^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2
*d^2 + a^4*b*c*d^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3*a^3*b^2*c^3*d +
3*a^4*b*c^2*d^2 - a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x + a)/((b^4*c^4 - 4*
a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2) - 6*b*d
^2*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d
^3 + a^4*d^4)*g^3*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2 + A*B*((6*b^
2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a*b*d^2)*x)/((
b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*g^3*i^2*x^3 +
(b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 - 2*a^4*b*d^4)
*g^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 + a^4*b*c
*d^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b*c^2*d^
2 - a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d +
6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2) - 6*b*d^2*log(d*x + c
)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*
g^3*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 1/4*B^2*(2*(b^3*c^3 - 12*a
*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3 - 6*(b^3*c*d^2 - a*b^2*d^3)*x^2 + 6
*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^
2 + a^2*b*d^3)*x)*log(b*x + a)^2 + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^
2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(d*x + c)^2 - 3*(3
```

```
*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b*d^3)*x - 6*(b^3*d^3*x^3 + a^2*b*c*d^2 +
(b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a)
 + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*
c*d^2 + a^2*b*d^3)*x - 2*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*
d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a))*log(d*x + c))*log(b
*e*x/(d*x + c) + a*e/(d*x + c))/(a^2*b^4*c^5*g^3*i^2 - 4*a^3*b^3*c^4*d*g^3*
i^2 + 6*a^4*b^2*c^3*d^2*g^3*i^2 - 4*a^5*b*c^2*d^3*g^3*i^2 + a^6*c*d^4*g^3*i
^2 + (b^6*c^4*d*g^3*i^2 - 4*a*b^5*c^3*d^2*g^3*i^2 + 6*a^2*b^4*c^2*d^3*g^3*i
^2 - 4*a^3*b^3*c*d^4*g^3*i^2 + a^4*b^2*d^5*g^3*i^2)*x^3 + (b^6*c^5*g^3*i^2
- 2*a*b^5*c^4*d*g^3*i^2 - 2*a^2*b^4*c^3*d^2*g^3*i^2 + 8*a^3*b^3*c^2*d^3*g^3
*i^2 - 7*a^4*b^2*c*d^4*g^3*i^2 + 2*a^5*b*d^5*g^3*i^2)*x^2 + (2*a*b^5*c^5*g^
3*i^2 - 7*a^2*b^4*c^4*d*g^3*i^2 + 8*a^3*b^3*c^3*d^2*g^3*i^2 - 2*a^4*b^2*c^2
*d^3*g^3*i^2 - 2*a^5*b*c*d^4*g^3*i^2 + a^6*d^5*g^3*i^2)*x) + (b^3*c^3 - 24*
a*b^2*c^2*d + 15*a^2*b*c*d^2 + 8*a^3*d^3 - 4*(b^3*d^3*x^3 + a^2*b*c*d^2 + (
b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a)^
3 + 4*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2
*c*d^2 + a^2*b*d^3)*x)*log(d*x + c)^3 - 30*(b^3*c*d^2 - a*b^2*d^3)*x^2 + 6*
(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2
 + a^2*b*d^3)*x)*log(b*x + a)^2 + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2
 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x - 2*(b^3*d^3*x^3 + a^2*
b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*lo
g(b*x + a))*log(d*x + c)^2 - 3*(7*b^3*c^2*d + 6*a*b^2*c*d^2 - 13*a^2*b*d^3)
*x - 30*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b
^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a) + 6*(5*b^3*d^3*x^3 + 5*a^2*b*c*d^2 +
5*(b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + 2*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2
 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a)^2 + 5*(2*
a*b^2*c*d^2 + a^2*b*d^3)*x - 2*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*
a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a))*log(d*x + c))
/(a^2*b^4*c^5*g^3*i^2 - 4*a^3*b^3*c^4*d*g^3*i^2 + 6*a^4*b^2*c^3*d^2*g^3*i^2
 - 4*a^5*b*c^2*d^3*g^3*i^2 + a^6*c*d^4*g^3*i^2 + (b^6*c^4*d*g^3*i^2 - 4*a*b
^5*c^3*d^2*g^3*i^2 + 6*a^2*b^4*c^2*d^3*g^3*i^2 - 4*a^3*b^3*c*d^4*g^3*i^2 +
a^4*b^2*d^5*g^3*i^2)*x^3 + (b^6*c^5*g^3*i^2 - 2*a*b^5*c^4*d*g^3*i^2 - 2*a^2
*b^4*c^3*d^2*g^3*i^2 + 8*a^3*b^3*c^2*d^3*g^3*i^2 - 7*a^4*b^2*c*d^4*g^3*i^2
+ 2*a^5*b*d^5*g^3*i^2)*x^2 + (2*a*b^5*c^5*g^3*i^2 - 7*a^2*b^4*c^4*d*g^3*i^2
 + 8*a^3*b^3*c^3*d^2*g^3*i^2 - 2*a^4*b^2*c^2*d^3*g^3*i^2 - 2*a^5*b*c*d^4*g^
3*i^2 + a^6*d^5*g^3*i^2)*x)) + 1/2*A^2*((6*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*
d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a*b*d^2)*x)/((b^5*c^3*d - 3*a*b^4*c^2*d^2 +
3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*g^3*i^2*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2
*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*g^3*i^2*x^2 + (2*a*b^4*c^4 -
5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*g^3*i^2*x + (a
^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3)*g^3*i^2) + 6*b*
d^2*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*
d^3 + a^4*d^4)*g^3*i^2) - 6*b*d^2*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d +
6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2)) - 1/2*(b^3*c^3 - 12*
a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3 - 6*(b^3*c*d^2 - a*b^2*d^3)*x^2 +
```

```
6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d
^2 + a^2*b*d^3)*x)*log(b*x + a)^2 + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d
^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(d*x + c)^2 - 3*(
3*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b*d^3)*x - 6*(b^3*d^3*x^3 + a^2*b*c*d^2 +
 (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a
) + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2
*c*d^2 + a^2*b*d^3)*x - 2*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2
*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a))*log(d*x + c))*A*B/
(a^2*b^4*c^5*g^3*i^2 - 4*a^3*b^3*c^4*d*g^3*i^2 + 6*a^4*b^2*c^3*d^2*g^3*i^2
- 4*a^5*b*c^2*d^3*g^3*i^2 + a^6*c*d^4*g^3*i^2 + (b^6*c^4*d*g^3*i^2 - 4*a*b^
5*c^3*d^2*g^3*i^2 + 6*a^2*b^4*c^2*d^3*g^3*i^2 - 4*a^3*b^3*c*d^4*g^3*i^2 + a
^4*b^2*d^5*g^3*i^2)*x^3 + (b^6*c^5*g^3*i^2 - 2*a*b^5*c^4*d*g^3*i^2 - 2*a^2*
b^4*c^3*d^2*g^3*i^2 + 8*a^3*b^3*c^2*d^3*g^3*i^2 - 7*a^4*b^2*c*d^4*g^3*i^2 +
 2*a^5*b*d^5*g^3*i^2)*x^2 + (2*a*b^5*c^5*g^3*i^2 - 7*a^2*b^4*c^4*d*g^3*i^2
+ 8*a^3*b^3*c^3*d^2*g^3*i^2 - 2*a^4*b^2*c^2*d^3*g^3*i^2 - 2*a^5*b*c*d^4*g^3
*i^2 + a^6*d^5*g^3*i^2)*x)
```

Giac [A] (verification not implemented)

none

Time = 74.13 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.88

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = -\frac{1}{4}\left(\frac{2\left(B^2be^3-\frac{2\,(bex+ae)B^2de^2}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^2bcg^3i^2}{(dx+c)^2}-\frac{(bex+ae)^2adg^3i^2}{(dx+c)^2}}+\frac{2\left(2\,ABbe^3+B^2be^3-\frac{4\,(bex+ae)ABde^2}{dx+c}-\frac{4\,(bex+ae)B^2de^2}{dx+c}\right)\mathrm{l}}{\frac{(bex+ae)^2bcg^3i^2}{(dx+c)^2}-\frac{(bex+ae)^2adg^3i^2}{(dx+c)^2}}\right.$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, alg
orithm="giac")
```

```
[Out] -1/4*(2*(B^2*b*e^3 - 2*(b*e*x + a*e)*B^2*d*e^2/(d*x + c))*log((b*e*x + a*e)
/(d*x + c))^2/((b*e*x + a*e)^2*b*c*g^3*i^2/(d*x + c)^2 - (b*e*x + a*e)^2*a*
d*g^3*i^2/(d*x + c)^2) + 2*(2*A*B*b*e^3 + B^2*b*e^3 - 4*(b*e*x + a*e)*A*B*d
*e^2/(d*x + c) - 4*(b*e*x + a*e)*B^2*d*e^2/(d*x + c))*log((b*e*x + a*e)/(d*
x + c))/((b*e*x + a*e)^2*b*c*g^3*i^2/(d*x + c)^2 - (b*e*x + a*e)^2*a*d*g^3*
i^2/(d*x + c)^2) + (2*A^2*b*e^3 + 2*A*B*b*e^3 + B^2*b*e^3 - 4*(b*e*x + a*e)
*A^2*d*e^2/(d*x + c) - 8*(b*e*x + a*e)*A*B*d*e^2/(d*x + c) - 8*(b*e*x + a*e
)*B^2*d*e^2/(d*x + c))/((b*e*x + a*e)^2*b*c*g^3*i^2/(d*x + c)^2 - (b*e*x +
a*e)^2*a*d*g^3*i^2/(d*x + c)^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/(
(b*c*e - a*d*e)*(b*c - a*d)))^2
```

Mupad [B] (verification not implemented)

Time = 7.22 (sec) , antiderivative size = 1497, normalized size of antiderivative = 2.86

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^3*(c*i + d*i*x)^2
),x)
```

```
[Out] (B^2*b*d^2*log((e*(a + b*x))/(c + d*x))^3)/(g^3*i^2*(a*d - b*c)^2*(a^2*d^2
+ b^2*c^2 - 2*a*b*c*d)) - ((4*A^2*a^2*d^2 - 2*A^2*b^2*c^2 + 8*B^2*a^2*d^2 -
 B^2*b^2*c^2 - 8*A*B*a^2*d^2 - 2*A*B*b^2*c^2 + 10*A^2*a*b*c*d + 23*B^2*a*b*
c*d + 22*A*B*a*b*c*d)/(2*(a*d - b*c)) + (3*x^2*(2*A^2*b^2*d^2 + 5*B^2*b^2*d
^2 + 2*A*B*b^2*d^2))/(a*d - b*c) + (3*x*(6*A^2*a*b*d^2 + 13*B^2*a*b*d^2 + 2
*A^2*b^2*c*d + 7*B^2*b^2*c*d + 2*A*B*a*b*d^2 + 6*A*B*b^2*c*d))/(2*(a*d - b*
c)))/(x*(2*a^4*d^3*g^3*i^2 + 4*a*b^3*c^3*g^3*i^2 - 6*a^2*b^2*c^2*d*g^3*i^2)
 + x^2*(2*b^4*c^3*g^3*i^2 + 4*a^3*b*d^3*g^3*i^2 - 6*a^2*b^2*c*d^2*g^3*i^2)
+ x^3*(2*a^2*b^2*d^3*g^3*i^2 + 2*b^4*c^2*d*g^3*i^2 - 4*a*b^3*c*d^2*g^3*i^2)
 + 2*a^2*b^2*c^3*g^3*i^2 + 2*a^4*c*d^2*g^3*i^2 - 4*a^3*b*c^2*d*g^3*i^2) - (
log((e*(a + b*x))/(c + d*x))*((B^2*b*c - 4*B^2*a*d + 4*A*B*a*d + 2*A*B*b*c)
/(2*g^3*i^2*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) - x*((3*(B^2 - 2*A*B))
/(2*g^3*i^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (3*B*(2*A + B)*(a*d + b*c))/
(g^3*i^2*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) + (3*B*a*c*(2*A + B)
)/(g^3*i^2*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (3*B*b*d*x^2*(2*A
 + B))/(g^3*i^2*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))))/(b*x^3 + (a^
2*c)/(b*d) + (x^2*(b^2*c + 2*a*b*d))/(b*d) + (x*(a^2*d + 2*a*b*c))/(b*d)) -
 (b*d^2*atan((b*d^2*(2*A^2 + 5*B^2 + 2*A*B)*(2*a^4*d^4*g^3*i^2 - 2*b^4*c^4*
g^3*i^2 + 4*a*b^3*c^3*d*g^3*i^2 - 4*a^3*b*c*d^3*g^3*i^2)*3i)/(2*g^3*i^2*(a*
d - b*c)^4*(6*A^2*b*d^2 + 15*B^2*b*d^2 + 6*A*B*b*d^2)) + (b^2*d^3*x*(2*A^2
+ 5*B^2 + 2*A*B)*(a^3*d^3*g^3*i^2 - b^3*c^3*g^3*i^2 + 3*a*b^2*c^2*d*g^3*i^2
 - 3*a^2*b*c*d^2*g^3*i^2)*6i)/(g^3*i^2*(a*d - b*c)^4*(6*A^2*b*d^2 + 15*B^2*
b*d^2 + 6*A*B*b*d^2)))*(2*A^2 + 5*B^2 + 2*A*B)*3i)/(g^3*i^2*(a*d - b*c)^4)
- log((e*(a + b*x))/(c + d*x))^2*((x*((3*B^2)/(2*g^3*i^2*(a^2*d^2 + b^2*c^2
 - 2*a*b*c*d)) + (3*B^2*(a*d + b*c))/(g^3*i^2*(a*d - b*c)*(a^2*d^2 + b^2*c^
2 - 2*a*b*c*d))) + (B^2*(2*a*d + b*c))/(2*g^3*i^2*(a^2*b*d^3 + b^3*c^2*d -
2*a*b^2*c*d^2)) + (3*B^2*a*c)/(g^3*i^2*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a
*b*c*d)) + (3*B^2*b*d*x^2)/(g^3*i^2*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*
c*d)))/(b*x^3 + (a^2*c)/(b*d) + (x^2*(b^2*c + 2*a*b*d))/(b*d) + (x*(a^2*d +
 2*a*b*c))/(b*d)) - (3*B*b*d^2*(2*A + B))/(2*g^3*i^2*(a*d - b*c)^2*(a^2*d^2
 + b^2*c^2 - 2*a*b*c*d)))
```

3.99 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 682

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = -\frac{2ABd^4(a+bx)}{(bc-ad)^5g^4i^2(c+dx)}+\frac{2B^2d^4(a+bx)}{(bc-ad)^5g^4i^2(c+dx)} -\frac{12b^2B^2d^2(c+dx)}{(bc-ad)^5g^4i^2(a+bx)}+\frac{b^3B^2d(c+dx)^2}{(bc-ad)^5g^4i^2(a+bx)^2} -\frac{2b^4B^2(c+dx)^3}{27(bc-ad)^5g^4i^2(a+bx)^3}-\frac{2B^2d^4(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^5g^4i^2(c+dx)} -\frac{12b^2Bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(a+bx)} +\frac{2b^3Bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(a+bx)^2} -\frac{2b^4B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^5g^4i^2(a+bx)^3} +\frac{d^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^4i^2(c+dx)} -\frac{6b^2d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)} +\frac{2b^3d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)^2} -\frac{b^4(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^5g^4i^2(a+bx)^3} -\frac{4bd^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^5g^4i^2}$$

```
[Out] -2*A*B*d^4*(b*x+a)/(-a*d+b*c)^5/g^4/i^2/(d*x+c)+2*B^2*d^4*(b*x+a)/(-a*d+b*c
)^5/g^4/i^2/(d*x+c)-12*b^2*B^2*d^2*(d*x+c)/(-a*d+b*c)^5/g^4/i^2/(b*x+a)+b^3
*B^2*d*(d*x+c)^2/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-2/27*b^4*B^2*(d*x+c)^3/(-a*
d+b*c)^5/g^4/i^2/(b*x+a)^3-2*B^2*d^4*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(-a*d+b*
c)^5/g^4/i^2/(d*x+c)-12*b^2*B*d^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d
+b*c)^5/g^4/i^2/(b*x+a)+2*b^3*B*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a
*d+b*c)^5/g^4/i^2/(b*x+a)^2-2/9*b^4*B*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))
/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^3+d^4*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(
-a*d+b*c)^5/g^4/i^2/(d*x+c)-6*b^2*d^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2
```

```
/(-a*d+b*c)^5/g^4/i^2/(b*x+a)+2*b^3*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))
^2/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-1/3*b^4*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+
c)))^2/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^3-4/3*b*d^3*(A+B*ln(e*(b*x+a)/(d*x+c)))
^3/B/(-a*d+b*c)^5/g^4/i^2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 682, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2562, 2395, 2333, 2332, 2342, 2341, 2339, 30}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = -\frac{b^4(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3g^4i^2(a+bx)^3(bc-ad)^5} - \frac{2b^4B(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{9g^4i^2(a+bx)^3(bc-ad)^5} + \frac{2b^3d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^4i^2(a+bx)^2(bc-ad)^5} + \frac{2b^3Bd(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^4i^2(a+bx)^2(bc-ad)^5} - \frac{6b^2d^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^4i^2(a+bx)(bc-ad)^5} - \frac{12b^2Bd^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^4i^2(a+bx)(bc-ad)^5} + \frac{d^4(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^4i^2(c+dx)(bc-ad)^5} - \frac{2ABd^4(a+bx)}{g^4i^2(c+dx)(bc-ad)^5} - \frac{4bd^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^3}{3Bg^4i^2(bc-ad)^5} - \frac{2b^4B^2(c+dx)^3}{27g^4i^2(a+bx)^3(bc-ad)^5} + \frac{b^3B^2d(c+dx)^2}{g^4i^2(a+bx)^2(bc-ad)^5} - \frac{12b^2B^2d^2(c+dx)}{g^4i^2(a+bx)(bc-ad)^5} - \frac{2B^2d^4(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{g^4i^2(c+dx)(bc-ad)^5} + \frac{2B^2d^4(a+bx)}{g^4i^2(c+dx)(bc-ad)^5}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^4*(c*i + d*i*x)^2
),x]
```

```
[Out] (-2*A*B*d^4*(a + b*x))/((b*c - a*d)^5*g^4*i^2*(c + d*x)) + (2*B^2*d^4*(a +
b*x))/((b*c - a*d)^5*g^4*i^2*(c + d*x)) - (12*b^2*B^2*d^2*(c + d*x))/((b*c
- a*d)^5*g^4*i^2*(a + b*x)) + (b^3*B^2*d*(c + d*x)^2)/((b*c - a*d)^5*g^4*i^
2*(a + b*x)^2) - (2*b^4*B^2*(c + d*x)^3)/(27*(b*c - a*d)^5*g^4*i^2*(a + b*x
)^3) - (2*B^2*d^4*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/((b*c - a*d)^5*g^
4*i^2*(c + d*x)) - (12*b^2*B*d^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*
x)]))/((b*c - a*d)^5*g^4*i^2*(a + b*x)) + (2*b^3*B*d*(c + d*x)^2*(A + B*Log
[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^5*g^4*i^2*(a + b*x)^2) - (2*b^4*B*
(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(9*(b*c - a*d)^5*g^4*i^2*
(a + b*x)^3) + (d^4*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*c
 - a*d)^5*g^4*i^2*(c + d*x)) - (6*b^2*d^2*(c + d*x)*(A + B*Log[(e*(a + b*x)
)/(c + d*x)])^2)/((b*c - a*d)^5*g^4*i^2*(a + b*x)) + (2*b^3*d*(c + d*x)^2*(
A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*c - a*d)^5*g^4*i^2*(a + b*x)^2)
- (b^4*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*(b*c - a*d)^5
*g^4*i^2*(a + b*x)^3) - (4*b*d^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^3)/(3
*B*(b*c - a*d)^5*g^4*i^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)^4(A+B\log(ex))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2}$$

$$= \frac{\text{Subst}\left(\int \left(d^4(A+B\log(ex))^2 + \frac{b^4(A+B\log(ex))^2}{x^4} - \frac{4b^3d(A+B\log(ex))^2}{x^3} + \frac{6b^2d^2(A+B\log(ex))^2}{x^2} - \frac{4bd^3(A+B\log}{x}\right.\right.}{(bc-ad)^5g^4i^2}$$

$$= \frac{b^4\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2} - \frac{(4b^3d)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2}$$
$$+ \frac{(6b^2d^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2}$$
$$- \frac{(4bd^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2}$$
$$+ \frac{d^4\text{Subst}\left(\int (A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2}$$

$$= \frac{d^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^4i^2(c+dx)} - \frac{6b^2d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)} + \frac{2b^3d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{b^4(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^5g^4i^2(a+bx)^3} + \frac{(2b^4B)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{x^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{3(bc-ad)^5g^4i^2} - \frac{(4b^3Bd)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{x^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2} + \frac{(12b^2Bd^2)\,\text{Subst}\left(\int\frac{A+B\log(ex)}{x^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2} - \frac{(4bd^3)\,\text{Subst}\left(\int x^2\,dx,x,A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B(bc-ad)^5g^4i^2} - \frac{(2Bd^4)\,\text{Subst}\left(\int(A+B\log(ex))\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2}$$

$$= -\frac{2ABd^4(a+bx)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{12b^2B^2d^2(c+dx)}{(bc-ad)^5g^4i^2(a+bx)} + \frac{b^3B^2d(c+dx)^2}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{2b^4B^2(c+dx)^3}{27(bc-ad)^5g^4i^2(a+bx)^3} - \frac{12b^2Bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(a+bx)} + \frac{2b^3Bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{2b^4B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^5g^4i^2(a+bx)^3} + \frac{d^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^4i^2(c+dx)} - \frac{6b^2d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)} + \frac{2b^3d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{b^4(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^5g^4i^2(a+bx)^3} - \frac{4bd^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^5g^4i^2} - \frac{(2B^2d^4)\,\text{Subst}\left(\int\log(ex)\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2}$$

$$= -\frac{2ABd^4(a+bx)}{(bc-ad)^5g^4i^2(c+dx)} + \frac{2B^2d^4(a+bx)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{12b^2B^2d^2(c+dx)}{(bc-ad)^5g^4i^2(a+bx)} + \frac{b^3B^2d(c+dx)^2}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{2b^4B^2(c+dx)^3}{27(bc-ad)^5g^4i^2(a+bx)^3} - \frac{2B^2d^4(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{12b^2Bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(a+bx)} + \frac{2b^3Bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{2b^4B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^5g^4i^2(a+bx)^3} + \frac{d^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^4i^2(c+dx)} - \frac{6b^2d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)} + \frac{2b^3d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{b^4(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^5g^4i^2(a+bx)^3} - \frac{4bd^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^5g^4i^2}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 613, normalized size of antiderivative = 0.90

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = -\frac{-27(A^2-2AB+2B^2)\,d^3(-bc+ad)(a+bx)^3+b(9A^2+6AB+2B^2)\,(bc-ad)^3(c+dx)-3b(9A^2+}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^4*(c*i + d*
i*x)^2),x]
```

```
[Out] -1/27*(-27*(A^2 - 2*A*B + 2*B^2)*d^3*(-(b*c) + a*d)*(a + b*x)^3 + b*(9*A^2
+ 6*A*B + 2*B^2)*(b*c - a*d)^3*(c + d*x) - 3*b*(9*A^2 + 12*A*B + 7*B^2)*d*(
b*c - a*d)^2*(a + b*x)*(c + d*x) + 3*b*(27*A^2 + 78*A*B + 92*B^2)*d^2*(b*c
- a*d)*(a + b*x)^2*(c + d*x) + 6*b*(18*A^2 + 30*A*B + 55*B^2)*d^3*(a + b*x)
^3*(c + d*x)*Log[a + b*x] + 6*B*(b*c - a*d)*(9*(A - B)*d^3*(a + b*x)^3 + b*
(3*A + B)*(b*c - a*d)^2*(c + d*x) - 3*b*(3*A + 2*B)*d*(b*c - a*d)*(a + b*x)
*(c + d*x) + 3*b*(9*A + 13*B)*d^2*(a + b*x)^2*(c + d*x))*Log[(e*(a + b*x))/
(c + d*x)] + 9*B*(-3*a^4*B*d^4 + 12*a^3*b*d^3*(-(B*d*x) + A*(c + d*x)) + 18
*a^2*b^2*d^2*(2*A*d*x*(c + d*x) + B*c*(c + 2*d*x)) + 6*a*b^3*d*(6*A*d^2*x^2
*(c + d*x) + B*(-c^3 + 3*c^2*d*x + 9*c*d^2*x^2 + 3*d^3*x^3)) + b^4*(12*A*d^
3*x^3*(c + d*x) + B*(c^4 - 2*c^3*d*x + 6*c^2*d^2*x^2 + 22*c*d^3*x^3 + 10*d^
4*x^4)))*Log[(e*(a + b*x))/(c + d*x)]^2 + 36*b*B^2*d^3*(a + b*x)^3*(c + d*x
```

```
)*Log[(e*(a + b*x))/(c + d*x)]^3 - 6*b*(18*A^2 + 30*A*B + 55*B^2)*d^3*(a +
b*x)^3*(c + d*x)*Log[c + d*x])/((b*c - a*d)^5*g^4*i^2*(a + b*x)^3*(c + d*x)
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1385 vs. 2(674) = 1348.

Time = 2.94 (sec) , antiderivative size = 1386, normalized size of antiderivative = 2.03

method	result	size
parts	Expression too large to display	1386
derivativedivides	Expression too large to display	1588
default	Expression too large to display	1588
risch	Expression too large to display	2173
parallelrisch	Expression too large to display	2708
norman	Expression too large to display	2748

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x,method=_RET
URNVERBOSE)
```

```
[Out] A^2/g^4/i^2*(-d^3/(a*d-b*c)^4/(d*x+c)-4*d^3/(a*d-b*c)^5*b*ln(d*x+c)-1/3*b/(
a*d-b*c)^2/(b*x+a)^3+4*d^3/(a*d-b*c)^5*b*ln(b*x+a)-3*b/(a*d-b*c)^4*d^2/(b*x
+a)-b/(a*d-b*c)^3*d/(b*x+a)^2)-B^2/g^4/i^2/(a*d-b*c)/e*(d^4/(a*d-b*c)^4*((b
*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2*(b*e/d+(a*d
-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+2*(a*d-b*c)*e/d/(d*x+c)+
2*b*e/d)-4/3/(a*d-b*c)^4*b*d^3*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3+6/(a*d-b
*c)^4*b^2*d^2*e^2*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/
(d*x+c))^2-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-
2/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-4/(a*d-b*c)^4*b^3*d*e^3*(-1/2/(b*e/d+(a*d-
b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*
e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*
x+c))^2)+1/(a*d-b*c)^4*b^4*e^4*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e
/d+(a*d-b*c)*e/d/(d*x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a
*d-b*c)*e/d/(d*x+c))-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))-2*B*A/g^4/i^2/(
a*d-b*c)/e*(d^4/(a*d-b*c)^4*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*
c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)-2/(a*d-b*c)^4*b*d^3*e*ln(b*e/d
+(a*d-b*c)*e/d/(d*x+c))^2+6/(a*d-b*c)^4*b^2*d^2*e^2*(-1/(b*e/d+(a*d-b*c)*e/
d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))
-4/(a*d-b*c)^4*b^3*d*e^3*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*
d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+1/(a*d-b*c)^4*b^4*
e^4*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1
/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1534 vs. 2(674) = 1348.

Time = 0.36 (sec) , antiderivative size = 1534, normalized size of antiderivative = 2.25

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, alg
orithm="fricas")
```

```
[Out] -1/27*((9*A^2 + 6*A*B + 2*B^2)*b^4*c^4 - 27*(2*A^2 + 2*A*B + B^2)*a*b^3*c^3
*d + 162*(A^2 + 2*A*B + 2*B^2)*a^2*b^2*c^2*d^2 - 5*(18*A^2 + 66*A*B + 49*B^
2)*a^3*b*c*d^3 - 27*(A^2 - 2*A*B + 2*B^2)*a^4*d^4 + 6*((18*A^2 + 30*A*B + 5
5*B^2)*b^4*c*d^3 - (18*A^2 + 30*A*B + 55*B^2)*a*b^3*d^4)*x^3 + 36*(B^2*b^4*
d^4*x^4 + B^2*a^3*b*c*d^3 + (B^2*b^4*c*d^3 + 3*B^2*a*b^3*d^4)*x^3 + 3*(B^2*
a*b^3*c*d^3 + B^2*a^2*b^2*d^4)*x^2 + (3*B^2*a^2*b^2*c*d^3 + B^2*a^3*b*d^4)*
x)*log((b*e*x + a*e)/(d*x + c))^3 + 3*((18*A^2 + 66*A*B + 85*B^2)*b^4*c^2*d
^2 + 8*(9*A^2 + 6*A*B + 20*B^2)*a*b^3*c*d^3 - (90*A^2 + 114*A*B + 245*B^2)*
a^2*b^2*d^4)*x^2 + 9*(2*(6*A*B + 5*B^2)*b^4*d^4*x^4 + B^2*b^4*c^4 - 6*B^2*a
*b^3*c^3*d + 18*B^2*a^2*b^2*c^2*d^2 + 12*A*B*a^3*b*c*d^3 - 3*B^2*a^4*d^4 +
2*((6*A*B + 11*B^2)*b^4*c*d^3 + 9*(2*A*B + B^2)*a*b^3*d^4)*x^3 + 6*(B^2*b^4
*c^2*d^2 + 6*A*B*a^2*b^2*d^4 + 3*(2*A*B + 3*B^2)*a*b^3*c*d^3)*x^2 - 2*(B^2*
b^4*c^3*d - 9*B^2*a*b^3*c^2*d^2 - 18*(A*B + B^2)*a^2*b^2*c*d^3 - 6*(A*B - B
^2)*a^3*b*d^4)*x)*log((b*e*x + a*e)/(d*x + c))^2 - ((18*A^2 + 30*A*B + 19*B
^2)*b^4*c^3*d - 81*(2*A^2 + 6*A*B + 7*B^2)*a*b^3*c^2*d^2 - 3*(18*A^2 - 114*
A*B - 29*B^2)*a^2*b^2*c*d^3 + (198*A^2 + 114*A*B + 461*B^2)*a^3*b*d^4)*x +
6*((18*A^2 + 30*A*B + 55*B^2)*b^4*d^4*x^4 + 18*A^2*a^3*b*c*d^3 + (3*A*B + B
^2)*b^4*c^4 - 9*(2*A*B + B^2)*a*b^3*c^3*d + 54*(A*B + B^2)*a^2*b^2*c^2*d^2
- 9*(A*B - B^2)*a^4*d^4 + ((18*A^2 + 66*A*B + 85*B^2)*b^4*c*d^3 + 27*(2*A^2
 + 2*A*B + 5*B^2)*a*b^3*d^4)*x^3 + 3*((6*A*B + 11*B^2)*b^4*c^2*d^2 + 9*(2*A
^2 + 6*A*B + 7*B^2)*a*b^3*c*d^3 + 18*(A^2 + 2*B^2)*a^2*b^2*d^4)*x^2 - ((6*A
*B + 5*B^2)*b^4*c^3*d - 27*(2*A*B + 3*B^2)*a*b^3*c^2*d^2 - 54*(A^2 + 2*A*B
+ 2*B^2)*a^2*b^2*c*d^3 - 18*(A^2 - 2*A*B + 2*B^2)*a^3*b*d^4)*x)*log((b*e*x
+ a*e)/(d*x + c)))/((b^8*c^5*d - 5*a*b^7*c^4*d^2 + 10*a^2*b^6*c^3*d^3 - 10*
a^3*b^5*c^2*d^4 + 5*a^4*b^4*c*d^5 - a^5*b^3*d^6)*g^4*i^2*x^4 + (b^8*c^6 - 2
*a*b^7*c^5*d - 5*a^2*b^6*c^4*d^2 + 20*a^3*b^5*c^3*d^3 - 25*a^4*b^4*c^2*d^4
+ 14*a^5*b^3*c*d^5 - 3*a^6*b^2*d^6)*g^4*i^2*x^3 + 3*(a*b^7*c^6 - 4*a^2*b^6*
c^5*d + 5*a^3*b^5*c^4*d^2 - 5*a^5*b^3*c^2*d^4 + 4*a^6*b^2*c*d^5 - a^7*b*d^6
)*g^4*i^2*x^2 + (3*a^2*b^6*c^6 - 14*a^3*b^5*c^5*d + 25*a^4*b^4*c^4*d^2 - 20
*a^5*b^3*c^3*d^3 + 5*a^6*b^2*c^2*d^4 + 2*a^7*b*c*d^5 - a^8*d^6)*g^4*i^2*x +
 (a^3*b^5*c^6 - 5*a^4*b^4*c^5*d + 10*a^5*b^3*c^4*d^2 - 10*a^6*b^2*c^3*d^3 +
 5*a^7*b*c^2*d^4 - a^8*c*d^5)*g^4*i^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**4/(d*i*x+c*i)**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6160 vs. $2(674) = 1348$.

Time = 0.70 (sec) , antiderivative size = 6160, normalized size of antiderivative = 9.03

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, alg
orithm="maxima")
[Out] -1/3*B^2*((12*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*d + 13*a^2*b*c*d^2 + 3*a^
3*d^3 + 6*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*c^2*d - 8*a*b^2*c*d^2 - 11
*a^2*b*d^3)*x)/((b^7*c^4*d - 4*a*b^6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^
4*c*d^4 + a^4*b^3*d^5)*g^4*i^2*x^4 + (b^7*c^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3
*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a^5*b^2*d^5)*g^4*i^2*x^3 +
 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^4*b^3*c^2*d^3 - 3
*a^5*b^2*c*d^4 + a^6*b*d^5)*g^4*i^2*x^2 + (3*a^2*b^5*c^5 - 11*a^3*b^4*c^4*d
 + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)*g^4*i^2*
x + (a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 +
a^7*c*d^4)*g^4*i^2) + 12*b*d^3*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*
a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2) -
12*b*d^3*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a
^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2))*log(b*e*x/(d*x + c) + a
*e/(d*x + c))^2 - 2/3*A*B*((12*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*d + 13*a
^2*b*c*d^2 + 3*a^3*d^3 + 6*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*c^2*d - 8
*a*b^2*c*d^2 - 11*a^2*b*d^3)*x)/((b^7*c^4*d - 4*a*b^6*c^3*d^2 + 6*a^2*b^5*c
^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*g^4*i^2*x^4 + (b^7*c^5 - a*b^6*c^4*
d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a^5*b^2*d
^5)*g^4*i^2*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^
4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*g^4*i^2*x^2 + (3*a^2*b^5*c^5 -
 11*a^3*b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 +
a^7*d^5)*g^4*i^2*x + (a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6*a^5*b^2*c^3*d^2 - 4
```

*a^6*b*c^2*d^3 + a^7*c*d^4)*g^4*i^2) + 12*b*d^3*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2) - 12*b*d^3*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 1/27*B^2*(6*(b^4*c^4 - 9*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 - 55*a^3*b*c*d^3 + 9*a^4*d^4 + 30*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 3*(11*b^4*c^2*d^2 + 8*a*b^3*c*d^3 - 19*a^2*b^2*d^4)*x^2 - 18*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a)^2 - 18*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(d*x + c)^2 - (5*b^4*c^3*d - 81*a*b^3*c^2*d^2 + 57*a^2*b^2*c*d^3 + 19*a^3*b*d^4)*x + 30*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a) - 6*(5*b^4*d^4*x^4 + 5*a^3*b*c*d^3 + 5*(b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 15*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 5*(3*a^2*b^2*c*d^3 + a^3*b*d^4)*x - 6*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a))*log(d*x + c))*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(a^3*b^5*c^6*g^4*i^2 - 5*a^4*b^4*c^5*d*g^4*i^2 + 10*a^5*b^3*c^4*d^2*g^4*i^2 - 10*a^6*b^2*c^3*d^3*g^4*i^2 + 5*a^7*b*c^2*d^4*g^4*i^2 - a^8*c*d^5*g^4*i^2 + (b^8*c^5*d*g^4*i^2 - 5*a*b^7*c^4*d^2*g^4*i^2 + 10*a^2*b^6*c^3*d^3*g^4*i^2 - 10*a^3*b^5*c^2*d^4*g^4*i^2 + 5*a^4*b^4*c*d^5*g^4*i^2 - a^5*b^3*d^6*g^4*i^2)*x^4 + (b^8*c^6*g^4*i^2 - 2*a*b^7*c^5*d*g^4*i^2 - 5*a^2*b^6*c^4*d^2*g^4*i^2 + 20*a^3*b^5*c^3*d^3*g^4*i^2 - 25*a^4*b^4*c^2*d^4*g^4*i^2 + 14*a^5*b^3*c*d^5*g^4*i^2 - 3*a^6*b^2*d^6*g^4*i^2)*x^3 + 3*(a*b^7*c^6*g^4*i^2 - 4*a^2*b^6*c^5*d*g^4*i^2 + 5*a^3*b^5*c^4*d^2*g^4*i^2 - 5*a^5*b^3*c^2*d^4*g^4*i^2 + 4*a^6*b^2*c*d^5*g^4*i^2 - a^7*b*d^6*g^4*i^2)*x^2 + (3*a^2*b^6*c^6*g^4*i^2 - 14*a^3*b^5*c^5*d*g^4*i^2 + 25*a^4*b^4*c^4*d^2*g^4*i^2 - 20*a^5*b^3*c^3*d^3*g^4*i^2 + 5*a^6*b^2*c^2*d^4*g^4*i^2 + 2*a^7*b*c*d^5*g^4*i^2 - a^8*d^6*g^4*i^2)*x) + (2*b^4*c^4 - 27*a*b^3*c^3*d + 324*a^2*b^2*c^2*d^2 - 245*a^3*b*c*d^3 - 54*a^4*d^4 + 330*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 36*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a)^3 - 36*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(d*x + c)^3 + 15*(17*b^4*c^2*d^2 + 32*a*b^3*c*d^3 - 49*a^2*b^2*d^4)*x^2 - 90*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a)^2 - 18*(5*b^4*d^4*x^4 + 5*a^3*b*c*d^3 + 5*(b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 15*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 5*(3*a^2*b^2*c*d^3 + a^3*b*d^4)*x - 6*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a))*log(d*x + c)^2 - (19*b^4*c^3*d - 567*a*b^3*c^2*d^2 + 87*a^2*b^2*c*d^3 + 461*a^3*b*d^4)*x + 330*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a) - 6*(55*b^4*d^

4*x^4 + 55*a^3*b*c*d^3 + 55*(b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 165*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 18*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a)^2 + 55*(3*a^2*b^2*c*d^3 + a^3*b*d^4)*x - 30*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a))*log(d*x + c))/(a^3*b^5*c^6*g^4*i^2 - 5*a^4*b^4*c^5*d*g^4*i^2 + 10*a^5*b^3*c^4*d^2*g^4*i^2 - 10*a^6*b^2*c^3*d^3*g^4*i^2 + 5*a^7*b*c^2*d^4*g^4*i^2 - a^8*c*d^5*g^4*i^2 + (b^8*c^5*d*g^4*i^2 - 5*a*b^7*c^4*d^2*g^4*i^2 + 10*a^2*b^6*c^3*d^3*g^4*i^2 - 10*a^3*b^5*c^2*d^4*g^4*i^2 + 5*a^4*b^4*c*d^5*g^4*i^2 - a^5*b^3*d^6*g^4*i^2)*x^4 + (b^8*c^6*g^4*i^2 - 2*a*b^7*c^5*d*g^4*i^2 - 5*a^2*b^6*c^4*d^2*g^4*i^2 + 20*a^3*b^5*c^3*d^3*g^4*i^2 - 25*a^4*b^4*c^2*d^4*g^4*i^2 + 14*a^5*b^3*c*d^5*g^4*i^2 - 3*a^6*b^2*d^6*g^4*i^2)*x^3 + 3*(a*b^7*c^6*g^4*i^2 - 4*a^2*b^6*c^5*d*g^4*i^2 + 5*a^3*b^5*c^4*d^2*g^4*i^2 - 5*a^5*b^3*c^2*d^4*g^4*i^2 + 4*a^6*b^2*c*d^5*g^4*i^2 - a^7*b*d^6*g^4*i^2)*x^2 + (3*a^2*b^6*c^6*g^4*i^2 - 14*a^3*b^5*c^5*d*g^4*i^2 + 25*a^4*b^4*c^4*d^2*g^4*i^2 - 20*a^5*b^3*c^3*d^3*g^4*i^2 + 5*a^6*b^2*c^2*d^4*g^4*i^2 + 2*a^7*b*c*d^5*g^4*i^2 - a^8*d^6*g^4*i^2)*x)) - 1/3*A^2*((12*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*d + 13*a^2*b*c*d^2 + 3*a^3*d^3 + 6*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*c^2*d - 8*a*b^2*c*d^2 - 11*a^2*b*d^3)*x)/((b^7*c^4*d - 4*a*b^6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*g^4*i^2*x^4 + (b^7*c^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a^5*b^2*d^5)*g^4*i^2*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*g^4*i^2*x^2 + (3*a^2*b^5*c^5 - 11*a^3*b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)*g^4*i^2*x + (a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4)*g^4*i^2) + 12*b*d^3*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2) - 12*b*d^3*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2)) - 2/9*(b^4*c^4 - 9*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 - 55*a^3*b*c*d^3 + 9*a^4*d^4 + 30*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 3*(11*b^4*c^2*d^2 + 8*a*b^3*c*d^3 - 19*a^2*b^2*d^4)*x^2 - 18*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a)^2 - 18*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(d*x + c)^2 - (5*b^4*c^3*d - 81*a*b^3*c^2*d^2 + 57*a^2*b^2*c*d^3 + 19*a^3*b*d^4)*x + 30*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a) - 6*(5*b^4*d^4*x^4 + 5*a^3*b*c*d^3 + 5*(b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 15*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 5*(3*a^2*b^2*c*d^3 + a^3*b*d^4)*x - 6*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a))*log(d*x + c))*A*B/(a^3*b^5*c^6*g^4*i^2 - 5*a^4*b^4*c^5*d*g^4*i^2 + 10*a^5*b^3*c^4*d^2*g^4*i^2 - 10*a^6*b^2*c^3*d^3*g^4*i^2 + 5*a^7*b*c^2*d^4*g^4*i^2 - a^8*c*d^5

```
*g^4*i^2 + (b^8*c^5*d*g^4*i^2 - 5*a*b^7*c^4*d^2*g^4*i^2 + 10*a^2*b^6*c^3*d^
3*g^4*i^2 - 10*a^3*b^5*c^2*d^4*g^4*i^2 + 5*a^4*b^4*c*d^5*g^4*i^2 - a^5*b^3*
d^6*g^4*i^2)*x^4 + (b^8*c^6*g^4*i^2 - 2*a*b^7*c^5*d*g^4*i^2 - 5*a^2*b^6*c^4
*d^2*g^4*i^2 + 20*a^3*b^5*c^3*d^3*g^4*i^2 - 25*a^4*b^4*c^2*d^4*g^4*i^2 + 14
*a^5*b^3*c*d^5*g^4*i^2 - 3*a^6*b^2*d^6*g^4*i^2)*x^3 + 3*(a*b^7*c^6*g^4*i^2
- 4*a^2*b^6*c^5*d*g^4*i^2 + 5*a^3*b^5*c^4*d^2*g^4*i^2 - 5*a^5*b^3*c^2*d^4*g
^4*i^2 + 4*a^6*b^2*c*d^5*g^4*i^2 - a^7*b*d^6*g^4*i^2)*x^2 + (3*a^2*b^6*c^6*
g^4*i^2 - 14*a^3*b^5*c^5*d*g^4*i^2 + 25*a^4*b^4*c^4*d^2*g^4*i^2 - 20*a^5*b^
3*c^3*d^3*g^4*i^2 + 5*a^6*b^2*c^2*d^4*g^4*i^2 + 2*a^7*b*c*d^5*g^4*i^2 - a^8
*d^6*g^4*i^2)*x)
```

Giac [A] (verification not implemented)

none

Time = 88.80 (sec) , antiderivative size = 754, normalized size of antiderivative = 1.11

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx =$$

$$-\frac{1}{54}\left(\frac{18\left(B^2b^2e^4-\frac{3\,(bex+ae)B^2bde^3}{dx+c}+\frac{3\,(bex+ae)^2B^2d^2e^2}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^3b^2c^2g^4i^2}{(dx+c)^3}-\frac{2\,(bex+ae)^3abcdg^4i^2}{(dx+c)^3}+\frac{(bex+ae)^3a^2d^2g^4i^2}{(dx+c)^3}}+\frac{6\left(6\,ABb^2e^4+2\,B^2b^2e^4-\frac{18\,(bex+\ldots}{dx\ldots}\right.}{(bex+\ldots}\right.$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, alg
orithm="giac")
```

```
[Out] -1/54*(18*(B^2*b^2*e^4 - 3*(b*e*x + a*e)*B^2*b*d*e^3/(d*x + c) + 3*(b*e*x +
 a*e)^2*B^2*d^2*e^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a
*e)^3*b^2*c^2*g^4*i^2/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4*i^2/(d*x
+ c)^3 + (b*e*x + a*e)^3*a^2*d^2*g^4*i^2/(d*x + c)^3) + 6*(6*A*B*b^2*e^4 +
2*B^2*b^2*e^4 - 18*(b*e*x + a*e)*A*B*b*d*e^3/(d*x + c) - 9*(b*e*x + a*e)*B^
2*b*d*e^3/(d*x + c) + 18*(b*e*x + a*e)^2*A*B*d^2*e^2/(d*x + c)^2 + 18*(b*e*
x + a*e)^2*B^2*d^2*e^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/((b*e*x +
a*e)^3*b^2*c^2*g^4*i^2/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4*i^2/(d*x
 + c)^3 + (b*e*x + a*e)^3*a^2*d^2*g^4*i^2/(d*x + c)^3) + (18*A^2*b^2*e^4 +
12*A*B*b^2*e^4 + 4*B^2*b^2*e^4 - 54*(b*e*x + a*e)*A^2*b*d*e^3/(d*x + c) - 5
4*(b*e*x + a*e)*A*B*b*d*e^3/(d*x + c) - 27*(b*e*x + a*e)*B^2*b*d*e^3/(d*x +
 c) + 54*(b*e*x + a*e)^2*A^2*d^2*e^2/(d*x + c)^2 + 108*(b*e*x + a*e)^2*A*B*
d^2*e^2/(d*x + c)^2 + 108*(b*e*x + a*e)^2*B^2*d^2*e^2/(d*x + c)^2)/((b*e*x
+ a*e)^3*b^2*c^2*g^4*i^2/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4*i^2/(d
*x + c)^3 + (b*e*x + a*e)^3*a^2*d^2*g^4*i^2/(d*x + c)^3))*(b*c/((b*c*e - a*
d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

Mupad [B] (verification not implemented)

Time = 10.02 (sec) , antiderivative size = 2701, normalized size of antiderivative = 3.96

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx=\text{Too large to display}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^4*(c*i + d*i*x)^2
),x)
```

```
[Out] (log((e*(a + b*x))/(c + d*x))*(x^2*((4*B^2*b*d)/(g^4*i^2*(a*d - b*c)^3) - (
4*b*d^3*(b*d*((2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))
/(2*b*d^2)) + ((a*d + b*c)*(a*d - b*c))/d^2)*(5*B^2 + 6*A*B))/(3*g^4*i^2*(a
*d - b*c)^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) + x*((8*(
2*B^2 - 3*A*B))/(9*g^4*i^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (4*B^2*(a*d +
 b*c))/(g^4*i^2*(a*d - b*c)^3) - (4*b*d^3*(((2*a^2*d^2 + b^2*c^2 - 3*a*b*c*
d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2))*(a*d + b*c) + (a*c*(a*d - b*c))/d
^2)*(5*B^2 + 6*A*B))/(3*g^4*i^2*(a*d - b*c)^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*
c^2*d - 3*a^2*b*c*d^2))) - (2*(B^2*b*c - 9*B^2*a*d + 9*A*B*a*d + 3*A*B*b*c)
)/(9*g^4*i^2*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (4*B^2*a*c)/(g^4*i^
2*(a*d - b*c)^3) - (4*b^2*d^2*x^3*(5*B^2 + 6*A*B))/(3*g^4*i^2*(a*d - b*c)*(
a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (4*a*b*c*d^3*((2*a^2*
d^2 + b^2*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2))*(5*B^2 +
6*A*B))/(3*g^4*i^2*(a*d - b*c)^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2))))/(b^2*x^4 + (a^3*c)/(b*d) + (x*(a^3*d + 3*a^2*b*c))/(b*d) + (x^
3*(b^3*c + 3*a*b^2*d))/(b*d) + (x^2*(3*a*b^2*c + 3*a^2*b*d))/(b*d)) - ((27*
A^2*a^3*d^3 + 9*A^2*b^3*c^3 + 54*B^2*a^3*d^3 + 2*B^2*b^3*c^3 - 54*A*B*a^3*d
^3 + 6*A*B*b^3*c^3 - 45*A^2*a*b^2*c^2*d + 117*A^2*a^2*b*c*d^2 - 25*B^2*a*b^
2*c^2*d + 299*B^2*a^2*b*c*d^2 - 48*A*B*a*b^2*c^2*d + 276*A*B*a^2*b*c*d^2)/(
3*(a*d - b*c)) + (2*x^3*(18*A^2*b^3*d^3 + 55*B^2*b^3*d^3 + 30*A*B*b^3*d^3))
/(a*d - b*c) + (x*(198*A^2*a^2*b*d^3 + 461*B^2*a^2*b*d^3 - 18*A^2*b^3*c^2*d
 - 19*B^2*b^3*c^2*d + 144*A^2*a*b^2*c*d^2 + 548*B^2*a*b^2*c*d^2 + 114*A*B*a
^2*b*d^3 - 30*A*B*b^3*c^2*d + 456*A*B*a*b^2*c*d^2))/(3*(a*d - b*c)) + (x^2*
(90*A^2*a*b^2*d^3 + 245*B^2*a*b^2*d^3 + 18*A^2*b^3*c*d^2 + 85*B^2*b^3*c*d^2
 + 114*A*B*a*b^2*d^3 + 66*A*B*b^3*c*d^2))/(a*d - b*c))/(x*(9*a^6*d^4*g^4*i^
2 - 27*a^2*b^4*c^4*g^4*i^2 + 72*a^3*b^3*c^3*d*g^4*i^2 - 54*a^4*b^2*c^2*d^2*
g^4*i^2) - x^2*(27*a*b^5*c^4*g^4*i^2 - 27*a^5*b*d^4*g^4*i^2 - 54*a^2*b^4*c^
3*d*g^4*i^2 + 54*a^4*b^2*c*d^3*g^4*i^2) - x^3*(9*b^6*c^4*g^4*i^2 - 27*a^4*b
^2*d^4*g^4*i^2 + 72*a^3*b^3*c*d^3*g^4*i^2 - 54*a^2*b^4*c^2*d^2*g^4*i^2) + x
^4*(9*a^3*b^3*d^4*g^4*i^2 - 9*b^6*c^3*d*g^4*i^2 + 27*a*b^5*c^2*d^2*g^4*i^2
- 27*a^2*b^4*c*d^3*g^4*i^2) - 9*a^3*b^3*c^4*g^4*i^2 + 9*a^6*c*d^3*g^4*i^2 +
 27*a^4*b^2*c^3*d*g^4*i^2 - 27*a^5*b*c^2*d^2*g^4*i^2) - log((e*(a + b*x))/(
c + d*x))^2*((x*((4*B^2)/(3*g^4*i^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (4*B
^2*b*d^3*(((2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2
*b*d^2))*(a*d + b*c) + (a*c*(a*d - b*c))/d^2))/(g^4*i^2*(a*d - b*c)^2*(a^3*
```

```
d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) + (B^2*(3*a*d + b*c))/(3*g
^4*i^2*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (4*B^2*b^2*d^2*x^3)/(g^4*
i^2*(a*d - b*c)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (4*B
^2*b*d^3*x^2*(b*d*((2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d -
b*c))/(2*b*d^2)) + ((a*d + b*c)*(a*d - b*c))/d^2))/(g^4*i^2*(a*d - b*c)^2*(
a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (4*B^2*a*b*c*d^3*((2*
a^2*d^2 + b^2*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)))/(g^4
*i^2*(a*d - b*c)^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))/(b
^2*x^4 + (a^3*c)/(b*d) + (x*(a^3*d + 3*a^2*b*c))/(b*d) + (x^3*(b^3*c + 3*a*
b^2*d))/(b*d) + (x^2*(3*a*b^2*c + 3*a^2*b*d))/(b*d)) - (2*b*d^3*(5*B^2 + 6*
A*B))/(3*g^4*i^2*(a*d - b*c)^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2))) - (b*d^3*atan((b*d^3*(18*A^2 + 55*B^2 + 30*A*B)*(9*a^5*d^5*g^4*i^
2 + 9*b^5*c^5*g^4*i^2 - 27*a*b^4*c^4*d*g^4*i^2 - 27*a^4*b*c*d^4*g^4*i^2 + 1
8*a^2*b^3*c^3*d^2*g^4*i^2 + 18*a^3*b^2*c^2*d^3*g^4*i^2)*2i)/(9*g^4*i^2*(a*d
 - b*c)^5*(36*A^2*b*d^3 + 110*B^2*b*d^3 + 60*A*B*b*d^3)) + (b^2*d^4*x*(18*A
^2 + 55*B^2 + 30*A*B)*(a^4*d^4*g^4*i^2 + b^4*c^4*g^4*i^2 - 4*a*b^3*c^3*d*g^
4*i^2 - 4*a^3*b*c*d^3*g^4*i^2 + 6*a^2*b^2*c^2*d^2*g^4*i^2)*4i)/(g^4*i^2*(a*
d - b*c)^5*(36*A^2*b*d^3 + 110*B^2*b*d^3 + 60*A*B*b*d^3)))*(18*A^2 + 55*B^2
 + 30*A*B)*4i)/(9*g^4*i^2*(a*d - b*c)^5) + (4*B^2*b*d^3*log((e*(a + b*x))/(
c + d*x))^3)/(3*g^4*i^2*(a*d - b*c)^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
3*a^2*b*c*d^2))
```

3.100 $\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 635

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{B^2(bc-ad)g^3(a+bx)^2}{4d^2i^3(c+dx)^2} - \frac{4AbB(bc-ad)g^3(a+bx)}{d^3i^3(c+dx)} + \frac{4bB^2(bc-ad)g^3(a+bx)}{d^3i^3(c+dx)} - \frac{4bB^2(bc-ad)g^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^3i^3(c+dx)} - \frac{B(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i^3(c+dx)^2} + \frac{2b^2B(bc-ad)g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4i^3} + \frac{b^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^3} + \frac{(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^2i^3(c+dx)^2} + \frac{2b(bc-ad)g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^3(c+dx)} + \frac{3b^2(bc-ad)g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^4i^3} + \frac{2b^2B^2(bc-ad)g^3\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3} + \frac{6b^2B(bc-ad)g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3} - \frac{6b^2B^2(bc-ad)g^3\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}$$

```
[Out] 1/4*B^2*(-a*d+b*c)*g^3*(b*x+a)^2/d^2/i^3/(d*x+c)^2-4*A*b*B*(-a*d+b*c)*g^3*(
b*x+a)/d^3/i^3/(d*x+c)+4*b*B^2*(-a*d+b*c)*g^3*(b*x+a)/d^3/i^3/(d*x+c)-4*b*B
^2*(-a*d+b*c)*g^3*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/d^3/i^3/(d*x+c)-1/2*B*(-a*d
+b*c)*g^3*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^2/i^3/(d*x+c)^2+2*b^2*B*(
-a*d+b*c)*g^3*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4/i^3+
b^2*g^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i^3+1/2*(-a*d+b*c)*g^3*(b
*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^2/i^3/(d*x+c)^2+2*b*(-a*d+b*c)*g^3*
(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i^3/(d*x+c)+3*b^2*(-a*d+b*c)*g^3*
ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^4/i^3+2*b^2*B^2*(-
a*d+b*c)*g^3*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i^3+6*b^2*B*(-a*d+b*c)*g^3*
```

```
(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i^3-6*b^2*B^
2*(-a*d+b*c)*g^3*polylog(3,d*(b*x+a)/b/(d*x+c))/d^4/i^3
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.262$, Rules used = {2562, 2395, 2333, 2332, 2342, 2341, 2355, 2354, 2438, 2421, 6724}

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{6b^2Bg^3(bc-ad)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^4i^3}$$
$$+\frac{3b^2g^3(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^4i^3}$$
$$+\frac{2b^2Bg^3(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^4i^3}$$
$$+\frac{b^2g^3(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^3i^3}+\frac{2bg^3(a+bx)(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^3i^3(c+dx)}$$
$$-\frac{4AbBg^3(a+bx)(bc-ad)}{d^3i^3(c+dx)}+\frac{g^3(a+bx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d^2i^3(c+dx)^2}$$
$$-\frac{Bg^3(a+bx)^2(bc-ad)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d^2i^3(c+dx)^2}$$
$$+\frac{2b^2B^2g^3(bc-ad)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}$$
$$-\frac{6b^2B^2g^3(bc-ad)\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}-\frac{4bB^2g^3(a+bx)(bc-ad)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^3i^3(c+dx)}$$
$$+\frac{4bB^2g^3(a+bx)(bc-ad)}{d^3i^3(c+dx)}+\frac{B^2g^3(a+bx)^2(bc-ad)}{4d^2i^3(c+dx)^2}$$

```
[In] Int[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x)^
3,x]
```

```
[Out] (B^2*(b*c - a*d)*g^3*(a + b*x)^2)/(4*d^2*i^3*(c + d*x)^2) - (4*A*b*B*(b*c -
 a*d)*g^3*(a + b*x))/(d^3*i^3*(c + d*x)) + (4*b*B^2*(b*c - a*d)*g^3*(a + b*
x))/(d^3*i^3*(c + d*x)) - (4*b*B^2*(b*c - a*d)*g^3*(a + b*x)*Log[(e*(a + b*
x))/(c + d*x)])/(d^3*i^3*(c + d*x)) - (B*(b*c - a*d)*g^3*(a + b*x)^2*(A + B
*Log[(e*(a + b*x))/(c + d*x)]))/(2*d^2*i^3*(c + d*x)^2) + (2*b^2*B*(b*c - a
```

```
*d)*g^3*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x)])
)/(d^4*i^3) + (b^2*g^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d
^3*i^3) + ((b*c - a*d)*g^3*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
^2)/(2*d^2*i^3*(c + d*x)^2) + (2*b*(b*c - a*d)*g^3*(a + b*x)*(A + B*Log[(e*
(a + b*x))/(c + d*x)])^2)/(d^3*i^3*(c + d*x)) + (3*b^2*(b*c - a*d)*g^3*Log[
(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d^4*i^3
) + (2*b^2*B^2*(b*c - a*d)*g^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^
4*i^3) + (6*b^2*B*(b*c - a*d)*g^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Poly
Log[2, (d*(a + b*x))/(b*(c + d*x))])/(d^4*i^3) - (6*b^2*B^2*(b*c - a*d)*g^3
*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(d^4*i^3)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
```

```
, p}, x] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\left((bc-ad)g^3\right)\text{Subst}\left(\int \frac{x^3(A+B\log(ex))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{i^3}$$

$$= \frac{\left((bc-ad)g^3\right)\text{Subst}\left(\int \left(\frac{2b(A+B\log(ex))^2}{d^3} + \frac{x(A+B\log(ex))^2}{d^2} + \frac{b^3(A+B\log(ex))^2}{d^3(-b+dx)^2} + \frac{3b^2(A+B\log(ex))^2}{d^3(-b+dx)}\right)\,dx, x, \frac{a+}{c+}\right.}{i^3}$$

$$= \frac{(2b(bc-ad)g^3)\,\text{Subst}\left(\int (A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^3} + \frac{(3b^2(bc-ad)g^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{-b+dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^3} + \frac{(b^3(bc-ad)g^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{(-b+dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^3} + \frac{((bc-ad)g^3)\,\text{Subst}\left(\int x(A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^3}$$

$$= \frac{b^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^3} + \frac{(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^2i^3(c+dx)^2} + \frac{2b(bc-ad)g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^3(c+dx)} + \frac{3b^2(bc-ad)g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^4i^3} - \frac{(6b^2B(bc-ad)g^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex))\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4i^3} - \frac{(4bB(bc-ad)g^3)\,\text{Subst}\left(\int (A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^3} + \frac{(2b^2B(bc-ad)g^3)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{-b+dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^3} - \frac{(B(bc-ad)g^3)\,\text{Subst}\left(\int x(A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^3}$$

$$
\begin{aligned}
&= \frac{B^2(bc-ad)g^3(a+bx)^2}{4d^2i^3(c+dx)^2} - \frac{4AbB(bc-ad)g^3(a+bx)}{d^3i^3(c+dx)} \\
&- \frac{B(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i^3(c+dx)^2} \\
&+ \frac{2b^2B(bc-ad)g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4i^3} \\
&+ \frac{b^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^3} \\
&+ \frac{(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^2i^3(c+dx)^2} \\
&+ \frac{2b(bc-ad)g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^3(c+dx)} \\
&+ \frac{3b^2(bc-ad)g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^4i^3} \\
&+ \frac{6b^2B(bc-ad)g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3} \\
&- \frac{(2b^2B^2(bc-ad)g^3)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4i^3} \\
&- \frac{(6b^2B^2(bc-ad)g^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4i^3} \\
&- \frac{(4bB^2(bc-ad)g^3)\operatorname{Subst}\left(\int\log(ex)\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{B^2(bc-ad)g^3(a+bx)^2}{4d^2i^3(c+dx)^2} - \frac{4AbB(bc-ad)g^3(a+bx)}{d^3i^3(c+dx)} \\
&+ \frac{4bB^2(bc-ad)g^3(a+bx)}{d^3i^3(c+dx)} - \frac{4bB^2(bc-ad)g^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^3i^3(c+dx)} \\
&- \frac{B(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i^3(c+dx)^2} \\
&+ \frac{2b^2B(bc-ad)g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4i^3} \\
&+ \frac{b^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^3} \\
&+ \frac{(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^2i^3(c+dx)^2} \\
&+ \frac{2b(bc-ad)g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^3(c+dx)} \\
&+ \frac{3b^2(bc-ad)g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^4i^3} \\
&+ \frac{2b^2B^2(bc-ad)g^3\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3} \\
&+ \frac{6b^2B(bc-ad)g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3} \\
&- \frac{6b^2B^2(bc-ad)g^3\operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5929 vs. 2(635) = 1270.

Time = 7.36 (sec) , antiderivative size = 5929, normalized size of antiderivative = 9.34

$$
\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d
*i*x)^3,x]
```

```
[Out] Result too large to show
```

Maple [F]

$$\int \frac{(bgx+ag)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(dix+ci)^3}dx$$

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x)
```

```
[Out] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^3}\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, alg
orithm="fricas")
```

```
[Out] integral((A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a
^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a
^3*g^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g
^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log((b*e*x + a*e)/(d*x + c)))/(d^
3*i^3*x^3 + 3*c*d^2*i^3*x^2 + 3*c^2*d*i^3*x + c^3*i^3), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \frac{g^3\left(\int \frac{A^2a^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{A^2b^3x^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{B^2a^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{2ABa^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\right.}{}$$

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**3,x)
```

```
[Out] g**3*(Integral(A**2*a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x
) + Integral(A**2*b**3*x**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3)
, x) + Integral(B**2*a**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c**3 + 3
*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*a**3*log(a*e/(c
 + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
 x) + Integral(3*A**2*a*b**2*x**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3
*x**3), x) + Integral(3*A**2*a**2*b*x/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
```

```
d**3*x**3), x) + Integral(B**2*b**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x
))**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*
b**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**
2*x**2 + d**3*x**3), x) + Integral(3*B**2*a*b**2*x**2*log(a*e/(c + d*x) + b
*e*x/(c + d*x))**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + In
tegral(3*B**2*a**2*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c**3 + 3*c*
*2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(6*A*B*a*b**2*x**2*log(a*
e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x*
*3), x) + Integral(6*A*B*a**2*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**
3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/i**3
```

Maxima [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^3}\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, alg
orithm="maxima")
```

```
[Out] -3/2*A*B*a^2*b*g^3*(2*(2*d*x + c)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^4
*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - (b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a
*d^2)*x)/((b*c*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^
3*d^2 - a*c^2*d^3)*i^3) - 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 -
2*a*b*c*d^3 + a^2*d^4)*i^3) + 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^
2 - 2*a*b*c*d^3 + a^2*d^4)*i^3)) - 1/2*A^2*b^3*g^3*((6*c^2*d*x + 5*c^3)/(d^
6*i^3*x^2 + 2*c*d^5*i^3*x + c^2*d^4*i^3) - 2*x/(d^3*i^3) + 6*c*log(d*x + c)
/(d^4*i^3)) + 1/2*A*B*a^3*g^3*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i
^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) - 2*log
(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3)
 + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log
(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) + 3/2*A^2*a*b^2*g^3*((
4*c*d*x + 3*c^2)/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) + 2*log(d*x +
c)/(d^3*i^3)) - 3/2*(2*d*x + c)*A^2*a^2*b*g^3/(d^4*i^3*x^2 + 2*c*d^3*i^3*x
+ c^2*d^2*i^3) - 1/2*A^2*a^3*g^3/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3)
- 1/2*(2*((b^3*c*d^2*g^3 - a*b^2*d^3*g^3)*B^2*x^2 + 2*(b^3*c^2*d*g^3 - a*b^
2*c*d^2*g^3)*B^2*x + (b^3*c^3*g^3 - a*b^2*c^2*d*g^3)*B^2)*log(d*x + c)^3 -
(2*B^2*b^3*d^3*g^3*x^3 + 4*B^2*b^3*c*d^2*g^3*x^2 - 2*(2*b^3*c^2*d*g^3 - 6*a
*b^2*c*d^2*g^3 + 3*a^2*b*d^3*g^3)*B^2*x - (5*b^3*c^3*g^3 - 9*a*b^2*c^2*d*g^
3 + 3*a^2*b*c*d^2*g^3 + a^3*d^3*g^3)*B^2)*log(d*x + c)^2)/(d^6*i^3*x^2 + 2*
c*d^5*i^3*x + c^2*d^4*i^3) - integrate(-(3*B^2*a^2*b*d^3*g^3*x*log(e)^2 + B
^2*a^3*d^3*g^3*log(e)^2 + (B^2*b^3*d^3*g^3*log(e)^2 + 2*A*B*b^3*d^3*g^3*log
(e))*x^3 + 3*(B^2*a*b^2*d^3*g^3*log(e)^2 + 2*A*B*a*b^2*d^3*g^3*log(e))*x^2
+ (B^2*b^3*d^3*g^3*x^3 + 3*B^2*a*b^2*d^3*g^3*x^2 + 3*B^2*a^2*b*d^3*g^3*x +
```

```
B^2*a^3*d^3*g^3)*log(b*x + a)^2 + 2*(3*B^2*a^2*b*d^3*g^3*x*log(e) + B^2*a^3
*d^3*g^3*log(e) + (B^2*b^3*d^3*g^3*log(e) + A*B*b^3*d^3*g^3)*x^3 + 3*(B^2*a
*b^2*d^3*g^3*log(e) + A*B*a*b^2*d^3*g^3)*x^2)*log(b*x + a) + (2*(2*b^3*c^2*
d*g^3 - 6*a*b^2*c*d^2*g^3 - 3*(g^3*log(e) - g^3)*a^2*b*d^3)*B^2*x - 2*(A*B*
b^3*d^3*g^3 + (g^3*log(e) + g^3)*B^2*b^3*d^3)*x^3 + (5*b^3*c^3*g^3 - 9*a*b^
2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 - (2*g^3*log(e) - g^3)*a^3*d^3)*B^2 - 2*(3*
A*B*a*b^2*d^3*g^3 + (3*a*b^2*d^3*g^3*log(e) + 2*b^3*c*d^2*g^3)*B^2)*x^2 - 2
*(B^2*b^3*d^3*g^3*x^3 + 3*B^2*a*b^2*d^3*g^3*x^2 + 3*B^2*a^2*b*d^3*g^3*x + B
^2*a^3*d^3*g^3)*log(b*x + a))*log(d*x + c))/(d^6*i^3*x^3 + 3*c*d^5*i^3*x^2
+ 3*c^2*d^4*i^3*x + c^3*d^3*i^3), x)
```

Giac [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^3}\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, alg
orithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*i
)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx=\int \frac{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(c\,i+d\,i\,x)^3}\,dx$$

```
[In] int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)^
3,x)
```

```
[Out] int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)^
3, x)
```

3.101 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 410

$$\begin{aligned}\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx &= -\frac{B^2g^2(a+bx)^2}{4di^3(c+dx)^2}+\frac{2AbBg^2(a+bx)}{d^2i^3(c+dx)}-\frac{2bB^2g^2(a+bx)}{d^2i^3(c+dx)}+\frac{2bB^2g^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^2i^3(c+dx)}\\ &\quad+\frac{Bg^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di^3(c+dx)^2}-\frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2di^3(c+dx)^2}\\ &\quad-\frac{bg^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^3(c+dx)}-\frac{b^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^3}\\ &\quad-\frac{2b^2Bg^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3}+\frac{2b^2B^2g^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3}\end{aligned}$$

```
[Out] -1/4*B^2*g^2*(b*x+a)^2/d/i^3/(d*x+c)^2+2*A*b*B*g^2*(b*x+a)/d^2/i^3/(d*x+c)-
2*b*B^2*g^2*(b*x+a)/d^2/i^3/(d*x+c)+2*b*B^2*g^2*(b*x+a)*ln(e*(b*x+a)/(d*x+c
))/d^2/i^3/(d*x+c)+1/2*B*g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i^3/(d
*x+c)^2-1/2*g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d/i^3/(d*x+c)^2-b*g
^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^2/i^3/(d*x+c)-b^2*g^2*ln((-a*d+b
*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i^3-2*b^2*B*g^2*(A+B*ln(e*
(b*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i^3+2*b^2*B^2*g^2*poly
log(3,d*(b*x+a)/b/(d*x+c))/d^3/i^3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2562, 2395, 2333, 2332, 2342, 2341, 2354, 2421, 6724}

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$
$$=-\frac{2b^2Bg^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^3i^3}$$
$$-\frac{b^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^3i^3}-\frac{bg^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^2i^3(c+dx)}$$
$$+\frac{2AbBg^2(a+bx)}{d^2i^3(c+dx)}-\frac{g^2(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2di^3(c+dx)^2}$$
$$+\frac{Bg^2(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2di^3(c+dx)^2}+\frac{2b^2B^2g^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3}$$
$$+\frac{2bB^2g^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^2i^3(c+dx)}-\frac{2bB^2g^2(a+bx)}{d^2i^3(c+dx)}-\frac{B^2g^2(a+bx)^2}{4di^3(c+dx)^2}$$

```
[In] Int[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x)^
3,x]
```

```
[Out] -1/4*(B^2*g^2*(a + b*x)^2)/(d*i^3*(c + d*x)^2) + (2*A*b*B*g^2*(a + b*x))/(d
^2*i^3*(c + d*x)) - (2*b*B^2*g^2*(a + b*x))/(d^2*i^3*(c + d*x)) + (2*b*B^2*
g^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(d^2*i^3*(c + d*x)) + (B*g^2*(a
 + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d*i^3*(c + d*x)^2) - (g^
2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*d*i^3*(c + d*x)^2)
 - (b*g^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d^2*i^3*(c + d
*x)) - (b^2*g^2*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c
+ d*x)])^2)/(d^3*i^3) - (2*b^2*B*g^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*P
olyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^3*i^3) + (2*b^2*B^2*g^2*PolyLog[
3, (d*(a + b*x))/(b*(c + d*x))])/(d^3*i^3)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
```

```
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
```

```
tegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g^2\text{Subst}\left(\int \frac{x^2(A+B\log(ex))^2}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{i^3} \\
&= \frac{g^2\text{Subst}\left(\int \left(-\frac{b(A+B\log(ex))^2}{d^2}-\frac{x(A+B\log(ex))^2}{d}-\frac{b^2(A+B\log(ex))^2}{d^2(-b+dx)}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{i^3} \\
&= -\frac{(bg^2)\,\text{Subst}\left(\int (A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^3} \\
&\quad -\frac{(b^2g^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{-b+dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^3} \\
&\quad -\frac{g^2\text{Subst}\left(\int x(A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{di^3} \\
&= -\frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2di^3(c+dx)^2}-\frac{bg^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^3(c+dx)} \\
&\quad -\frac{b^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^3} \\
&\quad +\frac{(2b^2Bg^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^3} \\
&\quad +\frac{(2bBg^2)\,\text{Subst}\left(\int (A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^3} \\
&\quad +\frac{(Bg^2)\,\text{Subst}\left(\int x(A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{di^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{B^2g^2(a+bx)^2}{4di^3(c+dx)^2} + \frac{2AbBg^2(a+bx)}{d^2i^3(c+dx)} + \frac{Bg^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di^3(c+dx)^2} \\
&\quad - \frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2di^3(c+dx)^2} - \frac{bg^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^3(c+dx)} \\
&\quad - \frac{b^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^3} - \frac{2b^2Bg^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3} \\
&\quad + \frac{(2b^2B^2g^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^3} + \frac{(2bB^2g^2)\operatorname{Subst}\left(\int\log(ex)\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^3} \\
&= -\frac{B^2g^2(a+bx)^2}{4di^3(c+dx)^2} + \frac{2AbBg^2(a+bx)}{d^2i^3(c+dx)} - \frac{2bB^2g^2(a+bx)}{d^2i^3(c+dx)} \\
&\quad + \frac{2bB^2g^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^2i^3(c+dx)} + \frac{Bg^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di^3(c+dx)^2} \\
&\quad - \frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2di^3(c+dx)^2} - \frac{bg^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^3(c+dx)} \\
&\quad - \frac{b^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^3} \\
&\quad - \frac{2b^2Bg^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3} + \frac{2b^2B^2g^2\operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3384 vs. $2(410) = 820$.

Time = 4.08 (sec) , antiderivative size = 3384, normalized size of antiderivative = 8.25

$$
\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d
*i*x)^3,x]
```

```
[Out] (g^2*((-6*A^2*(b*c - a*d)^2)/(c + d*x)^2 + (24*A^2*b*(b*c - a*d))/(c + d*x)
 + 12*A^2*b^2*Log[c + d*x] - (12*a*A*b*B*d*(-(b^2*c^3) + 4*a*b*c^2*d - 3*a^
2*c*d^2 - 2*b^2*c^2*d*x + 6*a*b*c*d^2*x - 4*a^2*d^3*x - 2*b*(b*c - 2*a*d)*(
c + d*x)^2*Log[a + b*x] + 2*(b*c - a*d)^2*(c + 2*d*x)*Log[(e*(a + b*x))/(c
+ d*x)] + 2*b^2*c^3*Log[c + d*x] - 4*a*b*c^2*d*Log[c + d*x] + 4*b^2*c^2*d*x
*Log[c + d*x] - 8*a*b*c*d^2*x*Log[c + d*x] + 2*b^2*c*d^2*x^2*Log[c + d*x] -
 4*a*b*d^3*x^2*Log[c + d*x]))/((b*c - a*d)^2*(c + d*x)^2) - (6*a^2*A*B*d^2*
```

(-(b^2*c^2) + 4*a*b*c*d - a^2*d^2 + 2*b^2*c*d*x + 2*a*b*d^2*x + 2*b^2*d^2*x
^2 - 2*b^2*(c + d*x)^2*Log[a/b + x] + 2*(b*c - a*d)^2*Log[(e*(a + b*x))/(c
+ d*x)] + 2*b^2*c^2*Log[(b*(c + d*x))/(b*c - a*d)] + 4*b^2*c*d*x*Log[(b*(c
+ d*x))/(b*c - a*d)] + 2*b^2*d^2*x^2*Log[(b*(c + d*x))/(b*c - a*d)]))/((b*c
- a*d)^2*(c + d*x)^2) + 6*A*b^2*B*(-2*Log[c/d + x]^2 - (8*c*(1 + Log[c/d +
x]))/(c + d*x) + (c^2*(1 + 2*Log[c/d + x]))/(c + d*x)^2 + 8*c*(Log[a/b + x
]/(c + d*x) + (b*(Log[a + b*x] - Log[c + d*x]))/(-(b*c) + a*d)) + 2*(-Log[a
/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])*((c*(3*c + 4*d*x))/(
c + d*x)^2 + 2*Log[c + d*x]) + (2*c^2*(-Log[a/b + x] + (b*(c + d*x)*(b*c -
a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]))/(b*c - a*d)^2))
/(c + d*x)^2 + 4*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2,
(d*(a + b*x))/(-(b*c) + a*d)])) + (3*a^2*B^2*d^2*(-(b*c - a*d)^2 - 6*b*(b*c
- a*d)*(c + d*x) - 6*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[
a + b*x]^2 + 2*(b*c - a*d)^2*Log[(e*(a + b*x))/(c + d*x)] + 4*b*(b*c - a*d)
*(c + d*x)*Log[(e*(a + b*x))/(c + d*x)] + 4*b^2*(c + d*x)^2*Log[a + b*x]*Lo
g[(e*(a + b*x))/(c + d*x)] - 2*(b*c - a*d)^2*Log[(e*(a + b*x))/(c + d*x)]^2
+ 6*b^2*(c + d*x)^2*Log[c + d*x] + 4*b^2*(c + d*x)^2*Log[a + b*x]*Log[(b*(
c + d*x))/(b*c - a*d)] - 4*b^2*(c + d*x)^2*Log[(d*(a + b*x))/(-(b*c) + a*d)
]*Log[(b*c - a*d)/(b*c + b*d*x)] + 4*b^2*(c + d*x)^2*Log[(e*(a + b*x))/(c +
d*x)]*Log[(b*c - a*d)/(b*c + b*d*x)] - 2*b^2*(c + d*x)^2*Log[(b*c - a*d)/(
b*c + b*d*x)]^2 + 4*b^2*(c + d*x)^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)
] + 4*b^2*(c + d*x)^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)^
2*(c + d*x)^2) + (6*a*b*B^2*d*(-4*(b*c - a*d)^2*(c + d*x)*(2 + 2*Log[c/d +
x] + Log[c/d + x]^2) + c*(b*c - a*d)^2*(1 + 2*Log[c/d + x] + 2*Log[c/d + x]
^2) - 2*(b*c - a*d)^2*(c + 2*d*x)*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a
+ b*x))/(c + d*x)])^2 - 2*(Log[a/b + x] - Log[c/d + x] - Log[(e*(a + b*x))
/(c + d*x)])*(4*(b*c - a*d)^2*(c + d*x)*(1 + Log[c/d + x]) - c*(b*c - a*d)^
2*(1 + 2*Log[c/d + x]) - 4*(b*c - a*d)*(c + d*x)*((b*c - a*d)*Log[a/b + x]
- b*(c + d*x)*(Log[a + b*x] - Log[c + d*x])) + 2*c*((b*c - a*d)^2*Log[a/b +
x] - b*(c + d*x)*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c
+ d*x]))) + 4*(b*c - a*d)*(c + d*x)*(Log[a/b + x]*(d*(a + b*x)*Log[a/b + x
] - 2*b*(c + d*x)*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*b*(c + d*x)*PolyLog[2
, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*c*((b*c - a*d)^2*Log[a/b + x]^2 - b*(c
+ d*x)*(b*(c + d*x)*Log[a/b + x]^2 - 2*b*(c + d*x)*(Log[a + b*x] - Log[c +
d*x]) - 2*Log[a/b + x]*(-(b*c) + a*d + b*(c + d*x)*Log[(b*(c + d*x))/(b*c
- a*d)]) - 2*b*(c + d*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])) + 4*(b*
c - a*d)*(c + d*x)*(2*(b*c - a*d)*Log[a/b + x]*(1 + Log[c/d + x]) + b*(c +
d*x)*(Log[c/d + x]^2 - 2*Log[a + b*x] - 2*Log[c/d + x]*Log[(d*(a + b*x))/(-
(b*c) + a*d)] + 2*Log[c + d*x]) - 2*b*(c + d*x)*PolyLog[2, (b*(c + d*x))/(b
*c - a*d)]) + 2*c*(b*(b*c - a*d)*(c + d*x) - (b*c - a*d)^2*Log[a/b + x]*(1
+ 2*Log[c/d + x]) + b^2*(c + d*x)^2*Log[a + b*x] - b^2*(c + d*x)^2*Log[c +
d*x] - b*(c + d*x)*(b*(c + d*x)*Log[c/d + x]^2 - 2*(b*c - a*d)*(1 + Log[c/d
+ x]) - 2*b*(c + d*x)*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Po
lyLog[2, (b*(c + d*x))/(b*c - a*d)])))))/((b*c - a*d)^2*(c + d*x)^2) + b^2*
B^2*(4*Log[c/d + x]^3 + (24*c*(2 + 2*Log[c/d + x] + Log[c/d + x]^2))/(c + d

```
*x) - (3*c^2*(1 + 2*Log[c/d + x] + 2*Log[c/d + x]^2))/(c + d*x)^2 + (6*(-Lo
g[a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])^2*(c*(3*c + 4*d*x
) + 2*(c + d*x)^2*Log[c + d*x]))/(c + d*x)^2 + (24*c*(-(d*(a + b*x)*Log[a/b
 + x]^2) + 2*b*(c + d*x)*Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + 2*b*
(c + d*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/((b*c - a*d)*(c + d*x)
) + 6*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])*((-2*c^
2*Log[a/b + x])/(c + d*x)^2 + (8*c*Log[a/b + x])/(c + d*x) - 2*Log[c/d + x]
^2 - (8*c*(1 + Log[c/d + x]))/(c + d*x) + (c^2*(1 + 2*Log[c/d + x]))/(c + d
*x)^2 + (8*b*c*(-Log[a + b*x] + Log[c + d*x]))/(b*c - a*d) + (2*b*c^2*(b*c
- a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]))/((b*c - a*d)^
2*(c + d*x)) + 4*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2,
(d*(a + b*x))/(-(b*c) + a*d)])) + (6*c^2*(-Log[a/b + x]^2 + (b*(c + d*x)*(b
*(c + d*x)*Log[a/b + x]^2 - 2*b*(c + d*x)*(Log[a + b*x] - Log[c + d*x]) - 2
*Log[a/b + x]*(-(b*c) + a*d + b*(c + d*x)*Log[(b*(c + d*x))/(b*c - a*d)]) -
 2*b*(c + d*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/(b*c - a*d)^2))/(
c + d*x)^2 + 12*(Log[a/b + x]^2*Log[(b*(c + d*x))/(b*c - a*d)] + 2*Log[a/b
+ x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 2*PolyLog[3, (d*(a + b*x))/
(-(b*c) + a*d)]) + 6*(2*Log[c/d + x]^2*(-Log[a/b + x] + Log[(d*(a + b*x))/(
-(b*c) + a*d)]) + 4*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + (4
*c*(2*(b*c - a*d)*Log[a/b + x]*(1 + Log[c/d + x]) + b*(c + d*x)*(Log[c/d +
x]^2 - 2*Log[a + b*x] - 2*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] +
2*Log[c + d*x]) - 2*b*(c + d*x)*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((-
(b*c) + a*d)*(c + d*x)) + (c^2*(-(b*(b*c - a*d)*(c + d*x)) + (b*c - a*d)^2*
Log[a/b + x]*(1 + 2*Log[c/d + x]) - b^2*(c + d*x)^2*Log[a + b*x] + b^2*(c +
 d*x)^2*Log[c + d*x] + b*(c + d*x)*(b*(c + d*x)*Log[c/d + x]^2 - 2*(b*c - a
*d)*(1 + Log[c/d + x]) - 2*b*(c + d*x)*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b
*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))))/((b*c - a*d)^2*(c +
 d*x)^2) - 4*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]))))/(12*d^3*i^3)
```

Maple [F]

$$\int \frac{(bgx+ag)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(dix+ci)^3}dx$$

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x)
```

```
[Out] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^3}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, alg
orithm="fricas")
```

```
[Out] integral((A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^
2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*
b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log((b*e*x + a*e)/(d*x + c)))/
(d^3*i^3*x^3 + 3*c*d^2*i^3*x^2 + 3*c^2*d*i^3*x + c^3*i^3), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$
$$=\frac{g^2\left(\int \frac{A^2a^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{A^2b^2x^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{B^2a^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{2ABa^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\right.}{}$$

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**3,x)
```

```
[Out] g**2*(Integral(A**2*a**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x
) + Integral(A**2*b**2*x**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3)
, x) + Integral(B**2*a**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c**3 + 3
*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*a**2*log(a*e/(c
 + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
 x) + Integral(2*A**2*a*b*x/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3)
, x) + Integral(B**2*b**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c**
3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*b**2*x**2*
log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d
**3*x**3), x) + Integral(2*B**2*a*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))*
*2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(4*A*B*a*b
*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
+ d**3*x**3), x))/i**3
```

Maxima [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^3}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, alg
orithm="maxima")
```

```
[Out] -A*B*a*b*g^2*(2*(2*d*x + c)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^4*i^3*x
^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - (b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*
x)/((b*c*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2
- a*c^2*d^3)*i^3) - 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*b*
c*d^3 + a^2*d^4)*i^3) + 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*
a*b*c*d^3 + a^2*d^4)*i^3)) + 1/2*A*B*a^2*g^2*((2*b*d*x + 3*b*c - a*d)/((b*c
*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^
2)*i^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^3*i^3*x^2 + 2*c*d^2*i^3
*x + c^2*d*i^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i
^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) + 1/2*A
^2*b^2*g^2*((4*c*d*x + 3*c^2)/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) +
 2*log(d*x + c)/(d^3*i^3)) - (2*d*x + c)*A^2*a*b*g^2/(d^4*i^3*x^2 + 2*c*d^3
*i^3*x + c^2*d^2*i^3) - 1/2*A^2*a^2*g^2/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*
d*i^3) + 1/6*(2*(B^2*b^2*d^2*g^2*x^2 + 2*B^2*b^2*c*d*g^2*x + B^2*b^2*c^2*g^
2)*log(d*x + c)^3 + 3*(4*(b^2*c*d*g^2 - a*b*d^2*g^2)*B^2*x + (3*b^2*c^2*g^2
 - 2*a*b*c*d*g^2 - a^2*d^2*g^2)*B^2)*log(d*x + c)^2)/(d^5*i^3*x^2 + 2*c*d^4
*i^3*x + c^2*d^3*i^3) - integrate(-(2*B^2*a*b*d^2*g^2*x*log(e)^2 + B^2*a^2*
d^2*g^2*log(e)^2 + (B^2*b^2*d^2*g^2*log(e)^2 + 2*A*B*b^2*d^2*g^2*log(e))*x^
2 + (B^2*b^2*d^2*g^2*x^2 + 2*B^2*a*b*d^2*g^2*x + B^2*a^2*d^2*g^2)*log(b*x +
 a)^2 + 2*(2*B^2*a*b*d^2*g^2*x*log(e) + B^2*a^2*d^2*g^2*log(e) + (B^2*b^2*d
^2*g^2*log(e) + A*B*b^2*d^2*g^2)*x^2)*log(b*x + a) - (4*(b^2*c*d*g^2 + (g^2
*log(e) - g^2)*a*b*d^2)*B^2*x + (3*b^2*c^2*g^2 - 2*a*b*c*d*g^2 + (2*g^2*log
(e) - g^2)*a^2*d^2)*B^2 + 2*(B^2*b^2*d^2*g^2*log(e) + A*B*b^2*d^2*g^2)*x^2
+ 2*(B^2*b^2*d^2*g^2*x^2 + 2*B^2*a*b*d^2*g^2*x + B^2*a^2*d^2*g^2)*log(b*x +
 a))*log(d*x + c))/(d^5*i^3*x^3 + 3*c*d^4*i^3*x^2 + 3*c^2*d^3*i^3*x + c^3*d
^2*i^3), x)
```

Giac [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^3}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, alg
orithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*i
)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx=\int \frac{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(c\,i+d\,i\,x)^3}\,dx$$

```
[In] int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)^
3,x)
```

```
[Out] int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)^
3, x)
```

3.102 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 141

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \frac{B^2g(a+bx)^2}{4(bc-ad)i^3(c+dx)^2} - \frac{Bg(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)i^3(c+dx)^2} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)i^3(c+dx)^2}$$

```
[Out] 1/4*B^2*g*(b*x+a)^2/(-a*d+b*c)/i^3/(d*x+c)^2-1/2*B*g*(b*x+a)^2*(A+B*ln(e*(b
*x+a)/(d*x+c)))/(-a*d+b*c)/i^3/(d*x+c)^2+1/2*g*(b*x+a)^2*(A+B*ln(e*(b*x+a)/
(d*x+c)))^2/(-a*d+b*c)/i^3/(d*x+c)^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2562, 2342, 2341}

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \frac{g(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2i^3(c+dx)^2(bc-ad)} - \frac{Bg(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2i^3(c+dx)^2(bc-ad)} + \frac{B^2g(a+bx)^2}{4i^3(c+dx)^2(bc-ad)}$$

```
[In] Int[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x)^3,
x]
```

```
[Out] (B^2*g*(a + b*x)^2)/(4*(b*c - a*d)*i^3*(c + d*x)^2) - (B*g*(a + b*x)^2*(A +
 B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*c - a*d)*i^3*(c + d*x)^2) + (g*(a +
 b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(b*c - a*d)*i^3*(c + d*x
)^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{g\text{Subst}\left(\int x(A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^3}\\
&= \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)i^3(c+dx)^2} - \frac{(Bg)\text{Subst}\left(\int x(A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^3}\\
&= \frac{B^2g(a+bx)^2}{4(bc-ad)i^3(c+dx)^2} - \frac{Bg(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)i^3(c+dx)^2}\\
&\quad + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)i^3(c+dx)^2}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 767, normalized size of antiderivative = 5.44

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$
$$=\frac{g\left(2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-4b(bc-ad)(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+4bB(c+dx)\left(2(b\right.\right.}{}$$

```
[In] Integrate[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i
*x)^3,x]
```

```
[Out] (g*(2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - 4*b*(b*c - a*d
)*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 4*b*B*(c + d*x)*(2*(b*
c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*b*(c + d*x)*Log[a + b*x]*
(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b*(c + d*x)*(A + B*Log[(e*(a + b*x
))/(c + d*x)])*Log[c + d*x] - 2*B*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b
*(c + d*x)*Log[c + d*x]) - b*B*(c + d*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Lo
g[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])
 + b*B*(c + d*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[
c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - B*(2*(b*c - a*d)^2*(
A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*b*(b*c - a*d)*(c + d*x)*(A + B*Log[
(e*(a + b*x))/(c + d*x)]) + 4*b^2*(c + d*x)^2*Log[a + b*x]*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]) - 4*b^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)
])*Log[c + d*x] - 4*b*B*(c + d*x)*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b
*(c + d*x)*Log[c + d*x]) - B*((b*c - a*d)^2 + 2*b*(b*c - a*d)*(c + d*x) + 2
*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c + d*x]) - 2*b^2*B*(
c + d*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)])
- 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*b^2*B*(c + d*x)^2*((2*Log
[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2,
(b*(c + d*x))/(b*c - a*d)]))))/(4*d^2*(b*c - a*d)*i^3*(c + d*x)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(135) = 270.

Time = 0.90 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.44

method	result
norman	$$-\frac{-\frac{2A^2adg+2A^2bcg-2BadgA-2BbcgA+B^2adg+B^2bcg}{4i\,d^2}-\frac{(2A^2bg-2BbgA+B^2bg)x}{2id}-\frac{B^2a^2g\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{2i(ad-cb)}-\frac{B^2b^2g\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right.}{2(ad-cb)i}}{i^2(dx+c)}$$
derivativedivides	$$-\frac{e(ad-cb)\left(\frac{g\,d^2A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^2e^3i^3}+\frac{2g\,d^2AB\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2}-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{4}\right)}{(ad-cb)^2e^3i^3}+\frac{g\,d^2B^2\left(\left(\frac{be}{d}\right.\right.}{}\right.}{d^2}$$
default	$$-\frac{e(ad-cb)\left(\frac{g\,d^2A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^2e^3i^3}+\frac{2g\,d^2AB\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2}-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{4}\right)}{(ad-cb)^2e^3i^3}+\frac{g\,d^2B^2\left(\left(\frac{be}{d}\right.\right.}{}\right.}{d^2}$$
parallelrisch	$$-\frac{8ABx\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^2d^4g+2AB\,b^3c^2d^2g+2A^2a^2b\,d^4g-2A^2b^3c^2d^2g+B^2a^2b\,d^4g-B^2b^3c^2d^2g-2B^2x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3c}{}$$
parts	$$\frac{g\,A^2\left(-\frac{b}{d^2(dx+c)}-\frac{ad-cb}{2d^2(dx+c)^2}\right)}{i^3}-\frac{g\,B^2d\left(a\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2}-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2}\right.\right.}{}$$
risch	Expression too large to display

```
[In] int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x,method=_RETUR
NVERBOSE)
```

```
[Out] (-1/4*(2*A^2*a*d*g+2*A^2*b*c*g-2*A*B*a*d*g-2*A*B*b*c*g+B^2*a*d*g+B^2*b*c*g)
/i/d^2-1/2*(2*A^2*b*g-2*A*B*b*g+B^2*b*g)/i/d*x-1/2*B^2*a^2*g/i/(a*d-b*c)*ln
(e*(b*x+a)/(d*x+c))^2-1/2*B^2*b^2*g/(a*d-b*c)/i*x^2*ln(e*(b*x+a)/(d*x+c))^2
-1/2*(2*A-B)*g*a^2*B/i/(a*d-b*c)*ln(e*(b*x+a)/(d*x+c))-B^2*a*b*g/i/(a*d-b*c
)*x*ln(e*(b*x+a)/(d*x+c))^2-1/2*b^2/i*g*B*(2*A-B)/(a*d-b*c)*x^2*ln(e*(b*x+a
)/(d*x+c))-g*a*B/i*b*(2*A-B)/(a*d-b*c)*x*ln(e*(b*x+a)/(d*x+c)))/i^2/(d*x+c)
^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(135) = 270$.

Time = 0.30 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.09

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = -\frac{2\left((2A^2-2AB+B^2)b^2cd-(2A^2-2AB+B^2)abd^2\right)gx-2\left(B^2b^2d^2gx^2+2B^2abd^2gx+B^2a^2d^2g\right)\log}{4\left((bc\right.}$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, algor
ithm="fricas")
```

```
[Out] -1/4*(2*((2*A^2 - 2*A*B + B^2)*b^2*c*d - (2*A^2 - 2*A*B + B^2)*a*b*d^2)*g*x
 - 2*(B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log((b*e*x + a
*e)/(d*x + c))^2 + ((2*A^2 - 2*A*B + B^2)*b^2*c^2 - (2*A^2 - 2*A*B + B^2)*a
^2*d^2)*g - 2*((2*A*B - B^2)*b^2*d^2*g*x^2 + 2*(2*A*B - B^2)*a*b*d^2*g*x +
(2*A*B - B^2)*a^2*d^2*g)*log((b*e*x + a*e)/(d*x + c)))/((b*c*d^4 - a*d^5)*i
^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. $2(121) = 242$.

Time = 5.09 (sec) , antiderivative size = 712, normalized size of antiderivative = 5.05

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{Bb^2g(2A-B)\log\left(x+\frac{2ABab^2dg+2ABb^3cg-B^2ab^2dg-B^2b^3cg-\frac{Ba^2b^2d^2g(2A-B)}{ad-bc}+\frac{2Bab^3cdg(2A-B)}{ad-bc}-\frac{Bb^4c^2g(2A-B)}{ad-bc}}{4ABb^3dg-2B^2b^3dg}\right)}{2d^2i^3\,(ad-bc)}$$
$$-\frac{Bb^2g(2A-B)\log\left(x+\frac{2ABab^2dg+2ABb^3cg-B^2ab^2dg-B^2b^3cg+\frac{Ba^2b^2d^2g(2A-B)}{ad-bc}-\frac{2Bab^3cdg(2A-B)}{ad-bc}+\frac{Bb^4c^2g(2A-B)}{ad-bc}}{4ABb^3dg-2B^2b^3dg}\right)}{2d^2i^3\,(ad-bc)}$$
$$+\frac{(-B^2a^2g-2B^2abgx-B^2b^2gx^2)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2ac^2di^3+4acd^2i^3x+2ad^3i^3x^2-2bc^3i^3-4bc^2di^3x-2bcd^2i^3x^2}$$
$$+\frac{-2A^2adg-2A^2bcg+2ABadg+2ABbcg-B^2adg-B^2bcg+x(-4A^2bdg+4ABbdg-2B^2bdg)}{4c^2d^2i^3+8cd^3i^3x+4d^4i^3x^2}$$
$$+\frac{(-2ABadg-2ABbcg-4ABbdgx+B^2adg+B^2bcg+2B^2bdgx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2c^2d^2i^3+4cd^3i^3x+2d^4i^3x^2}$$

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**3,x)
```

```
[Out] B*b**2*g*(2*A - B)*log(x + (2*A*B*a*b**2*d*g + 2*A*B*b**3*c*g - B**2*a*b**2
*d*g - B**2*b**3*c*g - B*a**2*b**2*d**2*g*(2*A - B)/(a*d - b*c) + 2*B*a*b**
3*c*d*g*(2*A - B)/(a*d - b*c) - B*b**4*c**2*g*(2*A - B)/(a*d - b*c))/(4*A*B
*b**3*d*g - 2*B**2*b**3*d*g))/(2*d**2*i**3*(a*d - b*c)) - B*b**2*g*(2*A - B
)*log(x + (2*A*B*a*b**2*d*g + 2*A*B*b**3*c*g - B**2*a*b**2*d*g - B**2*b**3*
c*g + B*a**2*b**2*d**2*g*(2*A - B)/(a*d - b*c) - 2*B*a*b**3*c*d*g*(2*A - B)
/(a*d - b*c) + B*b**4*c**2*g*(2*A - B)/(a*d - b*c))/(4*A*B*b**3*d*g - 2*B**
2*b**3*d*g))/(2*d**2*i**3*(a*d - b*c)) + (-B**2*a**2*g - 2*B**2*a*b*g*x - B
**2*b**2*g*x**2)*log(e*(a + b*x)/(c + d*x))**2/(2*a*c**2*d*i**3 + 4*a*c*d**
2*i**3*x + 2*a*d**3*i**3*x**2 - 2*b*c**3*i**3 - 4*b*c**2*d*i**3*x - 2*b*c*d
**2*i**3*x**2) + (-2*A**2*a*d*g - 2*A**2*b*c*g + 2*A*B*a*d*g + 2*A*B*b*c*g
- B**2*a*d*g - B**2*b*c*g + x*(-4*A**2*b*d*g + 4*A*B*b*d*g - 2*B**2*b*d*g))
/(4*c**2*d**2*i**3 + 8*c*d**3*i**3*x + 4*d**4*i**3*x**2) + (-2*A*B*a*d*g -
2*A*B*b*c*g - 4*A*B*b*d*g*x + B**2*a*d*g + B**2*b*c*g + 2*B**2*b*d*g*x)*log
(e*(a + b*x)/(c + d*x))/(2*c**2*d**2*i**3 + 4*c*d**3*i**3*x + 2*d**4*i**3*x
**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1966 vs. $2(135) = 270$.

Time = 0.29 (sec) , antiderivative size = 1966, normalized size of antiderivative = 13.94

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, algor
ithm="maxima")
```

```
[Out] -1/2*(2*d*x + c)*B^2*b*g*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(d^4*i^3*x^
2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) + 1/4*(2*((2*b*d*x + 3*b*c - a*d)/((b*c*d^
3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*
i^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2
*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3))*log(b*e*x/(d*x + c
) + a*e/(d*x + c)) - (7*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^2 + 2*
b^2*c*d*x + b^2*c^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2)*log(d*x + c)^2 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*log(b*x + a) - 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*(b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x + c))/(b^2*c^4*d*
i^3 - 2*a*b*c^3*d^2*i^3 + a^2*c^2*d^3*i^3 + (b^2*c^2*d^3*i^3 - 2*a*b*c*d^4*
i^3 + a^2*d^5*i^3)*x^2 + 2*(b^2*c^3*d^2*i^3 - 2*a*b*c^2*d^3*i^3 + a^2*c*d^4
*i^3)*x))*B^2*a*g + 1/4*(2*((b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*x)/((b*c
*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*
```

```
d^3)*i^3) + 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*b*c*d^3 +
a^2*d^4)*i^3) - 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^
3 + a^2*d^4)*i^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - (b^2*c^3 - 8*a*b*
c^2*d + 7*a^2*c*d^2 + 2*(b^2*c^3 - 2*a*b*c^2*d + (b^2*c*d^2 - 2*a*b*d^3)*x^
2 + 2*(b^2*c^2*d - 2*a*b*c*d^2)*x)*log(b*x + a)^2 + 2*(b^2*c^3 - 2*a*b*c^2*
d + (b^2*c*d^2 - 2*a*b*d^3)*x^2 + 2*(b^2*c^2*d - 2*a*b*c*d^2)*x)*log(d*x +
c)^2 + 2*(b^2*c^2*d - 5*a*b*c*d^2 + 4*a^2*d^3)*x + 2*(b^2*c^3 - 4*a*b*c^2*d
 + (b^2*c*d^2 - 4*a*b*d^3)*x^2 + 2*(b^2*c^2*d - 4*a*b*c*d^2)*x)*log(b*x + a
) - 2*(b^2*c^3 - 4*a*b*c^2*d + (b^2*c*d^2 - 4*a*b*d^3)*x^2 + 2*(b^2*c^2*d -
 4*a*b*c*d^2)*x + 2*(b^2*c^3 - 2*a*b*c^2*d + (b^2*c*d^2 - 2*a*b*d^3)*x^2 +
2*(b^2*c^2*d - 2*a*b*c*d^2)*x)*log(b*x + a))*log(d*x + c))/(b^2*c^4*d^2*i^3
 - 2*a*b*c^3*d^3*i^3 + a^2*c^2*d^4*i^3 + (b^2*c^2*d^4*i^3 - 2*a*b*c*d^5*i^3
 + a^2*d^6*i^3)*x^2 + 2*(b^2*c^3*d^3*i^3 - 2*a*b*c^2*d^4*i^3 + a^2*c*d^5*i^
3)*x))*B^2*b*g - 1/2*A*B*b*g*(2*(2*d*x + c)*log(b*e*x/(d*x + c) + a*e/(d*x
+ c))/(d^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - (b*c^2 - 3*a*c*d + 2*(b
*c*d - 2*a*d^2)*x)/((b*c*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3
*x + (b*c^3*d^2 - a*c^2*d^3)*i^3) - 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*
c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*i^3) + 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((
b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*i^3)) + 1/2*A*B*a*g*((2*b*d*x + 3*b*c
- a*d)/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*
d - a*c^2*d^2)*i^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^3*i^3*x^2 +
 2*c*d^2*i^3*x + c^2*d*i^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2
+ a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i
^3)) - 1/2*B^2*a*g*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(d^3*i^3*x^2 + 2*
c*d^2*i^3*x + c^2*d*i^3) - 1/2*(2*d*x + c)*A^2*b*g/(d^4*i^3*x^2 + 2*c*d^3*i
^3*x + c^2*d^2*i^3) - 1/2*A^2*a*g/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3)
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.40

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(\frac{2\,(bex+ae)^2B^2g\log\left(\frac{bex+ae}{dx+c}\right)^2}{(dx+c)^2ei^3}+\frac{2\,(2\,ABg-B^2g)(bex+ae)^2\log\left(\frac{bex+ae}{dx+c}\right)}{(dx+c)^2ei^3}+\frac{(2\,A^2g-2\,ABg+B^2g)}{(dx+c)^2ei^3}\right.$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, algor
ithm="giac")
```

```
[Out] 1/4*(2*(b*e*x + a*e)^2*B^2*g*log((b*e*x + a*e)/(d*x + c))^2/((d*x + c)^2*e*
i^3) + 2*(2*A*B*g - B^2*g)*(b*e*x + a*e)^2*log((b*e*x + a*e)/(d*x + c))/((d
*x + c)^2*e*i^3) + (2*A^2*g - 2*A*B*g + B^2*g)*(b*e*x + a*e)^2/((d*x + c)^2
```

```
*e*i^3))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a
*d)))
```

Mupad [B] (verification not implemented)

Time = 2.86 (sec) , antiderivative size = 474, normalized size of antiderivative = 3.36

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx =$$

$$-\frac{x\,(2\,b\,d\,g\,A^2-2\,b\,d\,g\,A\,B+b\,d\,g\,B^2)+A^2\,a\,d\,g+A^2\,b\,c\,g+\frac{B^2\,a\,d\,g}{2}+\frac{B^2\,b\,c\,g}{2}-A\,B\,a\,d\,g-A\,B\,b\,c\,g}{2\,c^2\,d^2\,i^3+4\,c\,d^3\,i^3\,x+2\,d^4\,i^3\,x^2}$$
$$-\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{\frac{B^2\,a\,g}{2\,d^2\,i^3}+\frac{B^2\,b\,c\,g}{2\,d^3\,i^3}+\frac{B^2\,b\,g\,x}{d^2\,i^3}}{2\,c\,x+d\,x^2+\frac{c^2}{d}}+\frac{B^2\,b^2\,g}{2\,d^2\,i^3\,(a\,d-b\,c)}\right)$$
$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(\frac{A\,B\,c\,g}{d^3\,i^3}-x\left(\frac{B^2\,g}{d^2\,i^3}-\frac{2\,A\,B\,g}{d^2\,i^3}\right)+\frac{B\,g\,(A\,a\,d-B\,a\,d+B\,b\,c)}{b\,d^3\,i^3}+\frac{B^2\,b^2\,g\,\left(\frac{a^2\,d^2-3\,a\,b\,c\,d+2\,b^2\,c^2}{2\,b^3\,d}-\frac{c\,(a\,d-b\,c)}{2\,b^2\,d}\right)}{d^2\,i^3\,(a\,d-b\,c)}\right)}{\frac{d\,x^2}{b}+\frac{c^2}{b\,d}+\frac{2\,c\,x}{b}}$$
$$+\frac{B\,b^2\,g\,\mathrm{atan}\left(\frac{\left(\frac{2\,a\,d^3\,i^3+2\,b\,c\,d^2\,i^3}{2\,d^2\,i^3}+2\,b\,d\,x\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(2\,A-B)\,1\mathrm{i}}{d^2\,i^3\,(a\,d-b\,c)}$$

```
[In] int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)^3,
x)
```

```
[Out] (B*b^2*g*atan((((2*a*d^3*i^3 + 2*b*c*d^2*i^3)/(2*d^2*i^3) + 2*b*d*x)*1i)/(a
*d - b*c))*(2*A - B)*1i)/(d^2*i^3*(a*d - b*c)) - log((e*(a + b*x))/(c + d*x
))^2*(((B^2*a*g)/(2*d^2*i^3) + (B^2*b*c*g)/(2*d^3*i^3) + (B^2*b*g*x)/(d^2*i
^3))/(2*c*x + d*x^2 + c^2/d) + (B^2*b^2*g)/(2*d^2*i^3*(a*d - b*c))) - (log(
(e*(a + b*x))/(c + d*x))*((A*B*c*g)/(d^3*i^3) - x*((B^2*g)/(d^2*i^3) - (2*A
*B*g)/(d^2*i^3)) + (B*g*(A*a*d - B*a*d + B*b*c))/(b*d^3*i^3) + (B^2*b^2*g*(
(a^2*d^2 + 2*b^2*c^2 - 3*a*b*c*d)/(2*b^3*d) - (c*(a*d - b*c))/(2*b^2*d)))/(
d^2*i^3*(a*d - b*c))))/((d*x^2)/b + c^2/(b*d) + (2*c*x)/b) - (x*(2*A^2*b*d*
g + B^2*b*d*g - 2*A*B*b*d*g) + A^2*a*d*g + A^2*b*c*g + (B^2*a*d*g)/2 + (B^2
*b*c*g)/2 - A*B*a*d*g - A*B*b*c*g)/(2*c^2*d^2*i^3 + 2*d^4*i^3*x^2 + 4*c*d^3
*i^3*x)
```

3.103 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 296

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = -\frac{B^2d(a+bx)^2}{4(bc-ad)^2i^3(c+dx)^2}-\frac{2AbB(a+bx)}{(bc-ad)^2i^3(c+dx)} + \frac{2bB^2(a+bx)}{(bc-ad)^2i^3(c+dx)}-\frac{2bB^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^2i^3(c+dx)} + \frac{Bd(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^2i^3(c+dx)^2} - \frac{d(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^2i^3(c+dx)^2} + \frac{b(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^2i^3(c+dx)}$$

```
[Out] -1/4*B^2*d*(b*x+a)^2/(-a*d+b*c)^2/i^3/(d*x+c)^2-2*A*b*B*(b*x+a)/(-a*d+b*c)^
2/i^3/(d*x+c)+2*b*B^2*(b*x+a)/(-a*d+b*c)^2/i^3/(d*x+c)-2*b*B^2*(b*x+a)*ln(e
*(b*x+a)/(d*x+c))/(-a*d+b*c)^2/i^3/(d*x+c)+1/2*B*d*(b*x+a)^2*(A+B*ln(e*(b*x
+a)/(d*x+c)))/(-a*d+b*c)^2/i^3/(d*x+c)^2-1/2*d*(b*x+a)^2*(A+B*ln(e*(b*x+a)/
(d*x+c)))^2/(-a*d+b*c)^2/i^3/(d*x+c)^2+b*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c))
)^2/(-a*d+b*c)^2/i^3/(d*x+c)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2552, 2367, 2333, 2332, 2342, 2341}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \frac{Bd(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2i^3(c+dx)^2(bc-ad)^2} + \frac{b(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{i^3(c+dx)(bc-ad)^2} - \frac{d(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2i^3(c+dx)^2(bc-ad)^2} - \frac{2AbB(a+bx)}{i^3(c+dx)(bc-ad)^2} - \frac{2bB^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{i^3(c+dx)(bc-ad)^2} + \frac{2bB^2(a+bx)}{i^3(c+dx)(bc-ad)^2} - \frac{B^2d(a+bx)^2}{4i^3(c+dx)^2(bc-ad)^2}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(c*i + d*i*x)^3,x]
```

```
[Out] -1/4*(B^2*d*(a + b*x)^2)/((b*c - a*d)^2*i^3*(c + d*x)^2) - (2*A*b*B*(a + b*
x))/((b*c - a*d)^2*i^3*(c + d*x)) + (2*b*B^2*(a + b*x))/((b*c - a*d)^2*i^3*
(c + d*x)) - (2*b*B^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/((b*c - a*d)^
2*i^3*(c + d*x)) + (B*d*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(
2*(b*c - a*d)^2*i^3*(c + d*x)^2) - (d*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/
(c + d*x)])^2)/(2*(b*c - a*d)^2*i^3*(c + d*x)^2) + (b*(a + b*x)*(A + B*Log[
(e*(a + b*x))/(c + d*x)])^2)/((b*c - a*d)^2*i^3*(c + d*x))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2552

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(
m + 1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a
 + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n
 + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
- c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (b-dx)(A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 i^3} \\
&= \frac{\text{Subst}\left(\int \left(b(A+B\log(ex))^2 - dx(A+B\log(ex))^2\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 i^3} \\
&= \frac{b\text{Subst}\left(\int (A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 i^3} - \frac{d\text{Subst}\left(\int x(A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 i^3} \\
&= -\frac{d(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^2 i^3 (c+dx)^2} + \frac{b(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^2 i^3 (c+dx)} \\
&\quad - \frac{(2bB)\text{Subst}\left(\int (A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 i^3} \\
&\quad + \frac{(Bd)\text{Subst}\left(\int x(A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 i^3}
\end{aligned}$$

$$= -\frac{B^2 d(a+bx)^2}{4(bc-ad)^2 i^3 (c+dx)^2} - \frac{2AbB(a+bx)}{(bc-ad)^2 i^3 (c+dx)} + \frac{Bd(a+bx)^2 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^2 i^3 (c+dx)^2} - \frac{d(a+bx)^2 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^2 i^3 (c+dx)^2} + \frac{b(a+bx)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^2 i^3 (c+dx)} - \frac{(2bB^2)\,\mathrm{Subst}\left(\int \log(ex)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 i^3}$$

$$= -\frac{B^2 d(a+bx)^2}{4(bc-ad)^2 i^3 (c+dx)^2} - \frac{2AbB(a+bx)}{(bc-ad)^2 i^3 (c+dx)} + \frac{2bB^2(a+bx)}{(bc-ad)^2 i^3 (c+dx)} - \frac{2bB^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^2 i^3 (c+dx)} + \frac{Bd(a+bx)^2 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^2 i^3 (c+dx)^2} - \frac{d(a+bx)^2 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^2 i^3 (c+dx)^2} + \frac{b(a+bx)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^2 i^3 (c+dx)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.50

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$

$$= \frac{-2\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + \frac{B\left(2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) + 4b(bc-ad)(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) + 4b^2(c+dx)^2\log(a+bx)\left(A\right.\right.}{}}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(c*i + d*i*x)^3,x]
```

```
[Out] (-2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(2*(b*c - a*d)^2*(A + B*Log
[(e*(a + b*x))/(c + d*x)]) + 4*b*(b*c - a*d)*(c + d*x)*(A + B*Log[(e*(a + b
*x))/(c + d*x)]) + 4*b^2*(c + d*x)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/
(c + d*x)]) - 4*b^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c
+ d*x] - 4*b*B*(c + d*x)*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x
)*Log[c + d*x]) - B*((b*c - a*d)^2 + 2*b*(b*c - a*d)*(c + d*x) + 2*b^2*(c +
 d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c + d*x]) - 2*b^2*B*(c + d*x)^
2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyL
og[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*b^2*B*(c + d*x)^2*((2*Log[(d*(a +
b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d
*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(4*d*i^3*(c + d*x)^2)
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.63

method	result
norman	$\frac{\frac{Bb(2Abc-Bad-2Bbc)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{i\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{B^2b^2cx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{i\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{\left(2A^2ad-2A^2bc-2ABad+4ABbc+B^2ad-4B^2bc\right)x}{2ci(ad-cb)}-\frac{Ba(2Aad-4}{2}}{}$
parts	$-\frac{A^2}{2i^3(dx+c)^2d}-\frac{B^2d\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2}-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2}+\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{4}\right.}{i^3(ad-}$
parallelrisch	$-\frac{-8ABx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3cd^4-8AB\ln\left(\frac{e(bx+a)}{dx+c}\right)ab^2cd^4-4A^2ab^2cd^4-2ABa^2bd^5-6ABb^3c^2d^3-8B^2ab^2cd^4+8ABa}{}$
derivativedivides	$-\frac{e(ad-cb)\left(-\frac{d^2A^2b\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{(ad-cb)^3e^2i^3}+\frac{d^3A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^3e^3i^3}-\frac{2d^2ABb\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad-cb)e}{d(dx+c)}-\right.}{(ad-cb)^3e^2i^3}\right.}{}$
default	$-\frac{e(ad-cb)\left(-\frac{d^2A^2b\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{(ad-cb)^3e^2i^3}+\frac{d^3A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^3e^3i^3}-\frac{2d^2ABb\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad-cb)e}{d(dx+c)}-\right.}{(ad-cb)^3e^2i^3}\right.}{}$
risch	Expression too large to display

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (B/i*b*(2*A*b*c-B*a*d-2*B*b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)/(
d*x+c))+B^2*b^2*c/i/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)/(d*x+c))^2+1
/2*(2*A^2*a*d-2*A^2*b*c-2*A*B*a*d+4*A*B*b*c+B^2*a*d-4*B^2*b*c)/c/i/(a*d-b*c
)*x-1/2*B*a*(2*A*a*d-4*A*b*c-B*a*d+4*B*b*c)/i/(a^2*d^2-2*a*b*c*d+b^2*c^2)*l
n(e*(b*x+a)/(d*x+c))-1/2*B^2*a*(a*d-2*b*c)/i/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln
(e*(b*x+a)/(d*x+c))^2+1/4*(2*A^2*a*d-2*A^2*b*c-2*A*B*a*d+6*A*B*b*c+B^2*a*d-
7*B^2*b*c)/c^2*d/i/(a*d-b*c)*x^2+1/2*b^2*d*B^2/i/(a^2*d^2-2*a*b*c*d+b^2*c^2
)*x^2*ln(e*(b*x+a)/(d*x+c))^2+1/2*d*B/i*b^2*(2*A-3*B)/(a^2*d^2-2*a*b*c*d+b^
2*c^2)*x^2*ln(e*(b*x+a)/(d*x+c)))/i^2/(d*x+c)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.26

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = -\frac{(2A^2-6AB+7B^2)b^2c^2-4(A^2-2AB+2B^2)abcd+(2A^2-2AB+B^2)a^2d^2-2(B^2b^2d^2x^2+2B^2}{}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, algorithm="fricas
")
```

```
[Out] -1/4*((2*A^2 - 6*A*B + 7*B^2)*b^2*c^2 - 4*(A^2 - 2*A*B + 2*B^2)*a*b*c*d + (
2*A^2 - 2*A*B + B^2)*a^2*d^2 - 2*(B^2*b^2*d^2*x^2 + 2*B^2*b^2*c*d*x + 2*B^2
*a*b*c*d - B^2*a^2*d^2)*log((b*e*x + a*e)/(d*x + c))^2 - 2*((2*A*B - 3*B^2)
*b^2*c*d - (2*A*B - 3*B^2)*a*b*d^2)*x - 2*((2*A*B - 3*B^2)*b^2*d^2*x^2 + 4*
(A*B - B^2)*a*b*c*d - (2*A*B - B^2)*a^2*d^2 - 2*(B^2*a*b*d^2 - 2*(A*B - B^2
)*b^2*c*d)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^2*c^2*d^3 - 2*a*b*c*d^4 + a
^2*d^5)*i^3*x^2 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*i^3*x + (b^2*
c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*i^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 892 vs. 2(269) = 538.

Time = 2.21 (sec) , antiderivative size = 892, normalized size of antiderivative = 3.01

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx =$$

$$-\frac{Bb^2\cdot(2A-3B)\log\left(x+\frac{2ABab^2d+2ABb^3c-3B^2ab^2d-3B^2b^3c-\frac{Ba^3b^2d^3\cdot(2A-3B)}{(ad-bc)^2}+\frac{3Ba^2b^3cd^2\cdot(2A-3B)}{(ad-bc)^2}-\frac{3Bab^4c^2d(2A-3B)}{(ad-bc)^2}+\frac{Bb^5}{(}}{4ABb^3d-6B^2b^3d}\right.}{2di^3\,(ad-bc)^2}$$

$$+\frac{Bb^2\cdot(2A-3B)\log\left(x+\frac{2ABab^2d+2ABb^3c-3B^2ab^2d-3B^2b^3c+\frac{Ba^3b^2d^3\cdot(2A-3B)}{(ad-bc)^2}-\frac{3Ba^2b^3cd^2\cdot(2A-3B)}{(ad-bc)^2}+\frac{3Bab^4c^2d(2A-3B)}{(ad-bc)^2}-\frac{Bb^5}{(}}{4ABb^3d-6B^2b^3d}\right.}{2di^3\,(ad-bc)^2}$$

$$+\frac{(-B^2a^2d+2B^2abc+2B^2b^2cx+B^2b^2dx^2)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2a^2c^2d^2i^3+4a^2cd^3i^3x+2a^2d^4i^3x^2-4abc^3di^3-8abc^2d^2i^3x-4abcd^3i^3x^2+2b^2c^4i^3+4b^2c^3di^3x+2b^2c^2d}$$

$$+\frac{(-2ABad+2ABbc+B^2ad-3B^2bc-2B^2bdx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2ac^2d^2i^3+4acd^3i^3x+2ad^4i^3x^2-2bc^3di^3-4bc^2d^2i^3x-2bcd^3i^3x^2}$$

$$+\frac{-2A^2ad+2A^2bc+2ABad-6ABbc-B^2ad+7B^2bc+x(-4ABbd+6B^2bd)}{4ac^2d^2i^3-4bc^3di^3+x^2\cdot(4ad^4i^3-4bcd^3i^3)+x\,(8acd^3i^3-8bc^2d^2i^3)}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**3,x)
```

```
[Out] -B*b**2*(2*A - 3*B)*log(x + (2*A*B*a*b**2*d + 2*A*B*b**3*c - 3*B**2*a*b**2*
d - 3*B**2*b**3*c - B*a**3*b**2*d**3*(2*A - 3*B)/(a*d - b*c)**2 + 3*B*a**2*
b**3*c*d**2*(2*A - 3*B)/(a*d - b*c)**2 - 3*B*a*b**4*c**2*d*(2*A - 3*B)/(a*d
 - b*c)**2 + B*b**5*c**3*(2*A - 3*B)/(a*d - b*c)**2)/(4*A*B*b**3*d - 6*B**2
*b**3*d))/(2*d*i**3*(a*d - b*c)**2) + B*b**2*(2*A - 3*B)*log(x + (2*A*B*a*b
**2*d + 2*A*B*b**3*c - 3*B**2*a*b**2*d - 3*B**2*b**3*c + B*a**3*b**2*d**3*(
2*A - 3*B)/(a*d - b*c)**2 - 3*B*a**2*b**3*c*d**2*(2*A - 3*B)/(a*d - b*c)**2
 + 3*B*a*b**4*c**2*d*(2*A - 3*B)/(a*d - b*c)**2 - B*b**5*c**3*(2*A - 3*B)/(
a*d - b*c)**2)/(4*A*B*b**3*d - 6*B**2*b**3*d))/(2*d*i**3*(a*d - b*c)**2) +
(-B**2*a**2*d + 2*B**2*a*b*c + 2*B**2*b**2*c*x + B**2*b**2*d*x**2)*log(e*(a
 + b*x)/(c + d*x))**2/(2*a**2*c**2*d**2*i**3 + 4*a**2*c*d**3*i**3*x + 2*a**
2*d**4*i**3*x**2 - 4*a*b*c**3*d*i**3 - 8*a*b*c**2*d**2*i**3*x - 4*a*b*c*d**
3*i**3*x**2 + 2*b**2*c**4*i**3 + 4*b**2*c**3*d*i**3*x + 2*b**2*c**2*d**2*i*
*3*x**2) + (-2*A*B*a*d + 2*A*B*b*c + B**2*a*d - 3*B**2*b*c - 2*B**2*b*d*x)*
log(e*(a + b*x)/(c + d*x))/(2*a*c**2*d**2*i**3 + 4*a*c*d**3*i**3*x + 2*a*d*
*4*i**3*x**2 - 2*b*c**3*d*i**3 - 4*b*c**2*d**2*i**3*x - 2*b*c*d**3*i**3*x**
2) + (-2*A**2*a*d + 2*A**2*b*c + 2*A*B*a*d - 6*A*B*b*c - B**2*a*d + 7*B**2*
b*c + x*(-4*A*B*b*d + 6*B**2*b*d))/(4*a*c**2*d**2*i**3 - 4*b*c**3*d*i**3 +
x**2*(4*a*d**4*i**3 - 4*b*c*d**3*i**3) + x*(8*a*c*d**3*i**3 - 8*b*c**2*d**2
*i**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(290) = 580$.

Time = 0.23 (sec) , antiderivative size = 848, normalized size of antiderivative = 2.86

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{1}{4}\left(2\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)i^3x^2+2\,(bc^2d^2-acd^3)i^3x+(bc^3d-ac^2d^2)i^3}+\frac{2\,b^2\log(bx+a)}{(b^2c^2d-2\,abcd^2+a^2d^3)i^3}-\frac{2\,b\right.\right.$$
$$+\frac{1}{2}\,AB\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)i^3x^2+2\,(bc^2d^2-acd^3)i^3x+(bc^3d-ac^2d^2)i^3}-\frac{2\,\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{d^3i^3x^2+2\,cd^2i^3x+c^2di^3}+\frac{2}{(b^2c^2d}\right.$$
$$-\frac{B^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)^2}{2\,(d^3i^3x^2+2\,cd^2i^3x+c^2di^3)}-\frac{A^2}{2\,(d^3i^3x^2+2\,cd^2i^3x+c^2di^3)}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, algorithm="maxima
")
```

```
[Out] 1/4*(2*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 -
 a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b^2*log(b*x + a)/((b^2*c^2
```

```
*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*c
*d^2 + a^2*d^3)*i^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - (7*b^2*c^2 - 8
*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a)^2
 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x + c)^2 + 6*(b^2*c*d - a*
b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a) - 2*(3*b^2*
d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*log(b*x + a))*log(d*x + c))/(b^2*c^4*d*i^3 - 2*a*b*c^3*d^2*i^3 + a^2*c^2*d
^3*i^3 + (b^2*c^2*d^3*i^3 - 2*a*b*c*d^4*i^3 + a^2*d^5*i^3)*x^2 + 2*(b^2*c^3
*d^2*i^3 - 2*a*b*c^2*d^3*i^3 + a^2*c*d^4*i^3)*x))*B^2 + 1/2*A*B*((2*b*d*x +
 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x +
(b*c^3*d - a*c^2*d^2)*i^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^3*i^
3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b
*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2
*d^3)*i^3)) - 1/2*B^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(d^3*i^3*x^2 +
 2*c*d^2*i^3*x + c^2*d*i^3) - 1/2*A^2/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*
i^3)
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.25

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(2\left(\frac{2\,(bex+ae)B^2b}{(bci^3-adi^3)(dx+c)}-\frac{(bex+ae)^2B^2d}{(bcei^3-adei^3)(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2-2\left(\frac{(2\,ABd-B^2d)(bex+a}{(bcei^3-adei^3)(dx+c}\right.\right.$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*(2*(b*e*x + a*e)*B^2*b/((b*c*i^3 - a*d*i^3)*(d*x + c)) - (b*e*x + a*
e)^2*B^2*d/((b*c*e*i^3 - a*d*e*i^3)*(d*x + c)^2))*log((b*e*x + a*e)/(d*x +
c))^2 - 2*((2*A*B*d - B^2*d)*(b*e*x + a*e)^2/((b*c*e*i^3 - a*d*e*i^3)*(d*x
+ c)^2) - 4*(A*B*b - B^2*b)*(b*e*x + a*e)/((b*c*i^3 - a*d*i^3)*(d*x + c)))*
log((b*e*x + a*e)/(d*x + c)) - (2*A^2*d - 2*A*B*d + B^2*d)*(b*e*x + a*e)^2/
((b*c*e*i^3 - a*d*e*i^3)*(d*x + c)^2) + 4*(A^2*b - 2*A*B*b + 2*B^2*b)*(b*e*
x + a*e)/((b*c*i^3 - a*d*i^3)*(d*x + c)))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)
) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 2.88 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.71

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= -\frac{\frac{2A^2ad-2A^2bc+B^2ad-7B^2bc-2ABad+6ABbc}{2(ad-bc)}-\frac{x(3B^2bd-2ABbd)}{ad-bc}}{2c^2di^3+4cd^2i^3x+2d^3i^3x^2} - \ln\left(\frac{e(a+bx)}{c+dx}\right)^2\left(\frac{B^2}{2d^2i^3\left(2cx+dx^2+\frac{c^2}{d}\right)}-\frac{B^2b^2}{2di^3(a^2d^2-2abcd+b^2c^2)}\right) - \frac{\ln\left(\frac{e(a+bx)}{c+dx}\right)\left(\frac{AB}{bd^2i^3}+\frac{B^2x(ad-bc)}{di^3(a^2d^2-2abcd+b^2c^2)}-\frac{B^2b^2\left(\frac{a^2d^2-3abcd+2b^2c^2}{2b^3d}-\frac{c(ad-bc)}{2b^2d}\right)}{di^3(a^2d^2-2abcd+b^2c^2)}\right)}{\frac{dx^2}{b}+\frac{c^2}{bd}+\frac{2cx}{b}} + \frac{Bb^2\,\mathrm{atan}\left(\frac{Bb^2\left(2bdx+\frac{a^2d^3i^3-b^2c^2di^3}{di^3(ad-bc)}\right)(2A-3B)\,1i}{(ad-bc)(3B^2b^2-2ABb^2)}\right)(2A-3B)\,1i}{di^3(ad-bc)^2}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(c*i + d*i*x)^3,x)
```

```
[Out] (B*b^2*atan((B*b^2*(2*b*d*x + (a^2*d^3*i^3 - b^2*c^2*d*i^3)/(d*i^3*(a*d - b
*c)))*(2*A - 3*B)*1i)/((a*d - b*c)*(3*B^2*b^2 - 2*A*B*b^2)))*(2*A - 3*B)*1i
)/(d*i^3*(a*d - b*c)^2) - log((e*(a + b*x))/(c + d*x))^2*(B^2/(2*d^2*i^3*(2
*c*x + d*x^2 + c^2/d)) - (B^2*b^2)/(2*d*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)
)) - (log((e*(a + b*x))/(c + d*x))*((A*B)/(b*d^2*i^3) + (B^2*x*(a*d - b*c))
/(d*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (B^2*b^2*((a^2*d^2 + 2*b^2*c^2 -
 3*a*b*c*d)/(2*b^3*d) - (c*(a*d - b*c))/(2*b^2*d)))/(d*i^3*(a^2*d^2 + b^2*c
^2 - 2*a*b*c*d))))/((d*x^2)/b + c^2/(b*d) + (2*c*x)/b) - ((2*A^2*a*d - 2*A^
2*b*c + B^2*a*d - 7*B^2*b*c - 2*A*B*a*d + 6*A*B*b*c)/(2*(a*d - b*c)) - (x*(
3*B^2*b*d - 2*A*B*b*d))/(a*d - b*c))/(2*c^2*d*i^3 + 2*d^3*i^3*x^2 + 4*c*d^2
*i^3*x)
```

3.104 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 375

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx = \frac{B^2d^2(a+bx)^2}{4(bc-ad)^3gi^3(c+dx)^2}+\frac{4AbBd(a+bx)}{(bc-ad)^3gi^3(c+dx)} - \frac{4bB^2d(a+bx)}{(bc-ad)^3gi^3(c+dx)}+\frac{4bB^2d(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^3gi^3(c+dx)} - \frac{Bd^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3gi^3(c+dx)^2} + \frac{d^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^3gi^3(c+dx)^2} - \frac{2bd(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3gi^3(c+dx)} + \frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^3gi^3}$$

```
[Out] 1/4*B^2*d^2*(b*x+a)^2/(-a*d+b*c)^3/g/i^3/(d*x+c)^2+4*A*b*B*d*(b*x+a)/(-a*d+
b*c)^3/g/i^3/(d*x+c)-4*b*B^2*d*(b*x+a)/(-a*d+b*c)^3/g/i^3/(d*x+c)+4*b*B^2*d
*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(-a*d+b*c)^3/g/i^3/(d*x+c)-1/2*B*d^2*(b*x+a)
^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g/i^3/(d*x+c)^2+1/2*d^2*(b*x+a)
^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g/i^3/(d*x+c)^2-2*b*d*(b*x+a)
*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g/i^3/(d*x+c)+1/3*b^2*(A+B*ln(e
*(b*x+a)/(d*x+c)))^3/B/(-a*d+b*c)^3/g/i^3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2562, 2388, 2339, 30, 2333, 2332, 2367, 2342, 2341}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx=\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^3}{3Bgi^3(bc-ad)^3}+\frac{d^2(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2gi^3(c+dx)^2(bc-ad)^3}-\frac{Bd^2(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2gi^3(c+dx)^2(bc-ad)^3}-\frac{2bd(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{gi^3(c+dx)(bc-ad)^3}+\frac{4AbBd(a+bx)}{gi^3(c+dx)(bc-ad)^3}+\frac{B^2d^2(a+bx)^2}{4gi^3(c+dx)^2(bc-ad)^3}+\frac{4bB^2d(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{gi^3(c+dx)(bc-ad)^3}-\frac{4bB^2d(a+bx)}{gi^3(c+dx)(bc-ad)^3}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)*(c*i + d*i*x)^3),
x]
```

```
[Out] (B^2*d^2*(a + b*x)^2)/(4*(b*c - a*d)^3*g*i^3*(c + d*x)^2) + (4*A*b*B*d*(a +
 b*x))/((b*c - a*d)^3*g*i^3*(c + d*x)) - (4*b*B^2*d*(a + b*x))/((b*c - a*d)
^3*g*i^3*(c + d*x)) + (4*b*B^2*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/((
b*c - a*d)^3*g*i^3*(c + d*x)) - (B*d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))
/(c + d*x)]))/(2*(b*c - a*d)^3*g*i^3*(c + d*x)^2) + (d^2*(a + b*x)^2*(A + B
*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(b*c - a*d)^3*g*i^3*(c + d*x)^2) - (2*
b*d*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*c - a*d)^3*g*i^3*
(c + d*x)) + (b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^3)/(3*B*(b*c - a*d)^
3*g*i^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
```

```
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3}$$

$$= \frac{b\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3} - \frac{d\text{Subst}\left(\int (b-dx)(A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3}$$

$$= \frac{b^2\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3} - \frac{d\text{Subst}\left(\int \left(b(A+B\log(ex))^2 - dx(A+B\log(ex))^2\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3} - \frac{(bd)\text{Subst}\left(\int (A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3}$$

$$= -\frac{bd(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3gi^3(c+dx)} + \frac{b^2\text{Subst}\left(\int x^2\,dx, x, A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B(bc-ad)^3gi^3} - \frac{(bd)\text{Subst}\left(\int (A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3} + \frac{(2bBd)\text{Subst}\left(\int (A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3} + \frac{d^2\text{Subst}\left(\int x(A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3}$$

$$= \frac{2AbBd(a+bx)}{(bc-ad)^3gi^3(c+dx)} + \frac{d^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^3gi^3(c+dx)^2} - \frac{2bd(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3gi^3(c+dx)} + \frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^3gi^3} + \frac{(2bBd)\text{Subst}\left(\int (A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3} + \frac{\left(2bB^2d\right)\text{Subst}\left(\int \log(ex)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3} - \frac{\left(Bd^2\right)\text{Subst}\left(\int x(A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3}$$

$$= \frac{B^2d^2(a+bx)^2}{4(bc-ad)^3gi^3(c+dx)^2} + \frac{4AbBd(a+bx)}{(bc-ad)^3gi^3(c+dx)} - \frac{2bB^2d(a+bx)}{(bc-ad)^3gi^3(c+dx)} + \frac{2bB^2d(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^3gi^3(c+dx)} - \frac{Bd^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3gi^3(c+dx)^2} + \frac{d^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^3gi^3(c+dx)^2} - \frac{2bd(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3gi^3(c+dx)} + \frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^3gi^3} + \frac{(2bB^2d)\,\text{Subst}\left(\int\log(ex)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3}$$

$$= \frac{B^2d^2(a+bx)^2}{4(bc-ad)^3gi^3(c+dx)^2} + \frac{4AbBd(a+bx)}{(bc-ad)^3gi^3(c+dx)} - \frac{4bB^2d(a+bx)}{(bc-ad)^3gi^3(c+dx)} + \frac{4bB^2d(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^3gi^3(c+dx)} - \frac{Bd^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3gi^3(c+dx)^2} + \frac{d^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^3gi^3(c+dx)^2} - \frac{2bd(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3gi^3(c+dx)} + \frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^3gi^3}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.77

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx = \frac{\frac{3(2A^2-2AB+B^2)(bc-ad)^2}{(c+dx)^2} + \frac{6b(2A^2-6AB+7B^2)(bc-ad)}{c+dx} + 6b^2(2A^2-6AB+7B^2)\log(a+bx) + \frac{6B(bc-ad)(B(-7bc+ad-6}{}}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)*(c*i + d*i*
x)^3),x]
```

```
[Out] ((3*(2*A^2 - 2*A*B + B^2)*(b*c - a*d)^2)/(c + d*x)^2 + (6*b*(2*A^2 - 6*A*B
+ 7*B^2)*(b*c - a*d))/(c + d*x) + 6*b^2*(2*A^2 - 6*A*B + 7*B^2)*Log[a + b*x
] + (6*B*(b*c - a*d)*(B*(-7*b*c + a*d - 6*b*d*x) + A*(6*b*c - 2*a*d + 4*b*d
*x))*Log[(e*(a + b*x))/(c + d*x)])/(c + d*x)^2 + (6*B*(2*A*b^2*(c + d*x)^2
+ B*d*(a + b*x)*(-4*b*c + a*d - 3*b*d*x))*Log[(e*(a + b*x))/(c + d*x)]^2)/(
c + d*x)^2 + 4*b^2*B^2*Log[(e*(a + b*x))/(c + d*x)]^3 - 6*b^2*(2*A^2 - 6*A*
B + 7*B^2)*Log[c + d*x])/(12*(b*c - a*d)^3*g*i^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 768 vs. $2(367) = 734$.

Time = 1.37 (sec) , antiderivative size = 769, normalized size of antiderivative = 2.05

method	result
parts	$\frac{A^2\left(-\frac{1}{2(ad-cb)(dx+c)^2}+\frac{b^2\ln(dx+c)}{(ad-cb)^3}+\frac{b}{(ad-cb)^2(dx+c)}-\frac{b^2\ln(bx+a)}{(ad-cb)^3}\right)}{g\,i^3}-\frac{B^2 d\left(d\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2}\right.\right.}{}$
derivativedivides	$-\frac{e(ad-cb)\left(\frac{d^2A^2b^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^3(ad-cb)^4g}-\frac{2d^3A^2b\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^3(ad-cb)^4g}+\frac{d^4A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2e^3i^3(ad-cb)^4g}+\frac{d^2AB\,b^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{e\,i^3(ad-cb)^4g}\right.}{}$
default	$-\frac{e(ad-cb)\left(\frac{d^2A^2b^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^3(ad-cb)^4g}-\frac{2d^3A^2b\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^3(ad-cb)^4g}+\frac{d^4A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2e^3i^3(ad-cb)^4g}+\frac{d^2AB\,b^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{e\,i^3(ad-cb)^4g}\right.}{}$
norman	$-\frac{2A^2a\,d^3-6A^2bc\,d^2-2ABa\,d^3+14ABbc\,d^2+B^2a\,d^3-15B^2bc\,d^2}{4ig\,d^2\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{\left(2A^2b^2c^2+2AB\,a^2d^2-8ABabcd-B^2a^2d^2+8B^2abcd\right)\ln\left(\frac{e(bx+}{dx+}\right.}{2gi\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$
parallelrisch	Expression too large to display
risch	Expression too large to display

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x,method=_RETUR
NVERBOSE)
```

```
[Out] A^2/g/i^3*(-1/2/(a*d-b*c)/(d*x+c)^2+b^2/(a*d-b*c)^3*ln(d*x+c)+b/(a*d-b*c)^2
/(d*x+c)-b^2/(a*d-b*c)^3*ln(b*x+a))-B^2/g/i^3*d/(a*d-b*c)^2/e^2*(d/(a*d-b*c
)*(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/
2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+1/4*(b*e/
d+(a*d-b*c)*e/d/(d*x+c))^2)-2*b*e/(a*d-b*c)*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*
ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+
(a*d-b*c)*e/d/(d*x+c))+2*(a*d-b*c)*e/d/(d*x+c)+2*b*e/d)+1/3/d/(a*d-b*c)*e^2
*b^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-2*B*A/g/i^3*d/(a*d-b*c)^2/e^2*(d/(a
*d-b*c)*(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-2*b*e/(a*d-b*c)*((b*e/d+(a*d-b*c)*e/
d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)+1/2
/d/(a*d-b*c)*e^2*b^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.45

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$$
$$=\frac{3\,(6\,A^2-14\,AB+15\,B^2)b^2c^2-24\,(A^2-2\,AB+2\,B^2)abcd+3\,(2\,A^2-2\,AB+B^2)a^2d^2+4\,(B^2b^2d^2x^2+$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x, algor
ithm="fricas")
```

```
[Out] 1/12*(3*(6*A^2 - 14*A*B + 15*B^2)*b^2*c^2 - 24*(A^2 - 2*A*B + 2*B^2)*a*b*c*
d + 3*(2*A^2 - 2*A*B + B^2)*a^2*d^2 + 4*(B^2*b^2*d^2*x^2 + 2*B^2*b^2*c*d*x
+ B^2*b^2*c^2)*log((b*e*x + a*e)/(d*x + c))^3 + 6*((2*A*B - 3*B^2)*b^2*d^2*
x^2 + 2*A*B*b^2*c^2 - 4*B^2*a*b*c*d + B^2*a^2*d^2 - 2*(B^2*a*b*d^2 - 2*(A*B
 - B^2)*b^2*c*d)*x)*log((b*e*x + a*e)/(d*x + c))^2 + 6*((2*A^2 - 6*A*B + 7*
B^2)*b^2*c*d - (2*A^2 - 6*A*B + 7*B^2)*a*b*d^2)*x + 6*((2*A^2 - 6*A*B + 7*B
^2)*b^2*d^2*x^2 + 2*A^2*b^2*c^2 - 8*(A*B - B^2)*a*b*c*d + (2*A*B - B^2)*a^2
*d^2 + 2*(2*(A^2 - 2*A*B + 2*B^2)*b^2*c*d - (2*A*B - 3*B^2)*a*b*d^2)*x)*log
((b*e*x + a*e)/(d*x + c)))/((b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4
- a^3*d^5)*g*i^3*x^2 + 2*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a
^3*c*d^4)*g*i^3*x + (b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^
3)*g*i^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1488 vs. 2(333) = 666.

Time = 4.20 (sec) , antiderivative size = 1488, normalized size of antiderivative = 3.97

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)/(d*i*x+c*i)**3,x)
```

```
[Out] -B**2*b**2*log(e*(a + b*x)/(c + d*x))**3/(3*a**3*d**3*g*i**3 - 9*a**2*b*c*d
**2*g*i**3 + 9*a*b**2*c**2*d*g*i**3 - 3*b**3*c**3*g*i**3) + b**2*(2*A**2 -
6*A*B + 7*B**2)*log(x + (2*A**2*a*b**2*d + 2*A**2*b**3*c - 6*A*B*a*b**2*d -
 6*A*B*b**3*c + 7*B**2*a*b**2*d + 7*B**2*b**3*c - a**4*b**2*d**4*(2*A**2 -
6*A*B + 7*B**2)/(a*d - b*c)**3 + 4*a**3*b**3*c*d**3*(2*A**2 - 6*A*B + 7*B**
2)/(a*d - b*c)**3 - 6*a**2*b**4*c**2*d**2*(2*A**2 - 6*A*B + 7*B**2)/(a*d -
```

```
b*c)**3 + 4*a*b**5*c**3*d*(2*A**2 - 6*A*B + 7*B**2)/(a*d - b*c)**3 - b**6*c
**4*(2*A**2 - 6*A*B + 7*B**2)/(a*d - b*c)**3)/(4*A**2*b**3*d - 12*A*B*b**3*
d + 14*B**2*b**3*d))/(2*g*i**3*(a*d - b*c)**3) - b**2*(2*A**2 - 6*A*B + 7*B
**2)*log(x + (2*A**2*a*b**2*d + 2*A**2*b**3*c - 6*A*B*a*b**2*d - 6*A*B*b**3
*c + 7*B**2*a*b**2*d + 7*B**2*b**3*c + a**4*b**2*d**4*(2*A**2 - 6*A*B + 7*B
**2)/(a*d - b*c)**3 - 4*a**3*b**3*c*d**3*(2*A**2 - 6*A*B + 7*B**2)/(a*d - b
*c)**3 + 6*a**2*b**4*c**2*d**2*(2*A**2 - 6*A*B + 7*B**2)/(a*d - b*c)**3 - 4
*a*b**5*c**3*d*(2*A**2 - 6*A*B + 7*B**2)/(a*d - b*c)**3 + b**6*c**4*(2*A**2
 - 6*A*B + 7*B**2)/(a*d - b*c)**3)/(4*A**2*b**3*d - 12*A*B*b**3*d + 14*B**2
*b**3*d))/(2*g*i**3*(a*d - b*c)**3) + (-2*A*B*a*d + 6*A*B*b*c + 4*A*B*b*d*x
 + B**2*a*d - 7*B**2*b*c - 6*B**2*b*d*x)*log(e*(a + b*x)/(c + d*x))/(2*a**2
*c**2*d**2*g*i**3 + 4*a**2*c*d**3*g*i**3*x + 2*a**2*d**4*g*i**3*x**2 - 4*a*
b*c**3*d*g*i**3 - 8*a*b*c**2*d**2*g*i**3*x - 4*a*b*c*d**3*g*i**3*x**2 + 2*b
**2*c**4*g*i**3 + 4*b**2*c**3*d*g*i**3*x + 2*b**2*c**2*d**2*g*i**3*x**2) +
(-2*A*B*b**2*c**2 - 4*A*B*b**2*c*d*x - 2*A*B*b**2*d**2*x**2 - B**2*a**2*d**
2 + 4*B**2*a*b*c*d + 2*B**2*a*b*d**2*x + 4*B**2*b**2*c*d*x + 3*B**2*b**2*d*
*2*x**2)*log(e*(a + b*x)/(c + d*x))**2/(2*a**3*c**2*d**3*g*i**3 + 4*a**3*c*
d**4*g*i**3*x + 2*a**3*d**5*g*i**3*x**2 - 6*a**2*b*c**3*d**2*g*i**3 - 12*a*
*2*b*c**2*d**3*g*i**3*x - 6*a**2*b*c*d**4*g*i**3*x**2 + 6*a*b**2*c**4*d*g*i
**3 + 12*a*b**2*c**3*d**2*g*i**3*x + 6*a*b**2*c**2*d**3*g*i**3*x**2 - 2*b**
3*c**5*g*i**3 - 4*b**3*c**4*d*g*i**3*x - 2*b**3*c**3*d**2*g*i**3*x**2) + (-
2*A**2*a*d + 6*A**2*b*c + 2*A*B*a*d - 14*A*B*b*c - B**2*a*d + 15*B**2*b*c +
 x*(4*A**2*b*d - 12*A*B*b*d + 14*B**2*b*d))/(4*a**2*c**2*d**2*g*i**3 - 8*a*
b*c**3*d*g*i**3 + 4*b**2*c**4*g*i**3 + x**2*(4*a**2*d**4*g*i**3 - 8*a*b*c*d
**3*g*i**3 + 4*b**2*c**2*d**2*g*i**3) + x*(8*a**2*c*d**3*g*i**3 - 16*a*b*c*
*2*d**2*g*i**3 + 8*b**2*c**3*d*g*i**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2116 vs. $2(367) = 734$.

Time = 0.34 (sec) , antiderivative size = 2116, normalized size of antiderivative = 5.64

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x, algor
ithm="maxima")
```

```
[Out] 1/2*B^2*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*g*i
^3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3*x + (b^2*c^4 - 2*a
*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c)/((b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3))*log(b*e*x/(d*x + c) + a*e/(d*x
+ c))^2 + A*B*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^
```

```
4)*g*i^3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3*x + (b^2*c^4
 - 2*a*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c)/((b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3))*log(b*e*x/(d*x + c) + a*e
/(d*x + c)) - 1/12*B^2*(6*(7*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*log(d*x + c)^2 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*log(b*x + a) - 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 +
 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x + c))*log(b*
e*x/(d*x + c) + a*e/(d*x + c))/(b^3*c^5*g*i^3 - 3*a*b^2*c^4*d*g*i^3 + 3*a^2
*b*c^3*d^2*g*i^3 - a^3*c^2*d^3*g*i^3 + (b^3*c^3*d^2*g*i^3 - 3*a*b^2*c^2*d^3
*g*i^3 + 3*a^2*b*c*d^4*g*i^3 - a^3*d^5*g*i^3)*x^2 + 2*(b^3*c^4*d*g*i^3 - 3*
a*b^2*c^3*d^2*g*i^3 + 3*a^2*b*c^2*d^3*g*i^3 - a^3*c*d^4*g*i^3)*x) - (45*b^2
*c^2 - 48*a*b*c*d + 3*a^2*d^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log
(b*x + a)^3 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x + c)^3 + 18*(
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a)^2 + 6*(3*b^2*d^2*x^2 + 6*
b^2*c*d*x + 3*b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a
))*log(d*x + c)^2 + 42*(b^2*c*d - a*b*d^2)*x + 42*(b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*log(b*x + a) - 6*(7*b^2*d^2*x^2 + 14*b^2*c*d*x + 7*b^2*c^2 + 2
*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a)^2 + 6*(b^2*d^2*x^2 + 2*
b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x + c))/(b^3*c^5*g*i^3 - 3*a*b^2*c
^4*d*g*i^3 + 3*a^2*b*c^3*d^2*g*i^3 - a^3*c^2*d^3*g*i^3 + (b^3*c^3*d^2*g*i^3
 - 3*a*b^2*c^2*d^3*g*i^3 + 3*a^2*b*c*d^4*g*i^3 - a^3*d^5*g*i^3)*x^2 + 2*(b^
3*c^4*d*g*i^3 - 3*a*b^2*c^3*d^2*g*i^3 + 3*a^2*b*c^2*d^3*g*i^3 - a^3*c*d^4*g
*i^3)*x)) + 1/2*A^2*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c*d^3 +
a^2*d^4)*g*i^3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3*x + (b
^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c)/((b^3
*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3)) - 1/2*(7*b^2*c^2 -
8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a)^
2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x + c)^2 + 6*(b^2*c*d - a
*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a) - 2*(3*b^2
*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*log(b*x + a))*log(d*x + c))*A*B/(b^3*c^5*g*i^3 - 3*a*b^2*c^4*d*g*i^3 + 3*
a^2*b*c^3*d^2*g*i^3 - a^3*c^2*d^3*g*i^3 + (b^3*c^3*d^2*g*i^3 - 3*a*b^2*c^2*
d^3*g*i^3 + 3*a^2*b*c*d^4*g*i^3 - a^3*d^5*g*i^3)*x^2 + 2*(b^3*c^4*d*g*i^3 -
 3*a*b^2*c^3*d^2*g*i^3 + 3*a^2*b*c^2*d^3*g*i^3 - a^3*c*d^4*g*i^3)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.79

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$$
$$=\frac{1}{12}\left(\frac{4\,B^2b^2e\log\left(\frac{bex+ae}{dx+c}\right)^3}{b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3}+\frac{12\,A^2b^2e\log\left(\frac{bex+ae}{dx+c}\right)}{b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3}+6\left(\frac{2\,ABb^2e}{b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3}-\right.\right.$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x, algor
ithm="giac")
```

```
[Out] 1/12*(4*B^2*b^2*e*log((b*e*x + a*e)/(d*x + c))^3/(b^2*c^2*g*i^3 - 2*a*b*c*d
*g*i^3 + a^2*d^2*g*i^3) + 12*A^2*b^2*e*log((b*e*x + a*e)/(d*x + c))/(b^2*c^
2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3) + 6*(2*A*B*b^2*e/(b^2*c^2*g*i^3
- 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3) - 4*(b*e*x + a*e)*B^2*b*d/((b^2*c^2*g*i^
3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)) + (b*e*x + a*e)^2*B^2*d^2/(
(b^2*c^2*e*g*i^3 - 2*a*b*c*d*e*g*i^3 + a^2*d^2*e*g*i^3)*(d*x + c)^2))*log((
b*e*x + a*e)/(d*x + c))^2 + 6*((2*A*B*d^2 - B^2*d^2)*(b*e*x + a*e)^2/((b^2*
c^2*e*g*i^3 - 2*a*b*c*d*e*g*i^3 + a^2*d^2*e*g*i^3)*(d*x + c)^2) - 8*(A*B*b*
d - B^2*b*d)*(b*e*x + a*e)/((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^
3)*(d*x + c)))*log((b*e*x + a*e)/(d*x + c)) + 3*(2*A^2*d^2 - 2*A*B*d^2 + B^
2*d^2)*(b*e*x + a*e)^2/((b^2*c^2*e*g*i^3 - 2*a*b*c*d*e*g*i^3 + a^2*d^2*e*g*
i^3)*(d*x + c)^2) - 24*(A^2*b*d - 2*A*B*b*d + 2*B^2*b*d)*(b*e*x + a*e)/((b^
2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)))*(b*c/((b*c*e - a
*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

Mupad [B] (verification not implemented)

Time = 4.86 (sec) , antiderivative size = 984, normalized size of antiderivative = 2.62

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$$

$$= -\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{\frac{B^2\,b^2\,\left(\frac{a^2\,d^2-3\,a\,b\,c\,d+2\,b^2\,c^2}{2\,b^3\,d}-\frac{c\,(a\,d-b\,c)}{2\,b^2\,d}\right)}{g\,i^3\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}-\frac{B^2\,x\,(a\,d-b\,c)}{g\,i^3\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}}{\frac{d\,x^2}{b}+\frac{c^2}{b\,d}+\frac{2\,c\,x}{b}}+\frac{B\,b^2\,(2\,A-3\,B)}{2\,g\,i^3\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}\right)$$
$$-\frac{\frac{2\,A^2\,a\,d-6\,A^2\,b\,c+B^2\,a\,d-15\,B^2\,b\,c-2\,A\,B\,a\,d+14\,A\,B\,b\,c}{2\,(a\,d-b\,c)}-\frac{x\,(2\,b\,d\,A^2-6\,b\,d\,A\,B+7\,b\,d\,B^2)}{a\,d-b\,c}}{x^2\,(2\,a\,d^3\,g\,i^3-2\,b\,c\,d^2\,g\,i^3)+x\,(4\,a\,c\,d^2\,g\,i^3-4\,b\,c^2\,d\,g\,i^3)-2\,b\,c^3\,g\,i^3+2\,a\,c^2\,d\,g\,i^3}$$
$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(\frac{B^2}{b\,d\,g\,i^3\,(a\,d-b\,c)}+\frac{B\,b^2\,\left(\frac{a^2\,d^2-3\,a\,b\,c\,d+2\,b^2\,c^2}{2\,b^3\,d}-\frac{c\,(a\,d-b\,c)}{2\,b^2\,d}\right)(2\,A-3\,B)}{g\,i^3\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}-\frac{B\,x\,(2\,A-3\,B)\,(a\,d-b\,c)}{g\,i^3\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}\right)}{\frac{d\,x^2}{b}+\frac{c^2}{b\,d}+\frac{2\,c\,x}{b}}$$
$$-\frac{B^2\,b^2\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^3}{3\,g\,i^3\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}$$
$$+\frac{b^2\,\mathrm{atan}\left(\frac{b^2\,\left(A^2-3\,A\,B+\frac{7\,B^2}{2}\right)(2\,g\,a^3\,d^3\,i^3-2\,g\,a^2\,b\,c\,d^2\,i^3-2\,g\,a\,b^2\,c^2\,d\,i^3+2\,g\,b^3\,c^3\,i^3)\,1\mathrm{i}}{g\,i^3\,(a\,d-b\,c)^3\,(2\,A^2\,b^2-6\,A\,B\,b^2+7\,B^2\,b^2)}+\frac{b^3\,d\,x\,(g\,a^2\,d^2\,i^3-2\,g\,a\,b\,c\,d\,i^3+g\,b^2\,c^2\,i^3)\,(A^2-}{g\,i^3\,(a\,d-b\,c)^3\,(2\,A^2\,b^2-6\,A\,B\,b^2+7}\right.}{g\,i^3\,(a\,d-b\,c)^3}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)*(c*i + d*i*x)^3),
x)
```

```
[Out] (b^2*atan((b^2*(A^2 + (7*B^2)/2 - 3*A*B)*(2*a^3*d^3*g*i^3 + 2*b^3*c^3*g*i^3
 - 2*a*b^2*c^2*d*g*i^3 - 2*a^2*b*c*d^2*g*i^3)*1i)/(g*i^3*(a*d - b*c)^3*(2*A
^2*b^2 + 7*B^2*b^2 - 6*A*B*b^2)) + (b^3*d*x*(a^2*d^2*g*i^3 + b^2*c^2*g*i^3
- 2*a*b*c*d*g*i^3)*(A^2 + (7*B^2)/2 - 3*A*B)*4i)/(g*i^3*(a*d - b*c)^3*(2*A^
2*b^2 + 7*B^2*b^2 - 6*A*B*b^2)))*(A^2 + (7*B^2)/2 - 3*A*B)*2i)/(g*i^3*(a*d
- b*c)^3) - ((2*A^2*a*d - 6*A^2*b*c + B^2*a*d - 15*B^2*b*c - 2*A*B*a*d + 14
*A*B*b*c)/(2*(a*d - b*c)) - (x*(2*A^2*b*d + 7*B^2*b*d - 6*A*B*b*d))/(a*d -
b*c))/(x^2*(2*a*d^3*g*i^3 - 2*b*c*d^2*g*i^3) + x*(4*a*c*d^2*g*i^3 - 4*b*c^2
*d*g*i^3) - 2*b*c^3*g*i^3 + 2*a*c^2*d*g*i^3) - (log((e*(a + b*x))/(c + d*x)
)*(B^2/(b*d*g*i^3*(a*d - b*c)) + (B*b^2*((a^2*d^2 + 2*b^2*c^2 - 3*a*b*c*d)/
(2*b^3*d) - (c*(a*d - b*c))/(2*b^2*d))*(2*A - 3*B))/(g*i^3*(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (B*x*(2*A - 3*B)*(a*d - b*c))/(g*i^3
*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))))/((d*x^2)/b + c^2/(b
*d) + (2*c*x)/b) - log((e*(a + b*x))/(c + d*x))^2*(((B^2*b^2*((a^2*d^2 + 2*
```

```
b^2*c^2 - 3*a*b*c*d)/(2*b^3*d) - (c*(a*d - b*c))/(2*b^2*d)))/(g*i^3*(a^3*d^
3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (B^2*x*(a*d - b*c))/(g*i^3*
(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))/((d*x^2)/b + c^2/(b*d
) + (2*c*x)/b) + (B*b^2*(2*A - 3*B))/(2*g*i^3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*
c^2*d - 3*a^2*b*c*d^2))) - (B^2*b^2*log((e*(a + b*x))/(c + d*x))^3)/(3*g*i^
3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))
```

3.105 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 525

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = -\frac{B^2d^3(a+bx)^2}{4(bc-ad)^4g^2i^3(c+dx)^2}-\frac{6AbBd^2(a+bx)}{(bc-ad)^4g^2i^3(c+dx)}+\frac{6bB^2d^2(a+bx)}{(bc-ad)^4g^2i^3(c+dx)}-\frac{2b^3B^2(c+dx)}{(bc-ad)^4g^2i^3(a+bx)}-\frac{6bB^2d^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^4g^2i^3(c+dx)}+\frac{Bd^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^2i^3(c+dx)^2}-\frac{2b^3B(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3(a+bx)}-\frac{d^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^4g^2i^3(c+dx)^2}+\frac{3bd^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^2i^3(c+dx)}-\frac{b^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^2i^3(a+bx)}-\frac{b^2d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{B(bc-ad)^4g^2i^3}$$

```
[Out] -1/4*B^2*d^3*(b*x+a)^2/(-a*d+b*c)^4/g^2/i^3/(d*x+c)^2-6*A*b*B*d^2*(b*x+a)/(
-a*d+b*c)^4/g^2/i^3/(d*x+c)+6*b*B^2*d^2*(b*x+a)/(-a*d+b*c)^4/g^2/i^3/(d*x+c
)-2*b^3*B^2*(d*x+c)/(-a*d+b*c)^4/g^2/i^3/(b*x+a)-6*b*B^2*d^2*(b*x+a)*ln(e*(
b*x+a)/(d*x+c))/(-a*d+b*c)^4/g^2/i^3/(d*x+c)+1/2*B*d^3*(b*x+a)^2*(A+B*ln(e*
(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^2/i^3/(d*x+c)^2-2*b^3*B*(d*x+c)*(A+B*ln(e*
(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^2/i^3/(b*x+a)-1/2*d^3*(b*x+a)^2*(A+B*ln(e*
(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^2/i^3/(d*x+c)^2+3*b*d^2*(b*x+a)*(A+B*ln(
e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^2/i^3/(d*x+c)-b^3*(d*x+c)*(A+B*ln(e*(b
*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^2/i^3/(b*x+a)-b^2*d*(A+B*ln(e*(b*x+a)/(d*x
+c)))^3/B/(-a*d+b*c)^4/g^2/i^3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2562, 2395, 2333, 2332, 2342, 2341, 2339, 30}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = -\frac{b^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^2i^3(a+bx)(bc-ad)^4} - \frac{2b^3B(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2i^3(a+bx)(bc-ad)^4} - \frac{b^2d\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^3}{Bg^2i^3(bc-ad)^4} - \frac{d^3(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g^2i^3(c+dx)^2(bc-ad)^4} + \frac{Bd^3(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^2i^3(c+dx)^2(bc-ad)^4} + \frac{3bd^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^2i^3(c+dx)(bc-ad)^4} - \frac{6AbBd^2(a+bx)}{g^2i^3(c+dx)(bc-ad)^4} - \frac{2b^3B^2(c+dx)}{g^2i^3(a+bx)(bc-ad)^4} - \frac{B^2d^3(a+bx)^2}{4g^2i^3(c+dx)^2(bc-ad)^4} - \frac{6bB^2d^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{g^2i^3(c+dx)(bc-ad)^4} + \frac{6bB^2d^2(a+bx)}{g^2i^3(c+dx)(bc-ad)^4}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^2*(c*i + d*i*x)^3
),x]
```

```
[Out] -1/4*(B^2*d^3*(a + b*x)^2)/((b*c - a*d)^4*g^2*i^3*(c + d*x)^2) - (6*A*b*B*d
^2*(a + b*x))/((b*c - a*d)^4*g^2*i^3*(c + d*x)) + (6*b*B^2*d^2*(a + b*x))/(
(b*c - a*d)^4*g^2*i^3*(c + d*x)) - (2*b^3*B^2*(c + d*x))/((b*c - a*d)^4*g^2
*i^3*(a + b*x)) - (6*b*B^2*d^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/((b*
c - a*d)^4*g^2*i^3*(c + d*x)) + (B*d^3*(a + b*x)^2*(A + B*Log[(e*(a + b*x))
/(c + d*x)]))/(2*(b*c - a*d)^4*g^2*i^3*(c + d*x)^2) - (2*b^3*B*(c + d*x)*(A
 + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^4*g^2*i^3*(a + b*x)) - (d^
3*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(b*c - a*d)^4*g^2*
i^3*(c + d*x)^2) + (3*b*d^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^
2)/((b*c - a*d)^4*g^2*i^3*(c + d*x)) - (b^3*(c + d*x)*(A + B*Log[(e*(a + b*
x))/(c + d*x)])^2)/((b*c - a*d)^4*g^2*i^3*(a + b*x)) - (b^2*d*(A + B*Log[(e
*(a + b*x))/(c + d*x)])^3)/(B*(b*c - a*d)^4*g^2*i^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)^3(A+B\log(ex))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}\\
&= \frac{\text{Subst}\left(\int \left(3bd^2(A+B\log(ex))^2+\frac{b^3(A+B\log(ex))^2}{x^2}-\frac{3b^2d(A+B\log(ex))^2}{x}-d^3x(A+B\log(ex))^2\right)\,dx, x}{(bc-ad)^4g^2i^3}\\
&= \frac{b^3\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}-\frac{(3b^2d)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}\\
&\quad+\frac{(3bd^2)\,\text{Subst}\left(\int (A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}-\frac{d^3\text{Subst}\left(\int x(A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}\\
&= -\frac{d^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^4g^2i^3(c+dx)^2}+\frac{3bd^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^2i^3(c+dx)}\\
&\quad-\frac{b^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^2i^3(a+bx)}+\frac{(2b^3B)\,\text{Subst}\left(\int \frac{A+B\log(ex)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}\\
&\quad-\frac{(3b^2d)\,\text{Subst}\left(\int x^2\,dx, x, A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B(bc-ad)^4g^2i^3}\\
&\quad-\frac{(6bBd^2)\,\text{Subst}\left(\int (A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}\\
&\quad+\frac{(Bd^3)\,\text{Subst}\left(\int x(A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}
\end{aligned}
$$

$$= -\frac{B^2d^3(a+bx)^2}{4(bc-ad)^4g^2i^3(c+dx)^2} - \frac{6AbBd^2(a+bx)}{(bc-ad)^4g^2i^3(c+dx)} - \frac{2b^3B^2(c+dx)}{(bc-ad)^4g^2i^3(a+bx)} + \frac{Bd^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^2i^3(c+dx)^2} - \frac{2b^3B(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3(a+bx)} - \frac{d^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^4g^2i^3(c+dx)^2} + \frac{3bd^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^2i^3(c+dx)} - \frac{b^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^2i^3(a+bx)} - \frac{b^2d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{B(bc-ad)^4g^2i^3} - \frac{(6bB^2d^2)\,\text{Subst}\left(\int\log(ex)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}$$

$$= -\frac{B^2d^3(a+bx)^2}{4(bc-ad)^4g^2i^3(c+dx)^2} - \frac{6AbBd^2(a+bx)}{(bc-ad)^4g^2i^3(c+dx)} + \frac{6bB^2d^2(a+bx)}{(bc-ad)^4g^2i^3(c+dx)} - \frac{2b^3B^2(c+dx)}{(bc-ad)^4g^2i^3(a+bx)} - \frac{6bB^2d^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^4g^2i^3(c+dx)} + \frac{Bd^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^2i^3(c+dx)^2} - \frac{2b^3B(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3(a+bx)} - \frac{d^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^4g^2i^3(c+dx)^2} + \frac{3bd^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^2i^3(c+dx)} - \frac{b^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^2i^3(a+bx)} - \frac{b^2d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{B(bc-ad)^4g^2i^3}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 453, normalized size of antiderivative = 0.86

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = -\frac{(2A^2-2AB+B^2)\,d(bc-ad)^2(a+bx)+2b(4A^2-10AB+11B^2)\,d(bc-ad)(a+bx)(c+dx)+4b^2(A^2}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^2*(c*i + d*
i*x)^3),x]
```

```
[Out] -1/4*((2*A^2 - 2*A*B + B^2)*d*(b*c - a*d)^2*(a + b*x) + 2*b*(4*A^2 - 10*A*B
 + 11*B^2)*d*(b*c - a*d)*(a + b*x)*(c + d*x) + 4*b^2*(A^2 + 2*A*B + 2*B^2)*
(b*c - a*d)*(c + d*x)^2 + 6*b^2*(2*A^2 - 2*A*B + 5*B^2)*d*(a + b*x)*(c + d*
```

```
x)^2*Log[a + b*x] + 2*B*(b*c - a*d)*((2*A - B)*d*(b*c - a*d)*(a + b*x) + 2*
b*(4*A - 5*B)*d*(a + b*x)*(c + d*x) + 4*b^2*(A + B)*(c + d*x)^2)*Log[(e*(a
+ b*x))/(c + d*x)] + 2*B*(a^3*B*d^3 - 3*a^2*b*B*d^2*(2*c + d*x) + 3*a*b^2*d
*(2*A*(c + d*x)^2 - B*d*x*(4*c + 3*d*x)) + b^3*(6*A*d*x*(c + d*x)^2 + B*(2*
c^3 + 6*c^2*d*x - 3*d^3*x^3)))*Log[(e*(a + b*x))/(c + d*x)]^2 + 4*b^2*B^2*d
*(a + b*x)*(c + d*x)^2*Log[(e*(a + b*x))/(c + d*x)]^3 - 6*b^2*(2*A^2 - 2*A*
B + 5*B^2)*d*(a + b*x)*(c + d*x)^2*Log[c + d*x])/((b*c - a*d)^4*g^2*i^3*(a
+ b*x)*(c + d*x)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. 2(519) = 1038.

Time = 2.78 (sec) , antiderivative size = 1080, normalized size of antiderivative = 2.06

method	result	size
parts	Expression too large to display	1080
derivativedivides	Expression too large to display	1233
default	Expression too large to display	1233
parallelrisch	Expression too large to display	1762
norman	Expression too large to display	1852
risch	Expression too large to display	2114

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x,method=_RET
URNVERBOSE)
```

```
[Out] 1/g^2*A^2/i^3*(-1/2*d/(a*d-b*c)^2/(d*x+c)^2+3*d/(a*d-b*c)^4*b^2*ln(d*x+c)+2
*d/(a*d-b*c)^3*b/(d*x+c)+b^2/(a*d-b*c)^3/(b*x+a)-3*d/(a*d-b*c)^4*b^2*ln(b*x
+a))-B^2/g^2/i^3*d/(a*d-b*c)^2/e^2*(d^2/(a*d-b*c)^2*(1/2*(b*e/d+(a*d-b*c)*e
/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2*(b*e/d+(a*d-b*c)*e/d/(d
*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^
2)-3/(a*d-b*c)^2*b*d*e*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^2-2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)+2*(a*d-b*c)*e/d/(d*x+c)+2*b*e/d)+1/(a*d-b*c)^2*b^2*e^2*ln(b*e/d+(a*d-b*c)
*e/d/(d*x+c))^3-1/d/(a*d-b*c)^2*b^3*e^3*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(
a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))-2*B*A/g^2/i^3*d/(a*
d-b*c)^2/e^2*(d^2/(a*d-b*c)^2*(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d
+(a*d-b*c)*e/d/(d*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-3*d/(a*d-b*c)^
2*b*e*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b
*c)*e/d/(d*x+c)-b*e/d)+3/2/(a*d-b*c)^2*b^2*e^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+
c))^2-1/d/(a*d-b*c)^2*b^3*e^3*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a
*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 1008, normalized size of antiderivative = 1.92

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = -\frac{4\,(A^2+2\,AB+2\,B^2)b^3c^3+3\,(2\,A^2-10\,AB+5\,B^2)ab^2c^2d-12\,(A^2-2\,AB+2\,B^2)a^2bcd^2+(2\,A^2-2}{}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x, alg
orithm="fricas")
```

```
[Out] -1/4*(4*(A^2 + 2*A*B + 2*B^2)*b^3*c^3 + 3*(2*A^2 - 10*A*B + 5*B^2)*a*b^2*c^
2*d - 12*(A^2 - 2*A*B + 2*B^2)*a^2*b*c*d^2 + (2*A^2 - 2*A*B + B^2)*a^3*d^3
+ 4*(B^2*b^3*d^3*x^3 + B^2*a*b^2*c^2*d + (2*B^2*b^3*c*d^2 + B^2*a*b^2*d^3)*
x^2 + (B^2*b^3*c^2*d + 2*B^2*a*b^2*c*d^2)*x)*log((b*e*x + a*e)/(d*x + c))^3
 + 6*((2*A^2 - 2*A*B + 5*B^2)*b^3*c*d^2 - (2*A^2 - 2*A*B + 5*B^2)*a*b^2*d^3
)*x^2 + 2*(3*(2*A*B - B^2)*b^3*d^3*x^3 + 2*B^2*b^3*c^3 + 6*A*B*a*b^2*c^2*d
- 6*B^2*a^2*b*c*d^2 + B^2*a^3*d^3 + 3*(4*A*B*b^3*c*d^2 + (2*A*B - 3*B^2)*a*
b^2*d^3)*x^2 - 3*(B^2*a^2*b*d^3 - 2*(A*B + B^2)*b^3*c^2*d - 4*(A*B - B^2)*a
*b^2*c*d^2)*x)*log((b*e*x + a*e)/(d*x + c))^2 + 3*((6*A^2 - 2*A*B + 13*B^2)
*b^3*c^2*d - 2*(2*A^2 + 2*A*B + 3*B^2)*a*b^2*c*d^2 - (2*A^2 - 6*A*B + 7*B^2
)*a^2*b*d^3)*x + 2*(3*(2*A^2 - 2*A*B + 5*B^2)*b^3*d^3*x^3 + 6*A^2*a*b^2*c^2
*d + 4*(A*B + B^2)*b^3*c^3 - 12*(A*B - B^2)*a^2*b*c*d^2 + (2*A*B - B^2)*a^3
*d^3 + 3*(4*(A^2 + 2*B^2)*b^3*c*d^2 + (2*A^2 - 6*A*B + 7*B^2)*a*b^2*d^3)*x^
2 + 3*(2*(A^2 + 2*A*B + 2*B^2)*b^3*c^2*d + 4*(A^2 - 2*A*B + 2*B^2)*a*b^2*c*
d^2 - (2*A*B - 3*B^2)*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^5*c^4
*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b*d^6)*g
^2*i^3*x^3 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2
*c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*g^2*i^3*x^2 + (b^5*c^6 - 2*a*b^4*c^5*d
- 2*a^2*b^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*g^
2*i^3*x + (a*b^4*c^6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^
3 + a^5*c^2*d^4)*g^2*i^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2/(d*i*x+c*i)**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4188 vs. $2(519) = 1038$.

Time = 0.50 (sec) , antiderivative size = 4188, normalized size of antiderivative = 7.98

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x, alg
orithm="maxima")
[Out] -1/2*B^2*((6*b^2*d^2*x^2 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d +
 a*b*d^2)*x)/((b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)
*g^2*i^3*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c
*d^4 - a^4*d^5)*g^2*i^3*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 +
5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*g^2*i^3*x + (a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3
*a^3*b*c^3*d^2 - a^4*c^2*d^3)*g^2*i^3) + 6*b^2*d*log(b*x + a)/((b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3) - 6*b^
2*d*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*
d^3 + a^4*d^4)*g^2*i^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2 - A*B*((6*b
^2*d^2*x^2 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a*b*d^2)*x)/(
(b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*g^2*i^3*x^3 +
 (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5
)*g^2*i^3*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^
3 - 2*a^4*c*d^4)*g^2*i^3*x + (a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2
 - a^4*c^2*d^3)*g^2*i^3) + 6*b^2*d*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d +
 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3) - 6*b^2*d*log(d*x +
c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*g^2*i^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 1/4*B^2*(2*(4*b^3*c^3 - 1
5*a*b^2*c^2*d + 12*a^2*b*c*d^2 - a^3*d^3 - 6*(b^3*c*d^2 - a*b^2*d^3)*x^2 -
6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d +
 2*a*b^2*c*d^2)*x)*log(b*x + a)^2 - 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c
*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(d*x + c)^2 - 3*(
```

b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x - 6*(b^3*d^3*x^3 + a*b^2*c^2*d +
 (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(b*x + a
) + 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2
*d + 2*a*b^2*c*d^2)*x + 2*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2
*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(b*x + a))*log(d*x + c))*log(
b*e*x/(d*x + c) + a*e/(d*x + c))/(a*b^4*c^6*g^2*i^3 - 4*a^2*b^3*c^5*d*g^2*i
^3 + 6*a^3*b^2*c^4*d^2*g^2*i^3 - 4*a^4*b*c^3*d^3*g^2*i^3 + a^5*c^2*d^4*g^2*
i^3 + (b^5*c^4*d^2*g^2*i^3 - 4*a*b^4*c^3*d^3*g^2*i^3 + 6*a^2*b^3*c^2*d^4*g^
2*i^3 - 4*a^3*b^2*c*d^5*g^2*i^3 + a^4*b*d^6*g^2*i^3)*x^3 + (2*b^5*c^5*d*g^2
*i^3 - 7*a*b^4*c^4*d^2*g^2*i^3 + 8*a^2*b^3*c^3*d^3*g^2*i^3 - 2*a^3*b^2*c^2*
d^4*g^2*i^3 - 2*a^4*b*c*d^5*g^2*i^3 + a^5*d^6*g^2*i^3)*x^2 + (b^5*c^6*g^2*i
^3 - 2*a*b^4*c^5*d*g^2*i^3 - 2*a^2*b^3*c^4*d^2*g^2*i^3 + 8*a^3*b^2*c^3*d^3*
g^2*i^3 - 7*a^4*b*c^2*d^4*g^2*i^3 + 2*a^5*c*d^5*g^2*i^3)*x) + (8*b^3*c^3 +
15*a*b^2*c^2*d - 24*a^2*b*c*d^2 + a^3*d^3 + 4*(b^3*d^3*x^3 + a*b^2*c^2*d +
(2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(b*x + a)
^3 - 4*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^
2*d + 2*a*b^2*c*d^2)*x)*log(d*x + c)^3 + 30*(b^3*c*d^2 - a*b^2*d^3)*x^2 + 6
*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d +
2*a*b^2*c*d^2)*x)*log(b*x + a)^2 + 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*
d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x + 2*(b^3*d^3*x^3 + a*b
^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*l
og(b*x + a))*log(d*x + c)^2 + 3*(13*b^3*c^2*d - 6*a*b^2*c*d^2 - 7*a^2*b*d^3
)*x + 30*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*
c^2*d + 2*a*b^2*c*d^2)*x)*log(b*x + a) - 6*(5*b^3*d^3*x^3 + 5*a*b^2*c^2*d +
 5*(2*b^3*c*d^2 + a*b^2*d^3)*x^2 + 2*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*
d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(b*x + a)^2 + 5*(b
^3*c^2*d + 2*a*b^2*c*d^2)*x + 2*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 +
 a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(b*x + a))*log(d*x + c)
)/(a*b^4*c^6*g^2*i^3 - 4*a^2*b^3*c^5*d*g^2*i^3 + 6*a^3*b^2*c^4*d^2*g^2*i^3
 - 4*a^4*b*c^3*d^3*g^2*i^3 + a^5*c^2*d^4*g^2*i^3 + (b^5*c^4*d^2*g^2*i^3 - 4*
a*b^4*c^3*d^3*g^2*i^3 + 6*a^2*b^3*c^2*d^4*g^2*i^3 - 4*a^3*b^2*c*d^5*g^2*i^3
 + a^4*b*d^6*g^2*i^3)*x^3 + (2*b^5*c^5*d*g^2*i^3 - 7*a*b^4*c^4*d^2*g^2*i^3
+ 8*a^2*b^3*c^3*d^3*g^2*i^3 - 2*a^3*b^2*c^2*d^4*g^2*i^3 - 2*a^4*b*c*d^5*g^2
*i^3 + a^5*d^6*g^2*i^3)*x^2 + (b^5*c^6*g^2*i^3 - 2*a*b^4*c^5*d*g^2*i^3 - 2*
a^2*b^3*c^4*d^2*g^2*i^3 + 8*a^3*b^2*c^3*d^3*g^2*i^3 - 7*a^4*b*c^2*d^4*g^2*i
^3 + 2*a^5*c*d^5*g^2*i^3)*x)) - 1/2*A^2*((6*b^2*d^2*x^2 + 2*b^2*c^2 + 5*a*b
*c*d - a^2*d^2 + 3*(3*b^2*c*d + a*b*d^2)*x)/((b^4*c^3*d^2 - 3*a*b^3*c^2*d^3
 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*g^2*i^3*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^
2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*g^2*i^3*x^2 + (b^4*c^5 - a*b
^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*g^2*i^3*x + (
a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3)*g^2*i^3) + 6*b
^2*d*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c
*d^3 + a^4*d^4)*g^2*i^3) - 6*b^2*d*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d +
 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3)) - 1/2*(4*b^3*c^3 -
15*a*b^2*c^2*d + 12*a^2*b*c*d^2 - a^3*d^3 - 6*(b^3*c*d^2 - a*b^2*d^3)*x^2 -

```
6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d
+ 2*a*b^2*c*d^2)*x)*log(b*x + a)^2 - 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*
c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(d*x + c)^2 - 3*
(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x - 6*(b^3*d^3*x^3 + a*b^2*c^2*d
+ (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(b*x +
a) + 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^
2*d + 2*a*b^2*c*d^2)*x + 2*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^
2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(b*x + a))*log(d*x + c))*A*B
/(a*b^4*c^6*g^2*i^3 - 4*a^2*b^3*c^5*d*g^2*i^3 + 6*a^3*b^2*c^4*d^2*g^2*i^3 -
 4*a^4*b*c^3*d^3*g^2*i^3 + a^5*c^2*d^4*g^2*i^3 + (b^5*c^4*d^2*g^2*i^3 - 4*a
*b^4*c^3*d^3*g^2*i^3 + 6*a^2*b^3*c^2*d^4*g^2*i^3 - 4*a^3*b^2*c*d^5*g^2*i^3
+ a^4*b*d^6*g^2*i^3)*x^3 + (2*b^5*c^5*d*g^2*i^3 - 7*a*b^4*c^4*d^2*g^2*i^3 +
 8*a^2*b^3*c^3*d^3*g^2*i^3 - 2*a^3*b^2*c^2*d^4*g^2*i^3 - 2*a^4*b*c*d^5*g^2*
i^3 + a^5*d^6*g^2*i^3)*x^2 + (b^5*c^6*g^2*i^3 - 2*a*b^4*c^5*d*g^2*i^3 - 2*a
^2*b^3*c^4*d^2*g^2*i^3 + 8*a^3*b^2*c^3*d^3*g^2*i^3 - 7*a^4*b*c^2*d^4*g^2*i^
3 + 2*a^5*c*d^5*g^2*i^3)*x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2(dix+ci)^3}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x, alg
orithm="giac")
```

```
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/((b*g*x + a*g)^2*(d*i*x + c*
i)^3), x)
```

Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 1505, normalized size of antiderivative = 2.87

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx=\text{Too large to display}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^2*(c*i + d*i*x)^3
),x)
```

```
[Out] ((4*A^2*b^2*c^2 - 2*A^2*a^2*d^2 - B^2*a^2*d^2 + 8*B^2*b^2*c^2 + 2*A*B*a^2*d
^2 + 8*A*B*b^2*c^2 + 10*A^2*a*b*c*d + 23*B^2*a*b*c*d - 22*A*B*a*b*c*d)/(2*(
a*d - b*c)) + (3*x^2*(2*A^2*b^2*d^2 + 5*B^2*b^2*d^2 - 2*A*B*b^2*d^2))/(a*d
- b*c) + (3*x*(2*A^2*a*b*d^2 + 7*B^2*a*b*d^2 + 6*A^2*b^2*c*d + 13*B^2*b^2*c
```

```
*d - 6*A*B*a*b*d^2 - 2*A*B*b^2*c*d))/(2*(a*d - b*c)))/(x*(2*b^3*c^4*g^2*i^3
 + 4*a^3*c*d^3*g^2*i^3 - 6*a^2*b*c^2*d^2*g^2*i^3) + x^2*(2*a^3*d^4*g^2*i^3
+ 4*b^3*c^3*d*g^2*i^3 - 6*a*b^2*c^2*d^2*g^2*i^3) + x^3*(2*b^3*c^2*d^2*g^2*i
^3 + 2*a^2*b*d^4*g^2*i^3 - 4*a*b^2*c*d^3*g^2*i^3) + 2*a^3*c^2*d^2*g^2*i^3 +
 2*a*b^2*c^4*g^2*i^3 - 4*a^2*b*c^3*d*g^2*i^3) - log((e*(a + b*x))/(c + d*x)
)^2*((x*((3*B^2)/(2*g^2*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (3*B^2*(a*d
+ b*c))/(g^2*i^3*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) + (B^2*(a*d
+ 2*b*c))/(2*g^2*i^3*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) - (3*B^2*a*c)
/(g^2*i^3*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (3*B^2*b*d*x^2)/(g
^2*i^3*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(d*x^3 + (a*c^2)/(b*d)
 + (x^2*(a*d^2 + 2*b*c*d))/(b*d) + (x*(b*c^2 + 2*a*c*d))/(b*d)) + (3*B*b^2*
d*(2*A - B))/(2*g^2*i^3*(a*d - b*c)^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - (
log((e*(a + b*x))/(c + d*x))*(x*((3*(B^2 + 2*A*B))/(2*g^2*i^3*(a^2*d^2 + b^
2*c^2 - 2*a*b*c*d)) - (3*B*(2*A - B)*(a*d + b*c))/(g^2*i^3*(a*d - b*c)*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d))) + (4*B^2*b*c - B^2*a*d + 2*A*B*a*d + 4*A*B*b*
c)/(2*g^2*i^3*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) - (3*B*a*c*(2*A - B)
)/(g^2*i^3*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (3*B*b*d*x^2*(2*A
 - B))/(g^2*i^3*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))))/(d*x^3 + (a*
c^2)/(b*d) + (x^2*(a*d^2 + 2*b*c*d))/(b*d) + (x*(b*c^2 + 2*a*c*d))/(b*d)) +
 (b^2*d*atan((b^2*d*(2*A^2 + 5*B^2 - 2*A*B)*(2*a^4*d^4*g^2*i^3 - 2*b^4*c^4*
g^2*i^3 + 4*a*b^3*c^3*d*g^2*i^3 - 4*a^3*b*c*d^3*g^2*i^3)*3i)/(2*g^2*i^3*(a*
d - b*c)^4*(6*A^2*b^2*d + 15*B^2*b^2*d - 6*A*B*b^2*d)) + (b^3*d^2*x*(2*A^2
+ 5*B^2 - 2*A*B)*(a^3*d^3*g^2*i^3 - b^3*c^3*g^2*i^3 + 3*a*b^2*c^2*d*g^2*i^3
 - 3*a^2*b*c*d^2*g^2*i^3)*6i)/(g^2*i^3*(a*d - b*c)^4*(6*A^2*b^2*d + 15*B^2*
b^2*d - 6*A*B*b^2*d)))*(2*A^2 + 5*B^2 - 2*A*B)*3i)/(g^2*i^3*(a*d - b*c)^4)
- (B^2*b^2*d*log((e*(a + b*x))/(c + d*x))^3)/(g^2*i^3*(a*d - b*c)^2*(a^2*d^
2 + b^2*c^2 - 2*a*b*c*d))
```

3.106 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 685

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \frac{B^2d^4(a+bx)^2}{4(bc-ad)^5g^3i^3(c+dx)^2}+\frac{8AbBd^3(a+bx)}{(bc-ad)^5g^3i^3(c+dx)} - \frac{8bB^2d^3(a+bx)}{(bc-ad)^5g^3i^3(c+dx)}+\frac{8b^3B^2d(c+dx)}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4B^2(c+dx)^2}{4(bc-ad)^5g^3i^3(a+bx)^2}+\frac{8bB^2d^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^5g^3i^3(c+dx)} - \frac{Bd^4(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^5g^3i^3(c+dx)^2} + \frac{8b^3Bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4B(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^5g^3i^3(a+bx)^2} + \frac{d^4(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^5g^3i^3(c+dx)^2} - \frac{4bd^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^3i^3(c+dx)} + \frac{4b^3d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^5g^3i^3(a+bx)^2} + \frac{2b^2d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{B(bc-ad)^5g^3i^3}$$

```
[Out] 1/4*B^2*d^4*(b*x+a)^2/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2+8*A*b*B*d^3*(b*x+a)/(-
a*d+b*c)^5/g^3/i^3/(d*x+c)-8*b*B^2*d^3*(b*x+a)/(-a*d+b*c)^5/g^3/i^3/(d*x+c)
+8*b^3*B^2*d*(d*x+c)/(-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/4*b^4*B^2*(d*x+c)^2/(-a
*d+b*c)^5/g^3/i^3/(b*x+a)^2+8*b*B^2*d^3*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(-a*d
+b*c)^5/g^3/i^3/(d*x+c)-1/2*B*d^4*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a
*d+b*c)^5/g^3/i^3/(d*x+c)^2+8*b^3*B*d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(
-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/2*b^4*B*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))
/(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2+1/2*d^4*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)
))^2/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2-4*b*d^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+
```

```
c)))^2/(-a*d+b*c)^5/g^3/i^3/(d*x+c)+4*b^3*d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+
c)))^2/(-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/2*b^4*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*
x+c)))^2/(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2+2*b^2*d^2*(A+B*ln(e*(b*x+a)/(d*x+c)
))^3/B/(-a*d+b*c)^5/g^3/i^3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 685, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2562, 2395, 2333, 2332, 2342, 2341, 2339, 30}

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = -\frac{b^4(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g^3i^3(a+bx)^2(bc-ad)^5} - \frac{b^4B(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^3i^3(a+bx)^2(bc-ad)^5} + \frac{4b^3d(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^3i^3(a+bx)(bc-ad)^5} + \frac{8b^3Bd(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^3i^3(a+bx)(bc-ad)^5} + \frac{2b^2d^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^3}{Bg^3i^3(bc-ad)^5} + \frac{d^4(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2g^3i^3(c+dx)^2(bc-ad)^5} - \frac{Bd^4(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2g^3i^3(c+dx)^2(bc-ad)^5} - \frac{4bd^3(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^3i^3(c+dx)(bc-ad)^5} + \frac{8AbBd^3(a+bx)}{g^3i^3(c+dx)(bc-ad)^5} - \frac{b^4B^2(c+dx)^2}{4g^3i^3(a+bx)^2(bc-ad)^5} + \frac{8b^3B^2d(c+dx)}{g^3i^3(a+bx)(bc-ad)^5} + \frac{B^2d^4(a+bx)^2}{4g^3i^3(c+dx)^2(bc-ad)^5} + \frac{8bB^2d^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{g^3i^3(c+dx)(bc-ad)^5} - \frac{8bB^2d^3(a+bx)}{g^3i^3(c+dx)(bc-ad)^5}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^3*(c*i + d*i*x)^3
),x]
```

```
[Out] (B^2*d^4*(a + b*x)^2)/(4*(b*c - a*d)^5*g^3*i^3*(c + d*x)^2) + (8*A*b*B*d^3*
(a + b*x))/((b*c - a*d)^5*g^3*i^3*(c + d*x)) - (8*b*B^2*d^3*(a + b*x))/((b*
c - a*d)^5*g^3*i^3*(c + d*x)) + (8*b^3*B^2*d*(c + d*x))/((b*c - a*d)^5*g^3*
i^3*(a + b*x)) - (b^4*B^2*(c + d*x)^2)/(4*(b*c - a*d)^5*g^3*i^3*(a + b*x)^2
) + (8*b*B^2*d^3*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/((b*c - a*d)^5*g^3
*i^3*(c + d*x)) - (B*d^4*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/
(2*(b*c - a*d)^5*g^3*i^3*(c + d*x)^2) + (8*b^3*B*d*(c + d*x)*(A + B*Log[(e*
(a + b*x))/(c + d*x)]))/((b*c - a*d)^5*g^3*i^3*(a + b*x)) - (b^4*B*(c + d*x
)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*c - a*d)^5*g^3*i^3*(a + b*x
)^2) + (d^4*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(b*c - a
*d)^5*g^3*i^3*(c + d*x)^2) - (4*b*d^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c
 + d*x)])^2)/((b*c - a*d)^5*g^3*i^3*(c + d*x)) + (4*b^3*d*(c + d*x)*(A + B*
Log[(e*(a + b*x))/(c + d*x)])^2)/((b*c - a*d)^5*g^3*i^3*(a + b*x)) - (b^4*(
c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(b*c - a*d)^5*g^3*i^3
*(a + b*x)^2) + (2*b^2*d^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^3)/(B*(b*c
- a*d)^5*g^3*i^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)^4(A+B\log(ex))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}$$

$$= \frac{\text{Subst}\left(\int \left(-4bd^3(A+B\log(ex))^2 + \frac{b^4(A+B\log(ex))^2}{x^3} - \frac{4b^3d(A+B\log(ex))^2}{x^2} + \frac{6b^2d^2(A+B\log(ex))^2}{x} + d^4x(A$$

$$(bc-ad)^5g^3i^3$$

$$= \frac{b^4\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3} - \frac{(4b^3d)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}$$

$$+ \frac{(6b^2d^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}$$

$$- \frac{(4bd^3)\,\text{Subst}\left(\int (A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}$$

$$+ \frac{d^4\text{Subst}\left(\int x(A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}$$

$$= \frac{d^4(a+bx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^5 g^3 i^3 (c+dx)^2} - \frac{4bd^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5 g^3 i^3 (c+dx)} + \frac{4b^3 d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5 g^3 i^3 (a+bx)} - \frac{b^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^5 g^3 i^3 (a+bx)^2} + \frac{(b^4 B)\,\mathrm{Subst}\left(\int \frac{A+B\log(ex)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5 g^3 i^3} - \frac{(8b^3 Bd)\,\mathrm{Subst}\left(\int \frac{A+B\log(ex)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5 g^3 i^3} + \frac{(6b^2 d^2)\,\mathrm{Subst}\left(\int x^2\,dx, x, A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B(bc-ad)^5 g^3 i^3} + \frac{(8bBd^3)\,\mathrm{Subst}\left(\int (A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5 g^3 i^3} - \frac{(Bd^4)\,\mathrm{Subst}\left(\int x(A+B\log(ex))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5 g^3 i^3}$$

$$= \frac{B^2 d^4 (a+bx)^2}{4(bc-ad)^5 g^3 i^3 (c+dx)^2} + \frac{8AbBd^3(a+bx)}{(bc-ad)^5 g^3 i^3 (c+dx)} + \frac{8b^3 B^2 d(c+dx)}{(bc-ad)^5 g^3 i^3 (a+bx)} - \frac{b^4 B^2 (c+dx)^2}{4(bc-ad)^5 g^3 i^3 (a+bx)^2} - \frac{Bd^4(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^5 g^3 i^3 (c+dx)^2} + \frac{8b^3 Bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5 g^3 i^3 (a+bx)} - \frac{b^4 B(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^5 g^3 i^3 (a+bx)^2} + \frac{d^4(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^5 g^3 i^3 (c+dx)^2} - \frac{4bd^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5 g^3 i^3 (c+dx)} + \frac{4b^3 d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5 g^3 i^3 (a+bx)} - \frac{b^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^5 g^3 i^3 (a+bx)^2} + \frac{2b^2 d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{B(bc-ad)^5 g^3 i^3} + \frac{(8bB^2 d^3)\,\mathrm{Subst}\left(\int \log(ex)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5 g^3 i^3}$$

$$= \frac{B^2d^4(a+bx)^2}{4(bc-ad)^5g^3i^3(c+dx)^2} + \frac{8AbBd^3(a+bx)}{(bc-ad)^5g^3i^3(c+dx)} - \frac{8bB^2d^3(a+bx)}{(bc-ad)^5g^3i^3(c+dx)} + \frac{8b^3B^2d(c+dx)}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4B^2(c+dx)^2}{4(bc-ad)^5g^3i^3(a+bx)^2} + \frac{8bB^2d^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^5g^3i^3(c+dx)} - \frac{Bd^4(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^5g^3i^3(c+dx)^2} + \frac{8b^3Bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4B(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^5g^3i^3(a+bx)^2} + \frac{d^4(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^5g^3i^3(c+dx)^2} - \frac{4bd^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^3i^3(c+dx)} + \frac{4b^3d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^5g^3i^3(a+bx)^2} + \frac{2b^2d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{B(bc-ad)^5g^3i^3}$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 611, normalized size of antiderivative = 0.89

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \frac{(2A^2-2AB+B^2)\,d^2(bc-ad)^2(a+bx)^2 + 2b(6A^2-14AB+15B^2)\,d^2(bc-ad)(a+bx)^2(c+dx) - b^2(2$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^3*(c*i + d*
i*x)^3),x]
```

```
[Out] ((2*A^2 - 2*A*B + B^2)*d^2*(b*c - a*d)^2*(a + b*x)^2 + 2*b*(6*A^2 - 14*A*B
+ 15*B^2)*d^2*(b*c - a*d)*(a + b*x)^2*(c + d*x) - b^2*(2*A^2 + 2*A*B + B^2)
*(b*c - a*d)^2*(c + d*x)^2 + 2*b^2*(6*A^2 + 14*A*B + 15*B^2)*d*(b*c - a*d)*
(a + b*x)*(c + d*x)^2 + 12*b^2*(2*A^2 + 5*B^2)*d^2*(a + b*x)^2*(c + d*x)^2*
Log[a + b*x] + 2*B*(b*c - a*d)*((2*A - B)*d^2*(b*c - a*d)*(a + b*x)^2 + 2*b
*(6*A - 7*B)*d^2*(a + b*x)^2*(c + d*x) - b^2*(2*A + B)*(b*c - a*d)*(c + d*x
)^2 + 2*b^2*(6*A + 7*B)*d*(a + b*x)*(c + d*x)^2)*Log[(e*(a + b*x))/(c + d*x
)] - 2*B*(-(a^4*B*d^4) + 4*a^3*b*B*d^3*(2*c + d*x) - 6*a^2*b^2*d^2*(2*A*(c
+ d*x)^2 - B*d*x*(4*c + 3*d*x)) + b^4*(-12*A*d^2*x^2*(c + d*x)^2 + B*c*(c^3
 - 4*c^2*d*x - 18*c*d^2*x^2 - 12*d^3*x^3)) - 4*a*b^3*d*(6*A*d*x*(c + d*x)^2
 + B*(2*c^3 + 6*c^2*d*x - 3*d^3*x^3)))*Log[(e*(a + b*x))/(c + d*x)]^2 + 8*b
^2*B^2*d^2*(a + b*x)^2*(c + d*x)^2*Log[(e*(a + b*x))/(c + d*x)]^3 - 12*b^2*
```

```
(2*A^2 + 5*B^2)*d^2*(a + b*x)^2*(c + d*x)^2*Log[c + d*x])/(4*(b*c - a*d)^5*
g^3*i^3*(a + b*x)^2*(c + d*x)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1395 vs. 2(673) = 1346.

Time = 2.34 (sec) , antiderivative size = 1396, normalized size of antiderivative = 2.04

method	result	size
parts	Expression too large to display	1396
derivativedivides	Expression too large to display	1589
default	Expression too large to display	1589
risch	Expression too large to display	2582
parallelrisch	Expression too large to display	2645
norman	Expression too large to display	2758

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x,method=_RET
URNVERBOSE)
```

```
[Out] A^2/g^3/i^3*(-1/2*d^2/(a*d-b*c)^3/(d*x+c)^2+6*d^2/(a*d-b*c)^5*b^2*ln(d*x+c)
+3*d^2/(a*d-b*c)^4*b/(d*x+c)+1/2*b^2/(a*d-b*c)^3/(b*x+a)^2-6*d^2/(a*d-b*c)^
5*b^2*ln(b*x+a)+3*b^2/(a*d-b*c)^4*d/(b*x+a))-B^2/g^3/i^3*d/(a*d-b*c)^2/e^2*
(d^3/(a*d-b*c)^3*(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^2-1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*
x+c))+1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-4/(a*d-b*c)^3*b*d^2*e*((b*e/d+(a
*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2*(b*e/d+(a*d-b*c)*e
/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+2*(a*d-b*c)*e/d/(d*x+c)+2*b*e/d
)+2*d/(a*d-b*c)^3*b^2*e^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3-4/(a*d-b*c)^3*b
^3*e^3*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-
2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a
*d-b*c)*e/d/(d*x+c)))+1/d/(a*d-b*c)^3*b^4*e^4*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d
*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))-2
*B*A/g^3/i^3*d/(a*d-b*c)^2/e^2*(d^3/(a*d-b*c)^3*(1/2*(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))
^2)-4*d^2/(a*d-b*c)^3*b*e*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)
*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)+3*d/(a*d-b*c)^3*b^2*e^2*ln(b*e/d
+(a*d-b*c)*e/d/(d*x+c))^2-4/(a*d-b*c)^3*b^3*e^3*(-1/(b*e/d+(a*d-b*c)*e/d/(d
*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+1/d
/(a*d-b*c)^3*b^4*e^4*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*
c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1517 vs. 2(673) = 1346.

Time = 0.41 (sec) , antiderivative size = 1517, normalized size of antiderivative = 2.21

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x, alg
orithm="fricas")
```

```
[Out] -1/4*(60*A*B*a^2*b^2*c^2*d^2 + (2*A^2 + 2*A*B + B^2)*b^4*c^4 - 16*(A^2 + 2*
A*B + 2*B^2)*a*b^3*c^3*d + 16*(A^2 - 2*A*B + 2*B^2)*a^3*b*c*d^3 - (2*A^2 -
2*A*B + B^2)*a^4*d^4 - 12*((2*A^2 + 5*B^2)*b^4*c*d^3 - (2*A^2 + 5*B^2)*a*b^
3*d^4)*x^3 - 8*(B^2*b^4*d^4*x^4 + B^2*a^2*b^2*c^2*d^2 + 2*(B^2*b^4*c*d^3 +
B^2*a*b^3*d^4)*x^3 + (B^2*b^4*c^2*d^2 + 4*B^2*a*b^3*c*d^3 + B^2*a^2*b^2*d^4
)*x^2 + 2*(B^2*a*b^3*c^2*d^2 + B^2*a^2*b^2*c*d^3)*x)*log((b*e*x + a*e)/(d*x
 + c))^3 + 6*(8*A*B*a*b^3*c*d^3 - (6*A^2 + 4*A*B + 15*B^2)*b^4*c^2*d^2 + (6
*A^2 - 4*A*B + 15*B^2)*a^2*b^2*d^4)*x^2 - 2*(12*A*B*b^4*d^4*x^4 - B^2*b^4*c
^4 + 8*B^2*a*b^3*c^3*d + 12*A*B*a^2*b^2*c^2*d^2 - 8*B^2*a^3*b*c*d^3 + B^2*a
^4*d^4 + 12*((2*A*B + B^2)*b^4*c*d^3 + (2*A*B - B^2)*a*b^3*d^4)*x^3 + 6*(8*
A*B*a*b^3*c*d^3 + (2*A*B + 3*B^2)*b^4*c^2*d^2 + (2*A*B - 3*B^2)*a^2*b^2*d^4
)*x^2 + 4*(B^2*b^4*c^3*d - B^2*a^3*b*d^4 + 6*(A*B + B^2)*a*b^3*c^2*d^2 + 6*
(A*B - B^2)*a^2*b^2*c*d^3)*x)*log((b*e*x + a*e)/(d*x + c))^2 - 4*((2*A^2 +
6*A*B + 7*B^2)*b^4*c^3*d + 6*(2*A^2 - A*B + 4*B^2)*a*b^3*c^2*d^2 - 6*(2*A^2
 + A*B + 4*B^2)*a^2*b^2*c*d^3 - (2*A^2 - 6*A*B + 7*B^2)*a^3*b*d^4)*x - 2*(6
*(2*A^2 + 5*B^2)*b^4*d^4*x^4 + 12*A^2*a^2*b^2*c^2*d^2 - (2*A*B + B^2)*b^4*c
^4 + 16*(A*B + B^2)*a*b^3*c^3*d - 16*(A*B - B^2)*a^3*b*c*d^3 + (2*A*B - B^2
)*a^4*d^4 + 12*((2*A^2 + 2*A*B + 5*B^2)*b^4*c*d^3 + (2*A^2 - 2*A*B + 5*B^2)
*a*b^3*d^4)*x^3 + 6*((2*A^2 + 6*A*B + 7*B^2)*b^4*c^2*d^2 + 8*(A^2 + 2*B^2)*
a*b^3*c*d^3 + (2*A^2 - 6*A*B + 7*B^2)*a^2*b^2*d^4)*x^2 + 4*((2*A*B + 3*B^2)
*b^4*c^3*d + 6*(A^2 + 2*A*B + 2*B^2)*a*b^3*c^2*d^2 + 6*(A^2 - 2*A*B + 2*B^2
)*a^2*b^2*c*d^3 - (2*A*B - 3*B^2)*a^3*b*d^4)*x)*log((b*e*x + a*e)/(d*x + c)
))/((b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^2*d^
5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^7)*g^3*i^3*x^4 + 2*(b^7*c^6*d - 4*a*b^6*c^5
*d^2 + 5*a^2*b^5*c^4*d^3 - 5*a^4*b^3*c^2*d^5 + 4*a^5*b^2*c*d^6 - a^6*b*d^7)
*g^3*i^3*x^3 + (b^7*c^7 - a*b^6*c^6*d - 9*a^2*b^5*c^5*d^2 + 25*a^3*b^4*c^4*
d^3 - 25*a^4*b^3*c^3*d^4 + 9*a^5*b^2*c^2*d^5 + a^6*b*c*d^6 - a^7*d^7)*g^3*i
^3*x^2 + 2*(a*b^6*c^7 - 4*a^2*b^5*c^6*d + 5*a^3*b^4*c^5*d^2 - 5*a^5*b^2*c^3
*d^4 + 4*a^6*b*c^2*d^5 - a^7*c*d^6)*g^3*i^3*x + (a^2*b^5*c^7 - 5*a^3*b^4*c^
6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d
^5)*g^3*i^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3/(d*i*x+c*i)**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5583 vs. 2(673) = 1346.

Time = 0.55 (sec) , antiderivative size = 5583, normalized size of antiderivative = 8.15

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x, alg
orithm="maxima")
[Out] 1/2*B^2*((12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d^
3 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 + a^2*b*d
^3)*x)/((b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^
5 + a^4*b^2*d^6)*g^3*i^3*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4*c
^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*g^3*i^3*x^3 + (b^
6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d^
6)*g^3*i^3*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4
*b^2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*g^3*i^3*x + (a^2*b^4*c^6 - 4*a^
3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4)*g^3*i^3) +
 12*b^2*d^2*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 1
0*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3) - 12*b^2*d^2*log(d*x
+ c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 +
5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2 +
 A*B*((12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d^3 +
 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 + a^2*b*d^3)
*x)/((b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5 +
 a^4*b^2*d^6)*g^3*i^3*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4*c^3*
d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*g^3*i^3*x^3 + (b^6*c
^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d^6)*
g^3*i^3*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b^
2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*g^3*i^3*x + (a^2*b^4*c^6 - 4*a^3*b
^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4)*g^3*i^3) + 12
```

```
*b^2*d^2*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a
^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3) - 12*b^2*d^2*log(d*x + c
)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a
^4*b*c*d^4 - a^5*d^5)*g^3*i^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 1/4*
B^2*(2*(b^4*c^4 - 16*a*b^3*c^3*d + 30*a^2*b^2*c^2*d^2 - 16*a^3*b*c*d^3 + a^
4*d^4 - 12*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 12*(b^4*d^4*x^
4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^
3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x +
 a)^2 - 24*(b^4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 +
 (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b
^2*c*d^3)*x)*log(b*x + a)*log(d*x + c) + 12*(b^4*d^4*x^4 + a^2*b^2*c^2*d^2
+ 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^
4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(d*x + c)^2 - 12*(b^4*c^3*
d - a*b^3*c^2*d^2 - a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*e*x/(d*x + c) + a*e
/(d*x + c))/(a^2*b^5*c^7*g^3*i^3 - 5*a^3*b^4*c^6*d*g^3*i^3 + 10*a^4*b^3*c^5
*d^2*g^3*i^3 - 10*a^5*b^2*c^4*d^3*g^3*i^3 + 5*a^6*b*c^3*d^4*g^3*i^3 - a^7*c
^2*d^5*g^3*i^3 + (b^7*c^5*d^2*g^3*i^3 - 5*a*b^6*c^4*d^3*g^3*i^3 + 10*a^2*b^
5*c^3*d^4*g^3*i^3 - 10*a^3*b^4*c^2*d^5*g^3*i^3 + 5*a^4*b^3*c*d^6*g^3*i^3 -
a^5*b^2*d^7*g^3*i^3)*x^4 + 2*(b^7*c^6*d*g^3*i^3 - 4*a*b^6*c^5*d^2*g^3*i^3 +
 5*a^2*b^5*c^4*d^3*g^3*i^3 - 5*a^4*b^3*c^2*d^5*g^3*i^3 + 4*a^5*b^2*c*d^6*g^
3*i^3 - a^6*b*d^7*g^3*i^3)*x^3 + (b^7*c^7*g^3*i^3 - a*b^6*c^6*d*g^3*i^3 - 9
*a^2*b^5*c^5*d^2*g^3*i^3 + 25*a^3*b^4*c^4*d^3*g^3*i^3 - 25*a^4*b^3*c^3*d^4*
g^3*i^3 + 9*a^5*b^2*c^2*d^5*g^3*i^3 + a^6*b*c*d^6*g^3*i^3 - a^7*d^7*g^3*i^3
)*x^2 + 2*(a*b^6*c^7*g^3*i^3 - 4*a^2*b^5*c^6*d*g^3*i^3 + 5*a^3*b^4*c^5*d^2*
g^3*i^3 - 5*a^5*b^2*c^3*d^4*g^3*i^3 + 4*a^6*b*c^2*d^5*g^3*i^3 - a^7*c*d^6*g
^3*i^3)*x) + (b^4*c^4 - 32*a*b^3*c^3*d + 32*a^3*b*c*d^3 - a^4*d^4 - 60*(b^4
*c*d^3 - a*b^3*d^4)*x^3 - 8*(b^4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 +
 a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^
3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a)^3 - 24*(b^4*d^4*x^4 + a^2*b^2*c^
2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*
b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a)*log(d*x +
c)^2 + 8*(b^4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (
b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2
*c*d^3)*x)*log(d*x + c)^3 - 90*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^2 - 4*(7*b^4*c
^3*d + 24*a*b^3*c^2*d^2 - 24*a^2*b^2*c*d^3 - 7*a^3*b*d^4)*x - 60*(b^4*d^4*x
^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b
^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x
+ a) + 12*(5*b^4*d^4*x^4 + 5*a^2*b^2*c^2*d^2 + 10*(b^4*c*d^3 + a*b^3*d^4)*x
^3 + 5*(b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(b^4*d^4*x^4 + a
^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d
^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a)^2
 + 10*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(d*x + c))/(a^2*b^5*c^7*g^3*i^3
 - 5*a^3*b^4*c^6*d*g^3*i^3 + 10*a^4*b^3*c^5*d^2*g^3*i^3 - 10*a^5*b^2*c^4*d^
3*g^3*i^3 + 5*a^6*b*c^3*d^4*g^3*i^3 - a^7*c^2*d^5*g^3*i^3 + (b^7*c^5*d^2*g^
3*i^3 - 5*a*b^6*c^4*d^3*g^3*i^3 + 10*a^2*b^5*c^3*d^4*g^3*i^3 - 10*a^3*b^4*c
```

^2*d^5*g^3*i^3 + 5*a^4*b^3*c*d^6*g^3*i^3 - a^5*b^2*d^7*g^3*i^3)*x^4 + 2*(b^7*c^6*d*g^3*i^3 - 4*a*b^6*c^5*d^2*g^3*i^3 + 5*a^2*b^5*c^4*d^3*g^3*i^3 - 5*a^4*b^3*c^2*d^5*g^3*i^3 + 4*a^5*b^2*c*d^6*g^3*i^3 - a^6*b*d^7*g^3*i^3)*x^3 + (b^7*c^7*g^3*i^3 - a*b^6*c^6*d*g^3*i^3 - 9*a^2*b^5*c^5*d^2*g^3*i^3 + 25*a^3*b^4*c^4*d^3*g^3*i^3 - 25*a^4*b^3*c^3*d^4*g^3*i^3 + 9*a^5*b^2*c^2*d^5*g^3*i^3 + a^6*b*c*d^6*g^3*i^3 - a^7*d^7*g^3*i^3)*x^2 + 2*(a*b^6*c^7*g^3*i^3 - 4*a^2*b^5*c^6*d*g^3*i^3 + 5*a^3*b^4*c^5*d^2*g^3*i^3 - 5*a^5*b^2*c^3*d^4*g^3*i^3 + 4*a^6*b*c^2*d^5*g^3*i^3 - a^7*c*d^6*g^3*i^3)*x)) + 1/2*A^2*((12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d^3 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 + a^2*b*d^3)*x)/((b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5 + a^4*b^2*d^6)*g^3*i^3*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*g^3*i^3*x^3 + (b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d^6)*g^3*i^3*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b^2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*g^3*i^3*x + (a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4)*g^3*i^3) + 12*b^2*d^2*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3) - 12*b^2*d^2*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3)) - 1/2*(b^4*c^4 - 16*a*b^3*c^3*d + 30*a^2*b^2*c^2*d^2 - 16*a^3*b*c*d^3 + a^4*d^4 - 12*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 12*(b^4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a)^2 - 24*(b^4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a)*log(d*x + c) + 12*(b^4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(d*x + c)^2 - 12*(b^4*c^3*d - a*b^3*c^2*d^2 - a^2*b^2*c*d^3 + a^3*b*d^4)*x)*A*B/(a^2*b^5*c^7*g^3*i^3 - 5*a^3*b^4*c^6*d*g^3*i^3 + 10*a^4*b^3*c^5*d^2*g^3*i^3 - 10*a^5*b^2*c^4*d^3*g^3*i^3 + 5*a^6*b*c^3*d^4*g^3*i^3 - a^7*c^2*d^5*g^3*i^3 + (b^7*c^5*d^2*g^3*i^3 - 5*a*b^6*c^4*d^3*g^3*i^3 + 10*a^2*b^5*c^3*d^4*g^3*i^3 - 10*a^3*b^4*c^2*d^5*g^3*i^3 + 5*a^4*b^3*c*d^6*g^3*i^3 - a^5*b^2*d^7*g^3*i^3)*x^4 + 2*(b^7*c^6*d*g^3*i^3 - 4*a*b^6*c^5*d^2*g^3*i^3 + 5*a^2*b^5*c^4*d^3*g^3*i^3 - 5*a^4*b^3*c^2*d^5*g^3*i^3 + 4*a^5*b^2*c*d^6*g^3*i^3 - a^6*b*d^7*g^3*i^3)*x^3 + (b^7*c^7*g^3*i^3 - a*b^6*c^6*d*g^3*i^3 - 9*a^2*b^5*c^5*d^2*g^3*i^3 + 25*a^3*b^4*c^4*d^3*g^3*i^3 - 25*a^4*b^3*c^3*d^4*g^3*i^3 + 9*a^5*b^2*c^2*d^5*g^3*i^3 + a^6*b*c*d^6*g^3*i^3 - a^7*d^7*g^3*i^3)*x^2 + 2*(a*b^6*c^7*g^3*i^3 - 4*a^2*b^5*c^6*d*g^3*i^3 + 5*a^3*b^4*c^5*d^2*g^3*i^3 - 5*a^5*b^2*c^3*d^4*g^3*i^3 + 4*a^6*b*c^2*d^5*g^3*i^3 - a^7*c*d^6*g^3*i^3)*x)

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^3(dix+ci)^3}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x, alg
orithm="giac")
```

```
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/((b*g*x + a*g)^3*(d*i*x + c*
i)^3), x)
```

Mupad [B] (verification not implemented)

Time = 11.14 (sec) , antiderivative size = 2155, normalized size of antiderivative = 3.15

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx=\text{Too large to display}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^3*(c*i + d*i*x)^3
),x)
```

```
[Out] ((2*x*(2*A^2*a^2*b*d^3 + 7*B^2*a^2*b*d^3 + 2*A^2*b^3*c^2*d + 7*B^2*b^3*c^2*
d + 14*A^2*a*b^2*c*d^2 + 31*B^2*a*b^2*c*d^2 - 6*A*B*a^2*b*d^3 + 6*A*B*b^3*c
^2*d))/(a*d - b*c) - (2*A^2*a^3*d^3 + 2*A^2*b^3*c^3 + B^2*a^3*d^3 + B^2*b^3
*c^3 - 2*A*B*a^3*d^3 + 2*A*B*b^3*c^3 - 14*A^2*a*b^2*c^2*d - 14*A^2*a^2*b*c*
d^2 - 31*B^2*a*b^2*c^2*d - 31*B^2*a^2*b*c*d^2 - 30*A*B*a*b^2*c^2*d + 30*A*B
*a^2*b*c*d^2)/(2*(a*d - b*c)) + (6*x^3*(2*A^2*b^3*d^3 + 5*B^2*b^3*d^3))/(a*
d - b*c) + (3*x^2*(6*A^2*a*b^2*d^3 + 15*B^2*a*b^2*d^3 + 6*A^2*b^3*c*d^2 + 1
5*B^2*b^3*c*d^2 - 4*A*B*a*b^2*d^3 + 4*A*B*b^3*c*d^2))/(a*d - b*c))/(x^4*(2*
a^3*b^2*d^5*g^3*i^3 - 2*b^5*c^3*d^2*g^3*i^3 + 6*a*b^4*c^2*d^3*g^3*i^3 - 6*a
^2*b^3*c*d^4*g^3*i^3) - x*(4*a*b^4*c^5*g^3*i^3 - 4*a^5*c*d^4*g^3*i^3 - 8*a^
2*b^3*c^4*d*g^3*i^3 + 8*a^4*b*c^2*d^3*g^3*i^3) + x^3*(4*a^4*b*d^5*g^3*i^3 -
 4*b^5*c^4*d*g^3*i^3 + 8*a*b^4*c^3*d^2*g^3*i^3 - 8*a^3*b^2*c*d^4*g^3*i^3) +
 x^2*(2*a^5*d^5*g^3*i^3 - 2*b^5*c^5*g^3*i^3 - 2*a*b^4*c^4*d*g^3*i^3 + 2*a^4
*b*c*d^4*g^3*i^3 + 16*a^2*b^3*c^3*d^2*g^3*i^3 - 16*a^3*b^2*c^2*d^3*g^3*i^3)
 - 2*a^2*b^3*c^5*g^3*i^3 + 2*a^5*c^2*d^3*g^3*i^3 + 6*a^3*b^2*c^4*d*g^3*i^3
- 6*a^4*b*c^3*d^2*g^3*i^3) + log((e*(a + b*x))/(c + d*x))^2*((x*((3*B^2*(a*
d + b*c)^2)/(g^3*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) - B^2/(g^3*i^3*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d)) + (6*B^2*a*b*c*d)/(g^3*i^3*(a^2*d^2 + b^2*c^2
- 2*a*b*c*d)^2)) - (B^2*(a*d + b*c))/(2*g^3*i^3*(a^2*b*d^3 + b^3*c^2*d - 2*
a*b^2*c*d^2)) + (6*B^2*b^2*d^2*x^3)/(g^3*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d
)^2) + (3*B^2*a*c*(a*d + b*c))/(g^3*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2)
```

```
+ (9*B^2*b*d*x^2*(a*d + b*c))/(g^3*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2))/
(b*d*x^4 + (a^2*c^2)/(b*d) + (x*(2*a*b*c^2 + 2*a^2*c*d))/(b*d) + (x^2*(a^2*
d^2 + b^2*c^2 + 4*a*b*c*d))/(b*d) + (x^3*(2*a*b*d^2 + 2*b^2*c*d))/(b*d)) -
(6*A*B*b^2*d^2)/(g^3*i^3*(a*d - b*c)^5)) + (log((e*(a + b*x))/(c + d*x))*(x
^2*((6*b*d*(B^2*b*c - B^2*a*d + A*B*a*d + A*B*b*c))/(g^3*i^3*(a^2*d^2 + b^2
*c^2 - 2*a*b*c*d)^2) + (12*A*B*b*d*(a*d + b*c))/(g^3*i^3*(a^2*d^2 + b^2*c^2
 - 2*a*b*c*d)^2)) + x*((6*(a*d + b*c)*(B^2*b*c - B^2*a*d + A*B*a*d + A*B*b*
c))/(g^3*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) - (2*A*B)/(g^3*i^3*(a^2*d^2
 + b^2*c^2 - 2*a*b*c*d)) + (12*A*B*a*b*c*d)/(g^3*i^3*(a^2*d^2 + b^2*c^2 - 2
*a*b*c*d)^2)) - (B^2*b*c - B^2*a*d + 2*A*B*a*d + 2*A*B*b*c)/(2*g^3*i^3*(a^2
*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (6*a*c*(B^2*b*c - B^2*a*d + A*B*a*d
+ A*B*b*c))/(g^3*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) + (12*A*B*b^2*d^2*x
^3)/(g^3*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2)))/(b*d*x^4 + (a^2*c^2)/(b*d
) + (x*(2*a*b*c^2 + 2*a^2*c*d))/(b*d) + (x^2*(a^2*d^2 + b^2*c^2 + 4*a*b*c*d
))/(b*d) + (x^3*(2*a*b*d^2 + 2*b^2*c*d))/(b*d)) - (2*B^2*b^2*d^2*log((e*(a
+ b*x))/(c + d*x))^3)/(g^3*i^3*(a*d - b*c)^5) + (b^2*d^2*atan((b^2*d^2*((a^
5*d^5*g^3*i^3 + b^5*c^5*g^3*i^3 - 3*a*b^4*c^4*d*g^3*i^3 - 3*a^4*b*c*d^4*g^3
*i^3 + 2*a^2*b^3*c^3*d^2*g^3*i^3 + 2*a^3*b^2*c^2*d^3*g^3*i^3)/(a^4*d^4*g^3*
i^3 + b^4*c^4*g^3*i^3 - 4*a*b^3*c^3*d*g^3*i^3 - 4*a^3*b*c*d^3*g^3*i^3 + 6*a
^2*b^2*c^2*d^2*g^3*i^3) + 2*b*d*x)*(2*A^2 + 5*B^2)*(a^4*d^4*g^3*i^3 + b^4*c
^4*g^3*i^3 - 4*a*b^3*c^3*d*g^3*i^3 - 4*a^3*b*c*d^3*g^3*i^3 + 6*a^2*b^2*c^2*
d^2*g^3*i^3)*3i)/(g^3*i^3*(6*A^2*b^2*d^2 + 15*B^2*b^2*d^2)*(a*d - b*c)^5))*
(2*A^2 + 5*B^2)*6i)/(g^3*i^3*(a*d - b*c)^5)
```

3.107 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 851

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = -\frac{B^2d^5(a+bx)^2}{4(bc-ad)^6g^4i^3(c+dx)^2} - \frac{10AbBd^4(a+bx)}{(bc-ad)^6g^4i^3(c+dx)} + \frac{10bB^2d^4(a+bx)}{(bc-ad)^6g^4i^3(c+dx)} - \frac{20b^3B^2d^2(c+dx)}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4B^2d(c+dx)^2}{4(bc-ad)^6g^4i^3(a+bx)^2} - \frac{2b^5B^2(c+dx)^3}{27(bc-ad)^6g^4i^3(a+bx)^3} - \frac{10bB^2d^4(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^6g^4i^3(c+dx)} + \frac{Bd^5(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2} - \frac{20b^3Bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4Bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2} - \frac{2b^5B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^6g^4i^3(a+bx)^3} - \frac{d^5(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^6g^4i^3(c+dx)^2} + \frac{5bd^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^6g^4i^3(c+dx)} - \frac{10b^3d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^6g^4i^3(a+bx)^2} - \frac{b^5(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^6g^4i^3(a+bx)^3} - \frac{10b^2d^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^6g^4i^3}$$

```
[Out] -1/4*B^2*d^5*(b*x+a)^2/(-a*d+b*c)^6/g^4/i^3/(d*x+c)^2-10*A*b*B*d^4*(b*x+a)/
(-a*d+b*c)^6/g^4/i^3/(d*x+c)+10*b*B^2*d^4*(b*x+a)/(-a*d+b*c)^6/g^4/i^3/(d*x
```

```
+c)-20*b^3*B^2*d^2*(d*x+c)/(-a*d+b*c)^6/g^4/i^3/(b*x+a)+5/4*b^4*B^2*d*(d*x+
c)^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-2/27*b^5*B^2*(d*x+c)^3/(-a*d+b*c)^6/g^4
/i^3/(b*x+a)^3-10*b*B^2*d^4*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(-a*d+b*c)^6/g^4/
i^3/(d*x+c)+1/2*B*d^5*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^6/g^
4/i^3/(d*x+c)^2-20*b^3*B*d^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)
^6/g^4/i^3/(b*x+a)+5/2*b^4*B*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+
b*c)^6/g^4/i^3/(b*x+a)^2-2/9*b^5*B*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-
a*d+b*c)^6/g^4/i^3/(b*x+a)^3-1/2*d^5*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^
2/(-a*d+b*c)^6/g^4/i^3/(d*x+c)^2+5*b*d^4*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c))
)^2/(-a*d+b*c)^6/g^4/i^3/(d*x+c)-10*b^3*d^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+
c)))^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)+5/2*b^4*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(
d*x+c)))^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-1/3*b^5*(d*x+c)^3*(A+B*ln(e*(b*x+
a)/(d*x+c)))^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^3-10/3*b^2*d^3*(A+B*ln(e*(b*x+a
)/(d*x+c)))^3/B/(-a*d+b*c)^6/g^4/i^3
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 851, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used

$= \{2562, 2395, 2333, 2332, 2342, 2341, 2339, 30\}$

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = -\frac{2B^2(c+dx)^3b^5}{27(bc-ad)^6g^4i^3(a+bx)^3} - \frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^5}{3(bc-ad)^6g^4i^3(a+bx)^3} - \frac{2B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^5}{9(bc-ad)^6g^4i^3(a+bx)^3} + \frac{5B^2d(c+dx)^2b^4}{4(bc-ad)^6g^4i^3(a+bx)^2} + \frac{5d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^4}{2(bc-ad)^6g^4i^3(a+bx)^2} + \frac{5Bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^4}{2(bc-ad)^6g^4i^3(a+bx)^2} - \frac{10d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^3}{(bc-ad)^6g^4i^3(a+bx)} - \frac{20B^2d^2(c+dx)b^3}{(bc-ad)^6g^4i^3(a+bx)} - \frac{20Bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{(bc-ad)^6g^4i^3(a+bx)} - \frac{10d^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3b^2}{3B(bc-ad)^6g^4i^3} + \frac{5d^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b}{(bc-ad)^6g^4i^3(c+dx)} - \frac{10B^2d^4(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)b}{(bc-ad)^6g^4i^3(c+dx)} + \frac{10B^2d^4(a+bx)b}{(bc-ad)^6g^4i^3(c+dx)} - \frac{10ABd^4(a+bx)b}{(bc-ad)^6g^4i^3(c+dx)} - \frac{d^5(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^6g^4i^3(c+dx)^2} + \frac{Bd^5(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2} - \frac{B^2d^5(a+bx)^2}{4(bc-ad)^6g^4i^3(c+dx)^2}$$

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^4*(c*i + d*i*x)^3
),x]
```

```
[Out] -1/4*(B^2*d^5*(a + b*x)^2)/((b*c - a*d)^6*g^4*i^3*(c + d*x)^2) - (10*A*b*B*
d^4*(a + b*x))/((b*c - a*d)^6*g^4*i^3*(c + d*x)) + (10*b*B^2*d^4*(a + b*x))
/((b*c - a*d)^6*g^4*i^3*(c + d*x)) - (20*b^3*B^2*d^2*(c + d*x))/((b*c - a*d
)^6*g^4*i^3*(a + b*x)) + (5*b^4*B^2*d*(c + d*x)^2)/(4*(b*c - a*d)^6*g^4*i^3
*(a + b*x)^2) - (2*b^5*B^2*(c + d*x)^3)/(27*(b*c - a*d)^6*g^4*i^3*(a + b*x)
^3) - (10*b*B^2*d^4*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/((b*c - a*d)^6*
g^4*i^3*(c + d*x)) + (B*d^5*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/(2*(b*c - a*d)^6*g^4*i^3*(c + d*x)^2) - (20*b^3*B*d^2*(c + d*x)*(A + B*L
og[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^6*g^4*i^3*(a + b*x)) + (5*b^4*B*
d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*c - a*d)^6*g^4*i^
3*(a + b*x)^2) - (2*b^5*B*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))
/(9*(b*c - a*d)^6*g^4*i^3*(a + b*x)^3) - (d^5*(a + b*x)^2*(A + B*Log[(e*(a
+ b*x))/(c + d*x)])^2)/(2*(b*c - a*d)^6*g^4*i^3*(c + d*x)^2) + (5*b*d^4*(a
+ b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*c - a*d)^6*g^4*i^3*(c +
d*x)) - (10*b^3*d^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/((b*c
 - a*d)^6*g^4*i^3*(a + b*x)) + (5*b^4*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x)
)/(c + d*x)])^2)/(2*(b*c - a*d)^6*g^4*i^3*(a + b*x)^2) - (b^5*(c + d*x)^3*(
A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*(b*c - a*d)^6*g^4*i^3*(a + b*x)^3
) - (10*b^2*d^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^3)/(3*B*(b*c - a*d)^6*
g^4*i^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2562

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A +
B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGtQ
[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && In
tegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)^5(A+B\log(ex))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}$$

$$= \frac{\text{Subst}\left(\int \left(5bd^4(A+B\log(ex))^2 + \frac{b^5(A+B\log(ex))^2}{x^4} - \frac{5b^4d(A+B\log(ex))^2}{x^3} + \frac{10b^3d^2(A+B\log(ex))^2}{x^2} - \frac{10b^2d^3(A+\cdots}{\cdots}\right.\right.}{(bc-ad)^6g^4i^3}$$

$$= \frac{b^5\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3} - \frac{(5b^4d)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}$$
$$+ \frac{(10b^3d^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3} - \frac{(10b^2d^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}$$
$$+ \frac{(5bd^4)\,\text{Subst}\left(\int (A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3} - \frac{d^5\text{Subst}\left(\int x(A+B\log(ex))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}$$

$$= -\frac{d^5(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^6g^4i^3(c+dx)^2} + \frac{5bd^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^6g^4i^3(c+dx)}$$
$$- \frac{10b^3d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^6g^4i^3(a+bx)^2}$$
$$- \frac{b^5(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^6g^4i^3(a+bx)^3} + \frac{(2b^5B)\operatorname{Subst}\left(\int\frac{A+B\log(ex)}{x^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{3(bc-ad)^6g^4i^3}$$
$$- \frac{(5b^4Bd)\operatorname{Subst}\left(\int\frac{A+B\log(ex)}{x^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}$$
$$+ \frac{(20b^3Bd^2)\operatorname{Subst}\left(\int\frac{A+B\log(ex)}{x^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}$$
$$- \frac{(10b^2d^3)\operatorname{Subst}\left(\int x^2\,dx,x,A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B(bc-ad)^6g^4i^3}$$
$$- \frac{(10bBd^4)\operatorname{Subst}\left(\int(A+B\log(ex))\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}$$
$$+ \frac{(Bd^5)\operatorname{Subst}\left(\int x(A+B\log(ex))\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}$$

$$= -\frac{B^2d^5(a+bx)^2}{4(bc-ad)^6g^4i^3(c+dx)^2} - \frac{10AbBd^4(a+bx)}{(bc-ad)^6g^4i^3(c+dx)} - \frac{20b^3B^2d^2(c+dx)}{(bc-ad)^6g^4i^3(a+bx)}$$
$$+ \frac{5b^4B^2d(c+dx)^2}{4(bc-ad)^6g^4i^3(a+bx)^2} - \frac{2b^5B^2(c+dx)^3}{27(bc-ad)^6g^4i^3(a+bx)^3}$$
$$+ \frac{Bd^5(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2} - \frac{20b^3Bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(a+bx)}$$
$$+ \frac{5b^4Bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2} - \frac{2b^5B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^6g^4i^3(a+bx)^3}$$
$$- \frac{d^5(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^6g^4i^3(c+dx)^2} + \frac{5bd^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^6g^4i^3(c+dx)}$$
$$- \frac{10b^3d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^6g^4i^3(a+bx)^2}$$
$$- \frac{b^5(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^6g^4i^3(a+bx)^3} - \frac{10b^2d^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^6g^4i^3}$$
$$- \frac{(10bB^2d^4)\operatorname{Subst}\left(\int\log(ex)\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}$$

$$= -\frac{B^2d^5(a+bx)^2}{4(bc-ad)^6g^4i^3(c+dx)^2} - \frac{10AbBd^4(a+bx)}{(bc-ad)^6g^4i^3(c+dx)} + \frac{10bB^2d^4(a+bx)}{(bc-ad)^6g^4i^3(c+dx)} - \frac{20b^3B^2d^2(c+dx)}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4B^2d(c+dx)^2}{4(bc-ad)^6g^4i^3(a+bx)^2} - \frac{2b^5B^2(c+dx)^3}{27(bc-ad)^6g^4i^3(a+bx)^3} - \frac{10bB^2d^4(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^6g^4i^3(c+dx)} + \frac{Bd^5(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2} - \frac{20b^3Bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4Bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2} - \frac{2b^5B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^6g^4i^3(a+bx)^3} - \frac{d^5(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^6g^4i^3(c+dx)^2} + \frac{5bd^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^6g^4i^3(c+dx)} - \frac{10b^3d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^6g^4i^3(a+bx)^2} - \frac{b^5(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^6g^4i^3(a+bx)^3} - \frac{10b^2d^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^6g^4i^3}$$

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 793, normalized size of antiderivative = 0.93

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = -\frac{27(2A^2-2AB+B^2)\,d^3(bc-ad)^2(a+bx)^3+54b(8A^2-18AB+19B^2)\,d^3(bc-ad)(a+bx)^3(c+dx)+}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^4*(c*i + d*
i*x)^3),x]
```

```
[Out] -1/108*(27*(2*A^2 - 2*A*B + B^2)*d^3*(b*c - a*d)^2*(a + b*x)^3 + 54*b*(8*A^
2 - 18*A*B + 19*B^2)*d^3*(b*c - a*d)*(a + b*x)^3*(c + d*x) + 4*b^2*(9*A^2 +
 6*A*B + 2*B^2)*(b*c - a*d)^3*(c + d*x)^2 - 3*b^2*(54*A^2 + 66*A*B + 37*B^2
)*d*(b*c - a*d)^2*(a + b*x)*(c + d*x)^2 + 6*b^2*(108*A^2 + 282*A*B + 319*B^
2)*d^2*(b*c - a*d)*(a + b*x)^2*(c + d*x)^2 + 60*b^2*(18*A^2 + 12*A*B + 49*B
^2)*d^3*(a + b*x)^3*(c + d*x)^2*Log[a + b*x] + 6*B*(b*c - a*d)*(9*(2*A - B)
*d^3*(b*c - a*d)*(a + b*x)^3 + 18*b*(8*A - 9*B)*d^3*(a + b*x)^3*(c + d*x) +
 4*b^2*(3*A + B)*(b*c - a*d)^2*(c + d*x)^2 - 3*b^2*(18*A + 11*B)*d*(b*c - a
*d)*(a + b*x)*(c + d*x)^2 + 6*b^2*(36*A + 47*B)*d^2*(a + b*x)^2*(c + d*x)^2
```

```
)*Log[(e*(a + b*x))/(c + d*x)] + 18*B*(3*a^5*B*d^5 - 15*a^4*b*B*d^4*(2*c +
d*x) + 30*a^3*b^2*d^3*(2*A*(c + d*x)^2 - B*d*x*(4*c + 3*d*x)) + 30*a^2*b^3*
d^2*(6*A*d*x*(c + d*x)^2 + B*(2*c^3 + 6*c^2*d*x - 3*d^3*x^3)) + 15*a*b^4*d*
(12*A*d^2*x^2*(c + d*x)^2 + B*c*(-c^3 + 4*c^2*d*x + 18*c*d^2*x^2 + 12*d^3*x
^3)) + b^5*(60*A*d^3*x^3*(c + d*x)^2 + B*(2*c^5 - 5*c^4*d*x + 20*c^3*d^2*x^
2 + 110*c^2*d^3*x^3 + 100*c*d^4*x^4 + 20*d^5*x^5)))*Log[(e*(a + b*x))/(c +
d*x)]^2 + 360*b^2*B^2*d^3*(a + b*x)^3*(c + d*x)^2*Log[(e*(a + b*x))/(c + d*
x)]^3 - 60*b^2*(18*A^2 + 12*A*B + 49*B^2)*d^3*(a + b*x)^3*(c + d*x)^2*Log[c
 + d*x])/((b*c - a*d)^6*g^4*i^3*(a + b*x)^3*(c + d*x)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1707 vs. 2(831) = 1662.

Time = 7.75 (sec) , antiderivative size = 1708, normalized size of antiderivative = 2.01

method	result	size
parts	Expression too large to display	1708
derivativedivides	Expression too large to display	1951
default	Expression too large to display	1951
risch	Expression too large to display	3021
parallelrisch	Expression too large to display	3969
norman	Expression too large to display	4027

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x,method=_RET
URNVERBOSE)
```

```
[Out] A^2/g^4/i^3*(-1/2*d^3/(a*d-b*c)^4/(d*x+c)^2+10*d^3/(a*d-b*c)^6*b^2*ln(d*x+c
)+4*d^3/(a*d-b*c)^5*b/(d*x+c)+1/3*b^2/(a*d-b*c)^3/(b*x+a)^3-10*d^3/(a*d-b*c
)^6*b^2*ln(b*x+a)+6*b^2/(a*d-b*c)^5*d^2/(b*x+a)+3/2*b^2/(a*d-b*c)^4*d/(b*x+
a)^2)-B^2/g^4/i^3*d/(a*d-b*c)^2/e^2*(d^4/(a*d-b*c)^4*(1/2*(b*e/d+(a*d-b*c)*
e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2*(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))
^2)-5*d^3/(a*d-b*c)^4*b*e*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)
*e/d/(d*x+c))^2-2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x
+c))+2*(a*d-b*c)*e/d/(d*x+c)+2*b*e/d)+10/3*d^2/(a*d-b*c)^4*b^2*e^2*ln(b*e/d
+(a*d-b*c)*e/d/(d*x+c))^3-10*d/(a*d-b*c)^4*b^3*e^3*(-1/(b*e/d+(a*d-b*c)*e/d
/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))
*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+5/(a*d-b*
c)^4*b^4*e^4*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c
))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-1/d/(a*d-b*c)^4*b^5*e^5*(-1/3/(b*e/
d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/9/(b*e/d+(a*
d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/27/(b*e/d+(a*d-b*c)
*e/d/(d*x+c))^3))-2*B*A/g^4/i^3*d/(a*d-b*c)^2/e^2*(d^4/(a*d-b*c)^4*(1/2*(b*
```

```
e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4*(b*e/d+(a*
d-b*c)*e/d/(d*x+c))^2)-5*d^3/(a*d-b*c)^4*b*e*((b*e/d+(a*d-b*c)*e/d/(d*x+c))
*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)+5*d^2/(a*d-b*
c)^4*b^2*e^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-10*d/(a*d-b*c)^4*b^3*e^3*(-1
/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*
d-b*c)*e/d/(d*x+c)))+5/(a*d-b*c)^4*b^4*e^4*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+
c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-
1/d/(a*d-b*c)^4*b^5*e^5*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d
-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2257 vs. $2(831) = 1662$.

Time = 0.44 (sec) , antiderivative size = 2257, normalized size of antiderivative = 2.65

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x, alg
orithm="fricas")
```

```
[Out] -1/108*(4*(9*A^2 + 6*A*B + 2*B^2)*b^5*c^5 - 135*(2*A^2 + 2*A*B + B^2)*a*b^4
*c^4*d + 1080*(A^2 + 2*A*B + 2*B^2)*a^2*b^3*c^3*d^2 - 20*(18*A^2 + 147*A*B
+ 49*B^2)*a^3*b^2*c^2*d^3 - 540*(A^2 - 2*A*B + 2*B^2)*a^4*b*c*d^4 + 27*(2*A
^2 - 2*A*B + B^2)*a^5*d^5 + 60*((18*A^2 + 12*A*B + 49*B^2)*b^5*c*d^4 - (18*
A^2 + 12*A*B + 49*B^2)*a*b^4*d^5)*x^4 + 30*(3*(18*A^2 + 24*A*B + 53*B^2)*b^
5*c^2*d^3 + 2*(18*A^2 - 24*A*B + 37*B^2)*a*b^4*c*d^4 - (90*A^2 + 24*A*B + 2
33*B^2)*a^2*b^3*d^5)*x^3 + 360*(B^2*b^5*d^5*x^5 + B^2*a^3*b^2*c^2*d^3 + (2*
B^2*b^5*c*d^4 + 3*B^2*a*b^4*d^5)*x^4 + (B^2*b^5*c^2*d^3 + 6*B^2*a*b^4*c*d^4
 + 3*B^2*a^2*b^3*d^5)*x^3 + (3*B^2*a*b^4*c^2*d^3 + 6*B^2*a^2*b^3*c*d^4 + B^
2*a^3*b^2*d^5)*x^2 + (3*B^2*a^2*b^3*c^2*d^3 + 2*B^2*a^3*b^2*c*d^4)*x)*log((
b*e*x + a*e)/(d*x + c))^3 + 10*(2*(18*A^2 + 66*A*B + 85*B^2)*b^5*c^3*d^2 +
3*(126*A^2 + 84*A*B + 307*B^2)*a*b^4*c^2*d^3 - 12*(18*A^2 + 39*A*B + 49*B^2
)*a^2*b^3*c*d^4 - (198*A^2 - 84*A*B + 503*B^2)*a^3*b^2*d^5)*x^2 + 18*(20*(3
*A*B + B^2)*b^5*d^5*x^5 + 2*B^2*b^5*c^5 - 15*B^2*a*b^4*c^4*d + 60*B^2*a^2*b
^3*c^3*d^2 + 60*A*B*a^3*b^2*c^2*d^3 - 30*B^2*a^4*b*c*d^4 + 3*B^2*a^5*d^5 +
20*(9*A*B*a*b^4*d^5 + (6*A*B + 5*B^2)*b^5*c*d^4)*x^4 + 10*((6*A*B + 11*B^2)
*b^5*c^2*d^3 + 18*(2*A*B + B^2)*a*b^4*c*d^4 + 9*(2*A*B - B^2)*a^2*b^3*d^5)*
x^3 + 10*(2*B^2*b^5*c^3*d^2 + 36*A*B*a^2*b^3*c*d^4 + 9*(2*A*B + 3*B^2)*a*b^
4*c^2*d^3 + 3*(2*A*B - 3*B^2)*a^3*b^2*d^5)*x^2 - 5*(B^2*b^5*c^4*d - 12*B^2*
a*b^4*c^3*d^2 + 3*B^2*a^4*b*d^5 - 36*(A*B + B^2)*a^2*b^3*c^2*d^3 - 24*(A*B
- B^2)*a^3*b^2*c*d^4)*x)*log((b*e*x + a*e)/(d*x + c))^2 - 5*((18*A^2 + 30*A
*B + 19*B^2)*b^5*c^4*d - 108*(2*A^2 + 6*A*B + 7*B^2)*a*b^4*c^3*d^2 - 12*(36
*A^2 - 39*A*B + 59*B^2)*a^2*b^3*c^2*d^3 + 8*(72*A^2 + 39*A*B + 157*B^2)*a^3
```

```
*b^2*c*d^4 + 27*(2*A^2 - 6*A*B + 7*B^2)*a^4*b*d^5)*x + 6*(10*(18*A^2 + 12*A
*B + 49*B^2)*b^5*d^5*x^5 + 180*A^2*a^3*b^2*c^2*d^3 + 4*(3*A*B + B^2)*b^5*c^
5 - 45*(2*A*B + B^2)*a*b^4*c^4*d + 360*(A*B + B^2)*a^2*b^3*c^3*d^2 - 180*(A
*B - B^2)*a^4*b*c*d^4 + 9*(2*A*B - B^2)*a^5*d^5 + 10*(2*(18*A^2 + 30*A*B +
55*B^2)*b^5*c*d^4 + 27*(2*A^2 + 5*B^2)*a*b^4*d^5)*x^4 + 10*((18*A^2 + 66*A*
B + 85*B^2)*b^5*c^2*d^3 + 54*(2*A^2 + 2*A*B + 5*B^2)*a*b^4*c*d^4 + 27*(2*A^
2 - 2*A*B + 5*B^2)*a^2*b^3*d^5)*x^3 + 10*(2*(6*A*B + 11*B^2)*b^5*c^3*d^2 +
27*(2*A^2 + 6*A*B + 7*B^2)*a*b^4*c^2*d^3 + 108*(A^2 + 2*B^2)*a^2*b^3*c*d^4
+ 9*(2*A^2 - 6*A*B + 7*B^2)*a^3*b^2*d^5)*x^2 - 5*((6*A*B + 5*B^2)*b^5*c^4*d
 - 36*(2*A*B + 3*B^2)*a*b^4*c^3*d^2 - 108*(A^2 + 2*A*B + 2*B^2)*a^2*b^3*c^2
*d^3 - 72*(A^2 - 2*A*B + 2*B^2)*a^3*b^2*c*d^4 + 9*(2*A*B - 3*B^2)*a^4*b*d^5
)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^9*c^6*d^2 - 6*a*b^8*c^5*d^3 + 15*a^2
*b^7*c^4*d^4 - 20*a^3*b^6*c^3*d^5 + 15*a^4*b^5*c^2*d^6 - 6*a^5*b^4*c*d^7 +
a^6*b^3*d^8)*g^4*i^3*x^5 + (2*b^9*c^7*d - 9*a*b^8*c^6*d^2 + 12*a^2*b^7*c^5*
d^3 + 5*a^3*b^6*c^4*d^4 - 30*a^4*b^5*c^3*d^5 + 33*a^5*b^4*c^2*d^6 - 16*a^6*
b^3*c*d^7 + 3*a^7*b^2*d^8)*g^4*i^3*x^4 + (b^9*c^8 - 18*a^2*b^7*c^6*d^2 + 52
*a^3*b^6*c^5*d^3 - 60*a^4*b^5*c^4*d^4 + 24*a^5*b^4*c^3*d^5 + 10*a^6*b^3*c^2
*d^6 - 12*a^7*b^2*c*d^7 + 3*a^8*b*d^8)*g^4*i^3*x^3 + (3*a*b^8*c^8 - 12*a^2*
b^7*c^7*d + 10*a^3*b^6*c^6*d^2 + 24*a^4*b^5*c^5*d^3 - 60*a^5*b^4*c^4*d^4 +
52*a^6*b^3*c^3*d^5 - 18*a^7*b^2*c^2*d^6 + a^9*d^8)*g^4*i^3*x^2 + (3*a^2*b^7
*c^8 - 16*a^3*b^6*c^7*d + 33*a^4*b^5*c^6*d^2 - 30*a^5*b^4*c^5*d^3 + 5*a^6*b
^3*c^4*d^4 + 12*a^7*b^2*c^3*d^5 - 9*a^8*b*c^2*d^6 + 2*a^9*c*d^7)*g^4*i^3*x
+ (a^3*b^6*c^8 - 6*a^4*b^5*c^7*d + 15*a^5*b^4*c^6*d^2 - 20*a^6*b^3*c^5*d^3
+ 15*a^7*b^2*c^4*d^4 - 6*a^8*b*c^3*d^5 + a^9*c^2*d^6)*g^4*i^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**4/(d*i*x+c*i)**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9282 vs. $2(831) = 1662$.

Time = 1.08 (sec) , antiderivative size = 9282, normalized size of antiderivative = 10.91

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x, alg
orithm="maxima")

[Out] -1/6*B^2*((60*b^4*d^4*x^4 + 2*b^4*c^4 - 13*a*b^3*c^3*d + 47*a^2*b^2*c^2*d^2
 + 27*a^3*b*c*d^3 - 3*a^4*d^4 + 30*(3*b^4*c*d^3 + 5*a*b^3*d^4)*x^3 + 10*(2*
b^4*c^2*d^2 + 23*a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2 - 5*(b^4*c^3*d - 11*a*b^
3*c^2*d^2 - 35*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x)/((b^8*c^5*d^2 - 5*a*b^7*c^4*
d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*b^3*d
^7)*g^4*i^3*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 + 10*a
^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7)*g^4
*i^3*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c^4*d^3
 - 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d^7)*g^
4*i^3*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a^4*b^4*c
^4*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a^8*d^7)*g
^4*i^3*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5*d^2 - 10*a^
5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*d^6)*g^4*i^3*
x + (a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*a^6*b^2*c^4*d^
3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5)*g^4*i^3) + 60*b^2*d^3*log(b*x + a)/((b^6
*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2
*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3) - 60*b^2*d^3*log(d*x + c)/((b^
6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^
2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3))*log(b*e*x/(d*x + c) + a*e/(d
*x + c))^2 - 1/3*A*B*((60*b^4*d^4*x^4 + 2*b^4*c^4 - 13*a*b^3*c^3*d + 47*a^2
*b^2*c^2*d^2 + 27*a^3*b*c*d^3 - 3*a^4*d^4 + 30*(3*b^4*c*d^3 + 5*a*b^3*d^4)*
x^3 + 10*(2*b^4*c^2*d^2 + 23*a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2 - 5*(b^4*c^3
*d - 11*a*b^3*c^2*d^2 - 35*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x)/((b^8*c^5*d^2 -
5*a*b^7*c^4*d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6
 - a^5*b^3*d^7)*g^4*i^3*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^
4*d^3 + 10*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*
b^2*d^7)*g^4*i^3*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3
*b^5*c^4*d^3 - 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a
^7*b*d^7)*g^4*i^3*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 +
25*a^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6
- a^8*d^7)*g^4*i^3*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5
*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*d
^6)*g^4*i^3*x + (a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*a^
6*b^2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5)*g^4*i^3) + 60*b^2*d^3*log(b*
x + a)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3
+ 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3) - 60*b^2*d^3*log(d
*x + c)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3
 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3))*log(b*e*x/(d*x +
 c) + a*e/(d*x + c)) - 1/108*B^2*(6*(4*b^5*c^5 - 45*a*b^4*c^4*d + 360*a^2*b
^3*c^3*d^2 - 490*a^3*b^2*c^2*d^3 + 180*a^4*b*c*d^4 - 9*a^5*d^5 + 120*(b^5*c
*d^4 - a*b^4*d^5)*x^4 + 120*(3*b^5*c^2*d^3 - 2*a*b^4*c*d^4 - a^2*b^3*d^5)*x
^3 + 20*(11*b^5*c^3*d^2 + 21*a*b^4*c^2*d^3 - 39*a^2*b^3*c*d^4 + 7*a^3*b^2*d
```

^5)*x^2 - 180*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*
x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3
+ 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4
)*x)*log(b*x + a)^2 - 180*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3
*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*
b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a
^3*b^2*c*d^4)*x)*log(d*x + c)^2 - 5*(5*b^5*c^4*d - 108*a*b^4*c^3*d^2 + 78*a
^2*b^3*c^2*d^3 + 52*a^3*b^2*c*d^4 - 27*a^4*b*d^5)*x + 120*(b^5*d^5*x^5 + a^
3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*
d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5
)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a) - 120*(b^5*d^
5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 +
6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a
^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x - 3*(b^5*d^5*x^5
+ a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^
4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2
*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a))*log(d*x
+ c))*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(a^3*b^6*c^8*g^4*i^3 - 6*a^4*b^5
*c^7*d*g^4*i^3 + 15*a^5*b^4*c^6*d^2*g^4*i^3 - 20*a^6*b^3*c^5*d^3*g^4*i^3 +
15*a^7*b^2*c^4*d^4*g^4*i^3 - 6*a^8*b*c^3*d^5*g^4*i^3 + a^9*c^2*d^6*g^4*i^3
+ (b^9*c^6*d^2*g^4*i^3 - 6*a*b^8*c^5*d^3*g^4*i^3 + 15*a^2*b^7*c^4*d^4*g^4*i
^3 - 20*a^3*b^6*c^3*d^5*g^4*i^3 + 15*a^4*b^5*c^2*d^6*g^4*i^3 - 6*a^5*b^4*c*
d^7*g^4*i^3 + a^6*b^3*d^8*g^4*i^3)*x^5 + (2*b^9*c^7*d*g^4*i^3 - 9*a*b^8*c^6
*d^2*g^4*i^3 + 12*a^2*b^7*c^5*d^3*g^4*i^3 + 5*a^3*b^6*c^4*d^4*g^4*i^3 - 30*
a^4*b^5*c^3*d^5*g^4*i^3 + 33*a^5*b^4*c^2*d^6*g^4*i^3 - 16*a^6*b^3*c*d^7*g^4
*i^3 + 3*a^7*b^2*d^8*g^4*i^3)*x^4 + (b^9*c^8*g^4*i^3 - 18*a^2*b^7*c^6*d^2*g
^4*i^3 + 52*a^3*b^6*c^5*d^3*g^4*i^3 - 60*a^4*b^5*c^4*d^4*g^4*i^3 + 24*a^5*b
^4*c^3*d^5*g^4*i^3 + 10*a^6*b^3*c^2*d^6*g^4*i^3 - 12*a^7*b^2*c*d^7*g^4*i^3
+ 3*a^8*b*d^8*g^4*i^3)*x^3 + (3*a*b^8*c^8*g^4*i^3 - 12*a^2*b^7*c^7*d*g^4*i^
3 + 10*a^3*b^6*c^6*d^2*g^4*i^3 + 24*a^4*b^5*c^5*d^3*g^4*i^3 - 60*a^5*b^4*c^
4*d^4*g^4*i^3 + 52*a^6*b^3*c^3*d^5*g^4*i^3 - 18*a^7*b^2*c^2*d^6*g^4*i^3 + a
^9*d^8*g^4*i^3)*x^2 + (3*a^2*b^7*c^8*g^4*i^3 - 16*a^3*b^6*c^7*d*g^4*i^3 + 3
3*a^4*b^5*c^6*d^2*g^4*i^3 - 30*a^5*b^4*c^5*d^3*g^4*i^3 + 5*a^6*b^3*c^4*d^4*
g^4*i^3 + 12*a^7*b^2*c^3*d^5*g^4*i^3 - 9*a^8*b*c^2*d^6*g^4*i^3 + 2*a^9*c*d^
7*g^4*i^3)*x) + (8*b^5*c^5 - 135*a*b^4*c^4*d + 2160*a^2*b^3*c^3*d^2 - 980*a
^3*b^2*c^2*d^3 - 1080*a^4*b*c*d^4 + 27*a^5*d^5 + 2940*(b^5*c*d^4 - a*b^4*d^
5)*x^4 + 30*(159*b^5*c^2*d^3 + 74*a*b^4*c*d^4 - 233*a^2*b^3*d^5)*x^3 + 360*
(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2
*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*
d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x +
a)^3 - 360*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^
4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 +
6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*
x)*log(d*x + c)^3 + 10*(170*b^5*c^3*d^2 + 921*a*b^4*c^2*d^3 - 588*a^2*b^3*c
*d^4 - 503*a^3*b^2*d^5)*x^2 - 360*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c

*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a)^2 - 360*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x - 3*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a))*log(d*x + c)^2 - 5*(19*b^5*c^4*d - 756*a*b^4*c^3*d^2 - 708*a^2*b^3*c^2*d^3 + 1256*a^3*b^2*c*d^4 + 189*a^4*b*d^5)*x + 2940*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a) - 60*(49*b^5*d^5*x^5 + 49*a^3*b^2*c^2*d^3 + 49*(2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + 49*(b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + 49*(3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + 18*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a)^2 + 49*(3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x - 12*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a))*log(d*x + c))/(a^3*b^6*c^8*g^4*i^3 - 6*a^4*b^5*c^7*d*g^4*i^3 + 15*a^5*b^4*c^6*d^2*g^4*i^3 - 20*a^6*b^3*c^5*d^3*g^4*i^3 + 15*a^7*b^2*c^4*d^4*g^4*i^3 - 6*a^8*b*c^3*d^5*g^4*i^3 + a^9*c^2*d^6*g^4*i^3 + (b^9*c^6*d^2*g^4*i^3 - 6*a*b^8*c^5*d^3*g^4*i^3 + 15*a^2*b^7*c^4*d^4*g^4*i^3 - 20*a^3*b^6*c^3*d^5*g^4*i^3 + 15*a^4*b^5*c^2*d^6*g^4*i^3 - 6*a^5*b^4*c*d^7*g^4*i^3 + a^6*b^3*d^8*g^4*i^3)*x^5 + (2*b^9*c^7*d*g^4*i^3 - 9*a*b^8*c^6*d^2*g^4*i^3 + 12*a^2*b^7*c^5*d^3*g^4*i^3 + 5*a^3*b^6*c^4*d^4*g^4*i^3 - 30*a^4*b^5*c^3*d^5*g^4*i^3 + 33*a^5*b^4*c^2*d^6*g^4*i^3 - 16*a^6*b^3*c*d^7*g^4*i^3 + 3*a^7*b^2*d^8*g^4*i^3)*x^4 + (b^9*c^8*g^4*i^3 - 18*a^2*b^7*c^6*d^2*g^4*i^3 + 52*a^3*b^6*c^5*d^3*g^4*i^3 - 60*a^4*b^5*c^4*d^4*g^4*i^3 + 24*a^5*b^4*c^3*d^5*g^4*i^3 + 10*a^6*b^3*c^2*d^6*g^4*i^3 - 12*a^7*b^2*c*d^7*g^4*i^3 + 3*a^8*b*d^8*g^4*i^3)*x^3 + (3*a*b^8*c^8*g^4*i^3 - 12*a^2*b^7*c^7*d*g^4*i^3 + 10*a^3*b^6*c^6*d^2*g^4*i^3 + 24*a^4*b^5*c^5*d^3*g^4*i^3 - 60*a^5*b^4*c^4*d^4*g^4*i^3 + 52*a^6*b^3*c^3*d^5*g^4*i^3 - 18*a^7*b^2*c^2*d^6*g^4*i^3 + a^9*d^8*g^4*i^3)*x^2 + (3*a^2*b^7*c^8*g^4*i^3 - 16*a^3*b^6*c^7*d*g^4*i^3 + 33*a^4*b^5*c^6*d^2*g^4*i^3 - 30*a^5*b^4*c^5*d^3*g^4*i^3 + 5*a^6*b^3*c^4*d^4*g^4*i^3 + 12*a^7*b^2*c^3*d^5*g^4*i^3 - 9*a^8*b*c^2*d^6*g^4*i^3 + 2*a^9*c*d^7*g^4*i^3)*x)) - 1/6*A^2*((60*b^4*d^4*x^4 + 2*b^4*c^4 - 13*a*b^3*c^3*d + 47*a^2*b^2*c^2*d^2 + 27*a^3*b*c*d^3 - 3*a^4*d^4 + 30*(3*b^4*c*d^3 + 5*a*b^3*d^4)*x^3 + 10*(2*b^4*c^2*d^2 + 23*a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2 - 5*(b^4*c^3*d - 11*a*b^3*c^2*d^2 - 35*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x)/((b^8*c^5*d^2 - 5*a*b^7*c^4*d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*b^3*d^7)*g^4*i^3*x^5 + (2*b^8*c^6*d -

7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 + 10*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7)*g^4*i^3*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c^4*d^3 - 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d^7)*g^4*i^3*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a^8*d^7)*g^4*i^3*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*d^6)*g^4*i^3*x + (a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*a^6*b^2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5)*g^4*i^3) + 60*b^2*d^3*log(b*x + a)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3) - 60*b^2*d^3*log(d*x + c)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3)) - 1/18*(4*b^5*c^5 - 45*a*b^4*c^4*d + 360*a^2*b^3*c^3*d^2 - 490*a^3*b^2*c^2*d^3 + 180*a^4*b*c*d^4 - 9*a^5*d^5 + 120*(b^5*c*d^4 - a*b^4*d^5)*x^4 + 120*(3*b^5*c^2*d^3 - 2*a*b^4*c*d^4 - a^2*b^3*d^5)*x^3 + 20*(11*b^5*c^3*d^2 + 21*a*b^4*c^2*d^3 - 39*a^2*b^3*c*d^4 + 7*a^3*b^2*d^5)*x^2 - 180*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a)^2 - 180*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(d*x + c)^2 - 5*(5*b^5*c^4*d - 108*a*b^4*c^3*d^2 + 78*a^2*b^3*c^2*d^3 + 52*a^3*b^2*c*d^4 - 27*a^4*b*d^5)*x + 120*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a) - 120*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x - 3*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a))*log(d*x + c))*A*B/(a^3*b^6*c^8*g^4*i^3 - 6*a^4*b^5*c^7*d*g^4*i^3 + 15*a^5*b^4*c^6*d^2*g^4*i^3 - 20*a^6*b^3*c^5*d^3*g^4*i^3 + 15*a^7*b^2*c^4*d^4*g^4*i^3 - 6*a^8*b*c^3*d^5*g^4*i^3 + a^9*c^2*d^6*g^4*i^3 + (b^9*c^6*d^2*g^4*i^3 - 6*a*b^8*c^5*d^3*g^4*i^3 + 15*a^2*b^7*c^4*d^4*g^4*i^3 - 20*a^3*b^6*c^3*d^5*g^4*i^3 + 15*a^4*b^5*c^2*d^6*g^4*i^3 - 6*a^5*b^4*c*d^7*g^4*i^3 + a^6*b^3*d^8*g^4*i^3)*x^5 + (2*b^9*c^7*d*g^4*i^3 - 9*a*b^8*c^6*d^2*g^4*i^3 + 12*a^2*b^7*c^5*d^3*g^4*i^3 + 5*a^3*b^6*c^4*d^4*g^4*i^3 - 30*a^4*b^5*c^3*d^5*g^4*i^3 + 33*a^5*b^4*c^2*d^6*g^4*i^3 - 16*a^6*b^3*c*d^7*g^4*i^3 + 3*a^7*b^2*d^8*g^4*i^3)*x^4 + (b^9*c^8*g^4*i^3 - 18*a^2*b^7*c^6*d^2*g^4*i^3 + 52*a^3*b^6*c^5*d^3*g^4*i^3 - 60*a^4*b^5*c^4*d^4*g^4*i^3 + 24*a^5*b^4*c^3*d^5*g^4*i^3 + 10*a^6*b^3*c^2*d^6*g^4*i^3 - 12*a^7*b^2*c*d^7*g^4*i^3 + 3*a^8*b*d^8*g^4*i^3)*x^3 + (3*a*b^8*c^8*g^4*i^

```
3 - 12*a^2*b^7*c^7*d*g^4*i^3 + 10*a^3*b^6*c^6*d^2*g^4*i^3 + 24*a^4*b^5*c^5*
d^3*g^4*i^3 - 60*a^5*b^4*c^4*d^4*g^4*i^3 + 52*a^6*b^3*c^3*d^5*g^4*i^3 - 18*
a^7*b^2*c^2*d^6*g^4*i^3 + a^9*d^8*g^4*i^3)*x^2 + (3*a^2*b^7*c^8*g^4*i^3 - 1
6*a^3*b^6*c^7*d*g^4*i^3 + 33*a^4*b^5*c^6*d^2*g^4*i^3 - 30*a^5*b^4*c^5*d^3*g
^4*i^3 + 5*a^6*b^3*c^4*d^4*g^4*i^3 + 12*a^7*b^2*c^3*d^5*g^4*i^3 - 9*a^8*b*c
^2*d^6*g^4*i^3 + 2*a^9*c*d^7*g^4*i^3)*x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^4(dix+ci)^3}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x, alg
orithm="giac")
```

```
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/((b*g*x + a*g)^4*(d*i*x + c*
i)^3), x)
```

Mupad [B] (verification not implemented)

Time = 15.42 (sec) , antiderivative size = 3550, normalized size of antiderivative = 4.17

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx=\text{Too large to display}$$

```
[In] int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^4*(c*i + d*i*x)^3
),x)
```

```
[Out] ((36*A^2*b^4*c^4 - 54*A^2*a^4*d^4 - 27*B^2*a^4*d^4 + 8*B^2*b^4*c^4 + 54*A*B
*a^4*d^4 + 24*A*B*b^4*c^4 + 846*A^2*a^2*b^2*c^2*d^2 + 2033*B^2*a^2*b^2*c^2*
d^2 - 234*A^2*a*b^3*c^3*d + 486*A^2*a^3*b*c*d^3 - 127*B^2*a*b^3*c^3*d + 105
3*B^2*a^3*b*c*d^3 - 246*A*B*a*b^3*c^3*d - 1026*A*B*a^3*b*c*d^3 + 1914*A*B*a
^2*b^2*c^2*d^2)/(6*(a*d - b*c)) + (10*x^4*(18*A^2*b^4*d^4 + 49*B^2*b^4*d^4
+ 12*A*B*b^4*d^4))/(a*d - b*c) + (5*x*(54*A^2*a^3*b*d^4 + 189*B^2*a^3*b*d^4
 - 18*A^2*b^4*c^3*d - 19*B^2*b^4*c^3*d + 198*A^2*a*b^3*c^2*d^2 + 630*A^2*a^
2*b^2*c*d^3 + 737*B^2*a*b^3*c^2*d^2 + 1445*B^2*a^2*b^2*c*d^3 - 162*A*B*a^3*
b*d^4 - 30*A*B*b^4*c^3*d + 618*A*B*a*b^3*c^2*d^2 + 150*A*B*a^2*b^2*c*d^3))/
(6*(a*d - b*c)) + (5*x^2*(198*A^2*a^2*b^2*d^4 + 503*B^2*a^2*b^2*d^4 + 36*A^
2*b^4*c^2*d^2 + 170*B^2*b^4*c^2*d^2 - 84*A*B*a^2*b^2*d^4 + 132*A*B*b^4*c^2*
d^2 + 414*A^2*a*b^3*c*d^3 + 1091*B^2*a*b^3*c*d^3 + 384*A*B*a*b^3*c*d^3))/(3
*(a*d - b*c)) + (5*x^3*(90*A^2*a*b^3*d^4 + 233*B^2*a*b^3*d^4 + 54*A^2*b^4*c
*d^3 + 159*B^2*b^4*c*d^3 + 24*A*B*a*b^3*d^4 + 72*A*B*b^4*c*d^3))/(a*d - b*c
```

))/(x^5*(18*a^4*b^3*d^6*g^4*i^3 + 18*b^7*c^4*d^2*g^4*i^3 - 72*a*b^6*c^3*d^3
*g^4*i^3 - 72*a^3*b^4*c*d^5*g^4*i^3 + 108*a^2*b^5*c^2*d^4*g^4*i^3) + x*(54*
a^2*b^5*c^6*g^4*i^3 + 36*a^7*c*d^5*g^4*i^3 - 180*a^3*b^4*c^5*d*g^4*i^3 - 90
*a^6*b*c^2*d^4*g^4*i^3 + 180*a^4*b^3*c^4*d^2*g^4*i^3) + x^2*(18*a^7*d^6*g^4
*i^3 + 54*a*b^6*c^6*g^4*i^3 + 36*a^6*b*c*d^5*g^4*i^3 - 108*a^2*b^5*c^5*d*g^
4*i^3 - 90*a^3*b^4*c^4*d^2*g^4*i^3 + 360*a^4*b^3*c^3*d^3*g^4*i^3 - 270*a^5*
b^2*c^2*d^4*g^4*i^3) + x^3*(18*b^7*c^6*g^4*i^3 + 54*a^6*b*d^6*g^4*i^3 + 36*
a*b^6*c^5*d*g^4*i^3 - 108*a^5*b^2*c*d^5*g^4*i^3 - 270*a^2*b^5*c^4*d^2*g^4*i
^3 + 360*a^3*b^4*c^3*d^3*g^4*i^3 - 90*a^4*b^3*c^2*d^4*g^4*i^3) + x^4*(54*a^
5*b^2*d^6*g^4*i^3 + 36*b^7*c^5*d*g^4*i^3 - 90*a*b^6*c^4*d^2*g^4*i^3 - 180*a
^4*b^3*c*d^5*g^4*i^3 + 180*a^3*b^4*c^2*d^4*g^4*i^3) + 18*a^3*b^4*c^6*g^4*i^
3 + 18*a^7*c^2*d^4*g^4*i^3 - 72*a^4*b^3*c^5*d*g^4*i^3 - 72*a^6*b*c^3*d^3*g^
4*i^3 + 108*a^5*b^2*c^4*d^2*g^4*i^3) + log((e*(a + b*x))/(c + d*x))^2*((x*(
(5*B^2*(a*d + b*c)*(2*a*d + b*c))/(3*g^4*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d
)^2) - (5*B^2)/(6*g^4*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (5*B^2*a*b*c*d
)/(g^4*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) + (20*B^2*a*b*c*d*(a*d + b*c)
)/(g^4*i^3*(a*d - b*c)^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) + x^3*((5*B^2*b^
2*d^2)/(g^4*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) + (20*B^2*b^2*d^2*(a*d +
 b*c))/(g^4*i^3*(a*d - b*c)^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) + x^2*((5*B
^2*b*d*(a*d + b*c))/(g^4*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) + (5*B^2*b*
d*(2*a*d + b*c))/(3*g^4*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) + (10*B^2*b^
2*d^3*((2*a*c*(a*d - b*c))/d + ((a*d + b*c)^2*(a*d - b*c))/(b*d^2)))/(g^4*i
^3*(a*d - b*c)^4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - (B^2*(3*a*d + 2*b*c))/
(6*g^4*i^3*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (5*B^2*a*c*(2*a*d + b
*c))/(3*g^4*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) + (10*B^2*b^3*d^3*x^4)/(
g^4*i^3*(a*d - b*c)^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (10*B^2*a^2*b*c^2*
d)/(g^4*i^3*(a*d - b*c)^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(b^2*d*x^5 + (x
^4*(3*a*b^2*d^2 + 2*b^3*c*d))/(b*d) + (a^3*c^2)/(b*d) + (x^2*(a^3*d^2 + 3*a
*b^2*c^2 + 6*a^2*b*c*d))/(b*d) + (x^3*(b^3*c^2 + 3*a^2*b*d^2 + 6*a*b^2*c*d)
)/(b*d) + (x*(3*a^2*b*c^2 + 2*a^3*c*d))/(b*d)) - (10*B*b^2*d^3*(3*A + B))/(
3*g^4*i^3*(a*d - b*c)^4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) + (log((e*(a + b*
x))/(c + d*x))*(x^2*((10*b*d*(B^2*b*c - 7*B^2*a*d + 6*A*B*a*d + 3*A*B*b*c))
/(9*g^4*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) - (10*(a*d + b*c)*(2*B^2*b*d
 - 3*A*B*b*d))/(3*g^4*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) + (20*B*b^2*d^
3*(3*A + B)*((2*a*c*(a*d - b*c))/d + ((a*d + b*c)^2*(a*d - b*c))/(b*d^2)))/
(3*g^4*i^3*(a*d - b*c)^4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - x^3*((10*b*d*(
2*B^2*b*d - 3*A*B*b*d))/(3*g^4*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) - (40
*B*b^2*d^2*(3*A + B)*(a*d + b*c))/(3*g^4*i^3*(a*d - b*c)^3*(a^2*d^2 + b^2*c
^2 - 2*a*b*c*d))) + x*((5*(B^2 - 6*A*B))/(18*g^4*i^3*(a^2*d^2 + b^2*c^2 - 2
*a*b*c*d)) + (10*(a*d + b*c)*(B^2*b*c - 7*B^2*a*d + 6*A*B*a*d + 3*A*B*b*c))
/(9*g^4*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) - (10*a*c*(2*B^2*b*d - 3*A*B
*b*d))/(3*g^4*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) + (40*B*a*b*c*d*(3*A +
 B)*(a*d + b*c))/(3*g^4*i^3*(a*d - b*c)^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))
 - (4*B^2*b*c - 9*B^2*a*d + 18*A*B*a*d + 12*A*B*b*c)/(18*g^4*i^3*(a^2*b*d^3
 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (10*a*c*(B^2*b*c - 7*B^2*a*d + 6*A*B*a*d +

```
 3*A*B*b*c))/(9*g^4*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) + (20*B*b^3*d^3*
x^4*(3*A + B))/(3*g^4*i^3*(a*d - b*c)^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) +
(20*B*a^2*b*c^2*d*(3*A + B))/(3*g^4*i^3*(a*d - b*c)^3*(a^2*d^2 + b^2*c^2 -
2*a*b*c*d))))/(b^2*d*x^5 + (x^4*(3*a*b^2*d^2 + 2*b^3*c*d))/(b*d) + (a^3*c^2
)/(b*d) + (x^2*(a^3*d^2 + 3*a*b^2*c^2 + 6*a^2*b*c*d))/(b*d) + (x^3*(b^3*c^2
 + 3*a^2*b*d^2 + 6*a*b^2*c*d))/(b*d) + (x*(3*a^2*b*c^2 + 2*a^3*c*d))/(b*d))
 + (b^2*d^3*atan((b^2*d^3*(18*A^2 + 49*B^2 + 12*A*B)*(9*a^6*d^6*g^4*i^3 - 9
*b^6*c^6*g^4*i^3 + 36*a*b^5*c^5*d*g^4*i^3 - 36*a^5*b*c*d^5*g^4*i^3 - 45*a^2
*b^4*c^4*d^2*g^4*i^3 + 45*a^4*b^2*c^2*d^4*g^4*i^3)*5i)/(9*g^4*i^3*(a*d - b*
c)^6*(90*A^2*b^2*d^3 + 245*B^2*b^2*d^3 + 60*A*B*b^2*d^3)) + (b^3*d^4*x*(18*
A^2 + 49*B^2 + 12*A*B)*(a^5*d^5*g^4*i^3 - b^5*c^5*g^4*i^3 + 5*a*b^4*c^4*d*g
^4*i^3 - 5*a^4*b*c*d^4*g^4*i^3 - 10*a^2*b^3*c^3*d^2*g^4*i^3 + 10*a^3*b^2*c^
2*d^3*g^4*i^3)*10i)/(g^4*i^3*(a*d - b*c)^6*(90*A^2*b^2*d^3 + 245*B^2*b^2*d^
3 + 60*A*B*b^2*d^3)))*(18*A^2 + 49*B^2 + 12*A*B)*10i)/(9*g^4*i^3*(a*d - b*c
)^6) - (10*B^2*b^2*d^3*log((e*(a + b*x))/(c + d*x))^3)/(3*g^4*i^3*(a*d - b*
c)^4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
```

3.108 $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 223

$$\begin{aligned}&\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx\\&=-\frac{B(bc-ad)^4g^3inx}{20bd^3}+\frac{B(bc-ad)^3g^3in(a+bx)^2}{40b^2d^2}\\&\quad-\frac{B(bc-ad)^2g^3in(a+bx)^3}{60b^2d}+\frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b}\\&\quad+\frac{(bc-ad)g^3i(a+bx)^4\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^2}+\frac{B(bc-ad)^5g^3in\log(c+dx)}{20b^2d^4}\end{aligned}$$

```
[Out] -1/20*B*(-a*d+b*c)^4*g^3*i*n*x/b/d^3+1/40*B*(-a*d+b*c)^3*g^3*i*n*(b*x+a)^2/
b^2/d^2-1/60*B*(-a*d+b*c)^2*g^3*i*n*(b*x+a)^3/b^2/d+1/5*g^3*i*(b*x+a)^4*(d*
x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b+1/20*(-a*d+b*c)*g^3*i*(b*x+a)^4*(A-B
*n+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2+1/20*B*(-a*d+b*c)^5*g^3*i*n*ln(d*x+c)/b
^2/d^4
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used

$= \{2559, 2547, 21, 45\}$

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=\frac{g^3i(a+bx)^4(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{20b^2}+\frac{g^3i(a+bx)^4(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}+\frac{Bg^3in(bc-ad)^5\log(c+dx)}{20b^2d^4}+\frac{Bg^3in(a+bx)^2(bc-ad)^3}{40b^2d^2}-\frac{Bg^3in(a+bx)^3(bc-ad)^2}{60b^2d}-\frac{Bg^3inx(bc-ad)^4}{20bd^3}$$

```
[In] Int[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/20*(B*(b*c - a*d)^4*g^3*i*n*x)/(b*d^3) + (B*(b*c - a*d)^3*g^3*i*n*(a + b
*x)^2)/(40*b^2*d^2) - (B*(b*c - a*d)^2*g^3*i*n*(a + b*x)^3)/(60*b^2*d) + (g
^3*i*(a + b*x)^4*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*b) +
((b*c - a*d)*g^3*i*(a + b*x)^4*(A - B*n + B*Log[e*((a + b*x)/(c + d*x))^n])
)/(20*b^2) + (B*(b*c - a*d)^5*g^3*i*n*Log[c + d*x])/(20*b^2*d^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rule 2559

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Simp[(f
 + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 2
```

```
))), x] + Dist[i*((b*c - a*d)/(b*d*(m + 2))), Int[(f + g*x)^m*(A - B*n + B*
Log[e*((a + b*x)/(c + d*x))^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i
, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*
i, 0] && IGtQ[m, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g^3 i(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b} \\
&\quad+\frac{((bc-ad)i)\int (ag+bgx)^3\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx}{5b} \\
&= \frac{g^3 i(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b} \\
&\quad+\frac{(bc-ad)g^3 i(a+bx)^4\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^2} \\
&\quad-\frac{(B(bc-ad)^2 in)\int \frac{(ag+bgx)^4}{(a+bx)(c+dx)}\,dx}{20b^2 g} \\
&= \frac{g^3 i(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b} \\
&\quad+\frac{(bc-ad)g^3 i(a+bx)^4\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^2} \\
&\quad-\frac{(B(bc-ad)^2 g^3 in)\int \frac{(a+bx)^3}{c+dx}\,dx}{20b^2} \\
&= \frac{g^3 i(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b} \\
&\quad+\frac{(bc-ad)g^3 i(a+bx)^4\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^2} \\
&\quad-\frac{(B(bc-ad)^2 g^3 in)\int \left(\frac{b(bc-ad)^2}{d^3}-\frac{b(bc-ad)(a+bx)}{d^2}+\frac{b(a+bx)^2}{d}+\frac{(-bc+ad)^3}{d^3(c+dx)}\right)\,dx}{20b^2} \\
&= -\frac{B(bc-ad)^4 g^3 inx}{20bd^3}+\frac{B(bc-ad)^3 g^3 in(a+bx)^2}{40b^2d^2} \\
&\quad-\frac{B(bc-ad)^2 g^3 in(a+bx)^3}{60b^2 d}+\frac{g^3 i(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b} \\
&\quad+\frac{(bc-ad)g^3 i(a+bx)^4\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^2} \\
&\quad+\frac{B(bc-ad)^5 g^3 in\log(c+dx)}{20b^2 d^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.21

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{g^3i\Big(30(bc-ad)(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+24d(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{5B(bc-ad)^2n(6bd(}{}}{}$$

```
[In] Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^
n]),x]
```

```
[Out] (g^3*i*(30*(b*c - a*d)*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) +
 24*d*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (5*B*(b*c - a*d)
^2*n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a +
 b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/d^4 + (2*B*(b*c - a*d)*n*(12*b*d*(
b*c - a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c - a*d)*(a + b
*x)^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]))/d^4))/(120*b^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1194 vs. 2(211) = 422.

Time = 11.44 (sec) , antiderivative size = 1195, normalized size of antiderivative = 5.36

method	result	size
parallelrisch	Expression too large to display	1195

```
[In] int((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/120*(22*B*x^3*a^2*b^3*d^5*g^3*i*n^2-2*B*x^3*b^5*c^2*d^3*g^3*i*n^2+120*A*x
^3*a^2*b^3*d^5*g^3*i*n+27*B*x^2*a^3*b^2*d^5*g^3*i*n^2+3*B*x^2*b^5*c^3*d^2*g
^3*i*n^2+60*A*x^2*a^3*b^2*d^5*g^3*i*n+6*B*x*a^4*b*d^5*g^3*i*n^2-6*B*x*b^5*c
^4*d*g^3*i*n^2+24*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*b^5*d^5*g^3*i*n+6*B*x^4*a
*b^4*d^5*g^3*i*n^2-6*B*x^4*b^5*c*d^4*g^3*i*n^2+90*A*x^4*a*b^4*d^5*g^3*i*n+3
0*A*x^4*b^5*c*d^4*g^3*i*n+120*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c*d^4*g
^3*i*n+180*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*c*d^4*g^3*i*n+120*B*x*ln
(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c*d^4*g^3*i*n-6*B*ln(b*x+a)*a^5*d^5*g^3*i*n
^2+24*A*x^5*b^5*d^5*g^3*i*n-6*B*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^5*g^3*i*n+6
*B*ln(b*x+a)*b^5*c^5*g^3*i*n^2+30*B*x*a^3*b^2*c*d^4*g^3*i*n^2-60*B*x*a^2*b^
3*c^2*d^3*g^3*i*n^2+30*B*x*a*b^4*c^3*d^2*g^3*i*n^2+120*A*x*a^3*b^2*c*d^4*g^
3*i*n+60*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^2*d^3*g^3*i*n-60*B*ln(e*((b*
x+a)/(d*x+c))^n)*a^2*b^3*c^3*d^2*g^3*i*n+30*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b
^4*c^4*d*g^3*i*n+30*B*ln(b*x+a)*a^4*b*c*d^4*g^3*i*n^2-60*B*ln(b*x+a)*a^3*b^
```

```
2*c^2*d^3*g^3*i*n^2+60*B*ln(b*x+a)*a^2*b^3*c^3*d^2*g^3*i*n^2+90*B*x^4*ln(e*
((b*x+a)/(d*x+c))^n)*a*b^4*d^5*g^3*i*n+30*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b
^5*c*d^4*g^3*i*n+120*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*d^5*g^3*i*n-20
*B*x^3*a*b^4*c*d^4*g^3*i*n^2-30*B*ln(b*x+a)*a*b^4*c^4*d*g^3*i*n^2+120*A*x^3
*a*b^4*c*d^4*g^3*i*n+60*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*d^5*g^3*i*n
-15*B*x^2*a^2*b^3*c*d^4*g^3*i*n^2-15*B*x^2*a*b^4*c^2*d^3*g^3*i*n^2+180*A*x^
2*a^2*b^3*c*d^4*g^3*i*n-63*B*a^4*b*c*d^4*g^3*i*n^2+45*B*a^3*b^2*c^2*d^3*g^3
*i*n^2+45*B*a^2*b^3*c^3*d^2*g^3*i*n^2-27*B*a*b^4*c^4*d*g^3*i*n^2-180*A*a^4*
b*c*d^4*g^3*i*n-300*A*a^3*b^2*c^2*d^3*g^3*i*n-6*B*a^5*d^5*g^3*i*n^2+6*B*b^5
*c^5*g^3*i*n^2)/d^4/n/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(213) = 426$.

Time = 0.48 (sec) , antiderivative size = 720, normalized size of antiderivative = 3.23

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{24\,Ab^5d^5g^3ix^5+6\,(5\,Ba^4bcd^4-Ba^5d^5)g^3in\log{(bx+a)}+6\,(Bb^5c^5-5\,Bab^4c^4d+10\,Ba^2b^3c^3d^2-10\,Ba}{}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, alg
orithm="fricas")
```

```
[Out] 1/120*(24*A*b^5*d^5*g^3*i*x^5 + 6*(5*B*a^4*b*c*d^4 - B*a^5*d^5)*g^3*i*n*log
(b*x + a) + 6*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 - 10*B*a^
3*b^2*c^2*d^3)*g^3*i*n*log(d*x + c) - 6*((B*b^5*c*d^4 - B*a*b^4*d^5)*g^3*i*
n - 5*(A*b^5*c*d^4 + 3*A*a*b^4*d^5)*g^3*i)*x^4 - 2*((B*b^5*c^2*d^3 + 10*B*a
*b^4*c*d^4 - 11*B*a^2*b^3*d^5)*g^3*i*n - 60*(A*a*b^4*c*d^4 + A*a^2*b^3*d^5)
*g^3*i)*x^3 + 3*((B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 - 5*B*a^2*b^3*c*d^4 + 9
*B*a^3*b^2*d^5)*g^3*i*n + 20*(3*A*a^2*b^3*c*d^4 + A*a^3*b^2*d^5)*g^3*i)*x^2
 + 6*(20*A*a^3*b^2*c*d^4*g^3*i - (B*b^5*c^4*d - 5*B*a*b^4*c^3*d^2 + 10*B*a^
2*b^3*c^2*d^3 - 5*B*a^3*b^2*c*d^4 - B*a^4*b*d^5)*g^3*i*n)*x + 6*(4*B*b^5*d^
5*g^3*i*x^5 + 20*B*a^3*b^2*c*d^4*g^3*i*x + 5*(B*b^5*c*d^4 + 3*B*a*b^4*d^5)*
g^3*i*x^4 + 20*(B*a*b^4*c*d^4 + B*a^2*b^3*d^5)*g^3*i*x^3 + 10*(3*B*a^2*b^3*
c*d^4 + B*a^3*b^2*d^5)*g^3*i*x^2)*log(e) + 6*(4*B*b^5*d^5*g^3*i*n*x^5 + 20*
B*a^3*b^2*c*d^4*g^3*i*n*x + 5*(B*b^5*c*d^4 + 3*B*a*b^4*d^5)*g^3*i*n*x^4 + 2
0*(B*a*b^4*c*d^4 + B*a^2*b^3*d^5)*g^3*i*n*x^3 + 10*(3*B*a^2*b^3*c*d^4 + B*a
^3*b^2*d^5)*g^3*i*n*x^2)*log((b*x + a)/(d*x + c)))/(b^2*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**3*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1118 vs. $2(213) = 426$.

Time = 0.22 (sec) , antiderivative size = 1118, normalized size of antiderivative = 5.01

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, alg
orithm="maxima")
[Out] 1/5*B*b^3*d*g^3*i*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A*b^3*d*
g^3*i*x^5 + 1/4*B*b^3*c*g^3*i*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) +
3/4*B*a*b^2*d*g^3*i*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*b^3*
c*g^3*i*x^4 + 3/4*A*a*b^2*d*g^3*i*x^4 + B*a*b^2*c*g^3*i*x^3*log(e*(b*x/(d*x
 + c) + a/(d*x + c))^n) + B*a^2*b*d*g^3*i*x^3*log(e*(b*x/(d*x + c) + a/(d*x
 + c))^n) + A*a*b^2*c*g^3*i*x^3 + A*a^2*b*d*g^3*i*x^3 + 3/2*B*a^2*b*c*g^3*i
*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*B*a^3*d*g^3*i*x^2*log(e*(
b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*a^2*b*c*g^3*i*x^2 + 1/2*A*a^3*d*g^3
*i*x^2 + 1/60*B*b^3*d*g^3*i*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c
)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3
+ 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) - 1/
24*B*b^3*c*g^3*i*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b
^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^
3*d^3)*x)/(b^3*d^3)) - 1/8*B*a*b^2*d*g^3*i*n*(6*a^4*log(b*x + a)/b^4 - 6*c^
4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*
d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*a*b^2*c*g^3*i*n*(2*a
^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2
*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + 1/2*B*a^2*b*d*g^3*i*n*(2*a^3*log(b*x +
 a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 -
a^2*d^2)*x)/(b^2*d^2)) - 3/2*B*a^2*b*c*g^3*i*n*(a^2*log(b*x + a)/b^2 - c^2*
log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) - 1/2*B*a^3*d*g^3*i*n*(a^2*log(b*x
+ a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*a^3*c*g^3*i*n*(a
*log(b*x + a)/b - c*log(d*x + c)/d) + B*a^3*c*g^3*i*x*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n) + A*a^3*c*g^3*i*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3945 vs. 2(213) = 426.

Time = 1.23 (sec) , antiderivative size = 3945, normalized size of antiderivative = 17.69

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, alg
orithm="giac")
```

```
[Out] -1/120*(6*(B*b^9*c^6*g^3*i*n - 6*B*a*b^8*c^5*d*g^3*i*n - 5*(b*x + a)*B*b^8*
c^6*d*g^3*i*n/(d*x + c) + 15*B*a^2*b^7*c^4*d^2*g^3*i*n + 30*(b*x + a)*B*a*b
^7*c^5*d^2*g^3*i*n/(d*x + c) + 10*(b*x + a)^2*B*b^7*c^6*d^2*g^3*i*n/(d*x +
c)^2 - 20*B*a^3*b^6*c^3*d^3*g^3*i*n - 75*(b*x + a)*B*a^2*b^6*c^4*d^3*g^3*i*
n/(d*x + c) - 60*(b*x + a)^2*B*a*b^6*c^5*d^3*g^3*i*n/(d*x + c)^2 - 10*(b*x
+ a)^3*B*b^6*c^6*d^3*g^3*i*n/(d*x + c)^3 + 15*B*a^4*b^5*c^2*d^4*g^3*i*n + 1
00*(b*x + a)*B*a^3*b^5*c^3*d^4*g^3*i*n/(d*x + c) + 150*(b*x + a)^2*B*a^2*b^
5*c^4*d^4*g^3*i*n/(d*x + c)^2 + 60*(b*x + a)^3*B*a*b^5*c^5*d^4*g^3*i*n/(d*x
 + c)^3 - 6*B*a^5*b^4*c*d^5*g^3*i*n - 75*(b*x + a)*B*a^4*b^4*c^2*d^5*g^3*i*
n/(d*x + c) - 200*(b*x + a)^2*B*a^3*b^4*c^3*d^5*g^3*i*n/(d*x + c)^2 - 150*(
b*x + a)^3*B*a^2*b^4*c^4*d^5*g^3*i*n/(d*x + c)^3 + B*a^6*b^3*d^6*g^3*i*n +
30*(b*x + a)*B*a^5*b^3*c*d^6*g^3*i*n/(d*x + c) + 150*(b*x + a)^2*B*a^4*b^3*
c^2*d^6*g^3*i*n/(d*x + c)^2 + 200*(b*x + a)^3*B*a^3*b^3*c^3*d^6*g^3*i*n/(d*
x + c)^3 - 5*(b*x + a)*B*a^6*b^2*d^7*g^3*i*n/(d*x + c) - 60*(b*x + a)^2*B*a
^5*b^2*c*d^7*g^3*i*n/(d*x + c)^2 - 150*(b*x + a)^3*B*a^4*b^2*c^2*d^7*g^3*i*
n/(d*x + c)^3 + 10*(b*x + a)^2*B*a^6*b*d^8*g^3*i*n/(d*x + c)^2 + 60*(b*x +
a)^3*B*a^5*b*c*d^8*g^3*i*n/(d*x + c)^3 - 10*(b*x + a)^3*B*a^6*d^9*g^3*i*n/(
d*x + c)^3)*log((b*x + a)/(d*x + c))/(b^5*d^4 - 5*(b*x + a)*b^4*d^5/(d*x +
c) + 10*(b*x + a)^2*b^3*d^6/(d*x + c)^2 - 10*(b*x + a)^3*b^2*d^7/(d*x + c)^
3 + 5*(b*x + a)^4*b*d^8/(d*x + c)^4 - (b*x + a)^5*d^9/(d*x + c)^5) + (5*B*b
^10*c^6*g^3*i*n - 30*B*a*b^9*c^5*d*g^3*i*n - 19*(b*x + a)*B*b^9*c^6*d*g^3*i
*n/(d*x + c) + 75*B*a^2*b^8*c^4*d^2*g^3*i*n + 114*(b*x + a)*B*a*b^8*c^5*d^2
*g^3*i*n/(d*x + c) + 23*(b*x + a)^2*B*b^8*c^6*d^2*g^3*i*n/(d*x + c)^2 - 100
*B*a^3*b^7*c^3*d^3*g^3*i*n - 285*(b*x + a)*B*a^2*b^7*c^4*d^3*g^3*i*n/(d*x +
 c) - 138*(b*x + a)^2*B*a*b^7*c^5*d^3*g^3*i*n/(d*x + c)^2 - 3*(b*x + a)^3*B
*b^7*c^6*d^3*g^3*i*n/(d*x + c)^3 + 75*B*a^4*b^6*c^2*d^4*g^3*i*n + 380*(b*x
+ a)*B*a^3*b^6*c^3*d^4*g^3*i*n/(d*x + c) + 345*(b*x + a)^2*B*a^2*b^6*c^4*d^
4*g^3*i*n/(d*x + c)^2 + 18*(b*x + a)^3*B*a*b^6*c^5*d^4*g^3*i*n/(d*x + c)^3
- 6*(b*x + a)^4*B*b^6*c^6*d^4*g^3*i*n/(d*x + c)^4 - 30*B*a^5*b^5*c*d^5*g^3*
i*n - 285*(b*x + a)*B*a^4*b^5*c^2*d^5*g^3*i*n/(d*x + c) - 460*(b*x + a)^2*B
*a^3*b^5*c^3*d^5*g^3*i*n/(d*x + c)^2 - 45*(b*x + a)^3*B*a^2*b^5*c^4*d^5*g^3
*i*n/(d*x + c)^3 + 36*(b*x + a)^4*B*a*b^5*c^5*d^5*g^3*i*n/(d*x + c)^4 + 5*B
*a^6*b^4*d^6*g^3*i*n + 114*(b*x + a)*B*a^5*b^4*c*d^6*g^3*i*n/(d*x + c) + 34
5*(b*x + a)^2*B*a^4*b^4*c^2*d^6*g^3*i*n/(d*x + c)^2 + 60*(b*x + a)^3*B*a^3*
```

b^4*c^3*d^6*g^3*i*n/(d*x + c)^3 - 90*(b*x + a)^4*B*a^2*b^4*c^4*d^6*g^3*i*n/(d*x + c)^4 - 19*(b*x + a)*B*a^6*b^3*d^7*g^3*i*n/(d*x + c) - 138*(b*x + a)^2*B*a^5*b^3*c*d^7*g^3*i*n/(d*x + c)^2 - 45*(b*x + a)^3*B*a^4*b^3*c^2*d^7*g^3*i*n/(d*x + c)^3 + 120*(b*x + a)^4*B*a^3*b^3*c^3*d^7*g^3*i*n/(d*x + c)^4 + 23*(b*x + a)^2*B*a^6*b^2*d^8*g^3*i*n/(d*x + c)^2 + 18*(b*x + a)^3*B*a^5*b^2*c*d^8*g^3*i*n/(d*x + c)^3 - 90*(b*x + a)^4*B*a^4*b^2*c^2*d^8*g^3*i*n/(d*x + c)^4 - 3*(b*x + a)^3*B*a^6*b*d^9*g^3*i*n/(d*x + c)^3 + 36*(b*x + a)^4*B*a^5*b*c*d^9*g^3*i*n/(d*x + c)^4 - 6*(b*x + a)^4*B*a^6*d^10*g^3*i*n/(d*x + c)^4 + 6*B*b^10*c^6*g^3*i*log(e) - 36*B*a*b^9*c^5*d*g^3*i*log(e) - 30*(b*x + a)*B*b^9*c^6*d*g^3*i*log(e)/(d*x + c) + 90*B*a^2*b^8*c^4*d^2*g^3*i*log(e) + 180*(b*x + a)*B*a*b^8*c^5*d^2*g^3*i*log(e)/(d*x + c) + 60*(b*x + a)^2*B*b^8*c^6*d^2*g^3*i*log(e)/(d*x + c)^2 - 120*B*a^3*b^7*c^3*d^3*g^3*i*log(e) - 450*(b*x + a)*B*a^2*b^7*c^4*d^3*g^3*i*log(e)/(d*x + c) - 360*(b*x + a)^2*B*a*b^7*c^5*d^3*g^3*i*log(e)/(d*x + c)^2 - 60*(b*x + a)^3*B*b^7*c^6*d^3*g^3*i*log(e)/(d*x + c)^3 + 90*B*a^4*b^6*c^2*d^4*g^3*i*log(e) + 600*(b*x + a)*B*a^3*b^6*c^3*d^4*g^3*i*log(e)/(d*x + c) + 900*(b*x + a)^2*B*a^2*b^6*c^4*d^4*g^3*i*log(e)/(d*x + c)^2 + 360*(b*x + a)^3*B*a*b^6*c^5*d^4*g^3*i*log(e)/(d*x + c)^3 - 36*B*a^5*b^5*c*d^5*g^3*i*log(e) - 450*(b*x + a)*B*a^4*b^5*c^2*d^5*g^3*i*log(e)/(d*x + c) - 1200*(b*x + a)^2*B*a^3*b^5*c^3*d^5*g^3*i*log(e)/(d*x + c)^2 - 900*(b*x + a)^3*B*a^2*b^5*c^4*d^5*g^3*i*log(e)/(d*x + c)^3 + 6*B*a^6*b^4*d^6*g^3*i*log(e) + 180*(b*x + a)*B*a^5*b^4*c*d^6*g^3*i*log(e)/(d*x + c) + 900*(b*x + a)^2*B*a^4*b^4*c^2*d^6*g^3*i*log(e)/(d*x + c)^2 + 1200*(b*x + a)^3*B*a^3*b^4*c^3*d^6*g^3*i*log(e)/(d*x + c)^3 - 30*(b*x + a)*B*a^6*b^3*d^7*g^3*i*log(e)/(d*x + c) - 360*(b*x + a)^2*B*a^5*b^3*c*d^7*g^3*i*log(e)/(d*x + c)^2 - 900*(b*x + a)^3*B*a^4*b^3*c^2*d^7*g^3*i*log(e)/(d*x + c)^3 + 60*(b*x + a)^2*B*a^6*b^2*d^8*g^3*i*log(e)/(d*x + c)^2 + 360*(b*x + a)^3*B*a^5*b^2*c*d^8*g^3*i*log(e)/(d*x + c)^3 - 60*(b*x + a)^3*B*a^6*b*d^9*g^3*i*log(e)/(d*x + c)^3 + 6*A*b^10*c^6*g^3*i - 36*A*a*b^9*c^5*d*g^3*i - 30*(b*x + a)*A*b^9*c^6*d*g^3*i/(d*x + c) + 90*A*a^2*b^8*c^4*d^2*g^3*i + 180*(b*x + a)*A*a*b^8*c^5*d^2*g^3*i/(d*x + c) + 60*(b*x + a)^2*A*b^8*c^6*d^2*g^3*i/(d*x + c)^2 - 120*A*a^3*b^7*c^3*d^3*g^3*i - 450*(b*x + a)*A*a^2*b^7*c^4*d^3*g^3*i/(d*x + c) - 360*(b*x + a)^2*A*a*b^7*c^5*d^3*g^3*i/(d*x + c)^2 - 60*(b*x + a)^3*A*b^7*c^6*d^3*g^3*i/(d*x + c)^3 + 90*A*a^4*b^6*c^2*d^4*g^3*i + 600*(b*x + a)*A*a^3*b^6*c^3*d^4*g^3*i/(d*x + c) + 900*(b*x + a)^2*A*a^2*b^6*c^4*d^4*g^3*i/(d*x + c)^2 + 360*(b*x + a)^3*A*a*b^6*c^5*d^4*g^3*i/(d*x + c)^3 - 36*A*a^5*b^5*c*d^5*g^3*i - 450*(b*x + a)*A*a^4*b^5*c^2*d^5*g^3*i/(d*x + c) - 1200*(b*x + a)^2*A*a^3*b^5*c^3*d^5*g^3*i/(d*x + c)^2 - 900*(b*x + a)^3*A*a^2*b^5*c^4*d^5*g^3*i/(d*x + c)^3 + 6*A*a^6*b^4*d^6*g^3*i + 180*(b*x + a)*A*a^5*b^4*c*d^6*g^3*i/(d*x + c) + 900*(b*x + a)^2*A*a^4*b^4*c^2*d^6*g^3*i/(d*x + c)^2 + 1200*(b*x + a)^3*A*a^3*b^4*c^3*d^6*g^3*i/(d*x + c)^3 - 30*(b*x + a)*A*a^6*b^3*d^7*g^3*i/(d*x + c) - 360*(b*x + a)^2*A*a^5*b^3*c*d^7*g^3*i/(d*x + c)^2 - 900*(b*x + a)^3*A*a^4*b^3*c^2*d^7*g^3*i/(d*x + c)^3 + 60*(b*x + a)^2*A*a^6*b^2*d^8*g^3*i/(d*x + c)^2 + 360*(b*x + a)^3*A*a^5*b^2*c*d^8*g^3*i/(d*x + c)^3 - 60*(b*x + a)^3*A*a^6*b*d^9*g^3*i/(d*x + c)^3)/(b^6*d^4 - 5*(b*x + a)*b^5*d^5/(d*x + c) + 10*(b*x + a)^2*b^4*d^6/(d*x + c)^2 -

```
10*(b*x + a)^3*b^3*d^7/(d*x + c)^3 + 5*(b*x + a)^4*b^2*d^8/(d*x + c)^4 - (b
*x + a)^5*b*d^9/(d*x + c)^5) + 6*(B*b^6*c^6*g^3*i*n - 6*B*a*b^5*c^5*d*g^3*i
*n + 15*B*a^2*b^4*c^4*d^2*g^3*i*n - 20*B*a^3*b^3*c^3*d^3*g^3*i*n + 15*B*a^4
*b^2*c^2*d^4*g^3*i*n - 6*B*a^5*b*c*d^5*g^3*i*n + B*a^6*d^6*g^3*i*n)*log(b -
 (b*x + a)*d/(d*x + c))/(b^2*d^4) - 6*(B*b^6*c^6*g^3*i*n - 6*B*a*b^5*c^5*d*
g^3*i*n + 15*B*a^2*b^4*c^4*d^2*g^3*i*n - 20*B*a^3*b^3*c^3*d^3*g^3*i*n + 15*
B*a^4*b^2*c^2*d^4*g^3*i*n - 6*B*a^5*b*c*d^5*g^3*i*n + B*a^6*d^6*g^3*i*n)*lo
g((b*x + a)/(d*x + c))/(b^2*d^4))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 1237, normalized size of antiderivative = 5.55

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

```
[In] int((a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x*((a*c*(((20*a*d + 20*b*c)*((b^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d*n - B*
b*c*n))/5 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/20))/(20*b*d) - (b*g^3*i*(24*A*
a^2*d^2 + 4*A*b^2*c^2 + 3*B*a^2*d^2*n - B*b^2*c^2*n + 32*A*a*b*c*d - 2*B*a*
b*c*d*n))/(4*d) + A*a*b^2*c*g^3*i))/(b*d) - ((20*a*d + 20*b*c)*(((20*a*d +
20*b*c)*(((20*a*d + 20*b*c)*((b^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d*n - B*
b*c*n))/5 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/20))/(20*b*d) - (b*g^3*i*(24*A*
a^2*d^2 + 4*A*b^2*c^2 + 3*B*a^2*d^2*n - B*b^2*c^2*n + 32*A*a*b*c*d - 2*B*a*
b*c*d*n))/(4*d) + A*a*b^2*c*g^3*i))/(20*b*d) - (a*c*((b^2*g^3*i*(20*A*a*d +
 10*A*b*c + B*a*d*n - B*b*c*n))/5 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/20))/(b
*d) + (a*g^3*i*(4*A*a^2*d^2 + 4*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + 12*
A*a*b*c*d))/d))/(20*b*d) + (a^2*g^3*i*(2*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d
^2*n - 3*B*b^2*c^2*n + 16*A*a*b*c*d + 2*B*a*b*c*d*n))/(2*b*d)) + x^2*(((20*
a*d + 20*b*c)*(((20*a*d + 20*b*c)*((b^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d*
n - B*b*c*n))/5 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/20))/(20*b*d) - (b*g^3*i*
(24*A*a^2*d^2 + 4*A*b^2*c^2 + 3*B*a^2*d^2*n - B*b^2*c^2*n + 32*A*a*b*c*d -
2*B*a*b*c*d*n))/(4*d) + A*a*b^2*c*g^3*i))/(40*b*d) - (a*c*((b^2*g^3*i*(20*A
*a*d + 10*A*b*c + B*a*d*n - B*b*c*n))/5 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/2
0))/(2*b*d) + (a*g^3*i*(4*A*a^2*d^2 + 4*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2
*n + 12*A*a*b*c*d))/(2*d)) - x^3*(((20*a*d + 20*b*c)*((b^2*g^3*i*(20*A*a*d
+ 10*A*b*c + B*a*d*n - B*b*c*n))/5 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/20))/(
60*b*d) - (b*g^3*i*(24*A*a^2*d^2 + 4*A*b^2*c^2 + 3*B*a^2*d^2*n - B*b^2*c^2*
n + 32*A*a*b*c*d - 2*B*a*b*c*d*n))/(12*d) + (A*a*b^2*c*g^3*i)/3) + log(e*((
a + b*x)/(c + d*x))^n)*((B*a^2*g^3*i*x^2*(a*d + 3*b*c))/2 + (B*b^2*g^3*i*x^
4*(3*a*d + b*c))/4 + B*a^3*c*g^3*i*x + (B*b^3*d*g^3*i*x^5)/5 + B*a*b*g^3*i*
x^3*(a*d + b*c)) + x^4*((b^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d*n - B*b*c*n
))/20 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/80) - (log(a + b*x)*(B*a^5*d*g^3*i*
n - 5*B*a^4*b*c*g^3*i*n))/(20*b^2) + (log(c + d*x)*(B*b^3*c^5*g^3*i*n - 10*
```

```
B*a^3*c^2*d^3*g^3*i*n - 5*B*a*b^2*c^4*d*g^3*i*n + 10*B*a^2*b*c^3*d^2*g^3*i*
n))/(20*d^4) + (A*b^3*d*g^3*i*x^5)/5
```

3.109 $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 41, antiderivative size = 190

$$\begin{aligned}&\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx\\&=\frac{B(bc-ad)^3g^2inx}{12bd^2}-\frac{B(bc-ad)^2g^2in(a+bx)^2}{24b^2d}\\&\quad+\frac{g^2i(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b}\\&\quad+\frac{(bc-ad)g^2i(a+bx)^3\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12b^2}-\frac{B(bc-ad)^4g^2in\log(c+dx)}{12b^2d^3}\end{aligned}$$

```
[Out] 1/12*B*(-a*d+b*c)^3*g^2*i*n*x/b/d^2-1/24*B*(-a*d+b*c)^2*g^2*i*n*(b*x+a)^2/b
^2/d+1/4*g^2*i*(b*x+a)^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b+1/12*(-a
*d+b*c)*g^2*i*(b*x+a)^3*(A-B*n+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2-1/12*B*(-a*
d+b*c)^4*g^2*i*n*ln(d*x+c)/b^2/d^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {2559, 2547, 21, 45}

$$\begin{aligned}&\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx\\&=\frac{g^2i(a+bx)^3(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{12b^2}\\&\quad+\frac{g^2i(a+bx)^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}-\frac{Bg^2in(bc-ad)^4\log(c+dx)}{12b^2d^3}\\&\quad-\frac{Bg^2in(a+bx)^2(bc-ad)^2}{24b^2d}+\frac{Bg^2inx(bc-ad)^3}{12bd^2}\end{aligned}$$

```
[In] Int[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (B*(b*c - a*d)^3*g^2*i*n*x)/(12*b*d^2) - (B*(b*c - a*d)^2*g^2*i*n*(a + b*x)
^2)/(24*b^2*d) + (g^2*i*(a + b*x)^3*(c + d*x)*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(4*b) + ((b*c - a*d)*g^2*i*(a + b*x)^3*(A - B*n + B*Log[e*((a +
b*x)/(c + d*x))^n]))/(12*b^2) - (B*(b*c - a*d)^4*g^2*i*n*Log[c + d*x])/(12*
b^2*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rule 2559

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Simp[(f
 + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 2
))), x] + Dist[i*((b*c - a*d)/(b*d*(m + 2))), Int[(f + g*x)^m*(A - B*n + B*
Log[e*((a + b*x)/(c + d*x))^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i
, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*
i, 0] && IGtQ[m, -2]
```

Rubi steps

$$\text{integral} = \frac{g^2 i (a+bx)^3 (c+dx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b} + \frac{((bc-ad)i)\int (ag+bgx)^2 \left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx}{4b}$$

$$= \frac{g^2 i(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b} + \frac{(bc-ad)g^2 i(a+bx)^3\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12b^2} - \frac{(B(bc-ad)^2 in)\int \frac{(ag+bgx)^3}{(a+bx)(c+dx)}\,dx}{12b^2 g}$$

$$= \frac{g^2 i(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b} + \frac{(bc-ad)g^2 i(a+bx)^3\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12b^2} - \frac{(B(bc-ad)^2 g^2 in)\int \frac{(a+bx)^2}{c+dx}\,dx}{12b^2}$$

$$= \frac{g^2 i(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b} + \frac{(bc-ad)g^2 i(a+bx)^3\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12b^2} - \frac{(B(bc-ad)^2 g^2 in)\int \left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx}{12b^2}$$

$$= \frac{B(bc-ad)^3 g^2 inx}{12bd^2} - \frac{B(bc-ad)^2 g^2 in(a+bx)^2}{24b^2 d} + \frac{g^2 i(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b} + \frac{(bc-ad)g^2 i(a+bx)^3\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12b^2} - \frac{B(bc-ad)^4 g^2 in\log(c+dx)}{12b^2 d^3}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.18

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$= \frac{g^2 i\Big(8(bc-ad)(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+6d(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{4B(bc-ad)^2n(2bd(b\cdots}{\cdots}}{24b^2\cdots}$$

```
[In] Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^
n]),x]
```

```
[Out] (g^2*i*(8*(b*c - a*d)*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) +
6*d*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (4*B*(b*c - a*d)^2
```

```
*n*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]))/
d^3 - (B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b
*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/d^3))/(24*b^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 875 vs. 2(180) = 360.

Time = 5.00 (sec) , antiderivative size = 876, normalized size of antiderivative = 4.61

method	result
parallelrisch	$\frac{2B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4c^4g^2in+6A\,x^4b^4d^4g^2in-2B\ln(bx+a)b^4c^4g^2i\,n^2-2B\ln(bx+a)a^4d^4g^2i\,n^2+24B\,x^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a\,b^3c\,d^3}{}$

```
[In] int((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/24*(2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^4*g^2*i*n+6*A*x^4*b^4*d^4*g^2*i*n
-2*B*ln(b*x+a)*b^4*c^4*g^2*i*n^2-2*B*ln(b*x+a)*a^4*d^4*g^2*i*n^2+24*B*x^2*l
n(e*((b*x+a)/(d*x+c))^n)*a*b^3*c*d^3*g^2*i*n+24*B*x*ln(e*((b*x+a)/(d*x+c))^
n)*a^2*b^2*c*d^3*g^2*i*n+16*A*x^3*a*b^3*d^4*g^2*i*n+8*A*x^3*b^4*c*d^3*g^2*i
*n+5*B*x^2*a^2*b^2*d^4*g^2*i*n^2-B*x^2*b^4*c^2*d^2*g^2*i*n^2+12*A*x^2*a^2*b
^2*d^4*g^2*i*n+2*B*x*a^3*b*d^4*g^2*i*n^2+2*B*x*b^4*c^3*d*g^2*i*n^2+6*B*x^4*
ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^4*g^2*i*n-12*B*ln(b*x+a)*a^2*b^2*c^2*d^2*g^
2*i*n^2+8*B*ln(b*x+a)*a*b^3*c^3*d*g^2*i*n^2-8*B*x*a*b^3*c^2*d^2*g^2*i*n^2+2
4*A*x*a^2*b^2*c*d^3*g^2*i*n+8*B*ln(b*x+a)*a^3*b*c*d^3*g^2*i*n^2+12*B*ln(e*(
(b*x+a)/(d*x+c))^n)*a^2*b^2*c^2*d^2*g^2*i*n-8*B*ln(e*((b*x+a)/(d*x+c))^n)*a
*b^3*c^3*d*g^2*i*n+16*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*d^4*g^2*i*n+8*B
*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^3*g^2*i*n+12*B*x^2*ln(e*((b*x+a)/(d*
x+c))^n)*a^2*b^2*d^4*g^2*i*n-4*B*x^2*a*b^3*c*d^3*g^2*i*n^2+24*A*x^2*a*b^3*c
*d^3*g^2*i*n+4*B*x*a^2*b^2*c*d^3*g^2*i*n^2-11*B*a^3*b*c*d^3*g^2*i*n^2+8*B*a
^2*b^2*c^2*d^2*g^2*i*n^2+7*B*a*b^3*c^3*d*g^2*i*n^2-36*A*a^3*b*c*d^3*g^2*i*n
-48*A*a^2*b^2*c^2*d^2*g^2*i*n+2*B*x^3*a*b^3*d^4*g^2*i*n^2-2*B*x^3*b^4*c*d^3
*g^2*i*n^2-2*B*b^4*c^4*g^2*i*n^2-2*B*a^4*d^4*g^2*i*n^2)/b^2/d^3/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(182) = 364.

Time = 0.43 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.78

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{6\,Ab^4d^4g^2ix^4+2\,(4\,Ba^3bcd^3-Ba^4d^4)g^2in\log(bx+a)-2\,(Bb^4c^4-4\,Bab^3c^3d+6\,Ba^2b^2c^2d^2)g^2in\log(dx}{}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, alg
orithm="fricas")
```

```
[Out] 1/24*(6*A*b^4*d^4*g^2*i*x^4 + 2*(4*B*a^3*b*c*d^3 - B*a^4*d^4)*g^2*i*n*log(b
*x + a) - 2*(B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2)*g^2*i*n*log
(d*x + c) - 2*((B*b^4*c*d^3 - B*a*b^3*d^4)*g^2*i*n - 4*(A*b^4*c*d^3 + 2*A*a
*b^3*d^4)*g^2*i)*x^3 - ((B*b^4*c^2*d^2 + 4*B*a*b^3*c*d^3 - 5*B*a^2*b^2*d^4)
*g^2*i*n - 12*(2*A*a*b^3*c*d^3 + A*a^2*b^2*d^4)*g^2*i)*x^2 + 2*(12*A*a^2*b^
2*c*d^3*g^2*i + (B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 2*B*a^2*b^2*c*d^3 + B*a^
3*b*d^4)*g^2*i*n)*x + 2*(3*B*b^4*d^4*g^2*i*x^4 + 12*B*a^2*b^2*c*d^3*g^2*i*x
 + 4*(B*b^4*c*d^3 + 2*B*a*b^3*d^4)*g^2*i*x^3 + 6*(2*B*a*b^3*c*d^3 + B*a^2*b
^2*d^4)*g^2*i*x^2)*log(e) + 2*(3*B*b^4*d^4*g^2*i*n*x^4 + 12*B*a^2*b^2*c*d^3
*g^2*i*n*x + 4*(B*b^4*c*d^3 + 2*B*a*b^3*d^4)*g^2*i*n*x^3 + 6*(2*B*a*b^3*c*d
^3 + B*a^2*b^2*d^4)*g^2*i*n*x^2)*log((b*x + a)/(d*x + c)))/(b^2*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1013 vs. $2(173) = 346$.

Time = 84.02 (sec) , antiderivative size = 1013, normalized size of antiderivative = 5.33

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\begin{cases} a^2cg^2ix\left(A+B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) \\ a^2g^2\left(Acix+\frac{Adix^2}{2}+\frac{Bc^2i\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2d}+\frac{Bcinx}{2}+Bcix\log\left(e\left(\frac{a}{c+dx}\right)^n\right)+\frac{Bdinx^2}{4}+\frac{Bdix^2\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2}\right) \\ ci\left(Aa^2g^2x+Aabg^2x^2+\frac{Ab^2g^2x^3}{3}+\frac{Ba^3g^2\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{3b}-\frac{Ba^2g^2nx}{3}+Ba^2g^2x\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)-\frac{Babg^2nx^2}{3} \cdots\right. \\ Aa^2cg^2ix+\frac{Aa^2dg^2ix^2}{2}+Aabcg^2ix^2+\frac{2Aabdg^2ix^3}{3}+\frac{Ab^2cg^2ix^3}{3}+\frac{Ab^2dg^2ix^4}{4}-\frac{Ba^4dg^2in\log\left(\frac{c}{d}+x\right)}{12b^2}-\frac{Ba^4dg^2i\log\left(e\left(\cdots\right)\right)}{12b^2} \cdots \end{cases}$$

```
[In] integrate((b*g*x+a*g)**2*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Piecewise((a**2*c*g**2*i*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (
a**2*g**2*(A*c*i*x + A*d*i*x**2/2 + B*c**2*i*log(e*(a/(c + d*x))**n)/(2*d)
+ B*c*i*n*x/2 + B*c*i*x*log(e*(a/(c + d*x))**n) + B*d*i*n*x**2/4 + B*d*i*x*
*2*log(e*(a/(c + d*x))**n)/2), Eq(b, 0)), (c*i*(A*a**2*g**2*x + A*a*b*g**2*
x**2 + A*b**2*g**2*x**3/3 + B*a**3*g**2*log(e*(a/c + b*x/c)**n)/(3*b) - B*a
**2*g**2*n*x/3 + B*a**2*g**2*x*log(e*(a/c + b*x/c)**n) - B*a*b*g**2*n*x**2/
3 + B*a*b*g**2*x**2*log(e*(a/c + b*x/c)**n) - B*b**2*g**2*n*x**3/9 + B*b**2
*g**2*x**3*log(e*(a/c + b*x/c)**n)/3), Eq(d, 0)), (A*a**2*c*g**2*i*x + A*a*
*2*d*g**2*i*x**2/2 + A*a*b*c*g**2*i*x**2 + 2*A*a*b*d*g**2*i*x**3/3 + A*b**2
*c*g**2*i*x**3/3 + A*b**2*d*g**2*i*x**4/4 - B*a**4*d*g**2*i*n*log(c/d + x)/
(12*b**2) - B*a**4*d*g**2*i*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(12*b**
```

```
2) + B*a**3*c*g**2*i*n*log(c/d + x)/(3*b) + B*a**3*c*g**2*i*log(e*(a/(c + d
*x) + b*x/(c + d*x))**n)/(3*b) + B*a**3*d*g**2*i*n*x/(12*b) - B*a**2*c**2*g
**2*i*n*log(c/d + x)/(2*d) + B*a**2*c*g**2*i*n*x/6 + B*a**2*c*g**2*i*x*log(
e*(a/(c + d*x) + b*x/(c + d*x))**n) + 5*B*a**2*d*g**2*i*n*x**2/24 + B*a**2*
d*g**2*i*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/2 + B*a*b*c**3*g**2*i
*n*log(c/d + x)/(3*d**2) - B*a*b*c**2*g**2*i*n*x/(3*d) - B*a*b*c*g**2*i*n*x
**2/6 + B*a*b*c*g**2*i*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*a*b
*d*g**2*i*n*x**3/12 + 2*B*a*b*d*g**2*i*x**3*log(e*(a/(c + d*x) + b*x/(c + d
*x))**n)/3 - B*b**2*c**4*g**2*i*n*log(c/d + x)/(12*d**3) + B*b**2*c**3*g**2
*i*n*x/(12*d**2) - B*b**2*c**2*g**2*i*n*x**2/(24*d) - B*b**2*c*g**2*i*n*x**
3/12 + B*b**2*c*g**2*i*x**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/3 + B*b
**2*d*g**2*i*x**4*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(182) = 364$.

Time = 0.21 (sec) , antiderivative size = 740, normalized size of antiderivative = 3.89

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$\begin{aligned}
&= \frac{1}{4}\,Bb^2dg^2ix^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{4}\,Ab^2dg^2ix^4 \\
&\quad+\frac{1}{3}\,Bb^2cg^2ix^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{2}{3}\,Babdg^2ix^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) \\
&\quad+\frac{1}{3}\,Ab^2cg^2ix^3+\frac{2}{3}\,Aabdg^2ix^3+Babcg^2ix^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) \\
&\quad+\frac{1}{2}\,Ba^2dg^2ix^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aabcg^2ix^2+\frac{1}{2}\,Aa^2dg^2ix^2 \\
&\quad-\frac{1}{24}\,Bb^2dg^2in\left(\frac{6\,a^4\log(bx+a)}{b^4}-\frac{6\,c^4\log(dx+c)}{d^4}+\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2d-a^2bd^3)x^2+6\,(b^3c^3\right. \\
&\quad+\frac{1}{6}\,Bb^2cg^2in\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right) \\
&\quad+\frac{1}{3}\,Babdg^2in\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right) \\
&\quad-Babcg^2in\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right) \\
&\quad-\frac{1}{2}\,Ba^2dg^2in\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right) \\
&\quad+Ba^2cg^2in\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right) \\
&\quad+Ba^2cg^2ix\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aa^2cg^2ix
\end{aligned}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, alg
orithm="maxima")
```

```
[Out] 1/4*B*b^2*d*g^2*i*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*b^2*d*
g^2*i*x^4 + 1/3*B*b^2*c*g^2*i*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) +
2/3*B*a*b*d*g^2*i*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*b^2*c*
g^2*i*x^3 + 2/3*A*a*b*d*g^2*i*x^3 + B*a*b*c*g^2*i*x^2*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n) + 1/2*B*a^2*d*g^2*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c
))^n) + A*a*b*c*g^2*i*x^2 + 1/2*A*a^2*d*g^2*i*x^2 - 1/24*B*b^2*d*g^2*i*n*(6
*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)
*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))
+ 1/6*B*b^2*c*g^2*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((
b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + 1/3*B*a*b*d*
g^2*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*
d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - B*a*b*c*g^2*i*n*(a^2*log(b
*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) - 1/2*B*a^2*d*g^2
*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) +
B*a^2*c*g^2*i*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*a^2*c*g^2*i*x*log
(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a^2*c*g^2*i*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2571 vs. 2(182) = 364.

Time = 1.32 (sec) , antiderivative size = 2571, normalized size of antiderivative = 13.53

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, alg
orithm="giac")
```

```
[Out] 1/24*(2*(B*b^7*c^5*g^2*i*n - 5*B*a*b^6*c^4*d*g^2*i*n - 4*(b*x + a)*B*b^6*c^
5*d*g^2*i*n/(d*x + c) + 10*B*a^2*b^5*c^3*d^2*g^2*i*n + 20*(b*x + a)*B*a*b^5
*c^4*d^2*g^2*i*n/(d*x + c) + 6*(b*x + a)^2*B*b^5*c^5*d^2*g^2*i*n/(d*x + c)^
2 - 10*B*a^3*b^4*c^2*d^3*g^2*i*n - 40*(b*x + a)*B*a^2*b^4*c^3*d^3*g^2*i*n/(
d*x + c) - 30*(b*x + a)^2*B*a*b^4*c^4*d^3*g^2*i*n/(d*x + c)^2 + 5*B*a^4*b^3
*c*d^4*g^2*i*n + 40*(b*x + a)*B*a^3*b^3*c^2*d^4*g^2*i*n/(d*x + c) + 60*(b*x
 + a)^2*B*a^2*b^3*c^3*d^4*g^2*i*n/(d*x + c)^2 - B*a^5*b^2*d^5*g^2*i*n - 20*
(b*x + a)*B*a^4*b^2*c*d^5*g^2*i*n/(d*x + c) - 60*(b*x + a)^2*B*a^3*b^2*c^2*
d^5*g^2*i*n/(d*x + c)^2 + 4*(b*x + a)*B*a^5*b*d^6*g^2*i*n/(d*x + c) + 30*(b
*x + a)^2*B*a^4*b*c*d^6*g^2*i*n/(d*x + c)^2 - 6*(b*x + a)^2*B*a^5*d^7*g^2*i
*n/(d*x + c)^2)*log((b*x + a)/(d*x + c))/(b^4*d^3 - 4*(b*x + a)*b^3*d^4/(d*
x + c) + 6*(b*x + a)^2*b^2*d^5/(d*x + c)^2 - 4*(b*x + a)^3*b*d^6/(d*x + c)^
3 + (b*x + a)^4*d^7/(d*x + c)^4) + (B*b^8*c^5*g^2*i*n - 5*B*a*b^7*c^4*d*g^2
*i*n - 2*(b*x + a)*B*b^7*c^5*d*g^2*i*n/(d*x + c) + 10*B*a^2*b^6*c^3*d^2*g^2
```

*i*n + 10*(b*x + a)*B*a*b^6*c^4*d^2*g^2*i*n/(d*x + c) - (b*x + a)^2*B*b^6*c
^5*d^2*g^2*i*n/(d*x + c)^2 - 10*B*a^3*b^5*c^2*d^3*g^2*i*n - 20*(b*x + a)*B*
a^2*b^5*c^3*d^3*g^2*i*n/(d*x + c) + 5*(b*x + a)^2*B*a*b^5*c^4*d^3*g^2*i*n/(
d*x + c)^2 + 2*(b*x + a)^3*B*b^5*c^5*d^3*g^2*i*n/(d*x + c)^3 + 5*B*a^4*b^4*
c*d^4*g^2*i*n + 20*(b*x + a)*B*a^3*b^4*c^2*d^4*g^2*i*n/(d*x + c) - 10*(b*x
+ a)^2*B*a^2*b^4*c^3*d^4*g^2*i*n/(d*x + c)^2 - 10*(b*x + a)^3*B*a*b^4*c^4*d
^4*g^2*i*n/(d*x + c)^3 - B*a^5*b^3*d^5*g^2*i*n - 10*(b*x + a)*B*a^4*b^3*c*d
^5*g^2*i*n/(d*x + c) + 10*(b*x + a)^2*B*a^3*b^3*c^2*d^5*g^2*i*n/(d*x + c)^2
 + 20*(b*x + a)^3*B*a^2*b^3*c^3*d^5*g^2*i*n/(d*x + c)^3 + 2*(b*x + a)*B*a^5
*b^2*d^6*g^2*i*n/(d*x + c) - 5*(b*x + a)^2*B*a^4*b^2*c*d^6*g^2*i*n/(d*x + c
)^2 - 20*(b*x + a)^3*B*a^3*b^2*c^2*d^6*g^2*i*n/(d*x + c)^3 + (b*x + a)^2*B*
a^5*b*d^7*g^2*i*n/(d*x + c)^2 + 10*(b*x + a)^3*B*a^4*b*c*d^7*g^2*i*n/(d*x +
 c)^3 - 2*(b*x + a)^3*B*a^5*d^8*g^2*i*n/(d*x + c)^3 + 2*B*b^8*c^5*g^2*i*log
(e) - 10*B*a*b^7*c^4*d*g^2*i*log(e) - 8*(b*x + a)*B*b^7*c^5*d*g^2*i*log(e)/
(d*x + c) + 20*B*a^2*b^6*c^3*d^2*g^2*i*log(e) + 40*(b*x + a)*B*a*b^6*c^4*d^
2*g^2*i*log(e)/(d*x + c) + 12*(b*x + a)^2*B*b^6*c^5*d^2*g^2*i*log(e)/(d*x +
 c)^2 - 20*B*a^3*b^5*c^2*d^3*g^2*i*log(e) - 80*(b*x + a)*B*a^2*b^5*c^3*d^3*
g^2*i*log(e)/(d*x + c) - 60*(b*x + a)^2*B*a*b^5*c^4*d^3*g^2*i*log(e)/(d*x +
 c)^2 + 10*B*a^4*b^4*c*d^4*g^2*i*log(e) + 80*(b*x + a)*B*a^3*b^4*c^2*d^4*g^
2*i*log(e)/(d*x + c) + 120*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^2*i*log(e)/(d*x
+ c)^2 - 2*B*a^5*b^3*d^5*g^2*i*log(e) - 40*(b*x + a)*B*a^4*b^3*c*d^5*g^2*i*
log(e)/(d*x + c) - 120*(b*x + a)^2*B*a^3*b^3*c^2*d^5*g^2*i*log(e)/(d*x + c)
^2 + 8*(b*x + a)*B*a^5*b^2*d^6*g^2*i*log(e)/(d*x + c) + 60*(b*x + a)^2*B*a^
4*b^2*c*d^6*g^2*i*log(e)/(d*x + c)^2 - 12*(b*x + a)^2*B*a^5*b*d^7*g^2*i*log
(e)/(d*x + c)^2 + 2*A*b^8*c^5*g^2*i - 10*A*a*b^7*c^4*d*g^2*i - 8*(b*x + a)*
A*b^7*c^5*d*g^2*i/(d*x + c) + 20*A*a^2*b^6*c^3*d^2*g^2*i + 40*(b*x + a)*A*a
*b^6*c^4*d^2*g^2*i/(d*x + c) + 12*(b*x + a)^2*A*b^6*c^5*d^2*g^2*i/(d*x + c)
^2 - 20*A*a^3*b^5*c^2*d^3*g^2*i - 80*(b*x + a)*A*a^2*b^5*c^3*d^3*g^2*i/(d*x
 + c) - 60*(b*x + a)^2*A*a*b^5*c^4*d^3*g^2*i/(d*x + c)^2 + 10*A*a^4*b^4*c*d
^4*g^2*i + 80*(b*x + a)*A*a^3*b^4*c^2*d^4*g^2*i/(d*x + c) + 120*(b*x + a)^2
*A*a^2*b^4*c^3*d^4*g^2*i/(d*x + c)^2 - 2*A*a^5*b^3*d^5*g^2*i - 40*(b*x + a)
*A*a^4*b^3*c*d^5*g^2*i/(d*x + c) - 120*(b*x + a)^2*A*a^3*b^3*c^2*d^5*g^2*i/
(d*x + c)^2 + 8*(b*x + a)*A*a^5*b^2*d^6*g^2*i/(d*x + c) + 60*(b*x + a)^2*A*
a^4*b^2*c*d^6*g^2*i/(d*x + c)^2 - 12*(b*x + a)^2*A*a^5*b*d^7*g^2*i/(d*x + c
)^2)/(b^5*d^3 - 4*(b*x + a)*b^4*d^4/(d*x + c) + 6*(b*x + a)^2*b^3*d^5/(d*x
+ c)^2 - 4*(b*x + a)^3*b^2*d^6/(d*x + c)^3 + (b*x + a)^4*b*d^7/(d*x + c)^4)
 + 2*(B*b^5*c^5*g^2*i*n - 5*B*a*b^4*c^4*d*g^2*i*n + 10*B*a^2*b^3*c^3*d^2*g^
2*i*n - 10*B*a^3*b^2*c^2*d^3*g^2*i*n + 5*B*a^4*b*c*d^4*g^2*i*n - B*a^5*d^5*
g^2*i*n)*log(-b + (b*x + a)*d/(d*x + c))/(b^2*d^3) - 2*(B*b^5*c^5*g^2*i*n -
 5*B*a*b^4*c^4*d*g^2*i*n + 10*B*a^2*b^3*c^3*d^2*g^2*i*n - 10*B*a^3*b^2*c^2*
d^3*g^2*i*n + 5*B*a^4*b*c*d^4*g^2*i*n - B*a^5*d^5*g^2*i*n)*log((b*x + a)/(d
*x + c))/(b^2*d^3))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)

Mupad [B] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 663, normalized size of antiderivative = 3.49

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(B\,a^2\,c\,g^2\,i\,x+\frac{B\,a\,g^2\,i\,x^2\,(a\,d+2\,b\,c)}{2}+\frac{B\,b\,g^2\,i\,x^3\,(2\,a\,d+b\,c)}{3}+\frac{B\,b^2\,d\,g^2\,i\,x^4}{4}\right)$$
$$+x^3\left(\frac{b\,g^2\,i\,(12\,A\,a\,d+8\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{12}-\frac{A\,b\,g^2\,i\,(12\,a\,d+12\,b\,c)}{36}\right)$$
$$+x\left(\frac{(12\,a\,d+12\,b\,c)\left(\frac{(12\,a\,d+12\,b\,c)\left(\frac{b\,g^2\,i\,(12\,A\,a\,d+8\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4}-\frac{A\,b\,g^2\,i\,(12\,a\,d+12\,b\,c)}{12}\right)}{12\,b\,d}-\frac{g^2\,i\,(9\,A\,a^2\,d^2+3\,A\,b^2\,c^2+2\,B}{}\right.}{12\,b\,d}\right.$$
$$\left.-\frac{a\,c\left(\frac{b\,g^2\,i\,(12\,A\,a\,d+8\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4}-\frac{A\,b\,g^2\,i\,(12\,a\,d+12\,b\,c)}{12}\right)}{b\,d}+\frac{a\,g^2\,i\,(2\,A\,a^2\,d^2+6\,A\,b^2\,c^2+B\,a^2\,d^2\,n-2\,B\,b^2\,c^2\,n+12\,A\,a\,b\,c\,d+B\,a\,b\,c\,d\,n)}{2\,b\,d}\right)$$
$$-x^2\left(\frac{(12\,a\,d+12\,b\,c)\left(\frac{b\,g^2\,i\,(12\,A\,a\,d+8\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4}-\frac{A\,b\,g^2\,i\,(12\,a\,d+12\,b\,c)}{12}\right)}{24\,b\,d}-\frac{g^2\,i\,(9\,A\,a^2\,d^2+3\,A\,b^2\,c^2+2\,B\,a^2\,d^2\,n-B\,b^2\,c^2\,n+18\,A\,a\,b\,c\,d-B\,a\,b\,c\,d\,n)}{6\,d}+\frac{A\,a\,b\,c\,g^2\,i}{2}\right)-\frac{\ln(a+b\,x)\,(B\,a^4\,d\,g^2\,i\,n-4\,B\,a^3\,b\,c\,g^2\,i\,n)}{12\,b^2}$$
$$-\frac{\ln(c+d\,x)\,(6\,B\,i\,n\,a^2\,c^2\,d^2\,g^2-4\,B\,i\,n\,a\,b\,c^3\,d\,g^2+B\,i\,n\,b^2\,c^4\,g^2)}{12\,d^3}+\frac{A\,b^2\,d\,g^2\,i\,x^4}{4}$$

```
[In] int((a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] log(e*((a + b*x)/(c + d*x))^n)*(B*a^2*c*g^2*i*x + (B*a*g^2*i*x^2*(a*d + 2*b
*c))/2 + (B*b*g^2*i*x^3*(2*a*d + b*c))/3 + (B*b^2*d*g^2*i*x^4)/4) + x^3*((b
*g^2*i*(12*A*a*d + 8*A*b*c + B*a*d*n - B*b*c*n))/12 - (A*b*g^2*i*(12*a*d +
12*b*c))/36) + x*(((12*a*d + 12*b*c)*(((12*a*d + 12*b*c)*((b*g^2*i*(12*A*a*
d + 8*A*b*c + B*a*d*n - B*b*c*n))/4 - (A*b*g^2*i*(12*a*d + 12*b*c))/12))/(1
2*b*d) - (g^2*i*(9*A*a^2*d^2 + 3*A*b^2*c^2 + 2*B*a^2*d^2*n - B*b^2*c^2*n +
18*A*a*b*c*d - B*a*b*c*d*n))/(3*d) + A*a*b*c*g^2*i))/(12*b*d) - (a*c*((b*g^
```

```
2*i*(12*A*a*d + 8*A*b*c + B*a*d*n - B*b*c*n))/4 - (A*b*g^2*i*(12*a*d + 12*b
*c))/12))/(b*d) + (a*g^2*i*(2*A*a^2*d^2 + 6*A*b^2*c^2 + B*a^2*d^2*n - 2*B*b
^2*c^2*n + 12*A*a*b*c*d + B*a*b*c*d*n))/(2*b*d)) - x^2*(((12*a*d + 12*b*c)*
((b*g^2*i*(12*A*a*d + 8*A*b*c + B*a*d*n - B*b*c*n))/4 - (A*b*g^2*i*(12*a*d
+ 12*b*c))/12))/(24*b*d) - (g^2*i*(9*A*a^2*d^2 + 3*A*b^2*c^2 + 2*B*a^2*d^2*
n - B*b^2*c^2*n + 18*A*a*b*c*d - B*a*b*c*d*n))/(6*d) + (A*a*b*c*g^2*i)/2) -
 (log(a + b*x)*(B*a^4*d*g^2*i*n - 4*B*a^3*b*c*g^2*i*n))/(12*b^2) - (log(c +
 d*x)*(B*b^2*c^4*g^2*i*n + 6*B*a^2*c^2*d^2*g^2*i*n - 4*B*a*b*c^3*d*g^2*i*n)
)/(12*d^3) + (A*b^2*d*g^2*i*x^4)/4
```

3.110 $\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 39, antiderivative size = 149

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = -\frac{B(bc-ad)^2ginx}{6bd}+\frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b} + \frac{(bc-ad)gi(a+bx)^2\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^2}+\frac{B(bc-ad)^3gin\log(c+dx)}{6b^2d^2}$$

```
[Out] -1/6*B*(-a*d+b*c)^2*g*i*n*x/b/d+1/3*g*i*(b*x+a)^2*(d*x+c)*(A+B*ln(e*((b*x+a
)/(d*x+c))^n))/b+1/6*(-a*d+b*c)*g*i*(b*x+a)^2*(A-B*n+B*ln(e*((b*x+a)/(d*x+c
))^n))/b^2+1/6*B*(-a*d+b*c)^3*g*i*n*ln(d*x+c)/b^2/d^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2559, 2547, 21, 45}

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{gi(a+bx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{6b^2} + \frac{gi(a+bx)^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b} + \frac{Bgin(bc-ad)^3\log(c+dx)}{6b^2d^2}-\frac{Bginx(bc-ad)^2}{6bd}$$

```
[In] Int[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/6*(B*(b*c - a*d)^2*g*i*n*x)/(b*d) + (g*i*(a + b*x)^2*(c + d*x)*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]))/(3*b) + ((b*c - a*d)*g*i*(a + b*x)^2*(A - B*
n + B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*b^2) + (B*(b*c - a*d)^3*g*i*n*Log
[c + d*x])/(6*b^2*d^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rule 2559

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Simp[(f
 + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 2
))), x] + Dist[i*((b*c - a*d)/(b*d*(m + 2))), Int[(f + g*x)^m*(A - B*n + B*
Log[e*((a + b*x)/(c + d*x))^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i
, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*
i, 0] && IGtQ[m, -2]
```

Rubi steps

$$\text{integral} = \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b} + \frac{((bc-ad)i)\int(ag+bgx)\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx}{3b}$$

$$= \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b} + \frac{(bc-ad)gi(a+bx)^2\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^2} - \frac{(B(bc-ad)^2in)\int\frac{(ag+bgx)^2}{(a+bx)(c+dx)}\,dx}{6b^2g}$$

$$= \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b} + \frac{(bc-ad)gi(a+bx)^2\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^2} - \frac{(B(bc-ad)^2gin)\int\frac{a+bx}{c+dx}\,dx}{6b^2}$$

$$= \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b} + \frac{(bc-ad)gi(a+bx)^2\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^2} - \frac{(B(bc-ad)^2gin)\int\left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx}{6b^2}$$

$$= -\frac{B(bc-ad)^2ginx}{6bd} + \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b} + \frac{(bc-ad)gi(a+bx)^2\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^2} + \frac{B(bc-ad)^3gin\log(c+dx)}{6b^2d^2}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.27

$$\int(ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \frac{gi\left(-a^2Bd^2(3bc+ad)n\log(a+bx)+b\left(dx(a^2Bd^2n-b^2Bcn(c+dx)+Ab^2dx(3c+2dx)+abd(6Ac+3A\right.\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
),x]
```

```
[Out] (g*i*(-(a^2*B*d^2*(3*b*c + a*d)*n*Log[a + b*x]) + b*(d*x*(a^2*B*d^2*n - b^2
*B*c*n*(c + d*x) + A*b^2*d*x*(3*c + 2*d*x) + a*b*d*(6*A*c + 3*A*d*x + B*d*n
*x)) + B*d^2*(6*a^2*c + 3*a*b*x*(2*c + d*x) + b^2*x^2*(3*c + 2*d*x))*Log[e*
((a + b*x)/(c + d*x))^n] + B*c*(b^2*c^2 - 3*a*b*c*d + 6*a^2*d^2)*n*Log[c +
d*x])))/(6*b^2*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(141) = 282$.

Time = 1.72 (sec) , antiderivative size = 507, normalized size of antiderivative = 3.40

method	result
parallelrisch	$\frac{6Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a\,b^{2}c\,d^{2}gin+2B\,x^{3}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{3}d^{3}gin+3B\,x^{2}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{3}c\,d^{2}gin+6Axa\,b^{2}c\,d^{2}gin+3B\,x^{2}\ln\left(e\left(\right.\right.}{}$

```
[In] int((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETUR
NVERBOSE)
```

```
[Out] 1/6*(6*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*c*d^2*g*i*n+2*B*x^3*ln(e*((b*x+a
)/(d*x+c))^n)*b^3*d^3*g*i*n+3*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d^2*g*i
*n+6*A*x*a*b^2*c*d^2*g*i*n+3*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*d^3*g*i*
n+3*B*ln(b*x+a)*a^2*b*c*d^2*g*i*n^2-3*B*ln(b*x+a)*a*b^2*c^2*d*g*i*n^2+3*B*l
n(e*((b*x+a)/(d*x+c))^n)*a*b^2*c^2*d*g*i*n-2*B*a^2*b*c*d^2*g*i*n^2+2*B*a*b^
2*c^2*d*g*i*n^2-9*A*a^2*b*c*d^2*g*i*n-9*A*a*b^2*c^2*d*g*i*n+B*b^3*c^3*g*i*n
^2-B*a^3*d^3*g*i*n^2+B*x^2*a*b^2*d^3*g*i*n^2-B*x^2*b^3*c*d^2*g*i*n^2+3*A*x^
2*a*b^2*d^3*g*i*n+3*A*x^2*b^3*c*d^2*g*i*n+B*x*a^2*b*d^3*g*i*n^2-B*x*b^3*c^2
*d*g*i*n^2-B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c^3*g*i*n+2*A*x^3*b^3*d^3*g*i*n-
B*ln(b*x+a)*a^3*d^3*g*i*n^2+B*ln(b*x+a)*b^3*c^3*g*i*n^2)/n/b^2/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(143) = 286$.

Time = 0.37 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.07

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)\,dx$$
$$=\frac{2\,Ab^{3}d^{3}gix^{3}+(3\,Ba^{2}bcd^{2}-Ba^{3}d^{3})gin\log(bx+a)+(Bb^{3}c^{3}-3\,Bab^{2}c^{2}d)gin\log(dx+c)-((Bb^{3}cd^{2}-B}{}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algor
ithm="fricas")
```

```
[Out] 1/6*(2*A*b^3*d^3*g*i*x^3 + (3*B*a^2*b*c*d^2 - B*a^3*d^3)*g*i*n*log(b*x + a)
 + (B*b^3*c^3 - 3*B*a*b^2*c^2*d)*g*i*n*log(d*x + c) - ((B*b^3*c*d^2 - B*a*b
^2*d^3)*g*i*n - 3*(A*b^3*c*d^2 + A*a*b^2*d^3)*g*i)*x^2 + (6*A*a*b^2*c*d^2*g
*i - (B*b^3*c^2*d - B*a^2*b*d^3)*g*i*n)*x + (2*B*b^3*d^3*g*i*x^3 + 6*B*a*b^
2*c*d^2*g*i*x + 3*(B*b^3*c*d^2 + B*a*b^2*d^3)*g*i*x^2)*log(e) + (2*B*b^3*d^
3*g*i*n*x^3 + 6*B*a*b^2*c*d^2*g*i*n*x + 3*(B*b^3*c*d^2 + B*a*b^2*d^3)*g*i*n
*x^2)*log((b*x + a)/(d*x + c)))/(b^2*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(134) = 268$.

Time = 19.63 (sec) , antiderivative size = 643, normalized size of antiderivative = 4.32

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \begin{cases} acgix\left(A+B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) \\ ag\left(Acix+\frac{Adix^2}{2}+\frac{Bc^2i\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2d}+\frac{Bcinx}{2}+Bcix\log\left(e\left(\frac{a}{c+dx}\right)^n\right)+\frac{Bdinx^2}{4}+\frac{Bdix^2\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2}\right) \\ ci\left(Aagx+\frac{Abgx^2}{2}+\frac{Ba^2g\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{2b}-\frac{Bagnx}{2}+Bagx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)-\frac{Bbgnx^2}{4}+\frac{Bbgx^2\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{2}\right. \\ Aacgix+\frac{Aadgix^2}{2}+\frac{Abcgix^2}{2}+\frac{Abdgix^3}{3}-\frac{Ba^3dgin\log\left(\frac{c}{d}+x\right)}{6b^2}-\frac{Ba^3dgi\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{6b^2}+\frac{Ba^2cgin\log\left(\frac{c}{d}+x\right)}{2b}+\frac{Ba}{} \end{cases}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Piecewise((a*c*g*i*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (a*g*(A
*c*i*x + A*d*i*x**2/2 + B*c**2*i*log(e*(a/(c + d*x))**n)/(2*d) + B*c*i*n*x/
2 + B*c*i*x*log(e*(a/(c + d*x))**n) + B*d*i*n*x**2/4 + B*d*i*x**2*log(e*(a/
(c + d*x))**n)/2), Eq(b, 0)), (c*i*(A*a*g*x + A*b*g*x**2/2 + B*a**2*g*log(e
*(a/c + b*x/c)**n)/(2*b) - B*a*g*n*x/2 + B*a*g*x*log(e*(a/c + b*x/c)**n) -
B*b*g*n*x**2/4 + B*b*g*x**2*log(e*(a/c + b*x/c)**n)/2), Eq(d, 0)), (A*a*c*g
*i*x + A*a*d*g*i*x**2/2 + A*b*c*g*i*x**2/2 + A*b*d*g*i*x**3/3 - B*a**3*d*g*
i*n*log(c/d + x)/(6*b**2) - B*a**3*d*g*i*log(e*(a/(c + d*x) + b*x/(c + d*x)
)**n)/(6*b**2) + B*a**2*c*g*i*n*log(c/d + x)/(2*b) + B*a**2*c*g*i*log(e*(a/
(c + d*x) + b*x/(c + d*x))**n)/(2*b) + B*a**2*d*g*i*n*x/(6*b) - B*a*c**2*g*
i*n*log(c/d + x)/(2*d) + B*a*c*g*i*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n
) + B*a*d*g*i*n*x**2/6 + B*a*d*g*i*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))
**n)/2 + B*b*c**3*g*i*n*log(c/d + x)/(6*d**2) - B*b*c**2*g*i*n*x/(6*d) - B*
b*c*g*i*n*x**2/6 + B*b*c*g*i*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/2
 + B*b*d*g*i*x**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(143) = 286.

Time = 0.20 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.64

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{1}{3}\,Bbdgix^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{3}\,Abdgix^3$$
$$+\frac{1}{2}\,Bbcgix^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+\frac{1}{2}\,Badgix^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{2}\,Abcgix^2+\frac{1}{2}\,Aadgix^2$$
$$+\frac{1}{6}\,Bbdgin\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$-\frac{1}{2}\,Bbcgin\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$-\frac{1}{2}\,Badgin\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bacgin\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)$$
$$+Bacgix\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aacgix$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algor
ithm="maxima")
```

```
[Out] 1/3*B*b*d*g*i*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*b*d*g*i*x^
3 + 1/2*B*b*c*g*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*B*a*d*g*
i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*b*c*g*i*x^2 + 1/2*A*a*
d*g*i*x^2 + 1/6*B*b*d*g*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^
3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 1/2*B*
b*c*g*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d
)) - 1/2*B*a*d*g*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c -
a*d)*x/(b*d)) + B*a*c*g*i*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*a*c*g
*i*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a*c*g*i*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1276 vs. 2(143) = 286.

Time = 0.66 (sec) , antiderivative size = 1276, normalized size of antiderivative = 8.56

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algor
ithm="giac")
```

```
[Out] -1/6*((B*b^5*c^4*g*i*n - 4*B*a*b^4*c^3*d*g*i*n - 3*(b*x + a)*B*b^4*c^4*d*g*
i*n/(d*x + c) + 6*B*a^2*b^3*c^2*d^2*g*i*n + 12*(b*x + a)*B*a*b^3*c^3*d^2*g*
i*n/(d*x + c) - 4*B*a^3*b^2*c*d^3*g*i*n - 18*(b*x + a)*B*a^2*b^2*c^2*d^3*g*
i*n/(d*x + c) + B*a^4*b*d^4*g*i*n + 12*(b*x + a)*B*a^3*b*c*d^4*g*i*n/(d*x +
 c) - 3*(b*x + a)*B*a^4*d^5*g*i*n/(d*x + c))*log((b*x + a)/(d*x + c))/(b^3*
d^2 - 3*(b*x + a)*b^2*d^3/(d*x + c) + 3*(b*x + a)^2*b*d^4/(d*x + c)^2 - (b*
x + a)^3*d^5/(d*x + c)^3) + ((b*x + a)*B*b^5*c^4*d*g*i*n/(d*x + c) - 4*(b*x
 + a)*B*a*b^4*c^3*d^2*g*i*n/(d*x + c) - (b*x + a)^2*B*b^4*c^4*d^2*g*i*n/(d*
x + c)^2 + 6*(b*x + a)*B*a^2*b^3*c^2*d^3*g*i*n/(d*x + c) + 4*(b*x + a)^2*B*
a*b^3*c^3*d^3*g*i*n/(d*x + c)^2 - 4*(b*x + a)*B*a^3*b^2*c*d^4*g*i*n/(d*x +
c) - 6*(b*x + a)^2*B*a^2*b^2*c^2*d^4*g*i*n/(d*x + c)^2 + (b*x + a)*B*a^4*b*
d^5*g*i*n/(d*x + c) + 4*(b*x + a)^2*B*a^3*b*c*d^5*g*i*n/(d*x + c)^2 - (b*x
+ a)^2*B*a^4*d^6*g*i*n/(d*x + c)^2 + B*b^6*c^4*g*i*log(e) - 4*B*a*b^5*c^3*d
*g*i*log(e) - 3*(b*x + a)*B*b^5*c^4*d*g*i*log(e)/(d*x + c) + 6*B*a^2*b^4*c^
2*d^2*g*i*log(e) + 12*(b*x + a)*B*a*b^4*c^3*d^2*g*i*log(e)/(d*x + c) - 4*B*
a^3*b^3*c*d^3*g*i*log(e) - 18*(b*x + a)*B*a^2*b^3*c^2*d^3*g*i*log(e)/(d*x +
 c) + B*a^4*b^2*d^4*g*i*log(e) + 12*(b*x + a)*B*a^3*b^2*c*d^4*g*i*log(e)/(d
*x + c) - 3*(b*x + a)*B*a^4*b*d^5*g*i*log(e)/(d*x + c) + A*b^6*c^4*g*i - 4*
A*a*b^5*c^3*d*g*i - 3*(b*x + a)*A*b^5*c^4*d*g*i/(d*x + c) + 6*A*a^2*b^4*c^2
*d^2*g*i + 12*(b*x + a)*A*a*b^4*c^3*d^2*g*i/(d*x + c) - 4*A*a^3*b^3*c*d^3*g
*i - 18*(b*x + a)*A*a^2*b^3*c^2*d^3*g*i/(d*x + c) + A*a^4*b^2*d^4*g*i + 12*
(b*x + a)*A*a^3*b^2*c*d^4*g*i/(d*x + c) - 3*(b*x + a)*A*a^4*b*d^5*g*i/(d*x
+ c))/(b^4*d^2 - 3*(b*x + a)*b^3*d^3/(d*x + c) + 3*(b*x + a)^2*b^2*d^4/(d*x
 + c)^2 - (b*x + a)^3*b*d^5/(d*x + c)^3) + (B*b^4*c^4*g*i*n - 4*B*a*b^3*c^3
*d*g*i*n + 6*B*a^2*b^2*c^2*d^2*g*i*n - 4*B*a^3*b*c*d^3*g*i*n + B*a^4*d^4*g*
i*n)*log(b - (b*x + a)*d/(d*x + c))/(b^2*d^2) - (B*b^4*c^4*g*i*n - 4*B*a*b^
3*c^3*d*g*i*n + 6*B*a^2*b^2*c^2*d^2*g*i*n - 4*B*a^3*b*c*d^3*g*i*n + B*a^4*d
^4*g*i*n)*log((b*x + a)/(d*x + c))/(b^2*d^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c
 - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.98

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B\,b\,d\,g\,i\,x^3}{3}+\frac{B\,g\,i\,(a\,d+b\,c)\,x^2}{2}+B\,a\,c\,g\,i\,x\right)$$
$$-x\left(\frac{\left(\frac{g\,i\,(6\,A\,a\,d+6\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{3}-\frac{A\,g\,i\,(6\,a\,d+6\,b\,c)}{6}\right)(6\,a\,d+6\,b\,c)}{6\,b\,d}+A\,a\,c\,g\,i\right.$$
$$\left.-\frac{g\,i\,(2\,A\,a^2\,d^2+2\,A\,b^2\,c^2+B\,a^2\,d^2\,n-B\,b^2\,c^2\,n+8\,A\,a\,b\,c\,d)}{2\,b\,d}\right)$$
$$+x^2\left(\frac{g\,i\,(6\,A\,a\,d+6\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{6}-\frac{A\,g\,i\,(6\,a\,d+6\,b\,c)}{12}\right)$$
$$-\frac{\ln(a+b\,x)\,\left(B\,a^3\,d\,g\,i\,n-3\,B\,a^2\,b\,c\,g\,i\,n\right)}{6\,b^2}$$
$$+\frac{\ln(c+d\,x)\,\left(B\,b\,c^3\,g\,i\,n-3\,B\,a\,c^2\,d\,g\,i\,n\right)}{6\,d^2}+\frac{A\,b\,d\,g\,i\,x^3}{3}$$

```
[In] int((a*g + b*g*x)*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] log(e*((a + b*x)/(c + d*x))^n)*((B*g*i*x^2*(a*d + b*c))/2 + (B*b*d*g*i*x^3)
/3 + B*a*c*g*i*x) - x*((((g*i*(6*A*a*d + 6*A*b*c + B*a*d*n - B*b*c*n))/3 -
(A*g*i*(6*a*d + 6*b*c))/6)*(6*a*d + 6*b*c))/(6*b*d) + A*a*c*g*i - (g*i*(2*A
*a^2*d^2 + 2*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + 8*A*a*b*c*d))/(2*b*d))
 + x^2*((g*i*(6*A*a*d + 6*A*b*c + B*a*d*n - B*b*c*n))/6 - (A*g*i*(6*a*d + 6
*b*c))/12) - (log(a + b*x)*(B*a^3*d*g*i*n - 3*B*a^2*b*c*g*i*n))/(6*b^2) + (
log(c + d*x)*(B*b*c^3*g*i*n - 3*B*a*c^2*d*g*i*n))/(6*d^2) + (A*b*d*g*i*x^3)
/3
```

3.111 $\int (ci+dix) \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx$

Optimal result

Integrand size = 31, antiderivative size = 86

$$\int (ci+dix) \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx = -\frac{B(bc-ad)inx}{2b}-\frac{B(bc-ad)^2in\log(a+bx)}{2b^2d}+\frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d}$$

```
[Out] -1/2*B*(-a*d+b*c)*i*n*x/b-1/2*B*(-a*d+b*c)^2*i*n*ln(b*x+a)/b^2/d+1/2*i*(d*x
+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2547, 21, 45}

$$\int (ci+dix) \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx = \frac{i(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d}-\frac{Bin(bc-ad)^2\log(a+bx)}{2b^2d}-\frac{Binx(bc-ad)}{2b}$$

```
[In] Int[(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/2*(B*(b*c - a*d)*i*n*x)/b - (B*(b*c - a*d)^2*i*n*Log[a + b*x])/(2*b^2*d)
 + (i*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d} - \frac{(B(bc-ad)n)\int\frac{(ci+dix)^2}{(a+bx)(c+dx)}\,dx}{2di}\\ &= \frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d} - \frac{(B(bc-ad)in)\int\frac{c+dx}{a+bx}\,dx}{2d}\\ &= \frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d} - \frac{(B(bc-ad)in)\int\left(\frac{d}{b}+\frac{bc-ad}{b(a+bx)}\right)dx}{2d}\\ &= -\frac{B(bc-ad)inx}{2b} - \frac{B(bc-ad)^2in\log(a+bx)}{2b^2d} + \frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \frac{i\left(-\frac{B(bc-ad)n(bdx+(bc-ad)\log(a+bx))}{b^2}+(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{2d}$$

```
[In] Integrate[(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] (i*(-((B*(b*c - a*d)*n*(b*d*x + (b*c - a*d)*Log[a + b*x]))/b^2) + (c + d*x)
^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(2*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(80) = 160$.

Time = 0.68 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.91

method	result
parallelrisch	$\frac{B\,x^2 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) b^2 d^2 in + A\,x^2 b^2 d^2 in - B\ln(bx+a) a^2 d^2 i\,n^2 + 2B\ln(bx+a) abcdi\,n^2 - B\ln(bx+a) b^2 c^2 i\,n^2 + 2Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)\right.}{2b^2}$

```
[In] int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*i*n+A*x^2*b^2*d^2*i*n-B*ln(b*x
+a)*a^2*d^2*i*n^2+2*B*ln(b*x+a)*a*b*c*d*i*n^2-B*ln(b*x+a)*b^2*c^2*i*n^2+2*B
*x*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*i*n+B*x*a*b*d^2*i*n^2-B*x*b^2*c*d*i*n^
2+2*A*x*b^2*c*d*i*n+B*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c^2*i*n-B*a^2*d^2*i*n^2
+B*b^2*c^2*i*n^2-3*A*a*b*c*d*i*n-2*A*b^2*c^2*i*n)/b^2/n/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(80) = 160$.

Time = 0.35 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.88

$$\int (ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{Ab^2d^2ix^2 - Bb^2c^2in\log(dx+c) + (2\,Babcd - Ba^2d^2)in\log(bx+a) + (2\,Ab^2cdi - (Bb^2cd - Babd^2)in)x}{2\,b^2d}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas
")
```

```
[Out] 1/2*(A*b^2*d^2*i*x^2 - B*b^2*c^2*i*n*log(d*x + c) + (2*B*a*b*c*d - B*a^2*d^
2)*i*n*log(b*x + a) + (2*A*b^2*c*d*i - (B*b^2*c*d - B*a*b*d^2)*i*n)*x + (B*
b^2*d^2*i*x^2 + 2*B*b^2*c*d*i*x)*log(e) + (B*b^2*d^2*i*n*x^2 + 2*B*b^2*c*d*
i*n*x)*log((b*x + a)/(d*x + c)))/(b^2*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(73) = 146$.

Time = 5.09 (sec) , antiderivative size = 382, normalized size of antiderivative = 4.44

$$\int (ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \begin{cases} cix\left(A + B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) \\ Acix + \frac{Adix^2}{2} + \frac{Bc^2 i\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2d} + \frac{Bcinx}{2} + Bcix\log\left(e\left(\frac{a}{c+dx}\right)^n\right) + \frac{Bdinx^2}{4} + \frac{Bdix^2\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2} \\ ci\left(Ax + \frac{Ba\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{b} - Bnx + Bx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)\right) \\ Acix + \frac{Adix^2}{2} - \frac{Ba^2 din\log\left(\frac{c}{d}+x\right)}{2b^2} - \frac{Ba^2 di\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{2b^2} + \frac{Bacin\log\left(\frac{c}{d}+x\right)}{b} + \frac{Baci\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{b} + \frac{Badin}{2b} \end{cases}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Piecewise((c*i*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (A*c*i*x +
A*d*i*x**2/2 + B*c**2*i*log(e*(a/(c + d*x))**n)/(2*d) + B*c*i*n*x/2 + B*c*i
*x*log(e*(a/(c + d*x))**n) + B*d*i*n*x**2/4 + B*d*i*x**2*log(e*(a/(c + d*x)
)**n)/2, Eq(b, 0)), (c*i*(A*x + B*a*log(e*(a/c + b*x/c)**n)/b - B*n*x + B*x
*log(e*(a/c + b*x/c)**n)), Eq(d, 0)), (A*c*i*x + A*d*i*x**2/2 - B*a**2*d*i*
n*log(c/d + x)/(2*b**2) - B*a**2*d*i*log(e*(a/(c + d*x) + b*x/(c + d*x))**n
)/(2*b**2) + B*a*c*i*n*log(c/d + x)/b + B*a*c*i*log(e*(a/(c + d*x) + b*x/(c
 + d*x))**n)/b + B*a*d*i*n*x/(2*b) - B*c**2*i*n*log(c/d + x)/(2*d) - B*c*i*
n*x/2 + B*c*i*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*d*i*x**2*log(e*
(a/(c + d*x) + b*x/(c + d*x))**n)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.81

$$\int (ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{1}{2}Bdix^2\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right) + \frac{1}{2}Adix^2 - \frac{1}{2}Bdin\left(\frac{a^2\log(bx+a)}{b^2} - \frac{c^2\log(dx+c)}{d^2} + \frac{(bc-ad)x}{bd}\right) + Bcin\left(\frac{a\log(bx+a)}{b} - \frac{c\log(dx+c)}{d}\right) + Bcix\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right) + Acix$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima
")
```

```
[Out] 1/2*B*d*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*d*i*x^2 - 1/2*
B*d*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))
 + B*c*i*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*c*i*x*log(e*(b*x/(d*x
+ c) + a/(d*x + c))^n) + A*c*i*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(80) = 160$.

Time = 0.46 (sec) , antiderivative size = 580, normalized size of antiderivative = 6.74

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{1}{2}\left(\frac{(Bb^3c^3in-3\,Bab^2c^2din+3\,Ba^2bcd^2in-Ba^3d^3in)\log\left(\frac{bx+a}{dx+c}\right)}{b^2d-\frac{2\,(bx+a)bd^2}{dx+c}+\frac{(bx+a)^2d^3}{(dx+c)^2}}-\frac{Bb^4c^3in-3\,Bab^3c^2din-\frac{(bx+a)Bb^3c^3}{dx+c}}{}\right.$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] 1/2*((B*b^3*c^3*i*n - 3*B*a*b^2*c^2*d*i*n + 3*B*a^2*b*c*d^2*i*n - B*a^3*d^3
*i*n)*log((b*x + a)/(d*x + c))/(b^2*d - 2*(b*x + a)*b*d^2/(d*x + c) + (b*x
+ a)^2*d^3/(d*x + c)^2) - (B*b^4*c^3*i*n - 3*B*a*b^3*c^2*d*i*n - (b*x + a)*
B*b^3*c^3*d*i*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*i*n + 3*(b*x + a)*B*a*b^2*c^2
*d^2*i*n/(d*x + c) - B*a^3*b*d^3*i*n - 3*(b*x + a)*B*a^2*b*c*d^3*i*n/(d*x +
 c) + (b*x + a)*B*a^3*d^4*i*n/(d*x + c) - B*b^4*c^3*i*log(e) + 3*B*a*b^3*c^
2*d*i*log(e) - 3*B*a^2*b^2*c*d^2*i*log(e) + B*a^3*b*d^3*i*log(e) - A*b^4*c^
3*i + 3*A*a*b^3*c^2*d*i - 3*A*a^2*b^2*c*d^2*i + A*a^3*b*d^3*i)/(b^3*d - 2*(
b*x + a)*b^2*d^2/(d*x + c) + (b*x + a)^2*b*d^3/(d*x + c)^2) + (B*b^3*c^3*i*
n - 3*B*a*b^2*c^2*d*i*n + 3*B*a^2*b*c*d^2*i*n - B*a^3*d^3*i*n)*log(-b + (b*
x + a)*d/(d*x + c))/(b^2*d) - (B*b^3*c^3*i*n - 3*B*a*b^2*c^2*d*i*n + 3*B*a^
2*b*c*d^2*i*n - B*a^3*d^3*i*n)*log((b*x + a)/(d*x + c))/(b^2*d))*(b*c/(b*c
- a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.56

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=x\left(\frac{i\,(2\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{2\,b}-\frac{A\,i\,(2\,a\,d+2\,b\,c)}{2\,b}\right)$$
$$+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B\,d\,i\,x^2}{2}+B\,c\,i\,x\right)$$
$$-\frac{\ln(a+b\,x)\,\left(B\,a^2\,d\,i\,n-2\,B\,a\,b\,c\,i\,n\right)}{2\,b^2}+\frac{A\,d\,i\,x^2}{2}-\frac{B\,c^2\,i\,n\,\ln(c+d\,x)}{2\,d}$$

```
[In] int((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x*((i*(2*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(2*b) - (A*i*(2*a*d + 2*b*c)
)/(2*b)) + log(e*((a + b*x)/(c + d*x))^n)*((B*d*i*x^2)/2 + B*c*i*x) - (log(
a + b*x)*(B*a^2*d*i*n - 2*B*a*b*c*i*n))/(2*b^2) + (A*d*i*x^2)/2 - (B*c^2*i*
n*log(c + d*x))/(2*d)
```

3.112 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 141

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = \frac{i(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg} - \frac{(bc-ad)i\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g} + \frac{B(bc-ad)in\,\mathrm{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{b^2g}$$

```
[Out] i*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g-(-a*d+b*c)*i*ln((a*d-b*c)/d/(
b*x+a))*(A-B*n+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g+B*(-a*d+b*c)*i*n*polylog(
2,1+(-a*d+b*c)/d/(b*x+a))/b^2/g
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used

= {2559, 2541, 2458, 2378, 2370, 2352}

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$$
$$= -\frac{i(bc-ad)\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{b^2g}$$
$$+\frac{i(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}+\frac{Bin(bc-ad)\,\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{b^2g}$$

```
[In] Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x),x]
```

```
[Out] (i*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b*g) - ((b*c - a*d)*i
*Log[-((b*c - a*d)/(d*(a + b*x)))]*(A - B*n + B*Log[e*((a + b*x)/(c + d*x))
^n]))/(b^2*g) + (B*(b*c - a*d)*i*n*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))
])/(b^2*g)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2541

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a + b*
x))])*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/g), x] + Dist[B*n*((b*c - a*d
```

```
)/g), Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x], x] /;
FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[b*f -
a*g, 0]
```

Rule 2559

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Simp[(f
 + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 2
))), x] + Dist[i*((b*c - a*d)/(b*d*(m + 2))), Int[(f + g*x)^m*(A - B*n + B*
Log[e*((a + b*x)/(c + d*x))^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i
, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*
i, 0] && IGtQ[m, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg} + \frac{((bc-ad)i)\int \frac{A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx}\,dx}{b} \\
&= \frac{i(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg} \\
&\quad - \frac{(bc-ad)i\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g} \\
&\quad + \frac{(B(bc-ad)^2in)\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx}{b^2g} \\
&= \frac{i(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg} \\
&\quad - \frac{(bc-ad)i\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g} \\
&\quad + \frac{(B(bc-ad)^2in)\,\text{Subst}\left(\int \frac{\log\left(\frac{-bc+ad}{dx}\right)}{x\left(\frac{bc-ad}{b}+\frac{dx}{b}\right)}\,dx, x, a+bx\right)}{b^3g} \\
&= \frac{i(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg} \\
&\quad - \frac{(bc-ad)i\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g} \\
&\quad - \frac{(B(bc-ad)^2in)\,\text{Subst}\left(\int \frac{\log\left(\frac{(-bc+ad)x}{d}\right)}{\left(\frac{bc-ad}{b}+\frac{d}{bx}\right)x}\,dx, x, \frac{1}{a+bx}\right)}{b^3g}
\end{aligned}$$

$$= \frac{i(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg} - \frac{(bc-ad)i\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g} - \frac{(B(bc-ad)^2in)\,\text{Subst}\left(\int\frac{\log\left(\frac{(-bc+ad)x}{d}\right)}{\frac{d}{b}+\frac{(bc-ad)x}{b}}\,dx,x,\frac{1}{a+bx}\right)}{b^3g}$$

$$= \frac{i(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg} - \frac{(bc-ad)i\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g} + \frac{B(bc-ad)in\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.22

$$\int\frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = \frac{i\left(B(-bc+ad)n\log^2(a+bx)+2\left(Abdx+Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+B(-bc+ad)n\log(c+dx)\right)+2(bc\right.}{2b^2g}$$

```
[In] Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g
*x),x]

[Out] (i*(B*(-(b*c) + a*d)*n*Log[a + b*x]^2 + 2*(A*b*d*x + B*d*(a + b*x)*Log[e*((
a + b*x)/(c + d*x))^n] + B*(-(b*c) + a*d)*n*Log[c + d*x]) + 2*(b*c - a*d)*L
og[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n] + B*n*Log[(b*(c + d*x))/(
b*c - a*d)]) + 2*B*(b*c - a*d)*n*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))
/(2*b^2*g)
```

Maple [F]

$$\int \frac{(dix+ci)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}{bgx+ag}dx$$

```
[In] int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

```
[Out] int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)}{ag+bgx}\,dx = \int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algor
ithm="fricas")
```

```
[Out] integral((A*d*i*x + A*c*i + (B*d*i*x + B*c*i)*log(e*((b*x + a)/(d*x + c))^n
))/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)}{ag+bgx}\,dx = \frac{i\left(\int \frac{Ac}{a+bx}\,dx+\int \frac{Adx}{a+bx}\,dx+\int \frac{Bc\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}{a+bx}\,dx+\int \frac{Bdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}{a+bx}\,dx\right)}{g}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g),x)
```

```
[Out] i*(Integral(A*c/(a + b*x), x) + Integral(A*d*x/(a + b*x), x) + Integral(B*c
*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x) + Integral(B*d*x*log
(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x))/g
```

Maxima [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.96

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = Adi\left(\frac{x}{bg}-\frac{a\log(bx+a)}{b^2g}\right)-\frac{Bcin\log(dx+c)}{bg} + \frac{Aci\log(bgx+ag)}{bg} + \frac{(bcin-adin)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{b^2g} + \frac{2\,Bbdix\log(e)-(bcin-adin)B\log(bx+a)^2+2\,(bci\log(e)+(in-i\log(e))ad)B\log(bx+a)+2\,(Bb\ldots}{2\,b\ldots}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algor
ithm="maxima")
```

```
[Out] A*d*i*(x/(b*g) - a*log(b*x + a)/(b^2*g)) - B*c*i*n*log(d*x + c)/(b*g) + A*c
*i*log(b*g*x + a*g)/(b*g) + (b*c*i*n - a*d*i*n)*(log(b*x + a)*log((b*d*x +
a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(b^2*g) + 1/2*
(2*B*b*d*i*x*log(e) - (b*c*i*n - a*d*i*n)*B*log(b*x + a)^2 + 2*(b*c*i*log(e
) + (i*n - i*log(e))*a*d)*B*log(b*x + a) + 2*(B*b*d*i*x + (b*c*i - a*d*i)*B
*log(b*x + a))*log((b*x + a)^n) - 2*(B*b*d*i*x + (b*c*i - a*d*i)*B*log(b*x
+ a))*log((d*x + c)^n))/(b^2*g)
```

Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = \int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algor
ithm="giac")
```

```
[Out] integrate((d*i*x + c*i)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x + a*g
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = \int \frac{(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}{a\,g+b\,g\,x}\,dx$$

```
[In] int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x),x)
```

```
[Out] int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x), x
)
```

3.113 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 150

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = -\frac{Bin(c+dx)}{bg^2(a+bx)} - \frac{i(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a+bx)} - \frac{di\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} + \frac{Bdin\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}$$

```
[Out] -B*i*n*(d*x+c)/b/g^2/(b*x+a)-i*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g^
2/(b*x+a)-d*i*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^2
/g^2+B*d*i*n*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/g^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used

$= \{2561, 2380, 2341, 2379, 2438\}$

$$\int \frac{(ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag + bgx)^2}\,dx = -\frac{di\log\left(1 - \frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{b^2g^2} - \frac{i(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{bg^2(a+bx)} + \frac{Bdin\,\mathrm{PolyLog}\left(2, \frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} - \frac{Bin(c+dx)}{bg^2(a+bx)}$$

```
[In] Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^2,
x]
```

```
[Out] -((B*i*n*(c + d*x))/(b*g^2*(a + b*x))) - (i*(c + d*x)*(A + B*Log[e*((a + b*
x)/(c + d*x))^n]))/(b*g^2*(a + b*x)) - (d*i*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^2*g^2) + (B*d*i*n*PolyLog[
2, (b*(c + d*x))/(d*(a + b*x))])/(b^2*g^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
```

```
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^2}\\
&= \frac{i\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^2} + \frac{(di)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^2}\\
&= -\frac{Bin(c+dx)}{bg^2(a+bx)} - \frac{i(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a+bx)}\\
&\quad - \frac{di\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}\\
&\quad + \frac{(Bdin)\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2}\\
&= -\frac{Bin(c+dx)}{bg^2(a+bx)} - \frac{i(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a+bx)}\\
&\quad - \frac{di\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} + \frac{Bdin\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.26

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$$

$$= \frac{i\left(-\frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2(a+bx)} + \frac{d\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2} - \frac{Bn\left(\frac{bc-ad}{a+bx}+d\log(a+bx)-d\log(c+dx)\right)}{b^2} - \frac{Bdn\left(\log^2(a+\cdots\right.}{\cdots}\right.}{g^2}$$

```
[In] Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g
*x)^2,x]
```

```
[Out] (i*(-(((b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^2*(a + b*x)))
 + (d*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/b^2 - (B*n*((b*c
 - a*d)/(a + b*x) + d*Log[a + b*x] - d*Log[c + d*x]))/b^2 - (B*d*n*(Log[a +
 b*x]^2 - 2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 2*PolyLog[2, -((d
*(a + b*x))/(b*c - a*d))]))/(2*b^2)))/g^2
```

Maple [F]

$$\int \frac{(dix+ci)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}{\left(bgx+ag\right)^{2}}dx$$

```
[In] int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

```
[Out] int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

Fricas [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)}{(ag+bgx)^{2}}\,dx = \int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)}{\left(bgx+ag\right)^{2}}\,dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, alg
orithm="fricas")
```

```
[Out] integral((A*d*i*x + A*c*i + (B*d*i*x + B*c*i)*log(e*((b*x + a)/(d*x + c))^n
))/(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)}{(ag+bgx)^{2}}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)}{(ag+bgx)^{2}}\,dx = \int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)}{\left(bgx+ag\right)^{2}}\,dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, alg
orithm="maxima")
```

```
[Out] -B*c*i*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) -
d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) + B*d*i*((((b*x + a)*log(b*x + a) + a
)*log((b*x + a)^n) - ((b*x + a)*log(b*x + a) + a)*log((d*x + c)^n))/(b^3*g^
2*x + a*b^2*g^2) + integrate((b^2*d*x^2*log(e) + b^2*c*x*log(e) - a*b*c*n +
```

```
 a^2*d*n - (a*b*c*n - a^2*d*n + (b^2*c*n - a*b*d*n)*x)*log(b*x + a))/(b^4*d
*g^2*x^3 + a^2*b^2*c*g^2 + (b^4*c*g^2 + 2*a*b^3*d*g^2)*x^2 + (2*a*b^3*c*g^2
 + a^2*b^2*d*g^2)*x), x)) + A*d*i*(a/(b^3*g^2*x + a*b^2*g^2) + log(b*x + a)
/(b^2*g^2)) - B*c*i*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^2*g^2*x + a*b
*g^2) - A*c*i/(b^2*g^2*x + a*b*g^2)
```

Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, alg
orithm="giac")
```

```
[Out] integrate((d*i*x + c*i)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x + a*g
)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}{{\left(a\,g+b\,g\,x\right)}^2}\,dx$$

```
[In] int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^2,
x)
```

```
[Out] int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^2,
 x)
```

3.114 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 89

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = -\frac{Bin(c+dx)^2}{4(bc-ad)g^3(a+bx)^2} - \frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)g^3(a+bx)^2}$$

```
[Out] -1/4*B*i*n*(d*x+c)^2/(-a*d+b*c)/g^3/(b*x+a)^2-1/2*i*(d*x+c)^2*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))/(-a*d+b*c)/g^3/(b*x+a)^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.049, Rules used = {2561, 2341}

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = -\frac{i(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^3(a+bx)^2(bc-ad)} - \frac{Bin(c+dx)^2}{4g^3(a+bx)^2(bc-ad)}$$

```
[In] Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^3,
x]
```

```
[Out] -1/4*(B*i*n*(c + d*x)^2)/((b*c - a*d)*g^3*(a + b*x)^2) - (i*(c + d*x)^2*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*c - a*d)*g^3*(a + b*x)^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{i\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^3}$$
$$= -\frac{Bin(c+dx)^2}{4(bc-ad)g^3(a+bx)^2} - \frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)g^3(a+bx)^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 216 vs. $2(89) = 178$.

Time = 0.10 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.43

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$$
$$= \frac{i\left(-\frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2(a+bx)^2} - \frac{d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2(a+bx)} - \frac{Bdn\left(\frac{1}{a+bx}+\frac{d\log(a+bx)}{bc-ad}-\frac{d\log(c+dx)}{bc-ad}\right)}{b^2} - \frac{Bn\left(\frac{bc-ad}{(a+bx)^2}-\frac{2d}{a+bx}-\frac{2d^2}{\cdots}\right)}{4\cdots}\right.}{g^3}$$

```
[In] Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g
*x)^3,x]
```

```
[Out] (i*(-1/2*((b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^2*(a + b*x
)^2) - (d*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^2*(a + b*x)) - (B*d*n*
((a + b*x)^(-1) + (d*Log[a + b*x])/(b*c - a*d) - (d*Log[c + d*x])/(b*c - a*
d)))/b^2 - (B*n*((b*c - a*d)/(a + b*x)^2 - (2*d)/(a + b*x) - (2*d^2*Log[a +
 b*x])/(b*c - a*d) + (2*d^2*Log[c + d*x])/(b*c - a*d)))/(4*b^2)))/g^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(85) = 170.

Time = 5.16 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.66

method	result
parallelrisch	$-\frac{-4Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{4}c\,d^{2}in+B\,a^{2}b^{2}d^{3}i\,n^{2}-B\,b^{4}c^{2}di\,n^{2}+2A\,a^{2}b^{2}d^{3}in-2A\,b^{4}c^{2}din-2B\,x^{2}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{4}d^{3}in+2Bxa\,b}{4g^{3}(bx+a)^{2}b^{4}dn(ad-cb)}$

```
[In] int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x,method=_RET
URNVERBOSE)
```

```
[Out] -1/4*(-4*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^2*i*n+B*a^2*b^2*d^3*i*n^2-B*
b^4*c^2*d*i*n^2+2*A*a^2*b^2*d^3*i*n-2*A*b^4*c^2*d*i*n-2*B*x^2*ln(e*((b*x+a)
/(d*x+c))^n)*b^4*d^3*i*n+2*B*x*a*b^3*d^3*i*n^2-2*B*x*b^4*c*d^2*i*n^2+4*A*x*
a*b^3*d^3*i*n-4*A*x*b^4*c*d^2*i*n-2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2*d*i
*n)/g^3/(b*x+a)^2/b^4/d/n/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(85) = 170.

Time = 0.37 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.81

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)}{(ag+bgx)^{3}}\,dx = -\frac{(Bb^{2}c^{2}-Ba^{2}d^{2})in+2\,(Ab^{2}c^{2}-Aa^{2}d^{2})i+2\,((Bb^{2}cd-Babd^{2})in+2\,(Ab^{2}cd-Aabd^{2})i)x+2\,(2\,(Bb^{2}cd}{4\,((b^{5}c-ab^{4}d)g^{3}x^{2}+2\,(ab^{4}c-a^{2}b^{3}d}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, alg
orithm="fricas")
```

```
[Out] -1/4*((B*b^2*c^2 - B*a^2*d^2)*i*n + 2*(A*b^2*c^2 - A*a^2*d^2)*i + 2*((B*b^2
*c*d - B*a*b*d^2)*i*n + 2*(A*b^2*c*d - A*a*b*d^2)*i)*x + 2*(2*(B*b^2*c*d -
B*a*b*d^2)*i*x + (B*b^2*c^2 - B*a^2*d^2)*i)*log(e) + 2*(B*b^2*d^2*i*n*x^2 +
 2*B*b^2*c*d*i*n*x + B*b^2*c^2*i*n)*log((b*x + a)/(d*x + c)))/((b^5*c - a*b
^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. $2(76) = 152$.

Time = 80.89 (sec) , antiderivative size = 1360, normalized size of antiderivative = 15.28

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}(A+B\log(0^n e))\left(-\frac{ci}{2x^2}-\frac{di}{x}\right)}{g^3} \\ -\frac{Ad^2i}{b^3cg^3+b^3dg^3x}-\frac{Bd^2i\log\left(e\left(\frac{bc}{cd+d^2x}+\frac{bx}{c+dx}\right)^n\right)}{b^3cg^3+b^3dg^3x} \\ \frac{Acix+\frac{Adix^2}{2}+\frac{Bc^2i\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2d}+\frac{Bcinx}{2}+Bcix\log\left(e\left(\frac{a}{c+dx}\right)^n\right)+\frac{Bdinx^2}{4}+\frac{Bdix^2\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2}}{a^3g^3} \\ -\frac{2Aa^2d^2i}{4a^3b^2dg^3-4a^2b^3cg^3+8a^2b^3dg^3x-8ab^4cg^3x+4ab^4dg^3x^2-4b^5cg^3x^2}-\frac{4Aabd^2ix}{4a^3b^2dg^3-4a^2b^3cg^3+8a^2b^3dg^3x-8ab^4cg^3x+4ab^4dg^3x^2-4b^5cg^3x^2} \end{cases}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3,x)
```

```
[Out] Piecewise((zoo*(A + B*log(0**n*e))*(-c*i/(2*x**2) - d*i/x)/g**3, Eq(a, 0) &
 Eq(b, 0)), (-A*d**2*i/(b**3*c*g**3 + b**3*d*g**3*x) - B*d**2*i*log(e*(b*c/
(c*d + d**2*x) + b*x/(c + d*x))**n)/(b**3*c*g**3 + b**3*d*g**3*x), Eq(a, b*
c/d)), ((A*c*i*x + A*d*i*x**2/2 + B*c**2*i*log(e*(a/(c + d*x))**n)/(2*d) +
B*c*i*n*x/2 + B*c*i*x*log(e*(a/(c + d*x))**n) + B*d*i*n*x**2/4 + B*d*i*x**2
*log(e*(a/(c + d*x))**n)/2)/(a**3*g**3), Eq(b, 0)), (-2*A*a**2*d**2*i/(4*a*
*3*b**2*d*g**3 - 4*a**2*b**3*c*g**3 + 8*a**2*b**3*d*g**3*x - 8*a*b**4*c*g**
3*x + 4*a*b**4*d*g**3*x**2 - 4*b**5*c*g**3*x**2) - 4*A*a*b*d**2*i*x/(4*a**3
*b**2*d*g**3 - 4*a**2*b**3*c*g**3 + 8*a**2*b**3*d*g**3*x - 8*a*b**4*c*g**3*
x + 4*a*b**4*d*g**3*x**2 - 4*b**5*c*g**3*x**2) + 2*A*b**2*c**2*i/(4*a**3*b*
*2*d*g**3 - 4*a**2*b**3*c*g**3 + 8*a**2*b**3*d*g**3*x - 8*a*b**4*c*g**3*x +
 4*a*b**4*d*g**3*x**2 - 4*b**5*c*g**3*x**2) + 4*A*b**2*c*d*i*x/(4*a**3*b**2
*d*g**3 - 4*a**2*b**3*c*g**3 + 8*a**2*b**3*d*g**3*x - 8*a*b**4*c*g**3*x + 4
*a*b**4*d*g**3*x**2 - 4*b**5*c*g**3*x**2) - B*a**2*d**2*i*n/(4*a**3*b**2*d*
g**3 - 4*a**2*b**3*c*g**3 + 8*a**2*b**3*d*g**3*x - 8*a*b**4*c*g**3*x + 4*a*
b**4*d*g**3*x**2 - 4*b**5*c*g**3*x**2) - 2*B*a*b*d**2*i*n*x/(4*a**3*b**2*d*
g**3 - 4*a**2*b**3*c*g**3 + 8*a**2*b**3*d*g**3*x - 8*a*b**4*c*g**3*x + 4*a*
b**4*d*g**3*x**2 - 4*b**5*c*g**3*x**2) + B*b**2*c**2*i*n/(4*a**3*b**2*d*g**
3 - 4*a**2*b**3*c*g**3 + 8*a**2*b**3*d*g**3*x - 8*a*b**4*c*g**3*x + 4*a*b**
4*d*g**3*x**2 - 4*b**5*c*g**3*x**2) + 2*B*b**2*c**2*i*log(e*(a/(c + d*x) +
b*x/(c + d*x))**n)/(4*a**3*b**2*d*g**3 - 4*a**2*b**3*c*g**3 + 8*a**2*b**3*d
*g**3*x - 8*a*b**4*c*g**3*x + 4*a*b**4*d*g**3*x**2 - 4*b**5*c*g**3*x**2) +
2*B*b**2*c*d*i*n*x/(4*a**3*b**2*d*g**3 - 4*a**2*b**3*c*g**3 + 8*a**2*b**3*d
*g**3*x - 8*a*b**4*c*g**3*x + 4*a*b**4*d*g**3*x**2 - 4*b**5*c*g**3*x**2) +
4*B*b**2*c*d*i*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(4*a**3*b**2*d*g**
3 - 4*a**2*b**3*c*g**3 + 8*a**2*b**3*d*g**3*x - 8*a*b**4*c*g**3*x + 4*a*b**
```

```
4*d*g**3*x**2 - 4*b**5*c*g**3*x**2) + 2*B*b**2*d**2*i*x**2*log(e*(a/(c + d*
x) + b*x/(c + d*x))**n)/(4*a**3*b**2*d*g**3 - 4*a**2*b**3*c*g**3 + 8*a**2*b
**3*d*g**3*x - 8*a*b**4*c*g**3*x + 4*a*b**4*d*g**3*x**2 - 4*b**5*c*g**3*x**
2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. $2(85) = 170$.

Time = 0.20 (sec) , antiderivative size = 582, normalized size of antiderivative = 6.54

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx =$$

$$-\frac{1}{4}\,Bdin\left(\frac{3\,abc-a^2d+2\,(2\,b^2c-abd)x}{(b^5c-ab^4d)g^3x^2+2\,(ab^4c-a^2b^3d)g^3x+(a^2b^3c-a^3b^2d)g^3}+\frac{2\,(2\,bcd-ad^2)\log\left(bx+a\right)}{(b^4c^2-2\,ab^3cd+a^2b^2d^2)g^3}-\frac{2}{(b}\right.$$

$$+\frac{1}{4}\,Bcin\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}+\frac{2\,d^2\log\left(bx+a\right)}{(b^3c^2-2\,ab^2cd+a^2bd^2)g^3}-\frac{}{(b^3c}\right.$$

$$-\frac{(2\,bx+a)Bdi\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{2\,(b^4g^3x^2+2\,ab^3g^3x+a^2b^2g^3)}-\frac{(2\,bx+a)Adi}{2\,(b^4g^3x^2+2\,ab^3g^3x+a^2b^2g^3)}$$

$$-\frac{Bci\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}-\frac{Aci}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, alg
orithm="maxima")
```

```
[Out] -1/4*B*d*i*n*((3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*x)/((b^5*c - a*b^4*d)*
g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3) + 2*
(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3)
- 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g
^3)) + 1/4*B*c*i*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*
(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x +
a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2
 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*(2*b*x + a)*B*d*i*log(e*(b*x/(d*x +
 c) + a/(d*x + c))^n)/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) - 1/2*(2*
b*x + a)*A*d*i/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) - 1/2*B*c*i*log(
e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3
) - 1/2*A*c*i/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)
```

Giac [A] (verification not implemented)

none

Time = 0.78 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.12

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = -\frac{1}{4}\left(\frac{2\,(dx+c)^2Bin\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)^2g^3}+\frac{(Bin+2\,Bi\log(e)+2\,Ai)(dx+c)^2}{(bx+a)^2g^3}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, alg
orithm="giac")
```

```
[Out] -1/4*(2*(d*x + c)^2*B*i*n*log((b*x + a)/(d*x + c))/((b*x + a)^2*g^3) + (B*i
*n + 2*B*i*log(e) + 2*A*i)*(d*x + c)^2/((b*x + a)^2*g^3))*(b*c/(b*c - a*d)^
2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.29

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = -\frac{x\,(2\,A\,b\,d\,i+B\,b\,d\,i\,n)+A\,a\,d\,i+A\,b\,c\,i+\frac{B\,a\,d\,i\,n}{2}+\frac{B\,b\,c\,i\,n}{2}}{2\,a^2\,b^2\,g^3+4\,a\,b^3\,g^3\,x+2\,b^4\,g^3\,x^2} - \frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B\,c\,i}{2\,b}+\frac{B\,a\,d\,i}{2\,b^2}+\frac{B\,d\,i\,x}{b}\right)}{a^2\,g^3+2\,a\,b\,g^3\,x+b^2\,g^3\,x^2} - \frac{B\,d^2\,i\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,1\mathrm{i}}{b^2\,g^3\,(a\,d-b\,c)}$$

```
[In] int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^3,
x)
```

```
[Out] - (x*(2*A*b*d*i + B*b*d*i*n) + A*a*d*i + A*b*c*i + (B*a*d*i*n)/2 + (B*b*c*i
*n)/2)/(2*a^2*b^2*g^3 + 2*b^4*g^3*x^2 + 4*a*b^3*g^3*x) - (log(e*((a + b*x)/
(c + d*x))^n)*((B*c*i)/(2*b) + (B*a*d*i)/(2*b^2) + (B*d*i*x)/b))/(a^2*g^3 +
 b^2*g^3*x^2 + 2*a*b*g^3*x) - (B*d^2*i*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*
c) + 1i)*1i)/(b^2*g^3*(a*d - b*c))
```

3.115 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 181

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = \frac{Bdin(c+dx)^2}{4(bc-ad)^2g^4(a+bx)^2} - \frac{bBin(c+dx)^3}{9(bc-ad)^2g^4(a+bx)^3} + \frac{di(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2g^4(a+bx)^2} - \frac{bi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^2g^4(a+bx)^3}$$

```
[Out] 1/4*B*d*i*n*(d*x+c)^2/(-a*d+b*c)^2/g^4/(b*x+a)^2-1/9*b*B*i*n*(d*x+c)^3/(-a*
d+b*c)^2/g^4/(b*x+a)^3+1/2*d*i*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-
a*d+b*c)^2/g^4/(b*x+a)^2-1/3*b*i*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/
(-a*d+b*c)^2/g^4/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {2561, 45, 2372, 12}

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = -\frac{bi(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3g^4(a+bx)^3(bc-ad)^2} + \frac{di(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^4(a+bx)^2(bc-ad)^2} - \frac{bBin(c+dx)^3}{9g^4(a+bx)^3(bc-ad)^2} + \frac{Bdin(c+dx)^2}{4g^4(a+bx)^2(bc-ad)^2}$$

```
[In] Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^4,
x]
```

```
[Out] (B*d*i*n*(c + d*x)^2)/(4*(b*c - a*d)^2*g^4*(a + b*x)^2) - (b*B*i*n*(c + d*x
)^3)/(9*(b*c - a*d)^2*g^4*(a + b*x)^3) + (d*i*(c + d*x)^2*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n]))/(2*(b*c - a*d)^2*g^4*(a + b*x)^2) - (b*i*(c + d*x)^3
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(b*c - a*d)^2*g^4*(a + b*x)^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
```

```
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex^n))}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^4} \\
&= \frac{di(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2g^4(a+bx)^2} - \frac{bi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^2g^4(a+bx)^3} \\
&\quad - \frac{(Bin)\text{Subst}\left(\int \frac{-2b+3dx}{6x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^4} \\
&= \frac{di(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2g^4(a+bx)^2} - \frac{bi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^2g^4(a+bx)^3} \\
&\quad - \frac{(Bin)\text{Subst}\left(\int \frac{-2b+3dx}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{6(bc-ad)^2g^4} \\
&= \frac{di(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2g^4(a+bx)^2} - \frac{bi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^2g^4(a+bx)^3} \\
&\quad - \frac{(Bin)\text{Subst}\left(\int \left(-\frac{2b}{x^4}+\frac{3d}{x^3}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{6(bc-ad)^2g^4} \\
&= \frac{Bdin(c+dx)^2}{4(bc-ad)^2g^4(a+bx)^2} - \frac{bBin(c+dx)^3}{9(bc-ad)^2g^4(a+bx)^3} \\
&\quad + \frac{di(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2g^4(a+bx)^2} - \frac{bi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^2g^4(a+bx)^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.08

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx =$$

$$-\frac{i\left(\frac{12Abc}{(a+bx)^3}-\frac{12aAd}{(a+bx)^3}+\frac{4bBcn}{(a+bx)^3}-\frac{4aBdn}{(a+bx)^3}+\frac{18Ad}{(a+bx)^2}+\frac{3Bdn}{(a+bx)^2}-\frac{6Bd^2n}{(bc-ad)(a+bx)}-\frac{6Bd^3n\log(a+bx)}{(bc-ad)^2}+\frac{6B(2bc+ad+3bdx)}{(a+b}\right.}{36b^2g^4}$$

```
[In] Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g
*x)^4,x]
```

```
[Out] -1/36*(i*((12*A*b*c)/(a + b*x)^3 - (12*a*A*d)/(a + b*x)^3 + (4*b*B*c*n)/(a
+ b*x)^3 - (4*a*B*d*n)/(a + b*x)^3 + (18*A*d)/(a + b*x)^2 + (3*B*d*n)/(a +
b*x)^2 - (6*B*d^2*n)/((b*c - a*d)*(a + b*x)) - (6*B*d^3*n*Log[a + b*x])/(b*
c - a*d)^2 + (6*B*(2*b*c + a*d + 3*b*d*x)*Log[e*((a + b*x)/(c + d*x))^n])/(
a + b*x)^3 + (6*B*d^3*n*Log[c + d*x])/(b*c - a*d)^2))/(b^2*g^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(173) = 346$.

Time = 10.20 (sec) , antiderivative size = 455, normalized size of antiderivative = 2.51

method	result
parallelrisch	$-\frac{18Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{6}c^{2}d^{2}in-18Bxa\,b^{5}c\,d^{3}i\,n^{2}-36Axa\,b^{5}c\,d^{3}in-18B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a\,b^{5}c^{2}d^{2}in-18B\,x^{2}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right.}{}$

```
[In] int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x,method=_RET
URNVERBOSE)
```

```
[Out] -1/36*(18*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c^2*d^2*i*n-18*B*x*a*b^5*c*d^3*
i*n^2-36*A*x*a*b^5*c*d^3*i*n-18*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c^2*d^2*i
*n-18*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*d^4*i*n-36*B*x*ln(e*((b*x+a)/(d
*x+c))^n)*a*b^5*c*d^3*i*n+5*B*a^3*b^3*d^4*i*n^2+4*B*b^6*c^3*d*i*n^2+6*A*a^3
*b^3*d^4*i*n+12*A*b^6*c^3*d*i*n-6*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^6*d^4*i
*n+6*B*x^2*a*b^5*d^4*i*n^2-6*B*x^2*b^6*c*d^3*i*n^2+15*B*x*a^2*b^4*d^4*i*n^2
+3*B*x*b^6*c^2*d^2*i*n^2+18*A*x*a^2*b^4*d^4*i*n+18*A*x*b^6*c^2*d^2*i*n+12*B
*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c^3*d*i*n-9*B*a*b^5*c^2*d^2*i*n^2-18*A*a*b^5
*c^2*d^2*i*n)/g^4/(b*x+a)^3/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^5/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(173) = 346$.

Time = 0.38 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.64

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)}{(ag+bgx)^{4}}\,dx = \frac{6\,(Bb^{3}cd^{2}-Bab^{2}d^{3})inx^{2}-(4\,Bb^{3}c^{3}-9\,Bab^{2}c^{2}d+5\,Ba^{3}d^{3})in-6\,(2\,Ab^{3}c^{3}-3\,Aab^{2}c^{2}d+Aa^{3}d^{3})i-3\,(}{}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, alg
orithm="fricas")
```

```
[Out] 1/36*(6*(B*b^3*c*d^2 - B*a*b^2*d^3)*i*n*x^2 - (4*B*b^3*c^3 - 9*B*a*b^2*c^2*
d + 5*B*a^3*d^3)*i*n - 6*(2*A*b^3*c^3 - 3*A*a*b^2*c^2*d + A*a^3*d^3)*i - 3*
((B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*i*n + 6*(A*b^3*c^2*d - 2*A
```

```
*a*b^2*c*d^2 + A*a^2*b*d^3)*i)*x - 6*(3*(B*b^3*c^2*d - 2*B*a*b^2*c*d^2 + B*
a^2*b*d^3)*i*x + (2*B*b^3*c^3 - 3*B*a*b^2*c^2*d + B*a^3*d^3)*i)*log(e) + 6*
(B*b^3*d^3*i*n*x^3 + 3*B*a*b^2*d^3*i*n*x^2 - 3*(B*b^3*c^2*d - 2*B*a*b^2*c*d
^2)*i*n*x - (2*B*b^3*c^3 - 3*B*a*b^2*c^2*d)*i*n)*log((b*x + a)/(d*x + c)))/
((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c
*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g
^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 945 vs. 2(173) = 346.

Time = 0.22 (sec) , antiderivative size = 945, normalized size of antiderivative = 5.22

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = -\frac{1}{18}\,Bcin\left(\frac{6\,b^2d^2x^2+2\,b^2c^2-7\,abcd+11\,a^2d^2-3\,(b^2cd-5\,abd^2)}{(b^6c^2-2\,ab^5cd+a^2b^4d^2)g^4x^3+3\,(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2)g^4x^2+3\,(a^2b^4c^2-2\,a^3b^3cd+a^4b}\right. - \frac{1}{36}\,Bdin\left(\frac{5\,ab^2c^2-22\,a^2bcd+5\,a^3d^2-6\,(3\,b^3cd-ab^2d^2)x^2+3\,(3\,b^3c^2-16\,ab}{(b^7c^2-2\,ab^6cd+a^2b^5d^2)g^4x^3+3\,(ab^6c^2-2\,a^2b^5cd+a^3b^4d^2)g^4x^2+3\,(a^2b^5c^2-2\,a^3b^4cd+a^4b}\right. - \frac{(3\,bx+a)Bdi\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{6\,(b^5g^4x^3+3\,ab^4g^4x^2+3\,a^2b^3g^4x+a^3b^2g^4)} - \frac{(3\,bx+a)Adi}{6\,(b^5g^4x^3+3\,ab^4g^4x^2+3\,a^2b^3g^4x+a^3b^2g^4)} - \frac{Bci\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{3\,(b^4g^4x^3+3\,ab^3g^4x^2+3\,a^2b^2g^4x+a^3bg^4)} - \frac{Aci}{3\,(b^4g^4x^3+3\,ab^3g^4x^2+3\,a^2b^2g^4x+a^3bg^4)}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, alg
orithm="maxima")
[Out] -1/18*B*c*i*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2
*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*
b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3
*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4)
```

```
+ 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^
3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
a^3*b*d^3)*g^4)) - 1/36*B*d*i*n*((5*a*b^2*c^2 - 22*a^2*b*c*d + 5*a^3*d^2 -
6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c^2 - 16*a*b^2*c*d + 5*a^2*b*d^2)*
x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^
5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2
)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4) - 6*(3*b*c*d^2 -
 a*d^3)*log(b*x + a)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*
d^3)*g^4) + 6*(3*b*c*d^2 - a*d^3)*log(d*x + c)/((b^5*c^3 - 3*a*b^4*c^2*d +
3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4)) - 1/6*(3*b*x + a)*B*d*i*log(e*(b*x/(d*
x + c) + a/(d*x + c))^n)/(b^5*g^4*x^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x +
 a^3*b^2*g^4) - 1/6*(3*b*x + a)*A*d*i/(b^5*g^4*x^3 + 3*a*b^4*g^4*x^2 + 3*a^
2*b^3*g^4*x + a^3*b^2*g^4) - 1/3*B*c*i*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n)/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) - 1/3*A*c*
i/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Giac [A] (verification not implemented)

none

Time = 1.08 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.29

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx =$$

$$-\frac{1}{36}\left(\frac{6\left(2\,Bbin-\frac{3\,(bx+a)Bdin}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^3bcg^4}{(dx+c)^3}-\frac{(bx+a)^3adg^4}{(dx+c)^3}}+\frac{4\,Bbin-\frac{9\,(bx+a)Bdin}{dx+c}+12\,Bbi\log\left(e\right)-\frac{18\,(bx+a)Bdi\log(e)}{dx+c}+1}{\frac{(bx+a)^3bcg^4}{(dx+c)^3}-\frac{(bx+a)^3adg^4}{(dx+c)^3}}\right.$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, alg
orithm="giac")
```

```
[Out] -1/36*(6*(2*B*b*i*n - 3*(b*x + a)*B*d*i*n/(d*x + c))*log((b*x + a)/(d*x + c
))/((b*x + a)^3*b*c*g^4/(d*x + c)^3 - (b*x + a)^3*a*d*g^4/(d*x + c)^3) + (4
*B*b*i*n - 9*(b*x + a)*B*d*i*n/(d*x + c) + 12*B*b*i*log(e) - 18*(b*x + a)*B
*d*i*log(e)/(d*x + c) + 12*A*b*i - 18*(b*x + a)*A*d*i/(d*x + c))/((b*x + a)
^3*b*c*g^4/(d*x + c)^3 - (b*x + a)^3*a*d*g^4/(d*x + c)^3))*(b*c/(b*c - a*d)
^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.07

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx =$$

$$-\frac{\frac{6\,A\,a^2\,d^2\,i-12\,A\,b^2\,c^2\,i+5\,B\,a^2\,d^2\,i\,n-4\,B\,b^2\,c^2\,i\,n+6\,A\,a\,b\,c\,d\,i+5\,B\,a\,b\,c\,d\,i\,n}{6\,(a\,d-b\,c)}+\frac{x\,(6\,A\,a\,b\,d^2\,i-6\,A\,b^2\,c\,d\,i-B\,b^2\,c\,d\,i\,n+5\,B\,a\,b\,d^2\,i\,n)}{2\,(a\,d-b\,c)}+\frac{B\,b^2}{a}}{6\,a^3\,b^2\,g^4+18\,a^2\,b^3\,g^4\,x+18\,a\,b^4\,g^4\,x^2+6\,b^5\,g^4\,x^3}$$

$$-\frac{\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(\frac{B\,c\,i}{3\,b}+\frac{B\,a\,d\,i}{6\,b^2}+\frac{B\,d\,i\,x}{2\,b}\right)}{a^3\,g^4+3\,a^2\,b\,g^4\,x+3\,a\,b^2\,g^4\,x^2+b^3\,g^4\,x^3}-\frac{B\,d^3\,i\,n\,\mathrm{atanh}\left(\frac{6\,b^4\,c^2\,g^4-6\,a^2\,b^2\,d^2\,g^4}{6\,b^2\,g^4\,(a\,d-b\,c)^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{3\,b^2\,g^4\,{(a\,d-b\,c)}^2}$$

```
[In] int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^4,
x)
```

```
[Out] - ((6*A*a^2*d^2*i - 12*A*b^2*c^2*i + 5*B*a^2*d^2*i*n - 4*B*b^2*c^2*i*n + 6*
A*a*b*c*d*i + 5*B*a*b*c*d*i*n)/(6*(a*d - b*c)) + (x*(6*A*a*b*d^2*i - 6*A*b^
2*c*d*i - B*b^2*c*d*i*n + 5*B*a*b*d^2*i*n))/(2*(a*d - b*c)) + (B*b^2*d^2*i*
n*x^2)/(a*d - b*c))/(6*a^3*b^2*g^4 + 6*b^5*g^4*x^3 + 18*a^2*b^3*g^4*x + 18*
a*b^4*g^4*x^2) - (log(e*((a + b*x)/(c + d*x))^n)*((B*c*i)/(3*b) + (B*a*d*i)
/(6*b^2) + (B*d*i*x)/(2*b)))/(a^3*g^4 + b^3*g^4*x^3 + 3*a*b^2*g^4*x^2 + 3*a
^2*b*g^4*x) - (B*d^3*i*n*atanh((6*b^4*c^2*g^4 - 6*a^2*b^2*d^2*g^4)/(6*b^2*g
^4*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(3*b^2*g^4*(a*d - b*c)^2)
```

3.116 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 281

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx = -\frac{Bd^2in(c+dx)^2}{4(bc-ad)^3g^5(a+bx)^2} + \frac{2bBdin(c+dx)^3}{9(bc-ad)^3g^5(a+bx)^3} - \frac{b^2Bin(c+dx)^4}{16(bc-ad)^3g^5(a+bx)^4} - \frac{d^2i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^5(a+bx)^2} + \frac{2bdi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^3g^5(a+bx)^3} - \frac{b^2i(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bc-ad)^3g^5(a+bx)^4}$$

```
[Out] -1/4*B*d^2*i*n*(d*x+c)^2/(-a*d+b*c)^3/g^5/(b*x+a)^2+2/9*b*B*d*i*n*(d*x+c)^3
/(-a*d+b*c)^3/g^5/(b*x+a)^3-1/16*b^2*B*i*n*(d*x+c)^4/(-a*d+b*c)^3/g^5/(b*x+
a)^4-1/2*d^2*i*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^5/(
b*x+a)^2+2/3*b*d*i*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g
^5/(b*x+a)^3-1/4*b^2*i*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)
^3/g^5/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {2561, 45, 2372, 12, 14}

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx = -\frac{b^2i(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4g^5(a+bx)^4(bc-ad)^3} - \frac{d^2i(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^5(a+bx)^2(bc-ad)^3} + \frac{2bdi(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3g^5(a+bx)^3(bc-ad)^3} - \frac{b^2Bin(c+dx)^4}{16g^5(a+bx)^4(bc-ad)^3} - \frac{Bd^2in(c+dx)^2}{4g^5(a+bx)^2(bc-ad)^3} + \frac{2bBdin(c+dx)^3}{9g^5(a+bx)^3(bc-ad)^3}$$

```
[In] Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^5,
x]
```

```
[Out] -1/4*(B*d^2*i*n*(c + d*x)^2)/((b*c - a*d)^3*g^5*(a + b*x)^2) + (2*b*B*d*i*n
*(c + d*x)^3)/(9*(b*c - a*d)^3*g^5*(a + b*x)^3) - (b^2*B*i*n*(c + d*x)^4)/(
16*(b*c - a*d)^3*g^5*(a + b*x)^4) - (d^2*i*(c + d*x)^2*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(2*(b*c - a*d)^3*g^5*(a + b*x)^2) + (2*b*d*i*(c + d*x)^
3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(b*c - a*d)^3*g^5*(a + b*x)^3)
 - (b^2*i*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*(b*c - a*d
)^3*g^5*(a + b*x)^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex^n))}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^5} \\
&= -\frac{d^2i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^5(a+bx)^2}+\frac{2bdi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^3g^5(a+bx)^3} \\
&\quad -\frac{b^2i(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bc-ad)^3g^5(a+bx)^4}-\frac{(Bin)\text{Subst}\left(\int \frac{-3b^2+8bdx-6d^2x^2}{12x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^5} \\
&= -\frac{d^2i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^5(a+bx)^2}+\frac{2bdi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^3g^5(a+bx)^3} \\
&\quad -\frac{b^2i(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bc-ad)^3g^5(a+bx)^4}-\frac{(Bin)\text{Subst}\left(\int \frac{-3b^2+8bdx-6d^2x^2}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{12(bc-ad)^3g^5} \\
&= -\frac{d^2i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^5(a+bx)^2}+\frac{2bdi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^3g^5(a+bx)^3} \\
&\quad -\frac{b^2i(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bc-ad)^3g^5(a+bx)^4}-\frac{(Bin)\text{Subst}\left(\int \left(-\frac{3b^2}{x^5}+\frac{8bd}{x^4}-\frac{6d^2}{x^3}\right)dx, x, \frac{a+bx}{c+dx}\right)}{12(bc-ad)^3g^5}
\end{aligned}
$$

$$= -\frac{Bd^2in(c+dx)^2}{4(bc-ad)^3g^5(a+bx)^2} + \frac{2bBdin(c+dx)^3}{9(bc-ad)^3g^5(a+bx)^3} - \frac{b^2Bin(c+dx)^4}{16(bc-ad)^3g^5(a+bx)^4} - \frac{d^2i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^5(a+bx)^2} + \frac{2bdi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^3g^5(a+bx)^3} - \frac{b^2i(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bc-ad)^3g^5(a+bx)^4}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.78

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx = -\frac{i\left(\frac{36Abc}{(a+bx)^4} - \frac{36aAd}{(a+bx)^4} + \frac{9bBcn}{(a+bx)^4} - \frac{9aBdn}{(a+bx)^4} + \frac{48Ad}{(a+bx)^3} + \frac{4Bdn}{(a+bx)^3} - \frac{6Bd^2n}{(bc-ad)(a+bx)^2} + \frac{12Bd^3n}{(bc-ad)^2(a+bx)} + \frac{12Bd^4n\log(a+bx)}{(bc-ad)^3}\right.}{144b^2g^5}$$

```
[In] Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g
*x)^5,x]
```

```
[Out] -1/144*(i*((36*A*b*c)/(a + b*x)^4 - (36*a*A*d)/(a + b*x)^4 + (9*b*B*c*n)/(a
 + b*x)^4 - (9*a*B*d*n)/(a + b*x)^4 + (48*A*d)/(a + b*x)^3 + (4*B*d*n)/(a +
 b*x)^3 - (6*B*d^2*n)/((b*c - a*d)*(a + b*x)^2) + (12*B*d^3*n)/((b*c - a*d)
^2*(a + b*x)) + (12*B*d^4*n*Log[a + b*x])/(b*c - a*d)^3 + (12*B*(3*b*c + a*
d + 4*b*d*x)*Log[e*((a + b*x)/(c + d*x))^n])/(a + b*x)^4 - (12*B*d^4*n*Log[
c + d*x])/(b*c - a*d)^3))/(b^2*g^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 985 vs. 2(269) = 538.

Time = 27.71 (sec) , antiderivative size = 986, normalized size of antiderivative = 3.51

method	result
parallelrisch	$\frac{48Ax^3a^7bcd^4in-288Ax^3a^5b^3c^3d^2in-144Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^7bc^3d^2in+48Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^6b^2c^4din+384Ax^3a^4b^4c^4din+}{}$

```
[In] int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x,method=_RET
URNVERBOSE)
```

```
[Out] 1/144*(48*A*x^3*a^7*b*c*d^4*i*n-288*A*x^3*a^5*b^3*c^3*d^2*i*n-144*B*x*ln(e*
((b*x+a)/(d*x+c))^n)*a^7*b*c^3*d^2*i*n+48*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^6
*b^2*c^4*d*i*n+384*A*x^3*a^4*b^4*c^4*d*i*n+72*B*x^2*ln(e*((b*x+a)/(d*x+c))^
n)*a^8*c*d^4*i*n+48*B*x^2*a^7*b*c^2*d^3*i*n^2-222*B*x^2*a^6*b^2*c^3*d^2*i*n
```

```
^2+192*B*x^2*a^5*b^3*c^4*d*i*n^2-432*A*x^2*a^6*b^2*c^3*d^2*i*n+576*A*x^2*a^
5*b^3*c^4*d*i*n+144*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^8*c^2*d^3*i*n-168*B*x*a
^7*b*c^3*d^2*i*n^2+132*B*x*a^6*b^2*c^4*d*i*n^2-432*A*x*a^7*b*c^3*d^2*i*n+43
2*A*x*a^6*b^2*c^4*d*i*n-96*B*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c^4*d*i*n+12*B
*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c*d^4*i*n+48*B*x^3*ln(e*((b*x+a)/(d*
x+c))^n)*a^7*b*c*d^4*i*n+13*B*x^4*a^6*b^2*c*d^4*i*n^2-36*B*x^4*a^4*b^4*c^3*
d^2*i*n^2+32*B*x^4*a^3*b^5*c^4*d*i*n^2+12*A*x^4*a^6*b^2*c*d^4*i*n-72*A*x^4*
a^4*b^4*c^3*d^2*i*n+96*A*x^4*a^3*b^5*c^4*d*i*n+40*B*x^3*a^7*b*c*d^4*i*n^2+1
2*B*x^3*a^6*b^2*c^2*d^3*i*n^2-144*B*x^3*a^5*b^3*c^3*d^2*i*n^2+128*B*x^3*a^4
*b^4*c^4*d*i*n^2-9*B*x^4*a^2*b^6*c^5*i*n^2-36*A*x^4*a^2*b^6*c^5*i*n-36*B*x^
3*a^3*b^5*c^5*i*n^2-144*A*x^3*a^3*b^5*c^5*i*n+36*B*x^2*a^8*c*d^4*i*n^2-54*B
*x^2*a^4*b^4*c^5*i*n^2+72*A*x^2*a^8*c*d^4*i*n-216*A*x^2*a^4*b^4*c^5*i*n+72*
B*x*a^8*c^2*d^3*i*n^2-36*B*x*a^5*b^3*c^5*i*n^2+144*A*x*a^8*c^2*d^3*i*n-144*
A*x*a^5*b^3*c^5*i*n+72*B*ln(e*((b*x+a)/(d*x+c))^n)*a^8*c^3*d^2*i*n+36*B*ln(
e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^5*i*n)/g^5/(b*x+a)^4/n/(a^3*d^3-3*a^2*b*c*
d^2+3*a*b^2*c^2*d-b^3*c^3)/a^6/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(269) = 538$.

Time = 0.35 (sec) , antiderivative size = 773, normalized size of antiderivative = 2.75

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx =$$
$$-\frac{12\,(Bb^4cd^3-Bab^3d^4)inx^3-6\,(Bb^4c^2d^2-8\,Bab^3cd^3+7\,Ba^2b^2d^4)inx^2+(9\,Bb^4c^4-32\,Bab^3c^3d+36\,\ldots}{}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, alg
orithm="fricas")
```

```
[Out] -1/144*(12*(B*b^4*c*d^3 - B*a*b^3*d^4)*i*n*x^3 - 6*(B*b^4*c^2*d^2 - 8*B*a*b
^3*c*d^3 + 7*B*a^2*b^2*d^4)*i*n*x^2 + (9*B*b^4*c^4 - 32*B*a*b^3*c^3*d + 36*
B*a^2*b^2*c^2*d^2 - 13*B*a^4*d^4)*i*n + 12*(3*A*b^4*c^4 - 8*A*a*b^3*c^3*d +
 6*A*a^2*b^2*c^2*d^2 - A*a^4*d^4)*i + 4*((B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 +
 18*B*a^2*b^2*c*d^3 - 13*B*a^3*b*d^4)*i*n + 12*(A*b^4*c^3*d - 3*A*a*b^3*c^2
*d^2 + 3*A*a^2*b^2*c*d^3 - A*a^3*b*d^4)*i)*x + 12*(4*(B*b^4*c^3*d - 3*B*a*b
^3*c^2*d^2 + 3*B*a^2*b^2*c*d^3 - B*a^3*b*d^4)*i*x + (3*B*b^4*c^4 - 8*B*a*b^
3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - B*a^4*d^4)*i)*log(e) + 12*(B*b^4*d^4*i*n*x^
4 + 4*B*a*b^3*d^4*i*n*x^3 + 6*B*a^2*b^2*d^4*i*n*x^2 + 4*(B*b^4*c^3*d - 3*B*
a*b^3*c^2*d^2 + 3*B*a^2*b^2*c*d^3)*i*n*x + (3*B*b^4*c^4 - 8*B*a*b^3*c^3*d +
 6*B*a^2*b^2*c^2*d^2)*i*n)*log((b*x + a)/(d*x + c)))/((b^9*c^3 - 3*a*b^8*c^
2*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*g^5*x^4 + 4*(a*b^8*c^3 - 3*a^2*b^7*c^2
*d + 3*a^3*b^6*c*d^2 - a^4*b^5*d^3)*g^5*x^3 + 6*(a^2*b^7*c^3 - 3*a^3*b^6*c^
2*d + 3*a^4*b^5*c*d^2 - a^5*b^4*d^3)*g^5*x^2 + 4*(a^3*b^6*c^3 - 3*a^4*b^5*c
```

```
^2*d + 3*a^5*b^4*c*d^2 - a^6*b^3*d^3)*g^5*x + (a^4*b^5*c^3 - 3*a^5*b^4*c^2*
d + 3*a^6*b^3*c*d^2 - a^7*b^2*d^3)*g^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**5,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1398 vs. $2(269) = 538$.

Time = 0.25 (sec) , antiderivative size = 1398, normalized size of antiderivative = 4.98

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, alg
orithm="maxima")
[Out] 1/48*B*c*i*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2
 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*
d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^
5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4
*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^
3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b
^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^
3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2
 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*
b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/144*
B*d*i*n*((7*a*b^3*c^3 - 33*a^2*b^2*c^2*d + 75*a^3*b*c*d^2 - 13*a^4*d^3 + 12
*(4*b^4*c*d^2 - a*b^3*d^3)*x^3 - 6*(4*b^4*c^2*d - 29*a*b^3*c*d^2 + 7*a^2*b^
2*d^3)*x^2 + 4*(4*b^4*c^3 - 21*a*b^3*c^2*d + 57*a^2*b^2*c*d^2 - 13*a^3*b*d^
3)*x)/((b^9*c^3 - 3*a*b^8*c^2*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*g^5*x^4 +
4*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*a^3*b^6*c*d^2 - a^4*b^5*d^3)*g^5*x^3 + 6
*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*a^4*b^5*c*d^2 - a^5*b^4*d^3)*g^5*x^2 +
4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2 - a^6*b^3*d^3)*g^5*x + (
a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d^2 - a^7*b^2*d^3)*g^5) + 12*(4
*b*c*d^3 - a*d^4)*log(b*x + a)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^
2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5) - 12*(4*b*c*d^3 - a*d^4)*log(d*x +
```

```
c)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^
2*d^4)*g^5)) - 1/12*(4*b*x + a)*B*d*i*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
)/(b^6*g^5*x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^
4*b^2*g^5) - 1/12*(4*b*x + a)*A*d*i/(b^6*g^5*x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*
b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g^5) - 1/4*B*c*i*log(e*(b*x/(d*x +
c) + a/(d*x + c))^n)/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4
*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A*c*i/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*
a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

Giac [A] (verification not implemented)

none

Time = 1.36 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.40

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx = -\frac{1}{144}\left(\frac{12\left(3\,Bb^2in-\frac{8\,(bx+a)Bbdin}{dx+c}+\frac{6\,(bx+a)^2Bd^2in}{(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^4b^2c^2g^5}{(dx+c)^4}-\frac{2\,(bx+a)^4abcdg^5}{(dx+c)^4}+\frac{(bx+a)^4a^2d^2g^5}{(dx+c)^4}}+\frac{9\,Bb^2in-\frac{32\,(bx+a)Bbdin}{dx+c}+\frac{36\,(bx+a)^2Bd^2in}{(dx+c)^2}}{}\right.$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, alg
orithm="giac")
```

```
[Out] -1/144*(12*(3*B*b^2*i*n - 8*(b*x + a)*B*b*d*i*n/(d*x + c) + 6*(b*x + a)^2*B
*d^2*i*n/(d*x + c)^2)*log((b*x + a)/(d*x + c))/((b*x + a)^4*b^2*c^2*g^5/(d*
x + c)^4 - 2*(b*x + a)^4*a*b*c*d*g^5/(d*x + c)^4 + (b*x + a)^4*a^2*d^2*g^5/
(d*x + c)^4) + (9*B*b^2*i*n - 32*(b*x + a)*B*b*d*i*n/(d*x + c) + 36*(b*x +
a)^2*B*d^2*i*n/(d*x + c)^2 + 36*B*b^2*i*log(e) - 96*(b*x + a)*B*b*d*i*log(e
)/(d*x + c) + 72*(b*x + a)^2*B*d^2*i*log(e)/(d*x + c)^2 + 36*A*b^2*i - 96*(
b*x + a)*A*b*d*i/(d*x + c) + 72*(b*x + a)^2*A*d^2*i/(d*x + c)^2)/((b*x + a)
^4*b^2*c^2*g^5/(d*x + c)^4 - 2*(b*x + a)^4*a*b*c*d*g^5/(d*x + c)^4 + (b*x +
 a)^4*a^2*d^2*g^5/(d*x + c)^4))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.17

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx$$
$$= \frac{B\,d^4\,i\,n\,\mathrm{atanh}\left(\frac{12\,a^3\,b^2\,d^3\,g^5-12\,a^2\,b^3\,c\,d^2\,g^5-12\,a\,b^4\,c^2\,d\,g^5+12\,b^5\,c^3\,g^5}{12\,b^2\,g^5\,(a\,d-b\,c)^3}+\frac{2\,b\,d\,x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^3}\right)}{6\,b^2\,g^5\,(a\,d-b\,c)^3}$$
$$-\frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B\,c\,i}{4\,b}+\frac{B\,a\,d\,i}{12\,b^2}+\frac{B\,d\,i\,x}{3\,b}\right)}{a^4\,g^5+4\,a^3\,b\,g^5\,x+6\,a^2\,b^2\,g^5\,x^2+4\,a\,b^3\,g^5\,x^3+b^4\,g^5\,x^4}$$
$$-\frac{\frac{12\,A\,a^3\,d^3\,i+36\,A\,b^3\,c^3\,i+13\,B\,a^3\,d^3\,i\,n+9\,B\,b^3\,c^3\,i\,n-60\,A\,a\,b^2\,c^2\,d\,i+12\,A\,a^2\,b\,c\,d^2\,i-23\,B\,a\,b^2\,c^2\,d\,i\,n+13\,B\,a^2\,b\,c\,d^2\,i\,n}{12\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\frac{x\,(12\,A\,a^2\,b\,d^3\,i+1}{}}{12\,a^4\,b^2\,g^5+48\,a^3\,b^3\,g^5\,x+72\,a^2\,b^4\,g}$$

```
[In] int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^5,
x)
```

```
[Out] (B*d^4*i*n*atanh((12*b^5*c^3*g^5 + 12*a^3*b^2*d^3*g^5 - 12*a*b^4*c^2*d*g^5
- 12*a^2*b^3*c*d^2*g^5)/(12*b^2*g^5*(a*d - b*c)^3) + (2*b*d*x*(a^2*d^2 + b^
2*c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(6*b^2*g^5*(a*d - b*c)^3) - (log(e*((a
+ b*x)/(c + d*x))^n)*((B*c*i)/(4*b) + (B*a*d*i)/(12*b^2) + (B*d*i*x)/(3*b))
)/(a^4*g^5 + b^4*g^5*x^4 + 4*a*b^3*g^5*x^3 + 6*a^2*b^2*g^5*x^2 + 4*a^3*b*g^
5*x) - ((12*A*a^3*d^3*i + 36*A*b^3*c^3*i + 13*B*a^3*d^3*i*n + 9*B*b^3*c^3*i
*n - 60*A*a*b^2*c^2*d*i + 12*A*a^2*b*c*d^2*i - 23*B*a*b^2*c^2*d*i*n + 13*B*
a^2*b*c*d^2*i*n)/(12*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(12*A*a^2*b*d^3*
i + 12*A*b^3*c^2*d*i - 24*A*a*b^2*c*d^2*i + 13*B*a^2*b*d^3*i*n + B*b^3*c^2*
d*i*n - 5*B*a*b^2*c*d^2*i*n))/(3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (d*x^2*
(B*b^3*c*d*i*n - 7*B*a*b^2*d^2*i*n))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) +
(B*b^3*d^3*i*n*x^3)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(12*a^4*b^2*g^5 + 12*b
^6*g^5*x^4 + 48*a^3*b^3*g^5*x + 48*a*b^5*g^5*x^3 + 72*a^2*b^4*g^5*x^2)
```

3.117 $\int (ag+bgx)^3(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal result

Integrand size = 43, antiderivative size = 442

$$\begin{aligned}
&\int (ag+bgx)^3(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx \\
&= \frac{B(bc-ad)^5g^3i^2nx}{60b^2d^3}+\frac{B(bc-ad)^4g^3i^2n(c+dx)^2}{120bd^4}-\frac{19B(bc-ad)^3g^3i^2n(c+dx)^3}{180d^4} \\
&\quad+\frac{13bB(bc-ad)^2g^3i^2n(c+dx)^4}{120d^4}-\frac{b^2B(bc-ad)g^3i^2n(c+dx)^5}{30d^4} \\
&\quad-\frac{(bc-ad)^3g^3i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^4} \\
&\quad+\frac{3b(bc-ad)^2g^3i^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^4} \\
&\quad-\frac{3b^2(bc-ad)g^3i^2(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^4} \\
&\quad+\frac{b^3g^3i^2(c+dx)^6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^4} \\
&\quad+\frac{B(bc-ad)^6g^3i^2n\log\left(\frac{a+bx}{c+dx}\right)}{60b^3d^4}+\frac{B(bc-ad)^6g^3i^2n\log(c+dx)}{60b^3d^4}
\end{aligned}$$

```
[Out] 1/60*B*(-a*d+b*c)^5*g^3*i^2*n*x/b^2/d^3+1/120*B*(-a*d+b*c)^4*g^3*i^2*n*(d*x
+c)^2/b/d^4-19/180*B*(-a*d+b*c)^3*g^3*i^2*n*(d*x+c)^3/d^4+13/120*b*B*(-a*d+
b*c)^2*g^3*i^2*n*(d*x+c)^4/d^4-1/30*b^2*B*(-a*d+b*c)*g^3*i^2*n*(d*x+c)^5/d^
4-1/3*(-a*d+b*c)^3*g^3*i^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4+3/
4*b*(-a*d+b*c)^2*g^3*i^2*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4-3/5*
b^2*(-a*d+b*c)*g^3*i^2*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4+1/6*b^
3*g^3*i^2*(d*x+c)^6*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4+1/60*B*(-a*d+b*c)^6
*g^3*i^2*n*ln((b*x+a)/(d*x+c))/b^3/d^4+1/60*B*(-a*d+b*c)^6*g^3*i^2*n*ln(d*x
+c)/b^3/d^4
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2561, 45, 2382, 12, 1634}

$$\begin{aligned}&\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx\\&=\frac{b^3g^3i^2(c+dx)^6\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6d^4}\\&\quad-\frac{3b^2g^3i^2(c+dx)^5(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^4}\\&\quad-\frac{g^3i^2(c+dx)^3(bc-ad)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^4}\\&\quad+\frac{3bg^3i^2(c+dx)^4(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^4}\\&\quad+\frac{Bg^3i^2n(bc-ad)^6\log\left(\frac{a+bx}{c+dx}\right)}{60b^3d^4}+\frac{Bg^3i^2n(bc-ad)^6\log(c+dx)}{60b^3d^4}\\&\quad-\frac{b^2Bg^3i^2n(c+dx)^5(bc-ad)}{30d^4}+\frac{Bg^3i^2nx(bc-ad)^5}{60b^2d^3}+\frac{Bg^3i^2n(c+dx)^2(bc-ad)^4}{120bd^4}\\&\quad-\frac{19Bg^3i^2n(c+dx)^3(bc-ad)^3}{180d^4}+\frac{13bBg^3i^2n(c+dx)^4(bc-ad)^2}{120d^4}\end{aligned}$$

```
[In] Int[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),
x]
```

```
[Out] (B*(b*c - a*d)^5*g^3*i^2*n*x)/(60*b^2*d^3) + (B*(b*c - a*d)^4*g^3*i^2*n*(c
+ d*x)^2)/(120*b*d^4) - (19*B*(b*c - a*d)^3*g^3*i^2*n*(c + d*x)^3)/(180*d^4
) + (13*b*B*(b*c - a*d)^2*g^3*i^2*n*(c + d*x)^4)/(120*d^4) - (b^2*B*(b*c -
a*d)*g^3*i^2*n*(c + d*x)^5)/(30*d^4) - ((b*c - a*d)^3*g^3*i^2*(c + d*x)^3*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d^4) + (3*b*(b*c - a*d)^2*g^3*i^2
*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*d^4) - (3*b^2*(b*c
- a*d)*g^3*i^2*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*d^4)
+ (b^3*g^3*i^2*(c + d*x)^6*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*d^4)
+ (B*(b*c - a*d)^6*g^3*i^2*n*Log[(a + b*x)/(c + d*x)])/(60*b^3*d^4) + (B*(b
*c - a*d)^6*g^3*i^2*n*Log[c + d*x])/(60*b^3*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left((bc-ad)^6 g^3 i^2\right) \text{Subst}\left(\int \frac{x^3(A+B\log(ex^n))}{(b-dx)^7}\,dx, x, \frac{a+bx}{c+dx}\right)\\
&= -\frac{(bc-ad)^3 g^3 i^2 (c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^4}\\
&\quad+\frac{3b(bc-ad)^2 g^3 i^2 (c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^4}\\
&\quad-\frac{3b^2(bc-ad) g^3 i^2 (c+dx)^5 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^4}\\
&\quad+\frac{b^3 g^3 i^2 (c+dx)^6 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^4}\\
&\quad-\left(B(bc-ad)^6 g^3 i^2 n\right) \text{Subst}\left(\int \frac{-b^3+6b^2dx-15bd^2x^2+20d^3x^3}{60d^4x(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)\end{aligned}$$

$$= -\frac{(bc-ad)^3g^3i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^4} + \frac{3b(bc-ad)^2g^3i^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^4} - \frac{3b^2(bc-ad)g^3i^2(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^4} + \frac{b^3g^3i^2(c+dx)^6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^4} - \frac{(B(bc-ad)^6g^3i^2n)\,\text{Subst}\left(\int\frac{-b^3+6b^2dx-15bd^2x^2+20d^3x^3}{x(b-dx)^6}\,dx,x,\frac{a+bx}{c+dx}\right)}{60d^4}$$

$$= -\frac{(bc-ad)^3g^3i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^4} + \frac{3b(bc-ad)^2g^3i^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^4} - \frac{3b^2(bc-ad)g^3i^2(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^4} + \frac{b^3g^3i^2(c+dx)^6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^4} - \frac{(B(bc-ad)^6g^3i^2n)\,\text{Subst}\left(\int\left(-\frac{1}{b^3x}+\frac{10b^2d}{(b-dx)^6}-\frac{26bd}{(b-dx)^5}+\frac{19d}{(b-dx)^4}-\frac{d}{b(b-dx)^3}-\frac{d}{b^2(b-dx)^2}-\frac{d}{b^3(b-dx)}\right)\right.}{60d^4}$$

$$= \frac{B(bc-ad)^5g^3i^2nx}{60b^2d^3} + \frac{B(bc-ad)^4g^3i^2n(c+dx)^2}{120bd^4} - \frac{19B(bc-ad)^3g^3i^2n(c+dx)^3}{180d^4} + \frac{13bB(bc-ad)^2g^3i^2n(c+dx)^4}{120d^4} - \frac{b^2B(bc-ad)g^3i^2n(c+dx)^5}{30d^4} - \frac{(bc-ad)^3g^3i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^4} + \frac{3b(bc-ad)^2g^3i^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^4} - \frac{3b^2(bc-ad)g^3i^2(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^4} + \frac{b^3g^3i^2(c+dx)^6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^4} + \frac{B(bc-ad)^6g^3i^2n\log\left(\frac{a+bx}{c+dx}\right)}{60b^3d^4} + \frac{B(bc-ad)^6g^3i^2n\log(c+dx)}{60b^3d^4}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.00

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{g^3i^2\left(90d^4(bc-ad)^2(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+144d^5(bc-ad)(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]),x]
```

```
[Out] (g^3*i^2*(90*d^4*(b*c - a*d)^2*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]) + 144*d^5*(b*c - a*d)*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]) + 60*d^6*(a + b*x)^6*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 15*B*(b*
c - a*d)^3*n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*
d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^2*n*(12*
b*d*(b*c - a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c - a*d)*(
a + b*x)^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]) - B*(b*c -
a*d)*n*(60*b*d*(b*c - a*d)^4*x + 30*d^2*(-(b*c) + a*d)^3*(a + b*x)^2 + 20*d
^3*(b*c - a*d)^2*(a + b*x)^3 + 15*d^4*(-(b*c) + a*d)*(a + b*x)^4 + 12*d^5*(
a + b*x)^5 - 60*(b*c - a*d)^5*Log[c + d*x])))/(360*b^3*d^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1760 vs. 2(420) = 840.

Time = 24.52 (sec) , antiderivative size = 1761, normalized size of antiderivative = 3.98

method	result	size
parallelrisch	Expression too large to display	1761

```
[In] int((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/360*(540*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*c^2*d^4*g^3*i^2*n+360*B*
x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^3*c^2*d^4*g^3*i^2*n+540*B*x^4*ln(e*((b*x+
a)/(d*x+c))^n)*a*b^5*c*d^5*g^3*i^2*n+6*B*a^6*d^6*g^3*i^2*n^2+6*B*b^6*c^6*g^
3*i^2*n^2+120*A*x^3*a^3*b^3*d^6*g^3*i^2*n+720*B*x^3*ln(e*((b*x+a)/(d*x+c))^
n)*a^2*b^4*c*d^5*g^3*i^2*n+360*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c^2*d^
4*g^3*i^2*n+360*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^3*c*d^5*g^3*i^2*n+36*
B*x*a*b^5*c^4*d^2*g^3*i^2*n^2+360*A*x*a^3*b^3*c^2*d^4*g^3*i^2*n+120*B*ln(e*
((b*x+a)/(d*x+c))^n)*a^3*b^3*c^3*d^3*g^3*i^2*n-90*B*ln(e*((b*x+a)/(d*x+c))^
n)*a^2*b^4*c^4*d^2*g^3*i^2*n+36*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c^5*d*g^3
*i^2*n-36*B*ln(b*x+a)*a^5*b*c*d^5*g^3*i^2*n^2+90*B*ln(b*x+a)*a^4*b^2*c^2*d^
4*g^3*i^2*n^2-120*B*ln(b*x+a)*a^3*b^3*c^3*d^3*g^3*i^2*n^2+90*B*ln(b*x+a)*a^
```

```
2*b^4*c^4*d^2*g^3*i^2*n^2-36*B*ln(b*x+a)*a*b^5*c^5*d*g^3*i^2*n^2+360*A*x^3*
a*b^5*c^2*d^4*g^3*i^2*n+102*B*x^2*a^3*b^3*c*d^5*g^3*i^2*n^2-90*B*x^2*a^2*b^
4*c^2*d^4*g^3*i^2*n^2-18*B*x^2*a*b^5*c^3*d^3*g^3*i^2*n^2+360*A*x^2*a^3*b^3*
c*d^5*g^3*i^2*n+540*A*x^2*a^2*b^4*c^2*d^4*g^3*i^2*n+36*B*x*a^4*b^2*c*d^5*g^
3*i^2*n^2+30*B*x*a^3*b^3*c^2*d^4*g^3*i^2*n^2-90*B*x*a^2*b^4*c^3*d^3*g^3*i^2
*n^2+216*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*d^6*g^3*i^2*n+144*B*x^5*ln(e
*((b*x+a)/(d*x+c))^n)*b^6*c*d^5*g^3*i^2*n+270*B*x^4*ln(e*((b*x+a)/(d*x+c))^
n)*a^2*b^4*d^6*g^3*i^2*n+90*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c^2*d^4*g^3
*i^2*n-18*B*x^4*a*b^5*c*d^5*g^3*i^2*n^2+540*A*x^4*a*b^5*c*d^5*g^3*i^2*n+120
*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^3*d^6*g^3*i^2*n+42*B*x^3*a^2*b^4*c*d
^5*g^3*i^2*n^2-78*B*x^3*a*b^5*c^2*d^4*g^3*i^2*n^2+720*A*x^3*a^2*b^4*c*d^5*g
^3*i^2*n-12*B*x^5*b^6*c*d^5*g^3*i^2*n^2+216*A*x^5*a*b^5*d^6*g^3*i^2*n+144*A
*x^5*b^6*c*d^5*g^3*i^2*n+39*B*x^4*a^2*b^4*d^6*g^3*i^2*n^2-21*B*x^4*b^6*c^2*
d^4*g^3*i^2*n^2+270*A*x^4*a^2*b^4*d^6*g^3*i^2*n+90*A*x^4*b^6*c^2*d^4*g^3*i^
2*n+38*B*x^3*a^3*b^3*d^6*g^3*i^2*n^2-2*B*x^3*b^6*c^3*d^3*g^3*i^2*n^2+60*A*x
^6*b^6*d^6*g^3*i^2*n-6*B*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c^6*g^3*i^2*n+6*B*ln
(b*x+a)*b^6*c^6*g^3*i^2*n^2-33*B*a^5*b*c*d^5*g^3*i^2*n^2-168*B*a^4*b^2*c^2*
d^4*g^3*i^2*n^2+150*B*a^3*b^3*c^3*d^3*g^3*i^2*n^2+72*B*a^2*b^4*c^4*d^2*g^3*
i^2*n^2-33*B*a*b^5*c^5*d*g^3*i^2*n^2-720*A*a^4*b^2*c^2*d^4*g^3*i^2*n-900*A*
a^3*b^3*c^3*d^3*g^3*i^2*n+3*B*x^2*a^4*b^2*d^6*g^3*i^2*n^2+3*B*x^2*b^6*c^4*d
^2*g^3*i^2*n^2-6*B*x*a^5*b*d^6*g^3*i^2*n^2-6*B*x*b^6*c^5*d*g^3*i^2*n^2+60*B
*x^6*ln(e*((b*x+a)/(d*x+c))^n)*b^6*d^6*g^3*i^2*n+12*B*x^5*a*b^5*d^6*g^3*i^2
*n^2+6*B*ln(b*x+a)*a^6*d^6*g^3*i^2*n^2)/d^4/b^3/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1074 vs. $2(420) = 840$.

Time = 0.60 (sec) , antiderivative size = 1074, normalized size of antiderivative = 2.43

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{60\,Ab^6d^6g^3i^2x^6+6\,(15\,Ba^4b^2c^2d^4-6\,Ba^5bcd^5+Ba^6d^6)g^3i^2n\log\left(bx+a\right)+6\,(Bb^6c^6-6\,Bab^5c^5d+15\,Ba}{}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, a
lgorithm="fricas")
```

```
[Out] 1/360*(60*A*b^6*d^6*g^3*i^2*x^6 + 6*(15*B*a^4*b^2*c^2*d^4 - 6*B*a^5*b*c*d^5
 + B*a^6*d^6)*g^3*i^2*n*log(b*x + a) + 6*(B*b^6*c^6 - 6*B*a*b^5*c^5*d + 15*
B*a^2*b^4*c^4*d^2 - 20*B*a^3*b^3*c^3*d^3)*g^3*i^2*n*log(d*x + c) - 12*((B*b
^6*c*d^5 - B*a*b^5*d^6)*g^3*i^2*n - 6*(2*A*b^6*c*d^5 + 3*A*a*b^5*d^6)*g^3*i
^2)*x^5 - 3*((7*B*b^6*c^2*d^4 + 6*B*a*b^5*c*d^5 - 13*B*a^2*b^4*d^6)*g^3*i^2
*n - 30*(A*b^6*c^2*d^4 + 6*A*a*b^5*c*d^5 + 3*A*a^2*b^4*d^6)*g^3*i^2)*x^4 -
2*((B*b^6*c^3*d^3 + 39*B*a*b^5*c^2*d^4 - 21*B*a^2*b^4*c*d^5 - 19*B*a^3*b^3*
```

```
d^6)*g^3*i^2*n - 60*(3*A*a*b^5*c^2*d^4 + 6*A*a^2*b^4*c*d^5 + A*a^3*b^3*d^6)
*g^3*i^2)*x^3 + 3*((B*b^6*c^4*d^2 - 6*B*a*b^5*c^3*d^3 - 30*B*a^2*b^4*c^2*d^
4 + 34*B*a^3*b^3*c*d^5 + B*a^4*b^2*d^6)*g^3*i^2*n + 60*(3*A*a^2*b^4*c^2*d^4
 + 2*A*a^3*b^3*c*d^5)*g^3*i^2)*x^2 + 6*(60*A*a^3*b^3*c^2*d^4*g^3*i^2 - (B*b
^6*c^5*d - 6*B*a*b^5*c^4*d^2 + 15*B*a^2*b^4*c^3*d^3 - 5*B*a^3*b^3*c^2*d^4 -
 6*B*a^4*b^2*c*d^5 + B*a^5*b*d^6)*g^3*i^2*n)*x + 6*(10*B*b^6*d^6*g^3*i^2*x^
6 + 60*B*a^3*b^3*c^2*d^4*g^3*i^2*x + 12*(2*B*b^6*c*d^5 + 3*B*a*b^5*d^6)*g^3
*i^2*x^5 + 15*(B*b^6*c^2*d^4 + 6*B*a*b^5*c*d^5 + 3*B*a^2*b^4*d^6)*g^3*i^2*x
^4 + 20*(3*B*a*b^5*c^2*d^4 + 6*B*a^2*b^4*c*d^5 + B*a^3*b^3*d^6)*g^3*i^2*x^3
 + 30*(3*B*a^2*b^4*c^2*d^4 + 2*B*a^3*b^3*c*d^5)*g^3*i^2*x^2)*log(e) + 6*(10
*B*b^6*d^6*g^3*i^2*n*x^6 + 60*B*a^3*b^3*c^2*d^4*g^3*i^2*n*x + 12*(2*B*b^6*c
*d^5 + 3*B*a*b^5*d^6)*g^3*i^2*n*x^5 + 15*(B*b^6*c^2*d^4 + 6*B*a*b^5*c*d^5 +
 3*B*a^2*b^4*d^6)*g^3*i^2*n*x^4 + 20*(3*B*a*b^5*c^2*d^4 + 6*B*a^2*b^4*c*d^5
 + B*a^3*b^3*d^6)*g^3*i^2*n*x^3 + 30*(3*B*a^2*b^4*c^2*d^4 + 2*B*a^3*b^3*c*d
^5)*g^3*i^2*n*x^2)*log((b*x + a)/(d*x + c)))/(b^3*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**3*(d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1978 vs. $2(420) = 840$.

Time = 0.24 (sec) , antiderivative size = 1978, normalized size of antiderivative = 4.48

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, a
lgorithm="maxima")
[Out] 1/6*B*b^3*d^2*g^3*i^2*x^6*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/6*A*b^
3*d^2*g^3*i^2*x^6 + 2/5*B*b^3*c*d*g^3*i^2*x^5*log(e*(b*x/(d*x + c) + a/(d*x
 + c))^n) + 3/5*B*a*b^2*d^2*g^3*i^2*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))
^n) + 2/5*A*b^3*c*d*g^3*i^2*x^5 + 3/5*A*a*b^2*d^2*g^3*i^2*x^5 + 1/4*B*b^3*c
^2*g^3*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*B*a*b^2*c*d*g^3
*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/4*B*a^2*b*d^2*g^3*i^2*x
^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*b^3*c^2*g^3*i^2*x^4 + 3/2
```

```
*A*a*b^2*c*d*g^3*i^2*x^4 + 3/4*A*a^2*b*d^2*g^3*i^2*x^4 + B*a*b^2*c^2*g^3*i^
2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*B*a^2*b*c*d*g^3*i^2*x^3*lo
g(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*B*a^3*d^2*g^3*i^2*x^3*log(e*(b*x
/(d*x + c) + a/(d*x + c))^n) + A*a*b^2*c^2*g^3*i^2*x^3 + 2*A*a^2*b*c*d*g^3*
i^2*x^3 + 1/3*A*a^3*d^2*g^3*i^2*x^3 + 3/2*B*a^2*b*c^2*g^3*i^2*x^2*log(e*(b*
x/(d*x + c) + a/(d*x + c))^n) + B*a^3*c*d*g^3*i^2*x^2*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n) + 3/2*A*a^2*b*c^2*g^3*i^2*x^2 + A*a^3*c*d*g^3*i^2*x^2 - 1
/360*B*b^3*d^2*g^3*i^2*n*(60*a^6*log(b*x + a)/b^6 - 60*c^6*log(d*x + c)/d^6
 + (12*(b^5*c*d^4 - a*b^4*d^5)*x^5 - 15*(b^5*c^2*d^3 - a^2*b^3*d^5)*x^4 + 2
0*(b^5*c^3*d^2 - a^3*b^2*d^5)*x^3 - 30*(b^5*c^4*d - a^4*b*d^5)*x^2 + 60*(b^
5*c^5 - a^5*d^5)*x)/(b^5*d^5)) + 1/30*B*b^3*c*d*g^3*i^2*n*(12*a^5*log(b*x +
 a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4
*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 -
 a^4*d^4)*x)/(b^4*d^4)) + 1/20*B*a*b^2*d^2*g^3*i^2*n*(12*a^5*log(b*x + a)/b
^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*
d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*
d^4)*x)/(b^4*d^4)) - 1/24*B*b^3*c^2*g^3*i^2*n*(6*a^4*log(b*x + a)/b^4 - 6*c
^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b
*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 1/4*B*a*b^2*c*d*g^3*i^2*n
*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d
^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3
)) - 1/8*B*a^2*b*d^2*g^3*i^2*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)
/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(
b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*a*b^2*c^2*g^3*i^2*n*(2*a^3*log(b*x
 + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2
- a^2*d^2)*x)/(b^2*d^2)) + B*a^2*b*c*d*g^3*i^2*n*(2*a^3*log(b*x + a)/b^3 -
2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x
)/(b^2*d^2)) + 1/6*B*a^3*d^2*g^3*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(
d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2
)) - 3/2*B*a^2*b*c^2*g^3*i^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2
 + (b*c - a*d)*x/(b*d)) - B*a^3*c*d*g^3*i^2*n*(a^2*log(b*x + a)/b^2 - c^2*l
og(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*a^3*c^2*g^3*i^2*n*(a*log(b*x + a
)/b - c*log(d*x + c)/d) + B*a^3*c^2*g^3*i^2*x*log(e*(b*x/(d*x + c) + a/(d*x
 + c))^n) + A*a^3*c^2*g^3*i^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5133 vs. $2(420) = 840$.

Time = 1.80 (sec) , antiderivative size = 5133, normalized size of antiderivative = 11.61

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, a
lgorithm="giac")
```

[Out] -1/360*(6*(B*b^10*c^7*g^3*i^2*n - 7*B*a*b^9*c^6*d*g^3*i^2*n - 6*(b*x + a)*B
*b^9*c^7*d*g^3*i^2*n/(d*x + c) + 21*B*a^2*b^8*c^5*d^2*g^3*i^2*n + 42*(b*x +
 a)*B*a*b^8*c^6*d^2*g^3*i^2*n/(d*x + c) + 15*(b*x + a)^2*B*b^8*c^7*d^2*g^3*
i^2*n/(d*x + c)^2 - 35*B*a^3*b^7*c^4*d^3*g^3*i^2*n - 126*(b*x + a)*B*a^2*b^
7*c^5*d^3*g^3*i^2*n/(d*x + c) - 105*(b*x + a)^2*B*a*b^7*c^6*d^3*g^3*i^2*n/(
d*x + c)^2 - 20*(b*x + a)^3*B*b^7*c^7*d^3*g^3*i^2*n/(d*x + c)^3 + 35*B*a^4*
b^6*c^3*d^4*g^3*i^2*n + 210*(b*x + a)*B*a^3*b^6*c^4*d^4*g^3*i^2*n/(d*x + c)
 + 315*(b*x + a)^2*B*a^2*b^6*c^5*d^4*g^3*i^2*n/(d*x + c)^2 + 140*(b*x + a)^
3*B*a*b^6*c^6*d^4*g^3*i^2*n/(d*x + c)^3 - 21*B*a^5*b^5*c^2*d^5*g^3*i^2*n -
210*(b*x + a)*B*a^4*b^5*c^3*d^5*g^3*i^2*n/(d*x + c) - 525*(b*x + a)^2*B*a^3
*b^5*c^4*d^5*g^3*i^2*n/(d*x + c)^2 - 420*(b*x + a)^3*B*a^2*b^5*c^5*d^5*g^3*
i^2*n/(d*x + c)^3 + 7*B*a^6*b^4*c*d^6*g^3*i^2*n + 126*(b*x + a)*B*a^5*b^4*c
^2*d^6*g^3*i^2*n/(d*x + c) + 525*(b*x + a)^2*B*a^4*b^4*c^3*d^6*g^3*i^2*n/(d
x + c)^2 + 700(b*x + a)^3*B*a^3*b^4*c^4*d^6*g^3*i^2*n/(d*x + c)^3 - B*a^7
*b^3*d^7*g^3*i^2*n - 42*(b*x + a)*B*a^6*b^3*c*d^7*g^3*i^2*n/(d*x + c) - 315
*(b*x + a)^2*B*a^5*b^3*c^2*d^7*g^3*i^2*n/(d*x + c)^2 - 700*(b*x + a)^3*B*a^
4*b^3*c^3*d^7*g^3*i^2*n/(d*x + c)^3 + 6*(b*x + a)*B*a^7*b^2*d^8*g^3*i^2*n/(
d*x + c) + 105*(b*x + a)^2*B*a^6*b^2*c*d^8*g^3*i^2*n/(d*x + c)^2 + 420*(b*x
 + a)^3*B*a^5*b^2*c^2*d^8*g^3*i^2*n/(d*x + c)^3 - 15*(b*x + a)^2*B*a^7*b*d^
9*g^3*i^2*n/(d*x + c)^2 - 140*(b*x + a)^3*B*a^6*b*c*d^9*g^3*i^2*n/(d*x + c)
^3 + 20*(b*x + a)^3*B*a^7*d^10*g^3*i^2*n/(d*x + c)^3)*log((b*x + a)/(d*x +
c))/(b^6*d^4 - 6*(b*x + a)*b^5*d^5/(d*x + c) + 15*(b*x + a)^2*b^4*d^6/(d*x
+ c)^2 - 20*(b*x + a)^3*b^3*d^7/(d*x + c)^3 + 15*(b*x + a)^4*b^2*d^8/(d*x +
 c)^4 - 6*(b*x + a)^5*b*d^9/(d*x + c)^5 + (b*x + a)^6*d^10/(d*x + c)^6) + (
2*B*b^12*c^7*g^3*i^2*n - 14*B*a*b^11*c^6*d*g^3*i^2*n - 6*(b*x + a)*B*b^11*c
^7*d*g^3*i^2*n/(d*x + c) + 42*B*a^2*b^10*c^5*d^2*g^3*i^2*n + 42*(b*x + a)*B
*a*b^10*c^6*d^2*g^3*i^2*n/(d*x + c) - 3*(b*x + a)^2*B*b^10*c^7*d^2*g^3*i^2*
n/(d*x + c)^2 - 70*B*a^3*b^9*c^4*d^3*g^3*i^2*n - 126*(b*x + a)*B*a^2*b^9*c^
5*d^3*g^3*i^2*n/(d*x + c) + 21*(b*x + a)^2*B*a*b^9*c^6*d^3*g^3*i^2*n/(d*x +
 c)^2 + 34*(b*x + a)^3*B*b^9*c^7*d^3*g^3*i^2*n/(d*x + c)^3 + 70*B*a^4*b^8*c
^3*d^4*g^3*i^2*n + 210*(b*x + a)*B*a^3*b^8*c^4*d^4*g^3*i^2*n/(d*x + c) - 63
*(b*x + a)^2*B*a^2*b^8*c^5*d^4*g^3*i^2*n/(d*x + c)^2 - 238*(b*x + a)^3*B*a*
b^8*c^6*d^4*g^3*i^2*n/(d*x + c)^3 - 33*(b*x + a)^4*B*b^8*c^7*d^4*g^3*i^2*n/
(d*x + c)^4 - 42*B*a^5*b^7*c^2*d^5*g^3*i^2*n - 210*(b*x + a)*B*a^4*b^7*c^3*
d^5*g^3*i^2*n/(d*x + c) + 105*(b*x + a)^2*B*a^3*b^7*c^4*d^5*g^3*i^2*n/(d*x
+ c)^2 + 714*(b*x + a)^3*B*a^2*b^7*c^5*d^5*g^3*i^2*n/(d*x + c)^3 + 231*(b*x
 + a)^4*B*a*b^7*c^6*d^5*g^3*i^2*n/(d*x + c)^4 + 6*(b*x + a)^5*B*b^7*c^7*d^5
*g^3*i^2*n/(d*x + c)^5 + 14*B*a^6*b^6*c*d^6*g^3*i^2*n + 126*(b*x + a)*B*a^5
*b^6*c^2*d^6*g^3*i^2*n/(d*x + c) - 105*(b*x + a)^2*B*a^4*b^6*c^3*d^6*g^3*i^
2*n/(d*x + c)^2 - 1190*(b*x + a)^3*B*a^3*b^6*c^4*d^6*g^3*i^2*n/(d*x + c)^3
- 693*(b*x + a)^4*B*a^2*b^6*c^5*d^6*g^3*i^2*n/(d*x + c)^4 - 42*(b*x + a)^5*
B*a*b^6*c^6*d^6*g^3*i^2*n/(d*x + c)^5 - 2*B*a^7*b^5*d^7*g^3*i^2*n - 42*(b*x
 + a)*B*a^6*b^5*c*d^7*g^3*i^2*n/(d*x + c) + 63*(b*x + a)^2*B*a^5*b^5*c^2*d^
7*g^3*i^2*n/(d*x + c)^2 + 1190*(b*x + a)^3*B*a^4*b^5*c^3*d^7*g^3*i^2*n/(d*x
 + c)^3 + 1155*(b*x + a)^4*B*a^3*b^5*c^4*d^7*g^3*i^2*n/(d*x + c)^4 + 126*(b

*x + a)^5*B*a^2*b^5*c^5*d^7*g^3*i^2*n/(d*x + c)^5 + 6*(b*x + a)*B*a^7*b^4*d ^8*g^3*i^2*n/(d*x + c) - 21*(b*x + a)^2*B*a^6*b^4*c*d^8*g^3*i^2*n/(d*x + c) ^2 - 714*(b*x + a)^3*B*a^5*b^4*c^2*d^8*g^3*i^2*n/(d*x + c)^3 - 1155*(b*x + a)^4*B*a^4*b^4*c^3*d^8*g^3*i^2*n/(d*x + c)^4 - 210*(b*x + a)^5*B*a^3*b^4*c^ 4*d^8*g^3*i^2*n/(d*x + c)^5 + 3*(b*x + a)^2*B*a^7*b^3*d^9*g^3*i^2*n/(d*x + c)^2 + 238*(b*x + a)^3*B*a^6*b^3*c*d^9*g^3*i^2*n/(d*x + c)^3 + 693*(b*x + a)^4*B*a^5*b^3*c^2*d^9*g^3*i^2*n/(d*x + c)^4 + 210*(b*x + a)^5*B*a^4*b^3*c^3 *d^9*g^3*i^2*n/(d*x + c)^5 - 34*(b*x + a)^3*B*a^7*b^2*d^10*g^3*i^2*n/(d*x + c)^3 - 231*(b*x + a)^4*B*a^6*b^2*c*d^10*g^3*i^2*n/(d*x + c)^4 - 126*(b*x + a)^5*B*a^5*b^2*c^2*d^10*g^3*i^2*n/(d*x + c)^5 + 33*(b*x + a)^4*B*a^7*b*d^1 1*g^3*i^2*n/(d*x + c)^4 + 42*(b*x + a)^5*B*a^6*b*c*d^11*g^3*i^2*n/(d*x + c) ^5 - 6*(b*x + a)^5*B*a^7*d^12*g^3*i^2*n/(d*x + c)^5 + 6*B*b^12*c^7*g^3*i^2* log(e) - 42*B*a*b^11*c^6*d*g^3*i^2*log(e) - 36*(b*x + a)*B*b^11*c^7*d*g^3*i ^2*log(e)/(d*x + c) + 126*B*a^2*b^10*c^5*d^2*g^3*i^2*log(e) + 252*(b*x + a) *B*a*b^10*c^6*d^2*g^3*i^2*log(e)/(d*x + c) + 90*(b*x + a)^2*B*b^10*c^7*d^2* g^3*i^2*log(e)/(d*x + c)^2 - 210*B*a^3*b^9*c^4*d^3*g^3*i^2*log(e) - 756*(b* x + a)*B*a^2*b^9*c^5*d^3*g^3*i^2*log(e)/(d*x + c) - 630*(b*x + a)^2*B*a*b^9 *c^6*d^3*g^3*i^2*log(e)/(d*x + c)^2 - 120*(b*x + a)^3*B*b^9*c^7*d^3*g^3*i^2 *log(e)/(d*x + c)^3 + 210*B*a^4*b^8*c^3*d^4*g^3*i^2*log(e) + 1260*(b*x + a) *B*a^3*b^8*c^4*d^4*g^3*i^2*log(e)/(d*x + c) + 1890*(b*x + a)^2*B*a^2*b^8*c^ 5*d^4*g^3*i^2*log(e)/(d*x + c)^2 + 840*(b*x + a)^3*B*a*b^8*c^6*d^4*g^3*i^2* log(e)/(d*x + c)^3 - 126*B*a^5*b^7*c^2*d^5*g^3*i^2*log(e) - 1260*(b*x + a)* B*a^4*b^7*c^3*d^5*g^3*i^2*log(e)/(d*x + c) - 3150*(b*x + a)^2*B*a^3*b^7*c^4 *d^5*g^3*i^2*log(e)/(d*x + c)^2 - 2520*(b*x + a)^3*B*a^2*b^7*c^5*d^5*g^3*i^ 2*log(e)/(d*x + c)^3 + 42*B*a^6*b^6*c*d^6*g^3*i^2*log(e) + 756*(b*x + a)*B* a^5*b^6*c^2*d^6*g^3*i^2*log(e)/(d*x + c) + 3150*(b*x + a)^2*B*a^4*b^6*c^3*d ^6*g^3*i^2*log(e)/(d*x + c)^2 + 4200*(b*x + a)^3*B*a^3*b^6*c^4*d^6*g^3*i^2* log(e)/(d*x + c)^3 - 6*B*a^7*b^5*d^7*g^3*i^2*log(e) - 252*(b*x + a)*B*a^6*b ^5*c*d^7*g^3*i^2*log(e)/(d*x + c) - 1890*(b*x + a)^2*B*a^5*b^5*c^2*d^7*g^3* i^2*log(e)/(d*x + c)^2 - 4200*(b*x + a)^3*B*a^4*b^5*c^3*d^7*g^3*i^2*log(e)/ (d*x + c)^3 + 36*(b*x + a)*B*a^7*b^4*d^8*g^3*i^2*log(e)/(d*x + c) + 630*(b* x + a)^2*B*a^6*b^4*c*d^8*g^3*i^2*log(e)/(d*x + c)^2 + 2520*(b*x + a)^3*B*a^ 5*b^4*c^2*d^8*g^3*i^2*log(e)/(d*x + c)^3 - 90*(b*x + a)^2*B*a^7*b^3*d^9*g^3 *i^2*log(e)/(d*x + c)^2 - 840*(b*x + a)^3*B*a^6*b^3*c*d^9*g^3*i^2*log(e)/(d *x + c)^3 + 120*(b*x + a)^3*B*a^7*b^2*d^10*g^3*i^2*log(e)/(d*x + c)^3 + 6*A *b^12*c^7*g^3*i^2 - 42*A*a*b^11*c^6*d*g^3*i^2 - 36*(b*x + a)*A*b^11*c^7*d*g ^3*i^2/(d*x + c) + 126*A*a^2*b^10*c^5*d^2*g^3*i^2 + 252*(b*x + a)*A*a*b^10* c^6*d^2*g^3*i^2/(d*x + c) + 90*(b*x + a)^2*A*b^10*c^7*d^2*g^3*i^2/(d*x + c) ^2 - 210*A*a^3*b^9*c^4*d^3*g^3*i^2 - 756*(b*x + a)*A*a^2*b^9*c^5*d^3*g^3*i^ 2/(d*x + c) - 630*(b*x + a)^2*A*a*b^9*c^6*d^3*g^3*i^2/(d*x + c)^2 - 120*(b* x + a)^3*A*b^9*c^7*d^3*g^3*i^2/(d*x + c)^3 + 210*A*a^4*b^8*c^3*d^4*g^3*i^2 + 1260*(b*x + a)*A*a^3*b^8*c^4*d^4*g^3*i^2/(d*x + c) + 1890*(b*x + a)^2*A*a ^2*b^8*c^5*d^4*g^3*i^2/(d*x + c)^2 + 840*(b*x + a)^3*A*a*b^8*c^6*d^4*g^3*i^ 2/(d*x + c)^3 - 126*A*a^5*b^7*c^2*d^5*g^3*i^2 - 1260*(b*x + a)*A*a^4*b^7*c^ 3*d^5*g^3*i^2/(d*x + c) - 3150*(b*x + a)^2*A*a^3*b^7*c^4*d^5*g^3*i^2/(d*x +

```
c)^2 - 2520*(b*x + a)^3*A*a^2*b^7*c^5*d^5*g^3*i^2/(d*x + c)^3 + 42*A*a^6*b
^6*c*d^6*g^3*i^2 + 756*(b*x + a)*A*a^5*b^6*c^2*d^6*g^3*i^2/(d*x + c) + 3150
*(b*x + a)^2*A*a^4*b^6*c^3*d^6*g^3*i^2/(d*x + c)^2 + 4200*(b*x + a)^3*A*a^3
*b^6*c^4*d^6*g^3*i^2/(d*x + c)^3 - 6*A*a^7*b^5*d^7*g^3*i^2 - 252*(b*x + a)*
A*a^6*b^5*c*d^7*g^3*i^2/(d*x + c) - 1890*(b*x + a)^2*A*a^5*b^5*c^2*d^7*g^3*
i^2/(d*x + c)^2 - 4200*(b*x + a)^3*A*a^4*b^5*c^3*d^7*g^3*i^2/(d*x + c)^3 +
36*(b*x + a)*A*a^7*b^4*d^8*g^3*i^2/(d*x + c) + 630*(b*x + a)^2*A*a^6*b^4*c*
d^8*g^3*i^2/(d*x + c)^2 + 2520*(b*x + a)^3*A*a^5*b^4*c^2*d^8*g^3*i^2/(d*x +
 c)^3 - 90*(b*x + a)^2*A*a^7*b^3*d^9*g^3*i^2/(d*x + c)^2 - 840*(b*x + a)^3*
A*a^6*b^3*c*d^9*g^3*i^2/(d*x + c)^3 + 120*(b*x + a)^3*A*a^7*b^2*d^10*g^3*i^
2/(d*x + c)^3)/(b^8*d^4 - 6*(b*x + a)*b^7*d^5/(d*x + c) + 15*(b*x + a)^2*b^
6*d^6/(d*x + c)^2 - 20*(b*x + a)^3*b^5*d^7/(d*x + c)^3 + 15*(b*x + a)^4*b^4
*d^8/(d*x + c)^4 - 6*(b*x + a)^5*b^3*d^9/(d*x + c)^5 + (b*x + a)^6*b^2*d^10
/(d*x + c)^6) + 6*(B*b^7*c^7*g^3*i^2*n - 7*B*a*b^6*c^6*d*g^3*i^2*n + 21*B*a
^2*b^5*c^5*d^2*g^3*i^2*n - 35*B*a^3*b^4*c^4*d^3*g^3*i^2*n + 35*B*a^4*b^3*c^
3*d^4*g^3*i^2*n - 21*B*a^5*b^2*c^2*d^5*g^3*i^2*n + 7*B*a^6*b*c*d^6*g^3*i^2*
n - B*a^7*d^7*g^3*i^2*n)*log(-b + (b*x + a)*d/(d*x + c))/(b^3*d^4) - 6*(B*b
^7*c^7*g^3*i^2*n - 7*B*a*b^6*c^6*d*g^3*i^2*n + 21*B*a^2*b^5*c^5*d^2*g^3*i^2
*n - 35*B*a^3*b^4*c^4*d^3*g^3*i^2*n + 35*B*a^4*b^3*c^3*d^4*g^3*i^2*n - 21*B
*a^5*b^2*c^2*d^5*g^3*i^2*n + 7*B*a^6*b*c*d^6*g^3*i^2*n - B*a^7*d^7*g^3*i^2*
n)*log((b*x + a)/(d*x + c))/(b^3*d^4))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)
^2)
```

Mupad [B] (verification not implemented)

Time = 2.81 (sec) , antiderivative size = 2555, normalized size of antiderivative = 5.78

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

```
[In] int((a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),
x)
```

```
[Out] x^2*((a*c*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d*n - B*b*c*n))/6 -
(A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*b*d) - (b*g^
3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*d^2*n - 2*B*b^2*c^2*n + 60*A*a
*b*c*d - B*a*b*c*d*n))/5 + A*a*b^2*c*d*g^3*i^2))/(2*b*d) - ((60*a*d + 60*b*
c)*((g^3*i^2*(16*A*a^3*d^3 + 4*A*b^3*c^3 + 3*B*a^3*d^3*n - B*b^3*c^3*n + 48
*A*a*b^2*c^2*d + 72*A*a^2*b*c*d^2 - 5*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n))
/(4*d) + ((60*a*d + 60*b*c)*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d*
n - B*b*c*n))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c)
)/(60*b*d) - (b*g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*d^2*n - 2*B*
b^2*c^2*n + 60*A*a*b*c*d - B*a*b*c*d*n))/5 + A*a*b^2*c*d*g^3*i^2))/(60*b*d)
 - (a*c*((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d*n - B*b*c*n))/6 - (A*b
^2*d*g^3*i^2*(60*a*d + 60*b*c))/60))/(b*d)))/(120*b*d) + (a*g^3*i^2*(3*A*a^
```

3*d^3 + 12*A*b^3*c^3 + B*a^3*d^3*n - 3*B*b^3*c^3*n + 54*A*a*b^2*c^2*d + 36*
A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d*n + 5*B*a^2*b*c*d^2*n))/(6*b*d)) + x^3*((g^
3*i^2*(16*A*a^3*d^3 + 4*A*b^3*c^3 + 3*B*a^3*d^3*n - B*b^3*c^3*n + 48*A*a*b^
2*c^2*d + 72*A*a^2*b*c*d^2 - 5*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n))/(12*d)
 + ((60*a*d + 60*b*c)*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d*n - B*
b*c*n))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*
b*d) - (b*g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*d^2*n - 2*B*b^2*c^
2*n + 60*A*a*b*c*d - B*a*b*c*d*n))/5 + A*a*b^2*c*d*g^3*i^2))/(180*b*d) - (a
c((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d*n - B*b*c*n))/6 - (A*b^2*d*
g^3*i^2*(60*a*d + 60*b*c))/60))/(3*b*d)) - x^4*((((b^2*d*g^3*i^2*(24*A*a*d
+ 18*A*b*c + B*a*d*n - B*b*c*n))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60
)*(60*a*d + 60*b*c))/(240*b*d) - (b*g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 +
3*B*a^2*d^2*n - 2*B*b^2*c^2*n + 60*A*a*b*c*d - B*a*b*c*d*n))/20 + (A*a*b^2*
c*d*g^3*i^2)/4) + x^5*((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d*n - B*b*
c*n))/30 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/300) - x*(((60*a*d + 60*b*c)
*((a*c*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d*n - B*b*c*n))/6 - (A*
b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*b*d) - (b*g^3*i
^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*d^2*n - 2*B*b^2*c^2*n + 60*A*a*b*
c*d - B*a*b*c*d*n))/5 + A*a*b^2*c*d*g^3*i^2))/(b*d) - ((60*a*d + 60*b*c)*((
g^3*i^2*(16*A*a^3*d^3 + 4*A*b^3*c^3 + 3*B*a^3*d^3*n - B*b^3*c^3*n + 48*A*a*
b^2*c^2*d + 72*A*a^2*b*c*d^2 - 5*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n))/(4*d
) + ((60*a*d + 60*b*c)*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d*n - B
*b*c*n))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60
*b*d) - (b*g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*d^2*n - 2*B*b^2*c
^2*n + 60*A*a*b*c*d - B*a*b*c*d*n))/5 + A*a*b^2*c*d*g^3*i^2))/(60*b*d) - (a
c((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d*n - B*b*c*n))/6 - (A*b^2*d*
g^3*i^2*(60*a*d + 60*b*c))/60))/(b*d)))/(60*b*d) + (a*g^3*i^2*(3*A*a^3*d^3
+ 12*A*b^3*c^3 + B*a^3*d^3*n - 3*B*b^3*c^3*n + 54*A*a*b^2*c^2*d + 36*A*a^2*
b*c*d^2 - 3*B*a*b^2*c^2*d*n + 5*B*a^2*b*c*d^2*n))/(3*b*d)))/(60*b*d) + (a*c
*((g^3*i^2*(16*A*a^3*d^3 + 4*A*b^3*c^3 + 3*B*a^3*d^3*n - B*b^3*c^3*n + 48*A
*a*b^2*c^2*d + 72*A*a^2*b*c*d^2 - 5*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n))/(
4*d) + ((60*a*d + 60*b*c)*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d*n
- B*b*c*n))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/
(60*b*d) - (b*g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*d^2*n - 2*B*b^
2*c^2*n + 60*A*a*b*c*d - B*a*b*c*d*n))/5 + A*a*b^2*c*d*g^3*i^2))/(60*b*d) -
 (a*c*((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d*n - B*b*c*n))/6 - (A*b^2
*d*g^3*i^2*(60*a*d + 60*b*c))/60))/(b*d)))/(b*d) - (a^2*c*g^3*i^2*(6*A*a^2*
d^2 + 12*A*b^2*c^2 + 2*B*a^2*d^2*n - 3*B*b^2*c^2*n + 24*A*a*b*c*d + B*a*b*c
*d*n))/(2*b*d)) + log(e*((a + b*x)/(c + d*x))^n)*(B*a^3*c^2*g^3*i^2*x + (B*
a*g^3*i^2*x^3*(a^2*d^2 + 3*b^2*c^2 + 6*a*b*c*d))/3 + (B*b*g^3*i^2*x^4*(3*a^
2*d^2 + b^2*c^2 + 6*a*b*c*d))/4 + (B*b^3*d^2*g^3*i^2*x^6)/6 + (B*a^2*c*g^3*
i^2*x^2*(2*a*d + 3*b*c))/2 + (B*b^2*d*g^3*i^2*x^5*(3*a*d + 2*b*c))/5) + (lo
g(c + d*x)*(B*b^3*c^6*g^3*i^2*n - 20*B*a^3*c^3*d^3*g^3*i^2*n + 15*B*a^2*b*c
^4*d^2*g^3*i^2*n - 6*B*a*b^2*c^5*d*g^3*i^2*n))/(60*d^4) + (log(a + b*x)*(B*
a^6*d^2*g^3*i^2*n + 15*B*a^4*b^2*c^2*g^3*i^2*n - 6*B*a^5*b*c*d*g^3*i^2*n))/

```
(60*b^3) + (A*b^3*d^2*g^3*i^2*x^6)/6
```

3.118 $\int (ag+bgx)^2(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal result

Integrand size = 43, antiderivative size = 352

$$\begin{aligned}
&\int (ag+bgx)^2(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx \\
&= -\frac{B(bc-ad)^4g^2i^2nx}{30b^2d^2} - \frac{B(bc-ad)^3g^2i^2n(c+dx)^2}{60bd^3} + \frac{B(bc-ad)^2g^2i^2n(c+dx)^3}{10d^3} \\
&\quad - \frac{bB(bc-ad)g^2i^2n(c+dx)^4}{20d^3} + \frac{(bc-ad)^2g^2i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^3} \\
&\quad - \frac{b(bc-ad)g^2i^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^3} \\
&\quad + \frac{b^2g^2i^2(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^3} \\
&\quad - \frac{B(bc-ad)^5g^2i^2n\log\left(\frac{a+bx}{c+dx}\right)}{30b^3d^3} - \frac{B(bc-ad)^5g^2i^2n\log(c+dx)}{30b^3d^3}
\end{aligned}$$

```
[Out] -1/30*B*(-a*d+b*c)^4*g^2*i^2*n*x/b^2/d^2-1/60*B*(-a*d+b*c)^3*g^2*i^2*n*(d*x
+c)^2/b/d^3+1/10*B*(-a*d+b*c)^2*g^2*i^2*n*(d*x+c)^3/d^3-1/20*b*B*(-a*d+b*c)
*g^2*i^2*n*(d*x+c)^4/d^3+1/3*(-a*d+b*c)^2*g^2*i^2*(d*x+c)^3*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))/d^3-1/2*b*(-a*d+b*c)*g^2*i^2*(d*x+c)^4*(A+B*ln(e*((b*x+a)/
(d*x+c))^n))/d^3+1/5*b^2*g^2*i^2*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/
d^3-1/30*B*(-a*d+b*c)^5*g^2*i^2*n*ln((b*x+a)/(d*x+c))/b^3/d^3-1/30*B*(-a*d+
b*c)^5*g^2*i^2*n*ln(d*x+c)/b^3/d^3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2561, 45, 2382, 12, 907}

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=\frac{b^2g^2i^2(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^3}+\frac{g^2i^2(c+dx)^3(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^3}-\frac{bg^2i^2(c+dx)^4(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d^3}-\frac{Bg^2i^2n(bc-ad)^5\log\left(\frac{a+bx}{c+dx}\right)}{30b^3d^3}-\frac{Bg^2i^2n(bc-ad)^5\log(c+dx)}{30b^3d^3}-\frac{Bg^2i^2nx(bc-ad)^4}{30b^2d^2}-\frac{Bg^2i^2n(c+dx)^2(bc-ad)^3}{60bd^3}+\frac{Bg^2i^2n(c+dx)^3(bc-ad)^2}{10d^3}-\frac{bBg^2i^2n(c+dx)^4(bc-ad)}{20d^3}$$

```
[In] Int[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),
x]
```

```
[Out] -1/30*(B*(b*c - a*d)^4*g^2*i^2*n*x)/(b^2*d^2) - (B*(b*c - a*d)^3*g^2*i^2*n*
(c + d*x)^2)/(60*b*d^3) + (B*(b*c - a*d)^2*g^2*i^2*n*(c + d*x)^3)/(10*d^3)
- (b*B*(b*c - a*d)*g^2*i^2*n*(c + d*x)^4)/(20*d^3) + ((b*c - a*d)^2*g^2*i^2
*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d^3) - (b*(b*c - a*
d)*g^2*i^2*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d^3) + (b
^2*g^2*i^2*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*d^3) - (B
*(b*c - a*d)^5*g^2*i^2*n*Log[(a + b*x)/(c + d*x)])/(30*b^3*d^3) - (B*(b*c -
 a*d)^5*g^2*i^2*n*Log[c + d*x])/(30*b^3*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
```

```
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((bc-ad)^5 g^2 i^2\right) \operatorname{Subst}\left(\int \frac{x^2(A+B\log(ex^n))}{(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right) \\
&= \frac{(bc-ad)^2 g^2 i^2 (c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^3} \\
&\quad - \frac{b(bc-ad) g^2 i^2 (c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^3} \\
&\quad + \frac{b^2 g^2 i^2 (c+dx)^5 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^3} \\
&\quad - \left(B(bc-ad)^5 g^2 i^2 n\right) \operatorname{Subst}\left(\int \frac{b^2-5bdx+10d^2x^2}{30d^3 x(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right) \\
&= \frac{(bc-ad)^2 g^2 i^2 (c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^3} \\
&\quad - \frac{b(bc-ad) g^2 i^2 (c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^3} \\
&\quad + \frac{b^2 g^2 i^2 (c+dx)^5 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^3} \\
&\quad - \frac{\left(B(bc-ad)^5 g^2 i^2 n\right) \operatorname{Subst}\left(\int \frac{b^2-5bdx+10d^2x^2}{x(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{30d^3}
\end{aligned}$$

$$= \frac{(bc-ad)^2g^2i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^3} - \frac{b(bc-ad)g^2i^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^3} + \frac{b^2g^2i^2(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^3} - \frac{(B(bc-ad)^5g^2i^2n)\,\text{Subst}\left(\int\left(\frac{1}{b^3x}+\frac{6bd}{(b-dx)^5}-\frac{9d}{(b-dx)^4}+\frac{d}{b(b-dx)^3}+\frac{d}{b^2(b-dx)^2}+\frac{d}{b^3(b-dx)}\right)dx,x,\frac{a+b}{c+c}\right.}{30d^3}$$

$$= -\frac{B(bc-ad)^4g^2i^2nx}{30b^2d^2} - \frac{B(bc-ad)^3g^2i^2n(c+dx)^2}{60bd^3} + \frac{B(bc-ad)^2g^2i^2n(c+dx)^3}{10d^3} - \frac{bB(bc-ad)g^2i^2n(c+dx)^4}{20d^3} + \frac{(bc-ad)^2g^2i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^3} - \frac{b(bc-ad)g^2i^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^3} + \frac{b^2g^2i^2(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^3} - \frac{B(bc-ad)^5g^2i^2n\log\left(\frac{a+bx}{c+dx}\right)}{30b^3d^3} - \frac{B(bc-ad)^5g^2i^2n\log(c+dx)}{30b^3d^3}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.06

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \frac{g^2i^2\left(20d^3(bc-ad)^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+30d^4(bc-ad)(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]),x]

[Out] (g^2*i^2*(20*d^3*(b*c - a*d)^2*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]) + 30*d^4*(b*c - a*d)*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^
n]) + 12*d^5*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 10*B*(b*c
 - a*d)^3*n*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c
+ d*x]) - 5*B*(b*c - a*d)^2*n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)
*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + B*(b*c -
 a*d)*n*(12*b*d*(b*c - a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(
b*c - a*d)*(a + b*x)^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x])
))/(60*b^3*d^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1273 vs. 2(334) = 668.

Time = 11.13 (sec) , antiderivative size = 1274, normalized size of antiderivative = 3.62

method	result	size
parallelrisch	Expression too large to display	1274

```
[In] int((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/60*(60*A*x^2*a^2*b^3*c*d^4*g^2*i^2*n+60*A*x^2*a*b^4*c^2*d^3*g^2*i^2*n+2*B
*ln(b*x+a)*a^5*d^5*g^2*i^2*n^2+12*A*x^5*b^5*d^5*g^2*i^2*n+2*B*ln(e*((b*x+a)
/(d*x+c))^n)*b^5*c^5*g^2*i^2*n-2*B*ln(b*x+a)*b^5*c^5*g^2*i^2*n^2+12*B*x^5*l
n(e*((b*x+a)/(d*x+c))^n)*b^5*d^5*g^2*i^2*n+80*B*x^3*ln(e*((b*x+a)/(d*x+c))^
n)*a*b^4*c*d^4*g^2*i^2*n+60*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*c*d^4*g
^2*i^2*n+60*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^2*d^3*g^2*i^2*n+60*B*x*
ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*c^2*d^3*g^2*i^2*n+2*B*a^5*d^5*g^2*i^2*n^2
-2*B*b^5*c^5*g^2*i^2*n^2+10*B*x*a^3*b^2*c*d^4*g^2*i^2*n^2-10*B*x*a*b^4*c^3*
d^2*g^2*i^2*n^2+60*A*x*a^2*b^3*c^2*d^3*g^2*i^2*n+20*B*ln(e*((b*x+a)/(d*x+c)
)^n)*a^2*b^3*c^3*d^2*g^2*i^2*n-10*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^4*d*g
^2*i^2*n-10*B*ln(b*x+a)*a^4*b*c*d^4*g^2*i^2*n^2+20*B*ln(b*x+a)*a^3*b^2*c^2*
d^3*g^2*i^2*n^2-20*B*ln(b*x+a)*a^2*b^3*c^3*d^2*g^2*i^2*n^2+10*B*ln(b*x+a)*a
*b^4*c^4*d*g^2*i^2*n^2+30*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*d^5*g^2*i^2
*n+30*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c*d^4*g^2*i^2*n+20*B*x^3*ln(e*((b
*x+a)/(d*x+c))^n)*a^2*b^3*d^5*g^2*i^2*n+20*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*
b^5*c^2*d^3*g^2*i^2*n+80*A*x^3*a*b^4*c*d^4*g^2*i^2*n+15*B*x^2*a^2*b^3*c*d^4
*g^2*i^2*n^2-15*B*x^2*a*b^4*c^2*d^3*g^2*i^2*n^2-9*B*a^4*b*c*d^4*g^2*i^2*n^2
-25*B*a^3*b^2*c^2*d^3*g^2*i^2*n^2+25*B*a^2*b^3*c^3*d^2*g^2*i^2*n^2+9*B*a*b^
4*c^4*d*g^2*i^2*n^2-120*A*a^3*b^2*c^2*d^3*g^2*i^2*n-120*A*a^2*b^3*c^3*d^2*g
^2*i^2*n+3*B*x^4*a*b^4*d^5*g^2*i^2*n^2-3*B*x^4*b^5*c*d^4*g^2*i^2*n^2+30*A*x
^4*a*b^4*d^5*g^2*i^2*n+30*A*x^4*b^5*c*d^4*g^2*i^2*n+6*B*x^3*a^2*b^3*d^5*g^2
*i^2*n^2-6*B*x^3*b^5*c^2*d^3*g^2*i^2*n^2+20*A*x^3*a^2*b^3*d^5*g^2*i^2*n+20*
A*x^3*b^5*c^2*d^3*g^2*i^2*n+B*x^2*a^3*b^2*d^5*g^2*i^2*n^2-B*x^2*b^5*c^3*d^2
*g^2*i^2*n^2-2*B*x*a^4*b*d^5*g^2*i^2*n^2+2*B*x*b^5*c^4*d*g^2*i^2*n^2)/b^3/d
^3/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(334) = 668$.

Time = 0.44 (sec) , antiderivative size = 774, normalized size of antiderivative = 2.20

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$= \frac{12\,Ab^5d^5g^2i^2x^5+2\,(10\,Ba^3b^2c^2d^3-5\,Ba^4bcd^4+Ba^5d^5)g^2i^2n\log(bx+a)-2\,(Bb^5c^5-5\,Bab^4c^4d+10\,B}{}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, a
lgorithm="fricas")
```

```
[Out] 1/60*(12*A*b^5*d^5*g^2*i^2*x^5 + 2*(10*B*a^3*b^2*c^2*d^3 - 5*B*a^4*b*c*d^4
+ B*a^5*d^5)*g^2*i^2*n*log(b*x + a) - 2*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B
*a^2*b^3*c^3*d^2)*g^2*i^2*n*log(d*x + c) - 3*((B*b^5*c*d^4 - B*a*b^4*d^5)*g
^2*i^2*n - 10*(A*b^5*c*d^4 + A*a*b^4*d^5)*g^2*i^2)*x^4 - 2*(3*(B*b^5*c^2*d^
3 - B*a^2*b^3*d^5)*g^2*i^2*n - 10*(A*b^5*c^2*d^3 + 4*A*a*b^4*c*d^4 + A*a^2*
b^3*d^5)*g^2*i^2)*x^3 - ((B*b^5*c^3*d^2 + 15*B*a*b^4*c^2*d^3 - 15*B*a^2*b^3
*c*d^4 - B*a^3*b^2*d^5)*g^2*i^2*n - 60*(A*a*b^4*c^2*d^3 + A*a^2*b^3*c*d^4)*
g^2*i^2)*x^2 + 2*(30*A*a^2*b^3*c^2*d^3*g^2*i^2 + (B*b^5*c^4*d - 5*B*a*b^4*c
^3*d^2 + 5*B*a^3*b^2*c*d^4 - B*a^4*b*d^5)*g^2*i^2*n)*x + 2*(6*B*b^5*d^5*g^2
*i^2*x^5 + 30*B*a^2*b^3*c^2*d^3*g^2*i^2*x + 15*(B*b^5*c*d^4 + B*a*b^4*d^5)*
g^2*i^2*x^4 + 10*(B*b^5*c^2*d^3 + 4*B*a*b^4*c*d^4 + B*a^2*b^3*d^5)*g^2*i^2*
x^3 + 30*(B*a*b^4*c^2*d^3 + B*a^2*b^3*c*d^4)*g^2*i^2*x^2)*log(e) + 2*(6*B*b
^5*d^5*g^2*i^2*n*x^5 + 30*B*a^2*b^3*c^2*d^3*g^2*i^2*n*x + 15*(B*b^5*c*d^4 +
 B*a*b^4*d^5)*g^2*i^2*n*x^4 + 10*(B*b^5*c^2*d^3 + 4*B*a*b^4*c*d^4 + B*a^2*b
^3*d^5)*g^2*i^2*n*x^3 + 30*(B*a*b^4*c^2*d^3 + B*a^2*b^3*c*d^4)*g^2*i^2*n*x^
2)*log((b*x + a)/(d*x + c)))/(b^3*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**2*(d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1336 vs. $2(334) = 668$.

Time = 0.23 (sec) , antiderivative size = 1336, normalized size of antiderivative = 3.80

$$\int (ag + bgx)^2 (ci + dix)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, a
lgorithm="maxima")
```

```
[Out] 1/5*B*b^2*d^2*g^2*i^2*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A*b^
2*d^2*g^2*i^2*x^5 + 1/2*B*b^2*c*d*g^2*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x
 + c))^n) + 1/2*B*a*b*d^2*g^2*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
) + 1/2*A*b^2*c*d*g^2*i^2*x^4 + 1/2*A*a*b*d^2*g^2*i^2*x^4 + 1/3*B*b^2*c^2*g
^2*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 4/3*B*a*b*c*d*g^2*i^2*x
^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*B*a^2*d^2*g^2*i^2*x^3*log(e
*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*b^2*c^2*g^2*i^2*x^3 + 4/3*A*a*b*c
*d*g^2*i^2*x^3 + 1/3*A*a^2*d^2*g^2*i^2*x^3 + B*a*b*c^2*g^2*i^2*x^2*log(e*(b
*x/(d*x + c) + a/(d*x + c))^n) + B*a^2*c*d*g^2*i^2*x^2*log(e*(b*x/(d*x + c)
 + a/(d*x + c))^n) + A*a*b*c^2*g^2*i^2*x^2 + A*a^2*c*d*g^2*i^2*x^2 + 1/60*B
*b^2*d^2*g^2*i^2*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*
(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^
3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) - 1/12*B*b^2*c*
d*g^2*i^2*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^
2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*
x)/(b^3*d^3)) - 1/12*B*a*b*d^2*g^2*i^2*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*lo
g(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)
*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/6*B*b^2*c^2*g^2*i^2*n*(2*a^3
*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(
b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + 2/3*B*a*b*c*d*g^2*i^2*n*(2*a^3*log(b*x +
 a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 -
a^2*d^2)*x)/(b^2*d^2)) + 1/6*B*a^2*d^2*g^2*i^2*n*(2*a^3*log(b*x + a)/b^3 -
2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x
)/(b^2*d^2)) - B*a*b*c^2*g^2*i^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)
/d^2 + (b*c - a*d)*x/(b*d)) - B*a^2*c*d*g^2*i^2*n*(a^2*log(b*x + a)/b^2 - c
^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*a^2*c^2*g^2*i^2*n*(a*log(b*x
 + a)/b - c*log(d*x + c)/d) + B*a^2*c^2*g^2*i^2*x*log(e*(b*x/(d*x + c) + a/
(d*x + c))^n) + A*a^2*c^2*g^2*i^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3352 vs. 2(334) = 668.

Time = 1.27 (sec) , antiderivative size = 3352, normalized size of antiderivative = 9.52

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, a
lgorithm="giac")
```

```
[Out] 1/60*(2*(B*b^8*c^6*g^2*i^2*n - 6*B*a*b^7*c^5*d*g^2*i^2*n - 5*(b*x + a)*B*b^
7*c^6*d*g^2*i^2*n/(d*x + c) + 15*B*a^2*b^6*c^4*d^2*g^2*i^2*n + 30*(b*x + a)
*B*a*b^6*c^5*d^2*g^2*i^2*n/(d*x + c) + 10*(b*x + a)^2*B*b^6*c^6*d^2*g^2*i^2
*n/(d*x + c)^2 - 20*B*a^3*b^5*c^3*d^3*g^2*i^2*n - 75*(b*x + a)*B*a^2*b^5*c^
4*d^3*g^2*i^2*n/(d*x + c) - 60*(b*x + a)^2*B*a*b^5*c^5*d^3*g^2*i^2*n/(d*x +
 c)^2 + 15*B*a^4*b^4*c^2*d^4*g^2*i^2*n + 100*(b*x + a)*B*a^3*b^4*c^3*d^4*g^
2*i^2*n/(d*x + c) + 150*(b*x + a)^2*B*a^2*b^4*c^4*d^4*g^2*i^2*n/(d*x + c)^2
 - 6*B*a^5*b^3*c*d^5*g^2*i^2*n - 75*(b*x + a)*B*a^4*b^3*c^2*d^5*g^2*i^2*n/(
d*x + c) - 200*(b*x + a)^2*B*a^3*b^3*c^3*d^5*g^2*i^2*n/(d*x + c)^2 + B*a^6*
b^2*d^6*g^2*i^2*n + 30*(b*x + a)*B*a^5*b^2*c*d^6*g^2*i^2*n/(d*x + c) + 150*
(b*x + a)^2*B*a^4*b^2*c^2*d^6*g^2*i^2*n/(d*x + c)^2 - 5*(b*x + a)*B*a^6*b*d
^7*g^2*i^2*n/(d*x + c) - 60*(b*x + a)^2*B*a^5*b*c*d^7*g^2*i^2*n/(d*x + c)^2
 + 10*(b*x + a)^2*B*a^6*d^8*g^2*i^2*n/(d*x + c)^2)*log((b*x + a)/(d*x + c))
/(b^5*d^3 - 5*(b*x + a)*b^4*d^4/(d*x + c) + 10*(b*x + a)^2*b^3*d^5/(d*x + c
)^2 - 10*(b*x + a)^3*b^2*d^6/(d*x + c)^3 + 5*(b*x + a)^4*b*d^7/(d*x + c)^4
- (b*x + a)^5*d^8/(d*x + c)^5) + (2*(b*x + a)*B*b^9*c^6*d*g^2*i^2*n/(d*x +
c) - 12*(b*x + a)*B*a*b^8*c^5*d^2*g^2*i^2*n/(d*x + c) - 9*(b*x + a)^2*B*b^8
*c^6*d^2*g^2*i^2*n/(d*x + c)^2 + 30*(b*x + a)*B*a^2*b^7*c^4*d^3*g^2*i^2*n/(
d*x + c) + 54*(b*x + a)^2*B*a*b^7*c^5*d^3*g^2*i^2*n/(d*x + c)^2 + 9*(b*x +
a)^3*B*b^7*c^6*d^3*g^2*i^2*n/(d*x + c)^3 - 40*(b*x + a)*B*a^3*b^6*c^3*d^4*g
^2*i^2*n/(d*x + c) - 135*(b*x + a)^2*B*a^2*b^6*c^4*d^4*g^2*i^2*n/(d*x + c)^
2 - 54*(b*x + a)^3*B*a*b^6*c^5*d^4*g^2*i^2*n/(d*x + c)^3 - 2*(b*x + a)^4*B*
b^6*c^6*d^4*g^2*i^2*n/(d*x + c)^4 + 30*(b*x + a)*B*a^4*b^5*c^2*d^5*g^2*i^2*
n/(d*x + c) + 180*(b*x + a)^2*B*a^3*b^5*c^3*d^5*g^2*i^2*n/(d*x + c)^2 + 135
*(b*x + a)^3*B*a^2*b^5*c^4*d^5*g^2*i^2*n/(d*x + c)^3 + 12*(b*x + a)^4*B*a*b
^5*c^5*d^5*g^2*i^2*n/(d*x + c)^4 - 12*(b*x + a)*B*a^5*b^4*c*d^6*g^2*i^2*n/(
d*x + c) - 135*(b*x + a)^2*B*a^4*b^4*c^2*d^6*g^2*i^2*n/(d*x + c)^2 - 180*(b
*x + a)^3*B*a^3*b^4*c^3*d^6*g^2*i^2*n/(d*x + c)^3 - 30*(b*x + a)^4*B*a^2*b^
4*c^4*d^6*g^2*i^2*n/(d*x + c)^4 + 2*(b*x + a)*B*a^6*b^3*d^7*g^2*i^2*n/(d*x
+ c) + 54*(b*x + a)^2*B*a^5*b^3*c*d^7*g^2*i^2*n/(d*x + c)^2 + 135*(b*x + a)
^3*B*a^4*b^3*c^2*d^7*g^2*i^2*n/(d*x + c)^3 + 40*(b*x + a)^4*B*a^3*b^3*c^3*d
^7*g^2*i^2*n/(d*x + c)^4 - 9*(b*x + a)^2*B*a^6*b^2*d^8*g^2*i^2*n/(d*x + c)^
2 - 54*(b*x + a)^3*B*a^5*b^2*c*d^8*g^2*i^2*n/(d*x + c)^3 - 30*(b*x + a)^4*B
*a^4*b^2*c^2*d^8*g^2*i^2*n/(d*x + c)^4 + 9*(b*x + a)^3*B*a^6*b*d^9*g^2*i^2*
```

n/(d*x + c)^3 + 12*(b*x + a)^4*B*a^5*b*c*d^9*g^2*i^2*n/(d*x + c)^4 - 2*(b*x + a)^4*B*a^6*d^10*g^2*i^2*n/(d*x + c)^4 + 2*B*b^10*c^6*g^2*i^2*log(e) - 12*B*a*b^9*c^5*d*g^2*i^2*log(e) - 10*(b*x + a)*B*b^9*c^6*d*g^2*i^2*log(e)/(d*x + c) + 30*B*a^2*b^8*c^4*d^2*g^2*i^2*log(e) + 60*(b*x + a)*B*a*b^8*c^5*d^2*g^2*i^2*log(e)/(d*x + c) + 20*(b*x + a)^2*B*b^8*c^6*d^2*g^2*i^2*log(e)/(d*x + c)^2 - 40*B*a^3*b^7*c^3*d^3*g^2*i^2*log(e) - 150*(b*x + a)*B*a^2*b^7*c^4*d^3*g^2*i^2*log(e)/(d*x + c) - 120*(b*x + a)^2*B*a*b^7*c^5*d^3*g^2*i^2*log(e)/(d*x + c)^2 + 30*B*a^4*b^6*c^2*d^4*g^2*i^2*log(e) + 200*(b*x + a)*B*a^3*b^6*c^3*d^4*g^2*i^2*log(e)/(d*x + c) + 300*(b*x + a)^2*B*a^2*b^6*c^4*d^4*g^2*i^2*log(e)/(d*x + c)^2 - 12*B*a^5*b^5*c*d^5*g^2*i^2*log(e) - 150*(b*x + a)*B*a^4*b^5*c^2*d^5*g^2*i^2*log(e)/(d*x + c) - 400*(b*x + a)^2*B*a^3*b^5*c^3*d^5*g^2*i^2*log(e)/(d*x + c)^2 + 2*B*a^6*b^4*d^6*g^2*i^2*log(e) + 60*(b*x + a)*B*a^5*b^4*c*d^6*g^2*i^2*log(e)/(d*x + c) + 300*(b*x + a)^2*B*a^4*b^4*c^2*d^6*g^2*i^2*log(e)/(d*x + c)^2 - 10*(b*x + a)*B*a^6*b^3*d^7*g^2*i^2*log(e)/(d*x + c) - 120*(b*x + a)^2*B*a^5*b^3*c*d^7*g^2*i^2*log(e)/(d*x + c)^2 + 20*(b*x + a)^2*B*a^6*b^2*d^8*g^2*i^2*log(e)/(d*x + c)^2 + 2*A*b^10*c^6*g^2*i^2 - 12*A*a*b^9*c^5*d*g^2*i^2 - 10*(b*x + a)*A*b^9*c^6*d*g^2*i^2/(d*x + c) + 30*A*a^2*b^8*c^4*d^2*g^2*i^2 + 60*(b*x + a)*A*a*b^8*c^5*d^2*g^2*i^2/(d*x + c) + 20*(b*x + a)^2*A*b^8*c^6*d^2*g^2*i^2/(d*x + c)^2 - 40*A*a^3*b^7*c^3*d^3*g^2*i^2 - 150*(b*x + a)*A*a^2*b^7*c^4*d^3*g^2*i^2/(d*x + c) - 120*(b*x + a)^2*A*a*b^7*c^5*d^3*g^2*i^2/(d*x + c)^2 + 30*A*a^4*b^6*c^2*d^4*g^2*i^2 + 200*(b*x + a)*A*a^3*b^6*c^3*d^4*g^2*i^2/(d*x + c) + 300*(b*x + a)^2*A*a^2*b^6*c^4*d^4*g^2*i^2/(d*x + c)^2 - 12*A*a^5*b^5*c*d^5*g^2*i^2 - 150*(b*x + a)*A*a^4*b^5*c^2*d^5*g^2*i^2/(d*x + c) - 400*(b*x + a)^2*A*a^3*b^5*c^3*d^5*g^2*i^2/(d*x + c)^2 + 2*A*a^6*b^4*d^6*g^2*i^2 + 60*(b*x + a)*A*a^5*b^4*c*d^6*g^2*i^2/(d*x + c) + 300*(b*x + a)^2*A*a^4*b^4*c^2*d^6*g^2*i^2/(d*x + c)^2 - 10*(b*x + a)*A*a^6*b^3*d^7*g^2*i^2/(d*x + c) - 120*(b*x + a)^2*A*a^5*b^3*c*d^7*g^2*i^2/(d*x + c)^2 + 20*(b*x + a)^2*A*a^6*b^2*d^8*g^2*i^2/(d*x + c)^2)/(b^7*d^3 - 5*(b*x + a)*b^6*d^4/(d*x + c) + 10*(b*x + a)^2*b^5*d^5/(d*x + c)^2 - 10*(b*x + a)^3*b^4*d^6/(d*x + c)^3 + 5*(b*x + a)^4*b^3*d^7/(d*x + c)^4 - (b*x + a)^5*b^2*d^8/(d*x + c)^5) + 2*(B*b^6*c^6*g^2*i^2*n - 6*B*a*b^5*c^5*d*g^2*i^2*n + 15*B*a^2*b^4*c^4*d^2*g^2*i^2*n - 20*B*a^3*b^3*c^3*d^3*g^2*i^2*n + 15*B*a^4*b^2*c^2*d^4*g^2*i^2*n - 6*B*a^5*b*c*d^5*g^2*i^2*n + B*a^6*d^6*g^2*i^2*n)*log(b - (b*x + a)*d/(d*x + c))/(b^3*d^3) - 2*(B*b^6*c^6*g^2*i^2*n - 6*B*a*b^5*c^5*d*g^2*i^2*n + 15*B*a^2*b^4*c^4*d^2*g^2*i^2*n - 20*B*a^3*b^3*c^3*d^3*g^2*i^2*n + 15*B*a^4*b^2*c^2*d^4*g^2*i^2*n - 6*B*a^5*b*c*d^5*g^2*i^2*n + B*a^6*d^6*g^2*i^2*n)*log((b*x + a)/(d*x + c))/(b^3*d^3))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)

Mupad [B] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 1328, normalized size of antiderivative = 3.77

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

```
[In] int((a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),
x)
```

```
[Out] x^2*(((30*a*d + 30*b*c)*((((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d*n - B*
b*c*n))/5 - (A*b*d*g^2*i^2*(30*a*d + 30*b*c))/30)*(30*a*d + 30*b*c))/(30*b*
d) - (g^2*i^2*(6*A*a^2*d^2 + 6*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + 18*A
*a*b*c*d))/2 + A*a*b*c*d*g^2*i^2))/(60*b*d) - (a*c*((b*d*g^2*i^2*(15*A*a*d
+ 15*A*b*c + B*a*d*n - B*b*c*n))/5 - (A*b*d*g^2*i^2*(30*a*d + 30*b*c))/30))
/(2*b*d) + (g^2*i^2*(3*A*a^3*d^3 + 3*A*b^3*c^3 + B*a^3*d^3*n - B*b^3*c^3*n
+ 27*A*a*b^2*c^2*d + 27*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2
*n))/(6*b*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*g^2*i^2*x^3*(a^2*d^2 + b
^2*c^2 + 4*a*b*c*d))/3 + B*a^2*c^2*g^2*i^2*x + (B*b^2*d^2*g^2*i^2*x^5)/5 +
B*a*c*g^2*i^2*x^2*(a*d + b*c) + (B*b*d*g^2*i^2*x^4*(a*d + b*c))/2) - x^3*((
((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d*n - B*b*c*n))/5 - (A*b*d*g^2*i^2
*(30*a*d + 30*b*c))/30)*(30*a*d + 30*b*c))/(90*b*d) - (g^2*i^2*(6*A*a^2*d^2
 + 6*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + 18*A*a*b*c*d))/6 + (A*a*b*c*d*
g^2*i^2)/3) + x*((a*c*((((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d*n - B*b*
c*n))/5 - (A*b*d*g^2*i^2*(30*a*d + 30*b*c))/30)*(30*a*d + 30*b*c))/(30*b*d)
 - (g^2*i^2*(6*A*a^2*d^2 + 6*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + 18*A*a
*b*c*d))/2 + A*a*b*c*d*g^2*i^2))/(b*d) - ((30*a*d + 30*b*c)*(((30*a*d + 30*
b*c)*((((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d*n - B*b*c*n))/5 - (A*b*d*
g^2*i^2*(30*a*d + 30*b*c))/30)*(30*a*d + 30*b*c))/(30*b*d) - (g^2*i^2*(6*A*
a^2*d^2 + 6*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + 18*A*a*b*c*d))/2 + A*a*
b*c*d*g^2*i^2))/(30*b*d) - (a*c*((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d*
n - B*b*c*n))/5 - (A*b*d*g^2*i^2*(30*a*d + 30*b*c))/30))/(b*d) + (g^2*i^2*(
3*A*a^3*d^3 + 3*A*b^3*c^3 + B*a^3*d^3*n - B*b^3*c^3*n + 27*A*a*b^2*c^2*d +
27*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n))/(3*b*d)))/(30*b*
d) + (a*c*g^2*i^2*(3*A*a^2*d^2 + 3*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n +
9*A*a*b*c*d))/(b*d)) + x^4*((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d*n - B
*b*c*n))/20 - (A*b*d*g^2*i^2*(30*a*d + 30*b*c))/120) + (log(a + b*x)*(B*a^5
*d^2*g^2*i^2*n + 10*B*a^3*b^2*c^2*g^2*i^2*n - 5*B*a^4*b*c*d*g^2*i^2*n))/(30
*b^3) - (log(c + d*x)*(B*b^2*c^5*g^2*i^2*n + 10*B*a^2*c^3*d^2*g^2*i^2*n - 5
*B*a*b*c^4*d*g^2*i^2*n))/(30*d^3) + (A*b^2*d^2*g^2*i^2*x^5)/5
```

3.119 $\int (ag+bgx)(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal result

Integrand size = 41, antiderivative size = 250

$$\int (ag+bgx)(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \frac{B(bc-ad)^3gi^2nx}{12b^2d} + \frac{B(bc-ad)^2gi^2n(c+dx)^2}{24bd^2} - \frac{B(bc-ad)gi^2n(c+dx)^3}{12d^2} - \frac{(bc-ad)gi^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^2} + \frac{bgi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2} + \frac{B(bc-ad)^4gi^2n\log\left(\frac{a+bx}{c+dx}\right)}{12b^3d^2} + \frac{B(bc-ad)^4gi^2n\log(c+dx)}{12b^3d^2}$$

```
[Out] 1/12*B*(-a*d+b*c)^3*g*i^2*n*x/b^2/d+1/24*B*(-a*d+b*c)^2*g*i^2*n*(d*x+c)^2/b
/d^2-1/12*B*(-a*d+b*c)*g*i^2*n*(d*x+c)^3/d^2-1/3*(-a*d+b*c)*g*i^2*(d*x+c)^3
*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^2+1/4*b*g*i^2*(d*x+c)^4*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))/d^2+1/12*B*(-a*d+b*c)^4*g*i^2*n*ln((b*x+a)/(d*x+c))/b^3/d^2
+1/12*B*(-a*d+b*c)^4*g*i^2*n*ln(d*x+c)/b^3/d^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used

$= \{2561, 45, 2382, 12, 78\}$

$$\int (ag + bgx)(ci + dix)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$$
$$= -\frac{gi^2(c+dx)^3(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^2} + \frac{bgi^2(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^2}$$
$$+ \frac{Bgi^2n(bc-ad)^4\log\left(\frac{a+bx}{c+dx}\right)}{12b^3d^2} + \frac{Bgi^2n(bc-ad)^4\log(c+dx)}{12b^3d^2}$$
$$+ \frac{Bgi^2nx(bc-ad)^3}{12b^2d} + \frac{Bgi^2n(c+dx)^2(bc-ad)^2}{24bd^2} - \frac{Bgi^2n(c+dx)^3(bc-ad)}{12d^2}$$

```
[In] Int[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (B*(b*c - a*d)^3*g*i^2*n*x)/(12*b^2*d) + (B*(b*c - a*d)^2*g*i^2*n*(c + d*x)
^2)/(24*b*d^2) - (B*(b*c - a*d)*g*i^2*n*(c + d*x)^3)/(12*d^2) - ((b*c - a*d
)*g*i^2*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d^2) + (b*g*
i^2*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*d^2) + (B*(b*c -
 a*d)^4*g*i^2*n*Log[(a + b*x)/(c + d*x)])/(12*b^3*d^2) + (B*(b*c - a*d)^4*g
*i^2*n*Log[c + d*x])/(12*b^3*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
```

```
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \left((bc-ad)^4 gi^2\right) \text{Subst}\left(\int \frac{x(A+B\log(ex^n))}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= -\frac{(bc-ad)gi^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^2} + \frac{bgi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2} - \left(B(bc-ad)^4 gi^2 n\right)\text{Subst}\left(\int \frac{-b+4dx}{12d^2x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= -\frac{(bc-ad)gi^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^2} + \frac{bgi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2} - \frac{\left(B(bc-ad)^4 gi^2 n\right)\text{Subst}\left(\int \frac{-b+4dx}{x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{12d^2}$$

$$= -\frac{(bc-ad)gi^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^2} + \frac{bgi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2} - \frac{\left(B(bc-ad)^4 gi^2 n\right)\text{Subst}\left(\int \left(-\frac{1}{b^3x} + \frac{3d}{(b-dx)^4} - \frac{d}{b(b-dx)^3} - \frac{d}{b^2(b-dx)^2} - \frac{d}{b^3(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{12d^2}$$

$$= \frac{B(bc-ad)^3 gi^2 nx}{12b^2d} + \frac{B(bc-ad)^2 gi^2 n(c+dx)^2}{24bd^2} - \frac{B(bc-ad)gi^2 n(c+dx)^3}{12d^2} - \frac{(bc-ad)gi^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^2} + \frac{bgi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2} + \frac{B(bc-ad)^4 gi^2 n\log\left(\frac{a+bx}{c+dx}\right)}{12b^3d^2} + \frac{B(bc-ad)^4 gi^2 n\log(c+dx)}{12b^3d^2}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.90

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{gi^2\left(\frac{4B(bc-ad)^2n(2bd(bc-ad)x+b^2(c+dx)^2+2(bc-ad)^2\log(a+bx))}{b^3}-\frac{B(bc-ad)n(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-\right.}{24d^2}$$

```
[In] Integrate[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^
n]),x]
```

```
[Out] (g*i^2*((4*B*(b*c - a*d)^2*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*
c - a*d)^2*Log[a + b*x]))/b^3 - (B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3
*b^2*(b*c - a*d)*(c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a +
b*x]))/b^3 - 8*(b*c - a*d)*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]) + 6*b*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(24*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 836 vs. 2(236) = 472.

Time = 4.76 (sec) , antiderivative size = 837, normalized size of antiderivative = 3.35

method	result
parallelrisch	$\frac{24B\,x^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a\,b^3c\,d^3g\,i^2n+24Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a\,b^3c^2d^2g\,i^2n+6B\,x^4\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4d^4g\,i^2n-36Aa\,b^3c^3dg\,i^2n+}{}$

```
[In] int((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/24*(24*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*c*d^3*g*i^2*n+24*B*x*ln(e*((
b*x+a)/(d*x+c))^n)*a*b^3*c^2*d^2*g*i^2*n+6*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*
b^4*d^4*g*i^2*n-36*A*a*b^3*c^3*d*g*i^2*n+2*B*x^3*a*b^3*d^4*g*i^2*n^2-2*B*x^
3*b^4*c*d^3*g*i^2*n^2+8*A*x^3*a*b^3*d^4*g*i^2*n+16*A*x^3*b^4*c*d^3*g*i^2*n+
B*x^2*a^2*b^2*d^4*g*i^2*n^2-5*B*x^2*b^4*c^2*d^2*g*i^2*n^2+12*A*x^2*b^4*c^2*
d^2*g*i^2*n-2*B*x*a^3*b*d^4*g*i^2*n^2-2*B*x*b^4*c^3*d*g*i^2*n^2+8*B*ln(e*((
b*x+a)/(d*x+c))^n)*a*b^3*c^3*d*g*i^2*n+8*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*
b^3*d^4*g*i^2*n+16*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^3*g*i^2*n+12*B*x
^2*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2*d^2*g*i^2*n+24*A*x*a*b^3*c^2*d^2*g*i^2
*n-8*B*ln(b*x+a)*a^3*b*c*d^3*g*i^2*n^2+12*B*ln(b*x+a)*a^2*b^2*c^2*d^2*g*i^2
*n^2-8*B*ln(b*x+a)*a*b^3*c^3*d*g*i^2*n^2+4*B*x^2*a*b^3*c*d^3*g*i^2*n^2+24*A
*x^2*a*b^3*c*d^3*g*i^2*n+8*B*x*a^2*b^2*c*d^3*g*i^2*n^2-4*B*x*a*b^3*c^2*d^2*
g*i^2*n^2-7*B*a^3*b*c*d^3*g*i^2*n^2-8*B*a^2*b^2*c^2*d^2*g*i^2*n^2+11*B*a*b^
3*c^3*d*g*i^2*n^2-2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^4*g*i^2*n+6*A*x^4*b^4
```

```
*d^4*g*i^2*n+2*B*ln(b*x+a)*b^4*c^4*g*i^2*n^2+2*B*ln(b*x+a)*a^4*d^4*g*i^2*n^
2+2*B*b^4*c^4*g*i^2*n^2+2*B*a^4*d^4*g*i^2*n^2-48*A*a^2*b^2*c^2*d^2*g*i^2*n)
/b^3/d^2/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(236) = 472.

Time = 0.40 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.12

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \frac{6\,Ab^4d^4gi^2x^4+2\,(6\,Ba^2b^2c^2d^2-4\,Ba^3bcd^3+Ba^4d^4)gi^2n\log(bx+a)+2\,(Bb^4c^4-4\,Bab^3c^3d)gi^2n\log(dx}{}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, alg
orithm="fricas")
```

```
[Out] 1/24*(6*A*b^4*d^4*g*i^2*x^4 + 2*(6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3 + B*
a^4*d^4)*g*i^2*n*log(b*x + a) + 2*(B*b^4*c^4 - 4*B*a*b^3*c^3*d)*g*i^2*n*log
(d*x + c) - 2*((B*b^4*c*d^3 - B*a*b^3*d^4)*g*i^2*n - 4*(2*A*b^4*c*d^3 + A*a
*b^3*d^4)*g*i^2)*x^3 - ((5*B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 - B*a^2*b^2*d^4)
*g*i^2*n - 12*(A*b^4*c^2*d^2 + 2*A*a*b^3*c*d^3)*g*i^2)*x^2 + 2*(12*A*a*b^3*
c^2*d^2*g*i^2 - (B*b^4*c^3*d + 2*B*a*b^3*c^2*d^2 - 4*B*a^2*b^2*c*d^3 + B*a^
3*b*d^4)*g*i^2*n)*x + 2*(3*B*b^4*d^4*g*i^2*x^4 + 12*B*a*b^3*c^2*d^2*g*i^2*x
 + 4*(2*B*b^4*c*d^3 + B*a*b^3*d^4)*g*i^2*x^3 + 6*(B*b^4*c^2*d^2 + 2*B*a*b^3
*c*d^3)*g*i^2*x^2)*log(e) + 2*(3*B*b^4*d^4*g*i^2*n*x^4 + 12*B*a*b^3*c^2*d^2
*g*i^2*n*x + 4*(2*B*b^4*c*d^3 + B*a*b^3*d^4)*g*i^2*n*x^3 + 6*(B*b^4*c^2*d^2
 + 2*B*a*b^3*c*d^3)*g*i^2*n*x^2)*log((b*x + a)/(d*x + c)))/(b^3*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1054 vs. 2(233) = 466.

Time = 83.82 (sec) , antiderivative size = 1054, normalized size of antiderivative = 4.22

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Piecewise((a*c**2*g*i**2*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (
a*g*(A*c**2*i**2*x + A*c*d*i**2*x**2 + A*d**2*i**2*x**3/3 + B*c**3*i**2*log
(e*(a/(c + d*x))**n)/(3*d) + B*c**2*i**2*n*x/3 + B*c**2*i**2*x*log(e*(a/(c
+ d*x))**n) + B*c*d*i**2*n*x**2/3 + B*c*d*i**2*x**2*log(e*(a/(c + d*x))**n)
 + B*d**2*i**2*n*x**3/9 + B*d**2*i**2*x**3*log(e*(a/(c + d*x))**n)/3), Eq(b
```

```
, 0)), (c**2*i**2*(A*a*g*x + A*b*g*x**2/2 + B*a**2*g*log(e*(a/c + b*x/c)**n
)/(2*b) - B*a*g*n*x/2 + B*a*g*x*log(e*(a/c + b*x/c)**n) - B*b*g*n*x**2/4 +
B*b*g*x**2*log(e*(a/c + b*x/c)**n)/2), Eq(d, 0)), (A*a*c**2*g*i**2*x + A*a*
c*d*g*i**2*x**2 + A*a*d**2*g*i**2*x**3/3 + A*b*c**2*g*i**2*x**2/2 + 2*A*b*c
*d*g*i**2*x**3/3 + A*b*d**2*g*i**2*x**4/4 + B*a**4*d**2*g*i**2*n*log(c/d +
x)/(12*b**3) + B*a**4*d**2*g*i**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(
12*b**3) - B*a**3*c*d*g*i**2*n*log(c/d + x)/(3*b**2) - B*a**3*c*d*g*i**2*lo
g(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(3*b**2) - B*a**3*d**2*g*i**2*n*x/(12
*b**2) + B*a**2*c**2*g*i**2*n*log(c/d + x)/(2*b) + B*a**2*c**2*g*i**2*log(e
*(a/(c + d*x) + b*x/(c + d*x))**n)/(2*b) + B*a**2*c*d*g*i**2*n*x/(3*b) + B*
a**2*d**2*g*i**2*n*x**2/(24*b) - B*a*c**3*g*i**2*n*log(c/d + x)/(3*d) - B*a
*c**2*g*i**2*n*x/6 + B*a*c**2*g*i**2*x*log(e*(a/(c + d*x) + b*x/(c + d*x))*
*n) + B*a*c*d*g*i**2*n*x**2/6 + B*a*c*d*g*i**2*x**2*log(e*(a/(c + d*x) + b*
x/(c + d*x))**n) + B*a*d**2*g*i**2*n*x**3/12 + B*a*d**2*g*i**2*x**3*log(e*(
a/(c + d*x) + b*x/(c + d*x))**n)/3 + B*b*c**4*g*i**2*n*log(c/d + x)/(12*d**
2) - B*b*c**3*g*i**2*n*x/(12*d) - 5*B*b*c**2*g*i**2*n*x**2/24 + B*b*c**2*g*
i**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/2 - B*b*c*d*g*i**2*n*x**3
/12 + 2*B*b*c*d*g*i**2*x**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/3 + B*b
*d**2*g*i**2*x**4*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(236) = 472$.

Time = 0.20 (sec) , antiderivative size = 740, normalized size of antiderivative = 2.96

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{1}{4}Bbd^2gi^2x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{4}Abd^2gi^2x^4$$
$$+\frac{2}{3}Bbcdgi^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{3}Bad^2gi^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+\frac{2}{3}Abcdgi^2x^3+\frac{1}{3}Aad^2gi^2x^3+\frac{1}{2}Bbc^2gi^2x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+Bacdgi^2x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{2}Abc^2gi^2x^2+Aacdgi^2x^2$$
$$-\frac{1}{24}Bbd^2gi^2n\left(\frac{6a^4\log(bx+a)}{b^4}-\frac{6c^4\log(dx+c)}{d^4}+\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3c^2d-a^2bd^3)x^2+6(b^3c^3}{b^3d^3}\right.$$
$$+\frac{1}{3}Bbcdgi^2n\left(\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$+\frac{1}{6}Bad^2gi^2n\left(\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$-\frac{1}{2}Bbc^2gi^2n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$-Bacdgi^2n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bac^2gi^2n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)$$
$$+Bac^2gi^2x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aac^2gi^2x$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, alg
orithm="maxima")
```

```
[Out] 1/4*B*b*d^2*g*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*b*d^2*
g*i^2*x^4 + 2/3*B*b*c*d*g*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) +
1/3*B*a*d^2*g*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2/3*A*b*c*d*
g*i^2*x^3 + 1/3*A*a*d^2*g*i^2*x^3 + 1/2*B*b*c^2*g*i^2*x^2*log(e*(b*x/(d*x +
 c) + a/(d*x + c))^n) + B*a*c*d*g*i^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c
))^n) + 1/2*A*b*c^2*g*i^2*x^2 + A*a*c*d*g*i^2*x^2 - 1/24*B*b*d^2*g*i^2*n*(6
*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)
*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))
+ 1/3*B*b*c*d*g*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((
b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + 1/6*B*a*d^2*
g*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*
d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 1/2*B*b*c^2*g*i^2*n*(a^2*l
```

```
og(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) - B*a*c*d*g*i
^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) +
B*a*c^2*g*i^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*a*c^2*g*i^2*x*log
(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a*c^2*g*i^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1997 vs. $2(236) = 472$.

Time = 1.00 (sec) , antiderivative size = 1997, normalized size of antiderivative = 7.99

$$\int (ag + bgx)(ci + dix)^2 \left(A + B\log\left(e\left(\frac{a + bx}{c + dx}\right)^n\right)\right) dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, alg
orithm="giac")
```

```
[Out] -1/24*(2*(B*b^6*c^5*g*i^2*n - 5*B*a*b^5*c^4*d*g*i^2*n - 4*(b*x + a)*B*b^5*c
^5*d*g*i^2*n/(d*x + c) + 10*B*a^2*b^4*c^3*d^2*g*i^2*n + 20*(b*x + a)*B*a*b^
4*c^4*d^2*g*i^2*n/(d*x + c) - 10*B*a^3*b^3*c^2*d^3*g*i^2*n - 40*(b*x + a)*B
*a^2*b^3*c^3*d^3*g*i^2*n/(d*x + c) + 5*B*a^4*b^2*c*d^4*g*i^2*n + 40*(b*x +
a)*B*a^3*b^2*c^2*d^4*g*i^2*n/(d*x + c) - B*a^5*b*d^5*g*i^2*n - 20*(b*x + a)
*B*a^4*b*c*d^5*g*i^2*n/(d*x + c) + 4*(b*x + a)*B*a^5*d^6*g*i^2*n/(d*x + c))
*log((b*x + a)/(d*x + c))/(b^4*d^2 - 4*(b*x + a)*b^3*d^3/(d*x + c) + 6*(b*x
 + a)^2*b^2*d^4/(d*x + c)^2 - 4*(b*x + a)^3*b*d^5/(d*x + c)^3 + (b*x + a)^4
*d^6/(d*x + c)^4) - (B*b^8*c^5*g*i^2*n - 5*B*a*b^7*c^4*d*g*i^2*n - 6*(b*x +
 a)*B*b^7*c^5*d*g*i^2*n/(d*x + c) + 10*B*a^2*b^6*c^3*d^2*g*i^2*n + 30*(b*x
+ a)*B*a*b^6*c^4*d^2*g*i^2*n/(d*x + c) + 7*(b*x + a)^2*B*b^6*c^5*d^2*g*i^2*
n/(d*x + c)^2 - 10*B*a^3*b^5*c^2*d^3*g*i^2*n - 60*(b*x + a)*B*a^2*b^5*c^3*d
^3*g*i^2*n/(d*x + c) - 35*(b*x + a)^2*B*a*b^5*c^4*d^3*g*i^2*n/(d*x + c)^2 -
 2*(b*x + a)^3*B*b^5*c^5*d^3*g*i^2*n/(d*x + c)^3 + 5*B*a^4*b^4*c*d^4*g*i^2*
n + 60*(b*x + a)*B*a^3*b^4*c^2*d^4*g*i^2*n/(d*x + c) + 70*(b*x + a)^2*B*a^2
*b^4*c^3*d^4*g*i^2*n/(d*x + c)^2 + 10*(b*x + a)^3*B*a*b^4*c^4*d^4*g*i^2*n/(
d*x + c)^3 - B*a^5*b^3*d^5*g*i^2*n - 30*(b*x + a)*B*a^4*b^3*c*d^5*g*i^2*n/(
d*x + c) - 70*(b*x + a)^2*B*a^3*b^3*c^2*d^5*g*i^2*n/(d*x + c)^2 - 20*(b*x +
 a)^3*B*a^2*b^3*c^3*d^5*g*i^2*n/(d*x + c)^3 + 6*(b*x + a)*B*a^5*b^2*d^6*g*i
^2*n/(d*x + c) + 35*(b*x + a)^2*B*a^4*b^2*c*d^6*g*i^2*n/(d*x + c)^2 + 20*(b
*x + a)^3*B*a^3*b^2*c^2*d^6*g*i^2*n/(d*x + c)^3 - 7*(b*x + a)^2*B*a^5*b*d^7
*g*i^2*n/(d*x + c)^2 - 10*(b*x + a)^3*B*a^4*b*c*d^7*g*i^2*n/(d*x + c)^3 + 2
*(b*x + a)^3*B*a^5*d^8*g*i^2*n/(d*x + c)^3 - 2*B*b^8*c^5*g*i^2*log(e) + 10*
B*a*b^7*c^4*d*g*i^2*log(e) + 8*(b*x + a)*B*b^7*c^5*d*g*i^2*log(e)/(d*x + c)
 - 20*B*a^2*b^6*c^3*d^2*g*i^2*log(e) - 40*(b*x + a)*B*a*b^6*c^4*d^2*g*i^2*l
og(e)/(d*x + c) + 20*B*a^3*b^5*c^2*d^3*g*i^2*log(e) + 80*(b*x + a)*B*a^2*b^
5*c^3*d^3*g*i^2*log(e)/(d*x + c) - 10*B*a^4*b^4*c*d^4*g*i^2*log(e) - 80*(b*
x + a)*B*a^3*b^4*c^2*d^4*g*i^2*log(e)/(d*x + c) + 2*B*a^5*b^3*d^5*g*i^2*log
```

(e) + 40*(b*x + a)*B*a^4*b^3*c*d^5*g*i^2*log(e)/(d*x + c) - 8*(b*x + a)*B*a^5*b^2*d^6*g*i^2*log(e)/(d*x + c) - 2*A*b^8*c^5*g*i^2 + 10*A*a*b^7*c^4*d*g*i^2 + 8*(b*x + a)*A*b^7*c^5*d*g*i^2/(d*x + c) - 20*A*a^2*b^6*c^3*d^2*g*i^2 - 40*(b*x + a)*A*a*b^6*c^4*d^2*g*i^2/(d*x + c) + 20*A*a^3*b^5*c^2*d^3*g*i^2 + 80*(b*x + a)*A*a^2*b^5*c^3*d^3*g*i^2/(d*x + c) - 10*A*a^4*b^4*c*d^4*g*i^2 - 80*(b*x + a)*A*a^3*b^4*c^2*d^4*g*i^2/(d*x + c) + 2*A*a^5*b^3*d^5*g*i^2 + 40*(b*x + a)*A*a^4*b^3*c*d^5*g*i^2/(d*x + c) - 8*(b*x + a)*A*a^5*b^2*d^6*g*i^2/(d*x + c))/(b^6*d^2 - 4*(b*x + a)*b^5*d^3/(d*x + c) + 6*(b*x + a)^2*b^4*d^4/(d*x + c)^2 - 4*(b*x + a)^3*b^3*d^5/(d*x + c)^3 + (b*x + a)^4*b^2*d^6/(d*x + c)^4) + 2*(B*b^5*c^5*g*i^2*n - 5*B*a*b^4*c^4*d*g*i^2*n + 10*B*a^2*b^3*c^3*d^2*g*i^2*n - 10*B*a^3*b^2*c^2*d^3*g*i^2*n + 5*B*a^4*b*c*d^4*g*i^2*n - B*a^5*d^5*g*i^2*n)*log(-b + (b*x + a)*d/(d*x + c))/(b^3*d^2) - 2*(B*b^5*c^5*g*i^2*n - 5*B*a*b^4*c^4*d*g*i^2*n + 10*B*a^2*b^3*c^3*d^2*g*i^2*n - 10*B*a^3*b^2*c^2*d^3*g*i^2*n + 5*B*a^4*b*c*d^4*g*i^2*n - B*a^5*d^5*g*i^2*n)*log((b*x + a)/(d*x + c))/(b^3*d^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)

Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 661, normalized size of antiderivative = 2.64

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(B\,a\,c^2\,g\,i^2\,x+\frac{B\,c\,g\,i^2\,x^2\,(2\,a\,d+b\,c)}{2}+\frac{B\,d\,g\,i^2\,x^3\,(a\,d+2\,b\,c)}{3}+\frac{B\,b\,d^2\,g\,i^2\,x^4}{4}\right)+x^3\left(\frac{d\,g\,i^2\,(8\,A\,a\,d+12\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{12}-\frac{A\,d\,g\,i^2\,(12\,a\,d+12\,b\,c)}{36}\right)+x\left(\frac{(12\,a\,d+12\,b\,c)\left(\frac{\left(\frac{d\,g\,i^2\,(8\,A\,a\,d+12\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4}-\frac{A\,d\,g\,i^2\,(12\,a\,d+12\,b\,c)}{12}\right)(12\,a\,d+12\,b\,c)}{12\,b\,d}-\frac{g\,i^2\,(3\,A\,a^2\,d^2+9\,A\,b^2\,c^2+B\,a \cdots}{\cdots}\right.}{12\,b\,d}-\frac{a\,c\left(\frac{d\,g\,i^2\,(8\,A\,a\,d+12\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4}-\frac{A\,d\,g\,i^2\,(12\,a\,d+12\,b\,c)}{12}\right)}{b\,d}+\frac{c\,g\,i^2\,(6\,A\,a^2\,d^2+2\,A\,b^2\,c^2+2\,B\,a^2\,d^2\,n-B\,b^2\,c^2\,n+12\,A\,a\,b\,c\,d-B\,a\,b\,c\,d\,n)}{2\,b\,d}\right)-x^2\left(\frac{\left(\frac{d\,g\,i^2\,(8\,A\,a\,d+12\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4}-\frac{A\,d\,g\,i^2\,(12\,a\,d+12\,b\,c)}{12}\right)(12\,a\,d+12\,b\,c)}{24\,b\,d}-\frac{g\,i^2\,(3\,A\,a^2\,d^2+9\,A\,b^2\,c^2+B\,a^2\,d^2\,n-2\,B\,b^2\,c^2\,n+18\,A\,a\,b\,c\,d+B\,a\,b\,c\,d\,n)}{6\,b}+\frac{A\,a\,c\,d\,g\,i^2}{2}\right)+\frac{\ln(c+d\,x)\,(B\,b\,c^4\,g\,i^2\,n-4\,B\,a\,c^3\,d\,g\,i^2\,n)}{12\,d^2}+\frac{\ln(a+b\,x)\,(B\,g\,n\,a^4\,d^2\,i^2-4\,B\,g\,n\,a^3\,b\,c\,d\,i^2+6\,B\,g\,n\,a^2\,b^2\,c^2\,i^2)}{12\,b^3}+\frac{A\,b\,d^2\,g\,i^2\,x^4}{4}$$

```
[In] int((a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] log(e*((a + b*x)/(c + d*x))^n)*(B*a*c^2*g*i^2*x + (B*c*g*i^2*x^2*(2*a*d + b
*c))/2 + (B*d*g*i^2*x^3*(a*d + 2*b*c))/3 + (B*b*d^2*g*i^2*x^4)/4) + x^3*((d
*g*i^2*(8*A*a*d + 12*A*b*c + B*a*d*n - B*b*c*n))/12 - (A*d*g*i^2*(12*a*d +
12*b*c))/36) + x*(((12*a*d + 12*b*c)*((((d*g*i^2*(8*A*a*d + 12*A*b*c + B*a*
d*n - B*b*c*n))/4 - (A*d*g*i^2*(12*a*d + 12*b*c))/12)*(12*a*d + 12*b*c))/(1
2*b*d) - (g*i^2*(3*A*a^2*d^2 + 9*A*b^2*c^2 + B*a^2*d^2*n - 2*B*b^2*c^2*n +
18*A*a*b*c*d + B*a*b*c*d*n))/(3*b) + A*a*c*d*g*i^2))/(12*b*d) - (a*c*((d*g*
```

```
i^2*(8*A*a*d + 12*A*b*c + B*a*d*n - B*b*c*n))/4 - (A*d*g*i^2*(12*a*d + 12*b
*c))/12))/(b*d) + (c*g*i^2*(6*A*a^2*d^2 + 2*A*b^2*c^2 + 2*B*a^2*d^2*n - B*b
^2*c^2*n + 12*A*a*b*c*d - B*a*b*c*d*n))/(2*b*d)) - x^2*((((d*g*i^2*(8*A*a*d
 + 12*A*b*c + B*a*d*n - B*b*c*n))/4 - (A*d*g*i^2*(12*a*d + 12*b*c))/12)*(12
*a*d + 12*b*c))/(24*b*d) - (g*i^2*(3*A*a^2*d^2 + 9*A*b^2*c^2 + B*a^2*d^2*n
- 2*B*b^2*c^2*n + 18*A*a*b*c*d + B*a*b*c*d*n))/(6*b) + (A*a*c*d*g*i^2)/2) +
 (log(c + d*x)*(B*b*c^4*g*i^2*n - 4*B*a*c^3*d*g*i^2*n))/(12*d^2) + (log(a +
 b*x)*(B*a^4*d^2*g*i^2*n + 6*B*a^2*b^2*c^2*g*i^2*n - 4*B*a^3*b*c*d*g*i^2*n)
)/(12*b^3) + (A*b*d^2*g*i^2*x^4)/4
```

3.120 $\int (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal result

Integrand size = 33, antiderivative size = 124

$$\begin{aligned}\int (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx &= -\frac{B(bc-ad)^2i^2nx}{3b^2}-\frac{B(bc-ad)i^2n(c+dx)^2}{6bd}\\&\quad-\frac{B(bc-ad)^3i^2n\log(a+bx)}{3b^3d}+\frac{i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d}\end{aligned}$$

```
[Out] -1/3*B*(-a*d+b*c)^2*i^2*n*x/b^2-1/6*B*(-a*d+b*c)*i^2*n*(d*x+c)^2/b/d-1/3*B*
(-a*d+b*c)^3*i^2*n*ln(b*x+a)/b^3/d+1/3*i^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*
x+c))^n))/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2547, 21, 45}

$$\begin{aligned}\int (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx &= \frac{i^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d}-\frac{Bi^2n(bc-ad)^3\log(a+bx)}{3b^3d}\\&\quad-\frac{Bi^2nx(bc-ad)^2}{3b^2}-\frac{Bi^2n(c+dx)^2(bc-ad)}{6bd}\end{aligned}$$

```
[In] Int[(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/3*(B*(b*c - a*d)^2*i^2*n*x)/b^2 - (B*(b*c - a*d)*i^2*n*(c + d*x)^2)/(6*b
*d) - (B*(b*c - a*d)^3*i^2*n*Log[a + b*x])/(3*b^3*d) + (i^2*(c + d*x)^3*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d}-\frac{(B(bc-ad)n)\int\frac{(ci+dix)^3}{(a+bx)(c+dx)}\,dx}{3di}\\
&= \frac{i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d}-\frac{(B(bc-ad)i^2n)\int\frac{(c+dx)^2}{a+bx}\,dx}{3d}\\
&= \frac{i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d}-\frac{(B(bc-ad)i^2n)\int\left(\frac{d(bc-ad)}{b^2}+\frac{(bc-ad)^2}{b^2(a+bx)}+\frac{d(c+dx)}{b}\right)dx}{3d}\\
&= -\frac{B(bc-ad)^2i^2nx}{3b^2}-\frac{B(bc-ad)i^2n(c+dx)^2}{6bd}\\
&\quad -\frac{B(bc-ad)^3i^2n\log(a+bx)}{3b^3d}+\frac{i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.81

$$\int (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{i^2\left(-\frac{B(bc-ad)n\left(2bd(bc-ad)x+b^2(c+dx)^2+2(bc-ad)^2\log(a+bx)\right)}{2b^3}+(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{3d}$$

```
[In] Integrate[(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (i^2*(-1/2*(B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c
 - a*d)^2*Log[a + b*x]))/b^3 + (c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])))/(3*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. 2(116) = 232.

Time = 1.96 (sec) , antiderivative size = 463, normalized size of antiderivative = 3.73

method	result
parallelrisch	$2Ax^3b^3d^3i^2n+2B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3c^3i^2n+2B\ln(bx+a)a^3d^3i^2n^2-2B\ln(bx+a)b^3c^3i^2n^2+6B\,x^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3c\,d^2i^2n+2$

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*A*x^3*b^3*d^3*i^2*n+2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c^3*i^2*n+2*B*
ln(b*x+a)*a^3*d^3*i^2*n^2-2*B*ln(b*x+a)*b^3*c^3*i^2*n^2+6*B*x^2*ln(e*((b*x+
a)/(d*x+c))^n)*b^3*c*d^2*i^2*n+2*B*a^3*d^3*i^2*n^2+4*B*b^3*c^3*i^2*n^2-6*A*
b^3*c^3*i^2*n-6*B*ln(b*x+a)*a^2*b*c*d^2*i^2*n^2+6*B*ln(b*x+a)*a*b^2*c^2*d*i
^2*n^2+6*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c^2*d*i^2*n+6*B*x*a*b^2*c*d^2*i^
2*n^2-5*B*a^2*b*c*d^2*i^2*n^2-B*a*b^2*c^2*d*i^2*n^2-12*A*a*b^2*c^2*d*i^2*n+
2*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^3*i^2*n+B*x^2*a*b^2*d^3*i^2*n^2-B*x
^2*b^3*c*d^2*i^2*n^2+6*A*x^2*b^3*c*d^2*i^2*n-2*B*x*a^2*b*d^3*i^2*n^2-4*B*x*
b^3*c^2*d*i^2*n^2+6*A*x*b^3*c^2*d*i^2*n)/b^3/d/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. 2(116) = 232.

Time = 0.34 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.40

$$\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{2\,Ab^3d^3i^2x^3-2\,Bb^3c^3i^2n\log(dx+c)+2\,(3\,Bab^2c^2d-3\,Ba^2bcd^2+Ba^3d^3)i^2n\log(bx+a)+(6\,Ab^3cd^2i^2-$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] 1/6*(2*A*b^3*d^3*i^2*x^3 - 2*B*b^3*c^3*i^2*n*log(d*x + c) + 2*(3*B*a*b^2*c^
2*d - 3*B*a^2*b*c*d^2 + B*a^3*d^3)*i^2*n*log(b*x + a) + (6*A*b^3*c*d^2*i^2
- (B*b^3*c*d^2 - B*a*b^2*d^3)*i^2*n)*x^2 + 2*(3*A*b^3*c^2*d*i^2 - (2*B*b^3*
c^2*d - 3*B*a*b^2*c*d^2 + B*a^2*b*d^3)*i^2*n)*x + 2*(B*b^3*d^3*i^2*x^3 + 3*
B*b^3*c*d^2*i^2*x^2 + 3*B*b^3*c^2*d*i^2*x)*log(e) + 2*(B*b^3*d^3*i^2*n*x^3
+ 3*B*b^3*c*d^2*i^2*n*x^2 + 3*B*b^3*c^2*d*i^2*n*x)*log((b*x + a)/(d*x + c))
)/(b^3*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. 2(107) = 214.

Time = 18.64 (sec) , antiderivative size = 656, normalized size of antiderivative = 5.29

$$\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \begin{cases} c^2i^2x\left(A+B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) \\ Ac^2i^2x+Acdi^2x^2+\frac{Ad^2i^2x^3}{3}+\frac{Bc^3i^2\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{3d}+\frac{Bc^2i^2nx}{3}+Bc^2i^2x\log\left(e\left(\frac{a}{c+dx}\right)^n\right)+\frac{Bcdi^2nx^2}{3}+Bcdi^2x^2 \\ c^2i^2\left(Ax+\frac{Ba\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{b}-Bnx+Bx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)\right) \\ Ac^2i^2x+Acdi^2x^2+\frac{Ad^2i^2x^3}{3}+\frac{Ba^3d^2i^2n\log\left(\frac{c}{d}+x\right)}{3b^3}+\frac{Ba^3d^2i^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{3b^3}-\frac{Ba^2cdi^2n\log\left(\frac{c}{d}+x\right)}{b^2}-\frac{Ba^2cdi^2\log\left(\right.}{} \end{cases}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Piecewise(((c**2*i**2*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (A*c*
*2*i**2*x + A*c*d*i**2*x**2 + A*d**2*i**2*x**3/3 + B*c**3*i**2*log(e*(a/(c
+ d*x))**n)/(3*d) + B*c**2*i**2*n*x/3 + B*c**2*i**2*x*log(e*(a/(c + d*x))**
n) + B*c*d*i**2*n*x**2/3 + B*c*d*i**2*x**2*log(e*(a/(c + d*x))**n) + B*d**2
*i**2*n*x**3/9 + B*d**2*i**2*x**3*log(e*(a/(c + d*x))**n)/3, Eq(b, 0)), (c*
```

```
*2*i**2*(A*x + B*a*log(e*(a/c + b*x/c)**n)/b - B*n*x + B*x*log(e*(a/c + b*x
/c)**n)), Eq(d, 0)), (A*c**2*i**2*x + A*c*d*i**2*x**2 + A*d**2*i**2*x**3/3
+ B*a**3*d**2*i**2*n*log(c/d + x)/(3*b**3) + B*a**3*d**2*i**2*log(e*(a/(c +
 d*x) + b*x/(c + d*x))**n)/(3*b**3) - B*a**2*c*d*i**2*n*log(c/d + x)/b**2 -
 B*a**2*c*d*i**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/b**2 - B*a**2*d**2
*i**2*n*x/(3*b**2) + B*a*c**2*i**2*n*log(c/d + x)/b + B*a*c**2*i**2*log(e*(
a/(c + d*x) + b*x/(c + d*x))**n)/b + B*a*c*d*i**2*n*x/b + B*a*d**2*i**2*n*x
**2/(6*b) - B*c**3*i**2*n*log(c/d + x)/(3*d) - 2*B*c**2*i**2*n*x/3 + B*c**2
*i**2*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) - B*c*d*i**2*n*x**2/6 + B*c
*d*i**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*d**2*i**2*x**3*log
(e*(a/(c + d*x) + b*x/(c + d*x))**n)/3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(116) = 232$.

Time = 0.20 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.49

$$\int (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{1}{3}\,Bd^2i^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{3}\,Ad^2i^2x^3$$
$$+ Bcdi^2x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Acdi^2x^2$$
$$+ \frac{1}{6}\,Bd^2i^2n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$- Bcdi^2n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+ Bc^2i^2n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)$$
$$+ Bc^2i^2x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Ac^2i^2x$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] 1/3*B*d^2*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*d^2*i^2*x^
3 + B*c*d*i^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c*d*i^2*x^2 +
1/6*B*d^2*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*
d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - B*c*d*i^2*n*(a^2*l
og(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*c^2*i^2*n
*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*c^2*i^2*x*log(e*(b*x/(d*x + c) +
 a/(d*x + c))^n) + A*c^2*i^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 990 vs. 2(116) = 232.

Time = 0.65 (sec) , antiderivative size = 990, normalized size of antiderivative = 7.98

$$\int (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$=\frac{1}{6}\left(\frac{2\left(Bb^4c^4i^2n-4\,Bab^3c^3di^2n+6\,Ba^2b^2c^2d^2i^2n-4\,Ba^3bcd^3i^2n+Ba^4d^4i^2n\right)\log\left(\frac{bx+a}{dx+c}\right)}{b^3d-\frac{3\,(bx+a)b^2d^2}{dx+c}+\frac{3\,(bx+a)^2bd^3}{(dx+c)^2}-\frac{(bx+a)^3d^4}{(dx+c)^3}}-\frac{3\,Bb^6c^4i^2n-}{}\right.$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] 1/6*(2*(B*b^4*c^4*i^2*n - 4*B*a*b^3*c^3*d*i^2*n + 6*B*a^2*b^2*c^2*d^2*i^2*n
 - 4*B*a^3*b*c*d^3*i^2*n + B*a^4*d^4*i^2*n)*log((b*x + a)/(d*x + c))/(b^3*d
 - 3*(b*x + a)*b^2*d^2/(d*x + c) + 3*(b*x + a)^2*b*d^3/(d*x + c)^2 - (b*x +
 a)^3*d^4/(d*x + c)^3) - (3*B*b^6*c^4*i^2*n - 12*B*a*b^5*c^3*d*i^2*n - 5*(b
*x + a)*B*b^5*c^4*d*i^2*n/(d*x + c) + 18*B*a^2*b^4*c^2*d^2*i^2*n + 20*(b*x
+ a)*B*a*b^4*c^3*d^2*i^2*n/(d*x + c) + 2*(b*x + a)^2*B*b^4*c^4*d^2*i^2*n/(d
*x + c)^2 - 12*B*a^3*b^3*c*d^3*i^2*n - 30*(b*x + a)*B*a^2*b^3*c^2*d^3*i^2*n
/(d*x + c) - 8*(b*x + a)^2*B*a*b^3*c^3*d^3*i^2*n/(d*x + c)^2 + 3*B*a^4*b^2*
d^4*i^2*n + 20*(b*x + a)*B*a^3*b^2*c*d^4*i^2*n/(d*x + c) + 12*(b*x + a)^2*B
*a^2*b^2*c^2*d^4*i^2*n/(d*x + c)^2 - 5*(b*x + a)*B*a^4*b*d^5*i^2*n/(d*x + c
) - 8*(b*x + a)^2*B*a^3*b*c*d^5*i^2*n/(d*x + c)^2 + 2*(b*x + a)^2*B*a^4*d^6
*i^2*n/(d*x + c)^2 - 2*B*b^6*c^4*i^2*log(e) + 8*B*a*b^5*c^3*d*i^2*log(e) -
12*B*a^2*b^4*c^2*d^2*i^2*log(e) + 8*B*a^3*b^3*c*d^3*i^2*log(e) - 2*B*a^4*b^
2*d^4*i^2*log(e) - 2*A*b^6*c^4*i^2 + 8*A*a*b^5*c^3*d*i^2 - 12*A*a^2*b^4*c^2
*d^2*i^2 + 8*A*a^3*b^3*c*d^3*i^2 - 2*A*a^4*b^2*d^4*i^2)/(b^5*d - 3*(b*x + a
)*b^4*d^2/(d*x + c) + 3*(b*x + a)^2*b^3*d^3/(d*x + c)^2 - (b*x + a)^3*b^2*d
^4/(d*x + c)^3) + 2*(B*b^4*c^4*i^2*n - 4*B*a*b^3*c^3*d*i^2*n + 6*B*a^2*b^2*
c^2*d^2*i^2*n - 4*B*a^3*b*c*d^3*i^2*n + B*a^4*d^4*i^2*n)*log(b - (b*x + a)*
d/(d*x + c))/(b^3*d) - 2*(B*b^4*c^4*i^2*n - 4*B*a*b^3*c^3*d*i^2*n + 6*B*a^2
*b^2*c^2*d^2*i^2*n - 4*B*a^3*b*c*d^3*i^2*n + B*a^4*d^4*i^2*n)*log((b*x + a)
/(d*x + c))/(b^3*d))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.44

$$\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(B\,c^2\,i^2\,x+B\,c\,d\,i^2\,x^2+\frac{B\,d^2\,i^2\,x^3}{3}\right) - x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{d\,i^2\,(3\,A\,a\,d+9\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{3\,b}-\frac{A\,d\,i^2\,(3\,a\,d+3\,b\,c)}{3\,b}\right)}{3\,b\,d} - \frac{c\,i^2\,(3\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{b}+\frac{A\,a\,c\,d\,i^2}{b}\right) + x^2\left(\frac{d\,i^2\,(3\,A\,a\,d+9\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{6\,b}-\frac{A\,d\,i^2\,(3\,a\,d+3\,b\,c)}{6\,b}\right) + \frac{\ln(a+b\,x)\,\left(B\,n\,a^3\,d^2\,i^2-3\,B\,n\,a^2\,b\,c\,d\,i^2+3\,B\,n\,a\,b^2\,c^2\,i^2\right)}{3\,b^3} + \frac{A\,d^2\,i^2\,x^3}{3}-\frac{B\,c^3\,i^2\,n\,\ln(c+d\,x)}{3\,d}$$

```
[In] int((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] log(e*((a + b*x)/(c + d*x))^n)*((B*d^2*i^2*x^3)/3 + B*c^2*i^2*x + B*c*d*i^2
*x^2) - x*(((3*a*d + 3*b*c)*((d*i^2*(3*A*a*d + 9*A*b*c + B*a*d*n - B*b*c*n)
)/(3*b) - (A*d*i^2*(3*a*d + 3*b*c))/(3*b)))/(3*b*d) - (c*i^2*(3*A*a*d + 3*A
*b*c + B*a*d*n - B*b*c*n))/b + (A*a*c*d*i^2)/b) + x^2*((d*i^2*(3*A*a*d + 9*
A*b*c + B*a*d*n - B*b*c*n))/(6*b) - (A*d*i^2*(3*a*d + 3*b*c))/(6*b)) + (log
(a + b*x)*(B*a^3*d^2*i^2*n + 3*B*a*b^2*c^2*i^2*n - 3*B*a^2*b*c*d*i^2*n))/(3
*b^3) + (A*d^2*i^2*x^3)/3 - (B*c^3*i^2*n*log(c + d*x))/(3*d)
```

3.121 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 289

$$\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx \\
&= -\frac{Bd(bc-ad)i^2nx}{2b^2g}+\frac{d(bc-ad)i^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g} \\
&\quad+\frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg}-\frac{B(bc-ad)^2i^2n\log\left(\frac{a+bx}{c+dx}\right)}{2b^3g} \\
&\quad-\frac{3B(bc-ad)^2i^2n\log(c+dx)}{2b^3g}-\frac{(bc-ad)^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad+\frac{B(bc-ad)^2i^2n\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}
\end{aligned}$$

```
[Out] -1/2*B*d*(-a*d+b*c)*i^2*n*x/b^2/g+d*(-a*d+b*c)*i^2*(b*x+a)*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))/b^3/g+1/2*i^2*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g
-1/2*B*(-a*d+b*c)^2*i^2*n*ln((b*x+a)/(d*x+c))/b^3/g-3/2*B*(-a*d+b*c)^2*i^2*
n*ln(d*x+c)/b^3/g-(-a*d+b*c)^2*i^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(
d*x+c)/d/(b*x+a))/b^3/g+B*(-a*d+b*c)^2*i^2*n*polylog(2,b*(d*x+c)/d/(b*x+a))
/b^3/g
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {2561, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$$
$$= \frac{di^2(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3g} - \frac{i^2(bc-ad)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3g} + \frac{i^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2bg} + \frac{Bi^2n(bc-ad)^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} - \frac{Bi^2n(bc-ad)^2\log\left(\frac{a+bx}{c+dx}\right)}{2b^3g} - \frac{3Bi^2n(bc-ad)^2\log(c+dx)}{2b^3g} - \frac{Bdi^2nx(bc-ad)}{2b^2g}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x),
x]
```

```
[Out] -1/2*(B*d*(b*c - a*d)*i^2*n*x)/(b^2*g) + (d*(b*c - a*d)*i^2*(a + b*x)*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^3*g) + (i^2*(c + d*x)^2*(A + B*Log[e*
((a + b*x)/(c + d*x))^n]))/(2*b*g) - (B*(b*c - a*d)^2*i^2*n*Log[(a + b*x)/(
c + d*x)])/(2*b^3*g) - (3*B*(b*c - a*d)^2*i^2*n*Log[c + d*x])/(2*b^3*g) - (
(b*c - a*d)^2*i^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*
x))/(d*(a + b*x))])/(b^3*g) + (B*(b*c - a*d)^2*i^2*n*PolyLog[2, (b*(c + d*x
))/(d*(a + b*x))])/(b^3*g)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
```

```
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)^2 i^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{g}$$

$$= \frac{((bc-ad)^2i^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg} + \frac{(d(bc-ad)^2i^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg}$$

$$= \frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg} + \frac{((bc-ad)^2i^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g} + \frac{(d(bc-ad)^2i^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g} - \frac{(B(bc-ad)^2i^2n)\,\text{Subst}\left(\int \frac{1}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{2bg}$$

$$= \frac{d(bc-ad)i^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g} + \frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg} - \frac{(bc-ad)^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} + \frac{(B(bc-ad)^2i^2n)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g} - \frac{(B(bc-ad)^2i^2n)\,\text{Subst}\left(\int \left(\frac{1}{b^2x}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{2bg} - \frac{(Bd(bc-ad)^2i^2n)\,\text{Subst}\left(\int \frac{1}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g}$$

$$= -\frac{Bd(bc-ad)i^2nx}{2b^2g} + \frac{d(bc-ad)i^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g} + \frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg} - \frac{B(bc-ad)^2i^2n\log\left(\frac{a+bx}{c+dx}\right)}{2b^3g} - \frac{3B(bc-ad)^2i^2n\log(c+dx)}{2b^3g} - \frac{(bc-ad)^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} + \frac{B(bc-ad)^2i^2n\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.91

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$$
$$= \frac{i^2\Big(2Abd(bc-ad)x-B(bc-ad)n(bdx+(bc-ad)\log(a+bx))+2Bd(bc-ad)(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+}{}$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b
*g*x),x]
```

```
[Out] (i^2*(2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*n*(b*d*x + (b*c - a*d)*Log[a +
b*x]) + 2*B*d*(b*c - a*d)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + b^2*(c
 + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*(b*c - a*d)^2*Log[g*(a
 + b*x)]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*B*(b*c - a*d)^2*n*Log[c
 + d*x] + B*(b*c - a*d)^2*n*(-(Log[g*(a + b*x)]*(Log[g*(a + b*x)] - 2*Log[(
b*(c + d*x))/(b*c - a*d)])) + 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))
/(2*b^3*g)
```

Maple [F]

$$\int \frac{(dix+ci)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{bgx+ag}dx$$

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

```
[Out] int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, alg
orithm="fricas")
```

```
[Out] integral((A*d^2*i^2*x^2 + 2*A*c*d*i^2*x + A*c^2*i^2 + (B*d^2*i^2*x^2 + 2*B*
c*d*i^2*x + B*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n))/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = \frac{i^2\left(\int \frac{Ac^2}{a+bx}\,dx+\int \frac{Ad^2x^2}{a+bx}\,dx+\int \frac{Bc^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx+\int \frac{2Acdx}{a+bx}\,dx+\int \frac{Bd^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx+\int \frac{2}{}\right.}{g}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g),x)
```

```
[Out] i**2*(Integral(A*c**2/(a + b*x), x) + Integral(A*d**2*x**2/(a + b*x), x) +
Integral(B*c**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x) + Int
egral(2*A*c*d*x/(a + b*x), x) + Integral(B*d**2*x**2*log(e*(a/(c + d*x) + b
*x/(c + d*x))**n)/(a + b*x), x) + Integral(2*B*c*d*x*log(e*(a/(c + d*x) + b
*x/(c + d*x))**n)/(a + b*x), x))/g
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(280) = 560.

Time = 0.49 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.01

$$\begin{aligned}\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx &= 2\,Acdi^2\left(\frac{x}{bg}-\frac{a\log(bx+a)}{b^2g}\right)+\frac{1}{2}\,Ad^2i^2\left(\frac{2\,a^2\log(bx+a)}{b^3g}+\frac{bx^2-2\,ax}{b^2g}\right)\\ &\quad+\frac{Ac^2i^2\log(bgx+ag)}{bg}-\frac{(3\,bc^2i^2n-2\,acdi^2n)B\log(dx+c)}{2\,b^2g}\\ &\quad+\frac{(b^2c^2i^2n-2\,abcdi^2n+a^2d^2i^2n)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{b^3g}\\ &\quad+\frac{Bb^2d^2i^2x^2\log(e)-(b^2c^2i^2n-2\,abcdi^2n+a^2d^2i^2n)B\log(bx+a)^2-((i^2n-4\,i^2\log(e))b^2cd-(i^2n-}{}\end{aligned}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, alg
orithm="maxima")
```

```
[Out] 2*A*c*d*i^2*(x/(b*g) - a*log(b*x + a)/(b^2*g)) + 1/2*A*d^2*i^2*(2*a^2*log(b
*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A*c^2*i^2*log(b*g*x + a*g)/(b*
g) - 1/2*(3*b*c^2*i^2*n - 2*a*c*d*i^2*n)*B*log(d*x + c)/(b^2*g) + (b^2*c^2*
i^2*n - 2*a*b*c*d*i^2*n + a^2*d^2*i^2*n)*(log(b*x + a)*log((b*d*x + a*d)/(b
*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(b^3*g) + 1/2*(B*b^2*
d^2*i^2*x^2*log(e) - (b^2*c^2*i^2*n - 2*a*b*c*d*i^2*n + a^2*d^2*i^2*n)*B*lo
g(b*x + a)^2 - ((i^2*n - 4*i^2*log(e))*b^2*c*d - (i^2*n - 2*i^2*log(e))*a*b
```

```
*d^2)*B*x + (2*b^2*c^2*i^2*log(e) + 4*(i^2*n - i^2*log(e))*a*b*c*d - (3*i^2
*n - 2*i^2*log(e))*a^2*d^2)*B*log(b*x + a) + (B*b^2*d^2*i^2*x^2 + 2*(2*b^2*
c*d*i^2 - a*b*d^2*i^2)*B*x + 2*(b^2*c^2*i^2 - 2*a*b*c*d*i^2 + a^2*d^2*i^2)*
B*log(b*x + a))*log((b*x + a)^n) - (B*b^2*d^2*i^2*x^2 + 2*(2*b^2*c*d*i^2 -
a*b*d^2*i^2)*B*x + 2*(b^2*c^2*i^2 - 2*a*b*c*d*i^2 + a^2*d^2*i^2)*B*log(b*x
+ a))*log((d*x + c)^n))/(b^3*g)
```

Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, alg
orithm="giac")
```

```
[Out] integrate((d*i*x + c*i)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x + a
*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx=\int \frac{{\left(c\,i+d\,i\,x\right)}^2\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}{a\,g+b\,g\,x}\,dx$$

```
[In] int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x),
x)
```

```
[Out] int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x),
 x)
```

3.122 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 259

$$\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx \\
&= -\frac{B(bc-ad)i^2n(c+dx)}{b^2g^2(a+bx)} + \frac{d^2i^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^2} \\
&\quad - \frac{(bc-ad)i^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g^2(a+bx)} - \frac{Bd(bc-ad)i^2n\log(c+dx)}{b^3g^2} \\
&\quad - \frac{2d(bc-ad)i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} \\
&\quad + \frac{2Bd(bc-ad)i^2n\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}
\end{aligned}$$

```
[Out] -B*(-a*d+b*c)*i^2*n*(d*x+c)/b^2/g^2/(b*x+a)+d^2*i^2*(b*x+a)*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))/b^3/g^2-(-a*d+b*c)*i^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))
^n))/b^2/g^2/(b*x+a)-B*d*(-a*d+b*c)*i^2*n*ln(d*x+c)/b^3/g^2-2*d*(-a*d+b*c)*
i^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g^2+2*B*d
*(-a*d+b*c)*i^2*n*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.186$, Rules used = {2561, 46, 2393, 2341, 2351, 31, 2379, 2438}

$$\begin{aligned}&\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx\\&=\frac{d^2i^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3g^2}\\&-\frac{2di^2(bc-ad)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3g^2}\\&-\frac{i^2(c+dx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2g^2(a+bx)}+\frac{2Bdi^2n(bc-ad)\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}\\&-\frac{Bdi^2n(bc-ad)\log(c+dx)}{b^3g^2}-\frac{Bi^2n(c+dx)(bc-ad)}{b^2g^2(a+bx)}\end{aligned}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^
2,x]
```

```
[Out] -((B*(b*c - a*d)*i^2*n*(c + d*x))/(b^2*g^2*(a + b*x))) + (d^2*i^2*(a + b*x)
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^3*g^2) - ((b*c - a*d)*i^2*(c +
d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^2*g^2*(a + b*x)) - (B*d*(b*
c - a*d)*i^2*n*Log[c + d*x])/(b^3*g^2) - (2*d*(b*c - a*d)*i^2*(A + B*Log[e*
((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^3*g^2) +
 (2*B*d*(b*c - a*d)*i^2*n*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b^3*g^2
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule Rubi steps

$$\text{integral} = \frac{((bc-ad)i^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^2}$$

$$= \frac{((bc-ad)i^2)\,\text{Subst}\left(\int \left(\frac{A+B\log(ex^n)}{b^2x^2} + \frac{d^2(A+B\log(ex^n))}{b^2(b-dx)^2} + \frac{2d(A+B\log(ex^n))}{b^2x(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{g^2}$$

$$= \frac{((bc-ad)i^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2} + \frac{(2d(bc-ad)i^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2} + \frac{(d^2(bc-ad)i^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2}$$

$$= -\frac{B(bc-ad)i^2n(c+dx)}{b^2g^2(a+bx)} + \frac{d^2i^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^2} - \frac{(bc-ad)i^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g^2(a+bx)} - \frac{2d(bc-ad)i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} + \frac{(2Bd(bc-ad)i^2n)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^2} - \frac{(Bd^2(bc-ad)i^2n)\,\text{Subst}\left(\int \frac{1}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^2}$$

$$= -\frac{B(bc-ad)i^2n(c+dx)}{b^2g^2(a+bx)} + \frac{d^2i^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^2} - \frac{(bc-ad)i^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g^2(a+bx)} - \frac{Bd(bc-ad)i^2n\log(c+dx)}{b^3g^2} - \frac{2d(bc-ad)i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} + \frac{2Bd(bc-ad)i^2n\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.90

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$$

$$= \frac{i^2\left(Abd^2x - \frac{B(bc-ad)^2n}{a+bx} + Bd(-bc+ad)n\log(a+bx) + Bd^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - \frac{(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)\right.\right.}{a+bx}\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b
*g*x)^2,x]
```

```
[Out] (i^2*(A*b*d^2*x - (B*(b*c - a*d)^2*n)/(a + b*x) + B*d*(-(b*c) + a*d)*n*Log[
a + b*x] + B*d^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - ((b*c - a*d)^2*
(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + 2*d*(b*c - a*d)*Log[a +
 b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + B*d*(-(b*c) + a*d)*n*(Log[a
+ b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*
(a + b*x))/(-(b*c) + a*d)])))/(b^3*g^2)
```

Maple [F]

$$\int \frac{(dix+ci)^2 \left(A+B\ln \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(bgx+ag)^2}dx$$

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

```
[Out] int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^2 \left(B\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, a
lgorithm="fricas")
```

```
[Out] integral((A*d^2*i^2*x^2 + 2*A*c*d*i^2*x + A*c^2*i^2 + (B*d^2*i^2*x^2 + 2*B*
c*d*i^2*x + B*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n))/(b^2*g^2*x^2 + 2*a*b
*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1190 vs. $2(258) = 516$.

Time = 0.47 (sec) , antiderivative size = 1190, normalized size of antiderivative = 4.59

$$\int \frac{(ci + dix)^2 \left(A + B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag + bgx)^2} dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, a
lgorithm="maxima")
```

```
[Out] -B*c^2*i^2*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2
) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - A*(a^2/(b^4*g^2*x + a*b^3*g^2)
- x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*g^2))*d^2*i^2 + 2*A*c*d*i^2*(a/(b^3*g
^2*x + a*b^2*g^2) + log(b*x + a)/(b^2*g^2)) - B*c^2*i^2*log(e*(b*x/(d*x + c
) + a/(d*x + c))^n)/(b^2*g^2*x + a*b*g^2) - A*c^2*i^2/(b^2*g^2*x + a*b*g^2)
 - (b^2*c^2*d*i^2*n + a*b*c*d^2*i^2*n - a^2*d^3*i^2*n)*B*log(d*x + c)/(b^4*
c*g^2 - a*b^3*d*g^2) + ((b^3*c*d^2*i^2*log(e) - a*b^2*d^3*i^2*log(e))*B*x^2
 + (a*b^2*c*d^2*i^2*log(e) - a^2*b*d^3*i^2*log(e))*B*x - ((b^3*c^2*d*i^2*n
- 2*a*b^2*c*d^2*i^2*n + a^2*b*d^3*i^2*n)*B*x + (a*b^2*c^2*d*i^2*n - 2*a^2*b
*c*d^2*i^2*n + a^3*d^3*i^2*n)*B)*log(b*x + a)^2 + (2*(i^2*n + i^2*log(e))*a
*b^2*c^2*d - 3*(i^2*n + i^2*log(e))*a^2*b*c*d^2 + (i^2*n + i^2*log(e))*a^3*
d^3)*B + ((2*b^3*c^2*d*i^2*log(e) + (3*i^2*n - 4*i^2*log(e))*a*b^2*c*d^2 -
2*(i^2*n - i^2*log(e))*a^2*b*d^3)*B*x + (2*a*b^2*c^2*d*i^2*log(e) + (3*i^2*
n - 4*i^2*log(e))*a^2*b*c*d^2 - 2*(i^2*n - i^2*log(e))*a^3*d^3)*B)*log(b*x
+ a) + ((b^3*c*d^2*i^2 - a*b^2*d^3*i^2)*B*x^2 + (a*b^2*c*d^2*i^2 - a^2*b*d^
3*i^2)*B*x + (2*a*b^2*c^2*d*i^2 - 3*a^2*b*c*d^2*i^2 + a^3*d^3*i^2)*B + 2*((
b^3*c^2*d*i^2 - 2*a*b^2*c*d^2*i^2 + a^2*b*d^3*i^2)*B*x + (a*b^2*c^2*d*i^2 -
 2*a^2*b*c*d^2*i^2 + a^3*d^3*i^2)*B)*log(b*x + a))*log((b*x + a)^n) - ((b^3
*c*d^2*i^2 - a*b^2*d^3*i^2)*B*x^2 + (a*b^2*c*d^2*i^2 - a^2*b*d^3*i^2)*B*x +
 (2*a*b^2*c^2*d*i^2 - 3*a^2*b*c*d^2*i^2 + a^3*d^3*i^2)*B + 2*((b^3*c^2*d*i^
2 - 2*a*b^2*c*d^2*i^2 + a^2*b*d^3*i^2)*B*x + (a*b^2*c^2*d*i^2 - 2*a^2*b*c*d
^2*i^2 + a^3*d^3*i^2)*B)*log(b*x + a))*log((d*x + c)^n))/(a*b^4*c*g^2 - a^2
*b^3*d*g^2 + (b^5*c*g^2 - a*b^4*d*g^2)*x) + 2*(b*c*d*i^2*n - a*d^2*i^2*n)*(
log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c
 - a*d)))*B/(b^3*g^2)
```

Giac [F]

$$\int \frac{(ci + dix)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag + bgx)^2}\,dx = \int \frac{(dix + ci)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)}{(bgx + ag)^2}\,dx$$

[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, a lgorithm="giac")

[Out] integrate((d*i*x + c*i)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x + a *g)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci + dix)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag + bgx)^2}\,dx = \int \frac{(c\,i + d\,i\,x)^2 \left(A + B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(a\,g + b\,g\,x)^2}\,dx$$

[In] int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^ 2,x)

[Out] int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^ 2, x)

3.123 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 242

$$\begin{aligned}\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = &-\frac{Bdi^2n(c+dx)}{b^2g^3(a+bx)}-\frac{Bi^2n(c+dx)^2}{4bg^3(a+bx)^2} \\ &-\frac{di^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g^3(a+bx)} \\ &-\frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg^3(a+bx)^2} \\ &-\frac{d^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} \\ &+\frac{Bd^2i^2n\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3}\end{aligned}$$

```
[Out] -B*d*i^2*n*(d*x+c)/b^2/g^3/(b*x+a)-1/4*B*i^2*n*(d*x+c)^2/b/g^3/(b*x+a)^2-d*
i^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g^3/(b*x+a)-1/2*i^2*(d*x+c)
^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g^3/(b*x+a)^2-d^2*i^2*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g^3+B*d^2*i^2*n*polylog(2,b*(
d*x+c)/d/(b*x+a))/b^3/g^3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2561, 2380, 2341, 2379, 2438}

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = -\frac{d^2i^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3g^3} - \frac{di^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2g^3(a+bx)} - \frac{i^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2bg^3(a+bx)^2} + \frac{Bd^2i^2n\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} - \frac{Bdi^2n(c+dx)}{b^2g^3(a+bx)} - \frac{Bi^2n(c+dx)^2}{4bg^3(a+bx)^2}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^
3,x]
```

```
[Out] -((B*d*i^2*n*(c + d*x))/(b^2*g^3*(a + b*x))) - (B*i^2*n*(c + d*x)^2)/(4*b*g
^3*(a + b*x)^2) - (d*i^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/
(b^2*g^3*(a + b*x)) - (i^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]))/(2*b*g^3*(a + b*x)^2) - (d^2*i^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^3*g^3) + (B*d^2*i^2*n*PolyLog[2,
(b*(c + d*x))/(d*(a + b*x))])/(b^3*g^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
```

```
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{i^2\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^3}$$

$$= \frac{i^2\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^3} + \frac{(di^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^3}$$

$$= -\frac{Bi^2n(c+dx)^2}{4bg^3(a+bx)^2} - \frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg^3(a+bx)^2} + \frac{(di^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^3} + \frac{(d^2i^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^3}$$

$$= -\frac{Bdi^2n(c+dx)}{b^2g^3(a+bx)} - \frac{Bi^2n(c+dx)^2}{4bg^3(a+bx)^2} - \frac{di^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g^3(a+bx)} - \frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg^3(a+bx)^2} - \frac{d^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} + \frac{(Bd^2i^2n)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^3}$$

$$= -\frac{Bdi^2n(c+dx)}{b^2g^3(a+bx)} - \frac{Bi^2n(c+dx)^2}{4bg^3(a+bx)^2} - \frac{di^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g^3(a+bx)} - \frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg^3(a+bx)^2} - \frac{d^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} + \frac{Bd^2i^2n\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.07

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \frac{i^2\left(-\frac{B(bc-ad)^2n}{(a+bx)^2} + \frac{6Bd(-bc+ad)n}{a+bx} - 6Bd^2n\log(a+bx) - \frac{2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2} + \frac{8d(-bc+ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx}\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b
*g*x)^3,x]
```

```
[Out] (i^2*(-((B*(b*c - a*d)^2*n)/(a + b*x)^2) + (6*B*d*(-(b*c) + a*d)*n)/(a + b*
x) - 6*B*d^2*n*Log[a + b*x] - (2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n]))/(a + b*x)^2 + (8*d*(-(b*c) + a*d)*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(a + b*x) + 4*d^2*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]) + 6*B*d^2*n*Log[c + d*x] - 2*B*d^2*n*(Log[a + b*x]*(Log[a + b*x] - 2*
Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)
])))/(4*b^3*g^3)
```

Maple [F]

$$\int \frac{(dix+ci)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(bgx+ag)^3}dx$$

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x)
```

```
[Out] int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^3}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, a
lgorithm="fricas")
```

```
[Out] integral((A*d^2*i^2*x^2 + 2*A*c*d*i^2*x + A*c^2*i^2 + (B*d^2*i^2*x^2 + 2*B*
c*d*i^2*x + B*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n))/(b^3*g^3*x^3 + 3*a*b
^2*g^3*x^2 + 3*a^2*b*g^3*x + a^3*g^3), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$$
$$= \frac{i^2\left(\int \frac{Ac^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{Ad^2x^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{Bc^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{2Acdx}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\right.}{g^3}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3,x)
```

```
[Out] i**2*(Integral(A*c**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) +
 Integral(A*d**2*x**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) +
 Integral(B*c**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**3 + 3*a**2*b*x
 + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*c*d*x/(a**3 + 3*a**2*b*x +
 3*a*b**2*x**2 + b**3*x**3), x) + Integral(B*d**2*x**2*log(e*(a/(c + d*x) +
 b*x/(c + d*x))**n)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + I
ntegral(2*B*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**3 + 3*a**2*b*
x + 3*a*b**2*x**2 + b**3*x**3), x))/g**3
```

Maxima [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^3}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, a
lgorithm="maxima")
```

```
[Out] -1/2*B*c*d*i^2*n*((3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*x)/((b^5*c - a*b^4
*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3)
```

```
+ 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g
^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^
2)*g^3)) + 1/4*B*c^2*i^2*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*
x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*lo
g(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/(
(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) + 1/2*A*d^2*i^2*((4*a*b*x + 3*a^2
)/(b^5*g^3*x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(b^3*g^3)) +
 1/2*B*d^2*i^2*(((4*a*b*x + 3*a^2 + 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a
))*log((b*x + a)^n) - (4*a*b*x + 3*a^2 + 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*
x + a))*log((d*x + c)^n))/(b^5*g^3*x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) + 2*i
ntegrate(1/2*(2*b^3*d*x^3*log(e) + 2*b^3*c*x^2*log(e) - 3*a^2*b*c*n + 3*a^3
*d*n - 4*(a*b^2*c*n - a^2*b*d*n)*x - 2*(a^2*b*c*n - a^3*d*n + (b^3*c*n - a*
b^2*d*n)*x^2 + 2*(a*b^2*c*n - a^2*b*d*n)*x)*log(b*x + a))/(b^6*d*g^3*x^4 +
a^3*b^3*c*g^3 + (b^6*c*g^3 + 3*a*b^5*d*g^3)*x^3 + 3*(a*b^5*c*g^3 + a^2*b^4*
d*g^3)*x^2 + (3*a^2*b^4*c*g^3 + a^3*b^3*d*g^3)*x), x)) - (2*b*x + a)*B*c*d*
i^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a
^2*b^2*g^3) - (2*b*x + a)*A*c*d*i^2/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*
g^3) - 1/2*B*c^2*i^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^3*g^3*x^2 +
2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A*c^2*i^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a
^2*b*g^3)
```

Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^3}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, a
lgorithm="giac")
```

```
[Out] integrate((d*i*x + c*i)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x + a
*g)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx=\int \frac{(c\,i+d\,i\,x)^2\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}{(a\,g+b\,g\,x)^3}\,dx$$

```
[In] int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^
3,x)
```

```
[Out] int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^
3, x)
```

3.124 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 93

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = -\frac{Bi^2n(c+dx)^3}{9(bc-ad)g^4(a+bx)^3} - \frac{i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)g^4(a+bx)^3}$$

```
[Out] -1/9*B*i^2*n*(d*x+c)^3/(-a*d+b*c)/g^4/(b*x+a)^3-1/3*i^2*(d*x+c)^3*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/g^4/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {2561, 2341}

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = -\frac{i^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3g^4(a+bx)^3(bc-ad)} - \frac{Bi^2n(c+dx)^3}{9g^4(a+bx)^3(bc-ad)}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^
4,x]
```

```
[Out] -1/9*(B*i^2*n*(c + d*x)^3)/((b*c - a*d)*g^4*(a + b*x)^3) - (i^2*(c + d*x)^3
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(b*c - a*d)*g^4*(a + b*x)^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{i^2\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^4}$$

$$= -\frac{Bi^2n(c+dx)^3}{9(bc-ad)g^4(a+bx)^3} - \frac{i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)g^4(a+bx)^3}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 329 vs. 2(93) = 186.

Time = 0.19 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.54

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx =$$
$$-\frac{i^2\left(3Ab^3c^3-3a^3Ad^3+b^3Bc^3n-a^3Bd^3n+9Ab^3c^2dx-9a^2Abd^3x+3b^3Bc^2dnx-3a^2bBd^3nx+9Ab^3c\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b
*g*x)^4,x]
```

```
[Out] -1/9*(i^2*(3*A*b^3*c^3 - 3*a^3*A*d^3 + b^3*B*c^3*n - a^3*B*d^3*n + 9*A*b^3*
c^2*d*x - 9*a^2*A*b*d^3*x + 3*b^3*B*c^2*d*n*x - 3*a^2*b*B*d^3*n*x + 9*A*b^3
*c*d^2*x^2 - 9*a*A*b^2*d^3*x^2 + 3*b^3*B*c*d^2*n*x^2 - 3*a*b^2*B*d^3*n*x^2
+ 3*B*d^3*n*(a + b*x)^3*Log[a + b*x] + 3*B*(b*c - a*d)*(a^2*d^2 + a*b*d*(c
+ 3*d*x) + b^2*(c^2 + 3*c*d*x + 3*d^2*x^2))*Log[e*((a + b*x)/(c + d*x))^n]
- 3*a^3*B*d^3*n*Log[c + d*x] - 9*a^2*b*B*d^3*n*x*Log[c + d*x] - 9*a*b^2*B*d
^3*n*x^2*Log[c + d*x] - 3*b^3*B*d^3*n*x^3*Log[c + d*x]))/(b^3*(b*c - a*d)*g
^4*(a + b*x)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(89) = 178.

Time = 10.00 (sec) , antiderivative size = 376, normalized size of antiderivative = 4.04

method	result
parallelrisch	$-\frac{B a^3 b^2 d^4 i^2 n^2 - B b^5 c^3 d i^2 n^2 + 3 A a^3 b^2 d^4 i^2 n - 3 A b^5 c^3 d i^2 n - 3 B x^3 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) b^5 d^4 i^2 n + 3 B x^2 a b^4 d^4 i^2 n^2 - 3 B x^2 b^5 c d^3 i^2 n}{}$

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x,method=_R
ETURNVERBOSE)
```

```
[Out] -1/9*(B*a^3*b^2*d^4*i^2*n^2-B*b^5*c^3*d*i^2*n^2+3*A*a^3*b^2*d^4*i^2*n-3*A*b
^5*c^3*d*i^2*n-3*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^5*d^4*i^2*n+3*B*x^2*a*b^
4*d^4*i^2*n^2-3*B*x^2*b^5*c*d^3*i^2*n^2+9*A*x^2*a*b^4*d^4*i^2*n-9*A*x^2*b^5
*c*d^3*i^2*n+3*B*x*a^2*b^3*d^4*i^2*n^2-3*B*x*b^5*c^2*d^2*i^2*n^2+9*A*x*a^2*
b^3*d^4*i^2*n-9*A*x*b^5*c^2*d^2*i^2*n-3*B*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^3
*d*i^2*n-9*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c*d^3*i^2*n-9*B*x*ln(e*((b*x
+a)/(d*x+c))^n)*b^5*c^2*d^2*i^2*n)/g^4/(b*x+a)^3/b^5/d/n/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(89) = 178.

Time = 0.33 (sec) , antiderivative size = 409, normalized size of antiderivative = 4.40

$$\int \frac{(ci+dix)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = -\frac{(Bb^3c^3 - Ba^3d^3)i^2n + 3\,(Ab^3c^3 - Aa^3d^3)i^2 + 3\,((Bb^3cd^2 - Bab^2d^3)i^2n + 3\,(Ab^3cd^2 - Aab^2d^3)i^2)x^2 + 3\,(}{}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, a
lgorithm="fricas")
```

```
[Out] -1/9*((B*b^3*c^3 - B*a^3*d^3)*i^2*n + 3*(A*b^3*c^3 - A*a^3*d^3)*i^2 + 3*((B
*b^3*c*d^2 - B*a*b^2*d^3)*i^2*n + 3*(A*b^3*c*d^2 - A*a*b^2*d^3)*i^2)*x^2 +
3*((B*b^3*c^2*d - B*a^2*b*d^3)*i^2*n + 3*(A*b^3*c^2*d - A*a^2*b*d^3)*i^2)*x
 + 3*(3*(B*b^3*c*d^2 - B*a*b^2*d^3)*i^2*x^2 + 3*(B*b^3*c^2*d - B*a^2*b*d^3)
*i^2*x + (B*b^3*c^3 - B*a^3*d^3)*i^2)*log(e) + 3*(B*b^3*d^3*i^2*n*x^3 + 3*B
*b^3*c*d^2*i^2*n*x^2 + 3*B*b^3*c^2*d*i^2*n*x + B*b^3*c^3*i^2*n)*log((b*x +
a)/(d*x + c)))/((b^7*c - a*b^6*d)*g^4*x^3 + 3*(a*b^6*c - a^2*b^5*d)*g^4*x^2
 + 3*(a^2*b^5*c - a^3*b^4*d)*g^4*x + (a^3*b^4*c - a^4*b^3*d)*g^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx=\text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1544 vs. $2(89) = 178$.

Time = 0.25 (sec) , antiderivative size = 1544, normalized size of antiderivative = 16.60

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx=\text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, a
lgorithm="maxima")
[Out] -1/18*B*d^2*i^2*n*((11*a^2*b^2*c^2 - 7*a^3*b*c*d + 2*a^4*d^2 + 6*(3*b^4*c^2
 - 3*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 3*(9*a*b^3*c^2 - 7*a^2*b^2*c*d + 2*a^3*
b*d^2)*x)/((b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*g^4*x^3 + 3*(a*b^7*c^2 - 2
*a^2*b^6*c*d + a^3*b^5*d^2)*g^4*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d + a^4*
b^4*d^2)*g^4*x + (a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2)*g^4) + 6*(3*b^
2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^2*d + 3
*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4) - 6*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3
)*log(d*x + c)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g
^4)) - 1/18*B*c^2*i^2*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^
2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x
^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 -
 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*
d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2
- a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^
2*c*d^2 - a^3*b*d^3)*g^4)) - 1/18*B*c*d*i^2*n*((5*a*b^2*c^2 - 22*a^2*b*c*d
+ 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c^2 - 16*a*b^2*c*d +
 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3 + 3*(a*b^6*
c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*c*d
 + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4) -
6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*
d^2 - a^3*b^2*d^3)*g^4) + 6*(3*b*c*d^2 - a*d^3)*log(d*x + c)/((b^5*c^3 - 3*
a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4)) - 1/3*(3*b*x + a)*B*c*d*
i^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^5*g^4*x^3 + 3*a*b^4*g^4*x^2 +
```

```
 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - 1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*B*d^2*i^2
*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^6*g^4*x^3 + 3*a*b^5*g^4*x^2 + 3*
a^2*b^4*g^4*x + a^3*b^3*g^4) - 1/3*(3*b*x + a)*A*c*d*i^2/(b^5*g^4*x^3 + 3*a
*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - 1/3*(3*b^2*x^2 + 3*a*b*x +
a^2)*A*d^2*i^2/(b^6*g^4*x^3 + 3*a*b^5*g^4*x^2 + 3*a^2*b^4*g^4*x + a^3*b^3*g
^4) - 1/3*B*c^2*i^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^4*g^4*x^3 + 3
*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) - 1/3*A*c^2*i^2/(b^4*g^4*x^3
+ 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

Giac [A] (verification not implemented)

none

Time = 1.55 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.16

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = -\frac{1}{9}\left(\frac{3\,(dx+c)^3Bi^2n\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)^3g^4}+\frac{(Bi^2n+3\,Bi^2\log(e)+3\,Ai^2)(dx+c)^3}{(bx+a)^3g^4}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, a
lgorithm="giac")
```

```
[Out] -1/9*(3*(d*x + c)^3*B*i^2*n*log((b*x + a)/(d*x + c))/((b*x + a)^3*g^4) + (B
*i^2*n + 3*B*i^2*log(e) + 3*A*i^2)*(d*x + c)^3/((b*x + a)^3*g^4))*(b*c/(b*c
 - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 421, normalized size of antiderivative = 4.53

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = -\frac{x\,(3\,A\,a\,b\,d^2\,i^2+3\,A\,b^2\,c\,d\,i^2+B\,a\,b\,d^2\,i^2\,n+B\,b^2\,c\,d\,i^2\,n)+x^2\,(3\,A\,b^2\,d^2\,i^2+B\,b^2\,d^2\,i^2\,n)+A\,a^2\,d^2\,i^2+}{3\,a^3\,b^3\,g^4+9\,a^2\,b^4\,g^4\,x+9\,a\,b^5\,g^4\,x^2+3\,b^6\,g^4\,x^3} - \frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(a\left(\frac{B\,a\,d^2\,i^2}{3\,b^3}+\frac{B\,c\,d\,i^2}{3\,b^2}\right)+x\left(b\left(\frac{B\,a\,d^2\,i^2}{3\,b^3}+\frac{B\,c\,d\,i^2}{3\,b^2}\right)+\frac{2\,B\,a\,d^2\,i^2}{3\,b^2}+\frac{2\,B\,c\,d\,i^2}{3\,b}\right)+\frac{B\,c^2\,i^2}{3\,b}+\frac{B\,d^2\,i^2\,x^2}{b}\right)}{a^3\,g^4+3\,a^2\,b\,g^4\,x+3\,a\,b^2\,g^4\,x^2+b^3\,g^4\,x^3} - \frac{B\,d^3\,i^2\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{3\,b^3\,g^4\,(a\,d-b\,c)}$$

```
[In] int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^
4,x)
```

```
[Out] - (x*(3*A*a*b*d^2*i^2 + 3*A*b^2*c*d*i^2 + B*a*b*d^2*i^2*n + B*b^2*c*d*i^2*n
) + x^2*(3*A*b^2*d^2*i^2 + B*b^2*d^2*i^2*n) + A*a^2*d^2*i^2 + A*b^2*c^2*i^2
 + (B*a^2*d^2*i^2*n)/3 + (B*b^2*c^2*i^2*n)/3 + A*a*b*c*d*i^2 + (B*a*b*c*d*i
^2*n)/3)/(3*a^3*b^3*g^4 + 3*b^6*g^4*x^3 + 9*a^2*b^4*g^4*x + 9*a*b^5*g^4*x^2
) - (log(e*((a + b*x)/(c + d*x))^n)*(a*((B*a*d^2*i^2)/(3*b^3) + (B*c*d*i^2)
/(3*b^2)) + x*(b*((B*a*d^2*i^2)/(3*b^3) + (B*c*d*i^2)/(3*b^2)) + (2*B*a*d^2
*i^2)/(3*b^2) + (2*B*c*d*i^2)/(3*b)) + (B*c^2*i^2)/(3*b) + (B*d^2*i^2*x^2)/
b))/(a^3*g^4 + b^3*g^4*x^3 + 3*a*b^2*g^4*x^2 + 3*a^2*b*g^4*x) - (B*d^3*i^2*
n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(3*b^3*g^4*(a*d - b*c))
```

3.125 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 189

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx = \frac{Bdi^2n(c+dx)^3}{9(bc-ad)^2g^5(a+bx)^3} - \frac{bBi^2n(c+dx)^4}{16(bc-ad)^2g^5(a+bx)^4} + \frac{di^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^2g^5(a+bx)^3} - \frac{bi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bc-ad)^2g^5(a+bx)^4}$$

```
[Out] 1/9*B*d*i^2*n*(d*x+c)^3/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/16*b*B*i^2*n*(d*x+c)^4
/(-a*d+b*c)^2/g^5/(b*x+a)^4+1/3*d*i^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))
^n))/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/4*b*i^2*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x
+c))^n))/(-a*d+b*c)^2/g^5/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used

$= \{2561, 45, 2372, 12\}$

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx = -\frac{bi^2(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4g^5(a+bx)^4(bc-ad)^2} + \frac{di^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3g^5(a+bx)^3(bc-ad)^2} - \frac{bBi^2n(c+dx)^4}{16g^5(a+bx)^4(bc-ad)^2} + \frac{Bdi^2n(c+dx)^3}{9g^5(a+bx)^3(bc-ad)^2}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^
5,x]
```

```
[Out] (B*d*i^2*n*(c + d*x)^3)/(9*(b*c - a*d)^2*g^5*(a + b*x)^3) - (b*B*i^2*n*(c +
 d*x)^4)/(16*(b*c - a*d)^2*g^5*(a + b*x)^4) + (d*i^2*(c + d*x)^3*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]))/(3*(b*c - a*d)^2*g^5*(a + b*x)^3) - (b*i^2*(c
 + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*(b*c - a*d)^2*g^5*(a +
 b*x)^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
```

```
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{i^2 \text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex^n))}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 g^5}$$

$$= \frac{di^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^2 g^5 (a+bx)^3} - \frac{bi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bc-ad)^2 g^5 (a+bx)^4} - \frac{(Bi^2 n)\,\text{Subst}\left(\int \frac{-3b+4dx}{12x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 g^5}$$

$$= \frac{di^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^2 g^5 (a+bx)^3} - \frac{bi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bc-ad)^2 g^5 (a+bx)^4} - \frac{(Bi^2 n)\,\text{Subst}\left(\int \frac{-3b+4dx}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{12(bc-ad)^2 g^5}$$

$$= \frac{di^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^2 g^5 (a+bx)^3} - \frac{bi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bc-ad)^2 g^5 (a+bx)^4} - \frac{(Bi^2 n)\,\text{Subst}\left(\int \left(-\frac{3b}{x^5}+\frac{4d}{x^4}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{12(bc-ad)^2 g^5}$$

$$= \frac{Bdi^2 n(c+dx)^3}{9(bc-ad)^2 g^5 (a+bx)^3} - \frac{bBi^2 n(c+dx)^4}{16(bc-ad)^2 g^5 (a+bx)^4} + \frac{di^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^2 g^5 (a+bx)^3} - \frac{bi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bc-ad)^2 g^5 (a+bx)^4}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 474 vs. 2(189) = 378.

Time = 0.26 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.51

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx$$
$$= -\frac{(bc-ad)^2 i^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b^3g^5(a+bx)^4} - \frac{2d(bc-ad)i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^3g^5(a+bx)^3} - \frac{d^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^3g^5(a+bx)^2} - \frac{Bd^2i^2n\left(\frac{1}{(a+bx)^2}-\frac{2d}{(bc-ad)(a+bx)}-\frac{2d^2\log(a+bx)}{(bc-ad)^2}+\frac{2d^2\log(c+dx)}{(bc-ad)^2}\right)}{4b^3g^5} - \frac{Bdi^2n\left(\frac{2(bc-ad)}{(a+bx)^3}-\frac{3d}{(a+bx)^2}+\frac{6d^2}{(bc-ad)(a+bx)}+\frac{6d^3\log(a+bx)}{(bc-ad)^2}-\frac{6d^3\log(c+dx)}{(bc-ad)^2}\right)}{9b^3g^5} - \frac{Bi^2n\left(\frac{3(bc-ad)^2}{(a+bx)^4}-\frac{4d(bc-ad)}{(a+bx)^3}+\frac{6d^2}{(a+bx)^2}-\frac{12d^3}{(bc-ad)(a+bx)}-\frac{12d^4\log(a+bx)}{(bc-ad)^2}+\frac{12d^4\log(c+dx)}{(bc-ad)^2}\right)}{48b^3g^5}$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b
*g*x)^5,x]
```

```
[Out] -1/4*((b*c - a*d)^2*i^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^3*g^5*(a
 + b*x)^4) - (2*d*(b*c - a*d)*i^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(
3*b^3*g^5*(a + b*x)^3) - (d^2*i^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(
2*b^3*g^5*(a + b*x)^2) - (B*d^2*i^2*n*((a + b*x)^(-2) - (2*d)/((b*c - a*d)*
(a + b*x)) - (2*d^2*Log[a + b*x])/(b*c - a*d)^2 + (2*d^2*Log[c + d*x])/(b*c
 - a*d)^2))/(4*b^3*g^5) - (B*d*i^2*n*((2*(b*c - a*d))/(a + b*x)^3 - (3*d)/(
a + b*x)^2 + (6*d^2)/((b*c - a*d)*(a + b*x)) + (6*d^3*Log[a + b*x])/(b*c -
a*d)^2 - (6*d^3*Log[c + d*x])/(b*c - a*d)^2))/(9*b^3*g^5) - (B*i^2*n*((3*(b
*c - a*d)^2)/(a + b*x)^4 - (4*d*(b*c - a*d))/(a + b*x)^3 + (6*d^2)/(a + b*x
)^2 - (12*d^3)/((b*c - a*d)*(a + b*x)) - (12*d^4*Log[a + b*x])/(b*c - a*d)^
2 + (12*d^4*Log[c + d*x])/(b*c - a*d)^2))/(48*b^3*g^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 854 vs. 2(181) = 362.

Time = 27.07 (sec) , antiderivative size = 855, normalized size of antiderivative = 4.52

method	result
parallelrisch	$\frac{9B\,x^4a^2b^5c^5i^2n^2+36A\,x^4a^2b^5c^5i^2n+16B\,x^3a^7c\,d^4i^2n^2+36B\,x^3a^3b^4c^5i^2n^2+48A\,x^3a^7c\,d^4i^2n+144A\,x^3a^3b^4c^5i^2n+48B\,x^2a^7c^2}{}$

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/144*(9*B*x^4*a^2*b^5*c^5*i^2*n^2+36*A*x^4*a^2*b^5*c^5*i^2*n+16*B*x^3*a^7*
c*d^4*i^2*n^2+36*B*x^3*a^3*b^4*c^5*i^2*n^2+48*A*x^3*a^7*c*d^4*i^2*n+144*A*x
^3*a^3*b^4*c^5*i^2*n+48*B*x^2*a^7*c^2*d^3*i^2*n^2+54*B*x^2*a^4*b^3*c^5*i^2*
n^2+144*A*x^2*a^7*c^2*d^3*i^2*n+216*A*x^2*a^4*b^3*c^5*i^2*n+48*B*x*a^7*c^3*
d^2*i^2*n^2+36*B*x*a^5*b^2*c^5*i^2*n^2+144*A*x*a^7*c^3*d^2*i^2*n+144*A*x*a^
5*b^2*c^5*i^2*n+48*B*ln(e*((b*x+a)/(d*x+c))^n)*a^7*c^4*d*i^2*n-36*B*ln(e*((
b*x+a)/(d*x+c))^n)*a^6*b*c^5*i^2*n+48*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^7*c
*d^4*i^2*n+7*B*x^4*a^6*b*c*d^4*i^2*n^2-16*B*x^4*a^3*b^4*c^4*d*i^2*n^2+12*A*
x^4*a^6*b*c*d^4*i^2*n-48*A*x^4*a^3*b^4*c^4*d*i^2*n+144*B*x^2*ln(e*((b*x+a)/
(d*x+c))^n)*a^7*c^2*d^3*i^2*n-6*B*x^2*a^6*b*c^3*d^2*i^2*n^2-96*B*x^2*a^5*b^
2*c^4*d*i^2*n^2-72*A*x^2*a^6*b*c^3*d^2*i^2*n-288*A*x^2*a^5*b^2*c^4*d*i^2*n+
144*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^7*c^3*d^2*i^2*n-84*B*x*a^6*b*c^4*d*i^2*
n^2-288*A*x*a^6*b*c^4*d*i^2*n+12*B*x^3*a^6*b*c^2*d^3*i^2*n^2-64*B*x^3*a^4*b
^3*c^4*d*i^2*n^2-192*A*x^3*a^4*b^3*c^4*d*i^2*n+12*B*x^4*ln(e*((b*x+a)/(d*x+
c))^n)*a^6*b*c*d^4*i^2*n-72*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b*c^3*d^2*i
^2*n-96*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b*c^4*d*i^2*n)/g^5/(b*x+a)^4/n/(a
^2*d^2-2*a*b*c*d+b^2*c^2)/a^6/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. 2(181) = 362.

Time = 0.37 (sec) , antiderivative size = 710, normalized size of antiderivative = 3.76

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx = \frac{12\,(Bb^4cd^3-Bab^3d^4)i^2nx^3-(9\,Bb^4c^4-16\,Bab^3c^3d+7\,Ba^4d^4)i^2n-12\,(3\,Ab^4c^4-4\,Aab^3c^3d+Aa^4d^4)i^2}{}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, a
lgorithm="fricas")
```

```
[Out] 1/144*(12*(B*b^4*c*d^3 - B*a*b^3*d^4)*i^2*n*x^3 - (9*B*b^4*c^4 - 16*B*a*b^3
*c^3*d + 7*B*a^4*d^4)*i^2*n - 12*(3*A*b^4*c^4 - 4*A*a*b^3*c^3*d + A*a^4*d^4
```

```
)*i^2 - 6*((B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4)*i^2*n + 12*(
A*b^4*c^2*d^2 - 2*A*a*b^3*c*d^3 + A*a^2*b^2*d^4)*i^2)*x^2 - 4*((5*B*b^4*c^3
*d - 12*B*a*b^3*c^2*d^2 + 7*B*a^3*b*d^4)*i^2*n + 12*(2*A*b^4*c^3*d - 3*A*a*
b^3*c^2*d^2 + A*a^3*b*d^4)*i^2)*x - 12*(6*(B*b^4*c^2*d^2 - 2*B*a*b^3*c*d^3
+ B*a^2*b^2*d^4)*i^2*x^2 + 4*(2*B*b^4*c^3*d - 3*B*a*b^3*c^2*d^2 + B*a^3*b*d
^4)*i^2*x + (3*B*b^4*c^4 - 4*B*a*b^3*c^3*d + B*a^4*d^4)*i^2)*log(e) + 12*(B
*b^4*d^4*i^2*n*x^4 + 4*B*a*b^3*d^4*i^2*n*x^3 - 6*(B*b^4*c^2*d^2 - 2*B*a*b^3
*c*d^3)*i^2*n*x^2 - 4*(2*B*b^4*c^3*d - 3*B*a*b^3*c^2*d^2)*i^2*n*x - (3*B*b^
4*c^4 - 4*B*a*b^3*c^3*d)*i^2*n)*log((b*x + a)/(d*x + c)))/((b^9*c^2 - 2*a*b
^8*c*d + a^2*b^7*d^2)*g^5*x^4 + 4*(a*b^8*c^2 - 2*a^2*b^7*c*d + a^3*b^6*d^2)
*g^5*x^3 + 6*(a^2*b^7*c^2 - 2*a^3*b^6*c*d + a^4*b^5*d^2)*g^5*x^2 + 4*(a^3*b
^6*c^2 - 2*a^4*b^5*c*d + a^5*b^4*d^2)*g^5*x + (a^4*b^5*c^2 - 2*a^5*b^4*c*d
+ a^6*b^3*d^2)*g^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci + dix)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag + bgx)^5}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**5,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2247 vs. $2(181) = 362$.

Time = 0.30 (sec) , antiderivative size = 2247, normalized size of antiderivative = 11.89

$$\int \frac{(ci + dix)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag + bgx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, a
lgorithm="maxima")
[Out] 1/48*B*c^2*i^2*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c
*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^
2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^
3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4
*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^
5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a
^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*
b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2
*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 -
```

```
4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/
144*B*d^2*i^2*n*((13*a^2*b^3*c^3 - 75*a^3*b^2*c^2*d + 33*a^4*b*c*d^2 - 7*a^
5*d^3 - 12*(6*b^5*c^2*d - 4*a*b^4*c*d^2 + a^2*b^3*d^3)*x^3 + 6*(6*b^5*c^3 -
 46*a*b^4*c^2*d + 29*a^2*b^3*c*d^2 - 7*a^3*b^2*d^3)*x^2 + 4*(10*a*b^4*c^3 -
 63*a^2*b^3*c^2*d + 33*a^3*b^2*c*d^2 - 7*a^4*b*d^3)*x)/((b^10*c^3 - 3*a*b^9
*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7*d^3)*g^5*x^4 + 4*(a*b^9*c^3 - 3*a^2*b^8*
c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*d^3)*g^5*x^3 + 6*(a^2*b^8*c^3 - 3*a^3*b^7
*c^2*d + 3*a^4*b^6*c*d^2 - a^5*b^5*d^3)*g^5*x^2 + 4*(a^3*b^7*c^3 - 3*a^4*b^
6*c^2*d + 3*a^5*b^5*c*d^2 - a^6*b^4*d^3)*g^5*x + (a^4*b^6*c^3 - 3*a^5*b^5*c
^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d^3)*g^5) - 12*(6*b^2*c^2*d^2 - 4*a*b*c*d^
3 + a^2*d^4)*log(b*x + a)/((b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4
*a^3*b^4*c*d^3 + a^4*b^3*d^4)*g^5) + 12*(6*b^2*c^2*d^2 - 4*a*b*c*d^3 + a^2*
d^4)*log(d*x + c)/((b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4
*c*d^3 + a^4*b^3*d^4)*g^5)) - 1/72*B*c*d*i^2*n*((7*a*b^3*c^3 - 33*a^2*b^2*c
^2*d + 75*a^3*b*c*d^2 - 13*a^4*d^3 + 12*(4*b^4*c*d^2 - a*b^3*d^3)*x^3 - 6*(
4*b^4*c^2*d - 29*a*b^3*c*d^2 + 7*a^2*b^2*d^3)*x^2 + 4*(4*b^4*c^3 - 21*a*b^3
*c^2*d + 57*a^2*b^2*c*d^2 - 13*a^3*b*d^3)*x)/((b^9*c^3 - 3*a*b^8*c^2*d + 3*
a^2*b^7*c*d^2 - a^3*b^6*d^3)*g^5*x^4 + 4*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*a
^3*b^6*c*d^2 - a^4*b^5*d^3)*g^5*x^3 + 6*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*
a^4*b^5*c*d^2 - a^5*b^4*d^3)*g^5*x^2 + 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3
*a^5*b^4*c*d^2 - a^6*b^3*d^3)*g^5*x + (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^
6*b^3*c*d^2 - a^7*b^2*d^3)*g^5) + 12*(4*b*c*d^3 - a*d^4)*log(b*x + a)/((b^6
*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g
^5) - 12*(4*b*c*d^3 - a*d^4)*log(d*x + c)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2
*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5)) - 1/6*(4*b*x + a)*B*c*d
*i^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^6*g^5*x^4 + 4*a*b^5*g^5*x^3
+ 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g^5) - 1/12*(6*b^2*x^2 + 4*
a*b*x + a^2)*B*d^2*i^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^7*g^5*x^4
+ 4*a*b^6*g^5*x^3 + 6*a^2*b^5*g^5*x^2 + 4*a^3*b^4*g^5*x + a^4*b^3*g^5) - 1/
6*(4*b*x + a)*A*c*d*i^2/(b^6*g^5*x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2
+ 4*a^3*b^3*g^5*x + a^4*b^2*g^5) - 1/12*(6*b^2*x^2 + 4*a*b*x + a^2)*A*d^2*i
^2/(b^7*g^5*x^4 + 4*a*b^6*g^5*x^3 + 6*a^2*b^5*g^5*x^2 + 4*a^3*b^4*g^5*x + a
^4*b^3*g^5) - 1/4*B*c^2*i^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^5*g^5
*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) -
 1/4*A*c^2*i^2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b
^2*g^5*x + a^4*b*g^5)
```

Giac [A] (verification not implemented)

none

Time = 1.98 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.32

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx = -\frac{1}{144}\left(\frac{12\left(3\,Bbi^2n-\frac{4\,(bx+a)Bdi^2n}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^4bcg^5}{(dx+c)^4}-\frac{(bx+a)^4adg^5}{(dx+c)^4}}+\frac{9\,Bbi^2n-\frac{16\,(bx+a)Bdi^2n}{dx+c}+36\,Bbi^2\log\left(e\right)-\frac{48\,(bx+a)Bdi^2}{dx+c}}{\frac{(bx+a)^4bcg^5}{(dx+c)^4}-\frac{(bx+a)^4adg^5}{(dx+c)^4}}\right.$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, a
lgorithm="giac")
```

```
[Out] -1/144*(12*(3*B*b*i^2*n - 4*(b*x + a)*B*d*i^2*n/(d*x + c))*log((b*x + a)/(d
*x + c))/((b*x + a)^4*b*c*g^5/(d*x + c)^4 - (b*x + a)^4*a*d*g^5/(d*x + c)^4
) + (9*B*b*i^2*n - 16*(b*x + a)*B*d*i^2*n/(d*x + c) + 36*B*b*i^2*log(e) - 4
8*(b*x + a)*B*d*i^2*log(e)/(d*x + c) + 36*A*b*i^2 - 48*(b*x + a)*A*d*i^2/(d
*x + c))/((b*x + a)^4*b*c*g^5/(d*x + c)^4 - (b*x + a)^4*a*d*g^5/(d*x + c)^4
))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 652, normalized size of antiderivative = 3.45

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx = -\frac{\frac{12\,A\,a^3\,d^3\,i^2-36\,A\,b^3\,c^3\,i^2+7\,B\,a^3\,d^3\,i^2\,n-9\,B\,b^3\,c^3\,i^2\,n+12\,A\,a\,b^2\,c^2\,d\,i^2+12\,A\,a^2\,b\,c\,d^2\,i^2+7\,B\,a\,b^2\,c^2\,d\,i^2\,n+7\,B\,a^2\,b\,c\,d^2\,i^2\,n}{12\,(a\,d-b\,c)}+\frac{x\,(12\,A\,a^2}{}}{12\,a^4\,b^3\,g^5+48\,a^3\,b^4} - \frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(a\left(\frac{B\,a\,d^2\,i^2}{12\,b^3}+\frac{B\,c\,d\,i^2}{6\,b^2}\right)+x\left(b\left(\frac{B\,a\,d^2\,i^2}{12\,b^3}+\frac{B\,c\,d\,i^2}{6\,b^2}\right)+\frac{B\,a\,d^2\,i^2}{4\,b^2}+\frac{B\,c\,d\,i^2}{2\,b}\right)+\frac{B\,c^2\,i^2}{4\,b}+\frac{B\,d^2\,i^2\,x^2}{2\,b}\right)}{a^4\,g^5+4\,a^3\,b\,g^5\,x+6\,a^2\,b^2\,g^5\,x^2+4\,a\,b^3\,g^5\,x^3+b^4\,g^5\,x^4} - \frac{B\,d^4\,i^2\,n\,\mathrm{atanh}\left(\frac{12\,b^5\,c^2\,g^5-12\,a^2\,b^3\,d^2\,g^5}{12\,b^3\,g^5\,(a\,d-b\,c)^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{6\,b^3\,g^5\,(a\,d-b\,c)^2}$$

```
[In] int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^
5,x)
```

```
[Out] - ((12*A*a^3*d^3*i^2 - 36*A*b^3*c^3*i^2 + 7*B*a^3*d^3*i^2*n - 9*B*b^3*c^3*i
^2*n + 12*A*a*b^2*c^2*d*i^2 + 12*A*a^2*b*c*d^2*i^2 + 7*B*a*b^2*c^2*d*i^2*n
+ 7*B*a^2*b*c*d^2*i^2*n)/(12*(a*d - b*c)) + (x*(12*A*a^2*b*d^3*i^2 - 24*A*b
^3*c^2*d*i^2 + 12*A*a*b^2*c*d^2*i^2 + 7*B*a^2*b*d^3*i^2*n - 5*B*b^3*c^2*d*i
^2*n + 7*B*a*b^2*c*d^2*i^2*n))/(3*(a*d - b*c)) + (x^2*(12*A*a*b^2*d^3*i^2 -
```

12*A*b^3*c*d^2*i^2 + 7*B*a*b^2*d^3*i^2*n - B*b^3*c*d^2*i^2*n))/(2*(a*d - b*c)) + (B*b^3*d^3*i^2*n*x^3)/(a*d - b*c))/(12*a^4*b^3*g^5 + 12*b^7*g^5*x^4 + 48*a^3*b^4*g^5*x + 48*a*b^6*g^5*x^3 + 72*a^2*b^5*g^5*x^2) - (log(e*((a + b*x)/(c + d*x))^n)*(a*((B*a*d^2*i^2)/(12*b^3) + (B*c*d*i^2)/(6*b^2)) + x*(b*((B*a*d^2*i^2)/(12*b^3) + (B*c*d*i^2)/(6*b^2)) + (B*a*d^2*i^2)/(4*b^2) + (B*c*d*i^2)/(2*b)) + (B*c^2*i^2)/(4*b) + (B*d^2*i^2*x^2)/(2*b)))/(a^4*g^5 + b^4*g^5*x^4 + 4*a*b^3*g^5*x^3 + 6*a^2*b^2*g^5*x^2 + 4*a^3*b*g^5*x) - (B*d^4*i^2*n*atanh((12*b^5*c^2*g^5 - 12*a^2*b^3*d^2*g^5)/(12*b^3*g^5*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(6*b^3*g^5*(a*d - b*c)^2)

3.126 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 293

$$\begin{aligned}\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx = &-\frac{Bd^2i^2n(c+dx)^3}{9(bc-ad)^3g^6(a+bx)^3} \\ &+\frac{bBdi^2n(c+dx)^4}{8(bc-ad)^3g^6(a+bx)^4} \\ &-\frac{b^2Bi^2n(c+dx)^5}{25(bc-ad)^3g^6(a+bx)^5} \\ &-\frac{d^2i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^3g^6(a+bx)^3} \\ &+\frac{bdi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^6(a+bx)^4} \\ &-\frac{b^2i^2(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5(bc-ad)^3g^6(a+bx)^5}\end{aligned}$$

```
[Out] -1/9*B*d^2*i^2*n*(d*x+c)^3/(-a*d+b*c)^3/g^6/(b*x+a)^3+1/8*b*B*d*i^2*n*(d*x+
c)^4/(-a*d+b*c)^3/g^6/(b*x+a)^4-1/25*b^2*B*i^2*n*(d*x+c)^5/(-a*d+b*c)^3/g^6
/(b*x+a)^5-1/3*d^2*i^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)
^3/g^6/(b*x+a)^3+1/2*b*d*i^2*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*
d+b*c)^3/g^6/(b*x+a)^4-1/5*b^2*i^2*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n)
)/(-a*d+b*c)^3/g^6/(b*x+a)^5
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2561, 45, 2372, 12, 14}

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx = -\frac{b^2i^2(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5g^6(a+bx)^5(bc-ad)^3} - \frac{d^2i^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3g^6(a+bx)^3(bc-ad)^3} + \frac{bdi^2(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^6(a+bx)^4(bc-ad)^3} - \frac{b^2Bi^2n(c+dx)^5}{25g^6(a+bx)^5(bc-ad)^3} - \frac{Bd^2i^2n(c+dx)^3}{9g^6(a+bx)^3(bc-ad)^3} + \frac{bBdi^2n(c+dx)^4}{8g^6(a+bx)^4(bc-ad)^3}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^
6,x]
```

```
[Out] -1/9*(B*d^2*i^2*n*(c + d*x)^3)/((b*c - a*d)^3*g^6*(a + b*x)^3) + (b*B*d*i^2
*n*(c + d*x)^4)/(8*(b*c - a*d)^3*g^6*(a + b*x)^4) - (b^2*B*i^2*n*(c + d*x)^
5)/(25*(b*c - a*d)^3*g^6*(a + b*x)^5) - (d^2*i^2*(c + d*x)^3*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]))/(3*(b*c - a*d)^3*g^6*(a + b*x)^3) + (b*d*i^2*(c +
 d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*c - a*d)^3*g^6*(a + b
*x)^4) - (b^2*i^2*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*(b
*c - a*d)^3*g^6*(a + b*x)^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i^2\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex^n))}{x^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^6} \\
&= -\frac{d^2i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^3g^6(a+bx)^3}+\frac{bdi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^6(a+bx)^4} \\
&\quad -\frac{b^2i^2(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5(bc-ad)^3g^6(a+bx)^5}-\frac{(Bi^2n)\,\text{Subst}\left(\int \frac{-6b^2+15bdx-10d^2x^2}{30x^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^6} \\
&= -\frac{d^2i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^3g^6(a+bx)^3}+\frac{bdi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^6(a+bx)^4} \\
&\quad -\frac{b^2i^2(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5(bc-ad)^3g^6(a+bx)^5}-\frac{(Bi^2n)\,\text{Subst}\left(\int \frac{-6b^2+15bdx-10d^2x^2}{x^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{30(bc-ad)^3g^6} \\
&= -\frac{d^2i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^3g^6(a+bx)^3}+\frac{bdi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^6(a+bx)^4} \\
&\quad -\frac{b^2i^2(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5(bc-ad)^3g^6(a+bx)^5} \\
&\quad -\frac{(Bi^2n)\,\text{Subst}\left(\int \left(-\frac{6b^2}{x^6}+\frac{15bd}{x^5}-\frac{10d^2}{x^4}\right)dx, x, \frac{a+bx}{c+dx}\right)}{30(bc-ad)^3g^6}
\end{aligned}$$

$$= -\frac{Bd^2i^2n(c+dx)^3}{9(bc-ad)^3g^6(a+bx)^3} + \frac{bBdi^2n(c+dx)^4}{8(bc-ad)^3g^6(a+bx)^4} - \frac{b^2Bi^2n(c+dx)^5}{25(bc-ad)^3g^6(a+bx)^5} - \frac{d^2i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^3g^6(a+bx)^3} + \frac{bdi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^6(a+bx)^4} - \frac{b^2i^2(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5(bc-ad)^3g^6(a+bx)^5}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.22

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx = \frac{i^2\left(-\frac{360Ab^2c^2}{(a+bx)^5}+\frac{720aAbcd}{(a+bx)^5}-\frac{360a^2Ad^2}{(a+bx)^5}-\frac{72b^2Bc^2n}{(a+bx)^5}+\frac{144abBcdn}{(a+bx)^5}-\frac{72a^2Bd^2n}{(a+bx)^5}-\frac{900Abcd}{(a+bx)^4}+\frac{900aAd^2}{(a+bx)^4}-\frac{135bBcdn}{(a+bx)^4}+\frac{135aBd}{(a+bx)}\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b
*g*x)^6,x]
```

```
[Out] (i^2*((-360*A*b^2*c^2)/(a + b*x)^5 + (720*a*A*b*c*d)/(a + b*x)^5 - (360*a^2
*A*d^2)/(a + b*x)^5 - (72*b^2*B*c^2*n)/(a + b*x)^5 + (144*a*b*B*c*d*n)/(a +
 b*x)^5 - (72*a^2*B*d^2*n)/(a + b*x)^5 - (900*A*b*c*d)/(a + b*x)^4 + (900*a
*A*d^2)/(a + b*x)^4 - (135*b*B*c*d*n)/(a + b*x)^4 + (135*a*B*d^2*n)/(a + b*
x)^4 - (600*A*d^2)/(a + b*x)^3 - (20*B*d^2*n)/(a + b*x)^3 + (30*B*d^3*n)/((
b*c - a*d)*(a + b*x)^2) - (60*B*d^4*n)/((b*c - a*d)^2*(a + b*x)) - (60*B*d^
5*n*Log[a + b*x])/(b*c - a*d)^3 - (60*B*(a^2*d^2 + a*b*d*(3*c + 5*d*x) + b^
2*(6*c^2 + 15*c*d*x + 10*d^2*x^2))*Log[e*((a + b*x)/(c + d*x))^n])/(a + b*x
)^5 + (60*B*d^5*n*Log[c + d*x])/(b*c - a*d)^3))/(1800*b^3*g^6)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1048 vs. 2(281) = 562.

Time = 57.56 (sec) , antiderivative size = 1049, normalized size of antiderivative = 3.58

method	result	size
parallelrisch	Expression too large to display	1049

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^6,x,method=_R
ETURNVERBOSE)
```

```
[Out] -1/1800*(-1800*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^7*c*d^5*i^2*n+1800*B*x
^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c^2*d^4*i^2*n-1800*B*x*ln(e*((b*x+a)/(d*
```

```
x+c))^n)*a^2*b^7*c^2*d^4*i^2*n+2400*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c^3
*d^3*i^2*n-300*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*d^6*i^2*n-600*B*x^3*ln
(e*((b*x+a)/(d*x+c))^n)*a^2*b^7*d^6*i^2*n-300*B*x^3*a*b^8*c*d^5*i^2*n^2-600
*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^3*d^3*i^2*n-600*B*x^2*a^2*b^7*c*d^5*
i^2*n^2+150*B*x^2*a*b^8*c^2*d^4*i^2*n^2-1800*A*x^2*a^2*b^7*c*d^5*i^2*n+1800
*A*x^2*a*b^8*c^2*d^4*i^2*n-900*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^4*d^2*i^
2*n-600*B*x*a^2*b^7*c^2*d^4*i^2*n^2+500*B*x*a*b^8*c^3*d^3*i^2*n^2-1800*A*x*
a^2*b^7*c^2*d^4*i^2*n+2400*A*x*a*b^8*c^3*d^3*i^2*n-600*B*ln(e*((b*x+a)/(d*x
+c))^n)*a^2*b^7*c^3*d^3*i^2*n+900*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c^4*d^2
*i^2*n+47*B*a^5*b^4*d^6*i^2*n^2-72*B*b^9*c^5*d*i^2*n^2+60*A*a^5*b^4*d^6*i^2
*n-360*A*b^9*c^5*d*i^2*n-60*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*b^9*d^6*i^2*n+6
0*B*x^4*a*b^8*d^6*i^2*n^2-60*B*x^4*b^9*c*d^5*i^2*n^2+270*B*x^3*a^2*b^7*d^6*
i^2*n^2+30*B*x^3*b^9*c^2*d^4*i^2*n^2+470*B*x^2*a^3*b^6*d^6*i^2*n^2-20*B*x^2
*b^9*c^3*d^3*i^2*n^2+600*A*x^2*a^3*b^6*d^6*i^2*n-600*A*x^2*b^9*c^3*d^3*i^2*
n+235*B*x*a^4*b^5*d^6*i^2*n^2-135*B*x*b^9*c^4*d^2*i^2*n^2+300*A*x*a^4*b^5*d
^6*i^2*n-900*A*x*b^9*c^4*d^2*i^2*n-360*B*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^5*
d*i^2*n-200*B*a^2*b^7*c^3*d^3*i^2*n^2+225*B*a*b^8*c^4*d^2*i^2*n^2-600*A*a^2
*b^7*c^3*d^3*i^2*n+900*A*a*b^8*c^4*d^2*i^2*n)/g^6/(b*x+a)^5/n/(a^3*d^3-3*a^
2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^7/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1087 vs. $2(281) = 562$.

Time = 0.37 (sec) , antiderivative size = 1087, normalized size of antiderivative = 3.71

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx = -\frac{60\,(Bb^5cd^4 - Bab^4d^5)i^2nx^4 - 30\,(Bb^5c^2d^3 - 10\,Bab^4cd^4 + 9\,Ba^2b^3d^5)i^2nx^3 + (72\,Bb^5c^5 - 225\,Bab^4c^4d}{}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^6,x, a
lgorithm="fricas")
```

```
[Out] -1/1800*(60*(B*b^5*c*d^4 - B*a*b^4*d^5)*i^2*n*x^4 - 30*(B*b^5*c^2*d^3 - 10*
B*a*b^4*c*d^4 + 9*B*a^2*b^3*d^5)*i^2*n*x^3 + (72*B*b^5*c^5 - 225*B*a*b^4*c^
4*d + 200*B*a^2*b^3*c^3*d^2 - 47*B*a^5*d^5)*i^2*n + 60*(6*A*b^5*c^5 - 15*A*
a*b^4*c^4*d + 10*A*a^2*b^3*c^3*d^2 - A*a^5*d^5)*i^2 + 10*((2*B*b^5*c^3*d^2
- 15*B*a*b^4*c^2*d^3 + 60*B*a^2*b^3*c*d^4 - 47*B*a^3*b^2*d^5)*i^2*n + 60*(A
*b^5*c^3*d^2 - 3*A*a*b^4*c^2*d^3 + 3*A*a^2*b^3*c*d^4 - A*a^3*b^2*d^5)*i^2)*
x^2 + 5*((27*B*b^5*c^4*d - 100*B*a*b^4*c^3*d^2 + 120*B*a^2*b^3*c^2*d^3 - 47
*B*a^4*b*d^5)*i^2*n + 60*(3*A*b^5*c^4*d - 8*A*a*b^4*c^3*d^2 + 6*A*a^2*b^3*c
^2*d^3 - A*a^4*b*d^5)*i^2)*x + 60*(10*(B*b^5*c^3*d^2 - 3*B*a*b^4*c^2*d^3 +
3*B*a^2*b^3*c*d^4 - B*a^3*b^2*d^5)*i^2*x^2 + 5*(3*B*b^5*c^4*d - 8*B*a*b^4*c
^3*d^2 + 6*B*a^2*b^3*c^2*d^3 - B*a^4*b*d^5)*i^2*x + (6*B*b^5*c^5 - 15*B*a*b
```

```
^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 - B*a^5*d^5)*i^2)*log(e) + 60*(B*b^5*d^5*i^
2*n*x^5 + 5*B*a*b^4*d^5*i^2*n*x^4 + 10*B*a^2*b^3*d^5*i^2*n*x^3 + 10*(B*b^5*
c^3*d^2 - 3*B*a*b^4*c^2*d^3 + 3*B*a^2*b^3*c*d^4)*i^2*n*x^2 + 5*(3*B*b^5*c^4
*d - 8*B*a*b^4*c^3*d^2 + 6*B*a^2*b^3*c^2*d^3)*i^2*n*x + (6*B*b^5*c^5 - 15*B
*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2)*i^2*n)*log((b*x + a)/(d*x + c)))/((b^1
1*c^3 - 3*a*b^10*c^2*d + 3*a^2*b^9*c*d^2 - a^3*b^8*d^3)*g^6*x^5 + 5*(a*b^10
*c^3 - 3*a^2*b^9*c^2*d + 3*a^3*b^8*c*d^2 - a^4*b^7*d^3)*g^6*x^4 + 10*(a^2*b
^9*c^3 - 3*a^3*b^8*c^2*d + 3*a^4*b^7*c*d^2 - a^5*b^6*d^3)*g^6*x^3 + 10*(a^3
*b^8*c^3 - 3*a^4*b^7*c^2*d + 3*a^5*b^6*c*d^2 - a^6*b^5*d^3)*g^6*x^2 + 5*(a^
4*b^7*c^3 - 3*a^5*b^6*c^2*d + 3*a^6*b^5*c*d^2 - a^7*b^4*d^3)*g^6*x + (a^5*b
^6*c^3 - 3*a^6*b^5*c^2*d + 3*a^7*b^4*c*d^2 - a^8*b^3*d^3)*g^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx=\text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**6,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3058 vs. $2(281) = 562$.

Time = 0.35 (sec) , antiderivative size = 3058, normalized size of antiderivative = 10.44

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx=\text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^6,x, a
lgorithm="maxima")
```

```
[Out] -1/300*B*c^2*i^2*n*((60*b^4*d^4*x^4 + 12*b^4*c^4 - 63*a*b^3*c^3*d + 137*a^2
*b^2*c^2*d^2 - 163*a^3*b*c*d^3 + 137*a^4*d^4 - 30*(b^4*c*d^3 - 9*a*b^3*d^4)
*x^3 + 10*(2*b^4*c^2*d^2 - 13*a*b^3*c*d^3 + 47*a^2*b^2*d^4)*x^2 - 5*(3*b^4*
c^3*d - 17*a*b^3*c^2*d^2 + 43*a^2*b^2*c*d^3 - 77*a^3*b*d^4)*x)/((b^10*c^4 -
 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*g^6*x^5
 + 5*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 + a
^5*b^5*d^4)*g^6*x^4 + 10*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2
 - 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*g^6*x^3 + 10*(a^3*b^7*c^4 - 4*a^4*b^6*c^3
*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*g^6*x^2 + 5*(a^4*b^
6*c^4 - 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d^4
)*g^6*x + (a^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4*a^8*b^2*c*
```

d^3 + a^9*b*d^4)*g^6) + 60*d^5*log(b*x + a)/((b^6*c^5 - 5*a*b^5*c^4*d + 10*
a^2*b^4*c^3*d^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5)*g^6) -
60*d^5*log(d*x + c)/((b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2 - 10*a^3
*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5)*g^6)) - 1/1800*B*d^2*i^2*n*((47
*a^2*b^4*c^4 - 278*a^3*b^3*c^3*d + 822*a^4*b^2*c^2*d^2 - 278*a^5*b*c*d^3 +
47*a^6*d^4 + 60*(10*b^6*c^2*d^2 - 5*a*b^5*c*d^3 + a^2*b^4*d^4)*x^4 - 30*(10
*b^6*c^3*d - 95*a*b^5*c^2*d^2 + 46*a^2*b^4*c*d^3 - 9*a^3*b^3*d^4)*x^3 + 10*
(20*b^6*c^4 - 140*a*b^5*c^3*d + 537*a^2*b^4*c^2*d^2 - 248*a^3*b^3*c*d^3 + 4
7*a^4*b^2*d^4)*x^2 + 5*(35*a*b^5*c^4 - 218*a^2*b^4*c^3*d + 702*a^3*b^3*c^2*
d^2 - 278*a^4*b^2*c*d^3 + 47*a^5*b*d^4)*x)/((b^12*c^4 - 4*a*b^11*c^3*d + 6*
a^2*b^10*c^2*d^2 - 4*a^3*b^9*c*d^3 + a^4*b^8*d^4)*g^6*x^5 + 5*(a*b^11*c^4 -
4*a^2*b^10*c^3*d + 6*a^3*b^9*c^2*d^2 - 4*a^4*b^8*c*d^3 + a^5*b^7*d^4)*g^6*
x^4 + 10*(a^2*b^10*c^4 - 4*a^3*b^9*c^3*d + 6*a^4*b^8*c^2*d^2 - 4*a^5*b^7*c*
d^3 + a^6*b^6*d^4)*g^6*x^3 + 10*(a^3*b^9*c^4 - 4*a^4*b^8*c^3*d + 6*a^5*b^7*
c^2*d^2 - 4*a^6*b^6*c*d^3 + a^7*b^5*d^4)*g^6*x^2 + 5*(a^4*b^8*c^4 - 4*a^5*b
^7*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^7*b^5*c*d^3 + a^8*b^4*d^4)*g^6*x + (a^5*
b^7*c^4 - 4*a^6*b^6*c^3*d + 6*a^7*b^5*c^2*d^2 - 4*a^8*b^4*c*d^3 + a^9*b^3*d
^4)*g^6) + 60*(10*b^2*c^2*d^3 - 5*a*b*c*d^4 + a^2*d^5)*log(b*x + a)/((b^8*c
^5 - 5*a*b^7*c^4*d + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 + 5*a^4*b^4*c*
d^4 - a^5*b^3*d^5)*g^6) - 60*(10*b^2*c^2*d^3 - 5*a*b*c*d^4 + a^2*d^5)*log(d
*x + c)/((b^8*c^5 - 5*a*b^7*c^4*d + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3
+ 5*a^4*b^4*c*d^4 - a^5*b^3*d^5)*g^6)) - 1/600*B*c*d*i^2*n*((27*a*b^4*c^4
- 148*a^2*b^3*c^3*d + 352*a^3*b^2*c^2*d^2 - 548*a^4*b*c*d^3 + 77*a^5*d^4 -
60*(5*b^5*c*d^3 - a*b^4*d^4)*x^4 + 30*(5*b^5*c^2*d^2 - 46*a*b^4*c*d^3 + 9*a
^2*b^3*d^4)*x^3 - 10*(10*b^5*c^3*d - 67*a*b^4*c^2*d^2 + 248*a^2*b^3*c*d^3 -
47*a^3*b^2*d^4)*x^2 + 5*(15*b^5*c^4 - 88*a*b^4*c^3*d + 232*a^2*b^3*c^2*d^2
- 428*a^3*b^2*c*d^3 + 77*a^4*b*d^4)*x)/((b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2
*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4)*g^6*x^5 + 5*(a*b^10*c^4 - 4*a
^2*b^9*c^3*d + 6*a^3*b^8*c^2*d^2 - 4*a^4*b^7*c*d^3 + a^5*b^6*d^4)*g^6*x^4 +
10*(a^2*b^9*c^4 - 4*a^3*b^8*c^3*d + 6*a^4*b^7*c^2*d^2 - 4*a^5*b^6*c*d^3 +
a^6*b^5*d^4)*g^6*x^3 + 10*(a^3*b^8*c^4 - 4*a^4*b^7*c^3*d + 6*a^5*b^6*c^2*d^
2 - 4*a^6*b^5*c*d^3 + a^7*b^4*d^4)*g^6*x^2 + 5*(a^4*b^7*c^4 - 4*a^5*b^6*c^3
*d + 6*a^6*b^5*c^2*d^2 - 4*a^7*b^4*c*d^3 + a^8*b^3*d^4)*g^6*x + (a^5*b^6*c^
4 - 4*a^6*b^5*c^3*d + 6*a^7*b^4*c^2*d^2 - 4*a^8*b^3*c*d^3 + a^9*b^2*d^4)*g^
6) - 60*(5*b*c*d^4 - a*d^5)*log(b*x + a)/((b^7*c^5 - 5*a*b^6*c^4*d + 10*a^2
*b^5*c^3*d^2 - 10*a^3*b^4*c^2*d^3 + 5*a^4*b^3*c*d^4 - a^5*b^2*d^5)*g^6) + 6
0*(5*b*c*d^4 - a*d^5)*log(d*x + c)/((b^7*c^5 - 5*a*b^6*c^4*d + 10*a^2*b^5*c
^3*d^2 - 10*a^3*b^4*c^2*d^3 + 5*a^4*b^3*c*d^4 - a^5*b^2*d^5)*g^6)) - 1/10*(
5*b*x + a)*B*c*d*i^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^7*g^6*x^5 +
5*a*b^6*g^6*x^4 + 10*a^2*b^5*g^6*x^3 + 10*a^3*b^4*g^6*x^2 + 5*a^4*b^3*g^6*x
+ a^5*b^2*g^6) - 1/30*(10*b^2*x^2 + 5*a*b*x + a^2)*B*d^2*i^2*log(e*(b*x/(d
*x + c) + a/(d*x + c))^n)/(b^8*g^6*x^5 + 5*a*b^7*g^6*x^4 + 10*a^2*b^6*g^6*x
^3 + 10*a^3*b^5*g^6*x^2 + 5*a^4*b^4*g^6*x + a^5*b^3*g^6) - 1/10*(5*b*x + a)
*A*c*d*i^2/(b^7*g^6*x^5 + 5*a*b^6*g^6*x^4 + 10*a^2*b^5*g^6*x^3 + 10*a^3*b^4
*g^6*x^2 + 5*a^4*b^3*g^6*x + a^5*b^2*g^6) - 1/30*(10*b^2*x^2 + 5*a*b*x + a^

```
2)*A*d^2*i^2/(b^8*g^6*x^5 + 5*a*b^7*g^6*x^4 + 10*a^2*b^6*g^6*x^3 + 10*a^3*b
^5*g^6*x^2 + 5*a^4*b^4*g^6*x + a^5*b^3*g^6) - 1/5*B*c^2*i^2*log(e*(b*x/(d*x
 + c) + a/(d*x + c))^n)/(b^6*g^6*x^5 + 5*a*b^5*g^6*x^4 + 10*a^2*b^4*g^6*x^3
 + 10*a^3*b^3*g^6*x^2 + 5*a^4*b^2*g^6*x + a^5*b*g^6) - 1/5*A*c^2*i^2/(b^6*g
^6*x^5 + 5*a*b^5*g^6*x^4 + 10*a^2*b^4*g^6*x^3 + 10*a^3*b^3*g^6*x^2 + 5*a^4*
b^2*g^6*x + a^5*b*g^6)
```

Giac [A] (verification not implemented)

none

Time = 2.58 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.43

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx = -\frac{1}{1800}\left(\frac{60\left(6\,Bb^2i^2n-\frac{15\,(bx+a)Bbdi^2n}{dx+c}+\frac{10\,(bx+a)^2Bd^2i^2n}{(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^5b^2c^2g^6}{(dx+c)^5}-\frac{2\,(bx+a)^5abcdg^6}{(dx+c)^5}+\frac{(bx+a)^5a^2d^2g^6}{(dx+c)^5}}+\frac{72\,Bb^2i^2n-\frac{225\,(bx+a)Bbdi^2n}{dx+c}+\frac{200\,(bx}{(}}{}\right.$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^6,x, a
lgorithm="giac")
```

```
[Out] -1/1800*(60*(6*B*b^2*i^2*n - 15*(b*x + a)*B*b*d*i^2*n/(d*x + c) + 10*(b*x +
 a)^2*B*d^2*i^2*n/(d*x + c)^2)*log((b*x + a)/(d*x + c))/((b*x + a)^5*b^2*c^
2*g^6/(d*x + c)^5 - 2*(b*x + a)^5*a*b*c*d*g^6/(d*x + c)^5 + (b*x + a)^5*a^2
*d^2*g^6/(d*x + c)^5) + (72*B*b^2*i^2*n - 225*(b*x + a)*B*b*d*i^2*n/(d*x +
c) + 200*(b*x + a)^2*B*d^2*i^2*n/(d*x + c)^2 + 360*B*b^2*i^2*log(e) - 900*(
b*x + a)*B*b*d*i^2*log(e)/(d*x + c) + 600*(b*x + a)^2*B*d^2*i^2*log(e)/(d*x
 + c)^2 + 360*A*b^2*i^2 - 900*(b*x + a)*A*b*d*i^2/(d*x + c) + 600*(b*x + a)
^2*A*d^2*i^2/(d*x + c)^2)/((b*x + a)^5*b^2*c^2*g^6/(d*x + c)^5 - 2*(b*x + a
)^5*a*b*c*d*g^6/(d*x + c)^5 + (b*x + a)^5*a^2*d^2*g^6/(d*x + c)^5))*(b*c/(b
*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 3.37 (sec) , antiderivative size = 954, normalized size of antiderivative = 3.26

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx$$

$$= \frac{B\,d^5\,i^2\,n\,\mathrm{atanh}\left(\frac{30\,a^3\,b^3\,d^3\,g^6-30\,a^2\,b^4\,c\,d^2\,g^6-30\,a\,b^5\,c^2\,d\,g^6+30\,b^6\,c^3\,g^6}{30\,b^3\,g^6\,(a\,d-b\,c)^3}+\frac{2\,b\,d\,x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^3}\right)}{15\,b^3\,g^6\,(a\,d-b\,c)^3}$$

$$-\frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(a\left(\frac{B\,a\,d^2\,i^2}{30\,b^3}+\frac{B\,c\,d\,i^2}{10\,b^2}\right)+x\left(b\left(\frac{B\,a\,d^2\,i^2}{30\,b^3}+\frac{B\,c\,d\,i^2}{10\,b^2}\right)+\frac{2\,B\,a\,d^2\,i^2}{15\,b^2}+\frac{2\,B\,c\,d\,i^2}{5\,b}\right)+\frac{B\,c^2\,i^2}{5\,b}+\frac{B\,d^2\,i^2\,x^2}{3\,b}\right.}{a^5\,g^6+5\,a^4\,b\,g^6\,x+10\,a^3\,b^2\,g^6\,x^2+10\,a^2\,b^3\,g^6\,x^3+5\,a\,b^4\,g^6\,x^4+b^5\,g^6\,x^5}$$

$$-\frac{\frac{60\,A\,a^4\,d^4\,i^2+360\,A\,b^4\,c^4\,i^2+47\,B\,a^4\,d^4\,i^2\,n+72\,B\,b^4\,c^4\,i^2\,n+60\,A\,a^2\,b^2\,c^2\,d^2\,i^2-540\,A\,a\,b^3\,c^3\,d\,i^2+60\,A\,a^3\,b\,c\,d^3\,i^2-153\,B\,a\,b^3\,c^3\,d\,i^2\,n+47\,B}{60\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}}{}$$

```
[In] int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^
6,x)
```

```
[Out] (B*d^5*i^2*n*atanh((30*b^6*c^3*g^6 + 30*a^3*b^3*d^3*g^6 - 30*a*b^5*c^2*d*g^
6 - 30*a^2*b^4*c*d^2*g^6)/(30*b^3*g^6*(a*d - b*c)^3) + (2*b*d*x*(a^2*d^2 +
b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(15*b^3*g^6*(a*d - b*c)^3) - (log(e*(
(a + b*x)/(c + d*x))^n)*(a*((B*a*d^2*i^2)/(30*b^3) + (B*c*d*i^2)/(10*b^2))
+ x*(b*((B*a*d^2*i^2)/(30*b^3) + (B*c*d*i^2)/(10*b^2)) + (2*B*a*d^2*i^2)/(1
5*b^2) + (2*B*c*d*i^2)/(5*b)) + (B*c^2*i^2)/(5*b) + (B*d^2*i^2*x^2)/(3*b)))
/(a^5*g^6 + b^5*g^6*x^5 + 5*a*b^4*g^6*x^4 + 10*a^3*b^2*g^6*x^2 + 10*a^2*b^3
*g^6*x^3 + 5*a^4*b*g^6*x) - ((60*A*a^4*d^4*i^2 + 360*A*b^4*c^4*i^2 + 47*B*a
^4*d^4*i^2*n + 72*B*b^4*c^4*i^2*n + 60*A*a^2*b^2*c^2*d^2*i^2 - 540*A*a*b^3*
c^3*d*i^2 + 60*A*a^3*b*c*d^3*i^2 - 153*B*a*b^3*c^3*d*i^2*n + 47*B*a^3*b*c*d
^3*i^2*n + 47*B*a^2*b^2*c^2*d^2*i^2*n)/(60*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
 + (x^2*(60*A*a^2*b^2*d^4*i^2 + 60*A*b^4*c^2*d^2*i^2 + 47*B*a^2*b^2*d^4*i^2
*n + 2*B*b^4*c^2*d^2*i^2*n - 120*A*a*b^3*c*d^3*i^2 - 13*B*a*b^3*c*d^3*i^2*n
))/(6*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(60*A*a^3*b*d^4*i^2 + 180*A*b^4
*c^3*d*i^2 - 300*A*a*b^3*c^2*d^2*i^2 + 60*A*a^2*b^2*c*d^3*i^2 + 47*B*a^3*b*
d^4*i^2*n + 27*B*b^4*c^3*d*i^2*n - 73*B*a*b^3*c^2*d^2*i^2*n + 47*B*a^2*b^2*
c*d^3*i^2*n))/(12*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (d*x^3*(9*B*a*b^3*d^3*
i^2*n - B*b^4*c*d^2*i^2*n))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B*b^4*d^
4*i^2*n*x^4)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(30*a^5*b^3*g^6 + 30*b^8*g^6*
x^5 + 150*a^4*b^4*g^6*x + 150*a*b^7*g^6*x^4 + 300*a^3*b^5*g^6*x^2 + 300*a^2
*b^6*g^6*x^3)
```

3.127 $\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal result

Integrand size = 43, antiderivative size = 477

$$\begin{aligned}
&\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx \\
&= \frac{B(bc-ad)^6g^3i^3nx}{140b^3d^3} + \frac{B(bc-ad)^5g^3i^3n(c+dx)^2}{280b^2d^4} + \frac{B(bc-ad)^4g^3i^3n(c+dx)^3}{420bd^4} \\
&\quad - \frac{17B(bc-ad)^3g^3i^3n(c+dx)^4}{280d^4} + \frac{bB(bc-ad)^2g^3i^3n(c+dx)^5}{14d^4} \\
&\quad - \frac{b^2B(bc-ad)g^3i^3n(c+dx)^6}{42d^4} - \frac{(bc-ad)^3g^3i^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^4} \\
&\quad + \frac{3b(bc-ad)^2g^3i^3(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^4} \\
&\quad - \frac{b^2(bc-ad)g^3i^3(c+dx)^6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^4} \\
&\quad + \frac{b^3g^3i^3(c+dx)^7\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{7d^4} \\
&\quad + \frac{B(bc-ad)^7g^3i^3n\log\left(\frac{a+bx}{c+dx}\right)}{140b^4d^4} + \frac{B(bc-ad)^7g^3i^3n\log(c+dx)}{140b^4d^4}
\end{aligned}$$

```
[Out] 1/140*B*(-a*d+b*c)^6*g^3*i^3*n*x/b^3/d^3+1/280*B*(-a*d+b*c)^5*g^3*i^3*n*(d*
x+c)^2/b^2/d^4+1/420*B*(-a*d+b*c)^4*g^3*i^3*n*(d*x+c)^3/b/d^4-17/280*B*(-a*
d+b*c)^3*g^3*i^3*n*(d*x+c)^4/d^4+1/14*b*B*(-a*d+b*c)^2*g^3*i^3*n*(d*x+c)^5/
d^4-1/42*b^2*B*(-a*d+b*c)*g^3*i^3*n*(d*x+c)^6/d^4-1/4*(-a*d+b*c)^3*g^3*i^3*
(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4+3/5*b*(-a*d+b*c)^2*g^3*i^3*(d
*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4-1/2*b^2*(-a*d+b*c)*g^3*i^3*(d*x
+c)^6*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4+1/7*b^3*g^3*i^3*(d*x+c)^7*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))/d^4+1/140*B*(-a*d+b*c)^7*g^3*i^3*n*ln((b*x+a)/(d*x
+c))/b^4/d^4+1/140*B*(-a*d+b*c)^7*g^3*i^3*n*ln(d*x+c)/b^4/d^4
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2561, 45, 2382, 12, 1634}

$$\begin{aligned}&\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx\\&=\frac{b^3g^3i^3(c+dx)^7\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{7d^4}\\&-\frac{b^2g^3i^3(c+dx)^6(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d^4}\\&-\frac{g^3i^3(c+dx)^4(bc-ad)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^4}\\&+\frac{3bg^3i^3(c+dx)^5(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^4}+\frac{Bg^3i^3n(bc-ad)^7\log\left(\frac{a+bx}{c+dx}\right)}{140b^4d^4}\\&+\frac{Bg^3i^3n(bc-ad)^7\log(c+dx)}{140b^4d^4}+\frac{Bg^3i^3nx(bc-ad)^6}{140b^3d^3}+\frac{Bg^3i^3n(c+dx)^2(bc-ad)^5}{280b^2d^4}\\&-\frac{b^2Bg^3i^3n(c+dx)^6(bc-ad)}{42d^4}+\frac{Bg^3i^3n(c+dx)^3(bc-ad)^4}{420bd^4}\\&-\frac{17Bg^3i^3n(c+dx)^4(bc-ad)^3}{280d^4}+\frac{bBg^3i^3n(c+dx)^5(bc-ad)^2}{14d^4}\end{aligned}$$

```
[In] Int[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),
x]
```

```
[Out] (B*(b*c - a*d)^6*g^3*i^3*n*x)/(140*b^3*d^3) + (B*(b*c - a*d)^5*g^3*i^3*n*(c
 + d*x)^2)/(280*b^2*d^4) + (B*(b*c - a*d)^4*g^3*i^3*n*(c + d*x)^3)/(420*b*d
^4) - (17*B*(b*c - a*d)^3*g^3*i^3*n*(c + d*x)^4)/(280*d^4) + (b*B*(b*c - a*
d)^2*g^3*i^3*n*(c + d*x)^5)/(14*d^4) - (b^2*B*(b*c - a*d)*g^3*i^3*n*(c + d*
x)^6)/(42*d^4) - ((b*c - a*d)^3*g^3*i^3*(c + d*x)^4*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/(4*d^4) + (3*b*(b*c - a*d)^2*g^3*i^3*(c + d*x)^5*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]))/(5*d^4) - (b^2*(b*c - a*d)*g^3*i^3*(c + d*x)
^6*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d^4) + (b^3*g^3*i^3*(c + d*x)
^7*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(7*d^4) + (B*(b*c - a*d)^7*g^3*i
^3*n*Log[(a + b*x)/(c + d*x)])/(140*b^4*d^4) + (B*(b*c - a*d)^7*g^3*i^3*n*L
og[c + d*x])/(140*b^4*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((bc-ad)^7g^3i^3\right)\text{Subst}\left(\int \frac{x^3(A+B\log(ex^n))}{(b-dx)^8}\,dx, x, \frac{a+bx}{c+dx}\right)\\
&= -\frac{(bc-ad)^3g^3i^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^4}\\
&\quad+\frac{3b(bc-ad)^2g^3i^3(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^4}\\
&\quad-\frac{b^2(bc-ad)g^3i^3(c+dx)^6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^4}\\
&\quad+\frac{b^3g^3i^3(c+dx)^7\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{7d^4}\\
&\quad-\left(B(bc-ad)^7g^3i^3n\right)\text{Subst}\left(\int \frac{-b^3+7b^2dx-21bd^2x^2+35d^3x^3}{140d^4x(b-dx)^7}\,dx, x, \frac{a+bx}{c+dx}\right)
\end{aligned}$$

$$= -\frac{(bc-ad)^3 g^3 i^3 (c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^4} + \frac{3b(bc-ad)^2 g^3 i^3 (c+dx)^5 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^4} - \frac{b^2(bc-ad) g^3 i^3 (c+dx)^6 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^4} + \frac{b^3 g^3 i^3 (c+dx)^7 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{7d^4} - \frac{(B(bc-ad)^7 g^3 i^3 n) \operatorname{Subst}\left(\int \frac{-b^3+7b^2dx-21bd^2x^2+35d^3x^3}{x(b-dx)^7}\,dx, x, \frac{a+bx}{c+dx}\right)}{140d^4}$$

$$= -\frac{(bc-ad)^3 g^3 i^3 (c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^4} + \frac{3b(bc-ad)^2 g^3 i^3 (c+dx)^5 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^4} - \frac{b^2(bc-ad) g^3 i^3 (c+dx)^6 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^4} + \frac{b^3 g^3 i^3 (c+dx)^7 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{7d^4} - \frac{(B(bc-ad)^7 g^3 i^3 n) \operatorname{Subst}\left(\int \left(-\frac{1}{b^4x} + \frac{20b^2d}{(b-dx)^7} - \frac{50bd}{(b-dx)^6} + \frac{34d}{(b-dx)^5} - \frac{d}{b(b-dx)^4} - \frac{d}{b^2(b-dx)^3} - \frac{d}{b^3(b-dx)}\right.\right.}{140d^4}$$

$$= \frac{B(bc-ad)^6 g^3 i^3 n x}{140b^3d^3} + \frac{B(bc-ad)^5 g^3 i^3 n (c+dx)^2}{280b^2d^4} + \frac{B(bc-ad)^4 g^3 i^3 n (c+dx)^3}{420bd^4} - \frac{17B(bc-ad)^3 g^3 i^3 n (c+dx)^4}{280d^4} + \frac{bB(bc-ad)^2 g^3 i^3 n (c+dx)^5}{14d^4} - \frac{b^2B(bc-ad) g^3 i^3 n (c+dx)^6}{42d^4} - \frac{(bc-ad)^3 g^3 i^3 (c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^4} + \frac{3b(bc-ad)^2 g^3 i^3 (c+dx)^5 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^4} - \frac{b^2(bc-ad) g^3 i^3 (c+dx)^6 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^4} + \frac{b^3 g^3 i^3 (c+dx)^7 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{7d^4} + \frac{B(bc-ad)^7 g^3 i^3 n \log\left(\frac{a+bx}{c+dx}\right)}{140b^4d^4} + \frac{B(bc-ad)^7 g^3 i^3 n \log(c+dx)}{140b^4d^4}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.32

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=\frac{g^3i^3\left(210d^4(bc-ad)^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+504d^5(bc-ad)^2(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]),x]
```

```
[Out] (g^3*i^3*(210*d^4*(b*c - a*d)^3*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]) + 504*d^5*(b*c - a*d)^2*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]) + 420*d^6*(b*c - a*d)*(a + b*x)^6*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]) + 120*d^7*(a + b*x)^7*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 35*B*(
b*c - a*d)^4*n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 +
2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + 42*B*(b*c - a*d)^3*n*(1
2*b*d*(b*c - a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c - a*d)
*(a + b*x)^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]) - 7*B*(b*
c - a*d)^2*n*(60*b*d*(b*c - a*d)^4*x + 30*d^2*(-(b*c) + a*d)^3*(a + b*x)^2
+ 20*d^3*(b*c - a*d)^2*(a + b*x)^3 + 15*d^4*(-(b*c) + a*d)*(a + b*x)^4 + 12
*d^5*(a + b*x)^5 - 60*(b*c - a*d)^5*Log[c + d*x]) + 2*B*(b*c - a*d)*n*(60*b
*d*(b*c - a*d)^5*x - 30*d^2*(b*c - a*d)^4*(a + b*x)^2 + 20*d^3*(b*c - a*d)^
3*(a + b*x)^3 - 15*d^4*(b*c - a*d)^2*(a + b*x)^4 + 12*d^5*(b*c - a*d)*(a +
b*x)^5 - 10*d^6*(a + b*x)^6 - 60*(b*c - a*d)^6*Log[c + d*x])))/(840*b^4*d^4
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2178 vs. 2(453) = 906.

Time = 50.10 (sec) , antiderivative size = 2179, normalized size of antiderivative = 4.57

method	result	size
parallelrisch	Expression too large to display	2179

```
[In] int((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/840*(-6*B*x*b^7*c^6*d*g^3*i^3*n^2+120*B*x^7*ln(e*((b*x+a)/(d*x+c))^n)*b^7
*d^7*g^3*i^3*n+20*B*x^6*a*b^6*d^7*g^3*i^3*n^2-20*B*x^6*b^7*c*d^6*g^3*i^3*n^
2+420*A*x^6*a*b^6*d^7*g^3*i^3*n+420*A*x^6*b^7*c*d^6*g^3*i^3*n+60*B*x^5*a^2*
b^5*d^7*g^3*i^3*n^2-60*B*x^5*b^7*c^2*d^5*g^3*i^3*n^2+504*A*x^5*a^2*b^5*d^7*
g^3*i^3*n+504*A*x^5*b^7*c^2*d^5*g^3*i^3*n+51*B*x^4*a^3*b^4*d^7*g^3*i^3*n^2-
51*B*x^4*b^7*c^3*d^4*g^3*i^3*n^2+1512*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6
```

*c*d^6*g^3*i^3*n+1890*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^5*c*d^6*g^3*i^3
*n+1890*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*c^2*d^5*g^3*i^3*n+840*B*x^3*l
n(e*((b*x+a)/(d*x+c))^n)*a^3*b^4*c*d^6*g^3*i^3*n+2520*B*x^3*ln(e*((b*x+a)/(
d*x+c))^n)*a^2*b^5*c^2*d^5*g^3*i^3*n+840*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*
b^6*c^3*d^4*g^3*i^3*n+1260*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^4*c^2*d^5*
g^3*i^3*n+1260*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^5*c^3*d^4*g^3*i^3*n+84
0*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^4*c^3*d^4*g^3*i^3*n-21*B*x^2*a*b^6*c^
4*d^3*g^3*i^3*n^2+1260*A*x^2*a^3*b^4*c^2*d^5*g^3*i^3*n+1260*A*x^2*a^2*b^5*c
^3*d^4*g^3*i^3*n+420*B*x^6*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*d^7*g^3*i^3*n+42
0*B*x^6*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c*d^6*g^3*i^3*n+504*B*x^5*ln(e*((b*x+
a)/(d*x+c))^n)*a^2*b^5*d^7*g^3*i^3*n+39*B*a^6*b*c*d^6*g^3*i^3*n^2-105*B*a^5
*b^2*c^2*d^5*g^3*i^3*n^2-378*B*a^4*b^3*c^3*d^4*g^3*i^3*n^2+378*B*a^3*b^4*c^
4*d^3*g^3*i^3*n^2+504*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c^2*d^5*g^3*i^3*n
+1512*A*x^5*a*b^6*c*d^6*g^3*i^3*n+210*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b
^4*d^7*g^3*i^3*n+210*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c^3*d^4*g^3*i^3*n-
210*B*ln(b*x+a)*a^3*b^4*c^4*d^3*g^3*i^3*n^2+126*B*ln(b*x+a)*a^2*b^5*c^5*d^2
*g^3*i^3*n^2-42*B*ln(b*x+a)*a*b^6*c^6*d*g^3*i^3*n^2+147*B*x^4*a^2*b^5*c*d^6
*g^3*i^3*n^2-147*B*x^4*a*b^6*c^2*d^5*g^3*i^3*n^2+1890*A*x^4*a^2*b^5*c*d^6*g
^3*i^3*n+1890*A*x^4*a*b^6*c^2*d^5*g^3*i^3*n+196*B*x^3*a^3*b^4*c*d^6*g^3*i^3
*n^2-196*B*x^3*a*b^6*c^3*d^4*g^3*i^3*n^2+840*A*x^3*a^3*b^4*c*d^6*g^3*i^3*n+
2520*A*x^3*a^2*b^5*c^2*d^5*g^3*i^3*n+840*A*x^3*a*b^6*c^3*d^4*g^3*i^3*n+21*B
*x^2*a^4*b^3*c*d^6*g^3*i^3*n^2+252*B*x^2*a^3*b^4*c^2*d^5*g^3*i^3*n^2-252*B*
x^2*a^2*b^5*c^3*d^4*g^3*i^3*n^2-42*B*x*a^5*b^2*c*d^6*g^3*i^3*n^2+126*B*x*a^
4*b^3*c^2*d^5*g^3*i^3*n^2-126*B*x*a^2*b^5*c^4*d^3*g^3*i^3*n^2+42*B*x*a*b^6*
c^5*d^2*g^3*i^3*n^2+105*B*a^2*b^5*c^5*d^2*g^3*i^3*n^2+840*A*x*a^3*b^4*c^3*d
^4*g^3*i^3*n+210*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^4*c^4*d^3*g^3*i^3*n-126*
B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^5*c^5*d^2*g^3*i^3*n+42*B*ln(e*((b*x+a)/(d
*x+c))^n)*a*b^6*c^6*d*g^3*i^3*n+42*B*ln(b*x+a)*a^6*b*c*d^6*g^3*i^3*n^2-126*
B*ln(b*x+a)*a^5*b^2*c^2*d^5*g^3*i^3*n^2+210*B*ln(b*x+a)*a^4*b^3*c^3*d^4*g^3
*i^3*n^2-39*B*a*b^6*c^6*d*g^3*i^3*n^2-2100*A*a^4*b^3*c^3*d^4*g^3*i^3*n-2100
*A*a^3*b^4*c^4*d^3*g^3*i^3*n+210*A*x^4*a^3*b^4*d^7*g^3*i^3*n+210*A*x^4*b^7*
c^3*d^4*g^3*i^3*n+2*B*x^3*a^4*b^3*d^7*g^3*i^3*n^2-2*B*x^3*b^7*c^4*d^3*g^3*i
^3*n^2-3*B*x^2*a^5*b^2*d^7*g^3*i^3*n^2+3*B*x^2*b^7*c^5*d^2*g^3*i^3*n^2+6*B*
x*a^6*b*d^7*g^3*i^3*n^2+120*A*x^7*b^7*d^7*g^3*i^3*n-6*B*ln(e*((b*x+a)/(d*x+
c))^n)*b^7*c^7*g^3*i^3*n+6*B*ln(b*x+a)*b^7*c^7*g^3*i^3*n^2-6*B*ln(b*x+a)*a^
7*d^7*g^3*i^3*n^2-6*B*a^7*d^7*g^3*i^3*n^2+6*B*b^7*c^7*g^3*i^3*n^2)/b^4/d^4/
n

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1336 vs. $2(453) = 906$.

Time = 0.93 (sec) , antiderivative size = 1336, normalized size of antiderivative = 2.80

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, a
lgorithm="fricas")
```

```
[Out] 1/840*(120*A*b^7*d^7*g^3*i^3*x^7 + 6*(35*B*a^4*b^3*c^3*d^4 - 21*B*a^5*b^2*c
^2*d^5 + 7*B*a^6*b*c*d^6 - B*a^7*d^7)*g^3*i^3*n*log(b*x + a) + 6*(B*b^7*c^7
 - 7*B*a*b^6*c^6*d + 21*B*a^2*b^5*c^5*d^2 - 35*B*a^3*b^4*c^4*d^3)*g^3*i^3*n
*log(d*x + c) - 20*((B*b^7*c*d^6 - B*a*b^6*d^7)*g^3*i^3*n - 21*(A*b^7*c*d^6
 + A*a*b^6*d^7)*g^3*i^3)*x^6 - 12*(5*(B*b^7*c^2*d^5 - B*a^2*b^5*d^7)*g^3*i^
3*n - 42*(A*b^7*c^2*d^5 + 3*A*a*b^6*c*d^6 + A*a^2*b^5*d^7)*g^3*i^3)*x^5 - 3
*((17*B*b^7*c^3*d^4 + 49*B*a*b^6*c^2*d^5 - 49*B*a^2*b^5*c*d^6 - 17*B*a^3*b^
4*d^7)*g^3*i^3*n - 70*(A*b^7*c^3*d^4 + 9*A*a*b^6*c^2*d^5 + 9*A*a^2*b^5*c*d^
6 + A*a^3*b^4*d^7)*g^3*i^3)*x^4 - 2*((B*b^7*c^4*d^3 + 98*B*a*b^6*c^3*d^4 -
98*B*a^3*b^4*c*d^6 - B*a^4*b^3*d^7)*g^3*i^3*n - 420*(A*a*b^6*c^3*d^4 + 3*A*
a^2*b^5*c^2*d^5 + A*a^3*b^4*c*d^6)*g^3*i^3)*x^3 + 3*((B*b^7*c^5*d^2 - 7*B*a
*b^6*c^4*d^3 - 84*B*a^2*b^5*c^3*d^4 + 84*B*a^3*b^4*c^2*d^5 + 7*B*a^4*b^3*c*
d^6 - B*a^5*b^2*d^7)*g^3*i^3*n + 420*(A*a^2*b^5*c^3*d^4 + A*a^3*b^4*c^2*d^5
)*g^3*i^3)*x^2 + 6*(140*A*a^3*b^4*c^3*d^4*g^3*i^3 - (B*b^7*c^6*d - 7*B*a*b^
6*c^5*d^2 + 21*B*a^2*b^5*c^4*d^3 - 21*B*a^4*b^3*c^2*d^5 + 7*B*a^5*b^2*c*d^6
 - B*a^6*b*d^7)*g^3*i^3*n)*x + 6*(20*B*b^7*d^7*g^3*i^3*x^7 + 140*B*a^3*b^4*
c^3*d^4*g^3*i^3*x + 70*(B*b^7*c*d^6 + B*a*b^6*d^7)*g^3*i^3*x^6 + 84*(B*b^7*
c^2*d^5 + 3*B*a*b^6*c*d^6 + B*a^2*b^5*d^7)*g^3*i^3*x^5 + 35*(B*b^7*c^3*d^4
+ 9*B*a*b^6*c^2*d^5 + 9*B*a^2*b^5*c*d^6 + B*a^3*b^4*d^7)*g^3*i^3*x^4 + 140*
(B*a*b^6*c^3*d^4 + 3*B*a^2*b^5*c^2*d^5 + B*a^3*b^4*c*d^6)*g^3*i^3*x^3 + 210
*(B*a^2*b^5*c^3*d^4 + B*a^3*b^4*c^2*d^5)*g^3*i^3*x^2)*log(e) + 6*(20*B*b^7*
d^7*g^3*i^3*n*x^7 + 140*B*a^3*b^4*c^3*d^4*g^3*i^3*n*x + 70*(B*b^7*c*d^6 + B
*a*b^6*d^7)*g^3*i^3*n*x^6 + 84*(B*b^7*c^2*d^5 + 3*B*a*b^6*c*d^6 + B*a^2*b^5
*d^7)*g^3*i^3*n*x^5 + 35*(B*b^7*c^3*d^4 + 9*B*a*b^6*c^2*d^5 + 9*B*a^2*b^5*c
*d^6 + B*a^3*b^4*d^7)*g^3*i^3*n*x^4 + 140*(B*a*b^6*c^3*d^4 + 3*B*a^2*b^5*c^
2*d^5 + B*a^3*b^4*c*d^6)*g^3*i^3*n*x^3 + 210*(B*a^2*b^5*c^3*d^4 + B*a^3*b^4
*c^2*d^5)*g^3*i^3*n*x^2)*log((b*x + a)/(d*x + c)))/(b^4*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**3*(d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2901 vs. $2(453) = 906$.

Time = 0.28 (sec) , antiderivative size = 2901, normalized size of antiderivative = 6.08

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, a
lgorithm="maxima")
[Out] 1/7*B*b^3*d^3*g^3*i^3*x^7*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/7*A*b^
3*d^3*g^3*i^3*x^7 + 1/2*B*b^3*c*d^2*g^3*i^3*x^6*log(e*(b*x/(d*x + c) + a/(d
*x + c))^n) + 1/2*B*a*b^2*d^3*g^3*i^3*x^6*log(e*(b*x/(d*x + c) + a/(d*x + c
))^n) + 1/2*A*b^3*c*d^2*g^3*i^3*x^6 + 1/2*A*a*b^2*d^3*g^3*i^3*x^6 + 3/5*B*b
^3*c^2*d*g^3*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 9/5*B*a*b^2*c
*d^2*g^3*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/5*B*a^2*b*d^3*g
^3*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/5*A*b^3*c^2*d*g^3*i^3
*x^5 + 9/5*A*a*b^2*c*d^2*g^3*i^3*x^5 + 3/5*A*a^2*b*d^3*g^3*i^3*x^5 + 1/4*B*
b^3*c^3*g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 9/4*B*a*b^2*c^
2*d*g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 9/4*B*a^2*b*c*d^2*
g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*B*a^3*d^3*g^3*i^3*
x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*b^3*c^3*g^3*i^3*x^4 + 9/
4*A*a*b^2*c^2*d*g^3*i^3*x^4 + 9/4*A*a^2*b*c*d^2*g^3*i^3*x^4 + 1/4*A*a^3*d^3
*g^3*i^3*x^4 + B*a*b^2*c^3*g^3*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n) + 3*B*a^2*b*c^2*d*g^3*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + B
*a^3*c*d^2*g^3*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a*b^2*c^3
*g^3*i^3*x^3 + 3*A*a^2*b*c^2*d*g^3*i^3*x^3 + A*a^3*c*d^2*g^3*i^3*x^3 + 3/2*
B*a^2*b*c^3*g^3*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*B*a^3*
c^2*d*g^3*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*a^2*b*c^3*
g^3*i^3*x^2 + 3/2*A*a^3*c^2*d*g^3*i^3*x^2 + 1/420*B*b^3*d^3*g^3*i^3*n*(60*a
^7*log(b*x + a)/b^7 - 60*c^7*log(d*x + c)/d^7 - (10*(b^6*c*d^5 - a*b^5*d^6)
*x^6 - 12*(b^6*c^2*d^4 - a^2*b^4*d^6)*x^5 + 15*(b^6*c^3*d^3 - a^3*b^3*d^6)*
x^4 - 20*(b^6*c^4*d^2 - a^4*b^2*d^6)*x^3 + 30*(b^6*c^5*d - a^5*b*d^6)*x^2 -
```

60*(b^6*c^6 - a^6*d^6)*x)/(b^6*d^6)) - 1/120*B*b^3*c*d^2*g^3*i^3*n*(60*a^6*log(b*x + a)/b^6 - 60*c^6*log(d*x + c)/d^6 + (12*(b^5*c*d^4 - a*b^4*d^5)*x^5 - 15*(b^5*c^2*d^3 - a^2*b^3*d^5)*x^4 + 20*(b^5*c^3*d^2 - a^3*b^2*d^5)*x^3 - 30*(b^5*c^4*d - a^4*b*d^5)*x^2 + 60*(b^5*c^5 - a^5*d^5)*x)/(b^5*d^5)) - 1/120*B*a*b^2*d^3*g^3*i^3*n*(60*a^6*log(b*x + a)/b^6 - 60*c^6*log(d*x + c)/d^6 + (12*(b^5*c*d^4 - a*b^4*d^5)*x^5 - 15*(b^5*c^2*d^3 - a^2*b^3*d^5)*x^4 + 20*(b^5*c^3*d^2 - a^3*b^2*d^5)*x^3 - 30*(b^5*c^4*d - a^4*b*d^5)*x^2 + 60*(b^5*c^5 - a^5*d^5)*x)/(b^5*d^5)) + 1/20*B*b^3*c^2*d*g^3*i^3*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) + 3/20*B*a*b^2*c*d^2*g^3*i^3*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) + 1/20*B*a^2*b*d^3*g^3*i^3*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) - 1/24*B*b^3*c^3*g^3*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 3/8*B*a*b^2*c^2*d*g^3*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 3/8*B*a^2*b*c*d^2*g^3*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 1/24*B*a^3*d^3*g^3*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*a*b^2*c^3*g^3*i^3*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + 3/2*B*a^2*b*c^2*d*g^3*i^3*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + 1/2*B*a^3*c*d^2*g^3*i^3*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3/2*B*a^2*b*c^3*g^3*i^3*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) - 3/2*B*a^3*c^2*d*g^3*i^3*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*a^3*c^3*g^3*i^3*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*a^3*c^3*g^3*i^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a^3*c^3*g^3*i^3*x

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5934 vs. $2(453) = 906$.

Time = 2.50 (sec) , antiderivative size = 5934, normalized size of antiderivative = 12.44

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, a
lgorithm="giac")
```

```
[Out] -1/840*(6*(B*b^11*c^8*g^3*i^3*n - 8*B*a*b^10*c^7*d*g^3*i^3*n - 7*(b*x + a)*
B*b^10*c^8*d*g^3*i^3*n/(d*x + c) + 28*B*a^2*b^9*c^6*d^2*g^3*i^3*n + 56*(b*x
 + a)*B*a*b^9*c^7*d^2*g^3*i^3*n/(d*x + c) + 21*(b*x + a)^2*B*b^9*c^8*d^2*g^
3*i^3*n/(d*x + c)^2 - 56*B*a^3*b^8*c^5*d^3*g^3*i^3*n - 196*(b*x + a)*B*a^2*
b^8*c^6*d^3*g^3*i^3*n/(d*x + c) - 168*(b*x + a)^2*B*a*b^8*c^7*d^3*g^3*i^3*n
/(d*x + c)^2 - 35*(b*x + a)^3*B*b^8*c^8*d^3*g^3*i^3*n/(d*x + c)^3 + 70*B*a^
4*b^7*c^4*d^4*g^3*i^3*n + 392*(b*x + a)*B*a^3*b^7*c^5*d^4*g^3*i^3*n/(d*x +
c) + 588*(b*x + a)^2*B*a^2*b^7*c^6*d^4*g^3*i^3*n/(d*x + c)^2 + 280*(b*x + a
)^3*B*a*b^7*c^7*d^4*g^3*i^3*n/(d*x + c)^3 - 56*B*a^5*b^6*c^3*d^5*g^3*i^3*n
- 490*(b*x + a)*B*a^4*b^6*c^4*d^5*g^3*i^3*n/(d*x + c) - 1176*(b*x + a)^2*B*
a^3*b^6*c^5*d^5*g^3*i^3*n/(d*x + c)^2 - 980*(b*x + a)^3*B*a^2*b^6*c^6*d^5*g
^3*i^3*n/(d*x + c)^3 + 28*B*a^6*b^5*c^2*d^6*g^3*i^3*n + 392*(b*x + a)*B*a^5
*b^5*c^3*d^6*g^3*i^3*n/(d*x + c) + 1470*(b*x + a)^2*B*a^4*b^5*c^4*d^6*g^3*i
^3*n/(d*x + c)^2 + 1960*(b*x + a)^3*B*a^3*b^5*c^5*d^6*g^3*i^3*n/(d*x + c)^3
 - 8*B*a^7*b^4*c*d^7*g^3*i^3*n - 196*(b*x + a)*B*a^6*b^4*c^2*d^7*g^3*i^3*n/
(d*x + c) - 1176*(b*x + a)^2*B*a^5*b^4*c^3*d^7*g^3*i^3*n/(d*x + c)^2 - 2450
*(b*x + a)^3*B*a^4*b^4*c^4*d^7*g^3*i^3*n/(d*x + c)^3 + B*a^8*b^3*d^8*g^3*i^
3*n + 56*(b*x + a)*B*a^7*b^3*c*d^8*g^3*i^3*n/(d*x + c) + 588*(b*x + a)^2*B*
a^6*b^3*c^2*d^8*g^3*i^3*n/(d*x + c)^2 + 1960*(b*x + a)^3*B*a^5*b^3*c^3*d^8*
g^3*i^3*n/(d*x + c)^3 - 7*(b*x + a)*B*a^8*b^2*d^9*g^3*i^3*n/(d*x + c) - 168
*(b*x + a)^2*B*a^7*b^2*c*d^9*g^3*i^3*n/(d*x + c)^2 - 980*(b*x + a)^3*B*a^6*
b^2*c^2*d^9*g^3*i^3*n/(d*x + c)^3 + 21*(b*x + a)^2*B*a^8*b*d^10*g^3*i^3*n/(
d*x + c)^2 + 280*(b*x + a)^3*B*a^7*b*c*d^10*g^3*i^3*n/(d*x + c)^3 - 35*(b*x
 + a)^3*B*a^8*d^11*g^3*i^3*n/(d*x + c)^3)*log((b*x + a)/(d*x + c))/(b^7*d^4
 - 7*(b*x + a)*b^6*d^5/(d*x + c) + 21*(b*x + a)^2*b^5*d^6/(d*x + c)^2 - 35*
(b*x + a)^3*b^4*d^7/(d*x + c)^3 + 35*(b*x + a)^4*b^3*d^8/(d*x + c)^4 - 21*(
b*x + a)^5*b^2*d^9/(d*x + c)^5 + 7*(b*x + a)^6*b*d^10/(d*x + c)^6 - (b*x +
a)^7*d^11/(d*x + c)^7) + (6*(b*x + a)*B*b^13*c^8*d*g^3*i^3*n/(d*x + c) - 48
*(b*x + a)*B*a*b^12*c^7*d^2*g^3*i^3*n/(d*x + c) - 39*(b*x + a)^2*B*b^12*c^8
*d^2*g^3*i^3*n/(d*x + c)^2 + 168*(b*x + a)*B*a^2*b^11*c^6*d^3*g^3*i^3*n/(d*
x + c) + 312*(b*x + a)^2*B*a*b^11*c^7*d^3*g^3*i^3*n/(d*x + c)^2 + 107*(b*x
+ a)^3*B*b^11*c^8*d^3*g^3*i^3*n/(d*x + c)^3 - 336*(b*x + a)*B*a^3*b^10*c^5*
d^4*g^3*i^3*n/(d*x + c) - 1092*(b*x + a)^2*B*a^2*b^10*c^6*d^4*g^3*i^3*n/(d*
x + c)^2 - 856*(b*x + a)^3*B*a*b^10*c^7*d^4*g^3*i^3*n/(d*x + c)^3 - 107*(b*
```

x + a)^4*B*b^10*c^8*d^4*g^3*i^3*n/(d*x + c)^4 + 420*(b*x + a)*B*a^4*b^9*c^4
*d^5*g^3*i^3*n/(d*x + c) + 2184*(b*x + a)^2*B*a^3*b^9*c^5*d^5*g^3*i^3*n/(d*
x + c)^2 + 2996*(b*x + a)^3*B*a^2*b^9*c^6*d^5*g^3*i^3*n/(d*x + c)^3 + 856*(
b*x + a)^4*B*a*b^9*c^7*d^5*g^3*i^3*n/(d*x + c)^4 + 39*(b*x + a)^5*B*b^9*c^8
*d^5*g^3*i^3*n/(d*x + c)^5 - 336*(b*x + a)*B*a^5*b^8*c^3*d^6*g^3*i^3*n/(d*x
 + c) - 2730*(b*x + a)^2*B*a^4*b^8*c^4*d^6*g^3*i^3*n/(d*x + c)^2 - 5992*(b*
x + a)^3*B*a^3*b^8*c^5*d^6*g^3*i^3*n/(d*x + c)^3 - 2996*(b*x + a)^4*B*a^2*b
^8*c^6*d^6*g^3*i^3*n/(d*x + c)^4 - 312*(b*x + a)^5*B*a*b^8*c^7*d^6*g^3*i^3*
n/(d*x + c)^5 - 6*(b*x + a)^6*B*b^8*c^8*d^6*g^3*i^3*n/(d*x + c)^6 + 168*(b*
x + a)*B*a^6*b^7*c^2*d^7*g^3*i^3*n/(d*x + c) + 2184*(b*x + a)^2*B*a^5*b^7*c
^3*d^7*g^3*i^3*n/(d*x + c)^2 + 7490*(b*x + a)^3*B*a^4*b^7*c^4*d^7*g^3*i^3*n
/(d*x + c)^3 + 5992*(b*x + a)^4*B*a^3*b^7*c^5*d^7*g^3*i^3*n/(d*x + c)^4 + 1
092*(b*x + a)^5*B*a^2*b^7*c^6*d^7*g^3*i^3*n/(d*x + c)^5 + 48*(b*x + a)^6*B*
a*b^7*c^7*d^7*g^3*i^3*n/(d*x + c)^6 - 48*(b*x + a)*B*a^7*b^6*c*d^8*g^3*i^3*
n/(d*x + c) - 1092*(b*x + a)^2*B*a^6*b^6*c^2*d^8*g^3*i^3*n/(d*x + c)^2 - 59
92*(b*x + a)^3*B*a^5*b^6*c^3*d^8*g^3*i^3*n/(d*x + c)^3 - 7490*(b*x + a)^4*B
*a^4*b^6*c^4*d^8*g^3*i^3*n/(d*x + c)^4 - 2184*(b*x + a)^5*B*a^3*b^6*c^5*d^8
*g^3*i^3*n/(d*x + c)^5 - 168*(b*x + a)^6*B*a^2*b^6*c^6*d^8*g^3*i^3*n/(d*x +
 c)^6 + 6*(b*x + a)*B*a^8*b^5*d^9*g^3*i^3*n/(d*x + c) + 312*(b*x + a)^2*B*a
^7*b^5*c*d^9*g^3*i^3*n/(d*x + c)^2 + 2996*(b*x + a)^3*B*a^6*b^5*c^2*d^9*g^3
*i^3*n/(d*x + c)^3 + 5992*(b*x + a)^4*B*a^5*b^5*c^3*d^9*g^3*i^3*n/(d*x + c)
^4 + 2730*(b*x + a)^5*B*a^4*b^5*c^4*d^9*g^3*i^3*n/(d*x + c)^5 + 336*(b*x +
a)^6*B*a^3*b^5*c^5*d^9*g^3*i^3*n/(d*x + c)^6 - 39*(b*x + a)^2*B*a^8*b^4*d^1
0*g^3*i^3*n/(d*x + c)^2 - 856*(b*x + a)^3*B*a^7*b^4*c*d^10*g^3*i^3*n/(d*x +
 c)^3 - 2996*(b*x + a)^4*B*a^6*b^4*c^2*d^10*g^3*i^3*n/(d*x + c)^4 - 2184*(b
*x + a)^5*B*a^5*b^4*c^3*d^10*g^3*i^3*n/(d*x + c)^5 - 420*(b*x + a)^6*B*a^4*
b^4*c^4*d^10*g^3*i^3*n/(d*x + c)^6 + 107*(b*x + a)^3*B*a^8*b^3*d^11*g^3*i^3
*n/(d*x + c)^3 + 856*(b*x + a)^4*B*a^7*b^3*c*d^11*g^3*i^3*n/(d*x + c)^4 + 1
092*(b*x + a)^5*B*a^6*b^3*c^2*d^11*g^3*i^3*n/(d*x + c)^5 + 336*(b*x + a)^6*
B*a^5*b^3*c^3*d^11*g^3*i^3*n/(d*x + c)^6 - 107*(b*x + a)^4*B*a^8*b^2*d^12*g
^3*i^3*n/(d*x + c)^4 - 312*(b*x + a)^5*B*a^7*b^2*c*d^12*g^3*i^3*n/(d*x + c)
^5 - 168*(b*x + a)^6*B*a^6*b^2*c^2*d^12*g^3*i^3*n/(d*x + c)^6 + 39*(b*x + a
)^5*B*a^8*b*d^13*g^3*i^3*n/(d*x + c)^5 + 48*(b*x + a)^6*B*a^7*b*c*d^13*g^3*
i^3*n/(d*x + c)^6 - 6*(b*x + a)^6*B*a^8*d^14*g^3*i^3*n/(d*x + c)^6 + 6*B*b^
14*c^8*g^3*i^3*log(e) - 48*B*a*b^13*c^7*d*g^3*i^3*log(e) - 42*(b*x + a)*B*b
^13*c^8*d*g^3*i^3*log(e)/(d*x + c) + 168*B*a^2*b^12*c^6*d^2*g^3*i^3*log(e)
+ 336*(b*x + a)*B*a*b^12*c^7*d^2*g^3*i^3*log(e)/(d*x + c) + 126*(b*x + a)^2
*B*b^12*c^8*d^2*g^3*i^3*log(e)/(d*x + c)^2 - 336*B*a^3*b^11*c^5*d^3*g^3*i^3
log(e) - 1176(b*x + a)*B*a^2*b^11*c^6*d^3*g^3*i^3*log(e)/(d*x + c) - 1008
*(b*x + a)^2*B*a*b^11*c^7*d^3*g^3*i^3*log(e)/(d*x + c)^2 - 210*(b*x + a)^3*
B*b^11*c^8*d^3*g^3*i^3*log(e)/(d*x + c)^3 + 420*B*a^4*b^10*c^4*d^4*g^3*i^3*
log(e) + 2352*(b*x + a)*B*a^3*b^10*c^5*d^4*g^3*i^3*log(e)/(d*x + c) + 3528*
(b*x + a)^2*B*a^2*b^10*c^6*d^4*g^3*i^3*log(e)/(d*x + c)^2 + 1680*(b*x + a)^
3*B*a*b^10*c^7*d^4*g^3*i^3*log(e)/(d*x + c)^3 - 336*B*a^5*b^9*c^3*d^5*g^3*i
^3*log(e) - 2940*(b*x + a)*B*a^4*b^9*c^4*d^5*g^3*i^3*log(e)/(d*x + c) - 705

6*(b*x + a)^2*B*a^3*b^9*c^5*d^5*g^3*i^3*log(e)/(d*x + c)^2 - 5880*(b*x + a)^3*B*a^2*b^9*c^6*d^5*g^3*i^3*log(e)/(d*x + c)^3 + 168*B*a^6*b^8*c^2*d^6*g^3*i^3*log(e) + 2352*(b*x + a)*B*a^5*b^8*c^3*d^6*g^3*i^3*log(e)/(d*x + c) + 8820*(b*x + a)^2*B*a^4*b^8*c^4*d^6*g^3*i^3*log(e)/(d*x + c)^2 + 11760*(b*x + a)^3*B*a^3*b^8*c^5*d^6*g^3*i^3*log(e)/(d*x + c)^3 - 48*B*a^7*b^7*c*d^7*g^3*i^3*log(e) - 1176*(b*x + a)*B*a^6*b^7*c^2*d^7*g^3*i^3*log(e)/(d*x + c) - 7056*(b*x + a)^2*B*a^5*b^7*c^3*d^7*g^3*i^3*log(e)/(d*x + c)^2 - 14700*(b*x + a)^3*B*a^4*b^7*c^4*d^7*g^3*i^3*log(e)/(d*x + c)^3 + 6*B*a^8*b^6*d^8*g^3*i^3*log(e) + 336*(b*x + a)*B*a^7*b^6*c*d^8*g^3*i^3*log(e)/(d*x + c) + 3528*(b*x + a)^2*B*a^6*b^6*c^2*d^8*g^3*i^3*log(e)/(d*x + c)^2 + 11760*(b*x + a)^3*B*a^5*b^6*c^3*d^8*g^3*i^3*log(e)/(d*x + c)^3 - 42*(b*x + a)*B*a^8*b^5*d^9*g^3*i^3*log(e)/(d*x + c) - 1008*(b*x + a)^2*B*a^7*b^5*c*d^9*g^3*i^3*log(e)/(d*x + c)^2 - 5880*(b*x + a)^3*B*a^6*b^5*c^2*d^9*g^3*i^3*log(e)/(d*x + c)^3 + 126*(b*x + a)^2*B*a^8*b^4*d^10*g^3*i^3*log(e)/(d*x + c)^2 + 1680*(b*x + a)^3*B*a^7*b^4*c*d^10*g^3*i^3*log(e)/(d*x + c)^3 - 210*(b*x + a)^3*B*a^8*b^3*d^11*g^3*i^3*log(e)/(d*x + c)^3 + 6*A*b^14*c^8*g^3*i^3 - 48*A*a*b^13*c^7*d*g^3*i^3 - 42*(b*x + a)*A*b^13*c^8*d*g^3*i^3/(d*x + c) + 168*A*a^2*b^12*c^6*d^2*g^3*i^3 + 336*(b*x + a)*A*a*b^12*c^7*d^2*g^3*i^3/(d*x + c) + 126*(b*x + a)^2*A*b^12*c^8*d^2*g^3*i^3/(d*x + c)^2 - 336*A*a^3*b^11*c^5*d^3*g^3*i^3 - 1176*(b*x + a)*A*a^2*b^11*c^6*d^3*g^3*i^3/(d*x + c) - 1008*(b*x + a)^2*A*a*b^11*c^7*d^3*g^3*i^3/(d*x + c)^2 - 210*(b*x + a)^3*A*b^11*c^8*d^3*g^3*i^3/(d*x + c)^3 + 420*A*a^4*b^10*c^4*d^4*g^3*i^3 + 2352*(b*x + a)*A*a^3*b^10*c^5*d^4*g^3*i^3/(d*x + c) + 3528*(b*x + a)^2*A*a^2*b^10*c^6*d^4*g^3*i^3/(d*x + c)^2 + 1680*(b*x + a)^3*A*a*b^10*c^7*d^4*g^3*i^3/(d*x + c)^3 - 336*A*a^5*b^9*c^3*d^5*g^3*i^3 - 2940*(b*x + a)*A*a^4*b^9*c^4*d^5*g^3*i^3/(d*x + c) - 7056*(b*x + a)^2*A*a^3*b^9*c^5*d^5*g^3*i^3/(d*x + c)^2 - 5880*(b*x + a)^3*A*a^2*b^9*c^6*d^5*g^3*i^3/(d*x + c)^3 + 168*A*a^6*b^8*c^2*d^6*g^3*i^3 + 2352*(b*x + a)*A*a^5*b^8*c^3*d^6*g^3*i^3/(d*x + c) + 8820*(b*x + a)^2*A*a^4*b^8*c^4*d^6*g^3*i^3/(d*x + c)^2 + 11760*(b*x + a)^3*A*a^3*b^8*c^5*d^6*g^3*i^3/(d*x + c)^3 - 48*A*a^7*b^7*c*d^7*g^3*i^3 - 1176*(b*x + a)*A*a^6*b^7*c^2*d^7*g^3*i^3/(d*x + c) - 7056*(b*x + a)^2*A*a^5*b^7*c^3*d^7*g^3*i^3/(d*x + c)^2 - 14700*(b*x + a)^3*A*a^4*b^7*c^4*d^7*g^3*i^3/(d*x + c)^3 + 6*A*a^8*b^6*d^8*g^3*i^3 + 336*(b*x + a)*A*a^7*b^6*c*d^8*g^3*i^3/(d*x + c) + 3528*(b*x + a)^2*A*a^6*b^6*c^2*d^8*g^3*i^3/(d*x + c)^2 + 11760*(b*x + a)^3*A*a^5*b^6*c^3*d^8*g^3*i^3/(d*x + c)^3 - 42*(b*x + a)*A*a^8*b^5*d^9*g^3*i^3/(d*x + c) - 1008*(b*x + a)^2*A*a^7*b^5*c*d^9*g^3*i^3/(d*x + c)^2 - 5880*(b*x + a)^3*A*a^6*b^5*c^2*d^9*g^3*i^3/(d*x + c)^3 + 126*(b*x + a)^2*A*a^8*b^4*d^10*g^3*i^3/(d*x + c)^2 + 1680*(b*x + a)^3*A*a^7*b^4*c*d^10*g^3*i^3/(d*x + c)^3 - 210*(b*x + a)^3*A*a^8*b^3*d^11*g^3*i^3/(d*x + c)^3)/(b^10*d^4 - 7*(b*x + a)*b^9*d^5/(d*x + c) + 21*(b*x + a)^2*b^8*d^6/(d*x + c)^2 - 35*(b*x + a)^3*b^7*d^7/(d*x + c)^3 + 35*(b*x + a)^4*b^6*d^8/(d*x + c)^4 - 21*(b*x + a)^5*b^5*d^9/(d*x + c)^5 + 7*(b*x + a)^6*b^4*d^10/(d*x + c)^6 - (b*x + a)^7*b^3*d^11/(d*x + c)^7) + 6*(B*b^8*c^8*g^3*i^3*n - 8*B*a*b^7*c^7*d*g^3*i^3*n + 28*B*a^2*b^6*c^6*d^2*g^3*i^3*n - 56*B*a^3*b^5*c^5*d^3*g^3*i^3*n + 70*B*a^4*b^4*c^4*d^4*g^3*i^3*n - 56*B*a^5*b^3*c^3*d^5*g^3*i^3*n + 28*B*a^6*b^2*c^2*d^6*g^3*i^

```
3*n - 8*B*a^7*b*c*d^7*g^3*i^3*n + B*a^8*d^8*g^3*i^3*n)*log(b - (b*x + a)*d/
(d*x + c))/(b^4*d^4) - 6*(B*b^8*c^8*g^3*i^3*n - 8*B*a*b^7*c^7*d*g^3*i^3*n +
 28*B*a^2*b^6*c^6*d^2*g^3*i^3*n - 56*B*a^3*b^5*c^5*d^3*g^3*i^3*n + 70*B*a^4
*b^4*c^4*d^4*g^3*i^3*n - 56*B*a^5*b^3*c^3*d^5*g^3*i^3*n + 28*B*a^6*b^2*c^2*
d^6*g^3*i^3*n - 8*B*a^7*b*c*d^7*g^3*i^3*n + B*a^8*d^8*g^3*i^3*n)*log((b*x +
 a)/(d*x + c))/(b^4*d^4))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 3.06 (sec) , antiderivative size = 4476, normalized size of antiderivative = 9.38

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

```
[In] int((a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),
x)
```

```
[Out] x^4*((g^3*i^3*(20*A*a^3*d^3 + 20*A*b^3*c^3 + 3*B*a^3*d^3*n - 3*B*b^3*c^3*n
+ 120*A*a*b^2*c^2*d + 120*A*a^2*b*c*d^2 - 6*B*a*b^2*c^2*d*n + 6*B*a^2*b*c*d
^2*n))/20 + ((140*a*d + 140*b*c)*((((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c +
 B*a*d*n - B*b*c*n))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140)*(140*
a*d + 140*b*c))/(140*b*d) - (b*d*g^3*i^3*(12*A*a^2*d^2 + 12*A*b^2*c^2 + B*a
^2*d^2*n - B*b^2*c^2*n + 32*A*a*b*c*d))/2 + A*a*b^2*c*d^2*g^3*i^3))/(560*b*
d) - (a*c*((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d*n - B*b*c*n))/7 -
(A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140))/(4*b*d)) + x^3*((g^3*i^3*(4*A
*a^4*d^4 + 4*A*b^4*c^4 + B*a^4*d^4*n - B*b^4*c^4*n + 144*A*a^2*b^2*c^2*d^2
+ 64*A*a*b^3*c^3*d + 64*A*a^3*b*c*d^3 - 8*B*a*b^3*c^3*d*n + 8*B*a^3*b*c*d^3
*n))/(12*b*d) - ((140*a*d + 140*b*c)*((g^3*i^3*(20*A*a^3*d^3 + 20*A*b^3*c^3
 + 3*B*a^3*d^3*n - 3*B*b^3*c^3*n + 120*A*a*b^2*c^2*d + 120*A*a^2*b*c*d^2 -
6*B*a*b^2*c^2*d*n + 6*B*a^2*b*c*d^2*n))/5 + ((140*a*d + 140*b*c)*((((b^2*d^
2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d*n - B*b*c*n))/7 - (A*b^2*d^2*g^3*i^3
*(140*a*d + 140*b*c))/140)*(140*a*d + 140*b*c))/(140*b*d) - (b*d*g^3*i^3*(1
2*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + 32*A*a*b*c*d))/2 +
 A*a*b^2*c*d^2*g^3*i^3))/(140*b*d) - (a*c*((b^2*d^2*g^3*i^3*(28*A*a*d + 28*
A*b*c + B*a*d*n - B*b*c*n))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140
))/(b*d)))/(420*b*d) + (a*c*((((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*
d*n - B*b*c*n))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140)*(140*a*d +
 140*b*c))/(140*b*d) - (b*d*g^3*i^3*(12*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^
2*n - B*b^2*c^2*n + 32*A*a*b*c*d))/2 + A*a*b^2*c*d^2*g^3*i^3))/(3*b*d)) + x
^6*((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d*n - B*b*c*n))/42 - (A*b^2
*d^2*g^3*i^3*(140*a*d + 140*b*c))/840) - x^2*(((140*a*d + 140*b*c)*((g^3*i^
3*(4*A*a^4*d^4 + 4*A*b^4*c^4 + B*a^4*d^4*n - B*b^4*c^4*n + 144*A*a^2*b^2*c^
2*d^2 + 64*A*a*b^3*c^3*d + 64*A*a^3*b*c*d^3 - 8*B*a*b^3*c^3*d*n + 8*B*a^3*b
*c*d^3*n))/(4*b*d) - ((140*a*d + 140*b*c)*((g^3*i^3*(20*A*a^3*d^3 + 20*A*b^
3*c^3 + 3*B*a^3*d^3*n - 3*B*b^3*c^3*n + 120*A*a*b^2*c^2*d + 120*A*a^2*b*c*d
```

```
^2 - 6*B*a*b^2*c^2*d*n + 6*B*a^2*b*c*d^2*n))/5 + ((140*a*d + 140*b*c)*((((b
^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d*n - B*b*c*n))/7 - (A*b^2*d^2*g^
3*i^3*(140*a*d + 140*b*c))/140)*(140*a*d + 140*b*c))/(140*b*d) - (b*d*g^3*i
^3*(12*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + 32*A*a*b*c*d)
)/2 + A*a*b^2*c*d^2*g^3*i^3))/(140*b*d) - (a*c*((b^2*d^2*g^3*i^3*(28*A*a*d
+ 28*A*b*c + B*a*d*n - B*b*c*n))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c)
)/140))/(b*d)))/(140*b*d) + (a*c*((((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c +
 B*a*d*n - B*b*c*n))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140)*(140*
a*d + 140*b*c))/(140*b*d) - (b*d*g^3*i^3*(12*A*a^2*d^2 + 12*A*b^2*c^2 + B*a
^2*d^2*n - B*b^2*c^2*n + 32*A*a*b*c*d))/2 + A*a*b^2*c*d^2*g^3*i^3))/(b*d)))
/(280*b*d) + (a*c*((g^3*i^3*(20*A*a^3*d^3 + 20*A*b^3*c^3 + 3*B*a^3*d^3*n -
3*B*b^3*c^3*n + 120*A*a*b^2*c^2*d + 120*A*a^2*b*c*d^2 - 6*B*a*b^2*c^2*d*n +
 6*B*a^2*b*c*d^2*n))/5 + ((140*a*d + 140*b*c)*((((b^2*d^2*g^3*i^3*(28*A*a*d
 + 28*A*b*c + B*a*d*n - B*b*c*n))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c
))/140)*(140*a*d + 140*b*c))/(140*b*d) - (b*d*g^3*i^3*(12*A*a^2*d^2 + 12*A*
b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + 32*A*a*b*c*d))/2 + A*a*b^2*c*d^2*g^3*
i^3))/(140*b*d) - (a*c*((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d*n - B
*b*c*n))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140))/(b*d)))/(2*b*d)
- (a*c*g^3*i^3*(4*A*a^3*d^3 + 4*A*b^3*c^3 + B*a^3*d^3*n - B*b^3*c^3*n + 24*
A*a*b^2*c^2*d + 24*A*a^2*b*c*d^2 - 2*B*a*b^2*c^2*d*n + 2*B*a^2*b*c*d^2*n))/
(2*b*d)) - x^5*((((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d*n - B*b*c*n
))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140)*(140*a*d + 140*b*c))/(7
00*b*d) - (b*d*g^3*i^3*(12*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c
^2*n + 32*A*a*b*c*d))/10 + (A*a*b^2*c*d^2*g^3*i^3)/5) + x*(((140*a*d + 140*
b*c)*(((140*a*d + 140*b*c)*((g^3*i^3*(4*A*a^4*d^4 + 4*A*b^4*c^4 + B*a^4*d^4
*n - B*b^4*c^4*n + 144*A*a^2*b^2*c^2*d^2 + 64*A*a*b^3*c^3*d + 64*A*a^3*b*c*
d^3 - 8*B*a*b^3*c^3*d*n + 8*B*a^3*b*c*d^3*n))/(4*b*d) - ((140*a*d + 140*b*c
)*((g^3*i^3*(20*A*a^3*d^3 + 20*A*b^3*c^3 + 3*B*a^3*d^3*n - 3*B*b^3*c^3*n +
120*A*a*b^2*c^2*d + 120*A*a^2*b*c*d^2 - 6*B*a*b^2*c^2*d*n + 6*B*a^2*b*c*d^2
*n))/5 + ((140*a*d + 140*b*c)*((((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*
a*d*n - B*b*c*n))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140)*(140*a*d
 + 140*b*c))/(140*b*d) - (b*d*g^3*i^3*(12*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*
d^2*n - B*b^2*c^2*n + 32*A*a*b*c*d))/2 + A*a*b^2*c*d^2*g^3*i^3))/(140*b*d)
- (a*c*((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d*n - B*b*c*n))/7 - (A*
b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140))/(b*d)))/(140*b*d) + (a*c*((((b^2
*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d*n - B*b*c*n))/7 - (A*b^2*d^2*g^3*
i^3*(140*a*d + 140*b*c))/140)*(140*a*d + 140*b*c))/(140*b*d) - (b*d*g^3*i^3
*(12*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + 32*A*a*b*c*d))/
2 + A*a*b^2*c*d^2*g^3*i^3))/(b*d)))/(140*b*d) + (a*c*((g^3*i^3*(20*A*a^3*d^
3 + 20*A*b^3*c^3 + 3*B*a^3*d^3*n - 3*B*b^3*c^3*n + 120*A*a*b^2*c^2*d + 120*
A*a^2*b*c*d^2 - 6*B*a*b^2*c^2*d*n + 6*B*a^2*b*c*d^2*n))/5 + ((140*a*d + 140
*b*c)*((((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d*n - B*b*c*n))/7 - (A
*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140)*(140*a*d + 140*b*c))/(140*b*d) -
 (b*d*g^3*i^3*(12*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + 32
*A*a*b*c*d))/2 + A*a*b^2*c*d^2*g^3*i^3))/(140*b*d) - (a*c*((b^2*d^2*g^3*i^3
```

```
*(28*A*a*d + 28*A*b*c + B*a*d*n - B*b*c*n))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d
 + 140*b*c))/140))/(b*d)))/(b*d) - (a*c*g^3*i^3*(4*A*a^3*d^3 + 4*A*b^3*c^3
+ B*a^3*d^3*n - B*b^3*c^3*n + 24*A*a*b^2*c^2*d + 24*A*a^2*b*c*d^2 - 2*B*a*b
^2*c^2*d*n + 2*B*a^2*b*c*d^2*n))/(b*d)))/(140*b*d) - (a*c*((g^3*i^3*(4*A*a^
4*d^4 + 4*A*b^4*c^4 + B*a^4*d^4*n - B*b^4*c^4*n + 144*A*a^2*b^2*c^2*d^2 + 6
4*A*a*b^3*c^3*d + 64*A*a^3*b*c*d^3 - 8*B*a*b^3*c^3*d*n + 8*B*a^3*b*c*d^3*n)
)/(4*b*d) - ((140*a*d + 140*b*c)*((g^3*i^3*(20*A*a^3*d^3 + 20*A*b^3*c^3 + 3
*B*a^3*d^3*n - 3*B*b^3*c^3*n + 120*A*a*b^2*c^2*d + 120*A*a^2*b*c*d^2 - 6*B*
a*b^2*c^2*d*n + 6*B*a^2*b*c*d^2*n))/5 + ((140*a*d + 140*b*c)*((((b^2*d^2*g^
3*i^3*(28*A*a*d + 28*A*b*c + B*a*d*n - B*b*c*n))/7 - (A*b^2*d^2*g^3*i^3*(14
0*a*d + 140*b*c))/140)*(140*a*d + 140*b*c))/(140*b*d) - (b*d*g^3*i^3*(12*A*
a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + 32*A*a*b*c*d))/2 + A*a
*b^2*c*d^2*g^3*i^3))/(140*b*d) - (a*c*((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*
c + B*a*d*n - B*b*c*n))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140))/(
b*d)))/(140*b*d) + (a*c*((((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d*n
- B*b*c*n))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140)*(140*a*d + 140
*b*c))/(140*b*d) - (b*d*g^3*i^3*(12*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^2*n
- B*b^2*c^2*n + 32*A*a*b*c*d))/2 + A*a*b^2*c*d^2*g^3*i^3))/(b*d)))/(b*d) +
(a^2*c^2*g^3*i^3*(12*A*a^2*d^2 + 12*A*b^2*c^2 + 3*B*a^2*d^2*n - 3*B*b^2*c^2
*n + 32*A*a*b*c*d))/(2*b*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*g^3*i^3*x
^4*(a^3*d^3 + b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2))/4 + B*a^3*c^3*g^3*i
^3*x + (B*b^3*d^3*g^3*i^3*x^7)/7 + (3*B*a^2*c^2*g^3*i^3*x^2*(a*d + b*c))/2
+ (B*b^2*d^2*g^3*i^3*x^6*(a*d + b*c))/2 + B*a*c*g^3*i^3*x^3*(a^2*d^2 + b^2*
c^2 + 3*a*b*c*d) + (3*B*b*d*g^3*i^3*x^5*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/5)
 - (log(a + b*x)*(B*a^7*d^3*g^3*i^3*n - 35*B*a^4*b^3*c^3*g^3*i^3*n + 21*B*a
^5*b^2*c^2*d*g^3*i^3*n - 7*B*a^6*b*c*d^2*g^3*i^3*n))/(140*b^4) + (log(c + d
*x)*(B*b^3*c^7*g^3*i^3*n - 35*B*a^3*c^4*d^3*g^3*i^3*n + 21*B*a^2*b*c^5*d^2*
g^3*i^3*n - 7*B*a*b^2*c^6*d*g^3*i^3*n))/(140*d^4) + (A*b^3*d^3*g^3*i^3*x^7)
/7
```

3.128 $\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal result

Integrand size = 43, antiderivative size = 387

$$\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = -\frac{B(bc-ad)^5g^2i^3nx}{60b^3d^2} - \frac{B(bc-ad)^4g^2i^3n(c+dx)^2}{120b^2d^3} - \frac{B(bc-ad)^3g^2i^3n(c+dx)^3}{180bd^3} + \frac{7B(bc-ad)^2g^2i^3n(c+dx)^4}{120d^3} - \frac{bB(bc-ad)g^2i^3n(c+dx)^5}{30d^3} + \frac{(bc-ad)^2g^2i^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^3} - \frac{2b(bc-ad)g^2i^3(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^3} + \frac{b^2g^2i^3(c+dx)^6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^3} - \frac{B(bc-ad)^6g^2i^3n\log\left(\frac{a+bx}{c+dx}\right)}{60b^4d^3} - \frac{B(bc-ad)^6g^2i^3n\log(c+dx)}{60b^4d^3}$$

```
[Out] -1/60*B*(-a*d+b*c)^5*g^2*i^3*n*x/b^3/d^2-1/120*B*(-a*d+b*c)^4*g^2*i^3*n*(d*
x+c)^2/b^2/d^3-1/180*B*(-a*d+b*c)^3*g^2*i^3*n*(d*x+c)^3/b/d^3+7/120*B*(-a*d
+b*c)^2*g^2*i^3*n*(d*x+c)^4/d^3-1/30*b*B*(-a*d+b*c)*g^2*i^3*n*(d*x+c)^5/d^3
+1/4*(-a*d+b*c)^2*g^2*i^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^3-2/5
*b*(-a*d+b*c)*g^2*i^3*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^3+1/6*b^2
*g^2*i^3*(d*x+c)^6*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^3-1/60*B*(-a*d+b*c)^6*
g^2*i^3*n*ln((b*x+a)/(d*x+c))/b^4/d^3-1/60*B*(-a*d+b*c)^6*g^2*i^3*n*ln(d*x+
c)/b^4/d^3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2561, 45, 2382, 12, 907}

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=\frac{b^2g^2i^3(c+dx)^6\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6d^3}+\frac{g^2i^3(c+dx)^4(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^3}-\frac{2bg^2i^3(c+dx)^5(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^3}-\frac{Bg^2i^3n(bc-ad)^6\log\left(\frac{a+bx}{c+dx}\right)}{60b^4d^3}-\frac{Bg^2i^3n(bc-ad)^6\log(c+dx)}{60b^4d^3}-\frac{Bg^2i^3nx(bc-ad)^5}{60b^3d^2}-\frac{Bg^2i^3n(c+dx)^2(bc-ad)^4}{120b^2d^3}-\frac{Bg^2i^3n(c+dx)^3(bc-ad)^3}{180bd^3}+\frac{7Bg^2i^3n(c+dx)^4(bc-ad)^2}{120d^3}-\frac{bBg^2i^3n(c+dx)^5(bc-ad)}{30d^3}$$

```
[In] Int[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),
x]
```

```
[Out] -1/60*(B*(b*c - a*d)^5*g^2*i^3*n*x)/(b^3*d^2) - (B*(b*c - a*d)^4*g^2*i^3*n*
(c + d*x)^2)/(120*b^2*d^3) - (B*(b*c - a*d)^3*g^2*i^3*n*(c + d*x)^3)/(180*b
*d^3) + (7*B*(b*c - a*d)^2*g^2*i^3*n*(c + d*x)^4)/(120*d^3) - (b*B*(b*c - a
*d)*g^2*i^3*n*(c + d*x)^5)/(30*d^3) + ((b*c - a*d)^2*g^2*i^3*(c + d*x)^4*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*d^3) - (2*b*(b*c - a*d)*g^2*i^3*(c
 + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*d^3) + (b^2*g^2*i^3*(c
 + d*x)^6*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*d^3) - (B*(b*c - a*d)^
6*g^2*i^3*n*Log[(a + b*x)/(c + d*x)])/(60*b^4*d^3) - (B*(b*c - a*d)^6*g^2*i
^3*n*Log[c + d*x])/(60*b^4*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((bc-ad)^6 g^2 i^3\right) \text{Subst}\left(\int \frac{x^2(A+B\log(ex^n))}{(b-dx)^7}\,dx, x, \frac{a+bx}{c+dx}\right) \\
&= \frac{(bc-ad)^2 g^2 i^3 (c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^3} \\
&\quad - \frac{2b(bc-ad) g^2 i^3 (c+dx)^5 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^3} \\
&\quad + \frac{b^2 g^2 i^3 (c+dx)^6 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^3} \\
&\quad - \left(B(bc-ad)^6 g^2 i^3 n\right) \text{Subst}\left(\int \frac{b^2-6bdx+15d^2x^2}{60d^3x(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)
\end{aligned}$$

$$= \frac{(bc-ad)^2 g^2 i^3 (c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^3} - \frac{2b(bc-ad) g^2 i^3 (c+dx)^5 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^3} + \frac{b^2 g^2 i^3 (c+dx)^6 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^3} - \frac{(B(bc-ad)^6 g^2 i^3 n)\,\text{Subst}\left(\int \frac{b^2-6bdx+15d^2x^2}{x(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{60d^3}$$

$$= \frac{(bc-ad)^2 g^2 i^3 (c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^3} - \frac{2b(bc-ad) g^2 i^3 (c+dx)^5 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^3} + \frac{b^2 g^2 i^3 (c+dx)^6 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^3} - \frac{(B(bc-ad)^6 g^2 i^3 n)\,\text{Subst}\left(\int \left(\frac{1}{b^4x}+\frac{10bd}{(b-dx)^6}-\frac{14d}{(b-dx)^5}+\frac{d}{b(b-dx)^4}+\frac{d}{b^2(b-dx)^3}+\frac{d}{b^3(b-dx)^2}+\frac{d}{b^4(b-dx)}\right)\right.}{60d^3}$$

$$= -\frac{B(bc-ad)^5 g^2 i^3 nx}{60b^3d^2} - \frac{B(bc-ad)^4 g^2 i^3 n (c+dx)^2}{120b^2d^3} - \frac{B(bc-ad)^3 g^2 i^3 n (c+dx)^3}{180bd^3} + \frac{7B(bc-ad)^2 g^2 i^3 n (c+dx)^4}{120d^3} - \frac{bB(bc-ad) g^2 i^3 n (c+dx)^5}{30d^3} + \frac{(bc-ad)^2 g^2 i^3 (c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^3} - \frac{2b(bc-ad) g^2 i^3 (c+dx)^5 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^3} + \frac{b^2 g^2 i^3 (c+dx)^6 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^3} - \frac{B(bc-ad)^6 g^2 i^3 n \log\left(\frac{a+bx}{c+dx}\right)}{60b^4d^3} - \frac{B(bc-ad)^6 g^2 i^3 n \log(c+dx)}{60b^4d^3}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.14

$$\int (ag+bgx)^2 (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \frac{g^2 i^3 \left(-15B(bc-ad)^3 n (6bd(bc-ad)^2 x + 3b^2(bc-ad)(c+dx)^2 + 2b^3(c+dx)^3 + 6(bc-ad)^3 \log(a+bx)) \right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]),x]
```

```
[Out] (g^2*i^3*(-15*B*(b*c - a*d)^3*n*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*
(c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]) + 12*B*(b*c
 - a*d)^2*n*(12*b*d*(b*c - a*d)^3*x + 6*b^2*(b*c - a*d)^2*(c + d*x)^2 + 4*b
^3*(b*c - a*d)*(c + d*x)^3 + 3*b^4*(c + d*x)^4 + 12*(b*c - a*d)^4*Log[a + b
*x]) - B*(b*c - a*d)*n*(60*b*d*(b*c - a*d)^4*x + 30*b^2*(b*c - a*d)^3*(c +
d*x)^2 + 20*b^3*(b*c - a*d)^2*(c + d*x)^3 + 15*b^4*(b*c - a*d)*(c + d*x)^4
+ 12*b^5*(c + d*x)^5 + 60*(b*c - a*d)^5*Log[a + b*x]) + 90*b^4*(b*c - a*d)^
2*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 144*b^5*(b*c - a*d)*
(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 60*b^6*(c + d*x)^6*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n])))/(360*b^4*d^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1720 vs. $2(367) = 734$.

Time = 25.41 (sec) , antiderivative size = 1721, normalized size of antiderivative = 4.45

method	result	size
parallelrisch	Expression too large to display	1721

```
[In] int((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/360*(12*B*x^5*a*b^5*d^6*g^2*i^3*n^2+540*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a
*b^5*c*d^5*g^2*i^3*n+360*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*c*d^5*g^2*
i^3*n+720*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c^2*d^4*g^2*i^3*n+540*B*x^2
*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*c^2*d^4*g^2*i^3*n+360*B*x^2*ln(e*((b*x+a
)/(d*x+c))^n)*a*b^5*c^3*d^3*g^2*i^3*n+360*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2
*b^4*c^3*d^3*g^2*i^3*n+33*B*a^5*b*c*d^5*g^2*i^3*n^2-72*B*a^4*b^2*c^2*d^4*g^
2*i^3*n^2-150*B*a^3*b^3*c^3*d^3*g^2*i^3*n^2+168*B*a^2*b^4*c^4*d^2*g^2*i^3*n
^2+33*B*a*b^5*c^5*d*g^2*i^3*n^2-900*A*a^3*b^3*c^3*d^3*g^2*i^3*n-720*A*a^2*b
^4*c^4*d^2*g^2*i^3*n+90*B*x^2*a^2*b^4*c^2*d^4*g^2*i^3*n^2-102*B*x^2*a*b^5*c
^3*d^3*g^2*i^3*n^2+540*A*x^2*a^2*b^4*c^2*d^4*g^2*i^3*n+360*A*x^2*a*b^5*c^3*
d^3*g^2*i^3*n-36*B*x*a^4*b^2*c*d^5*g^2*i^3*n^2+90*B*x*a^3*b^3*c^2*d^4*g^2*i
^3*n^2-30*B*x*a^2*b^4*c^3*d^3*g^2*i^3*n^2-36*B*x*a*b^5*c^4*d^2*g^2*i^3*n^2+
360*A*x*a^2*b^4*c^3*d^3*g^2*i^3*n+90*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*c^
4*d^2*g^2*i^3*n+144*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*d^6*g^2*i^3*n+216
*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c*d^5*g^2*i^3*n+90*B*x^4*ln(e*((b*x+a)
/(d*x+c))^n)*a^2*b^4*d^6*g^2*i^3*n+270*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^6*
c^2*d^4*g^2*i^3*n+18*B*x^4*a*b^5*c*d^5*g^2*i^3*n^2+540*A*x^4*a*b^5*c*d^5*g^
2*i^3*n+120*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c^3*d^3*g^2*i^3*n+78*B*x^3*
a^2*b^4*c*d^5*g^2*i^3*n^2-42*B*x^3*a*b^5*c^2*d^4*g^2*i^3*n^2+360*A*x^3*a^2*
b^4*c*d^5*g^2*i^3*n+720*A*x^3*a*b^5*c^2*d^4*g^2*i^3*n+18*B*x^2*a^3*b^3*c*d^
5*g^2*i^3*n^2-36*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c^5*d*g^2*i^3*n+36*B*ln(
b*x+a)*a^5*b*c*d^5*g^2*i^3*n^2-90*B*ln(b*x+a)*a^4*b^2*c^2*d^4*g^2*i^3*n^2+1
20*B*ln(b*x+a)*a^3*b^3*c^3*d^3*g^2*i^3*n^2-90*B*ln(b*x+a)*a^2*b^4*c^4*d^2*g
```

```
^2*i^3*n^2+36*B*ln(b*x+a)*a*b^5*c^5*d*g^2*i^3*n^2+60*A*x^6*b^6*d^6*g^2*i^3*
n+6*B*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c^6*g^2*i^3*n-6*B*ln(b*x+a)*b^6*c^6*g^2
*i^3*n^2-6*B*ln(b*x+a)*a^6*d^6*g^2*i^3*n^2-6*B*a^6*d^6*g^2*i^3*n^2-6*B*b^6*
c^6*g^2*i^3*n^2-12*B*x^5*b^6*c*d^5*g^2*i^3*n^2+144*A*x^5*a*b^5*d^6*g^2*i^3*
n+216*A*x^5*b^6*c*d^5*g^2*i^3*n+21*B*x^4*a^2*b^4*d^6*g^2*i^3*n^2-39*B*x^4*b
^6*c^2*d^4*g^2*i^3*n^2+90*A*x^4*a^2*b^4*d^6*g^2*i^3*n+270*A*x^4*b^6*c^2*d^4
*g^2*i^3*n+2*B*x^3*a^3*b^3*d^6*g^2*i^3*n^2-38*B*x^3*b^6*c^3*d^3*g^2*i^3*n^2
+120*A*x^3*b^6*c^3*d^3*g^2*i^3*n-3*B*x^2*a^4*b^2*d^6*g^2*i^3*n^2-3*B*x^2*b^
6*c^4*d^2*g^2*i^3*n^2+6*B*x*a^5*b*d^6*g^2*i^3*n^2+6*B*x*b^6*c^5*d*g^2*i^3*n
^2+60*B*x^6*ln(e*((b*x+a)/(d*x+c))^n)*b^6*d^6*g^2*i^3*n)/d^3/b^4/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1075 vs. $2(367) = 734$.

Time = 0.66 (sec) , antiderivative size = 1075, normalized size of antiderivative = 2.78

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{60\,Ab^6d^6g^2i^3x^6+6\,(20\,Ba^3b^3c^3d^3-15\,Ba^4b^2c^2d^4+6\,Ba^5bcd^5-Ba^6d^6)g^2i^3n\log(bx+a)-6\,(Bb^6c^6-6\,B}{}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, a
lgorithm="fricas")
```

```
[Out] 1/360*(60*A*b^6*d^6*g^2*i^3*x^6 + 6*(20*B*a^3*b^3*c^3*d^3 - 15*B*a^4*b^2*c^
2*d^4 + 6*B*a^5*b*c*d^5 - B*a^6*d^6)*g^2*i^3*n*log(b*x + a) - 6*(B*b^6*c^6
- 6*B*a*b^5*c^5*d + 15*B*a^2*b^4*c^4*d^2)*g^2*i^3*n*log(d*x + c) - 12*((B*b
^6*c*d^5 - B*a*b^5*d^6)*g^2*i^3*n - 6*(3*A*b^6*c*d^5 + 2*A*a*b^5*d^6)*g^2*i
^3)*x^5 - 3*((13*B*b^6*c^2*d^4 - 6*B*a*b^5*c*d^5 - 7*B*a^2*b^4*d^6)*g^2*i^3
*n - 30*(3*A*b^6*c^2*d^4 + 6*A*a*b^5*c*d^5 + A*a^2*b^4*d^6)*g^2*i^3)*x^4 -
2*((19*B*b^6*c^3*d^3 + 21*B*a*b^5*c^2*d^4 - 39*B*a^2*b^4*c*d^5 - B*a^3*b^3*
d^6)*g^2*i^3*n - 60*(A*b^6*c^3*d^3 + 6*A*a*b^5*c^2*d^4 + 3*A*a^2*b^4*c*d^5)
*g^2*i^3)*x^3 - 3*((B*b^6*c^4*d^2 + 34*B*a*b^5*c^3*d^3 - 30*B*a^2*b^4*c^2*d
^4 - 6*B*a^3*b^3*c*d^5 + B*a^4*b^2*d^6)*g^2*i^3*n - 60*(2*A*a*b^5*c^3*d^3 +
 3*A*a^2*b^4*c^2*d^4)*g^2*i^3)*x^2 + 6*(60*A*a^2*b^4*c^3*d^3*g^2*i^3 + (B*b
^6*c^5*d - 6*B*a*b^5*c^4*d^2 - 5*B*a^2*b^4*c^3*d^3 + 15*B*a^3*b^3*c^2*d^4 -
 6*B*a^4*b^2*c*d^5 + B*a^5*b*d^6)*g^2*i^3*n)*x + 6*(10*B*b^6*d^6*g^2*i^3*x^
6 + 60*B*a^2*b^4*c^3*d^3*g^2*i^3*x + 12*(3*B*b^6*c*d^5 + 2*B*a*b^5*d^6)*g^2
*i^3*x^5 + 15*(3*B*b^6*c^2*d^4 + 6*B*a*b^5*c*d^5 + B*a^2*b^4*d^6)*g^2*i^3*x
^4 + 20*(B*b^6*c^3*d^3 + 6*B*a*b^5*c^2*d^4 + 3*B*a^2*b^4*c*d^5)*g^2*i^3*x^3
 + 30*(2*B*a*b^5*c^3*d^3 + 3*B*a^2*b^4*c^2*d^4)*g^2*i^3*x^2)*log(e) + 6*(10
*B*b^6*d^6*g^2*i^3*n*x^6 + 60*B*a^2*b^4*c^3*d^3*g^2*i^3*n*x + 12*(3*B*b^6*c
*d^5 + 2*B*a*b^5*d^6)*g^2*i^3*n*x^5 + 15*(3*B*b^6*c^2*d^4 + 6*B*a*b^5*c*d^5
 + B*a^2*b^4*d^6)*g^2*i^3*n*x^4 + 20*(B*b^6*c^3*d^3 + 6*B*a*b^5*c^2*d^4 + 3
```

```
*B*a^2*b^4*c*d^5)*g^2*i^3*n*x^3 + 30*(2*B*a*b^5*c^3*d^3 + 3*B*a^2*b^4*c^2*d
^4)*g^2*i^3*n*x^2)*log((b*x + a)/(d*x + c)))/(b^4*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**2*(d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1978 vs. $2(367) = 734$.

Time = 0.25 (sec) , antiderivative size = 1978, normalized size of antiderivative = 5.11

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, a
lgorithm="maxima")
[Out] 1/6*B*b^2*d^3*g^2*i^3*x^6*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/6*A*b^
2*d^3*g^2*i^3*x^6 + 3/5*B*b^2*c*d^2*g^2*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d
*x + c))^n) + 2/5*B*a*b*d^3*g^2*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))
^n) + 3/5*A*b^2*c*d^2*g^2*i^3*x^5 + 2/5*A*a*b*d^3*g^2*i^3*x^5 + 3/4*B*b^2*c
^2*d*g^2*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*B*a*b*c*d^2*g
^2*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*B*a^2*d^3*g^2*i^3*x
^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/4*A*b^2*c^2*d*g^2*i^3*x^4 + 3
/2*A*a*b*c*d^2*g^2*i^3*x^4 + 1/4*A*a^2*d^3*g^2*i^3*x^4 + 1/3*B*b^2*c^3*g^2*
i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*B*a*b*c^2*d*g^2*i^3*x^3*
log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + B*a^2*c*d^2*g^2*i^3*x^3*log(e*(b*x
/(d*x + c) + a/(d*x + c))^n) + 1/3*A*b^2*c^3*g^2*i^3*x^3 + 2*A*a*b*c^2*d*g^
2*i^3*x^3 + A*a^2*c*d^2*g^2*i^3*x^3 + B*a*b*c^3*g^2*i^3*x^2*log(e*(b*x/(d*x
 + c) + a/(d*x + c))^n) + 3/2*B*a^2*c^2*d*g^2*i^3*x^2*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n) + A*a*b*c^3*g^2*i^3*x^2 + 3/2*A*a^2*c^2*d*g^2*i^3*x^2 - 1
/360*B*b^2*d^3*g^2*i^3*n*(60*a^6*log(b*x + a)/b^6 - 60*c^6*log(d*x + c)/d^6
 + (12*(b^5*c*d^4 - a*b^4*d^5)*x^5 - 15*(b^5*c^2*d^3 - a^2*b^3*d^5)*x^4 + 2
0*(b^5*c^3*d^2 - a^3*b^2*d^5)*x^3 - 30*(b^5*c^4*d - a^4*b*d^5)*x^2 + 60*(b^
5*c^5 - a^5*d^5)*x)/(b^5*d^5)) + 1/20*B*b^2*c*d^2*g^2*i^3*n*(12*a^5*log(b*x
 + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b
^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4
```

```
 - a^4*d^4)*x)/(b^4*d^4)) + 1/30*B*a*b*d^3*g^2*i^3*n*(12*a^5*log(b*x + a)/b
^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*
d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*
d^4)*x)/(b^4*d^4)) - 1/8*B*b^2*c^2*d*g^2*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*
c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*
b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 1/4*B*a*b*c*d^2*g^2*i^3*
n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*
d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^
3)) - 1/24*B*a^2*d^3*g^2*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)
/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(
b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/6*B*b^2*c^3*g^2*i^3*n*(2*a^3*log(b*x +
 a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 -
a^2*d^2)*x)/(b^2*d^2)) + B*a*b*c^2*d*g^2*i^3*n*(2*a^3*log(b*x + a)/b^3 - 2*
c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/
(b^2*d^2)) + 1/2*B*a^2*c*d^2*g^2*i^3*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(
d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2
)) - B*a*b*c^3*g^2*i^3*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*
c - a*d)*x/(b*d)) - 3/2*B*a^2*c^2*d*g^2*i^3*n*(a^2*log(b*x + a)/b^2 - c^2*l
og(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*a^2*c^3*g^2*i^3*n*(a*log(b*x + a
)/b - c*log(d*x + c)/d) + B*a^2*c^3*g^2*i^3*x*log(e*(b*x/(d*x + c) + a/(d*x
 + c))^n) + A*a^2*c^3*g^2*i^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4300 vs. $2(367) = 734$.

Time = 1.78 (sec) , antiderivative size = 4300, normalized size of antiderivative = 11.11

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, a
lgorithm="giac")
```

```
[Out] 1/360*(6*(B*b^9*c^7*g^2*i^3*n - 7*B*a*b^8*c^6*d*g^2*i^3*n - 6*(b*x + a)*B*b
^8*c^7*d*g^2*i^3*n/(d*x + c) + 21*B*a^2*b^7*c^5*d^2*g^2*i^3*n + 42*(b*x + a
)*B*a*b^7*c^6*d^2*g^2*i^3*n/(d*x + c) + 15*(b*x + a)^2*B*b^7*c^7*d^2*g^2*i^
3*n/(d*x + c)^2 - 35*B*a^3*b^6*c^4*d^3*g^2*i^3*n - 126*(b*x + a)*B*a^2*b^6*
c^5*d^3*g^2*i^3*n/(d*x + c) - 105*(b*x + a)^2*B*a*b^6*c^6*d^3*g^2*i^3*n/(d*
x + c)^2 + 35*B*a^4*b^5*c^3*d^4*g^2*i^3*n + 210*(b*x + a)*B*a^3*b^5*c^4*d^4
*g^2*i^3*n/(d*x + c) + 315*(b*x + a)^2*B*a^2*b^5*c^5*d^4*g^2*i^3*n/(d*x + c
)^2 - 21*B*a^5*b^4*c^2*d^5*g^2*i^3*n - 210*(b*x + a)*B*a^4*b^4*c^3*d^5*g^2*
i^3*n/(d*x + c) - 525*(b*x + a)^2*B*a^3*b^4*c^4*d^5*g^2*i^3*n/(d*x + c)^2 +
 7*B*a^6*b^3*c*d^6*g^2*i^3*n + 126*(b*x + a)*B*a^5*b^3*c^2*d^6*g^2*i^3*n/(d
*x + c) + 525*(b*x + a)^2*B*a^4*b^3*c^3*d^6*g^2*i^3*n/(d*x + c)^2 - B*a^7*b
```

^2*d^7*g^2*i^3*n - 42*(b*x + a)*B*a^6*b^2*c*d^7*g^2*i^3*n/(d*x + c) - 315*(
b*x + a)^2*B*a^5*b^2*c^2*d^7*g^2*i^3*n/(d*x + c)^2 + 6*(b*x + a)*B*a^7*b*d^
8*g^2*i^3*n/(d*x + c) + 105*(b*x + a)^2*B*a^6*b*c*d^8*g^2*i^3*n/(d*x + c)^2
 - 15*(b*x + a)^2*B*a^7*d^9*g^2*i^3*n/(d*x + c)^2)*log((b*x + a)/(d*x + c))
/(b^6*d^3 - 6*(b*x + a)*b^5*d^4/(d*x + c) + 15*(b*x + a)^2*b^4*d^5/(d*x + c
)^2 - 20*(b*x + a)^3*b^3*d^6/(d*x + c)^3 + 15*(b*x + a)^4*b^2*d^7/(d*x + c)
^4 - 6*(b*x + a)^5*b*d^8/(d*x + c)^5 + (b*x + a)^6*d^9/(d*x + c)^6) - (2*B*
b^12*c^7*g^2*i^3*n - 14*B*a*b^11*c^6*d*g^2*i^3*n - 18*(b*x + a)*B*b^11*c^7*
d*g^2*i^3*n/(d*x + c) + 42*B*a^2*b^10*c^5*d^2*g^2*i^3*n + 126*(b*x + a)*B*a
*b^10*c^6*d^2*g^2*i^3*n/(d*x + c) + 63*(b*x + a)^2*B*b^10*c^7*d^2*g^2*i^3*n
/(d*x + c)^2 - 70*B*a^3*b^9*c^4*d^3*g^2*i^3*n - 378*(b*x + a)*B*a^2*b^9*c^5
*d^3*g^2*i^3*n/(d*x + c) - 441*(b*x + a)^2*B*a*b^9*c^6*d^3*g^2*i^3*n/(d*x +
 c)^2 - 74*(b*x + a)^3*B*b^9*c^7*d^3*g^2*i^3*n/(d*x + c)^3 + 70*B*a^4*b^8*c
^3*d^4*g^2*i^3*n + 630*(b*x + a)*B*a^3*b^8*c^4*d^4*g^2*i^3*n/(d*x + c) + 13
23*(b*x + a)^2*B*a^2*b^8*c^5*d^4*g^2*i^3*n/(d*x + c)^2 + 518*(b*x + a)^3*B*
a*b^8*c^6*d^4*g^2*i^3*n/(d*x + c)^3 + 33*(b*x + a)^4*B*b^8*c^7*d^4*g^2*i^3*
n/(d*x + c)^4 - 42*B*a^5*b^7*c^2*d^5*g^2*i^3*n - 630*(b*x + a)*B*a^4*b^7*c^
3*d^5*g^2*i^3*n/(d*x + c) - 2205*(b*x + a)^2*B*a^3*b^7*c^4*d^5*g^2*i^3*n/(d
x + c)^2 - 1554(b*x + a)^3*B*a^2*b^7*c^5*d^5*g^2*i^3*n/(d*x + c)^3 - 231*
(b*x + a)^4*B*a*b^7*c^6*d^5*g^2*i^3*n/(d*x + c)^4 - 6*(b*x + a)^5*B*b^7*c^7
*d^5*g^2*i^3*n/(d*x + c)^5 + 14*B*a^6*b^6*c*d^6*g^2*i^3*n + 378*(b*x + a)*B
*a^5*b^6*c^2*d^6*g^2*i^3*n/(d*x + c) + 2205*(b*x + a)^2*B*a^4*b^6*c^3*d^6*g
^2*i^3*n/(d*x + c)^2 + 2590*(b*x + a)^3*B*a^3*b^6*c^4*d^6*g^2*i^3*n/(d*x +
c)^3 + 693*(b*x + a)^4*B*a^2*b^6*c^5*d^6*g^2*i^3*n/(d*x + c)^4 + 42*(b*x +
a)^5*B*a*b^6*c^6*d^6*g^2*i^3*n/(d*x + c)^5 - 2*B*a^7*b^5*d^7*g^2*i^3*n - 12
6*(b*x + a)*B*a^6*b^5*c*d^7*g^2*i^3*n/(d*x + c) - 1323*(b*x + a)^2*B*a^5*b^
5*c^2*d^7*g^2*i^3*n/(d*x + c)^2 - 2590*(b*x + a)^3*B*a^4*b^5*c^3*d^7*g^2*i^
3*n/(d*x + c)^3 - 1155*(b*x + a)^4*B*a^3*b^5*c^4*d^7*g^2*i^3*n/(d*x + c)^4
 - 126*(b*x + a)^5*B*a^2*b^5*c^5*d^7*g^2*i^3*n/(d*x + c)^5 + 18*(b*x + a)*B*
a^7*b^4*d^8*g^2*i^3*n/(d*x + c) + 441*(b*x + a)^2*B*a^6*b^4*c*d^8*g^2*i^3*n
/(d*x + c)^2 + 1554*(b*x + a)^3*B*a^5*b^4*c^2*d^8*g^2*i^3*n/(d*x + c)^3 + 1
155*(b*x + a)^4*B*a^4*b^4*c^3*d^8*g^2*i^3*n/(d*x + c)^4 + 210*(b*x + a)^5*B
*a^3*b^4*c^4*d^8*g^2*i^3*n/(d*x + c)^5 - 63*(b*x + a)^2*B*a^7*b^3*d^9*g^2*i
^3*n/(d*x + c)^2 - 518*(b*x + a)^3*B*a^6*b^3*c*d^9*g^2*i^3*n/(d*x + c)^3 -
693*(b*x + a)^4*B*a^5*b^3*c^2*d^9*g^2*i^3*n/(d*x + c)^4 - 210*(b*x + a)^5*B
*a^4*b^3*c^3*d^9*g^2*i^3*n/(d*x + c)^5 + 74*(b*x + a)^3*B*a^7*b^2*d^10*g^2*
i^3*n/(d*x + c)^3 + 231*(b*x + a)^4*B*a^6*b^2*c*d^10*g^2*i^3*n/(d*x + c)^4
 + 126*(b*x + a)^5*B*a^5*b^2*c^2*d^10*g^2*i^3*n/(d*x + c)^5 - 33*(b*x + a)^4
*B*a^7*b*d^11*g^2*i^3*n/(d*x + c)^4 - 42*(b*x + a)^5*B*a^6*b*c*d^11*g^2*i^3
*n/(d*x + c)^5 + 6*(b*x + a)^5*B*a^7*d^12*g^2*i^3*n/(d*x + c)^5 - 6*B*b^12*
c^7*g^2*i^3*log(e) + 42*B*a*b^11*c^6*d*g^2*i^3*log(e) + 36*(b*x + a)*B*b^11
*c^7*d*g^2*i^3*log(e)/(d*x + c) - 126*B*a^2*b^10*c^5*d^2*g^2*i^3*log(e) - 2
52*(b*x + a)*B*a*b^10*c^6*d^2*g^2*i^3*log(e)/(d*x + c) - 90*(b*x + a)^2*B*b
^10*c^7*d^2*g^2*i^3*log(e)/(d*x + c)^2 + 210*B*a^3*b^9*c^4*d^3*g^2*i^3*log(
e) + 756*(b*x + a)*B*a^2*b^9*c^5*d^3*g^2*i^3*log(e)/(d*x + c) + 630*(b*x +

a)^2*B*a*b^9*c^6*d^3*g^2*i^3*log(e)/(d*x + c)^2 - 210*B*a^4*b^8*c^3*d^4*g^2 *i^3*log(e) - 1260*(b*x + a)*B*a^3*b^8*c^4*d^4*g^2*i^3*log(e)/(d*x + c) - 1 890*(b*x + a)^2*B*a^2*b^8*c^5*d^4*g^2*i^3*log(e)/(d*x + c)^2 + 126*B*a^5*b^ 7*c^2*d^5*g^2*i^3*log(e) + 1260*(b*x + a)*B*a^4*b^7*c^3*d^5*g^2*i^3*log(e)/ (d*x + c) + 3150*(b*x + a)^2*B*a^3*b^7*c^4*d^5*g^2*i^3*log(e)/(d*x + c)^2 - 42*B*a^6*b^6*c*d^6*g^2*i^3*log(e) - 756*(b*x + a)*B*a^5*b^6*c^2*d^6*g^2*i^ 3*log(e)/(d*x + c) - 3150*(b*x + a)^2*B*a^4*b^6*c^3*d^6*g^2*i^3*log(e)/(d*x + c)^2 + 6*B*a^7*b^5*d^7*g^2*i^3*log(e) + 252*(b*x + a)*B*a^6*b^5*c*d^7*g^ 2*i^3*log(e)/(d*x + c) + 1890*(b*x + a)^2*B*a^5*b^5*c^2*d^7*g^2*i^3*log(e)/ (d*x + c)^2 - 36*(b*x + a)*B*a^7*b^4*d^8*g^2*i^3*log(e)/(d*x + c) - 630*(b* x + a)^2*B*a^6*b^4*c*d^8*g^2*i^3*log(e)/(d*x + c)^2 + 90*(b*x + a)^2*B*a^7* b^3*d^9*g^2*i^3*log(e)/(d*x + c)^2 - 6*A*b^12*c^7*g^2*i^3 + 42*A*a*b^11*c^6 *d*g^2*i^3 + 36*(b*x + a)*A*b^11*c^7*d*g^2*i^3/(d*x + c) - 126*A*a^2*b^10*c ^5*d^2*g^2*i^3 - 252*(b*x + a)*A*a*b^10*c^6*d^2*g^2*i^3/(d*x + c) - 90*(b*x + a)^2*A*b^10*c^7*d^2*g^2*i^3/(d*x + c)^2 + 210*A*a^3*b^9*c^4*d^3*g^2*i^3 + 756*(b*x + a)*A*a^2*b^9*c^5*d^3*g^2*i^3/(d*x + c) + 630*(b*x + a)^2*A*a*b ^9*c^6*d^3*g^2*i^3/(d*x + c)^2 - 210*A*a^4*b^8*c^3*d^4*g^2*i^3 - 1260*(b*x + a)*A*a^3*b^8*c^4*d^4*g^2*i^3/(d*x + c) - 1890*(b*x + a)^2*A*a^2*b^8*c^5*d ^4*g^2*i^3/(d*x + c)^2 + 126*A*a^5*b^7*c^2*d^5*g^2*i^3 + 1260*(b*x + a)*A*a ^4*b^7*c^3*d^5*g^2*i^3/(d*x + c) + 3150*(b*x + a)^2*A*a^3*b^7*c^4*d^5*g^2*i ^3/(d*x + c)^2 - 42*A*a^6*b^6*c*d^6*g^2*i^3 - 756*(b*x + a)*A*a^5*b^6*c^2*d ^6*g^2*i^3/(d*x + c) - 3150*(b*x + a)^2*A*a^4*b^6*c^3*d^6*g^2*i^3/(d*x + c) ^2 + 6*A*a^7*b^5*d^7*g^2*i^3 + 252*(b*x + a)*A*a^6*b^5*c*d^7*g^2*i^3/(d*x + c) + 1890*(b*x + a)^2*A*a^5*b^5*c^2*d^7*g^2*i^3/(d*x + c)^2 - 36*(b*x + a) *A*a^7*b^4*d^8*g^2*i^3/(d*x + c) - 630*(b*x + a)^2*A*a^6*b^4*c*d^8*g^2*i^3/ (d*x + c)^2 + 90*(b*x + a)^2*A*a^7*b^3*d^9*g^2*i^3/(d*x + c)^2)/(b^9*d^3 - 6*(b*x + a)*b^8*d^4/(d*x + c) + 15*(b*x + a)^2*b^7*d^5/(d*x + c)^2 - 20*(b* x + a)^3*b^6*d^6/(d*x + c)^3 + 15*(b*x + a)^4*b^5*d^7/(d*x + c)^4 - 6*(b*x + a)^5*b^4*d^8/(d*x + c)^5 + (b*x + a)^6*b^3*d^9/(d*x + c)^6) + 6*(B*b^7*c^ 7*g^2*i^3*n - 7*B*a*b^6*c^6*d*g^2*i^3*n + 21*B*a^2*b^5*c^5*d^2*g^2*i^3*n - 35*B*a^3*b^4*c^4*d^3*g^2*i^3*n + 35*B*a^4*b^3*c^3*d^4*g^2*i^3*n - 21*B*a^5* b^2*c^2*d^5*g^2*i^3*n + 7*B*a^6*b*c*d^6*g^2*i^3*n - B*a^7*d^7*g^2*i^3*n)*lo g(-b + (b*x + a)*d/(d*x + c))/(b^4*d^3) - 6*(B*b^7*c^7*g^2*i^3*n - 7*B*a*b^ 6*c^6*d*g^2*i^3*n + 21*B*a^2*b^5*c^5*d^2*g^2*i^3*n - 35*B*a^3*b^4*c^4*d^3*g ^2*i^3*n + 35*B*a^4*b^3*c^3*d^4*g^2*i^3*n - 21*B*a^5*b^2*c^2*d^5*g^2*i^3*n + 7*B*a^6*b*c*d^6*g^2*i^3*n - B*a^7*d^7*g^2*i^3*n)*log((b*x + a)/(d*x + c)) /(b^4*d^3))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)

Mupad [B] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 2547, normalized size of antiderivative = 6.58

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

```
[In] int((a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),
x)
```

```
[Out] x^2*((a*c*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d*n - B*b*c*n))/6 -
(A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*b*d) - (d*g^
2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*d^2*n - 3*B*b^2*c^2*n + 60*A*a
*b*c*d + B*a*b*c*d*n))/5 + A*a*b*c*d^2*g^2*i^3))/(2*b*d) - ((60*a*d + 60*b*
c)*((g^2*i^3*(4*A*a^3*d^3 + 16*A*b^3*c^3 + B*a^3*d^3*n - 3*B*b^3*c^3*n + 72
*A*a*b^2*c^2*d + 48*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d*n + 5*B*a^2*b*c*d^2*n))
/(4*b) + ((60*a*d + 60*b*c)*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d*
n - B*b*c*n))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c)
)/(60*b*d) - (d*g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*d^2*n - 3*B*
b^2*c^2*n + 60*A*a*b*c*d + B*a*b*c*d*n))/5 + A*a*b*c*d^2*g^2*i^3))/(60*b*d)
 - (a*c*((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d*n - B*b*c*n))/6 - (A*b
*d^2*g^2*i^3*(60*a*d + 60*b*c))/60))/(b*d)))/(120*b*d) + (c*g^2*i^3*(12*A*a
^3*d^3 + 3*A*b^3*c^3 + 3*B*a^3*d^3*n - B*b^3*c^3*n + 36*A*a*b^2*c^2*d + 54*
A*a^2*b*c*d^2 - 5*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n))/(6*b*d)) + x^3*((g^
2*i^3*(4*A*a^3*d^3 + 16*A*b^3*c^3 + B*a^3*d^3*n - 3*B*b^3*c^3*n + 72*A*a*b^
2*c^2*d + 48*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d*n + 5*B*a^2*b*c*d^2*n))/(12*b)
 + ((60*a*d + 60*b*c)*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d*n - B*
b*c*n))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*
b*d) - (d*g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*d^2*n - 3*B*b^2*c^
2*n + 60*A*a*b*c*d + B*a*b*c*d*n))/5 + A*a*b*c*d^2*g^2*i^3))/(180*b*d) - (a
*c*((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d*n - B*b*c*n))/6 - (A*b*d^2*
g^2*i^3*(60*a*d + 60*b*c))/60))/(3*b*d)) - x^4*((((b*d^2*g^2*i^3*(18*A*a*d
+ 24*A*b*c + B*a*d*n - B*b*c*n))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60
)*(60*a*d + 60*b*c))/(240*b*d) - (d*g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 +
2*B*a^2*d^2*n - 3*B*b^2*c^2*n + 60*A*a*b*c*d + B*a*b*c*d*n))/20 + (A*a*b*c*
d^2*g^2*i^3)/4) + x^5*((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d*n - B*b*
c*n))/30 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/300) - x*(((60*a*d + 60*b*c)
*((a*c*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d*n - B*b*c*n))/6 - (A*
b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*b*d) - (d*g^2*i
^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*d^2*n - 3*B*b^2*c^2*n + 60*A*a*b*
c*d + B*a*b*c*d*n))/5 + A*a*b*c*d^2*g^2*i^3))/(b*d) - ((60*a*d + 60*b*c)*((
g^2*i^3*(4*A*a^3*d^3 + 16*A*b^3*c^3 + B*a^3*d^3*n - 3*B*b^3*c^3*n + 72*A*a*
b^2*c^2*d + 48*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d*n + 5*B*a^2*b*c*d^2*n))/(4*b
) + ((60*a*d + 60*b*c)*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d*n - B
*b*c*n))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60
```

```
*b*d) - (d*g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*d^2*n - 3*B*b^2*c
^2*n + 60*A*a*b*c*d + B*a*b*c*d*n))/5 + A*a*b*c*d^2*g^2*i^3))/(60*b*d) - (a
*c*((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d*n - B*b*c*n))/6 - (A*b*d^2*
g^2*i^3*(60*a*d + 60*b*c))/60))/(b*d)))/(60*b*d) + (c*g^2*i^3*(12*A*a^3*d^3
 + 3*A*b^3*c^3 + 3*B*a^3*d^3*n - B*b^3*c^3*n + 36*A*a*b^2*c^2*d + 54*A*a^2*
b*c*d^2 - 5*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n))/(3*b*d)))/(60*b*d) + (a*c
*((g^2*i^3*(4*A*a^3*d^3 + 16*A*b^3*c^3 + B*a^3*d^3*n - 3*B*b^3*c^3*n + 72*A
*a*b^2*c^2*d + 48*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d*n + 5*B*a^2*b*c*d^2*n))/(
4*b) + ((60*a*d + 60*b*c)*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d*n
- B*b*c*n))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/
(60*b*d) - (d*g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*d^2*n - 3*B*b^
2*c^2*n + 60*A*a*b*c*d + B*a*b*c*d*n))/5 + A*a*b*c*d^2*g^2*i^3))/(60*b*d) -
 (a*c*((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d*n - B*b*c*n))/6 - (A*b*d
^2*g^2*i^3*(60*a*d + 60*b*c))/60))/(b*d)))/(b*d) - (a*c^2*g^2*i^3*(12*A*a^2
*d^2 + 6*A*b^2*c^2 + 3*B*a^2*d^2*n - 2*B*b^2*c^2*n + 24*A*a*b*c*d - B*a*b*c
*d*n))/(2*b*d)) + log(e*((a + b*x)/(c + d*x))^n)*(B*a^2*c^3*g^2*i^3*x + (B*
c*g^2*i^3*x^3*(3*a^2*d^2 + b^2*c^2 + 6*a*b*c*d))/3 + (B*d*g^2*i^3*x^4*(a^2*
d^2 + 3*b^2*c^2 + 6*a*b*c*d))/4 + (B*b^2*d^3*g^2*i^3*x^6)/6 + (B*a*c^2*g^2*
i^3*x^2*(3*a*d + 2*b*c))/2 + (B*b*d^2*g^2*i^3*x^5*(2*a*d + 3*b*c))/5) - (lo
g(a + b*x)*(B*a^6*d^3*g^2*i^3*n - 20*B*a^3*b^3*c^3*g^2*i^3*n + 15*B*a^4*b^2
*c^2*d*g^2*i^3*n - 6*B*a^5*b*c*d^2*g^2*i^3*n))/(60*b^4) - (log(c + d*x)*(B*
b^2*c^6*g^2*i^3*n + 15*B*a^2*c^4*d^2*g^2*i^3*n - 6*B*a*b*c^5*d*g^2*i^3*n))/
(60*d^3) + (A*b^2*d^3*g^2*i^3*x^6)/6
```

3.129 $\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal result

Integrand size = 41, antiderivative size = 283

$$\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \frac{B(bc-ad)^4gi^3nx}{20b^3d} + \frac{B(bc-ad)^3gi^3n(c+dx)^2}{40b^2d^2} + \frac{B(bc-ad)^2gi^3n(c+dx)^3}{60bd^2} - \frac{B(bc-ad)gi^3n(c+dx)^4}{20d^2} - \frac{(bc-ad)gi^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2} + \frac{bgi^3(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^2} + \frac{B(bc-ad)^5gi^3n\log\left(\frac{a+bx}{c+dx}\right)}{20b^4d^2} + \frac{B(bc-ad)^5gi^3n\log(c+dx)}{20b^4d^2}$$

```
[Out] 1/20*B*(-a*d+b*c)^4*g*i^3*n*x/b^3/d+1/40*B*(-a*d+b*c)^3*g*i^3*n*(d*x+c)^2/b
^2/d^2+1/60*B*(-a*d+b*c)^2*g*i^3*n*(d*x+c)^3/b/d^2-1/20*B*(-a*d+b*c)*g*i^3*
n*(d*x+c)^4/d^2-1/4*(-a*d+b*c)*g*i^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^
n))/d^2+1/5*b*g*i^3*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^2+1/20*B*(-
a*d+b*c)^5*g*i^3*n*ln((b*x+a)/(d*x+c))/b^4/d^2+1/20*B*(-a*d+b*c)^5*g*i^3*n*
ln(d*x+c)/b^4/d^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {2561, 45, 2382, 12, 78}

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=-\frac{gi^3(c+dx)^4(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^2}+\frac{bgi^3(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^2}$$
$$+\frac{Bgi^3n(bc-ad)^5\log\left(\frac{a+bx}{c+dx}\right)}{20b^4d^2}+\frac{Bgi^3n(bc-ad)^5\log(c+dx)}{20b^4d^2}+\frac{Bgi^3nx(bc-ad)^4}{20b^3d}$$
$$+\frac{Bgi^3n(c+dx)^2(bc-ad)^3}{40b^2d^2}+\frac{Bgi^3n(c+dx)^3(bc-ad)^2}{60bd^2}-\frac{Bgi^3n(c+dx)^4(bc-ad)}{20d^2}$$

```
[In] Int[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (B*(b*c - a*d)^4*g*i^3*n*x)/(20*b^3*d) + (B*(b*c - a*d)^3*g*i^3*n*(c + d*x)
^2)/(40*b^2*d^2) + (B*(b*c - a*d)^2*g*i^3*n*(c + d*x)^3)/(60*b*d^2) - (B*(b
*c - a*d)*g*i^3*n*(c + d*x)^4)/(20*d^2) - ((b*c - a*d)*g*i^3*(c + d*x)^4*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*d^2) + (b*g*i^3*(c + d*x)^5*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]))/(5*d^2) + (B*(b*c - a*d)^5*g*i^3*n*Log[(a
 + b*x)/(c + d*x)])/(20*b^4*d^2) + (B*(b*c - a*d)^5*g*i^3*n*Log[c + d*x])/(
20*b^4*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left((bc-ad)^5 gi^3\right) \text{Subst}\left(\int \frac{x(A+B\log(ex^n))}{(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)\\
&= -\frac{(bc-ad)gi^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2}\\
&\quad+\frac{bgi^3(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^2}\\
&\quad-\left(B(bc-ad)^5gi^3n\right)\text{Subst}\left(\int \frac{-b+5dx}{20d^2x(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)\\
&= -\frac{(bc-ad)gi^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2}\\
&\quad+\frac{bgi^3(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^2}\\
&\quad-\frac{\left(B(bc-ad)^5gi^3n\right)\text{Subst}\left(\int \frac{-b+5dx}{x(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{20d^2}\\
&= -\frac{(bc-ad)gi^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2}+\frac{bgi^3(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^2}\\
&\quad-\frac{\left(B(bc-ad)^5gi^3n\right)\text{Subst}\left(\int \left(-\frac{1}{b^4x}+\frac{4d}{(b-dx)^5}-\frac{d}{b(b-dx)^4}-\frac{d}{b^2(b-dx)^3}-\frac{d}{b^3(b-dx)^2}-\frac{d}{b^4(b-dx)}\right)dx, x,\right.}{20d^2}
\end{aligned}
$$

$$= \frac{B(bc-ad)^4gi^3nx}{20b^3d} + \frac{B(bc-ad)^3gi^3n(c+dx)^2}{40b^2d^2} + \frac{B(bc-ad)^2gi^3n(c+dx)^3}{60bd^2} - \frac{B(bc-ad)gi^3n(c+dx)^4}{20d^2} - \frac{(bc-ad)gi^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2} + \frac{bgi^3(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^2} + \frac{B(bc-ad)^5gi^3n\log\left(\frac{a+bx}{c+dx}\right)}{20b^4d^2} + \frac{B(bc-ad)^5gi^3n\log(c+dx)}{20b^4d^2}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.95

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \frac{gi^3\left(\frac{5B(bc-ad)^2n(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad)^3\log(a+bx))}{b^4} - \frac{2B(bc-ad)n(12bd(bc-ad)^3x+6b^2(bc-ad)^2(c+\cdots}{\cdots}\right)}{\cdots}$$

```
[In] Integrate[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^
n]),x]
```

```
[Out] (g*i^3*((5*B*(b*c - a*d)^2*n*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c
+ d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]))/b^4 - (2*B*(b
*c - a*d)*n*(12*b*d*(b*c - a*d)^3*x + 6*b^2*(b*c - a*d)^2*(c + d*x)^2 + 4*b
^3*(b*c - a*d)*(c + d*x)^3 + 3*b^4*(c + d*x)^4 + 12*(b*c - a*d)^4*Log[a + b
*x]))/b^4 - 30*(b*c - a*d)*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]) + 24*b*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(120*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1118 vs. 2(267) = 534.

Time = 11.62 (sec) , antiderivative size = 1119, normalized size of antiderivative = 3.95

method	result	size
parallelrisch	Expression too large to display	1119

```
[In] int((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/120*(6*B*ln(b*x+a)*b^5*c^5*g*i^3*n^2-6*B*ln(b*x+a)*a^5*d^5*g*i^3*n^2-6*B*
ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^5*g*i^3*n+24*A*x^5*b^5*d^5*g*i^3*n+24*B*x^5
*ln(e*((b*x+a)/(d*x+c))^n)*b^5*d^5*g*i^3*n+120*B*x^3*ln(e*((b*x+a)/(d*x+c))
^n)*a*b^4*c*d^4*g*i^3*n+180*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^2*d^3*g
*i^3*n+120*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^3*d^2*g*i^3*n+30*B*ln(e*((
```

```
b*x+a)/(d*x+c))^n)*a*b^4*c^4*d*g*i^3*n+30*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a
*b^4*d^5*g*i^3*n+90*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c*d^4*g*i^3*n+120*B
*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^2*d^3*g*i^3*n-27*B*x^2*b^5*c^3*d^2*g*i
^3*n^2+60*A*x^2*b^5*c^3*d^2*g*i^3*n+6*B*x*a^4*b*d^5*g*i^3*n^2-6*B*x*b^5*c^4
*d*g*i^3*n^2+6*B*b^5*c^5*g*i^3*n^2-6*B*a^5*d^5*g*i^3*n^2+20*B*x^3*a*b^4*c*d
^4*g*i^3*n^2+120*A*x^3*a*b^4*c*d^4*g*i^3*n+60*B*x^2*ln(e*((b*x+a)/(d*x+c))^
n)*b^5*c^3*d^2*g*i^3*n+15*B*x^2*a^2*b^3*c*d^4*g*i^3*n^2+15*B*x^2*a*b^4*c^2*
d^3*g*i^3*n^2+180*A*x^2*a*b^4*c^2*d^3*g*i^3*n-30*B*x*a^3*b^2*c*d^4*g*i^3*n^
2+60*B*x*a^2*b^3*c^2*d^3*g*i^3*n^2-30*B*x*a*b^4*c^3*d^2*g*i^3*n^2+120*A*x*a
*b^4*c^3*d^2*g*i^3*n+30*B*ln(b*x+a)*a^4*b*c*d^4*g*i^3*n^2-60*B*ln(b*x+a)*a^
3*b^2*c^2*d^3*g*i^3*n^2+60*B*ln(b*x+a)*a^2*b^3*c^3*d^2*g*i^3*n^2-30*B*ln(b*
x+a)*a*b^4*c^4*d*g*i^3*n^2+27*B*a^4*b*c*d^4*g*i^3*n^2-45*B*a^3*b^2*c^2*d^3*
g*i^3*n^2-45*B*a^2*b^3*c^3*d^2*g*i^3*n^2+63*B*a*b^4*c^4*d*g*i^3*n^2-300*A*a
^2*b^3*c^3*d^2*g*i^3*n-180*A*a*b^4*c^4*d*g*i^3*n+6*B*x^4*a*b^4*d^5*g*i^3*n^
2-6*B*x^4*b^5*c*d^4*g*i^3*n^2+30*A*x^4*a*b^4*d^5*g*i^3*n+90*A*x^4*b^5*c*d^4
*g*i^3*n+2*B*x^3*a^2*b^3*d^5*g*i^3*n^2-22*B*x^3*b^5*c^2*d^3*g*i^3*n^2+120*A
*x^3*b^5*c^2*d^3*g*i^3*n-3*B*x^2*a^3*b^2*d^5*g*i^3*n^2)/b^4/d^2/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(267) = 534$.

Time = 0.45 (sec) , antiderivative size = 721, normalized size of antiderivative = 2.55

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{24\,Ab^5d^5gi^3x^5+6\,(10\,Ba^2b^3c^3d^2-10\,Ba^3b^2c^2d^3+5\,Ba^4bcd^4-Ba^5d^5)gi^3n\log{(bx+a)}+6\,(Bb^5c^5-5\,B}{}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, alg
orithm="fricas")
```

```
[Out] 1/120*(24*A*b^5*d^5*g*i^3*x^5 + 6*(10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2*
d^3 + 5*B*a^4*b*c*d^4 - B*a^5*d^5)*g*i^3*n*log(b*x + a) + 6*(B*b^5*c^5 - 5*
B*a*b^4*c^4*d)*g*i^3*n*log(d*x + c) - 6*((B*b^5*c*d^4 - B*a*b^4*d^5)*g*i^3*
n - 5*(3*A*b^5*c*d^4 + A*a*b^4*d^5)*g*i^3)*x^4 - 2*((11*B*b^5*c^2*d^3 - 10*
B*a*b^4*c*d^4 - B*a^2*b^3*d^5)*g*i^3*n - 60*(A*b^5*c^2*d^3 + A*a*b^4*c*d^4)
*g*i^3)*x^3 - 3*((9*B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 - 5*B*a^2*b^3*c*d^4 +
 B*a^3*b^2*d^5)*g*i^3*n - 20*(A*b^5*c^3*d^2 + 3*A*a*b^4*c^2*d^3)*g*i^3)*x^2
 + 6*(20*A*a*b^4*c^3*d^2*g*i^3 - (B*b^5*c^4*d + 5*B*a*b^4*c^3*d^2 - 10*B*a^
2*b^3*c^2*d^3 + 5*B*a^3*b^2*c*d^4 - B*a^4*b*d^5)*g*i^3*n)*x + 6*(4*B*b^5*d^
5*g*i^3*x^5 + 20*B*a*b^4*c^3*d^2*g*i^3*x + 5*(3*B*b^5*c*d^4 + B*a*b^4*d^5)*
g*i^3*x^4 + 20*(B*b^5*c^2*d^3 + B*a*b^4*c*d^4)*g*i^3*x^3 + 10*(B*b^5*c^3*d^
2 + 3*B*a*b^4*c^2*d^3)*g*i^3*x^2)*log(e) + 6*(4*B*b^5*d^5*g*i^3*n*x^5 + 20*
B*a*b^4*c^3*d^2*g*i^3*n*x + 5*(3*B*b^5*c*d^4 + B*a*b^4*d^5)*g*i^3*n*x^4 + 2
```

```
0*(B*b^5*c^2*d^3 + B*a*b^4*c*d^4)*g*i^3*n*x^3 + 10*(B*b^5*c^3*d^2 + 3*B*a*b
^4*c^2*d^3)*g*i^3*n*x^2)*log((b*x + a)/(d*x + c)))/(b^4*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int (ag + bgx)(ci + dix)^3 \left(A + B \log \left(e\left(\frac{a + bx}{c + dx}\right)^n\right)\right) \, dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1118 vs. $2(267) = 534$.

Time = 0.23 (sec) , antiderivative size = 1118, normalized size of antiderivative = 3.95

$$\int (ag + bgx)(ci + dix)^3 \left(A + B \log \left(e\left(\frac{a + bx}{c + dx}\right)^n\right)\right) \, dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, alg
orithm="maxima")
[Out] 1/5*B*b*d^3*g*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A*b*d^3*
g*i^3*x^5 + 3/4*B*b*c*d^2*g*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
+ 1/4*B*a*d^3*g*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/4*A*b*c*
d^2*g*i^3*x^4 + 1/4*A*a*d^3*g*i^3*x^4 + B*b*c^2*d*g*i^3*x^3*log(e*(b*x/(d*x
 + c) + a/(d*x + c))^n) + B*a*c*d^2*g*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x
 + c))^n) + A*b*c^2*d*g*i^3*x^3 + A*a*c*d^2*g*i^3*x^3 + 1/2*B*b*c^3*g*i^3*x
^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*B*a*c^2*d*g*i^3*x^2*log(e*(
b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*b*c^3*g*i^3*x^2 + 3/2*A*a*c^2*d*g*i
^3*x^2 + 1/60*B*b*d^3*g*i^3*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c
)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3
+ 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) - 1/
8*B*b*c*d^2*g*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(
b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a
^3*d^3)*x)/(b^3*d^3)) - 1/24*B*a*d^3*g*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^
4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*
d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*b*c^2*d*g*i^3*n*(2*a
^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2
*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + 1/2*B*a*c*d^2*g*i^3*n*(2*a^3*log(b*x +
 a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 -
a^2*d^2)*x)/(b^2*d^2)) - 1/2*B*b*c^3*g*i^3*n*(a^2*log(b*x + a)/b^2 - c^2*lo
```

```
g(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) - 3/2*B*a*c^2*d*g*i^3*n*(a^2*log(b*x
+ a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*a*c^3*g*i^3*n*(a
*log(b*x + a)/b - c*log(d*x + c)/d) + B*a*c^3*g*i^3*x*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n) + A*a*c^3*g*i^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2584 vs. 2(267) = 534.

Time = 1.18 (sec) , antiderivative size = 2584, normalized size of antiderivative = 9.13

$$\int (ag + bgx)(ci + dix)^3 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, alg
orithm="giac")
```

```
[Out] -1/120*(6*(B*b^7*c^6*g*i^3*n - 6*B*a*b^6*c^5*d*g*i^3*n - 5*(b*x + a)*B*b^6*
c^6*d*g*i^3*n/(d*x + c) + 15*B*a^2*b^5*c^4*d^2*g*i^3*n + 30*(b*x + a)*B*a*b
^5*c^5*d^2*g*i^3*n/(d*x + c) - 20*B*a^3*b^4*c^3*d^3*g*i^3*n - 75*(b*x + a)*
B*a^2*b^4*c^4*d^3*g*i^3*n/(d*x + c) + 15*B*a^4*b^3*c^2*d^4*g*i^3*n + 100*(b
*x + a)*B*a^3*b^3*c^3*d^4*g*i^3*n/(d*x + c) - 6*B*a^5*b^2*c*d^5*g*i^3*n - 7
5*(b*x + a)*B*a^4*b^2*c^2*d^5*g*i^3*n/(d*x + c) + B*a^6*b*d^6*g*i^3*n + 30*
(b*x + a)*B*a^5*b*c*d^6*g*i^3*n/(d*x + c) - 5*(b*x + a)*B*a^6*d^7*g*i^3*n/(
d*x + c))*log((b*x + a)/(d*x + c))/(b^5*d^2 - 5*(b*x + a)*b^4*d^3/(d*x + c)
 + 10*(b*x + a)^2*b^3*d^4/(d*x + c)^2 - 10*(b*x + a)^3*b^2*d^5/(d*x + c)^3
+ 5*(b*x + a)^4*b*d^6/(d*x + c)^4 - (b*x + a)^5*d^7/(d*x + c)^5) - (5*B*b^1
0*c^6*g*i^3*n - 30*B*a*b^9*c^5*d*g*i^3*n - 31*(b*x + a)*B*b^9*c^6*d*g*i^3*n
/(d*x + c) + 75*B*a^2*b^8*c^4*d^2*g*i^3*n + 186*(b*x + a)*B*a*b^8*c^5*d^2*g
*i^3*n/(d*x + c) + 47*(b*x + a)^2*B*b^8*c^6*d^2*g*i^3*n/(d*x + c)^2 - 100*B
*a^3*b^7*c^3*d^3*g*i^3*n - 465*(b*x + a)*B*a^2*b^7*c^4*d^3*g*i^3*n/(d*x + c
) - 282*(b*x + a)^2*B*a*b^7*c^5*d^3*g*i^3*n/(d*x + c)^2 - 27*(b*x + a)^3*B*
b^7*c^6*d^3*g*i^3*n/(d*x + c)^3 + 75*B*a^4*b^6*c^2*d^4*g*i^3*n + 620*(b*x +
 a)*B*a^3*b^6*c^3*d^4*g*i^3*n/(d*x + c) + 705*(b*x + a)^2*B*a^2*b^6*c^4*d^4
*g*i^3*n/(d*x + c)^2 + 162*(b*x + a)^3*B*a*b^6*c^5*d^4*g*i^3*n/(d*x + c)^3
+ 6*(b*x + a)^4*B*b^6*c^6*d^4*g*i^3*n/(d*x + c)^4 - 30*B*a^5*b^5*c*d^5*g*i^
3*n - 465*(b*x + a)*B*a^4*b^5*c^2*d^5*g*i^3*n/(d*x + c) - 940*(b*x + a)^2*B
*a^3*b^5*c^3*d^5*g*i^3*n/(d*x + c)^2 - 405*(b*x + a)^3*B*a^2*b^5*c^4*d^5*g*
i^3*n/(d*x + c)^3 - 36*(b*x + a)^4*B*a*b^5*c^5*d^5*g*i^3*n/(d*x + c)^4 + 5*
B*a^6*b^4*d^6*g*i^3*n + 186*(b*x + a)*B*a^5*b^4*c*d^6*g*i^3*n/(d*x + c) + 7
05*(b*x + a)^2*B*a^4*b^4*c^2*d^6*g*i^3*n/(d*x + c)^2 + 540*(b*x + a)^3*B*a^
3*b^4*c^3*d^6*g*i^3*n/(d*x + c)^3 + 90*(b*x + a)^4*B*a^2*b^4*c^4*d^6*g*i^3*
n/(d*x + c)^4 - 31*(b*x + a)*B*a^6*b^3*d^7*g*i^3*n/(d*x + c) - 282*(b*x + a
)^2*B*a^5*b^3*c*d^7*g*i^3*n/(d*x + c)^2 - 405*(b*x + a)^3*B*a^4*b^3*c^2*d^7
*g*i^3*n/(d*x + c)^3 - 120*(b*x + a)^4*B*a^3*b^3*c^3*d^7*g*i^3*n/(d*x + c)^
```

```
4 + 47*(b*x + a)^2*B*a^6*b^2*d^8*g*i^3*n/(d*x + c)^2 + 162*(b*x + a)^3*B*a^
5*b^2*c*d^8*g*i^3*n/(d*x + c)^3 + 90*(b*x + a)^4*B*a^4*b^2*c^2*d^8*g*i^3*n/
(d*x + c)^4 - 27*(b*x + a)^3*B*a^6*b*d^9*g*i^3*n/(d*x + c)^3 - 36*(b*x + a)
^4*B*a^5*b*c*d^9*g*i^3*n/(d*x + c)^4 + 6*(b*x + a)^4*B*a^6*d^10*g*i^3*n/(d*
x + c)^4 - 6*B*b^10*c^6*g*i^3*log(e) + 36*B*a*b^9*c^5*d*g*i^3*log(e) + 30*(
b*x + a)*B*b^9*c^6*d*g*i^3*log(e)/(d*x + c) - 90*B*a^2*b^8*c^4*d^2*g*i^3*lo
g(e) - 180*(b*x + a)*B*a*b^8*c^5*d^2*g*i^3*log(e)/(d*x + c) + 120*B*a^3*b^7
*c^3*d^3*g*i^3*log(e) + 450*(b*x + a)*B*a^2*b^7*c^4*d^3*g*i^3*log(e)/(d*x +
 c) - 90*B*a^4*b^6*c^2*d^4*g*i^3*log(e) - 600*(b*x + a)*B*a^3*b^6*c^3*d^4*g
*i^3*log(e)/(d*x + c) + 36*B*a^5*b^5*c*d^5*g*i^3*log(e) + 450*(b*x + a)*B*a
^4*b^5*c^2*d^5*g*i^3*log(e)/(d*x + c) - 6*B*a^6*b^4*d^6*g*i^3*log(e) - 180*
(b*x + a)*B*a^5*b^4*c*d^6*g*i^3*log(e)/(d*x + c) + 30*(b*x + a)*B*a^6*b^3*d
^7*g*i^3*log(e)/(d*x + c) - 6*A*b^10*c^6*g*i^3 + 36*A*a*b^9*c^5*d*g*i^3 + 3
0*(b*x + a)*A*b^9*c^6*d*g*i^3/(d*x + c) - 90*A*a^2*b^8*c^4*d^2*g*i^3 - 180*
(b*x + a)*A*a*b^8*c^5*d^2*g*i^3/(d*x + c) + 120*A*a^3*b^7*c^3*d^3*g*i^3 + 4
50*(b*x + a)*A*a^2*b^7*c^4*d^3*g*i^3/(d*x + c) - 90*A*a^4*b^6*c^2*d^4*g*i^3
 - 600*(b*x + a)*A*a^3*b^6*c^3*d^4*g*i^3/(d*x + c) + 36*A*a^5*b^5*c*d^5*g*i
^3 + 450*(b*x + a)*A*a^4*b^5*c^2*d^5*g*i^3/(d*x + c) - 6*A*a^6*b^4*d^6*g*i^
3 - 180*(b*x + a)*A*a^5*b^4*c*d^6*g*i^3/(d*x + c) + 30*(b*x + a)*A*a^6*b^3*
d^7*g*i^3/(d*x + c))/(b^8*d^2 - 5*(b*x + a)*b^7*d^3/(d*x + c) + 10*(b*x + a
)^2*b^6*d^4/(d*x + c)^2 - 10*(b*x + a)^3*b^5*d^5/(d*x + c)^3 + 5*(b*x + a)^
4*b^4*d^6/(d*x + c)^4 - (b*x + a)^5*b^3*d^7/(d*x + c)^5) + 6*(B*b^6*c^6*g*i
^3*n - 6*B*a*b^5*c^5*d*g*i^3*n + 15*B*a^2*b^4*c^4*d^2*g*i^3*n - 20*B*a^3*b^
3*c^3*d^3*g*i^3*n + 15*B*a^4*b^2*c^2*d^4*g*i^3*n - 6*B*a^5*b*c*d^5*g*i^3*n
+ B*a^6*d^6*g*i^3*n)*log(b - (b*x + a)*d/(d*x + c))/(b^4*d^2) - 6*(B*b^6*c^
6*g*i^3*n - 6*B*a*b^5*c^5*d*g*i^3*n + 15*B*a^2*b^4*c^4*d^2*g*i^3*n - 20*B*a
^3*b^3*c^3*d^3*g*i^3*n + 15*B*a^4*b^2*c^2*d^4*g*i^3*n - 6*B*a^5*b*c*d^5*g*i
^3*n + B*a^6*d^6*g*i^3*n)*log((b*x + a)/(d*x + c))/(b^4*d^2))*(b*c/(b*c - a
*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 1234, normalized size of antiderivative = 4.36

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

```
[In] int((a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x*((a*c*((((20*a*d + 20*b*c)*((d^2*g*i^3*(10*A*a*d + 20*A*b*c + B*a*d*n - B*
b*c*n))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c))/20))/(20*b*d) - (d*g*i^3*(4*A*a
^2*d^2 + 24*A*b^2*c^2 + B*a^2*d^2*n - 3*B*b^2*c^2*n + 32*A*a*b*c*d + 2*B*a*
b*c*d*n))/(4*b) + A*a*c*d^2*g*i^3))/(b*d) - ((20*a*d + 20*b*c)*(((20*a*d +
20*b*c)*(((20*a*d + 20*b*c)*((d^2*g*i^3*(10*A*a*d + 20*A*b*c + B*a*d*n - B*
b*c*n))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c))/20))/(20*b*d) - (d*g*i^3*(4*A*a
```

^2*d^2 + 24*A*b^2*c^2 + B*a^2*d^2*n - 3*B*b^2*c^2*n + 32*A*a*b*c*d + 2*B*a*b*c*d*n))/(4*b) + A*a*c*d^2*g*i^3))/(20*b*d) - (a*c*((d^2*g*i^3*(10*A*a*d + 20*A*b*c + B*a*d*n - B*b*c*n))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c))/20))/(b*d) + (c*g*i^3*(4*A*a^2*d^2 + 4*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + 12*A*a*b*c*d))/b))/(20*b*d) + (c^2*g*i^3*(12*A*a^2*d^2 + 2*A*b^2*c^2 + 3*B*a^2*d^2*n - B*b^2*c^2*n + 16*A*a*b*c*d - 2*B*a*b*c*d*n))/(2*b*d)) - x^3*(((20*a*d + 20*b*c)*((d^2*g*i^3*(10*A*a*d + 20*A*b*c + B*a*d*n - B*b*c*n))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c))/20))/(60*b*d) - (d*g*i^3*(4*A*a^2*d^2 + 24*A*b^2*c^2 + B*a^2*d^2*n - 3*B*b^2*c^2*n + 32*A*a*b*c*d + 2*B*a*b*c*d*n))/(12*b) + (A*a*c*d^2*g*i^3)/3) + x^2*(((20*a*d + 20*b*c)*(((20*a*d + 20*b*c)*((d^2*g*i^3*(10*A*a*d + 20*A*b*c + B*a*d*n - B*b*c*n))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c))/20))/(20*b*d) - (d*g*i^3*(4*A*a^2*d^2 + 24*A*b^2*c^2 + B*a^2*d^2*n - 3*B*b^2*c^2*n + 32*A*a*b*c*d + 2*B*a*b*c*d*n))/(4*b) + A*a*c*d^2*g*i^3))/(40*b*d) - (a*c*((d^2*g*i^3*(10*A*a*d + 20*A*b*c + B*a*d*n - B*b*c*n))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c))/20))/(2*b*d) + (c*g*i^3*(4*A*a^2*d^2 + 4*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + 12*A*a*b*c*d))/(2*b)) + log(e*((a + b*x)/(c + d*x))^n)*((B*c^2*g*i^3*x^2*(3*a*d + b*c))/2 + (B*d^2*g*i^3*x^4*(a*d + 3*b*c))/4 + B*a*c^3*g*i^3*x + (B*b*d^3*g*i^3*x^5)/5 + B*c*d*g*i^3*x^3*(a*d + b*c)) + x^4*((d^2*g*i^3*(10*A*a*d + 20*A*b*c + B*a*d*n - B*b*c*n))/20 - (A*d^2*g*i^3*(20*a*d + 20*b*c))/80) + (log(c + d*x)*(B*b*c^5*g*i^3*n - 5*B*a*c^4*d*g*i^3*n))/(20*d^2) - (log(a + b*x)*(B*a^5*d^3*g*i^3*n - 10*B*a^2*b^3*c^3*g*i^3*n - 5*B*a^4*b*c*d^2*g*i^3*n + 10*B*a^3*b^2*c^2*d*g*i^3*n))/(20*b^4) + (A*b*d^3*g*i^3*x^5)/5

3.130 $\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 156

$$\begin{aligned}\int (ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx &= -\frac{B(bc-ad)^3i^3nx}{4b^3}-\frac{B(bc-ad)^2i^3n(c+dx)^2}{8b^2d}-\frac{B(bc-ad)i^3n(c+dx)^3}{12bd}\\&\quad-\frac{B(bc-ad)^4i^3n\log(a+bx)}{4b^4d}+\frac{i^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d}\end{aligned}$$

```
[Out] -1/4*B*(-a*d+b*c)^3*i^3*n*x/b^3-1/8*B*(-a*d+b*c)^2*i^3*n*(d*x+c)^2/b^2/d-1/
12*B*(-a*d+b*c)*i^3*n*(d*x+c)^3/b/d-1/4*B*(-a*d+b*c)^4*i^3*n*ln(b*x+a)/b^4/
d+1/4*i^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2547, 21, 45}

$$\begin{aligned}\int (ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx &= \frac{i^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d}-\frac{Bi^3n(bc-ad)^4\log(a+bx)}{4b^4d}\\&\quad-\frac{Bi^3nx(bc-ad)^3}{4b^3}-\frac{Bi^3n(c+dx)^2(bc-ad)^2}{8b^2d}-\frac{Bi^3n(c+dx)^3(bc-ad)}{12bd}\end{aligned}$$

```
[In] Int[(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -1/4*(B*(b*c - a*d)^3*i^3*n*x)/b^3 - (B*(b*c - a*d)^2*i^3*n*(c + d*x)^2)/(8
*b^2*d) - (B*(b*c - a*d)*i^3*n*(c + d*x)^3)/(12*b*d) - (B*(b*c - a*d)^4*i^3
```

```
*n*Log[a + b*x])/(4*b^4*d) + (i^3*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(4*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d} - \frac{(B(bc-ad)n)\int\frac{(ci+dix)^4}{(a+bx)(c+dx)}\,dx}{4di} \\
&= \frac{i^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d} - \frac{(B(bc-ad)i^3n)\int\frac{(c+dx)^3}{a+bx}\,dx}{4d} \\
&= \frac{i^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d} \\
&\quad - \frac{(B(bc-ad)i^3n)\int\left(\frac{d(bc-ad)^2}{b^3}+\frac{(bc-ad)^3}{b^3(a+bx)}+\frac{d(bc-ad)(c+dx)}{b^2}+\frac{d(c+dx)^2}{b}\right)dx}{4d} \\
&= -\frac{B(bc-ad)^3i^3nx}{4b^3} - \frac{B(bc-ad)^2i^3n(c+dx)^2}{8b^2d} - \frac{B(bc-ad)i^3n(c+dx)^3}{12bd} \\
&\quad - \frac{B(bc-ad)^4i^3n\log(a+bx)}{4b^4d} + \frac{i^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{i^3\left(-\frac{B(bc-ad)n\left(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad)^3\log(a+bx)\right)}{6b^4}+(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{4d}$$

```
[In] Integrate[(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] (i^3*(-1/6*(B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c +
 d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]))/b^4 + (c + d*x
)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(4*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. 2(146) = 292.

Time = 4.81 (sec) , antiderivative size = 652, normalized size of antiderivative = 4.18

method	result
parallelrisch	$\frac{24B\,x^3\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4c\,d^3i^3n+6A\,x^4b^4d^4i^3n+6B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4c^4i^3n-6B\ln(bx+a)a^4d^4i^3n^2-6B\ln(bx+a)b^4c^4i^3n^2+36}{}$

```
[In] int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(24*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^3*i^3*n+6*A*x^4*b^4*d^4*i^
3*n+6*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^4*i^3*n-6*B*ln(b*x+a)*a^4*d^4*i^3*n
^2-6*B*ln(b*x+a)*b^4*c^4*i^3*n^2+36*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2
*d^2*i^3*n+12*B*x^2*a*b^3*c*d^3*i^3*n^2+24*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^
4*c^3*d*i^3*n-24*B*x*a^2*b^2*c*d^3*i^3*n^2+36*B*x*a*b^3*c^2*d^2*i^3*n^2+24*
B*ln(b*x+a)*a^3*b*c*d^3*i^3*n^2-36*B*ln(b*x+a)*a^2*b^2*c^2*d^2*i^3*n^2+24*B
*ln(b*x+a)*a*b^3*c^3*d*i^3*n^2+21*B*a^3*b*c*d^3*i^3*n^2-24*B*a^2*b^2*c^2*d^
2*i^3*n^2-9*B*a*b^3*c^3*d*i^3*n^2-60*A*a*b^3*c^3*d*i^3*n+6*B*x^4*ln(e*((b*x
+a)/(d*x+c))^n)*b^4*d^4*i^3*n+2*B*x^3*a*b^3*d^4*i^3*n^2-2*B*x^3*b^4*c*d^3*i
^3*n^2+24*A*x^3*b^4*c*d^3*i^3*n-3*B*x^2*a^2*b^2*d^4*i^3*n^2-9*B*x^2*b^4*c^2
*d^2*i^3*n^2+36*A*x^2*b^4*c^2*d^2*i^3*n+6*B*x*a^3*b*d^4*i^3*n^2-18*B*x*b^4*
c^3*d*i^3*n^2+24*A*x*b^4*c^3*d*i^3*n-6*B*a^4*d^4*i^3*n^2+18*B*b^4*c^4*i^3*n
^2-24*A*b^4*c^4*i^3*n)/b^4/d/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(146) = 292$.

Time = 0.38 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.75

$$\int (ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{6\,Ab^4d^4i^3x^4-6\,Bb^4c^4i^3n\log(dx+c)+6\,(4\,Bab^3c^3d-6\,Ba^2b^2c^2d^2+4\,Ba^3bcd^3-Ba^4d^4)i^3n\log(bx+a)}{}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fric
as")
```

```
[Out] 1/24*(6*A*b^4*d^4*i^3*x^4 - 6*B*b^4*c^4*i^3*n*log(d*x + c) + 6*(4*B*a*b^3*c
^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*a^3*b*c*d^3 - B*a^4*d^4)*i^3*n*log(b*x + a
) + 2*(12*A*b^4*c*d^3*i^3 - (B*b^4*c*d^3 - B*a*b^3*d^4)*i^3*n)*x^3 + 3*(12*
A*b^4*c^2*d^2*i^3 - (3*B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*i^3
*n)*x^2 + 6*(4*A*b^4*c^3*d*i^3 - (3*B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 4*B*a
^2*b^2*c*d^3 - B*a^3*b*d^4)*i^3*n)*x + 6*(B*b^4*d^4*i^3*x^4 + 4*B*b^4*c*d^3
*i^3*x^3 + 6*B*b^4*c^2*d^2*i^3*x^2 + 4*B*b^4*c^3*d*i^3*x)*log(e) + 6*(B*b^4
*d^4*i^3*n*x^4 + 4*B*b^4*c*d^3*i^3*n*x^3 + 6*B*b^4*c^2*d^2*i^3*n*x^2 + 4*B*
b^4*c^3*d*i^3*n*x)*log((b*x + a)/(d*x + c)))/(b^4*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(136) = 272$.

Time = 78.68 (sec) , antiderivative size = 945, normalized size of antiderivative = 6.06

$$\int (ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \begin{cases} c^3i^3x\left(A+B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) \\ Ac^3i^3x+\frac{3Ac^2di^3x^2}{2}+Acd^2i^3x^3+\frac{Ad^3i^3x^4}{4}+\frac{Bc^4i^3\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{4d}+\frac{Bc^3i^3nx}{4}+Bc^3i^3x\log\left(e\left(\frac{a}{c+dx}\right)^n\right)+\frac{3Bc^2di^3}{8} \\ c^3i^3\left(Ax+\frac{Ba\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{b}-Bnx+Bx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)\right) \\ Ac^3i^3x+\frac{3Ac^2di^3x^2}{2}+Acd^2i^3x^3+\frac{Ad^3i^3x^4}{4}-\frac{Ba^4d^3i^3n\log\left(\frac{c}{d}+x\right)}{4b^4}-\frac{Ba^4d^3i^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{4b^4}+\frac{Ba^3cd^2i^3n\log\left(\frac{c}{d}+\right.}{b^3} \end{cases}$$

```
[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Piecewise((c**3*i**3*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (A*c*
*3*i**3*x + 3*A*c**2*d*i**3*x**2/2 + A*c*d**2*i**3*x**3 + A*d**3*i**3*x**4/
4 + B*c**4*i**3*log(e*(a/(c + d*x))**n)/(4*d) + B*c**3*i**3*n*x/4 + B*c**3*
```

```
i**3*x*log(e*(a/(c + d*x))**n) + 3*B*c**2*d*i**3*n*x**2/8 + 3*B*c**2*d*i**3
*x**2*log(e*(a/(c + d*x))**n)/2 + B*c*d**2*i**3*n*x**3/4 + B*c*d**2*i**3*x*
*3*log(e*(a/(c + d*x))**n) + B*d**3*i**3*n*x**4/16 + B*d**3*i**3*x**4*log(e
*(a/(c + d*x))**n)/4, Eq(b, 0)), (c**3*i**3*(A*x + B*a*log(e*(a/c + b*x/c)*
*n)/b - B*n*x + B*x*log(e*(a/c + b*x/c)**n)), Eq(d, 0)), (A*c**3*i**3*x + 3
*A*c**2*d*i**3*x**2/2 + A*c*d**2*i**3*x**3 + A*d**3*i**3*x**4/4 - B*a**4*d*
*3*i**3*n*log(c/d + x)/(4*b**4) - B*a**4*d**3*i**3*log(e*(a/(c + d*x) + b*x
/(c + d*x))**n)/(4*b**4) + B*a**3*c*d**2*i**3*n*log(c/d + x)/b**3 + B*a**3*
c*d**2*i**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/b**3 + B*a**3*d**3*i**3
*n*x/(4*b**3) - 3*B*a**2*c**2*d*i**3*n*log(c/d + x)/(2*b**2) - 3*B*a**2*c**
2*d*i**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(2*b**2) - B*a**2*c*d**2*i
**3*n*x/b**2 - B*a**2*d**3*i**3*n*x**2/(8*b**2) + B*a*c**3*i**3*n*log(c/d +
 x)/b + B*a*c**3*i**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/b + 3*B*a*c**
2*d*i**3*n*x/(2*b) + B*a*c*d**2*i**3*n*x**2/(2*b) + B*a*d**3*i**3*n*x**3/(1
2*b) - B*c**4*i**3*n*log(c/d + x)/(4*d) - 3*B*c**3*i**3*n*x/4 + B*c**3*i**3
*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) - 3*B*c**2*d*i**3*n*x**2/8 + 3*B
*c**2*d*i**3*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/2 - B*c*d**2*i**3
*n*x**3/12 + B*c*d**2*i**3*x**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B
*d**3*i**3*x**4*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(146) = 292$.

Time = 0.21 (sec) , antiderivative size = 479, normalized size of antiderivative = 3.07

$$\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{1}{4}\,Bd^3i^3x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+\frac{1}{4}\,Ad^3i^3x^4 + Bcd^2i^3x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) + Acd^2i^3x^3$$
$$+\frac{3}{2}\,Bc^2di^3x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) + \frac{3}{2}\,Ac^2di^3x^2$$
$$-\frac{1}{24}\,Bd^3i^3n\left(\frac{6\,a^4\log(bx+a)}{b^4} - \frac{6\,c^4\log(dx+c)}{d^4} + \frac{2\,(b^3cd^2-ab^2d^3)x^3 - 3\,(b^3c^2d-a^2bd^3)x^2 + 6\,(b^3c^3-}{b^3d^3}\right.$$
$$+\frac{1}{2}\,Bcd^2i^3n\left(\frac{2\,a^3\log(bx+a)}{b^3} - \frac{2\,c^3\log(dx+c)}{d^3} - \frac{(b^2cd-abd^2)x^2 - 2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$-\frac{3}{2}\,Bc^2di^3n\left(\frac{a^2\log(bx+a)}{b^2} - \frac{c^2\log(dx+c)}{d^2} + \frac{(bc-ad)x}{bd}\right)$$
$$+Bc^3i^3n\left(\frac{a\log(bx+a)}{b} - \frac{c\log(dx+c)}{d}\right)$$
$$+Bc^3i^3x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) + Ac^3i^3x$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*B*d^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*d^3*i^3*x^
4 + B*c*d^2*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c*d^2*i^3*x^
3 + 3/2*B*c^2*d*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*c^2*
d*i^3*x^2 - 1/24*B*d^3*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d
^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^
3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*c*d^2*i^3*n*(2*a^3*log(b*x + a)/b^3
- 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)
*x)/(b^2*d^2)) - 3/2*B*c^2*d*i^3*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)
/d^2 + (b*c - a*d)*x/(b*d)) + B*c^3*i^3*n*(a*log(b*x + a)/b - c*log(d*x + c
)/d) + B*c^3*i^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c^3*i^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. $2(146) = 292$.

Time = 0.80 (sec) , antiderivative size = 1402, normalized size of antiderivative = 8.99

$$\int (ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac
")
```

```
[Out] 1/24*(6*(B*b^5*c^5*i^3*n - 5*B*a*b^4*c^4*d*i^3*n + 10*B*a^2*b^3*c^3*d^2*i^3
*n - 10*B*a^3*b^2*c^2*d^3*i^3*n + 5*B*a^4*b*c*d^4*i^3*n - B*a^5*d^5*i^3*n)*
log((b*x + a)/(d*x + c))/(b^4*d - 4*(b*x + a)*b^3*d^2/(d*x + c) + 6*(b*x +
a)^2*b^2*d^3/(d*x + c)^2 - 4*(b*x + a)^3*b*d^4/(d*x + c)^3 + (b*x + a)^4*d^
5/(d*x + c)^4) - (11*B*b^8*c^5*i^3*n - 55*B*a*b^7*c^4*d*i^3*n - 26*(b*x + a
)*B*b^7*c^5*d*i^3*n/(d*x + c) + 110*B*a^2*b^6*c^3*d^2*i^3*n + 130*(b*x + a)
*B*a*b^6*c^4*d^2*i^3*n/(d*x + c) + 21*(b*x + a)^2*B*b^6*c^5*d^2*i^3*n/(d*x
+ c)^2 - 110*B*a^3*b^5*c^2*d^3*i^3*n - 260*(b*x + a)*B*a^2*b^5*c^3*d^3*i^3*
n/(d*x + c) - 105*(b*x + a)^2*B*a*b^5*c^4*d^3*i^3*n/(d*x + c)^2 - 6*(b*x +
a)^3*B*b^5*c^5*d^3*i^3*n/(d*x + c)^3 + 55*B*a^4*b^4*c*d^4*i^3*n + 260*(b*x
+ a)*B*a^3*b^4*c^2*d^4*i^3*n/(d*x + c) + 210*(b*x + a)^2*B*a^2*b^4*c^3*d^4*
i^3*n/(d*x + c)^2 + 30*(b*x + a)^3*B*a*b^4*c^4*d^4*i^3*n/(d*x + c)^3 - 11*B
*a^5*b^3*d^5*i^3*n - 130*(b*x + a)*B*a^4*b^3*c*d^5*i^3*n/(d*x + c) - 210*(b
*x + a)^2*B*a^3*b^3*c^2*d^5*i^3*n/(d*x + c)^2 - 60*(b*x + a)^3*B*a^2*b^3*c^
3*d^5*i^3*n/(d*x + c)^3 + 26*(b*x + a)*B*a^5*b^2*d^6*i^3*n/(d*x + c) + 105*
(b*x + a)^2*B*a^4*b^2*c*d^6*i^3*n/(d*x + c)^2 + 60*(b*x + a)^3*B*a^3*b^2*c^
2*d^6*i^3*n/(d*x + c)^3 - 21*(b*x + a)^2*B*a^5*b*d^7*i^3*n/(d*x + c)^2 - 30
*(b*x + a)^3*B*a^4*b*c*d^7*i^3*n/(d*x + c)^3 + 6*(b*x + a)^3*B*a^5*d^8*i^3*
n/(d*x + c)^3 - 6*B*b^8*c^5*i^3*log(e) + 30*B*a*b^7*c^4*d*i^3*log(e) - 60*B
*a^2*b^6*c^3*d^2*i^3*log(e) + 60*B*a^3*b^5*c^2*d^3*i^3*log(e) - 30*B*a^4*b^
```

```
4*c*d^4*i^3*log(e) + 6*B*a^5*b^3*d^5*i^3*log(e) - 6*A*b^8*c^5*i^3 + 30*A*a*
b^7*c^4*d*i^3 - 60*A*a^2*b^6*c^3*d^2*i^3 + 60*A*a^3*b^5*c^2*d^3*i^3 - 30*A*
a^4*b^4*c*d^4*i^3 + 6*A*a^5*b^3*d^5*i^3)/(b^7*d - 4*(b*x + a)*b^6*d^2/(d*x
+ c) + 6*(b*x + a)^2*b^5*d^3/(d*x + c)^2 - 4*(b*x + a)^3*b^4*d^4/(d*x + c)^
3 + (b*x + a)^4*b^3*d^5/(d*x + c)^4) + 6*(B*b^5*c^5*i^3*n - 5*B*a*b^4*c^4*d
*i^3*n + 10*B*a^2*b^3*c^3*d^2*i^3*n - 10*B*a^3*b^2*c^2*d^3*i^3*n + 5*B*a^4*
b*c*d^4*i^3*n - B*a^5*d^5*i^3*n)*log(-b + (b*x + a)*d/(d*x + c))/(b^4*d) -
6*(B*b^5*c^5*i^3*n - 5*B*a*b^4*c^4*d*i^3*n + 10*B*a^2*b^3*c^3*d^2*i^3*n - 1
0*B*a^3*b^2*c^2*d^3*i^3*n + 5*B*a^4*b*c*d^4*i^3*n - B*a^5*d^5*i^3*n)*log((b
*x + a)/(d*x + c))/(b^4*d))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 588, normalized size of antiderivative = 3.77

$$\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$= x^3\left(\frac{d^2 i^3 (4Aad+16Abc+Badn-Bbcn)}{12b}-\frac{Ad^2 i^3 (4ad+4bc)}{12b}\right) - x^2\left(\frac{\left(\frac{d^2 i^3 (4Aad+16Abc+Badn-Bbcn)}{4b}-\frac{Ad^2 i^3 (4ad+4bc)}{4b}\right)(4ad+4bc)}{8bd} - \frac{cdi^3 (4Aad+6Abc+Badn-Bbcn)}{2b}+\frac{Aacd^2 i^3}{2b}\right) + \ln\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\left(Bc^3 i^3 x+\frac{3Bc^2 d i^3 x^2}{2}+Bcd^2 i^3 x^3+\frac{Bd^3 i^3 x^4}{4}\right) + x\left(\frac{(4ad+4bc)\left(\frac{\left(\frac{d^2 i^3 (4Aad+16Abc+Badn-Bbcn)}{4b}-\frac{Ad^2 i^3 (4ad+4bc)}{4b}\right)(4ad+4bc)}{4bd}-\frac{cdi^3 (4Aad+6Abc+Badn-Bbcn)}{b}\right.}{4bd} + \frac{c^2 i^3 (12Aad+8Abc+3Badn-3Bbcn)}{2b} - \frac{ac\left(\frac{d^2 i^3 (4Aad+16Abc+Badn-Bbcn)}{4b}-\frac{Ad^2 i^3 (4ad+4bc)}{4b}\right)}{bd}\right) - \frac{\ln(a+bx)\left(Bna^4 d^3 i^3-4Bna^3 bcd^2 i^3+6Bna^2 b^2 c^2 d i^3-4Bnab^3 c^3 i^3\right)}{4b^4} + \frac{Ad^3 i^3 x^4}{4} - \frac{Bc^4 i^3 n\ln(c+dx)}{4d}$$

```
[In] int((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

```
[Out] x^3*((d^2*i^3*(4*A*a*d + 16*A*b*c + B*a*d*n - B*b*c*n))/(12*b) - (A*d^2*i^3
*(4*a*d + 4*b*c))/(12*b)) - x^2*((((d^2*i^3*(4*A*a*d + 16*A*b*c + B*a*d*n -
 B*b*c*n))/(4*b) - (A*d^2*i^3*(4*a*d + 4*b*c))/(4*b))*(4*a*d + 4*b*c))/(8*b
*d) - (c*d*i^3*(4*A*a*d + 6*A*b*c + B*a*d*n - B*b*c*n))/(2*b) + (A*a*c*d^2*
i^3)/(2*b)) + log(e*((a + b*x)/(c + d*x))^n)*((B*d^3*i^3*x^4)/4 + B*c^3*i^3
*x + (3*B*c^2*d*i^3*x^2)/2 + B*c*d^2*i^3*x^3) + x*(((4*a*d + 4*b*c)*((((d^2
*i^3*(4*A*a*d + 16*A*b*c + B*a*d*n - B*b*c*n))/(4*b) - (A*d^2*i^3*(4*a*d +
4*b*c))/(4*b))*(4*a*d + 4*b*c))/(4*b*d) - (c*d*i^3*(4*A*a*d + 6*A*b*c + B*a
*d*n - B*b*c*n))/b + (A*a*c*d^2*i^3)/b))/(4*b*d) + (c^2*i^3*(12*A*a*d + 8*A
*b*c + 3*B*a*d*n - 3*B*b*c*n))/(2*b) - (a*c*((d^2*i^3*(4*A*a*d + 16*A*b*c +
 B*a*d*n - B*b*c*n))/(4*b) - (A*d^2*i^3*(4*a*d + 4*b*c))/(4*b)))/(b*d)) - (
log(a + b*x)*(B*a^4*d^3*i^3*n - 4*B*a*b^3*c^3*i^3*n - 4*B*a^3*b*c*d^2*i^3*n
 + 6*B*a^2*b^2*c^2*d*i^3*n))/(4*b^4) + (A*d^3*i^3*x^4)/4 - (B*c^4*i^3*n*log
(c + d*x))/(4*d)
```

3.131 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 373

$$
\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx \\
&= -\frac{5Bd(bc-ad)^2i^3nx}{6b^3g}-\frac{B(bc-ad)i^3n(c+dx)^2}{6b^2g} \\
&\quad+\frac{d(bc-ad)^2i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^4g} \\
&\quad+\frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g}+\frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bg} \\
&\quad-\frac{5B(bc-ad)^3i^3n\log\left(\frac{a+bx}{c+dx}\right)}{6b^4g}-\frac{11B(bc-ad)^3i^3n\log(c+dx)}{6b^4g} \\
&\quad-\frac{(bc-ad)^3i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g} \\
&\quad+\frac{B(bc-ad)^3i^3n\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}
\end{aligned}
$$

```
[Out] -5/6*B*d*(-a*d+b*c)^2*i^3*n*x/b^3/g-1/6*B*(-a*d+b*c)*i^3*n*(d*x+c)^2/b^2/g+
d*(-a*d+b*c)^2*i^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/g+1/2*(-a*d+
b*c)*i^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g+1/3*i^3*(d*x+c)^3*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g-5/6*B*(-a*d+b*c)^3*i^3*n*ln((b*x+a)/(d*
x+c))/b^4/g-11/6*B*(-a*d+b*c)^3*i^3*n*ln(d*x+c)/b^4/g-(-a*d+b*c)^3*i^3*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g+B*(-a*d+b*c)^3*
i^3*n*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {2561, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = \frac{di^3(a+bx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4g} - \frac{i^3(bc-ad)^3\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4g} + \frac{i^3(c+dx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b^2g} + \frac{i^3(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3bg} + \frac{Bi^3n(bc-ad)^3\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g} - \frac{5Bi^3n(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{6b^4g} - \frac{11Bi^3n(bc-ad)^3\log(c+dx)}{6b^4g} - \frac{5Bdi^3nx(bc-ad)^2}{6b^3g} - \frac{Bi^3n(c+dx)^2(bc-ad)}{6b^2g}$$

```
[In] Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x),
x]
```

```
[Out] (-5*B*d*(b*c - a*d)^2*i^3*n*x)/(6*b^3*g) - (B*(b*c - a*d)*i^3*n*(c + d*x)^2
)/(6*b^2*g) + (d*(b*c - a*d)^2*i^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]))/(b^4*g) + ((b*c - a*d)*i^3*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]))/(2*b^2*g) + (i^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]))/(3*b*g) - (5*B*(b*c - a*d)^3*i^3*n*Log[(a + b*x)/(c + d*x)])/(6*b^4
*g) - (11*B*(b*c - a*d)^3*i^3*n*Log[c + d*x])/(6*b^4*g) - ((b*c - a*d)^3*i^
3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x)
)])/(b^4*g) + (B*(b*c - a*d)^3*i^3*n*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))
])/(b^4*g)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\left((bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{g}$$

$$= \frac{\left((bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg} + \frac{\left(d(bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg}$$

$$= \frac{i^3(c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bg} + \frac{\left((bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2 g} + \frac{\left(d(bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2 g} - \frac{\left(B(bc-ad)^3 i^3 n\right) \text{Subst}\left(\int \frac{1}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3bg}$$

$$= \frac{(bc-ad) i^3 (c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2 g} + \frac{i^3(c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bg} + \frac{\left((bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3 g} + \frac{\left(d(bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3 g} - \frac{\left(B(bc-ad)^3 i^3 n\right) \text{Subst}\left(\int \frac{1}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b^2 g} - \frac{\left(B(bc-ad)^3 i^3 n\right) \text{Subst}\left(\int \left(\frac{1}{b^3 x} + \frac{d}{b(b-dx)^3} + \frac{d}{b^2(b-dx)^2} + \frac{d}{b^3(b-dx)}\right) dx, x, \frac{a+bx}{c+dx}\right)}{3bg}$$

$$= -\frac{Bd(bc-ad)^2 i^3 nx}{3b^3 g} - \frac{B(bc-ad)i^3 n(c+dx)^2}{6b^2 g} + \frac{d(bc-ad)^2 i^3 (a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^4 g} + \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2 g} + \frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bg} - \frac{B(bc-ad)^3 i^3 n\log\left(\frac{a+bx}{c+dx}\right)}{3b^4 g} - \frac{B(bc-ad)^3 i^3 n\log(c+dx)}{3b^4 g} - \frac{(bc-ad)^3 i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4 g} + \frac{(B(bc-ad)^3 i^3 n)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4 g} - \frac{(B(bc-ad)^3 i^3 n)\operatorname{Subst}\left(\int \left(\frac{1}{b^2 x}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{2b^2 g} - \frac{(Bd(bc-ad)^3 i^3 n)\operatorname{Subst}\left(\int \frac{1}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4 g}$$

$$= -\frac{5Bd(bc-ad)^2 i^3 nx}{6b^3 g} - \frac{B(bc-ad)i^3 n(c+dx)^2}{6b^2 g} + \frac{d(bc-ad)^2 i^3 (a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^4 g} + \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2 g} + \frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bg} - \frac{5B(bc-ad)^3 i^3 n\log\left(\frac{a+bx}{c+dx}\right)}{6b^4 g} - \frac{11B(bc-ad)^3 i^3 n\log(c+dx)}{6b^4 g} - \frac{(bc-ad)^3 i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4 g} + \frac{B(bc-ad)^3 i^3 n\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4 g}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.99

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$$
$$= \frac{i^3\Big(6Abd(bc-ad)^2x-3B(bc-ad)^2n(bdx+(bc-ad)\log(a+bx))-B(bc-ad)n(2bd(bc-ad)x+b^2(c$$

```
[In] Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b
*g*x),x]
```

```
[Out] (i^3*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*n*(b*d*x + (b*c - a*d)*Lo
g[a + b*x]) - B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b
*c - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*x)/
(c + d*x))^n] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]) + 2*b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 6*(b
*c - a*d)^3*Log[g*(a + b*x)]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*B*(
b*c - a*d)^3*n*Log[c + d*x] - 3*B*(b*c - a*d)^3*n*(Log[g*(a + b*x)]*(Log[g*
(a + b*x)] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))
/(-(b*c) + a*d)])))/(6*b^4*g)
```

Maple [F]

$$\int \frac{(dix+ci)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{bgx+ag}\,dx$$

```
[In] int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

```
[Out] int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, alg
orithm="fricas")
```

```
[Out] integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3 +
 (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log(e*((
b*x + a)/(d*x + c))^n))/(b*g*x + a*g), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$$
$$= \frac{i^3\left(\int \frac{Ac^3}{a+bx}\,dx+\int \frac{Ad^3x^3}{a+bx}\,dx+\int \frac{Bc^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx+\int \frac{3Acd^2x^2}{a+bx}\,dx+\int \frac{3Ac^2dx}{a+bx}\,dx+\int \frac{Bd^3x^3\log\left(e\left(\frac{a}{c+dx}+\frac{b}{c+}\right.\right.}{a+bx}\right.}{g}$$

```
[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g),x)
```

```
[Out] i**3*(Integral(A*c**3/(a + b*x), x) + Integral(A*d**3*x**3/(a + b*x), x) +
Integral(B*c**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x) + Int
egral(3*A*c*d**2*x**2/(a + b*x), x) + Integral(3*A*c**2*d*x/(a + b*x), x) +
 Integral(B*d**3*x**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x)
 + Integral(3*B*c*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*
x), x) + Integral(3*B*c**2*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a +
 b*x), x))/g
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 935 vs. 2(360) = 720.

Time = 0.50 (sec) , antiderivative size = 935, normalized size of antiderivative = 2.51

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$$
$$= 3\,Ac^2di^3\left(\frac{x}{bg}-\frac{a\log(bx+a)}{b^2g}\right)-\frac{1}{6}\,Ad^3i^3\left(\frac{6\,a^3\log(bx+a)}{b^4g}-\frac{2\,b^2x^3-3\,abx^2+6\,a^2x}{b^3g}\right)$$
$$+\frac{3}{2}\,Acd^2i^3\left(\frac{2\,a^2\log(bx+a)}{b^3g}+\frac{bx^2-2\,ax}{b^2g}\right)+\frac{Ac^3i^3\log(bgx+ag)}{bg}$$
$$-\frac{(11\,b^2c^3i^3n-15\,abc^2di^3n+6\,a^2cd^2i^3n)B\log(dx+c)}{6\,b^3g}$$
$$+\frac{(b^3c^3i^3n-3\,ab^2c^2di^3n+3\,a^2bcd^2i^3n-a^3d^3i^3n)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{b^4g}$$
$$+\frac{2\,Bb^3d^3i^3x^3\log(e)-((i^3n-9\,i^3\log(e))b^3cd^2-(i^3n-3\,i^3\log(e))ab^2d^3)Bx^2-3\,(b^3c^3i^3n-3\,ab^2c^2di^3n}{}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, alg
orithm="maxima")
```

```
[Out] 3*A*c^2*d*i^3*(x/(b*g) - a*log(b*x + a)/(b^2*g)) - 1/6*A*d^3*i^3*(6*a^3*log
(b*x + a)/(b^4*g) - (2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/(b^3*g)) + 3/2*A*c*d^
2*i^3*(2*a^2*log(b*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A*c^3*i^3*lo
```

```
g(b*g*x + a*g)/(b*g) - 1/6*(11*b^2*c^3*i^3*n - 15*a*b*c^2*d*i^3*n + 6*a^2*c
*d^2*i^3*n)*B*log(d*x + c)/(b^3*g) + (b^3*c^3*i^3*n - 3*a*b^2*c^2*d*i^3*n +
 3*a^2*b*c*d^2*i^3*n - a^3*d^3*i^3*n)*(log(b*x + a)*log((b*d*x + a*d)/(b*c
- a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(b^4*g) + 1/6*(2*B*b^3*d
^3*i^3*x^3*log(e) - ((i^3*n - 9*i^3*log(e))*b^3*c*d^2 - (i^3*n - 3*i^3*log(
e))*a*b^2*d^3)*B*x^2 - 3*(b^3*c^3*i^3*n - 3*a*b^2*c^2*d*i^3*n + 3*a^2*b*c*d
^2*i^3*n - a^3*d^3*i^3*n)*B*log(b*x + a)^2 - ((7*i^3*n - 18*i^3*log(e))*b^3
*c^2*d - 6*(2*i^3*n - 3*i^3*log(e))*a*b^2*c*d^2 + (5*i^3*n - 6*i^3*log(e))*
a^2*b*d^3)*B*x + (6*b^3*c^3*i^3*log(e) + 18*(i^3*n - i^3*log(e))*a*b^2*c^2*
d - 9*(3*i^3*n - 2*i^3*log(e))*a^2*b*c*d^2 + (11*i^3*n - 6*i^3*log(e))*a^3*
d^3)*B*log(b*x + a) + (2*B*b^3*d^3*i^3*x^3 + 3*(3*b^3*c*d^2*i^3 - a*b^2*d^3
*i^3)*B*x^2 + 6*(3*b^3*c^2*d*i^3 - 3*a*b^2*c*d^2*i^3 + a^2*b*d^3*i^3)*B*x +
 6*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^2*b*c*d^2*i^3 - a^3*d^3*i^3)*B*lo
g(b*x + a))*log((b*x + a)^n) - (2*B*b^3*d^3*i^3*x^3 + 3*(3*b^3*c*d^2*i^3 -
a*b^2*d^3*i^3)*B*x^2 + 6*(3*b^3*c^2*d*i^3 - 3*a*b^2*c*d^2*i^3 + a^2*b*d^3*i
^3)*B*x + 6*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^2*b*c*d^2*i^3 - a^3*d^3*
i^3)*B*log(b*x + a))*log((d*x + c)^n))/(b^4*g)
```

Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, alg
orithm="giac")
```

```
[Out] integrate((d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x + a
*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx=\int \frac{(c\,i+d\,i\,x)^3\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}{a\,g+b\,g\,x}\,dx$$

```
[In] int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x),
x)
```

```
[Out] int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x),
 x)
```

3.132 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 390

$$\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx \\
&= -\frac{Bd^2(bc-ad)i^3nx}{2b^3g^2}-\frac{B(bc-ad)^2i^3n(c+dx)}{b^3g^2(a+bx)} \\
&\quad+\frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^4g^2} \\
&\quad-\frac{(bc-ad)^2i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^2(a+bx)}+\frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^2} \\
&\quad-\frac{Bd(bc-ad)^2i^3n\log\left(\frac{a+bx}{c+dx}\right)}{2b^4g^2}-\frac{5Bd(bc-ad)^2i^3n\log(c+dx)}{2b^4g^2} \\
&\quad-\frac{3d(bc-ad)^2i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} \\
&\quad+\frac{3Bd(bc-ad)^2i^3n\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}
\end{aligned}$$

```
[Out] -1/2*B*d^2*(-a*d+b*c)*i^3*n*x/b^3/g^2-B*(-a*d+b*c)^2*i^3*n*(d*x+c)/b^3/g^2/
(b*x+a)+2*d^2*(-a*d+b*c)*i^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/g^
2-(-a*d+b*c)^2*i^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/g^2/(b*x+a)+
1/2*d*i^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g^2-1/2*B*d*(-a*d+b
*c)^2*i^3*n*ln((b*x+a)/(d*x+c))/b^4/g^2-5/2*B*d*(-a*d+b*c)^2*i^3*n*ln(d*x+c
)/b^4/g^2-3*d*(-a*d+b*c)^2*i^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+
c)/d/(b*x+a))/b^4/g^2+3*B*d*(-a*d+b*c)^2*i^3*n*polylog(2,b*(d*x+c)/d/(b*x+a
))/b^4/g^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {2561, 46, 2393, 2341, 2356, 2351, 31, 2379, 2438}

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$$
$$= \frac{2d^2i^3(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4g^2}$$
$$-\frac{3di^3(bc-ad)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4g^2}$$
$$-\frac{i^3(c+dx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3g^2(a+bx)}+\frac{di^3(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b^2g^2}$$
$$+\frac{3Bdi^3n(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}-\frac{Bdi^3n(bc-ad)^2\log\left(\frac{a+bx}{c+dx}\right)}{2b^4g^2}$$
$$-\frac{5Bdi^3n(bc-ad)^2\log(c+dx)}{2b^4g^2}-\frac{Bd^2i^3nx(bc-ad)}{2b^3g^2}-\frac{Bi^3n(c+dx)(bc-ad)^2}{b^3g^2(a+bx)}$$

```
[In] Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^
2,x]
```

```
[Out] -1/2*(B*d^2*(b*c - a*d)*i^3*n*x)/(b^3*g^2) - (B*(b*c - a*d)^2*i^3*n*(c + d*
x))/(b^3*g^2*(a + b*x)) + (2*d^2*(b*c - a*d)*i^3*(a + b*x)*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n]))/(b^4*g^2) - ((b*c - a*d)^2*i^3*(c + d*x)*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]))/(b^3*g^2*(a + b*x)) + (d*i^3*(c + d*x)^2*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*b^2*g^2) - (B*d*(b*c - a*d)^2*i^3*n*
Log[(a + b*x)/(c + d*x)])/(2*b^4*g^2) - (5*B*d*(b*c - a*d)^2*i^3*n*Log[c +
d*x])/(2*b^4*g^2) - (3*d*(b*c - a*d)^2*i^3*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^4*g^2) + (3*B*d*(b*c - a*d)
^2*i^3*n*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b^4*g^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
```

```
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)^2 i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^2}$$

$$= \frac{((bc-ad)^2 i^3)\,\text{Subst}\left(\int \left(\frac{A+B\log(ex^n)}{b^3x^2} + \frac{d^2(A+B\log(ex^n))}{b^2(b-dx)^3} + \frac{2d^2(A+B\log(ex^n))}{b^3(b-dx)^2} + \frac{3d(A+B\log(ex^n))}{b^3x(b-dx)}\right)\,dx, x, \frac{a}{c}\right.}{g^2}$$

$$= \frac{((bc-ad)^2 i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^2} + \frac{(3d(bc-ad)^2 i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^2} + \frac{(2d^2(bc-ad)^2 i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^2} + \frac{(d^2(bc-ad)^2 i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2}$$

$$= -\frac{B(bc-ad)^2 i^3 n(c+dx)}{b^3g^2(a+bx)} + \frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^4g^2} - \frac{(bc-ad)^2 i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^2(a+bx)} + \frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^2} - \frac{3d(bc-ad)^2 i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} + \frac{(3Bd(bc-ad)^2 i^3 n)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4g^2} - \frac{(Bd(bc-ad)^2 i^3 n)\,\text{Subst}\left(\int \frac{1}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b^2g^2} - \frac{(2Bd^2(bc-ad)^2 i^3 n)\,\text{Subst}\left(\int \frac{1}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4g^2}$$

$$= -\frac{B(bc-ad)^2 i^3 n(c+dx)}{b^3 g^2 (a+bx)} + \frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^4 g^2} - \frac{(bc-ad)^2 i^3 (c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3 g^2 (a+bx)} + \frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2 g^2} - \frac{2Bd(bc-ad)^2 i^3 n\log(c+dx)}{b^4 g^2} - \frac{3d(bc-ad)^2 i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4 g^2} + \frac{3Bd(bc-ad)^2 i^3 n \operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4 g^2} - \frac{(Bd(bc-ad)^2 i^3 n)\operatorname{Subst}\left(\int\left(\frac{1}{b^2 x}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{2b^2 g^2}$$

$$= -\frac{Bd^2(bc-ad)i^3 nx}{2b^3 g^2} - \frac{B(bc-ad)^2 i^3 n(c+dx)}{b^3 g^2 (a+bx)} + \frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^4 g^2} - \frac{(bc-ad)^2 i^3 (c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3 g^2 (a+bx)} + \frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2 g^2} - \frac{Bd(bc-ad)^2 i^3 n\log\left(\frac{a+bx}{c+dx}\right)}{2b^4 g^2} - \frac{5Bd(bc-ad)^2 i^3 n\log(c+dx)}{2b^4 g^2} - \frac{3d(bc-ad)^2 i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4 g^2} + \frac{3Bd(bc-ad)^2 i^3 n \operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4 g^2}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.01

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$$

$$= \frac{i^3\left(2Abd^2(3bc-2ad)x - bBd^2(bc-ad)nx - \frac{2B(bc-ad)^3 n}{a+bx} - a^2Bd^3n\log(a+bx) - 2Bd(bc-ad)^2 n\log(a+b\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b
*g*x)^2,x]
```

```
[Out] (i^3*(2*A*b*d^2*(3*b*c - 2*a*d)*x - b*B*d^2*(b*c - a*d)*n*x - (2*B*(b*c - a
*d)^3*n)/(a + b*x) - a^2*B*d^3*n*Log[a + b*x] - 2*B*d*(b*c - a*d)^2*n*Log[a
 + b*x] + 2*B*d^2*(3*b*c - 2*a*d)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n]
+ b^2*d^3*x^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (2*(b*c - a*d)^3*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + 6*d*(b*c - a*d)^2*Log[a +
b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + b^2*B*c^2*d*n*Log[c + d*x] +
2*B*d*(b*c - a*d)^2*n*Log[c + d*x] - 2*B*d*(-(b*c) + a*d)*(-3*b*c + 2*a*d)*
n*Log[c + d*x] - 3*B*d*(b*c - a*d)^2*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[
(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))
/(2*b^4*g^2)
```

Maple [F]

$$\int \frac{(dix+ci)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(bgx+ag)^2}\,dx$$

```
[In] int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

```
[Out] int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, a
lgorithm="fricas")
```

```
[Out] integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3 +
 (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log(e*((
b*x + a)/(d*x + c))^n))/(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1785 vs. $2(381) = 762$.

Time = 0.50 (sec) , antiderivative size = 1785, normalized size of antiderivative = 4.58

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, a
lgorithm="maxima")
```

```
[Out] -B*c^3*i^3*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2
) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - 3*A*(a^2/(b^4*g^2*x + a*b^3*g^2
) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*g^2))*c*d^2*i^3 + 1/2*(2*a^3/(b^5*g
^2*x + a*b^4*g^2) + 6*a^2*log(b*x + a)/(b^4*g^2) + (b*x^2 - 4*a*x)/(b^3*g^2
))*A*d^3*i^3 + 3*A*c^2*d*i^3*(a/(b^3*g^2*x + a*b^2*g^2) + log(b*x + a)/(b^2
*g^2)) - B*c^3*i^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^2*g^2*x + a*b*
g^2) - A*c^3*i^3/(b^2*g^2*x + a*b*g^2) - 1/2*(5*b^3*c^3*d*i^3*n - 3*a*b^2*c
^2*d^2*i^3*n - 2*a^2*b*c*d^3*i^3*n + 2*a^3*d^4*i^3*n)*B*log(d*x + c)/(b^5*c
*g^2 - a*b^4*d*g^2) + 1/2*((b^4*c*d^3*i^3*log(e) - a*b^3*d^4*i^3*log(e))*B*
x^3 - ((i^3*n - 6*i^3*log(e))*b^4*c^2*d^2 - (2*i^3*n - 9*i^3*log(e))*a*b^3*
c*d^3 + (i^3*n - 3*i^3*log(e))*a^2*b^2*d^4)*B*x^2 - ((i^3*n - 6*i^3*log(e))
*a*b^3*c^2*d^2 - 2*(i^3*n - 5*i^3*log(e))*a^2*b^2*c*d^3 + (i^3*n - 4*i^3*lo
g(e))*a^3*b*d^4)*B*x - 3*((b^4*c^3*d*i^3*n - 3*a*b^3*c^2*d^2*i^3*n + 3*a^2*
b^2*c*d^3*i^3*n - a^3*b*d^4*i^3*n)*B*x + (a*b^3*c^3*d*i^3*n - 3*a^2*b^2*c^2
*d^2*i^3*n + 3*a^3*b*c*d^3*i^3*n - a^4*d^4*i^3*n)*B)*log(b*x + a)^2 + 2*(3*
(i^3*n + i^3*log(e))*a*b^3*c^3*d - 6*(i^3*n + i^3*log(e))*a^2*b^2*c^2*d^2 +
 4*(i^3*n + i^3*log(e))*a^3*b*c*d^3 - (i^3*n + i^3*log(e))*a^4*d^4)*B + ((6
*b^4*c^3*d*i^3*log(e) + 6*(2*i^3*n - 3*i^3*log(e))*a*b^3*c^2*d^2 - (17*i^3*
n - 18*i^3*log(e))*a^2*b^2*c*d^3 + (7*i^3*n - 6*i^3*log(e))*a^3*b*d^4)*B*x
+ (6*a*b^3*c^3*d*i^3*log(e) + 6*(2*i^3*n - 3*i^3*log(e))*a^2*b^2*c^2*d^2 -
(17*i^3*n - 18*i^3*log(e))*a^3*b*c*d^3 + (7*i^3*n - 6*i^3*log(e))*a^4*d^4)*
B)*log(b*x + a) + ((b^4*c*d^3*i^3 - a*b^3*d^4*i^3)*B*x^3 + 3*(2*b^4*c^2*d^2
*i^3 - 3*a*b^3*c*d^3*i^3 + a^2*b^2*d^4*i^3)*B*x^2 + 2*(3*a*b^3*c^2*d^2*i^3
- 5*a^2*b^2*c*d^3*i^3 + 2*a^3*b*d^4*i^3)*B*x + 2*(3*a*b^3*c^3*d*i^3 - 6*a^2
*b^2*c^2*d^2*i^3 + 4*a^3*b*c*d^3*i^3 - a^4*d^4*i^3)*B + 6*((b^4*c^3*d*i^3 -
 3*a*b^3*c^2*d^2*i^3 + 3*a^2*b^2*c*d^3*i^3 - a^3*b*d^4*i^3)*B*x + (a*b^3*c^
3*d*i^3 - 3*a^2*b^2*c^2*d^2*i^3 + 3*a^3*b*c*d^3*i^3 - a^4*d^4*i^3)*B)*log(b
*x + a))*log((b*x + a)^n) - ((b^4*c*d^3*i^3 - a*b^3*d^4*i^3)*B*x^3 + 3*(2*b
^4*c^2*d^2*i^3 - 3*a*b^3*c*d^3*i^3 + a^2*b^2*d^4*i^3)*B*x^2 + 2*(3*a*b^3*c^
2*d^2*i^3 - 5*a^2*b^2*c*d^3*i^3 + 2*a^3*b*d^4*i^3)*B*x + 2*(3*a*b^3*c^3*d*i
^3 - 6*a^2*b^2*c^2*d^2*i^3 + 4*a^3*b*c*d^3*i^3 - a^4*d^4*i^3)*B + 6*((b^4*c
^3*d*i^3 - 3*a*b^3*c^2*d^2*i^3 + 3*a^2*b^2*c*d^3*i^3 - a^3*b*d^4*i^3)*B*x +
 (a*b^3*c^3*d*i^3 - 3*a^2*b^2*c^2*d^2*i^3 + 3*a^3*b*c*d^3*i^3 - a^4*d^4*i^3
)*B)*log(b*x + a))*log((d*x + c)^n))/(a*b^5*c*g^2 - a^2*b^4*d*g^2 + (b^6*c*
```

```
g^2 - a*b^5*d*g^2)*x) + 3*(b^2*c^2*d*i^3*n - 2*a*b*c*d^2*i^3*n + a^2*d^3*i^
3*n)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d
)/(b*c - a*d)))*B/(b^4*g^2)
```

Giac [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, a
lgorithm="giac")
```

```
[Out] integrate((d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x + a
*g)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(c\,i+d\,i\,x)^3\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}{(a\,g+b\,g\,x)^2}\,dx$$

```
[In] int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^
2,x)
```

```
[Out] int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^
2, x)
```

3.133 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 361

$$\begin{aligned}
\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx
&= -\frac{2Bd(bc-ad)i^3n(c+dx)}{b^3g^3(a+bx)} - \frac{B(bc-ad)i^3n(c+dx)^2}{4b^2g^3(a+bx)^2} \\
&\quad + \frac{d^3i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^4g^3} - \frac{2d(bc-ad)i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^3(a+bx)} \\
&\quad - \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^3(a+bx)^2} - \frac{Bd^2(bc-ad)i^3n\log(c+dx)}{b^4g^3} \\
&\quad - \frac{3d^2(bc-ad)i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} \\
&\quad + \frac{3Bd^2(bc-ad)i^3n\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}
\end{aligned}$$

```
[Out] -2*B*d*(-a*d+b*c)*i^3*n*(d*x+c)/b^3/g^3/(b*x+a)-1/4*B*(-a*d+b*c)*i^3*n*(d*x
+c)^2/b^2/g^3/(b*x+a)^2+d^3*i^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4
/g^3-2*d*(-a*d+b*c)*i^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/g^3/(b*
x+a)-1/2*(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g^3/(
b*x+a)^2-B*d^2*(-a*d+b*c)*i^3*n*ln(d*x+c)/b^4/g^3-3*d^2*(-a*d+b*c)*i^3*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^3+3*B*d^2*(-a*d
+b*c)*i^3*n*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {2561, 46, 2393, 2341, 2351, 31, 2379, 2438}

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$$
$$= \frac{d^3i^3(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4g^3} - \frac{3d^2i^3(bc-ad)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4g^3} - \frac{2di^3(c+dx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3g^3(a+bx)} - \frac{i^3(c+dx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b^2g^3(a+bx)^2} + \frac{3Bd^2i^3n(bc-ad)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} - \frac{Bd^2i^3n(bc-ad)\log(c+dx)}{b^4g^3} - \frac{2Bdi^3n(c+dx)(bc-ad)}{b^3g^3(a+bx)} - \frac{Bi^3n(c+dx)^2(bc-ad)}{4b^2g^3(a+bx)^2}$$

```
[In] Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^
3,x]
```

```
[Out] (-2*B*d*(b*c - a*d)*i^3*n*(c + d*x))/(b^3*g^3*(a + b*x)) - (B*(b*c - a*d)*i
^3*n*(c + d*x)^2)/(4*b^2*g^3*(a + b*x)^2) + (d^3*i^3*(a + b*x)*(A + B*Log[e
*((a + b*x)/(c + d*x))^n]))/(b^4*g^3) - (2*d*(b*c - a*d)*i^3*(c + d*x)*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^3*g^3*(a + b*x)) - ((b*c - a*d)*i^3*
(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*b^2*g^3*(a + b*x)^2)
 - (B*d^2*(b*c - a*d)*i^3*n*Log[c + d*x])/(b^4*g^3) - (3*d^2*(b*c - a*d)*i^
3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x)
)])/(b^4*g^3) + (3*B*d^2*(b*c - a*d)*i^3*n*PolyLog[2, (b*(c + d*x))/(d*(a +
 b*x))])/(b^4*g^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^3}$$

$$= \frac{((bc-ad)i^3)\,\text{Subst}\left(\int \left(\frac{A+B\log(ex^n)}{b^2x^3}+\frac{2d(A+B\log(ex^n))}{b^3x^2}+\frac{d^3(A+B\log(ex^n))}{b^3(b-dx)^2}+\frac{3d^2(A+B\log(ex^n))}{b^3x(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{g^3}$$

$$= \frac{((bc-ad)i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^2g^3} + \frac{(2d(bc-ad)i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3g^3} + \frac{(3d^2(bc-ad)i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3g^3} + \frac{(d^3(bc-ad)i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3g^3}$$

$$= -\frac{2Bd(bc-ad)i^3n(c+dx)}{b^3g^3(a+bx)} - \frac{B(bc-ad)i^3n(c+dx)^2}{4b^2g^3(a+bx)^2} + \frac{d^3i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^4g^3} - \frac{2d(bc-ad)i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^3(a+bx)} - \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^3(a+bx)^2} - \frac{3d^2(bc-ad)i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} + \frac{(3Bd^2(bc-ad)i^3n)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^4g^3} - \frac{(Bd^3(bc-ad)i^3n)\,\text{Subst}\left(\int \frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^4g^3}$$

$$= -\frac{2Bd(bc-ad)i^3n(c+dx)}{b^3g^3(a+bx)} - \frac{B(bc-ad)i^3n(c+dx)^2}{4b^2g^3(a+bx)^2} + \frac{d^3i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^4g^3} - \frac{2d(bc-ad)i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^3(a+bx)} - \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^3(a+bx)^2} - \frac{Bd^2(bc-ad)i^3n\log(c+dx)}{b^4g^3} - \frac{3d^2(bc-ad)i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} + \frac{3Bd^2(bc-ad)i^3n\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.92

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \frac{i^3\left(4Abd^3x - \frac{B(bc-ad)^3n}{(a+bx)^2} - \frac{10Bd(bc-ad)^2n}{a+bx} + 10Bd^2(-bc+ad)n\log(a+bx) + 4Bd^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - \right.}{}$$

```
[In] Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b
*g*x)^3,x]
```

```
[Out] (i^3*(4*A*b*d^3*x - (B*(b*c - a*d)^3*n)/(a + b*x)^2 - (10*B*d*(b*c - a*d)^2
*n)/(a + b*x) + 10*B*d^2*(-(b*c) + a*d)*n*Log[a + b*x] + 4*B*d^3*(a + b*x)*
Log[e*((a + b*x)/(c + d*x))^n] - (2*(b*c - a*d)^3*(A + B*Log[e*((a + b*x)/(
c + d*x))^n]))/(a + b*x)^2 - (12*d*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]))/(a + b*x) + 12*d^2*(b*c - a*d)*Log[a + b*x]*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n]) + 6*B*d^2*(b*c - a*d)*n*Log[c + d*x] + 6*B*d^2*(-(b*c
) + a*d)*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)])
- 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(4*b^4*g^3)
```

Maple [F]

$$\int \frac{(dix+ci)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(bgx+ag)^3}dx$$

```
[In] int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x)
[Out] int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^3}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, a
lgorithm="fricas")
[Out] integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3 +
 (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log(e*((
b*x + a)/(d*x + c))^n))/(b^3*g^3*x^3 + 3*a*b^2*g^3*x^2 + 3*a^2*b*g^3*x + a^
3*g^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2746 vs. $2(356) = 712$.

Time = 0.57 (sec) , antiderivative size = 2746, normalized size of antiderivative = 7.61

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, a
lgorithm="maxima")
```

[Out] -3/4*B*c^2*d*i^3*n*((3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*x)/((b^5*c - a*b
^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3
) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)
g^3) - 2(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*
d^2)*g^3)) + 1/4*B*c^3*i^3*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^
3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*
log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)
/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A*d^3*i^3*((6*a^2*b*x + 5
*a^3)/(b^6*g^3*x^2 + 2*a*b^5*g^3*x + a^2*b^4*g^3) - 2*x/(b^3*g^3) + 6*a*log
(b*x + a)/(b^4*g^3)) + 3/2*A*c*d^2*i^3*((4*a*b*x + 3*a^2)/(b^5*g^3*x^2 + 2*
a*b^4*g^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(b^3*g^3)) - 3/2*(2*b*x + a)*B*
c^2*d*i^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^4*g^3*x^2 + 2*a*b^3*g^3
*x + a^2*b^2*g^3) - 3/2*(2*b*x + a)*A*c^2*d*i^3/(b^4*g^3*x^2 + 2*a*b^3*g^3*
x + a^2*b^2*g^3) - 1/2*B*c^3*i^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^
3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A*c^3*i^3/(b^3*g^3*x^2 + 2*a*b
^2*g^3*x + a^2*b*g^3) - 1/2*(2*b^3*c^3*d^2*i^3*n + 8*a*b^2*c^2*d^3*i^3*n -
13*a^2*b*c*d^4*i^3*n + 5*a^3*d^5*i^3*n)*B*log(d*x + c)/(b^6*c^2*g^3 - 2*a*b
^5*c*d*g^3 + a^2*b^4*d^2*g^3) + 1/4*(4*(b^5*c^2*d^3*i^3*log(e) - 2*a*b^4*c*
d^4*i^3*log(e) + a^2*b^3*d^5*i^3*log(e))*B*x^3 + 8*(a*b^4*c^2*d^3*i^3*log(e
) - 2*a^2*b^3*c*d^4*i^3*log(e) + a^3*b^2*d^5*i^3*log(e))*B*x^2 + 2*(12*(i^3
*n + i^3*log(e))*a*b^4*c^3*d^2 - (27*i^3*n + 28*i^3*log(e))*a^2*b^3*c^2*d^3
 + 20*(i^3*n + i^3*log(e))*a^3*b^2*c*d^4 - (5*i^3*n + 4*i^3*log(e))*a^4*b*d
^5)*B*x - 6*((b^5*c^3*d^2*i^3*n - 3*a*b^4*c^2*d^3*i^3*n + 3*a^2*b^3*c*d^4*i
^3*n - a^3*b^2*d^5*i^3*n)*B*x^2 + 2*(a*b^4*c^3*d^2*i^3*n - 3*a^2*b^3*c^2*d^
3*i^3*n + 3*a^3*b^2*c*d^4*i^3*n - a^4*b*d^5*i^3*n)*B*x + (a^2*b^3*c^3*d^2*i
^3*n - 3*a^3*b^2*c^2*d^3*i^3*n + 3*a^4*b*c*d^4*i^3*n - a^5*d^5*i^3*n)*B)*lo
g(b*x + a)^2 + (3*(7*i^3*n + 6*i^3*log(e))*a^2*b^3*c^3*d^2 - (47*i^3*n + 46
*i^3*log(e))*a^3*b^2*c^2*d^3 + (35*i^3*n + 38*i^3*log(e))*a^4*b*c*d^4 - (9*
i^3*n + 10*i^3*log(e))*a^5*d^5)*B + 2*((6*b^5*c^3*d^2*i^3*log(e) + 2*(7*i^3
*n - 9*i^3*log(e))*a*b^4*c^2*d^3 - (19*i^3*n - 18*i^3*log(e))*a^2*b^3*c*d^4
 + (7*i^3*n - 6*i^3*log(e))*a^3*b^2*d^5)*B*x^2 + 2*(6*a*b^4*c^3*d^2*i^3*log
(e) + 2*(7*i^3*n - 9*i^3*log(e))*a^2*b^3*c^2*d^3 - (19*i^3*n - 18*i^3*log(e
))*a^3*b^2*c*d^4 + (7*i^3*n - 6*i^3*log(e))*a^4*b*d^5)*B*x + (6*a^2*b^3*c^3
*d^2*i^3*log(e) + 2*(7*i^3*n - 9*i^3*log(e))*a^3*b^2*c^2*d^3 - (19*i^3*n -
18*i^3*log(e))*a^4*b*c*d^4 + (7*i^3*n - 6*i^3*log(e))*a^5*d^5)*B)*log(b*x +
 a) + 2*(2*(b^5*c^2*d^3*i^3 - 2*a*b^4*c*d^4*i^3 + a^2*b^3*d^5*i^3)*B*x^3 +
4*(a*b^4*c^2*d^3*i^3 - 2*a^2*b^3*c*d^4*i^3 + a^3*b^2*d^5*i^3)*B*x^2 + 4*(3*
a*b^4*c^3*d^2*i^3 - 7*a^2*b^3*c^2*d^3*i^3 + 5*a^3*b^2*c*d^4*i^3 - a^4*b*d^5
*i^3)*B*x + (9*a^2*b^3*c^3*d^2*i^3 - 23*a^3*b^2*c^2*d^3*i^3 + 19*a^4*b*c*d^
4*i^3 - 5*a^5*d^5*i^3)*B + 6*((b^5*c^3*d^2*i^3 - 3*a*b^4*c^2*d^3*i^3 + 3*a^
2*b^3*c*d^4*i^3 - a^3*b^2*d^5*i^3)*B*x^2 + 2*(a*b^4*c^3*d^2*i^3 - 3*a^2*b^3
*c^2*d^3*i^3 + 3*a^3*b^2*c*d^4*i^3 - a^4*b*d^5*i^3)*B*x + (a^2*b^3*c^3*d^2*
i^3 - 3*a^3*b^2*c^2*d^3*i^3 + 3*a^4*b*c*d^4*i^3 - a^5*d^5*i^3)*B)*log(b*x +
 a))*log((b*x + a)^n) - 2*(2*(b^5*c^2*d^3*i^3 - 2*a*b^4*c*d^4*i^3 + a^2*b^3
*d^5*i^3)*B*x^3 + 4*(a*b^4*c^2*d^3*i^3 - 2*a^2*b^3*c*d^4*i^3 + a^3*b^2*d^5*

```
i^3)*B*x^2 + 4*(3*a*b^4*c^3*d^2*i^3 - 7*a^2*b^3*c^2*d^3*i^3 + 5*a^3*b^2*c*d
^4*i^3 - a^4*b*d^5*i^3)*B*x + (9*a^2*b^3*c^3*d^2*i^3 - 23*a^3*b^2*c^2*d^3*i
^3 + 19*a^4*b*c*d^4*i^3 - 5*a^5*d^5*i^3)*B + 6*((b^5*c^3*d^2*i^3 - 3*a*b^4*
c^2*d^3*i^3 + 3*a^2*b^3*c*d^4*i^3 - a^3*b^2*d^5*i^3)*B*x^2 + 2*(a*b^4*c^3*d
^2*i^3 - 3*a^2*b^3*c^2*d^3*i^3 + 3*a^3*b^2*c*d^4*i^3 - a^4*b*d^5*i^3)*B*x +
 (a^2*b^3*c^3*d^2*i^3 - 3*a^3*b^2*c^2*d^3*i^3 + 3*a^4*b*c*d^4*i^3 - a^5*d^5
*i^3)*B)*log(b*x + a))*log((d*x + c)^n))/(a^2*b^6*c^2*g^3 - 2*a^3*b^5*c*d*g
^3 + a^4*b^4*d^2*g^3 + (b^8*c^2*g^3 - 2*a*b^7*c*d*g^3 + a^2*b^6*d^2*g^3)*x^
2 + 2*(a*b^7*c^2*g^3 - 2*a^2*b^6*c*d*g^3 + a^3*b^5*d^2*g^3)*x) + 3*(b*c*d^2
*i^3*n - a*d^3*i^3*n)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + di
log(-(b*d*x + a*d)/(b*c - a*d)))*B/(b^4*g^3)
```

Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^3}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, a
lgorithm="giac")
```

```
[Out] integrate((d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x + a
*g)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx=\int \frac{(c\,i+d\,i\,x)^3\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}{(a\,g+b\,g\,x)^3}\,dx$$

```
[In] int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^
3,x)
```

```
[Out] int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^
3, x)
```

3.134 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 326

$$\begin{aligned}\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx =& -\frac{Bd^2i^3n(c+dx)}{b^3g^4(a+bx)} \\ & -\frac{Bdi^3n(c+dx)^2}{4b^2g^4(a+bx)^2}-\frac{Bi^3n(c+dx)^3}{9bg^4(a+bx)^3} \\ & -\frac{d^2i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^4(a+bx)} \\ & -\frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^4(a+bx)^2} \\ & -\frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bg^4(a+bx)^3} \\ & -\frac{d^3i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4} \\ & +\frac{Bd^3i^3n\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4}\end{aligned}$$

```
[Out] -B*d^2*i^3*n*(d*x+c)/b^3/g^4/(b*x+a)-1/4*B*d*i^3*n*(d*x+c)^2/b^2/g^4/(b*x+a
)^2-1/9*B*i^3*n*(d*x+c)^3/b/g^4/(b*x+a)^3-d^2*i^3*(d*x+c)*(A+B*ln(e*((b*x+a
)/(d*x+c))^n))/b^3/g^4/(b*x+a)-1/2*d*i^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+
c))^n))/b^2/g^4/(b*x+a)^2-1/3*i^3*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))
/b/g^4/(b*x+a)^3-d^3*i^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(
b*x+a))/b^4/g^4+B*d^3*i^3*n*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^4
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2561, 2380, 2341, 2379, 2438}

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = -\frac{d^3i^3\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4g^4} - \frac{d^2i^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3g^4(a+bx)} - \frac{di^3(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b^2g^4(a+bx)^2} - \frac{i^3(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3bg^4(a+bx)^3} + \frac{Bd^3i^3n\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4} - \frac{Bd^2i^3n(c+dx)}{b^3g^4(a+bx)} - \frac{Bdi^3n(c+dx)^2}{4b^2g^4(a+bx)^2} - \frac{Bi^3n(c+dx)^3}{9bg^4(a+bx)^3}$$

```
[In] Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^
4,x]
```

```
[Out] -((B*d^2*i^3*n*(c + d*x))/(b^3*g^4*(a + b*x))) - (B*d*i^3*n*(c + d*x)^2)/(4
*b^2*g^4*(a + b*x)^2) - (B*i^3*n*(c + d*x)^3)/(9*b*g^4*(a + b*x)^3) - (d^2*
i^3*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^3*g^4*(a + b*x)) -
 (d*i^3*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*b^2*g^4*(a +
 b*x)^2) - (i^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b*g^
4*(a + b*x)^3) - (d^3*i^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b
*(c + d*x))/(d*(a + b*x))])/(b^4*g^4) + (B*d^3*i^3*n*PolyLog[2, (b*(c + d*x
))/(d*(a + b*x))])/(b^4*g^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
```

```
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i^3\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^4(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^4} \\
&= \frac{i^3\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^4} + \frac{(di^3)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^4} \\
&= -\frac{Bi^3n(c+dx)^3}{9bg^4(a+bx)^3} - \frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bg^4(a+bx)^3} \\
&\quad + \frac{(di^3)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^4} + \frac{(d^2i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^4} \\
&= -\frac{Bdi^3n(c+dx)^2}{4b^2g^4(a+bx)^2} - \frac{Bi^3n(c+dx)^3}{9bg^4(a+bx)^3} - \frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^4(a+bx)^2} \\
&\quad - \frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bg^4(a+bx)^3} + \frac{(d^2i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^4} \\
&\quad + \frac{(d^3i^3)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^4}
\end{aligned}$$

$$= -\frac{Bd^2i^3n(c+dx)}{b^3g^4(a+bx)} - \frac{Bdi^3n(c+dx)^2}{4b^2g^4(a+bx)^2} - \frac{Bi^3n(c+dx)^3}{9bg^4(a+bx)^3} - \frac{d^2i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^4(a+bx)} - \frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^4(a+bx)^2} - \frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bg^4(a+bx)^3} - \frac{d^3i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4} + \frac{(Bd^3i^3n)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4g^4}$$

$$= -\frac{Bd^2i^3n(c+dx)}{b^3g^4(a+bx)} - \frac{Bdi^3n(c+dx)^2}{4b^2g^4(a+bx)^2} - \frac{Bi^3n(c+dx)^3}{9bg^4(a+bx)^3} - \frac{d^2i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^4(a+bx)} - \frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^4(a+bx)^2} - \frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bg^4(a+bx)^3} - \frac{d^3i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4} + \frac{Bd^3i^3n\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = \frac{i^3\left(-\frac{4B(bc-ad)^3n}{(a+bx)^3} - \frac{21Bd(bc-ad)^2n}{(a+bx)^2} + \frac{66Bd^2(-bc+ad)n}{a+bx} - 66Bd^3n\log(a+bx) - \frac{12(bc-ad)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3} - 54\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b
*g*x)^4,x]
```

```
[Out] (i^3*((-4*B*(b*c - a*d)^3*n)/(a + b*x)^3 - (21*B*d*(b*c - a*d)^2*n)/(a + b*
x)^2 + (66*B*d^2*(-(b*c) + a*d)*n)/(a + b*x) - 66*B*d^3*n*Log[a + b*x] - (1
2*(b*c - a*d)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^3 - (54*d
*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^2 + (108*d
^2*(-(b*c) + a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + 36*d^
3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 66*B*d^3*n*Log[c +
d*x] - 18*B*d^3*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c -
a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(36*b^4*g^4)
```

Maple [F]

$$\int \frac{(dix+ci)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(bgx+ag)^4}dx$$

[In] int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x)

[Out] int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x)

Fricas [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^4}\,dx$$

[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, a
lgorithm="fricas")

[Out] integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3 +
 (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log(e*((
b*x + a)/(d*x + c))^n))/(b^4*g^4*x^4 + 4*a*b^3*g^4*x^3 + 6*a^2*b^2*g^4*x^2
+ 4*a^3*b*g^4*x + a^4*g^4), x)

Sympy [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$$
$$=\frac{i^3\left(\int \frac{Ac^3}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx+\int \frac{Ad^3x^3}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx+\int \frac{Bc^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx+\dots\right.}{}$$

[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4,x)

[Out] i**3*(Integral(A*c**3/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3
 + b**4*x**4), x) + Integral(A*d**3*x**3/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x
2 + 4*a*b3*x**3 + b**4*x**4), x) + Integral(B*c**3*log(e*(a/(c + d*x) +
 b*x/(c + d*x))**n)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 +
 b**4*x**4), x) + Integral(3*A*c*d**2*x**2/(a**4 + 4*a**3*b*x + 6*a**2*b**2
*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(3*A*c**2*d*x/(a**4 + 4*a*
*3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(B*d**
3*x**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**4 + 4*a**3*b*x + 6*a**2*
b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(3*B*c*d**2*x**2*log(e
*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 +
4*a*b**3*x**3 + b**4*x**4), x) + Integral(3*B*c**2*d*x*log(e*(a/(c + d*x) +
 b*x/(c + d*x))**n)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 +
 b**4*x**4), x))/g**4

Maxima [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = \int \frac{(dix+ci)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^4}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, a
lgorithm="maxima")
```

```
[Out] -1/6*B*c*d^2*i^3*n*((11*a^2*b^2*c^2 - 7*a^3*b*c*d + 2*a^4*d^2 + 6*(3*b^4*c^
2 - 3*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 3*(9*a*b^3*c^2 - 7*a^2*b^2*c*d + 2*a^3
*b*d^2)*x)/((b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*g^4*x^3 + 3*(a*b^7*c^2 -
2*a^2*b^6*c*d + a^3*b^5*d^2)*g^4*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d + a^4
*b^4*d^2)*g^4*x + (a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2)*g^4) + 6*(3*b
^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^2*d +
3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4) - 6*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^
3)*log(d*x + c)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*
g^4)) - 1/18*B*c^3*i^3*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d
^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*
x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2
- 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b
*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2
 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b
^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/12*B*c^2*d*i^3*n*((5*a*b^2*c^2 - 22*a^2*b*c
*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c^2 - 16*a*b^2*c*
d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3 + 3*(a*b
^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*
c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4)
 - 6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3
*c*d^2 - a^3*b^2*d^3)*g^4) + 6*(3*b*c*d^2 - a*d^3)*log(d*x + c)/((b^5*c^3 -
 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4)) + 1/6*A*d^3*i^3*((18*
a*b^2*x^2 + 27*a^2*b*x + 11*a^3)/(b^7*g^4*x^3 + 3*a*b^6*g^4*x^2 + 3*a^2*b^5
*g^4*x + a^3*b^4*g^4) + 6*log(b*x + a)/(b^4*g^4)) + 1/6*B*d^3*i^3*(((18*a*b
^2*x^2 + 27*a^2*b*x + 11*a^3 + 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*
log(b*x + a))*log((b*x + a)^n) - (18*a*b^2*x^2 + 27*a^2*b*x + 11*a^3 + 6*(b
^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*log(b*x + a))*log((d*x + c)^n))/(b^
7*g^4*x^3 + 3*a*b^6*g^4*x^2 + 3*a^2*b^5*g^4*x + a^3*b^4*g^4) + 6*integrate(
1/6*(6*b^4*d*x^4*log(e) + 6*b^4*c*x^3*log(e) - 11*a^3*b*c*n + 11*a^4*d*n -
18*(a*b^3*c*n - a^2*b^2*d*n)*x^2 - 27*(a^2*b^2*c*n - a^3*b*d*n)*x - 6*(a^3*
b*c*n - a^4*d*n + (b^4*c*n - a*b^3*d*n)*x^3 + 3*(a*b^3*c*n - a^2*b^2*d*n)*x
^2 + 3*(a^2*b^2*c*n - a^3*b*d*n)*x)*log(b*x + a))/(b^8*d*g^4*x^5 + a^4*b^4*
c*g^4 + (b^8*c*g^4 + 4*a*b^7*d*g^4)*x^4 + 2*(2*a*b^7*c*g^4 + 3*a^2*b^6*d*g^
4)*x^3 + 2*(3*a^2*b^6*c*g^4 + 2*a^3*b^5*d*g^4)*x^2 + (4*a^3*b^5*c*g^4 + a^4
*b^4*d*g^4)*x), x)) - 1/2*(3*b*x + a)*B*c^2*d*i^3*log(e*(b*x/(d*x + c) + a/
(d*x + c))^n)/(b^5*g^4*x^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^
```

```
4) - (3*b^2*x^2 + 3*a*b*x + a^2)*B*c*d^2*i^3*log(e*(b*x/(d*x + c) + a/(d*x
+ c))^n)/(b^6*g^4*x^3 + 3*a*b^5*g^4*x^2 + 3*a^2*b^4*g^4*x + a^3*b^3*g^4) -
1/2*(3*b*x + a)*A*c^2*d*i^3/(b^5*g^4*x^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*
x + a^3*b^2*g^4) - (3*b^2*x^2 + 3*a*b*x + a^2)*A*c*d^2*i^3/(b^6*g^4*x^3 + 3
*a*b^5*g^4*x^2 + 3*a^2*b^4*g^4*x + a^3*b^3*g^4) - 1/3*B*c^3*i^3*log(e*(b*x/
(d*x + c) + a/(d*x + c))^n)/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*
x + a^3*b*g^4) - 1/3*A*c^3*i^3/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g
^4*x + a^3*b*g^4)
```

Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^4}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, a
lgorithm="giac")
```

```
[Out] integrate((d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x + a
*g)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx=\int \frac{(c\,i+d\,i\,x)^3\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}{(a\,g+b\,g\,x)^4}\,dx$$

```
[In] int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^
4,x)
```

```
[Out] int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^
4, x)
```

3.135 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 269

$$\begin{aligned}
&\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx \\
&= \frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3di} \\
&\quad - \frac{(bc-ad)g^3(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^2i} \\
&\quad + \frac{(bc-ad)^2g^3(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^3i} \\
&\quad + \frac{(bc-ad)^3g^3\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{6d^4i} \\
&\quad + \frac{B(bc-ad)^3g^3n\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}
\end{aligned}$$

```
[Out] 1/3*g^3*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d/i-1/6*(-a*d+b*c)*g^3*(b
*x+a)^2*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))/d^2/i+1/6*(-a*d+b*c)^2*g^3*
(b*x+a)*(6*A+5*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))/d^3/i+1/6*(-a*d+b*c)^3*g^
3*(6*A+11*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d^4/i
+B*(-a*d+b*c)^3*g^3*n*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {2561, 2384, 2354, 2438}

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$$
$$= \frac{g^3(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+6A+11Bn\right)}{6d^4i} + \frac{g^3(a+bx)(bc-ad)^2\left(6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+6A+5Bn\right)}{6d^3i} - \frac{g^3(a+bx)^2(bc-ad)\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{6d^2i} + \frac{g^3(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3di} + \frac{Bg^3n(bc-ad)^3\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}$$

```
[In] Int[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x),
x]
```

```
[Out] (g^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d*i) - ((b*c -
a*d)*g^3*(a + b*x)^2*(3*A + B*n + 3*B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*d
^2*i) + ((b*c - a*d)^2*g^3*(a + b*x)*(6*A + 5*B*n + 6*B*Log[e*((a + b*x)/(c
 + d*x))^n]))/(6*d^3*i) + ((b*c - a*d)^3*g^3*(6*A + 11*B*n + 6*B*Log[e*((a
+ b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(6*d^4*i) + (B*(b*c -
 a*d)^3*g^3*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^4*i)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{((bc-ad)^3g^3)\,\text{Subst}\left(\int \frac{x^3(A+B\log(ex^n))}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{i} \\
&= \frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3di} \\
&\quad - \frac{((bc-ad)^3g^3)\,\text{Subst}\left(\int \frac{x^2(3A+Bn+3B\log(ex^n))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3di} \\
&= \frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3di} \\
&\quad - \frac{(bc-ad)g^3(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^2i} \\
&\quad + \frac{((bc-ad)^3g^3)\,\text{Subst}\left(\int \frac{x(3Bn+2(3A+Bn)+6B\log(ex^n))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{6d^2i} \\
&= \frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3di} \\
&\quad - \frac{(bc-ad)g^3(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^2i} \\
&\quad + \frac{(bc-ad)^2g^3(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^3i} \\
&\quad - \frac{((bc-ad)^3g^3)\,\text{Subst}\left(\int \frac{9Bn+2(3A+Bn)+6B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{6d^3i}
\end{aligned}
$$

$$= \frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3di} - \frac{(bc-ad)g^3(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^2i} + \frac{(bc-ad)^2g^3(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^3i} + \frac{(bc-ad)^3g^3\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{6d^4i} - \frac{(B(bc-ad)^3g^3n)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^4i}$$

$$= \frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3di} - \frac{(bc-ad)g^3(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^2i} + \frac{(bc-ad)^2g^3(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^3i} + \frac{(bc-ad)^3g^3\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{6d^4i} + \frac{B(bc-ad)^3g^3n\,\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.38

$$\int\frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$$

$$= \frac{g^3\left(6Abd(bc-ad)^2x+6Bd(bc-ad)^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3d^2(-bc+ad)(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.\right.}{}$$

```
[In] Integrate[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d
*i*x),x]
```

```
[Out] (g^3*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b
*x)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
 - 6*B*(b*c - a*d)^3*n*Log[c + d*x] + B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x
- d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*n*(b*
d*x + (-(b*c) + a*d)*Log[c + d*x]) - 6*(b*c - a*d)^3*(A + B*Log[e*((a + b*x
)/(c + d*x))^n])*Log[i*(c + d*x)] + 3*B*(b*c - a*d)^3*n*((2*Log[(d*(a + b*x
))/(-(b*c) + a*d)] - Log[i*(c + d*x)])*Log[i*(c + d*x)] + 2*PolyLog[2, (b*(
c + d*x))/(b*c - a*d)])))/(6*d^4*i)
```

Maple [F]

$$\int \frac{(bgx+ag)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{dix+ci}dx$$

[In] int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)

[Out] int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)

Fricas [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = \int \frac{(bgx+ag)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{dix+ci}\,dx$$

[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algorithm="fricas")

[Out] integral((A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3 + (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log(e*((b*x + a)/(d*x + c))^n))/(d*i*x + c*i), x)

Sympy [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$$
$$= \frac{g^3\left(\int \frac{Aa^3}{c+dx}\,dx+\int \frac{Ab^3x^3}{c+dx}\,dx+\int \frac{Ba^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx+\int \frac{3Aab^2x^2}{c+dx}\,dx+\int \frac{3Aa^2bx}{c+dx}\,dx+\int \frac{Bb^3x^3\log\left(e\left(\frac{a}{c+dx}+\right.\right.}{c+dx}\right.}{i}$$

[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i),x)

[Out] g**3*(Integral(A*a**3/(c + d*x), x) + Integral(A*b**3*x**3/(c + d*x), x) + Integral(B*a**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x) + Integral(3*A*a*b**2*x**2/(c + d*x), x) + Integral(3*A*a**2*b*x/(c + d*x), x) + Integral(B*b**3*x**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x) + Integral(3*B*a*b**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x) + Integral(3*B*a**2*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x))/i

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1003 vs. 2(260) = 520.

Time = 0.49 (sec) , antiderivative size = 1003, normalized size of antiderivative = 3.73

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$$
$$=3\,Aa^2bg^3\left(\frac{x}{di}-\frac{c\log(dx+c)}{d^2i}\right)-\frac{1}{6}\,Ab^3g^3\left(\frac{6\,c^3\log(dx+c)}{d^4i}-\frac{2\,d^2x^3-3\,cdx^2+6\,c^2x}{d^3i}\right)$$
$$+\frac{3}{2}\,Aab^2g^3\left(\frac{2\,c^2\log(dx+c)}{d^3i}+\frac{dx^2-2\,cx}{d^2i}\right)+\frac{Aa^3g^3\log(dix+ci)}{di}$$
$$-\frac{(b^3c^3g^3n-3\,ab^2c^2dg^3n+3\,a^2bcd^2g^3n-a^3d^3g^3n)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{d^4i}$$
$$+\frac{(6\,a^3d^3g^3\log(e)-(11\,g^3n+6\,g^3\log(e))b^3c^3+9\,(3\,g^3n+2\,g^3\log(e))ab^2c^2d-18\,(g^3n+g^3\log(e))a^2bcd}{6\,d^4i}$$
$$+\frac{2\,Bb^3d^3g^3x^3\log(e)-((g^3n+3\,g^3\log(e))b^3cd^2-(g^3n+9\,g^3\log(e))ab^2d^3)Bx^2+6\,(b^3c^3g^3n-3\,ab^2c^2dg}{}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, alg
orithm="maxima")
```

```
[Out] 3*A*a^2*b*g^3*(x/(d*i) - c*log(d*x + c)/(d^2*i)) - 1/6*A*b^3*g^3*(6*c^3*log
(d*x + c)/(d^4*i) - (2*d^2*x^3 - 3*c*d*x^2 + 6*c^2*x)/(d^3*i)) + 3/2*A*a*b^
2*g^3*(2*c^2*log(d*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A*a^3*g^3*lo
g(d*i*x + c*i)/(d*i) - (b^3*c^3*g^3*n - 3*a*b^2*c^2*d*g^3*n + 3*a^2*b*c*d^2
*g^3*n - a^3*d^3*g^3*n)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) +
dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(d^4*i) + 1/6*(6*a^3*d^3*g^3*log(e) -
(11*g^3*n + 6*g^3*log(e))*b^3*c^3 + 9*(3*g^3*n + 2*g^3*log(e))*a*b^2*c^2*d
- 18*(g^3*n + g^3*log(e))*a^2*b*c*d^2)*B*log(d*x + c)/(d^4*i) + 1/6*(2*B*b^
3*d^3*g^3*x^3*log(e) - ((g^3*n + 3*g^3*log(e))*b^3*c*d^2 - (g^3*n + 9*g^3*l
og(e))*a*b^2*d^3)*B*x^2 + 6*(b^3*c^3*g^3*n - 3*a*b^2*c^2*d*g^3*n + 3*a^2*b*
c*d^2*g^3*n - a^3*d^3*g^3*n)*B*log(b*x + a)*log(d*x + c) - 3*(b^3*c^3*g^3*n
 - 3*a*b^2*c^2*d*g^3*n + 3*a^2*b*c*d^2*g^3*n - a^3*d^3*g^3*n)*B*log(d*x + c
)^2 + ((5*g^3*n + 6*g^3*log(e))*b^3*c^2*d - 6*(2*g^3*n + 3*g^3*log(e))*a*b^
2*c*d^2 + (7*g^3*n + 18*g^3*log(e))*a^2*b*d^3)*B*x + (6*a*b^2*c^2*d*g^3*n -
 15*a^2*b*c*d^2*g^3*n + 11*a^3*d^3*g^3*n)*B*log(b*x + a) + (2*B*b^3*d^3*g^3
*x^3 - 3*(b^3*c*d^2*g^3 - 3*a*b^2*d^3*g^3)*B*x^2 + 6*(b^3*c^2*d*g^3 - 3*a*b
^2*c*d^2*g^3 + 3*a^2*b*d^3*g^3)*B*x - 6*(b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 +
3*a^2*b*c*d^2*g^3 - a^3*d^3*g^3)*B*log(d*x + c))*log((b*x + a)^n) - (2*B*b^
3*d^3*g^3*x^3 - 3*(b^3*c*d^2*g^3 - 3*a*b^2*d^3*g^3)*B*x^2 + 6*(b^3*c^2*d*g^
3 - 3*a*b^2*c*d^2*g^3 + 3*a^2*b*d^3*g^3)*B*x - 6*(b^3*c^3*g^3 - 3*a*b^2*c^2
*d*g^3 + 3*a^2*b*c*d^2*g^3 - a^3*d^3*g^3)*B*log(d*x + c))*log((d*x + c)^n))
/(d^4*i)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3832 vs. 2(260) = 520.

Time = 189.49 (sec) , antiderivative size = 3832, normalized size of antiderivative = 14.25

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, alg
orithm="giac")
```

```
[Out] -1/120*(6*(B*b^9*c^6*g^3*n - 6*B*a*b^8*c^5*d*g^3*n - 5*(b*x + a)*B*b^8*c^6*
d*g^3*n/(d*x + c) + 15*B*a^2*b^7*c^4*d^2*g^3*n + 30*(b*x + a)*B*a*b^7*c^5*d
^2*g^3*n/(d*x + c) + 10*(b*x + a)^2*B*b^7*c^6*d^2*g^3*n/(d*x + c)^2 - 20*B*
a^3*b^6*c^3*d^3*g^3*n - 75*(b*x + a)*B*a^2*b^6*c^4*d^3*g^3*n/(d*x + c) - 60
*(b*x + a)^2*B*a*b^6*c^5*d^3*g^3*n/(d*x + c)^2 - 10*(b*x + a)^3*B*b^6*c^6*d
^3*g^3*n/(d*x + c)^3 + 15*B*a^4*b^5*c^2*d^4*g^3*n + 100*(b*x + a)*B*a^3*b^5
*c^3*d^4*g^3*n/(d*x + c) + 150*(b*x + a)^2*B*a^2*b^5*c^4*d^4*g^3*n/(d*x + c
)^2 + 60*(b*x + a)^3*B*a*b^5*c^5*d^4*g^3*n/(d*x + c)^3 - 6*B*a^5*b^4*c*d^5*
g^3*n - 75*(b*x + a)*B*a^4*b^4*c^2*d^5*g^3*n/(d*x + c) - 200*(b*x + a)^2*B*
a^3*b^4*c^3*d^5*g^3*n/(d*x + c)^2 - 150*(b*x + a)^3*B*a^2*b^4*c^4*d^5*g^3*n
/(d*x + c)^3 + B*a^6*b^3*d^6*g^3*n + 30*(b*x + a)*B*a^5*b^3*c*d^6*g^3*n/(d*
x + c) + 150*(b*x + a)^2*B*a^4*b^3*c^2*d^6*g^3*n/(d*x + c)^2 + 200*(b*x + a
)^3*B*a^3*b^3*c^3*d^6*g^3*n/(d*x + c)^3 - 5*(b*x + a)*B*a^6*b^2*d^7*g^3*n/(
d*x + c) - 60*(b*x + a)^2*B*a^5*b^2*c*d^7*g^3*n/(d*x + c)^2 - 150*(b*x + a)
^3*B*a^4*b^2*c^2*d^7*g^3*n/(d*x + c)^3 + 10*(b*x + a)^2*B*a^6*b*d^8*g^3*n/(
d*x + c)^2 + 60*(b*x + a)^3*B*a^5*b*c*d^8*g^3*n/(d*x + c)^3 - 10*(b*x + a)^
3*B*a^6*d^9*g^3*n/(d*x + c)^3)*log((b*x + a)/(d*x + c))/(b^5*d^4*i - 5*(b*x
 + a)*b^4*d^5*i/(d*x + c) + 10*(b*x + a)^2*b^3*d^6*i/(d*x + c)^2 - 10*(b*x
+ a)^3*b^2*d^7*i/(d*x + c)^3 + 5*(b*x + a)^4*b*d^8*i/(d*x + c)^4 - (b*x + a
)^5*d^9*i/(d*x + c)^5) + (5*B*b^10*c^6*g^3*n - 30*B*a*b^9*c^5*d*g^3*n - 19*
(b*x + a)*B*b^9*c^6*d*g^3*n/(d*x + c) + 75*B*a^2*b^8*c^4*d^2*g^3*n + 114*(b
*x + a)*B*a*b^8*c^5*d^2*g^3*n/(d*x + c) + 23*(b*x + a)^2*B*b^8*c^6*d^2*g^3*
n/(d*x + c)^2 - 100*B*a^3*b^7*c^3*d^3*g^3*n - 285*(b*x + a)*B*a^2*b^7*c^4*d
^3*g^3*n/(d*x + c) - 138*(b*x + a)^2*B*a*b^7*c^5*d^3*g^3*n/(d*x + c)^2 - 3*
(b*x + a)^3*B*b^7*c^6*d^3*g^3*n/(d*x + c)^3 + 75*B*a^4*b^6*c^2*d^4*g^3*n +
380*(b*x + a)*B*a^3*b^6*c^3*d^4*g^3*n/(d*x + c) + 345*(b*x + a)^2*B*a^2*b^6
*c^4*d^4*g^3*n/(d*x + c)^2 + 18*(b*x + a)^3*B*a*b^6*c^5*d^4*g^3*n/(d*x + c)
^3 - 6*(b*x + a)^4*B*b^6*c^6*d^4*g^3*n/(d*x + c)^4 - 30*B*a^5*b^5*c*d^5*g^3
*n - 285*(b*x + a)*B*a^4*b^5*c^2*d^5*g^3*n/(d*x + c) - 460*(b*x + a)^2*B*a^
3*b^5*c^3*d^5*g^3*n/(d*x + c)^2 - 45*(b*x + a)^3*B*a^2*b^5*c^4*d^5*g^3*n/(d
*x + c)^3 + 36*(b*x + a)^4*B*a*b^5*c^5*d^5*g^3*n/(d*x + c)^4 + 5*B*a^6*b^4*
d^6*g^3*n + 114*(b*x + a)*B*a^5*b^4*c*d^6*g^3*n/(d*x + c) + 345*(b*x + a)^2
*B*a^4*b^4*c^2*d^6*g^3*n/(d*x + c)^2 + 60*(b*x + a)^3*B*a^3*b^4*c^3*d^6*g^3
*n/(d*x + c)^3 - 90*(b*x + a)^4*B*a^2*b^4*c^4*d^6*g^3*n/(d*x + c)^4 - 19*(b
```

```
*x + a)*B*a^6*b^3*d^7*g^3*n/(d*x + c) - 138*(b*x + a)^2*B*a^5*b^3*c*d^7*g^3
*n/(d*x + c)^2 - 45*(b*x + a)^3*B*a^4*b^3*c^2*d^7*g^3*n/(d*x + c)^3 + 120*(
b*x + a)^4*B*a^3*b^3*c^3*d^7*g^3*n/(d*x + c)^4 + 23*(b*x + a)^2*B*a^6*b^2*d
^8*g^3*n/(d*x + c)^2 + 18*(b*x + a)^3*B*a^5*b^2*c*d^8*g^3*n/(d*x + c)^3 - 9
0*(b*x + a)^4*B*a^4*b^2*c^2*d^8*g^3*n/(d*x + c)^4 - 3*(b*x + a)^3*B*a^6*b*d
^9*g^3*n/(d*x + c)^3 + 36*(b*x + a)^4*B*a^5*b*c*d^9*g^3*n/(d*x + c)^4 - 6*(
b*x + a)^4*B*a^6*d^10*g^3*n/(d*x + c)^4 + 6*B*b^10*c^6*g^3*log(e) - 36*B*a*
b^9*c^5*d*g^3*log(e) - 30*(b*x + a)*B*b^9*c^6*d*g^3*log(e)/(d*x + c) + 90*B
*a^2*b^8*c^4*d^2*g^3*log(e) + 180*(b*x + a)*B*a*b^8*c^5*d^2*g^3*log(e)/(d*x
 + c) + 60*(b*x + a)^2*B*b^8*c^6*d^2*g^3*log(e)/(d*x + c)^2 - 120*B*a^3*b^7
*c^3*d^3*g^3*log(e) - 450*(b*x + a)*B*a^2*b^7*c^4*d^3*g^3*log(e)/(d*x + c)
- 360*(b*x + a)^2*B*a*b^7*c^5*d^3*g^3*log(e)/(d*x + c)^2 - 60*(b*x + a)^3*B
*b^7*c^6*d^3*g^3*log(e)/(d*x + c)^3 + 90*B*a^4*b^6*c^2*d^4*g^3*log(e) + 600
*(b*x + a)*B*a^3*b^6*c^3*d^4*g^3*log(e)/(d*x + c) + 900*(b*x + a)^2*B*a^2*b
^6*c^4*d^4*g^3*log(e)/(d*x + c)^2 + 360*(b*x + a)^3*B*a*b^6*c^5*d^4*g^3*log
(e)/(d*x + c)^3 - 36*B*a^5*b^5*c*d^5*g^3*log(e) - 450*(b*x + a)*B*a^4*b^5*c
^2*d^5*g^3*log(e)/(d*x + c) - 1200*(b*x + a)^2*B*a^3*b^5*c^3*d^5*g^3*log(e)
/(d*x + c)^2 - 900*(b*x + a)^3*B*a^2*b^5*c^4*d^5*g^3*log(e)/(d*x + c)^3 + 6
*B*a^6*b^4*d^6*g^3*log(e) + 180*(b*x + a)*B*a^5*b^4*c*d^6*g^3*log(e)/(d*x +
 c) + 900*(b*x + a)^2*B*a^4*b^4*c^2*d^6*g^3*log(e)/(d*x + c)^2 + 1200*(b*x
+ a)^3*B*a^3*b^4*c^3*d^6*g^3*log(e)/(d*x + c)^3 - 30*(b*x + a)*B*a^6*b^3*d^
7*g^3*log(e)/(d*x + c) - 360*(b*x + a)^2*B*a^5*b^3*c*d^7*g^3*log(e)/(d*x +
c)^2 - 900*(b*x + a)^3*B*a^4*b^3*c^2*d^7*g^3*log(e)/(d*x + c)^3 + 60*(b*x +
 a)^2*B*a^6*b^2*d^8*g^3*log(e)/(d*x + c)^2 + 360*(b*x + a)^3*B*a^5*b^2*c*d^
8*g^3*log(e)/(d*x + c)^3 - 60*(b*x + a)^3*B*a^6*b*d^9*g^3*log(e)/(d*x + c)^
3 + 6*A*b^10*c^6*g^3 - 36*A*a*b^9*c^5*d*g^3 - 30*(b*x + a)*A*b^9*c^6*d*g^3/
(d*x + c) + 90*A*a^2*b^8*c^4*d^2*g^3 + 180*(b*x + a)*A*a*b^8*c^5*d^2*g^3/(d
*x + c) + 60*(b*x + a)^2*A*b^8*c^6*d^2*g^3/(d*x + c)^2 - 120*A*a^3*b^7*c^3*
d^3*g^3 - 450*(b*x + a)*A*a^2*b^7*c^4*d^3*g^3/(d*x + c) - 360*(b*x + a)^2*A
*a*b^7*c^5*d^3*g^3/(d*x + c)^2 - 60*(b*x + a)^3*A*b^7*c^6*d^3*g^3/(d*x + c)
^3 + 90*A*a^4*b^6*c^2*d^4*g^3 + 600*(b*x + a)*A*a^3*b^6*c^3*d^4*g^3/(d*x +
c) + 900*(b*x + a)^2*A*a^2*b^6*c^4*d^4*g^3/(d*x + c)^2 + 360*(b*x + a)^3*A*
a*b^6*c^5*d^4*g^3/(d*x + c)^3 - 36*A*a^5*b^5*c*d^5*g^3 - 450*(b*x + a)*A*a^
4*b^5*c^2*d^5*g^3/(d*x + c) - 1200*(b*x + a)^2*A*a^3*b^5*c^3*d^5*g^3/(d*x +
 c)^2 - 900*(b*x + a)^3*A*a^2*b^5*c^4*d^5*g^3/(d*x + c)^3 + 6*A*a^6*b^4*d^6
*g^3 + 180*(b*x + a)*A*a^5*b^4*c*d^6*g^3/(d*x + c) + 900*(b*x + a)^2*A*a^4*
b^4*c^2*d^6*g^3/(d*x + c)^2 + 1200*(b*x + a)^3*A*a^3*b^4*c^3*d^6*g^3/(d*x +
 c)^3 - 30*(b*x + a)*A*a^6*b^3*d^7*g^3/(d*x + c) - 360*(b*x + a)^2*A*a^5*b^
3*c*d^7*g^3/(d*x + c)^2 - 900*(b*x + a)^3*A*a^4*b^3*c^2*d^7*g^3/(d*x + c)^3
 + 60*(b*x + a)^2*A*a^6*b^2*d^8*g^3/(d*x + c)^2 + 360*(b*x + a)^3*A*a^5*b^2
*c*d^8*g^3/(d*x + c)^3 - 60*(b*x + a)^3*A*a^6*b*d^9*g^3/(d*x + c)^3)/(b^6*d
^4*i - 5*(b*x + a)*b^5*d^5*i/(d*x + c) + 10*(b*x + a)^2*b^4*d^6*i/(d*x + c)
^2 - 10*(b*x + a)^3*b^3*d^7*i/(d*x + c)^3 + 5*(b*x + a)^4*b^2*d^8*i/(d*x +
c)^4 - (b*x + a)^5*b*d^9*i/(d*x + c)^5) + 6*(B*b^6*c^6*g^3*n - 6*B*a*b^5*c^
5*d*g^3*n + 15*B*a^2*b^4*c^4*d^2*g^3*n - 20*B*a^3*b^3*c^3*d^3*g^3*n + 15*B*
```

```
a^4*b^2*c^2*d^4*g^3*n - 6*B*a^5*b*c*d^5*g^3*n + B*a^6*d^6*g^3*n)*log(b - (b
*x + a)*d/(d*x + c))/(b^2*d^4*i) - 6*(B*b^6*c^6*g^3*n - 6*B*a*b^5*c^5*d*g^3
*n + 15*B*a^2*b^4*c^4*d^2*g^3*n - 20*B*a^3*b^3*c^3*d^3*g^3*n + 15*B*a^4*b^2
*c^2*d^4*g^3*n - 6*B*a^5*b*c*d^5*g^3*n + B*a^6*d^6*g^3*n)*log((b*x + a)/(d*
x + c))/(b^2*d^4*i))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx=\int \frac{{\left(a\,g+b\,g\,x\right)}^3\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}{c\,i+d\,i\,x}\,dx$$

```
[In] int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x),
x)
[Out] int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x),
 x)
```

3.136 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 211

$$\begin{aligned}\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx &= \frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di}\\ &\quad-\frac{(bc-ad)g^2(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i}\\ &\quad-\frac{(bc-ad)^2g^2\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2d^3i}\\ &\quad-\frac{B(bc-ad)^2g^2n\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}\end{aligned}$$

```
[Out] 1/2*g^2*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d/i-1/2*(-a*d+b*c)*g^2*(b
*x+a)*(2*A+B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))/d^2/i-1/2*(-a*d+b*c)^2*g^2*(2
*A+3*B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d^3/i-B*(-
a*d+b*c)^2*g^2*n*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {2561, 2384, 2354, 2438}

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$$
$$= -\frac{g^2(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+3Bn\right)}{2d^3i}$$
$$-\frac{g^2(a+bx)(bc-ad)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{2d^2i}$$
$$+\frac{g^2(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2di}-\frac{Bg^2n(bc-ad)^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}$$

```
[In] Int[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x),
x]
```

```
[Out] (g^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d*i) - ((b*c -
a*d)*g^2*(a + b*x)*(2*A + B*n + 2*B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d^2
*i) - ((b*c - a*d)^2*g^2*(2*A + 3*B*n + 2*B*Log[e*((a + b*x)/(c + d*x))^n])
*Log[(b*c - a*d)/(b*(c + d*x))])/(2*d^3*i) - (B*(b*c - a*d)^2*g^2*n*PolyLog
[2, (d*(a + b*x))/(b*(c + d*x))])/(d^3*i)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)^2g^2)\,\text{Subst}\left(\int \frac{x^2(A+B\log(ex^n))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{i}$$

$$= \frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di} - \frac{((bc-ad)^2g^2)\,\text{Subst}\left(\int \frac{x(2A+Bn+2B\log(ex^n))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{2di}$$

$$= \frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di} - \frac{(bc-ad)g^2(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i} + \frac{((bc-ad)^2g^2)\,\text{Subst}\left(\int \frac{2A+3Bn+2B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{2d^2i}$$

$$= \frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di} - \frac{(bc-ad)g^2(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i} - \frac{(bc-ad)^2g^2\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2d^3i} + \frac{(B(bc-ad)^2g^2n)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i}$$

$$= \frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di} - \frac{(bc-ad)g^2(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i} - \frac{(bc-ad)^2g^2\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2d^3i} - \frac{B(bc-ad)^2g^2n\,\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.26

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$$
$$=\frac{g^2\left(-2Abd(bc-ad)x+2Bd(-bc+ad)(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+d^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2\right.}{}$$

```
[In] Integrate[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d
*i*x),x]
```

```
[Out] (g^2*(-2*A*b*d*(b*c - a*d)*x + 2*B*d*(-(b*c) + a*d)*(a + b*x)*Log[e*((a + b
*x)/(c + d*x))^n] + d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
+ 2*B*(b*c - a*d)^2*n*Log[c + d*x] - B*(b*c - a*d)*n*(b*d*x + (-(b*c) + a*d
)*Log[c + d*x]) + 2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Lo
g[i*(c + d*x)] - B*(b*c - a*d)^2*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] -
Log[i*(c + d*x)])*Log[i*(c + d*x)] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)
])))/(2*d^3*i)
```

Maple [F]

$$\int \frac{(bgx+ag)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{dix+ci}dx$$

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

```
[Out] int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{dix+ci}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, alg
orithm="fricas")
```

```
[Out] integral((A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*B*
a*b*g^2*x + B*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n))/(d*i*x + c*i), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = \frac{g^2\left(\int \frac{Aa^2}{c+dx}\,dx + \int \frac{Ab^2x^2}{c+dx}\,dx + \int \frac{Ba^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx + \int \frac{2Aabx}{c+dx}\,dx + \int \frac{Bb^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx + \int \frac{2B}{}\right.}{i}$$

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i),x)
```

```
[Out] g**2*(Integral(A*a**2/(c + d*x), x) + Integral(A*b**2*x**2/(c + d*x), x) +
Integral(B*a**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x) + Int
egral(2*A*a*b*x/(c + d*x), x) + Integral(B*b**2*x**2*log(e*(a/(c + d*x) + b
*x/(c + d*x))**n)/(c + d*x), x) + Integral(2*B*a*b*x*log(e*(a/(c + d*x) + b
*x/(c + d*x))**n)/(c + d*x), x))/i
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(204) = 408$.

Time = 0.50 (sec) , antiderivative size = 627, normalized size of antiderivative = 2.97

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = 2\,Aabg^2\left(\frac{x}{di} - \frac{c\log(dx+c)}{d^2i}\right) + \frac{1}{2}\,Ab^2g^2\left(\frac{2\,c^2\log(dx+c)}{d^3i} + \frac{dx^2-2\,cx}{d^2i}\right) + \frac{Aa^2g^2\log(dix+ci)}{di} + \frac{(b^2c^2g^2n - 2\,abcdg^2n + a^2d^2g^2n)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right) + \mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{d^3i} + \frac{(2\,a^2d^2g^2\log(e) + (3\,g^2n + 2\,g^2\log(e))b^2c^2 - 4\,(g^2n + g^2\log(e))abcd)B\log(dx+c)}{2\,d^3i} + \frac{Bb^2d^2g^2x^2\log(e) - 2\,(b^2c^2g^2n - 2\,abcdg^2n + a^2d^2g^2n)B\log(bx+a)\log(dx+c) + (b^2c^2g^2n - 2\,abcdg^2}{}$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, alg
orithm="maxima")
```

```
[Out] 2*A*a*b*g^2*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + 1/2*A*b^2*g^2*(2*c^2*log(d
*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A*a^2*g^2*log(d*i*x + c*i)/(d*
i) + (b^2*c^2*g^2*n - 2*a*b*c*d*g^2*n + a^2*d^2*g^2*n)*(log(b*x + a)*log((b
*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(d^3*i)
 + 1/2*(2*a^2*d^2*g^2*log(e) + (3*g^2*n + 2*g^2*log(e))*b^2*c^2 - 4*(g^2*n
+ g^2*log(e))*a*b*c*d)*B*log(d*x + c)/(d^3*i) + 1/2*(B*b^2*d^2*g^2*x^2*log(
e) - 2*(b^2*c^2*g^2*n - 2*a*b*c*d*g^2*n + a^2*d^2*g^2*n)*B*log(b*x + a)*log
(d*x + c) + (b^2*c^2*g^2*n - 2*a*b*c*d*g^2*n + a^2*d^2*g^2*n)*B*log(d*x + c
```

```
)^2 - ((g^2*n + 2*g^2*log(e))*b^2*c*d - (g^2*n + 4*g^2*log(e))*a*b*d^2)*B*x
 - (2*a*b*c*d*g^2*n - 3*a^2*d^2*g^2*n)*B*log(b*x + a) + (B*b^2*d^2*g^2*x^2
- 2*(b^2*c*d*g^2 - 2*a*b*d^2*g^2)*B*x + 2*(b^2*c^2*g^2 - 2*a*b*c*d*g^2 + a^
2*d^2*g^2)*B*log(d*x + c))*log((b*x + a)^n) - (B*b^2*d^2*g^2*x^2 - 2*(b^2*c
*d*g^2 - 2*a*b*d^2*g^2)*B*x + 2*(b^2*c^2*g^2 - 2*a*b*c*d*g^2 + a^2*d^2*g^2)
*B*log(d*x + c))*log((d*x + c)^n))/(d^3*i)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2499 vs. $2(204) = 408$.

Time = 130.46 (sec) , antiderivative size = 2499, normalized size of antiderivative = 11.84

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, alg
orithm="giac")
```

```
[Out] 1/24*(2*(B*b^7*c^5*g^2*n - 5*B*a*b^6*c^4*d*g^2*n - 4*(b*x + a)*B*b^6*c^5*d*
g^2*n/(d*x + c) + 10*B*a^2*b^5*c^3*d^2*g^2*n + 20*(b*x + a)*B*a*b^5*c^4*d^2
*g^2*n/(d*x + c) + 6*(b*x + a)^2*B*b^5*c^5*d^2*g^2*n/(d*x + c)^2 - 10*B*a^3
*b^4*c^2*d^3*g^2*n - 40*(b*x + a)*B*a^2*b^4*c^3*d^3*g^2*n/(d*x + c) - 30*(b
*x + a)^2*B*a*b^4*c^4*d^3*g^2*n/(d*x + c)^2 + 5*B*a^4*b^3*c*d^4*g^2*n + 40*
(b*x + a)*B*a^3*b^3*c^2*d^4*g^2*n/(d*x + c) + 60*(b*x + a)^2*B*a^2*b^3*c^3*
d^4*g^2*n/(d*x + c)^2 - B*a^5*b^2*d^5*g^2*n - 20*(b*x + a)*B*a^4*b^2*c*d^5*
g^2*n/(d*x + c) - 60*(b*x + a)^2*B*a^3*b^2*c^2*d^5*g^2*n/(d*x + c)^2 + 4*(b
*x + a)*B*a^5*b*d^6*g^2*n/(d*x + c) + 30*(b*x + a)^2*B*a^4*b*c*d^6*g^2*n/(d
*x + c)^2 - 6*(b*x + a)^2*B*a^5*d^7*g^2*n/(d*x + c)^2)*log((b*x + a)/(d*x +
 c))/(b^4*d^3*i - 4*(b*x + a)*b^3*d^4*i/(d*x + c) + 6*(b*x + a)^2*b^2*d^5*i
/(d*x + c)^2 - 4*(b*x + a)^3*b*d^6*i/(d*x + c)^3 + (b*x + a)^4*d^7*i/(d*x +
 c)^4) + (B*b^8*c^5*g^2*n - 5*B*a*b^7*c^4*d*g^2*n - 2*(b*x + a)*B*b^7*c^5*d
*g^2*n/(d*x + c) + 10*B*a^2*b^6*c^3*d^2*g^2*n + 10*(b*x + a)*B*a*b^6*c^4*d^
2*g^2*n/(d*x + c) - (b*x + a)^2*B*b^6*c^5*d^2*g^2*n/(d*x + c)^2 - 10*B*a^3*
b^5*c^2*d^3*g^2*n - 20*(b*x + a)*B*a^2*b^5*c^3*d^3*g^2*n/(d*x + c) + 5*(b*x
 + a)^2*B*a*b^5*c^4*d^3*g^2*n/(d*x + c)^2 + 2*(b*x + a)^3*B*b^5*c^5*d^3*g^2
*n/(d*x + c)^3 + 5*B*a^4*b^4*c*d^4*g^2*n + 20*(b*x + a)*B*a^3*b^4*c^2*d^4*g
^2*n/(d*x + c) - 10*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^2*n/(d*x + c)^2 - 10*(b
*x + a)^3*B*a*b^4*c^4*d^4*g^2*n/(d*x + c)^3 - B*a^5*b^3*d^5*g^2*n - 10*(b*x
 + a)*B*a^4*b^3*c*d^5*g^2*n/(d*x + c) + 10*(b*x + a)^2*B*a^3*b^3*c^2*d^5*g^
2*n/(d*x + c)^2 + 20*(b*x + a)^3*B*a^2*b^3*c^3*d^5*g^2*n/(d*x + c)^3 + 2*(b
*x + a)*B*a^5*b^2*d^6*g^2*n/(d*x + c) - 5*(b*x + a)^2*B*a^4*b^2*c*d^6*g^2*n
/(d*x + c)^2 - 20*(b*x + a)^3*B*a^3*b^2*c^2*d^6*g^2*n/(d*x + c)^3 + (b*x +
a)^2*B*a^5*b*d^7*g^2*n/(d*x + c)^2 + 10*(b*x + a)^3*B*a^4*b*c*d^7*g^2*n/(d*
x + c)^3 - 2*(b*x + a)^3*B*a^5*d^8*g^2*n/(d*x + c)^3 + 2*B*b^8*c^5*g^2*log(
```

```
e) - 10*B*a*b^7*c^4*d*g^2*log(e) - 8*(b*x + a)*B*b^7*c^5*d*g^2*log(e)/(d*x
+ c) + 20*B*a^2*b^6*c^3*d^2*g^2*log(e) + 40*(b*x + a)*B*a*b^6*c^4*d^2*g^2*l
og(e)/(d*x + c) + 12*(b*x + a)^2*B*b^6*c^5*d^2*g^2*log(e)/(d*x + c)^2 - 20*
B*a^3*b^5*c^2*d^3*g^2*log(e) - 80*(b*x + a)*B*a^2*b^5*c^3*d^3*g^2*log(e)/(d
*x + c) - 60*(b*x + a)^2*B*a*b^5*c^4*d^3*g^2*log(e)/(d*x + c)^2 + 10*B*a^4*
b^4*c*d^4*g^2*log(e) + 80*(b*x + a)*B*a^3*b^4*c^2*d^4*g^2*log(e)/(d*x + c)
+ 120*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^2*log(e)/(d*x + c)^2 - 2*B*a^5*b^3*d^
5*g^2*log(e) - 40*(b*x + a)*B*a^4*b^3*c*d^5*g^2*log(e)/(d*x + c) - 120*(b*x
 + a)^2*B*a^3*b^3*c^2*d^5*g^2*log(e)/(d*x + c)^2 + 8*(b*x + a)*B*a^5*b^2*d^
6*g^2*log(e)/(d*x + c) + 60*(b*x + a)^2*B*a^4*b^2*c*d^6*g^2*log(e)/(d*x + c
)^2 - 12*(b*x + a)^2*B*a^5*b*d^7*g^2*log(e)/(d*x + c)^2 + 2*A*b^8*c^5*g^2 -
 10*A*a*b^7*c^4*d*g^2 - 8*(b*x + a)*A*b^7*c^5*d*g^2/(d*x + c) + 20*A*a^2*b^
6*c^3*d^2*g^2 + 40*(b*x + a)*A*a*b^6*c^4*d^2*g^2/(d*x + c) + 12*(b*x + a)^2
*A*b^6*c^5*d^2*g^2/(d*x + c)^2 - 20*A*a^3*b^5*c^2*d^3*g^2 - 80*(b*x + a)*A*
a^2*b^5*c^3*d^3*g^2/(d*x + c) - 60*(b*x + a)^2*A*a*b^5*c^4*d^3*g^2/(d*x + c
)^2 + 10*A*a^4*b^4*c*d^4*g^2 + 80*(b*x + a)*A*a^3*b^4*c^2*d^4*g^2/(d*x + c)
 + 120*(b*x + a)^2*A*a^2*b^4*c^3*d^4*g^2/(d*x + c)^2 - 2*A*a^5*b^3*d^5*g^2
- 40*(b*x + a)*A*a^4*b^3*c*d^5*g^2/(d*x + c) - 120*(b*x + a)^2*A*a^3*b^3*c^
2*d^5*g^2/(d*x + c)^2 + 8*(b*x + a)*A*a^5*b^2*d^6*g^2/(d*x + c) + 60*(b*x +
 a)^2*A*a^4*b^2*c*d^6*g^2/(d*x + c)^2 - 12*(b*x + a)^2*A*a^5*b*d^7*g^2/(d*x
 + c)^2)/(b^5*d^3*i - 4*(b*x + a)*b^4*d^4*i/(d*x + c) + 6*(b*x + a)^2*b^3*d
^5*i/(d*x + c)^2 - 4*(b*x + a)^3*b^2*d^6*i/(d*x + c)^3 + (b*x + a)^4*b*d^7*
i/(d*x + c)^4) + 2*(B*b^5*c^5*g^2*n - 5*B*a*b^4*c^4*d*g^2*n + 10*B*a^2*b^3*
c^3*d^2*g^2*n - 10*B*a^3*b^2*c^2*d^3*g^2*n + 5*B*a^4*b*c*d^4*g^2*n - B*a^5*
d^5*g^2*n)*log(-b + (b*x + a)*d/(d*x + c))/(b^2*d^3*i) - 2*(B*b^5*c^5*g^2*n
 - 5*B*a*b^4*c^4*d*g^2*n + 10*B*a^2*b^3*c^3*d^2*g^2*n - 10*B*a^3*b^2*c^2*d^
3*g^2*n + 5*B*a^4*b*c*d^4*g^2*n - B*a^5*d^5*g^2*n)*log((b*x + a)/(d*x + c))
/(b^2*d^3*i))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx=\int \frac{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{c\,i+d\,i\,x}\,dx$$

```
[In] int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x),
x)
```

```
[Out] int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x),
 x)
```

3.137 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 134

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = \frac{g(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{di} + \frac{(bc-ad)g\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i} + \frac{B(bc-ad)gn\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}$$

```
[Out] g*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d/i+(-a*d+b*c)*g*(A+B*n+B*ln(e*((
b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d^2/i+B*(-a*d+b*c)*g*n*polylog
(2,d*(b*x+a)/b/(d*x+c))/d^2/i
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {2561, 2384, 2354, 2438}

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = \frac{g(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A+Bn\right)}{d^2i} + \frac{g(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{di} + \frac{Bgn(bc-ad)\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}$$

```
[In] Int[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x),x]
```

```
[Out] (g*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*i) + ((b*c - a*d)*g
*(A + B*n + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))
])/(d^2*i) + (B*(b*c - a*d)*g*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d
^2*i)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)g)\text{Subst}\left(\int \frac{x(A+B\log(ex^n))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{i}$$

$$= \frac{g(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{di} - \frac{((bc-ad)g)\text{Subst}\left(\int \frac{A+Bn+B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{di}$$

$$= \frac{g(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{di} + \frac{(bc-ad)g\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i} - \frac{(B(bc-ad)gn)\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i}$$

$$= \frac{g(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{di} + \frac{(bc-ad)g\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i} + \frac{B(bc-ad)gn\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.27

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = \frac{g\left(2Abdx+2Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-2B(bc-ad)n\log(c+dx)-2(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.\right.}{2d^2i}$$

```
[In] Integrate[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i
*x),x]
```

```
[Out] (g*(2*A*b*d*x + 2*B*d*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 2*B*(b*c -
 a*d)*n*Log[c + d*x] - 2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
*Log[c + d*x] + B*(b*c - a*d)*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log
[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(2*d^2
*i)
```

Maple [F]

$$\int \frac{(bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{dix+ci}dx$$

```
[In] int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

```
[Out] int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

Fricas [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx=\int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{dix+ci}\,dx$$

[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algor ithm="fricas")

[Out] integral((A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log(e*((b*x + a)/(d*x + c))^n))/(d*i*x + c*i), x)

Sympy [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = \frac{g\left(\int \frac{Aa}{c+dx}\,dx+\int \frac{Abx}{c+dx}\,dx+\int \frac{Ba\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx+\int \frac{Bbx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx\right)}{i}$$

[In] integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i),x)

[Out] g*(Integral(A*a/(c + d*x), x) + Integral(A*b*x/(c + d*x), x) + Integral(B*a *log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x) + Integral(B*b*x*log (e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x))/i

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(133) = 266.

Time = 0.51 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.28

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = Abg\left(\frac{x}{di}-\frac{c\log(dx+c)}{d^2i}\right) + \frac{Aag\log(dix+ci)}{di} - \frac{(bcgn-adgn)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{d^2i} + \frac{(adg\log(e)-(gn+g\log(e))bc)B\log(dx+c)}{d^2i} + \frac{2\,Badgn\log(bx+a)+2\,Bbdgx\log(e)+2\,(bcgn-adgn)B\log(bx+a)\log(dx+c)-(bcgn-adgn)B\,\mathrm{l}}{}$$

[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algor ithm="maxima")

```
[Out] A*b*g*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + A*a*g*log(d*i*x + c*i)/(d*i) - (
b*c*g*n - a*d*g*n)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog
(-(b*d*x + a*d)/(b*c - a*d)))*B/(d^2*i) + (a*d*g*log(e) - (g*n + g*log(e))*
b*c)*B*log(d*x + c)/(d^2*i) + 1/2*(2*B*a*d*g*n*log(b*x + a) + 2*B*b*d*g*x*l
og(e) + 2*(b*c*g*n - a*d*g*n)*B*log(b*x + a)*log(d*x + c) - (b*c*g*n - a*d*
g*n)*B*log(d*x + c)^2 + 2*(B*b*d*g*x - (b*c*g - a*d*g)*B*log(d*x + c))*log(
(b*x + a)^n) - 2*(B*b*d*g*x - (b*c*g - a*d*g)*B*log(d*x + c))*log((d*x + c)
^n))/(d^2*i)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. $2(133) = 266$.

Time = 78.29 (sec) , antiderivative size = 1242, normalized size of antiderivative = 9.27

$$\int \frac{(ag + bgx)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci + dix}\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algor
ithm="giac")
```

```
[Out] -1/6*((B*b^5*c^4*g*n - 4*B*a*b^4*c^3*d*g*n - 3*(b*x + a)*B*b^4*c^4*d*g*n/(d
*x + c) + 6*B*a^2*b^3*c^2*d^2*g*n + 12*(b*x + a)*B*a*b^3*c^3*d^2*g*n/(d*x +
 c) - 4*B*a^3*b^2*c*d^3*g*n - 18*(b*x + a)*B*a^2*b^2*c^2*d^3*g*n/(d*x + c)
+ B*a^4*b*d^4*g*n + 12*(b*x + a)*B*a^3*b*c*d^4*g*n/(d*x + c) - 3*(b*x + a)*
B*a^4*d^5*g*n/(d*x + c))*log((b*x + a)/(d*x + c))/(b^3*d^2*i - 3*(b*x + a)*
b^2*d^3*i/(d*x + c) + 3*(b*x + a)^2*b*d^4*i/(d*x + c)^2 - (b*x + a)^3*d^5*i
/(d*x + c)^3) + ((b*x + a)*B*b^5*c^4*d*g*n/(d*x + c) - 4*(b*x + a)*B*a*b^4*
c^3*d^2*g*n/(d*x + c) - (b*x + a)^2*B*b^4*c^4*d^2*g*n/(d*x + c)^2 + 6*(b*x
+ a)*B*a^2*b^3*c^2*d^3*g*n/(d*x + c) + 4*(b*x + a)^2*B*a*b^3*c^3*d^3*g*n/(d
*x + c)^2 - 4*(b*x + a)*B*a^3*b^2*c*d^4*g*n/(d*x + c) - 6*(b*x + a)^2*B*a^2
*b^2*c^2*d^4*g*n/(d*x + c)^2 + (b*x + a)*B*a^4*b*d^5*g*n/(d*x + c) + 4*(b*x
 + a)^2*B*a^3*b*c*d^5*g*n/(d*x + c)^2 - (b*x + a)^2*B*a^4*d^6*g*n/(d*x + c)
^2 + B*b^6*c^4*g*log(e) - 4*B*a*b^5*c^3*d*g*log(e) - 3*(b*x + a)*B*b^5*c^4*
d*g*log(e)/(d*x + c) + 6*B*a^2*b^4*c^2*d^2*g*log(e) + 12*(b*x + a)*B*a*b^4*
c^3*d^2*g*log(e)/(d*x + c) - 4*B*a^3*b^3*c*d^3*g*log(e) - 18*(b*x + a)*B*a^
2*b^3*c^2*d^3*g*log(e)/(d*x + c) + B*a^4*b^2*d^4*g*log(e) + 12*(b*x + a)*B*
a^3*b^2*c*d^4*g*log(e)/(d*x + c) - 3*(b*x + a)*B*a^4*b*d^5*g*log(e)/(d*x +
c) + A*b^6*c^4*g - 4*A*a*b^5*c^3*d*g - 3*(b*x + a)*A*b^5*c^4*d*g/(d*x + c)
+ 6*A*a^2*b^4*c^2*d^2*g + 12*(b*x + a)*A*a*b^4*c^3*d^2*g/(d*x + c) - 4*A*a^
3*b^3*c*d^3*g - 18*(b*x + a)*A*a^2*b^3*c^2*d^3*g/(d*x + c) + A*a^4*b^2*d^4*
g + 12*(b*x + a)*A*a^3*b^2*c*d^4*g/(d*x + c) - 3*(b*x + a)*A*a^4*b*d^5*g/(d
*x + c))/(b^4*d^2*i - 3*(b*x + a)*b^3*d^3*i/(d*x + c) + 3*(b*x + a)^2*b^2*d
^4*i/(d*x + c)^2 - (b*x + a)^3*b*d^5*i/(d*x + c)^3) + (B*b^4*c^4*g*n - 4*B*
a*b^3*c^3*d*g*n + 6*B*a^2*b^2*c^2*d^2*g*n - 4*B*a^3*b*c*d^3*g*n + B*a^4*d^4
```

```
*g*n)*log(b - (b*x + a)*d/(d*x + c))/(b^2*d^2*i) - (B*b^4*c^4*g*n - 4*B*a*b
^3*c^3*d*g*n + 6*B*a^2*b^2*c^2*d^2*g*n - 4*B*a^3*b*c*d^3*g*n + B*a^4*d^4*g*
n)*log((b*x + a)/(d*x + c))/(b^2*d^2*i))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*
d)^2)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx=\int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}{c\,i+d\,i\,x}\,dx$$

```
[In] int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x),x)
[Out] int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x), x
)
```

3.138 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci+dix}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 80

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci+dix}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{di} - \frac{Bn\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{di}$$

```
[Out] -(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d/i-B*n*polylog(2
,d*(b*x+a)/b/(d*x+c))/d/i
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2543, 2458, 2378, 2370, 2352}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci+dix}\,dx = -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{di} - \frac{Bn\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{di}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*i + d*i*x),x]
```

```
[Out] -(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(
d*i)) - (B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d*i)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2370

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2378

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2543

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c + d*x
))])*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/g), x] + Dist[B*n*((b*c - a*d)
/g), Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[d*f - c*
g, 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{di}+\frac{(B(bc-ad)n)\int\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}\,dx}{di}$$

$$=-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{di}+\frac{(B(bc-ad)n)\text{Subst}\left(\int\frac{\log\left(\frac{bc-ad}{bx}\right)}{x\left(\frac{-bc+ad}{d}+\frac{bx}{d}\right)}\,dx,x,c+dx\right)}{d^2i}$$

$$=-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{di}-\frac{(B(bc-ad)n)\text{Subst}\left(\int\frac{\log\left(\frac{(bc-ad)x}{b}\right)}{\left(\frac{-bc+ad}{d}+\frac{b}{dx}\right)x}\,dx,x,\frac{1}{c+dx}\right)}{d^2i}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{di} - \frac{(B(bc-ad)n)\text{Subst}\left(\int \frac{\log\left(\frac{(bc-ad)x}{b}\right)}{\frac{b}{d}+\frac{(-bc+ad)x}{d}}\,dx, x, \frac{1}{c+dx}\right)}{d^2 i}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{di} - \frac{Bn\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{di}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci+dix}\,dx$$
$$= \frac{\log(i(c+dx))\left(2A-2Bn\log\left(\frac{d(a+bx)}{-bc+ad}\right)+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+Bn\log(i(c+dx))\right)-2Bn\,\text{PolyLog}\left(2,\frac{b(c}{bc}\right.}{2di}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*i + d*i*x),x]
```

```
[Out] (Log[i*(c + d*x)]*(2*A - 2*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*B*Log[
e*((a + b*x)/(c + d*x))^n] + B*n*Log[i*(c + d*x)]) - 2*B*n*PolyLog[2, (b*(c
 + d*x))/(b*c - a*d)])/(2*d*i)
```

Maple [F]

$$\int \frac{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{dix+ci}dx$$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

Fricas [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci+dix}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{dix+ci}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algorithm="fricas
")
```

```
[Out] integral((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(d*i*x + c*i), x)
```

Sympy [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci+dix}\,dx = \frac{\int \frac{A}{c+dx}\,dx + \int \frac{B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx}{i}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i),x)
```

```
[Out] (Integral(A/(c + d*x), x) + Integral(B*log(e*(a/(c + d*x) + b*x/(c + d*x))*
*n)/(c + d*x), x))/i
```

Maxima [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci+dix}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{dix+ci}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algorithm="maxima
")
```

```
[Out] -1/2*B*((2*n*log(b*x + a)*log(d*x + c) - n*log(d*x + c)^2 - 2*log(d*x + c)*
log((b*x + a)^n) + 2*log(d*x + c)*log((d*x + c)^n))/(d*i) - 2*integrate((n*
log(b*x + a) + log(e))/(d*i*x + c*i), x)) + A*log(d*i*x + c*i)/(d*i)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(79) = 158$.

Time = 54.29 (sec) , antiderivative size = 566, normalized size of antiderivative = 7.08

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci+dix}\,dx = \frac{1}{2}\left(\frac{\left(Bb^3c^3n-3\,Bab^2c^2dn+3\,Ba^2bcd^2n-Ba^3d^3n\right)\log\left(\frac{bx+a}{dx+c}\right)}{b^2di-\frac{2\,(bx+a)bd^2i}{dx+c}+\frac{(bx+a)^2d^3i}{(dx+c)^2}} - \frac{Bb^4c^3n-3\,Bab^3c^2dn-\frac{(bx+a)Bb^3c^3dn}{dx+c}+3\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algorithm="giac")
```

```
[Out] 1/2*((B*b^3*c^3*n - 3*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n - B*a^3*d^3*n)*lo
g((b*x + a)/(d*x + c))/(b^2*d*i - 2*(b*x + a)*b*d^2*i/(d*x + c) + (b*x + a)
^2*d^3*i/(d*x + c)^2) - (B*b^4*c^3*n - 3*B*a*b^3*c^2*d*n - (b*x + a)*B*b^3*
c^3*d*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*n + 3*(b*x + a)*B*a*b^2*c^2*d^2*n/(d*
x + c) - B*a^3*b*d^3*n - 3*(b*x + a)*B*a^2*b*c*d^3*n/(d*x + c) + (b*x + a)*
B*a^3*d^4*n/(d*x + c) - B*b^4*c^3*log(e) + 3*B*a*b^3*c^2*d*log(e) - 3*B*a^2
```

```
*b^2*c*d^2*log(e) + B*a^3*b*d^3*log(e) - A*b^4*c^3 + 3*A*a*b^3*c^2*d - 3*A*
a^2*b^2*c*d^2 + A*a^3*b*d^3)/(b^3*d*i - 2*(b*x + a)*b^2*d^2*i/(d*x + c) + (
b*x + a)^2*b*d^3*i/(d*x + c)^2) + (B*b^3*c^3*n - 3*B*a*b^2*c^2*d*n + 3*B*a^
2*b*c*d^2*n - B*a^3*d^3*n)*log(-b + (b*x + a)*d/(d*x + c))/(b^2*d*i) - (B*b
^3*c^3*n - 3*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n - B*a^3*d^3*n)*log((b*x +
a)/(d*x + c))/(b^2*d*i))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci + dix}\,dx = \int \frac{A + B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{c\,i + d\,i\,x}\,dx$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*i + d*i*x),x)
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*i + d*i*x), x)
```

3.139 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 50

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx=\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2B(bc-ad)gin}$$

```
[Out] 1/2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/B/(-a*d+b*c)/g/i/n
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {2561, 2338}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx=\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2Bgin(bc-ad)}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)*(c*i + d*i*x)),x]
[Out] (A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(2*B*(b*c - a*d)*g*i*n)
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
```

```
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)gi}$$

$$= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2B(bc-ad)gin}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 219, normalized size of antiderivative = 4.38

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx = \frac{2A\log(a+bx)-Bn\log^2(a+bx)+2B\log(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-2A\log(c+dx)+2Bn\log\left(\frac{d(a+bx)}{-bc+ad}\right)}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)*(c*i + d*i*
x)),x]
```

```
[Out] (2*A*Log[a + b*x] - B*n*Log[a + b*x]^2 + 2*B*Log[a + b*x]*Log[e*((a + b*x)/
(c + d*x))^n] - 2*A*Log[c + d*x] + 2*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)]*
Log[c + d*x] - 2*B*Log[e*((a + b*x)/(c + d*x))^n]*Log[c + d*x] - B*n*Log[c
+ d*x]^2 + 2*B*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 2*B*n*PolyLo
g[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2*B*n*PolyLog[2, (b*(c + d*x))/(b*c -
a*d)])/(2*(b*c - a*d)*g*i)
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.68

method	result	size
parallelrisch	$-\frac{B\,a^{2} c^{2} \ln\left(e \left(\frac{b x +a}{d x +c}\right)^{n}\right)^{2}+2 A \ln\left(e \left(\frac{b x +a}{d x +c}\right)^{n}\right) a^{2} c^{2}}{2 i g\,c^{2} a^{2} n \left(a d -c b \right)}$	84
default	$\frac{A \left(\frac{\ln(d x +c)}{a d -c b}-\frac{\ln(b x +a)}{a d -c b}\right)}{g i}-\frac{B \ln\left(e \left(\frac{b x +a}{d x +c}\right)^{n}\right)^{2}}{2 g i n \left(a d -c b \right)}$	88
parts	$\frac{A \left(\frac{\ln(d x +c)}{a d -c b}-\frac{\ln(b x +a)}{a d -c b}\right)}{g i}-\frac{B \ln\left(e \left(\frac{b x +a}{d x +c}\right)^{n}\right)^{2}}{2 g i n \left(a d -c b \right)}$	88

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i),x,method=_RETUR
NVERBOSE)
```

```
[Out] -1/2*(B*a^2*c^2*ln(e*((b*x+a)/(d*x+c))^n)^2+2*A*ln(e*((b*x+a)/(d*x+c))^n)*a
^2*c^2)/i/g/c^2/a^2/n/(a*d-b*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.48

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx = \frac{Bn\log\left(\frac{bx+a}{dx+c}\right)^2+2\,B\log\left(e\right)\log\left(\frac{bx+a}{dx+c}\right)+2\,A\log\left(\frac{bx+a}{dx+c}\right)}{2\,(bc-ad)gi}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i),x, algor
ithm="fricas")
```

```
[Out] 1/2*(B*n*log((b*x + a)/(d*x + c))^2 + 2*B*log(e)*log((b*x + a)/(d*x + c)) +
 2*A*log((b*x + a)/(d*x + c)))/((b*c - a*d)*g*i)
```

Sympy [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx = \frac{\int \frac{A}{ac+adx+bcx+bdx^2}\,dx+\int \frac{B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{ac+adx+bcx+bdx^2}\,dx}{gi}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)/(d*i*x+c*i),x)
```

```
[Out] (Integral(A/(a*c + a*d*x + b*c*x + b*d*x**2), x) + Integral(B*log(e*(a/(c +
 d*x) + b*x/(c + d*x))**n)/(a*c + a*d*x + b*c*x + b*d*x**2), x))/(g*i)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(48) = 96$.

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 3.50

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx = B\left(\frac{\log(bx+a)}{(bc-ad)gi}-\frac{\log(dx+c)}{(bc-ad)gi}\right)\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) - \frac{\left(\log(bx+a)^2-2\log(bx+a)\log(dx+c)+\log(dx+c)^2\right)Bn}{2(bcgi-adgi)} + A\left(\frac{\log(bx+a)}{(bc-ad)gi}-\frac{\log(dx+c)}{(bc-ad)gi}\right)$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i),x, algor
ithm="maxima")
```

```
[Out] B*(log(b*x + a)/((b*c - a*d)*g*i) - log(d*x + c)/((b*c - a*d)*g*i))*log(e*(
b*x/(d*x + c) + a/(d*x + c))^n) - 1/2*(log(b*x + a)^2 - 2*log(b*x + a)*log(
d*x + c) + log(d*x + c)^2)*B*n/(b*c*g*i - a*d*g*i) + A*(log(b*x + a)/((b*c
- a*d)*g*i) - log(d*x + c)/((b*c - a*d)*g*i))
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.84

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx = \frac{\left(Bn\log\left(\frac{bx+a}{dx+c}\right)^2+2\,B\log(e)\log\left(\frac{bx+a}{dx+c}\right)+2\,A\log\left(\frac{bx+a}{dx+c}\right)\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)}{2\,gi}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i),x, algor
ithm="giac")
```

```
[Out] 1/2*(B*n*log((b*x + a)/(d*x + c))^2 + 2*B*log(e)*log((b*x + a)/(d*x + c)) +
 2*A*log((b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(g*i
)
```

Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.52

$$\int \frac{A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx=-\frac{B\ln \left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)^2-A\,n\,\mathrm{atan}\left(\frac{b\,c\,2{}\mathrm{i}+b\,d\,x\,2{}\mathrm{i}}{a\,d-b\,c}+1{}\mathrm{i}\right)\,4{}\mathrm{i}}{2\,g\,i\,n\,\left(a\,d-b\,c\right)}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)*(c*i + d*i*x)),x)
```

```
[Out] -(B*log(e*((a + b*x)/(c + d*x))^n)^2 - A*n*atan((b*c*2i + b*d*x*2i)/(a*d -
b*c) + 1i)*4i)/(2*g*i*n*(a*d - b*c))
```

3.140 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 181

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{bBn(c+dx)}{(bc-ad)^2g^2i(a+bx)} - \frac{b(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2g^2i(a+bx)} - \frac{d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^2i} + \frac{Bdn\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^2g^2i}$$

```
[Out] -b*B*n*(d*x+c)/(-a*d+b*c)^2/g^2/i/(b*x+a)-b*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x
+c))^n))/(-a*d+b*c)^2/g^2/i/(b*x+a)-d*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b
*x+a)/(d*x+c))/(-a*d+b*c)^2/g^2/i+1/2*B*d*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c
)^2/g^2/i
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2561, 45, 2372, 14, 2338}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^2i(bc-ad)^2} - \frac{b(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^2i(a+bx)(bc-ad)^2} - \frac{bBn(c+dx)}{g^2i(a+bx)(bc-ad)^2} + \frac{Bdn\log^2\left(\frac{a+bx}{c+dx}\right)}{2g^2i(bc-ad)^2}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^2*(c*i + d*i*x)),
x]
```

```
[Out] -((b*B*n*(c + d*x))/((b*c - a*d)^2*g^2*i*(a + b*x))) - (b*(c + d*x)*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^2*g^2*i*(a + b*x)) - (d*(A +
B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)])/((b*c - a*d)^2*
g^2*i) + (B*d*n*Log[(a + b*x)/(c + d*x)]^2)/(2*(b*c - a*d)^2*g^2*i)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex^n))}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^2i}$$

$$= -\frac{b(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2g^2i(a+bx)} - \frac{d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^2i} - \frac{(Bn)\text{Subst}\left(\int \frac{-b-dx\log(x)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^2i}$$

$$= -\frac{b(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2g^2i(a+bx)} - \frac{d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^2i} - \frac{(Bn)\text{Subst}\left(\int \left(-\frac{b}{x^2}-\frac{d\log(x)}{x}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^2i}$$

$$= -\frac{bBn(c+dx)}{(bc-ad)^2g^2i(a+bx)} - \frac{b(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2g^2i(a+bx)} - \frac{d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^2i} + \frac{(Bdn)\text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^2i}$$

$$= -\frac{bBn(c+dx)}{(bc-ad)^2g^2i(a+bx)} - \frac{b(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2g^2i(a+bx)} - \frac{d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^2i} + \frac{Bdn\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^2g^2i}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.17 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.68

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{2(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2d(a+bx)\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2d(a+bx)\left(A+\right.}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^2*(c*i + d*
i*x)),x]
```

```
[Out] -1/2*(2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d*(a + b*x)*
Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*d*(a + b*x)*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 2*B*n*(b*c - a*d + d*(a + b*
x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*d*n*(a + b*x)*(Log[a + b*x]
*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b
*x))/(-(b*c) + a*d)]) + B*d*n*(a + b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d
)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))
/((b*c - a*d)^2*g^2*i*(a + b*x))
```

Maple [A] (verified)

Time = 4.79 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.49

method	result
parallelrisch	$-\frac{-2Ba\,b^{3}d^{3}n^{2}+2B\,b^{4}c\,d^{2}n^{2}-2Aa\,b^{3}d^{3}n+2A\,b^{4}c\,d^{2}n+Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}b^{4}d^{3}+2Ax\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{4}d^{3}+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{2i\,g^{2}(bx+a)n(a^{2}d^{2}-2abcd+b^{2}c^{2})b^{3}}$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i),x,method=_RET
URNVERBOSE)
```

```
[Out] -1/2*(-2*B*a*b^3*d^3*n^2+2*B*b^4*c*d^2*n^2-2*A*a*b^3*d^3*n+2*A*b^4*c*d^2*n+
B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*d^3+2*A*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4
*d^3+B*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^3*d^3+2*A*ln(e*((b*x+a)/(d*x+c))^n)*
a*b^3*d^3+2*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^3*n+2*B*ln(e*((b*x+a)/(d*x+
c))^n)*b^4*c*d^2*n)/i/g^2/(b*x+a)/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.07

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{(ag+bgx)^{2}(ci+dix)}\,dx = -\frac{2\,Abc-2\,Aad+(Bbdnx+Badn)\log\left(\frac{bx+a}{dx+c}\right)^{2}+2\,(Bbc-Bad)n+2\,\left(Bbc-Bad+(Bbdx+Bad)\log\left(\right.\right.}{2\left((b^{3}c^{2}-2\,ab^{2}cd+a^{2}bd^{2})g^{2}ix+(ab^{2}c^{2}-2\,a^{2}bcd+\right.}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i),x, alg
orithm="fricas")
```

```
[Out] -1/2*(2*A*b*c - 2*A*a*d + (B*b*d*n*x + B*a*d*n)*log((b*x + a)/(d*x + c))^2
+ 2*(B*b*c - B*a*d)*n + 2*(B*b*c - B*a*d + (B*b*d*x + B*a*d)*log((b*x + a)/
(d*x + c)))*log(e) + 2*(B*b*c*n + A*a*d + (B*b*d*n + A*b*d)*x)*log((b*x + a
)/(d*x + c)))/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^2*i*x + (a*b^2*c^2 - 2
*a^2*b*c*d + a^3*d^2)*g^2*i)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2/(d*i*x+c*i),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(179) = 358$.

Time = 0.22 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.36

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx =$$
$$-B\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i}+\frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}-\frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)\log\left(e\right.$$
$$+\frac{\left((bdx+ad)\log(bx+a)^2+(bdx+ad)\log(dx+c)^2-2\,bc+2\,ad-2\,(bdx+ad)\log(bx+a)+2\,(bdx\right.}{2\,(ab^2c^2g^2i-2\,a^2bcdg^2i+a^3d^2g^2i+(b^3c^2g^2i-2\,ab^2cdg^2i+a^2b}$$
$$-A\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i}+\frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}-\frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i),x, alg
orithm="maxima")
[Out] -B*(1/((b^2*c - a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/((
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*c*
d + a^2*d^2)*g^2*i))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*((b*d*x +
 a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d - 2*(b*
d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a))*log(
d*x + c))*B*n/(a*b^2*c^2*g^2*i - 2*a^2*b*c*d*g^2*i + a^3*d^2*g^2*i + (b^3*c
^2*g^2*i - 2*a*b^2*c*d*g^2*i + a^2*b*d^2*g^2*i)*x) - A*(1/((b^2*c - a*b*d)*
g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/((b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g^2*i))
```

Giac [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx=\int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{(bgx+ag)^2(dix+ci)}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i),x, alg
orithm="giac")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/((b*g*x + a*g)^2*(d*i*x +
c*i)), x)
```

Mupad [B] (verification not implemented)

Time = 2.80 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.32

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx=\frac{A}{g^2\,i\,\left(a\,d-b\,c\right)\,\left(a+b\,x\right)}+\frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{g^2\,i\,\left(a\,d-b\,c\right)\,\left(a+b\,x\right)}+\frac{B\,n}{g^2\,i\,\left(a\,d-b\,c\right)\,\left(a+b\,x\right)}-\frac{B\,d\,{\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}^2}{2\,g^2\,i\,n\,{\left(a\,d-b\,c\right)}^2}+\frac{A\,d\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{g^2\,i\,{\left(a\,d-b\,c\right)}^2}+\frac{B\,d\,n\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{g^2\,i\,{\left(a\,d-b\,c\right)}^2}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^2*(c*i + d*i*x)),
x)
```

```
[Out] A/(g^2*i*(a*d - b*c)*(a + b*x)) + (B*log(e*((a + b*x)/(c + d*x))^n))/(g^2*i
*(a*d - b*c)*(a + b*x)) + (A*d*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c
))*2i)/(g^2*i*(a*d - b*c)^2) + (B*n)/(g^2*i*(a*d - b*c)*(a + b*x)) + (B*d*n
*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(g^2*i*(a*d - b*c)^2) -
 (B*d*log(e*((a + b*x)/(c + d*x))^n)^2)/(2*g^2*i*n*(a*d - b*c)^2)
```

3.141 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 266

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx = -\frac{Bn(c+dx)^2\left(b-\frac{4d(a+bx)}{c+dx}\right)^2}{4(bc-ad)^3g^3i(a+bx)^2} + \frac{2bd(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2} + \frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i} - \frac{Bd^2n\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^3g^3i}$$

```
[Out] -1/4*B*n*(d*x+c)^2*(b-4*d*(b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g^3/i/(b*x+a)^2+2
*b*d*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^3/i/(b*x+a)-1/2
*b^2*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^3/i/(b*x+a)^2
+d^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d+b*c)^3/g^3/i
-1/2*B*d^2*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g^3/i
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {2561, 45, 2372, 12, 14, 37, 2338}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx = -\frac{b^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^3i(a+bx)^2(bc-ad)^3} + \frac{d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^3i(bc-ad)^3} + \frac{2bd(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^3i(a+bx)(bc-ad)^3} - \frac{Bd^2n\log^2\left(\frac{a+bx}{c+dx}\right)}{2g^3i(bc-ad)^3} - \frac{Bn(c+dx)^2\left(b-\frac{4d(a+bx)}{c+dx}\right)^2}{4g^3i(a+bx)^2(bc-ad)^3}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^3*(c*i + d*i*x)),
x]
```

```
[Out] -1/4*(B*n*(c + d*x)^2*(b - (4*d*(a + b*x))/(c + d*x))^2)/((b*c - a*d)^3*g^3
*i*(a + b*x)^2) + (2*b*d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/
((b*c - a*d)^3*g^3*i*(a + b*x)) - (b^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]))/(2*(b*c - a*d)^3*g^3*i*(a + b*x)^2) + (d^2*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)])/((b*c - a*d)^3*g^3*i) - (B
*d^2*n*Log[(a + b*x)/(c + d*x)]^2)/(2*(b*c - a*d)^3*g^3*i)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex^n))}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i}\\
&= \frac{2bd(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2}\\
&\quad + \frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i} - \frac{(Bn)\text{Subst}\left(\int \frac{-b^2+4bdx+2d^2x^2\log(x)}{2x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i}\\
&= \frac{2bd(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2}\\
&\quad + \frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i} - \frac{(Bn)\text{Subst}\left(\int \frac{-b^2+4bdx+2d^2x^2\log(x)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{2(bc-ad)^3g^3i}
\end{aligned}$$

$$= \frac{2bd(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2} + \frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i} - \frac{(Bn)\text{Subst}\left(\int\left(-\frac{b(b-4dx)}{x^3}+\frac{2d^2\log(x)}{x}\right)dx, x, \frac{a+bx}{c+dx}\right)}{2(bc-ad)^3g^3i}$$

$$= \frac{2bd(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2} + \frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i} + \frac{(bBn)\text{Subst}\left(\int\frac{b-4dx}{x^3}dx, x, \frac{a+bx}{c+dx}\right)}{2(bc-ad)^3g^3i} - \frac{(Bd^2n)\text{Subst}\left(\int\frac{\log(x)}{x}dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i}$$

$$= -\frac{Bn(c+dx)^2\left(b-\frac{4d(a+bx)}{c+dx}\right)^2}{4(bc-ad)^3g^3i(a+bx)^2} + \frac{2bd(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2} + \frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i} - \frac{Bd^2n\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^3g^3i}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.22 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.63

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}dx$$
$$= \frac{-2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4d(bc-ad)(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4d^2(a+bx)^2\log(a+}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^3*(c*i + d*
i*x)),x]
```

```
[Out] (-2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*d*(b*c - a*d)*
(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*d^2*(a + b*x)^2*Log[a
+ b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 4*d^2*(a + b*x)^2*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 4*B*d*n*(a + b*x)*(b*c - a*d +
 d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*n*((b*c - a*d)^2
+ 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a
```

```
+ b*x)^2*Log[c + d*x]) - 2*B*d^2*n*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x]
- 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) +
a*d)]) + 2*B*d^2*n*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[
c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(4*(b*c
- a*d)^3*g^3*i*(a + b*x)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(260) = 520$.

Time = 11.92 (sec) , antiderivative size = 628, normalized size of antiderivative = 2.36

method	result
parallelrisch	$-\frac{6Bx^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^4b^2c^2d^2n+8Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^5bc^2d^2n+4Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^4b^2c^3dn-2B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^4b^2c^4}{}$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i),x,method=_RET
URNVERBOSE)
```

```
[Out] -1/4*(6*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^2*c^2*d^2*n+8*B*x*ln(e*((b*x+
a)/(d*x+c))^n)*a^5*b*c^2*d^2*n+4*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^2*c^3*
d*n-2*B*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^2*c^4*n+B*x^2*a^2*b^4*c^4*n^2+2*A*x
^2*a^2*b^4*c^4*n+2*B*x*a^3*b^3*c^4*n^2+4*A*x*a^3*b^3*c^4*n+2*B*ln(e*((b*x+a
)/(d*x+c))^n)^2*a^6*c^2*d^2+4*A*ln(e*((b*x+a)/(d*x+c))^n)*a^6*c^2*d^2+2*B*x
^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b^2*c^2*d^2+7*B*x^2*a^4*b^2*c^2*d^2*n^2-
8*B*x^2*a^3*b^3*c^3*d*n^2+4*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^2*c^2*d^2
+6*A*x^2*a^4*b^2*c^2*d^2*n-8*A*x^2*a^3*b^3*c^3*d*n+4*B*x*ln(e*((b*x+a)/(d*x
+c))^n)^2*a^5*b*c^2*d^2+8*B*x*a^5*b*c^2*d^2*n^2-10*B*x*a^4*b^2*c^3*d*n^2+8*
A*x*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b*c^2*d^2+8*A*x*a^5*b*c^2*d^2*n-12*A*x*a^
4*b^2*c^3*d*n+8*B*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b*c^3*d*n)/i/g^3/(b*x+a)^2/
(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/a^4/c^2/n
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.82

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx = -\frac{2\,Ab^2c^2-8\,Aabcd+6\,Aa^2d^2-2\left(Bb^2d^2nx^2+2\,Babd^2nx+Ba^2d^2n\right)\log\left(\frac{bx+a}{dx+c}\right)^2+\left(Bb^2c^2-8\,Babcd+\right.}{}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i),x, alg
orithm="fricas")
```

```
[Out] -1/4*(2*A*b^2*c^2 - 8*A*a*b*c*d + 6*A*a^2*d^2 - 2*(B*b^2*d^2*n*x^2 + 2*B*a*
b*d^2*n*x + B*a^2*d^2*n)*log((b*x + a)/(d*x + c))^2 + (B*b^2*c^2 - 8*B*a*b*
c*d + 7*B*a^2*d^2)*n - 2*(2*A*b^2*c*d - 2*A*a*b*d^2 + 3*(B*b^2*c*d - B*a*b*
d^2)*n)*x + 2*(B*b^2*c^2 - 4*B*a*b*c*d + 3*B*a^2*d^2 - 2*(B*b^2*c*d - B*a*b
*d^2)*x - 2*(B*b^2*d^2*x^2 + 2*B*a*b*d^2*x + B*a^2*d^2)*log((b*x + a)/(d*x
+ c)))*log(e) - 2*(2*A*a^2*d^2 + (3*B*b^2*d^2*n + 2*A*b^2*d^2)*x^2 - (B*b^2
*c^2 - 4*B*a*b*c*d)*n + 2*(2*A*a*b*d^2 + (B*b^2*c*d + 2*B*a*b*d^2)*n)*x)*lo
g((b*x + a)/(d*x + c)))/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b
^2*d^3)*g^3*i*x^2 + 2*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*
b*d^3)*g^3*i*x + (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*
g^3*i)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3/(d*i*x+c*i),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 888 vs. 2(260) = 520.

Time = 0.25 (sec) , antiderivative size = 888, normalized size of antiderivative = 3.34

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx$$
$$=\frac{1}{2}B\left(\frac{2\,bdx-bc+3\,ad}{(b^4c^2-2\,ab^3cd+a^2b^2d^2)g^3ix^2+2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2)g^3ix+(a^2b^2c^2-2\,a^3bcd+a^4d^2)g^3i}+\frac{}{(}\right.$$
$$-\frac{\left(b^2c^2-8\,abcd+7\,a^2d^2+2\,(b^2d^2x^2+2\,abd^2x+a^2d^2)\log(bx+a)^2+2\,(b^2d^2x^2+2\,abd^2x+a^2d^2)\log(dx\right.}{4\,(a^2b^3c^3g^3i-3\,a^3b^2c^2dg^3i+3\,a^4bcd^2g^3i-a^5d^3g^3i+}$$
$$+\frac{1}{2}A\left(\frac{2\,bdx-bc+3\,ad}{(b^4c^2-2\,ab^3cd+a^2b^2d^2)g^3ix^2+2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2)g^3ix+(a^2b^2c^2-2\,a^3bcd+a^4d^2)g^3i}+\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i),x, alg
orithm="maxima")
```

```
[Out] 1/2*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3*i
*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i*x + (a^2*b^2*c^2 - 2
*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2*d
 + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c)/((b^3*c^3 - 3*a*b^2
```

```
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i))*log(e*(b*x/(d*x + c) + a/(d*x + c
))^n) - 1/4*(b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x
 + a^2*d^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*
x + c)^2 - 6*(b^2*c*d - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2
)*log(b*x + a) + 2*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^
2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))*log(d*x + c))*B*n/(a^2*b^3*c^3*g^3
*i - 3*a^3*b^2*c^2*d*g^3*i + 3*a^4*b*c*d^2*g^3*i - a^5*d^3*g^3*i + (b^5*c^3
*g^3*i - 3*a*b^4*c^2*d*g^3*i + 3*a^2*b^3*c*d^2*g^3*i - a^3*b^2*d^3*g^3*i)*x
^2 + 2*(a*b^4*c^3*g^3*i - 3*a^2*b^3*c^2*d*g^3*i + 3*a^3*b^2*c*d^2*g^3*i - a
^4*b*d^3*g^3*i)*x) + 1/2*A*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d
 + a^2*b^2*d^2)*g^3*i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i
*x + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c
)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i))
```

Giac [A] (verification not implemented)

none

Time = 104.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.39

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx = -\frac{1}{4}\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)^2\left(\frac{2\,(dx+c)^2Bn\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)^2g^3i}+\frac{(Bn+2\,B\log\left(e\right)+2\,A)(dx+c)^2}{(bx+a)^2g^3i}\right)$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i),x, alg
orithm="giac")
```

```
[Out] -1/4*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)^2*(2*(d*x + c)^2*B*n*log((b*x
+ a)/(d*x + c))/((b*x + a)^2*g^3*i) + (B*n + 2*B*log(e) + 2*A)*(d*x + c)^2/
((b*x + a)^2*g^3*i))
```

Mupad [B] (verification not implemented)

Time = 3.08 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.15

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx$$
$$= \frac{B\,d^2\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(\frac{g^3\,i\,n\,(a\,d-b\,c)\,(2\,a\,d-b\,c)}{2\,d^2}+\frac{a\,g^3\,i\,n\,(a\,d-b\,c)}{2\,d}+\frac{b\,g^3\,i\,n\,x\,(a\,d-b\,c)}{d}\right)}{g^3\,i\,n\,\left(a\,d-b\,c\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(i\,a^2\,g^3+2\,i\,a\,b\,g^3\,x+i\,b^2\,g^3\,x^2\right)}$$
$$-\frac{B\,d^2\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{2\,g^3\,i\,n\,\left(a\,d-b\,c\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}$$
$$-\frac{\frac{6\,A\,a\,d-2\,A\,b\,c+7\,B\,a\,d\,n-B\,b\,c\,n}{2\,(a\,d-b\,c)}+\frac{d\,x\,(2\,A\,b+3\,B\,b\,n)}{a\,d-b\,c}}{x^2\,\left(2\,b^3\,c\,g^3\,i-2\,a\,b^2\,d\,g^3\,i\right)+x\,\left(4\,a\,b^2\,c\,g^3\,i-4\,a^2\,b\,d\,g^3\,i\right)-2\,a^3\,d\,g^3\,i+2\,a^2\,b\,c\,g^3\,i}$$
$$+\frac{d^2\,\mathrm{atan}\left(\frac{d^2\,\left(A+\frac{3\,B\,n}{2}\right)\,\left(2\,i\,a^3\,d^3\,g^3-2\,i\,a^2\,b\,c\,d^2\,g^3-2\,i\,a\,b^2\,c^2\,d\,g^3+2\,i\,b^3\,c^3\,g^3\right)\,1\mathrm{i}}{g^3\,i\,(2\,A\,d^2+3\,B\,d^2\,n)\,(a\,d-b\,c)^3}+\frac{b\,d^3\,x\,\left(A+\frac{3\,B\,n}{2}\right)\,\left(i\,a^2\,d^2\,g^3-2\,i\,a\,b\,c\,d\,g^3+i\,b^2\,c^2\,g^3\right)\,4}{g^3\,i\,(2\,A\,d^2+3\,B\,d^2\,n)\,(a\,d-b\,c)^3}\right.}{g^3\,i\,\left(a\,d-b\,c\right)^3}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^3*(c*i + d*i*x)),
x)
```

```
[Out] (d^2*atan((d^2*(A + (3*B*n)/2)*(2*a^3*d^3*g^3*i + 2*b^3*c^3*g^3*i - 2*a*b^2
*c^2*d*g^3*i - 2*a^2*b*c*d^2*g^3*i)*1i)/(g^3*i*(2*A*d^2 + 3*B*d^2*n)*(a*d -
 b*c)^3) + (b*d^3*x*(A + (3*B*n)/2)*(a^2*d^2*g^3*i + b^2*c^2*g^3*i - 2*a*b*
c*d*g^3*i)*4i)/(g^3*i*(2*A*d^2 + 3*B*d^2*n)*(a*d - b*c)^3))*(A + (3*B*n)/2)
*2i)/(g^3*i*(a*d - b*c)^3) - ((6*A*a*d - 2*A*b*c + 7*B*a*d*n - B*b*c*n)/(2*
(a*d - b*c)) + (d*x*(2*A*b + 3*B*b*n))/(a*d - b*c))/(x^2*(2*b^3*c*g^3*i - 2
*a*b^2*d*g^3*i) + x*(4*a*b^2*c*g^3*i - 4*a^2*b*d*g^3*i) - 2*a^3*d*g^3*i + 2
*a^2*b*c*g^3*i) - (B*d^2*log(e*((a + b*x)/(c + d*x))^n)^2)/(2*g^3*i*n*(a*d
- b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B*d^2*log(e*((a + b*x)/(c + d*x)
)^n)*((g^3*i*n*(a*d - b*c)*(2*a*d - b*c))/(2*d^2) + (a*g^3*i*n*(a*d - b*c))
/(2*d) + (b*g^3*i*n*x*(a*d - b*c))/d))/(g^3*i*n*(a*d - b*c)*(a^2*d^2 + b^2*
c^2 - 2*a*b*c*d)*(a^2*g^3*i + b^2*g^3*i*x^2 + 2*a*b*g^3*i*x))
```

3.142 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 389

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx &= -\frac{3bBd^2n(c+dx)}{(bc-ad)^4g^4i(a+bx)}+\frac{3b^2Bdn(c+dx)^2}{4(bc-ad)^4g^4i(a+bx)^2}\\ &\quad-\frac{b^3Bn(c+dx)^3}{9(bc-ad)^4g^4i(a+bx)^3}\\ &\quad-\frac{3bd^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^4i(a+bx)}\\ &\quad+\frac{3b^2d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2}\\ &\quad-\frac{b^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^4g^4i(a+bx)^3}\\ &\quad-\frac{d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i}+\frac{Bd^3n\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^4i}\end{aligned}$$

```
[Out] -3*b*B*d^2*n*(d*x+c)/(-a*d+b*c)^4/g^4/i/(b*x+a)+3/4*b^2*B*d*n*(d*x+c)^2/(-a
*d+b*c)^4/g^4/i/(b*x+a)^2-1/9*b^3*B*n*(d*x+c)^3/(-a*d+b*c)^4/g^4/i/(b*x+a)^
3-3*b*d^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^4/i/(b*x+a
)+3/2*b^2*d*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^4/i/(b
*x+a)^2-1/3*b^3*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^4/
i/(b*x+a)^3-d^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d+b
*c)^4/g^4/i+1/2*B*d^3*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^4/g^4/i
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {2561, 45, 2372, 12, 14, 2338}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx = -\frac{b^3(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3g^4i(a+bx)^3(bc-ad)^4} + \frac{3b^2d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^4i(a+bx)^2(bc-ad)^4} - \frac{d^3\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^4i(bc-ad)^4} - \frac{3bd^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^4i(a+bx)(bc-ad)^4} - \frac{b^3Bn(c+dx)^3}{9g^4i(a+bx)^3(bc-ad)^4} + \frac{3b^2Bdn(c+dx)^2}{4g^4i(a+bx)^2(bc-ad)^4} + \frac{Bd^3n\log^2\left(\frac{a+bx}{c+dx}\right)}{2g^4i(bc-ad)^4} - \frac{3bBd^2n(c+dx)}{g^4i(a+bx)(bc-ad)^4}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^4*(c*i + d*i*x)),
x]
```

```
[Out] (-3*b*B*d^2*n*(c + d*x))/((b*c - a*d)^4*g^4*i*(a + b*x)) + (3*b^2*B*d*n*(c
+ d*x)^2)/(4*(b*c - a*d)^4*g^4*i*(a + b*x)^2) - (b^3*B*n*(c + d*x)^3)/(9*(b
*c - a*d)^4*g^4*i*(a + b*x)^3) - (3*b*d^2*(c + d*x)*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/((b*c - a*d)^4*g^4*i*(a + b*x)) + (3*b^2*d*(c + d*x)^2*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*c - a*d)^4*g^4*i*(a + b*x)^2) -
(b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(b*c - a*d)^4*g
^4*i*(a + b*x)^3) - (d^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*
x)/(c + d*x)])/((b*c - a*d)^4*g^4*i) + (B*d^3*n*Log[(a + b*x)/(c + d*x)]^2)
/(2*(b*c - a*d)^4*g^4*i)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)^3(A+B\log(ex^n))}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i} \\
&= -\frac{3bd^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^4i(a+bx)} + \frac{3b^2d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2} \\
&\quad - \frac{b^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^4g^4i(a+bx)^3} - \frac{d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i} \\
&\quad - \frac{(Bn)\text{Subst}\left(\int \frac{-2b^3+9b^2dx-18bd^2x^2-6d^3x^3\log(x)}{6x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i}
\end{aligned}
$$

$$= -\frac{3bd^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^4i(a+bx)} + \frac{3b^2d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2} - \frac{b^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^4g^4i(a+bx)^3} - \frac{d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i} - \frac{(Bn)\text{Subst}\left(\int \frac{-2b^3+9b^2dx-18bd^2x^2-6d^3x^3\log(x)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{6(bc-ad)^4g^4i}$$

$$= -\frac{3bd^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^4i(a+bx)} + \frac{3b^2d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2} - \frac{b^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^4g^4i(a+bx)^3} - \frac{d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i} - \frac{(Bn)\text{Subst}\left(\int \left(-\frac{b(2b^2-9bdx+18d^2x^2)}{x^4} - \frac{6d^3\log(x)}{x}\right)dx, x, \frac{a+bx}{c+dx}\right)}{6(bc-ad)^4g^4i}$$

$$= -\frac{3bd^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^4i(a+bx)} + \frac{3b^2d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2} - \frac{b^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^4g^4i(a+bx)^3} - \frac{d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i} + \frac{(bBn)\text{Subst}\left(\int \frac{2b^2-9bdx+18d^2x^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{6(bc-ad)^4g^4i} + \frac{(Bd^3n)\,\text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i}$$

$$= -\frac{3bd^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^4i(a+bx)} + \frac{3b^2d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2} - \frac{b^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^4g^4i(a+bx)^3} - \frac{d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i} + \frac{Bd^3n\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^4i} + \frac{(bBn)\text{Subst}\left(\int \left(\frac{2b^2}{x^4} - \frac{9bd}{x^3} + \frac{18d^2}{x^2}\right)dx, x, \frac{a+bx}{c+dx}\right)}{6(bc-ad)^4g^4i}$$

$$= -\frac{3bBd^2n(c+dx)}{(bc-ad)^4g^4i(a+bx)} + \frac{3b^2Bdn(c+dx)^2}{4(bc-ad)^4g^4i(a+bx)^2} - \frac{b^3Bn(c+dx)^3}{9(bc-ad)^4g^4i(a+bx)^3} - \frac{3bd^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^4i(a+bx)} + \frac{3b^2d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2} - \frac{b^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^4g^4i(a+bx)^3} - \frac{d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i} + \frac{Bd^3n\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^4i}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.33

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx$$
$$= \frac{-\frac{12A(bc-ad)^3}{(a+bx)^3} - \frac{4B(bc-ad)^3 n}{(a+bx)^3} + \frac{18Ad(bc-ad)^2}{(a+bx)^2} + \frac{15Bd(bc-ad)^2 n}{(a+bx)^2} + \frac{36Ad^2(-bc+ad)}{a+bx} + \frac{66Bd^2(-bc+ad)n}{a+bx} - 36Ad^3\log(a+b}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^4*(c*i + d*
i*x)),x]
```

```
[Out] ((-12*A*(b*c - a*d)^3)/(a + b*x)^3 - (4*B*(b*c - a*d)^3*n)/(a + b*x)^3 + (1
8*A*d*(b*c - a*d)^2)/(a + b*x)^2 + (15*B*d*(b*c - a*d)^2*n)/(a + b*x)^2 + (
36*A*d^2*(-(b*c) + a*d))/(a + b*x) + (66*B*d^2*(-(b*c) + a*d)*n)/(a + b*x)
- 36*A*d^3*Log[a + b*x] - 66*B*d^3*n*Log[a + b*x] + 18*B*d^3*n*Log[a + b*x]
^2 - (12*B*(b*c - a*d)^3*Log[e*((a + b*x)/(c + d*x))^n])/(a + b*x)^3 + (18*
B*d*(b*c - a*d)^2*Log[e*((a + b*x)/(c + d*x))^n])/(a + b*x)^2 + (36*B*d^2*(
-(b*c) + a*d)*Log[e*((a + b*x)/(c + d*x))^n])/(a + b*x) - 36*B*d^3*Log[a +
b*x]*Log[e*((a + b*x)/(c + d*x))^n] + 36*A*d^3*Log[c + d*x] + 66*B*d^3*n*Lo
g[c + d*x] - 36*B*d^3*n*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] + 36
*B*d^3*Log[e*((a + b*x)/(c + d*x))^n]*Log[c + d*x] + 18*B*d^3*n*Log[c + d*x
]^2 - 36*B*d^3*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 36*B*d^3*n*P
olyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 36*B*d^3*n*PolyLog[2, (b*(c + d*x
))/(b*c - a*d)])/(36*(b*c - a*d)^4*g^4*i)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. 2(379) = 758.

Time = 25.47 (sec) , antiderivative size = 1072, normalized size of antiderivative = 2.76

method	result	size
parallelrisch	Expression too large to display	1072

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i),x,method=_RET
URNVERBOSE)
```

```
[Out] -1/36*(18*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a^8*c^2*d^3+36*A*ln(e*((b*x+a)/(d*x
+c))^n)*a^8*c^2*d^3-4*B*x^3*a^2*b^6*c^5*n^2-12*A*x^3*a^2*b^6*c^5*n-12*B*x^2
*a^3*b^5*c^5*n^2-36*A*x^2*a^3*b^5*c^5*n-12*B*x*a^4*b^4*c^5*n^2-36*A*x*a^4*b
^4*c^5*n+12*B*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^3*c^5*n+18*B*x^3*ln(e*((b*x+a
)/(d*x+c))^n)^2*a^5*b^3*c^2*d^3+85*B*x^3*a^5*b^3*c^2*d^3*n^2-108*B*x^3*a^4*
```

```
b^4*c^3*d^2*n^2+27*B*x^3*a^3*b^5*c^4*d*n^2+36*A*x^3*ln(e*((b*x+a)/(d*x+c))^
n)*a^5*b^3*c^2*d^3+66*A*x^3*a^5*b^3*c^2*d^3*n-108*A*x^3*a^4*b^4*c^3*d^2*n+5
4*A*x^3*a^3*b^5*c^4*d*n+54*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b^2*c^2*d^
3+189*B*x^2*a^6*b^2*c^2*d^3*n^2-258*B*x^2*a^5*b^3*c^3*d^2*n^2+81*B*x^2*a^4*
b^4*c^4*d*n^2+108*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^2*d^3+162*A*x^2
*a^6*b^2*c^2*d^3*n-288*A*x^2*a^5*b^3*c^3*d^2*n+162*A*x^2*a^4*b^4*c^4*d*n+54
*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^7*b*c^2*d^3+108*B*x*a^7*b*c^2*d^3*n^2-16
2*B*x*a^6*b^2*c^3*d^2*n^2+66*B*x*a^5*b^3*c^4*d*n^2+108*A*x*ln(e*((b*x+a)/(d
*x+c))^n)*a^7*b*c^2*d^3+108*A*x*a^7*b*c^2*d^3*n-216*A*x*a^6*b^2*c^3*d^2*n+1
44*A*x*a^5*b^3*c^4*d*n+108*B*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c^3*d^2*n-54*B
*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^4*d*n+66*B*x^3*ln(e*((b*x+a)/(d*x+c))^
n)*a^5*b^3*c^2*d^3*n+162*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^2*d^3*n+
36*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^3*c^3*d^2*n+108*B*x*ln(e*((b*x+a)/
(d*x+c))^n)*a^7*b*c^2*d^3*n+108*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^3*d
^2*n-18*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^3*c^4*d*n)/i/g^4/(b*x+a)^3/(a*d
-b*c)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/n/a^5/c^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(379) = 758$.

Time = 0.39 (sec) , antiderivative size = 859, normalized size of antiderivative = 2.21

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx = -\frac{12\,Ab^3c^3-54\,Aab^2c^2d+108\,Aa^2bcd^2-66\,Aa^3d^3+6\left(6\,Ab^3cd^2-6\,Aab^2d^3+11\left(Bb^3cd^2-Bab^2d^3\right)n\right)x^2}{}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i),x, alg
orithm="fricas")
```

```
[Out] -1/36*(12*A*b^3*c^3 - 54*A*a*b^2*c^2*d + 108*A*a^2*b*c*d^2 - 66*A*a^3*d^3 +
 6*(6*A*b^3*c*d^2 - 6*A*a*b^2*d^3 + 11*(B*b^3*c*d^2 - B*a*b^2*d^3)*n)*x^2 +
 18*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x + B*a^3*d^3*
n)*log((b*x + a)/(d*x + c))^2 + (4*B*b^3*c^3 - 27*B*a*b^2*c^2*d + 108*B*a^2
*b*c*d^2 - 85*B*a^3*d^3)*n - 3*(6*A*b^3*c^2*d - 36*A*a*b^2*c*d^2 + 30*A*a^2
*b*d^3 + (5*B*b^3*c^2*d - 54*B*a*b^2*c*d^2 + 49*B*a^2*b*d^3)*n)*x + 6*(2*B*
b^3*c^3 - 9*B*a*b^2*c^2*d + 18*B*a^2*b*c*d^2 - 11*B*a^3*d^3 + 6*(B*b^3*c*d^
2 - B*a*b^2*d^3)*x^2 - 3*(B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x
+ 6*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3*x^2 + 3*B*a^2*b*d^3*x + B*a^3*d^3)*log((
b*x + a)/(d*x + c)))*log(e) + 6*(6*A*a^3*d^3 + (11*B*b^3*d^3*n + 6*A*b^3*d^
3)*x^3 + 3*(6*A*a*b^2*d^3 + (2*B*b^3*c*d^2 + 9*B*a*b^2*d^3)*n)*x^2 + (2*B*b
^3*c^3 - 9*B*a*b^2*c^2*d + 18*B*a^2*b*c*d^2)*n + 3*(6*A*a^2*b*d^3 - (B*b^3*
c^2*d - 6*B*a*b^2*c*d^2 - 6*B*a^2*b*d^3)*n)*x)*log((b*x + a)/(d*x + c)))/((
b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4
```

```
)*g^4*i*x^3 + 3*(a*b^6*c^4 - 4*a^2*b^5*c^3*d + 6*a^3*b^4*c^2*d^2 - 4*a^4*b^
3*c*d^3 + a^5*b^2*d^4)*g^4*i*x^2 + 3*(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 6*a^4
*b^3*c^2*d^2 - 4*a^5*b^2*c*d^3 + a^6*b*d^4)*g^4*i*x + (a^3*b^4*c^4 - 4*a^4*
b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)*g^4*i)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4/(d*i*x+c*i),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1472 vs. $2(379) = 758$.

Time = 0.29 (sec) , antiderivative size = 1472, normalized size of antiderivative = 3.78

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i),x, alg
orithm="maxima")
[Out] -1/6*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
5*a*b*d^2)*x)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^
4*i*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*g
^4*i*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*
g^4*i*x + (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*g^4*i)
+ 6*d^3*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4)*g^4*i) - 6*d^3*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d +
6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i))*log(e*(b*x/(d*x + c) +
 a/(d*x + c))^n) - 1/36*(4*b^3*c^3 - 27*a*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*
a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^
2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3
*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*
c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x
 + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*
d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3
*d^3)*log(b*x + a))*log(d*x + c))*B*n/(a^3*b^4*c^4*g^4*i - 4*a^4*b^3*c^3*d*
g^4*i + 6*a^5*b^2*c^2*d^2*g^4*i - 4*a^6*b*c*d^3*g^4*i + a^7*d^4*g^4*i + (b^
7*c^4*g^4*i - 4*a*b^6*c^3*d*g^4*i + 6*a^2*b^5*c^2*d^2*g^4*i - 4*a^3*b^4*c*d
```

```
^3*g^4*i + a^4*b^3*d^4*g^4*i)*x^3 + 3*(a*b^6*c^4*g^4*i - 4*a^2*b^5*c^3*d*g^
4*i + 6*a^3*b^4*c^2*d^2*g^4*i - 4*a^4*b^3*c*d^3*g^4*i + a^5*b^2*d^4*g^4*i)*
x^2 + 3*(a^2*b^5*c^4*g^4*i - 4*a^3*b^4*c^3*d*g^4*i + 6*a^4*b^3*c^2*d^2*g^4*
i - 4*a^5*b^2*c*d^3*g^4*i + a^6*b*d^4*g^4*i)*x) - 1/6*A*((6*b^2*d^2*x^2 + 2
*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^3 -
3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4*i*x^3 + 3*(a*b^5*c^3 - 3
*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*g^4*i*x^2 + 3*(a^2*b^4*c^3
- 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*g^4*i*x + (a^3*b^3*c^3 - 3
*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*g^4*i) + 6*d^3*log(b*x + a)/((b^4
*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i)
- 6*d^3*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4)*g^4*i))
```

Giac [A] (verification not implemented)

none

Time = 139.85 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.60

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx = -\frac{1}{36}\left(\frac{6\left(2\,Bbn-\frac{3\,(bx+a)Bdn}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^3bcg^4i}{(dx+c)^3}-\frac{(bx+a)^3adg^4i}{(dx+c)^3}}+\frac{4\,Bbn-\frac{9\,(bx+a)Bdn}{dx+c}+12\,Bb\log\left(e\right)-\frac{18\,(bx+a)Bd\log(e)}{dx+c}+12\,Ab}{\frac{(bx+a)^3bcg^4i}{(dx+c)^3}-\frac{(bx+a)^3adg^4i}{(dx+c)^3}}\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i),x, alg
orithm="giac")
```

```
[Out] -1/36*(6*(2*B*b*n - 3*(b*x + a)*B*d*n/(d*x + c))*log((b*x + a)/(d*x + c))/(
(b*x + a)^3*b*c*g^4*i/(d*x + c)^3 - (b*x + a)^3*a*d*g^4*i/(d*x + c)^3) + (4
*B*b*n - 9*(b*x + a)*B*d*n/(d*x + c) + 12*B*b*log(e) - 18*(b*x + a)*B*d*log
(e)/(d*x + c) + 12*A*b - 18*(b*x + a)*A*d/(d*x + c))/((b*x + a)^3*b*c*g^4*i
/(d*x + c)^3 - (b*x + a)^3*a*d*g^4*i/(d*x + c)^3))*(b*c/(b*c - a*d)^2 - a*d
/(b*c - a*d)^2)^2
```

Mupad [B] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 986, normalized size of antiderivative = 2.53

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx$$

$$= \frac{\frac{66\,A\,a^2\,d^2+12\,A\,b^2\,c^2+85\,B\,a^2\,d^2\,n+4\,B\,b^2\,c^2\,n-42\,A\,a\,b\,c\,d-23\,B\,a\,b\,c\,d\,n}{6\,(a\,d-b\,c)}+\frac{x\,(30\,A\,a\,b}{}}{x\,\left(18\,i\,a^4\,b\,d^2\,g^4-36\,i\,a^3\,b^2\,c\,d\,g^4+18\,i\,a^2\,b^3\,c^2\,g^4\right)+x^2\,\left(18\,i\,a^3\,b^2\,d^2\,g^4-36\,i\,a^2\,b^3\,c\,d\,g^4+18\,i\,a\,b^4\,c^2\,g^4\right)}$$

$$-\frac{B\,d^3\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{2\,g^4\,i\,n\,(a\,d-b\,c)\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}$$

$$+\frac{B\,d^3\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(x\left(b\left(\frac{g^4\,i\,n\,(a\,d-b\,c)\,(3\,a\,d-b\,c)}{6\,d^2}+\frac{a\,g^4\,i\,n\,(a\,d-b\,c)}{3\,d}\right)+\frac{2\,a\,b\,g^4\,i\,n\,(a\,d-b\,c)}{3\,d}+\frac{b\,g^4\,i\,n\,(a\,d-b\,c)\,(3\,a\,d-b\,c}{3\,d^2}\right.\right.}{g^4\,i\,n\,(a\,d-b\,c)\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}$$

$$+\frac{d^3\,\mathrm{atan}\left(\frac{d^3\,\left(\frac{i\,a^4\,d^4\,g^4-2\,i\,a^3\,b\,c\,d^3\,g^4+2\,i\,a\,b^3\,c^3\,d\,g^4-i\,b^4\,c^4\,g^4}{i\,a^3\,d^3\,g^4-3\,i\,a^2\,b\,c\,d^2\,g^4+3\,i\,a\,b^2\,c^2\,d\,g^4-i\,b^3\,c^3\,g^4}+2\,b\,d\,x\right)\,\left(A+\frac{11\,B\,n}{6}\right)\,\left(i\,a^3\,d^3\,g^4-3\,i\,a^2\,b\,c\,d^2\,g^4+3\,i\,a\,b^2\,c^2\,d\,g^4-i\,b^3\,c^3\,g^4\right)\,6i}{g^4\,i\,\left(6\,A\,d^3+11\,B\,d^3\,n\right)\,(a\,d-b\,c)^4}\right.}{g^4\,i\,(a\,d-b\,c)^4}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^4*(c*i + d*i*x)),
x)

[Out] ((66*A*a^2*d^2 + 12*A*b^2*c^2 + 85*B*a^2*d^2*n + 4*B*b^2*c^2*n - 42*A*a*b*c
*d - 23*B*a*b*c*d*n)/(6*(a*d - b*c)) + (x*(30*A*a*b*d^2 - 6*A*b^2*c*d + 49*
B*a*b*d^2*n - 5*B*b^2*c*d*n))/(2*(a*d - b*c)) + (d*x^2*(6*A*b^2*d + 11*B*b^
2*d*n))/(a*d - b*c))/(x*(18*a^4*b*d^2*g^4*i + 18*a^2*b^3*c^2*g^4*i - 36*a^3
*b^2*c*d*g^4*i) + x^2*(18*a*b^4*c^2*g^4*i + 18*a^3*b^2*d^2*g^4*i - 36*a^2*b
^3*c*d*g^4*i) + x^3*(6*b^5*c^2*g^4*i + 6*a^2*b^3*d^2*g^4*i - 12*a*b^4*c*d*g
^4*i) + 6*a^5*d^2*g^4*i + 6*a^3*b^2*c^2*g^4*i - 12*a^4*b*c*d*g^4*i) + (d^3*
atan((d^3*((a^4*d^4*g^4*i - b^4*c^4*g^4*i + 2*a*b^3*c^3*d*g^4*i - 2*a^3*b*c
*d^3*g^4*i)/(a^3*d^3*g^4*i - b^3*c^3*g^4*i + 3*a*b^2*c^2*d*g^4*i - 3*a^2*b*
c*d^2*g^4*i) + 2*b*d*x)*(A + (11*B*n)/6)*(a^3*d^3*g^4*i - b^3*c^3*g^4*i + 3
*a*b^2*c^2*d*g^4*i - 3*a^2*b*c*d^2*g^4*i)*6i)/(g^4*i*(6*A*d^3 + 11*B*d^3*n)
*(a*d - b*c)^4))*(A + (11*B*n)/6)*2i)/(g^4*i*(a*d - b*c)^4) - (B*d^3*log(e*
((a + b*x)/(c + d*x))^n)^2)/(2*g^4*i*n*(a*d - b*c)*(a^3*d^3 - b^3*c^3 + 3*a
*b^2*c^2*d - 3*a^2*b*c*d^2)) + (B*d^3*log(e*((a + b*x)/(c + d*x))^n)*(x*(b*
((g^4*i*n*(a*d - b*c)*(3*a*d - b*c))/(6*d^2) + (a*g^4*i*n*(a*d - b*c))/(3*d
)) + (2*a*b*g^4*i*n*(a*d - b*c))/(3*d) + (b*g^4*i*n*(a*d - b*c)*(3*a*d - b*
c))/(3*d^2)) + a*((g^4*i*n*(a*d - b*c)*(3*a*d - b*c))/(6*d^2) + (a*g^4*i*n*
(a*d - b*c))/(3*d)) + (g^4*i*n*(a*d - b*c)*(3*a^2*d^2 + b^2*c^2 - 3*a*b*c*d
))/(3*d^3) + (b^2*g^4*i*n*x^2*(a*d - b*c))/d))/(g^4*i*n*(a*d - b*c)*(a^3*d^
3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^3*g^4*i + b^3*g^4*i*x^3 + 3
*a^2*b*g^4*i*x + 3*a*b^2*g^4*i*x^2))
```

3.143 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 359

$$\begin{aligned}&\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx\\&=\frac{3B(bc-ad)^2g^3n(a+bx)}{d^3i^2(c+dx)}-\frac{(bc-ad)^2g^3(6A+5Bn)(a+bx)}{2d^3i^2(c+dx)}\\&\quad-\frac{3B(bc-ad)^2g^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^3i^2(c+dx)}+\frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di^2(c+dx)}\\&\quad-\frac{(bc-ad)g^3(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i^2(c+dx)}\\&\quad-\frac{b(bc-ad)^2g^3\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2d^4i^2}\\&\quad-\frac{3bB(bc-ad)^2g^3n\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}\end{aligned}$$

```
[Out] 3*B*(-a*d+b*c)^2*g^3*n*(b*x+a)/d^3/i^2/(d*x+c)-1/2*(-a*d+b*c)^2*g^3*(5*B*n+
6*A)*(b*x+a)/d^3/i^2/(d*x+c)-3*B*(-a*d+b*c)^2*g^3*(b*x+a)*ln(e*((b*x+a)/(d*
x+c))^n)/d^3/i^2/(d*x+c)+1/2*g^3*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/
d/i^2/(d*x+c)-1/2*(-a*d+b*c)*g^3*(b*x+a)^2*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+
c))^n))/d^2/i^2/(d*x+c)-1/2*b*(-a*d+b*c)^2*g^3*(6*A+5*B*n+6*B*ln(e*((b*x+a)
/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d^4/i^2-3*b*B*(-a*d+b*c)^2*g^3*n*pol
ylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {2561, 2384, 45, 2393, 2332, 2354, 2438}

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$$
$$= -\frac{bg^3(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+6A+5Bn\right)}{2d^4i^2} - \frac{g^3(a+bx)(6A+5Bn)(bc-ad)^2}{2d^3i^2(c+dx)} - \frac{g^3(a+bx)^2(bc-ad)\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d^2i^2(c+dx)} + \frac{g^3(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2di^2(c+dx)} - \frac{3bBg^3n(bc-ad)^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2} - \frac{3Bg^3(a+bx)(bc-ad)^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^3i^2(c+dx)} + \frac{3Bg^3n(a+bx)(bc-ad)^2}{d^3i^2(c+dx)}$$

```
[In] Int[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)^
2,x]
```

```
[Out] (3*B*(b*c - a*d)^2*g^3*n*(a + b*x))/(d^3*i^2*(c + d*x)) - ((b*c - a*d)^2*g^
3*(6*A + 5*B*n)*(a + b*x))/(2*d^3*i^2*(c + d*x)) - (3*B*(b*c - a*d)^2*g^3*(
a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(d^3*i^2*(c + d*x)) + (g^3*(a + b*
x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d*i^2*(c + d*x)) - ((b*c -
a*d)*g^3*(a + b*x)^2*(3*A + B*n + 3*B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d
^2*i^2*(c + d*x)) - (b*(b*c - a*d)^2*g^3*(6*A + 5*B*n + 6*B*Log[e*((a + b*x
)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(2*d^4*i^2) - (3*b*B*(b*c
- a*d)^2*g^3*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^4*i^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)^2g^3)\,\text{Subst}\left(\int \frac{x^3(A+B\log(ex^n))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{i^2}$$

$$= \frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di^2(c+dx)} - \frac{((bc-ad)^2g^3)\,\text{Subst}\left(\int \frac{x^2(3A+Bn+3B\log(ex^n))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{2di^2}$$

$$= \frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di^2(c+dx)} - \frac{(bc-ad)g^3(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i^2(c+dx)} + \frac{((bc-ad)^2g^3)\operatorname{Subst}\left(\int\frac{x(3Bn+2(3A+Bn)+6B\log(ex^n))}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{2d^2i^2}$$

$$= \frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di^2(c+dx)} - \frac{(bc-ad)g^3(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i^2(c+dx)} + \frac{((bc-ad)^2g^3)\operatorname{Subst}\left(\int\left(-\frac{3Bn+2(3A+Bn)+6B\log(ex^n)}{d}-\frac{b(3Bn+2(3A+Bn)+6B\log(ex^n))}{d(-b+dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{2d^2i^2}$$

$$= \frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di^2(c+dx)} - \frac{(bc-ad)g^3(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i^2(c+dx)} - \frac{((bc-ad)^2g^3)\operatorname{Subst}\left(\int(3Bn+2(3A+Bn)+6B\log(ex^n))\,dx,x,\frac{a+bx}{c+dx}\right)}{2d^3i^2} - \frac{(b(bc-ad)^2g^3)\operatorname{Subst}\left(\int\frac{3Bn+2(3A+Bn)+6B\log(ex^n)}{-b+dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{2d^3i^2}$$

$$= -\frac{(bc-ad)^2g^3(6A+5Bn)(a+bx)}{2d^3i^2(c+dx)} + \frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di^2(c+dx)} - \frac{(bc-ad)g^3(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i^2(c+dx)} - \frac{b(bc-ad)^2g^3\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2d^4i^2} - \frac{(3B(bc-ad)^2g^3)\operatorname{Subst}\left(\int\log(ex^n)\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i^2} + \frac{(3bB(bc-ad)^2g^3n)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^4i^2}$$

$$= \frac{3B(bc-ad)^2g^3n(a+bx)}{d^3i^2(c+dx)} - \frac{(bc-ad)^2g^3(6A+5Bn)(a+bx)}{2d^3i^2(c+dx)} - \frac{3B(bc-ad)^2g^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^3i^2(c+dx)} + \frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di^2(c+dx)} - \frac{(bc-ad)g^3(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i^2(c+dx)} - \frac{b(bc-ad)^2g^3\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2d^4i^2} - \frac{3bB(bc-ad)^2g^3n\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.04

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$$

$$= \frac{g^3\left(-2Ab^2d(2bc-3ad)x - 2bBd(2bc-3ad)(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + b^3d^2x^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) + \frac{2(b}{}\right.}{}$$

```
[In] Integrate[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d
*i*x)^2,x]
```

```
[Out] (g^3*(-2*A*b^2*d*(2*b*c - 3*a*d)*x - 2*b*B*d*(2*b*c - 3*a*d)*(a + b*x)*Log[
e*((a + b*x)/(c + d*x))^n] + b^3*d^2*x^2*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]) + (2*(b*c - a*d)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) +
 2*b*B*(2*b*c - 3*a*d)*(b*c - a*d)*n*Log[c + d*x] + 6*b*(b*c - a*d)^2*(A +
B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 2*B*(b*c - a*d)^2*n*((b*c
- a*d)/(c + d*x) + b*Log[a + b*x] - b*Log[c + d*x]) + b*B*n*(-(a^2*d^2*Log[
a + b*x]) + b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - 3*b*B*(b*c - a*d
)^2*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] +
2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(2*d^4*i^2)
```

Maple [F]

$$\int \frac{(bgx+ag)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(dix+ci)^2}\,dx$$

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x)
[Out] int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, a
lgorithm="fricas")
[Out] integral((A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3 +
 (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log(e*((
b*x + a)/(d*x + c))^n))/(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1892 vs. 2(350) = 700.

Time = 0.49 (sec) , antiderivative size = 1892, normalized size of antiderivative = 5.27

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, a
lgorithm="maxima")
[Out] B*a^3*g^3*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*i^2)
 - b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) + 1/2*(2*c^3/(d^5*i^2*x + c*d^4*i^
```

```
2) + 6*c^2*log(d*x + c)/(d^4*i^2) + (d*x^2 - 4*c*x)/(d^3*i^2))*A*b^3*g^3 -
3*A*a*b^2*(c^2/(d^4*i^2*x + c*d^3*i^2) - x/(d^2*i^2) + 2*c*log(d*x + c)/(d^
3*i^2))*g^3 + 3*A*a^2*b*g^3*(c/(d^3*i^2*x + c*d^2*i^2) + log(d*x + c)/(d^2*
i^2)) - B*a^3*g^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^2*i^2*x + c*d*i
^2) - A*a^3*g^3/(d^2*i^2*x + c*d*i^2) - 1/2*(6*a^3*b*d^3*g^3*log(e) - (7*g^
3*n + 6*g^3*log(e))*b^4*c^3 + (17*g^3*n + 18*g^3*log(e))*a*b^3*c^2*d - 6*(2
*g^3*n + 3*g^3*log(e))*a^2*b^2*c*d^2)*B*log(d*x + c)/(b*c*d^4*i^2 - a*d^5*i
^2) + 1/2*((b^4*c*d^3*g^3*log(e) - a*b^3*d^4*g^3*log(e))*B*x^3 - ((g^3*n +
3*g^3*log(e))*b^4*c^2*d^2 - (2*g^3*n + 9*g^3*log(e))*a*b^3*c*d^3 + (g^3*n +
 6*g^3*log(e))*a^2*b^2*d^4)*B*x^2 - ((g^3*n + 4*g^3*log(e))*b^4*c^3*d - 2*(
g^3*n + 5*g^3*log(e))*a*b^3*c^2*d^2 + (g^3*n + 6*g^3*log(e))*a^2*b^2*c*d^3)
*B*x - 6*((b^4*c^3*d*g^3*n - 3*a*b^3*c^2*d^2*g^3*n + 3*a^2*b^2*c*d^3*g^3*n
- a^3*b*d^4*g^3*n)*B*x + (b^4*c^4*g^3*n - 3*a*b^3*c^3*d*g^3*n + 3*a^2*b^2*c
^2*d^2*g^3*n - a^3*b*c*d^3*g^3*n)*B)*log(b*x + a)*log(d*x + c) + 3*((b^4*c^
3*d*g^3*n - 3*a*b^3*c^2*d^2*g^3*n + 3*a^2*b^2*c*d^3*g^3*n - a^3*b*d^4*g^3*n
)*B*x + (b^4*c^4*g^3*n - 3*a*b^3*c^3*d*g^3*n + 3*a^2*b^2*c^2*d^2*g^3*n - a^
3*b*c*d^3*g^3*n)*B)*log(d*x + c)^2 - 2*((g^3*n - g^3*log(e))*b^4*c^4 - 4*(g
^3*n - g^3*log(e))*a*b^3*c^3*d + 6*(g^3*n - g^3*log(e))*a^2*b^2*c^2*d^2 - 3
*(g^3*n - g^3*log(e))*a^3*b*c*d^3)*B - ((2*b^4*c^3*d*g^3*n - 2*a*b^3*c^2*d^
2*g^3*n - 3*a^2*b^2*c*d^3*g^3*n + 5*a^3*b*d^4*g^3*n)*B*x + (2*b^4*c^4*g^3*n
 - 2*a*b^3*c^3*d*g^3*n - 3*a^2*b^2*c^2*d^2*g^3*n + 5*a^3*b*c*d^3*g^3*n)*B)*
log(b*x + a) + ((b^4*c*d^3*g^3 - a*b^3*d^4*g^3)*B*x^3 - 3*(b^4*c^2*d^2*g^3
- 3*a*b^3*c*d^3*g^3 + 2*a^2*b^2*d^4*g^3)*B*x^2 - 2*(2*b^4*c^3*d*g^3 - 5*a*b
^3*c^2*d^2*g^3 + 3*a^2*b^2*c*d^3*g^3)*B*x + 2*(b^4*c^4*g^3 - 4*a*b^3*c^3*d*
g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 3*a^3*b*c*d^3*g^3)*B + 6*((b^4*c^3*d*g^3 - 3*
a*b^3*c^2*d^2*g^3 + 3*a^2*b^2*c*d^3*g^3 - a^3*b*d^4*g^3)*B*x + (b^4*c^4*g^3
 - 3*a*b^3*c^3*d*g^3 + 3*a^2*b^2*c^2*d^2*g^3 - a^3*b*c*d^3*g^3)*B)*log(d*x
+ c))*log((b*x + a)^n) - ((b^4*c*d^3*g^3 - a*b^3*d^4*g^3)*B*x^3 - 3*(b^4*c^
2*d^2*g^3 - 3*a*b^3*c*d^3*g^3 + 2*a^2*b^2*d^4*g^3)*B*x^2 - 2*(2*b^4*c^3*d*g
^3 - 5*a*b^3*c^2*d^2*g^3 + 3*a^2*b^2*c*d^3*g^3)*B*x + 2*(b^4*c^4*g^3 - 4*a*
b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 3*a^3*b*c*d^3*g^3)*B + 6*((b^4*c^3*
d*g^3 - 3*a*b^3*c^2*d^2*g^3 + 3*a^2*b^2*c*d^3*g^3 - a^3*b*d^4*g^3)*B*x + (b
^4*c^4*g^3 - 3*a*b^3*c^3*d*g^3 + 3*a^2*b^2*c^2*d^2*g^3 - a^3*b*c*d^3*g^3)*B
)*log(d*x + c))*log((d*x + c)^n))/(b*c^2*d^4*i^2 - a*c*d^5*i^2 + (b*c*d^5*i
^2 - a*d^6*i^2)*x) + 3*(b^3*c^2*g^3*n - 2*a*b^2*c*d*g^3*n + a^2*b*d^2*g^3*n
)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(
b*c - a*d)))*B/(d^4*i^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3072 vs. 2(350) = 700.

Time = 295.16 (sec) , antiderivative size = 3072, normalized size of antiderivative = 8.56

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, a
lgorithm="giac")
```

```
[Out] -1/24*(6*(B*b^8*c^5*g^3*n - 5*B*a*b^7*c^4*d*g^3*n - 4*(b*x + a)*B*b^7*c^5*d
*g^3*n/(d*x + c) + 10*B*a^2*b^6*c^3*d^2*g^3*n + 20*(b*x + a)*B*a*b^6*c^4*d^
2*g^3*n/(d*x + c) + 6*(b*x + a)^2*B*b^6*c^5*d^2*g^3*n/(d*x + c)^2 - 10*B*a^
3*b^5*c^2*d^3*g^3*n - 40*(b*x + a)*B*a^2*b^5*c^3*d^3*g^3*n/(d*x + c) - 30*(
b*x + a)^2*B*a*b^5*c^4*d^3*g^3*n/(d*x + c)^2 - 4*(b*x + a)^3*B*b^5*c^5*d^3*
g^3*n/(d*x + c)^3 + 5*B*a^4*b^4*c*d^4*g^3*n + 40*(b*x + a)*B*a^3*b^4*c^2*d^
4*g^3*n/(d*x + c) + 60*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^3*n/(d*x + c)^2 + 20
*(b*x + a)^3*B*a*b^4*c^4*d^4*g^3*n/(d*x + c)^3 - B*a^5*b^3*d^5*g^3*n - 20*(
b*x + a)*B*a^4*b^3*c*d^5*g^3*n/(d*x + c) - 60*(b*x + a)^2*B*a^3*b^3*c^2*d^5
*g^3*n/(d*x + c)^2 - 40*(b*x + a)^3*B*a^2*b^3*c^3*d^5*g^3*n/(d*x + c)^3 + 4
*(b*x + a)*B*a^5*b^2*d^6*g^3*n/(d*x + c) + 30*(b*x + a)^2*B*a^4*b^2*c*d^6*g
^3*n/(d*x + c)^2 + 40*(b*x + a)^3*B*a^3*b^2*c^2*d^6*g^3*n/(d*x + c)^3 - 6*(
b*x + a)^2*B*a^5*b*d^7*g^3*n/(d*x + c)^2 - 20*(b*x + a)^3*B*a^4*b*c*d^7*g^3
*n/(d*x + c)^3 + 4*(b*x + a)^3*B*a^5*d^8*g^3*n/(d*x + c)^3)*log((b*x + a)/(
d*x + c))/(b^4*d^4*i^2 - 4*(b*x + a)*b^3*d^5*i^2/(d*x + c) + 6*(b*x + a)^2*
b^2*d^6*i^2/(d*x + c)^2 - 4*(b*x + a)^3*b*d^7*i^2/(d*x + c)^3 + (b*x + a)^4
*d^8*i^2/(d*x + c)^4) + (11*B*b^8*c^5*g^3*n - 55*B*a*b^7*c^4*d*g^3*n - 38*(
b*x + a)*B*b^7*c^5*d*g^3*n/(d*x + c) + 110*B*a^2*b^6*c^3*d^2*g^3*n + 190*(b
*x + a)*B*a*b^6*c^4*d^2*g^3*n/(d*x + c) + 45*(b*x + a)^2*B*b^6*c^5*d^2*g^3*
n/(d*x + c)^2 - 110*B*a^3*b^5*c^2*d^3*g^3*n - 380*(b*x + a)*B*a^2*b^5*c^3*d
^3*g^3*n/(d*x + c) - 225*(b*x + a)^2*B*a*b^5*c^4*d^3*g^3*n/(d*x + c)^2 - 18
*(b*x + a)^3*B*b^5*c^5*d^3*g^3*n/(d*x + c)^3 + 55*B*a^4*b^4*c*d^4*g^3*n + 3
80*(b*x + a)*B*a^3*b^4*c^2*d^4*g^3*n/(d*x + c) + 450*(b*x + a)^2*B*a^2*b^4*
c^3*d^4*g^3*n/(d*x + c)^2 + 90*(b*x + a)^3*B*a*b^4*c^4*d^4*g^3*n/(d*x + c)^
3 - 11*B*a^5*b^3*d^5*g^3*n - 190*(b*x + a)*B*a^4*b^3*c*d^5*g^3*n/(d*x + c)
- 450*(b*x + a)^2*B*a^3*b^3*c^2*d^5*g^3*n/(d*x + c)^2 - 180*(b*x + a)^3*B*a
^2*b^3*c^3*d^5*g^3*n/(d*x + c)^3 + 38*(b*x + a)*B*a^5*b^2*d^6*g^3*n/(d*x +
c) + 225*(b*x + a)^2*B*a^4*b^2*c*d^6*g^3*n/(d*x + c)^2 + 180*(b*x + a)^3*B*
a^3*b^2*c^2*d^6*g^3*n/(d*x + c)^3 - 45*(b*x + a)^2*B*a^5*b*d^7*g^3*n/(d*x +
 c)^2 - 90*(b*x + a)^3*B*a^4*b*c*d^7*g^3*n/(d*x + c)^3 + 18*(b*x + a)^3*B*a
^5*d^8*g^3*n/(d*x + c)^3 + 6*B*b^8*c^5*g^3*log(e) - 30*B*a*b^7*c^4*d*g^3*lo
g(e) - 24*(b*x + a)*B*b^7*c^5*d*g^3*log(e)/(d*x + c) + 60*B*a^2*b^6*c^3*d^2
*g^3*log(e) + 120*(b*x + a)*B*a*b^6*c^4*d^2*g^3*log(e)/(d*x + c) + 36*(b*x
+ a)^2*B*b^6*c^5*d^2*g^3*log(e)/(d*x + c)^2 - 60*B*a^3*b^5*c^2*d^3*g^3*log(
```

```
e) - 240*(b*x + a)*B*a^2*b^5*c^3*d^3*g^3*log(e)/(d*x + c) - 180*(b*x + a)^2
*B*a*b^5*c^4*d^3*g^3*log(e)/(d*x + c)^2 - 24*(b*x + a)^3*B*b^5*c^5*d^3*g^3*
log(e)/(d*x + c)^3 + 30*B*a^4*b^4*c*d^4*g^3*log(e) + 240*(b*x + a)*B*a^3*b^
4*c^2*d^4*g^3*log(e)/(d*x + c) + 360*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^3*log(
e)/(d*x + c)^2 + 120*(b*x + a)^3*B*a*b^4*c^4*d^4*g^3*log(e)/(d*x + c)^3 - 6
*B*a^5*b^3*d^5*g^3*log(e) - 120*(b*x + a)*B*a^4*b^3*c*d^5*g^3*log(e)/(d*x +
 c) - 360*(b*x + a)^2*B*a^3*b^3*c^2*d^5*g^3*log(e)/(d*x + c)^2 - 240*(b*x +
 a)^3*B*a^2*b^3*c^3*d^5*g^3*log(e)/(d*x + c)^3 + 24*(b*x + a)*B*a^5*b^2*d^6
*g^3*log(e)/(d*x + c) + 180*(b*x + a)^2*B*a^4*b^2*c*d^6*g^3*log(e)/(d*x + c
)^2 + 240*(b*x + a)^3*B*a^3*b^2*c^2*d^6*g^3*log(e)/(d*x + c)^3 - 36*(b*x +
a)^2*B*a^5*b*d^7*g^3*log(e)/(d*x + c)^2 - 120*(b*x + a)^3*B*a^4*b*c*d^7*g^3
*log(e)/(d*x + c)^3 + 24*(b*x + a)^3*B*a^5*d^8*g^3*log(e)/(d*x + c)^3 + 6*A
*b^8*c^5*g^3 - 30*A*a*b^7*c^4*d*g^3 - 24*(b*x + a)*A*b^7*c^5*d*g^3/(d*x + c
) + 60*A*a^2*b^6*c^3*d^2*g^3 + 120*(b*x + a)*A*a*b^6*c^4*d^2*g^3/(d*x + c)
+ 36*(b*x + a)^2*A*b^6*c^5*d^2*g^3/(d*x + c)^2 - 60*A*a^3*b^5*c^2*d^3*g^3 -
 240*(b*x + a)*A*a^2*b^5*c^3*d^3*g^3/(d*x + c) - 180*(b*x + a)^2*A*a*b^5*c^
4*d^3*g^3/(d*x + c)^2 - 24*(b*x + a)^3*A*b^5*c^5*d^3*g^3/(d*x + c)^3 + 30*A
*a^4*b^4*c*d^4*g^3 + 240*(b*x + a)*A*a^3*b^4*c^2*d^4*g^3/(d*x + c) + 360*(b
*x + a)^2*A*a^2*b^4*c^3*d^4*g^3/(d*x + c)^2 + 120*(b*x + a)^3*A*a*b^4*c^4*d
^4*g^3/(d*x + c)^3 - 6*A*a^5*b^3*d^5*g^3 - 120*(b*x + a)*A*a^4*b^3*c*d^5*g^
3/(d*x + c) - 360*(b*x + a)^2*A*a^3*b^3*c^2*d^5*g^3/(d*x + c)^2 - 240*(b*x
+ a)^3*A*a^2*b^3*c^3*d^5*g^3/(d*x + c)^3 + 24*(b*x + a)*A*a^5*b^2*d^6*g^3/(
d*x + c) + 180*(b*x + a)^2*A*a^4*b^2*c*d^6*g^3/(d*x + c)^2 + 240*(b*x + a)^
3*A*a^3*b^2*c^2*d^6*g^3/(d*x + c)^3 - 36*(b*x + a)^2*A*a^5*b*d^7*g^3/(d*x +
 c)^2 - 120*(b*x + a)^3*A*a^4*b*c*d^7*g^3/(d*x + c)^3 + 24*(b*x + a)^3*A*a^
5*d^8*g^3/(d*x + c)^3)/(b^4*d^4*i^2 - 4*(b*x + a)*b^3*d^5*i^2/(d*x + c) + 6
*(b*x + a)^2*b^2*d^6*i^2/(d*x + c)^2 - 4*(b*x + a)^3*b*d^7*i^2/(d*x + c)^3
+ (b*x + a)^4*d^8*i^2/(d*x + c)^4) + 6*(B*b^5*c^5*g^3*n - 5*B*a*b^4*c^4*d*g
^3*n + 10*B*a^2*b^3*c^3*d^2*g^3*n - 10*B*a^3*b^2*c^2*d^3*g^3*n + 5*B*a^4*b*
c*d^4*g^3*n - B*a^5*d^5*g^3*n)*log(-b + (b*x + a)*d/(d*x + c))/(b*d^4*i^2)
- 6*(B*b^5*c^5*g^3*n - 5*B*a*b^4*c^4*d*g^3*n + 10*B*a^2*b^3*c^3*d^2*g^3*n -
 10*B*a^3*b^2*c^2*d^3*g^3*n + 5*B*a^4*b*c*d^4*g^3*n - B*a^5*d^5*g^3*n)*log(
(b*x + a)/(d*x + c))/(b*d^4*i^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx=\int \frac{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(c\,i+d\,i\,x)^2}\,dx$$

```
[In] int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)^
2,x)
[Out] int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)^
2, x)
```

3.144 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 275

$$\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx \\
&= -\frac{2B(bc-ad)g^2n(a+bx)}{d^2i^2(c+dx)}+\frac{(bc-ad)g^2(2A+Bn)(a+bx)}{d^2i^2(c+dx)} \\
&\quad+\frac{2B(bc-ad)g^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^2i^2(c+dx)}+\frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{di^2(c+dx)} \\
&\quad+\frac{b(bc-ad)g^2\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^2} \\
&\quad+\frac{2bB(bc-ad)g^2n\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2}
\end{aligned}$$

```
[Out] -2*B*(-a*d+b*c)*g^2*n*(b*x+a)/d^2/i^2/(d*x+c)+(-a*d+b*c)*g^2*(B*n+2*A)*(b*x
+a)/d^2/i^2/(d*x+c)+2*B*(-a*d+b*c)*g^2*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/d^
2/i^2/(d*x+c)+g^2*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d/i^2/(d*x+c)+b
*(-a*d+b*c)*g^2*(2*A+B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*
x+c))/d^3/i^2+2*b*B*(-a*d+b*c)*g^2*n*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {2561, 2384, 45, 2393, 2332, 2354, 2438}

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$$
$$=\frac{bg^2(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d^3i^2}+\frac{g^2(a+bx)(2A+Bn)(bc-ad)}{d^2i^2(c+dx)}+\frac{g^2(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{di^2(c+dx)}+\frac{2bBg^2n(bc-ad)\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2}+\frac{2Bg^2(a+bx)(bc-ad)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^2i^2(c+dx)}-\frac{2Bg^2n(a+bx)(bc-ad)}{d^2i^2(c+dx)}$$

```
[In] Int[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)^
2,x]
```

```
[Out] (-2*B*(b*c - a*d)*g^2*n*(a + b*x))/(d^2*i^2*(c + d*x)) + ((b*c - a*d)*g^2*(
2*A + B*n)*(a + b*x))/(d^2*i^2*(c + d*x)) + (2*B*(b*c - a*d)*g^2*(a + b*x)*
Log[e*((a + b*x)/(c + d*x))^n])/(d^2*i^2*(c + d*x)) + (g^2*(a + b*x)^2*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*i^2*(c + d*x)) + (b*(b*c - a*d)*g^2*
(2*A + B*n + 2*B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*
x))])/(d^3*i^2) + (2*b*B*(b*c - a*d)*g^2*n*PolyLog[2, (d*(a + b*x))/(b*(c +
 d*x))])/(d^3*i^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
```

```
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)g^2)\,\text{Subst}\left(\int \frac{x^2(A+B\log(ex^n))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{i^2}$$

$$= \frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{di^2(c+dx)} - \frac{((bc-ad)g^2)\,\text{Subst}\left(\int \frac{x(2A+Bn+2B\log(ex^n))}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{di^2}$$

$$= \frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{di^2(c+dx)} - \frac{((bc-ad)g^2)\,\text{Subst}\left(\int \left(-\frac{2A+Bn+2B\log(ex^n)}{d} - \frac{b(2A+Bn+2B\log(ex^n))}{d(-b+dx)}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{di^2}$$

$$= \frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{di^2(c+dx)} + \frac{((bc-ad)g^2)\operatorname{Subst}\left(\int(2A+Bn+2B\log(ex^n))\,dx,x,\frac{a+bx}{c+dx}\right)}{d^2i^2} + \frac{(b(bc-ad)g^2)\operatorname{Subst}\left(\int\frac{2A+Bn+2B\log(ex^n)}{-b+dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^2i^2}$$

$$= \frac{(bc-ad)g^2(2A+Bn)(a+bx)}{d^2i^2(c+dx)} + \frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{di^2(c+dx)} + \frac{b(bc-ad)g^2\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^2} + \frac{(2B(bc-ad)g^2)\operatorname{Subst}\left(\int\log(ex^n)\,dx,x,\frac{a+bx}{c+dx}\right)}{d^2i^2} - \frac{(2bB(bc-ad)g^2n)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i^2}$$

$$= -\frac{2B(bc-ad)g^2n(a+bx)}{d^2i^2(c+dx)} + \frac{(bc-ad)g^2(2A+Bn)(a+bx)}{d^2i^2(c+dx)} + \frac{2B(bc-ad)g^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^2i^2(c+dx)} + \frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{di^2(c+dx)} + \frac{b(bc-ad)g^2\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^2} + \frac{2bB(bc-ad)g^2n\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.92

$$\int\frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \frac{g^2\left(Ab^2dx+\frac{B(bc-ad)^2n}{c+dx}+bB(bc-ad)n\log(a+bx)+bBd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-\frac{(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{c+dx}\right.}{}$$

```
[In] Integrate[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d
*i*x)^2,x]
```

```
[Out] (g^2*(A*b^2*d*x + (B*(b*c - a*d)^2*n)/(c + d*x) + b*B*(b*c - a*d)*n*Log[a +
 b*x] + b*B*d*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - ((b*c - a*d)^2*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) - 2*b*B*(b*c - a*d)*n*Log[c
```

```
+ d*x] - 2*b*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x
] + b*B*(b*c - a*d)*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])
*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(d^3*i^2)
```

Maple [F]

$$\int \frac{(bgx+ag)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(dix+ci)^2}\,dx$$

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x)
[Out] int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, a
lgorithm="fricas")
[Out] integral((A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*B*
a*b*g^2*x + B*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n))/(d^2*i^2*x^2 + 2*c*d
*i^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1273 vs. $2(274) = 548$.

Time = 0.48 (sec) , antiderivative size = 1273, normalized size of antiderivative = 4.63

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, a
lgorithm="maxima")
```

```
[Out] B*a^2*g^2*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*i^2)
 - b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) - A*b^2*(c^2/(d^4*i^2*x + c*d^3*i^
2) - x/(d^2*i^2) + 2*c*log(d*x + c)/(d^3*i^2))*g^2 + 2*A*a*b*g^2*(c/(d^3*i^
2*x + c*d^2*i^2) + log(d*x + c)/(d^2*i^2)) - B*a^2*g^2*log(e*(b*x/(d*x + c)
 + a/(d*x + c))^n)/(d^2*i^2*x + c*d*i^2) - A*a^2*g^2/(d^2*i^2*x + c*d*i^2)
- (2*a^2*b*d^2*g^2*log(e) + 2*(g^2*n + g^2*log(e))*b^3*c^2 - (3*g^2*n + 4*g
^2*log(e))*a*b^2*c*d)*B*log(d*x + c)/(b*c*d^3*i^2 - a*d^4*i^2) + ((b^3*c*d^
2*g^2*log(e) - a*b^2*d^3*g^2*log(e))*B*x^2 + (b^3*c^2*d*g^2*log(e) - a*b^2*
c*d^2*g^2*log(e))*B*x + 2*((b^3*c^2*d*g^2*n - 2*a*b^2*c*d^2*g^2*n + a^2*b*d
^3*g^2*n)*B*x + (b^3*c^3*g^2*n - 2*a*b^2*c^2*d*g^2*n + a^2*b*c*d^2*g^2*n)*B
)*log(b*x + a)*log(d*x + c) - ((b^3*c^2*d*g^2*n - 2*a*b^2*c*d^2*g^2*n + a^2
*b*d^3*g^2*n)*B*x + (b^3*c^3*g^2*n - 2*a*b^2*c^2*d*g^2*n + a^2*b*c*d^2*g^2*
n)*B)*log(d*x + c)^2 + ((g^2*n - g^2*log(e))*b^3*c^3 - 3*(g^2*n - g^2*log(e
))*a*b^2*c^2*d + 2*(g^2*n - g^2*log(e))*a^2*b*c*d^2)*B + ((b^3*c^2*d*g^2*n
- a*b^2*c*d^2*g^2*n - a^2*b*d^3*g^2*n)*B*x + (b^3*c^3*g^2*n - a*b^2*c^2*d*g
^2*n - a^2*b*c*d^2*g^2*n)*B)*log(b*x + a) + ((b^3*c*d^2*g^2 - a*b^2*d^3*g^2
)*B*x^2 + (b^3*c^2*d*g^2 - a*b^2*c*d^2*g^2)*B*x - (b^3*c^3*g^2 - 3*a*b^2*c^
2*d*g^2 + 2*a^2*b*c*d^2*g^2)*B - 2*((b^3*c^2*d*g^2 - 2*a*b^2*c*d^2*g^2 + a^
2*b*d^3*g^2)*B*x + (b^3*c^3*g^2 - 2*a*b^2*c^2*d*g^2 + a^2*b*c*d^2*g^2)*B)*l
og(d*x + c))*log((b*x + a)^n) - ((b^3*c*d^2*g^2 - a*b^2*d^3*g^2)*B*x^2 + (b
^3*c^2*d*g^2 - a*b^2*c*d^2*g^2)*B*x - (b^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 2*
a^2*b*c*d^2*g^2)*B - 2*((b^3*c^2*d*g^2 - 2*a*b^2*c*d^2*g^2 + a^2*b*d^3*g^2)
*B*x + (b^3*c^3*g^2 - 2*a*b^2*c^2*d*g^2 + a^2*b*c*d^2*g^2)*B)*log(d*x + c))
*log((d*x + c)^n))/(b*c^2*d^3*i^2 - a*c*d^4*i^2 + (b*c*d^4*i^2 - a*d^5*i^2)
*x) - 2*(b^2*c*g^2*n - a*b*d*g^2*n)*(log(b*x + a)*log((b*d*x + a*d)/(b*c -
a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(d^3*i^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1898 vs. 2(274) = 548.

Time = 221.56 (sec) , antiderivative size = 1898, normalized size of antiderivative = 6.90

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, a
lgorithm="giac")
```

```
[Out] 1/6*(2*(B*b^6*c^4*g^2*n - 4*B*a*b^5*c^3*d*g^2*n - 3*(b*x + a)*B*b^5*c^4*d*g
^2*n/(d*x + c) + 6*B*a^2*b^4*c^2*d^2*g^2*n + 12*(b*x + a)*B*a*b^4*c^3*d^2*g
^2*n/(d*x + c) + 3*(b*x + a)^2*B*b^4*c^4*d^2*g^2*n/(d*x + c)^2 - 4*B*a^3*b^
3*c*d^3*g^2*n - 18*(b*x + a)*B*a^2*b^3*c^2*d^3*g^2*n/(d*x + c) - 12*(b*x +
a)^2*B*a*b^3*c^3*d^3*g^2*n/(d*x + c)^2 + B*a^4*b^2*d^4*g^2*n + 12*(b*x + a)
*B*a^3*b^2*c*d^4*g^2*n/(d*x + c) + 18*(b*x + a)^2*B*a^2*b^2*c^2*d^4*g^2*n/(
d*x + c)^2 - 3*(b*x + a)*B*a^4*b*d^5*g^2*n/(d*x + c) - 12*(b*x + a)^2*B*a^3
*b*c*d^5*g^2*n/(d*x + c)^2 + 3*(b*x + a)^2*B*a^4*d^6*g^2*n/(d*x + c)^2)*log
((b*x + a)/(d*x + c))/(b^3*d^3*i^2 - 3*(b*x + a)*b^2*d^4*i^2/(d*x + c) + 3*
(b*x + a)^2*b*d^5*i^2/(d*x + c)^2 - (b*x + a)^3*d^6*i^2/(d*x + c)^3) + (3*B
*b^6*c^4*g^2*n - 12*B*a*b^5*c^3*d*g^2*n - 7*(b*x + a)*B*b^5*c^4*d*g^2*n/(d*
x + c) + 18*B*a^2*b^4*c^2*d^2*g^2*n + 28*(b*x + a)*B*a*b^4*c^3*d^2*g^2*n/(d
*x + c) + 4*(b*x + a)^2*B*b^4*c^4*d^2*g^2*n/(d*x + c)^2 - 12*B*a^3*b^3*c*d^
3*g^2*n - 42*(b*x + a)*B*a^2*b^3*c^2*d^3*g^2*n/(d*x + c) - 16*(b*x + a)^2*B
*a*b^3*c^3*d^3*g^2*n/(d*x + c)^2 + 3*B*a^4*b^2*d^4*g^2*n + 28*(b*x + a)*B*a
^3*b^2*c*d^4*g^2*n/(d*x + c) + 24*(b*x + a)^2*B*a^2*b^2*c^2*d^4*g^2*n/(d*x
+ c)^2 - 7*(b*x + a)*B*a^4*b*d^5*g^2*n/(d*x + c) - 16*(b*x + a)^2*B*a^3*b*c
*d^5*g^2*n/(d*x + c)^2 + 4*(b*x + a)^2*B*a^4*d^6*g^2*n/(d*x + c)^2 + 2*B*b^
6*c^4*g^2*log(e) - 8*B*a*b^5*c^3*d*g^2*log(e) - 6*(b*x + a)*B*b^5*c^4*d*g^2
*log(e)/(d*x + c) + 12*B*a^2*b^4*c^2*d^2*g^2*log(e) + 24*(b*x + a)*B*a*b^4*
c^3*d^2*g^2*log(e)/(d*x + c) + 6*(b*x + a)^2*B*b^4*c^4*d^2*g^2*log(e)/(d*x
+ c)^2 - 8*B*a^3*b^3*c*d^3*g^2*log(e) - 36*(b*x + a)*B*a^2*b^3*c^2*d^3*g^2*
log(e)/(d*x + c) - 24*(b*x + a)^2*B*a*b^3*c^3*d^3*g^2*log(e)/(d*x + c)^2 +
2*B*a^4*b^2*d^4*g^2*log(e) + 24*(b*x + a)*B*a^3*b^2*c*d^4*g^2*log(e)/(d*x +
 c) + 36*(b*x + a)^2*B*a^2*b^2*c^2*d^4*g^2*log(e)/(d*x + c)^2 - 6*(b*x + a)
*B*a^4*b*d^5*g^2*log(e)/(d*x + c) - 24*(b*x + a)^2*B*a^3*b*c*d^5*g^2*log(e)
/(d*x + c)^2 + 6*(b*x + a)^2*B*a^4*d^6*g^2*log(e)/(d*x + c)^2 + 2*A*b^6*c^4
*g^2 - 8*A*a*b^5*c^3*d*g^2 - 6*(b*x + a)*A*b^5*c^4*d*g^2/(d*x + c) + 12*A*a
^2*b^4*c^2*d^2*g^2 + 24*(b*x + a)*A*a*b^4*c^3*d^2*g^2/(d*x + c) + 6*(b*x +
a)^2*A*b^4*c^4*d^2*g^2/(d*x + c)^2 - 8*A*a^3*b^3*c*d^3*g^2 - 36*(b*x + a)*A
*a^2*b^3*c^2*d^3*g^2/(d*x + c) - 24*(b*x + a)^2*A*a*b^3*c^3*d^3*g^2/(d*x +
c)^2 + 2*A*a^4*b^2*d^4*g^2 + 24*(b*x + a)*A*a^3*b^2*c*d^4*g^2/(d*x + c) + 3
6*(b*x + a)^2*A*a^2*b^2*c^2*d^4*g^2/(d*x + c)^2 - 6*(b*x + a)*A*a^4*b*d^5*g
^2/(d*x + c) - 24*(b*x + a)^2*A*a^3*b*c*d^5*g^2/(d*x + c)^2 + 6*(b*x + a)^2
```

```
*A*a^4*d^6*g^2/(d*x + c)^2)/(b^3*d^3*i^2 - 3*(b*x + a)*b^2*d^4*i^2/(d*x + c
) + 3*(b*x + a)^2*b*d^5*i^2/(d*x + c)^2 - (b*x + a)^3*d^6*i^2/(d*x + c)^3)
+ 2*(B*b^4*c^4*g^2*n - 4*B*a*b^3*c^3*d*g^2*n + 6*B*a^2*b^2*c^2*d^2*g^2*n -
4*B*a^3*b*c*d^3*g^2*n + B*a^4*d^4*g^2*n)*log(b - (b*x + a)*d/(d*x + c))/(b*
d^3*i^2) - 2*(B*b^4*c^4*g^2*n - 4*B*a*b^3*c^3*d*g^2*n + 6*B*a^2*b^2*c^2*d^2
*g^2*n - 4*B*a^3*b*c*d^3*g^2*n + B*a^4*d^4*g^2*n)*log((b*x + a)/(d*x + c))/
(b*d^3*i^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \int \frac{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}{{(c\,i+d\,i\,x)}^2}\,dx$$

```
[In] int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)^
2,x)
```

```
[Out] int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)^
2, x)
```

3.145 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 168

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = -\frac{Ag(a+bx)}{di^2(c+dx)}+\frac{Bgn(a+bx)}{di^2(c+dx)} - \frac{Bg(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{di^2(c+dx)} - \frac{bg\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i^2} - \frac{bBgn\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2}$$

```
[Out] -A*g*(b*x+a)/d/i^2/(d*x+c)+B*g*n*(b*x+a)/d/i^2/(d*x+c)-B*g*(b*x+a)*ln(e*((b
*x+a)/(d*x+c))^n)/d/i^2/(d*x+c)-b*g*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*
d+b*c)/b/(d*x+c))/d^2/i^2-b*B*g*n*polylog(2,d*(b*x+a)/b/(d*x+c))/d^2/i^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used

$= \{2561, 45, 2393, 2332, 2354, 2438\}$

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = -\frac{bg\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^2i^2} - \frac{Ag(a+bx)}{di^2(c+dx)} - \frac{bBgn\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2} - \frac{Bg(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{di^2(c+dx)} + \frac{Bgn(a+bx)}{di^2(c+dx)}$$

```
[In] Int[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)^2,
x]
```

```
[Out] -((A*g*(a + b*x))/(d*i^2*(c + d*x))) + (B*g*n*(a + b*x))/(d*i^2*(c + d*x))
- (B*g*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(d*i^2*(c + d*x)) - (b*g*(
A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(d^2*
i^2) - (b*B*g*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^2*i^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g\text{Subst}\left(\int \frac{x(A+B\log(ex^n))}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{i^2}\\
&= \frac{g\text{Subst}\left(\int \left(-\frac{A+B\log(ex^n)}{d}-\frac{b(A+B\log(ex^n))}{d(-b+dx)}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{i^2}\\
&= -\frac{g\text{Subst}\left(\int (A+B\log(ex^n))\,dx, x, \frac{a+bx}{c+dx}\right)}{di^2}-\frac{(bg)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{-b+dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{di^2}\\
&= -\frac{Ag(a+bx)}{di^2(c+dx)}-\frac{bg\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i^2}\\
&\quad -\frac{(Bg)\text{Subst}\left(\int \log(ex^n)\,dx, x, \frac{a+bx}{c+dx}\right)}{di^2}+\frac{(bBgn)\text{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^2}\\
&= -\frac{Ag(a+bx)}{di^2(c+dx)}+\frac{Bgn(a+bx)}{di^2(c+dx)}-\frac{Bg(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{di^2(c+dx)}\\
&\quad -\frac{bg\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i^2}-\frac{bBgn\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.09

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$$

$$= \frac{g\left(\frac{2(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{c+dx}+2b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(c+dx)-2Bn\left(\frac{bc-ad}{c+dx}+b\log(a+bx)-b\log\right.\right.}{2d^2i^2}$$

```
[In] Integrate[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i
*x)^2,x]
```

```
[Out] (g*((2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) + 2*b*
(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 2*B*n*((b*c - a*d)/(c
 + d*x) + b*Log[a + b*x] - b*Log[c + d*x]) - b*B*n*((2*Log[(d*(a + b*x))/(-
(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*
c - a*d)])))/(2*d^2*i^2)
```

Maple [F]

$$\int \frac{(bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}{(dix+ci)^{2}}dx$$

```
[In] int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x)
```

```
[Out] int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)}{(ci+dix)^{2}}\,dx=\int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)}{(dix+ci)^{2}}\,dx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, alg
orithm="fricas")
```

```
[Out] integral((A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log(e*((b*x + a)/(d*x + c))^n
))/(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)}{(ci+dix)^{2}}\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, alg
orithm="maxima")
```

```
[Out] B*a*g*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*i^2) - b
*log(d*x + c)/((b*c*d - a*d^2)*i^2)) - 1/2*B*b*g*((2*(d*n*x + c*n)*log(b*x
+ a)*log(d*x + c) - (d*n*x + c*n)*log(d*x + c)^2 - 2*((d*x + c)*log(d*x + c
) + c)*log((b*x + a)^n) + 2*((d*x + c)*log(d*x + c) + c)*log((d*x + c)^n))/
(d^3*i^2*x + c*d^2*i^2) - 2*integrate((b*d^2*x^2*log(e) + a*d^2*x*log(e) -
b*c^2*n + a*c*d*n + (b*d^2*n*x^2 + a*c*d*n + (b*c*d*n + a*d^2*n)*x)*log(b*x
 + a))/(b*d^4*i^2*x^3 + a*c^2*d^2*i^2 + (2*b*c*d^3*i^2 + a*d^4*i^2)*x^2 + (
b*c^2*d^2*i^2 + 2*a*c*d^3*i^2)*x), x)) + A*b*g*(c/(d^3*i^2*x + c*d^2*i^2) +
 log(d*x + c)/(d^2*i^2)) - B*a*g*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^
2*i^2*x + c*d*i^2) - A*a*g/(d^2*i^2*x + c*d*i^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 906 vs. $2(167) = 334$.

Time = 142.03 (sec) , antiderivative size = 906, normalized size of antiderivative = 5.39

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx=-\frac{1}{2}\left(\frac{\left(Bb^4c^3gn-3\,Bab^3c^2dgn-\frac{2\,(bx+a)Bb^3c^3dgn}{dx+c}+3\,Ba^2b^2cd^2gn+\frac{6\,(bx+a)Bab^2c^2d^2gn}{dx+c}-Ba^3bd^3gn-\frac{6\,(bx+\cdots}{\cdots}\right.}{b^2d^2i^2-\frac{2\,(bx+a)bd^3i^2}{dx+c}+\frac{(bx+a)^2d^4i^2}{(dx+c)^2}}\right.$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, alg
orithm="giac")
```

```
[Out] -1/2*((B*b^4*c^3*g*n - 3*B*a*b^3*c^2*d*g*n - 2*(b*x + a)*B*b^3*c^3*d*g*n/(d
*x + c) + 3*B*a^2*b^2*c*d^2*g*n + 6*(b*x + a)*B*a*b^2*c^2*d^2*g*n/(d*x + c)
 - B*a^3*b*d^3*g*n - 6*(b*x + a)*B*a^2*b*c*d^3*g*n/(d*x + c) + 2*(b*x + a)*
B*a^3*d^4*g*n/(d*x + c))*log((b*x + a)/(d*x + c))/(b^2*d^2*i^2 - 2*(b*x + a
)*b*d^3*i^2/(d*x + c) + (b*x + a)^2*d^4*i^2/(d*x + c)^2) + (B*b^4*c^3*g*n -
 3*B*a*b^3*c^2*d*g*n - (b*x + a)*B*b^3*c^3*d*g*n/(d*x + c) + 3*B*a^2*b^2*c*
d^2*g*n + 3*(b*x + a)*B*a*b^2*c^2*d^2*g*n/(d*x + c) - B*a^3*b*d^3*g*n - 3*(
b*x + a)*B*a^2*b*c*d^3*g*n/(d*x + c) + (b*x + a)*B*a^3*d^4*g*n/(d*x + c) +
B*b^4*c^3*g*log(e) - 3*B*a*b^3*c^2*d*g*log(e) - 2*(b*x + a)*B*b^3*c^3*d*g*l
og(e)/(d*x + c) + 3*B*a^2*b^2*c*d^2*g*log(e) + 6*(b*x + a)*B*a*b^2*c^2*d^2*
```

```
g*log(e)/(d*x + c) - B*a^3*b*d^3*g*log(e) - 6*(b*x + a)*B*a^2*b*c*d^3*g*log
(e)/(d*x + c) + 2*(b*x + a)*B*a^3*d^4*g*log(e)/(d*x + c) + A*b^4*c^3*g - 3*
A*a*b^3*c^2*d*g - 2*(b*x + a)*A*b^3*c^3*d*g/(d*x + c) + 3*A*a^2*b^2*c*d^2*g
 + 6*(b*x + a)*A*a*b^2*c^2*d^2*g/(d*x + c) - A*a^3*b*d^3*g - 6*(b*x + a)*A*
a^2*b*c*d^3*g/(d*x + c) + 2*(b*x + a)*A*a^3*d^4*g/(d*x + c))/(b^2*d^2*i^2 -
 2*(b*x + a)*b*d^3*i^2/(d*x + c) + (b*x + a)^2*d^4*i^2/(d*x + c)^2) + (B*b^
3*c^3*g*n - 3*B*a*b^2*c^2*d*g*n + 3*B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log(
-b + (b*x + a)*d/(d*x + c))/(b*d^2*i^2) - (B*b^3*c^3*g*n - 3*B*a*b^2*c^2*d*
g*n + 3*B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log((b*x + a)/(d*x + c))/(b*d^2*
i^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx=\int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}{{(c\,i+d\,i\,x)}^2}\,dx$$

```
[In] int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)^2,
x)

[Out] int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)^2,
 x)
```

3.146 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2} dx$

Optimal result

Integrand size = 33, antiderivative size = 102

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2} dx = \frac{A(a+bx)}{(bc-ad)i^2(c+dx)} - \frac{Bn(a+bx)}{(bc-ad)i^2(c+dx)} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)i^2(c+dx)}$$

```
[Out] A*(b*x+a)/(-a*d+b*c)/i^2/(d*x+c)-B*n*(b*x+a)/(-a*d+b*c)/i^2/(d*x+c)+B*(b*x+
a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/i^2/(d*x+c)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2551, 2332}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2} dx = \frac{A(a+bx)}{i^2(c+dx)(bc-ad)} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{i^2(c+dx)(bc-ad)} - \frac{Bn(a+bx)}{i^2(c+dx)(bc-ad)}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*i + d*i*x)^2,x]
```

```
[Out] (A*(a + b*x))/((b*c - a*d)*i^2*(c + d*x)) - (B*n*(a + b*x))/((b*c - a*d)*i^
2*(c + d*x)) + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/((b*c - a*d)*i^
2*(c + d*x))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int (A+B\log(ex^n))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}\\ &= \frac{A(a+bx)}{(bc-ad)i^2(c+dx)} + \frac{B\text{Subst}\left(\int \log(ex^n)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}\\ &= \frac{A(a+bx)}{(bc-ad)i^2(c+dx)} - \frac{Bn(a+bx)}{(bc-ad)i^2(c+dx)} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)i^2(c+dx)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{di(ci+dix)} + \frac{B(bc-ad)n\left(\frac{1}{(bc-ad)(c+dx)}+\frac{b\log(a+bx)}{(bc-ad)^2}-\frac{b\log(c+dx)}{(bc-ad)^2}\right)}{di^2}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*i + d*i*x)^2,x]
```

```
[Out] -((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(d*i*(c*i + d*i*x))) + (B*(b*c - a
*d)*n*(1/((b*c - a*d)*(c + d*x)) + (b*Log[a + b*x])/(b*c - a*d)^2 - (b*Log[
c + d*x])/(b*c - a*d)^2))/(d*i^2)
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{A}{i^{2}\left(dx+c\right)d}-\frac{B\left(\frac{\left(bx+a\right)\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{dx+c}-\frac{n\left(bx+a\right)}{dx+c}\right)}{i^{2}\left(ad-cb\right)}$	81
parts	$-\frac{A}{i^{2}\left(dx+c\right)d}-\frac{B\left(\frac{\left(bx+a\right)\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{dx+c}-\frac{n\left(bx+a\right)}{dx+c}\right)}{i^{2}\left(ad-cb\right)}$	81
parallelrisch	$-\frac{-Bab\,d^{3}n^{2}+B\,b^{2}c\,d^{2}n^{2}+Aab\,d^{3}n-A\,b^{2}c\,d^{2}n+Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{2}d^{3}n+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)ab\,d^{3}n}{i^{2}\left(dx+c\right)b\,d^{3}n\left(ad-cb\right)}$	129

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -A/i^2/(d*x+c)/d-1/i^2*B/(a*d-b*c)*((b*x+a)/(d*x+c)*ln(e*((b*x+a)/(d*x+c))^
n)-n*(b*x+a)/(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx = -\frac{Abc-Aad-(Bbc-Bad)n+(Bbc-Bad)\log(e)-(Bbdnx+Badn)\log\left(\frac{bx+a}{dx+c}\right)}{(bcd^2-ad^3)i^2x+(bc^2d-acd^2)i^2}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, algorithm="fric
as")
```

```
[Out] -(A*b*c - A*a*d - (B*b*c - B*a*d)*n + (B*b*c - B*a*d)*log(e) - (B*b*d*n*x +
 B*a*d*n)*log((b*x + a)/(d*x + c)))/((b*c*d^2 - a*d^3)*i^2*x + (b*c^2*d - a
*c*d^2)*i^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(82) = 164$.

Time = 11.44 (sec) , antiderivative size = 444, normalized size of antiderivative = 4.35

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx$$
$$= \begin{cases} -\frac{A}{cdi^2+d^2i^2x} - \frac{B\log\left(e\left(\frac{bc}{cd+d^2x}+\frac{bx}{c+dx}\right)^n\right)}{cdi^2+d^2i^2x} \\ \frac{Ax+\frac{Ba\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{b}-Bnx+Bx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{c^2i^2} \\ -\frac{Aad}{acd^2i^2+ad^3i^2x-bc^2di^2-bcd^2i^2x} + \frac{Abc}{acd^2i^2+ad^3i^2x-bc^2di^2-bcd^2i^2x} + \frac{Badn}{acd^2i^2+ad^3i^2x-bc^2di^2-bcd^2i^2x} - \frac{Bad\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)\right.}{acd^2i^2+ad^3i^2x-bc^2di^2-bcd} \end{cases}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**2,x)
```

```
[Out] Piecewise((-A/(c*d*i**2 + d**2*i**2*x) - B*log(e*(b*c/(c*d + d**2*x) + b*x/
(c + d*x))**n)/(c*d*i**2 + d**2*i**2*x), Eq(a, b*c/d)), ((A*x + B*a*log(e*(
a/c + b*x/c)**n)/b - B*n*x + B*x*log(e*(a/c + b*x/c)**n))/(c**2*i**2), Eq(d
, 0)), (-A*a*d/(a*c*d**2*i**2 + a*d**3*i**2*x - b*c**2*d*i**2 - b*c*d**2*i*
*2*x) + A*b*c/(a*c*d**2*i**2 + a*d**3*i**2*x - b*c**2*d*i**2 - b*c*d**2*i**
2*x) + B*a*d*n/(a*c*d**2*i**2 + a*d**3*i**2*x - b*c**2*d*i**2 - b*c*d**2*i*
*2*x) - B*a*d*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a*c*d**2*i**2 + a*d*
*3*i**2*x - b*c**2*d*i**2 - b*c*d**2*i**2*x) - B*b*c*n/(a*c*d**2*i**2 + a*d
**3*i**2*x - b*c**2*d*i**2 - b*c*d**2*i**2*x) - B*b*d*x*log(e*(a/(c + d*x)
+ b*x/(c + d*x))**n)/(a*c*d**2*i**2 + a*d**3*i**2*x - b*c**2*d*i**2 - b*c*d
**2*i**2*x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.33

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx = Bn\left(\frac{1}{d^2i^2x+cdi^2}+\frac{b\log(bx+a)}{(bcd-ad^2)i^2}-\frac{b\log(dx+c)}{(bcd-ad^2)i^2}\right) - \frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{d^2i^2x+cdi^2} - \frac{A}{d^2i^2x+cdi^2}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, algorithm="maxi
ma")
```

```
[Out] B*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*i^2) - b*log
(d*x + c)/((b*c*d - a*d^2)*i^2)) - B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
/(d^2*i^2*x + c*d*i^2) - A/(d^2*i^2*x + c*d*i^2)
```

Giac [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx$$
$$=\left(\frac{(bx+a)Bn\log\left(\frac{bx+a}{dx+c}\right)}{(dx+c)i^2}-\frac{(Bn-B\log\left(e\right)-A)(bx+a)}{(dx+c)i^2}\right)\left(\frac{bc}{\left(bc-ad\right)^2}-\frac{ad}{\left(bc-ad\right)^2}\right)$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, algorithm="giac
")
```

```
[Out] ((b*x + a)*B*n*log((b*x + a)/(d*x + c))/((d*x + c)*i^2) - (B*n - B*log(e) -
 A)*(b*x + a)/((d*x + c)*i^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.11

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx=-\frac{A-B\,n}{x\,d^2\,i^2+c\,d\,i^2}-\frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{d\,\left(c\,i^2+d\,i^2\,x\right)}$$
$$+\frac{B\,b\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{d\,i^2\,\left(a\,d-b\,c\right)}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*i + d*i*x)^2,x)
```

```
[Out] (B*b*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(d*i^2*(a*d - b*c)) -
 (B*log(e*((a + b*x)/(c + d*x))^n))/(d*(c*i^2 + d*i^2*x)) - (A - B*n)/(d^2*
i^2*x + c*d*i^2)
```

3.147 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 166

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx = -\frac{Ad(a+bx)}{(bc-ad)^2gi^2(c+dx)}+\frac{Bdn(a+bx)}{(bc-ad)^2gi^2(c+dx)} - \frac{Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^2gi^2(c+dx)}+\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2B(bc-ad)^2gi^2n}$$

```
[Out] -A*d*(b*x+a)/(-a*d+b*c)^2/g/i^2/(d*x+c)+B*d*n*(b*x+a)/(-a*d+b*c)^2/g/i^2/(d
*x+c)-B*d*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^2/g/i^2/(d*x+c)+1/2*
b*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/B/(-a*d+b*c)^2/g/i^2/n
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {2561, 2388, 2338, 2332}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx = \frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2Bgi^2n(bc-ad)^2}-\frac{Ad(a+bx)}{gi^2(c+dx)(bc-ad)^2} - \frac{Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{gi^2(c+dx)(bc-ad)^2}+\frac{Bdn(a+bx)}{gi^2(c+dx)(bc-ad)^2}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)*(c*i + d*i*x)^2),
x]

[Out] -((A*d*(a + b*x))/((b*c - a*d)^2*g*i^2*(c + d*x))) + (B*d*n*(a + b*x))/((b*
c - a*d)^2*g*i^2*(c + d*x)) - (B*d*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n]
```

```
)/((b*c - a*d)^2*g*i^2*(c + d*x)) + (b*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])^2)/(2*B*(b*c - a*d)^2*g*i^2*n)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex^n))}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 gi^2}\\
&= \frac{b\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 gi^2} - \frac{d\text{Subst}\left(\int (A+B\log(ex^n))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 gi^2}\\
&= -\frac{Ad(a+bx)}{(bc-ad)^2 gi^2(c+dx)} + \frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2B(bc-ad)^2 gi^2 n} - \frac{(Bd)\text{Subst}\left(\int \log(ex^n)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 gi^2}\\
&= -\frac{Ad(a+bx)}{(bc-ad)^2 gi^2(c+dx)} + \frac{Bdn(a+bx)}{(bc-ad)^2 gi^2(c+dx)}\\
&\quad - \frac{Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^2 gi^2(c+dx)} + \frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2B(bc-ad)^2 gi^2 n}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.83

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)(ci + dix)^2}\,dx$$
$$= \frac{2(bc-ad)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) + 2b(c+dx)\log(a+bx)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) - 2b(c+dx)\left(A + B\right.}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)*(c*i + d*i*
x)^2),x]
```

```
[Out] (2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*b*(c + d*x)*Log[a
 + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b*(c + d*x)*(A + B*Log[e
*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 2*B*n*(b*c - a*d + b*(c + d*x)*Lo
g[a + b*x] - b*(c + d*x)*Log[c + d*x]) - b*B*n*(c + d*x)*(Log[a + b*x]*(Log
[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/
(-(b*c) + a*d)]) + b*B*n*(c + d*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] -
Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(2*(
b*c - a*d)^2*g*i^2*(c + d*x))
```

Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.62

method	result
parallelrisch	$\frac{2Ba\,b^2d^4n^2-2B\,b^3c\,d^3n^2-2Aa\,b^2d^4n+2A\,b^3c\,d^3n+Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2b^3d^4+2Ax\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3d^4+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2b^3c}{2i^2g(dx+c)(a^2d^2-2abcd+b^2c^2)b^2d^3n}$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^2,x,method=_RET
URNVERBOSE)
```

```
[Out] 1/2*(2*B*a*b^2*d^4*n^2-2*B*b^3*c*d^3*n^2-2*A*a*b^2*d^4*n+2*A*b^3*c*d^3*n+B*
x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*d^4+2*A*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d
^4+B*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*c*d^3+2*A*ln(e*((b*x+a)/(d*x+c))^n)*b^
3*c*d^3-2*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^4*n-2*B*ln(e*((b*x+a)/(d*x+c)
)^n)*a*b^2*d^4*n)/i^2/g/(d*x+c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/d^3/n
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.18

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx = \frac{2\,Abc-2\,Aad+(Bbdnx+Bbcn)\log\left(\frac{bx+a}{dx+c}\right)^2-2\,(Bbc-Bad)n+2\,\left(Bbc-Bad+(Bbdx+Bbc)\log\left(\frac{bx}{dx}\right.\right.}{2\,((b^2c^2d-2\,abcd^2+a^2d^3)gi^2x+(b^2c^3-2\,abc^2d+a^2}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^2,x, alg
orithm="fricas")
```

```
[Out] 1/2*(2*A*b*c - 2*A*a*d + (B*b*d*n*x + B*b*c*n)*log((b*x + a)/(d*x + c))^2 -
 2*(B*b*c - B*a*d)*n + 2*(B*b*c - B*a*d + (B*b*d*x + B*b*c)*log((b*x + a)/(
d*x + c)))*log(e) - 2*(B*a*d*n - A*b*c + (B*b*d*n - A*b*d)*x)*log((b*x + a)
/(d*x + c)))/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g*i^2*x + (b^2*c^3 - 2*a*
b*c^2*d + a^2*c*d^2)*g*i^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)/(d*i*x+c*i)**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(164) = 328.

Time = 0.21 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.55

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx = B\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2}+\frac{b\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}-\frac{b\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}\right)\log\left(e\left(\right.\right.$$
$$-\frac{\left((bdx+bc)\log(bx+a)^2+(bdx+bc)\log(dx+c)^2+2\,bc-2\,ad+2\,(bdx+bc)\log(bx+a)-2\,(bdx+\right.}{2\,(b^2c^3gi^2-2\,abc^2dgi^2+a^2cd^2gi^2+(b^2c^2dgi^2-2\,abcd^2gi^2+a^2}$$
$$+A\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2}+\frac{b\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}-\frac{b\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}\right)$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^2,x, alg
orithm="maxima")
```

```
[Out] B*(1/((b*c*d - a*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*g*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1/2*((b*d*x +
b*c)*log(b*x + a)^2 + (b*d*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*a*d + 2*(b*d
*x + b*c)*log(b*x + a) - 2*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a))*log(d
*x + c))*B*n/(b^2*c^3*g*i^2 - 2*a*b*c^2*d*g*i^2 + a^2*c*d^2*g*i^2 + (b^2*c^
2*d*g*i^2 - 2*a*b*c*d^2*g*i^2 + a^2*d^3*g*i^2)*x) + A*(1/((b*c*d - a*d^2)*g
*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/((b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g*i^2))
```

Giac [A] (verification not implemented)

none

Time = 1.05 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.25

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx = \frac{1}{2}\left(\frac{Bbn\log\left(\frac{bx+a}{dx+c}\right)^2}{bcgi^2-adgi^2} - \frac{2\,(bx+a)Bdn\log\left(\frac{bx+a}{dx+c}\right)}{(bcgi^2-adgi^2)(dx+c)} + \frac{2\,(Bb\log(e)+Ab)\log\left(\frac{bx+a}{dx+c}\right)}{bcgi^2-adgi^2} + \frac{2\,(Bdn-Bd\log(e)-\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^2,x, alg
orithm="giac")
```

```
[Out] 1/2*(B*b*n*log((b*x + a)/(d*x + c))^2/(b*c*g*i^2 - a*d*g*i^2) - 2*(b*x + a)
*B*d*n*log((b*x + a)/(d*x + c))/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)) + 2*(B*
b*log(e) + A*b)*log((b*x + a)/(d*x + c))/(b*c*g*i^2 - a*d*g*i^2) + 2*(B*d*n
 - B*d*log(e) - A*d)*(b*x + a)/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)))*(b*c/(b
*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.45

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx = \frac{B\,n}{g\,i^2\,\left(a\,d-b\,c\right)\,\left(c+d\,x\right)} - \frac{A}{g\,i^2\,\left(a\,d-b\,c\right)\,\left(c+d\,x\right)} - \frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{g\,i^2\,\left(a\,d-b\,c\right)\,\left(c+d\,x\right)} + \frac{B\,b\,{\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}^2}{2\,g\,i^2\,n\,{\left(a\,d-b\,c\right)}^2} - \frac{A\,b\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{g\,i^2\,{\left(a\,d-b\,c\right)}^2} + \frac{B\,b\,n\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{g\,i^2\,{\left(a\,d-b\,c\right)}^2}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)*(c*i + d*i*x)^2),
x)
```

```
[Out] (B*n)/(g*i^2*(a*d - b*c)*(c + d*x)) - A/(g*i^2*(a*d - b*c)*(c + d*x)) - (B*
log(e*((a + b*x)/(c + d*x))^n))/(g*i^2*(a*d - b*c)*(c + d*x)) - (A*b*atan((
a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(g*i^2*(a*d - b*c)^2) + (B*b*n
*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(g*i^2*(a*d - b*c)^2) +
 (B*b*log(e*((a + b*x)/(c + d*x))^n)^2)/(2*g*i^2*n*(a*d - b*c)^2)
```

3.148 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 273

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx = &-\frac{Bd^2n(a+bx)}{(bc-ad)^3g^2i^2(c+dx)}-\frac{b^2Bn(c+dx)}{(bc-ad)^3g^2i^2(a+bx)}\\ &+\frac{d^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^2i^2(c+dx)}\\ &-\frac{b^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^2i^2(a+bx)}\\ &-\frac{2bd\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}+\frac{bBdn\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}\end{aligned}$$

```
[Out] -B*d^2*n*(b*x+a)/(-a*d+b*c)^3/g^2/i^2/(d*x+c)-b^2*B*n*(d*x+c)/(-a*d+b*c)^3/
g^2/i^2/(b*x+a)+d^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^
2/i^2/(d*x+c)-b^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^2/
i^2/(b*x+a)-2*b*d*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d
+b*c)^3/g^2/i^2+b*B*d*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g^2/i^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used

$= \{2561, 45, 2372, 2338\}$

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx = -\frac{b^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^2i^2(a+bx)(bc-ad)^3} + \frac{d^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^2i^2(c+dx)(bc-ad)^3} - \frac{2bd\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^2i^2(bc-ad)^3} - \frac{b^2Bn(c+dx)}{g^2i^2(a+bx)(bc-ad)^3} - \frac{Bd^2n(a+bx)}{g^2i^2(c+dx)(bc-ad)^3} + \frac{bBdn\log^2\left(\frac{a+bx}{c+dx}\right)}{g^2i^2(bc-ad)^3}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^2*(c*i + d*i*x)^2
),x]
```

```
[Out] -((B*d^2*n*(a + b*x))/((b*c - a*d)^3*g^2*i^2*(c + d*x))) - (b^2*B*n*(c + d*
x))/((b*c - a*d)^3*g^2*i^2*(a + b*x)) + (d^2*(a + b*x)*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/((b*c - a*d)^3*g^2*i^2*(c + d*x)) - (b^2*(c + d*x)*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^3*g^2*i^2*(a + b*x)) - (2*
b*d*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)])/((b*c
- a*d)^3*g^2*i^2) + (b*B*d*n*Log[(a + b*x)/(c + d*x)]^2)/((b*c - a*d)^3*g^2
*i^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex^n))}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}\\
&= \frac{d^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^2i^2(c+dx)} - \frac{b^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^2i^2(a+bx)}\\
&\quad - \frac{2bd\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}\\
&\quad - \frac{(Bn)\text{Subst}\left(\int\left(d^2-\frac{b^2}{x^2}-\frac{2bd\log(x)}{x}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}\\
&= -\frac{Bd^2n(a+bx)}{(bc-ad)^3g^2i^2(c+dx)} - \frac{b^2Bn(c+dx)}{(bc-ad)^3g^2i^2(a+bx)}\\
&\quad + \frac{d^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^2i^2(c+dx)} - \frac{b^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^2i^2(a+bx)}\\
&\quad - \frac{2bd\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2} + \frac{(2bBdn)\text{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}\\
&= -\frac{Bd^2n(a+bx)}{(bc-ad)^3g^2i^2(c+dx)} - \frac{b^2Bn(c+dx)}{(bc-ad)^3g^2i^2(a+bx)}\\
&\quad + \frac{d^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^2i^2(c+dx)} - \frac{b^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^2i^2(a+bx)}\\
&\quad - \frac{2bd\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2} + \frac{bBdn\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.25 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.25

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx$$
$$= \frac{-\frac{b^2Bcn}{a+bx}+\frac{abBdn}{a+bx}+\frac{bBcdn}{c+dx}-\frac{aBd^2n}{c+dx}-\frac{b(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx}+\frac{d(-bc+ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{c+dx}-2bd\log(a+}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^2*(c*i + d*
i*x)^2),x]
```

```
[Out] (-((b^2*B*c*n)/(a + b*x)) + (a*b*B*d*n)/(a + b*x) + (b*B*c*d*n)/(c + d*x) -
 (a*B*d^2*n)/(c + d*x) - (b*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^
n]))/(a + b*x) + (d*(-(b*c) + a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/
(c + d*x) - 2*b*d*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*b
*d*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + b*B*d*n*(Log[a + b
*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a
+ b*x))/(-(b*c) + a*d)]) - b*B*d*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] -
Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b*
c - a*d)^3*g^2*i^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. 2(273) = 546.

Time = 9.42 (sec) , antiderivative size = 666, normalized size of antiderivative = 2.44

method	result
parallelrisch	$\frac{-2Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^4bc^3d^2n+2Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^3b^2c^4dn+Bx^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2a^3b^2c^3d^2-Bx^2a^4bc^2d^3n^2+2Bx^2a^3b^2}{}$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x,method=_R
ETURNVERBOSE)
```

```
[Out] (-2*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b*c^3*d^2*n+2*B*x*ln(e*((b*x+a)/(d*x+
c))^n)*a^3*b^2*c^4*d*n+B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^2*c^3*d^2-B*
x^2*a^4*b*c^2*d^3*n^2+2*B*x^2*a^3*b^2*c^3*d^2*n^2-B*x^2*a^2*b^3*c^4*d*n^2+2
*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^3*d^2+A*x^2*a^4*b*c^2*d^3*n-A*x^
2*a^2*b^3*c^4*d*n+B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b*c^3*d^2+B*x*ln(e*((
b*x+a)/(d*x+c))^n)^2*a^3*b^2*c^4*d+B*x*a^4*b*c^3*d^2*n^2+B*x*a^3*b^2*c^4*d*
n^2+2*A*x*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b*c^3*d^2+2*A*x*ln(e*((b*x+a)/(d*x+
c))^n)*a^3*b^2*c^4*d-A*x*a^4*b*c^3*d^2*n+A*x*a^3*b^2*c^4*d*n-B*x*a^5*c^2*d^
```

```
3*n^2-B*x*a^2*b^3*c^5*n^2+A*x*a^5*c^2*d^3*n-A*x*a^2*b^3*c^5*n+B*ln(e*((b*x+
a)/(d*x+c))^n)^2*a^4*b*c^4*d-B*ln(e*((b*x+a)/(d*x+c))^n)*a^5*c^3*d^2*n+B*ln
(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^5*n+2*A*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b*c
^4*d)/i^2/g^2/(d*x+c)/(b*x+a)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/c^3/a^3
/n
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.65

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx = -\frac{Ab^2c^2-Aa^2d^2+(Bb^2d^2nx^2+Babcdn+(Bb^2cd+Babd^2)nx)\log\left(\frac{bx+a}{dx+c}\right)^2+(Bb^2c^2-2\,Babcd+Ba^2d^2}{(b^4}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, a
lgorithm="fricas")
```

```
[Out] -(A*b^2*c^2 - A*a^2*d^2 + (B*b^2*d^2*n*x^2 + B*a*b*c*d*n + (B*b^2*c*d + B*a
*b*d^2)*n*x)*log((b*x + a)/(d*x + c))^2 + (B*b^2*c^2 - 2*B*a*b*c*d + B*a^2*
d^2)*n + 2*(A*b^2*c*d - A*a*b*d^2)*x + (B*b^2*c^2 - B*a^2*d^2 + 2*(B*b^2*c*
d - B*a*b*d^2)*x + 2*(B*b^2*d^2*x^2 + B*a*b*c*d + (B*b^2*c*d + B*a*b*d^2)*x
)*log((b*x + a)/(d*x + c)))*log(e) + (2*A*b^2*d^2*x^2 + 2*A*a*b*c*d + (B*b^
2*c^2 - B*a^2*d^2)*n + 2*(A*b^2*c*d + A*a*b*d^2 + (B*b^2*c*d - B*a*b*d^2)*n
)*x)*log((b*x + a)/(d*x + c)))/((b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*
d^3 - a^3*b*d^4)*g^2*i^2*x^2 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a
^4*d^4)*g^2*i^2*x + (a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*
d^3)*g^2*i^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2/(d*i*x+c*i)**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 862 vs. $2(273) = 546$.

Time = 0.22 (sec) , antiderivative size = 862, normalized size of antiderivative = 3.16

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx =$$
$$-B\Bigg(\frac{2\,bdx + bc + ad}{(b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)g^2i^2x^2 + (b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3)g^2i^2x + (ab^2c^3 - 2\,a^2bc^2d + a^3cd$$
$$-\frac{\left(b^2c^2 - 2\,abcd + a^2d^2 - (b^2d^2x^2 + abcd + (b^2cd + abd^2)x)\log(bx+a)^2 + 2\,(b^2d^2x^2 + abcd + (b^2cd + ab\right.}{ab^3c^4g^2i^2 - 3\,a^2b^2c^3dg^2i^2 + 3\,a^3bc^2d^2g^2i^2 - a^4cd^3g^2i^2 + (b^4c^3dg^2i^2 - 3\,ab^3c^2d^2g^2i^2 + 3\,a^2b^2cd^3}$$
$$-A\Bigg(\frac{2\,bdx + bc + ad}{(b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)g^2i^2x^2 + (b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3)g^2i^2x + (ab^2c^3 - 2\,a^2bc^2d + a^3cd$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, a
lgorithm="maxima")
```

```
[Out] -B*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*g^2*i^2*
x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*g^2*i^2*x + (a*b^2*c^
3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2) + 2*b*d*log(b*x + a)/((b^3*c^3 - 3*
a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2) - 2*b*d*log(d*x + c)/((b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2))*log(e*(b*x/(d*x +
c) + a/(d*x + c))^n) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*d^2*x^2 + a*b*
c*d + (b^2*c*d + a*b*d^2)*x)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + a*b*c*d + (b
^2*c*d + a*b*d^2)*x)*log(b*x + a)*log(d*x + c) - (b^2*d^2*x^2 + a*b*c*d + (
b^2*c*d + a*b*d^2)*x)*log(d*x + c)^2)*B*n/(a*b^3*c^4*g^2*i^2 - 3*a^2*b^2*c^
3*d*g^2*i^2 + 3*a^3*b*c^2*d^2*g^2*i^2 - a^4*c*d^3*g^2*i^2 + (b^4*c^3*d*g^2*
i^2 - 3*a*b^3*c^2*d^2*g^2*i^2 + 3*a^2*b^2*c*d^3*g^2*i^2 - a^3*b*d^4*g^2*i^2
)*x^2 + (b^4*c^4*g^2*i^2 - 2*a*b^3*c^3*d*g^2*i^2 + 2*a^3*b*c*d^3*g^2*i^2 -
a^4*d^4*g^2*i^2)*x) - A*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2*a*b^2*c*d^2
+ a^2*b*d^3)*g^2*i^2*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*
g^2*i^2*x + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2) + 2*b*d*log(b*
x + a)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2) - 2*b*
d*log(d*x + c)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2
))
```

Giac [A] (verification not implemented)

none

Time = 106.59 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.35

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx$$
$$=-\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)^2\left(\frac{(dx+c)Bn\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)g^2i^2}+\frac{(Bn+B\log(e)+A)(dx+c)}{(bx+a)g^2i^2}\right)$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, a
lgorithm="giac")
```

```
[Out] -(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)^2*((d*x + c)*B*n*log((b*x + a)/(d*
x + c))/((b*x + a)*g^2*i^2) + (B*n + B*log(e) + A)*(d*x + c)/((b*x + a)*g^2
*i^2))
```

Mupad [B] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.58

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx=\frac{B\,b\,d\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{g^2\,i^2\,n\,(a\,d-b\,c)^3}-\frac{A\,b\,c}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)}$$
$$-\frac{A\,a\,d}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)}$$
$$+\frac{B\,a\,d\,n}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)}$$
$$-\frac{B\,b\,c\,n}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)}$$
$$-\frac{2\,A\,b\,d\,x}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)}$$
$$-\frac{B\,a\,d\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)}$$
$$-\frac{B\,b\,c\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)}$$
$$-\frac{2\,B\,b\,d\,x\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)}$$
$$-\frac{A\,b\,d\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,4\mathrm{i}}{g^2\,i^2\,(a\,d-b\,c)^3}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^2*(c*i + d*i*x)^2
),x)
```

```
[Out] (B*b*d*log(e*((a + b*x)/(c + d*x))^n)^2)/(g^2*i^2*n*(a*d - b*c)^3) - (A*a*d
)/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (A*b*c)/(g^2*i^2*(a*d - b*c
)^2*(a + b*x)*(c + d*x)) - (A*b*d*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d -
b*c))*4i)/(g^2*i^2*(a*d - b*c)^3) + (B*a*d*n)/(g^2*i^2*(a*d - b*c)^2*(a + b
*x)*(c + d*x)) - (B*b*c*n)/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (2
*A*b*d*x)/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (B*a*d*log(e*((a +
b*x)/(c + d*x))^n))/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (B*b*c*lo
g(e*((a + b*x)/(c + d*x))^n))/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) -
 (2*B*b*d*x*log(e*((a + b*x)/(c + d*x))^n))/(g^2*i^2*(a*d - b*c)^2*(a + b*x
)*(c + d*x))
```

3.149 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 380

$$\begin{aligned}
\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx &= \frac{Bd^3n(a+bx)}{(bc-ad)^4g^3i^2(c+dx)}+\frac{3b^2Bdn(c+dx)}{(bc-ad)^4g^3i^2(a+bx)} \\
&\quad-\frac{b^3Bn(c+dx)^2}{4(bc-ad)^4g^3i^2(a+bx)^2} \\
&\quad-\frac{d^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(c+dx)} \\
&\quad+\frac{3b^2d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} \\
&\quad-\frac{b^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} \\
&\quad+\frac{3bd^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2} \\
&\quad-\frac{3bBd^2n\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^3i^2}
\end{aligned}$$

```
[Out] B*d^3*n*(b*x+a)/(-a*d+b*c)^4/g^3/i^2/(d*x+c)+3*b^2*B*d*n*(d*x+c)/(-a*d+b*c)
^4/g^3/i^2/(b*x+a)-1/4*b^3*B*n*(d*x+c)^2/(-a*d+b*c)^4/g^3/i^2/(b*x+a)^2-d^3
*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^3/i^2/(d*x+c)+3*b^2
*d*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^3/i^2/(b*x+a)-1/2
*b^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^3/i^2/(b*x+a)
^2+3*b*d^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d+b*c)^4
/g^3/i^2-3/2*b*B*d^2*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^4/g^3/i^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {2561, 45, 2372, 12, 14, 2338}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}dx = -\frac{b^3(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^3i^2(a+bx)^2(bc-ad)^4} + \frac{3b^2d(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^3i^2(a+bx)(bc-ad)^4} - \frac{d^3(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^3i^2(c+dx)(bc-ad)^4} + \frac{3bd^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^3i^2(bc-ad)^4} - \frac{b^3Bn(c+dx)^2}{4g^3i^2(a+bx)^2(bc-ad)^4} + \frac{3b^2Bdn(c+dx)}{g^3i^2(a+bx)(bc-ad)^4} + \frac{Bd^3n(a+bx)}{g^3i^2(c+dx)(bc-ad)^4} - \frac{3bBd^2n\log^2\left(\frac{a+bx}{c+dx}\right)}{2g^3i^2(bc-ad)^4}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^3*(c*i + d*i*x)^2
),x]
```

```
[Out] (B*d^3*n*(a + b*x))/((b*c - a*d)^4*g^3*i^2*(c + d*x)) + (3*b^2*B*d*n*(c + d
*x))/((b*c - a*d)^4*g^3*i^2*(a + b*x)) - (b^3*B*n*(c + d*x)^2)/(4*(b*c - a*
d)^4*g^3*i^2*(a + b*x)^2) - (d^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]))/((b*c - a*d)^4*g^3*i^2*(c + d*x)) + (3*b^2*d*(c + d*x)*(A + B*Log[e
*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^4*g^3*i^2*(a + b*x)) - (b^3*(c + d
*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*c - a*d)^4*g^3*i^2*(a +
 b*x)^2) + (3*b*d^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c
 + d*x)])/((b*c - a*d)^4*g^3*i^2) - (3*b*B*d^2*n*Log[(a + b*x)/(c + d*x)]^2
)/(2*(b*c - a*d)^4*g^3*i^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)^3(A+B\log(ex^n))}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2} \\
&= -\frac{d^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(c+dx)} + \frac{3b^2d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} \\
&\quad - \frac{b^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} + \frac{3bd^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2} \\
&\quad - \frac{(Bn)\text{Subst}\left(\int \frac{-b^3+6b^2dx-2d^3x^3+6bd^2x^2\log(x)}{2x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2}
\end{aligned}
$$

$$= -\frac{d^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(c+dx)} + \frac{3b^2d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} + \frac{3bd^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2} - \frac{(Bn)\text{Subst}\left(\int\frac{-b^3+6b^2dx-2d^3x^3+6bd^2x^2\log(x)}{x^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^3i^2}$$

$$= -\frac{d^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(c+dx)} + \frac{3b^2d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} + \frac{3bd^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2} - \frac{(Bn)\text{Subst}\left(\int\left(\frac{-b^3+6b^2dx-2d^3x^3}{x^3}+\frac{6bd^2\log(x)}{x}\right)dx,x,\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^3i^2}$$

$$= -\frac{d^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(c+dx)} + \frac{3b^2d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} + \frac{3bd^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2} - \frac{(Bn)\text{Subst}\left(\int\frac{-b^3+6b^2dx-2d^3x^3}{x^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^3i^2} - \frac{(3bBd^2n)\,\text{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2}$$

$$= -\frac{d^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(c+dx)} + \frac{3b^2d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} + \frac{3bd^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2} - \frac{3bBd^2n\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^3i^2} - \frac{(Bn)\text{Subst}\left(\int\left(-2d^3-\frac{b^3}{x^3}+\frac{6b^2d}{x^2}\right)dx,x,\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^3i^2}$$

$$= \frac{Bd^3n(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} + \frac{3b^2Bdn(c+dx)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3Bn(c+dx)^2}{4(bc-ad)^4g^3i^2(a+bx)^2} - \frac{d^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(c+dx)} + \frac{3b^2d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} + \frac{3bd^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2} - \frac{3bBd^2n\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^3i^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.26

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx$$
$$= \frac{-\frac{bB(bc-ad)^2n}{(a+bx)^2}+\frac{8b^2Bcdn}{a+bx}-\frac{8abBd^2n}{a+bx}+\frac{2bBd(bc-ad)n}{a+bx}-\frac{4bBcd^2n}{c+dx}+\frac{4aBd^3n}{c+dx}+6bBd^2n\log(a+bx)-\frac{2b(bc-ad)^2\left(A+B\log\right.}{(a+bx)}}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^3*(c*i + d*
i*x)^2),x]
```

```
[Out] (-((b*B*(b*c - a*d)^2*n)/(a + b*x)^2) + (8*b^2*B*c*d*n)/(a + b*x) - (8*a*b*
B*d^2*n)/(a + b*x) + (2*b*B*d*(b*c - a*d)*n)/(a + b*x) - (4*b*B*c*d^2*n)/(c
 + d*x) + (4*a*B*d^3*n)/(c + d*x) + 6*b*B*d^2*n*Log[a + b*x] - (2*b*(b*c -
a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^2 + (8*b*d*(b*c -
a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + (4*d^2*(b*c - a*d)
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) + 12*b*d^2*Log[a + b*x]*
(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*b*B*d^2*n*Log[c + d*x] - 12*b*d^
2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 6*b*B*d^2*n*(Log[a
+ b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*
(a + b*x))/(-(b*c) + a*d)]) + 6*b*B*d^2*n*((2*Log[(d*(a + b*x))/(-(b*c) + a
*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]
))/(4*(b*c - a*d)^4*g^3*i^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 976 vs. 2(374) = 748.

Time = 28.77 (sec) , antiderivative size = 977, normalized size of antiderivative = 2.57

method	result
parallelrisch	$\frac{24Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a\,b^6c\,d^5n+12A\,x^2b^7c\,d^5n+6Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2a^2b^5d^6-3Bx\,a^2b^5d^6n^2+9Bx\,b^7c^2d^4n^2+12Ax\ln\left(e\left(\frac{bx+a}{dx+c}\right.\right.}{}$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/4*(24*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*c*d^5*n+12*A*x^2*b^7*c*d^5*n+6*
B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^5*d^6-3*B*x*a^2*b^5*d^6*n^2+9*B*x*b^7
*c^2*d^4*n^2+12*A*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^5*d^6-18*A*x*a^2*b^5*d^
6*n+6*A*x*b^7*c^2*d^4*n+6*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^5*c*d^5-4*B*l
n(e*((b*x+a)/(d*x+c))^n)*a^3*b^4*d^6*n-2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c^
```

```
3*d^3*n+12*A*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^5*c*d^5-15*B*a^2*b^5*c*d^5*n^2
+12*B*a*b^6*c^2*d^4*n^2-6*A*a^2*b^5*c*d^5*n+12*A*a*b^6*c^2*d^4*n+4*B*a^3*b^
4*d^6*n^2-B*b^7*c^3*d^3*n^2-4*A*a^3*b^4*d^6*n-2*A*b^7*c^3*d^3*n+6*B*x^3*ln(
e*((b*x+a)/(d*x+c))^n)^2*b^7*d^6+12*A*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^7*d^6
+6*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^7*d^6*n+12*B*x^2*ln(e*((b*x+a)/(d*x+c)
)^n)^2*a*b^6*d^6+6*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^7*c*d^5-6*B*x^2*a*b^
6*d^6*n^2+6*B*x^2*b^7*c*d^5*n^2+24*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*d^
6+12*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c*d^5-12*A*x^2*a*b^6*d^6*n+18*B*x^
2*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c*d^5*n+12*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*
a*b^6*c*d^5-12*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^5*d^6*n+6*B*x*ln(e*((b*x
+a)/(d*x+c))^n)*b^7*c^2*d^4*n-6*B*x*a*b^6*c*d^5*n^2+24*A*x*ln(e*((b*x+a)/(d
*x+c))^n)*a*b^6*c*d^5+12*A*x*a*b^6*c*d^5*n+12*B*ln(e*((b*x+a)/(d*x+c))^n)*a
*b^6*c^2*d^4*n)/i^2/g^3/(d*x+c)/(b*x+a)^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^
2*d-b^3*c^3)/(a*d-b*c)/b^4/d^3/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 946 vs. $2(374) = 748$.

Time = 0.37 (sec) , antiderivative size = 946, normalized size of antiderivative = 2.49

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx = -\frac{2\,Ab^3c^3 - 12\,Aab^2c^2d + 6\,Aa^2bcd^2 + 4\,Aa^3d^3 - 6\,(2\,Ab^3cd^2 - 2\,Aab^2d^3 + (Bb^3cd^2 - Bab^2d^3)n)x^2 - 6\,(}{}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, a
lgorithm="fricas")
```

```
[Out] -1/4*(2*A*b^3*c^3 - 12*A*a*b^2*c^2*d + 6*A*a^2*b*c*d^2 + 4*A*a^3*d^3 - 6*(2
*A*b^3*c*d^2 - 2*A*a*b^2*d^3 + (B*b^3*c*d^2 - B*a*b^2*d^3)*n)*x^2 - 6*(B*b^
3*d^3*n*x^3 + B*a^2*b*c*d^2*n + (B*b^3*c*d^2 + 2*B*a*b^2*d^3)*n*x^2 + (2*B*
a*b^2*c*d^2 + B*a^2*b*d^3)*n*x)*log((b*x + a)/(d*x + c))^2 + (B*b^3*c^3 - 1
2*B*a*b^2*c^2*d + 15*B*a^2*b*c*d^2 - 4*B*a^3*d^3)*n - 3*(2*A*b^3*c^2*d + 4*
A*a*b^2*c*d^2 - 6*A*a^2*b*d^3 + (3*B*b^3*c^2*d - 2*B*a*b^2*c*d^2 - B*a^2*b*
d^3)*n)*x + 2*(B*b^3*c^3 - 6*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2 + 2*B*a^3*d^3
- 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 - 3*(B*b^3*c^2*d + 2*B*a*b^2*c*d^2 - 3*
B*a^2*b*d^3)*x - 6*(B*b^3*d^3*x^3 + B*a^2*b*c*d^2 + (B*b^3*c*d^2 + 2*B*a*b^
2*d^3)*x^2 + (2*B*a*b^2*c*d^2 + B*a^2*b*d^3)*x)*log((b*x + a)/(d*x + c)))*l
og(e) - 2*(6*A*a^2*b*c*d^2 + 3*(B*b^3*d^3*n + 2*A*b^3*d^3)*x^3 + 3*(3*B*b^3
*c*d^2*n + 2*A*b^3*c*d^2 + 4*A*a*b^2*d^3)*x^2 - (B*b^3*c^3 - 6*B*a*b^2*c^2*
d + 2*B*a^3*d^3)*n + 3*(4*A*a*b^2*c*d^2 + 2*A*a^2*b*d^3 + (B*b^3*c^2*d + 4*
B*a*b^2*c*d^2 - 2*B*a^2*b*d^3)*n)*x)*log((b*x + a)/(d*x + c)))/((b^6*c^4*d
- 4*a*b^5*c^3*d^2 + 6*a^2*b^4*c^2*d^3 - 4*a^3*b^3*c*d^4 + a^4*b^2*d^5)*g^3*
i^2*x^3 + (b^6*c^5 - 2*a*b^5*c^4*d - 2*a^2*b^4*c^3*d^2 + 8*a^3*b^3*c^2*d^3
```

```
- 7*a^4*b^2*c*d^4 + 2*a^5*b*d^5)*g^3*i^2*x^2 + (2*a*b^5*c^5 - 7*a^2*b^4*c^4
*d + 8*a^3*b^3*c^3*d^2 - 2*a^4*b^2*c^2*d^3 - 2*a^5*b*c*d^4 + a^6*d^5)*g^3*i
^2*x + (a^2*b^4*c^5 - 4*a^3*b^3*c^4*d + 6*a^4*b^2*c^3*d^2 - 4*a^5*b*c^2*d^3
 + a^6*c*d^4)*g^3*i^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3/(d*i*x+c*i)**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1724 vs. 2(374) = 748.

Time = 0.29 (sec) , antiderivative size = 1724, normalized size of antiderivative = 4.54

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, a
lgorithm="maxima")
[Out] 1/2*B*((6*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a*
b*d^2)*x)/((b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*g^
3*i^2*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 -
2*a^4*b*d^4)*g^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d
^2 + a^4*b*c*d^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*
a^4*b*c^2*d^2 - a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x + a)/((b^4*c^4 - 4*a*
b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2) - 6*b*d^2
*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*g^3*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1/4*(b^3*c^3
 - 12*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3 - 6*(b^3*c*d^2 - a*b^2*d^3)*
x^2 + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b
^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a)^2 + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b
^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(d*x + c)^2
 - 3*(3*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b*d^3)*x - 6*(b^3*d^3*x^3 + a^2*b*c
*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b
*x + a) + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2
*a*b^2*c*d^2 + a^2*b*d^3)*x - 2*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2
*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a))*log(d*x + c)
```

)*B*n/(a^2*b^4*c^5*g^3*i^2 - 4*a^3*b^3*c^4*d*g^3*i^2 + 6*a^4*b^2*c^3*d^2*g^3*i^2 - 4*a^5*b*c^2*d^3*g^3*i^2 + a^6*c*d^4*g^3*i^2 + (b^6*c^4*d*g^3*i^2 - 4*a*b^5*c^3*d^2*g^3*i^2 + 6*a^2*b^4*c^2*d^3*g^3*i^2 - 4*a^3*b^3*c*d^4*g^3*i^2 + a^4*b^2*d^5*g^3*i^2)*x^3 + (b^6*c^5*g^3*i^2 - 2*a*b^5*c^4*d*g^3*i^2 - 2*a^2*b^4*c^3*d^2*g^3*i^2 + 8*a^3*b^3*c^2*d^3*g^3*i^2 - 7*a^4*b^2*c*d^4*g^3*i^2 + 2*a^5*b*d^5*g^3*i^2)*x^2 + (2*a*b^5*c^5*g^3*i^2 - 7*a^2*b^4*c^4*d*g^3*i^2 + 8*a^3*b^3*c^3*d^2*g^3*i^2 - 2*a^4*b^2*c^2*d^3*g^3*i^2 - 2*a^5*b*c*d^4*g^3*i^2 + a^6*d^5*g^3*i^2)*x) + 1/2*A*((6*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a*b*d^2)*x)/((b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*g^3*i^2*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*g^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2) - 6*b*d^2*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2))

Giac [A] (verification not implemented)

none

Time = 137.13 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.63

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx = -\frac{1}{4}\left(\frac{2\left(Bbn-\frac{2\,(bx+a)Bdn}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^2bcg^3i^2}{(dx+c)^2}-\frac{(bx+a)^2adg^3i^2}{(dx+c)^2}}+\frac{Bbn-\frac{4\,(bx+a)Bdn}{dx+c}+2\,Bb\log\left(e\right)-\frac{4\,(bx+a)Bd\log(e)}{dx+c}+2\,Ab-\frac{4\,(bx\cdots}{dx\cdots}}{\frac{(bx+a)^2bcg^3i^2}{(dx+c)^2}-\frac{(bx+a)^2adg^3i^2}{(dx+c)^2}}\right.$$

[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, algorithm="giac")

[Out] -1/4*(2*(B*b*n - 2*(b*x + a)*B*d*n/(d*x + c))*log((b*x + a)/(d*x + c))/((b*x + a)^2*b*c*g^3*i^2/(d*x + c)^2 - (b*x + a)^2*a*d*g^3*i^2/(d*x + c)^2) + (B*b*n - 4*(b*x + a)*B*d*n/(d*x + c) + 2*B*b*log(e) - 4*(b*x + a)*B*d*log(e)/(d*x + c) + 2*A*b - 4*(b*x + a)*A*d/(d*x + c))/((b*x + a)^2*b*c*g^3*i^2/(d*x + c)^2 - (b*x + a)^2*a*d*g^3*i^2/(d*x + c)^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)^2

Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 1016, normalized size of antiderivative = 2.67

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx = \frac{3\,B\,b\,d^2\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{2\,g^3\,i^2\,n\,(a\,d-b\,c)^4}$$

$$-\frac{\frac{4\,A\,a^2\,d^2-2\,A\,b^2\,c^2-4\,B\,a^2\,d^2\,n-B\,b^2\,c^2\,n+10\,A\,a\,b\,c\,d+11\,B\,a\,b\,c\,d\,n}{2\,(a\,d-b\,c)}+\frac{3\,x^2\,(2\,\cdots}{\cdots}}{x\,(2\,a^4\,d^3\,g^3\,i^2-6\,a^2\,b^2\,c^2\,d\,g^3\,i^2+4\,a\,b^3\,c^3\,g^3\,i^2)+x^2\,(4\,a^3\,b\,d^3\,g^3\,i^2-6\,a^2\,b^2\,c\,d^2\,g^3\,i^2+2\,b^4\,c^3\,g^3\,i^2)+x\cdots}$$

$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{\frac{B\,(2\,a\,d+b\,c)}{2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\frac{3\,B\,b\,d\,x}{2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}}{x\,(d\,a^2\,g^3\,i^2+2\,b\,c\,a\,g^3\,i^2)+x^2\,(c\,b^2\,g^3\,i^2+2\,a\,d\,b\,g^3\,i^2)+a^2\,c\,g^3\,i^2+b^2\,d\,g^3\,i^2\,x^3}\right.$$

$$\left.+\frac{3\,B\,b\,d^2\left(b\,g^3\,i^2\,n\,x^2\,(a\,d-b\,c)+\frac{a\,c\,g^3\,i^2\,n\,(a\,d-b\,c)}{d}+\frac{g^3\,i^2\,n\,x\,(a\,d+b\,c)\,(a\,d-b\,c)}{d}\right)}{g^3\,i^2\,n\,(a\,d-b\,c)^4\,\left(x\,(d\,a^2\,g^3\,i^2+2\,b\,c\,a\,g^3\,i^2)+x^2\,(c\,b^2\,g^3\,i^2+2\,a\,d\,b\,g^3\,i^2)+a^2\,c\,g^3\,i^2+b^2\,d\,g^3\,i^2\,x^3\right)}\right)$$

$$-\frac{b\,d^2\,\mathrm{atan}\left(\frac{b\,d^2\,(2\,A+B\,n)\left(\frac{a^4\,d^4\,g^3\,i^2-2\,a^3\,b\,c\,d^3\,g^3\,i^2+2\,a\,b^3\,c^3\,d\,g^3\,i^2-b^4\,c^4\,g^3\,i^2}{a^3\,d^3\,g^3\,i^2-3\,a^2\,b\,c\,d^2\,g^3\,i^2+3\,a\,b^2\,c^2\,d\,g^3\,i^2-b^3\,c^3\,g^3\,i^2}+2\,b\,d\,x\right)\,(a^3\,d^3\,g^3\,i^2-3\,a^2\,b\,c\,d^2\,g^3\,i^2+3\,a\,b^2\,c^2\,d\,g^3\,i^2-b^3\,c^3\cdots}{g^3\,i^2\,(6\,A\,b\,d^2+3\,B\,b\,d^2\,n)\,(a\,d-b\,c)^4}\right.\cdots}{g^3\,i^2\,(a\,d-b\,c)^4}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^3*(c*i + d*i*x)^2
),x)
```

```
[Out] (3*B*b*d^2*log(e*((a + b*x)/(c + d*x))^n)^2)/(2*g^3*i^2*n*(a*d - b*c)^4) -
((4*A*a^2*d^2 - 2*A*b^2*c^2 - 4*B*a^2*d^2*n - B*b^2*c^2*n + 10*A*a*b*c*d +
11*B*a*b*c*d*n)/(2*(a*d - b*c)) + (3*x^2*(2*A*b^2*d^2 + B*b^2*d^2*n))/(a*d
- b*c) + (3*x*(6*A*a*b*d^2 + 2*A*b^2*c*d + B*a*b*d^2*n + 3*B*b^2*c*d*n))/(2
*(a*d - b*c)))/(x*(2*a^4*d^3*g^3*i^2 + 4*a*b^3*c^3*g^3*i^2 - 6*a^2*b^2*c^2*
d*g^3*i^2) + x^2*(2*b^4*c^3*g^3*i^2 + 4*a^3*b*d^3*g^3*i^2 - 6*a^2*b^2*c*d^2
*g^3*i^2) + x^3*(2*a^2*b^2*d^3*g^3*i^2 + 2*b^4*c^2*d*g^3*i^2 - 4*a*b^3*c*d^
2*g^3*i^2) + 2*a^2*b^2*c^3*g^3*i^2 + 2*a^4*c*d^2*g^3*i^2 - 4*a^3*b*c^2*d*g^
3*i^2) - (b*d^2*atan((b*d^2*(2*A + B*n)*((a^4*d^4*g^3*i^2 - b^4*c^4*g^3*i^2
 + 2*a*b^3*c^3*d*g^3*i^2 - 2*a^3*b*c*d^3*g^3*i^2)/(a^3*d^3*g^3*i^2 - b^3*c^
3*g^3*i^2 + 3*a*b^2*c^2*d*g^3*i^2 - 3*a^2*b*c*d^2*g^3*i^2) + 2*b*d*x)*(a^3*
d^3*g^3*i^2 - b^3*c^3*g^3*i^2 + 3*a*b^2*c^2*d*g^3*i^2 - 3*a^2*b*c*d^2*g^3*i
^2)*3i)/(g^3*i^2*(6*A*b*d^2 + 3*B*b*d^2*n)*(a*d - b*c)^4))*(2*A + B*n)*3i)/
(g^3*i^2*(a*d - b*c)^4) - log(e*((a + b*x)/(c + d*x))^n)*(((B*(2*a*d + b*c)
)/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (3*B*b*d*x)/(2*(a^2*d^2 + b^2*c^2 -
 2*a*b*c*d)))/(x*(a^2*d*g^3*i^2 + 2*a*b*c*g^3*i^2) + x^2*(b^2*c*g^3*i^2 + 2
*a*b*d*g^3*i^2) + a^2*c*g^3*i^2 + b^2*d*g^3*i^2*x^3) + (3*B*b*d^2*(b*g^3*i^
2*n*x^2*(a*d - b*c) + (a*c*g^3*i^2*n*(a*d - b*c))/d + (g^3*i^2*n*x*(a*d + b
*c)*(a*d - b*c))/d))/(g^3*i^2*n*(a*d - b*c)^4*(x*(a^2*d*g^3*i^2 + 2*a*b*c*g
^3*i^2) + x^2*(b^2*c*g^3*i^2 + 2*a*b*d*g^3*i^2) + a^2*c*g^3*i^2 + b^2*d*g^3
*i^2*x^3)))
```

3.150 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 477

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = & -\frac{Bd^4n(a+bx)}{(bc-ad)^5g^4i^2(c+dx)}-\frac{6b^2Bd^2n(c+dx)}{(bc-ad)^5g^4i^2(a+bx)} \\ & +\frac{b^3Bdn(c+dx)^2}{(bc-ad)^5g^4i^2(a+bx)^2}-\frac{b^4Bn(c+dx)^3}{9(bc-ad)^5g^4i^2(a+bx)^3} \\ & +\frac{d^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(c+dx)} \\ & -\frac{6b^2d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(a+bx)} \\ & +\frac{2b^3d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(a+bx)^2} \\ & -\frac{b^4(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^5g^4i^2(a+bx)^3} \\ & -\frac{4bd^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2} \\ & +\frac{2bBd^3n\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2}\end{aligned}$$

```
[Out] -B*d^4*n*(b*x+a)/(-a*d+b*c)^5/g^4/i^2/(d*x+c)-6*b^2*B*d^2*n*(d*x+c)/(-a*d+b
*c)^5/g^4/i^2/(b*x+a)+b^3*B*d*n*(d*x+c)^2/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-1/
9*b^4*B*n*(d*x+c)^3/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^3+d^4*(b*x+a)*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^4/i^2/(d*x+c)-6*b^2*d^2*(d*x+c)*(A+B*ln(
e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^4/i^2/(b*x+a)+2*b^3*d*(d*x+c)^2*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-1/3*b^4*(d*x+c)^
```

```
3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^3-4*b*d^3*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d+b*c)^5/g^4/i^2+2*b*
B*d^3*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^5/g^4/i^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {2561, 45, 2372, 2338}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = -\frac{b^4(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3g^4i^2(a+bx)^3(bc-ad)^5} + \frac{2b^3d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^4i^2(a+bx)^2(bc-ad)^5} - \frac{6b^2d^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^4i^2(a+bx)(bc-ad)^5} + \frac{d^4(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^4i^2(c+dx)(bc-ad)^5} - \frac{4bd^3\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^4i^2(bc-ad)^5} - \frac{b^4Bn(c+dx)^3}{9g^4i^2(a+bx)^3(bc-ad)^5} + \frac{b^3Bdn(c+dx)^2}{g^4i^2(a+bx)^2(bc-ad)^5} - \frac{6b^2Bd^2n(c+dx)}{g^4i^2(a+bx)(bc-ad)^5} - \frac{Bd^4n(a+bx)}{g^4i^2(c+dx)(bc-ad)^5} + \frac{2bBd^3n\log^2\left(\frac{a+bx}{c+dx}\right)}{g^4i^2(bc-ad)^5}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^4*(c*i + d*i*x)^2
),x]
```

```
[Out] -((B*d^4*n*(a + b*x))/((b*c - a*d)^5*g^4*i^2*(c + d*x))) - (6*b^2*B*d^2*n*(
c + d*x))/((b*c - a*d)^5*g^4*i^2*(a + b*x)) + (b^3*B*d*n*(c + d*x)^2)/((b*c
 - a*d)^5*g^4*i^2*(a + b*x)^2) - (b^4*B*n*(c + d*x)^3)/(9*(b*c - a*d)^5*g^4
*i^2*(a + b*x)^3) + (d^4*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/
((b*c - a*d)^5*g^4*i^2*(c + d*x)) - (6*b^2*d^2*(c + d*x)*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/((b*c - a*d)^5*g^4*i^2*(a + b*x)) + (2*b^3*d*(c + d*x
)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^5*g^4*i^2*(a + b*x
)^2) - (b^4*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(b*c - a
*d)^5*g^4*i^2*(a + b*x)^3) - (4*b*d^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)*Log[(a + b*x)/(c + d*x)])/((b*c - a*d)^5*g^4*i^2) + (2*b*B*d^3*n*Log[(a +
 b*x)/(c + d*x)]^2)/((b*c - a*d)^5*g^4*i^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)^4(A+B\log(ex^n))}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2}\\
&= \frac{d^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{6b^2d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(a+bx)}\\
&\quad + \frac{2b^3d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{b^4(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^5g^4i^2(a+bx)^3}\\
&\quad - \frac{4bd^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2}\\
&\quad - \frac{(Bn)\text{Subst}\left(\int\left(d^4-\frac{b^4}{3x^4}+\frac{2b^3d}{x^3}-\frac{6b^2d^2}{x^2}-\frac{4bd^3\log(x)}{x}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2}
\end{aligned}
$$

$$= -\frac{Bd^4n(a+bx)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{6b^2Bd^2n(c+dx)}{(bc-ad)^5g^4i^2(a+bx)} + \frac{b^3Bdn(c+dx)^2}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{b^4Bn(c+dx)^3}{9(bc-ad)^5g^4i^2(a+bx)^3} + \frac{d^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{6b^2d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(a+bx)} + \frac{2b^3d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{b^4(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^5g^4i^2(a+bx)^3} - \frac{4bd^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2} + \frac{(4bBd^3n)\operatorname{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2}$$

$$= -\frac{Bd^4n(a+bx)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{6b^2Bd^2n(c+dx)}{(bc-ad)^5g^4i^2(a+bx)} + \frac{b^3Bdn(c+dx)^2}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{b^4Bn(c+dx)^3}{9(bc-ad)^5g^4i^2(a+bx)^3} + \frac{d^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{6b^2d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(a+bx)} + \frac{2b^3d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{b^4(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^5g^4i^2(a+bx)^3} - \frac{4bd^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2} + \frac{2bBd^3n\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.72 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.15

$$\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx =$$

$$-\frac{\frac{bB(bc-ad)^3n}{(a+bx)^3} - \frac{6bBd(bc-ad)^2n}{(a+bx)^2} + \frac{27b^2Bcd^2n}{a+bx} - \frac{27abBd^3n}{a+bx} + \frac{12bBd^2(bc-ad)n}{a+bx} - \frac{9bBcd^3n}{c+dx} + \frac{9aBd^4n}{c+dx} + 30bBd^3n\log(a+b}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^4*(c*i + d*
i*x)^2),x]
```

```
[Out] -1/9*((b*B*(b*c - a*d)^3*n)/(a + b*x)^3 - (6*b*B*d*(b*c - a*d)^2*n)/(a + b*
x)^2 + (27*b^2*B*c*d^2*n)/(a + b*x) - (27*a*b*B*d^3*n)/(a + b*x) + (12*b*B*
d^2*(b*c - a*d)*n)/(a + b*x) - (9*b*B*c*d^3*n)/(c + d*x) + (9*a*B*d^4*n)/(c
 + d*x) + 30*b*B*d^3*n*Log[a + b*x] + (3*b*(b*c - a*d)^3*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/(a + b*x)^3 - (9*b*d*(b*c - a*d)^2*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/(a + b*x)^2 + (27*b*d^2*(b*c - a*d)*(A + B*Log[e*((a
```

```
+ b*x)/(c + d*x))^n]))/(a + b*x) - (9*d^3*(-(b*c) + a*d)*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/(c + d*x) + 36*b*d^3*Log[a + b*x]*(A + B*Log[e*((a +
b*x)/(c + d*x))^n]) - 30*b*B*d^3*n*Log[c + d*x] - 36*b*d^3*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n])*Log[c + d*x] - 18*b*B*d^3*n*(Log[a + b*x]*(Log[a + b
*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c
) + a*d)]) + 18*b*B*d^3*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d
*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)^
5*g^4*i^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1909 vs. $2(473) = 946$.

Time = 44.86 (sec) , antiderivative size = 1910, normalized size of antiderivative = 4.00

method	result	size
parallelrisch	Expression too large to display	1910

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/9*(-90*A*x*a^6*b^3*c^5*d^2*n+45*A*x*a^5*b^4*c^6*d*n+54*B*ln(e*((b*x+a)/(d
*x+c))^n)*a^7*b^2*c^5*d^2*n-18*B*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^3*c^6*d*n+
18*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*a^5*b^4*c^4*d^3-27*B*x^3*a^7*b^2*c^2*d
^5*n^2+126*B*x^3*a^6*b^3*c^3*d^4*n^2-77*B*x^3*a^5*b^4*c^4*d^3*n^2-27*B*x^3*
a^4*b^5*c^5*d^2*n^2+6*B*x^3*a^3*b^6*c^6*d*n^2+108*A*x^3*ln(e*((b*x+a)/(d*x+
c))^n)*a^6*b^3*c^3*d^4+36*A*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^4*c^4*d^3+2
7*A*x^3*a^7*b^2*c^2*d^5*n+63*A*x^3*a^6*b^3*c^3*d^4*n-96*A*x^3*a^5*b^4*c^4*d
^3*n+9*A*x^3*a^3*b^6*c^6*d*n+9*A*x*a^9*c^2*d^5*n-9*A*x*a^4*b^5*c^7*n+18*B*l
n(e*((b*x+a)/(d*x+c))^n)^2*a^8*b*c^4*d^3-9*B*ln(e*((b*x+a)/(d*x+c))^n)*a^9*
c^3*d^4*n+3*B*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^4*c^7*n+36*A*ln(e*((b*x+a)/(d
*x+c))^n)*a^8*b*c^4*d^3-B*x^3*a^2*b^7*c^7*n^2-3*A*x^3*a^2*b^7*c^7*n-3*B*x^2
*a^3*b^6*c^7*n^2-9*A*x^2*a^3*b^6*c^7*n-9*B*x*a^9*c^2*d^5*n^2-3*B*x*a^4*b^5*
c^7*n^2+54*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^7*b^2*c^3*d^4+54*B*x^2*ln(e*
((b*x+a)/(d*x+c))^n)^2*a^6*b^3*c^4*d^3-27*B*x^2*a^8*b*c^2*d^5*n^2+81*B*x^2*
a^7*b^2*c^3*d^4*n^2+27*B*x^2*a^6*b^3*c^4*d^3*n^2-102*B*x^2*a^5*b^4*c^5*d^2*
n^2+24*B*x^2*a^4*b^5*c^6*d*n^2+108*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b^2*
c^3*d^4+108*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^3*c^4*d^3+27*A*x^2*a^8*b*
c^2*d^5*n+27*A*x^2*a^7*b^2*c^3*d^4*n-90*A*x^2*a^5*b^4*c^5*d^2*n+45*A*x^2*a^
4*b^5*c^6*d*n+18*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^8*b*c^3*d^4+54*B*x*ln(e*
((b*x+a)/(d*x+c))^n)^2*a^7*b^2*c^4*d^3+9*B*x*a^8*b*c^3*d^4*n^2+54*B*x*a^7*b
^2*c^4*d^3*n^2-72*B*x*a^6*b^3*c^5*d^2*n^2+21*B*x*a^5*b^4*c^6*d*n^2+18*B*x^4
*ln(e*((b*x+a)/(d*x+c))^n)^2*a^5*b^4*c^3*d^4-9*B*x^4*a^6*b^3*c^2*d^5*n^2+55
*B*x^4*a^5*b^4*c^3*d^4*n^2-54*B*x^4*a^4*b^5*c^4*d^3*n^2+9*B*x^4*a^3*b^6*c^5
*d^2*n^2-B*x^4*a^2*b^7*c^6*d*n^2+36*A*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^4
*c^3*d^4+9*A*x^4*a^6*b^3*c^2*d^5*n+30*A*x^4*a^5*b^4*c^3*d^4*n-54*A*x^4*a^4*
```

```
b^5*c^4*d^3*n+18*A*x^4*a^3*b^6*c^5*d^2*n-3*A*x^4*a^2*b^7*c^6*d*n+54*B*x^3*l
n(e*((b*x+a)/(d*x+c))^n)^2*a^6*b^3*c^3*d^4+36*A*x*ln(e*((b*x+a)/(d*x+c))^n)
*a^8*b*c^3*d^4+108*A*x*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b^2*c^4*d^3-9*A*x*a^8*
b*c^3*d^4*n+54*A*x*a^7*b^2*c^4*d^3*n+54*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^6
*b^3*c^3*d^4*n+66*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^4*c^4*d^3*n+162*B*x
^2*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^3*c^4*d^3*n+18*B*x^2*ln(e*((b*x+a)/(d*x+
c))^n)*a^5*b^4*c^5*d^2*n-36*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^8*b*c^3*d^4*n+1
08*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b^2*c^4*d^3*n+54*B*x*ln(e*((b*x+a)/(d*
x+c))^n)*a^6*b^3*c^5*d^2*n-6*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^4*c^6*d*n+
30*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^4*c^3*d^4*n)/i^2/g^4/(d*x+c)/(b*x+
a)^3/(a*d-b*c)^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/c^3/a^5/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1458 vs. $2(473) = 946$.

Time = 0.43 (sec) , antiderivative size = 1458, normalized size of antiderivative = 3.06

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, a
lgorithm="fricas")
```

```
[Out] -1/9*(3*A*b^4*c^4 - 18*A*a*b^3*c^3*d + 54*A*a^2*b^2*c^2*d^2 - 30*A*a^3*b*c*
d^3 - 9*A*a^4*d^4 + 6*(6*A*b^4*c*d^3 - 6*A*a*b^3*d^4 + 5*(B*b^4*c*d^3 - B*a
*b^3*d^4)*n)*x^3 + 3*(6*A*b^4*c^2*d^2 + 24*A*a*b^3*c*d^3 - 30*A*a^2*b^2*d^4
 + (11*B*b^4*c^2*d^2 + 8*B*a*b^3*c*d^3 - 19*B*a^2*b^2*d^4)*n)*x^2 + 18*(B*b
^4*d^4*n*x^4 + B*a^3*b*c*d^3*n + (B*b^4*c*d^3 + 3*B*a*b^3*d^4)*n*x^3 + 3*(B
*a*b^3*c*d^3 + B*a^2*b^2*d^4)*n*x^2 + (3*B*a^2*b^2*c*d^3 + B*a^3*b*d^4)*n*x
)*log((b*x + a)/(d*x + c))^2 + (B*b^4*c^4 - 9*B*a*b^3*c^3*d + 54*B*a^2*b^2*
c^2*d^2 - 55*B*a^3*b*c*d^3 + 9*B*a^4*d^4)*n - (6*A*b^4*c^3*d - 54*A*a*b^3*c
^2*d^2 - 18*A*a^2*b^2*c*d^3 + 66*A*a^3*b*d^4 + (5*B*b^4*c^3*d - 81*B*a*b^3*
c^2*d^2 + 57*B*a^2*b^2*c*d^3 + 19*B*a^3*b*d^4)*n)*x + 3*(B*b^4*c^4 - 6*B*a*
b^3*c^3*d + 18*B*a^2*b^2*c^2*d^2 - 10*B*a^3*b*c*d^3 - 3*B*a^4*d^4 + 12*(B*b
^4*c*d^3 - B*a*b^3*d^4)*x^3 + 6*(B*b^4*c^2*d^2 + 4*B*a*b^3*c*d^3 - 5*B*a^2*
b^2*d^4)*x^2 - 2*(B*b^4*c^3*d - 9*B*a*b^3*c^2*d^2 - 3*B*a^2*b^2*c*d^3 + 11*
B*a^3*b*d^4)*x + 12*(B*b^4*d^4*x^4 + B*a^3*b*c*d^3 + (B*b^4*c*d^3 + 3*B*a*b
^3*d^4)*x^3 + 3*(B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*x^2 + (3*B*a^2*b^2*c*d^3 +
B*a^3*b*d^4)*x)*log((b*x + a)/(d*x + c)))*log(e) + 3*(12*A*a^3*b*c*d^3 + 2*
(5*B*b^4*d^4*n + 6*A*b^4*d^4)*x^4 + 2*(6*A*b^4*c*d^3 + 18*A*a*b^3*d^4 + (11
*B*b^4*c*d^3 + 9*B*a*b^3*d^4)*n)*x^3 + 6*(6*A*a*b^3*c*d^3 + 6*A*a^2*b^2*d^4
 + (B*b^4*c^2*d^2 + 9*B*a*b^3*c*d^3)*n)*x^2 + (B*b^4*c^4 - 6*B*a*b^3*c^3*d
+ 18*B*a^2*b^2*c^2*d^2 - 3*B*a^4*d^4)*n + 2*(18*A*a^2*b^2*c*d^3 + 6*A*a^3*b
*d^4 - (B*b^4*c^3*d - 9*B*a*b^3*c^2*d^2 - 18*B*a^2*b^2*c*d^3 + 6*B*a^3*b*d^
```

```
4)*n)*x)*log((b*x + a)/(d*x + c)))/((b^8*c^5*d - 5*a*b^7*c^4*d^2 + 10*a^2*b
^6*c^3*d^3 - 10*a^3*b^5*c^2*d^4 + 5*a^4*b^4*c*d^5 - a^5*b^3*d^6)*g^4*i^2*x^
4 + (b^8*c^6 - 2*a*b^7*c^5*d - 5*a^2*b^6*c^4*d^2 + 20*a^3*b^5*c^3*d^3 - 25*
a^4*b^4*c^2*d^4 + 14*a^5*b^3*c*d^5 - 3*a^6*b^2*d^6)*g^4*i^2*x^3 + 3*(a*b^7*
c^6 - 4*a^2*b^6*c^5*d + 5*a^3*b^5*c^4*d^2 - 5*a^5*b^3*c^2*d^4 + 4*a^6*b^2*c
*d^5 - a^7*b*d^6)*g^4*i^2*x^2 + (3*a^2*b^6*c^6 - 14*a^3*b^5*c^5*d + 25*a^4*
b^4*c^4*d^2 - 20*a^5*b^3*c^3*d^3 + 5*a^6*b^2*c^2*d^4 + 2*a^7*b*c*d^5 - a^8*
d^6)*g^4*i^2*x + (a^3*b^5*c^6 - 5*a^4*b^4*c^5*d + 10*a^5*b^3*c^4*d^2 - 10*a
^6*b^2*c^3*d^3 + 5*a^7*b*c^2*d^4 - a^8*c*d^5)*g^4*i^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4/(d*i*x+c*i)**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2563 vs. $2(473) = 946$.

Time = 0.36 (sec) , antiderivative size = 2563, normalized size of antiderivative = 5.37

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, a
lgorithm="maxima")
[Out] -1/3*B*((12*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*d + 13*a^2*b*c*d^2 + 3*a^3*
d^3 + 6*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*c^2*d - 8*a*b^2*c*d^2 - 11*a
^2*b*d^3)*x)/((b^7*c^4*d - 4*a*b^6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*
c*d^4 + a^4*b^3*d^5)*g^4*i^2*x^4 + (b^7*c^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d
^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a^5*b^2*d^5)*g^4*i^2*x^3 + 3
*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^4*b^3*c^2*d^3 - 3*a
^5*b^2*c*d^4 + a^6*b*d^5)*g^4*i^2*x^2 + (3*a^2*b^5*c^5 - 11*a^3*b^4*c^4*d +
 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)*g^4*i^2*x
+ (a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^
7*c*d^4)*g^4*i^2) + 12*b*d^3*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^
2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2) - 12
*b*d^3*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3
*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2))*log(e*(b*x/(d*x + c) + a/
```

```
(d*x + c))^n) - 1/9*(b^4*c^4 - 9*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 - 55*a^3*
b*c*d^3 + 9*a^4*d^4 + 30*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 3*(11*b^4*c^2*d^2 +
8*a*b^3*c*d^3 - 19*a^2*b^2*d^4)*x^2 - 18*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*
c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c
*d^3 + a^3*b*d^4)*x)*log(b*x + a)^2 - 18*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*
c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c
*d^3 + a^3*b*d^4)*x)*log(d*x + c)^2 - (5*b^4*c^3*d - 81*a*b^3*c^2*d^2 + 57*
a^2*b^2*c*d^3 + 19*a^3*b*d^4)*x + 30*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^
3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3
 + a^3*b*d^4)*x)*log(b*x + a) - 6*(5*b^4*d^4*x^4 + 5*a^3*b*c*d^3 + 5*(b^4*c
*d^3 + 3*a*b^3*d^4)*x^3 + 15*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 5*(3*a^2*b^2
*c*d^3 + a^3*b*d^4)*x - 6*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3
*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^
4)*x)*log(b*x + a))*log(d*x + c))*B*n/(a^3*b^5*c^6*g^4*i^2 - 5*a^4*b^4*c^5*
d*g^4*i^2 + 10*a^5*b^3*c^4*d^2*g^4*i^2 - 10*a^6*b^2*c^3*d^3*g^4*i^2 + 5*a^7
*b*c^2*d^4*g^4*i^2 - a^8*c*d^5*g^4*i^2 + (b^8*c^5*d*g^4*i^2 - 5*a*b^7*c^4*d
^2*g^4*i^2 + 10*a^2*b^6*c^3*d^3*g^4*i^2 - 10*a^3*b^5*c^2*d^4*g^4*i^2 + 5*a^
4*b^4*c*d^5*g^4*i^2 - a^5*b^3*d^6*g^4*i^2)*x^4 + (b^8*c^6*g^4*i^2 - 2*a*b^7
*c^5*d*g^4*i^2 - 5*a^2*b^6*c^4*d^2*g^4*i^2 + 20*a^3*b^5*c^3*d^3*g^4*i^2 - 2
5*a^4*b^4*c^2*d^4*g^4*i^2 + 14*a^5*b^3*c*d^5*g^4*i^2 - 3*a^6*b^2*d^6*g^4*i^
2)*x^3 + 3*(a*b^7*c^6*g^4*i^2 - 4*a^2*b^6*c^5*d*g^4*i^2 + 5*a^3*b^5*c^4*d^2
*g^4*i^2 - 5*a^5*b^3*c^2*d^4*g^4*i^2 + 4*a^6*b^2*c*d^5*g^4*i^2 - a^7*b*d^6*
g^4*i^2)*x^2 + (3*a^2*b^6*c^6*g^4*i^2 - 14*a^3*b^5*c^5*d*g^4*i^2 + 25*a^4*b
^4*c^4*d^2*g^4*i^2 - 20*a^5*b^3*c^3*d^3*g^4*i^2 + 5*a^6*b^2*c^2*d^4*g^4*i^2
 + 2*a^7*b*c*d^5*g^4*i^2 - a^8*d^6*g^4*i^2)*x) - 1/3*A*((12*b^3*d^3*x^3 + b
^3*c^3 - 5*a*b^2*c^2*d + 13*a^2*b*c*d^2 + 3*a^3*d^3 + 6*(b^3*c*d^2 + 5*a*b^
2*d^3)*x^2 - 2*(b^3*c^2*d - 8*a*b^2*c*d^2 - 11*a^2*b*d^3)*x)/((b^7*c^4*d -
4*a*b^6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*g^4*i^
2*x^4 + (b^7*c^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 1
1*a^4*b^3*c*d^4 + 3*a^5*b^2*d^5)*g^4*i^2*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4
*d + 2*a^3*b^4*c^3*d^2 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*g
^4*i^2*x^2 + (3*a^2*b^5*c^5 - 11*a^3*b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5
*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)*g^4*i^2*x + (a^3*b^4*c^5 - 4*a^4*b^3*
c^4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4)*g^4*i^2) + 12*b*d^
3*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*
c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2) - 12*b*d^3*log(d*x + c)/((b^5*c
^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^
4 - a^5*d^5)*g^4*i^2))
```

Giac [A] (verification not implemented)

none

Time = 180.85 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.84

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = -\frac{1}{18}\left(\frac{6\left(Bb^2n-\frac{3\,(bx+a)Bbdn}{dx+c}+\frac{3\,(bx+a)^2Bd^2n}{(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^3b^2c^2g^4i^2}{(dx+c)^3}-\frac{2\,(bx+a)^3abcdg^4i^2}{(dx+c)^3}+\frac{(bx+a)^3a^2d^2g^4i^2}{(dx+c)^3}}+\frac{2\,Bb^2n-\frac{9\,(bx+a)Bbdn}{dx+c}+\frac{18\,(bx+a)^2Bd^2n}{(dx+c)^2}+6\,Bb^2}{\frac{(bx+a)}{(dx}}\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, a
lgorithm="giac")
```

```
[Out] -1/18*(6*(B*b^2*n - 3*(b*x + a)*B*b*d*n/(d*x + c) + 3*(b*x + a)^2*B*d^2*n/(
d*x + c)^2)*log((b*x + a)/(d*x + c))/((b*x + a)^3*b^2*c^2*g^4*i^2/(d*x + c)
^3 - 2*(b*x + a)^3*a*b*c*d*g^4*i^2/(d*x + c)^3 + (b*x + a)^3*a^2*d^2*g^4*i^
2/(d*x + c)^3) + (2*B*b^2*n - 9*(b*x + a)*B*b*d*n/(d*x + c) + 18*(b*x + a)^
2*B*d^2*n/(d*x + c)^2 + 6*B*b^2*log(e) - 18*(b*x + a)*B*b*d*log(e)/(d*x + c
) + 18*(b*x + a)^2*B*d^2*log(e)/(d*x + c)^2 + 6*A*b^2 - 18*(b*x + a)*A*b*d/
(d*x + c) + 18*(b*x + a)^2*A*d^2/(d*x + c)^2)/((b*x + a)^3*b^2*c^2*g^4*i^2/
(d*x + c)^3 - 2*(b*x + a)^3*a*b*c*d*g^4*i^2/(d*x + c)^3 + (b*x + a)^3*a^2*d
^2*g^4*i^2/(d*x + c)^3))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)^2
```

Mupad [B] (verification not implemented)

Time = 6.58 (sec) , antiderivative size = 1665, normalized size of antiderivative = 3.49

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^4*(c*i + d*i*x)^2
),x)
```

```
[Out] (2*B*b*d^3*log(e*((a + b*x)/(c + d*x))^n)^2)/(g^4*i^2*n*(a*d - b*c)^3*(a^2*
d^2 + b^2*c^2 - 2*a*b*c*d)) - log(e*((a + b*x)/(c + d*x))^n)*(((B*(3*a*d +
b*c))/(3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (4*B*b*d*x)/(3*(a^2*d^2 + b^2*c
^2 - 2*a*b*c*d)))/(x^3*(b^3*c*g^4*i^2 + 3*a*b^2*d*g^4*i^2) + x^2*(3*a*b^2*c
*g^4*i^2 + 3*a^2*b*d*g^4*i^2) + x*(a^3*d*g^4*i^2 + 3*a^2*b*c*g^4*i^2) + a^3
*c*g^4*i^2 + b^3*d*g^4*i^2*x^4) + (4*B*b*d^3*(x*((a*d + b*c)*((a*g^4*i^2*n*
(a*d - b*c))/(2*d) + (g^4*i^2*n*(a*d - b*c)*(2*a*d - b*c))/(2*d^2)) + (a*b*
c*g^4*i^2*n*(a*d - b*c))/d) + x^2*(b*d*((a*g^4*i^2*n*(a*d - b*c))/(2*d) + (
g^4*i^2*n*(a*d - b*c)*(2*a*d - b*c))/(2*d^2)) + (b*g^4*i^2*n*(a*d + b*c)*(a
*d - b*c))/d) + a*c*((a*g^4*i^2*n*(a*d - b*c))/(2*d) + (g^4*i^2*n*(a*d - b*
```

```
c)*(2*a*d - b*c))/(2*d^2)) + b^2*g^4*i^2*n*x^3*(a*d - b*c)))/(g^4*i^2*n*(a*
d - b*c)^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(x^3*(b^3*c*g^4*i^2 + 3*a*b^2*d*
g^4*i^2) + x^2*(3*a*b^2*c*g^4*i^2 + 3*a^2*b*d*g^4*i^2) + x*(a^3*d*g^4*i^2 +
 3*a^2*b*c*g^4*i^2) + a^3*c*g^4*i^2 + b^3*d*g^4*i^2*x^4))) - (b*d^3*atan((b
*d^3*((a^5*d^5*g^4*i^2 + b^5*c^5*g^4*i^2 - 3*a*b^4*c^4*d*g^4*i^2 - 3*a^4*b*
c*d^4*g^4*i^2 + 2*a^2*b^3*c^3*d^2*g^4*i^2 + 2*a^3*b^2*c^2*d^3*g^4*i^2)/(a^4
*d^4*g^4*i^2 + b^4*c^4*g^4*i^2 - 4*a*b^3*c^3*d*g^4*i^2 - 4*a^3*b*c*d^3*g^4*
i^2 + 6*a^2*b^2*c^2*d^2*g^4*i^2) + 2*b*d*x)*(6*A + 5*B*n)*(a^4*d^4*g^4*i^2
+ b^4*c^4*g^4*i^2 - 4*a*b^3*c^3*d*g^4*i^2 - 4*a^3*b*c*d^3*g^4*i^2 + 6*a^2*b
^2*c^2*d^2*g^4*i^2)*2i)/(g^4*i^2*(12*A*b*d^3 + 10*B*b*d^3*n)*(a*d - b*c)^5)
)*(6*A + 5*B*n)*4i)/(3*g^4*i^2*(a*d - b*c)^5) - ((9*A*a^3*d^3 + 3*A*b^3*c^3
 - 9*B*a^3*d^3*n + B*b^3*c^3*n - 15*A*a*b^2*c^2*d + 39*A*a^2*b*c*d^2 - 8*B*
a*b^2*c^2*d*n + 46*B*a^2*b*c*d^2*n)/(3*(a*d - b*c)) + (2*x^3*(6*A*b^3*d^3 +
 5*B*b^3*d^3*n))/(a*d - b*c) + (x*(66*A*a^2*b*d^3 - 6*A*b^3*c^2*d + 48*A*a*
b^2*c*d^2 + 19*B*a^2*b*d^3*n - 5*B*b^3*c^2*d*n + 76*B*a*b^2*c*d^2*n))/(3*(a
*d - b*c)) + (x^2*(30*A*a*b^2*d^3 + 6*A*b^3*c*d^2 + 19*B*a*b^2*d^3*n + 11*B
*b^3*c*d^2*n))/(a*d - b*c))/(x*(3*a^6*d^4*g^4*i^2 - 9*a^2*b^4*c^4*g^4*i^2 +
 24*a^3*b^3*c^3*d*g^4*i^2 - 18*a^4*b^2*c^2*d^2*g^4*i^2) - x^2*(9*a*b^5*c^4*
g^4*i^2 - 9*a^5*b*d^4*g^4*i^2 - 18*a^2*b^4*c^3*d*g^4*i^2 + 18*a^4*b^2*c*d^3
*g^4*i^2) - x^3*(3*b^6*c^4*g^4*i^2 - 9*a^4*b^2*d^4*g^4*i^2 + 24*a^3*b^3*c*d
^3*g^4*i^2 - 18*a^2*b^4*c^2*d^2*g^4*i^2) + x^4*(3*a^3*b^3*d^4*g^4*i^2 - 3*b
^6*c^3*d*g^4*i^2 + 9*a*b^5*c^2*d^2*g^4*i^2 - 9*a^2*b^4*c*d^3*g^4*i^2) - 3*a
^3*b^3*c^4*g^4*i^2 + 3*a^6*c*d^3*g^4*i^2 + 9*a^4*b^2*c^3*d*g^4*i^2 - 9*a^5*
b*c^2*d^2*g^4*i^2)
```

3.151 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 382

$$\begin{aligned}
&\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx \\
&= -\frac{3B(bc-ad)g^3n(a+bx)^2}{4d^2i^3(c+dx)^2}-\frac{3bB(bc-ad)g^3n(a+bx)}{d^3i^3(c+dx)} \\
&\quad+\frac{b(bc-ad)g^3(3A+Bn)(a+bx)}{d^3i^3(c+dx)}+\frac{3bB(bc-ad)g^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^3i^3(c+dx)} \\
&\quad+\frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{di^3(c+dx)^2} \\
&\quad+\frac{(bc-ad)g^3(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i^3(c+dx)^2} \\
&\quad+\frac{b^2(bc-ad)g^3\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^3} \\
&\quad+\frac{3b^2B(bc-ad)g^3n\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}
\end{aligned}$$

```
[Out] -3/4*B*(-a*d+b*c)*g^3*n*(b*x+a)^2/d^2/i^3/(d*x+c)^2-3*b*B*(-a*d+b*c)*g^3*n*
(b*x+a)/d^3/i^3/(d*x+c)+b*(-a*d+b*c)*g^3*(B*n+3*A)*(b*x+a)/d^3/i^3/(d*x+c)+
3*b*B*(-a*d+b*c)*g^3*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/d^3/i^3/(d*x+c)+g^3*
(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d/i^3/(d*x+c)^2+1/2*(-a*d+b*c)*g^
3*(b*x+a)^2*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))/d^2/i^3/(d*x+c)^2+b^2*(
-a*d+b*c)*g^3*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+
c))/d^4/i^3+3*b^2*B*(-a*d+b*c)*g^3*n*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i^3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {2561, 2384, 45, 2393, 2332, 2341, 2354, 2438}

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$=\frac{b^2g^3(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{d^4i^3}+\frac{bg^3(a+bx)(3A+Bn)(bc-ad)}{d^3i^3(c+dx)}+\frac{g^3(a+bx)^2(bc-ad)\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d^2i^3(c+dx)^2}+\frac{g^3(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{di^3(c+dx)^2}+\frac{3b^2Bg^3n(bc-ad)\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}+\frac{3bBg^3(a+bx)(bc-ad)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^3i^3(c+dx)}-\frac{3bBg^3n(a+bx)(bc-ad)}{d^3i^3(c+dx)}-\frac{3Bg^3n(a+bx)^2(bc-ad)}{4d^2i^3(c+dx)^2}$$

```
[In] Int[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)^
3,x]
```

```
[Out] (-3*B*(b*c - a*d)*g^3*n*(a + b*x)^2)/(4*d^2*i^3*(c + d*x)^2) - (3*b*B*(b*c
- a*d)*g^3*n*(a + b*x))/(d^3*i^3*(c + d*x)) + (b*(b*c - a*d)*g^3*(3*A + B*n
)*(a + b*x))/(d^3*i^3*(c + d*x)) + (3*b*B*(b*c - a*d)*g^3*(a + b*x)*Log[e*(
(a + b*x)/(c + d*x))^n])/(d^3*i^3*(c + d*x)) + (g^3*(a + b*x)^3*(A + B*Log[
e*((a + b*x)/(c + d*x))^n]))/(d*i^3*(c + d*x)^2) + ((b*c - a*d)*g^3*(a + b*
x)^2*(3*A + B*n + 3*B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d^2*i^3*(c + d*x)
^2) + (b^2*(b*c - a*d)*g^3*(3*A + B*n + 3*B*Log[e*((a + b*x)/(c + d*x))^n])
*Log[(b*c - a*d)/(b*(c + d*x))])/(d^4*i^3) + (3*b^2*B*(b*c - a*d)*g^3*n*Pol
yLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^4*i^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)g^3)\,\text{Subst}\left(\int \frac{x^3(A+B\log(ex^n))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{i^3}$$

$$= \frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{di^3(c+dx)^2} - \frac{((bc-ad)g^3)\,\text{Subst}\left(\int \frac{x^2(3A+Bn+3B\log(ex^n))}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{di^3}$$

$$= \frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{di^3(c+dx)^2}$$
$$- \frac{((bc-ad)g^3)\,\text{Subst}\left(\int \left(-\frac{b(3A+Bn+3B\log(ex^n))}{d^2} - \frac{x(3A+Bn+3B\log(ex^n))}{d} - \frac{b^2(3A+Bn+3B\log(ex^n))}{d^2(-b+dx)}\right)\,dx, x}{di^3}$$

$$= \frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{di^3(c+dx)^2}$$
$$+ \frac{(b(bc-ad)g^3)\,\text{Subst}\left(\int (3A+Bn+3B\log(ex^n))\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^3}$$
$$+ \frac{(b^2(bc-ad)g^3)\,\text{Subst}\left(\int \frac{3A+Bn+3B\log(ex^n)}{-b+dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^3}$$
$$+ \frac{((bc-ad)g^3)\,\text{Subst}\left(\int x(3A+Bn+3B\log(ex^n))\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^3}$$

$$= -\frac{3B(bc-ad)g^3n(a+bx)^2}{4d^2i^3(c+dx)^2} + \frac{b(bc-ad)g^3(3A+Bn)(a+bx)}{d^3i^3(c+dx)}$$
$$+ \frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{di^3(c+dx)^2}$$
$$+ \frac{(bc-ad)g^3(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i^3(c+dx)^2}$$
$$+ \frac{b^2(bc-ad)g^3\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^3}$$
$$+ \frac{(3bB(bc-ad)g^3)\,\text{Subst}\left(\int \log(ex^n)\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^3}$$
$$- \frac{(3b^2B(bc-ad)g^3n)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4i^3}$$

$$= -\frac{3B(bc-ad)g^3n(a+bx)^2}{4d^2i^3(c+dx)^2} - \frac{3bB(bc-ad)g^3n(a+bx)}{d^3i^3(c+dx)} + \frac{b(bc-ad)g^3(3A+Bn)(a+bx)}{d^3i^3(c+dx)} + \frac{3bB(bc-ad)g^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^3i^3(c+dx)} + \frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{di^3(c+dx)^2} + \frac{(bc-ad)g^3(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i^3(c+dx)^2} + \frac{b^2(bc-ad)g^3\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^3} + \frac{3b^2B(bc-ad)g^3n\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.87

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \frac{g^3\left(4Ab^3dx - \frac{B(bc-ad)^3n}{(c+dx)^2} + \frac{10bB(bc-ad)^2n}{c+dx} + 10b^2B(bc-ad)n\log(a+bx) + 4b^2Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + \right.}{}$$

```
[In] Integrate[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d
*i*x)^3,x]
```

```
[Out] (g^3*(4*A*b^3*d*x - (B*(b*c - a*d)^3*n)/(c + d*x)^2 + (10*b*B*(b*c - a*d)^2
*n)/(c + d*x) + 10*b^2*B*(b*c - a*d)*n*Log[a + b*x] + 4*b^2*B*d*(a + b*x)*L
og[e*((a + b*x)/(c + d*x))^n] + (2*(b*c - a*d)^3*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]))/(c + d*x)^2 - (12*b*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n]))/(c + d*x) - 14*b^2*B*(b*c - a*d)*n*Log[c + d*x] - 12*b^2*(b*c
- a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 6*b^2*B*(b*c -
 a*d)*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x]
+ 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(4*d^4*i^3)
```

Maple [F]

$$\int \frac{(bgx+ag)^{3} \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}{(dix+ci)^{3}}dx$$

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x)
[Out] int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^{3}\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)}{(dix+ci)^{3}}\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, a
lgorithm="fricas")
[Out] integral((A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3 +
 (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log(e*((
b*x + a)/(d*x + c))^n))/(d^3*i^3*x^3 + 3*c*d^2*i^3*x^2 + 3*c^2*d*i^3*x + c^
3*i^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2894 vs. $2(377) = 754$.

Time = 0.55 (sec) , antiderivative size = 2894, normalized size of antiderivative = 7.58

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, a
lgorithm="maxima")
```

```
[Out] 3/4*B*a^2*b*g^3*n*((b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*x)/((b*c*d^4 - a*
d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^3)
 + 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*
i^3) - 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d
^4)*i^3)) + 1/4*B*a^3*g^3*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3
*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b^2*l
og(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/
((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) - 1/2*A*b^3*g^3*((6*c^2*d*x + 5*
c^3)/(d^6*i^3*x^2 + 2*c*d^5*i^3*x + c^2*d^4*i^3) - 2*x/(d^3*i^3) + 6*c*log(
d*x + c)/(d^4*i^3)) + 3/2*A*a*b^2*g^3*((4*c*d*x + 3*c^2)/(d^5*i^3*x^2 + 2*c
*d^4*i^3*x + c^2*d^3*i^3) + 2*log(d*x + c)/(d^3*i^3)) - 3/2*(2*d*x + c)*B*a
^2*b*g^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^4*i^3*x^2 + 2*c*d^3*i^3*
x + c^2*d^2*i^3) - 3/2*(2*d*x + c)*A*a^2*b*g^3/(d^4*i^3*x^2 + 2*c*d^3*i^3*x
 + c^2*d^2*i^3) - 1/2*B*a^3*g^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^3
*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - 1/2*A*a^3*g^3/(d^3*i^3*x^2 + 2*c*d^
2*i^3*x + c^2*d*i^3) + 1/2*(6*a^3*b^2*d^3*g^3*log(e) - (7*g^3*n + 6*g^3*log
(e))*b^5*c^3 + (19*g^3*n + 18*g^3*log(e))*a*b^4*c^2*d - 2*(7*g^3*n + 9*g^3*
log(e))*a^2*b^3*c*d^2)*B*log(d*x + c)/(b^2*c^2*d^4*i^3 - 2*a*b*c*d^5*i^3 +
a^2*d^6*i^3) + 1/4*(4*(b^5*c^2*d^3*g^3*log(e) - 2*a*b^4*c*d^4*g^3*log(e) +
a^2*b^3*d^5*g^3*log(e))*B*x^3 + 8*(b^5*c^3*d^2*g^3*log(e) - 2*a*b^4*c^2*d^3
*g^3*log(e) + a^2*b^3*c*d^4*g^3*log(e))*B*x^2 + 2*((5*g^3*n - 4*g^3*log(e))
*b^5*c^4*d - 20*(g^3*n - g^3*log(e))*a*b^4*c^3*d^2 + (27*g^3*n - 28*g^3*log
(e))*a^2*b^3*c^2*d^3 - 12*(g^3*n - g^3*log(e))*a^3*b^2*c*d^4)*B*x + 12*((b^
5*c^3*d^2*g^3*n - 3*a*b^4*c^2*d^3*g^3*n + 3*a^2*b^3*c*d^4*g^3*n - a^3*b^2*d
^5*g^3*n)*B*x^2 + 2*(b^5*c^4*d*g^3*n - 3*a*b^4*c^3*d^2*g^3*n + 3*a^2*b^3*c^
2*d^3*g^3*n - a^3*b^2*c*d^4*g^3*n)*B*x + (b^5*c^5*g^3*n - 3*a*b^4*c^4*d*g^3
*n + 3*a^2*b^3*c^3*d^2*g^3*n - a^3*b^2*c^2*d^3*g^3*n)*B)*log(b*x + a)*log(d
*x + c) - 6*((b^5*c^3*d^2*g^3*n - 3*a*b^4*c^2*d^3*g^3*n + 3*a^2*b^3*c*d^4*g
^3*n - a^3*b^2*d^5*g^3*n)*B*x^2 + 2*(b^5*c^4*d*g^3*n - 3*a*b^4*c^3*d^2*g^3*
n + 3*a^2*b^3*c^2*d^3*g^3*n - a^3*b^2*c*d^4*g^3*n)*B*x + (b^5*c^5*g^3*n - 3
*a*b^4*c^4*d*g^3*n + 3*a^2*b^3*c^3*d^2*g^3*n - a^3*b^2*c^2*d^3*g^3*n)*B)*lo
g(d*x + c)^2 + ((9*g^3*n - 10*g^3*log(e))*b^5*c^5 - (35*g^3*n - 38*g^3*log(
e))*a*b^4*c^4*d + (47*g^3*n - 46*g^3*log(e))*a^2*b^3*c^3*d^2 - 3*(7*g^3*n -
 6*g^3*log(e))*a^3*b^2*c^2*d^3)*B + 2*((5*b^5*c^3*d^2*g^3*n - 13*a*b^4*c^2*
d^3*g^3*n + 8*a^2*b^3*c*d^4*g^3*n + 2*a^3*b^2*d^5*g^3*n)*B*x^2 + 2*(5*b^5*c
^4*d*g^3*n - 13*a*b^4*c^3*d^2*g^3*n + 8*a^2*b^3*c^2*d^3*g^3*n + 2*a^3*b^2*c
*d^4*g^3*n)*B*x + (5*b^5*c^5*g^3*n - 13*a*b^4*c^4*d*g^3*n + 8*a^2*b^3*c^3*d
^2*g^3*n + 2*a^3*b^2*c^2*d^3*g^3*n)*B)*log(b*x + a) + 2*(2*(b^5*c^2*d^3*g^3
 - 2*a*b^4*c*d^4*g^3 + a^2*b^3*d^5*g^3)*B*x^3 + 4*(b^5*c^3*d^2*g^3 - 2*a*b^
4*c^2*d^3*g^3 + a^2*b^3*c*d^4*g^3)*B*x^2 - 4*(b^5*c^4*d*g^3 - 5*a*b^4*c^3*d
^2*g^3 + 7*a^2*b^3*c^2*d^3*g^3 - 3*a^3*b^2*c*d^4*g^3)*B*x - (5*b^5*c^5*g^3
- 19*a*b^4*c^4*d*g^3 + 23*a^2*b^3*c^3*d^2*g^3 - 9*a^3*b^2*c^2*d^3*g^3)*B -
6*((b^5*c^3*d^2*g^3 - 3*a*b^4*c^2*d^3*g^3 + 3*a^2*b^3*c*d^4*g^3 - a^3*b^2*d
^5*g^3)*B*x^2 + 2*(b^5*c^4*d*g^3 - 3*a*b^4*c^3*d^2*g^3 + 3*a^2*b^3*c^2*d^3*
g^3 - a^3*b^2*c*d^4*g^3)*B*x + (b^5*c^5*g^3 - 3*a*b^4*c^4*d*g^3 + 3*a^2*b^3
```

```
*c^3*d^2*g^3 - a^3*b^2*c^2*d^3*g^3)*B)*log(d*x + c))*log((b*x + a)^n) - 2*(
2*(b^5*c^2*d^3*g^3 - 2*a*b^4*c*d^4*g^3 + a^2*b^3*d^5*g^3)*B*x^3 + 4*(b^5*c^
3*d^2*g^3 - 2*a*b^4*c^2*d^3*g^3 + a^2*b^3*c*d^4*g^3)*B*x^2 - 4*(b^5*c^4*d*g
^3 - 5*a*b^4*c^3*d^2*g^3 + 7*a^2*b^3*c^2*d^3*g^3 - 3*a^3*b^2*c*d^4*g^3)*B*x
 - (5*b^5*c^5*g^3 - 19*a*b^4*c^4*d*g^3 + 23*a^2*b^3*c^3*d^2*g^3 - 9*a^3*b^2
*c^2*d^3*g^3)*B - 6*((b^5*c^3*d^2*g^3 - 3*a*b^4*c^2*d^3*g^3 + 3*a^2*b^3*c*d
^4*g^3 - a^3*b^2*d^5*g^3)*B*x^2 + 2*(b^5*c^4*d*g^3 - 3*a*b^4*c^3*d^2*g^3 +
3*a^2*b^3*c^2*d^3*g^3 - a^3*b^2*c*d^4*g^3)*B*x + (b^5*c^5*g^3 - 3*a*b^4*c^4
*d*g^3 + 3*a^2*b^3*c^3*d^2*g^3 - a^3*b^2*c^2*d^3*g^3)*B)*log(d*x + c))*log(
(d*x + c)^n))/(b^2*c^4*d^4*i^3 - 2*a*b*c^3*d^5*i^3 + a^2*c^2*d^6*i^3 + (b^2
*c^2*d^6*i^3 - 2*a*b*c*d^7*i^3 + a^2*d^8*i^3)*x^2 + 2*(b^2*c^3*d^5*i^3 - 2*
a*b*c^2*d^6*i^3 + a^2*c*d^7*i^3)*x) - 3*(b^3*c*g^3*n - a*b^2*d*g^3*n)*(log(
b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a
*d)))*B/(d^4*i^3)
```

Giac [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^3}\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, a
lgorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(d*i*x + c
*i)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx=\int \frac{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}{(c\,i+d\,i\,x)^3}\,dx$$

```
[In] int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)^
3,x)
```

```
[Out] int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)^
3, x)
```

3.152 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 263

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \frac{Bg^2n(a+bx)^2}{4di^3(c+dx)^2}-\frac{Abg^2(a+bx)}{d^2i^3(c+dx)}+\frac{bBg^2n(a+bx)}{d^2i^3(c+dx)}-\frac{bBg^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^2i^3(c+dx)}-\frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di^3(c+dx)^2}-\frac{b^2g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^3}-\frac{b^2Bg^2n\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3}$$

```
[Out] 1/4*B*g^2*n*(b*x+a)^2/d/i^3/(d*x+c)^2-A*b*g^2*(b*x+a)/d^2/i^3/(d*x+c)+b*B*g
^2*n*(b*x+a)/d^2/i^3/(d*x+c)-b*B*g^2*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/d^2/
i^3/(d*x+c)-1/2*g^2*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d/i^3/(d*x+c)
^2-b^2*g^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d^3/i^3
-b^2*B*g^2*n*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {2561, 45, 2393, 2332, 2341, 2354, 2438}

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = -\frac{b^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^3i^3} - \frac{g^2(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2di^3(c+dx)^2} - \frac{Abg^2(a+bx)}{d^2i^3(c+dx)} - \frac{b^2Bg^2n\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3} - \frac{bBg^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^2i^3(c+dx)} + \frac{bBg^2n(a+bx)}{d^2i^3(c+dx)} + \frac{Bg^2n(a+bx)^2}{4di^3(c+dx)^2}$$

```
[In] Int[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)^
3,x]
```

```
[Out] (B*g^2*n*(a + b*x)^2)/(4*d*i^3*(c + d*x)^2) - (A*b*g^2*(a + b*x))/(d^2*i^3*
(c + d*x)) + (b*B*g^2*n*(a + b*x))/(d^2*i^3*(c + d*x)) - (b*B*g^2*(a + b*x)
*Log[e*((a + b*x)/(c + d*x))^n])/(d^2*i^3*(c + d*x)) - (g^2*(a + b*x)^2*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d*i^3*(c + d*x)^2) - (b^2*g^2*(A +
B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(d^3*i^3)
 - (b^2*B*g^2*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^3*i^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g^2\text{Subst}\left(\int \frac{x^2(A+B\log(ex^n))}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{i^3}\\
&= \frac{g^2\text{Subst}\left(\int \left(-\frac{b(A+B\log(ex^n))}{d^2}-\frac{x(A+B\log(ex^n))}{d}-\frac{b^2(A+B\log(ex^n))}{d^2(-b+dx)}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{i^3}\\
&= -\frac{(bg^2)\,\text{Subst}\left(\int (A+B\log(ex^n))\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^3}\\
&\quad-\frac{(b^2g^2)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{-b+dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^3}\\
&\quad-\frac{g^2\text{Subst}\left(\int x(A+B\log(ex^n))\,dx, x, \frac{a+bx}{c+dx}\right)}{di^3}
\end{aligned}$$

$$= \frac{Bg^2n(a+bx)^2}{4di^3(c+dx)^2} - \frac{Abg^2(a+bx)}{d^2i^3(c+dx)} - \frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di^3(c+dx)^2} - \frac{b^2g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^3} - \frac{(bBg^2)\,\mathrm{Subst}\left(\int\log(ex^n)\,dx,x,\frac{a+bx}{c+dx}\right)}{d^2i^3} + \frac{(b^2Bg^2n)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i^3}$$

$$= \frac{Bg^2n(a+bx)^2}{4di^3(c+dx)^2} - \frac{Abg^2(a+bx)}{d^2i^3(c+dx)} + \frac{bBg^2n(a+bx)}{d^2i^3(c+dx)} - \frac{bBg^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^2i^3(c+dx)} - \frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di^3(c+dx)^2} - \frac{b^2g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^3} - \frac{b^2Bg^2n\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.98

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$

$$= \frac{g^2\left(\frac{B(bc-ad)^2n}{(c+dx)^2} - \frac{6bB(bc-ad)n}{c+dx} - 6b^2Bn\log(a+bx) - \frac{2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2} + \frac{8b(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{c+dx}\right.}{}$$

```
[In] Integrate[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d
*i*x)^3,x]
```

```
[Out] (g^2*((B*(b*c - a*d)^2*n)/(c + d*x)^2 - (6*b*B*(b*c - a*d)*n)/(c + d*x) - 6
*b^2*B*n*Log[a + b*x] - (2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]))/(c + d*x)^2 + (8*b*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
)/(c + d*x) + 6*b^2*B*n*Log[c + d*x] + 4*b^2*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])*Log[c + d*x] - 2*b^2*B*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - L
og[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(4*d
^3*i^3)
```

Maple [F]

$$\int \frac{(bgx+ag)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(dix+ci)^3}dx$$

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x)
[Out] int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^3}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, a
lgorithm="fricas")
[Out] integral((A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*B*
a*b*g^2*x + B*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n))/(d^3*i^3*x^3 + 3*c*d
^2*i^3*x^2 + 3*c^2*d*i^3*x + c^3*i^3), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$= \frac{g^2\left(\int \frac{Aa^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{Ab^2x^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{Ba^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{2Aabx}{c^3+3c^2dx+3cd^2x^2+d}\right.}{i^3}$$

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**3,x)
[Out] g**2*(Integral(A*a**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) +
 Integral(A*b**2*x**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) +
 Integral(B*a**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**3 + 3*c**2*d*x
 + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*a*b*x/(c**3 + 3*c**2*d*x +
 3*c*d**2*x**2 + d**3*x**3), x) + Integral(B*b**2*x**2*log(e*(a/(c + d*x) +
 b*x/(c + d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + I
ntegral(2*B*a*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**3 + 3*c**2*d*
x + 3*c*d**2*x**2 + d**3*x**3), x))/i**3
```

Maxima [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^3}\,dx$$

[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, algorithm="maxima")

[Out] 1/2*B*a*b*g^2*n*((b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*x)/((b*c*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^3) + 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*i^3) - 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*i^3)) + 1/4*B*a^2*g^2*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) + 1/2*A*b^2*g^2*((4*c*d*x + 3*c^2)/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) + 2*log(d*x + c)/(d^3*i^3)) - 1/2*B*b^2*g^2*((2*(d^2*n*x^2 + 2*c*d*n*x + c^2*n)*log(b*x + a)*log(d*x + c) - (d^2*n*x^2 + 2*c*d*n*x + c^2*n)*log(d*x + c)^2 - (4*c*d*x + 3*c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*log(d*x + c))*log((b*x + a)^n) + (4*c*d*x + 3*c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*log(d*x + c))*log((d*x + c)^n))/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) - 2*integrate(1/2*(2*b*d^3*x^3*log(e) + 2*a*d^3*x^2*log(e) - 3*b*c^3*n + 3*a*c^2*d*n - 4*(b*c^2*d*n - a*c*d^2*n)*x + 2*(b*d^3*n*x^3 + a*c^2*d*n + (2*b*c*d^2*n + a*d^3*n)*x^2 + (b*c^2*d*n + 2*a*c*d^2*n)*x)*log(b*x + a))/(b*d^6*i^3*x^4 + a*c^3*d^3*i^3 + (3*b*c*d^5*i^3 + a*d^6*i^3)*x^3 + 3*(b*c^2*d^4*i^3 + a*c*d^5*i^3)*x^2 + (b*c^3*d^3*i^3 + 3*a*c^2*d^4*i^3)*x), x)) - (2*d*x + c)*B*a*b*g^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - (2*d*x + c)*A*a*b*g^2/(d^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - 1/2*B*a^2*g^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - 1/2*A*a^2*g^2/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3)

Giac [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^3}\,dx$$

[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, algorithm="giac")

[Out] integrate((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(d*i*x + c*i)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx=\int \frac{{\left(a\,g+b\,g\,x\right)}^2\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}{{\left(c\,i+d\,i\,x\right)}^3}\,dx$$

```
[In] int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)^
3,x)
```

```
[Out] int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)^
3, x)
```

3.153 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 89

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = -\frac{Bgn(a+bx)^2}{4(bc-ad)i^3(c+dx)^2} + \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)i^3(c+dx)^2}$$

```
[Out] -1/4*B*g*n*(b*x+a)^2/(-a*d+b*c)/i^3/(d*x+c)^2+1/2*g*(b*x+a)^2*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))/(-a*d+b*c)/i^3/(d*x+c)^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$, Rules used = {2561, 2341}

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \frac{g(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2i^3(c+dx)^2(bc-ad)} - \frac{Bgn(a+bx)^2}{4i^3(c+dx)^2(bc-ad)}$$

```
[In] Int[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)^3,
x]
```

```
[Out] -1/4*(B*g*n*(a + b*x)^2)/((b*c - a*d)*i^3*(c + d*x)^2) + (g*(a + b*x)^2*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*c - a*d)*i^3*(c + d*x)^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{g\text{Subst}\left(\int x(A+B\log\left(ex^n\right))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^3}\\ &= -\frac{Bgn(a+bx)^2}{4(bc-ad)i^3(c+dx)^2}+\frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)i^3(c+dx)^2}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 215 vs. 2(89) = 178.

Time = 0.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.42

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$= \frac{g\left(\frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2(c+dx)^2}-\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d^2(c+dx)}+\frac{bBn\left(\frac{1}{c+dx}+\frac{b\log(a+bx)}{bc-ad}-\frac{b\log(c+dx)}{bc-ad}\right)}{d^2}-\frac{Bn\left(\frac{bc-ad}{(c+dx)^2}+\frac{2b}{c+dx}+\frac{2b^2\log\cdots}{bc-\cdots}\right.}{4d^2}\cdots}{i^3}$$

```
[In] Integrate[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i
*x)^3,x]
```

```
[Out] (g*(((b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d^2*(c + d*x)^2
) - (b*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(d^2*(c + d*x)) + (b*B*n*((c
 + d*x)^(-1) + (b*Log[a + b*x])/(b*c - a*d) - (b*Log[c + d*x])/(b*c - a*d))
)/d^2 - (B*n*((b*c - a*d)/(c + d*x)^2 + (2*b)/(c + d*x) + (2*b^2*Log[a + b*
x])/(b*c - a*d) - (2*b^2*Log[c + d*x])/(b*c - a*d)))/(4*d^2)))/i^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(85) = 170.

Time = 4.64 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.66

method	result
parallelrisch	$-\frac{2B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a^{2}b\,d^{4}gn+2B\,x^{2}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{3}d^{4}gn-2Bxa\,b^{2}d^{4}g\,n^{2}+2Bx\,b^{3}c\,d^{3}g\,n^{2}+4Axa\,b^{2}d^{4}gn-4Ax\,b^{3}c\,d^{3}gn+4}{4i^{3}(dx+c)^{2}b\,d^{4}n(ad-cb)}$

[In] int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x,method=_RETURNVERBOSE)

```
[Out] -1/4*(2*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b*d^4*g*n+2*B*x^2*ln(e*((b*x+a)/(d*
x+c))^n)*b^3*d^4*g*n-2*B*x*a*b^2*d^4*g*n^2+2*B*x*b^3*c*d^3*g*n^2+4*A*x*a*b^
2*d^4*g*n-4*A*x*b^3*c*d^3*g*n+4*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*d^4*g*n
-B*a^2*b*d^4*g*n^2+B*b^3*c^2*d^2*g*n^2+2*A*a^2*b*d^4*g*n-2*A*b^3*c^2*d^2*g*
n)/i^3/(d*x+c)^2/b/d^4/n/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(85) = 170.

Time = 0.32 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.81

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)}{(ci+dix)^{3}}\,dx$$
$$=\frac{(Bb^{2}c^{2}-Ba^{2}d^{2})gn-2\,(Ab^{2}c^{2}-Aa^{2}d^{2})g+2\,((Bb^{2}cd-Babd^{2})gn-2\,(Ab^{2}cd-Aabd^{2})g)x-2\,(2\,(Bb^{2}cd}{4\,((bcd^{4}-ad^{5})i^{3}x^{2}+2\,(bc^{2}d^{3}-acd}$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, alg
orithm="fricas")
```

```
[Out] 1/4*((B*b^2*c^2 - B*a^2*d^2)*g*n - 2*(A*b^2*c^2 - A*a^2*d^2)*g + 2*((B*b^2*
c*d - B*a*b*d^2)*g*n - 2*(A*b^2*c*d - A*a*b*d^2)*g)*x - 2*(2*(B*b^2*c*d - B
*a*b*d^2)*g*x + (B*b^2*c^2 - B*a^2*d^2)*g)*log(e) + 2*(B*b^2*d^2*g*n*x^2 +
2*B*a*b*d^2*g*n*x + B*a^2*d^2*g*n)*log((b*x + a)/(d*x + c)))/((b*c*d^4 - a*
d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1328 vs. $2(75) = 150$.

Time = 111.17 (sec) , antiderivative size = 1328, normalized size of antiderivative = 14.92

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$= \begin{cases} -\frac{Abg}{cd^2i^3+d^3i^3x} - \frac{Bbg\log\left(e\left(\frac{bc}{cd+d^2x}+\frac{bx}{c+dx}\right)^n\right)}{cd^2i^3+d^3i^3x} \\ \frac{Aagx+\frac{Abgx^2}{2}+\frac{Ba^2g\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{2b}-\frac{Bagnx}{2}+Bagx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)-\frac{Bbgnx^2}{4}+\frac{Bbgx^2\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{2}}{c^3i^3} \\ -\frac{2Aa^2d^2g}{4ac^2d^3i^3+8acd^4i^3x+4ad^5i^3x^2-4bc^3d^2i^3-8bc^2d^3i^3x-4bcd^4i^3x^2} - \frac{4Aabd^2gx}{4ac^2d^3i^3+8acd^4i^3x+4ad^5i^3x^2-4bc^3d^2i^3-8bc^2d^3i^3x-4bcd^4i^3x^2} + \end{cases}$$

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**3,x)
```

```
[Out] Piecewise((-A*b*g/(c*d**2*i**3 + d**3*i**3*x) - B*b*g*log(e*(b*c/(c*d + d**
2*x) + b*x/(c + d*x))**n)/(c*d**2*i**3 + d**3*i**3*x), Eq(a, b*c/d)), ((A*a
*g*x + A*b*g*x**2/2 + B*a**2*g*log(e*(a/c + b*x/c)**n)/(2*b) - B*a*g*n*x/2
+ B*a*g*x*log(e*(a/c + b*x/c)**n) - B*b*g*n*x**2/4 + B*b*g*x**2*log(e*(a/c
+ b*x/c)**n)/2)/(c**3*i**3), Eq(d, 0)), (-2*A*a**2*d**2*g/(4*a*c**2*d**3*i*
*3 + 8*a*c*d**4*i**3*x + 4*a*d**5*i**3*x**2 - 4*b*c**3*d**2*i**3 - 8*b*c**2
*d**3*i**3*x - 4*b*c*d**4*i**3*x**2) - 4*A*a*b*d**2*g*x/(4*a*c**2*d**3*i**3
 + 8*a*c*d**4*i**3*x + 4*a*d**5*i**3*x**2 - 4*b*c**3*d**2*i**3 - 8*b*c**2*d
**3*i**3*x - 4*b*c*d**4*i**3*x**2) + 2*A*b**2*c**2*g/(4*a*c**2*d**3*i**3 +
8*a*c*d**4*i**3*x + 4*a*d**5*i**3*x**2 - 4*b*c**3*d**2*i**3 - 8*b*c**2*d**3
*i**3*x - 4*b*c*d**4*i**3*x**2) + 4*A*b**2*c*d*g*x/(4*a*c**2*d**3*i**3 + 8*
a*c*d**4*i**3*x + 4*a*d**5*i**3*x**2 - 4*b*c**3*d**2*i**3 - 8*b*c**2*d**3*i
**3*x - 4*b*c*d**4*i**3*x**2) + B*a**2*d**2*g*n/(4*a*c**2*d**3*i**3 + 8*a*c
*d**4*i**3*x + 4*a*d**5*i**3*x**2 - 4*b*c**3*d**2*i**3 - 8*b*c**2*d**3*i**3
*x - 4*b*c*d**4*i**3*x**2) - 2*B*a**2*d**2*g*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)/(4*a*c**2*d**3*i**3 + 8*a*c*d**4*i**3*x + 4*a*d**5*i**3*x**2 - 4*
b*c**3*d**2*i**3 - 8*b*c**2*d**3*i**3*x - 4*b*c*d**4*i**3*x**2) + 2*B*a*b*d
**2*g*n*x/(4*a*c**2*d**3*i**3 + 8*a*c*d**4*i**3*x + 4*a*d**5*i**3*x**2 - 4*
b*c**3*d**2*i**3 - 8*b*c**2*d**3*i**3*x - 4*b*c*d**4*i**3*x**2) - 4*B*a*b*d
**2*g*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(4*a*c**2*d**3*i**3 + 8*a*c
*d**4*i**3*x + 4*a*d**5*i**3*x**2 - 4*b*c**3*d**2*i**3 - 8*b*c**2*d**3*i**3
*x - 4*b*c*d**4*i**3*x**2) - B*b**2*c**2*g*n/(4*a*c**2*d**3*i**3 + 8*a*c*d*
*4*i**3*x + 4*a*d**5*i**3*x**2 - 4*b*c**3*d**2*i**3 - 8*b*c**2*d**3*i**3*x
- 4*b*c*d**4*i**3*x**2) - 2*B*b**2*c*d*g*n*x/(4*a*c**2*d**3*i**3 + 8*a*c*d*
*4*i**3*x + 4*a*d**5*i**3*x**2 - 4*b*c**3*d**2*i**3 - 8*b*c**2*d**3*i**3*x
- 4*b*c*d**4*i**3*x**2) - 2*B*b**2*d**2*g*x**2*log(e*(a/(c + d*x) + b*x/(c
+ d*x))**n)/(4*a*c**2*d**3*i**3 + 8*a*c*d**4*i**3*x + 4*a*d**5*i**3*x**2 -
4*b*c**3*d**2*i**3 - 8*b*c**2*d**3*i**3*x - 4*b*c*d**4*i**3*x**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(85) = 170$.

Time = 0.20 (sec) , antiderivative size = 578, normalized size of antiderivative = 6.49

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\,Bbgn\left(\frac{bc^2-3\,acd+2\,(bcd-2\,ad^2)x}{(bcd^4-ad^5)i^3x^2+2\,(bc^2d^3-acd^4)i^3x+(bc^3d^2-ac^2d^3)i^3}+\frac{2\,(b^2c-2\,abd)\log(bx+a)}{(b^2c^2d^2-2\,abcd^3+a^2d^4)i^3}-\frac{2\,(b^2c}{(b^2c^2}\right.$$
$$+\frac{1}{4}\,Bagn\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)i^3x^2+2\,(bc^2d^2-acd^3)i^3x+(bc^3d-ac^2d^2)i^3}+\frac{2\,b^2\log(bx+a)}{(b^2c^2d-2\,abcd^2+a^2d^3)i^3}-\frac{}{(b^2c^2}\right.$$
$$-\frac{(2\,dx+c)Bbg\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{2\,(d^4i^3x^2+2\,cd^3i^3x+c^2d^2i^3)}-\frac{(2\,dx+c)Abg}{2\,(d^4i^3x^2+2\,cd^3i^3x+c^2d^2i^3)}$$
$$-\frac{Bag\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{2\,(d^3i^3x^2+2\,cd^2i^3x+c^2di^3)}-\frac{Aag}{2\,(d^3i^3x^2+2\,cd^2i^3x+c^2di^3)}$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, alg
orithm="maxima")
```

```
[Out] 1/4*B*b*g*n*((b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*x)/((b*c*d^4 - a*d^5)*i
^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^3) + 2*(
b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*i^3) -
 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*i^
3)) + 1/4*B*a*g*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(
b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b^2*log(b*x + a
)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) - 1/2*(2*d*x + c)*B*b*g*log(e*(b*x/(d*x +
c) + a/(d*x + c))^n)/(d^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - 1/2*(2*d
*x + c)*A*b*g/(d^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - 1/2*B*a*g*log(e
*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3)
 - 1/2*A*a*g/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3)
```

Giac [A] (verification not implemented)

none

Time = 0.80 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.13

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(\frac{2\,(bx+a)^2Bgn\log\left(\frac{bx+a}{dx+c}\right)}{(dx+c)^2i^3}-\frac{(Bgn-2\,Bg\log(e)-2\,Ag)(bx+a)^2}{(dx+c)^2i^3}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, alg
orithm="giac")
```

```
[Out] 1/4*(2*(b*x + a)^2*B*g*n*log((b*x + a)/(d*x + c))/((d*x + c)^2*i^3) - (B*g*
n - 2*B*g*log(e) - 2*A*g)*(b*x + a)^2/((d*x + c)^2*i^3))*(b*c/(b*c - a*d)^2
 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.30

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$= -\frac{x\,(2\,A\,b\,d\,g-B\,b\,d\,g\,n)+A\,a\,d\,g+A\,b\,c\,g-\frac{B\,a\,d\,g\,n}{2}-\frac{B\,b\,c\,g\,n}{2}}{2\,c^2\,d^2\,i^3+4\,c\,d^3\,i^3\,x+2\,d^4\,i^3\,x^2}$$
$$-\frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(\frac{B\,a\,g}{2\,d}+\frac{B\,b\,c\,g}{2\,d^2}+\frac{B\,b\,g\,x}{d}\right)}{c^2\,i^3+2\,c\,d\,i^3\,x+d^2\,i^3\,x^2}+\frac{B\,b^2\,g\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,1\mathrm{i}}{d^2\,i^3\,(a\,d-b\,c)}$$

```
[In] int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)^3,
x)
```

```
[Out] (B*b^2*g*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*1i)/(d^2*i^3*(a*d - b
*c)) - (log(e*((a + b*x)/(c + d*x))^n)*((B*a*g)/(2*d) + (B*b*c*g)/(2*d^2) +
 (B*b*g*x)/d))/(c^2*i^3 + d^2*i^3*x^2 + 2*c*d*i^3*x) - (x*(2*A*b*d*g - B*b*
d*g*n) + A*a*d*g + A*b*c*g - (B*a*d*g*n)/2 - (B*b*c*g*n)/2)/(2*c^2*d^2*i^3
+ 2*d^4*i^3*x^2 + 4*c*d^3*i^3*x)
```

3.154 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 151

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx = \frac{Bn}{4di^3(c+dx)^2}+\frac{bBn}{2d(bc-ad)i^3(c+dx)}+\frac{b^2Bn\log(a+bx)}{2d(bc-ad)^2i^3}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2di^3(c+dx)^2}-\frac{b^2Bn\log(c+dx)}{2d(bc-ad)^2i^3}$$

```
[Out] 1/4*B*n/d/i^3/(d*x+c)^2+1/2*b*B*n/d/(-a*d+b*c)/i^3/(d*x+c)+1/2*b^2*B*n*ln(b
*x+a)/d/(-a*d+b*c)^2/i^3+1/2*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/d/i^3/(d*x+c)
^2-1/2*b^2*B*n*ln(d*x+c)/d/(-a*d+b*c)^2/i^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2547, 21, 46}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx = -\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2di^3(c+dx)^2}+\frac{b^2Bn\log(a+bx)}{2di^3(bc-ad)^2}-\frac{b^2Bn\log(c+dx)}{2di^3(bc-ad)^2}+\frac{bBn}{2di^3(c+dx)(bc-ad)}+\frac{Bn}{4di^3(c+dx)^2}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*i + d*i*x)^3,x]
```

```
[Out] (B*n)/(4*d*i^3*(c + d*x)^2) + (b*B*n)/(2*d*(b*c - a*d)*i^3*(c + d*x)) + (b^
2*B*n*Log[a + b*x])/(2*d*(b*c - a*d)^2*i^3) - (A + B*Log[e*((a + b*x)/(c +
d*x))^n])/(2*d*i^3*(c + d*x)^2) - (b^2*B*n*Log[c + d*x])/(2*d*(b*c - a*d)^2
*i^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2547

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Dist[B*n*((b*c - a*d)
/(g*(m + 1))), Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ
[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, -2]
```

Rubi steps

$$\text{integral} = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2di^3(c+dx)^2}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)(c+dx)(ci+dix)^2}\,dx}{2di}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2di^3(c+dx)^2}+\frac{(B(bc-ad)n)\int\frac{1}{(a+bx)(c+dx)^3}\,dx}{2di^3}$$

$$= -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2di^3(c+dx)^2} + \frac{(B(bc-ad)n)\int\left(\frac{b^3}{(bc-ad)^3(a+bx)}-\frac{d}{(bc-ad)(c+dx)^3}-\frac{bd}{(bc-ad)^2(c+dx)^2}-\frac{b^2d}{(bc-ad)^3(c+dx)}\right)dx}{2di^3}$$

$$= \frac{Bn}{4di^3(c+dx)^2}+\frac{bBn}{2d(bc-ad)i^3(c+dx)}+\frac{b^2Bn\log(a+bx)}{2d(bc-ad)^2i^3} - \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2di^3(c+dx)^2}-\frac{b^2Bn\log(c+dx)}{2d(bc-ad)^2i^3}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.76

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx = \frac{-2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{Bn\left((bc-ad)(3bc-ad+2bdx)+2b^2(c+dx)^2\log(a+bx)-2b^2(c+dx)^2\log(c+dx)\right)}{(bc-ad)^2}}{4di^3(c+dx)^2}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*i + d*i*x)^3,x]
```

```
[Out] (-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(3*b*c - a*d
 + 2*b*d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c + d*
x]))/(b*c - a*d)^2)/(4*d*i^3*(c + d*x)^2)
```

Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.80

method	result
parallelrisch	$-\frac{2A\,a^2b\,d^5n+2A\,b^3c^2d^3n-B\,a^2b\,d^5n^2-3B\,b^3c^2d^3n^2+4Ba\,b^2c\,d^4n^2-4Aa\,b^2c\,d^4n-2B\,x^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3d^5n+2Bxa\,b^2d^5n^2}{4i^3(dx+c)^2n(a^2d^2-2abcd+b^2}$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(2*A*a^2*b*d^5*n+2*A*b^3*c^2*d^3*n-B*a^2*b*d^5*n^2-3*B*b^3*c^2*d^3*n^2
+4*B*a*b^2*c*d^4*n^2-4*A*a*b^2*c*d^4*n-2*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^
3*d^5*n+2*B*x*a*b^2*d^5*n^2-2*B*x*b^3*c*d^4*n^2+2*B*ln(e*((b*x+a)/(d*x+c))^
n)*a^2*b*d^5*n-4*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d^4*n-4*B*ln(e*((b*x+a
)/(d*x+c))^n)*a*b^2*c*d^4*n)/i^3/(d*x+c)^2/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/
d^4
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.76

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx = -\frac{2\,Ab^2c^2-4\,Aabcd+2\,Aa^2d^2-2\,(Bb^2cd-Babd^2)nx-(3\,Bb^2c^2-4\,Babcd+Ba^2d^2)n+2\,(Bb^2c^2-2\,B}{4\,((b^2c^2d^3-2\,abcd^4+a^2d^5)i^3x^2+2\,(b^2c^3d^2-2\,abc^2d^3+a^2}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, algorithm="fric
as")
```

```
[Out] -1/4*(2*A*b^2*c^2 - 4*A*a*b*c*d + 2*A*a^2*d^2 - 2*(B*b^2*c*d - B*a*b*d^2)*n
*x - (3*B*b^2*c^2 - 4*B*a*b*c*d + B*a^2*d^2)*n + 2*(B*b^2*c^2 - 2*B*a*b*c*d
 + B*a^2*d^2)*log(e) - 2*(B*b^2*d^2*n*x^2 + 2*B*b^2*c*d*n*x + (2*B*a*b*c*d
- B*a^2*d^2)*n)*log((b*x + a)/(d*x + c)))/((b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2
*d^5)*i^3*x^2 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*i^3*x + (b^2*c^
4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*i^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2103 vs. $2(133) = 266$.

Time = 108.22 (sec) , antiderivative size = 2103, normalized size of antiderivative = 13.93

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci + dix)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**3,x)
```

```
[Out] Piecewise((-A/(2*c**2*d*i**3 + 4*c*d**2*i**3*x + 2*d**3*i**3*x**2) - B*log(
e*(b*c/(c*d + d**2*x) + b*x/(c + d*x))**n)/(2*c**2*d*i**3 + 4*c*d**2*i**3*x
 + 2*d**3*i**3*x**2), Eq(a, b*c/d)), ((A*x + B*a*log(e*(a/c + b*x/c)**n)/b
- B*n*x + B*x*log(e*(a/c + b*x/c)**n))/(c**3*i**3), Eq(d, 0)), (-2*A*a**2*d
**2/(4*a**2*c**2*d**3*i**3 + 8*a**2*c*d**4*i**3*x + 4*a**2*d**5*i**3*x**2 -
 8*a*b*c**3*d**2*i**3 - 16*a*b*c**2*d**3*i**3*x - 8*a*b*c*d**4*i**3*x**2 +
4*b**2*c**4*d*i**3 + 8*b**2*c**3*d**2*i**3*x + 4*b**2*c**2*d**3*i**3*x**2)
+ 4*A*a*b*c*d/(4*a**2*c**2*d**3*i**3 + 8*a**2*c*d**4*i**3*x + 4*a**2*d**5*i
**3*x**2 - 8*a*b*c**3*d**2*i**3 - 16*a*b*c**2*d**3*i**3*x - 8*a*b*c*d**4*i*
*3*x**2 + 4*b**2*c**4*d*i**3 + 8*b**2*c**3*d**2*i**3*x + 4*b**2*c**2*d**3*i
**3*x**2) - 2*A*b**2*c**2/(4*a**2*c**2*d**3*i**3 + 8*a**2*c*d**4*i**3*x + 4
*a**2*d**5*i**3*x**2 - 8*a*b*c**3*d**2*i**3 - 16*a*b*c**2*d**3*i**3*x - 8*a
*b*c*d**4*i**3*x**2 + 4*b**2*c**4*d*i**3 + 8*b**2*c**3*d**2*i**3*x + 4*b**2
*c**2*d**3*i**3*x**2) + B*a**2*d**2*n/(4*a**2*c**2*d**3*i**3 + 8*a**2*c*d**
4*i**3*x + 4*a**2*d**5*i**3*x**2 - 8*a*b*c**3*d**2*i**3 - 16*a*b*c**2*d**3*
i**3*x - 8*a*b*c*d**4*i**3*x**2 + 4*b**2*c**4*d*i**3 + 8*b**2*c**3*d**2*i**
3*x + 4*b**2*c**2*d**3*i**3*x**2) - 2*B*a**2*d**2*log(e*(a/(c + d*x) + b*x/
(c + d*x))**n)/(4*a**2*c**2*d**3*i**3 + 8*a**2*c*d**4*i**3*x + 4*a**2*d**5*
i**3*x**2 - 8*a*b*c**3*d**2*i**3 - 16*a*b*c**2*d**3*i**3*x - 8*a*b*c*d**4*i
**3*x**2 + 4*b**2*c**4*d*i**3 + 8*b**2*c**3*d**2*i**3*x + 4*b**2*c**2*d**3*
i**3*x**2) - 4*B*a*b*c*d*n/(4*a**2*c**2*d**3*i**3 + 8*a**2*c*d**4*i**3*x +
4*a**2*d**5*i**3*x**2 - 8*a*b*c**3*d**2*i**3 - 16*a*b*c**2*d**3*i**3*x - 8*
a*b*c*d**4*i**3*x**2 + 4*b**2*c**4*d*i**3 + 8*b**2*c**3*d**2*i**3*x + 4*b**
2*c**2*d**3*i**3*x**2) + 4*B*a*b*c*d*log(e*(a/(c + d*x) + b*x/(c + d*x))**n
)/(4*a**2*c**2*d**3*i**3 + 8*a**2*c*d**4*i**3*x + 4*a**2*d**5*i**3*x**2 - 8
*a*b*c**3*d**2*i**3 - 16*a*b*c**2*d**3*i**3*x - 8*a*b*c*d**4*i**3*x**2 + 4*
b**2*c**4*d*i**3 + 8*b**2*c**3*d**2*i**3*x + 4*b**2*c**2*d**3*i**3*x**2) -
```

```
2*B*a*b*d**2*n*x/(4*a**2*c**2*d**3*i**3 + 8*a**2*c*d**4*i**3*x + 4*a**2*d**
5*i**3*x**2 - 8*a*b*c**3*d**2*i**3 - 16*a*b*c**2*d**3*i**3*x - 8*a*b*c*d**4
*i**3*x**2 + 4*b**2*c**4*d*i**3 + 8*b**2*c**3*d**2*i**3*x + 4*b**2*c**2*d**
3*i**3*x**2) + 3*B*b**2*c**2*n/(4*a**2*c**2*d**3*i**3 + 8*a**2*c*d**4*i**3*
x + 4*a**2*d**5*i**3*x**2 - 8*a*b*c**3*d**2*i**3 - 16*a*b*c**2*d**3*i**3*x
- 8*a*b*c*d**4*i**3*x**2 + 4*b**2*c**4*d*i**3 + 8*b**2*c**3*d**2*i**3*x + 4
*b**2*c**2*d**3*i**3*x**2) + 2*B*b**2*c*d*n*x/(4*a**2*c**2*d**3*i**3 + 8*a*
*2*c*d**4*i**3*x + 4*a**2*d**5*i**3*x**2 - 8*a*b*c**3*d**2*i**3 - 16*a*b*c*
*2*d**3*i**3*x - 8*a*b*c*d**4*i**3*x**2 + 4*b**2*c**4*d*i**3 + 8*b**2*c**3*
d**2*i**3*x + 4*b**2*c**2*d**3*i**3*x**2) + 4*B*b**2*c*d*x*log(e*(a/(c + d*
x) + b*x/(c + d*x))**n)/(4*a**2*c**2*d**3*i**3 + 8*a**2*c*d**4*i**3*x + 4*a
**2*d**5*i**3*x**2 - 8*a*b*c**3*d**2*i**3 - 16*a*b*c**2*d**3*i**3*x - 8*a*b
*c*d**4*i**3*x**2 + 4*b**2*c**4*d*i**3 + 8*b**2*c**3*d**2*i**3*x + 4*b**2*c
**2*d**3*i**3*x**2) + 2*B*b**2*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x)
)**n)/(4*a**2*c**2*d**3*i**3 + 8*a**2*c*d**4*i**3*x + 4*a**2*d**5*i**3*x**2
 - 8*a*b*c**3*d**2*i**3 - 16*a*b*c**2*d**3*i**3*x - 8*a*b*c*d**4*i**3*x**2
+ 4*b**2*c**4*d*i**3 + 8*b**2*c**3*d**2*i**3*x + 4*b**2*c**2*d**3*i**3*x**2
), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.72

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx$$
$$= \frac{1}{4}\,Bn\left(\frac{2\,bdx + 3\,bc - ad}{(bcd^3 - ad^4)i^3x^2 + 2\,(bc^2d^2 - acd^3)i^3x + (bc^3d - ac^2d^2)i^3} + \frac{2\,b^2\log\left(bx + a\right)}{(b^2c^2d - 2\,abcd^2 + a^2d^3)i^3} - \frac{2\,b^2}{(b^2c^2d -}\right.$$
$$- \frac{B\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)}{2\,(d^3i^3x^2 + 2\,cd^2i^3x + c^2di^3)} - \frac{A}{2\,(d^3i^3x^2 + 2\,cd^2i^3x + c^2di^3)}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, algorithm="maxi
ma")
```

```
[Out] 1/4*B*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2
- a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b^2*log(b*x + a)/((b^2*c^
2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3)*i^3)) - 1/2*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^3*
i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - 1/2*A/(d^3*i^3*x^2 + 2*c*d^2*i^3*x +
 c^2*d*i^3)
```

Giac [A] (verification not implemented)

none

Time = 0.77 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.37

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(2\left(\frac{2\,(bx+a)Bbn}{(bci^3-adi^3)(dx+c)}-\frac{(bx+a)^2Bdn}{(bci^3-adi^3)(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)+\frac{(Bdn-2\,Bd\log(e)-2\,Ad)(bx}{(bci^3-adi^3)(dx+c)^2}\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, algorithm="giac
")
```

```
[Out] 1/4*(2*(2*(b*x + a)*B*b*n/((b*c*i^3 - a*d*i^3)*(d*x + c)) - (b*x + a)^2*B*d
*n/((b*c*i^3 - a*d*i^3)*(d*x + c)^2))*log((b*x + a)/(d*x + c)) + (B*d*n - 2
*B*d*log(e) - 2*A*d)*(b*x + a)^2/((b*c*i^3 - a*d*i^3)*(d*x + c)^2) - 4*(B*b
*n - B*b*log(e) - A*b)*(b*x + a)/((b*c*i^3 - a*d*i^3)*(d*x + c)))*(b*c/(b*c
 - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.46

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx=\frac{B\,b^2\,n\,\mathrm{atanh}\left(\frac{2\,a^2\,d^3\,i^3-2\,b^2\,c^2\,d\,i^3}{2\,d\,i^3\,(a\,d-b\,c)^2}+\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{d\,i^3\,(a\,d-b\,c)^2}-\frac{B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{2\,d\,\left(c^2\,i^3+2\,c\,d\,i^3\,x+d^2\,i^3\,x^2\right)}-\frac{\frac{2\,A\,a\,d-2\,A\,b\,c-B\,a\,d\,n+3\,B\,b\,c\,n}{2\,(a\,d-b\,c)}+\frac{B\,b\,d\,n\,x}{a\,d-b\,c}}{2\,c^2\,d\,i^3+4\,c\,d^2\,i^3\,x+2\,d^3\,i^3\,x^2}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*i + d*i*x)^3,x)
```

```
[Out] (B*b^2*n*atanh((2*a^2*d^3*i^3 - 2*b^2*c^2*d*i^3)/(2*d*i^3*(a*d - b*c)^2) +
(2*b*d*x)/(a*d - b*c)))/(d*i^3*(a*d - b*c)^2) - (B*log(e*((a + b*x)/(c + d*
x))^n))/(2*d*(c^2*i^3 + d^2*i^3*x^2 + 2*c*d*i^3*x)) - ((2*A*a*d - 2*A*b*c -
 B*a*d*n + 3*B*b*c*n)/(2*(a*d - b*c)) + (B*b*d*n*x)/(a*d - b*c))/(2*c^2*d*i
^3 + 2*d^3*i^3*x^2 + 4*c*d^2*i^3*x)
```

3.155 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 254

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx = -\frac{Bn\left(4b-\frac{d(a+bx)}{c+dx}\right)^2}{4(bc-ad)^3gi^3}+\frac{d^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3gi^3(c+dx)^2} - \frac{2bd(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3gi^3(c+dx)} + \frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3} - \frac{b^2Bn\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^3gi^3}$$

```
[Out] -1/4*B*n*(4*b-d*(b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g/i^3+1/2*d^2*(b*x+a)^2*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g/i^3/(d*x+c)^2-2*b*d*(b*x+a)*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g/i^3/(d*x+c)+b^2*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d+b*c)^3/g/i^3-1/2*b^2*B*n*ln((b*x
+a)/(d*x+c))^2/(-a*d+b*c)^3/g/i^3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used

$= \{2561, 45, 2372, 2338\}$

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx = \frac{b^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{gi^3(bc-ad)^3} + \frac{d^2(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2gi^3(c+dx)^2(bc-ad)^3} - \frac{2bd(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{gi^3(c+dx)(bc-ad)^3} - \frac{b^2Bn\log^2\left(\frac{a+bx}{c+dx}\right)}{2gi^3(bc-ad)^3} - \frac{Bn\left(4b-\frac{d(a+bx)}{c+dx}\right)^2}{4gi^3(bc-ad)^3}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)*(c*i + d*i*x)^3),
x]
```

```
[Out] -1/4*(B*n*(4*b - (d*(a + b*x))/(c + d*x))^2)/((b*c - a*d)^3*g*i^3) + (d^2*(
a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*c - a*d)^3*g*i^3*(
c + d*x)^2) - (2*b*d*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*
c - a*d)^3*g*i^3*(c + d*x)) + (b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*L
og[(a + b*x)/(c + d*x)])/((b*c - a*d)^3*g*i^3) - (b^2*B*n*Log[(a + b*x)/(c
+ d*x)]^2)/(2*(b*c - a*d)^3*g*i^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
```

```
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex^n))}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3}\\
&= \frac{d^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3gi^3(c+dx)^2}-\frac{2bd(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3gi^3(c+dx)}\\
&\quad+\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3}\\
&\quad-\frac{(Bn)\text{Subst}\left(\int\left(\frac{1}{2}d(-4b+dx)+\frac{b^2\log(x)}{x}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3}\\
&= -\frac{Bn\left(4b-\frac{d(a+bx)}{c+dx}\right)^2}{4(bc-ad)^3gi^3}+\frac{d^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3gi^3(c+dx)^2}\\
&\quad-\frac{2bd(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3gi^3(c+dx)}+\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3}\\
&\quad-\frac{(b^2Bn)\,\text{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3}\\
&= -\frac{Bn\left(4b-\frac{d(a+bx)}{c+dx}\right)^2}{4(bc-ad)^3gi^3}+\frac{d^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3gi^3(c+dx)^2}\\
&\quad-\frac{2bd(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3gi^3(c+dx)}\\
&\quad+\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3}-\frac{b^2Bn\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^3gi^3}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.21 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.71

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx$$
$$=\frac{2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4b(bc-ad)(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4b^2(c+dx)^2\log(a+}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)*(c*i + d*i*
x)^3),x]
```

```
[Out] (2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*b*(b*c - a*d)*(
c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*b^2*(c + d*x)^2*Log[a +
 b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 4*b^2*(c + d*x)^2*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 4*b*B*n*(c + d*x)*(b*c - a*d +
b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]) - B*n*((b*c - a*d)^2 +
 2*b*(b*c - a*d)*(c + d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*
x)^2*Log[c + d*x]) - 2*b^2*B*n*(c + d*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*
Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)
]) + 2*b^2*B*n*(c + d*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c +
d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(4*(b*c - a*
d)^3*g*i^3*(c + d*x)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. 2(248) = 496.

Time = 11.28 (sec) , antiderivative size = 628, normalized size of antiderivative = 2.47

method	result
parallelrisch	$-\frac{2Bx\,a^4c^3d^3n^2-4Ax\,a^4c^3d^3n+2B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^4c^4d^2n-6B\,x^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^2b^2c^4d^2n-4Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^3b\,c^4d^2}{}$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^3,x,method=_RET
URNVERBOSE)
```

```
[Out] -1/4*(2*B*x*a^4*c^3*d^3*n^2-4*A*x*a^4*c^3*d^3*n+2*B*ln(e*((b*x+a)/(d*x+c))^
n)*a^4*c^4*d^2*n-6*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^4*d^2*n-4*B*x*
ln(e*((b*x+a)/(d*x+c))^n)*a^3*b*c^4*d^2*n-8*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a
^2*b^2*c^5*d*n+B*x^2*a^4*c^2*d^4*n^2-2*A*x^2*a^4*c^2*d^4*n+2*B*ln(e*((b*x+a
)/(d*x+c))^n)^2*a^2*b^2*c^6+4*A*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^6+2*B*x
^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^2*c^4*d^2-8*B*x^2*a^3*b*c^3*d^3*n^2+7*
```

```
B*x^2*a^2*b^2*c^4*d^2*n^2+4*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^4*d^2
+8*A*x^2*a^3*b*c^3*d^3*n-6*A*x^2*a^2*b^2*c^4*d^2*n+4*B*x*ln(e*((b*x+a)/(d*x
+c))^n)^2*a^2*b^2*c^5*d-10*B*x*a^3*b*c^4*d^2*n^2+8*B*x*a^2*b^2*c^5*d*n^2+8*
A*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^5*d+12*A*x*a^3*b*c^4*d^2*n-8*A*x*a^
2*b^2*c^5*d*n-8*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b*c^5*d*n)/i^3/g/(d*x+c)^2/
(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/a^2/c^4/n
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.92

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx = \frac{6\,Ab^2c^2-8\,Aabcd+2\,Aa^2d^2+2\left(Bb^2d^2nx^2+2\,Bb^2cdnx+Bb^2c^2n\right)\log\left(\frac{bx+a}{dx+c}\right)^2-\left(7\,Bb^2c^2-8\,Babcd+B\right.}{}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^3,x, alg
orithm="fricas")
```

```
[Out] 1/4*(6*A*b^2*c^2 - 8*A*a*b*c*d + 2*A*a^2*d^2 + 2*(B*b^2*d^2*n*x^2 + 2*B*b^2
*c*d*n*x + B*b^2*c^2*n)*log((b*x + a)/(d*x + c))^2 - (7*B*b^2*c^2 - 8*B*a*b
*c*d + B*a^2*d^2)*n + 2*(2*A*b^2*c*d - 2*A*a*b*d^2 - 3*(B*b^2*c*d - B*a*b*d
^2)*n)*x + 2*(3*B*b^2*c^2 - 4*B*a*b*c*d + B*a^2*d^2 + 2*(B*b^2*c*d - B*a*b*
d^2)*x + 2*(B*b^2*d^2*x^2 + 2*B*b^2*c*d*x + B*b^2*c^2)*log((b*x + a)/(d*x +
 c)))*log(e) + 2*(2*A*b^2*c^2 - (3*B*b^2*d^2*n - 2*A*b^2*d^2)*x^2 - (4*B*a*
b*c*d - B*a^2*d^2)*n + 2*(2*A*b^2*c*d - (2*B*b^2*c*d + B*a*b*d^2)*n)*x)*log
((b*x + a)/(d*x + c)))/((b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^
3*d^5)*g*i^3*x^2 + 2*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c
*d^4)*g*i^3*x + (b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*g
*i^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)/(d*i*x+c*i)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 888 vs. 2(248) = 496.

Time = 0.23 (sec) , antiderivative size = 888, normalized size of antiderivative = 3.50

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx$$
$$=\frac{1}{2}B\left(\frac{2\,bdx+3\,bc-ad}{(b^2c^2d^2-2\,abcd^3+a^2d^4)gi^3x^2+2\,(b^2c^3d-2\,abc^2d^2+a^2cd^3)gi^3x+(b^2c^4-2\,abc^3d+a^2c^2d^2)gi^3}+\right.$$
$$-\frac{\left(7\,b^2c^2-8\,abcd+a^2d^2+2\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\log(bx+a)^2+2\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\log(dx\right.}{4\,(b^3c^5gi^3-3\,ab^2c^4dgi^3+3\,a^2bc^3d^2gi^3-a^3c^2d^3gi^3+(}$$
$$+\frac{1}{2}A\left(\frac{2\,bdx+3\,bc-ad}{(b^2c^2d^2-2\,abcd^3+a^2d^4)gi^3x^2+2\,(b^2c^3d-2\,abc^2d^2+a^2cd^3)gi^3x+(b^2c^4-2\,abc^3d+a^2c^2d^2)gi^3}\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^3,x, alg
orithm="maxima")
```

```
[Out] 1/2*B*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*g*i^3
*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3*x + (b^2*c^4 - 2*a*b
*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2*d
 + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c)/((b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3))*log(e*(b*x/(d*x + c) + a/(d*x + c
))^n) - 1/4*(7*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*
x + c)^2 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*log(b*x + a) - 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x + c))*B*n/(b^3*c^5*g*i^3 -
 3*a*b^2*c^4*d*g*i^3 + 3*a^2*b*c^3*d^2*g*i^3 - a^3*c^2*d^3*g*i^3 + (b^3*c^3
*d^2*g*i^3 - 3*a*b^2*c^2*d^3*g*i^3 + 3*a^2*b*c*d^4*g*i^3 - a^3*d^5*g*i^3)*x
^2 + 2*(b^3*c^4*d*g*i^3 - 3*a*b^2*c^3*d^2*g*i^3 + 3*a^2*b*c^2*d^3*g*i^3 - a
^3*c*d^4*g*i^3)*x) + 1/2*A*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c
*d^3 + a^2*d^4)*g*i^3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3
*x + (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c
)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3))
```

Giac [A] (verification not implemented)

none

Time = 1.52 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.65

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(\frac{2\,Bb^2n\log\left(\frac{bx+a}{dx+c}\right)^2}{b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3}-2\left(\frac{4\,(bx+a)Bbdn}{(b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3)(dx+c)}-\frac{(bx+a)^2B}{(b^2c^2gi^3-2\,abcdgi^3+a}\right.\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^3,x, alg
orithm="giac")
```

```
[Out] 1/4*(2*B*b^2*n*log((b*x + a)/(d*x + c))^2/(b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3
+ a^2*d^2*g*i^3) - 2*(4*(b*x + a)*B*b*d*n/((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3
 + a^2*d^2*g*i^3)*(d*x + c)) - (b*x + a)^2*B*d^2*n/((b^2*c^2*g*i^3 - 2*a*b*
c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)^2))*log((b*x + a)/(d*x + c)) + 4*(B*b^
2*log(e) + A*b^2)*log((b*x + a)/(d*x + c))/(b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3
 + a^2*d^2*g*i^3) - (B*d^2*n - 2*B*d^2*log(e) - 2*A*d^2)*(b*x + a)^2/((b^2*
c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)^2) + 8*(B*b*d*n - B*
b*d*log(e) - A*b*d)*(b*x + a)/((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g
*i^3)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 3.37 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.26

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx$$
$$=\frac{B\,b^2\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(\frac{c\,g\,i^3\,n\,(a\,d-b\,c)}{2\,b}-\frac{g\,i^3\,n\,(a\,d-b\,c)\,(a\,d-2\,b\,c)}{2\,b^2}+\frac{d\,g\,i^3\,n\,x\,(a\,d-b\,c)}{b}\right)}{g\,i^3\,n\,(a\,d-b\,c)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)\,(g\,c^2\,i^3+2\,g\,c\,d\,i^3\,x+g\,d^2\,i^3\,x^2)}$$
$$-\frac{B\,b^2\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{2\,g\,i^3\,n\,(a\,d-b\,c)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}$$
$$-\frac{\frac{2\,A\,a\,d-6\,A\,b\,c-B\,a\,d\,n+7\,B\,b\,c\,n}{2\,(a\,d-b\,c)}-\frac{b\,x\,(2\,A\,d-3\,B\,d\,n)}{a\,d-b\,c}}{x^2\,(2\,a\,d^3\,g\,i^3-2\,b\,c\,d^2\,g\,i^3)+x\,(4\,a\,c\,d^2\,g\,i^3-4\,b\,c^2\,d\,g\,i^3)-2\,b\,c^3\,g\,i^3+2\,a\,c^2\,d\,g\,i^3}$$
$$+\frac{b^2\,\mathrm{atan}\left(\frac{b^2\,\left(A-\frac{3\,B\,n}{2}\right)\,(2\,g\,a^3\,d^3\,i^3-2\,g\,a^2\,b\,c\,d^2\,i^3-2\,g\,a\,b^2\,c^2\,d\,i^3+2\,g\,b^3\,c^3\,i^3)\,1\mathrm{i}}{g\,i^3\,(2\,A\,b^2-3\,B\,b^2\,n)\,(a\,d-b\,c)^3}+\frac{b^3\,d\,x\,\left(A-\frac{3\,B\,n}{2}\right)\,(g\,a^2\,d^2\,i^3-2\,g\,a\,b\,c\,d\,i^3+g\,b^2\,c^2\,i^3)\,4}{g\,i^3\,(2\,A\,b^2-3\,B\,b^2\,n)\,(a\,d-b\,c)^3}\right.}{g\,i^3\,(a\,d-b\,c)^3}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)*(c*i + d*i*x)^3),
x)
```

```
[Out] (b^2*atan((b^2*(A - (3*B*n)/2)*(2*a^3*d^3*g*i^3 + 2*b^3*c^3*g*i^3 - 2*a*b^2
*c^2*d*g*i^3 - 2*a^2*b*c*d^2*g*i^3)*1i)/(g*i^3*(2*A*b^2 - 3*B*b^2*n)*(a*d -
 b*c)^3) + (b^3*d*x*(A - (3*B*n)/2)*(a^2*d^2*g*i^3 + b^2*c^2*g*i^3 - 2*a*b*
c*d*g*i^3)*4i)/(g*i^3*(2*A*b^2 - 3*B*b^2*n)*(a*d - b*c)^3))*(A - (3*B*n)/2)
*2i)/(g*i^3*(a*d - b*c)^3) - ((2*A*a*d - 6*A*b*c - B*a*d*n + 7*B*b*c*n)/(2*
(a*d - b*c)) - (b*x*(2*A*d - 3*B*d*n))/(a*d - b*c))/(x^2*(2*a*d^3*g*i^3 - 2
*b*c*d^2*g*i^3) + x*(4*a*c*d^2*g*i^3 - 4*b*c^2*d*g*i^3) - 2*b*c^3*g*i^3 + 2
*a*c^2*d*g*i^3) - (B*b^2*log(e*((a + b*x)/(c + d*x))^n)^2)/(2*g*i^3*n*(a*d
- b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B*b^2*log(e*((a + b*x)/(c + d*x)
)^n)*((c*g*i^3*n*(a*d - b*c))/(2*b) - (g*i^3*n*(a*d - b*c)*(a*d - 2*b*c))/(
2*b^2) + (d*g*i^3*n*x*(a*d - b*c))/b))/(g*i^3*n*(a*d - b*c)*(a^2*d^2 + b^2*
c^2 - 2*a*b*c*d)*(c^2*g*i^3 + d^2*g*i^3*x^2 + 2*c*d*g*i^3*x))
```

3.156 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 381

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx &= \frac{Bd^3n(a+bx)^2}{4(bc-ad)^4g^2i^3(c+dx)^2}-\frac{3bBd^2n(a+bx)}{(bc-ad)^4g^2i^3(c+dx)}\\ &-\frac{b^3Bn(c+dx)}{(bc-ad)^4g^2i^3(a+bx)}\\ &-\frac{d^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^2i^3(c+dx)^2}\\ &+\frac{3bd^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^2i^3(c+dx)}\\ &-\frac{b^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^2i^3(a+bx)}\\ &-\frac{3b^2d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}\\ &+\frac{3b^2Bdn\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^2i^3}\end{aligned}$$

```
[Out] 1/4*B*d^3*n*(b*x+a)^2/(-a*d+b*c)^4/g^2/i^3/(d*x+c)^2-3*b*B*d^2*n*(b*x+a)/(-
a*d+b*c)^4/g^2/i^3/(d*x+c)-b^3*B*n*(d*x+c)/(-a*d+b*c)^4/g^2/i^3/(b*x+a)-1/2
*d^3*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^2/i^3/(d*x+c)
^2+3*b*d^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^2/i^3/(d*
x+c)-b^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^2/i^3/(b*x+
a)-3*b^2*d*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d+b*c)^4
/g^2/i^3+3/2*b^2*B*d*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^4/g^2/i^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {2561, 45, 2372, 2338}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = -\frac{b^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^2i^3(a+bx)(bc-ad)^4} - \frac{3b^2d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^2i^3(bc-ad)^4} - \frac{d^3(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^2i^3(c+dx)^2(bc-ad)^4} + \frac{3bd^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^2i^3(c+dx)(bc-ad)^4} - \frac{b^3Bn(c+dx)}{g^2i^3(a+bx)(bc-ad)^4} + \frac{3b^2Bdn\log^2\left(\frac{a+bx}{c+dx}\right)}{2g^2i^3(bc-ad)^4} + \frac{Bd^3n(a+bx)^2}{4g^2i^3(c+dx)^2(bc-ad)^4} - \frac{3bBd^2n(a+bx)}{g^2i^3(c+dx)(bc-ad)^4}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^2*(c*i + d*i*x)^3
),x]
```

```
[Out] (B*d^3*n*(a + b*x)^2)/(4*(b*c - a*d)^4*g^2*i^3*(c + d*x)^2) - (3*b*B*d^2*n*
(a + b*x))/((b*c - a*d)^4*g^2*i^3*(c + d*x)) - (b^3*B*n*(c + d*x))/((b*c -
a*d)^4*g^2*i^3*(a + b*x)) - (d^3*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]))/(2*(b*c - a*d)^4*g^2*i^3*(c + d*x)^2) + (3*b*d^2*(a + b*x)*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^4*g^2*i^3*(c + d*x)) - (b^3*
(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^4*g^2*i^3*(a
 + b*x)) - (3*b^2*d*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c
 + d*x)])/((b*c - a*d)^4*g^2*i^3) + (3*b^2*B*d*n*Log[(a + b*x)/(c + d*x)]^2
)/(2*(b*c - a*d)^4*g^2*i^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)^3(A+B\log(ex^n))}{x^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}\\
&= -\frac{d^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^2i^3(c+dx)^2}+\frac{3bd^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^2i^3(c+dx)}\\
&\quad -\frac{b^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^2i^3(a+bx)}-\frac{3b^2d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}\\
&\quad -\frac{(Bn)\text{Subst}\left(\int\left(3bd^2-\frac{b^3}{x^2}-\frac{d^3x}{2}-\frac{3b^2d\log(x)}{x}\right)dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}\\
&= \frac{Bd^3n(a+bx)^2}{4(bc-ad)^4g^2i^3(c+dx)^2}-\frac{3bBd^2n(a+bx)}{(bc-ad)^4g^2i^3(c+dx)}\\
&\quad -\frac{b^3Bn(c+dx)}{(bc-ad)^4g^2i^3(a+bx)}-\frac{d^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^2i^3(c+dx)^2}\\
&\quad +\frac{3bd^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^2i^3(c+dx)}-\frac{b^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^2i^3(a+bx)}\\
&\quad -\frac{3b^2d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}+\frac{(3b^2Bdn)\,\text{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{Bd^3n(a+bx)^2}{4(bc-ad)^4g^2i^3(c+dx)^2} - \frac{3bBd^2n(a+bx)}{(bc-ad)^4g^2i^3(c+dx)} \\
&\quad - \frac{b^3Bn(c+dx)}{(bc-ad)^4g^2i^3(a+bx)} - \frac{d^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^2i^3(c+dx)^2} \\
&\quad + \frac{3bd^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^2i^3(c+dx)} - \frac{b^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^2i^3(a+bx)} \\
&\quad - \frac{3b^2d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3} + \frac{3b^2Bdn\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^2i^3}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.39 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.25

$$
\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx
$$

$$
= \frac{-\frac{4b^3Bcn}{a+bx} + \frac{4ab^2Bdn}{a+bx} + \frac{Bd(bc-ad)^2n}{(c+dx)^2} + \frac{8b^2Bcdn}{c+dx} - \frac{8abBd^2n}{c+dx} + \frac{2bBd(bc-ad)n}{c+dx} + 6b^2Bdn\log(a+bx) - \frac{4b^2(bc-ad)\left(A+B\right.}{a+b}}{}
$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^2*(c*i + d*
i*x)^3),x]
```

```
[Out] ((-4*b^3*B*c*n)/(a + b*x) + (4*a*b^2*B*d*n)/(a + b*x) + (B*d*(b*c - a*d)^2*
n)/(c + d*x)^2 + (8*b^2*B*c*d*n)/(c + d*x) - (8*a*b*B*d^2*n)/(c + d*x) + (2
*b*B*d*(b*c - a*d)*n)/(c + d*x) + 6*b^2*B*d*n*Log[a + b*x] - (4*b^2*(b*c -
a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) - (2*d*(b*c - a*d)^2
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x)^2 - (8*b*d*(b*c - a*d)*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) - 12*b^2*d*Log[a + b*x]*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*b^2*B*d*n*Log[c + d*x] + 12*b^2*d*
(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 6*b^2*B*d*n*(Log[a +
b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)]) - 6*b^2*B*d*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d
)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))
/(4*(b*c - a*d)^4*g^2*i^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(375) = 750.

Time = 28.55 (sec) , antiderivative size = 962, normalized size of antiderivative = 2.52

method	result
parallelrisch	$-\frac{-24Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a\,b^{5}c\,d^{6}n+6B\,x^{3}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}b^{6}d^{7}+12A\,x^{3}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{6}d^{7}-18B\,x^{2}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a\,b^{5}d^{7}n+}{}$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x,method=_R
ETURNVERBOSE)
```

```
[Out] -1/4*(-24*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c*d^6*n+6*B*x^3*ln(e*((b*x+a)
/(d*x+c))^n)^2*b^6*d^7+12*A*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^6*d^7-18*B*x^2*
ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*d^7*n+12*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*
b^5*c*d^6-6*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*d^7*n+12*B*x*ln(e*((b*x+a
)/(d*x+c))^n)*b^6*c^2*d^5*n-6*B*x*a*b^5*c*d^6*n^2+24*A*x*ln(e*((b*x+a)/(d*x
+c))^n)*a*b^5*c*d^6-12*A*x*a*b^5*c*d^6*n-12*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2
*b^4*c*d^6*n-6*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^6*d^7*n+6*B*x^2*ln(e*((b*x
+a)/(d*x+c))^n)^2*a*b^5*d^7+12*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^6*c*d^6+
6*B*x^2*a*b^5*d^7*n^2-6*B*x^2*b^6*c*d^6*n^2+12*A*x^2*ln(e*((b*x+a)/(d*x+c))
^n)*a*b^5*d^7+24*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c*d^6-12*A*x^2*a*b^5*d
^7*n+12*A*x^2*b^6*c*d^6*n+6*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^6*c^2*d^5+9*B
*x*a^2*b^4*d^7*n^2-3*B*x*b^6*c^2*d^5*n^2+12*A*x*ln(e*((b*x+a)/(d*x+c))^n)*b
^6*c^2*d^5-6*A*x*a^2*b^4*d^7*n+18*A*x*b^6*c^2*d^5*n+6*B*ln(e*((b*x+a)/(d*x+
c))^n)^2*a*b^5*c^2*d^5+2*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^3*d^7*n+4*B*ln(e
*((b*x+a)/(d*x+c))^n)*b^6*c^3*d^4*n+12*A*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c^
2*d^5-B*a^3*b^3*d^7*n^2+4*B*b^6*c^3*d^4*n^2+2*A*a^3*b^3*d^7*n+4*A*b^6*c^3*d
^4*n+12*B*a^2*b^4*c*d^6*n^2-15*B*a*b^5*c^2*d^5*n^2-12*A*a^2*b^4*c*d^6*n+6*A
*a*b^5*c^2*d^5*n)/i^3/g^2/(d*x+c)^2/(b*x+a)/(a*d-b*c)^2/(a^2*d^2-2*a*b*c*d+
b^2*c^2)/b^3/d^4/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 949 vs. 2(375) = 750.

Time = 0.38 (sec) , antiderivative size = 949, normalized size of antiderivative = 2.49

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{(ag+bgx)^{2}(ci+dix)^{3}}\,dx = -\frac{4\,Ab^{3}c^{3}+6\,Aab^{2}c^{2}d-12\,Aa^{2}bcd^{2}+2\,Aa^{3}d^{3}+6\,(2\,Ab^{3}cd^{2}-2\,Aab^{2}d^{3}-(Bb^{3}cd^{2}-Bab^{2}d^{3})n)x^{2}+6\,(B}{}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x, a
lgorithm="fricas")
```

```
[Out] -1/4*(4*A*b^3*c^3 + 6*A*a*b^2*c^2*d - 12*A*a^2*b*c*d^2 + 2*A*a^3*d^3 + 6*(2
*A*b^3*c*d^2 - 2*A*a*b^2*d^3 - (B*b^3*c*d^2 - B*a*b^2*d^3)*n)*x^2 + 6*(B*b^
3*d^3*n*x^3 + B*a*b^2*c^2*d*n + (2*B*b^3*c*d^2 + B*a*b^2*d^3)*n*x^2 + (B*b^
3*c^2*d + 2*B*a*b^2*c*d^2)*n*x)*log((b*x + a)/(d*x + c))^2 + (4*B*b^3*c^3 -
 15*B*a*b^2*c^2*d + 12*B*a^2*b*c*d^2 - B*a^3*d^3)*n + 3*(6*A*b^3*c^2*d - 4*
A*a*b^2*c*d^2 - 2*A*a^2*b*d^3 - (B*b^3*c^2*d + 2*B*a*b^2*c*d^2 - 3*B*a^2*b*
d^3)*n)*x + 2*(2*B*b^3*c^3 + 3*B*a*b^2*c^2*d - 6*B*a^2*b*c*d^2 + B*a^3*d^3
+ 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 + 3*(3*B*b^3*c^2*d - 2*B*a*b^2*c*d^2 -
B*a^2*b*d^3)*x + 6*(B*b^3*d^3*x^3 + B*a*b^2*c^2*d + (2*B*b^3*c*d^2 + B*a*b^
2*d^3)*x^2 + (B*b^3*c^2*d + 2*B*a*b^2*c*d^2)*x)*log((b*x + a)/(d*x + c)))*l
og(e) + 2*(6*A*a*b^2*c^2*d - 3*(B*b^3*d^3*n - 2*A*b^3*d^3)*x^3 - 3*(3*B*a*b
^2*d^3*n - 4*A*b^3*c*d^2 - 2*A*a*b^2*d^3)*x^2 + (2*B*b^3*c^3 - 6*B*a^2*b*c*
d^2 + B*a^3*d^3)*n + 3*(2*A*b^3*c^2*d + 4*A*a*b^2*c*d^2 + (2*B*b^3*c^2*d -
4*B*a*b^2*c*d^2 - B*a^2*b*d^3)*n)*x)*log((b*x + a)/(d*x + c)))/((b^5*c^4*d^
2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b*d^6)*g^2*
i^3*x^3 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2*c^
2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*g^2*i^3*x^2 + (b^5*c^6 - 2*a*b^4*c^5*d - 2
*a^2*b^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*g^2*i
^3*x + (a*b^4*c^6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 +
 a^5*c^2*d^4)*g^2*i^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2/(d*i*x+c*i)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1724 vs. 2(375) = 750.

Time = 0.30 (sec) , antiderivative size = 1724, normalized size of antiderivative = 4.52

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x, a
lgorithm="maxima")
```

```
[Out] -1/2*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a
*b*d^2)*x)/((b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*g
```

```
^2*i^3*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d
^4 - a^4*d^5)*g^2*i^3*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*
a^3*b*c^2*d^3 - 2*a^4*c*d^4)*g^2*i^3*x + (a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a
^3*b*c^3*d^2 - a^4*c^2*d^3)*g^2*i^3) + 6*b^2*d*log(b*x + a)/((b^4*c^4 - 4*a
*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3) - 6*b^2*
d*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^
3 + a^4*d^4)*g^2*i^3))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1/4*(4*b^3*
c^3 - 15*a*b^2*c^2*d + 12*a^2*b*c*d^2 - a^3*d^3 - 6*(b^3*c*d^2 - a*b^2*d^3)
*x^2 - 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*
c^2*d + 2*a*b^2*c*d^2)*x)*log(b*x + a)^2 - 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (
2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(d*x + c)^
2 - 3*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x - 6*(b^3*d^3*x^3 + a*b^2*
c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(
b*x + a) + 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (
b^3*c^2*d + 2*a*b^2*c*d^2)*x + 2*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2
+ a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(b*x + a))*log(d*x + c
))*B*n/(a*b^4*c^6*g^2*i^3 - 4*a^2*b^3*c^5*d*g^2*i^3 + 6*a^3*b^2*c^4*d^2*g^2
*i^3 - 4*a^4*b*c^3*d^3*g^2*i^3 + a^5*c^2*d^4*g^2*i^3 + (b^5*c^4*d^2*g^2*i^3
 - 4*a*b^4*c^3*d^3*g^2*i^3 + 6*a^2*b^3*c^2*d^4*g^2*i^3 - 4*a^3*b^2*c*d^5*g^
2*i^3 + a^4*b*d^6*g^2*i^3)*x^3 + (2*b^5*c^5*d*g^2*i^3 - 7*a*b^4*c^4*d^2*g^2
*i^3 + 8*a^2*b^3*c^3*d^3*g^2*i^3 - 2*a^3*b^2*c^2*d^4*g^2*i^3 - 2*a^4*b*c*d^
5*g^2*i^3 + a^5*d^6*g^2*i^3)*x^2 + (b^5*c^6*g^2*i^3 - 2*a*b^4*c^5*d*g^2*i^3
 - 2*a^2*b^3*c^4*d^2*g^2*i^3 + 8*a^3*b^2*c^3*d^3*g^2*i^3 - 7*a^4*b*c^2*d^4*
g^2*i^3 + 2*a^5*c*d^5*g^2*i^3)*x) - 1/2*A*((6*b^2*d^2*x^2 + 2*b^2*c^2 + 5*a
*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a*b*d^2)*x)/((b^4*c^3*d^2 - 3*a*b^3*c^2*d
^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*g^2*i^3*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*
d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*g^2*i^3*x^2 + (b^4*c^5 - a
*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*g^2*i^3*x +
 (a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3)*g^2*i^3) + 6
*b^2*d*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b
*c*d^3 + a^4*d^4)*g^2*i^3) - 6*b^2*d*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d
 + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3))
```

Giac [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx=\int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{(bgx+ag)^2(dix+ci)^3}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x, a
lgorithm="giac")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/((b*g*x + a*g)^2*(d*i*x +
c*i)^3), x)
```

Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 1018, normalized size of antiderivative = 2.67

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx$$

$$= \frac{\frac{4\,A\,b^2\,c^2-2\,A\,a^2\,d^2+B\,a^2\,d^2\,n+4\,B\,b^2\,c^2\,n+10\,A\,a\,b\,c\,d-11\,B\,a\,b\,c\,d\,n}{2\,(a\,d-b\,c)}+\frac{3\,x^2\,(2\,A}{}}{x\,\left(4\,a^3\,c\,d^3\,g^2\,i^3-6\,a^2\,b\,c^2\,d^2\,g^2\,i^3+2\,b^3\,c^4\,g^2\,i^3\right)+x^2\,\left(2\,a^3\,d^4\,g^2\,i^3-6\,a\,b^2\,c^2\,d^2\,g^2\,i^3+4\,b^3\,c^3\,d\,g^2\,i^3\right)+x^3}$$

$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{\frac{B\,(a\,d+2\,b\,c)}{2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\frac{3\,B\,b\,d\,x}{2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}}{x\,\left(b\,c^2\,g^2\,i^3+2\,a\,d\,c\,g^2\,i^3\right)+x^2\,\left(a\,d^2\,g^2\,i^3+2\,b\,c\,d\,g^2\,i^3\right)+a\,c^2\,g^2\,i^3+b\,d^2\,g^2\,i^3\,x^3}\right.$$

$$\left.-\frac{3\,B\,b^2\,d\left(d\,g^2\,i^3\,n\,x^2\,(a\,d-b\,c)+\frac{a\,c\,g^2\,i^3\,n\,(a\,d-b\,c)}{b}+\frac{g^2\,i^3\,n\,x\,(a\,d+b\,c)\,(a\,d-b\,c)}{b}\right)}{g^2\,i^3\,n\,(a\,d-b\,c)^4\,\left(x\,\left(b\,c^2\,g^2\,i^3+2\,a\,d\,c\,g^2\,i^3\right)+x^2\,\left(a\,d^2\,g^2\,i^3+2\,b\,c\,d\,g^2\,i^3\right)+a\,c^2\,g^2\,i^3+b\,d^2\,g^2\,i^3\,x^3\right)}\right)$$

$$-\frac{3\,B\,b^2\,d\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{2\,g^2\,i^3\,n\,(a\,d-b\,c)^4}$$

$$+\frac{b^2\,d\,\mathrm{atan}\left(\frac{b^2\,d\,(2\,A-B\,n)\left(\frac{a^4\,d^4\,g^2\,i^3-2\,a^3\,b\,c\,d^3\,g^2\,i^3+2\,a\,b^3\,c^3\,d\,g^2\,i^3-b^4\,c^4\,g^2\,i^3}{a^3\,d^3\,g^2\,i^3-3\,a^2\,b\,c\,d^2\,g^2\,i^3+3\,a\,b^2\,c^2\,d\,g^2\,i^3-b^3\,c^3\,g^2\,i^3}+2\,b\,d\,x\right)\left(a^3\,d^3\,g^2\,i^3-3\,a^2\,b\,c\,d^2\,g^2\,i^3+3\,a\,b^2\,c^2\,d\,g^2\,i^3-b^3\right.}{g^2\,i^3\,(6\,A\,b^2\,d-3\,B\,b^2\,d\,n)\,(a\,d-b\,c)^4}\right.}{g^2\,i^3\,(a\,d-b\,c)^4}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^2*(c*i + d*i*x)^3
),x)
```

```
[Out] ((4*A*b^2*c^2 - 2*A*a^2*d^2 + B*a^2*d^2*n + 4*B*b^2*c^2*n + 10*A*a*b*c*d -
11*B*a*b*c*d*n)/(2*(a*d - b*c)) + (3*x^2*(2*A*b^2*d^2 - B*b^2*d^2*n))/(a*d
- b*c) + (3*x*(2*A*a*b*d^2 + 6*A*b^2*c*d - 3*B*a*b*d^2*n - B*b^2*c*d*n))/(2
*(a*d - b*c)))/(x*(2*b^3*c^4*g^2*i^3 + 4*a^3*c*d^3*g^2*i^3 - 6*a^2*b*c^2*d^
2*g^2*i^3) + x^2*(2*a^3*d^4*g^2*i^3 + 4*b^3*c^3*d*g^2*i^3 - 6*a*b^2*c^2*d^2
*g^2*i^3) + x^3*(2*b^3*c^2*d^2*g^2*i^3 + 2*a^2*b*d^4*g^2*i^3 - 4*a*b^2*c*d^
3*g^2*i^3) + 2*a^3*c^2*d^2*g^2*i^3 + 2*a*b^2*c^4*g^2*i^3 - 4*a^2*b*c^3*d*g^
2*i^3) - log(e*((a + b*x)/(c + d*x))^n)*(((B*(a*d + 2*b*c))/(2*(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d)) + (3*B*b*d*x)/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(x*
(b*c^2*g^2*i^3 + 2*a*c*d*g^2*i^3) + x^2*(a*d^2*g^2*i^3 + 2*b*c*d*g^2*i^3) +
 a*c^2*g^2*i^3 + b*d^2*g^2*i^3*x^3) - (3*B*b^2*d*(d*g^2*i^3*n*x^2*(a*d - b*
c) + (a*c*g^2*i^3*n*(a*d - b*c))/b + (g^2*i^3*n*x*(a*d + b*c)*(a*d - b*c))/
b))/(g^2*i^3*n*(a*d - b*c)^4*(x*(b*c^2*g^2*i^3 + 2*a*c*d*g^2*i^3) + x^2*(a*
d^2*g^2*i^3 + 2*b*c*d*g^2*i^3) + a*c^2*g^2*i^3 + b*d^2*g^2*i^3*x^3))) + (b^
2*d*atan((b^2*d*(2*A - B*n)*((a^4*d^4*g^2*i^3 - b^4*c^4*g^2*i^3 + 2*a*b^3*c
^3*d*g^2*i^3 - 2*a^3*b*c*d^3*g^2*i^3)/(a^3*d^3*g^2*i^3 - b^3*c^3*g^2*i^3 +
3*a*b^2*c^2*d*g^2*i^3 - 3*a^2*b*c*d^2*g^2*i^3) + 2*b*d*x)*(a^3*d^3*g^2*i^3
- b^3*c^3*g^2*i^3 + 3*a*b^2*c^2*d*g^2*i^3 - 3*a^2*b*c*d^2*g^2*i^3)*3i)/(g^2
*i^3*(6*A*b^2*d - 3*B*b^2*d*n)*(a*d - b*c)^4))*(2*A - B*n)*3i)/(g^2*i^3*(a*
```

```
d - b*c)^4) - (3*B*b^2*d*log(e*((a + b*x)/(c + d*x))^n)^2)/(2*g^2*i^3*n*(a*
d - b*c)^4)
```

3.157 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 483

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx &= -\frac{Bd^4n(a+bx)^2}{4(bc-ad)^5g^3i^3(c+dx)^2}+\frac{4bBd^3n(a+bx)}{(bc-ad)^5g^3i^3(c+dx)} \\ &+\frac{4b^3Bdn(c+dx)}{(bc-ad)^5g^3i^3(a+bx)}-\frac{b^4Bn(c+dx)^2}{4(bc-ad)^5g^3i^3(a+bx)^2} \\ &+\frac{d^4(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^5g^3i^3(c+dx)^2} \\ &-\frac{4bd^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^3i^3(c+dx)} \\ &+\frac{4b^3d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^3i^3(a+bx)} \\ &-\frac{b^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^5g^3i^3(a+bx)^2} \\ &+\frac{6b^2d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3} \\ &-\frac{3b^2Bd^2n\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}\end{aligned}$$

```
[Out] -1/4*B*d^4*n*(b*x+a)^2/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2+4*b*B*d^3*n*(b*x+a)/(
-a*d+b*c)^5/g^3/i^3/(d*x+c)+4*b^3*B*d*n*(d*x+c)/(-a*d+b*c)^5/g^3/i^3/(b*x+a
)-1/4*b^4*B*n*(d*x+c)^2/(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2+1/2*d^4*(b*x+a)^2*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2-4*b*d^3*(b*x+a
)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^3/i^3/(d*x+c)+4*b^3*d*(d*x
+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/2*b^4*(d
```

```
*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2+6*b^
2*d^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d+b*c)^5/g^3/
i^3-3*b^2*B*d^2*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^5/g^3/i^3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {2561, 45, 2372, 2338}

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}dx=&-\frac{b^4(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^3i^3(a+bx)^2(bc-ad)^5}\\&+\frac{4b^3d(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^3i^3(a+bx)(bc-ad)^5}\\&+\frac{6b^2d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^3i^3(bc-ad)^5}\\&+\frac{d^4(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^3i^3(c+dx)^2(bc-ad)^5}\\&-\frac{4bd^3(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^3i^3(c+dx)(bc-ad)^5}\\&-\frac{b^4Bn(c+dx)^2}{4g^3i^3(a+bx)^2(bc-ad)^5}\\&+\frac{4b^3Bdn(c+dx)}{g^3i^3(a+bx)(bc-ad)^5}-\frac{3b^2Bd^2n\log^2\left(\frac{a+bx}{c+dx}\right)}{g^3i^3(bc-ad)^5}\\&-\frac{Bd^4n(a+bx)^2}{4g^3i^3(c+dx)^2(bc-ad)^5}+\frac{4bBd^3n(a+bx)}{g^3i^3(c+dx)(bc-ad)^5}\end{aligned}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^3*(c*i + d*i*x)^3
),x]
```

```
[Out] -1/4*(B*d^4*n*(a + b*x)^2)/((b*c - a*d)^5*g^3*i^3*(c + d*x)^2) + (4*b*B*d^3
*n*(a + b*x))/((b*c - a*d)^5*g^3*i^3*(c + d*x)) + (4*b^3*B*d*n*(c + d*x))/(
(b*c - a*d)^5*g^3*i^3*(a + b*x)) - (b^4*B*n*(c + d*x)^2)/(4*(b*c - a*d)^5*g
^3*i^3*(a + b*x)^2) + (d^4*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]))/(2*(b*c - a*d)^5*g^3*i^3*(c + d*x)^2) - (4*b*d^3*(a + b*x)*(A + B*Log[e
*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^5*g^3*i^3*(c + d*x)) + (4*b^3*d*(c
 + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^5*g^3*i^3*(a +
 b*x)) - (b^4*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*c -
 a*d)^5*g^3*i^3*(a + b*x)^2) + (6*b^2*d^2*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])*Log[(a + b*x)/(c + d*x)])/((b*c - a*d)^5*g^3*i^3) - (3*b^2*B*d^2*n*Lo
g[(a + b*x)/(c + d*x)]^2)/((b*c - a*d)^5*g^3*i^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)^4(A+B\log(ex^n))}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3} \\
&= \frac{d^4(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^5g^3i^3(c+dx)^2} - \frac{4bd^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^3i^3(c+dx)} \\
&\quad + \frac{4b^3d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^5g^3i^3(a+bx)^2} \\
&\quad + \frac{6b^2d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3} \\
&\quad - \frac{(Bn)\text{Subst}\left(\int \left(\frac{1}{2}\left(-8bd^3-\frac{b^4}{x^3}+\frac{8b^3d}{x^2}+d^4x\right)+\frac{6b^2d^2\log(x)}{x}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}
\end{aligned}
$$

$$= \frac{d^4(a+bx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^5g^3i^3(c+dx)^2} - \frac{4bd^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^3i^3(c+dx)}$$
$$+ \frac{4b^3d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^5g^3i^3(a+bx)^2}$$
$$+ \frac{6b^2d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}$$
$$- \frac{(Bn)\text{Subst}\left(\int\left(-8bd^3-\frac{b^4}{x^3}+\frac{8b^3d}{x^2}+d^4x\right)dx,x,\frac{a+bx}{c+dx}\right)}{2(bc-ad)^5g^3i^3}$$
$$- \frac{(6b^2Bd^2n)\,\text{Subst}\left(\int\frac{\log(x)}{x}dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}$$

$$= -\frac{Bd^4n(a+bx)^2}{4(bc-ad)^5g^3i^3(c+dx)^2} + \frac{4bBd^3n(a+bx)}{(bc-ad)^5g^3i^3(c+dx)}$$
$$+ \frac{4b^3Bdn(c+dx)}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4Bn(c+dx)^2}{4(bc-ad)^5g^3i^3(a+bx)^2}$$
$$+ \frac{d^4(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^5g^3i^3(c+dx)^2} - \frac{4bd^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^3i^3(c+dx)}$$
$$+ \frac{4b^3d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^5g^3i^3(a+bx)^2}$$
$$+ \frac{6b^2d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3} - \frac{3b^2Bd^2n\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.68 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.16

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx =$$

$$-\frac{\frac{b^2B(bc-ad)^2n}{(a+bx)^2} - \frac{12b^3Bcdn}{a+bx} + \frac{12ab^2Bd^2n}{a+bx} - \frac{2b^2Bd(bc-ad)n}{a+bx} + \frac{Bd^2(bc-ad)^2n}{(c+dx)^2} + \frac{12b^2Bcd^2n}{c+dx} - \frac{12abBd^3n}{c+dx} + \frac{2bBd^2(bc-ad)n}{c+dx} + \cdots}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^3*(c*i + d*
i*x)^3),x]
```

```
[Out] -1/4*((b^2*B*(b*c - a*d)^2*n)/(a + b*x)^2 - (12*b^3*B*c*d*n)/(a + b*x) + (1
2*a*b^2*B*d^2*n)/(a + b*x) - (2*b^2*B*d*(b*c - a*d)*n)/(a + b*x) + (B*d^2*(
b*c - a*d)^2*n)/(c + d*x)^2 + (12*b^2*B*c*d^2*n)/(c + d*x) - (12*a*b*B*d^3*
n)/(c + d*x) + (2*b*B*d^2*(b*c - a*d)*n)/(c + d*x) + (2*b^2*(b*c - a*d)^2*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^2 - (12*b^2*d*(b*c - a*d)*
```

```
(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) - (2*d^2*(b*c - a*d)^2*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x)^2 - (12*b*d^2*(b*c - a*d)*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) - 24*b^2*d^2*Log[a + b*x]*
(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 24*b^2*d^2*(A + B*Log[e*((a + b*x)
/(c + d*x))^n])*Log[c + d*x] + 12*b^2*B*d^2*n*(Log[a + b*x]*(Log[a + b*x] -
 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a
*d)]) - 12*b^2*B*d^2*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x]
)*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)^5*g
^3*i^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1449 vs. $2(475) = 950$.

Time = 54.00 (sec) , antiderivative size = 1450, normalized size of antiderivative = 3.00

method	result	size
parallelrisch	Expression too large to display	1450

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x,method=_R
ETURNVERBOSE)
```

```
[Out] -1/4*(12*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)^2*b^8*d^8+24*A*x^4*ln(e*((b*x+a)/(
d*x+c))^n)*b^8*d^8-24*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^7*d^8*n+24*B*x^3*
ln(e*((b*x+a)/(d*x+c))^n)*b^8*c*d^7*n+48*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*
a*b^7*c*d^7-36*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^6*d^8*n+36*B*x^2*ln(e*
((b*x+a)/(d*x+c))^n)*b^8*c^2*d^6*n-24*B*x^2*a*b^7*c*d^7*n^2+96*A*x^2*ln(e*(
(b*x+a)/(d*x+c))^n)*a*b^7*c*d^7+24*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^6*
c*d^7+24*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^7*c^2*d^6-8*B*x*ln(e*((b*x+a)/
(d*x+c))^n)*a^3*b^5*d^8*n+8*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^8*c^3*d^5*n-12*
B*x*a^2*b^6*c*d^7*n^2-12*B*x*a*b^7*c^2*d^6*n^2+48*A*x*ln(e*((b*x+a)/(d*x+c)
)^n)*a^2*b^6*c*d^7+48*A*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^7*c^2*d^6-48*A*x*a^
2*b^6*c*d^7*n+48*A*x*a*b^7*c^2*d^6*n-16*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^5
*c*d^7*n+16*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^7*c^3*d^5*n+24*A*x^2*ln(e*((b*x
+a)/(d*x+c))^n)*a^2*b^6*d^8+24*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^8*c^2*d^6-
36*A*x^2*a^2*b^6*d^8*n+36*A*x^2*b^8*c^2*d^6*n+12*B*x*a^3*b^5*d^8*n^2+12*B*x
*b^8*c^3*d^5*n^2-8*A*x*a^3*b^5*d^8*n+8*A*x*b^8*c^3*d^5*n+12*B*ln(e*((b*x+a)
/(d*x+c))^n)^2*a^2*b^6*c^2*d^6+2*B*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^4*d^8*n-
2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^8*c^4*d^4*n+24*A*ln(e*((b*x+a)/(d*x+c))^n)*
a^2*b^6*c^2*d^6+16*B*a^3*b^5*c*d^7*n^2-30*B*a^2*b^6*c^2*d^6*n^2+16*B*a*b^7*
c^3*d^5*n^2-16*A*a^3*b^5*c*d^7*n+16*A*a*b^7*c^3*d^5*n-48*B*x*ln(e*((b*x+a)/
(d*x+c))^n)*a^2*b^6*c*d^7*n+48*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^7*c^2*d^6*
n-B*a^4*b^4*d^8*n^2-B*b^8*c^4*d^4*n^2+2*A*a^4*b^4*d^8*n-2*A*b^8*c^4*d^4*n+2
4*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^7*d^8+24*B*x^3*ln(e*((b*x+a)/(d*x+c
))^n)^2*b^8*c*d^7+48*A*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^7*d^8+48*A*x^3*ln(
e*((b*x+a)/(d*x+c))^n)*b^8*c*d^7-24*A*x^3*a*b^7*d^8*n+24*A*x^3*b^8*c*d^7*n+
```

```
12*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^6*d^8+12*B*x^2*ln(e*((b*x+a)/(d*
x+c))^n)^2*b^8*c^2*d^6+12*B*x^2*a^2*b^6*d^8*n^2+12*B*x^2*b^8*c^2*d^6*n^2)/i
^3/g^3/(d*x+c)^2/(b*x+a)^2/n/(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a
^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^4/d^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1416 vs. $2(475) = 950$.

Time = 0.43 (sec) , antiderivative size = 1416, normalized size of antiderivative = 2.93

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^3(ci + dix)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x, a
lgorithm="fricas")
```

```
[Out] -1/4*(2*A*b^4*c^4 - 16*A*a*b^3*c^3*d + 16*A*a^3*b*c*d^3 - 2*A*a^4*d^4 - 24*
(A*b^4*c*d^3 - A*a*b^3*d^4)*x^3 - 12*(3*A*b^4*c^2*d^2 - 3*A*a^2*b^2*d^4 + (
B*b^4*c^2*d^2 - 2*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*n)*x^2 - 12*(B*b^4*d^4*n*x
^4 + B*a^2*b^2*c^2*d^2*n + 2*(B*b^4*c*d^3 + B*a*b^3*d^4)*n*x^3 + (B*b^4*c^2
*d^2 + 4*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*n*x^2 + 2*(B*a*b^3*c^2*d^2 + B*a^2*
b^2*c*d^3)*n*x)*log((b*x + a)/(d*x + c))^2 + (B*b^4*c^4 - 16*B*a*b^3*c^3*d
+ 30*B*a^2*b^2*c^2*d^2 - 16*B*a^3*b*c*d^3 + B*a^4*d^4)*n - 4*(2*A*b^4*c^3*d
 + 12*A*a*b^3*c^2*d^2 - 12*A*a^2*b^2*c*d^3 - 2*A*a^3*b*d^4 + 3*(B*b^4*c^3*d
 - B*a*b^3*c^2*d^2 - B*a^2*b^2*c*d^3 + B*a^3*b*d^4)*n)*x + 2*(B*b^4*c^4 - 8
*B*a*b^3*c^3*d + 8*B*a^3*b*c*d^3 - B*a^4*d^4 - 12*(B*b^4*c*d^3 - B*a*b^3*d^
4)*x^3 - 18*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*x^2 - 4*(B*b^4*c^3*d + 6*B*a*b^
3*c^2*d^2 - 6*B*a^2*b^2*c*d^3 - B*a^3*b*d^4)*x - 12*(B*b^4*d^4*x^4 + B*a^2*
b^2*c^2*d^2 + 2*(B*b^4*c*d^3 + B*a*b^3*d^4)*x^3 + (B*b^4*c^2*d^2 + 4*B*a*b^
3*c*d^3 + B*a^2*b^2*d^4)*x^2 + 2*(B*a*b^3*c^2*d^2 + B*a^2*b^2*c*d^3)*x)*log
((b*x + a)/(d*x + c)))*log(e) - 2*(12*A*b^4*d^4*x^4 + 12*A*a^2*b^2*c^2*d^2
+ 12*(2*A*b^4*c*d^3 + 2*A*a*b^3*d^4 + (B*b^4*c*d^3 - B*a*b^3*d^4)*n)*x^3 +
6*(2*A*b^4*c^2*d^2 + 8*A*a*b^3*c*d^3 + 2*A*a^2*b^2*d^4 + 3*(B*b^4*c^2*d^2 -
 B*a^2*b^2*d^4)*n)*x^2 - (B*b^4*c^4 - 8*B*a*b^3*c^3*d + 8*B*a^3*b*c*d^3 - B
*a^4*d^4)*n + 4*(6*A*a*b^3*c^2*d^2 + 6*A*a^2*b^2*c*d^3 + (B*b^4*c^3*d + 6*B
*a*b^3*c^2*d^2 - 6*B*a^2*b^2*c*d^3 - B*a^3*b*d^4)*n)*x)*log((b*x + a)/(d*x
+ c)))/((b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^
2*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^7)*g^3*i^3*x^4 + 2*(b^7*c^6*d - 4*a*b^6
*c^5*d^2 + 5*a^2*b^5*c^4*d^3 - 5*a^4*b^3*c^2*d^5 + 4*a^5*b^2*c*d^6 - a^6*b*
d^7)*g^3*i^3*x^3 + (b^7*c^7 - a*b^6*c^6*d - 9*a^2*b^5*c^5*d^2 + 25*a^3*b^4*
c^4*d^3 - 25*a^4*b^3*c^3*d^4 + 9*a^5*b^2*c^2*d^5 + a^6*b*c*d^6 - a^7*d^7)*g
^3*i^3*x^2 + 2*(a*b^6*c^7 - 4*a^2*b^5*c^6*d + 5*a^3*b^4*c^5*d^2 - 5*a^5*b^2
*c^3*d^4 + 4*a^6*b*c^2*d^5 - a^7*c*d^6)*g^3*i^3*x + (a^2*b^5*c^7 - 5*a^3*b^
4*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c
^2*d^5)*g^3*i^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3/(d*i*x+c*i)**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2383 vs. $2(475) = 950$.

Time = 0.33 (sec), antiderivative size = 2383, normalized size of antiderivative = 4.93

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x, a
lgorithm="maxima")
[Out] 1/2*B*((12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d^3
+ 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 + a^2*b*d^3
)*x)/((b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5
+ a^4*b^2*d^6)*g^3*i^3*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4*c^3
*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*g^3*i^3*x^3 + (b^6*
c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d^6)
*g^3*i^3*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b
^2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*g^3*i^3*x + (a^2*b^4*c^6 - 4*a^3*
b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4)*g^3*i^3) + 1
2*b^2*d^2*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*
a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3) - 12*b^2*d^2*log(d*x +
c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*
a^4*b*c*d^4 - a^5*d^5)*g^3*i^3))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1
/4*(b^4*c^4 - 16*a*b^3*c^3*d + 30*a^2*b^2*c^2*d^2 - 16*a^3*b*c*d^3 + a^4*d^
4 - 12*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 12*(b^4*d^4*x^4 +
a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*
d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a)^
2 - 24*(b^4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^
4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c
*d^3)*x)*log(b*x + a)*log(d*x + c) + 12*(b^4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*
(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x
^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(d*x + c)^2 - 12*(b^4*c^3*d -
a*b^3*c^2*d^2 - a^2*b^2*c*d^3 + a^3*b*d^4)*x)*B*n/(a^2*b^5*c^7*g^3*i^3 - 5*
```

```
a^3*b^4*c^6*d*g^3*i^3 + 10*a^4*b^3*c^5*d^2*g^3*i^3 - 10*a^5*b^2*c^4*d^3*g^3
*i^3 + 5*a^6*b*c^3*d^4*g^3*i^3 - a^7*c^2*d^5*g^3*i^3 + (b^7*c^5*d^2*g^3*i^3
 - 5*a*b^6*c^4*d^3*g^3*i^3 + 10*a^2*b^5*c^3*d^4*g^3*i^3 - 10*a^3*b^4*c^2*d^
5*g^3*i^3 + 5*a^4*b^3*c*d^6*g^3*i^3 - a^5*b^2*d^7*g^3*i^3)*x^4 + 2*(b^7*c^6
*d*g^3*i^3 - 4*a*b^6*c^5*d^2*g^3*i^3 + 5*a^2*b^5*c^4*d^3*g^3*i^3 - 5*a^4*b^
3*c^2*d^5*g^3*i^3 + 4*a^5*b^2*c*d^6*g^3*i^3 - a^6*b*d^7*g^3*i^3)*x^3 + (b^7
*c^7*g^3*i^3 - a*b^6*c^6*d*g^3*i^3 - 9*a^2*b^5*c^5*d^2*g^3*i^3 + 25*a^3*b^4
*c^4*d^3*g^3*i^3 - 25*a^4*b^3*c^3*d^4*g^3*i^3 + 9*a^5*b^2*c^2*d^5*g^3*i^3 +
 a^6*b*c*d^6*g^3*i^3 - a^7*d^7*g^3*i^3)*x^2 + 2*(a*b^6*c^7*g^3*i^3 - 4*a^2*
b^5*c^6*d*g^3*i^3 + 5*a^3*b^4*c^5*d^2*g^3*i^3 - 5*a^5*b^2*c^3*d^4*g^3*i^3 +
 4*a^6*b*c^2*d^5*g^3*i^3 - a^7*c*d^6*g^3*i^3)*x) + 1/2*A*((12*b^3*d^3*x^3 -
 b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d^3 + 18*(b^3*c*d^2 + a*b^2*
d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 + a^2*b*d^3)*x)/((b^6*c^4*d^2 - 4*a
*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5 + a^4*b^2*d^6)*g^3*i^3*x
^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4
 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*g^3*i^3*x^3 + (b^6*c^6 - 9*a^2*b^4*c^4*d^2
+ 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d^6)*g^3*i^3*x^2 + 2*(a*b^5*
c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b^2*c^3*d^3 - 3*a^5*b*c^2
*d^4 + a^6*c*d^5)*g^3*i^3*x + (a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^
4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4)*g^3*i^3) + 12*b^2*d^2*log(b*x + a)/(
(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*
b*c*d^4 - a^5*d^5)*g^3*i^3) - 12*b^2*d^2*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c
^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g
^3*i^3))
```

Giac [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx=\int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{(bgx+ag)^3(dix+ci)^3}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x, a
lgorithm="giac")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/((b*g*x + a*g)^3*(d*i*x +
c*i)^3), x)
```

Mupad [B] (verification not implemented)

Time = 4.60 (sec) , antiderivative size = 1341, normalized size of antiderivative = 2.78

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx$$

$$= \frac{2\,x\,(\cdots}{x^4\,\left(2\,a^3\,b^2\,d^5\,g^3\,i^3-6\,a^2\,b^3\,c\,d^4\,g^3\,i^3+6\,a\,b^4\,c^2\,d^3\,g^3\,i^3-2\,b^5\,c^3\,d^2\,g^3\,i^3\right)-x\,\left(-4\,a^5\,c\,d^4\,g^3\,i^3+8\,a^4\,b\,c^2\,d^3\,g\cdots\right.}$$
$$+\frac{\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(x\,\left(\frac{3\,B\,b\,d\,(a\,d+b\,c)^2}{(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)^2}-\frac{B\,b\,d}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}+\frac{6\,B\,a\,b^2\,c\,d^2}{(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)^2}\right)-\frac{B\,(a\,d+b\,c)}{2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\cdots\right.}{x\,\left(2\,d\,a^2\,c\,g^3\,i^3+2\,b\,a\,c^2\,g^3\,i^3\right)+x^3\,\left(2\,c\,b^2\,d\,g^3\,i^3+2\,a\,b\,d^2\,g^3\,i^3\right)+x^2\,\left(a^2\,d^2\,g^3\,i^3+4\,a\,b\,c\cdots\right.}$$
$$-\frac{3\,B\,b^2\,d^2\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{g^3\,i^3\,n\,(a\,d-b\,c)^5}$$
$$+\frac{A\,b^2\,d^2\,\mathrm{atan}\left(\frac{(a^5\,d^5\,g^3\,i^3-3\,a^4\,b\,c\,d^4\,g^3\,i^3+2\,a^3\,b^2\,c^2\,d^3\,g^3\,i^3+2\,a^2\,b^3\,c^3\,d^2\,g^3\,i^3-3\,a\,b^4\,c^4\,d\,g^3\,i^3+b^5\,c^5\,g^3\,i^3)\,1\mathrm{i}}{g^3\,i^3\,(a\,d-b\,c)^5}+\frac{b\,d\,x\,(a^4\,d^4\,g^3\,i^3-4\,a^3\cdots}{\cdots}\right.}{g^3\,i^3\,(a\,d-b\,c)^5}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^3*(c*i + d*i*x)^3
),x)
```

```
[Out] ((2*x*(2*A*a^2*b*d^3 + 2*A*b^3*c^2*d + 14*A*a*b^2*c*d^2 - 3*B*a^2*b*d^3*n +
 3*B*b^3*c^2*d*n))/(a*d - b*c) - (2*A*a^3*d^3 + 2*A*b^3*c^3 - B*a^3*d^3*n +
 B*b^3*c^3*n - 14*A*a*b^2*c^2*d - 14*A*a^2*b*c*d^2 - 15*B*a*b^2*c^2*d*n + 1
5*B*a^2*b*c*d^2*n)/(2*(a*d - b*c)) + (6*x^2*(3*A*a*b^2*d^3 + 3*A*b^3*c*d^2
- B*a*b^2*d^3*n + B*b^3*c*d^2*n))/(a*d - b*c) + (12*A*b^3*d^3*x^3)/(a*d - b
*c))/(x^4*(2*a^3*b^2*d^5*g^3*i^3 - 2*b^5*c^3*d^2*g^3*i^3 + 6*a*b^4*c^2*d^3*
g^3*i^3 - 6*a^2*b^3*c*d^4*g^3*i^3) - x*(4*a*b^4*c^5*g^3*i^3 - 4*a^5*c*d^4*g
^3*i^3 - 8*a^2*b^3*c^4*d*g^3*i^3 + 8*a^4*b*c^2*d^3*g^3*i^3) + x^3*(4*a^4*b*
d^5*g^3*i^3 - 4*b^5*c^4*d*g^3*i^3 + 8*a*b^4*c^3*d^2*g^3*i^3 - 8*a^3*b^2*c*d
^4*g^3*i^3) + x^2*(2*a^5*d^5*g^3*i^3 - 2*b^5*c^5*g^3*i^3 - 2*a*b^4*c^4*d*g^
3*i^3 + 2*a^4*b*c*d^4*g^3*i^3 + 16*a^2*b^3*c^3*d^2*g^3*i^3 - 16*a^3*b^2*c^2
*d^3*g^3*i^3) - 2*a^2*b^3*c^5*g^3*i^3 + 2*a^5*c^2*d^3*g^3*i^3 + 6*a^3*b^2*c
^4*d*g^3*i^3 - 6*a^4*b*c^3*d^2*g^3*i^3) + (log(e*((a + b*x)/(c + d*x))^n)*(
x*((3*B*b*d*(a*d + b*c)^2)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2 - (B*b*d)/(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d) + (6*B*a*b^2*c*d^2)/(a^2*d^2 + b^2*c^2 - 2*a*b*
c*d)^2) - (B*(a*d + b*c))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (6*B*b^3*d^
3*x^3)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2 + (9*B*b^2*d^2*x^2*(a*d + b*c))/(a
^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2 + (3*B*a*b*c*d*(a*d + b*c))/(a^2*d^2 + b^2*
c^2 - 2*a*b*c*d)^2))/(x*(2*a*b*c^2*g^3*i^3 + 2*a^2*c*d*g^3*i^3) + x^3*(2*a*
b*d^2*g^3*i^3 + 2*b^2*c*d*g^3*i^3) + x^2*(a^2*d^2*g^3*i^3 + b^2*c^2*g^3*i^3
 + 4*a*b*c*d*g^3*i^3) + a^2*c^2*g^3*i^3 + b^2*d^2*g^3*i^3*x^4) + (A*b^2*d^2
*atan(((a^5*d^5*g^3*i^3 + b^5*c^5*g^3*i^3 - 3*a*b^4*c^4*d*g^3*i^3 - 3*a^4*b
*c*d^4*g^3*i^3 + 2*a^2*b^3*c^3*d^2*g^3*i^3 + 2*a^3*b^2*c^2*d^3*g^3*i^3)*1i)
/(g^3*i^3*(a*d - b*c)^5) + (b*d*x*(a^4*d^4*g^3*i^3 + b^4*c^4*g^3*i^3 - 4*a*
```

```
b^3*c^3*d*g^3*i^3 - 4*a^3*b*c*d^3*g^3*i^3 + 6*a^2*b^2*c^2*d^2*g^3*i^3)*2i)/
(g^3*i^3*(a*d - b*c)^5))*12i)/(g^3*i^3*(a*d - b*c)^5) - (3*B*b^2*d^2*log(e*
((a + b*x)/(c + d*x))^n)^2)/(g^3*i^3*n*(a*d - b*c)^5)
```

3.158 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 587

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx &= \frac{Bd^5n(a+bx)^2}{4(bc-ad)^6g^4i^3(c+dx)^2} - \frac{5bBd^4n(a+bx)}{(bc-ad)^6g^4i^3(c+dx)} \\ &- \frac{10b^3Bd^2n(c+dx)}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4Bdn(c+dx)^2}{4(bc-ad)^6g^4i^3(a+bx)^2} \\ &- \frac{b^5Bn(c+dx)^3}{9(bc-ad)^6g^4i^3(a+bx)^3} \\ &- \frac{d^5(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2} \\ &+ \frac{5bd^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(c+dx)} \\ &- \frac{10b^3d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(a+bx)} \\ &+ \frac{5b^4d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2} \\ &- \frac{b^5(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^6g^4i^3(a+bx)^3} \\ &- \frac{10b^2d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3} \\ &+ \frac{5b^2Bd^3n\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}\end{aligned}$$

```
[Out] 1/4*B*d^5*n*(b*x+a)^2/(-a*d+b*c)^6/g^4/i^3/(d*x+c)^2-5*b*B*d^4*n*(b*x+a)/(-
a*d+b*c)^6/g^4/i^3/(d*x+c)-10*b^3*B*d^2*n*(d*x+c)/(-a*d+b*c)^6/g^4/i^3/(b*x
```

```
+a)+5/4*b^4*B*d*n*(d*x+c)^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-1/9*b^5*B*n*(d*x
+c)^3/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^3-1/2*d^5*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(
d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(d*x+c)^2+5*b*d^4*(b*x+a)*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(d*x+c)-10*b^3*d^2*(d*x+c)*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(b*x+a)+5/2*b^4*d*(d*x+c)^2*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-1/3*b^5*(d*x+c)^3
*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^3-10*b^2*d^3*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d+b*c)^6/g^4/i^3+5*
b^2*B*d^3*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^6/g^4/i^3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {2561, 45, 2372, 12, 14, 2338}

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = -\frac{b^5(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3g^4i^3(a+bx)^3(bc-ad)^6} + \frac{5b^4d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^4i^3(a+bx)^2(bc-ad)^6} - \frac{10b^3d^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^4i^3(a+bx)(bc-ad)^6} - \frac{10b^2d^3\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^4i^3(bc-ad)^6} - \frac{d^5(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^4i^3(c+dx)^2(bc-ad)^6} + \frac{5bd^4(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^4i^3(c+dx)(bc-ad)^6} - \frac{b^5Bn(c+dx)^3}{9g^4i^3(a+bx)^3(bc-ad)^6} + \frac{5b^4Bdn(c+dx)^2}{4g^4i^3(a+bx)^2(bc-ad)^6} - \frac{10b^3Bd^2n(c+dx)}{g^4i^3(a+bx)(bc-ad)^6} + \frac{5b^2Bd^3n\log^2\left(\frac{a+bx}{c+dx}\right)}{g^4i^3(bc-ad)^6} + \frac{Bd^5n(a+bx)^2}{4g^4i^3(c+dx)^2(bc-ad)^6} - \frac{5bBd^4n(a+bx)}{g^4i^3(c+dx)(bc-ad)^6}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^4*(c*i + d*i*x)^3
),x]
```

```
[Out] (B*d^5*n*(a + b*x)^2)/(4*(b*c - a*d)^6*g^4*i^3*(c + d*x)^2) - (5*b*B*d^4*n*
(a + b*x))/((b*c - a*d)^6*g^4*i^3*(c + d*x)) - (10*b^3*B*d^2*n*(c + d*x))/(
(b*c - a*d)^6*g^4*i^3*(a + b*x)) + (5*b^4*B*d*n*(c + d*x)^2)/(4*(b*c - a*d)
^6*g^4*i^3*(a + b*x)^2) - (b^5*B*n*(c + d*x)^3)/(9*(b*c - a*d)^6*g^4*i^3*(a
```

```
 + b*x)^3) - (d^5*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b
*c - a*d)^6*g^4*i^3*(c + d*x)^2) + (5*b*d^4*(a + b*x)*(A + B*Log[e*((a + b*
x)/(c + d*x))^n]))/((b*c - a*d)^6*g^4*i^3*(c + d*x)) - (10*b^3*d^2*(c + d*x
)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^6*g^4*i^3*(a + b*x))
 + (5*b^4*d*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*c - a
*d)^6*g^4*i^3*(a + b*x)^2) - (b^5*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(3*(b*c - a*d)^6*g^4*i^3*(a + b*x)^3) - (10*b^2*d^3*(A + B*Log[e
*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)])/((b*c - a*d)^6*g^4*i^3
) + (5*b^2*B*d^3*n*Log[(a + b*x)/(c + d*x)]^2)/((b*c - a*d)^6*g^4*i^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
```

```
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)^5(A+B\log(ex^n))}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}$$

$$= -\frac{d^5(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2} + \frac{5bd^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(c+dx)} - \frac{10b^3d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2} - \frac{b^5(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^6g^4i^3(a+bx)^3} - \frac{10b^2d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3} - \frac{(Bn)\text{Subst}\left(\int \frac{-2b^5+15b^4dx-60b^3d^2x^2+30bd^4x^4-3d^5x^5-60b^2d^3x^3\log(x)}{6x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}$$

$$= -\frac{d^5(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2} + \frac{5bd^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(c+dx)} - \frac{10b^3d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2} - \frac{b^5(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^6g^4i^3(a+bx)^3} - \frac{10b^2d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3} - \frac{(Bn)\text{Subst}\left(\int \frac{-2b^5+15b^4dx-60b^3d^2x^2+30bd^4x^4-3d^5x^5-60b^2d^3x^3\log(x)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{6(bc-ad)^6g^4i^3}$$

$$= -\frac{d^5(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2} + \frac{5bd^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(c+dx)} - \frac{10b^3d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2} - \frac{b^5(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^6g^4i^3(a+bx)^3} - \frac{10b^2d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3} - \frac{(Bn)\text{Subst}\left(\int \left(\frac{-2b^5+15b^4dx-60b^3d^2x^2+30bd^4x^4-3d^5x^5}{x^4} - \frac{60b^2d^3\log(x)}{x}\right)dx, x, \frac{a+bx}{c+dx}\right)}{6(bc-ad)^6g^4i^3}$$

$$= -\frac{d^5(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2}+\frac{5bd^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(c+dx)}$$
$$-\frac{10b^3d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(a+bx)}+\frac{5b^4d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2}$$
$$-\frac{b^5(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^6g^4i^3(a+bx)^3}-\frac{10b^2d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}$$
$$-\frac{(Bn)\text{Subst}\left(\int\frac{-2b^5+15b^4dx-60b^3d^2x^2+30bd^4x^4-3d^5x^5}{x^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{6(bc-ad)^6g^4i^3}$$
$$+\frac{(10b^2Bd^3n)\,\text{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}$$

$$= -\frac{d^5(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2}+\frac{5bd^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(c+dx)}$$
$$-\frac{10b^3d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(a+bx)}+\frac{5b^4d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2}$$
$$-\frac{b^5(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^6g^4i^3(a+bx)^3}-\frac{10b^2d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}$$
$$+\frac{5b^2Bd^3n\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}-\frac{(Bn)\text{Subst}\left(\int\left(30bd^4-\frac{2b^5}{x^4}+\frac{15b^4d}{x^3}-\frac{60b^3d^2}{x^2}-3d^5x\right)dx,x,\frac{a+bx}{c+dx}\right)}{6(bc-ad)^6g^4i^3}$$

$$= \frac{Bd^5n(a+bx)^2}{4(bc-ad)^6g^4i^3(c+dx)^2}-\frac{5bBd^4n(a+bx)}{(bc-ad)^6g^4i^3(c+dx)}$$
$$-\frac{10b^3Bd^2n(c+dx)}{(bc-ad)^6g^4i^3(a+bx)}+\frac{5b^4Bdn(c+dx)^2}{4(bc-ad)^6g^4i^3(a+bx)^2}$$
$$-\frac{b^5Bn(c+dx)^3}{9(bc-ad)^6g^4i^3(a+bx)^3}-\frac{d^5(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2}$$
$$+\frac{5bd^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(c+dx)}-\frac{10b^3d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(a+bx)}$$
$$+\frac{5b^4d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2}-\frac{b^5(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^6g^4i^3(a+bx)^3}$$
$$-\frac{10b^2d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}+\frac{5b^2Bd^3n\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.91 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.14

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx =$$

$$-\frac{\frac{4b^2B(bc-ad)^3n}{(a+bx)^3}-\frac{33b^2Bd(bc-ad)^2n}{(a+bx)^2}+\frac{216b^3Bcd^2n}{a+bx}-\frac{216ab^2Bd^3n}{a+bx}+\frac{66b^2Bd^2(bc-ad)n}{a+bx}-\frac{9Bd^3(bc-ad)^2n}{(c+dx)^2}-\frac{144b^2Bcd^3n}{c+dx}+\frac{144}{c}}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^4*(c*i + d*
i*x)^3),x]
```

```
[Out] -1/36*((4*b^2*B*(b*c - a*d)^3*n)/(a + b*x)^3 - (33*b^2*B*d*(b*c - a*d)^2*n)
/(a + b*x)^2 + (216*b^3*B*c*d^2*n)/(a + b*x) - (216*a*b^2*B*d^3*n)/(a + b*x
) + (66*b^2*B*d^2*(b*c - a*d)*n)/(a + b*x) - (9*B*d^3*(b*c - a*d)^2*n)/(c +
 d*x)^2 - (144*b^2*B*c*d^3*n)/(c + d*x) + (144*a*b*B*d^4*n)/(c + d*x) - (18
*b*B*d^3*(b*c - a*d)*n)/(c + d*x) + 120*b^2*B*d^3*n*Log[a + b*x] + (12*b^2*
(b*c - a*d)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^3 - (54*b^2
*d*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^2 + (216
*b^2*d^2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + (1
8*d^3*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x)^2 + (
144*b*d^3*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) + 3
60*b^2*d^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 120*b^2*B*
d^3*n*Log[c + d*x] - 360*b^2*d^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log
[c + d*x] - 180*b^2*B*d^3*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x
))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 180*b^2*B*
d^3*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] +
2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)^6*g^4*i^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2148 vs. 2(575) = 1150.

Time = 206.92 (sec) , antiderivative size = 2149, normalized size of antiderivative = 3.66

method	result	size
parallelrisch	Expression too large to display	2149

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x,method=_R
ETURNVERBOSE)
```

```
[Out] -1/36*(-180*B*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^6*c*d^8*n+360*B*ln(e*((b*x+a)
/(d*x+c))^n)*a^2*b^8*c^3*d^6*n-90*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9*c^4*d^5
```

*n+1080*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9*c*d^8*n+1620*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9*c^2*d^7*n-720*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^7*c*d^8*n+1080*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^8*c^2*d^7*n+360*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9*c^3*d^6*n-9*B*a^5*b^5*d^9*n^2+4*B*b^10*c^5*d^4*n^2+18*A*a^5*b^5*d^9*n+12*A*b^10*c^5*d^4*n+180*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)^2*b^10*d^9+360*A*x^5*ln(e*((b*x+a)/(d*x+c))^n)*b^10*d^9+600*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^10*c*d^8*n+1080*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^9*c*d^8-540*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^8*d^9*n+660*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^10*c^2*d^7*n-240*B*x^3*a*b^9*c*d^8*n^2+2160*A*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9*c*d^8+1260*A*x^2*a*b^9*c^2*d^7*n+360*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^7*c*d^8+540*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^8*c^2*d^7-780*B*x^2*a^2*b^8*c*d^8*n^2+420*B*x^2*a*b^9*c^2*d^7*n^2+2160*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^8*c*d^8+1080*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9*c^2*d^7-720*A*x^2*a^2*b^8*c*d^8*n-90*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^6*d^9*n-30*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^10*c^4*d^5*n+360*A*x^3*a*b^9*c*d^8*n+1080*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^8*c*d^8+540*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^9*c^2*d^7-540*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^7*d^9*n+120*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^10*c^3*d^6*n-260*B*x*a^3*b^7*c*d^8*n^2-390*B*x*a^2*b^8*c^2*d^7*n^2+540*B*x*a*b^9*c^3*d^6*n^2+720*A*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^7*c*d^8+1080*A*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^8*c^2*d^7-960*A*x*a^3*b^7*c*d^8*n+720*A*x*a^2*b^8*c^2*d^7*n+360*A*x*a*b^9*c^3*d^6*n-120*B*x^4*a*b^9*d^9*n^2+120*B*x^4*b^10*c*d^8*n^2+1080*A*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9*d^9+720*A*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^10*c*d^8-360*A*x^4*a*b^9*d^9*n+360*A*x^4*b^10*c*d^8*n+540*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^8*d^9+180*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b^10*c^2*d^7-120*B*x^3*a^2*b^8*d^9*n^2+360*B*x^3*b^10*c^2*d^7*n^2+1080*A*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^8*d^9+360*A*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^10*c^2*d^7-900*A*x^3*a^2*b^8*d^9*n+540*A*x^3*b^10*c^2*d^7*n+180*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^7*d^9+140*B*x^2*a^3*b^7*d^9*n^2+220*B*x^2*b^10*c^3*d^6*n^2+360*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^7*d^9-660*A*x^2*a^3*b^7*d^9*n+120*A*x^2*b^10*c^3*d^6*n+135*B*x*a^4*b^6*d^9*n^2-25*B*x*b^10*c^4*d^5*n^2-90*A*x*a^4*b^6*d^9*n-30*A*x*b^10*c^4*d^5*n+180*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^7*c^2*d^7+18*B*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^5*d^9*n+12*B*ln(e*((b*x+a)/(d*x+c))^n)*b^10*c^5*d^4*n+360*A*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^7*c^2*d^7+120*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*b^10*d^9*n+540*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^9*d^9+360*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)^2*b^10*c*d^8+180*B*a^4*b^6*c*d^8*n^2-490*B*a^3*b^7*c^2*d^7*n^2+360*B*a^2*b^8*c^3*d^6*n^2-45*B*a*b^9*c^4*d^5*n^2-180*A*a^4*b^6*c*d^8*n-120*A*a^3*b^7*c^2*d^7*n+360*A*a^2*b^8*c^3*d^6*n-90*A*a*b^9*c^4*d^5*n)/i^3/g^4/(d*x+c)^2/(b*x+a)^3/(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(a*d-b*c)/b^5/d^4/n

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2181 vs. $2(575) = 1150$.

Time = 0.46 (sec) , antiderivative size = 2181, normalized size of antiderivative = 3.72

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x, a
lgorithm="fricas")
```

```
[Out] -1/36*(12*A*b^5*c^5 - 90*A*a*b^4*c^4*d + 360*A*a^2*b^3*c^3*d^2 - 120*A*a^3*
b^2*c^2*d^3 - 180*A*a^4*b*c*d^4 + 18*A*a^5*d^5 + 120*(3*A*b^5*c*d^4 - 3*A*a
*b^4*d^5 + (B*b^5*c*d^4 - B*a*b^4*d^5)*n)*x^4 + 60*(9*A*b^5*c^2*d^3 + 6*A*a
*b^4*c*d^4 - 15*A*a^2*b^3*d^5 + 2*(3*B*b^5*c^2*d^3 - 2*B*a*b^4*c*d^4 - B*a^
2*b^3*d^5)*n)*x^3 + 20*(6*A*b^5*c^3*d^2 + 63*A*a*b^4*c^2*d^3 - 36*A*a^2*b^3
*c*d^4 - 33*A*a^3*b^2*d^5 + (11*B*b^5*c^3*d^2 + 21*B*a*b^4*c^2*d^3 - 39*B*a
^2*b^3*c*d^4 + 7*B*a^3*b^2*d^5)*n)*x^2 + 180*(B*b^5*d^5*n*x^5 + B*a^3*b^2*c
^2*d^3*n + (2*B*b^5*c*d^4 + 3*B*a*b^4*d^5)*n*x^4 + (B*b^5*c^2*d^3 + 6*B*a*b
^4*c*d^4 + 3*B*a^2*b^3*d^5)*n*x^3 + (3*B*a*b^4*c^2*d^3 + 6*B*a^2*b^3*c*d^4
+ B*a^3*b^2*d^5)*n*x^2 + (3*B*a^2*b^3*c^2*d^3 + 2*B*a^3*b^2*c*d^4)*n*x)*log
((b*x + a)/(d*x + c))^2 + (4*B*b^5*c^5 - 45*B*a*b^4*c^4*d + 360*B*a^2*b^3*c
^3*d^2 - 490*B*a^3*b^2*c^2*d^3 + 180*B*a^4*b*c*d^4 - 9*B*a^5*d^5)*n - 5*(6*
A*b^5*c^4*d - 72*A*a*b^4*c^3*d^2 - 144*A*a^2*b^3*c^2*d^3 + 192*A*a^3*b^2*c*
d^4 + 18*A*a^4*b*d^5 + (5*B*b^5*c^4*d - 108*B*a*b^4*c^3*d^2 + 78*B*a^2*b^3*
c^2*d^3 + 52*B*a^3*b^2*c*d^4 - 27*B*a^4*b*d^5)*n)*x + 6*(2*B*b^5*c^5 - 15*B
*a*b^4*c^4*d + 60*B*a^2*b^3*c^3*d^2 - 20*B*a^3*b^2*c^2*d^3 - 30*B*a^4*b*c*d
^4 + 3*B*a^5*d^5 + 60*(B*b^5*c*d^4 - B*a*b^4*d^5)*x^4 + 30*(3*B*b^5*c^2*d^3
 + 2*B*a*b^4*c*d^4 - 5*B*a^2*b^3*d^5)*x^3 + 10*(2*B*b^5*c^3*d^2 + 21*B*a*b^
4*c^2*d^3 - 12*B*a^2*b^3*c*d^4 - 11*B*a^3*b^2*d^5)*x^2 - 5*(B*b^5*c^4*d - 1
2*B*a*b^4*c^3*d^2 - 24*B*a^2*b^3*c^2*d^3 + 32*B*a^3*b^2*c*d^4 + 3*B*a^4*b*d
^5)*x + 60*(B*b^5*d^5*x^5 + B*a^3*b^2*c^2*d^3 + (2*B*b^5*c*d^4 + 3*B*a*b^4*
d^5)*x^4 + (B*b^5*c^2*d^3 + 6*B*a*b^4*c*d^4 + 3*B*a^2*b^3*d^5)*x^3 + (3*B*a
*b^4*c^2*d^3 + 6*B*a^2*b^3*c*d^4 + B*a^3*b^2*d^5)*x^2 + (3*B*a^2*b^3*c^2*d^
3 + 2*B*a^3*b^2*c*d^4)*x)*log((b*x + a)/(d*x + c)))*log(e) + 6*(60*A*a^3*b^
2*c^2*d^3 + 20*(B*b^5*d^5*n + 3*A*b^5*d^5)*x^5 + 20*(5*B*b^5*c*d^4*n + 6*A*
b^5*c*d^4 + 9*A*a*b^4*d^5)*x^4 + 10*(6*A*b^5*c^2*d^3 + 36*A*a*b^4*c*d^4 + 1
8*A*a^2*b^3*d^5 + (11*B*b^5*c^2*d^3 + 18*B*a*b^4*c*d^4 - 9*B*a^2*b^3*d^5)*n
)*x^3 + 10*(18*A*a*b^4*c^2*d^3 + 36*A*a^2*b^3*c*d^4 + 6*A*a^3*b^2*d^5 + (2*
B*b^5*c^3*d^2 + 27*B*a*b^4*c^2*d^3 - 9*B*a^3*b^2*d^5)*n)*x^2 + (2*B*b^5*c^5
 - 15*B*a*b^4*c^4*d + 60*B*a^2*b^3*c^3*d^2 - 30*B*a^4*b*c*d^4 + 3*B*a^5*d^5
)*n + 5*(36*A*a^2*b^3*c^2*d^3 + 24*A*a^3*b^2*c*d^4 - (B*b^5*c^4*d - 12*B*a*
b^4*c^3*d^2 - 36*B*a^2*b^3*c^2*d^3 + 24*B*a^3*b^2*c*d^4 + 3*B*a^4*b*d^5)*n)
*x)*log((b*x + a)/(d*x + c)))/((b^9*c^6*d^2 - 6*a*b^8*c^5*d^3 + 15*a^2*b^7*
c^4*d^4 - 20*a^3*b^6*c^3*d^5 + 15*a^4*b^5*c^2*d^6 - 6*a^5*b^4*c*d^7 + a^6*b
```

```
^3*d^8)*g^4*i^3*x^5 + (2*b^9*c^7*d - 9*a*b^8*c^6*d^2 + 12*a^2*b^7*c^5*d^3 +
 5*a^3*b^6*c^4*d^4 - 30*a^4*b^5*c^3*d^5 + 33*a^5*b^4*c^2*d^6 - 16*a^6*b^3*c
*d^7 + 3*a^7*b^2*d^8)*g^4*i^3*x^4 + (b^9*c^8 - 18*a^2*b^7*c^6*d^2 + 52*a^3*
b^6*c^5*d^3 - 60*a^4*b^5*c^4*d^4 + 24*a^5*b^4*c^3*d^5 + 10*a^6*b^3*c^2*d^6
- 12*a^7*b^2*c*d^7 + 3*a^8*b*d^8)*g^4*i^3*x^3 + (3*a*b^8*c^8 - 12*a^2*b^7*c
^7*d + 10*a^3*b^6*c^6*d^2 + 24*a^4*b^5*c^5*d^3 - 60*a^5*b^4*c^4*d^4 + 52*a^
6*b^3*c^3*d^5 - 18*a^7*b^2*c^2*d^6 + a^9*d^8)*g^4*i^3*x^2 + (3*a^2*b^7*c^8
- 16*a^3*b^6*c^7*d + 33*a^4*b^5*c^6*d^2 - 30*a^5*b^4*c^5*d^3 + 5*a^6*b^3*c^
4*d^4 + 12*a^7*b^2*c^3*d^5 - 9*a^8*b*c^2*d^6 + 2*a^9*c*d^7)*g^4*i^3*x + (a^
3*b^6*c^8 - 6*a^4*b^5*c^7*d + 15*a^5*b^4*c^6*d^2 - 20*a^6*b^3*c^5*d^3 + 15*
a^7*b^2*c^4*d^4 - 6*a^8*b*c^3*d^5 + a^9*c^2*d^6)*g^4*i^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4/(d*i*x+c*i)**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3819 vs. $2(575) = 1150$.

Time = 0.53 (sec) , antiderivative size = 3819, normalized size of antiderivative = 6.51

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x, a
lgorithm="maxima")
[Out] -1/6*B*((60*b^4*d^4*x^4 + 2*b^4*c^4 - 13*a*b^3*c^3*d + 47*a^2*b^2*c^2*d^2 +
 27*a^3*b*c*d^3 - 3*a^4*d^4 + 30*(3*b^4*c*d^3 + 5*a*b^3*d^4)*x^3 + 10*(2*b^
4*c^2*d^2 + 23*a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2 - 5*(b^4*c^3*d - 11*a*b^3*
c^2*d^2 - 35*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x)/((b^8*c^5*d^2 - 5*a*b^7*c^4*d^
3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*b^3*d^7
)*g^4*i^3*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 + 10*a^3
*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7)*g^4*i
^3*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c^4*d^3 -
 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d^7)*g^4*
i^3*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a^4*b^4*c^4
*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a^8*d^7)*g^4
```

*i^3*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5*d^2 - 10*a^5*
b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*d^6)*g^4*i^3*x
+ (a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*a^6*b^2*c^4*d^3
+ 5*a^7*b*c^3*d^4 - a^8*c^2*d^5)*g^4*i^3) + 60*b^2*d^3*log(b*x + a)/((b^6*c
^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c
^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3) - 60*b^2*d^3*log(d*x + c)/((b^6*
c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*
c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3))*log(e*(b*x/(d*x + c) + a/(d*x
+ c))^n) - 1/36*(4*b^5*c^5 - 45*a*b^4*c^4*d + 360*a^2*b^3*c^3*d^2 - 490*a^3
*b^2*c^2*d^3 + 180*a^4*b*c*d^4 - 9*a^5*d^5 + 120*(b^5*c*d^4 - a*b^4*d^5)*x^
4 + 120*(3*b^5*c^2*d^3 - 2*a*b^4*c*d^4 - a^2*b^3*d^5)*x^3 + 20*(11*b^5*c^3*
d^2 + 21*a*b^4*c^2*d^3 - 39*a^2*b^3*c*d^4 + 7*a^3*b^2*d^5)*x^2 - 180*(b^5*d
^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 +
6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 +
a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a)^2
- 180*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b
^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*
b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log
(d*x + c)^2 - 5*(5*b^5*c^4*d - 108*a*b^4*c^3*d^2 + 78*a^2*b^3*c^2*d^3 + 52*
a^3*b^2*c*d^4 - 27*a^4*b*d^5)*x + 120*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b
^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)
*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c
^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a) - 120*(b^5*d^5*x^5 + a^3*b^2*c^2*
d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^
2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3
*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x - 3*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 +
(2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*
d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b
^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a))*log(d*x + c))*B*n/(a^3*b^6*c
^8*g^4*i^3 - 6*a^4*b^5*c^7*d*g^4*i^3 + 15*a^5*b^4*c^6*d^2*g^4*i^3 - 20*a^6*
b^3*c^5*d^3*g^4*i^3 + 15*a^7*b^2*c^4*d^4*g^4*i^3 - 6*a^8*b*c^3*d^5*g^4*i^3
+ a^9*c^2*d^6*g^4*i^3 + (b^9*c^6*d^2*g^4*i^3 - 6*a*b^8*c^5*d^3*g^4*i^3 + 15
*a^2*b^7*c^4*d^4*g^4*i^3 - 20*a^3*b^6*c^3*d^5*g^4*i^3 + 15*a^4*b^5*c^2*d^6*
g^4*i^3 - 6*a^5*b^4*c*d^7*g^4*i^3 + a^6*b^3*d^8*g^4*i^3)*x^5 + (2*b^9*c^7*d
*g^4*i^3 - 9*a*b^8*c^6*d^2*g^4*i^3 + 12*a^2*b^7*c^5*d^3*g^4*i^3 + 5*a^3*b^6
*c^4*d^4*g^4*i^3 - 30*a^4*b^5*c^3*d^5*g^4*i^3 + 33*a^5*b^4*c^2*d^6*g^4*i^3
- 16*a^6*b^3*c*d^7*g^4*i^3 + 3*a^7*b^2*d^8*g^4*i^3)*x^4 + (b^9*c^8*g^4*i^3
- 18*a^2*b^7*c^6*d^2*g^4*i^3 + 52*a^3*b^6*c^5*d^3*g^4*i^3 - 60*a^4*b^5*c^4*
d^4*g^4*i^3 + 24*a^5*b^4*c^3*d^5*g^4*i^3 + 10*a^6*b^3*c^2*d^6*g^4*i^3 - 12*
a^7*b^2*c*d^7*g^4*i^3 + 3*a^8*b*d^8*g^4*i^3)*x^3 + (3*a*b^8*c^8*g^4*i^3 - 1
2*a^2*b^7*c^7*d*g^4*i^3 + 10*a^3*b^6*c^6*d^2*g^4*i^3 + 24*a^4*b^5*c^5*d^3*g
^4*i^3 - 60*a^5*b^4*c^4*d^4*g^4*i^3 + 52*a^6*b^3*c^3*d^5*g^4*i^3 - 18*a^7*b
^2*c^2*d^6*g^4*i^3 + a^9*d^8*g^4*i^3)*x^2 + (3*a^2*b^7*c^8*g^4*i^3 - 16*a^3
*b^6*c^7*d*g^4*i^3 + 33*a^4*b^5*c^6*d^2*g^4*i^3 - 30*a^5*b^4*c^5*d^3*g^4*i^
3 + 5*a^6*b^3*c^4*d^4*g^4*i^3 + 12*a^7*b^2*c^3*d^5*g^4*i^3 - 9*a^8*b*c^2*d^

```
6*g^4*i^3 + 2*a^9*c*d^7*g^4*i^3)*x) - 1/6*A*((60*b^4*d^4*x^4 + 2*b^4*c^4 -
13*a*b^3*c^3*d + 47*a^2*b^2*c^2*d^2 + 27*a^3*b*c*d^3 - 3*a^4*d^4 + 30*(3*b^
4*c*d^3 + 5*a*b^3*d^4)*x^3 + 10*(2*b^4*c^2*d^2 + 23*a*b^3*c*d^3 + 11*a^2*b^
2*d^4)*x^2 - 5*(b^4*c^3*d - 11*a*b^3*c^2*d^2 - 35*a^2*b^2*c*d^3 - 3*a^3*b*d
^4)*x)/((b^8*c^5*d^2 - 5*a*b^7*c^4*d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^
2*d^5 + 5*a^4*b^4*c*d^6 - a^5*b^3*d^7)*g^4*i^3*x^5 + (2*b^8*c^6*d - 7*a*b^7
*c^5*d^2 + 5*a^2*b^6*c^4*d^3 + 10*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13
*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7)*g^4*i^3*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a
^2*b^6*c^5*d^2 + 35*a^3*b^5*c^4*d^3 - 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5
+ 9*a^6*b^2*c*d^6 - 3*a^7*b*d^7)*g^4*i^3*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6
*d + a^3*b^5*c^5*d^2 + 25*a^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2
*c^2*d^5 - a^7*b*c*d^6 - a^8*d^7)*g^4*i^3*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5
*c^6*d + 20*a^4*b^4*c^5*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^
7*b*c^2*d^5 - 2*a^8*c*d^6)*g^4*i^3*x + (a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*
a^5*b^3*c^5*d^2 - 10*a^6*b^2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5)*g^4*i
^3) + 60*b^2*d^3*log(b*x + a)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^
2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*
i^3) - 60*b^2*d^3*log(d*x + c)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d
^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4
*i^3))
```

Giac [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx=\int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{(bgx+ag)^4(dix+ci)^3}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x, a
lgorithm="giac")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/((b*g*x + a*g)^4*(d*i*x +
c*i)^3), x)
```

Mupad [B] (verification not implemented)

Time = 7.38 (sec) , antiderivative size = 2400, normalized size of antiderivative = 4.09

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx=\text{Too large to display}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^4*(c*i + d*i*x)^3
),x)
```

```
[Out] log(e*((a + b*x)/(c + d*x))^n)*((x*((5*B*(2*a*b*d^2 + b^2*c*d)*(a*d + b*c))
/(3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) - (5*B*b*d)/(6*(a^2*d^2 + b^2*c^2 -
```

2*a*b*c*d)) + (5*B*a*b^2*c*d^2)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) + x^2*((
5*B*b*d*(2*a*b*d^2 + b^2*c*d))/(3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) + (5*B
*b^2*d^2*(a*d + b*c))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) - (B*(3*a*d + 2*b*
c))/(6*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (5*B*a*c*(2*a*b*d^2 + b^2*c*d))/(
3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) + (5*B*b^3*d^3*x^3)/(a^2*d^2 + b^2*c^2
 - 2*a*b*c*d)^2)/(x*(2*a^3*c*d*g^4*i^3 + 3*a^2*b*c^2*g^4*i^3) + x^2*(a^3*d^
2*g^4*i^3 + 3*a*b^2*c^2*g^4*i^3 + 6*a^2*b*c*d*g^4*i^3) + x^3*(b^3*c^2*g^4*i
^3 + 3*a^2*b*d^2*g^4*i^3 + 6*a*b^2*c*d*g^4*i^3) + x^4*(2*b^3*c*d*g^4*i^3 +
3*a*b^2*d^2*g^4*i^3) + a^3*c^2*g^4*i^3 + b^3*d^2*g^4*i^3*x^5) + (10*B*b^2*d
^3*(x^2*((g^4*i^3*n*(a*d + b*c)^2*(a*d - b*c))/d + 2*a*b*c*g^4*i^3*n*(a*d -
 b*c)) + b^2*d*g^4*i^3*n*x^4*(a*d - b*c) + (a^2*c^2*g^4*i^3*n*(a*d - b*c))/
d + 2*b*g^4*i^3*n*x^3*(a*d + b*c)*(a*d - b*c) + (2*a*c*g^4*i^3*n*x*(a*d + b
c)(a*d - b*c))/d))/(g^4*i^3*n*(a*d - b*c)^6*(x*(2*a^3*c*d*g^4*i^3 + 3*a^2
*b*c^2*g^4*i^3) + x^2*(a^3*d^2*g^4*i^3 + 3*a*b^2*c^2*g^4*i^3 + 6*a^2*b*c*d*
g^4*i^3) + x^3*(b^3*c^2*g^4*i^3 + 3*a^2*b*d^2*g^4*i^3 + 6*a*b^2*c*d*g^4*i^3
) + x^4*(2*b^3*c*d*g^4*i^3 + 3*a*b^2*d^2*g^4*i^3) + a^3*c^2*g^4*i^3 + b^3*d
^2*g^4*i^3*x^5))) + ((12*A*b^4*c^4 - 18*A*a^4*d^4 + 9*B*a^4*d^4*n + 4*B*b^4
*c^4*n + 282*A*a^2*b^2*c^2*d^2 - 78*A*a*b^3*c^3*d + 162*A*a^3*b*c*d^3 + 319
*B*a^2*b^2*c^2*d^2*n - 41*B*a*b^3*c^3*d*n - 171*B*a^3*b*c*d^3*n)/(6*(a*d -
b*c)) + (5*x*(18*A*a^3*b*d^4 - 6*A*b^4*c^3*d + 66*A*a*b^3*c^2*d^2 + 210*A*a
^2*b^2*c*d^3 - 27*B*a^3*b*d^4*n - 5*B*b^4*c^3*d*n + 103*B*a*b^3*c^2*d^2*n +
 25*B*a^2*b^2*c*d^3*n))/(6*(a*d - b*c)) + (20*x^4*(3*A*b^4*d^4 + B*b^4*d^4*
n))/(a*d - b*c) + (10*x^2*(33*A*a^2*b^2*d^4 + 6*A*b^4*c^2*d^2 - 7*B*a^2*b^2
*d^4*n + 11*B*b^4*c^2*d^2*n + 69*A*a*b^3*c*d^3 + 32*B*a*b^3*c*d^3*n))/(3*(a
*d - b*c)) + (10*x^3*(15*A*a*b^3*d^4 + 9*A*b^4*c*d^3 + 2*B*a*b^3*d^4*n + 6*
B*b^4*c*d^3*n))/(a*d - b*c))/(x^5*(6*a^4*b^3*d^6*g^4*i^3 + 6*b^7*c^4*d^2*g^
4*i^3 - 24*a*b^6*c^3*d^3*g^4*i^3 - 24*a^3*b^4*c*d^5*g^4*i^3 + 36*a^2*b^5*c^
2*d^4*g^4*i^3) + x*(18*a^2*b^5*c^6*g^4*i^3 + 12*a^7*c*d^5*g^4*i^3 - 60*a^3*
b^4*c^5*d*g^4*i^3 - 30*a^6*b*c^2*d^4*g^4*i^3 + 60*a^4*b^3*c^4*d^2*g^4*i^3)
+ x^2*(6*a^7*d^6*g^4*i^3 + 18*a*b^6*c^6*g^4*i^3 + 12*a^6*b*c*d^5*g^4*i^3 -
36*a^2*b^5*c^5*d*g^4*i^3 - 30*a^3*b^4*c^4*d^2*g^4*i^3 + 120*a^4*b^3*c^3*d^3
*g^4*i^3 - 90*a^5*b^2*c^2*d^4*g^4*i^3) + x^3*(6*b^7*c^6*g^4*i^3 + 18*a^6*b*
d^6*g^4*i^3 + 12*a*b^6*c^5*d*g^4*i^3 - 36*a^5*b^2*c*d^5*g^4*i^3 - 90*a^2*b^
5*c^4*d^2*g^4*i^3 + 120*a^3*b^4*c^3*d^3*g^4*i^3 - 30*a^4*b^3*c^2*d^4*g^4*i^
3) + x^4*(18*a^5*b^2*d^6*g^4*i^3 + 12*b^7*c^5*d*g^4*i^3 - 30*a*b^6*c^4*d^2*
g^4*i^3 - 60*a^4*b^3*c*d^5*g^4*i^3 + 60*a^3*b^4*c^2*d^4*g^4*i^3) + 6*a^3*b^
4*c^6*g^4*i^3 + 6*a^7*c^2*d^4*g^4*i^3 - 24*a^4*b^3*c^5*d*g^4*i^3 - 24*a^6*b
*c^3*d^3*g^4*i^3 + 36*a^5*b^2*c^4*d^2*g^4*i^3) + (b^2*d^3*atan((b^2*d^3*(3*
A + B*n)*((a^6*d^6*g^4*i^3 - b^6*c^6*g^4*i^3 + 4*a*b^5*c^5*d*g^4*i^3 - 4*a^
5*b*c*d^5*g^4*i^3 - 5*a^2*b^4*c^4*d^2*g^4*i^3 + 5*a^4*b^2*c^2*d^4*g^4*i^3)/
(a^5*d^5*g^4*i^3 - b^5*c^5*g^4*i^3 + 5*a*b^4*c^4*d*g^4*i^3 - 5*a^4*b*c*d^4*
g^4*i^3 - 10*a^2*b^3*c^3*d^2*g^4*i^3 + 10*a^3*b^2*c^2*d^3*g^4*i^3) + 2*b*d*
x)*(a^5*d^5*g^4*i^3 - b^5*c^5*g^4*i^3 + 5*a*b^4*c^4*d*g^4*i^3 - 5*a^4*b*c*d
^4*g^4*i^3 - 10*a^2*b^3*c^3*d^2*g^4*i^3 + 10*a^3*b^2*c^2*d^3*g^4*i^3)*10i)/
(g^4*i^3*(30*A*b^2*d^3 + 10*B*b^2*d^3*n)*(a*d - b*c)^6))*(3*A + B*n)*20i)/(

```
3*g^4*i^3*(a*d - b*c)^6) - (5*B*b^2*d^3*log(e*((a + b*x)/(c + d*x))^n)^2)/(
g^4*i^3*n*(a*d - b*c)^6)
```

3.159 $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 43, antiderivative size = 584

$$\begin{aligned}
&\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= \frac{3B^2(bc-ad)^4g^3in^2x}{10bd^3}-\frac{3B^2(bc-ad)^3g^3in^2(c+dx)^2}{20d^4} \\
&+\frac{bB^2(bc-ad)^2g^3in^2(c+dx)^3}{30d^4}-\frac{B(bc-ad)^2g^3in(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^2d} \\
&-\frac{B(bc-ad)g^3in(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^2} \\
&+\frac{(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^2} \\
&+\frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} \\
&+\frac{B(bc-ad)^3g^3in(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^2d^2} \\
&-\frac{B(bc-ad)^4g^3in(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^2d^3} \\
&-\frac{B(bc-ad)^5g^3in\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{60b^2d^4} \\
&-\frac{B^2(bc-ad)^5g^3in^2\log(c+dx)}{10b^2d^4}-\frac{B^2(bc-ad)^5g^3in^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{10b^2d^4}
\end{aligned}$$

```
[Out] 3/10*B^2*(-a*d+b*c)^4*g^3*i*n^2*x/b/d^3-3/20*B^2*(-a*d+b*c)^3*g^3*i*n^2*(d*
x+c)^2/d^4+1/30*b*B^2*(-a*d+b*c)^2*g^3*i*n^2*(d*x+c)^3/d^4-1/30*B*(-a*d+b*c
)^2*g^3*i*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d-1/10*B*(-a*d+b*
```

```
c)*g^3*i*n*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2+1/20*(-a*d+b*c)*g^
3*i*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2+1/5*g^3*i*(b*x+a)^4*(d*
x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/60*B*(-a*d+b*c)^3*g^3*i*n*(b*x+a
)^2*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d^2-1/60*B*(-a*d+b*c)^4*g^3
*i*n*(b*x+a)*(6*A+5*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d^3-1/60*B*(-a*d
+b*c)^5*g^3*i*n*(6*A+11*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/
(d*x+c))/b^2/d^4-1/10*B^2*(-a*d+b*c)^5*g^3*i*n^2*ln(d*x+c)/b^2/d^4-1/10*B^2
*(-a*d+b*c)^5*g^3*i*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/d^4
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.186$, Rules used = {2561, 2383, 2381, 2384, 2354, 2438, 2373, 45}

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= -\frac{Bg^3in(bc-ad)^5\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+6A+11Bn\right)}{60b^2d^4} - \frac{Bg^3in(a+bx)(bc-ad)^4\left(6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+6A+5Bn\right)}{60b^2d^3} + \frac{Bg^3in(a+bx)^2(bc-ad)^3\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{60b^2d^2} - \frac{Bg^3in(a+bx)^3(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{30b^2d} + \frac{g^3i(a+bx)^4(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{20b^2} - \frac{Bg^3in(a+bx)^4(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{10b^2} + \frac{g^3i(a+bx)^4(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b} - \frac{B^2g^3in^2(bc-ad)^5\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{10b^2d^4} - \frac{B^2g^3in^2(bc-ad)^5\log(c+dx)}{10b^2d^4} - \frac{3B^2g^3in^2(c+dx)^2(bc-ad)^3}{20d^4} + \frac{bB^2g^3in^2(c+dx)^3(bc-ad)^2}{30d^4} + \frac{3B^2g^3in^2x(bc-ad)^4}{10bd^3}$$

```
[In] Int[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,
x]
```

```
[Out] (3*B^2*(b*c - a*d)^4*g^3*i*n^2*x)/(10*b*d^3) - (3*B^2*(b*c - a*d)^3*g^3*i*n
^2*(c + d*x)^2)/(20*d^4) + (b*B^2*(b*c - a*d)^2*g^3*i*n^2*(c + d*x)^3)/(30*
d^4) - (B*(b*c - a*d)^2*g^3*i*n*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*
```

```
x))^n]))/(30*b^2*d) - (B*(b*c - a*d)*g^3*i*n*(a + b*x)^4*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/(10*b^2) + ((b*c - a*d)*g^3*i*(a + b*x)^4*(A + B*Log[
e*((a + b*x)/(c + d*x))^n])^2)/(20*b^2) + (g^3*i*(a + b*x)^4*(c + d*x)*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(5*b) + (B*(b*c - a*d)^3*g^3*i*n*(a +
 b*x)^2*(3*A + B*n + 3*B*Log[e*((a + b*x)/(c + d*x))^n]))/(60*b^2*d^2) - (B
*(b*c - a*d)^4*g^3*i*n*(a + b*x)*(6*A + 5*B*n + 6*B*Log[e*((a + b*x)/(c + d
*x))^n]))/(60*b^2*d^3) - (B*(b*c - a*d)^5*g^3*i*n*(6*A + 11*B*n + 6*B*Log[e
*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(60*b^2*d^4) - (
B^2*(b*c - a*d)^5*g^3*i*n^2*Log[c + d*x])/(10*b^2*d^4) - (B^2*(b*c - a*d)^5
*g^3*i*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(10*b^2*d^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2383

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Dist[(m + q + 2)/(d*(q + 1)), Int[(
f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Dist[b*n*(p/(d*(q +
```

```
1))), Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x]) /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ
[q, -1] && GtQ[m, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left((bc-ad)^5g^3i\right)\text{Subst}\left(\int\frac{x^3(A+B\log(ex^n))^2}{(b-dx)^6}\,dx,x,\frac{a+bx}{c+dx}\right)\\ &= \frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b}\\ &\quad+\frac{\left((bc-ad)^5g^3i\right)\text{Subst}\left(\int\frac{x^3(A+B\log(ex^n))^2}{(b-dx)^5}\,dx,x,\frac{a+bx}{c+dx}\right)}{5b}\\ &\quad-\frac{\left(2B(bc-ad)^5g^3in\right)\text{Subst}\left(\int\frac{x^3(A+B\log(ex^n))}{(b-dx)^5}\,dx,x,\frac{a+bx}{c+dx}\right)}{5b}\end{aligned}$$

$$=-\frac{B(bc-ad)g^3in(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^2}+\frac{(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^2}+\frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b}-\frac{(B(bc-ad)^5g^3in)\,\text{Subst}\left(\int\frac{x^3(A+B\log(ex^n))}{(b-dx)^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{10b^2}+\frac{(B^2(bc-ad)^5g^3in^2)\,\text{Subst}\left(\int\frac{x^3}{(b-dx)^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{10b^2}$$

$$=-\frac{B(bc-ad)^2g^3in(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^2d}-\frac{B(bc-ad)g^3in(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^2}+\frac{(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^2}+\frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b}+\frac{(B(bc-ad)^5g^3in)\,\text{Subst}\left(\int\frac{x^2(3A+Bn+3B\log(ex^n))}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{30b^2d}+\frac{(B^2(bc-ad)^5g^3in^2)\,\text{Subst}\left(\int\left(\frac{b^3}{d^3(b-dx)^4}-\frac{3b^2}{d^3(b-dx)^3}+\frac{3b}{d^3(b-dx)^2}-\frac{1}{d^3(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{10b^2}$$

$$= \frac{3B^2(bc-ad)^4g^3in^2x}{10bd^3} - \frac{3B^2(bc-ad)^3g^3in^2(c+dx)^2}{20d^4} + \frac{bB^2(bc-ad)^2g^3in^2(c+dx)^3}{30d^4} - \frac{B(bc-ad)^2g^3in(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^2d} - \frac{B(bc-ad)g^3in(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^2} + \frac{(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^2} + \frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} + \frac{B(bc-ad)^3g^3in(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^2d^2} - \frac{B^2(bc-ad)^5g^3in^2\log(c+dx)}{10b^2d^4} - \frac{(B(bc-ad)^5g^3in)\,\text{Subst}\left(\int\frac{x(3Bn+2(3A+Bn)+6B\log(ex^n))}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{60b^2d^2}$$

$$= \frac{3B^2(bc-ad)^4g^3in^2x}{10bd^3} - \frac{3B^2(bc-ad)^3g^3in^2(c+dx)^2}{20d^4} + \frac{bB^2(bc-ad)^2g^3in^2(c+dx)^3}{30d^4} - \frac{B(bc-ad)^2g^3in(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^2d} - \frac{B(bc-ad)g^3in(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^2} + \frac{(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^2} + \frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} + \frac{B(bc-ad)^3g^3in(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^2d^2} - \frac{B(bc-ad)^4g^3in(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^2d^3} - \frac{B^2(bc-ad)^5g^3in^2\log(c+dx)}{10b^2d^4} + \frac{(B(bc-ad)^5g^3in)\,\text{Subst}\left(\int\frac{9Bn+2(3A+Bn)+6B\log(ex^n)}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{60b^2d^3}$$

$$
\begin{aligned}
&= \frac{3B^2(bc-ad)^4g^3in^2x}{10bd^3} - \frac{3B^2(bc-ad)^3g^3in^2(c+dx)^2}{20d^4} \\
&+ \frac{bB^2(bc-ad)^2g^3in^2(c+dx)^3}{30d^4} \\
&- \frac{B(bc-ad)^2g^3in(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^2d} \\
&- \frac{B(bc-ad)g^3in(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^2} \\
&+ \frac{(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^2} \\
&+ \frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} \\
&+ \frac{B(bc-ad)^3g^3in(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^2d^2} \\
&- \frac{B(bc-ad)^4g^3in(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^2d^3} \\
&- \frac{B(bc-ad)^5g^3in\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{60b^2d^4} \\
&- \frac{B^2(bc-ad)^5g^3in^2\log(c+dx)}{10b^2d^4} \\
&+ \frac{(B^2(bc-ad)^5g^3in^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{10b^2d^4}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3B^2(bc-ad)^4g^3in^2x}{10bd^3} - \frac{3B^2(bc-ad)^3g^3in^2(c+dx)^2}{20d^4} \\
&+ \frac{bB^2(bc-ad)^2g^3in^2(c+dx)^3}{30d^4} \\
&- \frac{B(bc-ad)^2g^3in(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^2d} \\
&- \frac{B(bc-ad)g^3in(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^2} \\
&+ \frac{(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^2} \\
&+ \frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} \\
&+ \frac{B(bc-ad)^3g^3in(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^2d^2} \\
&- \frac{B(bc-ad)^4g^3in(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^2d^3} \\
&- \frac{B(bc-ad)^5g^3in\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{60b^2d^4} \\
&- \frac{B^2(bc-ad)^5g^3in^2\log(c+dx)}{10b^2d^4} - \frac{B^2(bc-ad)^5g^3in^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{10b^2d^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 949, normalized size of antiderivative = 1.62

$$
\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx
$$

$$
= \frac{g^3i\left(5(bc-ad)(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + 4d(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - \frac{5B(bc-ad)^2n\left(6A\right.}{}\right.}{}
$$

```
[In] Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2,x]
```

```
[Out] (g^3*i*(5*(b*c - a*d)*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
+ 4*d*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (5*B*(b*c - a*
d)^2*n*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a +
 b*x)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[e*((a +
b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x - d
^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*n*(b*d*x
```

```
 + (-(b*c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*n*((2*Log[(d*(a + b*x))
/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/
(b*c - a*d)])))/(3*d^4) + (B*(b*c - a*d)*n*(24*A*b*d*(b*c - a*d)^3*x + 24*B
*d*(b*c - a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 12*d^2*(b*c - a
*d)^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 8*d^3*(b*c - a*d
)*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*d^4*(a + b*x)^4*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n]) - 24*B*(b*c - a*d)^4*n*Log[c + d*x] -
24*(b*c - a*d)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 4*B*
(b*c - a*d)^2*n*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Lo
g[c + d*x]) + B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)
*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + 12*B*(b*
c - a*d)^3*n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 12*B*(b*c - a*d)^4*n*(
(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyL
og[2, (b*(c + d*x))/(b*c - a*d)])))/(3*d^4)))/(20*b^2)
```

Maple [F]

$$\int (bgx+ag)^3 (dix+ci) \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (bgx+ag)^3(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, a
lgorithm="fricas")
```

```
[Out] integral(A^2*b^3*d*g^3*i*x^4 + A^2*a^3*c*g^3*i + (A^2*b^3*c + 3*A^2*a*b^2*d
)*g^3*i*x^3 + 3*(A^2*a*b^2*c + A^2*a^2*b*d)*g^3*i*x^2 + (3*A^2*a^2*b*c + A^
2*a^3*d)*g^3*i*x + (B^2*b^3*d*g^3*i*x^4 + B^2*a^3*c*g^3*i + (B^2*b^3*c + 3*
B^2*a*b^2*d)*g^3*i*x^3 + 3*(B^2*a*b^2*c + B^2*a^2*b*d)*g^3*i*x^2 + (3*B^2*a
^2*b*c + B^2*a^3*d)*g^3*i*x)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^3*
d*g^3*i*x^4 + A*B*a^3*c*g^3*i + (A*B*b^3*c + 3*A*B*a*b^2*d)*g^3*i*x^3 + 3*(
A*B*a*b^2*c + A*B*a^2*b*d)*g^3*i*x^2 + (3*A*B*a^2*b*c + A*B*a^3*d)*g^3*i*x)
*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**3*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3764 vs. $2(559) = 1118$.

Time = 0.75 (sec) , antiderivative size = 3764, normalized size of antiderivative = 6.45

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, a
lgorithm="maxima")
[Out] 2/5*A*B*b^3*d*g^3*i*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A^2*b^
3*d*g^3*i*x^5 + 1/2*A*B*b^3*c*g^3*i*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))
^n) + 3/2*A*B*a*b^2*d*g^3*i*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/
4*A^2*b^3*c*g^3*i*x^4 + 3/4*A^2*a*b^2*d*g^3*i*x^4 + 2*A*B*a*b^2*c*g^3*i*x^3
*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*B*a^2*b*d*g^3*i*x^3*log(e*(b*
x/(d*x + c) + a/(d*x + c))^n) + A^2*a*b^2*c*g^3*i*x^3 + A^2*a^2*b*d*g^3*i*x
^3 + 3*A*B*a^2*b*c*g^3*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*B*a
^3*d*g^3*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A^2*a^2*b*c*g^3
*i*x^2 + 1/2*A^2*a^3*d*g^3*i*x^2 + 1/30*A*B*b^3*d*g^3*i*n*(12*a^5*log(b*x +
 a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4
*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 -
 a^4*d^4)*x)/(b^4*d^4)) - 1/12*A*B*b^3*c*g^3*i*n*(6*a^4*log(b*x + a)/b^4 -
6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^
2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 1/4*A*B*a*b^2*d*g^3*i*
n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*
d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^
3)) + A*B*a*b^2*c*g^3*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3
- ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + A*B*a^2*
b*d*g^3*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d -
a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3*A*B*a^2*b*c*g^3*i*n*
(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) - A*B*a
^3*d*g^3*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(
b*d)) + 2*A*B*a^3*c*g^3*i*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*a
```

^3*c*g^3*i*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a^3*c*g^3*i*x - 1
/60*(6*a^4*c*d^4*g^3*i*n^2 - (5*g^3*i*n^2 + 6*g^3*i*n*log(e))*b^4*c^5 + (19
*g^3*i*n^2 + 30*g^3*i*n*log(e))*a*b^3*c^4*d - (23*g^3*i*n^2 + 60*g^3*i*n*lo
g(e))*a^2*b^2*c^3*d^2 + 3*(g^3*i*n^2 + 20*g^3*i*n*log(e))*a^3*b*c^2*d^3)*B^
2*log(d*x + c)/(b*d^4) + 1/10*(b^5*c^5*g^3*i*n^2 - 5*a*b^4*c^4*d*g^3*i*n^2
+ 10*a^2*b^3*c^3*d^2*g^3*i*n^2 - 10*a^3*b^2*c^2*d^3*g^3*i*n^2 + 5*a^4*b*c*d
^4*g^3*i*n^2 - a^5*d^5*g^3*i*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*
d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^4) + 1/60*(12*B^2*b
^5*d^5*g^3*i*x^5*log(e)^2 - 3*((2*g^3*i*n*log(e) - 5*g^3*i*log(e)^2)*b^5*c*
d^4 - (2*g^3*i*n*log(e) + 15*g^3*i*log(e)^2)*a*b^4*d^5)*B^2*x^4 + 2*((g^3*i
*n^2 - g^3*i*n*log(e))*b^5*c^2*d^3 - 2*(g^3*i*n^2 + 5*g^3*i*n*log(e) - 15*g
^3*i*log(e)^2)*a*b^4*c*d^4 + (g^3*i*n^2 + 11*g^3*i*n*log(e) + 30*g^3*i*log(
e)^2)*a^2*b^3*d^5)*B^2*x^3 - ((2*g^3*i*n^2 - 3*g^3*i*n*log(e))*b^5*c^3*d^2
- 3*(4*g^3*i*n^2 - 5*g^3*i*n*log(e))*a*b^4*c^2*d^3 + 3*(6*g^3*i*n^2 + 5*g^3
*i*n*log(e) - 30*g^3*i*log(e)^2)*a^2*b^3*c*d^4 - (8*g^3*i*n^2 + 27*g^3*i*n*
log(e) + 30*g^3*i*log(e)^2)*a^3*b^2*d^5)*B^2*x^2 - 3*(5*a^4*b*c*d^4*g^3*i*n
^2 - a^5*d^5*g^3*i*n^2)*B^2*log(b*x + a)^2 - 6*(b^5*c^5*g^3*i*n^2 - 5*a*b^4
*c^4*d*g^3*i*n^2 + 10*a^2*b^3*c^3*d^2*g^3*i*n^2 - 10*a^3*b^2*c^2*d^3*g^3*i*
n^2)*B^2*log(b*x + a)*log(d*x + c) + 3*(b^5*c^5*g^3*i*n^2 - 5*a*b^4*c^4*d*g
^3*i*n^2 + 10*a^2*b^3*c^3*d^2*g^3*i*n^2 - 10*a^3*b^2*c^2*d^3*g^3*i*n^2)*B^2
*log(d*x + c)^2 + ((g^3*i*n^2 - 6*g^3*i*n*log(e))*b^5*c^4*d - 2*(4*g^3*i*n^
2 - 15*g^3*i*n*log(e))*a*b^4*c^3*d^2 + 12*(2*g^3*i*n^2 - 5*g^3*i*n*log(e))*
a^2*b^3*c^2*d^3 - 2*(14*g^3*i*n^2 - 15*g^3*i*n*log(e) - 30*g^3*i*log(e)^2)*
a^3*b^2*c*d^4 + (11*g^3*i*n^2 + 6*g^3*i*n*log(e))*a^4*b*d^5)*B^2*x - (6*a*b
^4*c^4*d*g^3*i*n^2 - 27*a^2*b^3*c^3*d^2*g^3*i*n^2 + 47*a^3*b^2*c^2*d^3*g^3*
i*n^2 - (31*g^3*i*n^2 + 30*g^3*i*n*log(e))*a^4*b*c*d^4 + (5*g^3*i*n^2 + 6*g
^3*i*n*log(e))*a^5*d^5)*B^2*log(b*x + a) + 3*(4*B^2*b^5*d^5*g^3*i*x^5 + 20*
B^2*a^3*b^2*c*d^4*g^3*i*x + 5*(b^5*c*d^4*g^3*i + 3*a*b^4*d^5*g^3*i)*B^2*x^4
+ 20*(a*b^4*c*d^4*g^3*i + a^2*b^3*d^5*g^3*i)*B^2*x^3 + 10*(3*a^2*b^3*c*d^4
*g^3*i + a^3*b^2*d^5*g^3*i)*B^2*x^2)*log((b*x + a)^n)^2 + 3*(4*B^2*b^5*d^5*
g^3*i*x^5 + 20*B^2*a^3*b^2*c*d^4*g^3*i*x + 5*(b^5*c*d^4*g^3*i + 3*a*b^4*d^5
*g^3*i)*B^2*x^4 + 20*(a*b^4*c*d^4*g^3*i + a^2*b^3*d^5*g^3*i)*B^2*x^3 + 10*(
3*a^2*b^3*c*d^4*g^3*i + a^3*b^2*d^5*g^3*i)*B^2*x^2)*log((d*x + c)^n)^2 + (2
4*B^2*b^5*d^5*g^3*i*x^5*log(e) - 6*((g^3*i*n - 5*g^3*i*log(e))*b^5*c*d^4 -
(g^3*i*n + 15*g^3*i*log(e))*a*b^4*d^5)*B^2*x^4 - 2*(b^5*c^2*d^3*g^3*i*n + 1
0*(g^3*i*n - 6*g^3*i*log(e))*a*b^4*c*d^4 - (11*g^3*i*n + 60*g^3*i*log(e))*a
^2*b^3*d^5)*B^2*x^3 + 3*(b^5*c^3*d^2*g^3*i*n - 5*a*b^4*c^2*d^3*g^3*i*n - 5*
(g^3*i*n - 12*g^3*i*log(e))*a^2*b^3*c*d^4 + (9*g^3*i*n + 20*g^3*i*log(e))*a
^3*b^2*d^5)*B^2*x^2 - 6*(b^5*c^4*d*g^3*i*n - 5*a*b^4*c^3*d^2*g^3*i*n + 10*a
^2*b^3*c^2*d^3*g^3*i*n - a^4*b*d^5*g^3*i*n - 5*(g^3*i*n + 4*g^3*i*log(e))*a
^3*b^2*c*d^4)*B^2*x + 6*(5*a^4*b*c*d^4*g^3*i*n - a^5*d^5*g^3*i*n)*B^2*log(b
x + a) + 6(b^5*c^5*g^3*i*n - 5*a*b^4*c^4*d*g^3*i*n + 10*a^2*b^3*c^3*d^2*g
^3*i*n - 10*a^3*b^2*c^2*d^3*g^3*i*n)*B^2*log(d*x + c))*log((b*x + a)^n) - (
24*B^2*b^5*d^5*g^3*i*x^5*log(e) - 6*((g^3*i*n - 5*g^3*i*log(e))*b^5*c*d^4 -
(g^3*i*n + 15*g^3*i*log(e))*a*b^4*d^5)*B^2*x^4 - 2*(b^5*c^2*d^3*g^3*i*n +

```
10*(g^3*i*n - 6*g^3*i*log(e))*a*b^4*c*d^4 - (11*g^3*i*n + 60*g^3*i*log(e))*
a^2*b^3*d^5)*B^2*x^3 + 3*(b^5*c^3*d^2*g^3*i*n - 5*a*b^4*c^2*d^3*g^3*i*n - 5
*(g^3*i*n - 12*g^3*i*log(e))*a^2*b^3*c*d^4 + (9*g^3*i*n + 20*g^3*i*log(e))*
a^3*b^2*d^5)*B^2*x^2 - 6*(b^5*c^4*d*g^3*i*n - 5*a*b^4*c^3*d^2*g^3*i*n + 10*
a^2*b^3*c^2*d^3*g^3*i*n - a^4*b*d^5*g^3*i*n - 5*(g^3*i*n + 4*g^3*i*log(e))*
a^3*b^2*c*d^4)*B^2*x + 6*(5*a^4*b*c*d^4*g^3*i*n - a^5*d^5*g^3*i*n)*B^2*log(
b*x + a) + 6*(b^5*c^5*g^3*i*n - 5*a*b^4*c^4*d*g^3*i*n + 10*a^2*b^3*c^3*d^2*
g^3*i*n - 10*a^3*b^2*c^2*d^3*g^3*i*n)*B^2*log(d*x + c) + 6*(4*B^2*b^5*d^5*g
^3*i*x^5 + 20*B^2*a^3*b^2*c*d^4*g^3*i*x + 5*(b^5*c*d^4*g^3*i + 3*a*b^4*d^5*
g^3*i)*B^2*x^4 + 20*(a*b^4*c*d^4*g^3*i + a^2*b^3*d^5*g^3*i)*B^2*x^3 + 10*(3
*a^2*b^3*c*d^4*g^3*i + a^3*b^2*d^5*g^3*i)*B^2*x^2)*log((b*x + a)^n))*log((d
*x + c)^n))/(b^2*d^4)
```

Giac [F]

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (bgx+ag)^3(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, a
lgorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^3*(d*i*x + c*i)*(B*log(e*((b*x + a)/(d*x + c))^n) +
 A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^3\,(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

```
[In] int((a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,
x)
```

```
[Out] int((a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,
 x)
```

3.160 $\int (ag+bgx)^2(ci+dix) \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

Optimal result

Integrand size = 43, antiderivative size = 487

$$\begin{aligned}
&\int (ag+bgx)^2(ci+dix) \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx \\
&= -\frac{B^2(bc-ad)^3g^2in^2x}{3bd^2}+\frac{B^2(bc-ad)^2g^2in^2(c+dx)^2}{12d^3} \\
&\quad -\frac{B(bc-ad)^2g^2in(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12b^2d} \\
&\quad -\frac{B(bc-ad)g^2in(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^2} \\
&\quad +\frac{(bc-ad)g^2i(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{12b^2} \\
&\quad +\frac{g^2i(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} \\
&\quad +\frac{B(bc-ad)^3g^2in(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12b^2d^2} \\
&\quad +\frac{B(bc-ad)^4g^2in\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{12b^2d^3} \\
&\quad +\frac{B^2(bc-ad)^4g^2in^2\log(c+dx)}{6b^2d^3}+\frac{B^2(bc-ad)^4g^2in^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{6b^2d^3}
\end{aligned}$$

```
[Out] -1/3*B^2*(-a*d+b*c)^3*g^2*i*n^2*x/b/d^2+1/12*B^2*(-a*d+b*c)^2*g^2*i*n^2*(d*
x+c)^2/d^3-1/12*B*(-a*d+b*c)^2*g^2*i*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c)
)^n))/b^2/d-1/6*B*(-a*d+b*c)*g^2*i*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^
n))/b^2+1/12*(-a*d+b*c)*g^2*i*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b
^2+1/4*g^2*i*(b*x+a)^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/12*B*(
-a*d+b*c)^3*g^2*i*n*(b*x+a)*(2*A+B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d^2
```

```
+1/12*B*(-a*d+b*c)^4*g^2*i*n*(2*A+3*B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((
-a*d+b*c)/b/(d*x+c))/b^2/d^3+1/6*B^2*(-a*d+b*c)^4*g^2*i*n^2*ln(d*x+c)/b^2/d
^3+1/6*B^2*(-a*d+b*c)^4*g^2*i*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/d^3
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {2561, 2383, 2381, 2384, 2354, 2438, 2373, 45}

$$\begin{aligned}
&\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx \\
&=\frac{Bg^2in(bc-ad)^4\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+3Bn\right)}{12b^2d^3} \\
&+\frac{Bg^2in(a+bx)(bc-ad)^3\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{12b^2d^2} \\
&-\frac{Bg^2in(a+bx)^2(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{12b^2d} \\
&+\frac{g^2i(a+bx)^3(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{12b^2} \\
&-\frac{Bg^2in(a+bx)^3(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6b^2} \\
&+\frac{g^2i(a+bx)^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b} \\
&+\frac{B^2g^2in^2(bc-ad)^4\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{6b^2d^3}+\frac{B^2g^2in^2(bc-ad)^4\log(c+dx)}{6b^2d^3} \\
&+\frac{B^2g^2in^2(c+dx)^2(bc-ad)^2}{12d^3}-\frac{B^2g^2in^2x(bc-ad)^3}{3bd^2}
\end{aligned}$$

```
[In] Int[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,
x]
```

```
[Out] -1/3*(B^2*(b*c - a*d)^3*g^2*i*n^2*x)/(b*d^2) + (B^2*(b*c - a*d)^2*g^2*i*n^2
*(c + d*x)^2)/(12*d^3) - (B*(b*c - a*d)^2*g^2*i*n*(a + b*x)^2*(A + B*Log[e*
((a + b*x)/(c + d*x))^n]))/(12*b^2*d) - (B*(b*c - a*d)*g^2*i*n*(a + b*x)^3*
(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*b^2) + ((b*c - a*d)*g^2*i*(a + b
*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(12*b^2) + (g^2*i*(a + b*x)
^3*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*b) + (B*(b*c - a*
d)^3*g^2*i*n*(a + b*x)*(2*A + B*n + 2*B*Log[e*((a + b*x)/(c + d*x))^n]))/(1
2*b^2*d^2) + (B*(b*c - a*d)^4*g^2*i*n*(2*A + 3*B*n + 2*B*Log[e*((a + b*x)/(
c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(12*b^2*d^3) + (B^2*(b*c - a*
d)^4*g^2*i*n^2*Log[c + d*x])/(6*b^2*d^3) + (B^2*(b*c - a*d)^4*g^2*i*n^2*Pol
yLog[2, (d*(a + b*x))/(b*(c + d*x))])/(6*b^2*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2383

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Dist[(m + q + 2)/(d*(q + 1)), Int[(
f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Dist[b*n*(p/(d*(q +
 1))), Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x]) /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ
[q, -1] && GtQ[m, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
```

```
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((bc-ad)^4g^2i\right)\text{Subst}\left(\int\frac{x^2(A+B\log(ex^n))^2}{(b-dx)^5}\,dx,x,\frac{a+bx}{c+dx}\right)\\
&= \frac{g^2i(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b}\\
&\quad+\frac{\left((bc-ad)^4g^2i\right)\text{Subst}\left(\int\frac{x^2(A+B\log(ex^n))^2}{(b-dx)^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{4b}\\
&\quad-\frac{\left(B(bc-ad)^4g^2in\right)\text{Subst}\left(\int\frac{x^2(A+B\log(ex^n))}{(b-dx)^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{2b}\\
&= -\frac{B(bc-ad)g^2in(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^2}\\
&\quad+\frac{(bc-ad)g^2i(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{12b^2}\\
&\quad+\frac{g^2i(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b}\\
&\quad-\frac{\left(B(bc-ad)^4g^2in\right)\text{Subst}\left(\int\frac{x^2(A+B\log(ex^n))}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{6b^2}\\
&\quad+\frac{\left(B^2(bc-ad)^4g^2in^2\right)\text{Subst}\left(\int\frac{x^2}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{6b^2}
\end{aligned}$$

$$= -\frac{B(bc-ad)^2g^2in(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12b^2d} - \frac{B(bc-ad)g^2in(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^2} + \frac{(bc-ad)g^2i(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{12b^2} + \frac{g^2i(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} + \frac{(B(bc-ad)^4g^2in)\,\text{Subst}\left(\int\frac{x(2A+Bn+2B\log(ex^n))}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{12b^2d} + \frac{(B^2(bc-ad)^4g^2in^2)\,\text{Subst}\left(\int\left(\frac{b^2}{d^2(b-dx)^3}-\frac{2b}{d^2(b-dx)^2}+\frac{1}{d^2(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{6b^2}$$

$$= -\frac{B^2(bc-ad)^3g^2in^2x}{3bd^2} + \frac{B^2(bc-ad)^2g^2in^2(c+dx)^2}{12d^3} - \frac{B(bc-ad)^2g^2in(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12b^2d} - \frac{B(bc-ad)g^2in(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^2} + \frac{(bc-ad)g^2i(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{12b^2} + \frac{g^2i(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} + \frac{B(bc-ad)^3g^2in(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12b^2d^2} + \frac{B^2(bc-ad)^4g^2in^2\log(c+dx)}{6b^2d^3} - \frac{(B(bc-ad)^4g^2in)\,\text{Subst}\left(\int\frac{2A+3Bn+2B\log(ex^n)}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{12b^2d^2}$$

$$= -\frac{B^2(bc-ad)^3g^2in^2x}{3bd^2} + \frac{B^2(bc-ad)^2g^2in^2(c+dx)^2}{12d^3} - \frac{B(bc-ad)^2g^2in(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12b^2d} - \frac{B(bc-ad)g^2in(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^2} + \frac{(bc-ad)g^2i(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{12b^2} + \frac{g^2i(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} + \frac{B(bc-ad)^3g^2in(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12b^2d^2} + \frac{B(bc-ad)^4g^2in\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{12b^2d^3} + \frac{B^2(bc-ad)^4g^2in^2\log(c+dx)}{6b^2d^3} - \frac{(B^2(bc-ad)^4g^2in^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{6b^2d^3}$$

$$= -\frac{B^2(bc-ad)^3g^2in^2x}{3bd^2} + \frac{B^2(bc-ad)^2g^2in^2(c+dx)^2}{12d^3} - \frac{B(bc-ad)^2g^2in(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12b^2d} - \frac{B(bc-ad)g^2in(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^2} + \frac{(bc-ad)g^2i(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{12b^2} + \frac{g^2i(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} + \frac{B(bc-ad)^3g^2in(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12b^2d^2} + \frac{B(bc-ad)^4g^2in\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{12b^2d^3} + \frac{B^2(bc-ad)^4g^2in^2\log(c+dx)}{6b^2d^3} + \frac{B^2(bc-ad)^4g^2in^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{6b^2d^3}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.47

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx$$

$$=\frac{g^2i\left(4(bc-ad)(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+3d(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{4B(bc-ad)^2n\left(2Ab\right.}{}\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2,x]
```

```
[Out] (g^2*i*(4*(b*c - a*d)*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
+ 3*d*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (4*B*(b*c - a*
d)^2*n*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[e*((a + b*x
)/(c + d*x))^n] - d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) -
2*B*(b*c - a*d)^2*n*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/
(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(b*d*x + (-(b*c) + a*d)*Log[c
 + d*x]) + B*(b*c - a*d)^2*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c
+ d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3 - (B*
(b*c - a*d)*n*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[
e*((a + b*x)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[e*
((a + b*x)/(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[e
*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(2*b*d*(b*c - a*d
)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*n
*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*n*((2*Log[(d*(a
+ b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c +
 d*x))/(b*c - a*d)])))/d^3))/(12*b^2)
```

Maple [F]

$$\int (bgx+ag)^2(dix+ci)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2dx$$

```
[In] int((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (bgx+ag)^2(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, a
lgorithm="fricas")
```

```
[Out] integral(A^2*b^2*d*g^2*i*x^3 + A^2*a^2*c*g^2*i + (A^2*b^2*c + 2*A^2*a*b*d)*
g^2*i*x^2 + (2*A^2*a*b*c + A^2*a^2*d)*g^2*i*x + (B^2*b^2*d*g^2*i*x^3 + B^2*
a^2*c*g^2*i + (B^2*b^2*c + 2*B^2*a*b*d)*g^2*i*x^2 + (2*B^2*a*b*c + B^2*a^2*
d)*g^2*i*x)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^2*d*g^2*i*x^3 + A*B
*a^2*c*g^2*i + (A*B*b^2*c + 2*A*B*a*b*d)*g^2*i*x^2 + (2*A*B*a*b*c + A*B*a^2
*d)*g^2*i*x)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**2*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2691 vs. $2(466) = 932$.

Time = 0.73 (sec) , antiderivative size = 2691, normalized size of antiderivative = 5.53

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, a
lgorithm="maxima")
```

```
[Out] 1/2*A*B*b^2*d*g^2*i*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*b^
2*d*g^2*i*x^4 + 2/3*A*B*b^2*c*g^2*i*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))
^n) + 4/3*A*B*a*b*d*g^2*i*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*
A^2*b^2*c*g^2*i*x^3 + 2/3*A^2*a*b*d*g^2*i*x^3 + 2*A*B*a*b*c*g^2*i*x^2*log(e
```

```
*(b*x/(d*x + c) + a/(d*x + c))^n) + A*B*a^2*d*g^2*i*x^2*log(e*(b*x/(d*x + c
) + a/(d*x + c))^n) + A^2*a*b*c*g^2*i*x^2 + 1/2*A^2*a^2*d*g^2*i*x^2 - 1/12*
A*B*b^2*d*g^2*i*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^
3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3
*d^3)*x)/(b^3*d^3)) + 1/3*A*B*b^2*c*g^2*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3
*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^
2*d^2)) + 2/3*A*B*a*b*d*g^2*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c
)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*
A*B*a*b*c*g^2*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d
)*x/(b*d)) - A*B*a^2*d*g^2*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2
 + (b*c - a*d)*x/(b*d)) + 2*A*B*a^2*c*g^2*i*n*(a*log(b*x + a)/b - c*log(d*x
 + c)/d) + 2*A*B*a^2*c*g^2*i*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2
*a^2*c*g^2*i*x - 1/12*(2*a^3*c*d^3*g^2*i*n^2 + (g^2*i*n^2 + 2*g^2*i*n*log(e
))*b^3*c^4 - 2*(g^2*i*n^2 + 4*g^2*i*n*log(e))*a*b^2*c^3*d - (g^2*i*n^2 - 12
*g^2*i*n*log(e))*a^2*b*c^2*d^2)*B^2*log(d*x + c)/(b*d^3) - 1/6*(b^4*c^4*g^2
*i*n^2 - 4*a*b^3*c^3*d*g^2*i*n^2 + 6*a^2*b^2*c^2*d^2*g^2*i*n^2 - 4*a^3*b*c*
d^3*g^2*i*n^2 + a^4*d^4*g^2*i*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a
*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^3) + 1/12*(3*B^2*b
^4*d^4*g^2*i*x^4*log(e)^2 - 2*((g^2*i*n*log(e) - 2*g^2*i*log(e)^2)*b^4*c*d^
3 - (g^2*i*n*log(e) + 4*g^2*i*log(e)^2)*a*b^3*d^4)*B^2*x^3 + ((g^2*i*n^2 -
g^2*i*n*log(e))*b^4*c^2*d^2 - 2*(g^2*i*n^2 + 2*g^2*i*n*log(e) - 6*g^2*i*log
(e)^2)*a*b^3*c*d^3 + (g^2*i*n^2 + 5*g^2*i*n*log(e) + 6*g^2*i*log(e)^2)*a^2*
b^2*d^4)*B^2*x^2 - (4*a^3*b*c*d^3*g^2*i*n^2 - a^4*d^4*g^2*i*n^2)*B^2*log(b*
x + a)^2 + 2*(b^4*c^4*g^2*i*n^2 - 4*a*b^3*c^3*d*g^2*i*n^2 + 6*a^2*b^2*c^2*d
^2*g^2*i*n^2)*B^2*log(b*x + a)*log(d*x + c) - (b^4*c^4*g^2*i*n^2 - 4*a*b^3*
c^3*d*g^2*i*n^2 + 6*a^2*b^2*c^2*d^2*g^2*i*n^2)*B^2*log(d*x + c)^2 - ((g^2*i
*n^2 - 2*g^2*i*n*log(e))*b^4*c^3*d - (5*g^2*i*n^2 - 8*g^2*i*n*log(e))*a*b^3
*c^2*d^2 + (7*g^2*i*n^2 - 4*g^2*i*n*log(e) - 12*g^2*i*log(e)^2)*a^2*b^2*c*d
^3 - (3*g^2*i*n^2 + 2*g^2*i*n*log(e))*a^3*b*d^4)*B^2*x + (2*a*b^3*c^3*d*g^2
*i*n^2 - 7*a^2*b^2*c^2*d^2*g^2*i*n^2 + 2*(3*g^2*i*n^2 + 4*g^2*i*n*log(e))*a
^3*b*c*d^3 - (g^2*i*n^2 + 2*g^2*i*n*log(e))*a^4*d^4)*B^2*log(b*x + a) + (3*
B^2*b^4*d^4*g^2*i*x^4 + 12*B^2*a^2*b^2*c*d^3*g^2*i*x + 4*(b^4*c*d^3*g^2*i +
 2*a*b^3*d^4*g^2*i)*B^2*x^3 + 6*(2*a*b^3*c*d^3*g^2*i + a^2*b^2*d^4*g^2*i)*B
^2*x^2)*log((b*x + a)^n)^2 + (3*B^2*b^4*d^4*g^2*i*x^4 + 12*B^2*a^2*b^2*c*d^
3*g^2*i*x + 4*(b^4*c*d^3*g^2*i + 2*a*b^3*d^4*g^2*i)*B^2*x^3 + 6*(2*a*b^3*c*
d^3*g^2*i + a^2*b^2*d^4*g^2*i)*B^2*x^2)*log((d*x + c)^n)^2 + (6*B^2*b^4*d^4
*g^2*i*x^4*log(e) - 2*((g^2*i*n - 4*g^2*i*log(e))*b^4*c*d^3 - (g^2*i*n + 8*
g^2*i*log(e))*a*b^3*d^4)*B^2*x^3 - (b^4*c^2*d^2*g^2*i*n + 4*(g^2*i*n - 6*g^
2*i*log(e))*a*b^3*c*d^3 - (5*g^2*i*n + 12*g^2*i*log(e))*a^2*b^2*d^4)*B^2*x^
2 + 2*(b^4*c^3*d*g^2*i*n - 4*a*b^3*c^2*d^2*g^2*i*n + a^3*b*d^4*g^2*i*n + 2*
(g^2*i*n + 6*g^2*i*log(e))*a^2*b^2*c*d^3)*B^2*x + 2*(4*a^3*b*c*d^3*g^2*i*n
- a^4*d^4*g^2*i*n)*B^2*log(b*x + a) - 2*(b^4*c^4*g^2*i*n - 4*a*b^3*c^3*d*g^
2*i*n + 6*a^2*b^2*c^2*d^2*g^2*i*n)*B^2*log(d*x + c))*log((b*x + a)^n) - (6*
B^2*b^4*d^4*g^2*i*x^4*log(e) - 2*((g^2*i*n - 4*g^2*i*log(e))*b^4*c*d^3 - (g
^2*i*n + 8*g^2*i*log(e))*a*b^3*d^4)*B^2*x^3 - (b^4*c^2*d^2*g^2*i*n + 4*(g^2
```

```
*i*n - 6*g^2*i*log(e))*a*b^3*c*d^3 - (5*g^2*i*n + 12*g^2*i*log(e))*a^2*b^2*
d^4)*B^2*x^2 + 2*(b^4*c^3*d*g^2*i*n - 4*a*b^3*c^2*d^2*g^2*i*n + a^3*b*d^4*g
^2*i*n + 2*(g^2*i*n + 6*g^2*i*log(e))*a^2*b^2*c*d^3)*B^2*x + 2*(4*a^3*b*c*d
^3*g^2*i*n - a^4*d^4*g^2*i*n)*B^2*log(b*x + a) - 2*(b^4*c^4*g^2*i*n - 4*a*b
^3*c^3*d*g^2*i*n + 6*a^2*b^2*c^2*d^2*g^2*i*n)*B^2*log(d*x + c) + 2*(3*B^2*b
^4*d^4*g^2*i*x^4 + 12*B^2*a^2*b^2*c*d^3*g^2*i*x + 4*(b^4*c*d^3*g^2*i + 2*a*
b^3*d^4*g^2*i)*B^2*x^3 + 6*(2*a*b^3*c*d^3*g^2*i + a^2*b^2*d^4*g^2*i)*B^2*x^
2)*log((b*x + a)^n))*log((d*x + c)^n))/(b^2*d^3)
```

Giac [F]

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (bgx+ag)^2(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, a
lgorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^2*(d*i*x + c*i)*(B*log(e*((b*x + a)/(d*x + c))^n) +
 A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^2\,(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

```
[In] int((a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,
x)
```

```
[Out] int((a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,
 x)
```

3.161 $\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 41, antiderivative size = 372

$$\begin{aligned}
&\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx \\
&= \frac{B^2(bc-ad)^2gin^2x}{3bd}-\frac{B(bc-ad)^2gin(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^2d} \\
&\quad -\frac{B(bc-ad)gin(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^2} \\
&\quad +\frac{(bc-ad)gi(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b^2} \\
&\quad +\frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} \\
&\quad -\frac{B(bc-ad)^3gin\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3b^2d^2} \\
&\quad -\frac{B^2(bc-ad)^3gin^2\log(c+dx)}{3b^2d^2}-\frac{B^2(bc-ad)^3gin^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^2d^2}
\end{aligned}$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*g*i*n^2*x/b/d-1/3*B*(-a*d+b*c)^2*g*i*n*(b*x+a)*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))/b^2/d-1/3*B*(-a*d+b*c)*g*i*n*(b*x+a)^2*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))/b^2+1/6*(-a*d+b*c)*g*i*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d
*x+c))^n))^2/b^2+1/3*g*i*(b*x+a)^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^
2/b-1/3*B*(-a*d+b*c)^3*g*i*n*(A+B*n+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b
*c)/b/(d*x+c))/b^2/d^2-1/3*B^2*(-a*d+b*c)^3*g*i*n^2*ln(d*x+c)/b^2/d^2-1/3*B
^2*(-a*d+b*c)^3*g*i*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/d^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {2561, 2383, 2381, 2384, 2354, 2438, 2373, 45}

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$= -\frac{Bgin(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A+Bn\right)}{3b^2d^2}$$
$$-\frac{Bgin(a+bx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b^2d}$$
$$+\frac{gi(a+bx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{6b^2}$$
$$-\frac{Bgin(a+bx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b^2}$$
$$+\frac{gi(a+bx)^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b}-\frac{B^2gin^2(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^2d^2}$$
$$-\frac{B^2gin^2(bc-ad)^3\log(c+dx)}{3b^2d^2}+\frac{B^2gin^2x(bc-ad)^2}{3bd}$$

```
[In] Int[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^2*g*i*n^2*x)/(3*b*d) - (B*(b*c - a*d)^2*g*i*n*(a + b*x)*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b^2*d) - (B*(b*c - a*d)*g*i*n*(a +
 b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b^2) + ((b*c - a*d)*g*i*
(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(6*b^2) + (g*i*(a + b
*x)^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*b) - (B*(b*c -
 a*d)^3*g*i*n*(A + B*n + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/
(b*(c + d*x))])/(3*b^2*d^2) - (B^2*(b*c - a*d)^3*g*i*n^2*Log[c + d*x])/(3*b
^2*d^2) - (B^2*(b*c - a*d)^3*g*i*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))
])/(3*b^2*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
```

```
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2383

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Dist[(m + q + 2)/(d*(q + 1)), Int[(
f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Dist[b*n*(p/(d*(q +
 1))), Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x]) /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ
[q, -1] && GtQ[m, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
```

```
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \left((bc-ad)^3 gi\right) \text{Subst}\left(\int \frac{x(A+B\log(ex^n))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} + \frac{((bc-ad)^3 gi)\,\text{Subst}\left(\int \frac{x(A+B\log(ex^n))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b} - \frac{(2B(bc-ad)^3 gin)\,\text{Subst}\left(\int \frac{x(A+B\log(ex^n))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b}$$

$$= -\frac{B(bc-ad)gin(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^2} + \frac{(bc-ad)gi(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b^2} + \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} - \frac{(B(bc-ad)^3 gin)\,\text{Subst}\left(\int \frac{x(A+B\log(ex^n))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b^2} + \frac{(B^2(bc-ad)^3 gin^2)\,\text{Subst}\left(\int \frac{x}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b^2}$$

$$= -\frac{B(bc-ad)^2 gin(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^2d} - \frac{B(bc-ad)gin(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^2} + \frac{(bc-ad)gi(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b^2} + \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} + \frac{(B(bc-ad)^3 gin)\,\text{Subst}\left(\int \frac{A+Bn+B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b^2d} + \frac{(B^2(bc-ad)^3 gin^2)\,\text{Subst}\left(\int \left(\frac{b}{d(-b+dx)^2}+\frac{1}{d(-b+dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{3b^2}$$

$$
\begin{aligned}
&= \frac{B^2(bc-ad)^2gin^2x}{3bd} - \frac{B(bc-ad)^2gin(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^2d} \\
&\quad - \frac{B(bc-ad)gin(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^2} \\
&\quad + \frac{(bc-ad)gi(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b^2} \\
&\quad + \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} \\
&\quad - \frac{B(bc-ad)^3gin\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3b^2d^2} \\
&\quad - \frac{B^2(bc-ad)^3gin^2\log(c+dx)}{3b^2d^2} \\
&\quad + \frac{(B^2(bc-ad)^3gin^2)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^2d^2} \\
&= \frac{B^2(bc-ad)^2gin^2x}{3bd} - \frac{B(bc-ad)^2gin(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^2d} \\
&\quad - \frac{B(bc-ad)gin(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^2} \\
&\quad + \frac{(bc-ad)gi(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b^2} \\
&\quad + \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} \\
&\quad - \frac{B(bc-ad)^3gin\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3b^2d^2} \\
&\quad - \frac{B^2(bc-ad)^3gin^2\log(c+dx)}{3b^2d^2} - \frac{B^2(bc-ad)^3gin^2\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{3b^2d^2}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 937 vs. 2(372) = 744.

Time = 0.44 (sec) , antiderivative size = 937, normalized size of antiderivative = 2.52

$$
\int(ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx
$$

$$
= \frac{gi\left(-6Ab^2Bcd(bc-ad)nx+6aAbBd^2(-bc+ad)nx+4AbBd(bc-ad)(bc+ad)nx-6bB^2cd(bc-ad)n(a\right.}{}
$$

```
[In] Integrate[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2,x]
```

```
[Out] (g*i*(-6*A*b^2*B*c*d*(b*c - a*d)*n*x + 6*a*A*b*B*d^2*(-(b*c) + a*d)*n*x + 4
*A*b*B*d*(b*c - a*d)*(b*c + a*d)*n*x - 6*b*B^2*c*d*(b*c - a*d)*n*(a + b*x)*
Log[e*((a + b*x)/(c + d*x))^n] + 6*a*B^2*d^2*(-(b*c) + a*d)*n*(a + b*x)*Log
[e*((a + b*x)/(c + d*x))^n] + 4*B^2*d*(b*c - a*d)*(b*c + a*d)*n*(a + b*x)*L
og[e*((a + b*x)/(c + d*x))^n] - 2*b^2*B*d^2*(b*c - a*d)*n*x^2*(A + B*Log[e*
((a + b*x)/(c + d*x))^n]) + 6*a^2*b*B*c*d^2*n*Log[a + b*x]*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n]) - 2*a^3*B*d^3*n*Log[a + b*x]*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]) + 6*a*b^2*c*d^2*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
+ 3*b^2*d^2*(b*c + a*d)*x^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 2*b^
3*d^3*x^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 6*b*B^2*c*(b*c - a*d)^
2*n^2*Log[c + d*x] + 6*a*B^2*d*(b*c - a*d)^2*n^2*Log[c + d*x] - 4*B^2*(b*c
- a*d)^2*(b*c + a*d)*n^2*Log[c + d*x] + 2*b^3*B*c^3*n*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])*Log[c + d*x] - 6*a*b^2*B*c^2*d*n*(A + B*Log[e*((a + b*x)/
(c + d*x))^n])*Log[c + d*x] + 2*B^2*(b*c - a*d)*n^2*(a^2*d^2*Log[a + b*x] -
 b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - 3*a^2*b*B^2*c*d^2*n^2*(Log[
a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (
d*(a + b*x))/(-(b*c) + a*d)]) + a^3*B^2*d^3*n^2*(Log[a + b*x]*(Log[a + b*x]
 - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) +
 a*d)]) - b^3*B^2*c^3*n^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d
*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 3*a*b^2*B^2*
c^2*d*n^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x
] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(6*b^2*d^2)
```

Maple [F]

$$\int (bgx+ag)\,(dix+ci)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}dx$$

```
[In] int((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}\,dx = \int (bgx+ag)(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}\,dx$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, alg
orithm="fricas")
```

```
[Out] integral(A^2*b*d*g*i*x^2 + A^2*a*c*g*i + (A^2*b*c + A^2*a*d)*g*i*x + (B^2*b
*d*g*i*x^2 + B^2*a*c*g*i + (B^2*b*c + B^2*a*d)*g*i*x)*log(e*((b*x + a)/(d*x
```

```
 + c))^n)^2 + 2*(A*B*b*d*g*i*x^2 + A*B*a*c*g*i + (A*B*b*c + A*B*a*d)*g*i*x)
*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1542 vs. $2(355) = 710$.

Time = 0.71 (sec) , antiderivative size = 1542, normalized size of antiderivative = 4.15

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, alg
orithm="maxima")
```

```
[Out] 2/3*A*B*b*d*g*i*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*b*d*g*
i*x^3 + A*B*b*c*g*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*B*a*d*g*
i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*b*c*g*i*x^2 + 1/2*A^
2*a*d*g*i*x^2 + 1/3*A*B*b*d*g*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x +
 c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) -
A*B*b*c*g*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/
(b*d)) - A*B*a*d*g*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c
- a*d)*x/(b*d)) + 2*A*B*a*c*g*i*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2
*A*B*a*c*g*i*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*c*g*i*x - 1/3
*(a^2*c*d^2*g*i*n^2 - b^2*c^3*g*i*n*log(e) - (g*i*n^2 - 3*g*i*n*log(e))*a*b
*c^2*d)*B^2*log(d*x + c)/(b*d^2) + 1/3*(b^3*c^3*g*i*n^2 - 3*a*b^2*c^2*d*g*i
*n^2 + 3*a^2*b*c*d^2*g*i*n^2 - a^3*d^3*g*i*n^2)*(log(b*x + a)*log((b*d*x +
a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^2) +
1/6*(2*B^2*b^3*d^3*g*i*x^3*log(e)^2 - ((2*g*i*n*log(e) - 3*g*i*log(e)^2)*b^
3*c*d^2 - (2*g*i*n*log(e) + 3*g*i*log(e)^2)*a*b^2*d^3)*B^2*x^2 - (3*a^2*b*c
*d^2*g*i*n^2 - a^3*d^3*g*i*n^2)*B^2*log(b*x + a)^2 - 2*(b^3*c^3*g*i*n^2 - 3
*a*b^2*c^2*d*g*i*n^2)*B^2*log(b*x + a)*log(d*x + c) + (b^3*c^3*g*i*n^2 - 3*
a*b^2*c^2*d*g*i*n^2)*B^2*log(d*x + c)^2 + 2*((g*i*n^2 - g*i*n*log(e))*b^3*c
^2*d - (2*g*i*n^2 - 3*g*i*log(e)^2)*a*b^2*c*d^2 + (g*i*n^2 + g*i*n*log(e))*
```

```
a^2*b*d^3)*B^2*x - 2*(a*b^2*c^2*d*g*i*n^2 + a^3*d^3*g*i*n*log(e) - (g*i*n^2
 + 3*g*i*n*log(e))*a^2*b*c*d^2)*B^2*log(b*x + a) + (2*B^2*b^3*d^3*g*i*x^3 +
 6*B^2*a*b^2*c*d^2*g*i*x + 3*(b^3*c*d^2*g*i + a*b^2*d^3*g*i)*B^2*x^2)*log((
b*x + a)^n)^2 + (2*B^2*b^3*d^3*g*i*x^3 + 6*B^2*a*b^2*c*d^2*g*i*x + 3*(b^3*c
*d^2*g*i + a*b^2*d^3*g*i)*B^2*x^2)*log((d*x + c)^n)^2 + 2*(2*B^2*b^3*d^3*g*
i*x^3*log(e) - ((g*i*n - 3*g*i*log(e))*b^3*c*d^2 - (g*i*n + 3*g*i*log(e))*a
*b^2*d^3)*B^2*x^2 - (b^3*c^2*d*g*i*n - a^2*b*d^3*g*i*n - 6*a*b^2*c*d^2*g*i*
log(e))*B^2*x + (3*a^2*b*c*d^2*g*i*n - a^3*d^3*g*i*n)*B^2*log(b*x + a) + (b
^3*c^3*g*i*n - 3*a*b^2*c^2*d*g*i*n)*B^2*log(d*x + c))*log((b*x + a)^n) - 2*
(2*B^2*b^3*d^3*g*i*x^3*log(e) - ((g*i*n - 3*g*i*log(e))*b^3*c*d^2 - (g*i*n
+ 3*g*i*log(e))*a*b^2*d^3)*B^2*x^2 - (b^3*c^2*d*g*i*n - a^2*b*d^3*g*i*n - 6
*a*b^2*c*d^2*g*i*log(e))*B^2*x + (3*a^2*b*c*d^2*g*i*n - a^3*d^3*g*i*n)*B^2*
log(b*x + a) + (b^3*c^3*g*i*n - 3*a*b^2*c^2*d*g*i*n)*B^2*log(d*x + c) + (2*
B^2*b^3*d^3*g*i*x^3 + 6*B^2*a*b^2*c*d^2*g*i*x + 3*(b^3*c*d^2*g*i + a*b^2*d^
3*g*i)*B^2*x^2)*log((b*x + a)^n))*log((d*x + c)^n))/(b^2*d^2)
```

Giac [F]

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \int (bgx+ag)(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, alg
orithm="giac")
```

```
[Out] integrate((b*g*x + a*g)*(d*i*x + c*i)*(B*log(e*((b*x + a)/(d*x + c))^n) + A
)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \int (a\,g+b\,g\,x)\,(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2\,dx$$

```
[In] int((a*g + b*g*x)*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((a*g + b*g*x)*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x
)
```

3.162 $\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 33, antiderivative size = 220

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = -\frac{B(bc-ad)in(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2}+\frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d}+\frac{B^2(bc-ad)^2in^2\log(c+dx)}{b^2d}+\frac{B(bc-ad)^2in\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d}-\frac{B^2(bc-ad)^2in^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d}$$

```
[Out] -B*(-a*d+b*c)*i*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2+1/2*i*(d*x+c)
^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d+B^2*(-a*d+b*c)^2*i*n^2*ln(d*x+c)/b^2
/d+B*(-a*d+b*c)^2*i*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x
+a))/b^2/d-B^2*(-a*d+b*c)^2*i*n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used

$= \{2551, 2356, 2389, 2379, 2438, 2351, 31\}$

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \frac{Bin(bc-ad)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2d} - \frac{Bin(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2} + \frac{i(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d} - \frac{B^2in^2(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d} + \frac{B^2in^2(bc-ad)^2\log(c+dx)}{b^2d}$$

```
[In] Int[(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] -((B*(b*c - a*d)*i*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/b^2)
 + (i*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*d) + (B^2*(b
*c - a*d)^2*i*n^2*Log[c + d*x])/(b^2*d) + (B*(b*c - a*d)^2*i*n*(A + B*Log[e
*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^2*d) -
(B^2*(b*c - a*d)^2*i*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b^2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
```

```
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left((bc-ad)^2 i\right)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)\\
&= \frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d} - \frac{\left(B(bc-ad)^2 in\right)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{d}\\
&= \frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d}\\
&\quad - \frac{\left(B(bc-ad)^2 in\right)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b}\\
&\quad - \frac{\left(B(bc-ad)^2 in\right)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bd}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{B(bc-ad)in(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2} \\
&\quad + \frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d} \\
&\quad + \frac{B(bc-ad)^2in\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d} \\
&\quad + \frac{(B^2(bc-ad)^2in^2)\,\mathrm{Subst}\left(\int\frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^2} \\
&\quad - \frac{(B^2(bc-ad)^2in^2)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^2d} \\
&= -\frac{B(bc-ad)in(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2} \\
&\quad + \frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d} + \frac{B^2(bc-ad)^2in^2\log(c+dx)}{b^2d} \\
&\quad + \frac{B(bc-ad)^2in\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d} \\
&\quad - \frac{B^2(bc-ad)^2in^2\mathrm{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.98

$$
\int(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx
$$

$$
= \frac{i\left((c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + \frac{B(bc-ad)n\left(B(bc-ad)n\log^2(a+bx)-2\left(Abdx+Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+B(-bc+ad)n\right.\right.}{}\right.}{2d}
$$

```
[In] Integrate[(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (i*((c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*(b*c - a*d)*n
*(B*(b*c - a*d)*n*Log[a + b*x]^2 - 2*(A*b*d*x + B*d*(a + b*x)*Log[e*((a + b
*x)/(c + d*x))^n] + B*(-(b*c) + a*d)*n*Log[c + d*x]) - 2*(b*c - a*d)*Log[a
+ b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n] + B*n*Log[(b*(c + d*x))/(b*c -
 a*d)]) + 2*B*(-(b*c) + a*d)*n*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/b
^2))/(2*d)
```

Maple [F]

$$\int (dix + ci) \left(A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

```
[In] int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fric
as")
```

```
[Out] integral(A^2*d*i*x + A^2*c*i + (B^2*d*i*x + B^2*c*i)*log(e*((b*x + a)/(d*x
+ c))^n)^2 + 2*(A*B*d*i*x + A*B*c*i)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 825 vs. $2(217) = 434$.

Time = 0.69 (sec) , antiderivative size = 825, normalized size of antiderivative = 3.75

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = ABdix^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) + \frac{1}{2}A^2dix^2 - ABdin\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right) + 2ABcin\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right) + 2ABcix\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) + A^2cix - \frac{(acdin^2-(in^2-in\log(e))bc^2)B^2\log(dx+c)}{bd} - \frac{(b^2c^2in^2-2abcdin^2+a^2d^2in^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{b^2d} + \frac{2B^2b^2c^2in^2\log(bx+a)\log(dx+c)-B^2b^2c^2in^2\log(dx+c)^2+B^2b^2d^2ix^2\log(e)^2-(2abcdin^2-a^2d^2}{}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxi
ma")
```

```
[Out] A*B*d*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*d*i*x^2 - A*B*
d*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) +
 2*A*B*c*i*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*c*i*x*log(e*(b*x
/(d*x + c) + a/(d*x + c))^n) + A^2*c*i*x - (a*c*d*i*n^2 - (i*n^2 - i*n*log(
e))*b*c^2)*B^2*log(d*x + c)/(b*d) - (b^2*c^2*i*n^2 - 2*a*b*c*d*i*n^2 + a^2*
d^2*i*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x
 + a*d)/(b*c - a*d)))*B^2/(b^2*d) + 1/2*(2*B^2*b^2*c^2*i*n^2*log(b*x + a)*l
og(d*x + c) - B^2*b^2*c^2*i*n^2*log(d*x + c)^2 + B^2*b^2*d^2*i*x^2*log(e)^2
 - (2*a*b*c*d*i*n^2 - a^2*d^2*i*n^2)*B^2*log(b*x + a)^2 + 2*(a*b*d^2*i*n*lo
g(e) - (i*n*log(e) - i*log(e)^2)*b^2*c*d)*B^2*x - 2*((i*n^2 - 2*i*n*log(e))
*a*b*c*d - (i*n^2 - i*n*log(e))*a^2*d^2)*B^2*log(b*x + a) + (B^2*b^2*d^2*i*
x^2 + 2*B^2*b^2*c*d*i*x)*log((b*x + a)^n)^2 + (B^2*b^2*d^2*i*x^2 + 2*B^2*b^
2*c*d*i*x)*log((d*x + c)^n)^2 + 2*(B^2*b^2*d^2*i*x^2*log(e) - B^2*b^2*c^2*i
*n*log(d*x + c) + (a*b*d^2*i*n - (i*n - 2*i*log(e))*b^2*c*d)*B^2*x + (2*a*b
*c*d*i*n - a^2*d^2*i*n)*B^2*log(b*x + a))*log((b*x + a)^n) - 2*(B^2*b^2*d^2
*i*x^2*log(e) - B^2*b^2*c^2*i*n*log(d*x + c) + (a*b*d^2*i*n - (i*n - 2*i*lo
g(e))*b^2*c*d)*B^2*x + (2*a*b*c*d*i*n - a^2*d^2*i*n)*B^2*log(b*x + a) + (B^
2*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b
^2*d)
```

Giac [F]

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac
")
[Out] integrate((d*i*x + c*i)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (c\,i+d\,i\,x)\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

```
[In] int((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
[Out] int((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.163 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 306

$$\begin{aligned}
&\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx \\
&= \frac{di(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2g} \\
&\quad+ \frac{2B(bc-ad)in\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^2g} \\
&\quad- \frac{(bc-ad)i\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g} \\
&\quad+ \frac{2B^2(bc-ad)in^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2g} \\
&\quad+ \frac{2B(bc-ad)in\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g} \\
&\quad+ \frac{2B^2(bc-ad)in^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g}
\end{aligned}$$

```
[Out] d*i*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2/g+2*B*(-a*d+b*c)*i*n*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^2/g-(-a*d+b*c)*i*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^2/g+2*B^2*(-a*d+
b*c)*i*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/g+2*B*(-a*d+b*c)*i*n*(A+B*ln(
e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/g+2*B^2*(-a*d+b*
c)*i*n^2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^2/g
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {2561, 2389, 2379, 2421, 6724, 2355, 2354, 2438}

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$$
$$=\frac{2Bin(bc-ad)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2g}+\frac{2Bin(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2g}+\frac{di(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^2g}-\frac{i(bc-ad)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^2g}+\frac{2B^2in^2(bc-ad)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2g}+\frac{2B^2in^2(bc-ad)\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g}$$

```
[In] Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x),
x]
```

```
[Out] (d*i*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(b^2*g) + (2*B*(b*
c - a*d)*i*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c +
 d*x))])/(b^2*g) - ((b*c - a*d)*i*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*
Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^2*g) + (2*B^2*(b*c - a*d)*i*n^2*Po
lyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^2*g) + (2*B*(b*c - a*d)*i*n*(A +
B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/
(b^2*g) + (2*B^2*(b*c - a*d)*i*n^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))])
/(b^2*g)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
```

```
, p}, x] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)i)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{g}$$

$$= \frac{((bc-ad)i)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg} + \frac{(d(bc-ad)i)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg}$$

$$= \frac{di(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2g} - \frac{(bc-ad)i\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g} + \frac{(2B(bc-ad)in)\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)(A+B\log(ex^n))}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g} - \frac{(2Bd(bc-ad)in)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g}$$

$$= \frac{di(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2g} + \frac{2B(bc-ad)in\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^2g} - \frac{(bc-ad)i\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g} + \frac{2B(bc-ad)in\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g} - \frac{(2B^2(bc-ad)in^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g} - \frac{(2B^2(bc-ad)in^2)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g}$$

$$
=\frac{di(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2g}+\frac{2B(bc-ad)in\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^2g}
-\frac{(bc-ad)i\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g}+\frac{2B^2(bc-ad)in^2\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^2g}
+\frac{2B(bc-ad)in\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\mathrm{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g}+\frac{2B^2(bc-ad)in^2\mathrm{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1372 vs. $2(306) = 612$.

Time = 0.59 (sec) , antiderivative size = 1372, normalized size of antiderivative = 4.48

$$
\int\frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx
=\frac{i\Big(3bdx\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-Bn\log\left(\frac{a+bx}{c+dx}\right)\right)^2+3(bc-ad)\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-Bn\log\right.}{}
$$

```
[In] Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b
*g*x),x]

[Out] (i*(3*b*d*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c +
d*x)])^2 + 3*(b*c - a*d)*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
 - B*n*Log[(a + b*x)/(c + d*x)])^2 - 3*B*n*(A + B*Log[e*((a + b*x)/(c + d*x
))^n] - B*n*Log[(a + b*x)/(c + d*x)])*(a*d*Log[a/b + x]^2 - 2*a*d*Log[a/b +
 x]*(1 + Log[a + b*x]) + 2*(-(b*c) + a*d + Log[c/d + x]*(b*c + a*d*Log[a +
b*x] - a*d*Log[(d*(a + b*x))/(-(b*c) + a*d)]) + (-(b*d*x) + a*d*Log[a + b*x
])*Log[(a + b*x)/(c + d*x)]) - 2*a*d*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
 + 3*b*B*c*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c +
 d*x)])*(Log[a/b + x]^2 - 2*Log[a + b*x]*(Log[a/b + x] - Log[c/d + x] - Log
[(a + b*x)/(c + d*x)]) - 2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]
+ PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - B^2*n^2*(a*d*Log[a/b + x]^3 - 3
*d*(2*b*x - 2*(a + b*x)*Log[a/b + x] + (a + b*x)*Log[a/b + x]^2) - 3*b*(2*d
*x - 2*(c + d*x)*Log[c/d + x] + (c + d*x)*Log[c/d + x]^2) - 3*d*(b*x - a*Lo
g[a + b*x])*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])^2 + 6
*(a*d + 2*b*d*x - b*d*x*Log[c/d + x] - b*c*Log[c + d*x] + Log[a/b + x]*(-(d
*(a + b*x)) + d*(a + b*x)*Log[c/d + x] + (b*c - a*d)*Log[(b*(c + d*x))/(b*c
 - a*d)]) + (b*c - a*d)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 3*(Log[
a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)])*(-2*b*c + 2*a*d - 2*d*(
a + b*x)*Log[a/b + x] + a*d*Log[a/b + x]^2 + 2*Log[c/d + x]*(b*(c + d*x) -
a*d*Log[(d*(a + b*x))/(-(b*c) + a*d)]) - 2*a*d*PolyLog[2, (b*(c + d*x))/(b*
c - a*d)]) - 3*a*d*(Log[a/b + x]^2*(Log[c/d + x] - Log[(b*(c + d*x))/(b*c -
 a*d)]) - 2*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2*PolyL
```

```
og[3, (d*(a + b*x))/(-(b*c) + a*d)]) + 3*a*d*(Log[c/d + x]^2*Log[(d*(a + b*
x))/(-(b*c) + a*d)] + 2*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]
- 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])) + b*B^2*c*n^2*(Log[a/b + x]^3 +
 3*Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 3*Log[a + b*x]*(-Log[
a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])^2 + 3*Log[a/b + x]^2*(-
Log[c/d + x] + Log[(b*(c + d*x))/(b*c - a*d)]) + 6*Log[a/b + x]*PolyLog[2,
(d*(a + b*x))/(-(b*c) + a*d)] + 6*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*
c - a*d)] - 3*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)])*(Log
[a/b + x]^2 - 2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2
, (b*(c + d*x))/(b*c - a*d)])) - 6*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)]
 - 6*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])))/(3*b^2*g)
```

Maple [F]

$$\int \frac{(dix + ci)\left(A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{bgx + ag}dx$$

```
[In] int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
[Out] int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{(ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{ag + bgx}\,dx = \int \frac{(dix + ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right) + A\right)^{2}}{bgx + ag}\,dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, alg
orithm="fricas")
[Out] integral((A^2*d*i*x + A^2*c*i + (B^2*d*i*x + B^2*c*i)*log(e*((b*x + a)/(d*x
 + c))^n)^2 + 2*(A*B*d*i*x + A*B*c*i)*log(e*((b*x + a)/(d*x + c))^n))/(b*g*
x + a*g), x)
```

Sympy [F]

$$\int \frac{(ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{ag + bgx}\,dx = \frac{i\left(\int \frac{A^2c}{a+bx}\,dx + \int \frac{A^2dx}{a+bx}\,dx + \int \frac{B^2c\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)^{2}}{a+bx}\,dx + \int \frac{2ABc\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}{a+bx}\,dx + \int \frac{B^2dx\log\left(e\left(\frac{a}{c+dx}+\frac{b}{c+}\right.\right.}{a+bx}\right.}{g}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g),x)
```

```
[Out] i*(Integral(A**2*c/(a + b*x), x) + Integral(A**2*d*x/(a + b*x), x) + Integr
al(B**2*c*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a + b*x), x) + Integr
al(2*A*B*c*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x) + Integral
(B**2*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a + b*x), x) + Integr
al(2*A*B*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x))/g
```

Maxima [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, alg
orithm="maxima")
```

```
[Out] A^2*d*i*(x/(b*g) - a*log(b*x + a)/(b^2*g)) + A^2*c*i*log(b*g*x + a*g)/(b*g)
 + (B^2*b*d*i*x + (b*c*i - a*d*i)*B^2*log(b*x + a))*log((d*x + c)^n)^2/(b^2
*g) - integrate(-(B^2*b^2*c^2*i*log(e)^2 + 2*A*B*b^2*c^2*i*log(e) + (B^2*b^
2*d^2*i*log(e)^2 + 2*A*B*b^2*d^2*i*log(e))*x^2 + (B^2*b^2*d^2*i*x^2 + 2*B^2
*b^2*c*d*i*x + B^2*b^2*c^2*i)*log((b*x + a)^n)^2 + 2*(B^2*b^2*c*d*i*log(e)^
2 + 2*A*B*b^2*c*d*i*log(e))*x + 2*(B^2*b^2*c^2*i*log(e) + A*B*b^2*c^2*i + (
B^2*b^2*d^2*i*log(e) + A*B*b^2*d^2*i)*x^2 + 2*(B^2*b^2*c*d*i*log(e) + A*B*b
^2*c*d*i)*x)*log((b*x + a)^n) - 2*(B^2*b^2*c^2*i*log(e) + A*B*b^2*c^2*i + (
(i*n + i*log(e))*B^2*b^2*d^2 + A*B*b^2*d^2*i)*x^2 + (2*A*B*b^2*c*d*i + (a*b
*d^2*i*n + 2*b^2*c*d*i*log(e))*B^2)*x + ((b^2*c*d*i*n - a*b*d^2*i*n)*B^2*x
+ (a*b*c*d*i*n - a^2*d^2*i*n)*B^2)*log(b*x + a) + (B^2*b^2*d^2*i*x^2 + 2*B^
2*b^2*c*d*i*x + B^2*b^2*c^2*i)*log((b*x + a)^n))*log((d*x + c)^n))/(b^3*d*g
*x^2 + a*b^2*c*g + (b^3*c*g + a*b^2*d*g)*x), x)
```

Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, alg
orithm="giac")
```

```
[Out] integrate((d*i*x + c*i)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x + a
*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag + bgx}\,dx = \int \frac{(c\,i + d\,i\,x)\ \left(A + B\ \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{a\,g + b\,g\,x}\,dx$$

```
[In] int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x),
x)
```

```
[Out] int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x),
 x)
```

3.164 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 261

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = -\frac{2B^2in^2(c+dx)}{bg^2(a+bx)}-\frac{2Bin(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a+bx)} - \frac{i(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{bg^2(a+bx)}-\frac{di\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} + \frac{2Bdin\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}+\frac{2B^2din^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}$$

```
[Out] -2*B^2*i*n^2*(d*x+c)/b/g^2/(b*x+a)-2*B*i*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+
c))^n))/b/g^2/(b*x+a)-i*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b/g^2/(b*
x+a)-d*i*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^2/g^
2+2*B*d*i*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/
b^2/g^2+2*B^2*d*i*n^2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^2/g^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {2561, 2380, 2342, 2341, 2379, 2421, 6724}

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$$
$$=\frac{2Bdin\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2g^2}-\frac{di\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^2g^2}-\frac{2Bin(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg^2(a+bx)}-\frac{i(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{bg^2(a+bx)}+\frac{2B^2din^2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}-\frac{2B^2in^2(c+dx)}{bg^2(a+bx)}$$

```
[In] Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x)^
2,x]
```

```
[Out] (-2*B^2*i*n^2*(c + d*x))/(b*g^2*(a + b*x)) - (2*B*i*n*(c + d*x)*(A + B*Log[
e*((a + b*x)/(c + d*x))^n]))/(b*g^2*(a + b*x)) - (i*(c + d*x)*(A + B*Log[e*
((a + b*x)/(c + d*x))^n])^2)/(b*g^2*(a + b*x)) - (d*i*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^2*g^2) + (2*B*
d*i*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (b*(c + d*x))/(d*(a
 + b*x))])/(b^2*g^2) + (2*B^2*d*i*n^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x)
)])/(b^2*g^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
```

```
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{i\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^2}$$

$$= \frac{i\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^2} + \frac{(di)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^2}$$

$$= -\frac{i(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{bg^2(a+bx)} - \frac{di\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} + \frac{(2Bin)\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x^2}dx, x, \frac{a+bx}{c+dx}\right)}{bg^2} + \frac{(2Bdin)\text{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)(A+B\log(ex^n))}{x}dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2}$$

$$= -\frac{2B^2in^2(c+dx)}{bg^2(a+bx)} - \frac{2Bin(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a+bx)} - \frac{i(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{bg^2(a+bx)} - \frac{di\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} + \frac{2Bdin\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} - \frac{(2B^2din^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{b}{dx}\right)}{x}dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2}$$

$$= -\frac{2B^2in^2(c+dx)}{bg^2(a+bx)} - \frac{2Bin(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a+bx)} - \frac{i(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{bg^2(a+bx)} - \frac{di\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} + \frac{2Bdin\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} + \frac{2B^2din^2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1564 vs. $2(261) = 522$.

Time = 1.09 (sec) , antiderivative size = 1564, normalized size of antiderivative = 5.99

$$\int\frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}dx$$

$$= \frac{i\left(-\frac{3(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-Bn\log\left(\frac{a+bx}{c+dx}\right)\right)^2}{a+bx} + 3d\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-Bn\log\left(\frac{a+bx}{c+dx}\right)\right)^2 + \frac{6b}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b
*g*x)^2,x]
```

```
[Out] (i*((-3*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*
x)/(c + d*x)])^2)/(a + b*x) + 3*d*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2 + (6*b*B*c*n*(-A - B*Log[e*((a
+ b*x)/(c + d*x))^n] + B*n*Log[(a + b*x)/(c + d*x)])*(-(d*(a + b*x)*Log[c/d
 + x]) + d*(a + b*x)*Log[(d*(a + b*x))/(-(b*c) + a*d)] + (b*c - a*d)*(1 + L
og[(a + b*x)/(c + d*x)])))/((b*c - a*d)*(a + b*x)) + (3*b*B^2*c*n^2*(-2*b*c
 + 2*a*d - 2*d*(a + b*x)*Log[a + b*x] - 2*(b*c - a*d)*Log[(a + b*x)/(c + d*
x)] - 2*d*(a + b*x)*Log[a + b*x]*Log[(a + b*x)/(c + d*x)] - (b*c - a*d)*Log
[(a + b*x)/(c + d*x)]^2 + 2*d*(a + b*x)*Log[c + d*x] - 2*d*(a + b*x)*Log[(a
 + b*x)/(c + d*x)]*Log[(b*c - a*d)/(b*c + b*d*x)] + d*(a + b*x)*(Log[a + b*
x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a +
 b*x))/(-(b*c) + a*d)]) + d*(a + b*x)*(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*Lo
g[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c - a*d)/(b*c + b*d*x)]) - 2*PolyL
og[2, (b*(c + d*x))/(b*c - a*d)])))/((b*c - a*d)*(a + b*x)) + 3*B*d*n*(A +
B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])*(Log[a/b +
 x]^2 - 2*Log[a/b + x]*Log[a + b*x] - 2*Log[c/d + x]*Log[(d*(a + b*x))/(-(b
*c) + a*d)] + 2*Log[a + b*x]*((a*d)/(b*c - a*d) + Log[c/d + x] + Log[(a + b
*x)/(c + d*x)]) + 2*a*((a + b*x)^(-1) + Log[(a + b*x)/(c + d*x)]/(a + b*x)
+ (d*Log[c + d*x])/(-(b*c) + a*d)) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)
]) + (B^2*d*n^2*((b*c - a*d)*(a + b*x)*Log[a/b + x]^3 + 3*a*(b*c - a*d)*(2
+ 2*Log[a/b + x] + Log[a/b + x]^2) + 3*(b*c - a*d)*(a + (a + b*x)*Log[a + b
*x])*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])^2 + 3*a*(d*(
a + b*x)*Log[a/b + x]^2 + 2*((-(b*c) + a*d)*Log[c/d + x] + d*(a + b*x)*(Log
[a + b*x] - Log[c + d*x])) - 2*Log[a/b + x]*((b*c - a*d)*Log[c/d + x] + d*(
a + b*x)*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*d*(a + b*x)*PolyLog[2, (d*(a +
 b*x))/(-(b*c) + a*d)]) + 3*a*(Log[c/d + x]*(b*(c + d*x)*Log[c/d + x] - 2*d
*(a + b*x)*Log[(d*(a + b*x))/(-(b*c) + a*d)]) - 2*d*(a + b*x)*PolyLog[2, (b
*(c + d*x))/(b*c - a*d)]) - 3*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/
(c + d*x)])*((b*c - a*d)*(a + b*x)*Log[a/b + x]^2 + 2*a*(b*c - a*d)*(1 + Lo
g[a/b + x]) + 2*a*(-(b*c) + a*d)*Log[c/d + x] + 2*a*d*(a + b*x)*(Log[a + b*
x] - Log[c + d*x]) - 2*(b*c - a*d)*(a + b*x)*(Log[c/d + x]*Log[(d*(a + b*x)
)/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - 3*(b*c - a*d)
*(a + b*x)*(Log[a/b + x]^2*(Log[c/d + x] - Log[(b*(c + d*x))/(b*c - a*d)])
- 2*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2*PolyLog[3, (d
*(a + b*x))/(-(b*c) + a*d)]) + 3*(b*c - a*d)*(a + b*x)*(Log[c/d + x]^2*Log[
(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*
c - a*d)] - 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])))/((b*c - a*d)*(a + b*
x))))/(3*b^2*g^2)
```

Maple [F]

$$\int \frac{(dix+ci)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(bgx+ag)^2}dx$$

```
[In] int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
[Out] int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

Fricas [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x, a
lgorithm="fricas")
[Out] integral((A^2*d*i*x + A^2*c*i + (B^2*d*i*x + B^2*c*i)*log(e*((b*x + a)/(d*x
 + c))^n)^2 + 2*(A*B*d*i*x + A*B*c*i)*log(e*((b*x + a)/(d*x + c))^n))/(b^2*
g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x, a
lgorithm="maxima")
[Out] -2*A*B*c*i*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2
) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) + A^2*d*i*(a/(b^3*g^2*x + a*b^2*g
^2) + log(b*x + a)/(b^2*g^2)) - 2*A*B*c*i*log(e*(b*x/(d*x + c) + a/(d*x + c
```

```
))^n)/(b^2*g^2*x + a*b*g^2) - A^2*c*i/(b^2*g^2*x + a*b*g^2) - ((b*c*i - a*d
*i)*B^2 - (B^2*b*d*i*x + B^2*a*d*i)*log(b*x + a))*log((d*x + c)^n)^2/(b^3*g
^2*x + a*b^2*g^2) - integrate(-(B^2*b^2*c^2*i*log(e)^2 + (B^2*b^2*d^2*i*log
(e)^2 + 2*A*B*b^2*d^2*i*log(e))*x^2 + (B^2*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i*
x + B^2*b^2*c^2*i)*log((b*x + a)^n)^2 + 2*(B^2*b^2*c*d*i*log(e)^2 + A*B*b^2
*c*d*i*log(e))*x + 2*(B^2*b^2*c^2*i*log(e) + (B^2*b^2*d^2*i*log(e) + A*B*b^
2*d^2*i)*x^2 + (2*B^2*b^2*c*d*i*log(e) + A*B*b^2*c*d*i)*x)*log((b*x + a)^n)
 + 2*((a*b*c*d*i*n - a^2*d^2*i*n - b^2*c^2*i*log(e))*B^2 - (B^2*b^2*d^2*i*l
og(e) + A*B*b^2*d^2*i)*x^2 - (A*B*b^2*c*d*i + (a*b*d^2*i*n - (i*n - 2*i*log
(e))*b^2*c*d)*B^2)*x - (B^2*b^2*d^2*i*n*x^2 + 2*B^2*a*b*d^2*i*n*x + B^2*a^2
*d^2*i*n)*log(b*x + a) - (B^2*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i*x + B^2*b^2*c
^2*i)*log((b*x + a)^n))*log((d*x + c)^n))/(b^4*d*g^2*x^3 + a^2*b^2*c*g^2 +
(b^4*c*g^2 + 2*a*b^3*d*g^2)*x^2 + (2*a*b^3*c*g^2 + a^2*b^2*d*g^2)*x), x)
```

Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x, a
lgorithm="giac")
```

```
[Out] integrate((d*i*x + c*i)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x + a
*g)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(a\,g+b\,g\,x)^2}\,dx$$

```
[In] int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x)^
2,x)
```

```
[Out] int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x)^
2, x)
```

3.165 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 151

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{B^2in^2(c+dx)^2}{4(bc-ad)g^3(a+bx)^2} - \frac{Bin(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)g^3(a+bx)^2} - \frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)g^3(a+bx)^2}$$

```
[Out] -1/4*B^2*i*n^2*(d*x+c)^2/(-a*d+b*c)/g^3/(b*x+a)^2-1/2*B*i*n*(d*x+c)^2*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/g^3/(b*x+a)^2-1/2*i*(d*x+c)^2*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/g^3/(b*x+a)^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.070, Rules used = {2561, 2342, 2341}

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{i(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g^3(a+bx)^2(bc-ad)} - \frac{Bin(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^3(a+bx)^2(bc-ad)} - \frac{B^2in^2(c+dx)^2}{4g^3(a+bx)^2(bc-ad)}$$

```
[In] Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x)^
3,x]
```

```
[Out] -1/4*(B^2*i*n^2*(c + d*x)^2)/((b*c - a*d)*g^3*(a + b*x)^2) - (B*i*n*(c + d*
x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*c - a*d)*g^3*(a + b*x)^2
) - (i*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(b*c - a*d)
*g^3*(a + b*x)^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^3}\\
&= -\frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)g^3(a+bx)^2}+\frac{(Bin)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^3}\\
&= -\frac{B^2in^2(c+dx)^2}{4(bc-ad)g^3(a+bx)^2}-\frac{Bin(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)g^3(a+bx)^2}\\
&\quad -\frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)g^3(a+bx)^2}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 801, normalized size of antiderivative = 5.30

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx =$$
$$-\frac{i\left(2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-4d(-bc+ad)(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+4Bdn(a+bx\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b
*g*x)^3,x]
```

```
[Out] -1/4*(i*(2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - 4*d*(-(
b*c) + a*d)*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 4*B*d*n*(a
 + b*x)*(2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d*(a + b*
x)*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*d*(a + b*x)*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 2*B*n*(b*c - a*d + d*(a +
 b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*d*n*(a + b*x)*(Log[a + b
*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a
+ b*x))/(-(b*c) + a*d)]) + B*d*n*(a + b*x)*((2*Log[(d*(a + b*x))/(-(b*c) +
a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)
])) + B*n*(2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*d*(-(
b*c) + a*d)*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 4*d^2*(a + b
*x)^2*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*d^2*(a + b*x)
^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 4*B*d*n*(a + b*x)*
(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) + B*n*((b
*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x]
 + 2*d^2*(a + b*x)^2*Log[c + d*x]) + 2*B*d^2*n*(a + b*x)^2*(Log[a + b*x]*(L
og[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x)
)/(-(b*c) + a*d)]) - 2*B*d^2*n*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) +
a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)
]))))/(b^2*(b*c - a*d)*g^3*(a + b*x)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(145) = 290.

Time = 4.83 (sec) , antiderivative size = 530, normalized size of antiderivative = 3.51

method	result
parallelrisch	$-\frac{-8ABx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4cd^2in-4B^2x\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4cd^2in^2+4ABxab^3d^3in^2-4ABxb^4cd^2in^2-4AB\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b}{}$

```
[In] int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x,method=_R
ETURNVERBOSE)
```

```
[Out] -1/4*(-8*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^2*i*n-4*B^2*x*ln(e*((b*x+a
)/(d*x+c))^n)*b^4*c*d^2*i*n^2+4*A*B*x*a*b^3*d^3*i*n^2-4*A*B*x*b^4*c*d^2*i*n
^2-4*A*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2*d*i*n-4*A*B*x^2*ln(e*((b*x+a)/(d
*x+c))^n)*b^4*d^3*i*n-4*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*c*d^2*i*n+2*A
*B*a^2*b^2*d^3*i*n^2+4*A^2*x*a*b^3*d^3*i*n-4*A^2*x*b^4*c*d^2*i*n-2*B^2*x^2*
ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*d^3*i*n-2*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)
*b^4*d^3*i*n^2-2*A*B*b^4*c^2*d*i*n^2+2*B^2*x*a*b^3*d^3*i*n^3-2*B^2*x*b^4*c*
d^2*i*n^3-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*c^2*d*i*n-2*B^2*ln(e*((b*x+
a)/(d*x+c))^n)*b^4*c^2*d*i*n^2+B^2*a^2*b^2*d^3*i*n^3-B^2*b^4*c^2*d*i*n^3+2*
A^2*a^2*b^2*d^3*i*n-2*A^2*b^4*c^2*d*i*n)/g^3/(b*x+a)^2/b^4/d/n/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(145) = 290$.

Time = 0.35 (sec) , antiderivative size = 600, normalized size of antiderivative = 3.97

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{(B^2b^2c^2-B^2a^2d^2)in^2+2\,(ABb^2c^2-ABa^2d^2)in+2\,(2\,(B^2b^2cd-B^2abd^2)ix+(B^2b^2c^2-B^2a^2d^2)i)\,\mathrm{lo}}{}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x, a
lgorithm="fricas")
```

```
[Out] -1/4*((B^2*b^2*c^2 - B^2*a^2*d^2)*i*n^2 + 2*(A*B*b^2*c^2 - A*B*a^2*d^2)*i*n
 + 2*(2*(B^2*b^2*c*d - B^2*a*b*d^2)*i*x + (B^2*b^2*c^2 - B^2*a^2*d^2)*i)*lo
g(e)^2 + 2*(B^2*b^2*d^2*i*n^2*x^2 + 2*B^2*b^2*c*d*i*n^2*x + B^2*b^2*c^2*i*n
^2)*log((b*x + a)/(d*x + c))^2 + 2*(A^2*b^2*c^2 - A^2*a^2*d^2)*i + 2*((B^2*
b^2*c*d - B^2*a*b*d^2)*i*n^2 + 2*(A*B*b^2*c*d - A*B*a*b*d^2)*i*n + 2*(A^2*b
^2*c*d - A^2*a*b*d^2)*i)*x + 2*((B^2*b^2*c^2 - B^2*a^2*d^2)*i*n + 2*(A*B*b^
2*c^2 - A*B*a^2*d^2)*i + 2*((B^2*b^2*c*d - B^2*a*b*d^2)*i*n + 2*(A*B*b^2*c*
d - A*B*a*b*d^2)*i)*x + 2*(B^2*b^2*d^2*i*n*x^2 + 2*B^2*b^2*c*d*i*n*x + B^2*
b^2*c^2*i*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*(B^2*b^2*c^2*i*n^2 + 2*A*
B*b^2*c^2*i*n + (B^2*b^2*d^2*i*n^2 + 2*A*B*b^2*d^2*i*n)*x^2 + 2*(B^2*b^2*c*
d*i*n^2 + 2*A*B*b^2*c*d*i*n)*x)*log((b*x + a)/(d*x + c)))/((b^5*c - a*b^4*d
)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3)
```

Sympy [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= \frac{i\left(\int \frac{A^2c}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{A^2dx}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{B^2c\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{2ABc\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+d}\right.\right.}{a^3+3a^2bx+3ab^2x^2+b}\right.}{g^3}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**3,x)
```

```
[Out] i*(Integral(A**2*c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + In
tegral(A**2*d*x/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integ
ral(B**2*c*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a**3 + 3*a**2*b*x +
3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*c*log(e*(a/(c + d*x) + b*x/
(c + d*x))**n)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integr
al(B**2*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a**3 + 3*a**2*b*x +
 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*d*x*log(e*(a/(c + d*x) + b
*x/(c + d*x))**n)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x))/g**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2017 vs. $2(145) = 290$.

Time = 0.31 (sec) , antiderivative size = 2017, normalized size of antiderivative = 13.36

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x, a
lgorithm="maxima")
```

```
[Out] -1/2*A*B*d*i*n*((3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*x)/((b^5*c - a*b^4*d
)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3) +
2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3
) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)
*g^3)) + 1/2*A*B*c*i*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2
+ 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*
x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3
*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*(2*b*x + a)*B^2*d*i*log(e*(b*x/
(d*x + c) + a/(d*x + c))^n)^2/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) +
 1/4*(2*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c
- a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*
c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^
```

```
2*c*d + a^2*b*d^2)*g^3))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - (b^2*c^2
- 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x +
 a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d
 - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3
*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2
*d^2)*log(b*x + a))*log(d*x + c))*n^2/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3
+ a^4*b*d^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2
*(a*b^4*c^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2*c*i - 1/4*(2
*n*((3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*x)/((b^5*c - a*b^4*d)*g^3*x^2 +
2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3) + 2*(2*b*c*d -
 a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3) - 2*(2*b*c
*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3))*log(e
*(b*x/(d*x + c) + a/(d*x + c))^n) + (7*a*b^2*c^2 - 8*a^2*b*c*d + a^3*d^2 -
2*(2*a^2*b*c*d - a^3*d^2 + (2*b^3*c*d - a*b^2*d^2)*x^2 + 2*(2*a*b^2*c*d - a
^2*b*d^2)*x)*log(b*x + a)^2 - 2*(2*a^2*b*c*d - a^3*d^2 + (2*b^3*c*d - a*b^2
*d^2)*x^2 + 2*(2*a*b^2*c*d - a^2*b*d^2)*x)*log(d*x + c)^2 + 2*(4*b^3*c^2 -
5*a*b^2*c*d + a^2*b*d^2)*x + 2*(4*a^2*b*c*d - a^3*d^2 + (4*b^3*c*d - a*b^2*
d^2)*x^2 + 2*(4*a*b^2*c*d - a^2*b*d^2)*x)*log(b*x + a) - 2*(4*a^2*b*c*d - a
^3*d^2 + (4*b^3*c*d - a*b^2*d^2)*x^2 + 2*(4*a*b^2*c*d - a^2*b*d^2)*x - 2*(2
*a^2*b*c*d - a^3*d^2 + (2*b^3*c*d - a*b^2*d^2)*x^2 + 2*(2*a*b^2*c*d - a^2*b
*d^2)*x)*log(b*x + a))*log(d*x + c))*n^2/(a^2*b^4*c^2*g^3 - 2*a^3*b^3*c*d*g
^3 + a^4*b^2*d^2*g^3 + (b^6*c^2*g^3 - 2*a*b^5*c*d*g^3 + a^2*b^4*d^2*g^3)*x^
2 + 2*(a*b^5*c^2*g^3 - 2*a^2*b^4*c*d*g^3 + a^3*b^3*d^2*g^3)*x))*B^2*d*i - (
2*b*x + a)*A*B*d*i*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^4*g^3*x^2 + 2*
a*b^3*g^3*x + a^2*b^2*g^3) - 1/2*B^2*c*i*log(e*(b*x/(d*x + c) + a/(d*x + c)
)^n)^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*(2*b*x + a)*A^2*d*i/
(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) - A*B*c*i*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n)/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2*c*i/(
b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)
```

Giac [A] (verification not implemented)

none

Time = 1.70 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.29

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx =$$
$$-\frac{1}{4}\left(\frac{2\,(dx+c)^2B^2in^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(bx+a)^2g^3}+\frac{2\,(B^2in^2+2\,B^2in\log\left(e\right)+2\,ABin)(dx+c)^2\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)^2g^3}+\frac{(B^2in^2}{}\right.$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x, a
lgorithm="giac")
```

```
[Out] -1/4*(2*(d*x + c)^2*B^2*i*n^2*log((b*x + a)/(d*x + c))^2/((b*x + a)^2*g^3)
+ 2*(B^2*i*n^2 + 2*B^2*i*n*log(e) + 2*A*B*i*n)*(d*x + c)^2*log((b*x + a)/(d
*x + c))/((b*x + a)^2*g^3) + (B^2*i*n^2 + 2*B^2*i*n*log(e) + 2*B^2*i*log(e)
^2 + 2*A*B*i*n + 4*A*B*i*log(e) + 2*A^2*i)*(d*x + c)^2/((b*x + a)^2*g^3))*(
b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 3.31 (sec) , antiderivative size = 561, normalized size of antiderivative = 3.72

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= -\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{\frac{B^2\,c\,i}{2\,b}+\frac{B^2\,d\,i\,x}{b}+\frac{B^2\,a\,d\,i}{2\,b^2}}{a^2\,g^3+2\,a\,b\,g^3\,x+b^2\,g^3\,x^2}-\frac{B^2\,d^2\,i}{2\,b^2\,g^3\,\left(a\,d-b\,c\right)}\right)$$
$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{A\,B\,a\,d\,i+A\,B\,b\,c\,i-B^2\,a\,d\,i\,n+B^2\,b\,c\,i\,n+2\,A\,B\,b\,d\,i\,x}{a^2\,b^2\,g^3+2\,a\,b^3\,g^3\,x+b^4\,g^3\,x^2}\right.$$
$$\left.+\frac{B^2\,d^2\,i\,\left(\frac{a\,b^2\,g^3\,n\,\left(a\,d-b\,c\right)}{2\,d}+\frac{b^3\,g^3\,n\,x\,\left(a\,d-b\,c\right)}{d}+\frac{b^2\,g^3\,n\,\left(a\,d-b\,c\right)\,\left(2\,a\,d-b\,c\right)}{2\,d^2}\right)}{b^2\,g^3\,\left(a\,d-b\,c\right)\,\left(a^2\,b^2\,g^3+2\,a\,b^3\,g^3\,x+b^4\,g^3\,x^2\right)}\right)$$
$$-\frac{x\,\left(2\,b\,d\,i\,A^2+2\,b\,d\,i\,A\,B\,n+b\,d\,i\,B^2\,n^2\right)+A^2\,a\,d\,i+A^2\,b\,c\,i+\frac{B^2\,a\,d\,i\,n^2}{2}+\frac{B^2\,b\,c\,i\,n^2}{2}+A\,B\,a\,d\,i\,n+A\,B\,b\,c}{2\,a^2\,b^2\,g^3+4\,a\,b^3\,g^3\,x+2\,b^4\,g^3\,x^2}$$
$$-\frac{B\,d^2\,i\,n\,\mathrm{atan}\left(\frac{B\,d^2\,i\,n\,\left(2\,A+B\,n\right)\,\left(\frac{c\,b^3\,g^3+a\,d\,b^2\,g^3}{b^2\,g^3}+2\,b\,d\,x\right)\,1{}\mathrm{i}}{\left(a\,d-b\,c\right)\,\left(i\,B^2\,d^2\,n^2+2\,A\,i\,B\,d^2\,n\right)}\right)\,\left(2\,A+B\,n\right)\,1{}\mathrm{i}}{b^2\,g^3\,\left(a\,d-b\,c\right)}$$

```
[In] int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x)^
3,x)
```

```
[Out] - log(e*((a + b*x)/(c + d*x))^n)^2*(((B^2*c*i)/(2*b) + (B^2*d*i*x)/b + (B^2
*a*d*i)/(2*b^2))/(a^2*g^3 + b^2*g^3*x^2 + 2*a*b*g^3*x) - (B^2*d^2*i)/(2*b^2
*g^3*(a*d - b*c))) - log(e*((a + b*x)/(c + d*x))^n)*((A*B*a*d*i + A*B*b*c*i
 - B^2*a*d*i*n + B^2*b*c*i*n + 2*A*B*b*d*i*x)/(a^2*b^2*g^3 + b^4*g^3*x^2 +
2*a*b^3*g^3*x) + (B^2*d^2*i*((a*b^2*g^3*n*(a*d - b*c))/(2*d) + (b^3*g^3*n*x
*(a*d - b*c))/d + (b^2*g^3*n*(a*d - b*c)*(2*a*d - b*c))/(2*d^2)))/(b^2*g^3*
(a*d - b*c)*(a^2*b^2*g^3 + b^4*g^3*x^2 + 2*a*b^3*g^3*x))) - (x*(2*A^2*b*d*i
 + B^2*b*d*i*n^2 + 2*A*B*b*d*i*n) + A^2*a*d*i + A^2*b*c*i + (B^2*a*d*i*n^2)
/2 + (B^2*b*c*i*n^2)/2 + A*B*a*d*i*n + A*B*b*c*i*n)/(2*a^2*b^2*g^3 + 2*b^4*
g^3*x^2 + 4*a*b^3*g^3*x) - (B*d^2*i*n*atan((B*d^2*i*n*(2*A + B*n)*((b^3*c*g
^3 + a*b^2*d*g^3)/(b^2*g^3) + 2*b*d*x)*1i)/((a*d - b*c)*(B^2*d^2*i*n^2 + 2*
A*B*d^2*i*n)))*(2*A + B*n)*1i)/(b^2*g^3*(a*d - b*c))
```

3.166 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 307

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \frac{B^2din^2(c+dx)^2}{4(bc-ad)^2g^4(a+bx)^2} - \frac{2bB^2in^2(c+dx)^3}{27(bc-ad)^2g^4(a+bx)^3} + \frac{Bdin(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2g^4(a+bx)^2} - \frac{2bBin(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^2g^4(a+bx)^3} + \frac{di(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^2g^4(a+bx)^2} - \frac{bi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^2g^4(a+bx)^3}$$

```
[Out] 1/4*B^2*d*i*n^2*(d*x+c)^2/(-a*d+b*c)^2/g^4/(b*x+a)^2-2/27*b*B^2*i*n^2*(d*x+
c)^3/(-a*d+b*c)^2/g^4/(b*x+a)^3+1/2*B*d*i*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d
*x+c))^n))/(-a*d+b*c)^2/g^4/(b*x+a)^2-2/9*b*B*i*n*(d*x+c)^3*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))/(-a*d+b*c)^2/g^4/(b*x+a)^3+1/2*d*i*(d*x+c)^2*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^4/(b*x+a)^2-1/3*b*i*(d*x+c)^3*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^4/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {2561, 2395, 2342, 2341}

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}dx = -\frac{bi(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g^4(a+bx)^3(bc-ad)^2} - \frac{2bBin(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9g^4(a+bx)^3(bc-ad)^2} + \frac{di(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g^4(a+bx)^2(bc-ad)^2} + \frac{Bdin(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^4(a+bx)^2(bc-ad)^2} - \frac{2bB^2in^2(c+dx)^3}{27g^4(a+bx)^3(bc-ad)^2} + \frac{B^2din^2(c+dx)^2}{4g^4(a+bx)^2(bc-ad)^2}$$

```
[In] Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x)^
4,x]
```

```
[Out] (B^2*d*i*n^2*(c + d*x)^2)/(4*(b*c - a*d)^2*g^4*(a + b*x)^2) - (2*b*B^2*i*n^
2*(c + d*x)^3)/(27*(b*c - a*d)^2*g^4*(a + b*x)^3) + (B*d*i*n*(c + d*x)^2*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*c - a*d)^2*g^4*(a + b*x)^2) - (
2*b*B*i*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(b*c - a*d
)^2*g^4*(a + b*x)^3) + (d*i*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2)/(2*(b*c - a*d)^2*g^4*(a + b*x)^2) - (b*i*(c + d*x)^3*(A + B*Log[e*((
a + b*x)/(c + d*x))^n])^2)/(3*(b*c - a*d)^2*g^4*(a + b*x)^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex^n))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^4}\\
&= \frac{i\text{Subst}\left(\int \left(\frac{b(A+B\log(ex^n))^2}{x^4}-\frac{d(A+B\log(ex^n))^2}{x^3}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^4}\\
&= \frac{(bi)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^4}-\frac{(di)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^4}\\
&= \frac{di(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^2g^4(a+bx)^2}-\frac{bi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^2g^4(a+bx)^3}\\
&\quad+\frac{(2bBin)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bc-ad)^2g^4}-\frac{(Bdin)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^4}\\
&= \frac{B^2din^2(c+dx)^2}{4(bc-ad)^2g^4(a+bx)^2}-\frac{2bB^2in^2(c+dx)^3}{27(bc-ad)^2g^4(a+bx)^3}\\
&\quad+\frac{Bdin(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2g^4(a+bx)^2}\\
&\quad-\frac{2bBin(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^2g^4(a+bx)^3}\\
&\quad+\frac{di(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^2g^4(a+bx)^2}-\frac{bi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^2g^4(a+bx)^3}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.63 (sec) , antiderivative size = 1082, normalized size of antiderivative = 3.52

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{i\left(36(bc-ad)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+54d(bc-ad)^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+2Bn\left(12A(\right.\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b
*g*x)^4,x]
```

```
[Out] -1/108*(i*(36*(b*c - a*d)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 54*d
*(b*c - a*d)^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 2*B*n*(
12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3*n - 18*A*d*(b*c - a*d)^2*(a + b*x) -
 15*B*d*(b*c - a*d)^2*n*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B
*d^2*(b*c - a*d)*n*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x] + 66*B*d
^3*n*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*n*(a + b*x)^3*Log[a + b*x]^2 + 12*
B*(b*c - a*d)^3*Log[e*((a + b*x)/(c + d*x))^n] - 18*B*d*(b*c - a*d)^2*(a +
b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[
e*((a + b*x)/(c + d*x))^n] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[e*((a +
b*x)/(c + d*x))^n] - 36*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3*n*(a + b*
x)^3*Log[c + d*x] + 36*B*d^3*n*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)
]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[e*((a + b*x)/(c + d*x))^n]*Log[c
+ d*x] - 18*B*d^3*n*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*n*(a + b*x)^3*Log
[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*n*(a + b*x)^3*PolyLog[2
, (d*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3*n*(a + b*x)^3*PolyLog[2, (b*(c +
 d*x))/(b*c - a*d)]) + 27*B*d*n*(a + b*x)*(2*(b*c - a*d)^2*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n]) + 4*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[e*((a + b*
x)/(c + d*x))^n]) - 4*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]) + 4*d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*L
og[c + d*x] - 4*B*d*n*(a + b*x)*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(
a + b*x)*Log[c + d*x]) + B*n*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x)
- 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) + 2*B*d^
2*n*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*
d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2*B*d^2*n*(a + b*x)^2*
((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*Poly
Log[2, (b*(c + d*x))/(b*c - a*d)]))))/(b^2*(b*c - a*d)^2*g^4*(a + b*x)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. 2(295) = 590.

Time = 10.49 (sec) , antiderivative size = 1099, normalized size of antiderivative = 3.58

method	result	size
parallelrisch	Expression too large to display	1099

```
[In] int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x,method=_R
ETURNVERBOSE)
```

```
[Out] -1/108*(-108*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*d^4*i*n-108*B^2*x*ln(e
*((b*x+a)/(d*x+c))^n)^2*a*b^5*c*d^3*i*n-108*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)
*a*b^5*c*d^3*i*n^2+108*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c^2*d^2*i*n-108*
A*B*x*a*b^5*c*d^3*i*n^2-108*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c^2*d^2*i*n
-216*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c*d^3*i*n-54*B^2*x*a*b^5*c*d^3*i
*n^3+90*A*B*x*a^2*b^4*d^4*i*n^2+18*A*B*x*b^6*c^2*d^2*i*n^2-54*B^2*ln(e*((b*
x+a)/(d*x+c))^n)^2*a*b^5*c^2*d^2*i*n-54*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5
*c^2*d^2*i*n^2-108*A^2*x*a*b^5*c*d^3*i*n+72*A*B*ln(e*((b*x+a)/(d*x+c))^n)*b
^6*c^3*d*i*n+19*B^2*a^3*b^3*d^4*i*n^3+8*B^2*b^6*c^3*d*i*n^3+18*A^2*a^3*b^3*
d^4*i*n+36*A^2*b^6*c^3*d*i*n-36*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^6*d^4*i
*n-54*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^5*d^4*i*n-54*B^2*x^2*ln(e*((b
*x+a)/(d*x+c))^n)*a*b^5*d^4*i*n^2-36*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^6*
c*d^3*i*n^2+36*A*B*x^2*a*b^5*d^4*i*n^2-36*A*B*x^2*b^6*c*d^3*i*n^2+54*B^2*x*
ln(e*((b*x+a)/(d*x+c))^n)^2*b^6*c^2*d^2*i*n+18*B^2*x*ln(e*((b*x+a)/(d*x+c))
^n)*b^6*c^2*d^2*i*n^2-18*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b^6*d^4*i*n-30
*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^6*d^4*i*n^2+30*B^2*x^2*a*b^5*d^4*i*n^3
-30*B^2*x^2*b^6*c*d^3*i*n^3+57*B^2*x*a^2*b^4*d^4*i*n^3-3*B^2*x*b^6*c^2*d^2*
i*n^3+36*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^6*c^3*d*i*n+24*B^2*ln(e*((b*x+a)
/(d*x+c))^n)*b^6*c^3*d*i*n^2+54*A^2*x*a^2*b^4*d^4*i*n+54*A^2*x*b^6*c^2*d^2*
i*n-54*A*B*a*b^5*c^2*d^2*i*n^2-27*B^2*a*b^5*c^2*d^2*i*n^3+30*A*B*a^3*b^3*d^
4*i*n^2+24*A*B*b^6*c^3*d*i*n^2-54*A^2*a*b^5*c^2*d^2*i*n)/g^4/(b*x+a)^3/n/(a
^2*d^2-2*a*b*c*d+b^2*c^2)/b^5/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1167 vs. 2(295) = 590.

Time = 0.40 (sec) , antiderivative size = 1167, normalized size of antiderivative = 3.80

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x, a
lgorithm="fricas")
```

```
[Out] -1/108*((8*B^2*b^3*c^3 - 27*B^2*a*b^2*c^2*d + 19*B^2*a^3*d^3)*i*n^2 + 6*(4*
A*B*b^3*c^3 - 9*A*B*a*b^2*c^2*d + 5*A*B*a^3*d^3)*i*n - 6*(5*(B^2*b^3*c*d^2
- B^2*a*b^2*d^3)*i*n^2 + 6*(A*B*b^3*c*d^2 - A*B*a*b^2*d^3)*i*n)*x^2 + 18*(3
*(B^2*b^3*c^2*d - 2*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3)*i*x + (2*B^2*b^3*c^3 -
 3*B^2*a*b^2*c^2*d + B^2*a^3*d^3)*i)*log(e)^2 - 18*(B^2*b^3*d^3*i*n^2*x^3 +
 3*B^2*a*b^2*d^3*i*n^2*x^2 - 3*(B^2*b^3*c^2*d - 2*B^2*a*b^2*c*d^2)*i*n^2*x
- (2*B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d)*i*n^2)*log((b*x + a)/(d*x + c))^2 + 1
8*(2*A^2*b^3*c^3 - 3*A^2*a*b^2*c^2*d + A^2*a^3*d^3)*i - 3*((B^2*b^3*c^2*d +
 18*B^2*a*b^2*c*d^2 - 19*B^2*a^2*b*d^3)*i*n^2 - 6*(A*B*b^3*c^2*d - 6*A*B*a*
b^2*c*d^2 + 5*A*B*a^2*b*d^3)*i*n - 18*(A^2*b^3*c^2*d - 2*A^2*a*b^2*c*d^2 +
A^2*a^2*b*d^3)*i)*x - 6*(6*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*i*n*x^2 - (4*B^2
*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 5*B^2*a^3*d^3)*i*n - 6*(2*A*B*b^3*c^3 - 3*A*
B*a*b^2*c^2*d + A*B*a^3*d^3)*i - 3*((B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 + 5*
B^2*a^2*b*d^3)*i*n + 6*(A*B*b^3*c^2*d - 2*A*B*a*b^2*c*d^2 + A*B*a^2*b*d^3)*
i)*x + 6*(B^2*b^3*d^3*i*n*x^3 + 3*B^2*a*b^2*d^3*i*n*x^2 - 3*(B^2*b^3*c^2*d
- 2*B^2*a*b^2*c*d^2)*i*n*x - (2*B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d)*i*n)*log((
b*x + a)/(d*x + c)))*log(e) + 6*((4*B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d)*i*n^2
- (5*B^2*b^3*d^3*i*n^2 + 6*A*B*b^3*d^3*i*n)*x^3 + 6*(2*A*B*b^3*c^3 - 3*A*B*
a*b^2*c^2*d)*i*n - 3*(6*A*B*a*b^2*d^3*i*n + (2*B^2*b^3*c*d^2 + 3*B^2*a*b^2*
d^3)*i*n^2)*x^2 + 3*((B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2)*i*n^2 + 6*(A*B*b^3
*c^2*d - 2*A*B*a*b^2*c*d^2)*i*n)*x)*log((b*x + a)/(d*x + c)))/((b^7*c^2 - 2
*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*
d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b
^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci + dix) \left(A + B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^4} \, dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**4,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3312 vs. $2(295) = 590$.

Time = 0.41 (sec) , antiderivative size = 3312, normalized size of antiderivative = 10.79

$$\int \frac{(ci + dix) \left(A + B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^4} \, dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x, a
lgorithm="maxima")
```

```
[Out] -1/9*A*B*c*i*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^
2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a
*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^
3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4)
 + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d
^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
 a^3*b*d^3)*g^4)) - 1/18*A*B*d*i*n*((5*a*b^2*c^2 - 22*a^2*b*c*d + 5*a^3*d^2
 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c^2 - 16*a*b^2*c*d + 5*a^2*b*d^
2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2
*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*
d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4) - 6*(3*b*c*d^
2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b
^2*d^3)*g^4) + 6*(3*b*c*d^2 - a*d^3)*log(d*x + c)/((b^5*c^3 - 3*a*b^4*c^2*d
 + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4)) - 1/6*(3*b*x + a)*B^2*d*i*log(e*(b*
x/(d*x + c) + a/(d*x + c))^n)^2/(b^5*g^4*x^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*
g^4*x + a^3*b^2*g^4) - 1/54*(6*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d +
11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d
^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*
b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d
 + a^5*b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b
^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d +
 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
+ (4*b^3*c^3 - 27*a*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^
2 - a*b^2*d^3)*x^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^
3*d^3)*log(b*x + a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x +
 a^3*d^3)*log(d*x + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*
x + 66*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x +
a) - 6*(11*b^3*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6
*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*lo
g(d*x + c))*n^2/(a^3*b^4*c^3*g^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^
4 - a^6*b*d^3*g^4 + (b^7*c^3*g^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4
- a^3*b^4*d^3*g^4)*x^3 + 3*(a*b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4
*c*d^2*g^4 - a^4*b^3*d^3*g^4)*x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^
4 + 3*a^4*b^3*c*d^2*g^4 - a^5*b^2*d^3*g^4)*x))*B^2*c*i - 1/108*(6*n*((5*a*b
^2*c^2 - 22*a^2*b*c*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^
3*c^2 - 16*a*b^2*c*d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^
2)*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b
^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d
+ a^5*b^2*d^2)*g^4) - 6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*a*b^
4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4) + 6*(3*b*c*d^2 - a*d^3)*log(d
*x + c)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4))*lo
g(e*(b*x/(d*x + c) + a/(d*x + c))^n) + (19*a*b^3*c^3 - 189*a^2*b^2*c^2*d +
189*a^3*b*c*d^2 - 19*a^4*d^3 - 6*(27*b^4*c^2*d - 32*a*b^3*c*d^2 + 5*a^2*b^2
*d^3)*x^2 + 18*(3*a^3*b*c*d^2 - a^4*d^3 + (3*b^4*c*d^2 - a*b^3*d^3)*x^3 + 3
*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^2 + 3*(3*a^2*b^2*c*d^2 - a^3*b*d^3)*x)*log
```

(b*x + a)^2 + 18*(3*a^3*b*c*d^2 - a^4*d^3 + (3*b^4*c*d^2 - a*b^3*d^3)*x^3 +
 3*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^2 + 3*(3*a^2*b^2*c*d^2 - a^3*b*d^3)*x)*l
og(d*x + c)^2 + 3*(9*b^4*c^3 - 125*a*b^3*c^2*d + 135*a^2*b^2*c*d^2 - 19*a^3
*b*d^3)*x - 6*(27*a^3*b*c*d^2 - 5*a^4*d^3 + (27*b^4*c*d^2 - 5*a*b^3*d^3)*x^
3 + 3*(27*a*b^3*c*d^2 - 5*a^2*b^2*d^3)*x^2 + 3*(27*a^2*b^2*c*d^2 - 5*a^3*b*
d^3)*x)*log(b*x + a) + 6*(27*a^3*b*c*d^2 - 5*a^4*d^3 + (27*b^4*c*d^2 - 5*a*
b^3*d^3)*x^3 + 3*(27*a*b^3*c*d^2 - 5*a^2*b^2*d^3)*x^2 + 3*(27*a^2*b^2*c*d^2
 - 5*a^3*b*d^3)*x - 6*(3*a^3*b*c*d^2 - a^4*d^3 + (3*b^4*c*d^2 - a*b^3*d^3)*
x^3 + 3*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^2 + 3*(3*a^2*b^2*c*d^2 - a^3*b*d^3)
*x)*log(b*x + a))*log(d*x + c))*n^2/(a^3*b^5*c^3*g^4 - 3*a^4*b^4*c^2*d*g^4
+ 3*a^5*b^3*c*d^2*g^4 - a^6*b^2*d^3*g^4 + (b^8*c^3*g^4 - 3*a*b^7*c^2*d*g^4
+ 3*a^2*b^6*c*d^2*g^4 - a^3*b^5*d^3*g^4)*x^3 + 3*(a*b^7*c^3*g^4 - 3*a^2*b^6
*c^2*d*g^4 + 3*a^3*b^5*c*d^2*g^4 - a^4*b^4*d^3*g^4)*x^2 + 3*(a^2*b^6*c^3*g^
4 - 3*a^3*b^5*c^2*d*g^4 + 3*a^4*b^4*c*d^2*g^4 - a^5*b^3*d^3*g^4)*x))*B^2*d*
i - 1/3*(3*b*x + a)*A*B*d*i*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^5*g^4
*x^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - 1/3*B^2*c*i*log(e
*(b*x/(d*x + c) + a/(d*x + c))^n)^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*
b^2*g^4*x + a^3*b*g^4) - 1/6*(3*b*x + a)*A^2*d*i/(b^5*g^4*x^3 + 3*a*b^4*g^4
*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - 2/3*A*B*c*i*log(e*(b*x/(d*x + c) +
a/(d*x + c))^n)/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^
4) - 1/3*A^2*c*i/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g
^4)

Giac [A] (verification not implemented)

none

Time = 2.31 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.63

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{1}{108}\left(\frac{18\left(2B^2bin^2-\frac{3(bx+a)B^2din^2}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)^2}{\frac{(bx+a)^3bcg^4}{(dx+c)^3}-\frac{(bx+a)^3adg^4}{(dx+c)^3}}+\frac{6\left(4B^2bin^2-\frac{9(bx+a)B^2din^2}{dx+c}+12B^2bin\log(e)-\frac{18(bx}{}\right.}{\frac{(bx+a)^3bcg^4}{(dx+c)^3}-}\right.$$

[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x, a
lgorithm="giac")

[Out] -1/108*(18*(2*B^2*b*i*n^2 - 3*(b*x + a)*B^2*d*i*n^2/(d*x + c))*log((b*x + a
)/(d*x + c))^2/((b*x + a)^3*b*c*g^4/(d*x + c)^3 - (b*x + a)^3*a*d*g^4/(d*x
+ c)^3) + 6*(4*B^2*b*i*n^2 - 9*(b*x + a)*B^2*d*i*n^2/(d*x + c) + 12*B^2*b*i
*n*log(e) - 18*(b*x + a)*B^2*d*i*n*log(e)/(d*x + c) + 12*A*B*b*i*n - 18*(b*
x + a)*A*B*d*i*n/(d*x + c))*log((b*x + a)/(d*x + c))/((b*x + a)^3*b*c*g^4/(
d*x + c)^3 - (b*x + a)^3*a*d*g^4/(d*x + c)^3) + (8*B^2*b*i*n^2 - 27*(b*x +
a)*B^2*d*i*n^2/(d*x + c) + 24*B^2*b*i*n*log(e) - 54*(b*x + a)*B^2*d*i*n*log

```
(e)/(d*x + c) + 36*B^2*b*i*log(e)^2 - 54*(b*x + a)*B^2*d*i*log(e)^2/(d*x +
c) + 24*A*B*b*i*n - 54*(b*x + a)*A*B*d*i*n/(d*x + c) + 72*A*B*b*i*log(e) -
108*(b*x + a)*A*B*d*i*log(e)/(d*x + c) + 36*A^2*b*i - 54*(b*x + a)*A^2*d*i/
(d*x + c))/((b*x + a)^3*b*c*g^4/(d*x + c)^3 - (b*x + a)^3*a*d*g^4/(d*x + c)
^3))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 4.26 (sec) , antiderivative size = 993, normalized size of antiderivative = 3.23

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{\frac{18\,i\,A^2\,a^2\,d^2+18\,i\,A^2\,a\,b\,c\,d-36\,i\,A^2\,b^2\,c^2+30\,i\,A\,B\,a^2\,d^2\,n+30\,i\,A\,B\,a\,b\,c\,d\,n-24\,i\,A\,B\,b^2\,c^2\,n+19\,i\,B^2\,a^2\,d^2\,n^2+19\,i\,B^2\,a\,b\,c\,d\,n^2-8\,i\,B^2\,b^2\,c^2}{6\,(a\,d-b\,c)}}{18\,a^3\,b^2\,g^4+54\,a^2\,b^3\,g^4\,x}$$

$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{\frac{B^2\,c\,i}{3\,b}+\frac{B^2\,d\,i\,x}{2\,b}+\frac{B^2\,a\,d\,i}{6\,b^2}}{a^3\,g^4+3\,a^2\,b\,g^4\,x+3\,a\,b^2\,g^4\,x^2+b^3\,g^4\,x^3}\right.$$

$$\left.-\frac{B^2\,d^3\,i}{6\,b^2\,g^4\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)$$

$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{A\,B\,a\,d\,i+2\,A\,B\,b\,c\,i-B^2\,a\,d\,i\,n+B^2\,b\,c\,i\,n+3\,A\,B\,b\,d\,i\,x}{3\,a^3\,b^2\,g^4+9\,a^2\,b^3\,g^4\,x+9\,a\,b^4\,g^4\,x^2+3\,b^5\,g^4\,x^3}\right.$$

$$+\frac{B^2\,d^3\,i\,\left(x\,\left(b\,\left(\frac{a\,b^2\,g^4\,n\,(a\,d-b\,c)}{d}+\frac{b^2\,g^4\,n\,(a\,d-b\,c)\,(3\,a\,d-b\,c)}{2\,d^2}\right)+\frac{2\,a\,b^3\,g^4\,n\,(a\,d-b\,c)}{d}+\frac{b^3\,g^4\,n\,(a\,d-b\,c)\,(3\,a\,d-b\,c)}{d^2}\right)+a\,\left(\frac{a\,b^2}{}\right.\right.}{3\,b^2\,g^4\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(3\,a^3\,b^2\,g^4+9\,a^2\right.}$$

$$-\frac{B\,d^3\,i\,n\,\mathrm{atan}\left(\frac{B\,d^3\,i\,n\,(6\,A+5\,B\,n)\,\left(2\,b\,d\,x-\frac{b^4\,c^2\,g^4-a^2\,b^2\,d^2\,g^4}{b^2\,g^4\,(a\,d-b\,c)}\right)\,1{}\mathrm{i}}{(a\,d-b\,c)\,\left(5\,i\,B^2\,d^3\,n^2+6\,A\,i\,B\,d^3\,n\right)}\right)\,(6\,A+5\,B\,n)\,1{}\mathrm{i}}{9\,b^2\,g^4\,{(a\,d-b\,c)}^2}$$

```
[In] int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x)^
4,x)
```

```
[Out] - ((18*A^2*a^2*d^2*i - 36*A^2*b^2*c^2*i + 19*B^2*a^2*d^2*i*n^2 - 8*B^2*b^2*
c^2*i*n^2 + 30*A*B*a^2*d^2*i*n - 24*A*B*b^2*c^2*i*n + 18*A^2*a*b*c*d*i + 19
*B^2*a*b*c*d*i*n^2 + 30*A*B*a*b*c*d*i*n)/(6*(a*d - b*c)) + (x*(18*A^2*a*b*d
^2*i - 18*A^2*b^2*c*d*i + 19*B^2*a*b*d^2*i*n^2 + B^2*b^2*c*d*i*n^2 + 30*A*B
*a*b*d^2*i*n - 6*A*B*b^2*c*d*i*n))/(2*(a*d - b*c)) + (x^2*(5*B^2*b^2*d^2*i*
n^2 + 6*A*B*b^2*d^2*i*n))/(a*d - b*c))/(18*a^3*b^2*g^4 + 18*b^5*g^4*x^3 + 5
4*a^2*b^3*g^4*x + 54*a*b^4*g^4*x^2) - log(e*((a + b*x)/(c + d*x))^n)^2*(((B
^2*c*i)/(3*b) + (B^2*d*i*x)/(2*b) + (B^2*a*d*i)/(6*b^2))/(a^3*g^4 + b^3*g^4
*x^3 + 3*a*b^2*g^4*x^2 + 3*a^2*b*g^4*x) - (B^2*d^3*i)/(6*b^2*g^4*(a^2*d^2 +
 b^2*c^2 - 2*a*b*c*d))) - log(e*((a + b*x)/(c + d*x))^n)*((A*B*a*d*i + 2*A*
```

```
B*b*c*i - B^2*a*d*i*n + B^2*b*c*i*n + 3*A*B*b*d*i*x)/(3*a^3*b^2*g^4 + 3*b^5
*g^4*x^3 + 9*a^2*b^3*g^4*x + 9*a*b^4*g^4*x^2) + (B^2*d^3*i*(x*(b*((a*b^2*g^
4*n*(a*d - b*c))/d + (b^2*g^4*n*(a*d - b*c)*(3*a*d - b*c))/(2*d^2)) + (2*a*
b^3*g^4*n*(a*d - b*c))/d + (b^3*g^4*n*(a*d - b*c)*(3*a*d - b*c))/d^2) + a*(
(a*b^2*g^4*n*(a*d - b*c))/d + (b^2*g^4*n*(a*d - b*c)*(3*a*d - b*c))/(2*d^2)
) + (3*b^4*g^4*n*x^2*(a*d - b*c))/d + (b^2*g^4*n*(a*d - b*c)*(3*a^2*d^2 + b
^2*c^2 - 3*a*b*c*d))/d^3))/(3*b^2*g^4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(3*a^
3*b^2*g^4 + 3*b^5*g^4*x^3 + 9*a^2*b^3*g^4*x + 9*a*b^4*g^4*x^2))) - (B*d^3*i
*n*atan((B*d^3*i*n*(6*A + 5*B*n)*(2*b*d*x - (b^4*c^2*g^4 - a^2*b^2*d^2*g^4)
/(b^2*g^4*(a*d - b*c)))*1i)/((a*d - b*c)*(5*B^2*d^3*i*n^2 + 6*A*B*d^3*i*n))
)*(6*A + 5*B*n)*1i)/(9*b^2*g^4*(a*d - b*c)^2)
```

3.167 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 475

$$\begin{aligned}\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = & -\frac{B^2d^2in^2(c+dx)^2}{4(bc-ad)^3g^5(a+bx)^2} \\ & +\frac{4bB^2din^2(c+dx)^3}{27(bc-ad)^3g^5(a+bx)^3} \\ & -\frac{b^2B^2in^2(c+dx)^4}{32(bc-ad)^3g^5(a+bx)^4} \\ & -\frac{Bd^2in(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^5(a+bx)^2} \\ & +\frac{4bBdin(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^3g^5(a+bx)^3} \\ & -\frac{b^2Bin(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8(bc-ad)^3g^5(a+bx)^4} \\ & -\frac{d^2i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^3g^5(a+bx)^2} \\ & +\frac{2bdi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^3g^5(a+bx)^3} \\ & -\frac{b^2i(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4(bc-ad)^3g^5(a+bx)^4}\end{aligned}$$

```
[Out] -1/4*B^2*d^2*i*n^2*(d*x+c)^2/(-a*d+b*c)^3/g^5/(b*x+a)^2+4/27*b*B^2*d*i*n^2*
(d*x+c)^3/(-a*d+b*c)^3/g^5/(b*x+a)^3-1/32*b^2*B^2*i*n^2*(d*x+c)^4/(-a*d+b*c
)^3/g^5/(b*x+a)^4-1/2*B*d^2*i*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(
```

```
-a*d+b*c)^3/g^5/(b*x+a)^2+4/9*b*B*d*i*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))/(-a*d+b*c)^3/g^5/(b*x+a)^3-1/8*b^2*B*i*n*(d*x+c)^4*(A+B*ln(e*((b*x+a
)/(d*x+c))^n))/(-a*d+b*c)^3/g^5/(b*x+a)^4-1/2*d^2*i*(d*x+c)^2*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^5/(b*x+a)^2+2/3*b*d*i*(d*x+c)^3*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^5/(b*x+a)^3-1/4*b^2*i*(d*x+c)^4*(
A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^5/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {2561, 2395, 2342, 2341}

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{b^2i(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4g^5(a+bx)^4(bc-ad)^3} - \frac{b^2Bin(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{8g^5(a+bx)^4(bc-ad)^3} - \frac{d^2i(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g^5(a+bx)^2(bc-ad)^3} - \frac{Bd^2in(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^5(a+bx)^2(bc-ad)^3} + \frac{2bdi(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g^5(a+bx)^3(bc-ad)^3} + \frac{4bBdin(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9g^5(a+bx)^3(bc-ad)^3} - \frac{b^2B^2in^2(c+dx)^4}{32g^5(a+bx)^4(bc-ad)^3} - \frac{B^2d^2in^2(c+dx)^2}{4g^5(a+bx)^2(bc-ad)^3} + \frac{4bB^2din^2(c+dx)^3}{27g^5(a+bx)^3(bc-ad)^3}$$

```
[In] Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x)^
5,x]
```

```
[Out] -1/4*(B^2*d^2*i*n^2*(c + d*x)^2)/((b*c - a*d)^3*g^5*(a + b*x)^2) + (4*b*B^2
*d*i*n^2*(c + d*x)^3)/(27*(b*c - a*d)^3*g^5*(a + b*x)^3) - (b^2*B^2*i*n^2*(
c + d*x)^4)/(32*(b*c - a*d)^3*g^5*(a + b*x)^4) - (B*d^2*i*n*(c + d*x)^2*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*c - a*d)^3*g^5*(a + b*x)^2) + (4
*b*B*d*i*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(b*c - a*
d)^3*g^5*(a + b*x)^3) - (b^2*B*i*n*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n]))/(8*(b*c - a*d)^3*g^5*(a + b*x)^4) - (d^2*i*(c + d*x)^2*(A + B*L
```

```
og[e*((a + b*x)/(c + d*x))^n])^2)/(2*(b*c - a*d)^3*g^5*(a + b*x)^2) + (2*b*
d*i*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*(b*c - a*d)^3*
g^5*(a + b*x)^3) - (b^2*i*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2)/(4*(b*c - a*d)^3*g^5*(a + b*x)^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{i\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex^n))^2}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^5}$$

$$= \frac{i\text{Subst}\left(\int \left(\frac{b^2(A+B\log(ex^n))^2}{x^5} - \frac{2bd(A+B\log(ex^n))^2}{x^4} + \frac{d^2(A+B\log(ex^n))^2}{x^3}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^5}$$

$$= \frac{(b^2i)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^5} - \frac{(2bdi)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^5} + \frac{(d^2i)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^5}$$

$$= -\frac{d^2i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^3g^5(a+bx)^2} + \frac{2bdi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^3g^5(a+bx)^3} - \frac{b^2i(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4(bc-ad)^3g^5(a+bx)^4} + \frac{(b^2Bin)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{2(bc-ad)^3g^5} - \frac{(4bBdin)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bc-ad)^3g^5} + \frac{(Bd^2in)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^5}$$

$$= -\frac{B^2d^2in^2(c+dx)^2}{4(bc-ad)^3g^5(a+bx)^2} + \frac{4bB^2din^2(c+dx)^3}{27(bc-ad)^3g^5(a+bx)^3} - \frac{b^2B^2in^2(c+dx)^4}{32(bc-ad)^3g^5(a+bx)^4} - \frac{Bd^2in(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^5(a+bx)^2} + \frac{4bBdin(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^3g^5(a+bx)^3} - \frac{b^2Bin(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8(bc-ad)^3g^5(a+bx)^4} - \frac{d^2i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^3g^5(a+bx)^2} + \frac{2bdi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^3g^5(a+bx)^3} - \frac{b^2i(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4(bc-ad)^3g^5(a+bx)^4}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.71 (sec) , antiderivative size = 1319, normalized size of antiderivative = 2.78

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{i\left(216(bc-ad)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - 288d(-bc+ad)^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + 16Bdn\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b
*g*x)^5,x]
```

```
[Out] -1/864*(i*(216*(b*c - a*d)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - 288
*d*(-(b*c) + a*d)^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 16
*B*d*n*(a + b*x)*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3*n - 18*A*d*(b*c -
a*d)^2*(a + b*x) - 15*B*d*(b*c - a*d)^2*n*(a + b*x) + 36*A*d^2*(b*c - a*d)*
(a + b*x)^2 + 66*B*d^2*(b*c - a*d)*n*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log
[a + b*x] + 66*B*d^3*n*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*n*(a + b*x)^3*Lo
```

```
g[a + b*x]^2 + 12*B*(b*c - a*d)^3*Log[e*((a + b*x)/(c + d*x))^n] - 18*B*d*(
b*c - a*d)^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d^2*(b*c - a*d
)*(a + b*x)^2*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d^3*(a + b*x)^3*Log[a +
 b*x]*Log[e*((a + b*x)/(c + d*x))^n] - 36*A*d^3*(a + b*x)^3*Log[c + d*x] -
66*B*d^3*n*(a + b*x)^3*Log[c + d*x] + 36*B*d^3*n*(a + b*x)^3*Log[(d*(a + b*
x))/(-(b*c) + a*d)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[e*((a + b*x)/(c
 + d*x))^n]*Log[c + d*x] - 18*B*d^3*n*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3
*n*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*n*(a
+ b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3*n*(a + b*x)^3*
PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 3*B*n*(36*A*(b*c - a*d)^4 + 9*B*(b
*c - a*d)^4*n + 48*A*d*(-(b*c) + a*d)^3*(a + b*x) + 28*B*d*(-(b*c) + a*d)^3
*n*(a + b*x) + 72*A*d^2*(b*c - a*d)^2*(a + b*x)^2 + 78*B*d^2*(b*c - a*d)^2*
n*(a + b*x)^2 + 144*A*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 300*B*d^3*(-(b*c) +
a*d)*n*(a + b*x)^3 - 144*A*d^4*(a + b*x)^4*Log[a + b*x] - 300*B*d^4*n*(a +
b*x)^4*Log[a + b*x] + 72*B*d^4*n*(a + b*x)^4*Log[a + b*x]^2 + 36*B*(b*c - a
*d)^4*Log[e*((a + b*x)/(c + d*x))^n] + 48*B*d*(-(b*c) + a*d)^3*(a + b*x)*Lo
g[e*((a + b*x)/(c + d*x))^n] + 72*B*d^2*(b*c - a*d)^2*(a + b*x)^2*Log[e*((a
 + b*x)/(c + d*x))^n] + 144*B*d^3*(-(b*c) + a*d)*(a + b*x)^3*Log[e*((a + b*
x)/(c + d*x))^n] - 144*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[e*((a + b*x)/(c +
 d*x))^n] + 144*A*d^4*(a + b*x)^4*Log[c + d*x] + 300*B*d^4*n*(a + b*x)^4*Lo
g[c + d*x] - 144*B*d^4*n*(a + b*x)^4*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[
c + d*x] + 144*B*d^4*(a + b*x)^4*Log[e*((a + b*x)/(c + d*x))^n]*Log[c + d*x
] + 72*B*d^4*n*(a + b*x)^4*Log[c + d*x]^2 - 144*B*d^4*n*(a + b*x)^4*Log[a +
 b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 144*B*d^4*n*(a + b*x)^4*PolyLog[2, (
d*(a + b*x))/(-(b*c) + a*d)] - 144*B*d^4*n*(a + b*x)^4*PolyLog[2, (b*(c + d
*x))/(b*c - a*d)])))/(b^2*(b*c - a*d)^3*g^5*(a + b*x)^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2196 vs. $2(457) = 914$.

Time = 28.62 (sec) , antiderivative size = 2197, normalized size of antiderivative = 4.63

method	result	size
parallelrisch	Expression too large to display	2197

```
[In] int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/864*(480*A*B*x^3*a^7*b*c*d^4*i*n^2+144*A*B*x^3*a^6*b^2*c^2*d^3*i*n^2-1728
*A*B*x^3*a^5*b^3*c^3*d^2*i*n^2+1536*A*B*x^3*a^4*b^4*c^4*d*i*n^2+576*B^2*x^2
*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c^2*d^3*i*n^2-72*B^2*x^2*ln(e*((b*x+a)/(d*
x+c))^n)*a^6*b^2*c^3*d^2*i*n^2+864*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^8*c*
d^4*i*n+576*A*B*x^2*a^7*b*c^2*d^3*i*n^2-2664*A*B*x^2*a^6*b^2*c^3*d^2*i*n^2+
2304*A*B*x^2*a^5*b^3*c^4*d*i*n^2-864*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^7*
b*c^3*d^2*i*n+288*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b^2*c^4*d*i*n-288*B
```

```
^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c^3*d^2*i*n^2+48*B^2*x*ln(e*((b*x+a)/(
d*x+c))^n)*a^6*b^2*c^4*d*i*n^2+1728*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^8*c^2
*d^3*i*n-2016*A*B*x*a^7*b*c^3*d^2*i*n^2+1584*A*B*x*a^6*b^2*c^4*d*i*n^2-1152
*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c^4*d*i*n+72*B^2*x^4*ln(e*((b*x+a)/(d*
x+c))^n)^2*a^6*b^2*c*d^4*i*n+156*B^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*
c*d^4*i*n^2+156*A*B*x^4*a^6*b^2*c*d^4*i*n^2-432*A*B*x^4*a^4*b^4*c^3*d^2*i*n
^2+384*A*B*x^4*a^3*b^5*c^4*d*i*n^2+288*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*
a^7*b*c*d^4*i*n+480*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c*d^4*i*n^2+144
*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^2*d^3*i*n^2-1728*A*B*x*ln(e*((
b*x+a)/(d*x+c))^n)*a^7*b*c^3*d^2*i*n+576*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^
6*b^2*c^4*d*i*n+144*A*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c*d^4*i*n+576
*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c*d^4*i*n+288*A^2*x^3*a^7*b*c*d^4*
i*n-1728*A^2*x^3*a^5*b^3*c^3*d^2*i*n+2304*A^2*x^3*a^4*b^4*c^4*d*i*n+432*A*B
*x^2*a^8*c*d^4*i*n^2-648*A*B*x^2*a^4*b^4*c^5*i*n^2+864*B^2*x*ln(e*((b*x+a)/
(d*x+c))^n)^2*a^8*c^2*d^3*i*n+864*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^8*c^2*d
^3*i*n^2-816*B^2*x*a^7*b*c^3*d^2*i*n^3+492*B^2*x*a^6*b^2*c^4*d*i*n^3-2592*A
^2*x^2*a^6*b^2*c^3*d^2*i*n+3456*A^2*x^2*a^5*b^3*c^4*d*i*n+864*A*B*x*a^8*c^2
*d^3*i*n^2-432*A*B*x*a^5*b^3*c^5*i*n^2-576*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*
a^7*b*c^4*d*i*n-384*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c^4*d*i*n^2-2592*A^
2*x*a^7*b*c^3*d^2*i*n+2592*A^2*x*a^6*b^2*c^4*d*i*n+864*A*B*ln(e*((b*x+a)/(d
*x+c))^n)*a^8*c^3*d^2*i*n+432*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^5*i*n
-162*B^2*x^2*a^4*b^4*c^5*i*n^3-864*A^2*x^3*a^3*b^5*c^5*i*n+432*B^2*x*a^8*c^
2*d^3*i*n^3-108*B^2*x*a^5*b^3*c^5*i*n^3+432*A^2*x^2*a^8*c*d^4*i*n-1296*A^2*
x^2*a^4*b^4*c^5*i*n+432*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^8*c^3*d^2*i*n+216
*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b^2*c^5*i*n+432*B^2*ln(e*((b*x+a)/(d*x
+c))^n)*a^8*c^3*d^2*i*n^2+108*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^5*i*n
^2+864*A^2*x*a^8*c^2*d^3*i*n-864*A^2*x*a^5*b^3*c^5*i*n-27*B^2*x^4*a^2*b^6*c
^5*i*n^3-108*B^2*x^3*a^3*b^5*c^5*i*n^3-216*A^2*x^4*a^2*b^6*c^5*i*n+216*B^2*
x^2*a^8*c*d^4*i*n^3+576*A^2*x^4*a^3*b^5*c^4*d*i*n-432*A*B*x^3*a^3*b^5*c^5*i
*n^2+432*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^8*c*d^4*i*n+432*B^2*x^2*ln(e
*((b*x+a)/(d*x+c))^n)*a^8*c*d^4*i*n^2+480*B^2*x^2*a^7*b*c^2*d^3*i*n^3-1302*
B^2*x^2*a^6*b^2*c^3*d^2*i*n^3+768*B^2*x^2*a^5*b^3*c^4*d*i*n^3+115*B^2*x^4*a
^6*b^2*c*d^4*i*n^3-216*B^2*x^4*a^4*b^4*c^3*d^2*i*n^3+128*B^2*x^4*a^3*b^5*c^
4*d*i*n^3-108*A*B*x^4*a^2*b^6*c^5*i*n^2+304*B^2*x^3*a^7*b*c*d^4*i*n^3+156*B
^2*x^3*a^6*b^2*c^2*d^3*i*n^3-864*B^2*x^3*a^5*b^3*c^3*d^2*i*n^3+512*B^2*x^3*
a^4*b^4*c^4*d*i*n^3+72*A^2*x^4*a^6*b^2*c*d^4*i*n-432*A^2*x^4*a^4*b^4*c^3*d^
2*i*n)/g^5/(b*x+a)^4/(a*d-b*c)^3/c/n/a^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1868 vs. 2(457) = 914.

Time = 0.42 (sec) , antiderivative size = 1868, normalized size of antiderivative = 3.93

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x, a
lgorithm="fricas")
```

```
[Out] -1/864*((27*B^2*b^4*c^4 - 128*B^2*a*b^3*c^3*d + 216*B^2*a^2*b^2*c^2*d^2 - 1
15*B^2*a^4*d^4)*i*n^2 + 12*(13*(B^2*b^4*c*d^3 - B^2*a*b^3*d^4)*i*n^2 + 12*(
A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*i*n)*x^3 + 12*(9*A*B*b^4*c^4 - 32*A*B*a*b^3*
c^3*d + 36*A*B*a^2*b^2*c^2*d^2 - 13*A*B*a^4*d^4)*i*n - 6*((B^2*b^4*c^2*d^2
 - 80*B^2*a*b^3*c*d^3 + 79*B^2*a^2*b^2*d^4)*i*n^2 + 12*(A*B*b^4*c^2*d^2 - 8*
A*B*a*b^3*c*d^3 + 7*A*B*a^2*b^2*d^4)*i*n)*x^2 + 72*(4*(B^2*b^4*c^3*d - 3*B^
2*a*b^3*c^2*d^2 + 3*B^2*a^2*b^2*c*d^3 - B^2*a^3*b*d^4)*i*x + (3*B^2*b^4*c^4
 - 8*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - B^2*a^4*d^4)*i)*log(e)^2 + 7
2*(B^2*b^4*d^4*i*n^2*x^4 + 4*B^2*a*b^3*d^4*i*n^2*x^3 + 6*B^2*a^2*b^2*d^4*i*
n^2*x^2 + 4*(B^2*b^4*c^3*d - 3*B^2*a*b^3*c^2*d^2 + 3*B^2*a^2*b^2*c*d^3)*i*n
^2*x + (3*B^2*b^4*c^4 - 8*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2)*i*n^2)*l
og((b*x + a)/(d*x + c))^2 + 72*(3*A^2*b^4*c^4 - 8*A^2*a*b^3*c^3*d + 6*A^2*a
^2*b^2*c^2*d^2 - A^2*a^4*d^4)*i - 4*((5*B^2*b^4*c^3*d - 12*B^2*a*b^3*c^2*d^
2 - 108*B^2*a^2*b^2*c*d^3 + 115*B^2*a^3*b*d^4)*i*n^2 - 12*(A*B*b^4*c^3*d -
6*A*B*a*b^3*c^2*d^2 + 18*A*B*a^2*b^2*c*d^3 - 13*A*B*a^3*b*d^4)*i*n - 72*(A^
2*b^4*c^3*d - 3*A^2*a*b^3*c^2*d^2 + 3*A^2*a^2*b^2*c*d^3 - A^2*a^3*b*d^4)*i)
*x + 12*(12*(B^2*b^4*c*d^3 - B^2*a*b^3*d^4)*i*n*x^3 - 6*(B^2*b^4*c^2*d^2 -
8*B^2*a*b^3*c*d^3 + 7*B^2*a^2*b^2*d^4)*i*n*x^2 + (9*B^2*b^4*c^4 - 32*B^2*a*
b^3*c^3*d + 36*B^2*a^2*b^2*c^2*d^2 - 13*B^2*a^4*d^4)*i*n + 12*(3*A*B*b^4*c^
4 - 8*A*B*a*b^3*c^3*d + 6*A*B*a^2*b^2*c^2*d^2 - A*B*a^4*d^4)*i + 4*((B^2*b^
4*c^3*d - 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 - 13*B^2*a^3*b*d^4)*i*
n + 12*(A*B*b^4*c^3*d - 3*A*B*a*b^3*c^2*d^2 + 3*A*B*a^2*b^2*c*d^3 - A*B*a^3
*b*d^4)*i)*x + 12*(B^2*b^4*d^4*i*n*x^4 + 4*B^2*a*b^3*d^4*i*n*x^3 + 6*B^2*a^
2*b^2*d^4*i*n*x^2 + 4*(B^2*b^4*c^3*d - 3*B^2*a*b^3*c^2*d^2 + 3*B^2*a^2*b^2*
c*d^3)*i*n*x + (3*B^2*b^4*c^4 - 8*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2)*
i*n)*log((b*x + a)/(d*x + c)))*log(e) + 12*((13*B^2*b^4*d^4*i*n^2 + 12*A*B*
b^4*d^4*i*n)*x^4 + (9*B^2*b^4*c^4 - 32*B^2*a*b^3*c^3*d + 36*B^2*a^2*b^2*c^2
*d^2)*i*n^2 + 4*(12*A*B*a*b^3*d^4*i*n + (3*B^2*b^4*c*d^3 + 10*B^2*a*b^3*d^4
)*i*n^2)*x^3 + 12*(3*A*B*b^4*c^4 - 8*A*B*a*b^3*c^3*d + 6*A*B*a^2*b^2*c^2*d^
2)*i*n + 6*(12*A*B*a^2*b^2*d^4*i*n - (B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 -
 6*B^2*a^2*b^2*d^4)*i*n^2)*x^2 + 4*((B^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 +
18*B^2*a^2*b^2*c*d^3)*i*n^2 + 12*(A*B*b^4*c^3*d - 3*A*B*a*b^3*c^2*d^2 + 3*A
*B*a^2*b^2*c*d^3)*i*n)*x)*log((b*x + a)/(d*x + c)))/((b^9*c^3 - 3*a*b^8*c^2
*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*g^5*x^4 + 4*(a*b^8*c^3 - 3*a^2*b^7*c^2*
```

```
d + 3*a^3*b^6*c*d^2 - a^4*b^5*d^3)*g^5*x^3 + 6*(a^2*b^7*c^3 - 3*a^3*b^6*c^2
*d + 3*a^4*b^5*c*d^2 - a^5*b^4*d^3)*g^5*x^2 + 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^
2*d + 3*a^5*b^4*c*d^2 - a^6*b^3*d^3)*g^5*x + (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d
 + 3*a^6*b^3*c*d^2 - a^7*b^2*d^3)*g^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**5,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4838 vs. 2(457) = 914.

Time = 0.54 (sec) , antiderivative size = 4838, normalized size of antiderivative = 10.19

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x, a
lgorithm="maxima")
```

```
[Out] 1/24*A*B*c*i*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d
^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*
c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*
b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b
^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*
b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6
*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*
d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d
^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*
a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/72
*A*B*d*i*n*((7*a*b^3*c^3 - 33*a^2*b^2*c^2*d + 75*a^3*b*c*d^2 - 13*a^4*d^3 +
 12*(4*b^4*c*d^2 - a*b^3*d^3)*x^3 - 6*(4*b^4*c^2*d - 29*a*b^3*c*d^2 + 7*a^2
*b^2*d^3)*x^2 + 4*(4*b^4*c^3 - 21*a*b^3*c^2*d + 57*a^2*b^2*c*d^2 - 13*a^3*b
*d^3)*x)/((b^9*c^3 - 3*a*b^8*c^2*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*g^5*x^4
 + 4*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*a^3*b^6*c*d^2 - a^4*b^5*d^3)*g^5*x^3
+ 6*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*a^4*b^5*c*d^2 - a^5*b^4*d^3)*g^5*x^2
 + 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2 - a^6*b^3*d^3)*g^5*x
+ (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d^2 - a^7*b^2*d^3)*g^5) + 12
```

*(4*b*c*d^3 - a*d^4)*log(b*x + a)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2
*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5) - 12*(4*b*c*d^3 - a*d^4)*log(d*x
 + c)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4
*b^2*d^4)*g^5)) - 1/12*(4*b*x + a)*B^2*d*i*log(e*(b*x/(d*x + c) + a/(d*x +
c))^n)^2/(b^6*g^5*x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5
*x + a^4*b^2*g^5) + 1/288*(12*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2
*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3
*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2
*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*
b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4
*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^
5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b
^2*c*d^2 - a^7*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d
+ 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x +
c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*
d^4)*g^5))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - (9*b^4*c^4 - 64*a*b^3*c
^3*d + 216*a^2*b^2*c^2*d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3
 - a*b^3*d^4)*x^3 + 6*(13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*
x^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x
 + a^4*d^4)*log(b*x + a)^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*
d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b
^3*c^2*d^2 + 324*a^2*b^2*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*
b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 1
2*(25*b^4*d^4*x^4 + 100*a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4
*x + 25*a^4*d^4 - 12*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4
*a^3*b*d^4*x + a^4*d^4)*log(b*x + a))*log(d*x + c))*n^2/(a^4*b^5*c^4*g^5 -
4*a^5*b^4*c^3*d*g^5 + 6*a^6*b^3*c^2*d^2*g^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d
^4*g^5 + (b^9*c^4*g^5 - 4*a*b^8*c^3*d*g^5 + 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b
^6*c*d^3*g^5 + a^4*b^5*d^4*g^5)*x^4 + 4*(a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^
5 + 6*a^3*b^6*c^2*d^2*g^5 - 4*a^4*b^5*c*d^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*
(a^2*b^7*c^4*g^5 - 4*a^3*b^6*c^3*d*g^5 + 6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*
c*d^3*g^5 + a^6*b^3*d^4*g^5)*x^2 + 4*(a^3*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5
 + 6*a^5*b^4*c^2*d^2*g^5 - 4*a^6*b^3*c*d^3*g^5 + a^7*b^2*d^4*g^5)*x))*B^2*c
i - 1/864(12*n*((7*a*b^3*c^3 - 33*a^2*b^2*c^2*d + 75*a^3*b*c*d^2 - 13*a^4
d^3 + 12(4*b^4*c*d^2 - a*b^3*d^3)*x^3 - 6*(4*b^4*c^2*d - 29*a*b^3*c*d^2 +
 7*a^2*b^2*d^3)*x^2 + 4*(4*b^4*c^3 - 21*a*b^3*c^2*d + 57*a^2*b^2*c*d^2 - 13
*a^3*b*d^3)*x)/((b^9*c^3 - 3*a*b^8*c^2*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*g
^5*x^4 + 4*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*a^3*b^6*c*d^2 - a^4*b^5*d^3)*g^
5*x^3 + 6*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*a^4*b^5*c*d^2 - a^5*b^4*d^3)*g
^5*x^2 + 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2 - a^6*b^3*d^3)*
g^5*x + (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d^2 - a^7*b^2*d^3)*g^5
) + 12*(4*b*c*d^3 - a*d^4)*log(b*x + a)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b
^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5) - 12*(4*b*c*d^3 - a*d^4)*l
og(d*x + c)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3
 + a^4*b^2*d^4)*g^5))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + (37*a*b^4*c^

4 - 304*a^2*b^3*c^3*d + 1512*a^3*b^2*c^2*d^2 - 1360*a^4*b*c*d^3 + 115*a^5*d
^4 + 12*(88*b^5*c^2*d^2 - 101*a*b^4*c*d^3 + 13*a^2*b^3*d^4)*x^3 - 6*(40*b^5
*c^3*d - 609*a*b^4*c^2*d^2 + 648*a^2*b^3*c*d^3 - 79*a^3*b^2*d^4)*x^2 - 72*(
4*a^4*b*c*d^3 - a^5*d^4 + (4*b^5*c*d^3 - a*b^4*d^4)*x^4 + 4*(4*a*b^4*c*d^3
- a^2*b^3*d^4)*x^3 + 6*(4*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^2 + 4*(4*a^3*b^2*c
*d^3 - a^4*b*d^4)*x)*log(b*x + a)^2 - 72*(4*a^4*b*c*d^3 - a^5*d^4 + (4*b^5*
c*d^3 - a*b^4*d^4)*x^4 + 4*(4*a*b^4*c*d^3 - a^2*b^3*d^4)*x^3 + 6*(4*a^2*b^3
*c*d^3 - a^3*b^2*d^4)*x^2 + 4*(4*a^3*b^2*c*d^3 - a^4*b*d^4)*x)*log(d*x + c)
^2 + 4*(16*b^5*c^4 - 163*a*b^4*c^3*d + 1068*a^2*b^3*c^2*d^2 - 1036*a^3*b^2*
c*d^3 + 115*a^4*b*d^4)*x + 12*(88*a^4*b*c*d^3 - 13*a^5*d^4 + (88*b^5*c*d^3
- 13*a*b^4*d^4)*x^4 + 4*(88*a*b^4*c*d^3 - 13*a^2*b^3*d^4)*x^3 + 6*(88*a^2*b
^3*c*d^3 - 13*a^3*b^2*d^4)*x^2 + 4*(88*a^3*b^2*c*d^3 - 13*a^4*b*d^4)*x)*log
(b*x + a) - 12*(88*a^4*b*c*d^3 - 13*a^5*d^4 + (88*b^5*c*d^3 - 13*a*b^4*d^4)
x^4 + 4(88*a*b^4*c*d^3 - 13*a^2*b^3*d^4)*x^3 + 6*(88*a^2*b^3*c*d^3 - 13*a
^3*b^2*d^4)*x^2 + 4*(88*a^3*b^2*c*d^3 - 13*a^4*b*d^4)*x - 12*(4*a^4*b*c*d^3
 - a^5*d^4 + (4*b^5*c*d^3 - a*b^4*d^4)*x^4 + 4*(4*a*b^4*c*d^3 - a^2*b^3*d^4
)*x^3 + 6*(4*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^2 + 4*(4*a^3*b^2*c*d^3 - a^4*b*
d^4)*x)*log(b*x + a))*log(d*x + c))*n^2/(a^4*b^6*c^4*g^5 - 4*a^5*b^5*c^3*d*
g^5 + 6*a^6*b^4*c^2*d^2*g^5 - 4*a^7*b^3*c*d^3*g^5 + a^8*b^2*d^4*g^5 + (b^10
*c^4*g^5 - 4*a*b^9*c^3*d*g^5 + 6*a^2*b^8*c^2*d^2*g^5 - 4*a^3*b^7*c*d^3*g^5
+ a^4*b^6*d^4*g^5)*x^4 + 4*(a*b^9*c^4*g^5 - 4*a^2*b^8*c^3*d*g^5 + 6*a^3*b^7
*c^2*d^2*g^5 - 4*a^4*b^6*c*d^3*g^5 + a^5*b^5*d^4*g^5)*x^3 + 6*(a^2*b^8*c^4*
g^5 - 4*a^3*b^7*c^3*d*g^5 + 6*a^4*b^6*c^2*d^2*g^5 - 4*a^5*b^5*c*d^3*g^5 + a
^6*b^4*d^4*g^5)*x^2 + 4*(a^3*b^7*c^4*g^5 - 4*a^4*b^6*c^3*d*g^5 + 6*a^5*b^5*
c^2*d^2*g^5 - 4*a^6*b^4*c*d^3*g^5 + a^7*b^3*d^4*g^5)*x))*B^2*d*i - 1/6*(4*b
*x + a)*A*B*d*i*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^6*g^5*x^4 + 4*a*b
^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g^5) - 1/4*B^2*c
*i*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3
+ 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/12*(4*b*x + a)*A^2*d
*i/(b^6*g^5*x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a
^4*b^2*g^5) - 1/2*A*B*c*i*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^5*g^5*x
^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1
/4*A^2*c*i/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g
^5*x + a^4*b*g^5)

Giac [A] (verification not implemented)

none

Time = 3.07 (sec) , antiderivative size = 871, normalized size of antiderivative = 1.83

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{1}{864}\left(\frac{72\left(3\,B^2b^2in^2-\frac{8\,(bx+a)B^2bdin^2}{dx+c}+\frac{6\,(bx+a)^2B^2d^2in^2}{(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)^2}{\frac{(bx+a)^4b^2c^2g^5}{(dx+c)^4}-\frac{2\,(bx+a)^4abcdg^5}{(dx+c)^4}+\frac{(bx+a)^4a^2d^2g^5}{(dx+c)^4}}+\frac{12\left(9\,B^2b^2in^2-\frac{32\,(bx+a)B^2bdin^2}{dx+c}+\right.}{}\right.$$

```
[In] integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x, a
lgorithm="giac")
```

```
[Out] -1/864*(72*(3*B^2*b^2*i*n^2 - 8*(b*x + a)*B^2*b*d*i*n^2/(d*x + c) + 6*(b*x
+ a)^2*B^2*d^2*i*n^2/(d*x + c)^2)*log((b*x + a)/(d*x + c))^2/((b*x + a)^4*b
^2*c^2*g^5/(d*x + c)^4 - 2*(b*x + a)^4*a*b*c*d*g^5/(d*x + c)^4 + (b*x + a)^
4*a^2*d^2*g^5/(d*x + c)^4) + 12*(9*B^2*b^2*i*n^2 - 32*(b*x + a)*B^2*b*d*i*n
^2/(d*x + c) + 36*(b*x + a)^2*B^2*d^2*i*n^2/(d*x + c)^2 + 36*B^2*b^2*i*n*lo
g(e) - 96*(b*x + a)*B^2*b*d*i*n*log(e)/(d*x + c) + 72*(b*x + a)^2*B^2*d^2*i
*n*log(e)/(d*x + c)^2 + 36*A*B*b^2*i*n - 96*(b*x + a)*A*B*b*d*i*n/(d*x + c)
 + 72*(b*x + a)^2*A*B*d^2*i*n/(d*x + c)^2)*log((b*x + a)/(d*x + c))/((b*x +
 a)^4*b^2*c^2*g^5/(d*x + c)^4 - 2*(b*x + a)^4*a*b*c*d*g^5/(d*x + c)^4 + (b*
x + a)^4*a^2*d^2*g^5/(d*x + c)^4) + (27*B^2*b^2*i*n^2 - 128*(b*x + a)*B^2*b
*d*i*n^2/(d*x + c) + 216*(b*x + a)^2*B^2*d^2*i*n^2/(d*x + c)^2 + 108*B^2*b^
2*i*n*log(e) - 384*(b*x + a)*B^2*b*d*i*n*log(e)/(d*x + c) + 432*(b*x + a)^2
*B^2*d^2*i*n*log(e)/(d*x + c)^2 + 216*B^2*b^2*i*log(e)^2 - 576*(b*x + a)*B^
2*b*d*i*log(e)^2/(d*x + c) + 432*(b*x + a)^2*B^2*d^2*i*log(e)^2/(d*x + c)^2
 + 108*A*B*b^2*i*n - 384*(b*x + a)*A*B*b*d*i*n/(d*x + c) + 432*(b*x + a)^2*
A*B*d^2*i*n/(d*x + c)^2 + 432*A*B*b^2*i*log(e) - 1152*(b*x + a)*A*B*b*d*i*l
og(e)/(d*x + c) + 864*(b*x + a)^2*A*B*d^2*i*log(e)/(d*x + c)^2 + 216*A^2*b^
2*i - 576*(b*x + a)*A^2*b*d*i/(d*x + c) + 432*(b*x + a)^2*A^2*d^2*i/(d*x +
c)^2)/((b*x + a)^4*b^2*c^2*g^5/(d*x + c)^4 - 2*(b*x + a)^4*a*b*c*d*g^5/(d*x
 + c)^4 + (b*x + a)^4*a^2*d^2*g^5/(d*x + c)^4))*(b*c/(b*c - a*d)^2 - a*d/(b
*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 6.36 (sec) , antiderivative size = 1794, normalized size of antiderivative = 3.78

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

```
[In] int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x)^
5,x)
```

```
[Out] ((72*A^2*a^3*d^3*i + 216*A^2*b^3*c^3*i + 115*B^2*a^3*d^3*i*n^2 + 27*B^2*b^3
*c^3*i*n^2 + 156*A*B*a^3*d^3*i*n + 108*A*B*b^3*c^3*i*n - 360*A^2*a*b^2*c^2*
```

```
d*i + 72*A^2*a^2*b*c*d^2*i - 101*B^2*a*b^2*c^2*d*i*n^2 + 115*B^2*a^2*b*c*d^
2*i*n^2 - 276*A*B*a*b^2*c^2*d*i*n + 156*A*B*a^2*b*c*d^2*i*n)/(12*(a*d - b*c
)) + (x^2*(79*B^2*a*b^2*d^3*i*n^2 - B^2*b^3*c*d^2*i*n^2 + 84*A*B*a*b^2*d^3*
i*n - 12*A*B*b^3*c*d^2*i*n))/(2*(a*d - b*c)) + (x*(72*A^2*a^2*b*d^3*i + 72*
A^2*b^3*c^2*d*i + 115*B^2*a^2*b*d^3*i*n^2 - 5*B^2*b^3*c^2*d*i*n^2 - 144*A^2
*a*b^2*c*d^2*i + 7*B^2*a*b^2*c*d^2*i*n^2 + 156*A*B*a^2*b*d^3*i*n + 12*A*B*b
^3*c^2*d*i*n - 60*A*B*a*b^2*c*d^2*i*n))/(3*(a*d - b*c)) + (d*x^3*(13*B^2*b^
3*d^2*i*n^2 + 12*A*B*b^3*d^2*i*n))/(a*d - b*c))/(x*(288*a^3*b^4*c*g^5 - 288
*a^4*b^3*d*g^5) - x^3*(288*a^2*b^5*d*g^5 - 288*a*b^6*c*g^5) + x^4*(72*b^7*c
*g^5 - 72*a*b^6*d*g^5) + x^2*(432*a^2*b^5*c*g^5 - 432*a^3*b^4*d*g^5) + 72*a
^4*b^3*c*g^5 - 72*a^5*b^2*d*g^5) - log(e*((a + b*x)/(c + d*x))^n)^2*(((B^2*
c*i)/(4*b) + (B^2*d*i*x)/(3*b) + (B^2*a*d*i)/(12*b^2))/(a^4*g^5 + b^4*g^5*x
^4 + 4*a*b^3*g^5*x^3 + 6*a^2*b^2*g^5*x^2 + 4*a^3*b*g^5*x) - (B^2*d^4*i)/(12
*b^2*g^5*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - log(e*((a
+ b*x)/(c + d*x))^n)*((A*B*a*d*i + 3*A*B*b*c*i - B^2*a*d*i*n + B^2*b*c*i*n
+ 4*A*B*b*d*i*x)/(6*a^4*b^2*g^5 + 6*b^6*g^5*x^4 + 24*a^3*b^3*g^5*x + 24*a*b
^5*g^5*x^3 + 36*a^2*b^4*g^5*x^2) + (B^2*d^4*i*(x^2*(b*(b*((3*a*b^2*g^5*n*(a
*d - b*c))/(2*d) + (b^2*g^5*n*(a*d - b*c)*(4*a*d - b*c))/(2*d^2)) + (3*a*b^
3*g^5*n*(a*d - b*c))/d + (b^3*g^5*n*(a*d - b*c)*(4*a*d - b*c))/d^2) + (9*a*
b^4*g^5*n*(a*d - b*c))/(2*d) + (3*b^4*g^5*n*(a*d - b*c)*(4*a*d - b*c))/(2*d
^2)) + a*(a*((3*a*b^2*g^5*n*(a*d - b*c))/(2*d) + (b^2*g^5*n*(a*d - b*c)*(4*
a*d - b*c))/(2*d^2)) + (b^2*g^5*n*(a*d - b*c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*
c*d))/(2*d^3)) + x*(a*(b*((3*a*b^2*g^5*n*(a*d - b*c))/(2*d) + (b^2*g^5*n*(a
*d - b*c)*(4*a*d - b*c))/(2*d^2)) + (3*a*b^3*g^5*n*(a*d - b*c))/d + (b^3*g^
5*n*(a*d - b*c)*(4*a*d - b*c))/d^2) + b*(a*((3*a*b^2*g^5*n*(a*d - b*c))/(2*
d) + (b^2*g^5*n*(a*d - b*c)*(4*a*d - b*c))/(2*d^2)) + (b^2*g^5*n*(a*d - b*c
)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(2*d^3)) + (3*b^3*g^5*n*(a*d - b*c)*(6
*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(2*d^3)) + (3*b^2*g^5*n*(a*d - b*c)*(4*a^3
*d^3 - b^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d^2))/(2*d^4) + (6*b^5*g^5*n*x^3
*(a*d - b*c))/d))/(6*b^2*g^5*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2)*(6*a^4*b^2*g^5 + 6*b^6*g^5*x^4 + 24*a^3*b^3*g^5*x + 24*a*b^5*g^5*x^3
+ 36*a^2*b^4*g^5*x^2))) - (B*d^4*i*n*atan((B*d^4*i*n*(12*A + 13*B*n)*((b^5*
c^3*g^5 + a^3*b^2*d^3*g^5 - a*b^4*c^2*d*g^5 - a^2*b^3*c*d^2*g^5)/(b^4*c^2*g
^5 + a^2*b^2*d^2*g^5 - 2*a*b^3*c*d*g^5) + 2*b*d*x)*(b^4*c^2*g^5 + a^2*b^2*d
^2*g^5 - 2*a*b^3*c*d*g^5)*1i)/(b^2*g^5*(a*d - b*c)^3*(13*B^2*d^4*i*n^2 + 12
*A*B*d^4*i*n)))*(12*A + 13*B*n)*1i)/(36*b^2*g^5*(a*d - b*c)^3)
```

3.168 $\int (ag+bgx)^3(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 45, antiderivative size = 766

$$\begin{aligned}
&\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= \frac{3B^2(bc-ad)^5g^3i^2n^2x}{20b^2d^3}+\frac{B^2(bc-ad)^2g^3i^2n^2(a+bx)^4}{60b^3}-\frac{3B^2(bc-ad)^4g^3i^2n^2(c+dx)^2}{40bd^4} \\
&+\frac{B^2(bc-ad)^3g^3i^2n^2(c+dx)^3}{60d^4}-\frac{B(bc-ad)^3g^3i^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{90b^3d} \\
&-\frac{B(bc-ad)^2g^3i^2n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^3} \\
&-\frac{B(bc-ad)g^3i^2n(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^2} \\
&+\frac{(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{60b^3} \\
&+\frac{(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{15b^2} \\
&+\frac{g^3i^2(a+bx)^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b} \\
&+\frac{B(bc-ad)^4g^3i^2n(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{180b^3d^2} \\
&-\frac{B(bc-ad)^5g^3i^2n(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{180b^3d^3} \\
&-\frac{B(bc-ad)^6g^3i^2n\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{180b^3d^4} \\
&-\frac{B^2(bc-ad)^6g^3i^2n^2\log(c+dx)}{20b^3d^4}-\frac{B^2(bc-ad)^6g^3i^2n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{30b^3d^4}
\end{aligned}$$

```
[Out] 3/20*B^2*(-a*d+b*c)^5*g^3*i^2*n^2*x/b^2/d^3+1/60*B^2*(-a*d+b*c)^2*g^3*i^2*n
^2*(b*x+a)^4/b^3-3/40*B^2*(-a*d+b*c)^4*g^3*i^2*n^2*(d*x+c)^2/b/d^4+1/60*B^2
*(-a*d+b*c)^3*g^3*i^2*n^2*(d*x+c)^3/d^4-1/90*B*(-a*d+b*c)^3*g^3*i^2*n*(b*x+
a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/d-1/20*B*(-a*d+b*c)^2*g^3*i^2*n*(b
*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3-1/15*B*(-a*d+b*c)*g^3*i^2*n*(b*
x+a)^4*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2+1/60*(-a*d+b*c)^2*g^3*i^
2*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^3+1/15*(-a*d+b*c)*g^3*i^2*(
b*x+a)^4*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2+1/6*g^3*i^2*(b*x+a)^
4*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/180*B*(-a*d+b*c)^4*g^3*i^
2*n*(b*x+a)^2*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/d^2-1/180*B*(-a*d
+b*c)^5*g^3*i^2*n*(b*x+a)*(6*A+5*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/d^3
-1/180*B*(-a*d+b*c)^6*g^3*i^2*n*(6*A+11*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))*
ln((-a*d+b*c)/b/(d*x+c))/b^3/d^4-1/20*B^2*(-a*d+b*c)^6*g^3*i^2*n^2*ln(d*x+c
)/b^3/d^4-1/30*B^2*(-a*d+b*c)^6*g^3*i^2*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/
b^3/d^4
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 766, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules

used = {2561, 2383, 2381, 2384, 2354, 2438, 2373, 45, 47, 37, 2382, 12, 79}

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= -\frac{Bg^3i^2n(bc-ad)^6\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+6A+11Bn\right)}{180b^3d^4}$$
$$-\frac{Bg^3i^2n(a+bx)(bc-ad)^5\left(6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+6A+5Bn\right)}{180b^3d^3}$$
$$+\frac{Bg^3i^2n(a+bx)^2(bc-ad)^4\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{180b^3d^2}$$
$$-\frac{Bg^3i^2n(a+bx)^3(bc-ad)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{90b^3d}$$
$$+\frac{g^3i^2(a+bx)^4(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{60b^3}$$
$$-\frac{Bg^3i^2n(a+bx)^4(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{20b^3}$$
$$+\frac{g^3i^2(a+bx)^4(c+dx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{15b^2}$$
$$-\frac{Bg^3i^2n(a+bx)^4(c+dx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{15b^2}$$
$$+\frac{g^3i^2(a+bx)^4(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{6b}$$
$$-\frac{B^2g^3i^2n^2(bc-ad)^6\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{30b^3d^4}-\frac{B^2g^3i^2n^2(bc-ad)^6\log(c+dx)}{20b^3d^4}$$
$$+\frac{B^2g^3i^2n^2(a+bx)^4(bc-ad)^2}{60b^3}+\frac{3B^2g^3i^2n^2x(bc-ad)^5}{20b^2d^3}$$
$$-\frac{3B^2g^3i^2n^2(c+dx)^2(bc-ad)^4}{40bd^4}+\frac{B^2g^3i^2n^2(c+dx)^3(bc-ad)^3}{60d^4}$$

```
[In] Int[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2,x]
```

```
[Out] (3*B^2*(b*c - a*d)^5*g^3*i^2*n^2*x)/(20*b^2*d^3) + (B^2*(b*c - a*d)^2*g^3*i
^2*n^2*(a + b*x)^4)/(60*b^3) - (3*B^2*(b*c - a*d)^4*g^3*i^2*n^2*(c + d*x)^2
)/(40*b*d^4) + (B^2*(b*c - a*d)^3*g^3*i^2*n^2*(c + d*x)^3)/(60*d^4) - (B*(b
*c - a*d)^3*g^3*i^2*n*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(
90*b^3*d) - (B*(b*c - a*d)^2*g^3*i^2*n*(a + b*x)^4*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]))/(20*b^3) - (B*(b*c - a*d)*g^3*i^2*n*(a + b*x)^4*(c + d*x)*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(15*b^2) + ((b*c - a*d)^2*g^3*i^2*(a
 + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(60*b^3) + ((b*c - a*d)
*g^3*i^2*(a + b*x)^4*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(1
5*b^2) + (g^3*i^2*(a + b*x)^4*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x)
```

```
)^n])^2)/(6*b) + (B*(b*c - a*d)^4*g^3*i^2*n*(a + b*x)^2*(3*A + B*n + 3*B*Lo
g[e*((a + b*x)/(c + d*x))^n]))/(180*b^3*d^2) - (B*(b*c - a*d)^5*g^3*i^2*n*(
a + b*x)*(6*A + 5*B*n + 6*B*Log[e*((a + b*x)/(c + d*x))^n]))/(180*b^3*d^3)
- (B*(b*c - a*d)^6*g^3*i^2*n*(6*A + 11*B*n + 6*B*Log[e*((a + b*x)/(c + d*x)
)^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(180*b^3*d^4) - (B^2*(b*c - a*d)^6*g^
3*i^2*n^2*Log[c + d*x])/(20*b^3*d^4) - (B^2*(b*c - a*d)^6*g^3*i^2*n^2*PolyL
og[2, (d*(a + b*x))/(b*(c + d*x))])/(30*b^3*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2383

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Dist[(m + q + 2)/(d*(q + 1)), Int[(
f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Dist[b*n*(p/(d*(q +
 1))), Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x]) /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ
[q, -1] && GtQ[m, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
```

```
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \left((bc-ad)^6 g^3 i^2\right) \text{Subst}\left(\int \frac{x^3 (A+B\log(ex^n))^2}{(b-dx)^7}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{g^3 i^2 (a+bx)^4 (c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b} + \frac{\left((bc-ad)^6 g^3 i^2\right) \text{Subst}\left(\int \frac{x^3 (A+B\log(ex^n))^2}{(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b} - \frac{\left(B(bc-ad)^6 g^3 i^2 n\right) \text{Subst}\left(\int \frac{x^3 (A+B\log(ex^n))}{(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b}$$

$$= -\frac{B(bc-ad)^2 g^3 i^2 n (a+bx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^3} - \frac{B(bc-ad) g^3 i^2 n (a+bx)^4 (c+dx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^2} + \frac{(bc-ad) g^3 i^2 (a+bx)^4 (c+dx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{15b^2} + \frac{g^3 i^2 (a+bx)^4 (c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b} + \frac{\left((bc-ad)^6 g^3 i^2\right) \text{Subst}\left(\int \frac{x^3 (A+B\log(ex^n))^2}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{15b^2} - \frac{\left(2B(bc-ad)^6 g^3 i^2 n\right) \text{Subst}\left(\int \frac{x^3 (A+B\log(ex^n))}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{15b^2} + \frac{\left(B^2 (bc-ad)^6 g^3 i^2 n^2\right) \text{Subst}\left(\int \frac{x^3 (5b-dx)}{20b^2 (b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b}$$

$$= -\frac{B(bc-ad)^2g^3i^2n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^3} - \frac{B(bc-ad)g^3i^2n(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^2} + \frac{(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{60b^3} + \frac{(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{15b^2} + \frac{g^3i^2(a+bx)^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b} - \frac{(B(bc-ad)^6g^3i^2n)\,\text{Subst}\left(\int\frac{x^3(A+B\log(ex^n))}{(b-dx)^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{30b^3} + \frac{(B^2(bc-ad)^6g^3i^2n^2)\,\text{Subst}\left(\int\frac{x^3(5b-dx)}{(b-dx)^5}\,dx,x,\frac{a+bx}{c+dx}\right)}{60b^3} + \frac{(B^2(bc-ad)^6g^3i^2n^2)\,\text{Subst}\left(\int\frac{x^3}{(b-dx)^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{30b^3}$$

$$= \frac{B^2(bc-ad)^2g^3i^2n^2(a+bx)^4}{60b^3} - \frac{B(bc-ad)^3g^3i^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{90b^3d} - \frac{B(bc-ad)^2g^3i^2n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^3} - \frac{B(bc-ad)g^3i^2n(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^2} + \frac{(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{60b^3} + \frac{(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{15b^2} + \frac{g^3i^2(a+bx)^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b} + \frac{(B(bc-ad)^6g^3i^2n)\,\text{Subst}\left(\int\frac{x^2(3A+Bn+3B\log(ex^n))}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{90b^3d} + \frac{(B^2(bc-ad)^6g^3i^2n^2)\,\text{Subst}\left(\int\frac{x^3}{(b-dx)^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{60b^3} + \frac{(B^2(bc-ad)^6g^3i^2n^2)\,\text{Subst}\left(\int\left(\frac{b^3}{d^3(b-dx)^4}-\frac{3b^2}{d^3(b-dx)^3}+\frac{3b}{d^3(b-dx)^2}-\frac{1}{d^3(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{30b^3}$$

$$= \frac{B^2(bc-ad)^5 g^3 i^2 n^2 x}{10b^2 d^3} + \frac{B^2(bc-ad)^2 g^3 i^2 n^2 (a+bx)^4}{60b^3} - \frac{B^2(bc-ad)^4 g^3 i^2 n^2 (c+dx)^2}{20bd^4} + \frac{B^2(bc-ad)^3 g^3 i^2 n^2 (c+dx)^3}{90d^4} - \frac{B(bc-ad)^3 g^3 i^2 n (a+bx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{90b^3 d} - \frac{B(bc-ad)^2 g^3 i^2 n (a+bx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^3} - \frac{B(bc-ad) g^3 i^2 n (a+bx)^4 (c+dx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^2} + \frac{(bc-ad)^2 g^3 i^2 (a+bx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{60b^3} + \frac{(bc-ad) g^3 i^2 (a+bx)^4 (c+dx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{15b^2} + \frac{g^3 i^2 (a+bx)^4 (c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b} + \frac{B(bc-ad)^4 g^3 i^2 n (a+bx)^2 \left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{180b^3 d^2} - \frac{B^2(bc-ad)^6 g^3 i^2 n^2 \log(c+dx)}{30b^3 d^4} - \frac{\left(B(bc-ad)^6 g^3 i^2 n\right) \operatorname{Subst}\left(\int \frac{x(3Bn+2(3A+Bn)+6B\log(ex^n))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{180b^3 d^2} + \frac{\left(B^2(bc-ad)^6 g^3 i^2 n^2\right) \operatorname{Subst}\left(\int \left(\frac{b^3}{d^3(b-dx)^4} - \frac{3b^2}{d^3(b-dx)^3} + \frac{3b}{d^3(b-dx)^2} - \frac{1}{d^3(b-dx)}\right) dx, x, \frac{a+bx}{c+dx}\right)}{60b^3}$$

$$
\begin{aligned}
&= \frac{3B^2(bc-ad)^5g^3i^2n^2x}{20b^2d^3} + \frac{B^2(bc-ad)^2g^3i^2n^2(a+bx)^4}{60b^3} \\
&\quad - \frac{3B^2(bc-ad)^4g^3i^2n^2(c+dx)^2}{40bd^4} + \frac{B^2(bc-ad)^3g^3i^2n^2(c+dx)^3}{60d^4} \\
&\quad - \frac{B(bc-ad)^3g^3i^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{90b^3d} \\
&\quad - \frac{B(bc-ad)^2g^3i^2n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^3} \\
&\quad - \frac{B(bc-ad)g^3i^2n(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^2} \\
&\quad + \frac{(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{60b^3} \\
&\quad + \frac{(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{15b^2} \\
&\quad + \frac{g^3i^2(a+bx)^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b} \\
&\quad + \frac{B(bc-ad)^4g^3i^2n(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{180b^3d^2} \\
&\quad - \frac{B(bc-ad)^5g^3i^2n(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{180b^3d^3} \\
&\quad - \frac{B^2(bc-ad)^6g^3i^2n^2\log(c+dx)}{20b^3d^4} \\
&\quad + \frac{\left(B(bc-ad)^6g^3i^2n\right)\operatorname{Subst}\left(\int \frac{9Bn+2(3A+Bn)+6B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{180b^3d^3}
\end{aligned}
$$

$$= \frac{3B^2(bc-ad)^5g^3i^2n^2x}{20b^2d^3}+\frac{B^2(bc-ad)^2g^3i^2n^2(a+bx)^4}{60b^3} - \frac{3B^2(bc-ad)^4g^3i^2n^2(c+dx)^2}{40bd^4}+\frac{B^2(bc-ad)^3g^3i^2n^2(c+dx)^3}{60d^4} - \frac{B(bc-ad)^3g^3i^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{90b^3d} - \frac{B(bc-ad)^2g^3i^2n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^3} - \frac{B(bc-ad)g^3i^2n(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^2} + \frac{(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{60b^3} + \frac{(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{15b^2} + \frac{g^3i^2(a+bx)^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b} + \frac{B(bc-ad)^4g^3i^2n(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{180b^3d^2} - \frac{B(bc-ad)^5g^3i^2n(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{180b^3d^3} - \frac{B(bc-ad)^6g^3i^2n\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{180b^3d^4} - \frac{B^2(bc-ad)^6g^3i^2n^2\log(c+dx)}{20b^3d^4} + \frac{(B^2(bc-ad)^6g^3i^2n^2)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{30b^3d^4}$$

$$= \frac{3B^2(bc-ad)^5g^3i^2n^2x}{20b^2d^3} + \frac{B^2(bc-ad)^2g^3i^2n^2(a+bx)^4}{60b^3} - \frac{3B^2(bc-ad)^4g^3i^2n^2(c+dx)^2}{40bd^4} + \frac{B^2(bc-ad)^3g^3i^2n^2(c+dx)^3}{60d^4} - \frac{B(bc-ad)^3g^3i^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{90b^3d} - \frac{B(bc-ad)^2g^3i^2n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^3} - \frac{B(bc-ad)g^3i^2n(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^2} + \frac{(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{60b^3} + \frac{(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{15b^2} + \frac{g^3i^2(a+bx)^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b} + \frac{B(bc-ad)^4g^3i^2n(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{180b^3d^2} - \frac{B(bc-ad)^5g^3i^2n(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{180b^3d^3} - \frac{B(bc-ad)^6g^3i^2n\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{180b^3d^4} - \frac{B^2(bc-ad)^6g^3i^2n^2\log(c+dx)}{20b^3d^4} - \frac{B^2(bc-ad)^6g^3i^2n^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{30b^3d^4}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1634 vs. $2(766) = 1532$.

Time = 0.84 (sec) , antiderivative size = 1634, normalized size of antiderivative = 2.13

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$

$$= \frac{g^3i^2\left(15(bc-ad)^2(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + 24d(bc-ad)(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + \right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^2,x]
```

```
[Out] (g^3*i^2*(15*(b*c - a*d)^2*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])^2 + 24*d*(b*c - a*d)*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2 + 10*d^2*(a + b*x)^6*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (5*B*(b*c
```

```
 - a*d)^3*n*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*
((a + b*x)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[e*((
a + b*x)/(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*
x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*n*(
b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*n*((2*Log[(d*(a +
b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d
*x))/(b*c - a*d)])))/d^4 + (2*B*(b*c - a*d)^2*n*(24*A*b*d*(b*c - a*d)^3*x +
 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 12*d^2*(b*
c - a*d)^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 8*d^3*(b*c
- a*d)*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*d^4*(a + b*x)
^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 24*B*(b*c - a*d)^4*n*Log[c + d*
x] - 24*(b*c - a*d)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] +
 4*B*(b*c - a*d)^2*n*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)
^2*Log[c + d*x]) + B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) +
 a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + 12*
B*(b*c - a*d)^3*n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 12*B*(b*c - a*d)^
4*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*
PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^4 - (B*(b*c - a*d)*n*(120*A*b*d*
(b*c - a*d)^4*x + 120*B*d*(b*c - a*d)^4*(a + b*x)*Log[e*((a + b*x)/(c + d*x
))^n] + 60*d^2*(-(b*c) + a*d)^3*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]) + 40*d^3*(b*c - a*d)^2*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]) + 30*d^4*(-(b*c) + a*d)*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]) + 24*d^5*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 120*B*
(b*c - a*d)^5*n*Log[c + d*x] - 120*(b*c - a*d)^5*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])*Log[c + d*x] + 20*B*(b*c - a*d)^3*n*(2*b*d*(b*c - a*d)*x - d^2
*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 5*B*(b*c - a*d)^2*n*(6*b*d*(
b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(
b*c - a*d)^3*Log[c + d*x]) + 2*B*(b*c - a*d)*n*(12*b*d*(b*c - a*d)^3*x - 6*
d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c - a*d)*(a + b*x)^3 - 3*d^4*(a +
b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]) + 60*B*(b*c - a*d)^4*n*(b*d*x + (-(
b*c) + a*d)*Log[c + d*x]) + 60*B*(b*c - a*d)^5*n*((2*Log[(d*(a + b*x))/(-(b
*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c
- a*d)])))/(6*d^4)))/(60*b^3)
```

Maple [F]

$$\int (bgx+ag)^3 (dix+ci)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$=\int (bgx+ag)^3(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
 algorithm="fricas")
```

```
[Out] integral(A^2*b^3*d^2*g^3*i^2*x^5 + A^2*a^3*c^2*g^3*i^2 + (2*A^2*b^3*c*d + 3
*A^2*a*b^2*d^2)*g^3*i^2*x^4 + (A^2*b^3*c^2 + 6*A^2*a*b^2*c*d + 3*A^2*a^2*b*
d^2)*g^3*i^2*x^3 + (3*A^2*a*b^2*c^2 + 6*A^2*a^2*b*c*d + A^2*a^3*d^2)*g^3*i^
2*x^2 + (3*A^2*a^2*b*c^2 + 2*A^2*a^3*c*d)*g^3*i^2*x + (B^2*b^3*d^2*g^3*i^2*
x^5 + B^2*a^3*c^2*g^3*i^2 + (2*B^2*b^3*c*d + 3*B^2*a*b^2*d^2)*g^3*i^2*x^4 +
 (B^2*b^3*c^2 + 6*B^2*a*b^2*c*d + 3*B^2*a^2*b*d^2)*g^3*i^2*x^3 + (3*B^2*a*b
^2*c^2 + 6*B^2*a^2*b*c*d + B^2*a^3*d^2)*g^3*i^2*x^2 + (3*B^2*a^2*b*c^2 + 2*
B^2*a^3*c*d)*g^3*i^2*x)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^3*d^2*g
^3*i^2*x^5 + A*B*a^3*c^2*g^3*i^2 + (2*A*B*b^3*c*d + 3*A*B*a*b^2*d^2)*g^3*i^
2*x^4 + (A*B*b^3*c^2 + 6*A*B*a*b^2*c*d + 3*A*B*a^2*b*d^2)*g^3*i^2*x^3 + (3*
A*B*a*b^2*c^2 + 6*A*B*a^2*b*c*d + A*B*a^3*d^2)*g^3*i^2*x^2 + (3*A*B*a^2*b*c
^2 + 2*A*B*a^3*c*d)*g^3*i^2*x)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**3*(d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2
,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5952 vs. $2(735) = 1470$.

Time = 0.82 (sec) , antiderivative size = 5952, normalized size of antiderivative = 7.77

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
 algorithm="maxima")

[Out] 1/3*A*B*b^3*d^2*g^3*i^2*x^6*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/6*A^
2*b^3*d^2*g^3*i^2*x^6 + 4/5*A*B*b^3*c*d*g^3*i^2*x^5*log(e*(b*x/(d*x + c) +
a/(d*x + c))^n) + 6/5*A*B*a*b^2*d^2*g^3*i^2*x^5*log(e*(b*x/(d*x + c) + a/(d
*x + c))^n) + 2/5*A^2*b^3*c*d*g^3*i^2*x^5 + 3/5*A^2*a*b^2*d^2*g^3*i^2*x^5 +
 1/2*A*B*b^3*c^2*g^3*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3*A*B
*a*b^2*c*d*g^3*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*B*a^2
*b*d^2*g^3*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*b^3*c^2
*g^3*i^2*x^4 + 3/2*A^2*a*b^2*c*d*g^3*i^2*x^4 + 3/4*A^2*a^2*b*d^2*g^3*i^2*x^
4 + 2*A*B*a*b^2*c^2*g^3*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 4*
A*B*a^2*b*c*d*g^3*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2/3*A*B*
a^3*d^2*g^3*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*b^2*c^2*
g^3*i^2*x^3 + 2*A^2*a^2*b*c*d*g^3*i^2*x^3 + 1/3*A^2*a^3*d^2*g^3*i^2*x^3 + 3
*A*B*a^2*b*c^2*g^3*i^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*B*a
^3*c*d*g^3*i^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A^2*a^2*b*c
^2*g^3*i^2*x^2 + A^2*a^3*c*d*g^3*i^2*x^2 - 1/180*A*B*b^3*d^2*g^3*i^2*n*(60*
a^6*log(b*x + a)/b^6 - 60*c^6*log(d*x + c)/d^6 + (12*(b^5*c*d^4 - a*b^4*d^5
)*x^5 - 15*(b^5*c^2*d^3 - a^2*b^3*d^5)*x^4 + 20*(b^5*c^3*d^2 - a^3*b^2*d^5)
*x^3 - 30*(b^5*c^4*d - a^4*b*d^5)*x^2 + 60*(b^5*c^5 - a^5*d^5)*x)/(b^5*d^5)
) + 1/15*A*B*b^3*c*d*g^3*i^2*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x +
c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3
 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) + 1
/10*A*B*a*b^2*d^2*g^3*i^2*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/
d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 +
6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) - 1/12
*A*B*b^3*c^2*g^3*i^2*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (
2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3
- a^3*d^3)*x)/(b^3*d^3)) - 1/2*A*B*a*b^2*c*d*g^3*i^2*n*(6*a^4*log(b*x + a)/
b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*
d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 1/4*A*B*a^2*b*d^
2*g^3*i^2*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^
2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*
x)/(b^3*d^3)) + A*B*a*b^2*c^2*g^3*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log
(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^
2)) + 2*A*B*a^2*b*c*d*g^3*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c
)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + 1/
3*A*B*a^3*d^2*g^3*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 -
((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3*A*B*a^2*
b*c^2*g^3*i^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*
x/(b*d)) - 2*A*B*a^3*c*d*g^3*i^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)
/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*a^3*c^2*g^3*i^2*n*(a*log(b*x + a)/b - c
*log(d*x + c)/d) + 2*A*B*a^3*c^2*g^3*i^2*x*log(e*(b*x/(d*x + c) + a/(d*x +
c))^n) + A^2*a^3*c^2*g^3*i^2*x - 1/180*(33*a^4*b*c^2*d^4*g^3*i^2*n^2 - 6*a^
```

5*c*d^5*g^3*i^2*n^2 - 2*(g^3*i^2*n^2 + 3*g^3*i^2*n*log(e))*b^5*c^6 + 6*(g^3
*i^2*n^2 + 6*g^3*i^2*n*log(e))*a*b^4*c^5*d + 3*(g^3*i^2*n^2 - 30*g^3*i^2*n*
log(e))*a^2*b^3*c^4*d^2 - 2*(17*g^3*i^2*n^2 - 60*g^3*i^2*n*log(e))*a^3*b^2*
c^3*d^3)*B^2*log(d*x + c)/(b^2*d^4) + 1/30*(b^6*c^6*g^3*i^2*n^2 - 6*a*b^5*c
^5*d*g^3*i^2*n^2 + 15*a^2*b^4*c^4*d^2*g^3*i^2*n^2 - 20*a^3*b^3*c^3*d^3*g^3*
i^2*n^2 + 15*a^4*b^2*c^2*d^4*g^3*i^2*n^2 - 6*a^5*b*c*d^5*g^3*i^2*n^2 + a^6*
d^6*g^3*i^2*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-
(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d^4) + 1/360*(60*B^2*b^6*d^6*g^3*i^2*x
^6*log(e)^2 - 24*((g^3*i^2*n*log(e) - 6*g^3*i^2*log(e)^2)*b^6*c*d^5 - (g^3*
i^2*n*log(e) + 9*g^3*i^2*log(e)^2)*a*b^5*d^6)*B^2*x^5 + 6*((g^3*i^2*n^2 - 7
*g^3*i^2*n*log(e) + 15*g^3*i^2*log(e)^2)*b^6*c^2*d^4 - 2*(g^3*i^2*n^2 + 3*g
^3*i^2*n*log(e) - 45*g^3*i^2*log(e)^2)*a*b^5*c*d^5 + (g^3*i^2*n^2 + 13*g^3*
i^2*n*log(e) + 45*g^3*i^2*log(e)^2)*a^2*b^4*d^6)*B^2*x^4 + 2*((3*g^3*i^2*n^
2 - 2*g^3*i^2*n*log(e))*b^6*c^3*d^3 + 3*(g^3*i^2*n^2 - 26*g^3*i^2*n*log(e)
+ 60*g^3*i^2*log(e)^2)*a*b^5*c^2*d^4 - 3*(5*g^3*i^2*n^2 - 14*g^3*i^2*n*log(
e) - 120*g^3*i^2*log(e)^2)*a^2*b^4*c*d^5 + (9*g^3*i^2*n^2 + 38*g^3*i^2*n*lo
g(e) + 60*g^3*i^2*log(e)^2)*a^3*b^3*d^6)*B^2*x^3 - ((7*g^3*i^2*n^2 - 6*g^3*
i^2*n*log(e))*b^6*c^4*d^2 - 2*(23*g^3*i^2*n^2 - 18*g^3*i^2*n*log(e))*a*b^5*
c^3*d^3 + 60*(g^3*i^2*n^2 + 3*g^3*i^2*n*log(e) - 9*g^3*i^2*log(e)^2)*a^2*b^
4*c^2*d^4 - 2*(5*g^3*i^2*n^2 + 102*g^3*i^2*n*log(e) + 180*g^3*i^2*log(e)^2)
*a^3*b^3*c*d^5 - (11*g^3*i^2*n^2 + 6*g^3*i^2*n*log(e))*a^4*b^2*d^6)*B^2*x^2
 - 6*(15*a^4*b^2*c^2*d^4*g^3*i^2*n^2 - 6*a^5*b*c*d^5*g^3*i^2*n^2 + a^6*d^6*
g^3*i^2*n^2)*B^2*log(b*x + a)^2 - 12*(b^6*c^6*g^3*i^2*n^2 - 6*a*b^5*c^5*d*g
^3*i^2*n^2 + 15*a^2*b^4*c^4*d^2*g^3*i^2*n^2 - 20*a^3*b^3*c^3*d^3*g^3*i^2*n^
2)*B^2*log(b*x + a)*log(d*x + c) + 6*(b^6*c^6*g^3*i^2*n^2 - 6*a*b^5*c^5*d*g
^3*i^2*n^2 + 15*a^2*b^4*c^4*d^2*g^3*i^2*n^2 - 20*a^3*b^3*c^3*d^3*g^3*i^2*n^
2)*B^2*log(d*x + c)^2 + 2*(2*(2*g^3*i^2*n^2 - 3*g^3*i^2*n*log(e))*b^6*c^5*d
 - 9*(3*g^3*i^2*n^2 - 4*g^3*i^2*n*log(e))*a*b^5*c^4*d^2 + (77*g^3*i^2*n^2 -
 90*g^3*i^2*n*log(e))*a^2*b^4*c^3*d^3 - (97*g^3*i^2*n^2 - 30*g^3*i^2*n*log(
e) - 180*g^3*i^2*log(e)^2)*a^3*b^3*c^2*d^4 + 3*(17*g^3*i^2*n^2 + 12*g^3*i^2
*n*log(e))*a^4*b^2*c*d^5 - 2*(4*g^3*i^2*n^2 + 3*g^3*i^2*n*log(e))*a^5*b*d^6
)*B^2*x - 2*(6*a*b^5*c^5*d*g^3*i^2*n^2 - 33*a^2*b^4*c^4*d^2*g^3*i^2*n^2 + 7
4*a^3*b^3*c^3*d^3*g^3*i^2*n^2 - 9*(7*g^3*i^2*n^2 + 10*g^3*i^2*n*log(e))*a^4
*b^2*c^2*d^4 + 18*(g^3*i^2*n^2 + 2*g^3*i^2*n*log(e))*a^5*b*c*d^5 - 2*(g^3*i
^2*n^2 + 3*g^3*i^2*n*log(e))*a^6*d^6)*B^2*log(b*x + a) + 6*(10*B^2*b^6*d^6*
g^3*i^2*x^6 + 60*B^2*a^3*b^3*c^2*d^4*g^3*i^2*x + 12*(2*b^6*c*d^5*g^3*i^2 +
3*a*b^5*d^6*g^3*i^2)*B^2*x^5 + 15*(b^6*c^2*d^4*g^3*i^2 + 6*a*b^5*c*d^5*g^3*
i^2 + 3*a^2*b^4*d^6*g^3*i^2)*B^2*x^4 + 20*(3*a*b^5*c^2*d^4*g^3*i^2 + 6*a^2*
b^4*c*d^5*g^3*i^2 + a^3*b^3*d^6*g^3*i^2)*B^2*x^3 + 30*(3*a^2*b^4*c^2*d^4*g^
3*i^2 + 2*a^3*b^3*c*d^5*g^3*i^2)*B^2*x^2)*log((b*x + a)^n)^2 + 6*(10*B^2*b^
6*d^6*g^3*i^2*x^6 + 60*B^2*a^3*b^3*c^2*d^4*g^3*i^2*x + 12*(2*b^6*c*d^5*g^3*
i^2 + 3*a*b^5*d^6*g^3*i^2)*B^2*x^5 + 15*(b^6*c^2*d^4*g^3*i^2 + 6*a*b^5*c*d^
5*g^3*i^2 + 3*a^2*b^4*d^6*g^3*i^2)*B^2*x^4 + 20*(3*a*b^5*c^2*d^4*g^3*i^2 +
6*a^2*b^4*c*d^5*g^3*i^2 + a^3*b^3*d^6*g^3*i^2)*B^2*x^3 + 30*(3*a^2*b^4*c^2*
d^4*g^3*i^2 + 2*a^3*b^3*c*d^5*g^3*i^2)*B^2*x^2)*log((d*x + c)^n)^2 + 2*(60*

```
B^2*b^6*d^6*g^3*i^2*x^6*log(e) - 12*((g^3*i^2*n - 12*g^3*i^2*log(e))*b^6*c*
d^5 - (g^3*i^2*n + 18*g^3*i^2*log(e))*a*b^5*d^6)*B^2*x^5 - 3*((7*g^3*i^2*n
- 30*g^3*i^2*log(e))*b^6*c^2*d^4 + 6*(g^3*i^2*n - 30*g^3*i^2*log(e))*a*b^5*
c*d^5 - (13*g^3*i^2*n + 90*g^3*i^2*log(e))*a^2*b^4*d^6)*B^2*x^4 - 2*(b^6*c^
3*d^3*g^3*i^2*n + 3*(13*g^3*i^2*n - 60*g^3*i^2*log(e))*a*b^5*c^2*d^4 - 3*(7
*g^3*i^2*n + 120*g^3*i^2*log(e))*a^2*b^4*c*d^5 - (19*g^3*i^2*n + 60*g^3*i^2
*log(e))*a^3*b^3*d^6)*B^2*x^3 + 3*(b^6*c^4*d^2*g^3*i^2*n - 6*a*b^5*c^3*d^3*
g^3*i^2*n + a^4*b^2*d^6*g^3*i^2*n - 30*(g^3*i^2*n - 6*g^3*i^2*log(e))*a^2*b
^4*c^2*d^4 + 2*(17*g^3*i^2*n + 60*g^3*i^2*log(e))*a^3*b^3*c*d^5)*B^2*x^2 -
6*(b^6*c^5*d*g^3*i^2*n - 6*a*b^5*c^4*d^2*g^3*i^2*n + 15*a^2*b^4*c^3*d^3*g^3
*i^2*n - 6*a^4*b^2*c*d^5*g^3*i^2*n + a^5*b*d^6*g^3*i^2*n - 5*(g^3*i^2*n + 1
2*g^3*i^2*log(e))*a^3*b^3*c^2*d^4)*B^2*x + 6*(15*a^4*b^2*c^2*d^4*g^3*i^2*n
- 6*a^5*b*c*d^5*g^3*i^2*n + a^6*d^6*g^3*i^2*n)*B^2*log(b*x + a) + 6*(b^6*c^
6*g^3*i^2*n - 6*a*b^5*c^5*d*g^3*i^2*n + 15*a^2*b^4*c^4*d^2*g^3*i^2*n - 20*a
^3*b^3*c^3*d^3*g^3*i^2*n)*B^2*log(d*x + c))*log((b*x + a)^n) - 2*(60*B^2*b^
6*d^6*g^3*i^2*x^6*log(e) - 12*((g^3*i^2*n - 12*g^3*i^2*log(e))*b^6*c*d^5 -
(g^3*i^2*n + 18*g^3*i^2*log(e))*a*b^5*d^6)*B^2*x^5 - 3*((7*g^3*i^2*n - 30*g
^3*i^2*log(e))*b^6*c^2*d^4 + 6*(g^3*i^2*n - 30*g^3*i^2*log(e))*a*b^5*c*d^5
- (13*g^3*i^2*n + 90*g^3*i^2*log(e))*a^2*b^4*d^6)*B^2*x^4 - 2*(b^6*c^3*d^3*
g^3*i^2*n + 3*(13*g^3*i^2*n - 60*g^3*i^2*log(e))*a*b^5*c^2*d^4 - 3*(7*g^3*i
^2*n + 120*g^3*i^2*log(e))*a^2*b^4*c*d^5 - (19*g^3*i^2*n + 60*g^3*i^2*log(e
))*a^3*b^3*d^6)*B^2*x^3 + 3*(b^6*c^4*d^2*g^3*i^2*n - 6*a*b^5*c^3*d^3*g^3*i^
2*n + a^4*b^2*d^6*g^3*i^2*n - 30*(g^3*i^2*n - 6*g^3*i^2*log(e))*a^2*b^4*c^2
*d^4 + 2*(17*g^3*i^2*n + 60*g^3*i^2*log(e))*a^3*b^3*c*d^5)*B^2*x^2 - 6*(b^6
*c^5*d*g^3*i^2*n - 6*a*b^5*c^4*d^2*g^3*i^2*n + 15*a^2*b^4*c^3*d^3*g^3*i^2*n
 - 6*a^4*b^2*c*d^5*g^3*i^2*n + a^5*b*d^6*g^3*i^2*n - 5*(g^3*i^2*n + 12*g^3*
i^2*log(e))*a^3*b^3*c^2*d^4)*B^2*x + 6*(15*a^4*b^2*c^2*d^4*g^3*i^2*n - 6*a^
5*b*c*d^5*g^3*i^2*n + a^6*d^6*g^3*i^2*n)*B^2*log(b*x + a) + 6*(b^6*c^6*g^3*
i^2*n - 6*a*b^5*c^5*d*g^3*i^2*n + 15*a^2*b^4*c^4*d^2*g^3*i^2*n - 20*a^3*b^3
*c^3*d^3*g^3*i^2*n)*B^2*log(d*x + c) + 6*(10*B^2*b^6*d^6*g^3*i^2*x^6 + 60*B
^2*a^3*b^3*c^2*d^4*g^3*i^2*x + 12*(2*b^6*c*d^5*g^3*i^2 + 3*a*b^5*d^6*g^3*i^
2)*B^2*x^5 + 15*(b^6*c^2*d^4*g^3*i^2 + 6*a*b^5*c*d^5*g^3*i^2 + 3*a^2*b^4*d^
6*g^3*i^2)*B^2*x^4 + 20*(3*a*b^5*c^2*d^4*g^3*i^2 + 6*a^2*b^4*c*d^5*g^3*i^2
+ a^3*b^3*d^6*g^3*i^2)*B^2*x^3 + 30*(3*a^2*b^4*c^2*d^4*g^3*i^2 + 2*a^3*b^3*
c*d^5*g^3*i^2)*B^2*x^2)*log((b*x + a)^n))*log((d*x + c)^n))/(b^3*d^4)
```

Giac [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
 algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$=\int (a\,g+b\,g\,x)^3\,(c\,i+d\,i\,x)^2\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2 dx$$

```
[In] int((a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^
2,x)
[Out] int((a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^
2, x)
```

3.169 $\int (ag+bgx)^2(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 45, antiderivative size = 819

$$\begin{aligned}
&\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx\\
&=-\frac{B^2(bc-ad)^4g^2i^2n^2x}{10b^2d^2}-\frac{B^2(bc-ad)^3g^2i^2n^2(c+dx)^2}{20bd^3}\\
&+\frac{B^2(bc-ad)^2g^2i^2n^2(c+dx)^3}{30d^3}-\frac{B(bc-ad)^3g^2i^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^3d}\\
&-\frac{B(bc-ad)^2g^2i^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^3}\\
&-\frac{B(bc-ad)^3g^2i^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5bd^3}\\
&+\frac{4B(bc-ad)^2g^2i^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15d^3}\\
&-\frac{bB(bc-ad)g^2i^2n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10d^3}\\
&+\frac{(bc-ad)^2g^2i^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{30b^3}\\
&+\frac{(bc-ad)g^2i^2(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^2}\\
&+\frac{g^2i^2(a+bx)^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b}\\
&+\frac{B(bc-ad)^4g^2i^2n(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^3d^2}\\
&+\frac{B(bc-ad)^5g^2i^2n\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{30b^3d^3}\\
&+\frac{B^2(bc-ad)^5g^2i^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{30b^3d^3}+\frac{B^2(bc-ad)^5g^2i^2n^2\log(c+dx)}{10b^3d^3}\\
&+\frac{B^2(bc-ad)^5g^2i^2n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{15b^3d^3}
\end{aligned}$$

```
[Out] -1/10*B^2*(-a*d+b*c)^4*g^2*i^2*n^2*x/b^2/d^2-1/20*B^2*(-a*d+b*c)^3*g^2*i^2*
n^2*(d*x+c)^2/b/d^3+1/30*B^2*(-a*d+b*c)^2*g^2*i^2*n^2*(d*x+c)^3/d^3-1/30*B*
(-a*d+b*c)^3*g^2*i^2*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/d-1/15
*B*(-a*d+b*c)^2*g^2*i^2*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3-1/5
*B*(-a*d+b*c)^3*g^2*i^2*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^3+4
/15*B*(-a*d+b*c)^2*g^2*i^2*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^3-
1/10*b*B*(-a*d+b*c)*g^2*i^2*n*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^3
+1/30*(-a*d+b*c)^2*g^2*i^2*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^3+
1/10*(-a*d+b*c)*g^2*i^2*(b*x+a)^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2
```

```
/b^2+1/5*g^2*i^2*(b*x+a)^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/
30*B*(-a*d+b*c)^4*g^2*i^2*n*(b*x+a)*(2*A+B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))
/b^3/d^2+1/30*B*(-a*d+b*c)^5*g^2*i^2*n*(2*A+3*B*n+2*B*ln(e*((b*x+a)/(d*x+c)
)^n))*ln((-a*d+b*c)/b/(d*x+c))/b^3/d^3+1/30*B^2*(-a*d+b*c)^5*g^2*i^2*n^2*ln
((b*x+a)/(d*x+c))/b^3/d^3+1/10*B^2*(-a*d+b*c)^5*g^2*i^2*n^2*ln(d*x+c)/b^3/d
^3+1/15*B^2*(-a*d+b*c)^5*g^2*i^2*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^3
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 819, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.244$, Rules used = {2561, 2383, 2381, 2384, 2354, 2438, 2373, 45, 2382, 12, 907}

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx$$

$$= \frac{Bg^2i^2n\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)(bc-ad)^5}{30b^3d^3} + \frac{B^2g^2i^2n^2\log\left(\frac{a+bx}{c+dx}\right)(bc-ad)^5}{30b^3d^3} + \frac{B^2g^2i^2n^2\log(c+dx)(bc-ad)^5}{10b^3d^3} + \frac{B^2g^2i^2n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)(bc-ad)^5}{15b^3d^3} - \frac{B^2g^2i^2n^2x(bc-ad)^4}{10b^2d^2} + \frac{Bg^2i^2n(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)^4}{30b^3d^2} - \frac{B^2g^2i^2n^2(c+dx)^2(bc-ad)^3}{20bd^3} - \frac{Bg^2i^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)^3}{30b^3d} - \frac{Bg^2i^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)^3}{5bd^3} + \frac{B^2g^2i^2n^2(c+dx)^3(bc-ad)^2}{30d^3} + \frac{g^2i^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(bc-ad)^2}{30b^3} - \frac{Bg^2i^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)^2}{15b^3} + \frac{4Bg^2i^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)^2}{15d^3} + \frac{g^2i^2(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(bc-ad)}{10b^2} - \frac{bBg^2i^2n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)}{10d^3} + \frac{g^2i^2(a+bx)^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b}$$

```
[In] Int[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2,x]
```

```
[Out] -1/10*(B^2*(b*c - a*d)^4*g^2*i^2*n^2*x)/(b^2*d^2) - (B^2*(b*c - a*d)^3*g^2*
i^2*n^2*(c + d*x)^2)/(20*b*d^3) + (B^2*(b*c - a*d)^2*g^2*i^2*n^2*(c + d*x)^
3)/(30*d^3) - (B*(b*c - a*d)^3*g^2*i^2*n*(a + b*x)^2*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]))/(30*b^3*d) - (B*(b*c - a*d)^2*g^2*i^2*n*(a + b*x)^3*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]))/(15*b^3) - (B*(b*c - a*d)^3*g^2*i^2*n*(c
 + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*b*d^3) + (4*B*(b*c - a
*d)^2*g^2*i^2*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(15*d^3
) - (b*B*(b*c - a*d)*g^2*i^2*n*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]))/(10*d^3) + ((b*c - a*d)^2*g^2*i^2*(a + b*x)^3*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])^2)/(30*b^3) + ((b*c - a*d)*g^2*i^2*(a + b*x)^3*(c + d*x)*
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(10*b^2) + (g^2*i^2*(a + b*x)^3*(
c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(5*b) + (B*(b*c - a*d)
^4*g^2*i^2*n*(a + b*x)*(2*A + B*n + 2*B*Log[e*((a + b*x)/(c + d*x))^n]))/(3
0*b^3*d^2) + (B*(b*c - a*d)^5*g^2*i^2*n*(2*A + 3*B*n + 2*B*Log[e*((a + b*x)
/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(30*b^3*d^3) + (B^2*(b*c -
a*d)^5*g^2*i^2*n^2*Log[(a + b*x)/(c + d*x)])/(30*b^3*d^3) + (B^2*(b*c - a*d
)^5*g^2*i^2*n^2*Log[c + d*x])/(10*b^3*d^3) + (B^2*(b*c - a*d)^5*g^2*i^2*n^2
*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(15*b^3*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2383

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Dist[(m + q + 2)/(d*(q + 1)), Int[(
f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Dist[b*n*(p/(d*(q +
 1))), Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x]) /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ
[q, -1] && GtQ[m, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \left((bc-ad)^5g^2i^2\right)\text{Subst}\left(\int\frac{x^2(A+B\log(ex^n))^2}{(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{g^2i^2(a+bx)^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} + \frac{\left(2(bc-ad)^5g^2i^2\right)\text{Subst}\left(\int\frac{x^2(A+B\log(ex^n))^2}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b} - \frac{\left(2B(bc-ad)^5g^2i^2n\right)\text{Subst}\left(\int\frac{x^2(A+B\log(ex^n))}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b}$$

$$= -\frac{B(bc-ad)^3g^2i^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5bd^3} + \frac{4B(bc-ad)^2g^2i^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15d^3} - \frac{bB(bc-ad)g^2i^2n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10d^3} + \frac{(bc-ad)g^2i^2(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^2} + \frac{g^2i^2(a+bx)^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} + \frac{\left((bc-ad)^5g^2i^2\right)\text{Subst}\left(\int\frac{x^2(A+B\log(ex^n))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{10b^2} - \frac{\left(B(bc-ad)^5g^2i^2n\right)\text{Subst}\left(\int\frac{x^2(A+B\log(ex^n))}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b^2} + \frac{\left(2B^2(bc-ad)^5g^2i^2n^2\right)\text{Subst}\left(\int\frac{b^2-4bdx+6d^2x^2}{12d^3x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b}$$

$$= -\frac{B(bc-ad)^2g^2i^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^3} - \frac{B(bc-ad)^3g^2i^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5bd^3} + \frac{4B(bc-ad)^2g^2i^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15d^3} - \frac{bB(bc-ad)g^2i^2n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10d^3} + \frac{(bc-ad)^2g^2i^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{30b^3} + \frac{(bc-ad)g^2i^2(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^2} + \frac{g^2i^2(a+bx)^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} - \frac{\left(B(bc-ad)^5g^2i^2n\right)\text{Subst}\left(\int\frac{x^2(A+B\log(ex^n))}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{15b^3} + \frac{\left(B^2(bc-ad)^5g^2i^2n^2\right)\text{Subst}\left(\int\frac{x^2}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{15b^3} + \frac{\left(B^2(bc-ad)^5g^2i^2n^2\right)\text{Subst}\left(\int\frac{b^2-4bdx+6d^2x^2}{x(b-dx)^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{30bd^3}$$

$$
\begin{aligned}
&= -\frac{B(bc-ad)^3 g^2 i^2 n (a+bx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^3 d} \\
&\quad -\frac{B(bc-ad)^2 g^2 i^2 n (a+bx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^3} \\
&\quad -\frac{B(bc-ad)^3 g^2 i^2 n (c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5bd^3} \\
&\quad +\frac{4B(bc-ad)^2 g^2 i^2 n (c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15d^3} \\
&\quad -\frac{bB(bc-ad) g^2 i^2 n (c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10d^3} \\
&\quad +\frac{(bc-ad)^2 g^2 i^2 (a+bx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{30b^3} \\
&\quad +\frac{(bc-ad) g^2 i^2 (a+bx)^3 (c+dx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^2} \\
&\quad +\frac{g^2 i^2 (a+bx)^3 (c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} \\
&\quad +\frac{\left(B(bc-ad)^5 g^2 i^2 n\right) \operatorname{Subst}\left(\int \frac{x(2A+Bn+2B\log(ex^n))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{30b^3 d} \\
&\quad +\frac{\left(B^2(bc-ad)^5 g^2 i^2 n^2\right) \operatorname{Subst}\left(\int \left(\frac{b^2}{d^2(b-dx)^3}-\frac{2b}{d^2(b-dx)^2}+\frac{1}{d^2(b-dx)}\right) dx, x, \frac{a+bx}{c+dx}\right)}{15b^3} \\
&\quad +\frac{\left(B^2(bc-ad)^5 g^2 i^2 n^2\right) \operatorname{Subst}\left(\int \left(\frac{1}{b^2 x}+\frac{3bd}{(b-dx)^4}-\frac{5d}{(b-dx)^3}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right) dx, x, \frac{a+bx}{c+dx}\right)}{30bd^3}
\end{aligned}
$$

$$= -\frac{B^2(bc-ad)^4g^2i^2n^2x}{10b^2d^2} - \frac{B^2(bc-ad)^3g^2i^2n^2(c+dx)^2}{20bd^3}$$
$$+ \frac{B^2(bc-ad)^2g^2i^2n^2(c+dx)^3}{30d^3}$$
$$- \frac{B(bc-ad)^3g^2i^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^3d}$$
$$- \frac{B(bc-ad)^2g^2i^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^3}$$
$$- \frac{B(bc-ad)^3g^2i^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5bd^3}$$
$$+ \frac{4B(bc-ad)^2g^2i^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15d^3}$$
$$- \frac{bB(bc-ad)g^2i^2n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10d^3}$$
$$+ \frac{(bc-ad)^2g^2i^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{30b^3}$$
$$+ \frac{(bc-ad)g^2i^2(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^2}$$
$$+ \frac{g^2i^2(a+bx)^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b}$$
$$+ \frac{B(bc-ad)^4g^2i^2n(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^3d^2}$$
$$+ \frac{B^2(bc-ad)^5g^2i^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{30b^3d^3} + \frac{B^2(bc-ad)^5g^2i^2n^2\log(c+dx)}{10b^3d^3}$$
$$- \frac{(B(bc-ad)^5g^2i^2n)\,\text{Subst}\left(\int\frac{2A+3Bn+2B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{30b^3d^2}$$

$$
\begin{aligned}
&= -\frac{B^2(bc-ad)^4g^2i^2n^2x}{10b^2d^2} - \frac{B^2(bc-ad)^3g^2i^2n^2(c+dx)^2}{20bd^3} \\
&+ \frac{B^2(bc-ad)^2g^2i^2n^2(c+dx)^3}{30d^3} \\
&- \frac{B(bc-ad)^3g^2i^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^3d} \\
&- \frac{B(bc-ad)^2g^2i^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^3} \\
&- \frac{B(bc-ad)^3g^2i^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5bd^3} \\
&+ \frac{4B(bc-ad)^2g^2i^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15d^3} \\
&- \frac{bB(bc-ad)g^2i^2n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10d^3} \\
&+ \frac{(bc-ad)^2g^2i^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{30b^3} \\
&+ \frac{(bc-ad)g^2i^2(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^2} \\
&+ \frac{g^2i^2(a+bx)^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} \\
&+ \frac{B(bc-ad)^4g^2i^2n(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^3d^2} \\
&+ \frac{B(bc-ad)^5g^2i^2n\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{30b^3d^3} \\
&+ \frac{B^2(bc-ad)^5g^2i^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{30b^3d^3} + \frac{B^2(bc-ad)^5g^2i^2n^2\log(c+dx)}{10b^3d^3} \\
&- \frac{(B^2(bc-ad)^5g^2i^2n^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{15b^3d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{B^2(bc-ad)^4g^2i^2n^2x}{10b^2d^2} - \frac{B^2(bc-ad)^3g^2i^2n^2(c+dx)^2}{20bd^3} \\
&+ \frac{B^2(bc-ad)^2g^2i^2n^2(c+dx)^3}{30d^3} \\
&- \frac{B(bc-ad)^3g^2i^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^3d} \\
&- \frac{B(bc-ad)^2g^2i^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^3} \\
&- \frac{B(bc-ad)^3g^2i^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5bd^3} \\
&+ \frac{4B(bc-ad)^2g^2i^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15d^3} \\
&- \frac{bB(bc-ad)g^2i^2n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10d^3} \\
&+ \frac{(bc-ad)^2g^2i^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{30b^3} \\
&+ \frac{(bc-ad)g^2i^2(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^2} \\
&+ \frac{g^2i^2(a+bx)^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} \\
&+ \frac{B(bc-ad)^4g^2i^2n(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^3d^2} \\
&+ \frac{B(bc-ad)^5g^2i^2n\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{30b^3d^3} \\
&+ \frac{B^2(bc-ad)^5g^2i^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{30b^3d^3} + \frac{B^2(bc-ad)^5g^2i^2n^2\log(c+dx)}{10b^3d^3} \\
&+ \frac{B^2(bc-ad)^5g^2i^2n^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{15b^3d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 1254, normalized size of antiderivative = 1.53

$$
\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx
$$

$$
= \frac{g^2i^2\left(20d^3(bc-ad)^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+30d^4(bc-ad)(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \cdots\right.}{}
$$

```
[In] Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^2,x]
```

```
[Out] (g^2*i^2*(20*d^3*(b*c - a*d)^2*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^2 + 30*d^4*(b*c - a*d)*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^2 + 12*d^5*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 20*
B*(b*c - a*d)^3*n*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[
e*((a + b*x)/(c + d*x))^n] - d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]) - 2*B*(b*c - a*d)^2*n*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[e*
((a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(b*d*x + (-(b*c) +
 a*d)*Log[c + d*x]) + B*(b*c - a*d)^2*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d
)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))
 - 10*B*(b*c - a*d)^2*n*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a +
 b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*Log[e*((a +
b*x)/(c + d*x))^n]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 6*(b*c - a*d)^3*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(2*b*d*
(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c
- a*d)^2*n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*n*((2*
Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[
2, (b*(c + d*x))/(b*c - a*d)])) + B*(b*c - a*d)*n*(24*A*b*d*(b*c - a*d)^3*x
 + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 12*d^2*(
b*c - a*d)^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 8*d^3*(b*
c - a*d)*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*d^4*(a + b*
x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 24*B*(b*c - a*d)^4*n*Log[c +
d*x] - 24*(b*c - a*d)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x]
 + 4*B*(b*c - a*d)^2*n*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*
d)^2*Log[c + d*x]) + B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c)
 + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + 1
2*B*(b*c - a*d)^3*n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 12*B*(b*c - a*d
)^4*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] +
2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))))/(60*b^3*d^3)
```

Maple [F]

$$\int \left(bgx+ag\right)^{2} \left(dix+ci\right)^{2} {\left(A+B\ln \left(e{\left(\frac{bx+a}{dx+c}\right)}^{n}\right)\right)}^{2}dx$$

```
[In] int((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (bgx+ag)^2(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
 algorithm="fricas")
```

```
[Out] integral(A^2*b^2*d^2*g^2*i^2*x^4 + A^2*a^2*c^2*g^2*i^2 + 2*(A^2*b^2*c*d + A
^2*a*b*d^2)*g^2*i^2*x^3 + (A^2*b^2*c^2 + 4*A^2*a*b*c*d + A^2*a^2*d^2)*g^2*i
^2*x^2 + 2*(A^2*a*b*c^2 + A^2*a^2*c*d)*g^2*i^2*x + (B^2*b^2*d^2*g^2*i^2*x^4
 + B^2*a^2*c^2*g^2*i^2 + 2*(B^2*b^2*c*d + B^2*a*b*d^2)*g^2*i^2*x^3 + (B^2*b
^2*c^2 + 4*B^2*a*b*c*d + B^2*a^2*d^2)*g^2*i^2*x^2 + 2*(B^2*a*b*c^2 + B^2*a^
2*c*d)*g^2*i^2*x)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^2*d^2*g^2*i^2
*x^4 + A*B*a^2*c^2*g^2*i^2 + 2*(A*B*b^2*c*d + A*B*a*b*d^2)*g^2*i^2*x^3 + (A
*B*b^2*c^2 + 4*A*B*a*b*c*d + A*B*a^2*d^2)*g^2*i^2*x^2 + 2*(A*B*a*b*c^2 + A*
B*a^2*c*d)*g^2*i^2*x)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**2*(d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2
,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4247 vs. $2(786) = 1572$.

Time = 0.78 (sec) , antiderivative size = 4247, normalized size of antiderivative = 5.19

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
 algorithm="maxima")
```

```
[Out] 2/5*A*B*b^2*d^2*g^2*i^2*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A^
2*b^2*d^2*g^2*i^2*x^5 + A*B*b^2*c*d*g^2*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d
*x + c))^n) + A*B*a*b*d^2*g^2*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
) + 1/2*A^2*b^2*c*d*g^2*i^2*x^4 + 1/2*A^2*a*b*d^2*g^2*i^2*x^4 + 2/3*A*B*b^2
*c^2*g^2*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 8/3*A*B*a*b*c*d*g
^2*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2/3*A*B*a^2*d^2*g^2*i^2
*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*b^2*c^2*g^2*i^2*x^3 +
 4/3*A^2*a*b*c*d*g^2*i^2*x^3 + 1/3*A^2*a^2*d^2*g^2*i^2*x^3 + 2*A*B*a*b*c^2*
g^2*i^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*B*a^2*c*d*g^2*i^2*
x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*b*c^2*g^2*i^2*x^2 + A^2*
a^2*c*d*g^2*i^2*x^2 + 1/30*A*B*b^2*d^2*g^2*i^2*n*(12*a^5*log(b*x + a)/b^5 -
 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2
- a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)
*x)/(b^4*d^4)) - 1/6*A*B*b^2*c*d*g^2*i^2*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*
log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^
3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 1/6*A*B*a*b*d^2*g^2*i^2*n*(6
*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)
*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))
+ 1/3*A*B*b^2*c^2*g^2*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^
3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + 4/3*A*
B*a*b*c*d*g^2*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^
2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + 1/3*A*B*a^2*d^
2*g^2*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d -
a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*A*B*a*b*c^2*g^2*i^2*
n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) - 2*A
*B*a^2*c*d*g^2*i^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c -
a*d)*x/(b*d)) + 2*A*B*a^2*c^2*g^2*i^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/
d) + 2*A*B*a^2*c^2*g^2*i^2*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a
^2*c^2*g^2*i^2*x - 1/30*(9*a^3*b*c^2*d^3*g^2*i^2*n^2 - 2*a^4*c*d^4*g^2*i^2*
n^2 + 2*b^4*c^5*g^2*i^2*n*log(e) + 2*(g^2*i^2*n^2 - 5*g^2*i^2*n*log(e))*a*b
^3*c^4*d - (9*g^2*i^2*n^2 - 20*g^2*i^2*n*log(e))*a^2*b^2*c^3*d^2)*B^2*log(d
*x + c)/(b^2*d^3) - 1/15*(b^5*c^5*g^2*i^2*n^2 - 5*a*b^4*c^4*d*g^2*i^2*n^2 +
 10*a^2*b^3*c^3*d^2*g^2*i^2*n^2 - 10*a^3*b^2*c^2*d^3*g^2*i^2*n^2 + 5*a^4*b*
c*d^4*g^2*i^2*n^2 - a^5*d^5*g^2*i^2*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b
*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d^3) + 1/60*(1
2*B^2*b^5*d^5*g^2*i^2*x^5*log(e)^2 - 6*((g^2*i^2*n*log(e) - 5*g^2*i^2*log(e
)^2)*b^5*c*d^4 - (g^2*i^2*n*log(e) + 5*g^2*i^2*log(e)^2)*a*b^4*d^5)*B^2*x^4
 + 2*((g^2*i^2*n^2 - 6*g^2*i^2*n*log(e) + 10*g^2*i^2*log(e)^2)*b^5*c^2*d^3
- 2*(g^2*i^2*n^2 - 20*g^2*i^2*log(e)^2)*a*b^4*c*d^4 + (g^2*i^2*n^2 + 6*g^2*
i^2*n*log(e) + 10*g^2*i^2*log(e)^2)*a^2*b^3*d^5)*B^2*x^3 + ((3*g^2*i^2*n^2
- 2*g^2*i^2*n*log(e))*b^5*c^3*d^2 - 3*(g^2*i^2*n^2 + 10*g^2*i^2*n*log(e) -
20*g^2*i^2*log(e)^2)*a*b^4*c^2*d^3 - 3*(g^2*i^2*n^2 - 10*g^2*i^2*n*log(e) -
 20*g^2*i^2*log(e)^2)*a^2*b^3*c*d^4 + (3*g^2*i^2*n^2 + 2*g^2*i^2*n*log(e))*
a^3*b^2*d^5)*B^2*x^2 - 2*(10*a^3*b^2*c^2*d^3*g^2*i^2*n^2 - 5*a^4*b*c*d^4*g^
2*i^2*n^2 + a^5*d^5*g^2*i^2*n^2)*B^2*log(b*x + a)^2 + 4*(b^5*c^5*g^2*i^2*n^
```

```
2 - 5*a*b^4*c^4*d*g^2*i^2*n^2 + 10*a^2*b^3*c^3*d^2*g^2*i^2*n^2)*B^2*log(b*x
 + a)*log(d*x + c) - 2*(b^5*c^5*g^2*i^2*n^2 - 5*a*b^4*c^4*d*g^2*i^2*n^2 + 1
0*a^2*b^3*c^3*d^2*g^2*i^2*n^2)*B^2*log(d*x + c)^2 - 2*(2*(g^2*i^2*n^2 - g^2
*i^2*n*log(e))*b^5*c^4*d - (11*g^2*i^2*n^2 - 10*g^2*i^2*n*log(e))*a*b^4*c^3
*d^2 + 6*(3*g^2*i^2*n^2 - 5*g^2*i^2*log(e)^2)*a^2*b^3*c^2*d^3 - (11*g^2*i^2
*n^2 + 10*g^2*i^2*n*log(e))*a^3*b^2*c*d^4 + 2*(g^2*i^2*n^2 + g^2*i^2*n*log(
e))*a^4*b*d^5)*B^2*x + 2*(2*a*b^4*c^4*d*g^2*i^2*n^2 - 9*a^2*b^3*c^3*d^2*g^2
*i^2*n^2 + 2*a^5*d^5*g^2*i^2*n*log(e) + (9*g^2*i^2*n^2 + 20*g^2*i^2*n*log(e
))*a^3*b^2*c^2*d^3 - 2*(g^2*i^2*n^2 + 5*g^2*i^2*n*log(e))*a^4*b*c*d^4)*B^2*
log(b*x + a) + 2*(6*B^2*b^5*d^5*g^2*i^2*x^5 + 30*B^2*a^2*b^3*c^2*d^3*g^2*i^
2*x + 15*(b^5*c*d^4*g^2*i^2 + a*b^4*d^5*g^2*i^2)*B^2*x^4 + 10*(b^5*c^2*d^3*
g^2*i^2 + 4*a*b^4*c*d^4*g^2*i^2 + a^2*b^3*d^5*g^2*i^2)*B^2*x^3 + 30*(a*b^4*
c^2*d^3*g^2*i^2 + a^2*b^3*c*d^4*g^2*i^2)*B^2*x^2)*log((b*x + a)^n)^2 + 2*(6
*B^2*b^5*d^5*g^2*i^2*x^5 + 30*B^2*a^2*b^3*c^2*d^3*g^2*i^2*x + 15*(b^5*c*d^4
*g^2*i^2 + a*b^4*d^5*g^2*i^2)*B^2*x^4 + 10*(b^5*c^2*d^3*g^2*i^2 + 4*a*b^4*c
*d^4*g^2*i^2 + a^2*b^3*d^5*g^2*i^2)*B^2*x^3 + 30*(a*b^4*c^2*d^3*g^2*i^2 + a
^2*b^3*c*d^4*g^2*i^2)*B^2*x^2)*log((d*x + c)^n)^2 + 2*(12*B^2*b^5*d^5*g^2*i
^2*x^5*log(e) - 3*((g^2*i^2*n - 10*g^2*i^2*log(e))*b^5*c*d^4 - (g^2*i^2*n +
 10*g^2*i^2*log(e))*a*b^4*d^5)*B^2*x^4 + 2*(40*a*b^4*c*d^4*g^2*i^2*log(e) -
 (3*g^2*i^2*n - 10*g^2*i^2*log(e))*b^5*c^2*d^3 + (3*g^2*i^2*n + 10*g^2*i^2*
log(e))*a^2*b^3*d^5)*B^2*x^3 - (b^5*c^3*d^2*g^2*i^2*n - a^3*b^2*d^5*g^2*i^2
*n + 15*(g^2*i^2*n - 4*g^2*i^2*log(e))*a*b^4*c^2*d^3 - 15*(g^2*i^2*n + 4*g^
2*i^2*log(e))*a^2*b^3*c*d^4)*B^2*x^2 + 2*(b^5*c^4*d*g^2*i^2*n - 5*a*b^4*c^3
*d^2*g^2*i^2*n + 5*a^3*b^2*c*d^4*g^2*i^2*n - a^4*b*d^5*g^2*i^2*n + 30*a^2*b
^3*c^2*d^3*g^2*i^2*log(e))*B^2*x + 2*(10*a^3*b^2*c^2*d^3*g^2*i^2*n - 5*a^4*
b*c*d^4*g^2*i^2*n + a^5*d^5*g^2*i^2*n)*B^2*log(b*x + a) - 2*(b^5*c^5*g^2*i^
2*n - 5*a*b^4*c^4*d*g^2*i^2*n + 10*a^2*b^3*c^3*d^2*g^2*i^2*n)*B^2*log(d*x +
 c))*log((b*x + a)^n) - 2*(12*B^2*b^5*d^5*g^2*i^2*x^5*log(e) - 3*((g^2*i^2*
n - 10*g^2*i^2*log(e))*b^5*c*d^4 - (g^2*i^2*n + 10*g^2*i^2*log(e))*a*b^4*d^
5)*B^2*x^4 + 2*(40*a*b^4*c*d^4*g^2*i^2*log(e) - (3*g^2*i^2*n - 10*g^2*i^2*l
og(e))*b^5*c^2*d^3 + (3*g^2*i^2*n + 10*g^2*i^2*log(e))*a^2*b^3*d^5)*B^2*x^3
 - (b^5*c^3*d^2*g^2*i^2*n - a^3*b^2*d^5*g^2*i^2*n + 15*(g^2*i^2*n - 4*g^2*i
^2*log(e))*a*b^4*c^2*d^3 - 15*(g^2*i^2*n + 4*g^2*i^2*log(e))*a^2*b^3*c*d^4)
*B^2*x^2 + 2*(b^5*c^4*d*g^2*i^2*n - 5*a*b^4*c^3*d^2*g^2*i^2*n + 5*a^3*b^2*c
*d^4*g^2*i^2*n - a^4*b*d^5*g^2*i^2*n + 30*a^2*b^3*c^2*d^3*g^2*i^2*log(e))*B
^2*x + 2*(10*a^3*b^2*c^2*d^3*g^2*i^2*n - 5*a^4*b*c*d^4*g^2*i^2*n + a^5*d^5*
g^2*i^2*n)*B^2*log(b*x + a) - 2*(b^5*c^5*g^2*i^2*n - 5*a*b^4*c^4*d*g^2*i^2*
n + 10*a^2*b^3*c^3*d^2*g^2*i^2*n)*B^2*log(d*x + c) + 2*(6*B^2*b^5*d^5*g^2*i
^2*x^5 + 30*B^2*a^2*b^3*c^2*d^3*g^2*i^2*x + 15*(b^5*c*d^4*g^2*i^2 + a*b^4*d
^5*g^2*i^2)*B^2*x^4 + 10*(b^5*c^2*d^3*g^2*i^2 + 4*a*b^4*c*d^4*g^2*i^2 + a^2
*b^3*d^5*g^2*i^2)*B^2*x^3 + 30*(a*b^4*c^2*d^3*g^2*i^2 + a^2*b^3*c*d^4*g^2*i
^2)*B^2*x^2)*log((b*x + a)^n))*log((d*x + c)^n))/(b^3*d^3)
```

Giac [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
 algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^2\,(c\,i+d\,i\,x)^2\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

```
[In] int((a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^
2,x)
[Out] int((a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^
2, x)
```

3.170 $\int (ag+bgx)(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 43, antiderivative size = 635

$$
\begin{aligned}
&\int (ag+bgx)(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= \frac{B^2(bc-ad)^3gi^2n^2x}{12b^2d} + \frac{B^2(bc-ad)^2gi^2n^2(c+dx)^2}{12bd^2} \\
&\quad - \frac{B(bc-ad)^3gi^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^3d} \\
&\quad - \frac{B(bc-ad)^2gi^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^3} \\
&\quad + \frac{B(bc-ad)^2gi^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4bd^2} \\
&\quad - \frac{B(bc-ad)gi^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^2} \\
&\quad + \frac{(bc-ad)^2gi^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{12b^3} \\
&\quad + \frac{(bc-ad)gi^2(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b^2} \\
&\quad + \frac{gi^2(a+bx)^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} \\
&\quad - \frac{B(bc-ad)^4gi^2n\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{6b^3d^2} \\
&\quad - \frac{B^2(bc-ad)^4gi^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{12b^3d^2} - \frac{B^2(bc-ad)^4gi^2n^2\log(c+dx)}{4b^3d^2} \\
&\quad - \frac{B^2(bc-ad)^4gi^2n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{6b^3d^2}
\end{aligned}
$$

```
[Out] 1/12*B^2*(-a*d+b*c)^3*g*i^2*n^2*x/b^2/d+1/12*B^2*(-a*d+b*c)^2*g*i^2*n^2*(d*
x+c)^2/b/d^2-1/6*B*(-a*d+b*c)^3*g*i^2*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))
^n))/b^3/d-1/6*B*(-a*d+b*c)^2*g*i^2*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))
^n))/b^3+1/4*B*(-a*d+b*c)^2*g*i^2*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n
))/b/d^2-1/6*B*(-a*d+b*c)*g*i^2*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))
/d^2+1/12*(-a*d+b*c)^2*g*i^2*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^
3+1/6*(-a*d+b*c)*g*i^2*(b*x+a)^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/
b^2+1/4*g*i^2*(b*x+a)^2*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b-1/6*B
*(-a*d+b*c)^4*g*i^2*n*(A+B*n+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(
d*x+c))/b^3/d^2-1/12*B^2*(-a*d+b*c)^4*g*i^2*n^2*ln((b*x+a)/(d*x+c))/b^3/d^2
-1/4*B^2*(-a*d+b*c)^4*g*i^2*n^2*ln(d*x+c)/b^3/d^2-1/6*B^2*(-a*d+b*c)^4*g*i^
2*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^2
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.256, Rules used = {2561, 2383, 2381, 2384, 2354, 2438, 2373, 45, 2382, 12, 78}

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx$$

$$= -\frac{Bgi^2n(bc-ad)^4\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A+Bn\right)}{6b^3d^2} - \frac{Bgi^2n(a+bx)(bc-ad)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6b^3d} + \frac{gi^2(a+bx)^2(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{12b^3} - \frac{Bgi^2n(a+bx)^2(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6b^3} + \frac{gi^2(a+bx)^2(c+dx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{6b^2} + \frac{Bgi^2n(c+dx)^2(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4bd^2} - \frac{Bgi^2n(c+dx)^3(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6d^2} + \frac{gi^2(a+bx)^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b} - \frac{B^2gi^2n^2(bc-ad)^4\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{6b^3d^2} - \frac{B^2gi^2n^2(bc-ad)^4\log\left(\frac{a+bx}{c+dx}\right)}{12b^3d^2} - \frac{B^2gi^2n^2(bc-ad)^4\log(c+dx)}{4b^3d^2} + \frac{B^2gi^2n^2x(bc-ad)^3}{12b^2d} + \frac{B^2gi^2n^2(c+dx)^2(bc-ad)^2}{12bd^2}$$

```
[In] Int[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,
x]
```

```
[Out] (B^2*(b*c - a*d)^3*g*i^2*n^2*x)/(12*b^2*d) + (B^2*(b*c - a*d)^2*g*i^2*n^2*(
c + d*x)^2)/(12*b*d^2) - (B*(b*c - a*d)^3*g*i^2*n*(a + b*x)*(A + B*Log[e*((
a + b*x)/(c + d*x))^n]))/(6*b^3*d) - (B*(b*c - a*d)^2*g*i^2*n*(a + b*x)^2*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*b^3) + (B*(b*c - a*d)^2*g*i^2*n*(
c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*b*d^2) - (B*(b*c - a*
d)*g*i^2*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*d^2) + ((
b*c - a*d)^2*g*i^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(1
2*b^3) + ((b*c - a*d)*g*i^2*(a + b*x)^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^2)/(6*b^2) + (g*i^2*(a + b*x)^2*(c + d*x)^2*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n])^2)/(4*b) - (B*(b*c - a*d)^4*g*i^2*n*(A + B*n + B*Log[
e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(6*b^3*d^2) - (
B^2*(b*c - a*d)^4*g*i^2*n^2*Log[(a + b*x)/(c + d*x)])/(12*b^3*d^2) - (B^2*(
b*c - a*d)^4*g*i^2*n^2*Log[c + d*x])/(4*b^3*d^2) - (B^2*(b*c - a*d)^4*g*i^2
*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(6*b^3*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2383

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Dist[(m + q + 2)/(d*(q + 1)), Int[(
f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Dist[b*n*(p/(d*(q +
 1))), Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x]) /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ
[q, -1] && GtQ[m, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \left((bc-ad)^4 gi^2\right) \text{Subst}\left(\int \frac{x(A+B\log(ex^n))^2}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{gi^2(a+bx)^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} + \frac{\left((bc-ad)^4 gi^2\right)\text{Subst}\left(\int \frac{x(A+B\log(ex^n))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b} - \frac{\left(B(bc-ad)^4 gi^2 n\right)\text{Subst}\left(\int \frac{x(A+B\log(ex^n))}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b}$$

$$= \frac{B(bc-ad)^2 gi^2 n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4bd^2} - \frac{B(bc-ad)gi^2 n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^2} + \frac{(bc-ad)gi^2(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b^2} + \frac{gi^2(a+bx)^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} + \frac{\left((bc-ad)^4 gi^2\right)\text{Subst}\left(\int \frac{x(A+B\log(ex^n))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{6b^2} - \frac{\left(B(bc-ad)^4 gi^2 n\right)\text{Subst}\left(\int \frac{x(A+B\log(ex^n))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b^2} + \frac{\left(B^2(bc-ad)^4 gi^2 n^2\right)\text{Subst}\left(\int \frac{-b+3dx}{6d^2x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b}$$

$$= -\frac{B(bc-ad)^2gi^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^3} + \frac{B(bc-ad)^2gi^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4bd^2} - \frac{B(bc-ad)gi^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^2} + \frac{(bc-ad)^2gi^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{12b^3} + \frac{(bc-ad)gi^2(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b^2} + \frac{gi^2(a+bx)^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} - \frac{(B(bc-ad)^4gi^2n)\,\text{Subst}\left(\int\frac{x(A+B\log(ex^n))}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{6b^3} + \frac{(B^2(bc-ad)^4gi^2n^2)\,\text{Subst}\left(\int\frac{x}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{6b^3} + \frac{(B^2(bc-ad)^4gi^2n^2)\,\text{Subst}\left(\int\frac{-b+3dx}{x(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{12bd^2}$$

$$= -\frac{B(bc-ad)^3gi^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^3d} - \frac{B(bc-ad)^2gi^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^3} + \frac{B(bc-ad)^2gi^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4bd^2} - \frac{B(bc-ad)gi^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^2} + \frac{(bc-ad)^2gi^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{12b^3} + \frac{(bc-ad)gi^2(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b^2} + \frac{gi^2(a+bx)^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} + \frac{(B(bc-ad)^4gi^2n)\,\text{Subst}\left(\int\frac{A+Bn+B\log(ex^n)}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{6b^3d} + \frac{(B^2(bc-ad)^4gi^2n^2)\,\text{Subst}\left(\int\left(\frac{b}{d(-b+dx)^2}+\frac{1}{d(-b+dx)}\right)\,dx,x,\frac{a+bx}{c+dx}\right)}{6b^3} + \frac{(B^2(bc-ad)^4gi^2n^2)\,\text{Subst}\left(\int\left(-\frac{1}{b^2x}+\frac{2d}{(b-dx)^3}-\frac{d}{b(b-dx)^2}-\frac{d}{b^2(b-dx)}\right)\,dx,x,\frac{a+bx}{c+dx}\right)}{12bd^2}$$

$$
\begin{aligned}
&= \frac{B^2(bc-ad)^3gi^2n^2x}{12b^2d} + \frac{B^2(bc-ad)^2gi^2n^2(c+dx)^2}{12bd^2} \\
&\quad - \frac{B(bc-ad)^3gi^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^3d} \\
&\quad - \frac{B(bc-ad)^2gi^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^3} \\
&\quad + \frac{B(bc-ad)^2gi^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4bd^2} \\
&\quad - \frac{B(bc-ad)gi^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^2} \\
&\quad + \frac{(bc-ad)^2gi^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{12b^3} \\
&\quad + \frac{(bc-ad)gi^2(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b^2} \\
&\quad + \frac{gi^2(a+bx)^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} \\
&\quad - \frac{B(bc-ad)^4gi^2n\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{6b^3d^2} \\
&\quad - \frac{B^2(bc-ad)^4gi^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{12b^3d^2} - \frac{B^2(bc-ad)^4gi^2n^2\log(c+dx)}{4b^3d^2} \\
&\quad + \frac{(B^2(bc-ad)^4gi^2n^2)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{6b^3d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{B^2(bc-ad)^3 gi^2 n^2 x}{12b^2 d} + \frac{B^2(bc-ad)^2 gi^2 n^2 (c+dx)^2}{12bd^2} \\
&\quad - \frac{B(bc-ad)^3 gi^2 n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^3 d} \\
&\quad - \frac{B(bc-ad)^2 gi^2 n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^3} \\
&\quad + \frac{B(bc-ad)^2 gi^2 n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4bd^2} \\
&\quad - \frac{B(bc-ad) gi^2 n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^2} \\
&\quad + \frac{(bc-ad)^2 gi^2 (a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{12b^3} \\
&\quad + \frac{(bc-ad) gi^2 (a+bx)^2 (c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b^2} \\
&\quad + \frac{gi^2 (a+bx)^2 (c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} \\
&\quad - \frac{B(bc-ad)^4 gi^2 n\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{6b^3 d^2} \\
&\quad - \frac{B^2(bc-ad)^4 gi^2 n^2 \log\left(\frac{a+bx}{c+dx}\right)}{12b^3 d^2} - \frac{B^2(bc-ad)^4 gi^2 n^2 \log(c+dx)}{4b^3 d^2} \\
&\quad - \frac{B^2(bc-ad)^4 gi^2 n^2 \operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{6b^3 d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 713, normalized size of antiderivative = 1.12

$$
\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx
$$

$$
= \frac{gi^2\left(-4(bc-ad)(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+3b(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{4B(bc-ad)^2n(\cdots}{\cdots}\right.}{\cdots}
$$

```
[In] Integrate[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2,x]
```

```
[Out] (g*i^2*(-4*(b*c - a*d)*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
 + 3*b*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (4*B*(b*c - a
*d)^2*n*(2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*n*(b*d*x + (b*c - a*d)*Log[a
 + b*x]) + 2*B*d*(b*c - a*d)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + b^2
*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*(b*c - a*d)^2*Log[a
 + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*B*(b*c - a*d)^2*n*Log[c
```

```
+ d*x] - B*(b*c - a*d)^2*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x)
)/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/b^3 - (B*(b
*c - a*d)*n*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*n*(b*d*x + (b*c -
a*d)*Log[a + b*x]) - B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2
 + 2*(b*c - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a
+ b*x)/(c + d*x))^n] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]) + 2*b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
 + 6*(b*c - a*d)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*
B*(b*c - a*d)^3*n*Log[c + d*x] - 3*B*(b*c - a*d)^3*n*(Log[a + b*x]*(Log[a +
 b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b
*c) + a*d)])))/b^3))/(12*d^2)
```

Maple [F]

$$\int (bgx + ag)(dix + ci)^2 \left(A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)(ci+dix)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (bgx+ag)(dix+ci)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, a
lgorithm="fricas")
```

```
[Out] integral(A^2*b*d^2*g*i^2*x^3 + A^2*a*c^2*g*i^2 + (2*A^2*b*c*d + A^2*a*d^2)*
g*i^2*x^2 + (A^2*b*c^2 + 2*A^2*a*c*d)*g*i^2*x + (B^2*b*d^2*g*i^2*x^3 + B^2*
a*c^2*g*i^2 + (2*B^2*b*c*d + B^2*a*d^2)*g*i^2*x^2 + (B^2*b*c^2 + 2*B^2*a*c*
d)*g*i^2*x)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b*d^2*g*i^2*x^3 + A*B
*a*c^2*g*i^2 + (2*A*B*b*c*d + A*B*a*d^2)*g*i^2*x^2 + (A*B*b*c^2 + 2*A*B*a*c
*d)*g*i^2*x)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2662 vs. $2(608) = 1216$.

Time = 0.74 (sec) , antiderivative size = 2662, normalized size of antiderivative = 4.19

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, a
lgorithm="maxima")
[Out] 1/2*A*B*b*d^2*g*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*b*
d^2*g*i^2*x^4 + 4/3*A*B*b*c*d*g*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))
^n) + 2/3*A*B*a*d^2*g*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2/3*
A^2*b*c*d*g*i^2*x^3 + 1/3*A^2*a*d^2*g*i^2*x^3 + A*B*b*c^2*g*i^2*x^2*log(e*(
b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*B*a*c*d*g*i^2*x^2*log(e*(b*x/(d*x + c
) + a/(d*x + c))^n) + 1/2*A^2*b*c^2*g*i^2*x^2 + A^2*a*c*d*g*i^2*x^2 - 1/12*
A*B*b*d^2*g*i^2*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^
3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3
*d^3)*x)/(b^3*d^3)) + 2/3*A*B*b*c*d*g*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3
*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^
2*d^2)) + 1/3*A*B*a*d^2*g*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c
)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - A*
B*b*c^2*g*i^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*
x/(b*d)) - 2*A*B*a*c*d*g*i^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2
 + (b*c - a*d)*x/(b*d)) + 2*A*B*a*c^2*g*i^2*n*(a*log(b*x + a)/b - c*log(d*x
 + c)/d) + 2*A*B*a*c^2*g*i^2*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2
*a*c^2*g*i^2*x - 1/12*(7*a^2*b*c^2*d^2*g*i^2*n^2 - 2*a^3*c*d^3*g*i^2*n^2 +
(g*i^2*n^2 - 2*g*i^2*n*log(e))*b^3*c^4 - 2*(3*g*i^2*n^2 - 4*g*i^2*n*log(e))
*a*b^2*c^3*d)*B^2*log(d*x + c)/(b^2*d^2) + 1/6*(b^4*c^4*g*i^2*n^2 - 4*a*b^3
*c^3*d*g*i^2*n^2 + 6*a^2*b^2*c^2*d^2*g*i^2*n^2 - 4*a^3*b*c*d^3*g*i^2*n^2 +
a^4*d^4*g*i^2*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog
(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d^2) + 1/12*(3*B^2*b^4*d^4*g*i^2*x^4
*log(e)^2 - 2*((g*i^2*n*log(e) - 4*g*i^2*log(e)^2)*b^4*c*d^3 - (g*i^2*n*log
```

```
(e) + 2*g*i^2*log(e)^2)*a*b^3*d^4)*B^2*x^3 + ((g*i^2*n^2 - 5*g*i^2*n*log(e)
 + 6*g*i^2*log(e)^2)*b^4*c^2*d^2 - 2*(g*i^2*n^2 - 2*g*i^2*n*log(e) - 6*g*i^
2*log(e)^2)*a*b^3*c*d^3 + (g*i^2*n^2 + g*i^2*n*log(e))*a^2*b^2*d^4)*B^2*x^2
 - (6*a^2*b^2*c^2*d^2*g*i^2*n^2 - 4*a^3*b*c*d^3*g*i^2*n^2 + a^4*d^4*g*i^2*n
^2)*B^2*log(b*x + a)^2 - 2*(b^4*c^4*g*i^2*n^2 - 4*a*b^3*c^3*d*g*i^2*n^2)*B^
2*log(b*x + a)*log(d*x + c) + (b^4*c^4*g*i^2*n^2 - 4*a*b^3*c^3*d*g*i^2*n^2)
*B^2*log(d*x + c)^2 + ((3*g*i^2*n^2 - 2*g*i^2*n*log(e))*b^4*c^3*d - (7*g*i^
2*n^2 + 4*g*i^2*n*log(e) - 12*g*i^2*log(e)^2)*a*b^3*c^2*d^2 + (5*g*i^2*n^2
+ 8*g*i^2*n*log(e))*a^2*b^2*c*d^3 - (g*i^2*n^2 + 2*g*i^2*n*log(e))*a^3*b*d^
4)*B^2*x - (2*a*b^3*c^3*d*g*i^2*n^2 - (g*i^2*n^2 + 12*g*i^2*n*log(e))*a^2*b
^2*c^2*d^2 - 2*(g*i^2*n^2 - 4*g*i^2*n*log(e))*a^3*b*c*d^3 + (g*i^2*n^2 - 2*
g*i^2*n*log(e))*a^4*d^4)*B^2*log(b*x + a) + (3*B^2*b^4*d^4*g*i^2*x^4 + 12*B
^2*a*b^3*c^2*d^2*g*i^2*x + 4*(2*b^4*c*d^3*g*i^2 + a*b^3*d^4*g*i^2)*B^2*x^3
+ 6*(b^4*c^2*d^2*g*i^2 + 2*a*b^3*c*d^3*g*i^2)*B^2*x^2)*log((b*x + a)^n)^2 +
 (3*B^2*b^4*d^4*g*i^2*x^4 + 12*B^2*a*b^3*c^2*d^2*g*i^2*x + 4*(2*b^4*c*d^3*g
*i^2 + a*b^3*d^4*g*i^2)*B^2*x^3 + 6*(b^4*c^2*d^2*g*i^2 + 2*a*b^3*c*d^3*g*i^
2)*B^2*x^2)*log((d*x + c)^n)^2 + (6*B^2*b^4*d^4*g*i^2*x^4*log(e) - 2*((g*i^
2*n - 8*g*i^2*log(e))*b^4*c*d^3 - (g*i^2*n + 4*g*i^2*log(e))*a*b^3*d^4)*B^2
*x^3 + (a^2*b^2*d^4*g*i^2*n - (5*g*i^2*n - 12*g*i^2*log(e))*b^4*c^2*d^2 + 4
*(g*i^2*n + 6*g*i^2*log(e))*a*b^3*c*d^3)*B^2*x^2 - 2*(b^4*c^3*d*g*i^2*n - 4
*a^2*b^2*c*d^3*g*i^2*n + a^3*b*d^4*g*i^2*n + 2*(g*i^2*n - 6*g*i^2*log(e))*a
*b^3*c^2*d^2)*B^2*x + 2*(6*a^2*b^2*c^2*d^2*g*i^2*n - 4*a^3*b*c*d^3*g*i^2*n
+ a^4*d^4*g*i^2*n)*B^2*log(b*x + a) + 2*(b^4*c^4*g*i^2*n - 4*a*b^3*c^3*d*g*
i^2*n)*B^2*log(d*x + c))*log((b*x + a)^n) - (6*B^2*b^4*d^4*g*i^2*x^4*log(e)
 - 2*((g*i^2*n - 8*g*i^2*log(e))*b^4*c*d^3 - (g*i^2*n + 4*g*i^2*log(e))*a*b
^3*d^4)*B^2*x^3 + (a^2*b^2*d^4*g*i^2*n - (5*g*i^2*n - 12*g*i^2*log(e))*b^4*
c^2*d^2 + 4*(g*i^2*n + 6*g*i^2*log(e))*a*b^3*c*d^3)*B^2*x^2 - 2*(b^4*c^3*d*
g*i^2*n - 4*a^2*b^2*c*d^3*g*i^2*n + a^3*b*d^4*g*i^2*n + 2*(g*i^2*n - 6*g*i^
2*log(e))*a*b^3*c^2*d^2)*B^2*x + 2*(6*a^2*b^2*c^2*d^2*g*i^2*n - 4*a^3*b*c*d
^3*g*i^2*n + a^4*d^4*g*i^2*n)*B^2*log(b*x + a) + 2*(b^4*c^4*g*i^2*n - 4*a*b
^3*c^3*d*g*i^2*n)*B^2*log(d*x + c) + 2*(3*B^2*b^4*d^4*g*i^2*x^4 + 12*B^2*a*
b^3*c^2*d^2*g*i^2*x + 4*(2*b^4*c*d^3*g*i^2 + a*b^3*d^4*g*i^2)*B^2*x^3 + 6*(
b^4*c^2*d^2*g*i^2 + 2*a*b^3*c*d^3*g*i^2)*B^2*x^2)*log((b*x + a)^n))*log((d*
x + c)^n))/(b^3*d^2)
```

Giac [F]

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (bgx+ag)(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, a
```

```
lgorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)*(d*i*x + c*i)^2*(B*log(e*((b*x + a)/(d*x + c))^n) +
 A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)\,\left(c\,i+d\,i\,x\right)^2\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

```
[In] int((a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,
x)
```

```
[Out] int((a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,
 x)
```

3.171 $\int (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 35, antiderivative size = 361

$$\begin{aligned}
&\int (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= \frac{B^2(bc-ad)^2i^2n^2x}{3b^2} - \frac{2B(bc-ad)^2i^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^3} \\
&\quad - \frac{B(bc-ad)i^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd} \\
&\quad + \frac{i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d} \\
&\quad + \frac{B^2(bc-ad)^3i^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d} + \frac{B^2(bc-ad)^3i^2n^2\log(c+dx)}{b^3d} \\
&\quad + \frac{2B(bc-ad)^3i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d} \\
&\quad - \frac{2B^2(bc-ad)^3i^2n^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d}
\end{aligned}$$

```
[Out] 1/3*B^2*(-a*d+b*c)^2*i^2*n^2*x/b^2-2/3*B*(-a*d+b*c)^2*i^2*n*(b*x+a)*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))/b^3-1/3*B*(-a*d+b*c)*i^2*n*(d*x+c)^2*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))/b/d+1/3*i^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/
d+1/3*B^2*(-a*d+b*c)^3*i^2*n^2*ln((b*x+a)/(d*x+c))/b^3/d+B^2*(-a*d+b*c)^3*i
^2*n^2*ln(d*x+c)/b^3/d+2/3*B*(-a*d+b*c)^3*i^2*n*(A+B*ln(e*((b*x+a)/(d*x+c))
^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/d-2/3*B^2*(-a*d+b*c)^3*i^2*n^2*polylog(2
,b*(d*x+c)/d/(b*x+a))/b^3/d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {2551, 2356, 2389, 2379, 2438, 2351, 31, 46}

$$\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx$$
$$=\frac{2Bi^2n(bc-ad)^3\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b^3d}$$
$$-\frac{2Bi^2n(a+bx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b^3}$$
$$-\frac{Bi^2n(c+dx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3bd}$$
$$+\frac{i^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d}-\frac{2B^2i^2n^2(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d}$$
$$+\frac{B^2i^2n^2(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d}+\frac{B^2i^2n^2(bc-ad)^3\log(c+dx)}{b^3d}+\frac{B^2i^2n^2x(bc-ad)^2}{3b^2}$$

```
[In] Int[(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (B^2*(b*c - a*d)^2*i^2*n^2*x)/(3*b^2) - (2*B*(b*c - a*d)^2*i^2*n*(a + b*x)*
(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b^3) - (B*(b*c - a*d)*i^2*n*(c +
 d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b*d) + (i^2*(c + d*x)^3*
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*d) + (B^2*(b*c - a*d)^3*i^2*n^
2*Log[(a + b*x)/(c + d*x)])/(3*b^3*d) + (B^2*(b*c - a*d)^3*i^2*n^2*Log[c +
d*x])/(b^3*d) + (2*B*(b*c - a*d)^3*i^2*n*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(3*b^3*d) - (2*B^2*(b*c - a*d)^3
*i^2*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(3*b^3*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
```

```
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\text{integral} = \left((bc-ad)^3 i^2\right) \text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{i^2(c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d} - \frac{(2B(bc-ad)^3 i^2 n)\,\mathrm{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3d}$$

$$= \frac{i^2(c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d} - \frac{(2B(bc-ad)^3 i^2 n)\,\mathrm{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b} - \frac{(2B(bc-ad)^3 i^2 n)\,\mathrm{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{3bd}$$

$$= -\frac{B(bc-ad)i^2 n(c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd} + \frac{i^2(c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d} - \frac{(2B(bc-ad)^3 i^2 n)\,\mathrm{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b^2} - \frac{(2B(bc-ad)^3 i^2 n)\,\mathrm{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b^2 d} + \frac{(B^2(bc-ad)^3 i^2 n^2)\,\mathrm{Subst}\left(\int \frac{1}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{3bd}$$

$$= -\frac{2B(bc-ad)^2 i^2 n(a+bx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^3} - \frac{B(bc-ad)i^2 n(c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd} + \frac{i^2(c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d} + \frac{2B(bc-ad)^3 i^2 n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3 d} + \frac{(2B^2(bc-ad)^3 i^2 n^2)\,\mathrm{Subst}\left(\int \frac{1}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b^3} - \frac{(2B^2(bc-ad)^3 i^2 n^2)\,\mathrm{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b^3 d} + \frac{(B^2(bc-ad)^3 i^2 n^2)\,\mathrm{Subst}\left(\int \left(\frac{1}{b^2 x}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{3bd}$$

$$= \frac{B^2(bc-ad)^2i^2n^2x}{3b^2} - \frac{2B(bc-ad)^2i^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^3} - \frac{B(bc-ad)i^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd} + \frac{i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d} + \frac{B^2(bc-ad)^3i^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d} + \frac{B^2(bc-ad)^3i^2n^2\log(c+dx)}{b^3d} + \frac{2B(bc-ad)^3i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d} - \frac{2B^2(bc-ad)^3i^2n^2\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.84

$$\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$

$$= \frac{i^2\left((c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - \frac{B(bc-ad)n\left(2Abd(bc-ad)x-B(bc-ad)n(bdx+(bc-ad)\log(a+bx))+2Bd(bc-ad)(a+bx)\log\right.}{}\right.}{}$$

```
[In] Integrate[(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (i^2*((c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d)
*n*(2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*n*(b*d*x + (b*c - a*d)*Log[a + b*
x]) + 2*B*d*(b*c - a*d)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + b^2*(c +
 d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*(b*c - a*d)^2*Log[a + b*
x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*B*(b*c - a*d)^2*n*Log[c + d*x
] - B*(b*c - a*d)^2*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*
c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/b^3))/(3*d)
```

Maple [F]

$$\int (dix+ci)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2dx$$

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x^2
 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B
*d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n
), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1473 vs. $2(346) = 692$.

Time = 0.72 (sec) , antiderivative size = 1473, normalized size of antiderivative = 4.08

$$\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] 2/3*A*B*d^2*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*d^2*i^
2*x^3 + 2*A*B*c*d*i^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c*d*
i^2*x^2 + 1/3*A*B*d^2*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^
3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*A*B*
c*d*i^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d
)) + 2*A*B*c^2*i^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*c^2*i^2*
```

```
x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c^2*i^2*x - 1/3*(5*a*b*c^2*d
*i^2*n^2 - 2*a^2*c*d^2*i^2*n^2 - (3*i^2*n^2 - 2*i^2*n*log(e))*b^2*c^3)*B^2*
log(d*x + c)/(b^2*d) - 2/3*(b^3*c^3*i^2*n^2 - 3*a*b^2*c^2*d*i^2*n^2 + 3*a^2
*b*c*d^2*i^2*n^2 - a^3*d^3*i^2*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c -
a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d) + 1/3*(B^2*b^3*d
^3*i^2*x^3*log(e)^2 + 2*B^2*b^3*c^3*i^2*n^2*log(b*x + a)*log(d*x + c) - B^2
*b^3*c^3*i^2*n^2*log(d*x + c)^2 + (a*b^2*d^3*i^2*n*log(e) - (i^2*n*log(e) -
 3*i^2*log(e)^2)*b^3*c*d^2)*B^2*x^2 - (3*a*b^2*c^2*d*i^2*n^2 - 3*a^2*b*c*d^
2*i^2*n^2 + a^3*d^3*i^2*n^2)*B^2*log(b*x + a)^2 + ((i^2*n^2 - 4*i^2*n*log(e
) + 3*i^2*log(e)^2)*b^3*c^2*d - 2*(i^2*n^2 - 3*i^2*n*log(e))*a*b^2*c*d^2 +
(i^2*n^2 - 2*i^2*n*log(e))*a^2*b*d^3)*B^2*x - (2*(2*i^2*n^2 - 3*i^2*n*log(e
))*a*b^2*c^2*d - (7*i^2*n^2 - 6*i^2*n*log(e))*a^2*b*c*d^2 + (3*i^2*n^2 - 2*
i^2*n*log(e))*a^3*d^3)*B^2*log(b*x + a) + (B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*
c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i^2*x)*log((b*x + a)^n)^2 + (B^2*b^3*d^3*i^
2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i^2*x)*log((d*x + c)^n)^2
 + (2*B^2*b^3*d^3*i^2*x^3*log(e) - 2*B^2*b^3*c^3*i^2*n*log(d*x + c) + (a*b^
2*d^3*i^2*n - (i^2*n - 6*i^2*log(e))*b^3*c*d^2)*B^2*x^2 + 2*(3*a*b^2*c*d^2*
i^2*n - a^2*b*d^3*i^2*n - (2*i^2*n - 3*i^2*log(e))*b^3*c^2*d)*B^2*x + 2*(3*
a*b^2*c^2*d*i^2*n - 3*a^2*b*c*d^2*i^2*n + a^3*d^3*i^2*n)*B^2*log(b*x + a))*
log((b*x + a)^n) - (2*B^2*b^3*d^3*i^2*x^3*log(e) - 2*B^2*b^3*c^3*i^2*n*log(
d*x + c) + (a*b^2*d^3*i^2*n - (i^2*n - 6*i^2*log(e))*b^3*c*d^2)*B^2*x^2 + 2
*(3*a*b^2*c*d^2*i^2*n - a^2*b*d^3*i^2*n - (2*i^2*n - 3*i^2*log(e))*b^3*c^2*
d)*B^2*x + 2*(3*a*b^2*c^2*d*i^2*n - 3*a^2*b*c*d^2*i^2*n + a^3*d^3*i^2*n)*B^
2*log(b*x + a) + 2*(B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b
^3*c^2*d*i^2*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^3*d)
```

Giac [F]

$$\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((d*i*x + c*i)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (c\,i+d\,i\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

```
[In] int((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
[Out] int((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.172 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 572

$$
\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx \\
&= -\frac{Bd(bc-ad)i^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g} \\
&\quad+\frac{d(bc-ad)i^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g}+\frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg} \\
&\quad+\frac{2B(bc-ad)^2i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^3g}+\frac{B^2(bc-ad)^2i^2n^2\log(c+dx)}{b^3g} \\
&\quad+\frac{B(bc-ad)^2i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad-\frac{(bc-ad)^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad+\frac{2B^2(bc-ad)^2i^2n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^3g}-\frac{B^2(bc-ad)^2i^2n^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad+\frac{2B(bc-ad)^2i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad+\frac{2B^2(bc-ad)^2i^2n^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}
\end{aligned}
$$

```
[Out] -B*d*(-a*d+b*c)*i^2*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/g+d*(-a*d
+b*c)*i^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^3/g+1/2*i^2*(d*x+c)^2
```

```
*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b/g+2*B*(-a*d+b*c)^2*i^2*n*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^3/g+B^2*(-a*d+b*c)^2*i^2*n^2*
ln(d*x+c)/b^3/g+B*(-a*d+b*c)^2*i^2*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b
*(d*x+c)/d/(b*x+a))/b^3/g-(-a*d+b*c)^2*i^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^
2*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g+2*B^2*(-a*d+b*c)^2*i^2*n^2*polylog(2,d*(b
*x+a)/b/(d*x+c))/b^3/g-B^2*(-a*d+b*c)^2*i^2*n^2*polylog(2,b*(d*x+c)/d/(b*x+
a))/b^3/g+2*B*(-a*d+b*c)^2*i^2*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,
b*(d*x+c)/d/(b*x+a))/b^3/g+2*B^2*(-a*d+b*c)^2*i^2*n^2*polylog(3,b*(d*x+c)/d
/(b*x+a))/b^3/g
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 572, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.244$, Rules used = {2561, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31}

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{2Bi^2n(bc-ad)^2 \operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3g}$$
$$+\frac{di^2(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^3g}$$
$$-\frac{Bdi^2n(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3g}$$
$$+\frac{2Bi^2n(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3g}$$
$$-\frac{i^2(bc-ad)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^3g}$$
$$+\frac{Bi^2n(bc-ad)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3g}$$
$$+\frac{i^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2bg}+\frac{2B^2i^2n^2(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^3g}$$
$$-\frac{B^2i^2n^2(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}$$
$$+\frac{2B^2i^2n^2(bc-ad)^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}+\frac{B^2i^2n^2(bc-ad)^2\log(c+dx)}{b^3g}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x
),x]
```

```
[Out] -((B*d*(b*c - a*d)*i^2*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/
(b^3*g)) + (d*(b*c - a*d)*i^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2)/(b^3*g) + (i^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)
/(2*b*g) + (2*B*(b*c - a*d)^2*i^2*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*
Log[(b*c - a*d)/(b*(c + d*x))])/(b^3*g) + (B^2*(b*c - a*d)^2*i^2*n^2*Log[c
+ d*x])/(b^3*g) + (B*(b*c - a*d)^2*i^2*n*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^3*g) - ((b*c - a*d)^2*i^2*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])
/(b^3*g) + (2*B^2*(b*c - a*d)^2*i^2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*
x))])/(b^3*g) - (B^2*(b*c - a*d)^2*i^2*n^2*PolyLog[2, (b*(c + d*x))/(d*(a +
 b*x))])/(b^3*g) + (2*B*(b*c - a*d)^2*i^2*n*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b^3*g) + (2*B^2*(b*c - a*
d)^2*i^2*n^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))])/(b^3*g)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
```

```
NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)^2 i^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{g}$$

$$= \frac{((bc-ad)^2 i^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg} + \frac{(d(bc-ad)^2 i^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg}$$

$$= \frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg} + \frac{((bc-ad)^2 i^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2 g} + \frac{(d(bc-ad)^2 i^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2 g} - \frac{(B(bc-ad)^2 i^2 n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg}$$

$$= \frac{d(bc-ad)i^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3 g} + \frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg} - \frac{(bc-ad)^2 i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3 g} + \frac{(2B(bc-ad)^2 i^2 n)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)(A+B\log(ex^n))}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3 g} - \frac{(B(bc-ad)^2 i^2 n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2 g} - \frac{(2Bd(bc-ad)^2 i^2 n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3 g} - \frac{(Bd(bc-ad)^2 i^2 n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2 g}$$

$$= -\frac{Bd(bc-ad)i^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g} + \frac{d(bc-ad)i^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g} + \frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg} + \frac{2B(bc-ad)^2i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^3g} + \frac{B(bc-ad)^2i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} - \frac{(bc-ad)^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} + \frac{2B(bc-ad)^2i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} - \frac{(B^2(bc-ad)^2i^2n^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3g} - \frac{(2B^2(bc-ad)^2i^2n^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3g} - \frac{(2B^2(bc-ad)^2i^2n^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3g} + \frac{(B^2d(bc-ad)^2i^2n^2)\,\text{Subst}\left(\int\frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3g}$$

$$
\begin{aligned}
&= -\frac{Bd(bc-ad)i^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g} \\
&+ \frac{d(bc-ad)i^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g} \\
&+ \frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg} \\
&+ \frac{2B(bc-ad)^2i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^3g} \\
&+ \frac{B^2(bc-ad)^2i^2n^2\log(c+dx)}{b^3g} \\
&+ \frac{B(bc-ad)^2i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&- \frac{(bc-ad)^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&+ \frac{2B^2(bc-ad)^2i^2n^2\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^3g} - \frac{B^2(bc-ad)^2i^2n^2\mathrm{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&+ \frac{2B(bc-ad)^2i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\mathrm{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&+ \frac{2B^2(bc-ad)^2i^2n^2\mathrm{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2852 vs. $2(572) = 1144$.

Time = 2.09 (sec) , antiderivative size = 2852, normalized size of antiderivative = 4.99

$$
\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g +
 b*g*x),x]
```

```
[Out] (i^2*(12*b*d*(2*b*c - a*d)*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Lo
g[(a + b*x)/(c + d*x)])^2 + 6*b^2*d^2*x^2*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n] - B*n*Log[(a + b*x)/(c + d*x)])^2 + 12*(b*c - a*d)^2*Log[a + b*x]*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2 - 24*b*
B*c*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])
*(a*d*Log[a/b + x]^2 - 2*a*d*Log[a/b + x]*(1 + Log[a + b*x]) + 2*(-(b*c) +
a*d + Log[c/d + x]*(b*c + a*d*Log[a + b*x] - a*d*Log[(d*(a + b*x))/(-(b*c)
```

+ a*d)]) + (-(b*d*x) + a*d*Log[a + b*x])*Log[(a + b*x)/(c + d*x)]) - 2*a*d*
PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 12*b^2*B*c^2*n*(A + B*Log[e*((a +
b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])*(Log[a/b + x]^2 - 2*Log[
a + b*x]*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)]) - 2*(Log[
c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c
- a*d)])) + 6*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)
/(c + d*x)])*(-4*a*d^2*(a + b*x)*(-1 + Log[a/b + x]) + 2*a^2*d^2*Log[a/b +
x]^2 + 4*a*b*d*(c + d*x)*(-1 + Log[c/d + x]) + d^2*(b*x*(2*a - b*x) + 2*b^2
*x^2*Log[a/b + x] - 2*a^2*Log[a + b*x]) - 2*d^2*(b*x*(-2*a + b*x) + 2*a^2*L
og[a + b*x])*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)]) + b^2
*(d*x*(-2*c + d*x) - 2*d^2*x^2*Log[c/d + x] + 2*c^2*Log[c + d*x]) - 4*a^2*d
^2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x
))/(b*c - a*d)])) - 8*b*B^2*c*n^2*(a*d*Log[a/b + x]^3 - 3*d*(2*b*x - 2*(a +
b*x)*Log[a/b + x] + (a + b*x)*Log[a/b + x]^2) - 3*b*(2*d*x - 2*(c + d*x)*L
og[c/d + x] + (c + d*x)*Log[c/d + x]^2) - 3*d*(b*x - a*Log[a + b*x])*(-Log[
a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])^2 + 6*(a*d + 2*b*d*x -
b*d*x*Log[c/d + x] - b*c*Log[c + d*x] + Log[a/b + x]*(-(d*(a + b*x)) + d*(a
+ b*x)*Log[c/d + x] + (b*c - a*d)*Log[(b*(c + d*x))/(b*c - a*d)]) + (b*c -
a*d)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 3*(Log[a/b + x] - Log[c/d
+ x] - Log[(a + b*x)/(c + d*x)])*(-2*b*c + 2*a*d - 2*d*(a + b*x)*Log[a/b +
x] + a*d*Log[a/b + x]^2 + 2*Log[c/d + x]*(b*(c + d*x) - a*d*Log[(d*(a + b*
x))/(-(b*c) + a*d)]) - 2*a*d*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 3*a*d
(Log[a/b + x]^2(Log[c/d + x] - Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Log[a/
b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2*PolyLog[3, (d*(a + b*x)
)/(-(b*c) + a*d)]) + 3*a*d*(Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) + a*d)
] + 2*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 2*PolyLog[3, (b*
(c + d*x))/(b*c - a*d)])) + B^2*n^2*(4*a^2*d^2*Log[a/b + x]^3 - 12*a*d^2*(2
*b*x - 2*(a + b*x)*Log[a/b + x] + (a + b*x)*Log[a/b + x]^2) - 3*d^2*(b*x*(6
*a - b*x) + (-6*a^2 - 4*a*b*x + 2*b^2*x^2)*Log[a/b + x] + 2*(a^2 - b^2*x^2)
*Log[a/b + x]^2) - 12*a*b*d*(2*d*x - 2*(c + d*x)*Log[c/d + x] + (c + d*x)*L
og[c/d + x]^2) - 3*b^2*(d*x*(6*c - d*x) + (-6*c^2 - 4*c*d*x + 2*d^2*x^2)*Lo
g[c/d + x] + 2*(c^2 - d^2*x^2)*Log[c/d + x]^2) + 6*d^2*(b*x*(-2*a + b*x) +
2*a^2*Log[a + b*x])*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)
])^2 - 6*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)])*(-4*a*d^2
*(a + b*x)*(-1 + Log[a/b + x]) + 2*a^2*d^2*Log[a/b + x]^2 + 4*a*b*d*(c + d*
x)*(-1 + Log[c/d + x]) + d^2*(b*x*(2*a - b*x) + 2*b^2*x^2*Log[a/b + x] - 2*
a^2*Log[a + b*x]) + b^2*(d*x*(-2*c + d*x) - 2*d^2*x^2*Log[c/d + x] + 2*c^2*
Log[c + d*x]) - 4*a^2*d^2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] +
PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + 6*(2*a*b*c*d + 3*b^2*c*d*x + 3*a
*b*d^2*x - b^2*d^2*x^2 - 2*a*b*d^2*x*Log[c/d + x] + b^2*d^2*x^2*Log[c/d + x
] - a^2*d^2*Log[a + b*x] - b^2*c^2*Log[c + d*x] - 2*a*b*c*d*Log[c + d*x] -
Log[a/b + x]*(b*d*(2*a*c + b*x*(2*c - d*x)) - 2*d^2*(a^2 - b^2*x^2)*Log[c/d
+ x] + (-2*b^2*c^2 + 2*a^2*d^2)*Log[(b*(c + d*x))/(b*c - a*d)]) + 2*(b^2*c
^2 - a^2*d^2)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 4*a*d*(a*d + 2*b*d
*x - b*d*x*Log[c/d + x] - b*c*Log[c + d*x] + Log[a/b + x]*(-(d*(a + b*x)) +

```
 d*(a + b*x)*Log[c/d + x] + (b*c - a*d)*Log[(b*(c + d*x))/(b*c - a*d)]) + (
b*c - a*d)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2*a^2*d^2*(Log[a/b +
 x]^2*(Log[c/d + x] - Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Log[a/b + x]*Poly
Log[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2*PolyLog[3, (d*(a + b*x))/(-(b*c) +
 a*d)])) + 12*a^2*d^2*(Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2
*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 2*PolyLog[3, (b*(c +
d*x))/(b*c - a*d)])) + 4*b^2*B^2*c^2*n^2*(Log[a/b + x]^3 + 3*Log[c/d + x]^2
*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 3*Log[a + b*x]*(-Log[a/b + x] + Log[c/
d + x] + Log[(a + b*x)/(c + d*x)])^2 + 3*Log[a/b + x]^2*(-Log[c/d + x] + Lo
g[(b*(c + d*x))/(b*c - a*d)]) + 6*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(
b*c) + a*d)] + 6*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 3*(Lo
g[a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)])*(Log[a/b + x]^2 - 2*(
Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(
b*c - a*d)])) - 6*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*PolyLog[3, (
b*(c + d*x))/(b*c - a*d)])))/(12*b^3*g)
```

Maple [F]

$$\int \frac{(dix+ci)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{bgx+ag}dx$$

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

```
[Out] int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, a
lgorithm="fricas")
```

```
[Out] integral((A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x^
2 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*
B*d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^
n))/(b*g*x + a*g), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, a
lgorithm="maxima")
[Out] 2*A^2*c*d*i^2*(x/(b*g) - a*log(b*x + a)/(b^2*g)) + 1/2*A^2*d^2*i^2*(2*a^2*l
og(b*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A^2*c^2*i^2*log(b*g*x + a*
g)/(b*g) + 1/2*(B^2*b^2*d^2*i^2*x^2 + 2*(2*b^2*c*d*i^2 - a*b*d^2*i^2)*B^2*x
 + 2*(b^2*c^2*i^2 - 2*a*b*c*d*i^2 + a^2*d^2*i^2)*B^2*log(b*x + a))*log((d*x
 + c)^n)^2/(b^3*g) - integrate(-(B^2*b^3*c^3*i^2*log(e)^2 + 2*A*B*b^3*c^3*i
^2*log(e) + (B^2*b^3*d^3*i^2*log(e)^2 + 2*A*B*b^3*d^3*i^2*log(e))*x^3 + 3*(
B^2*b^3*c*d^2*i^2*log(e)^2 + 2*A*B*b^3*c*d^2*i^2*log(e))*x^2 + (B^2*b^3*d^3
*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i^2*x + B^2*b^3*c^3*i^
2)*log((b*x + a)^n)^2 + 3*(B^2*b^3*c^2*d*i^2*log(e)^2 + 2*A*B*b^3*c^2*d*i^2
*log(e))*x + 2*(B^2*b^3*c^3*i^2*log(e) + A*B*b^3*c^3*i^2 + (B^2*b^3*d^3*i^2
*log(e) + A*B*b^3*d^3*i^2)*x^3 + 3*(B^2*b^3*c*d^2*i^2*log(e) + A*B*b^3*c*d^
2*i^2)*x^2 + 3*(B^2*b^3*c^2*d*i^2*log(e) + A*B*b^3*c^2*d*i^2)*x)*log((b*x +
 a)^n) - (2*B^2*b^3*c^3*i^2*log(e) + 2*A*B*b^3*c^3*i^2 + (2*A*B*b^3*d^3*i^2
 + (i^2*n + 2*i^2*log(e))*B^2*b^3*d^3)*x^3 + (6*A*B*b^3*c*d^2*i^2 - (a*b^2*
d^3*i^2*n - 2*(2*i^2*n + 3*i^2*log(e))*b^3*c*d^2)*B^2)*x^2 + 2*(3*A*B*b^3*c
^2*d*i^2 + (2*a*b^2*c*d^2*i^2*n - a^2*b*d^3*i^2*n + 3*b^3*c^2*d*i^2*log(e))
*B^2)*x + 2*((b^3*c^2*d*i^2*n - 2*a*b^2*c*d^2*i^2*n + a^2*b*d^3*i^2*n)*B^2*
x + (a*b^2*c^2*d*i^2*n - 2*a^2*b*c*d^2*i^2*n + a^3*d^3*i^2*n)*B^2)*log(b*x
+ a) + 2*(B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i
^2*x + B^2*b^3*c^3*i^2)*log((b*x + a)^n))*log((d*x + c)^n))/(b^4*d*g*x^2 +
a*b^3*c*g + (b^4*c*g + a*b^3*d*g)*x), x)
```

Giac [F]

$$\int \frac{(ci + dix)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag + bgx}\,dx = \int \frac{(dix + ci)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}{bgx + ag}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, a
lgorithm="giac")
```

```
[Out] integrate((d*i*x + c*i)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x +
 a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci + dix)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag + bgx}\,dx = \int \frac{\left(c\,i + d\,i\,x\right)^2 \left(A + B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{a\,g + b\,g\,x}\,dx$$

```
[In] int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x
),x)
```

```
[Out] int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x
), x)
```

3.173 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 472

$$\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx \\
&= -\frac{2B^2(bc-ad)i^2n^2(c+dx)}{b^2g^2(a+bx)} - \frac{2B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g^2(a+bx)} \\
&\quad + \frac{d^2i^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g^2} - \frac{(bc-ad)i^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2g^2(a+bx)} \\
&\quad + \frac{2Bd(bc-ad)i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^3g^2} \\
&\quad - \frac{2d(bc-ad)i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} \\
&\quad + \frac{2B^2d(bc-ad)i^2n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^3g^2} \\
&\quad + \frac{4Bd(bc-ad)i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} \\
&\quad + \frac{4B^2d(bc-ad)i^2n^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}
\end{aligned}$$

```
[Out] -2*B^2*(-a*d+b*c)*i^2*n^2*(d*x+c)/b^2/g^2/(b*x+a)-2*B*(-a*d+b*c)*i^2*n*(d*x
+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g^2/(b*x+a)+d^2*i^2*(b*x+a)*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))^2/b^3/g^2-(-a*d+b*c)*i^2*(d*x+c)*(A+B*ln(e*((b*x+a
)/(d*x+c))^n))^2/b^2/g^2/(b*x+a)+2*B*d*(-a*d+b*c)*i^2*n*(A+B*ln(e*((b*x+a)/
```

```
(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^3/g^2-2*d*(-a*d+b*c)*i^2*(A+B*ln(e*
((b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g^2+2*B^2*d*(-a*d+b*c
)*i^2*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/g^2+4*B*d*(-a*d+b*c)*i^2*n*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g^2+4*B^2*d
*(-a*d+b*c)*i^2*n^2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^3/g^2
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2561, 2395, 2342, 2341, 2355, 2354, 2438, 2379, 2421, 6724}

$$\begin{aligned}
&\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx \\
&= \frac{d^2i^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^3g^2} \\
&+ \frac{4Bdi^2n(bc-ad)\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3g^2} \\
&+ \frac{2Bdi^2n(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3g^2} \\
&- \frac{2di^2(bc-ad)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^3g^2} \\
&- \frac{i^2(c+dx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^2g^2(a+bx)} \\
&- \frac{2Bi^2n(c+dx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2g^2(a+bx)} \\
&+ \frac{2B^2di^2n^2(bc-ad)\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^3g^2} \\
&+ \frac{4B^2di^2n^2(bc-ad)\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} - \frac{2B^2i^2n^2(c+dx)(bc-ad)}{b^2g^2(a+bx)}
\end{aligned}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x
)^2,x]
```

```
[Out] (-2*B^2*(b*c - a*d)*i^2*n^2*(c + d*x))/(b^2*g^2*(a + b*x)) - (2*B*(b*c - a*
d)*i^2*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^2*g^2*(a + b*
x)) + (d^2*i^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(b^3*g^2
) - ((b*c - a*d)*i^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(b
^2*g^2*(a + b*x)) + (2*B*d*(b*c - a*d)*i^2*n*(A + B*Log[e*((a + b*x)/(c + d
```

```
*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(b^3*g^2) - (2*d*(b*c - a*d)*i^2*(
A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))
])/(b^3*g^2) + (2*B^2*d*(b*c - a*d)*i^2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c
+ d*x))])/(b^3*g^2) + (4*B*d*(b*c - a*d)*i^2*n*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b^3*g^2) + (4*B^2*d*(b
*c - a*d)*i^2*n^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))])/(b^3*g^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
```

```
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left((bc-ad)i^2\right)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^2} \\
&= \frac{\left((bc-ad)i^2\right)\text{Subst}\left(\int \left(\frac{(A+B\log(ex^n))^2}{b^2x^2}+\frac{d^2(A+B\log(ex^n))^2}{b^2(b-dx)^2}+\frac{2d(A+B\log(ex^n))^2}{b^2x(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{g^2} \\
&= \frac{\left((bc-ad)i^2\right)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2} \\
&\quad+\frac{\left(2d(bc-ad)i^2\right)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2} \\
&\quad+\frac{\left(d^2(bc-ad)i^2\right)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2}
\end{aligned}
$$

$$= \frac{d^2 i^2 (a+bx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3 g^2} - \frac{(bc-ad) i^2 (c+dx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2 g^2 (a+bx)} - \frac{2d(bc-ad) i^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3 g^2} + \frac{(2B(bc-ad) i^2 n) \operatorname{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2 g^2} + \frac{(4Bd(bc-ad) i^2 n) \operatorname{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)(A+B\log(ex^n))}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3 g^2} - \frac{(2Bd^2(bc-ad) i^2 n) \operatorname{Subst}\left(\int \frac{A+B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3 g^2}$$

$$= -\frac{2B^2(bc-ad) i^2 n^2 (c+dx)}{b^2 g^2 (a+bx)} - \frac{2B(bc-ad) i^2 n (c+dx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2 g^2 (a+bx)} + \frac{d^2 i^2 (a+bx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3 g^2} - \frac{(bc-ad) i^2 (c+dx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2 g^2 (a+bx)} + \frac{2Bd(bc-ad) i^2 n \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^3 g^2} - \frac{2d(bc-ad) i^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3 g^2} + \frac{4Bd(bc-ad) i^2 n \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3 g^2} - \frac{(2B^2 d(bc-ad) i^2 n^2) \operatorname{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3 g^2} - \frac{(4B^2 d(bc-ad) i^2 n^2) \operatorname{Subst}\left(\int \frac{\operatorname{Li}_2\left(\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3 g^2}$$

$$= -\frac{2B^2(bc-ad)i^2n^2(c+dx)}{b^2g^2(a+bx)} - \frac{2B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g^2(a+bx)}$$
$$+ \frac{d^2i^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g^2}$$
$$- \frac{(bc-ad)i^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2g^2(a+bx)}$$
$$+ \frac{2Bd(bc-ad)i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^3g^2}$$
$$- \frac{2d(bc-ad)i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}$$
$$+ \frac{2B^2d(bc-ad)i^2n^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^3g^2}$$
$$+ \frac{4Bd(bc-ad)i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}$$
$$+ \frac{4B^2d(bc-ad)i^2n^2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2885 vs. $2(472) = 944$.

Time = 2.78 (sec) , antiderivative size = 2885, normalized size of antiderivative = 6.11

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g +
 b*g*x)^2,x]
```

```
[Out] (i^2*(3*b*d^2*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(
c + d*x)])^2 - (3*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n
*Log[(a + b*x)/(c + d*x)])^2)/(a + b*x) + 6*d*(b*c - a*d)*Log[a + b*x]*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2 + (6*b^
2*B*c^2*n*(-A - B*Log[e*((a + b*x)/(c + d*x))^n] + B*n*Log[(a + b*x)/(c + d
*x)])*(-(d*(a + b*x)*Log[c/d + x]) + d*(a + b*x)*Log[(d*(a + b*x))/(-(b*c)
+ a*d)] + (b*c - a*d)*(1 + Log[(a + b*x)/(c + d*x)])))/((b*c - a*d)*(a + b*
x)) + (3*b^2*B^2*c^2*n^2*(-2*b*c + 2*a*d - 2*d*(a + b*x)*Log[a + b*x] - 2*(
b*c - a*d)*Log[(a + b*x)/(c + d*x)] - 2*d*(a + b*x)*Log[a + b*x]*Log[(a + b
*x)/(c + d*x)] - (b*c - a*d)*Log[(a + b*x)/(c + d*x)]^2 + 2*d*(a + b*x)*Log
[c + d*x] - 2*d*(a + b*x)*Log[(a + b*x)/(c + d*x)]*Log[(b*c - a*d)/(b*c + b
```

```
*d*x)] + d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c
 - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + d*(a + b*x)*(Log[
(b*c - a*d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c
- a*d)/(b*c + b*d*x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/((b*c -
 a*d)*(a + b*x)) + 6*b*B*c*d*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*
Log[(a + b*x)/(c + d*x)])*(Log[a/b + x]^2 - 2*Log[a/b + x]*Log[a + b*x] - 2
*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[a + b*x]*((a*d)/(b*
c - a*d) + Log[c/d + x] + Log[(a + b*x)/(c + d*x)]) + 2*a*((a + b*x)^(-1) +
 Log[(a + b*x)/(c + d*x)]/(a + b*x) + (d*Log[c + d*x])/(-(b*c) + a*d)) - 2*
PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 6*B*d^2*n*(A + B*Log[e*((a + b*x)/
(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])*((a + b*x)*(-1 + Log[a/b + x]
) - a*Log[a/b + x]^2 - (a^2*(1 + Log[a/b + x]))/(a + b*x) - b*(c/d + x)*(-1
 + Log[c/d + x]) + (a^2*Log[c/d + x])/(a + b*x) + (b*x - a^2/(a + b*x) - 2*
a*Log[a + b*x])*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)]) +
 (a^2*d*(Log[a + b*x] - Log[c + d*x]))/(-(b*c) + a*d) + 2*a*(Log[c/d + x]*L
og[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))
+ B^2*d^2*n^2*(6*b*x - 6*(a + b*x)*Log[a/b + x] + 3*(a + b*x)*Log[a/b + x]^
2 - 2*a*Log[a/b + x]^3 - (3*a^2*(2 + 2*Log[a/b + x] + Log[a/b + x]^2))/(a +
 b*x) + (3*b*(2*d*x - 2*(c + d*x)*Log[c/d + x] + (c + d*x)*Log[c/d + x]^2))
/d + 3*(b*x - a^2/(a + b*x) - 2*a*Log[a + b*x])*(-Log[a/b + x] + Log[c/d +
x] + Log[(a + b*x)/(c + d*x)])^2 - (6*(a*d + 2*b*d*x - b*d*x*Log[c/d + x] -
 b*c*Log[c + d*x] + Log[a/b + x]*(-(d*(a + b*x)) + d*(a + b*x)*Log[c/d + x]
 + (b*c - a*d)*Log[(b*(c + d*x))/(b*c - a*d)]) + (b*c - a*d)*PolyLog[2, (d*
(a + b*x))/(-(b*c) + a*d)]))/d + (3*a^2*(d*(a + b*x)*Log[a/b + x]^2 + 2*((-
(b*c) + a*d)*Log[c/d + x] + d*(a + b*x)*(Log[a + b*x] - Log[c + d*x])) - 2*
Log[a/b + x]*((b*c - a*d)*Log[c/d + x] + d*(a + b*x)*Log[(b*(c + d*x))/(b*c
 - a*d)]) - 2*d*(a + b*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/((-(b*
c) + a*d)*(a + b*x)) + (3*a^2*(-(b*(c + d*x)*Log[c/d + x]^2) + 2*d*(a + b*x
)*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*d*(a + b*x)*PolyLog[2,
 (b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)*(a + b*x)) + 6*(-Log[a/b + x] +
Log[c/d + x] + Log[(a + b*x)/(c + d*x)])*((a + b*x)*(-1 + Log[a/b + x]) - a
*Log[a/b + x]^2 - (a^2*(1 + Log[a/b + x]))/(a + b*x) - b*(c/d + x)*(-1 + Lo
g[c/d + x]) + (a^2*Log[c/d + x])/(a + b*x) + (a^2*d*(Log[a + b*x] - Log[c +
 d*x]))/(-(b*c) + a*d) + 2*a*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)
] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + 6*a*(Log[a/b + x]^2*(Log[c/d
+ x] - Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Log[a/b + x]*PolyLog[2, (d*(a +
b*x))/(-(b*c) + a*d)] + 2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)]) - 6*a*(
Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[c/d + x]*PolyLog[2
, (b*(c + d*x))/(b*c - a*d)] - 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])) +
(2*b*B^2*c*d*n^2*((b*c - a*d)*(a + b*x)*Log[a/b + x]^3 + 3*a*(b*c - a*d)*(2
 + 2*Log[a/b + x] + Log[a/b + x]^2) + 3*(b*c - a*d)*(a + (a + b*x)*Log[a +
b*x])*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])^2 + 3*a*(d*
(a + b*x)*Log[a/b + x]^2 + 2*((-(b*c) + a*d)*Log[c/d + x] + d*(a + b*x)*(Lo
g[a + b*x] - Log[c + d*x])) - 2*Log[a/b + x]*((b*c - a*d)*Log[c/d + x] + d*
(a + b*x)*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*d*(a + b*x)*PolyLog[2, (d*(a
```

```
+ b*x))/(-(b*c) + a*d)]) + 3*a*(Log[c/d + x]*(b*(c + d*x)*Log[c/d + x] - 2*
d*(a + b*x)*Log[(d*(a + b*x))/(-(b*c) + a*d)]) - 2*d*(a + b*x)*PolyLog[2, (
b*(c + d*x))/(b*c - a*d)]) - 3*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)
/(c + d*x)])*((b*c - a*d)*(a + b*x)*Log[a/b + x]^2 + 2*a*(b*c - a*d)*(1 + L
og[a/b + x]) + 2*a*(-(b*c) + a*d)*Log[c/d + x] + 2*a*d*(a + b*x)*(Log[a + b
*x] - Log[c + d*x]) - 2*(b*c - a*d)*(a + b*x)*(Log[c/d + x]*Log[(d*(a + b*x
))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - 3*(b*c - a*d
)*(a + b*x)*(Log[a/b + x]^2*(Log[c/d + x] - Log[(b*(c + d*x))/(b*c - a*d)])
 - 2*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2*PolyLog[3, (
d*(a + b*x))/(-(b*c) + a*d)]) + 3*(b*c - a*d)*(a + b*x)*(Log[c/d + x]^2*Log
[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b
*c - a*d)] - 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])))/((b*c - a*d)*(a + b
*x))))/(3*b^3*g^2)
```

Maple [F]

$$\int \frac{(dix+ci)^2 \left(A+B\ln \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(bgx+ag)^2}dx$$

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

```
[Out] int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)^2\left(B\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x,
 algorithm="fricas")
```

```
[Out] integral((A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x^
2 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*
B*d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^
n))/(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx=\text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**2
,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x,
 algorithm="maxima")
[Out] -2*A*B*c^2*i^2*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)
*g^2) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - A^2*(a^2/(b^4*g^2*x + a*b^3
*g^2) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*g^2))*d^2*i^2 + 2*A^2*c*d*i^2*(
a/(b^3*g^2*x + a*b^2*g^2) + log(b*x + a)/(b^2*g^2)) - 2*A*B*c^2*i^2*log(e*(
b*x/(d*x + c) + a/(d*x + c))^n)/(b^2*g^2*x + a*b*g^2) - A^2*c^2*i^2/(b^2*g^
2*x + a*b*g^2) + (B^2*b^2*d^2*i^2*x^2 + B^2*a*b*d^2*i^2*x - (b^2*c^2*i^2 -
2*a*b*c*d*i^2 + a^2*d^2*i^2)*B^2 + 2*((b^2*c*d*i^2 - a*b*d^2*i^2)*B^2*x + (
a*b*c*d*i^2 - a^2*d^2*i^2)*B^2)*log(b*x + a))*log((d*x + c)^n)^2/(b^4*g^2*x
 + a*b^3*g^2) - integrate(-(B^2*b^3*c^3*i^2*log(e)^2 + (B^2*b^3*d^3*i^2*log
(e)^2 + 2*A*B*b^3*d^3*i^2*log(e))*x^3 + 3*(B^2*b^3*c*d^2*i^2*log(e)^2 + 2*A
*B*b^3*c*d^2*i^2*log(e))*x^2 + (B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x
^2 + 3*B^2*b^3*c^2*d*i^2*x + B^2*b^3*c^3*i^2)*log((b*x + a)^n)^2 + (3*B^2*b
^3*c^2*d*i^2*log(e)^2 + 4*A*B*b^3*c^2*d*i^2*log(e))*x + 2*(B^2*b^3*c^3*i^2*
log(e) + (B^2*b^3*d^3*i^2*log(e) + A*B*b^3*d^3*i^2)*x^3 + 3*(B^2*b^3*c*d^2*
i^2*log(e) + A*B*b^3*c*d^2*i^2)*x^2 + (3*B^2*b^3*c^2*d*i^2*log(e) + 2*A*B*b
^3*c^2*d*i^2)*x)*log((b*x + a)^n) - 2*((A*B*b^3*d^3*i^2 + (i^2*n + i^2*log(
e))*B^2*b^3*d^3)*x^3 - (a*b^2*c^2*d*i^2*n - 2*a^2*b*c*d^2*i^2*n + a^3*d^3*i
^2*n - b^3*c^3*i^2*log(e))*B^2 + (3*A*B*b^3*c*d^2*i^2 + (2*a*b^2*d^3*i^2*n
+ 3*b^3*c*d^2*i^2*log(e))*B^2)*x^2 + (2*A*B*b^3*c^2*d*i^2 + (2*a*b^2*c*d^2*
i^2*n - (i^2*n - 3*i^2*log(e))*b^3*c^2*d)*B^2)*x + 2*((b^3*c*d^2*i^2*n - a*
b^2*d^3*i^2*n)*B^2*x^2 + 2*(a*b^2*c*d^2*i^2*n - a^2*b*d^3*i^2*n)*B^2*x + (a
^2*b*c*d^2*i^2*n - a^3*d^3*i^2*n)*B^2)*log(b*x + a) + (B^2*b^3*d^3*i^2*x^3
+ 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i^2*x + B^2*b^3*c^3*i^2)*log((b
*x + a)^n))*log((d*x + c)^n))/(b^5*d*g^2*x^3 + a^2*b^3*c*g^2 + (b^5*c*g^2 +
 2*a*b^4*d*g^2)*x^2 + (2*a*b^4*c*g^2 + a^2*b^3*d*g^2)*x), x)
```

Giac [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x,
 algorithm="giac")
```

```
[Out] integrate((d*i*x + c*i)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x +
 a*g)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(a\,g+b\,g\,x)^2}\,dx$$

```
[In] int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x
)^2,x)
```

```
[Out] int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x
)^2, x)
```

3.174 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 417

$$\begin{aligned}\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx
&= -\frac{2B^2di^2n^2(c+dx)}{b^2g^3(a+bx)} - \frac{B^2i^2n^2(c+dx)^2}{4bg^3(a+bx)^2} - \frac{2Bdi^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g^3(a+bx)} \\
&\quad - \frac{Bi^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg^3(a+bx)^2} - \frac{di^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2g^3(a+bx)} \\
&\quad - \frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg^3(a+bx)^2} - \frac{d^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} \\
&\quad + \frac{2Bd^2i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} \\
&\quad + \frac{2B^2d^2i^2n^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3}\end{aligned}$$

```
[Out] -2*B^2*d*i^2*n^2*(d*x+c)/b^2/g^3/(b*x+a)-1/4*B^2*i^2*n^2*(d*x+c)^2/b/g^3/(b
*x+a)^2-2*B*d*i^2*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g^3/(b*x+a)
-1/2*B*i^2*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g^3/(b*x+a)^2-d*i^
2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2/g^3/(b*x+a)-1/2*i^2*(d*x+c)
^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b/g^3/(b*x+a)^2-d^2*i^2*(A+B*ln(e*((b*
x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g^3+2*B*d^2*i^2*n*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g^3+2*B^2*d^2*i
^2*n^2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^3/g^3
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2561, 2380, 2342, 2341, 2379, 2421, 6724}

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= \frac{2Bd^2i^2n\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3g^3}$$
$$-\frac{d^2i^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^3g^3}$$
$$-\frac{di^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^2g^3(a+bx)}-\frac{2Bdi^2n(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2g^3(a+bx)}$$
$$-\frac{i^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2bg^3(a+bx)^2}-\frac{Bi^2n(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2bg^3(a+bx)^2}$$
$$+\frac{2B^2d^2i^2n^2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3}-\frac{2B^2di^2n^2(c+dx)}{b^2g^3(a+bx)}-\frac{B^2i^2n^2(c+dx)^2}{4bg^3(a+bx)^2}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x
)^3,x]
```

```
[Out] (-2*B^2*d*i^2*n^2*(c + d*x))/(b^2*g^3*(a + b*x)) - (B^2*i^2*n^2*(c + d*x)^2
)/(4*b*g^3*(a + b*x)^2) - (2*B*d*i^2*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]))/(b^2*g^3*(a + b*x)) - (B*i^2*n*(c + d*x)^2*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/(2*b*g^3*(a + b*x)^2) - (d*i^2*(c + d*x)*(A + B*Log[e
*((a + b*x)/(c + d*x))^n])^2)/(b^2*g^3*(a + b*x)) - (i^2*(c + d*x)^2*(A + B
*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*b*g^3*(a + b*x)^2) - (d^2*i^2*(A + B
*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b
^3*g^3) + (2*B*d^2*i^2*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2,
(b*(c + d*x))/(d*(a + b*x))])/(b^3*g^3) + (2*B^2*d^2*i^2*n^2*PolyLog[3, (b*
(c + d*x))/(d*(a + b*x))])/(b^3*g^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
```

```
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{i^2\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^3(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^3}$$

$$= \frac{i^2 \text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^3} + \frac{(di^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^3}$$

$$= -\frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(di^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^3} + \frac{(d^2i^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^3} + \frac{(Bi^2n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^3}$$

$$= -\frac{B^2i^2n^2(c+dx)^2}{4bg^3(a+bx)^2} - \frac{Bi^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg^3(a+bx)^2} - \frac{di^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2g^3(a+bx)} - \frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg^3(a+bx)^2} - \frac{d^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} + \frac{(2Bdi^2n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^3} + \frac{(2Bd^2i^2n)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)(A+B\log(ex^n))}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^3}$$

$$= -\frac{2B^2di^2n^2(c+dx)}{b^2g^3(a+bx)} - \frac{B^2i^2n^2(c+dx)^2}{4bg^3(a+bx)^2} - \frac{2Bdi^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g^3(a+bx)} - \frac{Bi^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg^3(a+bx)^2} - \frac{di^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2g^3(a+bx)} - \frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg^3(a+bx)^2} - \frac{d^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} + \frac{2Bd^2i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} - \frac{(2B^2d^2i^2n^2)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^3}$$

$$
\begin{aligned}
&= -\frac{2B^2di^2n^2(c+dx)}{b^2g^3(a+bx)} - \frac{B^2i^2n^2(c+dx)^2}{4bg^3(a+bx)^2} - \frac{2Bdi^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g^3(a+bx)} \\
&\quad - \frac{Bi^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg^3(a+bx)^2} - \frac{di^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2g^3(a+bx)} \\
&\quad - \frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg^3(a+bx)^2} - \frac{d^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} \\
&\quad + \frac{2Bd^2i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\mathrm{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} + \frac{2B^2d^2i^2n^2\mathrm{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3662 vs. $2(417) = 834$.

Time = 4.48 (sec) , antiderivative size = 3662, normalized size of antiderivative = 8.78

$$
\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g +
 b*g*x)^3,x]
```

```
[Out] (i^2*((-6*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a
+ b*x)/(c + d*x)])^2)/(a + b*x)^2 + (24*d*(-(b*c) + a*d)*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2)/(a + b*x) + 12*d^2*L
og[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c +
d*x)])^2 + (6*b^2*B*c^2*n*(b^2*c^2 - 4*a*b*c*d + a^2*d^2 - 2*b^2*c*d*x - 2*
a*b*d^2*x - 2*b^2*d^2*x^2 + 2*d^2*(a + b*x)^2*Log[c/d + x] - 2*d^2*(a + b*x
)^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*b^2*c^2*Log[(a + b*x)/(c + d*x)]
- 4*a*b*c*d*Log[(a + b*x)/(c + d*x)] + 2*a^2*d^2*Log[(a + b*x)/(c + d*x)])*
(-A - B*Log[e*((a + b*x)/(c + d*x))^n] + B*n*Log[(a + b*x)/(c + d*x)]))/((b
*c - a*d)^2*(a + b*x)^2) + (12*b*B*c*d*n*(-A - B*Log[e*((a + b*x)/(c + d*x)
)^n] + B*n*Log[(a + b*x)/(c + d*x)])*(3*a*b^2*c^2 - 4*a^2*b*c*d + a^3*d^2 +
 4*b^3*c^2*x - 6*a*b^2*c*d*x + 2*a^2*b*d^2*x - 2*d*(-2*b*c + a*d)*(a + b*x)
^2*Log[a + b*x] + 2*(b*c - a*d)^2*(a + 2*b*x)*Log[(a + b*x)/(c + d*x)] - 4*
a^2*b*c*d*Log[c + d*x] + 2*a^3*d^2*Log[c + d*x] - 8*a*b^2*c*d*x*Log[c + d*x
] + 4*a^2*b*d^2*x*Log[c + d*x] - 4*b^3*c*d*x^2*Log[c + d*x] + 2*a*b^2*d^2*x
^2*Log[c + d*x]))/((b*c - a*d)^2*(a + b*x)^2) + (3*b^2*B^2*c^2*n^2*(-(b*c -
 a*d)^2 + 6*d*(b*c - a*d)*(a + b*x) + 6*d^2*(a + b*x)^2*Log[a + b*x] - 2*d^
2*(a + b*x)^2*Log[a + b*x]^2 - 2*(b*c - a*d)^2*Log[(a + b*x)/(c + d*x)] + 4
*d*(b*c - a*d)*(a + b*x)*Log[(a + b*x)/(c + d*x)] + 4*d^2*(a + b*x)^2*Log[a
 + b*x]*Log[(a + b*x)/(c + d*x)] - 2*(b*c - a*d)^2*Log[(a + b*x)/(c + d*x)]
^2 - 6*d^2*(a + b*x)^2*Log[c + d*x] + 4*d^2*(a + b*x)^2*Log[a + b*x]*Log[(b
*(c + d*x))/(b*c - a*d)] - 4*d^2*(a + b*x)^2*Log[(d*(a + b*x))/(-(b*c) + a*
```

```
d)]*Log[(b*c - a*d)/(b*c + b*d*x)] + 4*d^2*(a + b*x)^2*Log[(a + b*x)/(c + d
*x)]*Log[(b*c - a*d)/(b*c + b*d*x)] - 2*d^2*(a + b*x)^2*Log[(b*c - a*d)/(b*
c + b*d*x)]^2 + 4*d^2*(a + b*x)^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]
+ 4*d^2*(a + b*x)^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)^2*
(a + b*x)^2) + 6*B*d^2*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a
 + b*x)/(c + d*x)])*(2*Log[a/b + x]^2 + (8*a*(1 + Log[a/b + x]))/(a + b*x)
- (a^2*(1 + 2*Log[a/b + x]))/(a + b*x)^2 + 2*((a*(3*a + 4*b*x))/(a + b*x)^2
 + 2*Log[a + b*x])*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)]
) + (8*a*((-(b*c) + a*d)*Log[c/d + x] + d*(a + b*x)*(Log[a + b*x] - Log[c +
 d*x])))/((b*c - a*d)*(a + b*x)) + (2*a^2*(Log[c/d + x] + (d*(a + b*x)*(b*c
 - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]))/(b*c - a*d)^
2))/(a + b*x)^2 - 4*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyL
og[2, (b*(c + d*x))/(b*c - a*d)])) + (6*b*B^2*c*d*n^2*(-4*(b*c - a*d)^2*(a
+ b*x)*(2 + 2*Log[a/b + x] + Log[a/b + x]^2) + a*(b*c - a*d)^2*(1 + 2*Log[a
/b + x] + 2*Log[a/b + x]^2) - 2*(b*c - a*d)^2*(a + 2*b*x)*(-Log[a/b + x] +
Log[c/d + x] + Log[(a + b*x)/(c + d*x)])^2 + 2*(Log[a/b + x] - Log[c/d + x]
 - Log[(a + b*x)/(c + d*x)])*(4*(b*c - a*d)^2*(a + b*x)*(1 + Log[a/b + x])
- a*(b*c - a*d)^2*(1 + 2*Log[a/b + x]) - 4*(b*c - a*d)*(a + b*x)*((b*c - a*
d)*Log[c/d + x] - d*(a + b*x)*(Log[a + b*x] - Log[c + d*x])) + 2*a*((b*c -
a*d)^2*Log[c/d + x] + d*(a + b*x)*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d
*(a + b*x)*Log[c + d*x]))) - 4*(b*c - a*d)*(a + b*x)*(d*(a + b*x)*Log[a/b +
 x]^2 + 2*((-(b*c) + a*d)*Log[c/d + x] + d*(a + b*x)*(Log[a + b*x] - Log[c
+ d*x])) - 2*Log[a/b + x]*((b*c - a*d)*Log[c/d + x] + d*(a + b*x)*Log[(b*(c
 + d*x))/(b*c - a*d)]) - 2*d*(a + b*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a
*d)]) + 2*a*(d*(-(b*c) + a*d)*(a + b*x) - (b*c - a*d)^2*(1 + 2*Log[a/b + x]
)*Log[c/d + x] - d^2*(a + b*x)^2*Log[a + b*x] + d^2*(a + b*x)^2*Log[c + d*x
] - d*(a + b*x)*(d*(a + b*x)*Log[a/b + x]^2 + 2*(b*c - a*d)*(1 + Log[a/b +
x]) - 2*d*(a + b*x)*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[
2, (d*(a + b*x))/(-(b*c) + a*d)]))) - 4*(b*c - a*d)*(a + b*x)*(Log[c/d + x]
*(b*(c + d*x)*Log[c/d + x] - 2*d*(a + b*x)*Log[(d*(a + b*x))/(-(b*c) + a*d)
]) - 2*d*(a + b*x)*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 2*a*((b*c - a*d
)^2*Log[c/d + x]^2 - d*(a + b*x)*(d*(a + b*x)*Log[c/d + x]^2 - 2*Log[c/d +
x]*(b*c - a*d + d*(a + b*x)*Log[(d*(a + b*x))/(-(b*c) + a*d)]) + 2*d*(a + b
*x)*(Log[a + b*x] - Log[c + d*x]) - 2*d*(a + b*x)*PolyLog[2, (b*(c + d*x))/
(b*c - a*d)]))))/((b*c - a*d)^2*(a + b*x)^2) + B^2*d^2*n^2*(4*Log[a/b + x]^
3 + (24*a*(2 + 2*Log[a/b + x] + Log[a/b + x]^2))/(a + b*x) - (3*a^2*(1 + 2*
Log[a/b + x] + 2*Log[a/b + x]^2))/(a + b*x)^2 + (6*(a*(3*a + 4*b*x) + 2*(a
+ b*x)^2*Log[a + b*x])*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d
*x)])^2)/(a + b*x)^2 + (24*a*(-(b*(c + d*x)*Log[c/d + x]^2) + 2*d*(a + b*x)
*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*d*(a + b*x)*PolyLog[2,
(b*(c + d*x))/(b*c - a*d)]))/((-(b*c) + a*d)*(a + b*x)) + 6*(-Log[a/b + x]
+ Log[c/d + x] + Log[(a + b*x)/(c + d*x)])*(2*Log[a/b + x]^2 + (8*a*(1 + Lo
g[a/b + x]))/(a + b*x) - (a^2*(1 + 2*Log[a/b + x]))/(a + b*x)^2 + (2*a^2*Lo
g[c/d + x])/(a + b*x)^2 - (8*a*Log[c/d + x])/(a + b*x) + (8*a*d*(-Log[a + b
*x] + Log[c + d*x]))/(-(b*c) + a*d) + (2*a^2*d*(b*c - a*d + d*(a + b*x)*Log
```

```
[a + b*x] - d*(a + b*x)*Log[c + d*x]))/((b*c - a*d)^2*(a + b*x)) - 4*(Log[c
/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c -
 a*d)])) + (6*a^2*(-Log[c/d + x]^2 + (d*(a + b*x)*(d*(a + b*x)*Log[c/d + x]
^2 - 2*Log[c/d + x]*(b*c - a*d + d*(a + b*x)*Log[(d*(a + b*x))/(-(b*c) + a*
d)]) + 2*d*(a + b*x)*(Log[a + b*x] - Log[c + d*x]) - 2*d*(a + b*x)*PolyLog[
2, (b*(c + d*x))/(b*c - a*d)]))/(b*c - a*d)^2))/(a + b*x)^2 + 6*(2*Log[a/b
+ x]^2*(-Log[c/d + x] + Log[(b*(c + d*x))/(b*c - a*d)]) + 4*Log[a/b + x]*Po
lyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + (4*a*(d*(a + b*x)*Log[a/b + x]^2 +
 2*((-(b*c) + a*d)*Log[c/d + x] + d*(a + b*x)*(Log[a + b*x] - Log[c + d*x])
) - 2*Log[a/b + x]*((b*c - a*d)*Log[c/d + x] + d*(a + b*x)*Log[(b*(c + d*x)
)/(b*c - a*d)]) - 2*d*(a + b*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/
((b*c - a*d)*(a + b*x)) + (a^2*(-(d*(-(b*c) + a*d)*(a + b*x)) + (b*c - a*d)
^2*(1 + 2*Log[a/b + x])*Log[c/d + x] + d^2*(a + b*x)^2*Log[a + b*x] - d^2*(
a + b*x)^2*Log[c + d*x] + d*(a + b*x)*(d*(a + b*x)*Log[a/b + x]^2 + 2*(b*c
- a*d)*(1 + Log[a/b + x]) - 2*d*(a + b*x)*(Log[a/b + x]*Log[(b*(c + d*x))/(
b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))))/((b*c - a*d)^2*(
a + b*x)^2) - 4*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)]) + 12*(Log[c/d + x
]^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[c/d + x]*PolyLog[2, (b*(c + d
*x))/(b*c - a*d)] - 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]))))/(12*b^3*g^3
)
```

Maple [F]

$$\int \frac{(dix + ci)^2 \left(A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(bgx + ag)^3}\,dx$$

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x)
[Out] int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x)
```

Fricas [F]

$$\int \frac{(ci + dix)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^3}\,dx = \int \frac{(dix + ci)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}{(bgx + ag)^3}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x,
 algorithm="fricas")
[Out] integral((A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x^
2 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*
B*d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^
n))/(b^3*g^3*x^3 + 3*a*b^2*g^3*x^2 + 3*a^2*b*g^3*x + a^3*g^3), x)
```

Sympy [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= \frac{i^2\left(\int \frac{A^2c^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{A^2d^2x^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{B^2c^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{2ABc^2\log\left(e\left(\frac{a}{c+dx}+\right.\right.}{a^3+3a^2bx+3ab^2x^2}\right.}{}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**3
,x)
```

```
[Out] i**2*(Integral(A**2*c**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x
) + Integral(A**2*d**2*x**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)
, x) + Integral(B**2*c**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a**3
+ 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*c**2*log(e*(
a/(c + d*x) + b*x/(c + d*x))**n)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*
x**3), x) + Integral(2*A**2*c*d*x/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3
*x**3), x) + Integral(B**2*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n
)**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*d
**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**3 + 3*a**2*b*x + 3*a*b
**2*x**2 + b**3*x**3), x) + Integral(2*B**2*c*d*x*log(e*(a/(c + d*x) + b*x/
(c + d*x))**n)**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Int
egral(4*A*B*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**3 + 3*a**2*b*
x + 3*a*b**2*x**2 + b**3*x**3), x))/g**3
```

Maxima [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x,
 algorithm="maxima")
```

```
[Out] -A*B*c*d*i^2*n*((3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*x)/((b^5*c - a*b^4*d
)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3) +
2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3
) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)
*g^3)) + 1/2*A*B*c^2*i^2*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*
x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*lo
g(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/(
(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) + 1/2*A^2*d^2*i^2*((4*a*b*x + 3*a
```

```
^2)/(b^5*g^3*x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(b^3*g^3))
 - 2*(2*b*x + a)*A*B*c*d*i^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^4*g^
3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) - (2*b*x + a)*A^2*c*d*i^2/(b^4*g^3*x^2
 + 2*a*b^3*g^3*x + a^2*b^2*g^3) - A*B*c^2*i^2*log(e*(b*x/(d*x + c) + a/(d*x
 + c))^n)/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2*c^2*i^2/(b^3*
g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*(4*(b^2*c*d*i^2 - a*b*d^2*i^2)*B
^2*x + (b^2*c^2*i^2 + 2*a*b*c*d*i^2 - 3*a^2*d^2*i^2)*B^2 - 2*(B^2*b^2*d^2*i
^2*x^2 + 2*B^2*a*b*d^2*i^2*x + B^2*a^2*d^2*i^2)*log(b*x + a))*log((d*x + c)
^n)^2/(b^5*g^3*x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) - integrate(-(3*B^2*b^3*c
^2*d*i^2*x*log(e)^2 + B^2*b^3*c^3*i^2*log(e)^2 + (B^2*b^3*d^3*i^2*log(e)^2
+ 2*A*B*b^3*d^3*i^2*log(e))*x^3 + (3*B^2*b^3*c*d^2*i^2*log(e)^2 + 2*A*B*b^3
*c*d^2*i^2*log(e))*x^2 + (B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3
*B^2*b^3*c^2*d*i^2*x + B^2*b^3*c^3*i^2)*log((b*x + a)^n)^2 + 2*(3*B^2*b^3*c
^2*d*i^2*x*log(e) + B^2*b^3*c^3*i^2*log(e) + (B^2*b^3*d^3*i^2*log(e) + A*B*
b^3*d^3*i^2)*x^3 + (3*B^2*b^3*c*d^2*i^2*log(e) + A*B*b^3*c*d^2*i^2)*x^2)*lo
g((b*x + a)^n) + ((6*a*b^2*c*d^2*i^2*n - 7*a^2*b*d^3*i^2*n + (i^2*n - 6*i^2
*log(e))*b^3*c^2*d)*B^2*x - 2*(B^2*b^3*d^3*i^2*log(e) + A*B*b^3*d^3*i^2)*x^
3 + (a*b^2*c^2*d*i^2*n + 2*a^2*b*c*d^2*i^2*n - 3*a^3*d^3*i^2*n - 2*b^3*c^3*
i^2*log(e))*B^2 - 2*(A*B*b^3*c*d^2*i^2 + (2*a*b^2*d^3*i^2*n - (2*i^2*n - 3*
i^2*log(e))*b^3*c*d^2)*B^2)*x^2 - 2*(B^2*b^3*d^3*i^2*n*x^3 + 3*B^2*a*b^2*d^
3*i^2*n*x^2 + 3*B^2*a^2*b*d^3*i^2*n*x + B^2*a^3*d^3*i^2*n)*log(b*x + a) - 2
*(B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i^2*x + B
^2*b^3*c^3*i^2)*log((b*x + a)^n))*log((d*x + c)^n))/(b^6*d*g^3*x^4 + a^3*b^
3*c*g^3 + (b^6*c*g^3 + 3*a*b^5*d*g^3)*x^3 + 3*(a*b^5*c*g^3 + a^2*b^4*d*g^3)
*x^2 + (3*a^2*b^4*c*g^3 + a^3*b^3*d*g^3)*x), x)
```

Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x,
 algorithm="giac")
```

```
[Out] integrate((d*i*x + c*i)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x +
 a*g)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx=\int \frac{{\left(c\,i+d\,i\,x\right)}^2\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2}{{\left(a\,g+b\,g\,x\right)}^3}\,dx$$

```
[In] int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x
)^3,x)
[Out] int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x
)^3, x)
```

3.175 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 157

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{2B^2i^2n^2(c+dx)^3}{27(bc-ad)g^4(a+bx)^3} - \frac{2Bi^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)g^4(a+bx)^3} - \frac{i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)g^4(a+bx)^3}$$

```
[Out] -2/27*B^2*i^2*n^2*(d*x+c)^3/(-a*d+b*c)/g^4/(b*x+a)^3-2/9*B*i^2*n*(d*x+c)^3*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/g^4/(b*x+a)^3-1/3*i^2*(d*x+c)^3*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/g^4/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2561, 2342, 2341}

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{i^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g^4(a+bx)^3(bc-ad)} - \frac{2Bi^2n(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9g^4(a+bx)^3(bc-ad)} - \frac{2B^2i^2n^2(c+dx)^3}{27g^4(a+bx)^3(bc-ad)}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x
)^4,x]
```

```
[Out] (-2*B^2*i^2*n^2*(c + d*x)^3)/(27*(b*c - a*d)*g^4*(a + b*x)^3) - (2*B*i^2*n*
(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(b*c - a*d)*g^4*(a +
 b*x)^3) - (i^2*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*(b
*c - a*d)*g^4*(a + b*x)^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{i^2 \text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)g^4}$$

$$= -\frac{i^2(c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)g^4(a+bx)^3} + \frac{(2Bi^2n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bc-ad)g^4}$$

$$= -\frac{2B^2i^2n^2(c+dx)^3}{27(bc-ad)g^4(a+bx)^3} - \frac{2Bi^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)g^4(a+bx)^3} - \frac{i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)g^4(a+bx)^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.18 (sec) , antiderivative size = 1418, normalized size of antiderivative = 9.03

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{i^2\left(18(bc-ad)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+54d(bc-ad)^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-54d^2(-bc\right.}{}$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g +
 b*g*x)^4,x]
```

```
[Out] -1/54*(i^2*(18*(b*c - a*d)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 54*
d*(b*c - a*d)^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - 54*d^2
*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + B*n*
(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3*n - 18*A*d*(b*c - a*d)^2*(a + b*x)
- 15*B*d*(b*c - a*d)^2*n*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*
B*d^2*(b*c - a*d)*n*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x] + 66*B*
d^3*n*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*n*(a + b*x)^3*Log[a + b*x]^2 + 12
*B*(b*c - a*d)^3*Log[e*((a + b*x)/(c + d*x))^n] - 18*B*d*(b*c - a*d)^2*(a +
 b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d^2*(b*c - a*d)*(a + b*x)^2*Log
[e*((a + b*x)/(c + d*x))^n] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[e*((a +
 b*x)/(c + d*x))^n] - 36*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3*n*(a + b
*x)^3*Log[c + d*x] + 36*B*d^3*n*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d
)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[e*((a + b*x)/(c + d*x))^n]*Log[c
 + d*x] - 18*B*d^3*n*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*n*(a + b*x)^3*Lo
g[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*n*(a + b*x)^3*PolyLog[
2, (d*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3*n*(a + b*x)^3*PolyLog[2, (b*(c
+ d*x))/(b*c - a*d)]) + 54*B*d^2*n*(a + b*x)^2*(2*(b*c - a*d)*(A + B*Log[e*
((a + b*x)/(c + d*x))^n]) + 2*d*(a + b*x)*Log[a + b*x]*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]) - 2*d*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*L
og[c + d*x] + 2*B*n*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log
[c + d*x]) - B*d*n*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*
x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*n*(a
+ b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] +
 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + 27*B*d*n*(a + b*x)*(2*(b*c - a
*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*d*(-(b*c) + a*d)*(a + b*x)
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 4*d^2*(a + b*x)^2*Log[a + b*x]*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*d^2*(a + b*x)^2*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n])*Log[c + d*x] - 4*B*d*n*(a + b*x)*(b*c - a*d + d*(a + b
*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) + B*n*((b*c - a*d)^2 + 2*d*(-(
b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*
Log[c + d*x]) + 2*B*d^2*n*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(
```

```
b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) -
2*B*d^2*n*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])
*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))))/(b^3*(b*c - a*d
)*g^4*(a + b*x)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 824 vs. $2(151) = 302$.

Time = 9.98 (sec) , antiderivative size = 825, normalized size of antiderivative = 5.25

method	result
parallelrisch	$-\frac{-54AB\,x^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^5c\,d^3i^2n-54ABx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^5c^2d^2i^2n-9B^2x^3\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2b^5d^4i^2n-6B^2x^3\ln\left(e\left(\frac{bx+a}{dx+c}\right.\right.}{}$

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x,method=
_RETURNVERBOSE)
```

```
[Out] -1/27*(-54*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c*d^3*i^2*n-54*A*B*x*ln(e*
((b*x+a)/(d*x+c))^n)*b^5*c^2*d^2*i^2*n-9*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^
2*b^5*d^4*i^2*n-6*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^5*d^4*i^2*n^2+6*B^2*x
^2*a*b^4*d^4*i^2*n^3-6*B^2*x^2*b^5*c*d^3*i^2*n^3+6*B^2*x*a^2*b^3*d^4*i^2*n^
3-6*B^2*x*b^5*c^2*d^2*i^2*n^3+27*A^2*x^2*a*b^4*d^4*i^2*n-27*A^2*x^2*b^5*c*d
^3*i^2*n-9*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^5*c^3*d*i^2*n-18*A*B*x^3*ln(e*
((b*x+a)/(d*x+c))^n)*b^5*d^4*i^2*n-27*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b
^5*c*d^3*i^2*n-18*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c*d^3*i^2*n^2+18*A*
B*x^2*a*b^4*d^4*i^2*n^2-18*A*B*x^2*b^5*c*d^3*i^2*n^2-27*B^2*x*ln(e*((b*x+a)
/(d*x+c))^n)^2*b^5*c^2*d^2*i^2*n-18*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^2
*d^2*i^2*n^2+18*A*B*x*a^2*b^3*d^4*i^2*n^2-18*A*B*x*b^5*c^2*d^2*i^2*n^2-18*A
*B*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^3*d*i^2*n+6*A*B*a^3*b^2*d^4*i^2*n^2-6*A*
B*b^5*c^3*d*i^2*n^2-9*A^2*b^5*c^3*d*i^2*n-6*B^2*ln(e*((b*x+a)/(d*x+c))^n)*b
^5*c^3*d*i^2*n^2+27*A^2*x*a^2*b^3*d^4*i^2*n-27*A^2*x*b^5*c^2*d^2*i^2*n+2*B^
2*a^3*b^2*d^4*i^2*n^3-2*B^2*b^5*c^3*d*i^2*n^3+9*A^2*a^3*b^2*d^4*i^2*n)/g^4/
(b*x+a)^3/b^5/d/n/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 975 vs. $2(151) = 302$.

Time = 0.35 (sec) , antiderivative size = 975, normalized size of antiderivative = 6.21

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{2\,(B^2b^3c^3-B^2a^3d^3)i^2n^2+6\,(ABb^3c^3-ABa^3d^3)i^2n+9\,(A^2b^3c^3-A^2a^3d^3)i^2+3\,(2\,(B^2b^3cd^2-B^2ab^2d^{\cdots}}{}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x,
 algorithm="fricas")
```

```
[Out] -1/27*(2*(B^2*b^3*c^3 - B^2*a^3*d^3)*i^2*n^2 + 6*(A*B*b^3*c^3 - A*B*a^3*d^3
)*i^2*n + 9*(A^2*b^3*c^3 - A^2*a^3*d^3)*i^2 + 3*(2*(B^2*b^3*c*d^2 - B^2*a*b
^2*d^3)*i^2*n^2 + 6*(A*B*b^3*c*d^2 - A*B*a*b^2*d^3)*i^2*n + 9*(A^2*b^3*c*d^
2 - A^2*a*b^2*d^3)*i^2)*x^2 + 9*(3*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*i^2*x^2
+ 3*(B^2*b^3*c^2*d - B^2*a^2*b*d^3)*i^2*x + (B^2*b^3*c^3 - B^2*a^3*d^3)*i^2
)*log(e)^2 + 9*(B^2*b^3*d^3*i^2*n^2*x^3 + 3*B^2*b^3*c*d^2*i^2*n^2*x^2 + 3*B
^2*b^3*c^2*d*i^2*n^2*x + B^2*b^3*c^3*i^2*n^2)*log((b*x + a)/(d*x + c))^2 +
3*(2*(B^2*b^3*c^2*d - B^2*a^2*b*d^3)*i^2*n^2 + 6*(A*B*b^3*c^2*d - A*B*a^2*b
*d^3)*i^2*n + 9*(A^2*b^3*c^2*d - A^2*a^2*b*d^3)*i^2)*x + 6*((B^2*b^3*c^3 -
B^2*a^3*d^3)*i^2*n + 3*(A*B*b^3*c^3 - A*B*a^3*d^3)*i^2 + 3*((B^2*b^3*c*d^2
- B^2*a*b^2*d^3)*i^2*n + 3*(A*B*b^3*c*d^2 - A*B*a*b^2*d^3)*i^2)*x^2 + 3*((B
^2*b^3*c^2*d - B^2*a^2*b*d^3)*i^2*n + 3*(A*B*b^3*c^2*d - A*B*a^2*b*d^3)*i^2
)*x + 3*(B^2*b^3*d^3*i^2*n*x^3 + 3*B^2*b^3*c*d^2*i^2*n*x^2 + 3*B^2*b^3*c^2*
d*i^2*n*x + B^2*b^3*c^3*i^2*n)*log((b*x + a)/(d*x + c)))*log(e) + 6*(B^2*b^
3*c^3*i^2*n^2 + 3*A*B*b^3*c^3*i^2*n + (B^2*b^3*d^3*i^2*n^2 + 3*A*B*b^3*d^3*
i^2*n)*x^3 + 3*(B^2*b^3*c*d^2*i^2*n^2 + 3*A*B*b^3*c*d^2*i^2*n)*x^2 + 3*(B^2
*b^3*c^2*d*i^2*n^2 + 3*A*B*b^3*c^2*d*i^2*n)*x)*log((b*x + a)/(d*x + c)))/((
b^7*c - a*b^6*d)*g^4*x^3 + 3*(a*b^6*c - a^2*b^5*d)*g^4*x^2 + 3*(a^2*b^5*c -
 a^3*b^4*d)*g^4*x + (a^3*b^4*c - a^4*b^3*d)*g^4)
```

Sympy [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$$

$$= \frac{i^2\left(\int \frac{A^2c^2}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx + \int \frac{A^2d^2x^2}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx + \int \frac{B^2c^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx + \right.}{}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**4
,x)
```

```
[Out] i**2*(Integral(A**2*c**2/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x
**3 + b**4*x**4), x) + Integral(A**2*d**2*x**2/(a**4 + 4*a**3*b*x + 6*a**2*
b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(B**2*c**2*log(e*(a/(c
 + d*x) + b*x/(c + d*x))**n)**2/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a
*b**3*x**3 + b**4*x**4), x) + Integral(2*A*B*c**2*log(e*(a/(c + d*x) + b*x/
(c + d*x))**n)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4
*x**4), x) + Integral(2*A**2*c*d*x/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 +
4*a*b**3*x**3 + b**4*x**4), x) + Integral(B**2*d**2*x**2*log(e*(a/(c + d*x)
 + b*x/(c + d*x))**n)**2/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x
```

```
**3 + b**4*x**4), x) + Integral(2*A*B*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c
 + d*x))**n)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x
**4), x) + Integral(2*B**2*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2
/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + I
ntegral(4*A*B*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**4 + 4*a**3*
b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x))/g**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5588 vs. $2(151) = 302$.

Time = 0.52 (sec) , antiderivative size = 5588, normalized size of antiderivative = 35.59

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x,
 algorithm="maxima")
```

```
[Out] -1/9*A*B*d^2*i^2*n*((11*a^2*b^2*c^2 - 7*a^3*b*c*d + 2*a^4*d^2 + 6*(3*b^4*c^
2 - 3*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 3*(9*a*b^3*c^2 - 7*a^2*b^2*c*d + 2*a^3
*b*d^2)*x)/((b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*g^4*x^3 + 3*(a*b^7*c^2 -
2*a^2*b^6*c*d + a^3*b^5*d^2)*g^4*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d + a^4
*b^4*d^2)*g^4*x + (a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2)*g^4) + 6*(3*b
^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^2*d +
3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4) - 6*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^
3)*log(d*x + c)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*
g^4)) - 1/9*A*B*c^2*i^2*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*
d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4
*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2
 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*
b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^
2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*
b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/9*A*B*c*d*i^2*n*((5*a*b^2*c^2 - 22*a^2*b*c
*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c^2 - 16*a*b^2*c*
d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3 + 3*(a*b
^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*
c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4)
 - 6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3
*c*d^2 - a^3*b^2*d^3)*g^4) + 6*(3*b*c*d^2 - a*d^3)*log(d*x + c)/((b^5*c^3 -
 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4)) - 1/3*(3*b*x + a)*B^2
*c*d*i^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2/(b^5*g^4*x^3 + 3*a*b^4*g^
4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - 1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*B^
2*d^2*i^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2/(b^6*g^4*x^3 + 3*a*b^5*g
^4*x^2 + 3*a^2*b^4*g^4*x + a^3*b^3*g^4) - 1/54*(6*n*((6*b^2*d^2*x^2 + 2*b^2
```

```
*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*
b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2
)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*
c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*
b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^
3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4))*log(e*(b*x/(d*x + c)
 + a/(d*x + c))^n) + (4*b^3*c^3 - 27*a*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3
*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 +
 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^
2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*c*d
^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x +
a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*d^3
*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^
3)*log(b*x + a))*log(d*x + c))*n^2/(a^3*b^4*c^3*g^4 - 3*a^4*b^3*c^2*d*g^4 +
 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^4 + (b^7*c^3*g^4 - 3*a*b^6*c^2*d*g^4 + 3
*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g^4)*x^3 + 3*(a*b^6*c^3*g^4 - 3*a^2*b^5*c^
2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 - a^4*b^3*d^3*g^4)*x^2 + 3*(a^2*b^5*c^3*g^4 -
 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^3*c*d^2*g^4 - a^5*b^2*d^3*g^4)*x))*B^2*c^2*i
^2 - 1/54*(6*n*((5*a*b^2*c^2 - 22*a^2*b*c*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*
b^2*d^2)*x^2 + 3*(3*b^3*c^2 - 16*a*b^2*c*d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*
a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d
^2)*g^4*x^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^
4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4) - 6*(3*b*c*d^2 - a*d^3)*log(b*x +
 a)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4) + 6*(3*
b*c*d^2 - a*d^3)*log(d*x + c)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 -
 a^3*b^2*d^3)*g^4))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + (19*a*b^3*c^3
- 189*a^2*b^2*c^2*d + 189*a^3*b*c*d^2 - 19*a^4*d^3 - 6*(27*b^4*c^2*d - 32*a
*b^3*c*d^2 + 5*a^2*b^2*d^3)*x^2 + 18*(3*a^3*b*c*d^2 - a^4*d^3 + (3*b^4*c*d^
2 - a*b^3*d^3)*x^3 + 3*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^2 + 3*(3*a^2*b^2*c*d
^2 - a^3*b*d^3)*x)*log(b*x + a)^2 + 18*(3*a^3*b*c*d^2 - a^4*d^3 + (3*b^4*c*
d^2 - a*b^3*d^3)*x^3 + 3*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^2 + 3*(3*a^2*b^2*c
*d^2 - a^3*b*d^3)*x)*log(d*x + c)^2 + 3*(9*b^4*c^3 - 125*a*b^3*c^2*d + 135*
a^2*b^2*c*d^2 - 19*a^3*b*d^3)*x - 6*(27*a^3*b*c*d^2 - 5*a^4*d^3 + (27*b^4*c
*d^2 - 5*a*b^3*d^3)*x^3 + 3*(27*a*b^3*c*d^2 - 5*a^2*b^2*d^3)*x^2 + 3*(27*a^
2*b^2*c*d^2 - 5*a^3*b*d^3)*x)*log(b*x + a) + 6*(27*a^3*b*c*d^2 - 5*a^4*d^3
+ (27*b^4*c*d^2 - 5*a*b^3*d^3)*x^3 + 3*(27*a*b^3*c*d^2 - 5*a^2*b^2*d^3)*x^2
 + 3*(27*a^2*b^2*c*d^2 - 5*a^3*b*d^3)*x - 6*(3*a^3*b*c*d^2 - a^4*d^3 + (3*b
^4*c*d^2 - a*b^3*d^3)*x^3 + 3*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^2 + 3*(3*a^2*
b^2*c*d^2 - a^3*b*d^3)*x)*log(b*x + a))*log(d*x + c))*n^2/(a^3*b^5*c^3*g^4
- 3*a^4*b^4*c^2*d*g^4 + 3*a^5*b^3*c*d^2*g^4 - a^6*b^2*d^3*g^4 + (b^8*c^3*g^
4 - 3*a*b^7*c^2*d*g^4 + 3*a^2*b^6*c*d^2*g^4 - a^3*b^5*d^3*g^4)*x^3 + 3*(a*b
^7*c^3*g^4 - 3*a^2*b^6*c^2*d*g^4 + 3*a^3*b^5*c*d^2*g^4 - a^4*b^4*d^3*g^4)*x
^2 + 3*(a^2*b^6*c^3*g^4 - 3*a^3*b^5*c^2*d*g^4 + 3*a^4*b^4*c*d^2*g^4 - a^5*b
^3*d^3*g^4)*x))*B^2*c*d*i^2 - 1/54*(6*n*((11*a^2*b^2*c^2 - 7*a^3*b*c*d + 2*
a^4*d^2 + 6*(3*b^4*c^2 - 3*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 3*(9*a*b^3*c^2 -
```

7*a^2*b^2*c*d + 2*a^3*b*d^2)*x)/((b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*g^4*
x^3 + 3*(a*b^7*c^2 - 2*a^2*b^6*c*d + a^3*b^5*d^2)*g^4*x^2 + 3*(a^2*b^6*c^2
- 2*a^3*b^5*c*d + a^4*b^4*d^2)*g^4*x + (a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b
^3*d^2)*g^4) + 6*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(b*x + a)/((b^6*c
^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4) - 6*(3*b^2*c^2*d -
3*a*b*c*d^2 + a^2*d^3)*log(d*x + c)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*
c*d^2 - a^3*b^3*d^3)*g^4))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + (85*a^2
*b^3*c^3 - 108*a^3*b^2*c^2*d + 27*a^4*b*c*d^2 - 4*a^5*d^3 + 6*(18*b^5*c^3 -
27*a*b^4*c^2*d + 11*a^2*b^3*c*d^2 - 2*a^3*b^2*d^3)*x^2 - 18*(3*a^3*b^2*c^2
*d - 3*a^4*b*c*d^2 + a^5*d^3 + (3*b^5*c^2*d - 3*a*b^4*c*d^2 + a^2*b^3*d^3)*
x^3 + 3*(3*a*b^4*c^2*d - 3*a^2*b^3*c*d^2 + a^3*b^2*d^3)*x^2 + 3*(3*a^2*b^3*
c^2*d - 3*a^3*b^2*c*d^2 + a^4*b*d^3)*x)*log(b*x + a)^2 - 18*(3*a^3*b^2*c^2*
d - 3*a^4*b*c*d^2 + a^5*d^3 + (3*b^5*c^2*d - 3*a*b^4*c*d^2 + a^2*b^3*d^3)*x
^3 + 3*(3*a*b^4*c^2*d - 3*a^2*b^3*c*d^2 + a^3*b^2*d^3)*x^2 + 3*(3*a^2*b^3*c
^2*d - 3*a^3*b^2*c*d^2 + a^4*b*d^3)*x)*log(d*x + c)^2 + 3*(63*a*b^4*c^3 - 8
6*a^2*b^3*c^2*d + 27*a^3*b^2*c*d^2 - 4*a^4*b*d^3)*x + 6*(18*a^3*b^2*c^2*d -
9*a^4*b*c*d^2 + 2*a^5*d^3 + (18*b^5*c^2*d - 9*a*b^4*c*d^2 + 2*a^2*b^3*d^3)
x^3 + 3(18*a*b^4*c^2*d - 9*a^2*b^3*c*d^2 + 2*a^3*b^2*d^3)*x^2 + 3*(18*a^2
*b^3*c^2*d - 9*a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*log(b*x + a) - 6*(18*a^3*b^2
*c^2*d - 9*a^4*b*c*d^2 + 2*a^5*d^3 + (18*b^5*c^2*d - 9*a*b^4*c*d^2 + 2*a^2*
b^3*d^3)*x^3 + 3*(18*a*b^4*c^2*d - 9*a^2*b^3*c*d^2 + 2*a^3*b^2*d^3)*x^2 + 3
*(18*a^2*b^3*c^2*d - 9*a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x - 6*(3*a^3*b^2*c^2*d
- 3*a^4*b*c*d^2 + a^5*d^3 + (3*b^5*c^2*d - 3*a*b^4*c*d^2 + a^2*b^3*d^3)*x^3
+ 3*(3*a*b^4*c^2*d - 3*a^2*b^3*c*d^2 + a^3*b^2*d^3)*x^2 + 3*(3*a^2*b^3*c^2
*d - 3*a^3*b^2*c*d^2 + a^4*b*d^3)*x)*log(b*x + a))*log(d*x + c))*n^2/(a^3*b
^6*c^3*g^4 - 3*a^4*b^5*c^2*d*g^4 + 3*a^5*b^4*c*d^2*g^4 - a^6*b^3*d^3*g^4 +
(b^9*c^3*g^4 - 3*a*b^8*c^2*d*g^4 + 3*a^2*b^7*c*d^2*g^4 - a^3*b^6*d^3*g^4)*x
^3 + 3*(a*b^8*c^3*g^4 - 3*a^2*b^7*c^2*d*g^4 + 3*a^3*b^6*c*d^2*g^4 - a^4*b^5
*d^3*g^4)*x^2 + 3*(a^2*b^7*c^3*g^4 - 3*a^3*b^6*c^2*d*g^4 + 3*a^4*b^5*c*d^2*
g^4 - a^5*b^4*d^3*g^4)*x))*B^2*d^2*i^2 - 2/3*(3*b*x + a)*A*B*c*d*i^2*log(e*
(b*x/(d*x + c) + a/(d*x + c))^n)/(b^5*g^4*x^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3
*g^4*x + a^3*b^2*g^4) - 2/3*(3*b^2*x^2 + 3*a*b*x + a^2)*A*B*d^2*i^2*log(e*(
b*x/(d*x + c) + a/(d*x + c))^n)/(b^6*g^4*x^3 + 3*a*b^5*g^4*x^2 + 3*a^2*b^4*
g^4*x + a^3*b^3*g^4) - 1/3*B^2*c^2*i^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n)^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) - 1/3*(3
*b*x + a)*A^2*c*d*i^2/(b^5*g^4*x^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^
3*b^2*g^4) - 1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*A^2*d^2*i^2/(b^6*g^4*x^3 + 3*a
*b^5*g^4*x^2 + 3*a^2*b^4*g^4*x + a^3*b^3*g^4) - 2/3*A*B*c^2*i^2*log(e*(b*x/
(d*x + c) + a/(d*x + c))^n)/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*
x + a^3*b*g^4) - 1/3*A^2*c^2*i^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2
*g^4*x + a^3*b*g^4)

Giac [A] (verification not implemented)

none

Time = 3.77 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.38

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{1}{27}\left(\frac{9\,(dx+c)^3B^2i^2n^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(bx+a)^3g^4}+\frac{6\,(B^2i^2n^2+3\,B^2i^2n\log(e)+3\,ABi^2n)(dx+c)^3\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)^3g^4}+\frac{(2\right.$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x,
 algorithm="giac")
```

```
[Out] -1/27*(9*(d*x + c)^3*B^2*i^2*n^2*log((b*x + a)/(d*x + c))^2/((b*x + a)^3*g^
4) + 6*(B^2*i^2*n^2 + 3*B^2*i^2*n*log(e) + 3*A*B*i^2*n)*(d*x + c)^3*log((b*
x + a)/(d*x + c))/((b*x + a)^3*g^4) + (2*B^2*i^2*n^2 + 6*B^2*i^2*n*log(e) +
 9*B^2*i^2*log(e)^2 + 6*A*B*i^2*n + 18*A*B*i^2*log(e) + 9*A^2*i^2)*(d*x + c
)^3/((b*x + a)^3*g^4))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 1195, normalized size of antiderivative = 7.61

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{x\,(9\,c\,A^2\,b^2\,d\,i^2+9\,a\,A^2\,b\,d^2\,i^2+6\,c\,A\,B\,b^2\,d\,i^2\,n+6\,a\,A\,B\,b\,d^2\,i^2\,n+2\,c\,B^2\,b^2\,d\,i^2\,n^2+2\,a\,B^2\,b\,d^2\,i^2\,n^2)}{}$$

$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{a\,(-a\,n\,B^2\,d^2\,i^2+b\,c\,n\,B^2\,d\,i^2+2\,A\,a\,B\,d^2\,i^2+2\,A\,b\,c\,B\,d\,i^2)+x\,(b\,(-a\,n\,B^2\,d^2\,i}{}\right.$$

$$+\frac{2\,B^2\,d^3\,i^2\left(x\left(b\left(\frac{a\,b^3\,g^4\,n\,(a\,d-b\,c)}{d}+\frac{b^3\,g^4\,n\,(a\,d-b\,c)\,(3\,a\,d-b\,c)}{2\,d^2}\right)+\frac{2\,a\,b^4\,g^4\,n\,(a\,d-b\,c)}{d}+\frac{b^4\,g^4\,n\,(a\,d-b\,c)\,(3\,a\,d-b\,c)}{d^2}\right)+a\left(\right.\right.}{3\,b^3\,g^4\,(a\,d-b\,c)\,(3\,a^3\,b^3\,g^4+9\,a^2\,b^4\,g^4}$$

$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{a\left(\frac{B^2\,c\,d\,i^2}{3\,b^2}+\frac{B^2\,a\,d^2\,i^2}{3\,b^3}\right)+x\left(b\left(\frac{B^2\,c\,d\,i^2}{3\,b^2}+\frac{B^2\,a\,d^2\,i^2}{3\,b^3}\right)+\frac{2\,B^2\,c\,d\,i^2}{3\,b}+\frac{2\,B^2\,a\,d^2\,i^2}{3\,b^2}\right)+\frac{B^2\,c^2}{3\,b}}{a^3\,g^4+3\,a^2\,b\,g^4\,x+3\,a\,b^2\,g^4\,x^2+b^3\,g^4\,x^3}\right.$$

$$\left.-\frac{B^2\,d^3\,i^2}{3\,b^3\,g^4\,(a\,d-b\,c)}\right)-\frac{B\,d^3\,i^2\,n\,\mathrm{atan}\left(\frac{\left(\frac{9\,c\,b^4\,g^4+9\,a\,d\,b^3\,g^4}{9\,b^3\,g^4}+2\,b\,d\,x\right)1\mathrm{i}}{a\,d-b\,c}\right)\,(3\,A+B\,n)\,4\mathrm{i}}{9\,b^3\,g^4\,(a\,d-b\,c)}$$

```
[In] int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x
)^4,x)
```

```
[Out] - (x*(9*A^2*a*b*d^2*i^2 + 9*A^2*b^2*c*d*i^2 + 2*B^2*a*b*d^2*i^2*n^2 + 2*B^2
*b^2*c*d*i^2*n^2 + 6*A*B*a*b*d^2*i^2*n + 6*A*B*b^2*c*d*i^2*n) + x^2*(9*A^2*
b^2*d^2*i^2 + 2*B^2*b^2*d^2*i^2*n^2 + 6*A*B*b^2*d^2*i^2*n) + 3*A^2*a^2*d^2*
i^2 + 3*A^2*b^2*c^2*i^2 + (2*B^2*a^2*d^2*i^2*n^2)/3 + (2*B^2*b^2*c^2*i^2*n^
2)/3 + 3*A^2*a*b*c*d*i^2 + 2*A*B*a^2*d^2*i^2*n + 2*A*B*b^2*c^2*i^2*n + (2*B
^2*a*b*c*d*i^2*n^2)/3 + 2*A*B*a*b*c*d*i^2*n)/(9*a^3*b^3*g^4 + 9*b^6*g^4*x^3
 + 27*a^2*b^4*g^4*x + 27*a*b^5*g^4*x^2) - log(e*((a + b*x)/(c + d*x))^n)*((
a*(2*A*B*a*d^2*i^2 - B^2*a*d^2*i^2*n + B^2*b*c*d*i^2*n + 2*A*B*b*c*d*i^2) +
 x*(b*(2*A*B*a*d^2*i^2 - B^2*a*d^2*i^2*n + B^2*b*c*d*i^2*n + 2*A*B*b*c*d*i^
2) + 4*A*B*a*b*d^2*i^2 + 4*A*B*b^2*c*d*i^2 - 2*B^2*a*b*d^2*i^2*n + 2*B^2*b^
2*c*d*i^2*n) + 2*A*B*b^2*c^2*i^2 - 2*B^2*a^2*d^2*i^2*n + 6*A*B*b^2*d^2*i^2*
x^2 + 2*B^2*a*b*c*d*i^2*n)/(3*a^3*b^3*g^4 + 3*b^6*g^4*x^3 + 9*a^2*b^4*g^4*x
 + 9*a*b^5*g^4*x^2) + (2*B^2*d^3*i^2*(x*(b*((a*b^3*g^4*n*(a*d - b*c))/d + (
b^3*g^4*n*(a*d - b*c)*(3*a*d - b*c))/(2*d^2)) + (2*a*b^4*g^4*n*(a*d - b*c))
/d + (b^4*g^4*n*(a*d - b*c)*(3*a*d - b*c))/d^2) + a*((a*b^3*g^4*n*(a*d - b*
c))/d + (b^3*g^4*n*(a*d - b*c)*(3*a*d - b*c))/(2*d^2)) + (3*b^5*g^4*n*x^2*(
a*d - b*c))/d + (b^3*g^4*n*(a*d - b*c)*(3*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/d
^3))/(3*b^3*g^4*(a*d - b*c)*(3*a^3*b^3*g^4 + 3*b^6*g^4*x^3 + 9*a^2*b^4*g^4*
x + 9*a*b^5*g^4*x^2))) - log(e*((a + b*x)/(c + d*x))^n)^2*((a*((B^2*c*d*i^2
)/(3*b^2) + (B^2*a*d^2*i^2)/(3*b^3)) + x*(b*((B^2*c*d*i^2)/(3*b^2) + (B^2*a
*d^2*i^2)/(3*b^3)) + (2*B^2*c*d*i^2)/(3*b) + (2*B^2*a*d^2*i^2)/(3*b^2)) + (
B^2*c^2*i^2)/(3*b) + (B^2*d^2*i^2*x^2)/b)/(a^3*g^4 + b^3*g^4*x^3 + 3*a*b^2*
g^4*x^2 + 3*a^2*b*g^4*x) - (B^2*d^3*i^2)/(3*b^3*g^4*(a*d - b*c))) - (B*d^3*
i^2*n*atan((((9*b^4*c*g^4 + 9*a*b^3*d*g^4)/(9*b^3*g^4) + 2*b*d*x)*1i)/(a*d
- b*c))*(3*A + B*n)*4i)/(9*b^3*g^4*(a*d - b*c))
```

3.176 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 319

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \frac{2B^2di^2n^2(c+dx)^3}{27(bc-ad)^2g^5(a+bx)^3} - \frac{bB^2i^2n^2(c+dx)^4}{32(bc-ad)^2g^5(a+bx)^4} + \frac{2Bdi^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^2g^5(a+bx)^3} - \frac{bBi^2n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8(bc-ad)^2g^5(a+bx)^4} + \frac{di^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^2g^5(a+bx)^3} - \frac{bi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4(bc-ad)^2g^5(a+bx)^4}$$

```
[Out] 2/27*B^2*d*i^2*n^2*(d*x+c)^3/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/32*b*B^2*i^2*n^2*
(d*x+c)^4/(-a*d+b*c)^2/g^5/(b*x+a)^4+2/9*B*d*i^2*n*(d*x+c)^3*(A+B*ln(e*((b*
x+a)/(d*x+c))^n))/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/8*b*B*i^2*n*(d*x+c)^4*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^2/g^5/(b*x+a)^4+1/3*d*i^2*(d*x+c)^3*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/4*b*i^2*(d*x+c
)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^5/(b*x+a)^4
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {2561, 2395, 2342, 2341}

$$\int \frac{(ci+dix)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5} \, dx = -\frac{bi^2(c+dx)^4 \left(B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4g^5(a+bx)^4(bc-ad)^2} - \frac{bBi^2n(c+dx)^4 \left(B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{8g^5(a+bx)^4(bc-ad)^2} + \frac{di^2(c+dx)^3 \left(B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g^5(a+bx)^3(bc-ad)^2} + \frac{2Bdi^2n(c+dx)^3 \left(B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9g^5(a+bx)^3(bc-ad)^2} - \frac{bB^2i^2n^2(c+dx)^4}{32g^5(a+bx)^4(bc-ad)^2} + \frac{2B^2di^2n^2(c+dx)^3}{27g^5(a+bx)^3(bc-ad)^2}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x
)^5,x]
```

```
[Out] (2*B^2*d*i^2*n^2*(c + d*x)^3)/(27*(b*c - a*d)^2*g^5*(a + b*x)^3) - (b*B^2*i
^2*n^2*(c + d*x)^4)/(32*(b*c - a*d)^2*g^5*(a + b*x)^4) + (2*B*d*i^2*n*(c +
d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(b*c - a*d)^2*g^5*(a + b*
x)^3) - (b*B*i^2*n*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(8*(
b*c - a*d)^2*g^5*(a + b*x)^4) + (d*i^2*(c + d*x)^3*(A + B*Log[e*((a + b*x)/
(c + d*x))^n])^2)/(3*(b*c - a*d)^2*g^5*(a + b*x)^3) - (b*i^2*(c + d*x)^4*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*(b*c - a*d)^2*g^5*(a + b*x)^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{i^2\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex^n))^2}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^5}$$

$$= \frac{i^2\text{Subst}\left(\int \left(\frac{b(A+B\log(ex^n))^2}{x^5} - \frac{d(A+B\log(ex^n))^2}{x^4}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^5}$$

$$= \frac{(bi^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^5} - \frac{(di^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^5}$$

$$= \frac{di^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^2g^5(a+bx)^3} - \frac{bi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4(bc-ad)^2g^5(a+bx)^4} + \frac{(bBi^2n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{2(bc-ad)^2g^5} - \frac{(2Bdi^2n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bc-ad)^2g^5}$$

$$= \frac{2B^2di^2n^2(c+dx)^3}{27(bc-ad)^2g^5(a+bx)^3} - \frac{bB^2i^2n^2(c+dx)^4}{32(bc-ad)^2g^5(a+bx)^4} + \frac{2Bdi^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^2g^5(a+bx)^3} - \frac{bBi^2n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8(bc-ad)^2g^5(a+bx)^4} + \frac{di^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^2g^5(a+bx)^3} - \frac{bi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4(bc-ad)^2g^5(a+bx)^4}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.63 (sec) , antiderivative size = 1787, normalized size of antiderivative = 5.60

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{i^2\Big(216(bc-ad)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-576d(-bc+ad)^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+432d^2}{}$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g +
 b*g*x)^5,x]
```

```
[Out] -1/864*(i^2*(216*(b*c - a*d)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - 5
76*d*(-(b*c) + a*d)^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 +
432*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
+ 32*B*d*n*(a + b*x)*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3*n - 18*A*d*(b*
c - a*d)^2*(a + b*x) - 15*B*d*(b*c - a*d)^2*n*(a + b*x) + 36*A*d^2*(b*c - a
*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a*d)*n*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3
*Log[a + b*x] + 66*B*d^3*n*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*n*(a + b*x)^
3*Log[a + b*x]^2 + 12*B*(b*c - a*d)^3*Log[e*((a + b*x)/(c + d*x))^n] - 18*B
*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d^2*(b*c -
 a*d)*(a + b*x)^2*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d^3*(a + b*x)^3*Log
[a + b*x]*Log[e*((a + b*x)/(c + d*x))^n] - 36*A*d^3*(a + b*x)^3*Log[c + d*x
] - 66*B*d^3*n*(a + b*x)^3*Log[c + d*x] + 36*B*d^3*n*(a + b*x)^3*Log[(d*(a
+ b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[e*((a + b*x
)/(c + d*x))^n]*Log[c + d*x] - 18*B*d^3*n*(a + b*x)^3*Log[c + d*x]^2 + 36*B
*d^3*n*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*n
*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3*n*(a + b*x
)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 3*B*n*(36*A*(b*c - a*d)^4 + 9*
B*(b*c - a*d)^4*n + 48*A*d*(-(b*c) + a*d)^3*(a + b*x) + 28*B*d*(-(b*c) + a*
d)^3*n*(a + b*x) + 72*A*d^2*(b*c - a*d)^2*(a + b*x)^2 + 78*B*d^2*(b*c - a*d
)^2*n*(a + b*x)^2 + 144*A*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 300*B*d^3*(-(b*c
) + a*d)*n*(a + b*x)^3 - 144*A*d^4*(a + b*x)^4*Log[a + b*x] - 300*B*d^4*n*(
a + b*x)^4*Log[a + b*x] + 72*B*d^4*n*(a + b*x)^4*Log[a + b*x]^2 + 36*B*(b*c
 - a*d)^4*Log[e*((a + b*x)/(c + d*x))^n] + 48*B*d*(-(b*c) + a*d)^3*(a + b*x
)*Log[e*((a + b*x)/(c + d*x))^n] + 72*B*d^2*(b*c - a*d)^2*(a + b*x)^2*Log[e
*((a + b*x)/(c + d*x))^n] + 144*B*d^3*(-(b*c) + a*d)*(a + b*x)^3*Log[e*((a
+ b*x)/(c + d*x))^n] - 144*B*d^4*(a + b*x)^4*Log[a + b*x]*Log[e*((a + b*x)/
(c + d*x))^n] + 144*A*d^4*(a + b*x)^4*Log[c + d*x] + 300*B*d^4*n*(a + b*x)^
4*Log[c + d*x] - 144*B*d^4*n*(a + b*x)^4*Log[(d*(a + b*x))/(-(b*c) + a*d)]*
Log[c + d*x] + 144*B*d^4*(a + b*x)^4*Log[e*((a + b*x)/(c + d*x))^n]*Log[c +
 d*x] + 72*B*d^4*n*(a + b*x)^4*Log[c + d*x]^2 - 144*B*d^4*n*(a + b*x)^4*Log
[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 144*B*d^4*n*(a + b*x)^4*PolyLog[
```

```
2, (d*(a + b*x))/(-(b*c) + a*d)] - 144*B*d^4*n*(a + b*x)^4*PolyLog[2, (b*(c
 + d*x))/(b*c - a*d)]) + 216*B*d^2*n*(a + b*x)^2*(2*(b*c - a*d)^2*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]) + 4*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]) - 4*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n]) + 4*d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])*Log[c + d*x] - 4*B*d*n*(a + b*x)*(b*c - a*d + d*(a + b*x)*Log[a + b*x
] - d*(a + b*x)*Log[c + d*x]) + B*n*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a
+ b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) +
 2*B*d^2*n*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b
*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2*B*d^2*n*(a +
b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] +
 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))))/(b^3*(b*c - a*d)^2*g^5*(a + b*
x)^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1883 vs. $2(307) = 614$.

Time = 27.47 (sec) , antiderivative size = 1884, normalized size of antiderivative = 5.91

method	result	size
parallelrisch	Expression too large to display	1884

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x,method=
_RETURNVERBOSE)
```

```
[Out] 1/864*(-1152*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b*c^4*d*i^2*n+144*A*B*x^4*
ln(e*((b*x+a)/(d*x+c))^n)*a^6*b*c*d^4*i^2*n-864*A*B*x^2*ln(e*((b*x+a)/(d*x+
c))^n)*a^6*b*c^3*d^2*i^2*n-192*A*B*x^4*a^3*b^4*c^4*d*i^2*n^2+144*B^2*x^3*ln
(e*((b*x+a)/(d*x+c))^n)*a^6*b*c^2*d^3*i^2*n^2+576*A*B*x^3*ln(e*((b*x+a)/(d*
x+c))^n)*a^7*c*d^4*i^2*n+144*A*B*x^3*a^6*b*c^2*d^3*i^2*n^2-768*A*B*x^3*a^4*
b^3*c^4*d*i^2*n^2-432*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b*c^3*d^2*i^2
*n-72*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b*c^3*d^2*i^2*n^2+1728*A*B*x^2*
ln(e*((b*x+a)/(d*x+c))^n)*a^7*c^2*d^3*i^2*n-72*A*B*x^2*a^6*b*c^3*d^2*i^2*n^
2-1152*A*B*x^2*a^5*b^2*c^4*d*i^2*n^2-576*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*
a^6*b*c^4*d*i^2*n-240*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b*c^4*d*i^2*n^2+1
728*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^7*c^3*d^2*i^2*n-1008*A*B*x*a^6*b*c^4*
d*i^2*n^2+72*B^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b*c*d^4*i^2*n+84*B^2*x
^4*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b*c*d^4*i^2*n^2+84*A*B*x^4*a^6*b*c*d^4*i^2
*n^2+108*A*B*x^4*a^2*b^5*c^5*i^2*n^2+288*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^
2*a^7*c*d^4*i^2*n+192*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^7*c*d^4*i^2*n^2+8
4*B^2*x^3*a^6*b*c^2*d^3*i^2*n^3-256*B^2*x^3*a^4*b^3*c^4*d*i^2*n^3+72*A^2*x^
4*a^6*b*c*d^4*i^2*n-288*A^2*x^4*a^3*b^4*c^4*d*i^2*n+192*A*B*x^3*a^7*c*d^4*i
^2*n^2+432*A*B*x^3*a^3*b^4*c^5*i^2*n^2+864*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n
)^2*a^7*c^2*d^3*i^2*n+576*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^7*c^2*d^3*i^2
*n^2+30*B^2*x^2*a^6*b*c^3*d^2*i^2*n^3-384*B^2*x^2*a^5*b^2*c^4*d*i^2*n^3-115
```

```
2*A^2*x^3*a^4*b^3*c^4*d*i^2*n+576*A*B*x^2*a^7*c^2*d^3*i^2*n^2+648*A*B*x^2*a
^4*b^3*c^5*i^2*n^2+27*B^2*x^4*a^2*b^5*c^5*i^2*n^3+64*B^2*x^3*a^7*c*d^4*i^2*
n^3+108*B^2*x^3*a^3*b^4*c^5*i^2*n^3+864*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a
^7*c^3*d^2*i^2*n+576*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^7*c^3*d^2*i^2*n^2-30
0*B^2*x*a^6*b*c^4*d*i^2*n^3-432*A^2*x^2*a^6*b*c^3*d^2*i^2*n-1728*A^2*x^2*a^
5*b^2*c^4*d*i^2*n+576*A*B*x*a^7*c^3*d^2*i^2*n^2+432*A*B*x*a^5*b^2*c^5*i^2*n
^2-1728*A^2*x*a^6*b*c^4*d*i^2*n+576*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^7*c^4*d
*i^2*n-432*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b*c^5*i^2*n+37*B^2*x^4*a^6*b*c
*d^4*i^2*n^3-64*B^2*x^4*a^3*b^4*c^4*d*i^2*n^3+216*A^2*x^4*a^2*b^5*c^5*i^2*n
+192*B^2*x^2*a^7*c^2*d^3*i^2*n^3+162*B^2*x^2*a^4*b^3*c^5*i^2*n^3+288*A^2*x^
3*a^7*c*d^4*i^2*n+864*A^2*x^3*a^3*b^4*c^5*i^2*n+192*B^2*x*a^7*c^3*d^2*i^2*n
^3+108*B^2*x*a^5*b^2*c^5*i^2*n^3+864*A^2*x^2*a^7*c^2*d^3*i^2*n+1296*A^2*x^2
*a^4*b^3*c^5*i^2*n+288*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^7*c^4*d*i^2*n-216*
B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b*c^5*i^2*n+192*B^2*ln(e*((b*x+a)/(d*x+
c))^n)*a^7*c^4*d*i^2*n^2-108*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b*c^5*i^2*n^
2+864*A^2*x*a^7*c^3*d^2*i^2*n+864*A^2*x*a^5*b^2*c^5*i^2*n)/g^5/(b*x+a)^4/n/
(a^2*d^2-2*a*b*c*d+b^2*c^2)/a^6/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1729 vs. $2(307) = 614$.

Time = 0.38 (sec) , antiderivative size = 1729, normalized size of antiderivative = 5.42

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x,
 algorithm="fricas")
```

```
[Out] -1/864*((27*B^2*b^4*c^4 - 64*B^2*a*b^3*c^3*d + 37*B^2*a^4*d^4)*i^2*n^2 + 12
*(9*A*B*b^4*c^4 - 16*A*B*a*b^3*c^3*d + 7*A*B*a^4*d^4)*i^2*n - 12*(7*(B^2*b^
4*c*d^3 - B^2*a*b^3*d^4)*i^2*n^2 + 12*(A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*i^2*n
)*x^3 + 72*(3*A^2*b^4*c^4 - 4*A^2*a*b^3*c^3*d + A^2*a^4*d^4)*i^2 - 6*((5*B^
2*b^4*c^2*d^2 + 32*B^2*a*b^3*c*d^3 - 37*B^2*a^2*b^2*d^4)*i^2*n^2 - 12*(A*B*
b^4*c^2*d^2 - 8*A*B*a*b^3*c*d^3 + 7*A*B*a^2*b^2*d^4)*i^2*n - 72*(A^2*b^4*c^
2*d^2 - 2*A^2*a*b^3*c*d^3 + A^2*a^2*b^2*d^4)*i^2)*x^2 + 72*(6*(B^2*b^4*c^2*
d^2 - 2*B^2*a*b^3*c*d^3 + B^2*a^2*b^2*d^4)*i^2*x^2 + 4*(2*B^2*b^4*c^3*d - 3
*B^2*a*b^3*c^2*d^2 + B^2*a^3*b*d^4)*i^2*x + (3*B^2*b^4*c^4 - 4*B^2*a*b^3*c^
3*d + B^2*a^4*d^4)*i^2)*log(e)^2 - 72*(B^2*b^4*d^4*i^2*n^2*x^4 + 4*B^2*a*b^
3*d^4*i^2*n^2*x^3 - 6*(B^2*b^4*c^2*d^2 - 2*B^2*a*b^3*c*d^3)*i^2*n^2*x^2 - 4
*(2*B^2*b^4*c^3*d - 3*B^2*a*b^3*c^2*d^2)*i^2*n^2*x - (3*B^2*b^4*c^4 - 4*B^2
*a*b^3*c^3*d)*i^2*n^2)*log((b*x + a)/(d*x + c))^2 + 4*((11*B^2*b^4*c^3*d -
48*B^2*a*b^3*c^2*d^2 + 37*B^2*a^3*b*d^4)*i^2*n^2 + 12*(5*A*B*b^4*c^3*d - 12
*A*B*a*b^3*c^2*d^2 + 7*A*B*a^3*b*d^4)*i^2*n + 72*(2*A^2*b^4*c^3*d - 3*A^2*a
```

```
*b^3*c^2*d^2 + A^2*a^3*b*d^4)*i^2)*x - 12*(12*(B^2*b^4*c*d^3 - B^2*a*b^3*d^
4)*i^2*n*x^3 - (9*B^2*b^4*c^4 - 16*B^2*a*b^3*c^3*d + 7*B^2*a^4*d^4)*i^2*n -
 12*(3*A*B*b^4*c^4 - 4*A*B*a*b^3*c^3*d + A*B*a^4*d^4)*i^2 - 6*((B^2*b^4*c^2
*d^2 - 8*B^2*a*b^3*c*d^3 + 7*B^2*a^2*b^2*d^4)*i^2*n + 12*(A*B*b^4*c^2*d^2 -
 2*A*B*a*b^3*c*d^3 + A*B*a^2*b^2*d^4)*i^2)*x^2 - 4*((5*B^2*b^4*c^3*d - 12*B
^2*a*b^3*c^2*d^2 + 7*B^2*a^3*b*d^4)*i^2*n + 12*(2*A*B*b^4*c^3*d - 3*A*B*a*b
^3*c^2*d^2 + A*B*a^3*b*d^4)*i^2)*x + 12*(B^2*b^4*d^4*i^2*n*x^4 + 4*B^2*a*b^
3*d^4*i^2*n*x^3 - 6*(B^2*b^4*c^2*d^2 - 2*B^2*a*b^3*c*d^3)*i^2*n*x^2 - 4*(2*
B^2*b^4*c^3*d - 3*B^2*a*b^3*c^2*d^2)*i^2*n*x - (3*B^2*b^4*c^4 - 4*B^2*a*b^3
*c^3*d)*i^2*n)*log((b*x + a)/(d*x + c)))*log(e) + 12*((9*B^2*b^4*c^4 - 16*B
^2*a*b^3*c^3*d)*i^2*n^2 - (7*B^2*b^4*d^4*i^2*n^2 + 12*A*B*b^4*d^4*i^2*n)*x^
4 + 12*(3*A*B*b^4*c^4 - 4*A*B*a*b^3*c^3*d)*i^2*n - 4*(12*A*B*a*b^3*d^4*i^2*
n + (3*B^2*b^4*c*d^3 + 4*B^2*a*b^3*d^4)*i^2*n^2)*x^3 + 6*((B^2*b^4*c^2*d^2
- 8*B^2*a*b^3*c*d^3)*i^2*n^2 + 12*(A*B*b^4*c^2*d^2 - 2*A*B*a*b^3*c*d^3)*i^2
*n)*x^2 + 4*((5*B^2*b^4*c^3*d - 12*B^2*a*b^3*c^2*d^2)*i^2*n^2 + 12*(2*A*B*b
^4*c^3*d - 3*A*B*a*b^3*c^2*d^2)*i^2*n)*x)*log((b*x + a)/(d*x + c)))/((b^9*c
^2 - 2*a*b^8*c*d + a^2*b^7*d^2)*g^5*x^4 + 4*(a*b^8*c^2 - 2*a^2*b^7*c*d + a^
3*b^6*d^2)*g^5*x^3 + 6*(a^2*b^7*c^2 - 2*a^3*b^6*c*d + a^4*b^5*d^2)*g^5*x^2
+ 4*(a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b^4*d^2)*g^5*x + (a^4*b^5*c^2 - 2*a^
5*b^4*c*d + a^6*b^3*d^2)*g^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx=\text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**5
,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8087 vs. $2(307) = 614$.

Time = 0.73 (sec) , antiderivative size = 8087, normalized size of antiderivative = 25.35

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx=\text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x,
 algorithm="maxima")
```

[Out] 1/24*A*B*c^2*i^2*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/72*A*B*d^2*i^2*n*((13*a^2*b^3*c^3 - 75*a^3*b^2*c^2*d + 33*a^4*b*c*d^2 - 7*a^5*d^3 - 12*(6*b^5*c^2*d - 4*a*b^4*c*d^2 + a^2*b^3*d^3)*x^3 + 6*(6*b^5*c^3 - 46*a*b^4*c^2*d + 29*a^2*b^3*c*d^2 - 7*a^3*b^2*d^3)*x^2 + 4*(10*a*b^4*c^3 - 63*a^2*b^3*c^2*d + 33*a^3*b^2*c*d^2 - 7*a^4*b*d^3)*x)/((b^10*c^3 - 3*a*b^9*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7*d^3)*g^5*x^4 + 4*(a*b^9*c^3 - 3*a^2*b^8*c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*d^3)*g^5*x^3 + 6*(a^2*b^8*c^3 - 3*a^3*b^7*c^2*d + 3*a^4*b^6*c*d^2 - a^5*b^5*d^3)*g^5*x^2 + 4*(a^3*b^7*c^3 - 3*a^4*b^6*c^2*d + 3*a^5*b^5*c*d^2 - a^6*b^4*d^3)*g^5*x + (a^4*b^6*c^3 - 3*a^5*b^5*c^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d^3)*g^5) - 12*(6*b^2*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*log(b*x + a)/((b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4)*g^5) + 12*(6*b^2*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*log(d*x + c)/((b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4)*g^5)) - 1/36*A*B*c*d*i^2*n*((7*a*b^3*c^3 - 33*a^2*b^2*c^2*d + 75*a^3*b*c*d^2 - 13*a^4*d^3 + 12*(4*b^4*c*d^2 - a*b^3*d^3)*x^3 - 6*(4*b^4*c^2*d - 29*a*b^3*c*d^2 + 7*a^2*b^2*d^3)*x^2 + 4*(4*b^4*c^3 - 21*a*b^3*c^2*d + 57*a^2*b^2*c*d^2 - 13*a^3*b*d^3)*x)/((b^9*c^3 - 3*a*b^8*c^2*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*g^5*x^4 + 4*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*a^3*b^6*c*d^2 - a^4*b^5*d^3)*g^5*x^3 + 6*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*a^4*b^5*c*d^2 - a^5*b^4*d^3)*g^5*x^2 + 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2 - a^6*b^3*d^3)*g^5*x + (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d^2 - a^7*b^2*d^3)*g^5) + 12*(4*b*c*d^3 - a*d^4)*log(b*x + a)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5) - 12*(4*b*c*d^3 - a*d^4)*log(d*x + c)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5)) - 1/6*(4*b*x + a)*B^2*c*d*i^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2/(b^6*g^5*x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g^5) - 1/12*(6*b^2*x^2 + 4*a*b*x + a^2)*B^2*d^2*i^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2/(b^7*g^5*x^4 + 4*a*b^6*g^5*x^3 + 6*a^2*b^5*g^5*x^2 + 4*a^3*b^4*g^5*x + a^4*b^3*g^5) + 1/288*(12*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2

- a^7*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b
^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*
c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)
)*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - (9*b^4*c^4 - 64*a*b^3*c^3*d + 21
6*a^2*b^2*c^2*d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*
d^4)*x^3 + 6*(13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*
(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^
4)*log(b*x + a)^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 +
 4*a^3*b*d^4*x + a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d^
2 + 324*a^2*b^2*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4*x
^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*b^4
*d^4*x^4 + 100*a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x + 25*a
^4*d^4 - 12*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^
4*x + a^4*d^4)*log(b*x + a))*log(d*x + c))*n^2/(a^4*b^5*c^4*g^5 - 4*a^5*b^4
*c^3*d*g^5 + 6*a^6*b^3*c^2*d^2*g^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d^4*g^5 +
(b^9*c^4*g^5 - 4*a*b^8*c^3*d*g^5 + 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b^6*c*d^3*
g^5 + a^4*b^5*d^4*g^5)*x^4 + 4*(a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^5 + 6*a^3
*b^6*c^2*d^2*g^5 - 4*a^4*b^5*c*d^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*(a^2*b^7*
c^4*g^5 - 4*a^3*b^6*c^3*d*g^5 + 6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*c*d^3*g^5
 + a^6*b^3*d^4*g^5)*x^2 + 4*(a^3*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5 + 6*a^5*
b^4*c^2*d^2*g^5 - 4*a^6*b^3*c*d^3*g^5 + a^7*b^2*d^4*g^5)*x))*B^2*c^2*i^2 -
1/432*(12*n*((7*a*b^3*c^3 - 33*a^2*b^2*c^2*d + 75*a^3*b*c*d^2 - 13*a^4*d^3
+ 12*(4*b^4*c*d^2 - a*b^3*d^3)*x^3 - 6*(4*b^4*c^2*d - 29*a*b^3*c*d^2 + 7*a^
2*b^2*d^3)*x^2 + 4*(4*b^4*c^3 - 21*a*b^3*c^2*d + 57*a^2*b^2*c*d^2 - 13*a^3*
b*d^3)*x)/((b^9*c^3 - 3*a*b^8*c^2*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*g^5*x^
4 + 4*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*a^3*b^6*c*d^2 - a^4*b^5*d^3)*g^5*x^3
 + 6*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*a^4*b^5*c*d^2 - a^5*b^4*d^3)*g^5*x^
2 + 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2 - a^6*b^3*d^3)*g^5*x
 + (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d^2 - a^7*b^2*d^3)*g^5) + 1
2*(4*b*c*d^3 - a*d^4)*log(b*x + a)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^
2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5) - 12*(4*b*c*d^3 - a*d^4)*log(d*
x + c)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^
4*b^2*d^4)*g^5))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + (37*a*b^4*c^4 - 3
04*a^2*b^3*c^3*d + 1512*a^3*b^2*c^2*d^2 - 1360*a^4*b*c*d^3 + 115*a^5*d^4 +
12*(88*b^5*c^2*d^2 - 101*a*b^4*c*d^3 + 13*a^2*b^3*d^4)*x^3 - 6*(40*b^5*c^3*
d - 609*a*b^4*c^2*d^2 + 648*a^2*b^3*c*d^3 - 79*a^3*b^2*d^4)*x^2 - 72*(4*a^4
*b*c*d^3 - a^5*d^4 + (4*b^5*c*d^3 - a*b^4*d^4)*x^4 + 4*(4*a*b^4*c*d^3 - a^2
*b^3*d^4)*x^3 + 6*(4*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^2 + 4*(4*a^3*b^2*c*d^3
- a^4*b*d^4)*x)*log(b*x + a)^2 - 72*(4*a^4*b*c*d^3 - a^5*d^4 + (4*b^5*c*d^3
 - a*b^4*d^4)*x^4 + 4*(4*a*b^4*c*d^3 - a^2*b^3*d^4)*x^3 + 6*(4*a^2*b^3*c*d^
3 - a^3*b^2*d^4)*x^2 + 4*(4*a^3*b^2*c*d^3 - a^4*b*d^4)*x)*log(d*x + c)^2 +
4*(16*b^5*c^4 - 163*a*b^4*c^3*d + 1068*a^2*b^3*c^2*d^2 - 1036*a^3*b^2*c*d^3
 + 115*a^4*b*d^4)*x + 12*(88*a^4*b*c*d^3 - 13*a^5*d^4 + (88*b^5*c*d^3 - 13*
a*b^4*d^4)*x^4 + 4*(88*a*b^4*c*d^3 - 13*a^2*b^3*d^4)*x^3 + 6*(88*a^2*b^3*c*
d^3 - 13*a^3*b^2*d^4)*x^2 + 4*(88*a^3*b^2*c*d^3 - 13*a^4*b*d^4)*x)*log(b*x

+ a) - 12*(88*a^4*b*c*d^3 - 13*a^5*d^4 + (88*b^5*c*d^3 - 13*a*b^4*d^4)*x^4
+ 4*(88*a*b^4*c*d^3 - 13*a^2*b^3*d^4)*x^3 + 6*(88*a^2*b^3*c*d^3 - 13*a^3*b^
2*d^4)*x^2 + 4*(88*a^3*b^2*c*d^3 - 13*a^4*b*d^4)*x - 12*(4*a^4*b*c*d^3 - a^
5*d^4 + (4*b^5*c*d^3 - a*b^4*d^4)*x^4 + 4*(4*a*b^4*c*d^3 - a^2*b^3*d^4)*x^3
+ 6*(4*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^2 + 4*(4*a^3*b^2*c*d^3 - a^4*b*d^4)*
x)*log(b*x + a))*log(d*x + c))*n^2/(a^4*b^6*c^4*g^5 - 4*a^5*b^5*c^3*d*g^5 +
6*a^6*b^4*c^2*d^2*g^5 - 4*a^7*b^3*c*d^3*g^5 + a^8*b^2*d^4*g^5 + (b^10*c^4*
g^5 - 4*a*b^9*c^3*d*g^5 + 6*a^2*b^8*c^2*d^2*g^5 - 4*a^3*b^7*c*d^3*g^5 + a^4
*b^6*d^4*g^5)*x^4 + 4*(a*b^9*c^4*g^5 - 4*a^2*b^8*c^3*d*g^5 + 6*a^3*b^7*c^2*
d^2*g^5 - 4*a^4*b^6*c*d^3*g^5 + a^5*b^5*d^4*g^5)*x^3 + 6*(a^2*b^8*c^4*g^5 -
4*a^3*b^7*c^3*d*g^5 + 6*a^4*b^6*c^2*d^2*g^5 - 4*a^5*b^5*c*d^3*g^5 + a^6*b^
4*d^4*g^5)*x^2 + 4*(a^3*b^7*c^4*g^5 - 4*a^4*b^6*c^3*d*g^5 + 6*a^5*b^5*c^2*d
^2*g^5 - 4*a^6*b^4*c*d^3*g^5 + a^7*b^3*d^4*g^5)*x))*B^2*c*d*i^2 - 1/864*(12
n((13*a^2*b^3*c^3 - 75*a^3*b^2*c^2*d + 33*a^4*b*c*d^2 - 7*a^5*d^3 - 12*(6
*b^5*c^2*d - 4*a*b^4*c*d^2 + a^2*b^3*d^3)*x^3 + 6*(6*b^5*c^3 - 46*a*b^4*c^2
*d + 29*a^2*b^3*c*d^2 - 7*a^3*b^2*d^3)*x^2 + 4*(10*a*b^4*c^3 - 63*a^2*b^3*c
^2*d + 33*a^3*b^2*c*d^2 - 7*a^4*b*d^3)*x)/((b^10*c^3 - 3*a*b^9*c^2*d + 3*a^
2*b^8*c*d^2 - a^3*b^7*d^3)*g^5*x^4 + 4*(a*b^9*c^3 - 3*a^2*b^8*c^2*d + 3*a^3
*b^7*c*d^2 - a^4*b^6*d^3)*g^5*x^3 + 6*(a^2*b^8*c^3 - 3*a^3*b^7*c^2*d + 3*a^
4*b^6*c*d^2 - a^5*b^5*d^3)*g^5*x^2 + 4*(a^3*b^7*c^3 - 3*a^4*b^6*c^2*d + 3*a
^5*b^5*c*d^2 - a^6*b^4*d^3)*g^5*x + (a^4*b^6*c^3 - 3*a^5*b^5*c^2*d + 3*a^6*
b^4*c*d^2 - a^7*b^3*d^3)*g^5) - 12*(6*b^2*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*
log(b*x + a)/((b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^
3 + a^4*b^3*d^4)*g^5) + 12*(6*b^2*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*log(d*x
+ c)/((b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*
b^3*d^4)*g^5))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + (115*a^2*b^4*c^4 -
1360*a^3*b^3*c^3*d + 1512*a^4*b^2*c^2*d^2 - 304*a^5*b*c*d^3 + 37*a^6*d^4 -
12*(108*b^6*c^3*d - 148*a*b^5*c^2*d^2 + 47*a^2*b^4*c*d^3 - 7*a^3*b^3*d^4)*x
^3 + 6*(36*b^6*c^4 - 712*a*b^5*c^3*d + 903*a^2*b^4*c^2*d^2 - 264*a^3*b^3*c*
d^3 + 37*a^4*b^2*d^4)*x^2 + 72*(6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4
+ (6*b^6*c^2*d^2 - 4*a*b^5*c*d^3 + a^2*b^4*d^4)*x^4 + 4*(6*a*b^5*c^2*d^2 -
4*a^2*b^4*c*d^3 + a^3*b^3*d^4)*x^3 + 6*(6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^
3 + a^4*b^2*d^4)*x^2 + 4*(6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c*d^3 + a^5*b*d^4)*
x)*log(b*x + a)^2 + 72*(6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4 + (6*b^
6*c^2*d^2 - 4*a*b^5*c*d^3 + a^2*b^4*d^4)*x^4 + 4*(6*a*b^5*c^2*d^2 - 4*a^2*b
^4*c*d^3 + a^3*b^3*d^4)*x^3 + 6*(6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*
b^2*d^4)*x^2 + 4*(6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c*d^3 + a^5*b*d^4)*x)*log(d
x + c)^2 + 4(76*a*b^5*c^4 - 1057*a^2*b^4*c^3*d + 1248*a^3*b^3*c^2*d^2 - 3
04*a^4*b^2*c*d^3 + 37*a^5*b*d^4)*x - 12*(108*a^4*b^2*c^2*d^2 - 40*a^5*b*c*d
^3 + 7*a^6*d^4 + (108*b^6*c^2*d^2 - 40*a*b^5*c*d^3 + 7*a^2*b^4*d^4)*x^4 + 4
*(108*a*b^5*c^2*d^2 - 40*a^2*b^4*c*d^3 + 7*a^3*b^3*d^4)*x^3 + 6*(108*a^2*b^
4*c^2*d^2 - 40*a^3*b^3*c*d^3 + 7*a^4*b^2*d^4)*x^2 + 4*(108*a^3*b^3*c^2*d^2
- 40*a^4*b^2*c*d^3 + 7*a^5*b*d^4)*x)*log(b*x + a) + 12*(108*a^4*b^2*c^2*d^2
- 40*a^5*b*c*d^3 + 7*a^6*d^4 + (108*b^6*c^2*d^2 - 40*a*b^5*c*d^3 + 7*a^2*b
^4*d^4)*x^4 + 4*(108*a*b^5*c^2*d^2 - 40*a^2*b^4*c*d^3 + 7*a^3*b^3*d^4)*x^3

```
+ 6*(108*a^2*b^4*c^2*d^2 - 40*a^3*b^3*c*d^3 + 7*a^4*b^2*d^4)*x^2 + 4*(108*a
^3*b^3*c^2*d^2 - 40*a^4*b^2*c*d^3 + 7*a^5*b*d^4)*x - 12*(6*a^4*b^2*c^2*d^2
- 4*a^5*b*c*d^3 + a^6*d^4 + (6*b^6*c^2*d^2 - 4*a*b^5*c*d^3 + a^2*b^4*d^4)*x
^4 + 4*(6*a*b^5*c^2*d^2 - 4*a^2*b^4*c*d^3 + a^3*b^3*d^4)*x^3 + 6*(6*a^2*b^4
*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*x^2 + 4*(6*a^3*b^3*c^2*d^2 - 4*a^
4*b^2*c*d^3 + a^5*b*d^4)*x)*log(b*x + a))*log(d*x + c))*n^2/(a^4*b^7*c^4*g^
5 - 4*a^5*b^6*c^3*d*g^5 + 6*a^6*b^5*c^2*d^2*g^5 - 4*a^7*b^4*c*d^3*g^5 + a^8
*b^3*d^4*g^5 + (b^11*c^4*g^5 - 4*a*b^10*c^3*d*g^5 + 6*a^2*b^9*c^2*d^2*g^5 -
 4*a^3*b^8*c*d^3*g^5 + a^4*b^7*d^4*g^5)*x^4 + 4*(a*b^10*c^4*g^5 - 4*a^2*b^9
*c^3*d*g^5 + 6*a^3*b^8*c^2*d^2*g^5 - 4*a^4*b^7*c*d^3*g^5 + a^5*b^6*d^4*g^5)
*x^3 + 6*(a^2*b^9*c^4*g^5 - 4*a^3*b^8*c^3*d*g^5 + 6*a^4*b^7*c^2*d^2*g^5 - 4
*a^5*b^6*c*d^3*g^5 + a^6*b^5*d^4*g^5)*x^2 + 4*(a^3*b^8*c^4*g^5 - 4*a^4*b^7*
c^3*d*g^5 + 6*a^5*b^6*c^2*d^2*g^5 - 4*a^6*b^5*c*d^3*g^5 + a^7*b^4*d^4*g^5)*
x))*B^2*d^2*i^2 - 1/3*(4*b*x + a)*A*B*c*d*i^2*log(e*(b*x/(d*x + c) + a/(d*x
 + c))^n)/(b^6*g^5*x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^
5*x + a^4*b^2*g^5) - 1/6*(6*b^2*x^2 + 4*a*b*x + a^2)*A*B*d^2*i^2*log(e*(b*x
/(d*x + c) + a/(d*x + c))^n)/(b^7*g^5*x^4 + 4*a*b^6*g^5*x^3 + 6*a^2*b^5*g^5
*x^2 + 4*a^3*b^4*g^5*x + a^4*b^3*g^5) - 1/4*B^2*c^2*i^2*log(e*(b*x/(d*x + c
) + a/(d*x + c))^n)^2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 +
4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/6*(4*b*x + a)*A^2*c*d*i^2/(b^6*g^5*x^4 + 4
*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g^5) - 1/12*
(6*b^2*x^2 + 4*a*b*x + a^2)*A^2*d^2*i^2/(b^7*g^5*x^4 + 4*a*b^6*g^5*x^3 + 6*
a^2*b^5*g^5*x^2 + 4*a^3*b^4*g^5*x + a^4*b^3*g^5) - 1/2*A*B*c^2*i^2*log(e*(b
*x/(d*x + c) + a/(d*x + c))^n)/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g
^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A^2*c^2*i^2/(b^5*g^5*x^4 + 4*a*
b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

Giac [A] (verification not implemented)

none

Time = 4.55 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.70

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{1}{864}\left(\frac{72\left(3B^2bi^2n^2-\frac{4(bx+a)B^2di^2n^2}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)^2}{\frac{(bx+a)^4bcg^5}{(dx+c)^4}-\frac{(bx+a)^4adg^5}{(dx+c)^4}}+\frac{12\left(9B^2bi^2n^2-\frac{16(bx+a)B^2di^2n^2}{dx+c}+36B^2bi^2n\log(e)\right.}{\frac{(bx+a}{(dx}}\right.$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x,
 algorithm="giac")
```

```
[Out] -1/864*(72*(3*B^2*b*i^2*n^2 - 4*(b*x + a)*B^2*d*i^2*n^2/(d*x + c))*log((b*x
 + a)/(d*x + c))^2/((b*x + a)^4*b*c*g^5/(d*x + c)^4 - (b*x + a)^4*a*d*g^5/(
d*x + c)^4) + 12*(9*B^2*b*i^2*n^2 - 16*(b*x + a)*B^2*d*i^2*n^2/(d*x + c) +
```

```
36*B^2*b*i^2*n*log(e) - 48*(b*x + a)*B^2*d*i^2*n*log(e)/(d*x + c) + 36*A*B*
b*i^2*n - 48*(b*x + a)*A*B*d*i^2*n/(d*x + c))*log((b*x + a)/(d*x + c))/((b*
x + a)^4*b*c*g^5/(d*x + c)^4 - (b*x + a)^4*a*d*g^5/(d*x + c)^4) + (27*B^2*b
*i^2*n^2 - 64*(b*x + a)*B^2*d*i^2*n^2/(d*x + c) + 108*B^2*b*i^2*n*log(e) -
192*(b*x + a)*B^2*d*i^2*n*log(e)/(d*x + c) + 216*B^2*b*i^2*log(e)^2 - 288*(
b*x + a)*B^2*d*i^2*log(e)^2/(d*x + c) + 108*A*B*b*i^2*n - 192*(b*x + a)*A*B
*d*i^2*n/(d*x + c) + 432*A*B*b*i^2*log(e) - 576*(b*x + a)*A*B*d*i^2*log(e)/
(d*x + c) + 216*A^2*b*i^2 - 288*(b*x + a)*A^2*d*i^2/(d*x + c))/((b*x + a)^4
*b*c*g^5/(d*x + c)^4 - (b*x + a)^4*a*d*g^5/(d*x + c)^4))*(b*c/(b*c - a*d)^2
 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 5.31 (sec) , antiderivative size = 1934, normalized size of antiderivative = 6.06

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx=\text{Too large to display}$$

```
[In] int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x
)^5,x)
```

```
[Out] - log(e*((a + b*x)/(c + d*x))^n)*((a*(A*B*a*d^2*i^2 - (B^2*a*d^2*i^2*n)/2 +
 (B^2*b*c*d*i^2*n)/2 + 2*A*B*b*c*d*i^2) + x*(b*(A*B*a*d^2*i^2 - (B^2*a*d^2*
i^2*n)/2 + (B^2*b*c*d*i^2*n)/2 + 2*A*B*b*c*d*i^2) + 3*A*B*a*b*d^2*i^2 + 6*A
*B*b^2*c*d*i^2 - (3*B^2*a*b*d^2*i^2*n)/2 + (3*B^2*b^2*c*d*i^2*n)/2) + 3*A*B
*b^2*c^2*i^2 - B^2*a^2*d^2*i^2*n + (B^2*b^2*c^2*i^2*n)/2 + 6*A*B*b^2*d^2*i^
2*x^2 + (B^2*a*b*c*d*i^2*n)/2)/(6*a^4*b^3*g^5 + 6*b^7*g^5*x^4 + 24*a^3*b^4*
g^5*x + 24*a*b^6*g^5*x^3 + 36*a^2*b^5*g^5*x^2) + (B^2*d^4*i^2*(x^2*(b*(b*((
3*a*b^3*g^5*n*(a*d - b*c))/(2*d) + (b^3*g^5*n*(a*d - b*c)*(4*a*d - b*c))/(2
*d^2)) + (3*a*b^4*g^5*n*(a*d - b*c))/d + (b^4*g^5*n*(a*d - b*c)*(4*a*d - b*
c))/d^2) + (9*a*b^5*g^5*n*(a*d - b*c))/(2*d) + (3*b^5*g^5*n*(a*d - b*c)*(4*
a*d - b*c))/(2*d^2)) + a*(a*((3*a*b^3*g^5*n*(a*d - b*c))/(2*d) + (b^3*g^5*n
*(a*d - b*c)*(4*a*d - b*c))/(2*d^2)) + (b^3*g^5*n*(a*d - b*c)*(6*a^2*d^2 +
b^2*c^2 - 4*a*b*c*d))/(2*d^3)) + x*(a*(b*((3*a*b^3*g^5*n*(a*d - b*c))/(2*d)
 + (b^3*g^5*n*(a*d - b*c)*(4*a*d - b*c))/(2*d^2)) + (3*a*b^4*g^5*n*(a*d - b
*c))/d + (b^4*g^5*n*(a*d - b*c)*(4*a*d - b*c))/d^2) + b*(a*((3*a*b^3*g^5*n*
(a*d - b*c))/(2*d) + (b^3*g^5*n*(a*d - b*c)*(4*a*d - b*c))/(2*d^2)) + (b^3*
g^5*n*(a*d - b*c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(2*d^3)) + (3*b^4*g^5*
n*(a*d - b*c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(2*d^3)) + (3*b^3*g^5*n*(a
*d - b*c)*(4*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b*c*d^2))/(2*d^4) +
(6*b^6*g^5*n*x^3*(a*d - b*c))/d))/(6*b^3*g^5*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d
)*(6*a^4*b^3*g^5 + 6*b^7*g^5*x^4 + 24*a^3*b^4*g^5*x + 24*a*b^6*g^5*x^3 + 36
*a^2*b^5*g^5*x^2))) - ((72*A^2*a^3*d^3*i^2 - 216*A^2*b^3*c^3*i^2 + 37*B^2*a
^3*d^3*i^2*n^2 - 27*B^2*b^3*c^3*i^2*n^2 + 72*A^2*a*b^2*c^2*d*i^2 + 72*A^2*a
^2*b*c*d^2*i^2 + 84*A*B*a^3*d^3*i^2*n - 108*A*B*b^3*c^3*i^2*n + 37*B^2*a*b^
```

2*c^2*d*i^2*n^2 + 37*B^2*a^2*b*c*d^2*i^2*n^2 + 84*A*B*a*b^2*c^2*d*i^2*n + 84*A*B*a^2*b*c*d^2*i^2*n)/(12*(a*d - b*c)) + (x^3*(7*B^2*b^3*d^3*i^2*n^2 + 12*A*B*b^3*d^3*i^2*n))/(a*d - b*c) + (x*(72*A^2*a^2*b*d^3*i^2 - 144*A^2*b^3*c^2*d*i^2 + 72*A^2*a*b^2*c*d^2*i^2 + 37*B^2*a^2*b*d^3*i^2*n^2 - 11*B^2*b^3*c^2*d*i^2*n^2 - 60*A*B*b^3*c^2*d*i^2*n + 37*B^2*a*b^2*c*d^2*i^2*n^2 + 84*A*B*a^2*b*d^3*i^2*n + 84*A*B*a*b^2*c*d^2*i^2*n))/(3*(a*d - b*c)) + (x^2*(72*A^2*a*b^2*d^3*i^2 - 72*A^2*b^3*c*d^2*i^2 + 37*B^2*a*b^2*d^3*i^2*n^2 + 5*B^2*b^3*c*d^2*i^2*n^2 - 12*A*B*b^3*c*d^2*i^2*n + 84*A*B*a*b^2*d^3*i^2*n))/(2*(a*d - b*c)))/(72*a^4*b^3*g^5 + 72*b^7*g^5*x^4 + 288*a^3*b^4*g^5*x + 288*a*b^6*g^5*x^3 + 432*a^2*b^5*g^5*x^2) - log(e*((a + b*x)/(c + d*x))^n)^2*((a*((B^2*c*d*i^2)/(6*b^2) + (B^2*a*d^2*i^2)/(12*b^3)) + x*(b*((B^2*c*d*i^2)/(6*b^2) + (B^2*a*d^2*i^2)/(12*b^3)) + (B^2*c*d*i^2)/(2*b) + (B^2*a*d^2*i^2)/(4*b^2)) + (B^2*c^2*i^2)/(4*b) + (B^2*d^2*i^2*x^2)/(2*b))/(a^4*g^5 + b^4*g^5*x^4 + 4*a*b^3*g^5*x^3 + 6*a^2*b^2*g^5*x^2 + 4*a^3*b*g^5*x) - (B^2*d^4*i^2)/(12*b^3*g^5*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - (B*d^4*i^2*n*atan(((2*b*d*x - (72*b^5*c^2*g^5 - 72*a^2*b^3*d^2*g^5)/(72*b^3*g^5*(a*d - b*c)))*1i)/(a*d - b*c))*(12*A + 7*B*n)*1i)/(36*b^3*g^5*(a*d - b*c)^2)

3.177 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 493

$$\begin{aligned}\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx = & -\frac{2B^2d^2i^2n^2(c+dx)^3}{27(bc-ad)^3g^6(a+bx)^3} \\ & +\frac{bB^2di^2n^2(c+dx)^4}{16(bc-ad)^3g^6(a+bx)^4} \\ & -\frac{2b^2B^2i^2n^2(c+dx)^5}{125(bc-ad)^3g^6(a+bx)^5} \\ & -\frac{2Bd^2i^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^3g^6(a+bx)^3} \\ & +\frac{bBdi^2n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bc-ad)^3g^6(a+bx)^4} \\ & -\frac{2b^2Bi^2n(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{25(bc-ad)^3g^6(a+bx)^5} \\ & -\frac{d^2i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^3g^6(a+bx)^3} \\ & +\frac{bdi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^3g^6(a+bx)^4} \\ & -\frac{b^2i^2(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5(bc-ad)^3g^6(a+bx)^5}\end{aligned}$$

```
[Out] -2/27*B^2*d^2*i^2*n^2*(d*x+c)^3/(-a*d+b*c)^3/g^6/(b*x+a)^3+1/16*b*B^2*d*i^2
*n^2*(d*x+c)^4/(-a*d+b*c)^3/g^6/(b*x+a)^4-2/125*b^2*B^2*i^2*n^2*(d*x+c)^5/(
-a*d+b*c)^3/g^6/(b*x+a)^5-2/9*B*d^2*i^2*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x
```

```
+c))^n))/(-a*d+b*c)^3/g^6/(b*x+a)^3+1/4*b*B*d*i^2*n*(d*x+c)^4*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^6/(b*x+a)^4-2/25*b^2*B*i^2*n*(d*x+c)^5*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^6/(b*x+a)^5-1/3*d^2*i^2*(d*x+c
)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^6/(b*x+a)^3+1/2*b*d*i^
2*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^6/(b*x+a)^4-1/
5*b^2*i^2*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^6/(b*x
+a)^5
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$, Rules used = {2561, 2395, 2342, 2341}

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}dx = -\frac{b^2i^2(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5g^6(a+bx)^5(bc-ad)^3} - \frac{2b^2Bi^2n(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{25g^6(a+bx)^5(bc-ad)^3} - \frac{d^2i^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g^6(a+bx)^3(bc-ad)^3} - \frac{2Bd^2i^2n(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9g^6(a+bx)^3(bc-ad)^3} + \frac{bdi^2(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g^6(a+bx)^4(bc-ad)^3} + \frac{bBdi^2n(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4g^6(a+bx)^4(bc-ad)^3} - \frac{2b^2B^2i^2n^2(c+dx)^5}{125g^6(a+bx)^5(bc-ad)^3} - \frac{2B^2d^2i^2n^2(c+dx)^3}{27g^6(a+bx)^3(bc-ad)^3} + \frac{bB^2di^2n^2(c+dx)^4}{16g^6(a+bx)^4(bc-ad)^3}$$

```
[In] Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x
)^6,x]
```

```
[Out] (-2*B^2*d^2*i^2*n^2*(c + d*x)^3)/(27*(b*c - a*d)^3*g^6*(a + b*x)^3) + (b*B^
2*d*i^2*n^2*(c + d*x)^4)/(16*(b*c - a*d)^3*g^6*(a + b*x)^4) - (2*b^2*B^2*i^
2*n^2*(c + d*x)^5)/(125*(b*c - a*d)^3*g^6*(a + b*x)^5) - (2*B*d^2*i^2*n*(c
+ d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(b*c - a*d)^3*g^6*(a +
b*x)^3) + (b*B*d*i^2*n*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/
(4*(b*c - a*d)^3*g^6*(a + b*x)^4) - (2*b^2*B*i^2*n*(c + d*x)^5*(A + B*Log[e
```

```
*((a + b*x)/(c + d*x))^n]))/(25*(b*c - a*d)^3*g^6*(a + b*x)^5) - (d^2*i^2*(
c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*(b*c - a*d)^3*g^6*(
a + b*x)^3) + (b*d*i^2*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
)/(2*(b*c - a*d)^3*g^6*(a + b*x)^4) - (b^2*i^2*(c + d*x)^5*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n])^2)/(5*(b*c - a*d)^3*g^6*(a + b*x)^5)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule steps

$$\text{integral} = \frac{i^2\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex^n))^2}{x^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^6}$$

$$= \frac{i^2\text{Subst}\left(\int \left(\frac{b^2(A+B\log(ex^n))^2}{x^6} - \frac{2bd(A+B\log(ex^n))^2}{x^5} + \frac{d^2(A+B\log(ex^n))^2}{x^4}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^6}$$

$$
\begin{aligned}
&= \frac{(b^2i^2)\,\mathrm{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^6} \\
&\quad - \frac{(2bdi^2)\,\mathrm{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^6} \\
&\quad + \frac{(d^2i^2)\,\mathrm{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^6} \\
&= -\frac{d^2i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^3g^6(a+bx)^3} + \frac{bdi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^3g^6(a+bx)^4} \\
&\quad - \frac{b^2i^2(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5(bc-ad)^3g^6(a+bx)^5} + \frac{(2b^2Bi^2n)\,\mathrm{Subst}\left(\int \frac{A+B\log(ex^n)}{x^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{5(bc-ad)^3g^6} \\
&\quad - \frac{(bBdi^2n)\,\mathrm{Subst}\left(\int \frac{A+B\log(ex^n)}{x^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^6} + \frac{(2Bd^2i^2n)\,\mathrm{Subst}\left(\int \frac{A+B\log(ex^n)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bc-ad)^3g^6} \\
&= -\frac{2B^2d^2i^2n^2(c+dx)^3}{27(bc-ad)^3g^6(a+bx)^3} + \frac{bB^2di^2n^2(c+dx)^4}{16(bc-ad)^3g^6(a+bx)^4} - \frac{2b^2B^2i^2n^2(c+dx)^5}{125(bc-ad)^3g^6(a+bx)^5} \\
&\quad - \frac{2Bd^2i^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^3g^6(a+bx)^3} + \frac{bBdi^2n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bc-ad)^3g^6(a+bx)^4} \\
&\quad - \frac{2b^2Bi^2n(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{25(bc-ad)^3g^6(a+bx)^5} - \frac{d^2i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^3g^6(a+bx)^3} \\
&\quad + \frac{bdi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^3g^6(a+bx)^4} - \frac{b^2i^2(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5(bc-ad)^3g^6(a+bx)^5}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.80 (sec) , antiderivative size = 2112, normalized size of antiderivative = 4.28

$$
\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g +
 b*g*x)^6,x]
[Out] -1/54000*(i^2*(10800*(b*c - a*d)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
 + 27000*d*(b*c - a*d)^4*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
 - 18000*d^2*(-(b*c) + a*d)^3*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^2 + 1000*B*d^2*n*(a + b*x)^2*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3*
n - 18*A*d*(b*c - a*d)^2*(a + b*x) - 15*B*d*(b*c - a*d)^2*n*(a + b*x) + 36*
A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a*d)*n*(a + b*x)^2 + 36*A*d
```

^3*(a + b*x)^3*Log[a + b*x] + 66*B*d^3*n*(a + b*x)^3*Log[a + b*x] - 18*B*d^
3*n*(a + b*x)^3*Log[a + b*x]^2 + 12*B*(b*c - a*d)^3*Log[e*((a + b*x)/(c + d
*x))^n] - 18*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 3
6*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d^3*(
a + b*x)^3*Log[a + b*x]*Log[e*((a + b*x)/(c + d*x))^n] - 36*A*d^3*(a + b*x)
^3*Log[c + d*x] - 66*B*d^3*n*(a + b*x)^3*Log[c + d*x] + 36*B*d^3*n*(a + b*x
)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*L
og[e*((a + b*x)/(c + d*x))^n]*Log[c + d*x] - 18*B*d^3*n*(a + b*x)^3*Log[c +
 d*x]^2 + 36*B*d^3*n*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)
] + 36*B*d^3*n*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 36*B*
d^3*n*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 375*B*d*n*(a + b
x)(36*A*(b*c - a*d)^4 + 9*B*(b*c - a*d)^4*n + 48*A*d*(-(b*c) + a*d)^3*(a
+ b*x) + 28*B*d*(-(b*c) + a*d)^3*n*(a + b*x) + 72*A*d^2*(b*c - a*d)^2*(a +
b*x)^2 + 78*B*d^2*(b*c - a*d)^2*n*(a + b*x)^2 + 144*A*d^3*(-(b*c) + a*d)*(a
 + b*x)^3 + 300*B*d^3*(-(b*c) + a*d)*n*(a + b*x)^3 - 144*A*d^4*(a + b*x)^4*
Log[a + b*x] - 300*B*d^4*n*(a + b*x)^4*Log[a + b*x] + 72*B*d^4*n*(a + b*x)^
4*Log[a + b*x]^2 + 36*B*(b*c - a*d)^4*Log[e*((a + b*x)/(c + d*x))^n] + 48*B
d(-(b*c) + a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 72*B*d^2*(b*
c - a*d)^2*(a + b*x)^2*Log[e*((a + b*x)/(c + d*x))^n] + 144*B*d^3*(-(b*c) +
 a*d)*(a + b*x)^3*Log[e*((a + b*x)/(c + d*x))^n] - 144*B*d^4*(a + b*x)^4*Lo
g[a + b*x]*Log[e*((a + b*x)/(c + d*x))^n] + 144*A*d^4*(a + b*x)^4*Log[c + d
*x] + 300*B*d^4*n*(a + b*x)^4*Log[c + d*x] - 144*B*d^4*n*(a + b*x)^4*Log[(d
*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] + 144*B*d^4*(a + b*x)^4*Log[e*((a
+ b*x)/(c + d*x))^n]*Log[c + d*x] + 72*B*d^4*n*(a + b*x)^4*Log[c + d*x]^2 -
 144*B*d^4*n*(a + b*x)^4*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 144*
B*d^4*n*(a + b*x)^4*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 144*B*d^4*n*
(a + b*x)^4*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 6*B*n*(720*A*(b*c - a*
d)^5 + 144*B*(b*c - a*d)^5*n - 900*A*d*(b*c - a*d)^4*(a + b*x) - 405*B*d*(b
*c - a*d)^4*n*(a + b*x) + 1200*A*d^2*(b*c - a*d)^3*(a + b*x)^2 + 940*B*d^2*
(b*c - a*d)^3*n*(a + b*x)^2 - 1800*A*d^3*(b*c - a*d)^2*(a + b*x)^3 - 2310*B
d^3(b*c - a*d)^2*n*(a + b*x)^3 + 3600*A*d^4*(b*c - a*d)*(a + b*x)^4 + 822
0*B*d^4*(b*c - a*d)*n*(a + b*x)^4 + 3600*A*d^5*(a + b*x)^5*Log[a + b*x] + 8
220*B*d^5*n*(a + b*x)^5*Log[a + b*x] - 1800*B*d^5*n*(a + b*x)^5*Log[a + b*x
]^2 + 720*B*(b*c - a*d)^5*Log[e*((a + b*x)/(c + d*x))^n] - 900*B*d*(b*c - a
d)^4(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 1200*B*d^2*(b*c - a*d)^3*(
a + b*x)^2*Log[e*((a + b*x)/(c + d*x))^n] - 1800*B*d^3*(b*c - a*d)^2*(a + b
*x)^3*Log[e*((a + b*x)/(c + d*x))^n] + 3600*B*d^4*(b*c - a*d)*(a + b*x)^4*L
og[e*((a + b*x)/(c + d*x))^n] + 3600*B*d^5*(a + b*x)^5*Log[a + b*x]*Log[e*(
(a + b*x)/(c + d*x))^n] - 3600*A*d^5*(a + b*x)^5*Log[c + d*x] - 8220*B*d^5*
n*(a + b*x)^5*Log[c + d*x] + 3600*B*d^5*n*(a + b*x)^5*Log[(d*(a + b*x))/(-(
b*c) + a*d)]*Log[c + d*x] - 3600*B*d^5*(a + b*x)^5*Log[e*((a + b*x)/(c + d*
x))^n]*Log[c + d*x] - 1800*B*d^5*n*(a + b*x)^5*Log[c + d*x]^2 + 3600*B*d^5*
n*(a + b*x)^5*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 3600*B*d^5*n*(a
 + b*x)^5*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 3600*B*d^5*n*(a + b*x)
^5*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b^3*(b*c - a*d)^3*g^6*(a + b*x

)^5)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2527 vs. 2(475) = 950.

Time = 56.95 (sec) , antiderivative size = 2528, normalized size of antiderivative = 5.13

method	result	size
parallelrisch	Expression too large to display	2528

```
[In] int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^6,x,method=
_RETURNVERBOSE)
```

```
[Out] -1/54000*(9000*A*B*x^2*a*b^8*c^2*d^4*i^2*n^2-54000*B^2*x*ln(e*((b*x+a)/(d*x
+c))^n)^2*a^2*b^7*c^2*d^4*i^2*n+72000*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b
^8*c^3*d^3*i^2*n-36000*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^7*c^2*d^4*i^2*
n^2+30000*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c^3*d^3*i^2*n^2-54000*A*B*x
*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^4*d^2*i^2*n-36000*A*B*x*a^2*b^7*c^2*d^4*i^
2*n^2+30000*A*B*x*a*b^8*c^3*d^3*i^2*n^2-36000*A*B*ln(e*((b*x+a)/(d*x+c))^n)
*a^2*b^7*c^3*d^3*i^2*n+54000*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c^4*d^2*i^
2*n-18000*A*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*d^6*i^2*n-18000*B^2*x^3*l
n(e*((b*x+a)/(d*x+c))^n)*a*b^8*c*d^5*i^2*n^2-36000*A*B*x^3*ln(e*((b*x+a)/(d
*x+c))^n)*a^2*b^7*d^6*i^2*n-18000*A*B*x^3*a*b^8*c*d^5*i^2*n^2-54000*B^2*x^2
*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^7*c*d^5*i^2*n+54000*B^2*x^2*ln(e*((b*x+a
)/(d*x+c))^n)^2*a*b^8*c^2*d^4*i^2*n-36000*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)
*a^2*b^7*c*d^5*i^2*n^2+9000*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c^2*d^4
*i^2*n^2-36000*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^3*d^3*i^2*n-36000*A*
B*x^2*a^2*b^7*c*d^5*i^2*n^2-4000*B^2*a^2*b^7*c^3*d^3*i^2*n^3+3375*B^2*a*b^8
*c^4*d^2*i^2*n^3+2820*A*B*a^5*b^4*d^6*i^2*n^2-4320*A*B*b^9*c^5*d*i^2*n^2-18
000*A^2*a^2*b^7*c^3*d^3*i^2*n+27000*A^2*a*b^8*c^4*d^2*i^2*n-108000*A*B*x^2*
ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^7*c*d^5*i^2*n+108000*A*B*x^2*ln(e*((b*x+a)/
(d*x+c))^n)*a*b^8*c^2*d^4*i^2*n-108000*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*
b^7*c^2*d^4*i^2*n+144000*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c^3*d^3*i^2*
n-1800*B^2*x^5*ln(e*((b*x+a)/(d*x+c))^n)^2*b^9*d^6*i^2*n-2820*B^2*x^5*ln(e*
((b*x+a)/(d*x+c))^n)*b^9*d^6*i^2*n^2+2820*B^2*x^4*a*b^8*d^6*i^2*n^3-2820*B^
2*x^4*b^9*c*d^5*i^2*n^3+10890*B^2*x^3*a^2*b^7*d^6*i^2*n^3-390*B^2*x^3*b^9*c
^2*d^4*i^2*n^3+14890*B^2*x^2*a^3*b^6*d^6*i^2*n^3+860*B^2*x^2*b^9*c^3*d^3*i^
2*n^3+7445*B^2*x*a^4*b^5*d^6*i^2*n^3-945*B^2*x*b^9*c^4*d^2*i^2*n^3+18000*A^
2*x^2*a^3*b^6*d^6*i^2*n-18000*A^2*x^2*b^9*c^3*d^3*i^2*n-10800*B^2*ln(e*((b*
x+a)/(d*x+c))^n)^2*b^9*c^5*d*i^2*n-4320*B^2*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c
^5*d*i^2*n^2+9000*A^2*x*a^4*b^5*d^6*i^2*n-27000*A^2*x*b^9*c^4*d^2*i^2*n+360
0*A*B*x^4*a*b^8*d^6*i^2*n^2-3600*A*B*x^4*b^9*c*d^5*i^2*n^2-18000*B^2*x^3*ln
(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^7*d^6*i^2*n-12000*B^2*x^3*ln(e*((b*x+a)/(d*
x+c))^n)*a^2*b^7*d^6*i^2*n^2+1800*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^2
*d^4*i^2*n^2-10500*B^2*x^3*a*b^8*c*d^5*i^2*n^3+16200*A*B*x^3*a^2*b^7*d^6*i^
```

```
2*n^2+1800*A*B*x^3*b^9*c^2*d^4*i^2*n^2-18000*B^2*x^2*ln(e*((b*x+a)/(d*x+c))
^n)^2*b^9*c^3*d^3*i^2*n-1200*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^3*d^3*
i^2*n^2-12000*B^2*x^2*a^2*b^7*c*d^5*i^2*n^3-3750*B^2*x^2*a*b^8*c^2*d^4*i^2*
n^3+28200*A*B*x^2*a^3*b^6*d^6*i^2*n^2-1200*A*B*x^2*b^9*c^3*d^3*i^2*n^2-2700
0*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^9*c^4*d^2*i^2*n-8100*B^2*x*ln(e*((b*x
+a)/(d*x+c))^n)*b^9*c^4*d^2*i^2*n^2-12000*B^2*x*a^2*b^7*c^2*d^4*i^2*n^3+550
0*B^2*x*a*b^8*c^3*d^3*i^2*n^3-54000*A^2*x^2*a^2*b^7*c*d^5*i^2*n+54000*A^2*x
^2*a*b^8*c^2*d^4*i^2*n+14100*A*B*x*a^4*b^5*d^6*i^2*n^2-8100*A*B*x*b^9*c^4*d
^2*i^2*n^2-18000*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^7*c^3*d^3*i^2*n+2700
0*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^8*c^4*d^2*i^2*n-12000*B^2*ln(e*((b*x+
a)/(d*x+c))^n)*a^2*b^7*c^3*d^3*i^2*n^2+13500*B^2*ln(e*((b*x+a)/(d*x+c))^n)*
a*b^8*c^4*d^2*i^2*n^2-54000*A^2*x*a^2*b^7*c^2*d^4*i^2*n+72000*A^2*x*a*b^8*c
^3*d^3*i^2*n-21600*A*B*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^5*d*i^2*n-12000*A*B*
a^2*b^7*c^3*d^3*i^2*n^2+13500*A*B*a*b^8*c^4*d^2*i^2*n^2-3600*A*B*x^5*ln(e*(
(b*x+a)/(d*x+c))^n)*b^9*d^6*i^2*n-9000*B^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)^2*
a*b^8*d^6*i^2*n-10500*B^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*d^6*i^2*n^2-3
600*B^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c*d^5*i^2*n^2+1489*B^2*a^5*b^4*d^
6*i^2*n^3-864*B^2*b^9*c^5*d*i^2*n^3+1800*A^2*a^5*b^4*d^6*i^2*n-10800*A^2*b^
9*c^5*d*i^2*n)/g^6/(b*x+a)^5/(a*d-b*c)^3/d/b^7/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2633 vs. $2(475) = 950$.

Time = 0.47 (sec) , antiderivative size = 2633, normalized size of antiderivative = 5.34

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^6,x,
 algorithm="fricas")
```

```
[Out] -1/54000*((864*B^2*b^5*c^5 - 3375*B^2*a*b^4*c^4*d + 4000*B^2*a^2*b^3*c^3*d^
2 - 1489*B^2*a^5*d^5)*i^2*n^2 + 60*(47*(B^2*b^5*c*d^4 - B^2*a*b^4*d^5)*i^2*
n^2 + 60*(A*B*b^5*c*d^4 - A*B*a*b^4*d^5)*i^2*n)*x^4 + 60*(72*A*B*b^5*c^5 -
225*A*B*a*b^4*c^4*d + 200*A*B*a^2*b^3*c^3*d^2 - 47*A*B*a^5*d^5)*i^2*n + 30*
((13*B^2*b^5*c^2*d^3 + 350*B^2*a*b^4*c*d^4 - 363*B^2*a^2*b^3*d^5)*i^2*n^2 -
 60*(A*B*b^5*c^2*d^3 - 10*A*B*a*b^4*c*d^4 + 9*A*B*a^2*b^3*d^5)*i^2*n)*x^3 +
 1800*(6*A^2*b^5*c^5 - 15*A^2*a*b^4*c^4*d + 10*A^2*a^2*b^3*c^3*d^2 - A^2*a^
5*d^5)*i^2 - 10*((86*B^2*b^5*c^3*d^2 - 375*B^2*a*b^4*c^2*d^3 - 1200*B^2*a^2
*b^3*c*d^4 + 1489*B^2*a^3*b^2*d^5)*i^2*n^2 - 60*(2*A*B*b^5*c^3*d^2 - 15*A*B
*a*b^4*c^2*d^3 + 60*A*B*a^2*b^3*c*d^4 - 47*A*B*a^3*b^2*d^5)*i^2*n - 1800*(A
^2*b^5*c^3*d^2 - 3*A^2*a*b^4*c^2*d^3 + 3*A^2*a^2*b^3*c*d^4 - A^2*a^3*b^2*d^
5)*i^2)*x^2 + 1800*(10*(B^2*b^5*c^3*d^2 - 3*B^2*a*b^4*c^2*d^3 + 3*B^2*a^2*b
^3*c*d^4 - B^2*a^3*b^2*d^5)*i^2*x^2 + 5*(3*B^2*b^5*c^4*d - 8*B^2*a*b^4*c^3*
```

d^2 + 6*B^2*a^2*b^3*c^2*d^3 - B^2*a^4*b*d^5)*i^2*x + (6*B^2*b^5*c^5 - 15*B^2*a*b^4*c^4*d + 10*B^2*a^2*b^3*c^3*d^2 - B^2*a^5*d^5)*i^2)*log(e)^2 + 1800*(B^2*b^5*d^5*i^2*n^2*x^5 + 5*B^2*a*b^4*d^5*i^2*n^2*x^4 + 10*B^2*a^2*b^3*d^5*i^2*n^2*x^3 + 10*(B^2*b^5*c^3*d^2 - 3*B^2*a*b^4*c^2*d^3 + 3*B^2*a^2*b^3*c*d^4)*i^2*n^2*x^2 + 5*(3*B^2*b^5*c^4*d - 8*B^2*a*b^4*c^3*d^2 + 6*B^2*a^2*b^3*c^2*d^3)*i^2*n^2*x + (6*B^2*b^5*c^5 - 15*B^2*a*b^4*c^4*d + 10*B^2*a^2*b^3*c^3*d^2)*i^2*n^2)*log((b*x + a)/(d*x + c))^2 + 5*((189*B^2*b^5*c^4*d - 1100*B^2*a*b^4*c^3*d^2 + 2400*B^2*a^2*b^3*c^2*d^3 - 1489*B^2*a^4*b*d^5)*i^2*n^2 + 60*(27*A*B*b^5*c^4*d - 100*A*B*a*b^4*c^3*d^2 + 120*A*B*a^2*b^3*c^2*d^3 - 47*A*B*a^4*b*d^5)*i^2*n + 1800*(3*A^2*b^5*c^4*d - 8*A^2*a*b^4*c^3*d^2 + 6*A^2*a^2*b^3*c^2*d^3 - A^2*a^4*b*d^5)*i^2)*x + 60*(60*(B^2*b^5*c*d^4 - B^2*a*b^4*d^5)*i^2*n*x^4 - 30*(B^2*b^5*c^2*d^3 - 10*B^2*a*b^4*c*d^4 + 9*B^2*a^2*b^3*d^5)*i^2*n*x^3 + (72*B^2*b^5*c^5 - 225*B^2*a*b^4*c^4*d + 200*B^2*a^2*b^3*c^3*d^2 - 47*B^2*a^5*d^5)*i^2*n + 60*(6*A*B*b^5*c^5 - 15*A*B*a*b^4*c^4*d + 10*A*B*a^2*b^3*c^3*d^2 - A*B*a^5*d^5)*i^2 + 10*((2*B^2*b^5*c^3*d^2 - 15*B^2*a*b^4*c^2*d^3 + 60*B^2*a^2*b^3*c*d^4 - 47*B^2*a^3*b^2*d^5)*i^2*n + 60*(A*B*b^5*c^3*d^2 - 3*A*B*a*b^4*c^2*d^3 + 3*A*B*a^2*b^3*c*d^4 - A*B*a^3*b^2*d^5)*i^2)*x^2 + 5*((27*B^2*b^5*c^4*d - 100*B^2*a*b^4*c^3*d^2 + 120*B^2*a^2*b^3*c^2*d^3 - 47*B^2*a^4*b*d^5)*i^2*n + 60*(3*A*B*b^5*c^4*d - 8*A*B*a*b^4*c^3*d^2 + 6*A*B*a^2*b^3*c^2*d^3 - A*B*a^4*b*d^5)*i^2)*x + 60*(B^2*b^5*d^5*i^2*n*x^5 + 5*B^2*a*b^4*d^5*i^2*n*x^4 + 10*B^2*a^2*b^3*d^5*i^2*n*x^3 + 10*(B^2*b^5*c^3*d^2 - 3*B^2*a*b^4*c^2*d^3 + 3*B^2*a^2*b^3*c*d^4)*i^2*n*x^2 + 5*(3*B^2*b^5*c^4*d - 8*B^2*a*b^4*c^3*d^2 + 6*B^2*a^2*b^3*c^2*d^3)*i^2*n*x + (6*B^2*b^5*c^5 - 15*B^2*a*b^4*c^4*d + 10*B^2*a^2*b^3*c^3*d^2)*i^2*n)*log((b*x + a)/(d*x + c)))*log(e) + 60*((47*B^2*b^5*d^5*i^2*n^2 + 60*A*B*b^5*d^5*i^2*n)*x^5 + (72*B^2*b^5*c^5 - 225*B^2*a*b^4*c^4*d + 200*B^2*a^2*b^3*c^3*d^2)*i^2*n^2 + 5*(60*A*B*a*b^4*d^5*i^2*n + (12*B^2*b^5*c*d^4 + 35*B^2*a*b^4*d^5)*i^2*n^2)*x^4 + 60*(6*A*B*b^5*c^5 - 15*A*B*a*b^4*c^4*d + 10*A*B*a^2*b^3*c^3*d^2)*i^2*n + 10*(60*A*B*a^2*b^3*d^5*i^2*n - (3*B^2*b^5*c^2*d^3 - 30*B^2*a*b^4*c*d^4 - 20*B^2*a^2*b^3*d^5)*i^2*n^2)*x^3 + 10*((2*B^2*b^5*c^3*d^2 - 15*B^2*a*b^4*c^2*d^3 + 60*B^2*a^2*b^3*c*d^4)*i^2*n^2 + 60*(A*B*b^5*c^3*d^2 - 3*A*B*a*b^4*c^2*d^3 + 3*A*B*a^2*b^3*c*d^4)*i^2*n)*x^2 + 5*((27*B^2*b^5*c^4*d - 100*B^2*a*b^4*c^3*d^2 + 120*B^2*a^2*b^3*c^2*d^3)*i^2*n^2 + 60*(3*A*B*b^5*c^4*d - 8*A*B*a*b^4*c^3*d^2 + 6*A*B*a^2*b^3*c^2*d^3)*i^2*n)*x)*log((b*x + a)/(d*x + c)))/((b^11*c^3 - 3*a*b^10*c^2*d + 3*a^2*b^9*c*d^2 - a^3*b^8*d^3)*g^6*x^5 + 5*(a*b^10*c^3 - 3*a^2*b^9*c^2*d + 3*a^3*b^8*c*d^2 - a^4*b^7*d^3)*g^6*x^4 + 10*(a^2*b^9*c^3 - 3*a^3*b^8*c^2*d + 3*a^4*b^7*c*d^2 - a^5*b^6*d^3)*g^6*x^3 + 10*(a^3*b^8*c^3 - 3*a^4*b^7*c^2*d + 3*a^5*b^6*c*d^2 - a^6*b^5*d^3)*g^6*x^2 + 5*(a^4*b^7*c^3 - 3*a^5*b^6*c^2*d + 3*a^6*b^5*c*d^2 - a^7*b^4*d^3)*g^6*x + (a^5*b^6*c^3 - 3*a^6*b^5*c^2*d + 3*a^7*b^4*c*d^2 - a^8*b^3*d^3)*g^6)

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**6
,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10936 vs. 2(475) = 950.

Time = 1.02 (sec) , antiderivative size = 10936, normalized size of antiderivative = 22.18

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^6,x,
 algorithm="maxima")
```

```
[Out] -1/150*A*B*c^2*i^2*n*((60*b^4*d^4*x^4 + 12*b^4*c^4 - 63*a*b^3*c^3*d + 137*a
^2*b^2*c^2*d^2 - 163*a^3*b*c*d^3 + 137*a^4*d^4 - 30*(b^4*c*d^3 - 9*a*b^3*d^
4)*x^3 + 10*(2*b^4*c^2*d^2 - 13*a*b^3*c*d^3 + 47*a^2*b^2*d^4)*x^2 - 5*(3*b^
4*c^3*d - 17*a*b^3*c^2*d^2 + 43*a^2*b^2*c*d^3 - 77*a^3*b*d^4)*x)/((b^10*c^4
 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*g^6*x
^5 + 5*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 +
 a^5*b^5*d^4)*g^6*x^4 + 10*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d
^2 - 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*g^6*x^3 + 10*(a^3*b^7*c^4 - 4*a^4*b^6*c
^3*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*g^6*x^2 + 5*(a^4*
b^6*c^4 - 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d
^4)*g^6*x + (a^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4*a^8*b^2*
c*d^3 + a^9*b*d^4)*g^6) + 60*d^5*log(b*x + a)/((b^6*c^5 - 5*a*b^5*c^4*d + 1
0*a^2*b^4*c^3*d^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5)*g^6)
- 60*d^5*log(d*x + c)/((b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2 - 10*a
^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5)*g^6)) - 1/900*A*B*d^2*i^2*n*(
(47*a^2*b^4*c^4 - 278*a^3*b^3*c^3*d + 822*a^4*b^2*c^2*d^2 - 278*a^5*b*c*d^3
 + 47*a^6*d^4 + 60*(10*b^6*c^2*d^2 - 5*a*b^5*c*d^3 + a^2*b^4*d^4)*x^4 - 30*
(10*b^6*c^3*d - 95*a*b^5*c^2*d^2 + 46*a^2*b^4*c*d^3 - 9*a^3*b^3*d^4)*x^3 +
10*(20*b^6*c^4 - 140*a*b^5*c^3*d + 537*a^2*b^4*c^2*d^2 - 248*a^3*b^3*c*d^3
+ 47*a^4*b^2*d^4)*x^2 + 5*(35*a*b^5*c^4 - 218*a^2*b^4*c^3*d + 702*a^3*b^3*c
^2*d^2 - 278*a^4*b^2*c*d^3 + 47*a^5*b*d^4)*x)/((b^12*c^4 - 4*a*b^11*c^3*d +
 6*a^2*b^10*c^2*d^2 - 4*a^3*b^9*c*d^3 + a^4*b^8*d^4)*g^6*x^5 + 5*(a*b^11*c^
```

4 - 4*a^2*b^10*c^3*d + 6*a^3*b^9*c^2*d^2 - 4*a^4*b^8*c*d^3 + a^5*b^7*d^4)*g
^6*x^4 + 10*(a^2*b^10*c^4 - 4*a^3*b^9*c^3*d + 6*a^4*b^8*c^2*d^2 - 4*a^5*b^7
*c*d^3 + a^6*b^6*d^4)*g^6*x^3 + 10*(a^3*b^9*c^4 - 4*a^4*b^8*c^3*d + 6*a^5*b
^7*c^2*d^2 - 4*a^6*b^6*c*d^3 + a^7*b^5*d^4)*g^6*x^2 + 5*(a^4*b^8*c^4 - 4*a^
5*b^7*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^7*b^5*c*d^3 + a^8*b^4*d^4)*g^6*x + (a
^5*b^7*c^4 - 4*a^6*b^6*c^3*d + 6*a^7*b^5*c^2*d^2 - 4*a^8*b^4*c*d^3 + a^9*b^
3*d^4)*g^6) + 60*(10*b^2*c^2*d^3 - 5*a*b*c*d^4 + a^2*d^5)*log(b*x + a)/((b^
8*c^5 - 5*a*b^7*c^4*d + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 + 5*a^4*b^4
*c*d^4 - a^5*b^3*d^5)*g^6) - 60*(10*b^2*c^2*d^3 - 5*a*b*c*d^4 + a^2*d^5)*lo
g(d*x + c)/((b^8*c^5 - 5*a*b^7*c^4*d + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*
d^3 + 5*a^4*b^4*c*d^4 - a^5*b^3*d^5)*g^6)) - 1/300*A*B*c*d*i^2*n*((27*a*b^4
*c^4 - 148*a^2*b^3*c^3*d + 352*a^3*b^2*c^2*d^2 - 548*a^4*b*c*d^3 + 77*a^5*d
^4 - 60*(5*b^5*c*d^3 - a*b^4*d^4)*x^4 + 30*(5*b^5*c^2*d^2 - 46*a*b^4*c*d^3
+ 9*a^2*b^3*d^4)*x^3 - 10*(10*b^5*c^3*d - 67*a*b^4*c^2*d^2 + 248*a^2*b^3*c*
d^3 - 47*a^3*b^2*d^4)*x^2 + 5*(15*b^5*c^4 - 88*a*b^4*c^3*d + 232*a^2*b^3*c^
2*d^2 - 428*a^3*b^2*c*d^3 + 77*a^4*b*d^4)*x)/((b^11*c^4 - 4*a*b^10*c^3*d +
6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4)*g^6*x^5 + 5*(a*b^10*c^4
- 4*a^2*b^9*c^3*d + 6*a^3*b^8*c^2*d^2 - 4*a^4*b^7*c*d^3 + a^5*b^6*d^4)*g^6*
x^4 + 10*(a^2*b^9*c^4 - 4*a^3*b^8*c^3*d + 6*a^4*b^7*c^2*d^2 - 4*a^5*b^6*c*d
^3 + a^6*b^5*d^4)*g^6*x^3 + 10*(a^3*b^8*c^4 - 4*a^4*b^7*c^3*d + 6*a^5*b^6*c
^2*d^2 - 4*a^6*b^5*c*d^3 + a^7*b^4*d^4)*g^6*x^2 + 5*(a^4*b^7*c^4 - 4*a^5*b^
6*c^3*d + 6*a^6*b^5*c^2*d^2 - 4*a^7*b^4*c*d^3 + a^8*b^3*d^4)*g^6*x + (a^5*b
^6*c^4 - 4*a^6*b^5*c^3*d + 6*a^7*b^4*c^2*d^2 - 4*a^8*b^3*c*d^3 + a^9*b^2*d^
4)*g^6) - 60*(5*b*c*d^4 - a*d^5)*log(b*x + a)/((b^7*c^5 - 5*a*b^6*c^4*d + 1
0*a^2*b^5*c^3*d^2 - 10*a^3*b^4*c^2*d^3 + 5*a^4*b^3*c*d^4 - a^5*b^2*d^5)*g^6
) + 60*(5*b*c*d^4 - a*d^5)*log(d*x + c)/((b^7*c^5 - 5*a*b^6*c^4*d + 10*a^2*
b^5*c^3*d^2 - 10*a^3*b^4*c^2*d^3 + 5*a^4*b^3*c*d^4 - a^5*b^2*d^5)*g^6)) - 1
/10*(5*b*x + a)*B^2*c*d*i^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2/(b^7*g
^6*x^5 + 5*a*b^6*g^6*x^4 + 10*a^2*b^5*g^6*x^3 + 10*a^3*b^4*g^6*x^2 + 5*a^4*
b^3*g^6*x + a^5*b^2*g^6) - 1/30*(10*b^2*x^2 + 5*a*b*x + a^2)*B^2*d^2*i^2*lo
g(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2/(b^8*g^6*x^5 + 5*a*b^7*g^6*x^4 + 10*
a^2*b^6*g^6*x^3 + 10*a^3*b^5*g^6*x^2 + 5*a^4*b^4*g^6*x + a^5*b^3*g^6) - 1/9
000*(60*n*((60*b^4*d^4*x^4 + 12*b^4*c^4 - 63*a*b^3*c^3*d + 137*a^2*b^2*c^2*
d^2 - 163*a^3*b*c*d^3 + 137*a^4*d^4 - 30*(b^4*c*d^3 - 9*a*b^3*d^4)*x^3 + 10
*(2*b^4*c^2*d^2 - 13*a*b^3*c*d^3 + 47*a^2*b^2*d^4)*x^2 - 5*(3*b^4*c^3*d - 1
7*a*b^3*c^2*d^2 + 43*a^2*b^2*c*d^3 - 77*a^3*b*d^4)*x)/((b^10*c^4 - 4*a*b^9*
c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*g^6*x^5 + 5*(a*b
^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 + a^5*b^5*d^
4)*g^6*x^4 + 10*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2 - 4*a^5*
b^5*c*d^3 + a^6*b^4*d^4)*g^6*x^3 + 10*(a^3*b^7*c^4 - 4*a^4*b^6*c^3*d + 6*a^
5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*g^6*x^2 + 5*(a^4*b^6*c^4 - 4
*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d^4)*g^6*x +
(a^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4*a^8*b^2*c*d^3 + a^9
*b*d^4)*g^6) + 60*d^5*log(b*x + a)/((b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c
^3*d^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5)*g^6) - 60*d^5*lo

g(d*x + c)/((b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2 - 10*a^3*b^3*c^2*
d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5)*g^6))*log(e*(b*x/(d*x + c) + a/(d*x + c)
)^n) + (144*b^5*c^5 - 1125*a*b^4*c^4*d + 4000*a^2*b^3*c^3*d^2 - 9000*a^3*b^
2*c^2*d^3 + 18000*a^4*b*c*d^4 - 12019*a^5*d^5 + 8220*(b^5*c*d^4 - a*b^4*d^5
)*x^4 - 30*(77*b^5*c^2*d^3 - 1250*a*b^4*c*d^4 + 1173*a^2*b^3*d^5)*x^3 + 10*
(94*b^5*c^3*d^2 - 975*a*b^4*c^2*d^3 + 6600*a^2*b^3*c*d^4 - 5719*a^3*b^2*d^5
)*x^2 - 1800*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b
^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*log(b*x + a)^2 - 1800*(b^5*d^5*x^5 +
5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*d^5*x +
a^5*d^5)*log(d*x + c)^2 - 5*(81*b^5*c^4*d - 700*a*b^4*c^3*d^2 + 3000*a^2*b
^3*c^2*d^3 - 10800*a^3*b^2*c*d^4 + 8419*a^4*b*d^5)*x + 8220*(b^5*d^5*x^5 +
5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*d^5*x +
a^5*d^5)*log(b*x + a) - 60*(137*b^5*d^5*x^5 + 685*a*b^4*d^5*x^4 + 1370*a^2
*b^3*d^5*x^3 + 1370*a^3*b^2*d^5*x^2 + 685*a^4*b*d^5*x + 137*a^5*d^5 - 60*(b
^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b^2*d^5*x^2 + 5*
a^4*b*d^5*x + a^5*d^5)*log(b*x + a))*log(d*x + c))*n^2/(a^5*b^6*c^5*g^6 - 5
*a^6*b^5*c^4*d*g^6 + 10*a^7*b^4*c^3*d^2*g^6 - 10*a^8*b^3*c^2*d^3*g^6 + 5*a^
9*b^2*c*d^4*g^6 - a^10*b*d^5*g^6 + (b^11*c^5*g^6 - 5*a*b^10*c^4*d*g^6 + 10*
a^2*b^9*c^3*d^2*g^6 - 10*a^3*b^8*c^2*d^3*g^6 + 5*a^4*b^7*c*d^4*g^6 - a^5*b^
6*d^5*g^6)*x^5 + 5*(a*b^10*c^5*g^6 - 5*a^2*b^9*c^4*d*g^6 + 10*a^3*b^8*c^3*d
^2*g^6 - 10*a^4*b^7*c^2*d^3*g^6 + 5*a^5*b^6*c*d^4*g^6 - a^6*b^5*d^5*g^6)*x^
4 + 10*(a^2*b^9*c^5*g^6 - 5*a^3*b^8*c^4*d*g^6 + 10*a^4*b^7*c^3*d^2*g^6 - 10
*a^5*b^6*c^2*d^3*g^6 + 5*a^6*b^5*c*d^4*g^6 - a^7*b^4*d^5*g^6)*x^3 + 10*(a^3
*b^8*c^5*g^6 - 5*a^4*b^7*c^4*d*g^6 + 10*a^5*b^6*c^3*d^2*g^6 - 10*a^6*b^5*c^
2*d^3*g^6 + 5*a^7*b^4*c*d^4*g^6 - a^8*b^3*d^5*g^6)*x^2 + 5*(a^4*b^7*c^5*g^6
- 5*a^5*b^6*c^4*d*g^6 + 10*a^6*b^5*c^3*d^2*g^6 - 10*a^7*b^4*c^2*d^3*g^6 +
5*a^8*b^3*c*d^4*g^6 - a^9*b^2*d^5*g^6)*x))*B^2*c^2*i^2 - 1/18000*(60*n*((27
*a*b^4*c^4 - 148*a^2*b^3*c^3*d + 352*a^3*b^2*c^2*d^2 - 548*a^4*b*c*d^3 + 77
*a^5*d^4 - 60*(5*b^5*c*d^3 - a*b^4*d^4)*x^4 + 30*(5*b^5*c^2*d^2 - 46*a*b^4*
c*d^3 + 9*a^2*b^3*d^4)*x^3 - 10*(10*b^5*c^3*d - 67*a*b^4*c^2*d^2 + 248*a^2*
b^3*c*d^3 - 47*a^3*b^2*d^4)*x^2 + 5*(15*b^5*c^4 - 88*a*b^4*c^3*d + 232*a^2*
b^3*c^2*d^2 - 428*a^3*b^2*c*d^3 + 77*a^4*b*d^4)*x)/((b^11*c^4 - 4*a*b^10*c^
3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4)*g^6*x^5 + 5*(a*b^1
0*c^4 - 4*a^2*b^9*c^3*d + 6*a^3*b^8*c^2*d^2 - 4*a^4*b^7*c*d^3 + a^5*b^6*d^4
)*g^6*x^4 + 10*(a^2*b^9*c^4 - 4*a^3*b^8*c^3*d + 6*a^4*b^7*c^2*d^2 - 4*a^5*b
^6*c*d^3 + a^6*b^5*d^4)*g^6*x^3 + 10*(a^3*b^8*c^4 - 4*a^4*b^7*c^3*d + 6*a^5
*b^6*c^2*d^2 - 4*a^6*b^5*c*d^3 + a^7*b^4*d^4)*g^6*x^2 + 5*(a^4*b^7*c^4 - 4*
a^5*b^6*c^3*d + 6*a^6*b^5*c^2*d^2 - 4*a^7*b^4*c*d^3 + a^8*b^3*d^4)*g^6*x +
(a^5*b^6*c^4 - 4*a^6*b^5*c^3*d + 6*a^7*b^4*c^2*d^2 - 4*a^8*b^3*c*d^3 + a^9*
b^2*d^4)*g^6) - 60*(5*b*c*d^4 - a*d^5)*log(b*x + a)/((b^7*c^5 - 5*a*b^6*c^4
*d + 10*a^2*b^5*c^3*d^2 - 10*a^3*b^4*c^2*d^3 + 5*a^4*b^3*c*d^4 - a^5*b^2*d^
5)*g^6) + 60*(5*b*c*d^4 - a*d^5)*log(d*x + c)/((b^7*c^5 - 5*a*b^6*c^4*d + 1
0*a^2*b^5*c^3*d^2 - 10*a^3*b^4*c^2*d^3 + 5*a^4*b^3*c*d^4 - a^5*b^2*d^5)*g^6
))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + (549*a*b^5*c^5 - 4625*a^2*b^4*c
^4*d + 19000*a^3*b^3*c^3*d^2 - 63000*a^4*b^2*c^2*d^3 + 51875*a^5*b*c*d^4 -

3799*a^6*d^5 - 60*(625*b^6*c^2*d^3 - 702*a*b^5*c*d^4 + 77*a^2*b^4*d^5)*x^4
+ 30*(325*b^6*c^3*d^2 - 5667*a*b^5*c^2*d^3 + 5975*a^2*b^4*c*d^4 - 633*a^3*b
^3*d^5)*x^3 - 10*(350*b^6*c^4*d - 3949*a*b^5*c^3*d^2 + 29475*a^2*b^4*c^2*d^
3 - 28775*a^3*b^3*c*d^4 + 2899*a^4*b^2*d^5)*x^2 + 1800*(5*a^5*b*c*d^4 - a^6
*d^5 + (5*b^6*c*d^4 - a*b^5*d^5)*x^5 + 5*(5*a*b^5*c*d^4 - a^2*b^4*d^5)*x^4
+ 10*(5*a^2*b^4*c*d^4 - a^3*b^3*d^5)*x^3 + 10*(5*a^3*b^3*c*d^4 - a^4*b^2*d^
5)*x^2 + 5*(5*a^4*b^2*c*d^4 - a^5*b*d^5)*x)*log(b*x + a)^2 + 1800*(5*a^5*b*
c*d^4 - a^6*d^5 + (5*b^6*c*d^4 - a*b^5*d^5)*x^5 + 5*(5*a*b^5*c*d^4 - a^2*b^
4*d^5)*x^4 + 10*(5*a^2*b^4*c*d^4 - a^3*b^3*d^5)*x^3 + 10*(5*a^3*b^3*c*d^4 -
a^4*b^2*d^5)*x^2 + 5*(5*a^4*b^2*c*d^4 - a^5*b*d^5)*x)*log(d*x + c)^2 + 5*(
225*b^6*c^5 - 2201*a*b^5*c^4*d + 10900*a^2*b^4*c^3*d^2 - 46200*a^3*b^3*c^2*
d^3 + 41075*a^4*b^2*c*d^4 - 3799*a^5*b*d^5)*x - 60*(625*a^5*b*c*d^4 - 77*a^
6*d^5 + (625*b^6*c*d^4 - 77*a*b^5*d^5)*x^5 + 5*(625*a*b^5*c*d^4 - 77*a^2*b^
4*d^5)*x^4 + 10*(625*a^2*b^4*c*d^4 - 77*a^3*b^3*d^5)*x^3 + 10*(625*a^3*b^3*
c*d^4 - 77*a^4*b^2*d^5)*x^2 + 5*(625*a^4*b^2*c*d^4 - 77*a^5*b*d^5)*x)*log(b
x + a) + 60(625*a^5*b*c*d^4 - 77*a^6*d^5 + (625*b^6*c*d^4 - 77*a*b^5*d^5)
x^5 + 5(625*a*b^5*c*d^4 - 77*a^2*b^4*d^5)*x^4 + 10*(625*a^2*b^4*c*d^4 - 7
7*a^3*b^3*d^5)*x^3 + 10*(625*a^3*b^3*c*d^4 - 77*a^4*b^2*d^5)*x^2 + 5*(625*a
^4*b^2*c*d^4 - 77*a^5*b*d^5)*x - 60*(5*a^5*b*c*d^4 - a^6*d^5 + (5*b^6*c*d^4
- a*b^5*d^5)*x^5 + 5*(5*a*b^5*c*d^4 - a^2*b^4*d^5)*x^4 + 10*(5*a^2*b^4*c*d
^4 - a^3*b^3*d^5)*x^3 + 10*(5*a^3*b^3*c*d^4 - a^4*b^2*d^5)*x^2 + 5*(5*a^4*b
^2*c*d^4 - a^5*b*d^5)*x)*log(b*x + a))*log(d*x + c))*n^2/(a^5*b^7*c^5*g^6 -
5*a^6*b^6*c^4*d*g^6 + 10*a^7*b^5*c^3*d^2*g^6 - 10*a^8*b^4*c^2*d^3*g^6 + 5*
a^9*b^3*c*d^4*g^6 - a^10*b^2*d^5*g^6 + (b^12*c^5*g^6 - 5*a*b^11*c^4*d*g^6 +
10*a^2*b^10*c^3*d^2*g^6 - 10*a^3*b^9*c^2*d^3*g^6 + 5*a^4*b^8*c*d^4*g^6 - a
^5*b^7*d^5*g^6)*x^5 + 5*(a*b^11*c^5*g^6 - 5*a^2*b^10*c^4*d*g^6 + 10*a^3*b^9
*c^3*d^2*g^6 - 10*a^4*b^8*c^2*d^3*g^6 + 5*a^5*b^7*c*d^4*g^6 - a^6*b^6*d^5*g
^6)*x^4 + 10*(a^2*b^10*c^5*g^6 - 5*a^3*b^9*c^4*d*g^6 + 10*a^4*b^8*c^3*d^2*g
^6 - 10*a^5*b^7*c^2*d^3*g^6 + 5*a^6*b^6*c*d^4*g^6 - a^7*b^5*d^5*g^6)*x^3 +
10*(a^3*b^9*c^5*g^6 - 5*a^4*b^8*c^4*d*g^6 + 10*a^5*b^7*c^3*d^2*g^6 - 10*a^6
*b^6*c^2*d^3*g^6 + 5*a^7*b^5*c*d^4*g^6 - a^8*b^4*d^5*g^6)*x^2 + 5*(a^4*b^8*
c^5*g^6 - 5*a^5*b^7*c^4*d*g^6 + 10*a^6*b^6*c^3*d^2*g^6 - 10*a^7*b^5*c^2*d^3
*g^6 + 5*a^8*b^4*c*d^4*g^6 - a^9*b^3*d^5*g^6)*x))*B^2*c*d*i^2 - 1/54000*(60
n((47*a^2*b^4*c^4 - 278*a^3*b^3*c^3*d + 822*a^4*b^2*c^2*d^2 - 278*a^5*b*c
*d^3 + 47*a^6*d^4 + 60*(10*b^6*c^2*d^2 - 5*a*b^5*c*d^3 + a^2*b^4*d^4)*x^4 -
30*(10*b^6*c^3*d - 95*a*b^5*c^2*d^2 + 46*a^2*b^4*c*d^3 - 9*a^3*b^3*d^4)*x^
3 + 10*(20*b^6*c^4 - 140*a*b^5*c^3*d + 537*a^2*b^4*c^2*d^2 - 248*a^3*b^3*c*
d^3 + 47*a^4*b^2*d^4)*x^2 + 5*(35*a*b^5*c^4 - 218*a^2*b^4*c^3*d + 702*a^3*b
^3*c^2*d^2 - 278*a^4*b^2*c*d^3 + 47*a^5*b*d^4)*x)/((b^12*c^4 - 4*a*b^11*c^3
*d + 6*a^2*b^10*c^2*d^2 - 4*a^3*b^9*c*d^3 + a^4*b^8*d^4)*g^6*x^5 + 5*(a*b^1
1*c^4 - 4*a^2*b^10*c^3*d + 6*a^3*b^9*c^2*d^2 - 4*a^4*b^8*c*d^3 + a^5*b^7*d^
4)*g^6*x^4 + 10*(a^2*b^10*c^4 - 4*a^3*b^9*c^3*d + 6*a^4*b^8*c^2*d^2 - 4*a^5
*b^7*c*d^3 + a^6*b^6*d^4)*g^6*x^3 + 10*(a^3*b^9*c^4 - 4*a^4*b^8*c^3*d + 6*a
^5*b^7*c^2*d^2 - 4*a^6*b^6*c*d^3 + a^7*b^5*d^4)*g^6*x^2 + 5*(a^4*b^8*c^4 -
4*a^5*b^7*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^7*b^5*c*d^3 + a^8*b^4*d^4)*g^6*x

+ (a^5*b^7*c^4 - 4*a^6*b^6*c^3*d + 6*a^7*b^5*c^2*d^2 - 4*a^8*b^4*c*d^3 + a^
9*b^3*d^4)*g^6) + 60*(10*b^2*c^2*d^3 - 5*a*b*c*d^4 + a^2*d^5)*log(b*x + a)/
((b^8*c^5 - 5*a*b^7*c^4*d + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 + 5*a^4
*b^4*c*d^4 - a^5*b^3*d^5)*g^6) - 60*(10*b^2*c^2*d^3 - 5*a*b*c*d^4 + a^2*d^5
)*log(d*x + c)/((b^8*c^5 - 5*a*b^7*c^4*d + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*
c^2*d^3 + 5*a^4*b^4*c*d^4 - a^5*b^3*d^5)*g^6))*log(e*(b*x/(d*x + c) + a/(d*
x + c))^n) + (1489*a^2*b^5*c^5 - 14375*a^3*b^4*c^4*d + 85000*a^4*b^3*c^3*d^
2 - 85000*a^5*b^2*c^2*d^3 + 14375*a^6*b*c*d^4 - 1489*a^7*d^5 + 60*(1100*b^7
*c^3*d^2 - 1425*a*b^6*c^2*d^3 + 372*a^2*b^5*c*d^4 - 47*a^3*b^4*d^5)*x^4 - 3
0*(500*b^7*c^4*d - 9825*a*b^6*c^3*d^2 + 11937*a^2*b^5*c^2*d^3 - 2975*a^3*b^
4*c*d^4 + 363*a^4*b^3*d^5)*x^3 + 10*(400*b^7*c^5 - 5450*a*b^6*c^4*d + 49189
*a^2*b^5*c^3*d^2 - 55525*a^3*b^4*c^2*d^3 + 12875*a^4*b^3*c*d^4 - 1489*a^5*b
^2*d^5)*x^2 - 1800*(10*a^5*b^2*c^2*d^3 - 5*a^6*b*c*d^4 + a^7*d^5 + (10*b^7*
c^2*d^3 - 5*a*b^6*c*d^4 + a^2*b^5*d^5)*x^5 + 5*(10*a*b^6*c^2*d^3 - 5*a^2*b^
5*c*d^4 + a^3*b^4*d^5)*x^4 + 10*(10*a^2*b^5*c^2*d^3 - 5*a^3*b^4*c*d^4 + a^4
*b^3*d^5)*x^3 + 10*(10*a^3*b^4*c^2*d^3 - 5*a^4*b^3*c*d^4 + a^5*b^2*d^5)*x^2
 + 5*(10*a^4*b^3*c^2*d^3 - 5*a^5*b^2*c*d^4 + a^6*b*d^5)*x)*log(b*x + a)^2 -
 1800*(10*a^5*b^2*c^2*d^3 - 5*a^6*b*c*d^4 + a^7*d^5 + (10*b^7*c^2*d^3 - 5*a
*b^6*c*d^4 + a^2*b^5*d^5)*x^5 + 5*(10*a*b^6*c^2*d^3 - 5*a^2*b^5*c*d^4 + a^3
*b^4*d^5)*x^4 + 10*(10*a^2*b^5*c^2*d^3 - 5*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^3
 + 10*(10*a^3*b^4*c^2*d^3 - 5*a^4*b^3*c*d^4 + a^5*b^2*d^5)*x^2 + 5*(10*a^4*
b^3*c^2*d^3 - 5*a^5*b^2*c*d^4 + a^6*b*d^5)*x)*log(d*x + c)^2 + 5*(925*a*b^6
*c^5 - 9911*a^2*b^5*c^4*d + 67900*a^3*b^4*c^3*d^2 - 71800*a^4*b^3*c^2*d^3 +
 14375*a^5*b^2*c*d^4 - 1489*a^6*b*d^5)*x + 60*(1100*a^5*b^2*c^2*d^3 - 325*a
^6*b*c*d^4 + 47*a^7*d^5 + (1100*b^7*c^2*d^3 - 325*a*b^6*c*d^4 + 47*a^2*b^5*
d^5)*x^5 + 5*(1100*a*b^6*c^2*d^3 - 325*a^2*b^5*c*d^4 + 47*a^3*b^4*d^5)*x^4
+ 10*(1100*a^2*b^5*c^2*d^3 - 325*a^3*b^4*c*d^4 + 47*a^4*b^3*d^5)*x^3 + 10*(
1100*a^3*b^4*c^2*d^3 - 325*a^4*b^3*c*d^4 + 47*a^5*b^2*d^5)*x^2 + 5*(1100*a^
4*b^3*c^2*d^3 - 325*a^5*b^2*c*d^4 + 47*a^6*b*d^5)*x)*log(b*x + a) - 60*(110
0*a^5*b^2*c^2*d^3 - 325*a^6*b*c*d^4 + 47*a^7*d^5 + (1100*b^7*c^2*d^3 - 325*
a*b^6*c*d^4 + 47*a^2*b^5*d^5)*x^5 + 5*(1100*a*b^6*c^2*d^3 - 325*a^2*b^5*c*d
^4 + 47*a^3*b^4*d^5)*x^4 + 10*(1100*a^2*b^5*c^2*d^3 - 325*a^3*b^4*c*d^4 + 4
7*a^4*b^3*d^5)*x^3 + 10*(1100*a^3*b^4*c^2*d^3 - 325*a^4*b^3*c*d^4 + 47*a^5*
b^2*d^5)*x^2 + 5*(1100*a^4*b^3*c^2*d^3 - 325*a^5*b^2*c*d^4 + 47*a^6*b*d^5)*
x - 60*(10*a^5*b^2*c^2*d^3 - 5*a^6*b*c*d^4 + a^7*d^5 + (10*b^7*c^2*d^3 - 5*
a*b^6*c*d^4 + a^2*b^5*d^5)*x^5 + 5*(10*a*b^6*c^2*d^3 - 5*a^2*b^5*c*d^4 + a^
3*b^4*d^5)*x^4 + 10*(10*a^2*b^5*c^2*d^3 - 5*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^
3 + 10*(10*a^3*b^4*c^2*d^3 - 5*a^4*b^3*c*d^4 + a^5*b^2*d^5)*x^2 + 5*(10*a^4
*b^3*c^2*d^3 - 5*a^5*b^2*c*d^4 + a^6*b*d^5)*x)*log(b*x + a))*log(d*x + c))*
n^2/(a^5*b^8*c^5*g^6 - 5*a^6*b^7*c^4*d*g^6 + 10*a^7*b^6*c^3*d^2*g^6 - 10*a^
8*b^5*c^2*d^3*g^6 + 5*a^9*b^4*c*d^4*g^6 - a^10*b^3*d^5*g^6 + (b^13*c^5*g^6
- 5*a*b^12*c^4*d*g^6 + 10*a^2*b^11*c^3*d^2*g^6 - 10*a^3*b^10*c^2*d^3*g^6 +
5*a^4*b^9*c*d^4*g^6 - a^5*b^8*d^5*g^6)*x^5 + 5*(a*b^12*c^5*g^6 - 5*a^2*b^11
*c^4*d*g^6 + 10*a^3*b^10*c^3*d^2*g^6 - 10*a^4*b^9*c^2*d^3*g^6 + 5*a^5*b^8*c
*d^4*g^6 - a^6*b^7*d^5*g^6)*x^4 + 10*(a^2*b^11*c^5*g^6 - 5*a^3*b^10*c^4*d*g

```
^6 + 10*a^4*b^9*c^3*d^2*g^6 - 10*a^5*b^8*c^2*d^3*g^6 + 5*a^6*b^7*c*d^4*g^6
- a^7*b^6*d^5*g^6)*x^3 + 10*(a^3*b^10*c^5*g^6 - 5*a^4*b^9*c^4*d*g^6 + 10*a^
5*b^8*c^3*d^2*g^6 - 10*a^6*b^7*c^2*d^3*g^6 + 5*a^7*b^6*c*d^4*g^6 - a^8*b^5*
d^5*g^6)*x^2 + 5*(a^4*b^9*c^5*g^6 - 5*a^5*b^8*c^4*d*g^6 + 10*a^6*b^7*c^3*d^
2*g^6 - 10*a^7*b^6*c^2*d^3*g^6 + 5*a^8*b^5*c*d^4*g^6 - a^9*b^4*d^5*g^6)*x))
*B^2*d^2*i^2 - 1/5*(5*b*x + a)*A*B*c*d*i^2*log(e*(b*x/(d*x + c) + a/(d*x +
c))^n)/(b^7*g^6*x^5 + 5*a*b^6*g^6*x^4 + 10*a^2*b^5*g^6*x^3 + 10*a^3*b^4*g^6
*x^2 + 5*a^4*b^3*g^6*x + a^5*b^2*g^6) - 1/15*(10*b^2*x^2 + 5*a*b*x + a^2)*A
*B*d^2*i^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^8*g^6*x^5 + 5*a*b^7*g^
6*x^4 + 10*a^2*b^6*g^6*x^3 + 10*a^3*b^5*g^6*x^2 + 5*a^4*b^4*g^6*x + a^5*b^3
*g^6) - 1/5*B^2*c^2*i^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2/(b^6*g^6*x
^5 + 5*a*b^5*g^6*x^4 + 10*a^2*b^4*g^6*x^3 + 10*a^3*b^3*g^6*x^2 + 5*a^4*b^2*
g^6*x + a^5*b*g^6) - 1/10*(5*b*x + a)*A^2*c*d*i^2/(b^7*g^6*x^5 + 5*a*b^6*g^
6*x^4 + 10*a^2*b^5*g^6*x^3 + 10*a^3*b^4*g^6*x^2 + 5*a^4*b^3*g^6*x + a^5*b^2
*g^6) - 1/30*(10*b^2*x^2 + 5*a*b*x + a^2)*A^2*d^2*i^2/(b^8*g^6*x^5 + 5*a*b^
7*g^6*x^4 + 10*a^2*b^6*g^6*x^3 + 10*a^3*b^5*g^6*x^2 + 5*a^4*b^4*g^6*x + a^5
*b^3*g^6) - 2/5*A*B*c^2*i^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^6*g^6
*x^5 + 5*a*b^5*g^6*x^4 + 10*a^2*b^4*g^6*x^3 + 10*a^3*b^3*g^6*x^2 + 5*a^4*b^
2*g^6*x + a^5*b*g^6) - 1/5*A^2*c^2*i^2/(b^6*g^6*x^5 + 5*a*b^5*g^6*x^4 + 10*
a^2*b^4*g^6*x^3 + 10*a^3*b^3*g^6*x^2 + 5*a^4*b^2*g^6*x + a^5*b*g^6)
```

Giac [A] (verification not implemented)

none

Time = 6.30 (sec) , antiderivative size = 931, normalized size of antiderivative = 1.89

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^6,x,
 algorithm="giac")
```

```
[Out] -1/54000*(1800*(6*B^2*b^2*i^2*n^2 - 15*(b*x + a)*B^2*b*d*i^2*n^2/(d*x + c)
+ 10*(b*x + a)^2*B^2*d^2*i^2*n^2/(d*x + c)^2)*log((b*x + a)/(d*x + c))^2/((
b*x + a)^5*b^2*c^2*g^6/(d*x + c)^5 - 2*(b*x + a)^5*a*b*c*d*g^6/(d*x + c)^5
+ (b*x + a)^5*a^2*d^2*g^6/(d*x + c)^5) + 60*(72*B^2*b^2*i^2*n^2 - 225*(b*x
+ a)*B^2*b*d*i^2*n^2/(d*x + c) + 200*(b*x + a)^2*B^2*d^2*i^2*n^2/(d*x + c)^
2 + 360*B^2*b^2*i^2*n*log(e) - 900*(b*x + a)*B^2*b*d*i^2*n*log(e)/(d*x + c)
 + 600*(b*x + a)^2*B^2*d^2*i^2*n*log(e)/(d*x + c)^2 + 360*A*B*b^2*i^2*n - 9
00*(b*x + a)*A*B*b*d*i^2*n/(d*x + c) + 600*(b*x + a)^2*A*B*d^2*i^2*n/(d*x +
 c)^2)*log((b*x + a)/(d*x + c))/((b*x + a)^5*b^2*c^2*g^6/(d*x + c)^5 - 2*(b
*x + a)^5*a*b*c*d*g^6/(d*x + c)^5 + (b*x + a)^5*a^2*d^2*g^6/(d*x + c)^5) +
(864*B^2*b^2*i^2*n^2 - 3375*(b*x + a)*B^2*b*d*i^2*n^2/(d*x + c) + 4000*(b*x
 + a)^2*B^2*d^2*i^2*n^2/(d*x + c)^2 + 4320*B^2*b^2*i^2*n*log(e) - 13500*(b*
```

```
x + a)*B^2*b*d*i^2*n*log(e)/(d*x + c) + 12000*(b*x + a)^2*B^2*d^2*i^2*n*log
(e)/(d*x + c)^2 + 10800*B^2*b^2*i^2*log(e)^2 - 27000*(b*x + a)*B^2*b*d*i^2*
log(e)^2/(d*x + c) + 18000*(b*x + a)^2*B^2*d^2*i^2*log(e)^2/(d*x + c)^2 + 4
320*A*B*b^2*i^2*n - 13500*(b*x + a)*A*B*b*d*i^2*n/(d*x + c) + 12000*(b*x +
a)^2*A*B*d^2*i^2*n/(d*x + c)^2 + 21600*A*B*b^2*i^2*log(e) - 54000*(b*x + a)
*A*B*b*d*i^2*log(e)/(d*x + c) + 36000*(b*x + a)^2*A*B*d^2*i^2*log(e)/(d*x +
 c)^2 + 10800*A^2*b^2*i^2 - 27000*(b*x + a)*A^2*b*d*i^2/(d*x + c) + 18000*(
b*x + a)^2*A^2*d^2*i^2/(d*x + c)^2)/((b*x + a)^5*b^2*c^2*g^6/(d*x + c)^5 -
2*(b*x + a)^5*a*b*c*d*g^6/(d*x + c)^5 + (b*x + a)^5*a^2*d^2*g^6/(d*x + c)^5
))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 7.85 (sec) , antiderivative size = 3296, normalized size of antiderivative = 6.69

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

```
[In] int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x
)^6,x)
```

```
[Out] ((1800*A^2*a^4*d^4*i^2 + 10800*A^2*b^4*c^4*i^2 + 1489*B^2*a^4*d^4*i^2*n^2 +
 864*B^2*b^4*c^4*i^2*n^2 - 16200*A^2*a*b^3*c^3*d*i^2 + 1800*A^2*a^3*b*c*d^3
*i^2 + 2820*A*B*a^4*d^4*i^2*n + 4320*A*B*b^4*c^4*i^2*n + 1800*A^2*a^2*b^2*c
^2*d^2*i^2 + 1489*B^2*a^2*b^2*c^2*d^2*i^2*n^2 - 2511*B^2*a*b^3*c^3*d*i^2*n^
2 + 1489*B^2*a^3*b*c*d^3*i^2*n^2 + 2820*A*B*a^2*b^2*c^2*d^2*i^2*n - 9180*A*
B*a*b^3*c^3*d*i^2*n + 2820*A*B*a^3*b*c*d^3*i^2*n)/(60*(a*d - b*c)) + (x*(18
00*A^2*a^3*b*d^4*i^2 + 5400*A^2*b^4*c^3*d*i^2 - 9000*A^2*a*b^3*c^2*d^2*i^2
+ 1800*A^2*a^2*b^2*c*d^3*i^2 + 1489*B^2*a^3*b*d^4*i^2*n^2 + 189*B^2*b^4*c^3
*d*i^2*n^2 + 1620*A*B*b^4*c^3*d*i^2*n - 911*B^2*a*b^3*c^2*d^2*i^2*n^2 + 148
9*B^2*a^2*b^2*c*d^3*i^2*n^2 + 2820*A*B*a^3*b*d^4*i^2*n - 4380*A*B*a*b^3*c^2
*d^2*i^2*n + 2820*A*B*a^2*b^2*c*d^3*i^2*n))/(12*(a*d - b*c)) + (x^2*(1800*A
^2*a^2*b^2*d^4*i^2 + 1800*A^2*b^4*c^2*d^2*i^2 - 3600*A^2*a*b^3*c*d^3*i^2 +
1489*B^2*a^2*b^2*d^4*i^2*n^2 - 86*B^2*b^4*c^2*d^2*i^2*n^2 + 2820*A*B*a^2*b^
2*d^4*i^2*n + 120*A*B*b^4*c^2*d^2*i^2*n + 289*B^2*a*b^3*c*d^3*i^2*n^2 - 780
*A*B*a*b^3*c*d^3*i^2*n))/(6*(a*d - b*c)) + (x^3*(363*B^2*a*b^3*d^4*i^2*n^2
+ 13*B^2*b^4*c*d^3*i^2*n^2 - 60*A*B*b^4*c*d^3*i^2*n + 540*A*B*a*b^3*d^4*i^2
*n))/(2*(a*d - b*c)) + (d*x^4*(47*B^2*b^4*d^3*i^2*n^2 + 60*A*B*b^4*d^3*i^2*
n))/(a*d - b*c))/(x*(4500*a^4*b^5*c*g^6 - 4500*a^5*b^4*d*g^6) - x^4*(4500*a
^2*b^7*d*g^6 - 4500*a*b^8*c*g^6) + x^5*(900*b^9*c*g^6 - 900*a*b^8*d*g^6) +
x^2*(9000*a^3*b^6*c*g^6 - 9000*a^4*b^5*d*g^6) + x^3*(9000*a^2*b^7*c*g^6 - 9
000*a^3*b^6*d*g^6) + 900*a^5*b^4*c*g^6 - 900*a^6*b^3*d*g^6) - log(e*((a + b
*x)/(c + d*x))^n)^2*((a*((B^2*c*d*i^2)/(10*b^2) + (B^2*a*d^2*i^2)/(30*b^3))
 + x*(b*((B^2*c*d*i^2)/(10*b^2) + (B^2*a*d^2*i^2)/(30*b^3)) + (2*B^2*c*d*i^
2)/(5*b) + (2*B^2*a*d^2*i^2)/(15*b^2)) + (B^2*c^2*i^2)/(5*b) + (B^2*d^2*i^2
```

*x^2)/(3*b))/(a^5*g^6 + b^5*g^6*x^5 + 5*a*b^4*g^6*x^4 + 10*a^3*b^2*g^6*x^2 + 10*a^2*b^3*g^6*x^3 + 5*a^4*b*g^6*x) - (B^2*d^5*i^2)/(30*b^3*g^6*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - log(e*((a + b*x)/(c + d*x))^n)*((a*(A*B*a*d^2*i^2 - (B^2*a*d^2*i^2*n)/2 + (B^2*b*c*d*i^2*n)/2 + 3*A*B*b*c*d*i^2) + x*(b*(A*B*a*d^2*i^2 - (B^2*a*d^2*i^2*n)/2 + (B^2*b*c*d*i^2*n)/2 + 3*A*B*b*c*d*i^2) + 4*A*B*a*b*d^2*i^2 + 12*A*B*b^2*c*d*i^2 - 2*B^2*a*b*d^2*i^2*n + 2*B^2*b^2*c*d*i^2*n) + 6*A*B*b^2*c^2*i^2 - B^2*a^2*d^2*i^2*n + B^2*b^2*c^2*i^2*n + 10*A*B*b^2*d^2*i^2*x^2)/(15*a^5*b^3*g^6 + 15*b^8*g^6*x^5 + 75*a^4*b^4*g^6*x + 75*a*b^7*g^6*x^4 + 150*a^3*b^5*g^6*x^2 + 150*a^2*b^6*g^6*x^3) + (B^2*d^5*i^2*(x^3*(b*(b*(b*((3*a*b^3*g^6*n*(a*d - b*c))/d + (3*b^3*g^6*n*(a*d - b*c)*(5*a*d - b*c))/(4*d^2)) + (6*a*b^4*g^6*n*(a*d - b*c))/d + (3*b^4*g^6*n*(a*d - b*c)*(5*a*d - b*c))/(2*d^2)) + (9*a*b^5*g^6*n*(a*d - b*c))/d + (9*b^5*g^6*n*(a*d - b*c)*(5*a*d - b*c))/(4*d^2)) + (12*a*b^6*g^6*n*(a*d - b*c))/d + (3*b^6*g^6*n*(a*d - b*c)*(5*a*d - b*c))/d^2) + x*(a*(a*(b*((3*a*b^3*g^6*n*(a*d - b*c))/d + (3*b^3*g^6*n*(a*d - b*c)*(5*a*d - b*c))/(4*d^2)) + (6*a*b^4*g^6*n*(a*d - b*c))/d + (3*b^4*g^6*n*(a*d - b*c)*(5*a*d - b*c))/(2*d^2)) + b*(a*((3*a*b^3*g^6*n*(a*d - b*c))/d + (3*b^3*g^6*n*(a*d - b*c)*(5*a*d - b*c))/(4*d^2)) + (b^3*g^6*n*(a*d - b*c)*(10*a^2*d^2 + b^2*c^2 - 5*a*b*c*d))/(2*d^3)) + (3*b^4*g^6*n*(a*d - b*c)*(10*a^2*d^2 + b^2*c^2 - 5*a*b*c*d))/(2*d^3)) + b*(a*(a*((3*a*b^3*g^6*n*(a*d - b*c))/d + (3*b^3*g^6*n*(a*d - b*c)*(5*a*d - b*c))/(4*d^2)) + (b^3*g^6*n*(a*d - b*c)*(10*a^2*d^2 + b^2*c^2 - 5*a*b*c*d))/(2*d^3)) + (3*b^3*g^6*n*(a*d - b*c)*(10*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2))/(4*d^4)) + (3*b^4*g^6*n*(a*d - b*c)*(10*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2))/d^4) + x^2*(a*(b*(b*((3*a*b^3*g^6*n*(a*d - b*c))/d + (3*b^3*g^6*n*(a*d - b*c)*(5*a*d - b*c))/(4*d^2)) + (6*a*b^4*g^6*n*(a*d - b*c))/d + (3*b^4*g^6*n*(a*d - b*c)*(5*a*d - b*c))/(2*d^2)) + (9*a*b^5*g^6*n*(a*d - b*c))/d + (9*b^5*g^6*n*(a*d - b*c)*(5*a*d - b*c))/(4*d^2)) + b*(a*(b*((3*a*b^3*g^6*n*(a*d - b*c))/d + (3*b^3*g^6*n*(a*d - b*c)*(5*a*d - b*c))/(4*d^2)) + (6*a*b^4*g^6*n*(a*d - b*c))/d + (3*b^4*g^6*n*(a*d - b*c)*(5*a*d - b*c))/(2*d^2)) + b*(a*((3*a*b^3*g^6*n*(a*d - b*c))/d + (3*b^3*g^6*n*(a*d - b*c)*(5*a*d - b*c))/(4*d^2)) + (b^3*g^6*n*(a*d - b*c)*(10*a^2*d^2 + b^2*c^2 - 5*a*b*c*d))/(2*d^3)) + (3*b^4*g^6*n*(a*d - b*c)*(10*a^2*d^2 + b^2*c^2 - 5*a*b*c*d))/(2*d^3)) + (3*b^5*g^6*n*(a*d - b*c)*(10*a^2*d^2 + b^2*c^2 - 5*a*b*c*d))/d^3) + a*(a*(a*((3*a*b^3*g^6*n*(a*d - b*c))/d + (3*b^3*g^6*n*(a*d - b*c)*(5*a*d - b*c))/(4*d^2)) + (b^3*g^6*n*(a*d - b*c)*(10*a^2*d^2 + b^2*c^2 - 5*a*b*c*d))/(2*d^3)) + (3*b^3*g^6*n*(a*d - b*c)*(10*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2))/(4*d^4)) + (15*b^7*g^6*n*x^4*(a*d - b*c))/d + (3*b^3*g^6*n*(a*d - b*c)*(5*a^4*d^4 + b^4*c^4 + 10*a^2*b^2*c^2*d^2 - 5*a*b^3*c^3*d - 10*a^3*b*c*d^3))/d^5))/(15*b^3*g^6*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(15*a^5*b^3*g^6 + 15*b^8*g^6*x^5 + 75*a^4*b^4*g^6*x + 75*a*b^7*g^6*x^4 + 150*a^3*b^5*g^6*x^2 + 150*a^2*b^6*g^6*x^3))) - (B*d^5*i^2*n*atan((B*d^5*i^2*n*(60*A + 47*B*n)*((b^6*c^3*g^6 + a^3*b^3*d^3*g^6 - a*b^5*c^2*d*g^6 - a^2*b^4*c*d^2*g^6)/(b^5*c^2*g^6 + a^2*b^3*d^2*g^6 - 2*a*b^4*c*d*g^6) + 2*b*d*x)*(b^5*c^2*g^6 + a^2*b^3*d^2*g^6 - 2*a*b^4*c*d*g^6)*1i)/(b^3*g^6*(47*B^2*d^5*i^2*n^2

```
 + 60*A*B*d^5*i^2*n)*(a*d - b*c)^3))*(60*A + 47*B*n)*1i)/(450*b^3*g^6*(a*d
- b*c)^3)
```

3.178 $\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 45, antiderivative size = 1172

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx$$
$$=\frac{5B^2(bc-ad)^6g^3i^3n^2x}{84b^3d^3}+\frac{B^2(bc-ad)^3g^3i^3n^2(a+bx)^4}{140b^4}-\frac{29B^2(bc-ad)^5g^3i^3n^2(c+dx)^2}{840b^2d^4}$$
$$+\frac{47B^2(bc-ad)^4g^3i^3n^2(c+dx)^3}{1260bd^4}-\frac{13B^2(bc-ad)^3g^3i^3n^2(c+dx)^4}{420d^4}$$
$$+\frac{bB^2(bc-ad)^2g^3i^3n^2(c+dx)^5}{105d^4}-\frac{B(bc-ad)^4g^3i^3n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{210b^4d}$$
$$-\frac{3B(bc-ad)^3g^3i^3n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{140b^4}$$
$$-\frac{B(bc-ad)^2g^3i^3n(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{35b^3}$$
$$+\frac{2B(bc-ad)^4g^3i^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{21bd^4}$$
$$-\frac{3B(bc-ad)^3g^3i^3n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{14d^4}$$
$$+\frac{6bB(bc-ad)^2g^3i^3n(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{35d^4}$$
$$-\frac{b^2B(bc-ad)g^3i^3n(c+dx)^6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{21d^4}$$
$$+\frac{(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{140b^4}$$
$$+\frac{(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{35b^3}$$
$$+\frac{(bc-ad)g^3i^3(a+bx)^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{14b^2}$$
$$+\frac{g^3i^3(a+bx)^4(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{7b}$$
$$+\frac{B(bc-ad)^5g^3i^3n(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{420b^4d^2}$$
$$-\frac{B(bc-ad)^6g^3i^3n(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{420b^4d^3}$$
$$-\frac{B(bc-ad)^7g^3i^3n\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{420b^4d^4}$$
$$-\frac{B^2(bc-ad)^7g^3i^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{210b^4d^4}-\frac{11B^2(bc-ad)^7g^3i^3n^2\log(c+dx)}{420b^4d^4}$$
$$-\frac{B^2(bc-ad)^7g^3i^3n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{70b^4d^4}$$

```
[Out] 5/84*B^2*(-a*d+b*c)^6*g^3*i^3*n^2*x/b^3/d^3+1/140*B^2*(-a*d+b*c)^3*g^3*i^3*
n^2*(b*x+a)^4/b^4-29/840*B^2*(-a*d+b*c)^5*g^3*i^3*n^2*(d*x+c)^2/b^2/d^4+47/
1260*B^2*(-a*d+b*c)^4*g^3*i^3*n^2*(d*x+c)^3/b/d^4-13/420*B^2*(-a*d+b*c)^3*g
^3*i^3*n^2*(d*x+c)^4/d^4+1/105*b*B^2*(-a*d+b*c)^2*g^3*i^3*n^2*(d*x+c)^5/d^4
-1/210*B*(-a*d+b*c)^4*g^3*i^3*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b
^4/d-3/140*B*(-a*d+b*c)^3*g^3*i^3*n*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n
))/b^4-1/35*B*(-a*d+b*c)^2*g^3*i^3*n*(b*x+a)^4*(d*x+c)*(A+B*ln(e*((b*x+a)/(
d*x+c))^n))/b^3+2/21*B*(-a*d+b*c)^4*g^3*i^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/
(d*x+c))^n))/b/d^4-3/14*B*(-a*d+b*c)^3*g^3*i^3*n*(d*x+c)^4*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))/d^4+6/35*b*B*(-a*d+b*c)^2*g^3*i^3*n*(d*x+c)^5*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))/d^4-1/21*b^2*B*(-a*d+b*c)*g^3*i^3*n*(d*x+c)^6*(A+B*ln(e*
((b*x+a)/(d*x+c))^n))/d^4+1/140*(-a*d+b*c)^3*g^3*i^3*(b*x+a)^4*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))^2/b^4+1/35*(-a*d+b*c)^2*g^3*i^3*(b*x+a)^4*(d*x+c)*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))^2/b^3+1/14*(-a*d+b*c)*g^3*i^3*(b*x+a)^4*(d*x+c)^
2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2+1/7*g^3*i^3*(b*x+a)^4*(d*x+c)^3*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/420*B*(-a*d+b*c)^5*g^3*i^3*n*(b*x+a)^2*(
3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/d^2-1/420*B*(-a*d+b*c)^6*g^3*i^3
*n*(b*x+a)*(6*A+5*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/d^3-1/420*B*(-a*d+
b*c)^7*g^3*i^3*n*(6*A+11*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b
/(d*x+c))/b^4/d^4-1/210*B^2*(-a*d+b*c)^7*g^3*i^3*n^2*ln((b*x+a)/(d*x+c))/b^
4/d^4-11/420*B^2*(-a*d+b*c)^7*g^3*i^3*n^2*ln(d*x+c)/b^4/d^4-1/70*B^2*(-a*d+
b*c)^7*g^3*i^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/d^4
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 1172, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.311, Rules

used = {2561, 2383, 2381, 2384, 2354, 2438, 2373, 45, 47, 37, 2382, 12, 79, 1634}

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx$$
$$= -\frac{Bg^3i^3n\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)(bc-ad)^7}{420b^4d^4}$$
$$-\frac{B^2g^3i^3n^2\log\left(\frac{a+bx}{c+dx}\right)(bc-ad)^7}{210b^4d^4}-\frac{11B^2g^3i^3n^2\log(c+dx)(bc-ad)^7}{420b^4d^4}$$
$$-\frac{B^2g^3i^3n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)(bc-ad)^7}{70b^4d^4}+\frac{5B^2g^3i^3n^2x(bc-ad)^6}{84b^3d^3}$$
$$-\frac{Bg^3i^3n(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)^6}{420b^4d^3}$$
$$-\frac{29B^2g^3i^3n^2(c+dx)^2(bc-ad)^5}{840b^2d^4}$$
$$+\frac{Bg^3i^3n(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)^5}{420b^4d^2}$$
$$+\frac{47B^2g^3i^3n^2(c+dx)^3(bc-ad)^4}{1260bd^4}-\frac{Bg^3i^3n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)^4}{210b^4d}$$
$$+\frac{2Bg^3i^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)^4}{21bd^4}+\frac{B^2g^3i^3n^2(a+bx)^4(bc-ad)^3}{140b^4}$$
$$-\frac{13B^2g^3i^3n^2(c+dx)^4(bc-ad)^3}{420d^4}+\frac{g^3i^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(bc-ad)^3}{140b^4}$$
$$-\frac{3Bg^3i^3n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)^3}{140b^4}$$
$$-\frac{3Bg^3i^3n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)^3}{14d^4}+\frac{bB^2g^3i^3n^2(c+dx)^5(bc-ad)^2}{105d^4}$$
$$+\frac{g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(bc-ad)^2}{35b^3}$$
$$+\frac{6bBg^3i^3n(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)^2}{35d^4}$$
$$-\frac{Bg^3i^3n(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)^2}{35b^3}$$
$$+\frac{g^3i^3(a+bx)^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(bc-ad)}{14b^2}$$
$$-\frac{b^2Bg^3i^3n(c+dx)^6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)}{21d^4}$$
$$+\frac{g^3i^3(a+bx)^4(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{7b}$$

```
[In] Int[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2,x]
```

```
[Out] (5*B^2*(b*c - a*d)^6*g^3*i^3*n^2*x)/(84*b^3*d^3) + (B^2*(b*c - a*d)^3*g^3*i
^3*n^2*(a + b*x)^4)/(140*b^4) - (29*B^2*(b*c - a*d)^5*g^3*i^3*n^2*(c + d*x)
^2)/(840*b^2*d^4) + (47*B^2*(b*c - a*d)^4*g^3*i^3*n^2*(c + d*x)^3)/(1260*b*
d^4) - (13*B^2*(b*c - a*d)^3*g^3*i^3*n^2*(c + d*x)^4)/(420*d^4) + (b*B^2*(b
*c - a*d)^2*g^3*i^3*n^2*(c + d*x)^5)/(105*d^4) - (B*(b*c - a*d)^4*g^3*i^3*n
*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(210*b^4*d) - (3*B*(b*
c - a*d)^3*g^3*i^3*n*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(1
40*b^4) - (B*(b*c - a*d)^2*g^3*i^3*n*(a + b*x)^4*(c + d*x)*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n]))/(35*b^3) + (2*B*(b*c - a*d)^4*g^3*i^3*n*(c + d*x)^3
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(21*b*d^4) - (3*B*(b*c - a*d)^3*g^
3*i^3*n*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(14*d^4) + (6*b
*B*(b*c - a*d)^2*g^3*i^3*n*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]))/(35*d^4) - (b^2*B*(b*c - a*d)*g^3*i^3*n*(c + d*x)^6*(A + B*Log[e*((a +
b*x)/(c + d*x))^n]))/(21*d^4) + ((b*c - a*d)^3*g^3*i^3*(a + b*x)^4*(A + B*L
og[e*((a + b*x)/(c + d*x))^n])^2)/(140*b^4) + ((b*c - a*d)^2*g^3*i^3*(a + b
*x)^4*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(35*b^3) + ((b*c
- a*d)*g^3*i^3*(a + b*x)^4*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])^2)/(14*b^2) + (g^3*i^3*(a + b*x)^4*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^2)/(7*b) + (B*(b*c - a*d)^5*g^3*i^3*n*(a + b*x)^2*(3*A + B*n
+ 3*B*Log[e*((a + b*x)/(c + d*x))^n]))/(420*b^4*d^2) - (B*(b*c - a*d)^6*g^3
*i^3*n*(a + b*x)*(6*A + 5*B*n + 6*B*Log[e*((a + b*x)/(c + d*x))^n]))/(420*b
^4*d^3) - (B*(b*c - a*d)^7*g^3*i^3*n*(6*A + 11*B*n + 6*B*Log[e*((a + b*x)/(
c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(420*b^4*d^4) - (B^2*(b*c - a
*d)^7*g^3*i^3*n^2*Log[(a + b*x)/(c + d*x)])/(210*b^4*d^4) - (11*B^2*(b*c -
a*d)^7*g^3*i^3*n^2*Log[c + d*x])/(420*b^4*d^4) - (B^2*(b*c - a*d)^7*g^3*i^3
*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(70*b^4*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
```

```
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2383

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Dist[(m + q + 2)/(d*(q + 1)), Int[(
f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Dist[b*n*(p/(d*(q +
 1))), Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x]) /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ
[q, -1] && GtQ[m, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \left((bc-ad)^7 g^3 i^3\right) \text{Subst}\left(\int \frac{x^3 (A+B\log(ex^n))^2}{(b-dx)^8}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{g^3 i^3 (a+bx)^4 (c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{7b} + \frac{\left(3(bc-ad)^7 g^3 i^3\right) \text{Subst}\left(\int \frac{x^3(A+B\log(ex^n))^2}{(b-dx)^7}\,dx, x, \frac{a+bx}{c+dx}\right)}{7b} - \frac{\left(2B(bc-ad)^7 g^3 i^3 n\right) \text{Subst}\left(\int \frac{x^3(A+B\log(ex^n))}{(b-dx)^7}\,dx, x, \frac{a+bx}{c+dx}\right)}{7b}$$

$$= \frac{2B(bc-ad)^4 g^3 i^3 n (c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{21bd^4} - \frac{3B(bc-ad)^3 g^3 i^3 n (c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{14d^4} + \frac{6bB(bc-ad)^2 g^3 i^3 n (c+dx)^5 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{35d^4} - \frac{b^2 B(bc-ad) g^3 i^3 n (c+dx)^6 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{21d^4} + \frac{(bc-ad) g^3 i^3 (a+bx)^4 (c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{14b^2} + \frac{g^3 i^3 (a+bx)^4 (c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{7b} + \frac{\left((bc-ad)^7 g^3 i^3\right) \text{Subst}\left(\int \frac{x^3(A+B\log(ex^n))^2}{(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{7b^2} - \frac{\left(B(bc-ad)^7 g^3 i^3 n\right) \text{Subst}\left(\int \frac{x^3(A+B\log(ex^n))}{(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{7b^2} + \frac{\left(2B^2(bc-ad)^7 g^3 i^3 n^2\right) \text{Subst}\left(\int \frac{-b^3+6b^2dx-15bd^2x^2+20d^3x^3}{60d^4x(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{7b}$$

$$
\begin{aligned}
&= -\frac{B(bc-ad)^3g^3i^3n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{140b^4} \\
&- \frac{B(bc-ad)^2g^3i^3n(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{35b^3} \\
&+ \frac{2B(bc-ad)^4g^3i^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{21bd^4} \\
&- \frac{3B(bc-ad)^3g^3i^3n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{14d^4} \\
&+ \frac{6bB(bc-ad)^2g^3i^3n(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{35d^4} \\
&- \frac{b^2B(bc-ad)g^3i^3n(c+dx)^6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{21d^4} \\
&+ \frac{(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{35b^3} \\
&+ \frac{(bc-ad)g^3i^3(a+bx)^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{14b^2} \\
&+ \frac{g^3i^3(a+bx)^4(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{7b} \\
&+ \frac{((bc-ad)^7g^3i^3)\,\text{Subst}\left(\int\frac{x^3(A+B\log(ex^n))^2}{(b-dx)^5}\,dx,x,\frac{a+bx}{c+dx}\right)}{35b^3} \\
&- \frac{(2B(bc-ad)^7g^3i^3n)\,\text{Subst}\left(\int\frac{x^3(A+B\log(ex^n))}{(b-dx)^5}\,dx,x,\frac{a+bx}{c+dx}\right)}{35b^3} \\
&+ \frac{(B^2(bc-ad)^7g^3i^3n^2)\,\text{Subst}\left(\int\frac{x^3(5b-dx)}{20b^2(b-dx)^5}\,dx,x,\frac{a+bx}{c+dx}\right)}{7b^2} \\
&+ \frac{(B^2(bc-ad)^7g^3i^3n^2)\,\text{Subst}\left(\int\frac{-b^3+6b^2dx-15bd^2x^2+20d^3x^3}{x(b-dx)^6}\,dx,x,\frac{a+bx}{c+dx}\right)}{210bd^4}
\end{aligned}
$$

$$= -\frac{3B(bc-ad)^3 g^3 i^3 n (a+bx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{140b^4} - \frac{B(bc-ad)^2 g^3 i^3 n (a+bx)^4 (c+dx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{35b^3} + \frac{2B(bc-ad)^4 g^3 i^3 n (c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{21bd^4} - \frac{3B(bc-ad)^3 g^3 i^3 n (c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{14d^4} + \frac{6bB(bc-ad)^2 g^3 i^3 n (c+dx)^5 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{35d^4} - \frac{b^2 B(bc-ad) g^3 i^3 n (c+dx)^6 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{21d^4} + \frac{(bc-ad)^3 g^3 i^3 (a+bx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{140b^4} + \frac{(bc-ad)^2 g^3 i^3 (a+bx)^4 (c+dx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{35b^3} + \frac{(bc-ad) g^3 i^3 (a+bx)^4 (c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{14b^2} + \frac{g^3 i^3 (a+bx)^4 (c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{7b} - \frac{(B(bc-ad)^7 g^3 i^3 n)\,\mathrm{Subst}\left(\int \frac{x^3(A+B\log(ex^n))}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{70b^4} + \frac{(B^2(bc-ad)^7 g^3 i^3 n^2)\,\mathrm{Subst}\left(\int \frac{x^3(5b-dx)}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{140b^4} + \frac{(B^2(bc-ad)^7 g^3 i^3 n^2)\,\mathrm{Subst}\left(\int \frac{x^3}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{70b^4} + \frac{(B^2(bc-ad)^7 g^3 i^3 n^2)\,\mathrm{Subst}\left(\int \left(-\frac{1}{b^3x} + \frac{10b^2d}{(b-dx)^6} - \frac{26bd}{(b-dx)^5} + \frac{19d}{(b-dx)^4} - \frac{d}{b(b-dx)^3} - \frac{d}{b^2(b-dx)^2} - \frac{d}{b^3(b-dx)}\right.\right.}{210bd^4}$$

$$
\begin{aligned}
&= -\frac{B^2(bc-ad)^6g^3i^3n^2x}{210b^3d^3} + \frac{B^2(bc-ad)^3g^3i^3n^2(a+bx)^4}{140b^4} \\
&- \frac{B^2(bc-ad)^5g^3i^3n^2(c+dx)^2}{420b^2d^4} + \frac{19B^2(bc-ad)^4g^3i^3n^2(c+dx)^3}{630bd^4} \\
&- \frac{13B^2(bc-ad)^3g^3i^3n^2(c+dx)^4}{420d^4} + \frac{bB^2(bc-ad)^2g^3i^3n^2(c+dx)^5}{105d^4} \\
&- \frac{B(bc-ad)^4g^3i^3n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{210b^4d} \\
&- \frac{3B(bc-ad)^3g^3i^3n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{140b^4} \\
&- \frac{B(bc-ad)^2g^3i^3n(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{35b^3} \\
&+ \frac{2B(bc-ad)^4g^3i^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{21bd^4} \\
&- \frac{3B(bc-ad)^3g^3i^3n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{14d^4} \\
&+ \frac{6bB(bc-ad)^2g^3i^3n(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{35d^4} \\
&- \frac{b^2B(bc-ad)g^3i^3n(c+dx)^6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{21d^4} \\
&+ \frac{(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{140b^4} \\
&+ \frac{(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{35b^3} \\
&+ \frac{(bc-ad)g^3i^3(a+bx)^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{14b^2} \\
&+ \frac{g^3i^3(a+bx)^4(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{7b} \\
&- \frac{B^2(bc-ad)^7g^3i^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{210b^4d^4} - \frac{B^2(bc-ad)^7g^3i^3n^2\log(c+dx)}{210b^4d^4} \\
&+ \frac{(B(bc-ad)^7g^3i^3n)\,\text{Subst}\left(\int \frac{x^2(3A+Bn+3B\log(ex^n))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{210b^4d} \\
&+ \frac{(B^2(bc-ad)^7g^3i^3n^2)\,\text{Subst}\left(\int \frac{x^3}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{140b^4} \\
&+ \frac{(B^2(bc-ad)^7g^3i^3n^2)\,\text{Subst}\left(\int \left(\frac{b^3}{d^3(b-dx)^4} - \frac{3b^2}{d^3(b-dx)^3} + \frac{3b}{d^3(b-dx)^2} - \frac{1}{d^3(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{70b^4}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4B^2(bc-ad)^6g^3i^3n^2x}{105b^3d^3} + \frac{B^2(bc-ad)^3g^3i^3n^2(a+bx)^4}{140b^4} \\
&- \frac{B^2(bc-ad)^5g^3i^3n^2(c+dx)^2}{42b^2d^4} + \frac{11B^2(bc-ad)^4g^3i^3n^2(c+dx)^3}{315bd^4} \\
&- \frac{13B^2(bc-ad)^3g^3i^3n^2(c+dx)^4}{420d^4} + \frac{bB^2(bc-ad)^2g^3i^3n^2(c+dx)^5}{105d^4} \\
&- \frac{B(bc-ad)^4g^3i^3n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{210b^4d} \\
&- \frac{3B(bc-ad)^3g^3i^3n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{140b^4} \\
&- \frac{B(bc-ad)^2g^3i^3n(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{35b^3} \\
&+ \frac{2B(bc-ad)^4g^3i^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{21bd^4} \\
&- \frac{3B(bc-ad)^3g^3i^3n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{14d^4} \\
&+ \frac{6bB(bc-ad)^2g^3i^3n(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{35d^4} \\
&- \frac{b^2B(bc-ad)g^3i^3n(c+dx)^6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{21d^4} \\
&+ \frac{(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{140b^4} \\
&+ \frac{(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{35b^3} \\
&+ \frac{(bc-ad)g^3i^3(a+bx)^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{14b^2} \\
&+ \frac{g^3i^3(a+bx)^4(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{7b} \\
&+ \frac{B(bc-ad)^5g^3i^3n(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{420b^4d^2} \\
&- \frac{B^2(bc-ad)^7g^3i^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{210b^4d^4} - \frac{2B^2(bc-ad)^7g^3i^3n^2\log(c+dx)}{105b^4d^4} \\
&- \frac{\left(B(bc-ad)^7g^3i^3n\right)\text{Subst}\left(\int \frac{x(3Bn+2(3A+Bn)+6B\log(ex^n))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{420b^4d^2} \\
&+ \frac{\left(B^2(bc-ad)^7g^3i^3n^2\right)\text{Subst}\left(\int \left(\frac{b^3}{d^3(b-dx)^4} - \frac{3b^2}{d^3(b-dx)^3} + \frac{3b}{d^3(b-dx)^2} - \frac{1}{d^3(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{140b^4}
\end{aligned}
$$

$$= \frac{5B^2(bc-ad)^6g^3i^3n^2x}{84b^3d^3} + \frac{B^2(bc-ad)^3g^3i^3n^2(a+bx)^4}{140b^4} - \frac{29B^2(bc-ad)^5g^3i^3n^2(c+dx)^2}{840b^2d^4} + \frac{47B^2(bc-ad)^4g^3i^3n^2(c+dx)^3}{1260bd^4} - \frac{13B^2(bc-ad)^3g^3i^3n^2(c+dx)^4}{420d^4} + \frac{bB^2(bc-ad)^2g^3i^3n^2(c+dx)^5}{105d^4} - \frac{B(bc-ad)^4g^3i^3n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{210b^4d} - \frac{3B(bc-ad)^3g^3i^3n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{140b^4} - \frac{B(bc-ad)^2g^3i^3n(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{35b^3} + \frac{2B(bc-ad)^4g^3i^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{21bd^4} - \frac{3B(bc-ad)^3g^3i^3n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{14d^4} + \frac{6bB(bc-ad)^2g^3i^3n(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{35d^4} - \frac{b^2B(bc-ad)g^3i^3n(c+dx)^6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{21d^4} + \frac{(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{140b^4} + \frac{(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{35b^3} + \frac{(bc-ad)g^3i^3(a+bx)^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{14b^2} + \frac{g^3i^3(a+bx)^4(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{7b} + \frac{B(bc-ad)^5g^3i^3n(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{420b^4d^2} - \frac{B(bc-ad)^6g^3i^3n(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{420b^4d^3} - \frac{B^2(bc-ad)^7g^3i^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{210b^4d^4} - \frac{11B^2(bc-ad)^7g^3i^3n^2\log(c+dx)}{420b^4d^4} + \frac{(B(bc-ad)^7g^3i^3n)\,\mathrm{Subst}\left(\int\frac{9Bn+2(3A+Bn)+6B\log(ex^n)}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{420b^4d^3}$$

$$= \frac{5B^2(bc-ad)^6g^3i^3n^2x}{84b^3d^3} + \frac{B^2(bc-ad)^3g^3i^3n^2(a+bx)^4}{140b^4} - \frac{29B^2(bc-ad)^5g^3i^3n^2(c+dx)^2}{840b^2d^4} + \frac{47B^2(bc-ad)^4g^3i^3n^2(c+dx)^3}{1260bd^4} - \frac{13B^2(bc-ad)^3g^3i^3n^2(c+dx)^4}{420d^4} + \frac{bB^2(bc-ad)^2g^3i^3n^2(c+dx)^5}{105d^4} - \frac{B(bc-ad)^4g^3i^3n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{210b^4d} - \frac{3B(bc-ad)^3g^3i^3n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{140b^4} - \frac{B(bc-ad)^2g^3i^3n(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{35b^3} + \frac{2B(bc-ad)^4g^3i^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{21bd^4} - \frac{3B(bc-ad)^3g^3i^3n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{14d^4} + \frac{6bB(bc-ad)^2g^3i^3n(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{35d^4} - \frac{b^2B(bc-ad)g^3i^3n(c+dx)^6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{21d^4} + \frac{(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{140b^4} + \frac{(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{35b^3} + \frac{(bc-ad)g^3i^3(a+bx)^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{14b^2} + \frac{g^3i^3(a+bx)^4(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{7b} + \frac{B(bc-ad)^5g^3i^3n(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{420b^4d^2} - \frac{B(bc-ad)^6g^3i^3n(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{420b^4d^3} - \frac{B(bc-ad)^7g^3i^3n\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{420b^4d^4} - \frac{B^2(bc-ad)^7g^3i^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{210b^4d^4} - \frac{11B^2(bc-ad)^7g^3i^3n^2\log(c+dx)}{420b^4d^4} + \frac{(B^2(bc-ad)^7g^3i^3n^2)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{70b^4d^4}$$

$$
\begin{aligned}
&= \frac{5B^2(bc-ad)^6g^3i^3n^2x}{84b^3d^3} + \frac{B^2(bc-ad)^3g^3i^3n^2(a+bx)^4}{140b^4} \\
&- \frac{29B^2(bc-ad)^5g^3i^3n^2(c+dx)^2}{840b^2d^4} + \frac{47B^2(bc-ad)^4g^3i^3n^2(c+dx)^3}{1260bd^4} \\
&- \frac{13B^2(bc-ad)^3g^3i^3n^2(c+dx)^4}{420d^4} + \frac{bB^2(bc-ad)^2g^3i^3n^2(c+dx)^5}{105d^4} \\
&- \frac{B(bc-ad)^4g^3i^3n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{210b^4d} \\
&- \frac{3B(bc-ad)^3g^3i^3n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{140b^4} \\
&- \frac{B(bc-ad)^2g^3i^3n(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{35b^3} \\
&+ \frac{2B(bc-ad)^4g^3i^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{21bd^4} \\
&- \frac{3B(bc-ad)^3g^3i^3n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{14d^4} \\
&+ \frac{6bB(bc-ad)^2g^3i^3n(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{35d^4} \\
&- \frac{b^2B(bc-ad)g^3i^3n(c+dx)^6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{21d^4} \\
&+ \frac{(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{140b^4} \\
&+ \frac{(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{35b^3} \\
&+ \frac{(bc-ad)g^3i^3(a+bx)^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{14b^2} \\
&+ \frac{g^3i^3(a+bx)^4(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{7b} \\
&+ \frac{B(bc-ad)^5g^3i^3n(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{420b^4d^2} \\
&- \frac{B(bc-ad)^6g^3i^3n(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{420b^4d^3} \\
&- \frac{B(bc-ad)^7g^3i^3n\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{420b^4d^4} \\
&- \frac{B^2(bc-ad)^7g^3i^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{210b^4d^4} - \frac{11B^2(bc-ad)^7g^3i^3n^2\log(c+dx)}{420b^4d^4} \\
&- \frac{B^2(bc-ad)^7g^3i^3n^2\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{70b^4d^4}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2448 vs. 2(1172) = 2344.

Time = 1.85 (sec) , antiderivative size = 2448, normalized size of antiderivative = 2.09

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Result too large to show}$$

```
[In] Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^2,x]
```

```
[Out] (g^3*i^3*(35*(b*c - a*d)^3*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])^2 + 84*d*(b*c - a*d)^2*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2 + 70*d^2*(b*c - a*d)*(a + b*x)^6*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2 + 20*d^3*(a + b*x)^7*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (35*B*(b
*c - a*d)^4*n*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[
e*((a + b*x)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[e*
((a + b*x)/(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[e
*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(2*b*d*(b*c - a*d
)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*n
*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*n*((2*Log[(d*(a
+ b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c +
 d*x))/(b*c - a*d)])))/(3*d^4) + (7*B*(b*c - a*d)^3*n*(24*A*b*d*(b*c - a*d)
^3*x + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 12*d
^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 8*d^3
*(b*c - a*d)*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*d^4*(a
+ b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 24*B*(b*c - a*d)^4*n*Log[
c + d*x] - 24*(b*c - a*d)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c +
d*x] + 4*B*(b*c - a*d)^2*n*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c
- a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(
b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x])
 + 12*B*(b*c - a*d)^3*n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 12*B*(b*c -
 a*d)^4*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x
] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^4 - (7*B*(b*c - a*d)^2*n*(
120*A*b*d*(b*c - a*d)^4*x + 120*B*d*(b*c - a*d)^4*(a + b*x)*Log[e*((a + b*x
)/(c + d*x))^n] + 60*d^2*(-(b*c) + a*d)^3*(a + b*x)^2*(A + B*Log[e*((a + b*
x)/(c + d*x))^n]) + 40*d^3*(b*c - a*d)^2*(a + b*x)^3*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]) + 30*d^4*(-(b*c) + a*d)*(a + b*x)^4*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]) + 24*d^5*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
) - 120*B*(b*c - a*d)^5*n*Log[c + d*x] - 120*(b*c - a*d)^5*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n])*Log[c + d*x] + 20*B*(b*c - a*d)^3*n*(2*b*d*(b*c - a*
d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 5*B*(b*c - a*d)^2*
n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*
x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + 2*B*(b*c - a*d)*n*(12*b*d*(b*c - a*d
```

```
)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c - a*d)*(a + b*x)^3 - 3
*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]) + 60*B*(b*c - a*d)^4*n*(b
*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 60*B*(b*c - a*d)^5*n*((2*Log[(d*(a +
b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d
*x))/(b*c - a*d)])))/(6*d^4) + (B*(b*c - a*d)*n*(360*A*b*d*(b*c - a*d)^5*x
+ 60*b^2*B*c*d*(b*c - a*d)^4*n*x - 60*a*b*B*d^2*(b*c - a*d)^4*n*x + 462*b*B
*d*(b*c - a*d)^5*n*x - 30*b*B*c*d^2*(b*c - a*d)^3*n*(a + b*x)^2 + 30*a*B*d^
3*(b*c - a*d)^3*n*(a + b*x)^2 - 141*B*d^2*(b*c - a*d)^4*n*(a + b*x)^2 + 20*
b*B*c*d^3*(b*c - a*d)^2*n*(a + b*x)^3 - 20*a*B*d^4*(b*c - a*d)^2*n*(a + b*x
)^3 + 54*B*d^3*(b*c - a*d)^3*n*(a + b*x)^3 - 15*b*B*c*d^4*(b*c - a*d)*n*(a
+ b*x)^4 + 15*a*B*d^5*(b*c - a*d)*n*(a + b*x)^4 - 18*B*d^4*(b*c - a*d)^2*n*
(a + b*x)^4 + 12*b*B*c*d^5*n*(a + b*x)^5 - 12*a*B*d^6*n*(a + b*x)^5 + 360*B
*d*(b*c - a*d)^5*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 180*d^2*(b*c -
a*d)^4*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 120*d^3*(b*c -
a*d)^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 90*d^4*(b*c - a
*d)^2*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 72*d^5*(b*c - a*
d)*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 60*d^6*(a + b*x)^6*
(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 60*b*B*c*(b*c - a*d)^5*n*Log[c + d
*x] + 60*a*B*d*(b*c - a*d)^5*n*Log[c + d*x] - 822*B*(b*c - a*d)^6*n*Log[c +
 d*x] - 360*(b*c - a*d)^6*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*
x] + 180*B*(b*c - a*d)^6*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c +
d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(9*d^4)))/(
140*b^4)
```

Maple [F]

$$\int (bgx+ag)^3 (dix+ci)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
[Out] int((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (bgx+ag)^3(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
 algorithm="fricas")
```

```
[Out] integral(A^2*b^3*d^3*g^3*i^3*x^6 + A^2*a^3*c^3*g^3*i^3 + 3*(A^2*b^3*c*d^2 +
 A^2*a*b^2*d^3)*g^3*i^3*x^5 + 3*(A^2*b^3*c^2*d + 3*A^2*a*b^2*c*d^2 + A^2*a^
2*b*d^3)*g^3*i^3*x^4 + (A^2*b^3*c^3 + 9*A^2*a*b^2*c^2*d + 9*A^2*a^2*b*c*d^2
 + A^2*a^3*d^3)*g^3*i^3*x^3 + 3*(A^2*a*b^2*c^3 + 3*A^2*a^2*b*c^2*d + A^2*a^
3*c*d^2)*g^3*i^3*x^2 + 3*(A^2*a^2*b*c^3 + A^2*a^3*c^2*d)*g^3*i^3*x + (B^2*b
^3*d^3*g^3*i^3*x^6 + B^2*a^3*c^3*g^3*i^3 + 3*(B^2*b^3*c*d^2 + B^2*a*b^2*d^3
)*g^3*i^3*x^5 + 3*(B^2*b^3*c^2*d + 3*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3)*g^3*i
^3*x^4 + (B^2*b^3*c^3 + 9*B^2*a*b^2*c^2*d + 9*B^2*a^2*b*c*d^2 + B^2*a^3*d^3
)*g^3*i^3*x^3 + 3*(B^2*a*b^2*c^3 + 3*B^2*a^2*b*c^2*d + B^2*a^3*c*d^2)*g^3*i
^3*x^2 + 3*(B^2*a^2*b*c^3 + B^2*a^3*c^2*d)*g^3*i^3*x)*log(e*((b*x + a)/(d*x
 + c))^n)^2 + 2*(A*B*b^3*d^3*g^3*i^3*x^6 + A*B*a^3*c^3*g^3*i^3 + 3*(A*B*b^3
*c*d^2 + A*B*a*b^2*d^3)*g^3*i^3*x^5 + 3*(A*B*b^3*c^2*d + 3*A*B*a*b^2*c*d^2
+ A*B*a^2*b*d^3)*g^3*i^3*x^4 + (A*B*b^3*c^3 + 9*A*B*a*b^2*c^2*d + 9*A*B*a^2
*b*c*d^2 + A*B*a^3*d^3)*g^3*i^3*x^3 + 3*(A*B*a*b^2*c^3 + 3*A*B*a^2*b*c^2*d
+ A*B*a^3*c*d^2)*g^3*i^3*x^2 + 3*(A*B*a^2*b*c^3 + A*B*a^3*c^2*d)*g^3*i^3*x)
*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**3*(d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2
,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7845 vs. $2(1125) = 2250$.

Time = 0.83 (sec) , antiderivative size = 7845, normalized size of antiderivative = 6.69

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
 algorithm="maxima")
```

```
[Out] 2/7*A*B*b^3*d^3*g^3*i^3*x^7*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/7*A^
2*b^3*d^3*g^3*i^3*x^7 + A*B*b^3*c*d^2*g^3*i^3*x^6*log(e*(b*x/(d*x + c) + a/
(d*x + c))^n) + A*B*a*b^2*d^3*g^3*i^3*x^6*log(e*(b*x/(d*x + c) + a/(d*x + c
))^n) + 1/2*A^2*b^3*c*d^2*g^3*i^3*x^6 + 1/2*A^2*a*b^2*d^3*g^3*i^3*x^6 + 6/5
```

*A*B*b^3*c^2*d*g^3*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 18/5*A*B*a*b^2*c*d^2*g^3*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 6/5*A*B*a^2*b*d^3*g^3*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/5*A^2*b^3*c^2*d*g^3*i^3*x^5 + 9/5*A^2*a*b^2*c*d^2*g^3*i^3*x^5 + 3/5*A^2*a^2*b*d^3*g^3*i^3*x^5 + 1/2*A*B*b^3*c^3*g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 9/2*A*B*a*b^2*c^2*d*g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 9/2*A*B*a^2*b*c*d^2*g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*B*a^3*d^3*g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*b^3*c^3*g^3*i^3*x^4 + 9/4*A^2*a*b^2*c^2*d*g^3*i^3*x^4 + 9/4*A^2*a^2*b*c*d^2*g^3*i^3*x^4 + 1/4*A^2*a^3*d^3*g^3*i^3*x^4 + 2*A*B*a*b^2*c^3*g^3*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 6*A*B*a^2*b*c^2*d*g^3*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*B*a^3*c*d^2*g^3*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*b^2*c^3*g^3*i^3*x^3 + 3*A^2*a^2*b*c^2*d*g^3*i^3*x^3 + A^2*a^3*c*d^2*g^3*i^3*x^3 + 3*A*B*a^2*b*c^3*g^3*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3*A*B*a^3*c^2*d*g^3*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A^2*a^2*b*c^3*g^3*i^3*x^2 + 3/2*A^2*a^3*c^2*d*g^3*i^3*x^2 + 1/210*A*B*b^3*d^3*g^3*i^3*n*(60*a^7*log(b*x + a)/b^7 - 60*c^7*log(d*x + c)/d^7 - (10*(b^6*c*d^5 - a*b^5*d^6)*x^6 - 12*(b^6*c^2*d^4 - a^2*b^4*d^6)*x^5 + 15*(b^6*c^3*d^3 - a^3*b^3*d^6)*x^4 - 20*(b^6*c^4*d^2 - a^4*b^2*d^6)*x^3 + 30*(b^6*c^5*d - a^5*b*d^6)*x^2 - 60*(b^6*c^6 - a^6*d^6)*x)/(b^6*d^6)) - 1/60*A*B*b^3*c*d^2*g^3*i^3*n*(60*a^6*log(b*x + a)/b^6 - 60*c^6*log(d*x + c)/d^6 + (12*(b^5*c*d^4 - a*b^4*d^5)*x^5 - 15*(b^5*c^2*d^3 - a^2*b^3*d^5)*x^4 + 20*(b^5*c^3*d^2 - a^3*b^2*d^5)*x^3 - 30*(b^5*c^4*d - a^4*b*d^5)*x^2 + 60*(b^5*c^5 - a^5*d^5)*x)/(b^5*d^5)) - 1/60*A*B*a*b^2*d^3*g^3*i^3*n*(60*a^6*log(b*x + a)/b^6 - 60*c^6*log(d*x + c)/d^6 + (12*(b^5*c*d^4 - a*b^4*d^5)*x^5 - 15*(b^5*c^2*d^3 - a^2*b^3*d^5)*x^4 + 20*(b^5*c^3*d^2 - a^3*b^2*d^5)*x^3 - 30*(b^5*c^4*d - a^4*b*d^5)*x^2 + 60*(b^5*c^5 - a^5*d^5)*x)/(b^5*d^5)) + 1/10*A*B*b^3*c^2*d*g^3*i^3*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) + 3/10*A*B*a*b^2*c*d^2*g^3*i^3*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) + 1/10*A*B*a^2*b*d^3*g^3*i^3*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) - 1/12*A*B*b^3*c^3*g^3*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 3/4*A*B*a*b^2*c^2*d*g^3*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 3/4*A*B*a^2*b*c*d^2*g^3*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 1/12*A*B*a^3*d^3*g^3*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(

b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + A*B*a*b^2*c^3*g^3*i^3*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + 3*A*B*a^2*b*c^2*d*g^3*i^3*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + A*B*a^3*c*d^2*g^3*i^3*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3*A*B*a^2*b*c^3*g^3*i^3*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) - 3*A*B*a^3*c^2*d*g^3*i^3*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*a^3*c^3*g^3*i^3*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*a^3*c^3*g^3*i^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a^3*c^3*g^3*i^3*x - 1/420*(107*a^4*b^2*c^3*d^4*g^3*i^3*n^2 - 39*a^5*b*c^2*d^5*g^3*i^3*n^2 + 6*a^6*c*d^6*g^3*i^3*n^2 - 6*b^6*c^7*g^3*i^3*n*log(e) - 6*(g^3*i^3*n^2 - 7*g^3*i^3*n*log(e))*a*b^5*c^6*d + 3*(13*g^3*i^3*n^2 - 42*g^3*i^3*n*log(e))*a^2*b^4*c^5*d^2 - (107*g^3*i^3*n^2 - 210*g^3*i^3*n*log(e))*a^3*b^3*c^4*d^3)*B^2*log(d*x + c)/(b^3*d^4) + 1/70*(b^7*c^7*g^3*i^3*n^2 - 7*a*b^6*c^6*d*g^3*i^3*n^2 + 21*a^2*b^5*c^5*d^2*g^3*i^3*n^2 - 35*a^3*b^4*c^4*d^3*g^3*i^3*n^2 + 35*a^4*b^3*c^3*d^4*g^3*i^3*n^2 - 21*a^5*b^2*c^2*d^5*g^3*i^3*n^2 + 7*a^6*b*c*d^6*g^3*i^3*n^2 - a^7*d^7*g^3*i^3*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^4*d^4) + 1/2520*(360*B^2*b^7*d^7*g^3*i^3*x^7*log(e)^2 - 60*((2*g^3*i^3*n*log(e) - 21*g^3*i^3*log(e)^2)*b^7*c*d^6 - (2*g^3*i^3*n*log(e) + 21*g^3*i^3*log(e)^2)*a*b^6*d^7)*B^2*x^6 + 24*((g^3*i^3*n^2 - 15*g^3*i^3*n*log(e) + 63*g^3*i^3*log(e)^2)*b^7*c^2*d^5 - (2*g^3*i^3*n^2 - 189*g^3*i^3*log(e)^2)*a*b^6*c*d^6 + (g^3*i^3*n^2 + 15*g^3*i^3*n*log(e) + 63*g^3*i^3*log(e)^2)*a^2*b^5*d^7)*B^2*x^5 + 6*((10*g^3*i^3*n^2 - 51*g^3*i^3*n*log(e) + 105*g^3*i^3*log(e)^2)*b^7*c^3*d^4 - (10*g^3*i^3*n^2 + 147*g^3*i^3*n*log(e) - 945*g^3*i^3*log(e)^2)*a*b^6*c^2*d^5 - (10*g^3*i^3*n^2 - 147*g^3*i^3*n*log(e) - 945*g^3*i^3*log(e)^2)*a^2*b^5*c*d^6 + (10*g^3*i^3*n^2 + 51*g^3*i^3*n*log(e) + 105*g^3*i^3*log(e)^2)*a^3*b^4*d^7)*B^2*x^4 + 2*((11*g^3*i^3*n^2 - 6*g^3*i^3*n*log(e))*b^7*c^4*d^3 + 4*(19*g^3*i^3*n^2 - 147*g^3*i^3*n*log(e) + 315*g^3*i^3*log(e)^2)*a*b^6*c^3*d^4 - 6*(29*g^3*i^3*n^2 - 630*g^3*i^3*log(e)^2)*a^2*b^5*c^2*d^5 + 4*(19*g^3*i^3*n^2 + 147*g^3*i^3*n*log(e) + 315*g^3*i^3*log(e)^2)*a^3*b^4*c*d^6 + (11*g^3*i^3*n^2 + 6*g^3*i^3*n*log(e))*a^4*b^3*d^7)*B^2*x^3 - 3*(3*(3*g^3*i^3*n^2 - 2*g^3*i^3*n*log(e))*b^7*c^5*d^2 - (67*g^3*i^3*n^2 - 42*g^3*i^3*n*log(e))*a*b^6*c^4*d^3 + 2*(29*g^3*i^3*n^2 + 252*g^3*i^3*n*log(e) - 630*g^3*i^3*log(e)^2)*a^2*b^5*c^3*d^4 + 2*(29*g^3*i^3*n^2 - 252*g^3*i^3*n*log(e) - 630*g^3*i^3*log(e)^2)*a^3*b^4*c^2*d^5 - (67*g^3*i^3*n^2 + 42*g^3*i^3*n*log(e))*a^4*b^3*c*d^6 + 3*(3*g^3*i^3*n^2 + 2*g^3*i^3*n*log(e))*a^5*b^2*d^7)*B^2*x^2 - 18*(35*a^4*b^3*c^3*d^4*g^3*i^3*n^2 - 21*a^5*b^2*c^2*d^5*g^3*i^3*n^2 + 7*a^6*b*c*d^6*g^3*i^3*n^2 - a^7*d^7*g^3*i^3*n^2)*B^2*log(b*x + a)^2 - 36*(b^7*c^7*g^3*i^3*n^2 - 7*a*b^6*c^6*d*g^3*i^3*n^2 + 21*a^2*b^5*c^5*d^2*g^3*i^3*n^2 - 35*a^3*b^4*c^4*d^3*g^3*i^3*n^2)*B^2*log(b*x + a)*log(d*x + c) + 18*(b^7*c^7*g^3*i^3*n^2 - 7*a*b^6*c^6*d*g^3*i^3*n^2 + 21*a^2*b^5*c^5*d^2*g^3*i^3*n^2 - 35*a^3*b^4*c^4*d^3*g^3*i^3*n^2)*B^2*log(d*x + c)^2 + 6*(6*(g^3*i

^3*n^2 - g^3*i^3*n*log(e))*b^7*c^6*d - 3*(15*g^3*i^3*n^2 - 14*g^3*i^3*n*log
(e))*a*b^6*c^5*d^2 + 2*(73*g^3*i^3*n^2 - 63*g^3*i^3*n*log(e))*a^2*b^5*c^4*d
^3 - 2*(107*g^3*i^3*n^2 - 210*g^3*i^3*log(e)^2)*a^3*b^4*c^3*d^4 + 2*(73*g^3
*i^3*n^2 + 63*g^3*i^3*n*log(e))*a^4*b^3*c^2*d^5 - 3*(15*g^3*i^3*n^2 + 14*g^
3*i^3*n*log(e))*a^5*b^2*c*d^6 + 6*(g^3*i^3*n^2 + g^3*i^3*n*log(e))*a^6*b*d^
7)*B^2*x - 6*(6*a*b^6*c^6*d*g^3*i^3*n^2 - 39*a^2*b^5*c^5*d^2*g^3*i^3*n^2 +
107*a^3*b^4*c^4*d^3*g^3*i^3*n^2 + 6*a^7*d^7*g^3*i^3*n*log(e) - (107*g^3*i^3
*n^2 + 210*g^3*i^3*n*log(e))*a^4*b^3*c^3*d^4 + 3*(13*g^3*i^3*n^2 + 42*g^3*i
^3*n*log(e))*a^5*b^2*c^2*d^5 - 6*(g^3*i^3*n^2 + 7*g^3*i^3*n*log(e))*a^6*b*c
*d^6)*B^2*log(b*x + a) + 18*(20*B^2*b^7*d^7*g^3*i^3*x^7 + 140*B^2*a^3*b^4*c
^3*d^4*g^3*i^3*x + 70*(b^7*c*d^6*g^3*i^3 + a*b^6*d^7*g^3*i^3)*B^2*x^6 + 84*
(b^7*c^2*d^5*g^3*i^3 + 3*a*b^6*c*d^6*g^3*i^3 + a^2*b^5*d^7*g^3*i^3)*B^2*x^5
 + 35*(b^7*c^3*d^4*g^3*i^3 + 9*a*b^6*c^2*d^5*g^3*i^3 + 9*a^2*b^5*c*d^6*g^3*
i^3 + a^3*b^4*d^7*g^3*i^3)*B^2*x^4 + 140*(a*b^6*c^3*d^4*g^3*i^3 + 3*a^2*b^5
*c^2*d^5*g^3*i^3 + a^3*b^4*c*d^6*g^3*i^3)*B^2*x^3 + 210*(a^2*b^5*c^3*d^4*g^
3*i^3 + a^3*b^4*c^2*d^5*g^3*i^3)*B^2*x^2)*log((b*x + a)^n)^2 + 18*(20*B^2*b
^7*d^7*g^3*i^3*x^7 + 140*B^2*a^3*b^4*c^3*d^4*g^3*i^3*x + 70*(b^7*c*d^6*g^3*
i^3 + a*b^6*d^7*g^3*i^3)*B^2*x^6 + 84*(b^7*c^2*d^5*g^3*i^3 + 3*a*b^6*c*d^6*
g^3*i^3 + a^2*b^5*d^7*g^3*i^3)*B^2*x^5 + 35*(b^7*c^3*d^4*g^3*i^3 + 9*a*b^6*
c^2*d^5*g^3*i^3 + 9*a^2*b^5*c*d^6*g^3*i^3 + a^3*b^4*d^7*g^3*i^3)*B^2*x^4 +
140*(a*b^6*c^3*d^4*g^3*i^3 + 3*a^2*b^5*c^2*d^5*g^3*i^3 + a^3*b^4*c*d^6*g^3*
i^3)*B^2*x^3 + 210*(a^2*b^5*c^3*d^4*g^3*i^3 + a^3*b^4*c^2*d^5*g^3*i^3)*B^2*
x^2)*log((d*x + c)^n)^2 + 6*(120*B^2*b^7*d^7*g^3*i^3*x^7*log(e) - 20*((g^3*
i^3*n - 21*g^3*i^3*log(e))*b^7*c*d^6 - (g^3*i^3*n + 21*g^3*i^3*log(e))*a*b^
6*d^7)*B^2*x^6 + 12*(126*a*b^6*c*d^6*g^3*i^3*log(e) - (5*g^3*i^3*n - 42*g^3
*i^3*log(e))*b^7*c^2*d^5 + (5*g^3*i^3*n + 42*g^3*i^3*log(e))*a^2*b^5*d^7)*B
^2*x^5 - 3*((17*g^3*i^3*n - 70*g^3*i^3*log(e))*b^7*c^3*d^4 + 7*(7*g^3*i^3*n
 - 90*g^3*i^3*log(e))*a*b^6*c^2*d^5 - 7*(7*g^3*i^3*n + 90*g^3*i^3*log(e))*a
^2*b^5*c*d^6 - (17*g^3*i^3*n + 70*g^3*i^3*log(e))*a^3*b^4*d^7)*B^2*x^4 - 2*
(b^7*c^4*d^3*g^3*i^3*n - a^4*b^3*d^7*g^3*i^3*n - 1260*a^2*b^5*c^2*d^5*g^3*i
^3*log(e) + 14*(7*g^3*i^3*n - 30*g^3*i^3*log(e))*a*b^6*c^3*d^4 - 14*(7*g^3*
i^3*n + 30*g^3*i^3*log(e))*a^3*b^4*c*d^6)*B^2*x^3 + 3*(b^7*c^5*d^2*g^3*i^3*
n - 7*a*b^6*c^4*d^3*g^3*i^3*n + 7*a^4*b^3*c*d^6*g^3*i^3*n - a^5*b^2*d^7*g^3
*i^3*n - 84*(g^3*i^3*n - 5*g^3*i^3*log(e))*a^2*b^5*c^3*d^4 + 84*(g^3*i^3*n
+ 5*g^3*i^3*log(e))*a^3*b^4*c^2*d^5)*B^2*x^2 - 6*(b^7*c^6*d*g^3*i^3*n - 7*a
*b^6*c^5*d^2*g^3*i^3*n + 21*a^2*b^5*c^4*d^3*g^3*i^3*n - 21*a^4*b^3*c^2*d^5*
g^3*i^3*n + 7*a^5*b^2*c*d^6*g^3*i^3*n - a^6*b*d^7*g^3*i^3*n - 140*a^3*b^4*c
^3*d^4*g^3*i^3*log(e))*B^2*x + 6*(35*a^4*b^3*c^3*d^4*g^3*i^3*n - 21*a^5*b^2
*c^2*d^5*g^3*i^3*n + 7*a^6*b*c*d^6*g^3*i^3*n - a^7*d^7*g^3*i^3*n)*B^2*log(b
x + a) + 6(b^7*c^7*g^3*i^3*n - 7*a*b^6*c^6*d*g^3*i^3*n + 21*a^2*b^5*c^5*d
^2*g^3*i^3*n - 35*a^3*b^4*c^4*d^3*g^3*i^3*n)*B^2*log(d*x + c))*log((b*x + a
)^n) - 6*(120*B^2*b^7*d^7*g^3*i^3*x^7*log(e) - 20*((g^3*i^3*n - 21*g^3*i^3*
log(e))*b^7*c*d^6 - (g^3*i^3*n + 21*g^3*i^3*log(e))*a*b^6*d^7)*B^2*x^6 + 12
*(126*a*b^6*c*d^6*g^3*i^3*log(e) - (5*g^3*i^3*n - 42*g^3*i^3*log(e))*b^7*c^
2*d^5 + (5*g^3*i^3*n + 42*g^3*i^3*log(e))*a^2*b^5*d^7)*B^2*x^5 - 3*((17*g^3

```
*i^3*n - 70*g^3*i^3*log(e))*b^7*c^3*d^4 + 7*(7*g^3*i^3*n - 90*g^3*i^3*log(e
))*a*b^6*c^2*d^5 - 7*(7*g^3*i^3*n + 90*g^3*i^3*log(e))*a^2*b^5*c*d^6 - (17*
g^3*i^3*n + 70*g^3*i^3*log(e))*a^3*b^4*d^7)*B^2*x^4 - 2*(b^7*c^4*d^3*g^3*i^
3*n - a^4*b^3*d^7*g^3*i^3*n - 1260*a^2*b^5*c^2*d^5*g^3*i^3*log(e) + 14*(7*g
^3*i^3*n - 30*g^3*i^3*log(e))*a*b^6*c^3*d^4 - 14*(7*g^3*i^3*n + 30*g^3*i^3*
log(e))*a^3*b^4*c*d^6)*B^2*x^3 + 3*(b^7*c^5*d^2*g^3*i^3*n - 7*a*b^6*c^4*d^3
*g^3*i^3*n + 7*a^4*b^3*c*d^6*g^3*i^3*n - a^5*b^2*d^7*g^3*i^3*n - 84*(g^3*i^
3*n - 5*g^3*i^3*log(e))*a^2*b^5*c^3*d^4 + 84*(g^3*i^3*n + 5*g^3*i^3*log(e))
*a^3*b^4*c^2*d^5)*B^2*x^2 - 6*(b^7*c^6*d*g^3*i^3*n - 7*a*b^6*c^5*d^2*g^3*i^
3*n + 21*a^2*b^5*c^4*d^3*g^3*i^3*n - 21*a^4*b^3*c^2*d^5*g^3*i^3*n + 7*a^5*b
^2*c*d^6*g^3*i^3*n - a^6*b*d^7*g^3*i^3*n - 140*a^3*b^4*c^3*d^4*g^3*i^3*log(
e))*B^2*x + 6*(35*a^4*b^3*c^3*d^4*g^3*i^3*n - 21*a^5*b^2*c^2*d^5*g^3*i^3*n
+ 7*a^6*b*c*d^6*g^3*i^3*n - a^7*d^7*g^3*i^3*n)*B^2*log(b*x + a) + 6*(b^7*c^
7*g^3*i^3*n - 7*a*b^6*c^6*d*g^3*i^3*n + 21*a^2*b^5*c^5*d^2*g^3*i^3*n - 35*a
^3*b^4*c^4*d^3*g^3*i^3*n)*B^2*log(d*x + c) + 6*(20*B^2*b^7*d^7*g^3*i^3*x^7
+ 140*B^2*a^3*b^4*c^3*d^4*g^3*i^3*x + 70*(b^7*c*d^6*g^3*i^3 + a*b^6*d^7*g^3
*i^3)*B^2*x^6 + 84*(b^7*c^2*d^5*g^3*i^3 + 3*a*b^6*c*d^6*g^3*i^3 + a^2*b^5*d
^7*g^3*i^3)*B^2*x^5 + 35*(b^7*c^3*d^4*g^3*i^3 + 9*a*b^6*c^2*d^5*g^3*i^3 + 9
*a^2*b^5*c*d^6*g^3*i^3 + a^3*b^4*d^7*g^3*i^3)*B^2*x^4 + 140*(a*b^6*c^3*d^4*
g^3*i^3 + 3*a^2*b^5*c^2*d^5*g^3*i^3 + a^3*b^4*c*d^6*g^3*i^3)*B^2*x^3 + 210*
(a^2*b^5*c^3*d^4*g^3*i^3 + a^3*b^4*c^2*d^5*g^3*i^3)*B^2*x^2)*log((b*x + a)^
n))*log((d*x + c)^n))/(b^4*d^4)
```

Giac [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
 algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^3\,(c\,i+d\,i\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

```
[In] int((a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^
2,x)
```

```
[Out] int((a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^
2, x)
```

3.179 $\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 45, antiderivative size = 976

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=-\frac{7B^2(bc-ad)^5g^2i^3n^2x}{180b^3d^2}-\frac{7B^2(bc-ad)^4g^2i^3n^2(c+dx)^2}{360b^2d^3}-\frac{B^2(bc-ad)^3g^2i^3n^2(c+dx)^3}{60bd^3}$$
$$+\frac{B^2(bc-ad)^2g^2i^3n^2(c+dx)^4}{60d^3}-\frac{B(bc-ad)^4g^2i^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^4d}$$
$$-\frac{B(bc-ad)^3g^2i^3n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^4}$$
$$-\frac{B(bc-ad)^4g^2i^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^2d^3}$$
$$+\frac{B(bc-ad)^3g^2i^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{45bd^3}$$
$$+\frac{7B(bc-ad)^2g^2i^3n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60d^3}$$
$$-\frac{bB(bc-ad)g^2i^3n(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15d^3}$$
$$+\frac{(bc-ad)^3g^2i^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{60b^4}$$
$$+\frac{(bc-ad)^2g^2i^3(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^3}$$
$$+\frac{(bc-ad)g^2i^3(a+bx)^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^2}$$
$$+\frac{g^2i^3(a+bx)^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b}$$
$$+\frac{B(bc-ad)^5g^2i^3n(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^4d^2}$$
$$+\frac{B(bc-ad)^6g^2i^3n\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{60b^4d^3}$$
$$+\frac{B^2(bc-ad)^6g^2i^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{36b^4d^3}+\frac{11B^2(bc-ad)^6g^2i^3n^2\log(c+dx)}{180b^4d^3}$$
$$+\frac{B^2(bc-ad)^6g^2i^3n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{30b^4d^3}$$

```
[Out] -7/180*B^2*(-a*d+b*c)^5*g^2*i^3*n^2*x/b^3/d^2-7/360*B^2*(-a*d+b*c)^4*g^2*i^
3*n^2*(d*x+c)^2/b^2/d^3-1/60*B^2*(-a*d+b*c)^3*g^2*i^3*n^2*(d*x+c)^3/b/d^3+1
/60*B^2*(-a*d+b*c)^2*g^2*i^3*n^2*(d*x+c)^4/d^3-1/60*B*(-a*d+b*c)^4*g^2*i^3*
n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/d-1/30*B*(-a*d+b*c)^3*g^2*i
```

```
^3*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4-1/10*B*(-a*d+b*c)^4*g^2*
i^3*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d^3+1/45*B*(-a*d+b*c)^3
*g^2*i^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^3+7/60*B*(-a*d+b*c
)^2*g^2*i^3*n*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^3-1/15*b*B*(-a*d+
b*c)*g^2*i^3*n*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^3+1/60*(-a*d+b*c
)^3*g^2*i^3*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^4+1/20*(-a*d+b*c)
^2*g^2*i^3*(b*x+a)^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^3+1/10*(-a
*d+b*c)*g^2*i^3*(b*x+a)^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2+1
/6*g^2*i^3*(b*x+a)^3*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/60*B*(
-a*d+b*c)^5*g^2*i^3*n*(b*x+a)*(2*A+B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/d
^2+1/60*B*(-a*d+b*c)^6*g^2*i^3*n*(2*A+3*B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))*
ln((-a*d+b*c)/b/(d*x+c))/b^4/d^3+1/36*B^2*(-a*d+b*c)^6*g^2*i^3*n^2*ln((b*x+
a)/(d*x+c))/b^4/d^3+11/180*B^2*(-a*d+b*c)^6*g^2*i^3*n^2*ln(d*x+c)/b^4/d^3+1
/30*B^2*(-a*d+b*c)^6*g^2*i^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/d^3
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 976, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.244$, Rules

used = {2561, 2383, 2381, 2384, 2354, 2438, 2373, 45, 2382, 12, 907}

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \frac{Bg^2i^3n\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)(bc-ad)^6}{60b^4d^3}$$
$$+ \frac{B^2g^2i^3n^2\log\left(\frac{a+bx}{c+dx}\right)(bc-ad)^6}{36b^4d^3} + \frac{11B^2g^2i^3n^2\log(c+dx)(bc-ad)^6}{180b^4d^3}$$
$$+ \frac{B^2g^2i^3n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)(bc-ad)^6}{30b^4d^3} - \frac{7B^2g^2i^3n^2x(bc-ad)^5}{180b^3d^2}$$
$$+ \frac{Bg^2i^3n(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)^5}{60b^4d^2}$$
$$- \frac{7B^2g^2i^3n^2(c+dx)^2(bc-ad)^4}{360b^2d^3} - \frac{Bg^2i^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)^4}{60b^4d}$$
$$- \frac{Bg^2i^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)^4}{10b^2d^3}$$
$$- \frac{B^2g^2i^3n^2(c+dx)^3(bc-ad)^3}{60bd^3} + \frac{g^2i^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(bc-ad)^3}{60b^4}$$
$$- \frac{Bg^2i^3n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)^3}{30b^4}$$
$$+ \frac{Bg^2i^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)^3}{45bd^3} + \frac{B^2g^2i^3n^2(c+dx)^4(bc-ad)^2}{60d^3}$$
$$+ \frac{g^2i^3(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(bc-ad)^2}{20b^3}$$
$$+ \frac{7Bg^2i^3n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)^2}{60d^3}$$
$$+ \frac{g^2i^3(a+bx)^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(bc-ad)}{10b^2}$$
$$- \frac{bBg^2i^3n(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(bc-ad)}{15d^3}$$
$$+ \frac{g^2i^3(a+bx)^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b}$$

```
[In] Int[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2,x]
```

```
[Out] (-7*B^2*(b*c - a*d)^5*g^2*i^3*n^2*x)/(180*b^3*d^2) - (7*B^2*(b*c - a*d)^4*g
^2*i^3*n^2*(c + d*x)^2)/(360*b^2*d^3) - (B^2*(b*c - a*d)^3*g^2*i^3*n^2*(c +
 d*x)^3)/(60*b*d^3) + (B^2*(b*c - a*d)^2*g^2*i^3*n^2*(c + d*x)^4)/(60*d^3)
- (B*(b*c - a*d)^4*g^2*i^3*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]))/(60*b^4*d) - (B*(b*c - a*d)^3*g^2*i^3*n*(a + b*x)^3*(A + B*Log[e*((a
```

```
+ b*x)/(c + d*x))^n]))/(30*b^4) - (B*(b*c - a*d)^4*g^2*i^3*n*(c + d*x)^2*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n]))/(10*b^2*d^3) + (B*(b*c - a*d)^3*g^2*i
^3*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(45*b*d^3) + (7*B*
(b*c - a*d)^2*g^2*i^3*n*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
/(60*d^3) - (b*B*(b*c - a*d)*g^2*i^3*n*(c + d*x)^5*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]))/(15*d^3) + ((b*c - a*d)^3*g^2*i^3*(a + b*x)^3*(A + B*Log[e*
((a + b*x)/(c + d*x))^n])^2)/(60*b^4) + ((b*c - a*d)^2*g^2*i^3*(a + b*x)^3*
(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(20*b^3) + ((b*c - a*d)
*g^2*i^3*(a + b*x)^3*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/
(10*b^2) + (g^2*i^3*(a + b*x)^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])^2)/(6*b) + (B*(b*c - a*d)^5*g^2*i^3*n*(a + b*x)*(2*A + B*n + 2*B*Lo
g[e*((a + b*x)/(c + d*x))^n]))/(60*b^4*d^2) + (B*(b*c - a*d)^6*g^2*i^3*n*(2
*A + 3*B*n + 2*B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*
x))]))/(60*b^4*d^3) + (B^2*(b*c - a*d)^6*g^2*i^3*n^2*Log[(a + b*x)/(c + d*x)
])/(36*b^4*d^3) + (11*B^2*(b*c - a*d)^6*g^2*i^3*n^2*Log[c + d*x])/(180*b^4*
d^3) + (B^2*(b*c - a*d)^6*g^2*i^3*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x)
)])/(30*b^4*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2383

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Dist[(m + q + 2)/(d*(q + 1)), Int[(
f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Dist[b*n*(p/(d*(q +
 1))), Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x]) /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ
[q, -1] && GtQ[m, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((bc-ad)^6 g^2 i^3\right) \text{Subst}\left(\int \frac{x^2 (A+B\log(ex^n))^2}{(b-dx)^7}\,dx, x, \frac{a+bx}{c+dx}\right)\\
&= \frac{g^2 i^3 (a+bx)^3 (c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b}\\
&\quad+\frac{\left((bc-ad)^6 g^2 i^3\right)\text{Subst}\left(\int \frac{x^2(A+B\log(ex^n))^2}{(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b}\\
&\quad-\frac{\left(B(bc-ad)^6 g^2 i^3 n\right)\text{Subst}\left(\int \frac{x^2(A+B\log(ex^n))}{(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b}\\
&= -\frac{B(bc-ad)^3 g^2 i^3 n (c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9bd^3}\\
&\quad+\frac{B(bc-ad)^2 g^2 i^3 n (c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^3}\\
&\quad-\frac{bB(bc-ad) g^2 i^3 n (c+dx)^5 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15d^3}\\
&\quad+\frac{(bc-ad) g^2 i^3 (a+bx)^3 (c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^2}\\
&\quad+\frac{g^2 i^3 (a+bx)^3 (c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b}\\
&\quad+\frac{\left((bc-ad)^6 g^2 i^3\right)\text{Subst}\left(\int \frac{x^2(A+B\log(ex^n))^2}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b^2}\\
&\quad-\frac{\left(B(bc-ad)^6 g^2 i^3 n\right)\text{Subst}\left(\int \frac{x^2(A+B\log(ex^n))}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b^2}\\
&\quad+\frac{\left(B^2(bc-ad)^6 g^2 i^3 n^2\right)\text{Subst}\left(\int \frac{b^2-5bdx+10d^2x^2}{30d^3x(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{B(bc-ad)^4 g^2 i^3 n (c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^2d^3} \\
&+ \frac{B(bc-ad)^3 g^2 i^3 n (c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{45bd^3} \\
&+ \frac{7B(bc-ad)^2 g^2 i^3 n (c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60d^3} \\
&- \frac{bB(bc-ad) g^2 i^3 n (c+dx)^5 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15d^3} \\
&+ \frac{(bc-ad)^2 g^2 i^3 (a+bx)^3 (c+dx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^3} \\
&+ \frac{(bc-ad) g^2 i^3 (a+bx)^3 (c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^2} \\
&+ \frac{g^2 i^3 (a+bx)^3 (c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b} \\
&+ \frac{((bc-ad)^6 g^2 i^3)\,\mathrm{Subst}\left(\int \frac{x^2(A+B\log(ex^n))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{20b^3} \\
&- \frac{(B(bc-ad)^6 g^2 i^3 n)\,\mathrm{Subst}\left(\int \frac{x^2(A+B\log(ex^n))}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{10b^3} \\
&+ \frac{(B^2(bc-ad)^6 g^2 i^3 n^2)\,\mathrm{Subst}\left(\int \frac{b^2-4bdx+6d^2x^2}{12d^3x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b^2} \\
&+ \frac{(B^2(bc-ad)^6 g^2 i^3 n^2)\,\mathrm{Subst}\left(\int \frac{b^2-5bdx+10d^2x^2}{x(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{90bd^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{B(bc-ad)^3 g^2 i^3 n (a+bx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^4} \\
&- \frac{B(bc-ad)^4 g^2 i^3 n (c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^2d^3} \\
&+ \frac{B(bc-ad)^3 g^2 i^3 n (c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{45bd^3} \\
&+ \frac{7B(bc-ad)^2 g^2 i^3 n (c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60d^3} \\
&- \frac{bB(bc-ad) g^2 i^3 n (c+dx)^5 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15d^3} \\
&+ \frac{(bc-ad)^3 g^2 i^3 (a+bx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{60b^4} \\
&+ \frac{(bc-ad)^2 g^2 i^3 (a+bx)^3 (c+dx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^3} \\
&+ \frac{(bc-ad) g^2 i^3 (a+bx)^3 (c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^2} \\
&+ \frac{g^2 i^3 (a+bx)^3 (c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b} \\
&- \frac{\left(B(bc-ad)^6 g^2 i^3 n\right) \operatorname{Subst}\left(\int \frac{x^2(A+B\log(ex^n))}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{30b^4} \\
&+ \frac{\left(B^2(bc-ad)^6 g^2 i^3 n^2\right) \operatorname{Subst}\left(\int \frac{x^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{30b^4} \\
&+ \frac{\left(B^2(bc-ad)^6 g^2 i^3 n^2\right) \operatorname{Subst}\left(\int \frac{b^2-4bdx+6d^2x^2}{x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{60b^2d^3} \\
&+ \frac{\left(B^2(bc-ad)^6 g^2 i^3 n^2\right) \operatorname{Subst}\left(\int \left(\frac{1}{b^3x} + \frac{6bd}{(b-dx)^5} - \frac{9d}{(b-dx)^4} + \frac{d}{b(b-dx)^3} + \frac{d}{b^2(b-dx)^2} + \frac{d}{b^3(b-dx)}\right)\,dx, x, \frac{a+}{c+}\right.}{90bd^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{B^2(bc-ad)^5g^2i^3n^2x}{90b^3d^2} + \frac{B^2(bc-ad)^4g^2i^3n^2(c+dx)^2}{180b^2d^3} \\
&- \frac{B^2(bc-ad)^3g^2i^3n^2(c+dx)^3}{30bd^3} + \frac{B^2(bc-ad)^2g^2i^3n^2(c+dx)^4}{60d^3} \\
&- \frac{B(bc-ad)^4g^2i^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^4d} \\
&- \frac{B(bc-ad)^3g^2i^3n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^4} \\
&- \frac{B(bc-ad)^4g^2i^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^2d^3} \\
&+ \frac{B(bc-ad)^3g^2i^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{45bd^3} \\
&+ \frac{7B(bc-ad)^2g^2i^3n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60d^3} \\
&- \frac{bB(bc-ad)g^2i^3n(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15d^3} \\
&+ \frac{(bc-ad)^3g^2i^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{60b^4} \\
&+ \frac{(bc-ad)^2g^2i^3(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^3} \\
&+ \frac{(bc-ad)g^2i^3(a+bx)^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^2} \\
&+ \frac{g^2i^3(a+bx)^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b} \\
&+ \frac{B^2(bc-ad)^6g^2i^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{90b^4d^3} + \frac{B^2(bc-ad)^6g^2i^3n^2\log(c+dx)}{90b^4d^3} \\
&+ \frac{(B(bc-ad)^6g^2i^3n)\,\text{Subst}\left(\int \frac{x(2A+Bn+2B\log(ex^n))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{60b^4d} \\
&+ \frac{(B^2(bc-ad)^6g^2i^3n^2)\,\text{Subst}\left(\int \left(\frac{b^2}{d^2(b-dx)^3} - \frac{2b}{d^2(b-dx)^2} + \frac{1}{d^2(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{30b^4} \\
&+ \frac{(B^2(bc-ad)^6g^2i^3n^2)\,\text{Subst}\left(\int \left(\frac{1}{b^2x} + \frac{3bd}{(b-dx)^4} - \frac{5d}{(b-dx)^3} + \frac{d}{b(b-dx)^2} + \frac{d}{b^2(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{60b^2d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{7B^2(bc-ad)^5g^2i^3n^2x}{180b^3d^2} - \frac{7B^2(bc-ad)^4g^2i^3n^2(c+dx)^2}{360b^2d^3} \\
&- \frac{B^2(bc-ad)^3g^2i^3n^2(c+dx)^3}{60bd^3} + \frac{B^2(bc-ad)^2g^2i^3n^2(c+dx)^4}{60d^3} \\
&- \frac{B(bc-ad)^4g^2i^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^4d} \\
&- \frac{B(bc-ad)^3g^2i^3n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^4} \\
&- \frac{B(bc-ad)^4g^2i^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^2d^3} \\
&+ \frac{B(bc-ad)^3g^2i^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{45bd^3} \\
&+ \frac{7B(bc-ad)^2g^2i^3n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60d^3} \\
&- \frac{bB(bc-ad)g^2i^3n(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15d^3} \\
&+ \frac{(bc-ad)^3g^2i^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{60b^4} \\
&+ \frac{(bc-ad)^2g^2i^3(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^3} \\
&+ \frac{(bc-ad)g^2i^3(a+bx)^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^2} \\
&+ \frac{g^2i^3(a+bx)^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b} \\
&+ \frac{B(bc-ad)^5g^2i^3n(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^4d^2} \\
&+ \frac{B^2(bc-ad)^6g^2i^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{36b^4d^3} + \frac{11B^2(bc-ad)^6g^2i^3n^2\log(c+dx)}{180b^4d^3} \\
&- \frac{(B(bc-ad)^6g^2i^3n)\operatorname{Subst}\left(\int \frac{2A+3Bn+2B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{60b^4d^2}
\end{aligned}
$$

$$= -\frac{7B^2(bc-ad)^5g^2i^3n^2x}{180b^3d^2} - \frac{7B^2(bc-ad)^4g^2i^3n^2(c+dx)^2}{360b^2d^3} - \frac{B^2(bc-ad)^3g^2i^3n^2(c+dx)^3}{60bd^3} + \frac{B^2(bc-ad)^2g^2i^3n^2(c+dx)^4}{60d^3} - \frac{B(bc-ad)^4g^2i^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^4d} - \frac{B(bc-ad)^3g^2i^3n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^4} - \frac{B(bc-ad)^4g^2i^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^2d^3} + \frac{B(bc-ad)^3g^2i^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{45bd^3} + \frac{7B(bc-ad)^2g^2i^3n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60d^3} - \frac{bB(bc-ad)g^2i^3n(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15d^3} + \frac{(bc-ad)^3g^2i^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{60b^4} + \frac{(bc-ad)^2g^2i^3(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^3} + \frac{(bc-ad)g^2i^3(a+bx)^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^2} + \frac{g^2i^3(a+bx)^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b} + \frac{B(bc-ad)^5g^2i^3n(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^4d^2} + \frac{B(bc-ad)^6g^2i^3n\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{60b^4d^3} + \frac{B^2(bc-ad)^6g^2i^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{36b^4d^3} + \frac{11B^2(bc-ad)^6g^2i^3n^2\log(c+dx)}{180b^4d^3} - \frac{(B^2(bc-ad)^6g^2i^3n^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{30b^4d^3}$$

$$
\begin{aligned}
&= -\frac{7B^2(bc-ad)^5g^2i^3n^2x}{180b^3d^2} - \frac{7B^2(bc-ad)^4g^2i^3n^2(c+dx)^2}{360b^2d^3} \\
&- \frac{B^2(bc-ad)^3g^2i^3n^2(c+dx)^3}{60bd^3} + \frac{B^2(bc-ad)^2g^2i^3n^2(c+dx)^4}{60d^3} \\
&- \frac{B(bc-ad)^4g^2i^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^4d} \\
&- \frac{B(bc-ad)^3g^2i^3n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^4} \\
&- \frac{B(bc-ad)^4g^2i^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^2d^3} \\
&+ \frac{B(bc-ad)^3g^2i^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{45bd^3} \\
&+ \frac{7B(bc-ad)^2g^2i^3n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60d^3} \\
&- \frac{bB(bc-ad)g^2i^3n(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15d^3} \\
&+ \frac{(bc-ad)^3g^2i^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{60b^4} \\
&+ \frac{(bc-ad)^2g^2i^3(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^3} \\
&+ \frac{(bc-ad)g^2i^3(a+bx)^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^2} \\
&+ \frac{g^2i^3(a+bx)^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b} \\
&+ \frac{B(bc-ad)^5g^2i^3n(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^4d^2} \\
&+ \frac{B(bc-ad)^6g^2i^3n\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{60b^4d^3} \\
&+ \frac{B^2(bc-ad)^6g^2i^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{36b^4d^3} + \frac{11B^2(bc-ad)^6g^2i^3n^2\log(c+dx)}{180b^4d^3} \\
&+ \frac{B^2(bc-ad)^6g^2i^3n^2\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{30b^4d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 1627, normalized size of antiderivative = 1.67

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\frac{g^2i^3\left(15(bc-ad)^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-24b(bc-ad)(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^2,x]
```

```
[Out] (g^2*i^3*(15*(b*c - a*d)^2*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])^2 - 24*b*(b*c - a*d)*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2 + 10*b^2*(c + d*x)^6*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (5*B*(b*c
 - a*d)^3*n*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*n*(b*d*x + (b*c -
a*d)*Log[a + b*x]) - B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2
 + 2*(b*c - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a
+ b*x)/(c + d*x))^n] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]) + 2*b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
 + 6*(b*c - a*d)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*
B*(b*c - a*d)^3*n*Log[c + d*x] - 3*B*(b*c - a*d)^3*n*(Log[a + b*x]*(Log[a +
 b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b
*c) + a*d)])))/b^4 + (2*B*(b*c - a*d)^2*n*(24*A*b*d*(b*c - a*d)^3*x - 12*B*
(b*c - a*d)^3*n*(b*d*x + (b*c - a*d)*Log[a + b*x]) - 4*B*(b*c - a*d)^2*n*(2
*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c - a*d)^2*Log[a + b*x]) - B*(b
*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c + d*x)^2 + 2*b^3*
(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]) + 24*B*d*(b*c - a*d)^3*(a + b*x
)*Log[e*((a + b*x)/(c + d*x))^n] + 12*b^2*(b*c - a*d)^2*(c + d*x)^2*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]) + 8*b^3*(b*c - a*d)*(c + d*x)^3*(A + B*Log[
e*((a + b*x)/(c + d*x))^n]) + 6*b^4*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n]) + 24*(b*c - a*d)^4*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]) - 24*B*(b*c - a*d)^4*n*Log[c + d*x] - 12*B*(b*c - a*d)^4*n*(Log[a +
 b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(
a + b*x))/(-(b*c) + a*d)])))/b^4 - (B*(b*c - a*d)*n*(120*A*b*d*(b*c - a*d)^
4*x - 60*B*(b*c - a*d)^4*n*(b*d*x + (b*c - a*d)*Log[a + b*x]) - 20*B*(b*c -
 a*d)^3*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c - a*d)^2*Log[a +
b*x]) - 5*B*(b*c - a*d)^2*n*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c +
 d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]) - 2*B*(b*c - a*
d)*n*(12*b*d*(b*c - a*d)^3*x + 6*b^2*(b*c - a*d)^2*(c + d*x)^2 + 4*b^3*(b*c
 - a*d)*(c + d*x)^3 + 3*b^4*(c + d*x)^4 + 12*(b*c - a*d)^4*Log[a + b*x]) +
120*B*d*(b*c - a*d)^4*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 60*b^2*(b*
c - a*d)^3*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 40*b^3*(b*c
 - a*d)^2*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 30*b^4*(b*c
```

```
- a*d)*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 24*b^5*(c + d*x
)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 120*(b*c - a*d)^5*Log[a + b*x]
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 120*B*(b*c - a*d)^5*n*Log[c + d*x
] - 60*B*(b*c - a*d)^5*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/
(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(6*b^4)))/(60
*d^3)
```

Maple [F]

$$\int (bgx+ag)^2 (dix+ci)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (bgx+ag)^2(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
 algorithm="fricas")
```

```
[Out] integral(A^2*b^2*d^3*g^2*i^3*x^5 + A^2*a^2*c^3*g^2*i^3 + (3*A^2*b^2*c*d^2 +
 2*A^2*a*b*d^3)*g^2*i^3*x^4 + (3*A^2*b^2*c^2*d + 6*A^2*a*b*c*d^2 + A^2*a^2*
d^3)*g^2*i^3*x^3 + (A^2*b^2*c^3 + 6*A^2*a*b*c^2*d + 3*A^2*a^2*c*d^2)*g^2*i^
3*x^2 + (2*A^2*a*b*c^3 + 3*A^2*a^2*c^2*d)*g^2*i^3*x + (B^2*b^2*d^3*g^2*i^3*
x^5 + B^2*a^2*c^3*g^2*i^3 + (3*B^2*b^2*c*d^2 + 2*B^2*a*b*d^3)*g^2*i^3*x^4 +
 (3*B^2*b^2*c^2*d + 6*B^2*a*b*c*d^2 + B^2*a^2*d^3)*g^2*i^3*x^3 + (B^2*b^2*c
^3 + 6*B^2*a*b*c^2*d + 3*B^2*a^2*c*d^2)*g^2*i^3*x^2 + (2*B^2*a*b*c^3 + 3*B^
2*a^2*c^2*d)*g^2*i^3*x)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^2*d^3*g
^2*i^3*x^5 + A*B*a^2*c^3*g^2*i^3 + (3*A*B*b^2*c*d^2 + 2*A*B*a*b*d^3)*g^2*i^
3*x^4 + (3*A*B*b^2*c^2*d + 6*A*B*a*b*c*d^2 + A*B*a^2*d^3)*g^2*i^3*x^3 + (A*
B*b^2*c^3 + 6*A*B*a*b*c^2*d + 3*A*B*a^2*c*d^2)*g^2*i^3*x^2 + (2*A*B*a*b*c^3
 + 3*A*B*a^2*c^2*d)*g^2*i^3*x)*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**2*(d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2
,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5931 vs. 2(937) = 1874.

Time = 0.78 (sec) , antiderivative size = 5931, normalized size of antiderivative = 6.08

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
 algorithm="maxima")
```

```
[Out] 1/3*A*B*b^2*d^3*g^2*i^3*x^6*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/6*A^
2*b^2*d^3*g^2*i^3*x^6 + 6/5*A*B*b^2*c*d^2*g^2*i^3*x^5*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n) + 4/5*A*B*a*b*d^3*g^2*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d
*x + c))^n) + 3/5*A^2*b^2*c*d^2*g^2*i^3*x^5 + 2/5*A^2*a*b*d^3*g^2*i^3*x^5 +
 3/2*A*B*b^2*c^2*d*g^2*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3*A
*B*a*b*c*d^2*g^2*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*B*a
^2*d^3*g^2*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/4*A^2*b^2*c^2
*d*g^2*i^3*x^4 + 3/2*A^2*a*b*c*d^2*g^2*i^3*x^4 + 1/4*A^2*a^2*d^3*g^2*i^3*x^
4 + 2/3*A*B*b^2*c^3*g^2*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 4*
A*B*a*b*c^2*d*g^2*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*B*a^
2*c*d^2*g^2*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*b^2*c^
3*g^2*i^3*x^3 + 2*A^2*a*b*c^2*d*g^2*i^3*x^3 + A^2*a^2*c*d^2*g^2*i^3*x^3 + 2
*A*B*a*b*c^3*g^2*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3*A*B*a^2
*c^2*d*g^2*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*b*c^3*g^2
*i^3*x^2 + 3/2*A^2*a^2*c^2*d*g^2*i^3*x^2 - 1/180*A*B*b^2*d^3*g^2*i^3*n*(60*
a^6*log(b*x + a)/b^6 - 60*c^6*log(d*x + c)/d^6 + (12*(b^5*c*d^4 - a*b^4*d^5
)*x^5 - 15*(b^5*c^2*d^3 - a^2*b^3*d^5)*x^4 + 20*(b^5*c^3*d^2 - a^3*b^2*d^5)
*x^3 - 30*(b^5*c^4*d - a^4*b*d^5)*x^2 + 60*(b^5*c^5 - a^5*d^5)*x)/(b^5*d^5)
) + 1/10*A*B*b^2*c*d^2*g^2*i^3*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x
+ c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x
^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) +
 1/15*A*B*a*b*d^3*g^2*i^3*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/
```

d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) - 1/4*A*B*b^2*c^2*d*g^2*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 1/2*A*B*a*b*c*d^2*g^2*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 1/12*A*B*a^2*d^3*g^2*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/3*A*B*b^2*c^3*g^2*i^3*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + 2*A*B*a*b*c^2*d*g^2*i^3*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + A*B*a^2*c*d^2*g^2*i^3*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*A*B*a*b*c^3*g^2*i^3*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) - 3*A*B*a^2*c^2*d*g^2*i^3*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*a^2*c^3*g^2*i^3*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*a^2*c^3*g^2*i^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a^2*c^3*g^2*i^3*x - 1/180*(74*a^3*b^2*c^3*d^3*g^2*i^3*n^2 - 33*a^4*b*c^2*d^4*g^2*i^3*n^2 + 6*a^5*c*d^5*g^2*i^3*n^2 - 2*(g^2*i^3*n^2 - 3*g^2*i^3*n*log(e))*b^5*c^6 + 18*(g^2*i^3*n^2 - 2*g^2*i^3*n*log(e))*a*b^4*c^5*d - 9*(7*g^2*i^3*n^2 - 10*g^2*i^3*n*log(e))*a^2*b^3*c^4*d^2)*B^2*log(d*x + c)/(b^3*d^3) - 1/30*(b^6*c^6*g^2*i^3*n^2 - 6*a*b^5*c^5*d*g^2*i^3*n^2 + 15*a^2*b^4*c^4*d^2*g^2*i^3*n^2 - 20*a^3*b^3*c^3*d^3*g^2*i^3*n^2 + 15*a^4*b^2*c^2*d^4*g^2*i^3*n^2 - 6*a^5*b*c*d^5*g^2*i^3*n^2 + a^6*d^6*g^2*i^3*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^4*d^3) + 1/360*(60*B^2*b^6*d^6*g^2*i^3*x^6*log(e)^2 - 24*((g^2*i^3*n*log(e) - 9*g^2*i^3*log(e)^2)*b^6*c*d^5 - (g^2*i^3*n*log(e) + 6*g^2*i^3*log(e)^2)*a*b^5*d^6)*B^2*x^5 + 6*((g^2*i^3*n^2 - 13*g^2*i^3*n*log(e) + 45*g^2*i^3*log(e)^2)*b^6*c^2*d^4 - 2*(g^2*i^3*n^2 - 3*g^2*i^3*n*log(e) - 45*g^2*i^3*log(e)^2)*a*b^5*c*d^5 + (g^2*i^3*n^2 + 7*g^2*i^3*n*log(e) + 15*g^2*i^3*log(e)^2)*a^2*b^4*d^6)*B^2*x^4 + 2*((9*g^2*i^3*n^2 - 38*g^2*i^3*n*log(e) + 60*g^2*i^3*log(e)^2)*b^6*c^3*d^3 - 3*(5*g^2*i^3*n^2 + 14*g^2*i^3*n*log(e) - 120*g^2*i^3*log(e)^2)*a*b^5*c^2*d^4 + 3*(g^2*i^3*n^2 + 26*g^2*i^3*n*log(e) + 60*g^2*i^3*log(e)^2)*a^2*b^4*c*d^5 + (3*g^2*i^3*n^2 + 2*g^2*i^3*n*log(e))*a^3*b^3*d^6)*B^2*x^3 + ((11*g^2*i^3*n^2 - 6*g^2*i^3*n*log(e))*b^6*c^4*d^2 + 2*(5*g^2*i^3*n^2 - 102*g^2*i^3*n*log(e) + 180*g^2*i^3*log(e)^2)*a*b^5*c^3*d^3 - 60*(g^2*i^3*n^2 - 3*g^2*i^3*n*log(e) - 9*g^2*i^3*log(e)^2)*a^2*b^4*c^2*d^4 + 2*(23*g^2*i^3*n^2 + 18*g^2*i^3*n*log(e))*a^3*b^3*c*d^5 - (7*g^2*i^3*n^2 + 6*g^2*i^3*n*log(e))*a^4*b^2*d^6)*B^2*x^2 - 6*(20*a^3*b^3*c^3*d^3*g^2*i^3*n^2 - 15*a^4*b^2*c^2*d^4*g^2*i^3*n^2 + 6*a^5*b*c*d^5*g^2*i^3*n^2 - a^6*d^6*g^2*i^3*n^2)*B^2*log(b*x + a)^2 + 12*(b^6*c^6*g^2*i^3*n^2 - 6*a*b^5*c^5*d*g^2*i^3*n^2 + 15*a^2*b^4*c^4*d^2*g^2*i^3*n^2)*B^2*log(b*x + a)*log(d*x + c) - 6*(b^6*c^6*g^2*i^3*n^2 - 6*a*b^5*c^5*d*g^2*i^3*n^2 + 15*a^2*b^4

*c^4*d^2*g^2*i^3*n^2)*B^2*log(d*x + c)^2 - 2*(2*(4*g^2*i^3*n^2 - 3*g^2*i^3*
n*log(e))*b^6*c^5*d - 3*(17*g^2*i^3*n^2 - 12*g^2*i^3*n*log(e))*a*b^5*c^4*d^
2 + (97*g^2*i^3*n^2 + 30*g^2*i^3*n*log(e) - 180*g^2*i^3*log(e)^2)*a^2*b^4*c
^3*d^3 - (77*g^2*i^3*n^2 + 90*g^2*i^3*n*log(e))*a^3*b^3*c^2*d^4 + 9*(3*g^2*
i^3*n^2 + 4*g^2*i^3*n*log(e))*a^4*b^2*c*d^5 - 2*(2*g^2*i^3*n^2 + 3*g^2*i^3*
n*log(e))*a^5*b*d^6)*B^2*x + 2*(6*a*b^5*c^5*d*g^2*i^3*n^2 - 33*a^2*b^4*c^4*
d^2*g^2*i^3*n^2 + 2*(17*g^2*i^3*n^2 + 60*g^2*i^3*n*log(e))*a^3*b^3*c^3*d^3
- 3*(g^2*i^3*n^2 + 30*g^2*i^3*n*log(e))*a^4*b^2*c^2*d^4 - 6*(g^2*i^3*n^2 -
6*g^2*i^3*n*log(e))*a^5*b*c*d^5 + 2*(g^2*i^3*n^2 - 3*g^2*i^3*n*log(e))*a^6*
d^6)*B^2*log(b*x + a) + 6*(10*B^2*b^6*d^6*g^2*i^3*x^6 + 60*B^2*a^2*b^4*c^3*
d^3*g^2*i^3*x + 12*(3*b^6*c*d^5*g^2*i^3 + 2*a*b^5*d^6*g^2*i^3)*B^2*x^5 + 15
*(3*b^6*c^2*d^4*g^2*i^3 + 6*a*b^5*c*d^5*g^2*i^3 + a^2*b^4*d^6*g^2*i^3)*B^2*
x^4 + 20*(b^6*c^3*d^3*g^2*i^3 + 6*a*b^5*c^2*d^4*g^2*i^3 + 3*a^2*b^4*c*d^5*g
^2*i^3)*B^2*x^3 + 30*(2*a*b^5*c^3*d^3*g^2*i^3 + 3*a^2*b^4*c^2*d^4*g^2*i^3)*
B^2*x^2)*log((b*x + a)^n)^2 + 6*(10*B^2*b^6*d^6*g^2*i^3*x^6 + 60*B^2*a^2*b^
4*c^3*d^3*g^2*i^3*x + 12*(3*b^6*c*d^5*g^2*i^3 + 2*a*b^5*d^6*g^2*i^3)*B^2*x^
5 + 15*(3*b^6*c^2*d^4*g^2*i^3 + 6*a*b^5*c*d^5*g^2*i^3 + a^2*b^4*d^6*g^2*i^3
)*B^2*x^4 + 20*(b^6*c^3*d^3*g^2*i^3 + 6*a*b^5*c^2*d^4*g^2*i^3 + 3*a^2*b^4*c
*d^5*g^2*i^3)*B^2*x^3 + 30*(2*a*b^5*c^3*d^3*g^2*i^3 + 3*a^2*b^4*c^2*d^4*g^2
*i^3)*B^2*x^2)*log((d*x + c)^n)^2 + 2*(60*B^2*b^6*d^6*g^2*i^3*x^6*log(e) -
12*((g^2*i^3*n - 18*g^2*i^3*log(e))*b^6*c*d^5 - (g^2*i^3*n + 12*g^2*i^3*log
(e))*a*b^5*d^6)*B^2*x^5 - 3*((13*g^2*i^3*n - 90*g^2*i^3*log(e))*b^6*c^2*d^4
 - 6*(g^2*i^3*n + 30*g^2*i^3*log(e))*a*b^5*c*d^5 - (7*g^2*i^3*n + 30*g^2*i^
3*log(e))*a^2*b^4*d^6)*B^2*x^4 + 2*(a^3*b^3*d^6*g^2*i^3*n - (19*g^2*i^3*n -
 60*g^2*i^3*log(e))*b^6*c^3*d^3 - 3*(7*g^2*i^3*n - 120*g^2*i^3*log(e))*a*b^
5*c^2*d^4 + 3*(13*g^2*i^3*n + 60*g^2*i^3*log(e))*a^2*b^4*c*d^5)*B^2*x^3 - 3
*(b^6*c^4*d^2*g^2*i^3*n - 6*a^3*b^3*c*d^5*g^2*i^3*n + a^4*b^2*d^6*g^2*i^3*n
 + 2*(17*g^2*i^3*n - 60*g^2*i^3*log(e))*a*b^5*c^3*d^3 - 30*(g^2*i^3*n + 6*g
^2*i^3*log(e))*a^2*b^4*c^2*d^4)*B^2*x^2 + 6*(b^6*c^5*d*g^2*i^3*n - 6*a*b^5*
c^4*d^2*g^2*i^3*n + 15*a^3*b^3*c^2*d^4*g^2*i^3*n - 6*a^4*b^2*c*d^5*g^2*i^3*
n + a^5*b*d^6*g^2*i^3*n - 5*(g^2*i^3*n - 12*g^2*i^3*log(e))*a^2*b^4*c^3*d^3
)*B^2*x + 6*(20*a^3*b^3*c^3*d^3*g^2*i^3*n - 15*a^4*b^2*c^2*d^4*g^2*i^3*n +
6*a^5*b*c*d^5*g^2*i^3*n - a^6*d^6*g^2*i^3*n)*B^2*log(b*x + a) - 6*(b^6*c^6*
g^2*i^3*n - 6*a*b^5*c^5*d*g^2*i^3*n + 15*a^2*b^4*c^4*d^2*g^2*i^3*n)*B^2*log
(d*x + c))*log((b*x + a)^n) - 2*(60*B^2*b^6*d^6*g^2*i^3*x^6*log(e) - 12*((g
^2*i^3*n - 18*g^2*i^3*log(e))*b^6*c*d^5 - (g^2*i^3*n + 12*g^2*i^3*log(e))*a
*b^5*d^6)*B^2*x^5 - 3*((13*g^2*i^3*n - 90*g^2*i^3*log(e))*b^6*c^2*d^4 - 6*(
g^2*i^3*n + 30*g^2*i^3*log(e))*a*b^5*c*d^5 - (7*g^2*i^3*n + 30*g^2*i^3*log(
e))*a^2*b^4*d^6)*B^2*x^4 + 2*(a^3*b^3*d^6*g^2*i^3*n - (19*g^2*i^3*n - 60*g^
2*i^3*log(e))*b^6*c^3*d^3 - 3*(7*g^2*i^3*n - 120*g^2*i^3*log(e))*a*b^5*c^2*
d^4 + 3*(13*g^2*i^3*n + 60*g^2*i^3*log(e))*a^2*b^4*c*d^5)*B^2*x^3 - 3*(b^6*
c^4*d^2*g^2*i^3*n - 6*a^3*b^3*c*d^5*g^2*i^3*n + a^4*b^2*d^6*g^2*i^3*n + 2*(
17*g^2*i^3*n - 60*g^2*i^3*log(e))*a*b^5*c^3*d^3 - 30*(g^2*i^3*n + 6*g^2*i^3
*log(e))*a^2*b^4*c^2*d^4)*B^2*x^2 + 6*(b^6*c^5*d*g^2*i^3*n - 6*a*b^5*c^4*d^
2*g^2*i^3*n + 15*a^3*b^3*c^2*d^4*g^2*i^3*n - 6*a^4*b^2*c*d^5*g^2*i^3*n + a^

```
5*b*d^6*g^2*i^3*n - 5*(g^2*i^3*n - 12*g^2*i^3*log(e))*a^2*b^4*c^3*d^3)*B^2*
x + 6*(20*a^3*b^3*c^3*d^3*g^2*i^3*n - 15*a^4*b^2*c^2*d^4*g^2*i^3*n + 6*a^5*
b*c*d^5*g^2*i^3*n - a^6*d^6*g^2*i^3*n)*B^2*log(b*x + a) - 6*(b^6*c^6*g^2*i^
3*n - 6*a*b^5*c^5*d*g^2*i^3*n + 15*a^2*b^4*c^4*d^2*g^2*i^3*n)*B^2*log(d*x +
 c) + 6*(10*B^2*b^6*d^6*g^2*i^3*x^6 + 60*B^2*a^2*b^4*c^3*d^3*g^2*i^3*x + 12
*(3*b^6*c*d^5*g^2*i^3 + 2*a*b^5*d^6*g^2*i^3)*B^2*x^5 + 15*(3*b^6*c^2*d^4*g^
2*i^3 + 6*a*b^5*c*d^5*g^2*i^3 + a^2*b^4*d^6*g^2*i^3)*B^2*x^4 + 20*(b^6*c^3*
d^3*g^2*i^3 + 6*a*b^5*c^2*d^4*g^2*i^3 + 3*a^2*b^4*c*d^5*g^2*i^3)*B^2*x^3 +
30*(2*a*b^5*c^3*d^3*g^2*i^3 + 3*a^2*b^4*c^2*d^4*g^2*i^3)*B^2*x^2)*log((b*x
+ a)^n))*log((d*x + c)^n))/(b^4*d^3)
```

Giac [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
 algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^2\,(c\,i+d\,i\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

```
[In] int((a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^
2,x)
```

```
[Out] int((a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^
2, x)
```

3.180 $\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 43, antiderivative size = 786

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \frac{B^2(bc-ad)^4gi^3n^2x}{60b^3d}+\frac{B^2(bc-ad)^3gi^3n^2(c+dx)^2}{30b^2d^2}+\frac{B^2(bc-ad)^2gi^3n^2(c+dx)^3}{30bd^2}$$
$$-\frac{B(bc-ad)^4gi^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^4d}$$
$$-\frac{B(bc-ad)^3gi^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^4}$$
$$+\frac{3B(bc-ad)^3gi^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^2d^2}$$
$$+\frac{B(bc-ad)^2gi^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30bd^2}$$
$$-\frac{B(bc-ad)gi^3n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10d^2}$$
$$+\frac{(bc-ad)^3gi^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^4}$$
$$+\frac{(bc-ad)^2gi^3(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^3}$$
$$+\frac{3(bc-ad)gi^3(a+bx)^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^2}$$
$$+\frac{gi^3(a+bx)^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b}$$
$$-\frac{B(bc-ad)^5gi^3n\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{10b^4d^2}$$
$$-\frac{B^2(bc-ad)^5gi^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d^2}-\frac{11B^2(bc-ad)^5gi^3n^2\log(c+dx)}{60b^4d^2}$$
$$-\frac{B^2(bc-ad)^5gi^3n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{10b^4d^2}$$

```
[Out] 1/60*B^2*(-a*d+b*c)^4*g*i^3*n^2*x/b^3/d+1/30*B^2*(-a*d+b*c)^3*g*i^3*n^2*(d*
x+c)^2/b^2/d^2+1/30*B^2*(-a*d+b*c)^2*g*i^3*n^2*(d*x+c)^3/b/d^2-1/10*B*(-a*d
+b*c)^4*g*i^3*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/d-1/10*B*(-a*d+
b*c)^3*g*i^3*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4+3/20*B*(-a*d+b
*c)^3*g*i^3*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d^2+1/30*B*(-a*
d+b*c)^2*g*i^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^2-1/10*B*(-a
*d+b*c)*g*i^3*n*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^2+1/20*(-a*d+b*
c)^3*g*i^3*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^4+1/10*(-a*d+b*c)^
2*g*i^3*(b*x+a)^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^3+3/20*(-a*d+
```

```
b*c)*g*i^3*(b*x+a)^2*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2+1/5*g*
i^3*(b*x+a)^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b-1/10*B*(-a*d+b*
c)^5*g*i^3*n*(A+B*n+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b
^4/d^2-1/12*B^2*(-a*d+b*c)^5*g*i^3*n^2*ln((b*x+a)/(d*x+c))/b^4/d^2-11/60*B^
2*(-a*d+b*c)^5*g*i^3*n^2*ln(d*x+c)/b^4/d^2-1/10*B^2*(-a*d+b*c)^5*g*i^3*n^2*
polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/d^2
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 786, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.256$, Rules used = {2561, 2383, 2381, 2384, 2354, 2438, 2373, 45, 2382, 12, 78}

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx$$
$$= -\frac{Bgi^3n(bc-ad)^5\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A+Bn\right)}{10b^4d^2} - \frac{Bgi^3n(a+bx)(bc-ad)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{10b^4d} + \frac{gi^3(a+bx)^2(bc-ad)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{20b^4} - \frac{Bgi^3n(a+bx)^2(bc-ad)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{10b^4} + \frac{gi^3(a+bx)^2(c+dx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{10b^3} + \frac{3Bgi^3n(c+dx)^2(bc-ad)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{20b^2d^2} + \frac{3gi^3(a+bx)^2(c+dx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{20b^2} + \frac{Bgi^3n(c+dx)^3(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{30bd^2} - \frac{Bgi^3n(c+dx)^4(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{10d^2} + \frac{gi^3(a+bx)^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b} - \frac{B^2gi^3n^2(bc-ad)^5\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{10b^4d^2} - \frac{B^2gi^3n^2(bc-ad)^5\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d^2} - \frac{11B^2gi^3n^2(bc-ad)^5\log(c+dx)}{60b^4d^2} + \frac{B^2gi^3n^2x(bc-ad)^4}{60b^3d} + \frac{B^2gi^3n^2(c+dx)^2(bc-ad)^3}{30b^2d^2} + \frac{B^2gi^3n^2(c+dx)^3(bc-ad)^2}{30bd^2}$$

```
[In] Int[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,
x]
```

```
[Out] (B^2*(b*c - a*d)^4*g*i^3*n^2*x)/(60*b^3*d) + (B^2*(b*c - a*d)^3*g*i^3*n^2*(
c + d*x)^2)/(30*b^2*d^2) + (B^2*(b*c - a*d)^2*g*i^3*n^2*(c + d*x)^3)/(30*b*
d^2) - (B*(b*c - a*d)^4*g*i^3*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]))/(10*b^4*d) - (B*(b*c - a*d)^3*g*i^3*n*(a + b*x)^2*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/(10*b^4) + (3*B*(b*c - a*d)^3*g*i^3*n*(c + d*x)^2*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/(20*b^2*d^2) + (B*(b*c - a*d)^2*g*i^3*
n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(30*b*d^2) - (B*(b*c
- a*d)*g*i^3*n*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(10*d^2)
 + ((b*c - a*d)^3*g*i^3*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2)/(20*b^4) + ((b*c - a*d)^2*g*i^3*(a + b*x)^2*(c + d*x)*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n])^2)/(10*b^3) + (3*(b*c - a*d)*g*i^3*(a + b*x)^2*(c + d*
x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(20*b^2) + (g*i^3*(a + b*x)^
2*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(5*b) - (B*(b*c - a
*d)^5*g*i^3*n*(A + B*n + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/
(b*(c + d*x))])/(10*b^4*d^2) - (B^2*(b*c - a*d)^5*g*i^3*n^2*Log[(a + b*x)/(
c + d*x)])/(12*b^4*d^2) - (11*B^2*(b*c - a*d)^5*g*i^3*n^2*Log[c + d*x])/(60
*b^4*d^2) - (B^2*(b*c - a*d)^5*g*i^3*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d
*x))])/(10*b^4*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
```

```
Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rule 2381

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Dist[b*n*(p/(d*(q + 1))), Int[(f*x)^
m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2382

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Dist[a + b*Log[c*x
^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a,
b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

Rule 2383

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Dist[(m + q + 2)/(d*(q + 1)), Int[(
f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Dist[b*n*(p/(d*(q +
 1))), Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x]) /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ
[q, -1] && GtQ[m, 0]
```

Rule 2384

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Dist[f/(e*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x)^(q + 1
)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x
] && ILtQ[q, -1] && GtQ[m, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \left((bc-ad)^5 gi^3\right) \text{Subst}\left(\int \frac{x(A+B\log(ex^n))^2}{(b-dx)^6}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{gi^3(a+bx)^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} + \frac{\left(3(bc-ad)^5 gi^3\right)\text{Subst}\left(\int \frac{x(A+B\log(ex^n))^2}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b} - \frac{\left(2B(bc-ad)^5 gi^3 n\right)\text{Subst}\left(\int \frac{x(A+B\log(ex^n))}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b}$$

$$= \frac{2B(bc-ad)^2 gi^3 n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15bd^2} - \frac{B(bc-ad)gi^3 n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10d^2} + \frac{3(bc-ad)gi^3(a+bx)^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^2} + \frac{gi^3(a+bx)^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} + \frac{\left(3(bc-ad)^5 gi^3\right)\text{Subst}\left(\int \frac{x(A+B\log(ex^n))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{10b^2} - \frac{\left(3B(bc-ad)^5 gi^3 n\right)\text{Subst}\left(\int \frac{x(A+B\log(ex^n))}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{10b^2} + \frac{\left(2B^2(bc-ad)^5 gi^3 n^2\right)\text{Subst}\left(\int \frac{-b+4dx}{12d^2x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{5b}$$

$$
\begin{aligned}
&= \frac{3B(bc-ad)^3gi^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^2d^2} \\
&+ \frac{B(bc-ad)^2gi^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30bd^2} \\
&- \frac{B(bc-ad)gi^3n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10d^2} \\
&+ \frac{(bc-ad)^2gi^3(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^3} \\
&+ \frac{3(bc-ad)gi^3(a+bx)^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^2} \\
&+ \frac{gi^3(a+bx)^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} \\
&+ \frac{((bc-ad)^5gi^3)\operatorname{Subst}\left(\int\frac{x(A+B\log(ex^n))^2}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{10b^3} \\
&- \frac{(B(bc-ad)^5gi^3n)\operatorname{Subst}\left(\int\frac{x(A+B\log(ex^n))}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{5b^3} \\
&+ \frac{(3B^2(bc-ad)^5gi^3n^2)\operatorname{Subst}\left(\int\frac{-b+3dx}{6d^2x(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{10b^2} \\
&+ \frac{(B^2(bc-ad)^5gi^3n^2)\operatorname{Subst}\left(\int\frac{-b+4dx}{x(b-dx)^4}\,dx,x,\frac{a+bx}{c+dx}\right)}{30bd^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{B(bc-ad)^3 gi^3 n(a+bx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^4} \\
&+ \frac{3B(bc-ad)^3 gi^3 n(c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^2d^2} \\
&+ \frac{B(bc-ad)^2 gi^3 n(c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30bd^2} \\
&- \frac{B(bc-ad) gi^3 n(c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10d^2} \\
&+ \frac{(bc-ad)^3 gi^3 (a+bx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^4} \\
&+ \frac{(bc-ad)^2 gi^3 (a+bx)^2(c+dx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^3} \\
&+ \frac{3(bc-ad) gi^3 (a+bx)^2(c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^2} \\
&+ \frac{gi^3 (a+bx)^2(c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} \\
&- \frac{(B(bc-ad)^5 gi^3 n) \operatorname{Subst}\left(\int \frac{x(A+B\log(ex^n))}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{10b^4} \\
&+ \frac{(B^2(bc-ad)^5 gi^3 n^2) \operatorname{Subst}\left(\int \frac{x}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{10b^4} \\
&+ \frac{(B^2(bc-ad)^5 gi^3 n^2) \operatorname{Subst}\left(\int \frac{-b+3dx}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{20b^2d^2} \\
&+ \frac{(B^2(bc-ad)^5 gi^3 n^2) \operatorname{Subst}\left(\int \left(-\frac{1}{b^3x} + \frac{3d}{(b-dx)^4} - \frac{d}{b(b-dx)^3} - \frac{d}{b^2(b-dx)^2} - \frac{d}{b^3(b-dx)}\right) dx, x, \frac{a+bx}{c+dx}\right)}{30bd^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{B^2(bc-ad)^4 g i^3 n^2 x}{30b^3 d} - \frac{B^2(bc-ad)^3 g i^3 n^2 (c+dx)^2}{60b^2 d^2} \\
&+ \frac{B^2(bc-ad)^2 g i^3 n^2 (c+dx)^3}{30bd^2} - \frac{B(bc-ad)^4 g i^3 n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^4 d} \\
&- \frac{B(bc-ad)^3 g i^3 n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^4} \\
&+ \frac{3B(bc-ad)^3 g i^3 n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^2 d^2} \\
&+ \frac{B(bc-ad)^2 g i^3 n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30bd^2} \\
&- \frac{B(bc-ad) g i^3 n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10d^2} \\
&+ \frac{(bc-ad)^3 g i^3 (a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^4} \\
&+ \frac{(bc-ad)^2 g i^3 (a+bx)^2 (c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^3} \\
&+ \frac{3(bc-ad) g i^3 (a+bx)^2 (c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^2} \\
&+ \frac{g i^3 (a+bx)^2 (c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} \\
&- \frac{B^2(bc-ad)^5 g i^3 n^2 \log\left(\frac{a+bx}{c+dx}\right)}{30b^4 d^2} - \frac{B^2(bc-ad)^5 g i^3 n^2 \log(c+dx)}{30b^4 d^2} \\
&+ \frac{(B(bc-ad)^5 g i^3 n)\,\mathrm{Subst}\left(\int \frac{A+Bn+B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{10b^4 d} \\
&+ \frac{(B^2(bc-ad)^5 g i^3 n^2)\,\mathrm{Subst}\left(\int \left(\frac{b}{d(-b+dx)^2} + \frac{1}{d(-b+dx)}\right) dx, x, \frac{a+bx}{c+dx}\right)}{10b^4} \\
&+ \frac{(B^2(bc-ad)^5 g i^3 n^2)\,\mathrm{Subst}\left(\int \left(-\frac{1}{b^2 x} + \frac{2d}{(b-dx)^3} - \frac{d}{b(b-dx)^2} - \frac{d}{b^2(b-dx)}\right) dx, x, \frac{a+bx}{c+dx}\right)}{20b^2 d^2}
\end{aligned}
$$

$$= \frac{B^2(bc-ad)^4gi^3n^2x}{60b^3d} + \frac{B^2(bc-ad)^3gi^3n^2(c+dx)^2}{30b^2d^2} + \frac{B^2(bc-ad)^2gi^3n^2(c+dx)^3}{30bd^2}$$
$$- \frac{B(bc-ad)^4gi^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^4d}$$
$$- \frac{B(bc-ad)^3gi^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^4}$$
$$+ \frac{3B(bc-ad)^3gi^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^2d^2}$$
$$+ \frac{B(bc-ad)^2gi^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30bd^2}$$
$$- \frac{B(bc-ad)gi^3n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10d^2}$$
$$+ \frac{(bc-ad)^3gi^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^4}$$
$$+ \frac{(bc-ad)^2gi^3(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^3}$$
$$+ \frac{3(bc-ad)gi^3(a+bx)^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^2}$$
$$+ \frac{gi^3(a+bx)^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b}$$
$$- \frac{B(bc-ad)^5gi^3n\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{10b^4d^2}$$
$$- \frac{B^2(bc-ad)^5gi^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d^2} - \frac{11B^2(bc-ad)^5gi^3n^2\log(c+dx)}{60b^4d^2}$$
$$+ \frac{(B^2(bc-ad)^5gi^3n^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{10b^4d^2}$$

$$
\begin{aligned}
&= \frac{B^2(bc-ad)^4gi^3n^2x}{60b^3d} + \frac{B^2(bc-ad)^3gi^3n^2(c+dx)^2}{30b^2d^2} + \frac{B^2(bc-ad)^2gi^3n^2(c+dx)^3}{30bd^2} \\
&\quad - \frac{B(bc-ad)^4gi^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^4d} \\
&\quad - \frac{B(bc-ad)^3gi^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^4} \\
&\quad + \frac{3B(bc-ad)^3gi^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^2d^2} \\
&\quad + \frac{B(bc-ad)^2gi^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30bd^2} \\
&\quad - \frac{B(bc-ad)gi^3n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10d^2} \\
&\quad + \frac{(bc-ad)^3gi^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^4} \\
&\quad + \frac{(bc-ad)^2gi^3(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^3} \\
&\quad + \frac{3(bc-ad)gi^3(a+bx)^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^2} \\
&\quad + \frac{gi^3(a+bx)^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} \\
&\quad - \frac{B(bc-ad)^5gi^3n\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{10b^4d^2} \\
&\quad - \frac{B^2(bc-ad)^5gi^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d^2} - \frac{11B^2(bc-ad)^5gi^3n^2\log(c+dx)}{60b^4d^2} \\
&\quad - \frac{B^2(bc-ad)^5gi^3n^2\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{10b^4d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 945, normalized size of antiderivative = 1.20

$$
\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx
$$

$$
= \frac{gi^3\left(-5(bc-ad)(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+4b(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{5B(bc-ad)^2n(}{}\right.}{}
$$

```
[In] Integrate[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2,x]
```

```
[Out] (g*i^3*(-5*(b*c - a*d)*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
 + 4*b*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (5*B*(b*c - a
```

```
*d)^2*n*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*n*(b*d*x + (b*c - a*d)
*Log[a + b*x]) - B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2
*(b*c - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*
x)/(c + d*x))^n] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]) + 2*b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 6
*(b*c - a*d)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*B*(b
*c - a*d)^3*n*Log[c + d*x] - 3*B*(b*c - a*d)^3*n*(Log[a + b*x]*(Log[a + b*x
] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c)
+ a*d)])))/(3*b^4) - (B*(b*c - a*d)*n*(24*A*b*d*(b*c - a*d)^3*x - 12*B*(b*c
 - a*d)^3*n*(b*d*x + (b*c - a*d)*Log[a + b*x]) - 4*B*(b*c - a*d)^2*n*(2*b*d
*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c - a*d)^2*Log[a + b*x]) - B*(b*c -
 a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c + d*x)^2 + 2*b^3*(c +
 d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]) + 24*B*d*(b*c - a*d)^3*(a + b*x)*Lo
g[e*((a + b*x)/(c + d*x))^n] + 12*b^2*(b*c - a*d)^2*(c + d*x)^2*(A + B*Log[
e*((a + b*x)/(c + d*x))^n]) + 8*b^3*(b*c - a*d)*(c + d*x)^3*(A + B*Log[e*((
a + b*x)/(c + d*x))^n]) + 6*b^4*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]) + 24*(b*c - a*d)^4*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^
n]) - 24*B*(b*c - a*d)^4*n*Log[c + d*x] - 12*B*(b*c - a*d)^4*n*(Log[a + b*x
]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a +
b*x))/(-(b*c) + a*d)])))/(3*b^4)))/(20*d^2)
```

Maple [F]

$$\int (bgx + ag) (dix + ci)^3 \left(A + B\ln\left(e\left(\frac{bx + a}{dx + c}\right)^n\right)\right)^2 dx$$

```
[In] int((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
[Out] int((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ag + bgx)(ci + dix)^3 \left(A + B\log\left(e\left(\frac{a + bx}{c + dx}\right)^n\right)\right)^2 dx = \int (bgx + ag)(dix + ci)^3 \left(B\log\left(e\left(\frac{bx + a}{dx + c}\right)^n\right) + A\right)^2 dx$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, a
lgorithm="fricas")
[Out] integral(A^2*b*d^3*g*i^3*x^4 + A^2*a*c^3*g*i^3 + (3*A^2*b*c*d^2 + A^2*a*d^3
)*g*i^3*x^3 + 3*(A^2*b*c^2*d + A^2*a*c*d^2)*g*i^3*x^2 + (A^2*b*c^3 + 3*A^2*
a*c^2*d)*g*i^3*x + (B^2*b*d^3*g*i^3*x^4 + B^2*a*c^3*g*i^3 + (3*B^2*b*c*d^2
+ B^2*a*d^3)*g*i^3*x^3 + 3*(B^2*b*c^2*d + B^2*a*c*d^2)*g*i^3*x^2 + (B^2*b*c
```

```
^3 + 3*B^2*a*c^2*d)*g*i^3*x)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b*d^
3*g*i^3*x^4 + A*B*a*c^3*g*i^3 + (3*A*B*b*c*d^2 + A*B*a*d^3)*g*i^3*x^3 + 3*(
A*B*b*c^2*d + A*B*a*c*d^2)*g*i^3*x^2 + (A*B*b*c^3 + 3*A*B*a*c^2*d)*g*i^3*x)
*log(e*((b*x + a)/(d*x + c))^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3724 vs. $2(753) = 1506$.

Time = 0.76 (sec) , antiderivative size = 3724, normalized size of antiderivative = 4.74

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, a
lgorithm="maxima")
[Out] 2/5*A*B*b*d^3*g*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A^2*b*
d^3*g*i^3*x^5 + 3/2*A*B*b*c*d^2*g*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c
))^n) + 1/2*A*B*a*d^3*g*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/
4*A^2*b*c*d^2*g*i^3*x^4 + 1/4*A^2*a*d^3*g*i^3*x^4 + 2*A*B*b*c^2*d*g*i^3*x^3
*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*B*a*c*d^2*g*i^3*x^3*log(e*(b*
x/(d*x + c) + a/(d*x + c))^n) + A^2*b*c^2*d*g*i^3*x^3 + A^2*a*c*d^2*g*i^3*x
^3 + A*B*b*c^3*g*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3*A*B*a*c
^2*d*g*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*b*c^3*g*i^3
*x^2 + 3/2*A^2*a*c^2*d*g*i^3*x^2 + 1/30*A*B*b*d^3*g*i^3*n*(12*a^5*log(b*x +
 a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4
*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 -
 a^4*d^4)*x)/(b^4*d^4)) - 1/4*A*B*b*c*d^2*g*i^3*n*(6*a^4*log(b*x + a)/b^4 -
 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a
^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 1/12*A*B*a*d^3*g*i^3*
n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*
d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^
3)) + A*B*b*c^2*d*g*i^3*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3
```

- ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + A*B*a*c*d^2*g*i^3*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - A*B*b*c^3*g*i^3*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) - 3*A*B*a*c^2*d*g*i^3*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*a*c^3*g*i^3*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*a*c^3*g*i^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*c^3*g*i^3*x - 1/60*(47*a^2*b^2*c^3*d^2*g*i^3*n^2 - 27*a^3*b*c^2*d^3*g*i^3*n^2 + 6*a^4*c*d^4*g*i^3*n^2 + (5*g*i^3*n^2 - 6*g*i^3*n*log(e))*b^4*c^5 - (31*g*i^3*n^2 - 30*g*i^3*n*log(e))*a*b^3*c^4*d)*B^2*log(d*x + c)/(b^3*d^2) + 1/10*(b^5*c^5*g*i^3*n^2 - 5*a*b^4*c^4*d*g*i^3*n^2 + 10*a^2*b^3*c^3*d^2*g*i^3*n^2 - 10*a^3*b^2*c^2*d^3*g*i^3*n^2 + 5*a^4*b*c*d^4*g*i^3*n^2 - a^5*d^5*g*i^3*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^4*d^2) + 1/60*(12*B^2*b^5*d^5*g*i^3*x^5*log(e)^2 - 3*((2*g*i^3*n*log(e) - 15*g*i^3*log(e)^2)*b^5*c*d^4 - (2*g*i^3*n*log(e) + 5*g*i^3*log(e)^2)*a*b^4*d^5)*B^2*x^4 + 2*((g*i^3*n^2 - 11*g*i^3*n*log(e) + 30*g*i^3*log(e)^2)*b^5*c^2*d^3 - 2*(g*i^3*n^2 - 5*g*i^3*n*log(e) - 15*g*i^3*log(e)^2)*a*b^4*c*d^4 + (g*i^3*n^2 + g*i^3*n*log(e))*a^2*b^3*d^5)*B^2*x^3 + ((8*g*i^3*n^2 - 27*g*i^3*n*log(e) + 30*g*i^3*log(e)^2)*b^5*c^3*d^2 - 3*(6*g*i^3*n^2 - 5*g*i^3*n*log(e) - 30*g*i^3*log(e)^2)*a*b^4*c^2*d^3 + 3*(4*g*i^3*n^2 + 5*g*i^3*n*log(e))*a^2*b^3*c*d^4 - (2*g*i^3*n^2 + 3*g*i^3*n*log(e))*a^3*b^2*d^5)*B^2*x^2 - 3*(10*a^2*b^3*c^3*d^2*g*i^3*n^2 - 10*a^3*b^2*c^2*d^3*g*i^3*n^2 + 5*a^4*b*c*d^4*g*i^3*n^2 - a^5*d^5*g*i^3*n^2)*B^2*log(b*x + a)^2 - 6*(b^5*c^5*g*i^3*n^2 - 5*a*b^4*c^4*d*g*i^3*n^2)*B^2*log(b*x + a)*log(d*x + c) + 3*(b^5*c^5*g*i^3*n^2 - 5*a*b^4*c^4*d*g*i^3*n^2)*B^2*log(d*x + c)^2 + ((11*g*i^3*n^2 - 6*g*i^3*n*log(e))*b^5*c^4*d - 2*(14*g*i^3*n^2 + 15*g*i^3*n*log(e) - 30*g*i^3*log(e)^2)*a*b^4*c^3*d^2 + 12*(2*g*i^3*n^2 + 5*g*i^3*n*log(e))*a^2*b^3*c^2*d^3 - 2*(4*g*i^3*n^2 + 15*g*i^3*n*log(e))*a^3*b^2*c*d^4 + (g*i^3*n^2 + 6*g*i^3*n*log(e))*a^4*b*d^5)*B^2*x - (6*a*b^4*c^4*d*g*i^3*n^2 + 3*(g*i^3*n^2 - 20*g*i^3*n*log(e))*a^2*b^3*c^3*d^2 - (23*g*i^3*n^2 - 60*g*i^3*n*log(e))*a^3*b^2*c^2*d^3 + (19*g*i^3*n^2 - 30*g*i^3*n*log(e))*a^4*b*c*d^4 - (5*g*i^3*n^2 - 6*g*i^3*n*log(e))*a^5*d^5)*B^2*log(b*x + a) + 3*(4*B^2*b^5*d^5*g*i^3*x^5 + 20*B^2*a*b^4*c^3*d^2*g*i^3*x + 5*(3*b^5*c*d^4*g*i^3 + a*b^4*d^5*g*i^3)*B^2*x^4 + 20*(b^5*c^2*d^3*g*i^3 + a*b^4*c*d^4*g*i^3)*B^2*x^3 + 10*(b^5*c^3*d^2*g*i^3 + 3*a*b^4*c^2*d^3*g*i^3)*B^2*x^2)*log((b*x + a)^n)^2 + 3*(4*B^2*b^5*d^5*g*i^3*x^5 + 20*B^2*a*b^4*c^3*d^2*g*i^3*x + 5*(3*b^5*c*d^4*g*i^3 + a*b^4*d^5*g*i^3)*B^2*x^4 + 20*(b^5*c^2*d^3*g*i^3 + a*b^4*c*d^4*g*i^3)*B^2*x^3 + 10*(b^5*c^3*d^2*g*i^3 + 3*a*b^4*c^2*d^3*g*i^3)*B^2*x^2)*log((d*x + c)^n)^2 + (24*B^2*b^5*d^5*g*i^3*x^5*log(e) - 6*((g*i^3*n - 15*g*i^3*log(e))*b^5*c*d^4 - (g*i^3*n + 5*g*i^3*log(e))*a*b^4*d^5)*B^2*x^4 + 2*(a^2*b^3*d^5*g*i^3*n - (11*g*i^3*n - 60*g*i^3*log(e))*b^5*c^2*d^3 + 10*(g*i^3*n + 6*g*i^3*log(e))*a*b^4*c*d^4)*B^2*x^3 + 3*(5*a^2*b^3*c*d^4*g*i^3*n - a^3*b^2*d^5*g*i^3*n - (9*g*i^3*n - 20*g*i^3*log(e))*b^5*c^3*d^2 + 5*(g*i^3*n + 12*g*i^3*log(e))*a*b^4*c^2*d^3)*B^2*x^2 - 6*(b^5*c^4*d*g*i^3*n - 10*a^2*b^3*c^2*d^3*g*i^3*n + 5*a^3*b^2*c*d^4*g*i^3*n - a^4*b*d^5*g*i^3*n + 5*(g*i^3*n - 4*

```
g*i^3*log(e))*a*b^4*c^3*d^2)*B^2*x + 6*(10*a^2*b^3*c^3*d^2*g*i^3*n - 10*a^3
*b^2*c^2*d^3*g*i^3*n + 5*a^4*b*c*d^4*g*i^3*n - a^5*d^5*g*i^3*n)*B^2*log(b*x
 + a) + 6*(b^5*c^5*g*i^3*n - 5*a*b^4*c^4*d*g*i^3*n)*B^2*log(d*x + c))*log((
b*x + a)^n) - (24*B^2*b^5*d^5*g*i^3*x^5*log(e) - 6*((g*i^3*n - 15*g*i^3*log
(e))*b^5*c*d^4 - (g*i^3*n + 5*g*i^3*log(e))*a*b^4*d^5)*B^2*x^4 + 2*(a^2*b^3
*d^5*g*i^3*n - (11*g*i^3*n - 60*g*i^3*log(e))*b^5*c^2*d^3 + 10*(g*i^3*n + 6
*g*i^3*log(e))*a*b^4*c*d^4)*B^2*x^3 + 3*(5*a^2*b^3*c*d^4*g*i^3*n - a^3*b^2*
d^5*g*i^3*n - (9*g*i^3*n - 20*g*i^3*log(e))*b^5*c^3*d^2 + 5*(g*i^3*n + 12*g
*i^3*log(e))*a*b^4*c^2*d^3)*B^2*x^2 - 6*(b^5*c^4*d*g*i^3*n - 10*a^2*b^3*c^2
*d^3*g*i^3*n + 5*a^3*b^2*c*d^4*g*i^3*n - a^4*b*d^5*g*i^3*n + 5*(g*i^3*n - 4
*g*i^3*log(e))*a*b^4*c^3*d^2)*B^2*x + 6*(10*a^2*b^3*c^3*d^2*g*i^3*n - 10*a^
3*b^2*c^2*d^3*g*i^3*n + 5*a^4*b*c*d^4*g*i^3*n - a^5*d^5*g*i^3*n)*B^2*log(b*
x + a) + 6*(b^5*c^5*g*i^3*n - 5*a*b^4*c^4*d*g*i^3*n)*B^2*log(d*x + c) + 6*(
4*B^2*b^5*d^5*g*i^3*x^5 + 20*B^2*a*b^4*c^3*d^2*g*i^3*x + 5*(3*b^5*c*d^4*g*i
^3 + a*b^4*d^5*g*i^3)*B^2*x^4 + 20*(b^5*c^2*d^3*g*i^3 + a*b^4*c*d^4*g*i^3)*
B^2*x^3 + 10*(b^5*c^3*d^2*g*i^3 + 3*a*b^4*c^2*d^3*g*i^3)*B^2*x^2)*log((b*x
+ a)^n))*log((d*x + c)^n))/(b^4*d^2)
```

Giac [F]

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \int (bgx+ag)(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

```
[In] integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, a
lgorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)*(d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) +
 A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \int (a\,g+b\,g\,x)\,\left(c\,i+d\,i\,x\right)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

```
[In] int((a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,
x)
```

```
[Out] int((a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,
 x)
```

3.181 $\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 35, antiderivative size = 454

$$
\begin{aligned}
&\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= \frac{5B^2(bc-ad)^3i^3n^2x}{12b^3} + \frac{B^2(bc-ad)^2i^3n^2(c+dx)^2}{12b^2d} \\
&\quad - \frac{B(bc-ad)^3i^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^4} \\
&\quad - \frac{B(bc-ad)^2i^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b^2d} \\
&\quad - \frac{B(bc-ad)i^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd} \\
&\quad + \frac{i^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} \\
&\quad + \frac{5B^2(bc-ad)^4i^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d} + \frac{11B^2(bc-ad)^4i^3n^2\log(c+dx)}{12b^4d} \\
&\quad + \frac{B(bc-ad)^4i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d} \\
&\quad - \frac{B^2(bc-ad)^4i^3n^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d}
\end{aligned}
$$

```
[Out] 5/12*B^2*(-a*d+b*c)^3*i^3*n^2*x/b^3+1/12*B^2*(-a*d+b*c)^2*i^3*n^2*(d*x+c)^2
/b^2/d-1/2*B*(-a*d+b*c)^3*i^3*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4
-1/4*B*(-a*d+b*c)^2*i^3*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d-1
/6*B*(-a*d+b*c)*i^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/4*i^3
*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d+5/12*B^2*(-a*d+b*c)^4*i^3*n^
2*ln((b*x+a)/(d*x+c))/b^4/d+11/12*B^2*(-a*d+b*c)^4*i^3*n^2*ln(d*x+c)/b^4/d+
```

```
1/2*B*(-a*d+b*c)^4*i^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(
b*x+a))/b^4/d-1/2*B^2*(-a*d+b*c)^4*i^3*n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b
^4/d
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2551, 2356, 2389, 2379, 2438, 2351, 31, 46}

$$\begin{aligned}&\int (ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx\\&=\frac{Bi^3n(bc-ad)^4\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b^4d}\\&-\frac{Bi^3n(a+bx)(bc-ad)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b^4}\\&-\frac{Bi^3n(c+dx)^2(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b^2d}\\&-\frac{Bi^3n(c+dx)^3(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6bd}\\&+\frac{i^3(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d}-\frac{B^2i^3n^2(bc-ad)^4\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d}\\&+\frac{5B^2i^3n^2(bc-ad)^4\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d}+\frac{11B^2i^3n^2(bc-ad)^4\log(c+dx)}{12b^4d}\\&+\frac{5B^2i^3n^2x(bc-ad)^3}{12b^3}+\frac{B^2i^3n^2(c+dx)^2(bc-ad)^2}{12b^2d}\end{aligned}$$

```
[In] Int[(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (5*B^2*(b*c - a*d)^3*i^3*n^2*x)/(12*b^3) + (B^2*(b*c - a*d)^2*i^3*n^2*(c +
d*x)^2)/(12*b^2*d) - (B*(b*c - a*d)^3*i^3*n*(a + b*x)*(A + B*Log[e*((a + b*
x)/(c + d*x))^n]))/(2*b^4) - (B*(b*c - a*d)^2*i^3*n*(c + d*x)^2*(A + B*Log[
e*((a + b*x)/(c + d*x))^n]))/(4*b^2*d) - (B*(b*c - a*d)*i^3*n*(c + d*x)^3*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*b*d) + (i^3*(c + d*x)^4*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n])^2)/(4*d) + (5*B^2*(b*c - a*d)^4*i^3*n^2*Log[(
a + b*x)/(c + d*x)])/(12*b^4*d) + (11*B^2*(b*c - a*d)^4*i^3*n^2*Log[c + d*x
])/(12*b^4*d) + (B*(b*c - a*d)^4*i^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(2*b^4*d) - (B^2*(b*c - a*d)^4*i^3
*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(2*b^4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
```

```
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\text{integral} = \left((bc-ad)^4 i^3\right) \text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^5}\,dx, x, \frac{a+bx}{c+dx}\right)$$

$$= \frac{i^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} - \frac{\left(B(bc-ad)^4 i^3 n\right)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{2d}$$

$$= \frac{i^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} - \frac{\left(B(bc-ad)^4 i^3 n\right)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b} - \frac{\left(B(bc-ad)^4 i^3 n\right)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{2bd}$$

$$= -\frac{B(bc-ad)i^3 n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd} + \frac{i^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} - \frac{\left(B(bc-ad)^4 i^3 n\right)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b^2} - \frac{\left(B(bc-ad)^4 i^3 n\right)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b^2 d} + \frac{\left(B^2(bc-ad)^4 i^3 n^2\right)\text{Subst}\left(\int \frac{1}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{6bd}$$

$$= -\frac{B(bc-ad)^2 i^3 n(c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b^2 d} - \frac{B(bc-ad) i^3 n(c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd} + \frac{i^3(c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} - \frac{(B(bc-ad)^4 i^3 n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b^3} - \frac{(B(bc-ad)^4 i^3 n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b^3 d} + \frac{(B^2(bc-ad)^4 i^3 n^2)\,\text{Subst}\left(\int \frac{1}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{4b^2 d} + \frac{(B^2(bc-ad)^4 i^3 n^2)\,\text{Subst}\left(\int \left(\frac{1}{b^3 x} + \frac{d}{b(b-dx)^3} + \frac{d}{b^2(b-dx)^2} + \frac{d}{b^3(b-dx)}\right) dx, x, \frac{a+bx}{c+dx}\right)}{6bd}$$

$$= \frac{B^2(bc-ad)^3 i^3 n^2 x}{6b^3} + \frac{B^2(bc-ad)^2 i^3 n^2 (c+dx)^2}{12b^2 d} - \frac{B(bc-ad)^3 i^3 n(a+bx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^4} - \frac{B(bc-ad)^2 i^3 n(c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b^2 d} - \frac{B(bc-ad) i^3 n(c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd} + \frac{i^3(c+dx)^4 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} + \frac{B^2(bc-ad)^4 i^3 n^2 \log\left(\frac{a+bx}{c+dx}\right)}{6b^4 d} + \frac{B^2(bc-ad)^4 i^3 n^2 \log(c+dx)}{6b^4 d} + \frac{B(bc-ad)^4 i^3 n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4 d} + \frac{(B^2(bc-ad)^4 i^3 n^2)\,\text{Subst}\left(\int \frac{1}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b^4} - \frac{(B^2(bc-ad)^4 i^3 n^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{2b^4 d} + \frac{(B^2(bc-ad)^4 i^3 n^2)\,\text{Subst}\left(\int \left(\frac{1}{b^2 x} + \frac{d}{b(b-dx)^2} + \frac{d}{b^2(b-dx)}\right) dx, x, \frac{a+bx}{c+dx}\right)}{4b^2 d}$$

$$
\begin{aligned}
&= \frac{5B^2(bc-ad)^3 i^3 n^2 x}{12b^3} + \frac{B^2(bc-ad)^2 i^3 n^2 (c+dx)^2}{12b^2 d} \\
&- \frac{B(bc-ad)^3 i^3 n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^4} \\
&- \frac{B(bc-ad)^2 i^3 n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b^2 d} \\
&- \frac{B(bc-ad) i^3 n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd} \\
&+ \frac{i^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} \\
&+ \frac{5B^2(bc-ad)^4 i^3 n^2 \log\left(\frac{a+bx}{c+dx}\right)}{12b^4 d} + \frac{11B^2(bc-ad)^4 i^3 n^2 \log(c+dx)}{12b^4 d} \\
&+ \frac{B(bc-ad)^4 i^3 n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4 d} \\
&- \frac{B^2(bc-ad)^4 i^3 n^2 \operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4 d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.90

$$
\int (ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx
$$

$$
= \frac{i^3\left((c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - \frac{B(bc-ad)n\left(6Abd(bc-ad)^2x-3B(bc-ad)^2n(bdx+(bc-ad)\log(a+bx))-B(bc-ad)n(2bd\right.}{}\right.}{}
$$

```
[In] Integrate[(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (i^3*((c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d)
*n*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*n*(b*d*x + (b*c - a*d)*Log[
a + b*x]) - B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c
 - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*x)/(c
 + d*x))^n] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]) + 2*b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 6*(b*c
 - a*d)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*B*(b*c -
a*d)^3*n*Log[c + d*x] - 3*B*(b*c - a*d)^3*n*(Log[a + b*x]*(Log[a + b*x] - 2
*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d
)])))/(3*b^4)))/(4*d)
```

Maple [F]

$$\int (dix+ci)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

```
[In] int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [F]

$$\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\ = \int (dix+ci)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*c^
3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2*c^
3*i^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*d^2*
i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log(e*((b*x + a)/(d*x + c))^n),
x)
```

Sympy [F(-1)]

Timed out.

$$\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2129 vs. 2(433) = 866.

Time = 0.73 (sec) , antiderivative size = 2129, normalized size of antiderivative = 4.69

$$\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="ma
xima")
```

```
[Out] 1/2*A*B*d^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*d^3*i^
3*x^4 + 2*A*B*c*d^2*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c*
d^2*i^3*x^3 + 3*A*B*c^2*d*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) +
3/2*A^2*c^2*d*i^3*x^2 - 1/12*A*B*d^3*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*
log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^
3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + A*B*c*d^2*i^3*n*(2*a^3*log(b
*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^
2 - a^2*d^2)*x)/(b^2*d^2)) - 3*A*B*c^2*d*i^3*n*(a^2*log(b*x + a)/b^2 - c^2*
log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*c^3*i^3*n*(a*log(b*x + a)/b
 - c*log(d*x + c)/d) + 2*A*B*c^3*i^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n) + A^2*c^3*i^3*x - 1/12*(26*a*b^2*c^3*d*i^3*n^2 - 21*a^2*b*c^2*d^2*i^3*n^
2 + 6*a^3*c*d^3*i^3*n^2 - (11*i^3*n^2 - 6*i^3*n*log(e))*b^3*c^4)*B^2*log(d*
x + c)/(b^3*d) - 1/2*(b^4*c^4*i^3*n^2 - 4*a*b^3*c^3*d*i^3*n^2 + 6*a^2*b^2*c
^2*d^2*i^3*n^2 - 4*a^3*b*c*d^3*i^3*n^2 + a^4*d^4*i^3*n^2)*(log(b*x + a)*log
((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b
^4*d) + 1/12*(3*B^2*b^4*d^4*i^3*x^4*log(e)^2 + 6*B^2*b^4*c^4*i^3*n^2*log(b*
x + a)*log(d*x + c) - 3*B^2*b^4*c^4*i^3*n^2*log(d*x + c)^2 + 2*(a*b^3*d^4*i
^3*n*log(e) - (i^3*n*log(e) - 6*i^3*log(e)^2)*b^4*c*d^3)*B^2*x^3 + ((i^3*n^
2 - 9*i^3*n*log(e) + 18*i^3*log(e)^2)*b^4*c^2*d^2 - 2*(i^3*n^2 - 6*i^3*n*lo
g(e))*a*b^3*c*d^3 + (i^3*n^2 - 3*i^3*n*log(e))*a^2*b^2*d^4)*B^2*x^2 - 3*(4*
a*b^3*c^3*d*i^3*n^2 - 6*a^2*b^2*c^2*d^2*i^3*n^2 + 4*a^3*b*c*d^3*i^3*n^2 - a
^4*d^4*i^3*n^2)*B^2*log(b*x + a)^2 + ((7*i^3*n^2 - 18*i^3*n*log(e) + 12*i^3
*log(e)^2)*b^4*c^3*d - (19*i^3*n^2 - 36*i^3*n*log(e))*a*b^3*c^2*d^2 + (17*i
^3*n^2 - 24*i^3*n*log(e))*a^2*b^2*c*d^3 - (5*i^3*n^2 - 6*i^3*n*log(e))*a^3*
b*d^4)*B^2*x - (6*(3*i^3*n^2 - 4*i^3*n*log(e))*a*b^3*c^3*d - 9*(5*i^3*n^2 -
 4*i^3*n*log(e))*a^2*b^2*c^2*d^2 + 2*(19*i^3*n^2 - 12*i^3*n*log(e))*a^3*b*c
*d^3 - (11*i^3*n^2 - 6*i^3*n*log(e))*a^4*d^4)*B^2*log(b*x + a) + 3*(B^2*b^4
*d^4*i^3*x^4 + 4*B^2*b^4*c*d^3*i^3*x^3 + 6*B^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2*
b^4*c^3*d*i^3*x)*log((b*x + a)^n)^2 + 3*(B^2*b^4*d^4*i^3*x^4 + 4*B^2*b^4*c*
d^3*i^3*x^3 + 6*B^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2*b^4*c^3*d*i^3*x)*log((d*x +
 c)^n)^2 + (6*B^2*b^4*d^4*i^3*x^4*log(e) - 6*B^2*b^4*c^4*i^3*n*log(d*x + c)
 + 2*(a*b^3*d^4*i^3*n - (i^3*n - 12*i^3*log(e))*b^4*c*d^3)*B^2*x^3 + 3*(4*a
*b^3*c*d^3*i^3*n - a^2*b^2*d^4*i^3*n - 3*(i^3*n - 4*i^3*log(e))*b^4*c^2*d^2
)*B^2*x^2 + 6*(6*a*b^3*c^2*d^2*i^3*n - 4*a^2*b^2*c*d^3*i^3*n + a^3*b*d^4*i^
```

```
3*n - (3*i^3*n - 4*i^3*log(e))*b^4*c^3*d)*B^2*x + 6*(4*a*b^3*c^3*d*i^3*n -
6*a^2*b^2*c^2*d^2*i^3*n + 4*a^3*b*c*d^3*i^3*n - a^4*d^4*i^3*n)*B^2*log(b*x
+ a))*log((b*x + a)^n) - (6*B^2*b^4*d^4*i^3*x^4*log(e) - 6*B^2*b^4*c^4*i^3*
n*log(d*x + c) + 2*(a*b^3*d^4*i^3*n - (i^3*n - 12*i^3*log(e))*b^4*c*d^3)*B^
2*x^3 + 3*(4*a*b^3*c*d^3*i^3*n - a^2*b^2*d^4*i^3*n - 3*(i^3*n - 4*i^3*log(e
))*b^4*c^2*d^2)*B^2*x^2 + 6*(6*a*b^3*c^2*d^2*i^3*n - 4*a^2*b^2*c*d^3*i^3*n
+ a^3*b*d^4*i^3*n - (3*i^3*n - 4*i^3*log(e))*b^4*c^3*d)*B^2*x + 6*(4*a*b^3*
c^3*d*i^3*n - 6*a^2*b^2*c^2*d^2*i^3*n + 4*a^3*b*c*d^3*i^3*n - a^4*d^4*i^3*n
)*B^2*log(b*x + a) + 6*(B^2*b^4*d^4*i^3*x^4 + 4*B^2*b^4*c*d^3*i^3*x^3 + 6*B
^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2*b^4*c^3*d*i^3*x)*log((b*x + a)^n))*log((d*x
+ c)^n))/(b^4*d)
```

Giac [F]

$$\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (dix+ci)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (c\,i+d\,i\,x)^3 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

```
[In] int((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

```
[Out] int((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

3.182 $\int \frac{(ci+dix)^3 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx} \, dx$

Optimal result

Integrand size = 45, antiderivative size = 762

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{B^2d(bc-ad)^2i^3n^2x}{3b^3g} - \frac{5Bd(bc-ad)^2i^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^4g}$$
$$- \frac{B(bc-ad)i^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^2g}$$
$$+ \frac{d(bc-ad)^2i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^4g}$$
$$+ \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g} + \frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg}$$
$$+ \frac{2B(bc-ad)^3i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^4g}$$
$$+ \frac{B^2(bc-ad)^3i^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^4g} + \frac{2B^2(bc-ad)^3i^3n^2\log(c+dx)}{b^4g}$$
$$+ \frac{5B(bc-ad)^3i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^4g}$$
$$- \frac{(bc-ad)^3i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}$$
$$+ \frac{2B^2(bc-ad)^3i^3n^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g} - \frac{5B^2(bc-ad)^3i^3n^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3b^4g}$$
$$+ \frac{2B(bc-ad)^3i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}$$
$$+ \frac{2B^2(bc-ad)^3i^3n^2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}$$

```
[Out] 1/3*B^2*d*(-a*d+b*c)^2*i^3*n^2*x/b^3/g-5/3*B*d*(-a*d+b*c)^2*i^3*n*(b*x+a)*(
A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/g-1/3*B*(-a*d+b*c)*i^3*n*(d*x+c)^2*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))/b^2/g+d*(-a*d+b*c)^2*i^3*(b*x+a)*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))^2/b^4/g+1/2*(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d
*x+c))^n))^2/b^2/g+1/3*i^3*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b/g+
2*B*(-a*d+b*c)^3*i^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x
+c))/b^4/g+1/3*B^2*(-a*d+b*c)^3*i^3*n^2*ln((b*x+a)/(d*x+c))/b^4/g+2*B^2*(-a
*d+b*c)^3*i^3*n^2*ln(d*x+c)/b^4/g+5/3*B*(-a*d+b*c)^3*i^3*n*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g-(-a*d+b*c)^3*i^3*(A+B*ln(e*
```

```
((b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g+2*B^2*(-a*d+b*c)^3*
i^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/g-5/3*B^2*(-a*d+b*c)^3*i^3*n^2*p
olylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g+2*B*(-a*d+b*c)^3*i^3*n*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g+2*B^2*(-a*d+b*c)^3*i^3
*n^2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^4/g
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 762, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2561, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31, 46}

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{2Bi^3n(bc-ad)^3 \operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4g} + \frac{di^3(a+bx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^4g} - \frac{5Bdi^3n(a+bx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b^4g} + \frac{2Bi^3n(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4g} - \frac{i^3(bc-ad)^3\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^4g} + \frac{5Bi^3n(bc-ad)^3\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b^4g} + \frac{i^3(c+dx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b^2g} - \frac{Bi^3n(c+dx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b^2g} + \frac{i^3(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3bg} + \frac{2B^2i^3n^2(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g} - \frac{5B^2i^3n^2(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3b^4g} + \frac{2B^2i^3n^2(bc-ad)^3\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g} + \frac{B^2i^3n^2(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{3b^4g} + \frac{2B^2i^3n^2(bc-ad)^3\log(c+dx)}{b^4g} + \frac{B^2di^3n^2x(bc-ad)^2}{3b^3g}$$

```
[In] Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x
),x]
```

```
[Out] (B^2*d*(b*c - a*d)^2*i^3*n^2*x)/(3*b^3*g) - (5*B*d*(b*c - a*d)^2*i^3*n*(a +
 b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b^4*g) - (B*(b*c - a*d)*i^
3*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b^2*g) + (d*(b*c
 - a*d)^2*i^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(b^4*g) +
 ((b*c - a*d)*i^3*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*
b^2*g) + (i^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*b*g)
 + (2*B*(b*c - a*d)^3*i^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c
 - a*d)/(b*(c + d*x))])/(b^4*g) + (B^2*(b*c - a*d)^3*i^3*n^2*Log[(a + b*x)/
(c + d*x)])/(3*b^4*g) + (2*B^2*(b*c - a*d)^3*i^3*n^2*Log[c + d*x])/(b^4*g)
+ (5*B*(b*c - a*d)^3*i^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (
b*(c + d*x))/(d*(a + b*x))])/(3*b^4*g) - ((b*c - a*d)^3*i^3*(A + B*Log[e*((
a + b*x)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^4*g) + (
2*B^2*(b*c - a*d)^3*i^3*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^4*g
) - (5*B^2*(b*c - a*d)^3*i^3*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(
3*b^4*g) + (2*B*(b*c - a*d)^3*i^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*
PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b^4*g) + (2*B^2*(b*c - a*d)^3*i^3
*n^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))])/(b^4*g)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
```

```
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\left((bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{g}$$

$$= \frac{\left((bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg} + \frac{\left(d(bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg}$$

$$= \frac{i^3(c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg} + \frac{\left((bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2 g} + \frac{\left(d(bc-ad)^3 i^3\right) \text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2 g} - \frac{\left(2B(bc-ad)^3 i^3 n\right) \text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3bg}$$

$$= \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g} + \frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg} + \frac{((bc-ad)^3i^3)\operatorname{Subst}\left(\int\frac{(A+B\log(ex^n))^2}{x(b-dx)}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3g} + \frac{(d(bc-ad)^3i^3)\operatorname{Subst}\left(\int\frac{(A+B\log(ex^n))^2}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3g} - \frac{(2B(bc-ad)^3i^3n)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^2g} - \frac{(B(bc-ad)^3i^3n)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^2g} - \frac{(2Bd(bc-ad)^3i^3n)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^2g}$$

$$
\begin{aligned}
&= -\frac{B(bc-ad)i^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^2g} \\
&+ \frac{d(bc-ad)^2i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^4g} \\
&+ \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g} \\
&+ \frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg} \\
&- \frac{(bc-ad)^3i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g} \\
&+ \frac{(2B(bc-ad)^3i^3n)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)(A+B\log(ex^n))}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^4g} \\
&- \frac{(2B(bc-ad)^3i^3n)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^3g} \\
&- \frac{(B(bc-ad)^3i^3n)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3g} \\
&- \frac{(2Bd(bc-ad)^3i^3n)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^4g} \\
&- \frac{(2Bd(bc-ad)^3i^3n)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^3g} \\
&- \frac{(Bd(bc-ad)^3i^3n)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3g} \\
&+ \frac{(B^2(bc-ad)^3i^3n^2)\operatorname{Subst}\left(\int\frac{1}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{3b^2g}
\end{aligned}
$$

$$= -\frac{5Bd(bc-ad)^2 i^3 n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^4 g} - \frac{B(bc-ad)i^3 n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^2 g} + \frac{d(bc-ad)^2 i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^4 g} + \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2 g} + \frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg} + \frac{2B(bc-ad)^3 i^3 n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^4 g} + \frac{5B(bc-ad)^3 i^3 n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^4 g} - \frac{(bc-ad)^3 i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4 g} + \frac{2B(bc-ad)^3 i^3 n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4 g} - \frac{(2B^2(bc-ad)^3 i^3 n^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b^4 g} - \frac{(B^2(bc-ad)^3 i^3 n^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4 g} - \frac{(2B^2(bc-ad)^3 i^3 n^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4 g} - \frac{(2B^2(bc-ad)^3 i^3 n^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4 g} + \frac{(B^2(bc-ad)^3 i^3 n^2)\,\text{Subst}\left(\int\left(\frac{1}{b^2 x}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{3b^2 g} + \frac{(2B^2 d(bc-ad)^3 i^3 n^2)\,\text{Subst}\left(\int\frac{1}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{3b^4 g} + \frac{(B^2 d(bc-ad)^3 i^3 n^2)\,\text{Subst}\left(\int\frac{1}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4 g}$$

$$= \frac{B^2d(bc-ad)^2i^3n^2x}{3b^3g} - \frac{5Bd(bc-ad)^2i^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^4g}$$
$$- \frac{B(bc-ad)i^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^2g}$$
$$+ \frac{d(bc-ad)^2i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^4g}$$
$$+ \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g}$$
$$+ \frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg}$$
$$+ \frac{2B(bc-ad)^3i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^4g}$$
$$+ \frac{B^2(bc-ad)^3i^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^4g} + \frac{2B^2(bc-ad)^3i^3n^2\log(c+dx)}{b^4g}$$
$$+ \frac{5B(bc-ad)^3i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^4g}$$
$$- \frac{(bc-ad)^3i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}$$
$$+ \frac{2B^2(bc-ad)^3i^3n^2\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g} - \frac{5B^2(bc-ad)^3i^3n^2\mathrm{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{3b^4g}$$
$$+ \frac{2B(bc-ad)^3i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\mathrm{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}$$
$$+ \frac{2B^2(bc-ad)^3i^3n^2\mathrm{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4969 vs. 2(762) = 1524.

Time = 7.27 (sec) , antiderivative size = 4969, normalized size of antiderivative = 6.52

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g +
 b*g*x),x]
```

```
[Out] (i^3*(36*b*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2 + 18*b^2*d^2*(3*b*c - a*d)*x^
```

2*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2 +
12*b^3*d^3*x^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(
c + d*x)])^2 + 36*(b*c - a*d)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d
*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2 - 108*b^2*B*c^2*n*(A + B*Log[e*((
a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])*(a*d*Log[a/b + x]^2
- 2*a*d*Log[a/b + x]*(1 + Log[a + b*x]) + 2*(-(b*c) + a*d + Log[c/d + x]*(b
*c + a*d*Log[a + b*x] - a*d*Log[(d*(a + b*x))/(-(b*c) + a*d)]) + (-(b*d*x)
+ a*d*Log[a + b*x])*Log[(a + b*x)/(c + d*x)]) - 2*a*d*PolyLog[2, (b*(c + d*
x))/(b*c - a*d)]) - 12*B*n*(-A - B*Log[e*((a + b*x)/(c + d*x))^n] + B*n*Log
[(a + b*x)/(c + d*x)])*(6*a^2*b*c*d^2 - 6*a^3*d^3 + 2*b^3*c^2*d*x + 3*a*b^2
*c*d^2*x - 5*a^2*b*d^3*x - b^3*c*d^2*x^2 + a*b^2*d^3*x^2 - 3*a^3*d^3*Log[a/
b + x]^2 - 6*a^2*b*c*d^2*Log[c/d + x] + 5*a^3*d^3*Log[a + b*x] - 6*a^3*d^3*
Log[c/d + x]*Log[a + b*x] + 6*a^3*d^3*Log[a/b + x]*(1 + Log[a + b*x]) + 6*a
^3*d^3*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 6*a^2*b*d^3*x*Log[(
a + b*x)/(c + d*x)] - 3*a*b^2*d^3*x^2*Log[(a + b*x)/(c + d*x)] + 2*b^3*d^3*
x^3*Log[(a + b*x)/(c + d*x)] - 6*a^3*d^3*Log[a + b*x]*Log[(a + b*x)/(c + d*
x)] - 2*b^3*c^3*Log[c + d*x] - 3*a*b^2*c^2*d*Log[c + d*x] + 6*a^3*d^3*PolyL
og[2, (b*(c + d*x))/(b*c - a*d)]) + 36*b^3*B*c^3*n*(A + B*Log[e*((a + b*x)/
(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])*(Log[a/b + x]^2 - 2*Log[a + b
x](Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)]) - 2*(Log[c/d +
x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d
)])) + 54*b*B*c*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)
/(c + d*x)])*(-4*a*d^2*(a + b*x)*(-1 + Log[a/b + x]) + 2*a^2*d^2*Log[a/b +
x]^2 + 4*a*b*d*(c + d*x)*(-1 + Log[c/d + x]) + d^2*(b*x*(2*a - b*x) + 2*b^2
*x^2*Log[a/b + x] - 2*a^2*Log[a + b*x]) - 2*d^2*(b*x*(-2*a + b*x) + 2*a^2*L
og[a + b*x])*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)]) + b^2
*(d*x*(-2*c + d*x) - 2*d^2*x^2*Log[c/d + x] + 2*c^2*Log[c + d*x]) - 4*a^2*d
^2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x
))/(b*c - a*d)])) - 36*b^2*B^2*c^2*n^2*(a*d*Log[a/b + x]^3 - 3*d*(2*b*x - 2
*(a + b*x)*Log[a/b + x] + (a + b*x)*Log[a/b + x]^2) - 3*b*(2*d*x - 2*(c + d
*x)*Log[c/d + x] + (c + d*x)*Log[c/d + x]^2) - 3*d*(b*x - a*Log[a + b*x])*(
-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])^2 + 6*(a*d + 2*b*d
*x - b*d*x*Log[c/d + x] - b*c*Log[c + d*x] + Log[a/b + x]*(-(d*(a + b*x)) +
d*(a + b*x)*Log[c/d + x] + (b*c - a*d)*Log[(b*(c + d*x))/(b*c - a*d)]) + (
b*c - a*d)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 3*(Log[a/b + x] - Lo
g[c/d + x] - Log[(a + b*x)/(c + d*x)])*(-2*b*c + 2*a*d - 2*d*(a + b*x)*Log[
a/b + x] + a*d*Log[a/b + x]^2 + 2*Log[c/d + x]*(b*(c + d*x) - a*d*Log[(d*(a
+ b*x))/(-(b*c) + a*d)]) - 2*a*d*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) -
3*a*d*(Log[a/b + x]^2*(Log[c/d + x] - Log[(b*(c + d*x))/(b*c - a*d)]) - 2*L
og[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2*PolyLog[3, (d*(a +
b*x))/(-(b*c) + a*d)]) + 3*a*d*(Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) +
a*d)] + 2*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 2*PolyLog[3
, (b*(c + d*x))/(b*c - a*d)])) + 9*b*B^2*c*n^2*(4*a^2*d^2*Log[a/b + x]^3 -
12*a*d^2*(2*b*x - 2*(a + b*x)*Log[a/b + x] + (a + b*x)*Log[a/b + x]^2) - 3*
d^2*(b*x*(6*a - b*x) + (-6*a^2 - 4*a*b*x + 2*b^2*x^2)*Log[a/b + x] + 2*(a^2

```
 - b^2*x^2)*Log[a/b + x]^2) - 12*a*b*d*(2*d*x - 2*(c + d*x)*Log[c/d + x] +
(c + d*x)*Log[c/d + x]^2) - 3*b^2*(d*x*(6*c - d*x) + (-6*c^2 - 4*c*d*x + 2*
d^2*x^2)*Log[c/d + x] + 2*(c^2 - d^2*x^2)*Log[c/d + x]^2) + 6*d^2*(b*x*(-2*
a + b*x) + 2*a^2*Log[a + b*x])*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x
)/(c + d*x)])^2 - 6*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)]
)*(-4*a*d^2*(a + b*x)*(-1 + Log[a/b + x]) + 2*a^2*d^2*Log[a/b + x]^2 + 4*a*
b*d*(c + d*x)*(-1 + Log[c/d + x]) + d^2*(b*x*(2*a - b*x) + 2*b^2*x^2*Log[a/
b + x] - 2*a^2*Log[a + b*x]) + b^2*(d*x*(-2*c + d*x) - 2*d^2*x^2*Log[c/d +
x] + 2*c^2*Log[c + d*x]) - 4*a^2*d^2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c
) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + 6*(2*a*b*c*d + 3*b^2*
c*d*x + 3*a*b*d^2*x - b^2*d^2*x^2 - 2*a*b*d^2*x*Log[c/d + x] + b^2*d^2*x^2*
Log[c/d + x] - a^2*d^2*Log[a + b*x] - b^2*c^2*Log[c + d*x] - 2*a*b*c*d*Log[
c + d*x] - Log[a/b + x]*(b*d*(2*a*c + b*x*(2*c - d*x)) - 2*d^2*(a^2 - b^2*x
^2)*Log[c/d + x] + (-2*b^2*c^2 + 2*a^2*d^2)*Log[(b*(c + d*x))/(b*c - a*d)])
 + 2*(b^2*c^2 - a^2*d^2)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 4*a*d*(
a*d + 2*b*d*x - b*d*x*Log[c/d + x] - b*c*Log[c + d*x] + Log[a/b + x]*(-(d*(
a + b*x)) + d*(a + b*x)*Log[c/d + x] + (b*c - a*d)*Log[(b*(c + d*x))/(b*c -
 a*d)]) + (b*c - a*d)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2*a^2*d^2
*(Log[a/b + x]^2*(Log[c/d + x] - Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Log[a/
b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2*PolyLog[3, (d*(a + b*x)
)/(-(b*c) + a*d)])) + 12*a^2*d^2*(Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c)
+ a*d)] + 2*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 2*PolyLog[
3, (b*(c + d*x))/(b*c - a*d)])) + 12*b^3*B^2*c^3*n^2*(Log[a/b + x]^3 + 3*Lo
g[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 3*Log[a + b*x]*(-Log[a/b +
 x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])^2 + 3*Log[a/b + x]^2*(-Log[c
/d + x] + Log[(b*(c + d*x))/(b*c - a*d)]) + 6*Log[a/b + x]*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)] + 6*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a
*d)] - 3*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)])*(Log[a/b
+ x]^2 - 2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*
(c + d*x))/(b*c - a*d)])) - 6*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*
PolyLog[3, (b*(c + d*x))/(b*c - a*d)]) - 2*B^2*n^2*(12*a*b^2*c^2*d + 18*a^2
*b*c*d^2 + 36*a^3*d^3 - 6*b^3*c^2*d*x + 12*a*b^2*c*d^2*x - 6*a^2*b*d^3*x -
12*a*b^2*c^2*d*Log[a/b + x] + 18*a^2*b*c*d^2*Log[a/b + x] + 13*a^3*d^3*Log[
a/b + x] + 3*a^3*d^3*Log[a/b + x]^2 - 12*a^3*d^3*Log[a/b + x]^3 + 22*b^3*c^
3*Log[c/d + x] + 27*a*b^2*c^2*d*Log[c/d + x] + 36*a^3*d^3*Log[c/d + x] - 36
*a^2*b*c*d^2*Log[a/b + x]*Log[c/d + x] + 30*a^3*d^3*Log[a/b + x]*Log[c/d +
x] - 6*b^3*c^3*Log[c/d + x]^2 - 9*a*b^2*c^2*d*Log[c/d + x]^2 + 18*a^2*b*c*d
^2*Log[c/d + x]^2 - 6*a^2*b*c*d^2*Log[a + b*x] - 13*a^3*d^3*Log[a + b*x] +
30*a^3*d^3*Log[a/b + x]*Log[a + b*x] + 18*a^3*d^3*Log[a/b + x]^2*Log[a + b*
x] - 30*a^3*d^3*Log[c/d + x]*Log[a + b*x] - 36*a^3*d^3*Log[a/b + x]*Log[c/d
 + x]*Log[a + b*x] + 18*a^3*d^3*Log[c/d + x]^2*Log[a + b*x] + 36*a^3*d^3*Lo
g[a/b + x]*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] - 18*a^3*d^3*Log[
c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - 36*a^2*b*c*d^2*Log[(a + b*x)
/(c + d*x)] + 36*a^3*d^3*Log[(a + b*x)/(c + d*x)] - 12*b^3*c^2*d*x*Log[(a +
 b*x)/(c + d*x)] - 18*a*b^2*c*d^2*x*Log[(a + b*x)/(c + d*x)] + 30*a^2*b*d^3
```

```
*x*Log[(a + b*x)/(c + d*x)] + 6*b^3*c*d^2*x^2*Log[(a + b*x)/(c + d*x)] - 6*
a*b^2*d^3*x^2*Log[(a + b*x)/(c + d*x)] - 36*a^3*d^3*Log[a/b + x]*Log[(a + b
*x)/(c + d*x)] + 18*a^3*d^3*Log[a/b + x]^2*Log[(a + b*x)/(c + d*x)] + 36*a^
2*b*c*d^2*Log[c/d + x]*Log[(a + b*x)/(c + d*x)] - 30*a^3*d^3*Log[a + b*x]*L
og[(a + b*x)/(c + d*x)] - 36*a^3*d^3*Log[a/b + x]*Log[a + b*x]*Log[(a + b*x
)/(c + d*x)] + 36*a^3*d^3*Log[c/d + x]*Log[a + b*x]*Log[(a + b*x)/(c + d*x)
] - 36*a^3*d^3*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[(a + b*x)
/(c + d*x)] - 18*a^2*b*d^3*x*Log[(a + b*x)/(c + d*x)]^2 + 9*a*b^2*d^3*x^2*L
og[(a + b*x)/(c + d*x)]^2 - 6*b^3*d^3*x^3*Log[(a + b*x)/(c + d*x)]^2 + 18*a
^3*d^3*Log[a + b*x]*Log[(a + b*x)/(c + d*x)]^2 - 4*b^3*c^3*Log[c + d*x] - 1
5*a*b^2*c^2*d*Log[c + d*x] - 66*a^2*b*c*d^2*Log[c + d*x] - 12*b^3*c^3*Log[a
/b + x]*Log[c + d*x] - 18*a*b^2*c^2*d*Log[a/b + x]*Log[c + d*x] + 12*b^3*c^
3*Log[c/d + x]*Log[c + d*x] + 18*a*b^2*c^2*d*Log[c/d + x]*Log[c + d*x] + 12
*b^3*c^3*Log[(a + b*x)/(c + d*x)]*Log[c + d*x] + 18*a*b^2*c^2*d*Log[(a + b*
x)/(c + d*x)]*Log[c + d*x] + 12*b^3*c^3*Log[a/b + x]*Log[(b*(c + d*x))/(b*c
 - a*d)] + 18*a*b^2*c^2*d*Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*
a^2*b*c*d^2*Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] - 66*a^3*d^3*Log[a/
b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + 18*a^3*d^3*Log[a/b + x]^2*Log[(b*(c
 + d*x))/(b*c - a*d)] + 6*(2*b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 11*a
^3*d^3 + 6*a^3*d^3*Log[a/b + x])*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] +
 36*a^3*d^3*(Log[a/b + x] - Log[(a + b*x)/(c + d*x)])*PolyLog[2, (b*(c + d*
x))/(b*c - a*d)] - 36*a^3*d^3*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 36
*a^3*d^3*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])))/(36*b^4*g)
```

Maple [F]

$$\int \frac{(dix+ci)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{bgx+ag}dx$$

```
[In] int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

```
[Out] int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, a
lgorithm="fricas")
```

```
[Out] integral((A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*c
^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2*c
^3*i^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*d^2
```

```
*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log(e*((b*x + a)/(d*x + c))^n))
/(b*g*x + a*g), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, a
lgorithm="maxima")
[Out] 3*A^2*c^2*d*i^3*(x/(b*g) - a*log(b*x + a)/(b^2*g)) - 1/6*A^2*d^3*i^3*(6*a^3
*log(b*x + a)/(b^4*g) - (2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/(b^3*g)) + 3/2*A^
2*c*d^2*i^3*(2*a^2*log(b*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A^2*c^
3*i^3*log(b*g*x + a*g)/(b*g) + 1/6*(2*B^2*b^3*d^3*i^3*x^3 + 3*(3*b^3*c*d^2*
i^3 - a*b^2*d^3*i^3)*B^2*x^2 + 6*(3*b^3*c^2*d*i^3 - 3*a*b^2*c*d^2*i^3 + a^2
*b*d^3*i^3)*B^2*x + 6*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^2*b*c*d^2*i^3
- a^3*d^3*i^3)*B^2*log(b*x + a))*log((d*x + c)^n)^2/(b^4*g) - integrate(-1/
3*(3*B^2*b^4*c^4*i^3*log(e)^2 + 6*A*B*b^4*c^4*i^3*log(e) + 3*(B^2*b^4*d^4*i
^3*log(e)^2 + 2*A*B*b^4*d^4*i^3*log(e))*x^4 + 12*(B^2*b^4*c*d^3*i^3*log(e)^
2 + 2*A*B*b^4*c*d^3*i^3*log(e))*x^3 + 18*(B^2*b^4*c^2*d^2*i^3*log(e)^2 + 2*
A*B*b^4*c^2*d^2*i^3*log(e))*x^2 + 3*(B^2*b^4*d^4*i^3*x^4 + 4*B^2*b^4*c*d^3*
i^3*x^3 + 6*B^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2*b^4*c^3*d*i^3*x + B^2*b^4*c^4*i
^3)*log((b*x + a)^n)^2 + 12*(B^2*b^4*c^3*d*i^3*log(e)^2 + 2*A*B*b^4*c^3*d*i
^3*log(e))*x + 6*(B^2*b^4*c^4*i^3*log(e) + A*B*b^4*c^4*i^3 + (B^2*b^4*d^4*i
^3*log(e) + A*B*b^4*d^4*i^3)*x^4 + 4*(B^2*b^4*c*d^3*i^3*log(e) + A*B*b^4*c*
d^3*i^3)*x^3 + 6*(B^2*b^4*c^2*d^2*i^3*log(e) + A*B*b^4*c^2*d^2*i^3)*x^2 + 4
*(B^2*b^4*c^3*d*i^3*log(e) + A*B*b^4*c^3*d*i^3)*x)*log((b*x + a)^n) - (6*B^
2*b^4*c^4*i^3*log(e) + 6*A*B*b^4*c^4*i^3 + 2*(3*A*B*b^4*d^4*i^3 + (i^3*n +
3*i^3*log(e))*B^2*b^4*d^4)*x^4 + (24*A*B*b^4*c*d^3*i^3 - (a*b^3*d^4*i^3*n -
 3*(3*i^3*n + 8*i^3*log(e))*b^4*c*d^3)*B^2)*x^3 + 3*(12*A*B*b^4*c^2*d^2*i^3
 - (3*a*b^3*c*d^3*i^3*n - a^2*b^2*d^4*i^3*n - 6*(i^3*n + 2*i^3*log(e))*b^4*
c^2*d^2)*B^2)*x^2 + 6*(4*A*B*b^4*c^3*d*i^3 + (3*a*b^3*c^2*d^2*i^3*n - 3*a^2
```

```
*b^2*c*d^3*i^3*n + a^3*b*d^4*i^3*n + 4*b^4*c^3*d*i^3*log(e))*B^2)*x + 6*((b
^4*c^3*d*i^3*n - 3*a*b^3*c^2*d^2*i^3*n + 3*a^2*b^2*c*d^3*i^3*n - a^3*b*d^4*
i^3*n)*B^2*x + (a*b^3*c^3*d*i^3*n - 3*a^2*b^2*c^2*d^2*i^3*n + 3*a^3*b*c*d^3
*i^3*n - a^4*d^4*i^3*n)*B^2)*log(b*x + a) + 6*(B^2*b^4*d^4*i^3*x^4 + 4*B^2*
b^4*c*d^3*i^3*x^3 + 6*B^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2*b^4*c^3*d*i^3*x + B^2
*b^4*c^4*i^3)*log((b*x + a)^n))*log((d*x + c)^n))/(b^5*d*g*x^2 + a*b^4*c*g
+ (b^5*c*g + a*b^4*d*g)*x), x)
```

Giac [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, a
lgorithm="giac")

[Out] integrate((d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x +
 a*g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\int \frac{(c\,i+d\,i\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

```
[In] int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x
),x)

[Out] int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x
), x)
```

3.183 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 739

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$$
$$= -\frac{2B^2(bc-ad)^2i^3n^2(c+dx)}{b^3g^2(a+bx)} - \frac{Bd^2(bc-ad)i^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^4g^2}$$
$$- \frac{2B(bc-ad)^2i^3n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^2(a+bx)}$$
$$+ \frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^4g^2}$$
$$- \frac{(bc-ad)^2i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g^2(a+bx)} + \frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g^2}$$
$$+ \frac{4Bd(bc-ad)^2i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^4g^2}$$
$$+ \frac{B^2d(bc-ad)^2i^3n^2\log(c+dx)}{b^4g^2}$$
$$+ \frac{Bd(bc-ad)^2i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$- \frac{3d(bc-ad)^2i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$+ \frac{4B^2d(bc-ad)^2i^3n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g^2} - \frac{B^2d(bc-ad)^2i^3n^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$+ \frac{6Bd(bc-ad)^2i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$+ \frac{6B^2d(bc-ad)^2i^3n^2\,\mathrm{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$

```
[Out] -2*B^2*(-a*d+b*c)^2*i^3*n^2*(d*x+c)/b^3/g^2/(b*x+a)-B*d^2*(-a*d+b*c)*i^3*n*
(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/g^2-2*B*(-a*d+b*c)^2*i^3*n*(d*x
+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/g^2/(b*x+a)+2*d^2*(-a*d+b*c)*i^3*(b
*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^4/g^2-(-a*d+b*c)^2*i^3*(d*x+c)*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^3/g^2/(b*x+a)+1/2*d*i^3*(d*x+c)^2*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))^2/b^2/g^2+4*B*d*(-a*d+b*c)^2*i^3*n*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^4/g^2+B^2*d*(-a*d+b*c)^2*i^3*n^
2*ln(d*x+c)/b^4/g^2+B*d*(-a*d+b*c)^2*i^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*
ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^2-3*d*(-a*d+b*c)^2*i^3*(A+B*ln(e*((b*x+a)/(
```

```
d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^2+4*B^2*d*(-a*d+b*c)^2*i^3*n^
2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/g^2-B^2*d*(-a*d+b*c)^2*i^3*n^2*polylog
(2,b*(d*x+c)/d/(b*x+a))/b^4/g^2+6*B*d*(-a*d+b*c)^2*i^3*n*(A+B*ln(e*((b*x+a)
/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^2+6*B^2*d*(-a*d+b*c)^2*i
^3*n^2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^4/g^2
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 739, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.311$, Rules used = {2561, 2395, 2342, 2341, 2356, 2389, 2379, 2438, 2351, 31, 2355, 2354, 2421, 6724}

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$$
$$= \frac{2d^2i^3(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^4g^2} - \frac{Bd^2i^3n(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4g^2} + \frac{6Bdi^3n(bc-ad)^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4g^2} + \frac{4Bdi^3n(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4g^2} - \frac{3di^3(bc-ad)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^4g^2} + \frac{Bdi^3n(bc-ad)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4g^2} - \frac{i^3(c+dx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^3g^2(a+bx)} - \frac{2Bi^3n(c+dx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3g^2(a+bx)} + \frac{di^3(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b^2g^2} + \frac{4B^2di^3n^2(bc-ad)^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g^2} - \frac{B^2di^3n^2(bc-ad)^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} + \frac{6B^2di^3n^2(bc-ad)^2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} + \frac{B^2di^3n^2(bc-ad)^2\log(c+dx)}{b^4g^2} - \frac{2B^2i^3n^2(c+dx)(bc-ad)^2}{b^3g^2(a+bx)}$$

```
[In] Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x
)^2,x]
```

```
[Out] (-2*B^2*(b*c - a*d)^2*i^3*n^2*(c + d*x))/(b^3*g^2*(a + b*x)) - (B*d^2*(b*c
 - a*d)*i^3*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^4*g^2) -
(2*B*(b*c - a*d)^2*i^3*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/
(b^3*g^2*(a + b*x)) + (2*d^2*(b*c - a*d)*i^3*(a + b*x)*(A + B*Log[e*((a + b
*x)/(c + d*x))^n])^2)/(b^4*g^2) - ((b*c - a*d)^2*i^3*(c + d*x)*(A + B*Log[e
*((a + b*x)/(c + d*x))^n])^2)/(b^3*g^2*(a + b*x)) + (d*i^3*(c + d*x)^2*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*b^2*g^2) + (4*B*d*(b*c - a*d)^2*i^
3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/
(b^4*g^2) + (B^2*d*(b*c - a*d)^2*i^3*n^2*Log[c + d*x])/(b^4*g^2) + (B*d*(b*
c - a*d)^2*i^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x
))/(d*(a + b*x))])/(b^4*g^2) - (3*d*(b*c - a*d)^2*i^3*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^4*g^2) + (4*B^
2*d*(b*c - a*d)^2*i^3*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^4*g^2
) - (B^2*d*(b*c - a*d)^2*i^3*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(
b^4*g^2) + (6*B*d*(b*c - a*d)^2*i^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b^4*g^2) + (6*B^2*d*(b*c - a*d)
^2*i^3*n^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))])/(b^4*g^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
```

```
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)^2 i^3)\operatorname{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^2}$$

$$= \frac{((bc-ad)^2 i^3)\operatorname{Subst}\left(\int \left(\frac{(A+B\log(ex^n))^2}{b^3x^2} + \frac{d^2(A+B\log(ex^n))^2}{b^2(b-dx)^3} + \frac{2d^2(A+B\log(ex^n))^2}{b^3(b-dx)^2} + \frac{3d(A+B\log(ex^n))^2}{b^3x(b-dx)}\right) dx}{g^2}$$

$$= \frac{((bc-ad)^2 i^3)\operatorname{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^2} + \frac{(3d(bc-ad)^2 i^3)\operatorname{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^2} + \frac{(2d^2(bc-ad)^2 i^3)\operatorname{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^2} + \frac{(d^2(bc-ad)^2 i^3)\operatorname{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^2}$$

$$= \frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^4g^2} - \frac{(bc-ad)^2i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g^2(a+bx)} + \frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g^2} - \frac{3d(bc-ad)^2i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} + \frac{(2B(bc-ad)^2i^3n)\,\mathrm{Subst}\left(\int\frac{A+B\log(ex^n)}{x^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3g^2} + \frac{(6Bd(bc-ad)^2i^3n)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)(A+B\log(ex^n))}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^4g^2} - \frac{(Bd(bc-ad)^2i^3n)\,\mathrm{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^2g^2} - \frac{(4Bd^2(bc-ad)^2i^3n)\,\mathrm{Subst}\left(\int\frac{A+B\log(ex^n)}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^4g^2}$$

$$= -\frac{2B^2(bc-ad)^2 i^3 n^2 (c+dx)}{b^3 g^2 (a+bx)} - \frac{2B(bc-ad)^2 i^3 n (c+dx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3 g^2 (a+bx)}$$
$$+ \frac{2d^2(bc-ad) i^3 (a+bx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^4 g^2}$$
$$- \frac{(bc-ad)^2 i^3 (c+dx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3 g^2 (a+bx)}$$
$$+ \frac{d i^3 (c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2 g^2}$$
$$+ \frac{4Bd(bc-ad)^2 i^3 n \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^4 g^2}$$
$$- \frac{3d(bc-ad)^2 i^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4 g^2}$$
$$+ \frac{6Bd(bc-ad)^2 i^3 n \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4 g^2}$$
$$- \frac{(Bd(bc-ad)^2 i^3 n) \operatorname{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3 g^2}$$
$$- \frac{(Bd^2(bc-ad)^2 i^3 n) \operatorname{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3 g^2}$$
$$- \frac{(4B^2 d(bc-ad)^2 i^3 n^2) \operatorname{Subst}\left(\int \frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4 g^2}$$
$$- \frac{(6B^2 d(bc-ad)^2 i^3 n^2) \operatorname{Subst}\left(\int \frac{\operatorname{Li}_2\left(\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4 g^2}$$

$$= -\frac{2B^2(bc-ad)^2i^3n^2(c+dx)}{b^3g^2(a+bx)} - \frac{Bd^2(bc-ad)i^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^4g^2}$$
$$- \frac{2B(bc-ad)^2i^3n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^2(a+bx)}$$
$$+ \frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^4g^2}$$
$$- \frac{(bc-ad)^2i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g^2(a+bx)}$$
$$+ \frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g^2}$$
$$+ \frac{4Bd(bc-ad)^2i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^4g^2}$$
$$+ \frac{Bd(bc-ad)^2i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$- \frac{3d(bc-ad)^2i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$+ \frac{4B^2d(bc-ad)^2i^3n^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g^2}$$
$$+ \frac{6Bd(bc-ad)^2i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$+ \frac{6B^2d(bc-ad)^2i^3n^2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$- \frac{(B^2d(bc-ad)^2i^3n^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4g^2}$$
$$+ \frac{(B^2d^2(bc-ad)^2i^3n^2)\,\text{Subst}\left(\int\frac{1}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4g^2}$$

$$= -\frac{2B^2(bc-ad)^2i^3n^2(c+dx)}{b^3g^2(a+bx)} - \frac{Bd^2(bc-ad)i^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^4g^2}$$
$$- \frac{2B(bc-ad)^2i^3n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^2(a+bx)}$$
$$+ \frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^4g^2}$$
$$- \frac{(bc-ad)^2i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g^2(a+bx)}$$
$$+ \frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g^2}$$
$$+ \frac{4Bd(bc-ad)^2i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^4g^2}$$
$$+ \frac{B^2d(bc-ad)^2i^3n^2\log(c+dx)}{b^4g^2}$$
$$+ \frac{Bd(bc-ad)^2i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$- \frac{3d(bc-ad)^2i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$+ \frac{4B^2d(bc-ad)^2i^3n^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g^2} - \frac{B^2d(bc-ad)^2i^3n^2\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$+ \frac{6Bd(bc-ad)^2i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$+ \frac{6B^2d(bc-ad)^2i^3n^2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4818 vs. 2(739) = 1478.

Time = 6.33 (sec) , antiderivative size = 4818, normalized size of antiderivative = 6.52

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g +
 b*g*x)^2,x]
```

```
[Out] (i^3*(4*b*d^2*(3*b*c - 2*a*d)*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n
*Log[(a + b*x)/(c + d*x)])^2 + 2*b^2*d^3*x^2*(A + B*Log[e*((a + b*x)/(c + d
```

*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2 - (4*(b*c - a*d)^3*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2)/(a + b*x) + 12*d
*(b*c - a*d)^2*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log
[(a + b*x)/(c + d*x)])^2 + (8*b^3*B*c^3*n*(-A - B*Log[e*((a + b*x)/(c + d*x
))^n] + B*n*Log[(a + b*x)/(c + d*x)])*(-(d*(a + b*x)*Log[c/d + x]) + d*(a +
 b*x)*Log[(d*(a + b*x))/(-(b*c) + a*d)] + (b*c - a*d)*(1 + Log[(a + b*x)/(c
 + d*x)])))/((b*c - a*d)*(a + b*x)) + (4*b^3*B^2*c^3*n^2*(-2*b*c + 2*a*d -
2*d*(a + b*x)*Log[a + b*x] - 2*(b*c - a*d)*Log[(a + b*x)/(c + d*x)] - 2*d*(
a + b*x)*Log[a + b*x]*Log[(a + b*x)/(c + d*x)] - (b*c - a*d)*Log[(a + b*x)/
(c + d*x)]^2 + 2*d*(a + b*x)*Log[c + d*x] - 2*d*(a + b*x)*Log[(a + b*x)/(c
+ d*x)]*Log[(b*c - a*d)/(b*c + b*d*x)] + d*(a + b*x)*(Log[a + b*x]*(Log[a +
 b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b
*c) + a*d)]) + d*(a + b*x)*(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*Log[(d*(a + b
*x))/(-(b*c) + a*d)] + Log[(b*c - a*d)/(b*c + b*d*x)]) - 2*PolyLog[2, (b*(c
 + d*x))/(b*c - a*d)])))/((b*c - a*d)*(a + b*x)) + 12*b^2*B*c^2*d*n*(A + B*
Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])*(Log[a/b + x
]^2 - 2*Log[a/b + x]*Log[a + b*x] - 2*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c
) + a*d)] + 2*Log[a + b*x]*((a*d)/(b*c - a*d) + Log[c/d + x] + Log[(a + b*x
)/(c + d*x)]) + 2*a*((a + b*x)^(-1) + Log[(a + b*x)/(c + d*x)]/(a + b*x) +
(d*Log[c + d*x])/(-(b*c) + a*d)) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
 + 4*B*d^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c +
 d*x)])*(4*a^2 - (4*a*b*c)/d + a*b*x - (b^2*c*x)/d + (2*a^3)/(a + b*x) + 3*
a^2*Log[a/b + x]^2 + (4*a*b*c*Log[c/d + x])/d - a^2*Log[a + b*x] + (2*a^3*d
*Log[a + b*x])/(b*c - a*d) + 6*a^2*Log[c/d + x]*Log[a + b*x] - 2*a^2*Log[a/
b + x]*(2 + 3*Log[a + b*x]) - 6*a^2*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c)
+ a*d)] - 4*a*b*x*Log[(a + b*x)/(c + d*x)] + b^2*x^2*Log[(a + b*x)/(c + d*x
)] + (2*a^3*Log[(a + b*x)/(c + d*x)])/(a + b*x) + 6*a^2*Log[a + b*x]*Log[(a
 + b*x)/(c + d*x)] + (b^2*c^2*Log[c + d*x])/d^2 + (2*a^3*d*Log[c + d*x])/(-
(b*c) + a*d) - 6*a^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 24*b*B*c*d^2*
n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])*((a
 + b*x)*(-1 + Log[a/b + x]) - a*Log[a/b + x]^2 - (a^2*(1 + Log[a/b + x]))/(
a + b*x) - b*(c/d + x)*(-1 + Log[c/d + x]) + (a^2*Log[c/d + x])/(a + b*x) +
 (b*x - a^2/(a + b*x) - 2*a*Log[a + b*x])*(-Log[a/b + x] + Log[c/d + x] + L
og[(a + b*x)/(c + d*x)]) + (a^2*d*(Log[a + b*x] - Log[c + d*x]))/(-(b*c) +
a*d) + 2*a*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*
(c + d*x))/(b*c - a*d)])) + 4*b*B^2*c*d^2*n^2*(6*b*x - 6*(a + b*x)*Log[a/b
+ x] + 3*(a + b*x)*Log[a/b + x]^2 - 2*a*Log[a/b + x]^3 - (3*a^2*(2 + 2*Log[
a/b + x] + Log[a/b + x]^2))/(a + b*x) + (3*b*(2*d*x - 2*(c + d*x)*Log[c/d +
 x] + (c + d*x)*Log[c/d + x]^2))/d + 3*(b*x - a^2/(a + b*x) - 2*a*Log[a + b
x])(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])^2 - (6*(a*d
+ 2*b*d*x - b*d*x*Log[c/d + x] - b*c*Log[c + d*x] + Log[a/b + x]*(-(d*(a +
b*x)) + d*(a + b*x)*Log[c/d + x] + (b*c - a*d)*Log[(b*(c + d*x))/(b*c - a*d
)]) + (b*c - a*d)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/d + (3*a^2*(d*
(a + b*x)*Log[a/b + x]^2 + 2*((-(b*c) + a*d)*Log[c/d + x] + d*(a + b*x)*(Lo
g[a + b*x] - Log[c + d*x])) - 2*Log[a/b + x]*((b*c - a*d)*Log[c/d + x] + d*

(a + b*x)*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*d*(a + b*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/((-(b*c) + a*d)*(a + b*x)) + (3*a^2*(-(b*(c + d*x)*Log[c/d + x]^2) + 2*d*(a + b*x)*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*d*(a + b*x)*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)*(a + b*x)) + 6*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])*((a + b*x)*(-1 + Log[a/b + x]) - a*Log[a/b + x]^2 - (a^2*(1 + Log[a/b + x]))/(a + b*x) - b*(c/d + x)*(-1 + Log[c/d + x]) + (a^2*Log[c/d + x])/(a + b*x) + (a^2*d*(Log[a + b*x] - Log[c + d*x]))/(-(b*c) + a*d) + 2*a*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + 6*a*(Log[a/b + x]^2*(Log[c/d + x] - Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)]) - 6*a*(Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])) + B^2*d^3*n^2*(b*x*(-6*a + b*x) + (6*a^2 + 4*a*b*x - 2*b^2*x^2)*Log[a/b + x] - 2*(a^2 - b^2*x^2)*Log[a/b + x]^2 + 4*a^2*Log[a/b + x]^3 + (4*a^3*(2 + 2*Log[a/b + x] + Log[a/b + x]^2))/(a + b*x) - 8*a*(2*b*x - 2*(a + b*x)*Log[a/b + x] + (a + b*x)*Log[a/b + x]^2) - (8*a*b*(2*d*x - 2*(c + d*x)*Log[c/d + x] + (c + d*x)*Log[c/d + x]^2))/d + (b^2*(d*x*(-6*c + d*x) + (6*c^2 + 4*c*d*x - 2*d^2*x^2)*Log[c/d + x] - 2*(c^2 - d^2*x^2)*Log[c/d + x]^2))/d^2 + 2*(-4*a*b*x + b^2*x^2 + (2*a^3)/(a + b*x) + 6*a^2*Log[a + b*x])*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])^2 + 4*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])*(4*a^2 - (4*a*b*c)/d + a*b*x - (b^2*c*x)/d + (2*a^3)/(a + b*x) + (-4*a^2 - 4*a*b*x + b^2*x^2 + (2*a^3)/(a + b*x))*Log[a/b + x] + 3*a^2*Log[a/b + x]^2 + (4*a*b*c*Log[c/d + x])/d + 4*a*b*x*Log[c/d + x] - b^2*x^2*Log[c/d + x] - (2*a^3*Log[c/d + x])/(a + b*x) - a^2*Log[a + b*x] + (2*a^3*d*Log[a + b*x])/(b*c - a*d) - 6*a^2*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + (b^2*c^2*Log[c + d*x])/d^2 + (2*a^3*d*Log[c + d*x])/(-(b*c) + a*d) - 6*a^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + (4*a^3*(-(b*(c + d*x)*Log[c/d + x]^2) + 2*d*(a + b*x)*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*d*(a + b*x)*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((-(b*c) + a*d)*(a + b*x)) + 2*((8*a*(a*d + 2*b*d*x - b*d*x*Log[c/d + x] - b*c*Log[c + d*x] + Log[a/b + x]*(-(d*(a + b*x)) + d*(a + b*x)*Log[c/d + x] + (b*c - a*d)*Log[(b*(c + d*x))/(b*c - a*d)]) + (b*c - a*d)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/d - (-2*a*b*c*d - 3*b^2*c*d*x - 3*a*b*d^2*x + b^2*d^2*x^2 + 2*a*b*d^2*x*Log[c/d + x] - b^2*d^2*x^2*Log[c/d + x] + a^2*d^2*Log[a + b*x] + b^2*c^2*Log[c + d*x] + 2*a*b*c*d*Log[c + d*x] + Log[a/b + x]*(b*d*(2*a*c + b*x*(2*c - d*x)) - 2*d^2*(a^2 - b^2*x^2)*Log[c/d + x] + (-2*b^2*c^2 + 2*a^2*d^2)*Log[(b*(c + d*x))/(b*c - a*d)]) + (-2*b^2*c^2 + 2*a^2*d^2)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/d^2 + (2*a^3*(d*(a + b*x)*Log[a/b + x]^2 + 2*((-(b*c) + a*d)*Log[c/d + x] + d*(a + b*x)*(Log[a + b*x] - Log[c + d*x])) - 2*Log[a/b + x]*((b*c - a*d)*Log[c/d + x] + d*(a + b*x)*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*d*(a + b*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/((b*c - a*d)*(a + b*x)) - 6*a^2*(Log[a/b + x]^2*(Log[c/d + x] - Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2*PolyLog[3, (d*(a + b*x

```
))/(-(b*c) + a*d)])) + 12*a^2*(Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) + a
*d)] + 2*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 2*PolyLog[3,
(b*(c + d*x))/(b*c - a*d)])) + (4*b^2*B^2*c^2*d*n^2*((b*c - a*d)*(a + b*x)*
Log[a/b + x]^3 + 3*a*(b*c - a*d)*(2 + 2*Log[a/b + x] + Log[a/b + x]^2) + 3*
(b*c - a*d)*(a + (a + b*x)*Log[a + b*x])*(-Log[a/b + x] + Log[c/d + x] + Lo
g[(a + b*x)/(c + d*x)])^2 + 3*a*(d*(a + b*x)*Log[a/b + x]^2 + 2*((-(b*c) +
a*d)*Log[c/d + x] + d*(a + b*x)*(Log[a + b*x] - Log[c + d*x])) - 2*Log[a/b
+ x]*((b*c - a*d)*Log[c/d + x] + d*(a + b*x)*Log[(b*(c + d*x))/(b*c - a*d)]
) - 2*d*(a + b*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 3*a*(Log[c/d
+ x]*(b*(c + d*x)*Log[c/d + x] - 2*d*(a + b*x)*Log[(d*(a + b*x))/(-(b*c) +
a*d)]) - 2*d*(a + b*x)*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 3*(Log[a/b
+ x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)])*((b*c - a*d)*(a + b*x)*Log[
a/b + x]^2 + 2*a*(b*c - a*d)*(1 + Log[a/b + x]) + 2*a*(-(b*c) + a*d)*Log[c/
d + x] + 2*a*d*(a + b*x)*(Log[a + b*x] - Log[c + d*x]) - 2*(b*c - a*d)*(a +
 b*x)*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c +
d*x))/(b*c - a*d)])) - 3*(b*c - a*d)*(a + b*x)*(Log[a/b + x]^2*(Log[c/d + x
] - Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Log[a/b + x]*PolyLog[2, (d*(a + b*x
))/(-(b*c) + a*d)] + 2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)]) + 3*(b*c -
 a*d)*(a + b*x)*(Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[c
/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 2*PolyLog[3, (b*(c + d*x))/
(b*c - a*d)])))/((b*c - a*d)*(a + b*x))))/(4*b^4*g^2)
```

Maple [F]

$$\int \frac{(dix+ci)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(bgx+ag)^2}dx$$

```
[In] int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

```
[Out] int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x,
 algorithm="fricas")
```

```
[Out] integral((A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*c
^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2*c
^3*i^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*d^2
*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log(e*((b*x + a)/(d*x + c))^n))
/(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**2
,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x,
 algorithm="maxima")
```

```
[Out] -2*A*B*c^3*i^3*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)
*g^2) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - 3*A^2*(a^2/(b^4*g^2*x + a*b
^3*g^2) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*g^2))*c*d^2*i^3 + 1/2*(2*a^3/
(b^5*g^2*x + a*b^4*g^2) + 6*a^2*log(b*x + a)/(b^4*g^2) + (b*x^2 - 4*a*x)/(b
^3*g^2))*A^2*d^3*i^3 + 3*A^2*c^2*d*i^3*(a/(b^3*g^2*x + a*b^2*g^2) + log(b*x
 + a)/(b^2*g^2)) - 2*A*B*c^3*i^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^
2*g^2*x + a*b*g^2) - A^2*c^3*i^3/(b^2*g^2*x + a*b*g^2) + 1/2*(B^2*b^3*d^3*i
^3*x^3 + 3*(2*b^3*c*d^2*i^3 - a*b^2*d^3*i^3)*B^2*x^2 + 2*(3*a*b^2*c*d^2*i^3
 - 2*a^2*b*d^3*i^3)*B^2*x - 2*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^2*b*c*
d^2*i^3 - a^3*d^3*i^3)*B^2 + 6*((b^3*c^2*d*i^3 - 2*a*b^2*c*d^2*i^3 + a^2*b*
d^3*i^3)*B^2*x + (a*b^2*c^2*d*i^3 - 2*a^2*b*c*d^2*i^3 + a^3*d^3*i^3)*B^2)*l
og(b*x + a))*log((d*x + c)^n)^2/(b^5*g^2*x + a*b^4*g^2) - integrate(-(B^2*b
^4*c^4*i^3*log(e)^2 + (B^2*b^4*d^4*i^3*log(e)^2 + 2*A*B*b^4*d^4*i^3*log(e))
*x^4 + 4*(B^2*b^4*c*d^3*i^3*log(e)^2 + 2*A*B*b^4*c*d^3*i^3*log(e))*x^3 + 6*
(B^2*b^4*c^2*d^2*i^3*log(e)^2 + 2*A*B*b^4*c^2*d^2*i^3*log(e))*x^2 + (B^2*b^
4*d^4*i^3*x^4 + 4*B^2*b^4*c*d^3*i^3*x^3 + 6*B^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2
*b^4*c^3*d*i^3*x + B^2*b^4*c^4*i^3)*log((b*x + a)^n)^2 + 2*(2*B^2*b^4*c^3*d
*i^3*log(e)^2 + 3*A*B*b^4*c^3*d*i^3*log(e))*x + 2*(B^2*b^4*c^4*i^3*log(e) +
 (B^2*b^4*d^4*i^3*log(e) + A*B*b^4*d^4*i^3)*x^4 + 4*(B^2*b^4*c*d^3*i^3*log(
e) + A*B*b^4*c*d^3*i^3)*x^3 + 6*(B^2*b^4*c^2*d^2*i^3*log(e) + A*B*b^4*c^2*d
^2*i^3)*x^2 + (4*B^2*b^4*c^3*d*i^3*log(e) + 3*A*B*b^4*c^3*d*i^3)*x)*log((b*
x + a)^n) - ((2*A*B*b^4*d^4*i^3 + (i^3*n + 2*i^3*log(e))*B^2*b^4*d^4)*x^4 +
 2*(4*A*B*b^4*c*d^3*i^3 - (a*b^3*d^4*i^3*n - (3*i^3*n + 4*i^3*log(e))*b^4*c
*d^3)*B^2)*x^3 - 2*(a*b^3*c^3*d*i^3*n - 3*a^2*b^2*c^2*d^2*i^3*n + 3*a^3*b*c
```

```
*d^3*i^3*n - a^4*d^4*i^3*n - b^4*c^4*i^3*log(e))*B^2 + (12*A*B*b^4*c^2*d^2*
i^3 + (12*a*b^3*c*d^3*i^3*n - 7*a^2*b^2*d^4*i^3*n + 12*b^4*c^2*d^2*i^3*log(
e))*B^2)*x^2 + 2*(3*A*B*b^4*c^3*d*i^3 + (3*a*b^3*c^2*d^2*i^3*n - a^3*b*d^4*
i^3*n - (i^3*n - 4*i^3*log(e))*b^4*c^3*d)*B^2)*x + 6*((b^4*c^2*d^2*i^3*n -
2*a*b^3*c*d^3*i^3*n + a^2*b^2*d^4*i^3*n)*B^2*x^2 + 2*(a*b^3*c^2*d^2*i^3*n -
 2*a^2*b^2*c*d^3*i^3*n + a^3*b*d^4*i^3*n)*B^2*x + (a^2*b^2*c^2*d^2*i^3*n -
2*a^3*b*c*d^3*i^3*n + a^4*d^4*i^3*n)*B^2)*log(b*x + a) + 2*(B^2*b^4*d^4*i^3
*x^4 + 4*B^2*b^4*c*d^3*i^3*x^3 + 6*B^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2*b^4*c^3*
d*i^3*x + B^2*b^4*c^4*i^3)*log((b*x + a)^n))*log((d*x + c)^n))/(b^6*d*g^2*x
^3 + a^2*b^4*c*g^2 + (b^6*c*g^2 + 2*a*b^5*d*g^2)*x^2 + (2*a*b^5*c*g^2 + a^2
*b^4*d*g^2)*x), x)
```

Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x,
 algorithm="giac")
```

```
[Out] integrate((d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x +
 a*g)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(a\,g+b\,g\,x)^2}\,dx$$

```
[In] int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x
)^2,x)
```

```
[Out] int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x
)^2, x)
```

3.184 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 644

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= -\frac{4B^2d(bc-ad)i^3n^2(c+dx)}{b^3g^3(a+bx)} - \frac{B^2(bc-ad)i^3n^2(c+dx)^2}{4b^2g^3(a+bx)^2}$$
$$- \frac{4Bd(bc-ad)i^3n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^3(a+bx)}$$
$$- \frac{B(bc-ad)i^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^3(a+bx)^2}$$
$$+ \frac{d^3i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^4g^3}$$
$$- \frac{2d(bc-ad)i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g^3(a+bx)}$$
$$- \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g^3(a+bx)^2}$$
$$+ \frac{2Bd^2(bc-ad)i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^4g^3}$$
$$- \frac{3d^2(bc-ad)i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}$$
$$+ \frac{2B^2d^2(bc-ad)i^3n^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g^3}$$
$$+ \frac{6Bd^2(bc-ad)i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}$$
$$+ \frac{6B^2d^2(bc-ad)i^3n^2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}$$

```
[Out] -4*B^2*d*(-a*d+b*c)*i^3*n^2*(d*x+c)/b^3/g^3/(b*x+a)-1/4*B^2*(-a*d+b*c)*i^3*
n^2*(d*x+c)^2/b^2/g^3/(b*x+a)^2-4*B*d*(-a*d+b*c)*i^3*n*(d*x+c)*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))/b^3/g^3/(b*x+a)-1/2*B*(-a*d+b*c)*i^3*n*(d*x+c)^2*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))/b^2/g^3/(b*x+a)^2+d^3*i^3*(b*x+a)*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))^2/b^4/g^3-2*d*(-a*d+b*c)*i^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d
*x+c))^n))^2/b^3/g^3/(b*x+a)-1/2*(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*((b*x+a
)/(d*x+c))^n))^2/b^2/g^3/(b*x+a)^2+2*B*d^2*(-a*d+b*c)*i^3*n*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^4/g^3-3*d^2*(-a*d+b*c)*i^3*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^3+2*B^2*d^2*(
-a*d+b*c)*i^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/g^3+6*B*d^2*(-a*d+b*c)
```

```
*i^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g
^3+6*B^2*d^2*(-a*d+b*c)*i^3*n^2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^4/g^3
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2561, 2395, 2342, 2341, 2355, 2354, 2438, 2379, 2421, 6724}

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}dx$$
$$=\frac{d^3i^3(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^4g^3}$$
$$+\frac{6Bd^2i^3n(bc-ad)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4g^3}$$
$$+\frac{2Bd^2i^3n(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4g^3}$$
$$-\frac{3d^2i^3(bc-ad)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^4g^3}$$
$$-\frac{2di^3(c+dx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^3g^3(a+bx)}$$
$$-\frac{4Bdi^3n(c+dx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3g^3(a+bx)}$$
$$-\frac{i^3(c+dx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b^2g^3(a+bx)^2}$$
$$-\frac{Bi^3n(c+dx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b^2g^3(a+bx)^2}$$
$$+\frac{2B^2d^2i^3n^2(bc-ad)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g^3}+\frac{6B^2d^2i^3n^2(bc-ad)\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}$$
$$-\frac{4B^2di^3n^2(c+dx)(bc-ad)}{b^3g^3(a+bx)}-\frac{B^2i^3n^2(c+dx)^2(bc-ad)}{4b^2g^3(a+bx)^2}$$

```
[In] Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x
)^3,x]
```

```
[Out] (-4*B^2*d*(b*c - a*d)*i^3*n^2*(c + d*x))/(b^3*g^3*(a + b*x)) - (B^2*(b*c -
a*d)*i^3*n^2*(c + d*x)^2)/(4*b^2*g^3*(a + b*x)^2) - (4*B*d*(b*c - a*d)*i^3*
n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^3*g^3*(a + b*x)) - (
B*(b*c - a*d)*i^3*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*
```

```
b^2*g^3*(a + b*x)^2) + (d^3*i^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^2)/(b^4*g^3) - (2*d*(b*c - a*d)*i^3*(c + d*x)*(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^2)/(b^3*g^3*(a + b*x)) - ((b*c - a*d)*i^3*(c + d*x)^2*(A +
B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*b^2*g^3*(a + b*x)^2) + (2*B*d^2*(b*
c - a*d)*i^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c
 + d*x))])/(b^4*g^3) - (3*d^2*(b*c - a*d)*i^3*(A + B*Log[e*((a + b*x)/(c +
d*x))^n])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^4*g^3) + (2*B^2*d^2*(b
*c - a*d)*i^3*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^4*g^3) + (6*B
*d^2*(b*c - a*d)*i^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (b
*(c + d*x))/(d*(a + b*x))])/(b^4*g^3) + (6*B^2*d^2*(b*c - a*d)*i^3*n^2*Poly
Log[3, (b*(c + d*x))/(d*(a + b*x))])/(b^4*g^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)i^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^3(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^3}$$

$$= \frac{((bc-ad)i^3)\,\text{Subst}\left(\int \left(\frac{(A+B\log(ex^n))^2}{b^2x^3} + \frac{2d(A+B\log(ex^n))^2}{b^3x^2} + \frac{d^3(A+B\log(ex^n))^2}{b^3(b-dx)^2} + \frac{3d^2(A+B\log(ex^n))^2}{b^3x(b-dx)}\right)\,dx,\right.}{g^3}$$

$$= \frac{((bc-ad)i^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^3} + \frac{(2d(bc-ad)i^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^3} + \frac{(3d^2(bc-ad)i^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^3} + \frac{(d^3(bc-ad)i^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^3}$$

$$= \frac{d^3i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^4g^3} - \frac{2d(bc-ad)i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g^3(a+bx)} - \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g^3(a+bx)^2} - \frac{3d^2(bc-ad)i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} + \frac{(B(bc-ad)i^3n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^3} + \frac{(4Bd(bc-ad)i^3n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^3} + \frac{(6Bd^2(bc-ad)i^3n)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)(A+B\log(ex^n))}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4g^3} - \frac{(2Bd^3(bc-ad)i^3n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4g^3}$$

$$= -\frac{4B^2d(bc-ad)i^3n^2(c+dx)}{b^3g^3(a+bx)} - \frac{B^2(bc-ad)i^3n^2(c+dx)^2}{4b^2g^3(a+bx)^2}$$
$$- \frac{4Bd(bc-ad)i^3n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^3(a+bx)}$$
$$- \frac{B(bc-ad)i^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^3(a+bx)^2}$$
$$+ \frac{d^3i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^4g^3}$$
$$- \frac{2d(bc-ad)i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g^3(a+bx)}$$
$$- \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g^3(a+bx)^2}$$
$$+ \frac{2Bd^2(bc-ad)i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^4g^3}$$
$$- \frac{3d^2(bc-ad)i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}$$
$$+ \frac{6Bd^2(bc-ad)i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}$$
$$- \frac{(2B^2d^2(bc-ad)i^3n^2)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4g^3}$$
$$- \frac{(6B^2d^2(bc-ad)i^3n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^4g^3}$$

$$
\begin{aligned}
&= -\frac{4B^2d(bc-ad)i^3n^2(c+dx)}{b^3g^3(a+bx)} - \frac{B^2(bc-ad)i^3n^2(c+dx)^2}{4b^2g^3(a+bx)^2} \\
&- \frac{4Bd(bc-ad)i^3n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^3(a+bx)} \\
&- \frac{B(bc-ad)i^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^3(a+bx)^2} \\
&+ \frac{d^3i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^4g^3} \\
&- \frac{2d(bc-ad)i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g^3(a+bx)} \\
&- \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g^3(a+bx)^2} \\
&+ \frac{2Bd^2(bc-ad)i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^4g^3} \\
&- \frac{3d^2(bc-ad)i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} \\
&+ \frac{2B^2d^2(bc-ad)i^3n^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g^3} \\
&+ \frac{6Bd^2(bc-ad)i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} \\
&+ \frac{6B^2d^2(bc-ad)i^3n^2\text{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6938 vs. $2(644) = 1288$.

Time = 6.86 (sec) , antiderivative size = 6938, normalized size of antiderivative = 10.77

$$
\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g +
 b*g*x)^3,x]
```

```
[Out] Result too large to show
```

Maple [F]

$$\int \frac{(dix+ci)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(bgx+ag)^3}dx$$

```
[In] int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x)
```

```
[Out] int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x,
 algorithm="fricas")
```

```
[Out] integral((A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*c
^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2*c
^3*i^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*d^2
*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log(e*((b*x + a)/(d*x + c))^n))
/(b^3*g^3*x^3 + 3*a*b^2*g^3*x^2 + 3*a^2*b*g^3*x + a^3*g^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**3
,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x,
 algorithm="maxima")
```

```
[Out] -3/2*A*B*c^2*d*i^3*n*((3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*x)/((b^5*c - a
*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g
^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^
2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^
2*d^2)*g^3)) + 1/2*A*B*c^3*i^3*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d
)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*
d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x
+ c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A^2*d^3*i^3*((6*a^2*b
*x + 5*a^3)/(b^6*g^3*x^2 + 2*a*b^5*g^3*x + a^2*b^4*g^3) - 2*x/(b^3*g^3) + 6
*a*log(b*x + a)/(b^4*g^3)) + 3/2*A^2*c*d^2*i^3*((4*a*b*x + 3*a^2)/(b^5*g^3*
x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(b^3*g^3)) - 3*(2*b*x +
 a)*A*B*c^2*d*i^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^4*g^3*x^2 + 2*a
*b^3*g^3*x + a^2*b^2*g^3) - 3/2*(2*b*x + a)*A^2*c^2*d*i^3/(b^4*g^3*x^2 + 2*
a*b^3*g^3*x + a^2*b^2*g^3) - A*B*c^3*i^3*log(e*(b*x/(d*x + c) + a/(d*x + c)
)^n)/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2*c^3*i^3/(b^3*g^3*x
^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 1/2*(2*B^2*b^3*d^3*i^3*x^3 + 4*B^2*a*b^2*
d^3*i^3*x^2 - 2*(3*b^3*c^2*d*i^3 - 6*a*b^2*c*d^2*i^3 + 2*a^2*b*d^3*i^3)*B^2
*x - (b^3*c^3*i^3 + 3*a*b^2*c^2*d*i^3 - 9*a^2*b*c*d^2*i^3 + 5*a^3*d^3*i^3)*
B^2 + 6*((b^3*c*d^2*i^3 - a*b^2*d^3*i^3)*B^2*x^2 + 2*(a*b^2*c*d^2*i^3 - a^2
*b*d^3*i^3)*B^2*x + (a^2*b*c*d^2*i^3 - a^3*d^3*i^3)*B^2)*log(b*x + a))*log(
(d*x + c)^n)^2/(b^6*g^3*x^2 + 2*a*b^5*g^3*x + a^2*b^4*g^3) - integrate(-(4*
B^2*b^4*c^3*d*i^3*x*log(e)^2 + B^2*b^4*c^4*i^3*log(e)^2 + (B^2*b^4*d^4*i^3*
log(e)^2 + 2*A*B*b^4*d^4*i^3*log(e))*x^4 + 4*(B^2*b^4*c*d^3*i^3*log(e)^2 +
2*A*B*b^4*c*d^3*i^3*log(e))*x^3 + 6*(B^2*b^4*c^2*d^2*i^3*log(e)^2 + A*B*b^4
*c^2*d^2*i^3*log(e))*x^2 + (B^2*b^4*d^4*i^3*x^4 + 4*B^2*b^4*c*d^3*i^3*x^3 +
 6*B^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2*b^4*c^3*d*i^3*x + B^2*b^4*c^4*i^3)*log((
b*x + a)^n)^2 + 2*(4*B^2*b^4*c^3*d*i^3*x*log(e) + B^2*b^4*c^4*i^3*log(e) +
(B^2*b^4*d^4*i^3*log(e) + A*B*b^4*d^4*i^3)*x^4 + 4*(B^2*b^4*c*d^3*i^3*log(e
) + A*B*b^4*c*d^3*i^3)*x^3 + 3*(2*B^2*b^4*c^2*d^2*i^3*log(e) + A*B*b^4*c^2*
d^2*i^3)*x^2)*log((b*x + a)^n) - (2*(A*B*b^4*d^4*i^3 + (i^3*n + i^3*log(e))
*B^2*b^4*d^4)*x^4 - (9*a*b^3*c^2*d^2*i^3*n - 21*a^2*b^2*c*d^3*i^3*n + 9*a^3
*b*d^4*i^3*n + (i^3*n - 8*i^3*log(e))*b^4*c^3*d)*B^2*x + 2*(4*A*B*b^4*c*d^3
*i^3 + (3*a*b^3*d^4*i^3*n + 4*b^4*c*d^3*i^3*log(e))*B^2)*x^3 - (a*b^3*c^3*d
*i^3*n + 3*a^2*b^2*c^2*d^2*i^3*n - 9*a^3*b*c*d^3*i^3*n + 5*a^4*d^4*i^3*n -
2*b^4*c^4*i^3*log(e))*B^2 + 6*(A*B*b^4*c^2*d^2*i^3 + (2*a*b^3*c*d^3*i^3*n -
 (i^3*n - 2*i^3*log(e))*b^4*c^2*d^2)*B^2)*x^2 + 6*((b^4*c*d^3*i^3*n - a*b^3
*d^4*i^3*n)*B^2*x^3 + 3*(a*b^3*c*d^3*i^3*n - a^2*b^2*d^4*i^3*n)*B^2*x^2 + 3
*(a^2*b^2*c*d^3*i^3*n - a^3*b*d^4*i^3*n)*B^2*x + (a^3*b*c*d^3*i^3*n - a^4*d
^4*i^3*n)*B^2)*log(b*x + a) + 2*(B^2*b^4*d^4*i^3*x^4 + 4*B^2*b^4*c*d^3*i^3*
x^3 + 6*B^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2*b^4*c^3*d*i^3*x + B^2*b^4*c^4*i^3)*
log((b*x + a)^n))*log((d*x + c)^n))/(b^7*d*g^3*x^4 + a^3*b^4*c*g^3 + (b^7*c
*g^3 + 3*a*b^6*d*g^3)*x^3 + 3*(a*b^6*c*g^3 + a^2*b^5*d*g^3)*x^2 + (3*a^2*b^
5*c*g^3 + a^3*b^4*d*g^3)*x), x)
```

Giac [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x,
 algorithm="giac")
[Out] integrate((d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x +
 a*g)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx=\int \frac{(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(a\,g+b\,g\,x)^3}\,dx$$

```
[In] int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x
)^3,x)
[Out] int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x
)^3, x)
```

3.185 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 561

$$\begin{aligned}\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx &= -\frac{2B^2d^2i^3n^2(c+dx)}{b^3g^4(a+bx)}-\frac{B^2di^3n^2(c+dx)^2}{4b^2g^4(a+bx)^2}-\frac{2B^2i^3n^2(c+dx)^3}{27bg^4(a+bx)^3}\\&-\frac{2Bd^2i^3n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^4(a+bx)}-\frac{Bdi^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^4(a+bx)^2}\\&-\frac{2Bi^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9bg^4(a+bx)^3}-\frac{d^2i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g^4(a+bx)}\\&-\frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g^4(a+bx)^2}-\frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg^4(a+bx)^3}\\&-\frac{d^3i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4}\\&+\frac{2Bd^3i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4}\\&+\frac{2B^2d^3i^3n^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4}\end{aligned}$$

```
[Out] -2*B^2*d^2*i^3*n^2*(d*x+c)/b^3/g^4/(b*x+a)-1/4*B^2*d*i^3*n^2*(d*x+c)^2/b^2/
g^4/(b*x+a)^2-2/27*B^2*i^3*n^2*(d*x+c)^3/b/g^4/(b*x+a)^3-2*B*d^2*i^3*n*(d*x
+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/g^4/(b*x+a)-1/2*B*d*i^3*n*(d*x+c)^2
*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g^4/(b*x+a)^2-2/9*B*i^3*n*(d*x+c)^3*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g^4/(b*x+a)^3-d^2*i^3*(d*x+c)*(A+B*ln(e*((b
```

```
*x+a)/(d*x+c))^n))^2/b^3/g^4/(b*x+a)-1/2*d*i^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)
/(d*x+c))^n))^2/b^2/g^4/(b*x+a)^2-1/3*i^3*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x
+c))^n))^2/b/g^4/(b*x+a)^3-d^3*i^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln(1-b
*(d*x+c)/d/(b*x+a))/b^4/g^4+2*B*d^3*i^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*p
olylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^4+2*B^2*d^3*i^3*n^2*polylog(3,b*(d*x+c)
/d/(b*x+a))/b^4/g^4
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2561, 2380, 2342, 2341, 2379, 2421, 6724}

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$$
$$= \frac{2Bd^3i^3n\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4g^4} - \frac{d^3i^3\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^4g^4} - \frac{d^2i^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^3g^4(a+bx)} - \frac{2Bd^2i^3n(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3g^4(a+bx)} - \frac{di^3(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b^2g^4(a+bx)^2} - \frac{Bdi^3n(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b^2g^4(a+bx)^2} - \frac{i^3(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3bg^4(a+bx)^3} - \frac{2Bi^3n(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9bg^4(a+bx)^3} + \frac{2B^2d^3i^3n^2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4} - \frac{2B^2d^2i^3n^2(c+dx)}{b^3g^4(a+bx)} - \frac{B^2di^3n^2(c+dx)^2}{4b^2g^4(a+bx)^2} - \frac{2B^2i^3n^2(c+dx)^3}{27bg^4(a+bx)^3}$$

```
[In] Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x
)^4,x]
```

```
[Out] (-2*B^2*d^2*i^3*n^2*(c + d*x))/(b^3*g^4*(a + b*x)) - (B^2*d*i^3*n^2*(c + d*
x)^2)/(4*b^2*g^4*(a + b*x)^2) - (2*B^2*i^3*n^2*(c + d*x)^3)/(27*b*g^4*(a +
b*x)^3) - (2*B*d^2*i^3*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/
(b^3*g^4*(a + b*x)) - (B*d*i^3*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]))/(2*b^2*g^4*(a + b*x)^2) - (2*B*i^3*n*(c + d*x)^3*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n]))/(9*b*g^4*(a + b*x)^3) - (d^2*i^3*(c + d*x)*(A + B*L
og[e*((a + b*x)/(c + d*x))^n])^2)/(b^3*g^4*(a + b*x)) - (d*i^3*(c + d*x)^2*
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*b^2*g^4*(a + b*x)^2) - (i^3*(c
```

```
 + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*b*g^4*(a + b*x)^3) -
 (d^3*i^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x))/(d
*(a + b*x))])/(b^4*g^4) + (2*B*d^3*i^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b^4*g^4) + (2*B^2*d^3*i^3*n^
2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))])/(b^4*g^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2380

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Dist[1/d, Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
Dist[e/d, Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; FreeQ
[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
```

```
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{i^3\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^4(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{g^4}$$

$$= \frac{i^3\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^4} + \frac{(di^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^3(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bg^4}$$

$$= -\frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(di^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^4} + \frac{(d^2i^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^4} + \frac{(2Bi^3n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{3bg^4}$$

$$= -\frac{2B^2i^3n^2(c+dx)^3}{27bg^4(a+bx)^3} - \frac{2Bi^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9bg^4(a+bx)^3} - \frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g^4(a+bx)^2} - \frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(d^2i^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^4} + \frac{(d^3i^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^3g^4} + \frac{(Bdi^3n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{b^2g^4}$$

$$= -\frac{B^2di^3n^2(c+dx)^2}{4b^2g^4(a+bx)^2} - \frac{2B^2i^3n^2(c+dx)^3}{27bg^4(a+bx)^3} - \frac{Bdi^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^4(a+bx)^2} - \frac{2Bi^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9bg^4(a+bx)^3} - \frac{d^2i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g^4(a+bx)} - \frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g^4(a+bx)^2} - \frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg^4(a+bx)^3} - \frac{d^3i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4} + \frac{(2Bd^2i^3n)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{x^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^3g^4} + \frac{(2Bd^3i^3n)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)(A+B\log(ex^n))}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^4g^4}$$

$$= -\frac{2B^2d^2i^3n^2(c+dx)}{b^3g^4(a+bx)} - \frac{B^2di^3n^2(c+dx)^2}{4b^2g^4(a+bx)^2} - \frac{2B^2i^3n^2(c+dx)^3}{27bg^4(a+bx)^3} - \frac{2Bd^2i^3n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^4(a+bx)} - \frac{Bdi^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^4(a+bx)^2} - \frac{2Bi^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9bg^4(a+bx)^3} - \frac{d^2i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g^4(a+bx)} - \frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g^4(a+bx)^2} - \frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg^4(a+bx)^3} - \frac{d^3i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4} + \frac{2Bd^3i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4} - \frac{(2B^2d^3i^3n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{b^4g^4}$$

$$
\begin{aligned}
&= -\frac{2B^2d^2i^3n^2(c+dx)}{b^3g^4(a+bx)} - \frac{B^2di^3n^2(c+dx)^2}{4b^2g^4(a+bx)^2} - \frac{2B^2i^3n^2(c+dx)^3}{27bg^4(a+bx)^3} \\
&- \frac{2Bd^2i^3n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^4(a+bx)} \\
&- \frac{Bdi^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^4(a+bx)^2} \\
&- \frac{2Bi^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9bg^4(a+bx)^3} - \frac{d^2i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g^4(a+bx)} \\
&- \frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g^4(a+bx)^2} - \frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg^4(a+bx)^3} \\
&- \frac{d^3i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4} \\
&+ \frac{2Bd^3i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4} + \frac{2B^2d^3i^3n^2\operatorname{Li}_3\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 8570 vs. 2(561) = 1122.

Time = 7.50 (sec) , antiderivative size = 8570, normalized size of antiderivative = 15.28

$$
\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g +
 b*g*x)^4,x]
[Out] Result too large to show
```

Maple [F]

$$
\int \frac{(dix+ci)^3\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(bgx+ag)^4}\,dx
$$

```
[In] int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x)
[Out] int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x)
```

Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^4}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x,
 algorithm="fricas")
```

```
[Out] integral((A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*c
^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2*c
^3*i^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*d^2
*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log(e*((b*x + a)/(d*x + c))^n))
/(b^4*g^4*x^4 + 4*a*b^3*g^4*x^3 + 6*a^2*b^2*g^4*x^2 + 4*a^3*b*g^4*x + a^4*g
^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**4
,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^4}\,dx$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x,
 algorithm="maxima")
```

```
[Out] -1/3*A*B*c*d^2*i^3*n*((11*a^2*b^2*c^2 - 7*a^3*b*c*d + 2*a^4*d^2 + 6*(3*b^4*
c^2 - 3*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 3*(9*a*b^3*c^2 - 7*a^2*b^2*c*d + 2*a
^3*b*d^2)*x)/((b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*g^4*x^3 + 3*(a*b^7*c^2
- 2*a^2*b^6*c*d + a^3*b^5*d^2)*g^4*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d + a
^4*b^4*d^2)*g^4*x + (a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2)*g^4) + 6*(3
*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^2*d
+ 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4) - 6*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*
d^3)*log(d*x + c)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3
```

)*g^4)) - 1/9*A*B*c^3*i^3*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/6*A*B*c^2*d*i^3*n*((5*a*b^2*c^2 - 22*a^2*b*c*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c^2 - 16*a*b^2*c*d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4) - 6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4) + 6*(3*b*c*d^2 - a*d^3)*log(d*x + c)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4)) + 1/6*A^2*d^3*i^3*((18*a*b^2*x^2 + 27*a^2*b*x + 11*a^3)/(b^7*g^4*x^3 + 3*a*b^6*g^4*x^2 + 3*a^2*b^5*g^4*x + a^3*b^4*g^4) + 6*log(b*x + a)/(b^4*g^4)) - (3*b*x + a)*A*B*c^2*d*i^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^5*g^4*x^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - 2*(3*b^2*x^2 + 3*a*b*x + a^2)*A*B*c*d^2*i^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^6*g^4*x^3 + 3*a*b^5*g^4*x^2 + 3*a^2*b^4*g^4*x + a^3*b^3*g^4) - 1/2*(3*b*x + a)*A^2*c^2*d*i^3/(b^5*g^4*x^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - (3*b^2*x^2 + 3*a*b*x + a^2)*A^2*c*d^2*i^3/(b^6*g^4*x^3 + 3*a*b^5*g^4*x^2 + 3*a^2*b^4*g^4*x + a^3*b^3*g^4) - 2/3*A*B*c^3*i^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) - 1/3*A^2*c^3*i^3/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) - 1/6*(18*(b^3*c*d^2*i^3 - a*b^2*d^3*i^3)*B^2*x^2 + 9*(b^3*c^2*d*i^3 + 2*a*b^2*c*d^2*i^3 - 3*a^2*b*d^3*i^3)*B^2*x + (2*b^3*c^3*i^3 + 3*a*b^2*c^2*d*i^3 + 6*a^2*b*c*d^2*i^3 - 11*a^3*d^3*i^3)*B^2 - 6*(B^2*b^3*d^3*i^3*x^3 + 3*B^2*a*b^2*d^3*i^3*x^2 + 3*B^2*a^2*b*d^3*i^3*x + B^2*a^3*d^3*i^3)*log(b*x + a))*log((d*x + c)^n)^2/(b^7*g^4*x^3 + 3*a*b^6*g^4*x^2 + 3*a^2*b^5*g^4*x + a^3*b^4*g^4) - integrate(-1/3*(18*B^2*b^4*c^2*d^2*i^3*x^2*log(e)^2 + 12*B^2*b^4*c^3*d*i^3*x*log(e)^2 + 3*B^2*b^4*c^4*i^3*log(e)^2 + 3*(B^2*b^4*d^4*i^3*log(e)^2 + 2*A*B*b^4*d^4*i^3*log(e))*x^4 + 6*(2*B^2*b^4*c*d^3*i^3*log(e)^2 + A*B*b^4*c*d^3*i^3*log(e))*x^3 + 3*(B^2*b^4*d^4*i^3*x^4 + 4*B^2*b^4*c*d^3*i^3*x^3 + 6*B^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2*b^4*c^3*d*i^3*x + B^2*b^4*c^4*i^3)*log((b*x + a)^n)^2 + 6*(6*B^2*b^4*c^2*d^2*i^3*x^2*log(e) + 4*B^2*b^4*c^3*d*i^3*x*log(e) + B^2*b^4*c^4*i^3*log(e) + (B^2*b^4*d^4*i^3*log(e) + A*B*b^4*d^4*i^3)*x^4 + (4*B^2*b^4*c*d^3*i^3*log(e) + A*B*b^4*c*d^3*i^3)*x^3)*log((b*x + a)^n) + (9*(4*a*b^3*c*d^3*i^3*n - 5*a^2*b^2*d^4*i^3*n + (i^3*n - 4*i^3*log(e))*b^4*c^2*d^2)*B^2*x^2 - 6*(B^2*b^4*d^4*i^3*log(e) + A*B*b^4*d^4*i^3)*x^4 + 2*(6*a*b^3*c^2*d^2*i^3*n + 12*a^2*b^2*c*d^3*i^3*n - 19*a^3*b*d^4*i^3*n + (i^3*n - 12*i^3*log(e))*b^4*c^3*d)*B^2*x - 6*(A*B*b^4*c*d^3*i^3 + (3*a*b^3*d^4*i^3*n - (3*i^3*n - 4*i^3*log(e))*b^4*c*d^3)*B^2)*x^3 + (2*a*b^3*c^3*d*i^3*n + 3*a^2*b^2*c^2*d^2*i^3*n + 6*a^3*b*c*d^3*i^3*n - 11*a^4*d^4*i^3*n - 6*b^4*c^4*i^3*log(e))*B^2 - 6*(B^2*b^4*d^4*i^3*n*x^4 + 4*B^2*a*b^3*d^4*i^3*n*x^3

```
+ 6*B^2*a^2*b^2*d^4*i^3*n*x^2 + 4*B^2*a^3*b*d^4*i^3*n*x + B^2*a^4*d^4*i^3*n
)*log(b*x + a) - 6*(B^2*b^4*d^4*i^3*x^4 + 4*B^2*b^4*c*d^3*i^3*x^3 + 6*B^2*b
^4*c^2*d^2*i^3*x^2 + 4*B^2*b^4*c^3*d*i^3*x + B^2*b^4*c^4*i^3)*log((b*x + a)
^n))*log((d*x + c)^n))/(b^8*d*g^4*x^5 + a^4*b^4*c*g^4 + (b^8*c*g^4 + 4*a*b^
7*d*g^4)*x^4 + 2*(2*a*b^7*c*g^4 + 3*a^2*b^6*d*g^4)*x^3 + 2*(3*a^2*b^6*c*g^4
 + 2*a^3*b^5*d*g^4)*x^2 + (4*a^3*b^5*c*g^4 + a^4*b^4*d*g^4)*x), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Timed out}$$

```
[In] integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x,
 algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \int \frac{{\left(c\,i+d\,i\,x\right)}^3\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2}{{\left(a\,g+b\,g\,x\right)}^4}\,dx$$

```
[In] int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x
)^4,x)
```

```
[Out] int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x
)^4, x)
```

3.186 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 768

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$$
$$= \frac{bB^2(bc-ad)^2g^3n^2x}{3d^3i}+\frac{7B(bc-ad)^2g^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^3i}$$
$$-\frac{b^2B(bc-ad)g^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^4i}$$
$$+\frac{3(bc-ad)^2g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i}$$
$$-\frac{3b^2(bc-ad)g^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^4i}$$
$$+\frac{b^3g^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d^4i}$$
$$+\frac{6B(bc-ad)^3g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i}$$
$$+\frac{(bc-ad)^3g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i}$$
$$+\frac{B^2(bc-ad)^3g^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{3d^4i}-\frac{2B^2(bc-ad)^3g^3n^2\log(c+dx)}{d^4i}$$
$$-\frac{7B(bc-ad)^3g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3d^4i}$$
$$+\frac{6B^2(bc-ad)^3g^3n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}$$
$$+\frac{2B(bc-ad)^3g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}$$
$$+\frac{7B^2(bc-ad)^3g^3n^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3d^4i}-\frac{2B^2(bc-ad)^3g^3n^2\,\mathrm{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}$$

```
[Out] 1/3*b*B^2*(-a*d+b*c)^2*g^3*n^2*x/d^3/i+7/3*B*(-a*d+b*c)^2*g^3*n*(b*x+a)*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))/d^3/i-1/3*b^2*B*(-a*d+b*c)*g^3*n*(d*x+c)^2*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))/d^4/i+3*(-a*d+b*c)^2*g^3*(b*x+a)*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))^2/d^3/i-3/2*b^2*(-a*d+b*c)*g^3*(d*x+c)^2*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))^2/d^4/i+1/3*b^3*g^3*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^
n))^2/d^4/i+6*B*(-a*d+b*c)^3*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d
+b*c)/b/(d*x+c))/d^4/i+(-a*d+b*c)^3*g^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*l
n((-a*d+b*c)/b/(d*x+c))/d^4/i+1/3*B^2*(-a*d+b*c)^3*g^3*n^2*ln((b*x+a)/(d*x+
c))/d^4/i-2*B^2*(-a*d+b*c)^3*g^3*n^2*ln(d*x+c)/d^4/i-7/3*B*(-a*d+b*c)^3*g^3
```

```
*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/d^4/i+6*B^2*(-
a*d+b*c)^3*g^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i+2*B*(-a*d+b*c)^3*g^
3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i+7/
3*B^2*(-a*d+b*c)^3*g^3*n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/d^4/i-2*B^2*(-a*d
+b*c)^3*g^3*n^2*polylog(3,d*(b*x+a)/b/(d*x+c))/d^4/i
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 768, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {2561, 2395, 2356, 2389, 2379, 2438, 2351, 31, 46, 2355, 2354, 2421, 6724}

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$$
$$=\frac{b^3g^3(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d^4i}-\frac{3b^2g^3(c+dx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d^4i}-\frac{b^2Bg^3n(c+dx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^4i}+\frac{2Bg^3n(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^4i}+\frac{g^3(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^4i}+\frac{6Bg^3n(bc-ad)^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^4i}-\frac{7Bg^3n(bc-ad)^3\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^4i}+\frac{3g^3(a+bx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^3i}+\frac{7Bg^3n(a+bx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^3i}+\frac{6B^2g^3n^2(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}+\frac{7B^2g^3n^2(bc-ad)^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3d^4i}-\frac{2B^2g^3n^2(bc-ad)^3\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}+\frac{B^2g^3n^2(bc-ad)^3\log\left(\frac{a+bx}{c+dx}\right)}{3d^4i}-\frac{2B^2g^3n^2(bc-ad)^3\log(c+dx)}{d^4i}+\frac{bB^2g^3n^2x(bc-ad)^2}{3d^3i}$$

```
[In] Int[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*x
),x]
```

```
[Out] (b*B^2*(b*c - a*d)^2*g^3*n^2*x)/(3*d^3*i) + (7*B*(b*c - a*d)^2*g^3*n*(a + b
*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d^3*i) - (b^2*B*(b*c - a*d)*
g^3*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d^4*i) + (3*(b
*c - a*d)^2*g^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d^3*i)
 - (3*b^2*(b*c - a*d)*g^3*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2)/(2*d^4*i) + (b^3*g^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2)/(3*d^4*i) + (6*B*(b*c - a*d)^3*g^3*n*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(d^4*i) + ((b*c - a*d)^3*g^3*(A + B*L
og[e*((a + b*x)/(c + d*x))^n])^2*Log[(b*c - a*d)/(b*(c + d*x))])/(d^4*i) +
(B^2*(b*c - a*d)^3*g^3*n^2*Log[(a + b*x)/(c + d*x)])/(3*d^4*i) - (2*B^2*(b*
c - a*d)^3*g^3*n^2*Log[c + d*x])/(d^4*i) - (7*B*(b*c - a*d)^3*g^3*n*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(3*d^
4*i) + (6*B^2*(b*c - a*d)^3*g^3*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))]
)/(d^4*i) + (2*B*(b*c - a*d)^3*g^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^4*i) + (7*B^2*(b*c - a*d)^3*g^
3*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(3*d^4*i) - (2*B^2*(b*c - a*
d)^3*g^3*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(d^4*i)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)^3g^3)\,\text{Subst}\left(\int \frac{x^3(A+B\log(ex^n))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{i}$$

$$= \frac{((bc-ad)^3g^3)\,\text{Subst}\left(\int \left(\frac{b^3(A+B\log(ex^n))^2}{d^3(b-dx)^4} - \frac{3b^2(A+B\log(ex^n))^2}{d^3(b-dx)^3} + \frac{3b(A+B\log(ex^n))^2}{d^3(b-dx)^2} - \frac{(A+B\log(ex^n))^2}{d^3(b-dx)}\right)dx,\right.}{i}$$

$$= -\frac{((bc-ad)^3g^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i} + \frac{(3b(bc-ad)^3g^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i} - \frac{(3b^2(bc-ad)^3g^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i} + \frac{(b^3(bc-ad)^3g^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i}$$

$$= \frac{3(bc-ad)^2 g^3 (a+bx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3 i} - \frac{3b^2(bc-ad) g^3 (c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^4 i} + \frac{b^3 g^3 (c+dx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d^4 i} + \frac{(bc-ad)^3 g^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4 i} - \frac{(2B(bc-ad)^3 g^3 n)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4 i} + \frac{(3b^2 B(bc-ad)^3 g^3 n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4 i} - \frac{(2b^3 B(bc-ad)^3 g^3 n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3d^4 i} - \frac{(6B(bc-ad)^3 g^3 n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3 i}$$

$$= \frac{3(bc-ad)^2g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i}$$
$$- \frac{3b^2(bc-ad)g^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^4i}$$
$$+ \frac{b^3g^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d^4i}$$
$$+ \frac{6B(bc-ad)^3g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i}$$
$$+ \frac{(bc-ad)^3g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i}$$
$$+ \frac{2B(bc-ad)^3g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}$$
$$+ \frac{(3bB(bc-ad)^3g^3n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4i}$$
$$- \frac{(2b^2B(bc-ad)^3g^3n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{3d^4i}$$
$$+ \frac{(3bB(bc-ad)^3g^3n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i}$$
$$- \frac{(2b^2B(bc-ad)^3g^3n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{3d^3i}$$
$$- \frac{(2B^2(bc-ad)^3g^3n^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4i}$$
$$- \frac{(6B^2(bc-ad)^3g^3n^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4i}$$

$$
\begin{aligned}
&= \frac{3B(bc-ad)^2 g^3 n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d^3 i} \\
&- \frac{b^2 B(bc-ad) g^3 n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^4 i} \\
&+ \frac{3(bc-ad)^2 g^3 (a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3 i} \\
&- \frac{3b^2(bc-ad) g^3 (c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^4 i} \\
&+ \frac{b^3 g^3 (c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d^4 i} \\
&+ \frac{6B(bc-ad)^3 g^3 n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4 i} \\
&+ \frac{(bc-ad)^3 g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4 i} \\
&- \frac{3B(bc-ad)^3 g^3 n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{d^4 i} \\
&+ \frac{6B^2(bc-ad)^3 g^3 n^2 \operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4 i} \\
&+ \frac{2B(bc-ad)^3 g^3 n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4 i} \\
&- \frac{2B^2(bc-ad)^3 g^3 n^2 \operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4 i} \\
&- \frac{\left(2bB(bc-ad)^3 g^3 n\right)\operatorname{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{3d^4 i} \\
&- \frac{\left(2bB(bc-ad)^3 g^3 n\right)\operatorname{Subst}\left(\int \frac{A+B\log(ex^n)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{3d^3 i} \\
&+ \frac{\left(3B^2(bc-ad)^3 g^3 n^2\right)\operatorname{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4 i} \\
&+ \frac{\left(b^2 B^2(bc-ad)^3 g^3 n^2\right)\operatorname{Subst}\left(\int \frac{1}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{3d^4 i} \\
&- \frac{\left(3B^2(bc-ad)^3 g^3 n^2\right)\operatorname{Subst}\left(\int \frac{1}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3 i}
\end{aligned}
$$

$$= \frac{7B(bc-ad)^2g^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^3i} - \frac{b^2B(bc-ad)g^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^4i} + \frac{3(bc-ad)^2g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i} - \frac{3b^2(bc-ad)g^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^4i} + \frac{b^3g^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d^4i} + \frac{6B(bc-ad)^3g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i} + \frac{(bc-ad)^3g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i} - \frac{3B^2(bc-ad)^3g^3n^2\log(c+dx)}{d^4i} - \frac{7B(bc-ad)^3g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3d^4i} + \frac{6B^2(bc-ad)^3g^3n^2\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i} + \frac{2B(bc-ad)^3g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i} + \frac{3B^2(bc-ad)^3g^3n^2\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{d^4i} - \frac{2B^2(bc-ad)^3g^3n^2\operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i} - \frac{(2B^2(bc-ad)^3g^3n^2)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{3d^4i} + \frac{(b^2B^2(bc-ad)^3g^3n^2)\operatorname{Subst}\left(\int\left(\frac{1}{b^2x}+\frac{d}{b(b-dx)^2}+\frac{d}{b^2(b-dx)}\right)dx,x,\frac{a+bx}{c+dx}\right)}{3d^4i} + \frac{(2B^2(bc-ad)^3g^3n^2)\operatorname{Subst}\left(\int\frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{3d^3i}$$

$$
\begin{aligned}
&= \frac{bB^2(bc-ad)^2g^3n^2x}{3d^3i} + \frac{7B(bc-ad)^2g^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^3i} \\
&- \frac{b^2B(bc-ad)g^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^4i} \\
&+ \frac{3(bc-ad)^2g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i} \\
&- \frac{3b^2(bc-ad)g^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^4i} \\
&+ \frac{b^3g^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d^4i} \\
&+ \frac{6B(bc-ad)^3g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i} \\
&+ \frac{(bc-ad)^3g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i} \\
&+ \frac{B^2(bc-ad)^3g^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{3d^4i} - \frac{2B^2(bc-ad)^3g^3n^2\log(c+dx)}{d^4i} \\
&- \frac{7B(bc-ad)^3g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3d^4i} \\
&+ \frac{6B^2(bc-ad)^3g^3n^2\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i} \\
&+ \frac{2B(bc-ad)^3g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i} \\
&+ \frac{7B^2(bc-ad)^3g^3n^2\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{3d^4i} - \frac{2B^2(bc-ad)^3g^3n^2\operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 1072, normalized size of antiderivative = 1.40

$$
\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx
$$

$$
= \frac{g^3\left(6bd(bc-ad)^2x\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+3d^2(-bc+ad)(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+2d^3(a+\right.}{}
$$

```
[In] Integrate[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i +
 d*i*x),x]
```

```
[Out] (g^3*(6*b*d*(b*c - a*d)^2*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 3*d^
2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 2*d
```

```
^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - 6*A^2*(b*c - a*d)
^3*Log[c + d*x] + 12*A*B*(b*c - a*d)^3*Log[e*((a + b*x)/(c + d*x))^n]*Log[(
b*c - a*d)/(b*c + b*d*x)] + 6*B^2*(b*c - a*d)^3*Log[e*((a + b*x)/(c + d*x))
^n]^2*Log[(b*c - a*d)/(b*c + b*d*x)] - 6*A*B*(b*c - a*d)^3*n*(Log[(b*c - a*
d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c - a*d)/(b
*c + b*d*x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 6*B*(b*c - a*d)^
2*n*(2*a*d*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b*c*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - a*B*d*n*(Log[a + b*x]*(Lo
g[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))
/(-(b*c) + a*d)]) + b*B*c*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c +
 d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + 3*B*(b*c
- a*d)^2*n*(2*A*b*d*x + 2*B*d*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 2*
B*(b*c - a*d)*n*Log[c + d*x] - 2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d
)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))
 + 2*B*(b*c - a*d)*n*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*L
og[e*((a + b*x)/(c + d*x))^n] - d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n]) - 2*B*(b*c - a*d)^2*n*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(b*d*x + (-(b*c
) + a*d)*Log[c + d*x]) + B*(b*c - a*d)^2*n*((2*Log[(d*(a + b*x))/(-(b*c) +
a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)
])) + 12*B^2*(b*c - a*d)^3*n*(Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[2, (d*
(a + b*x))/(b*(c + d*x))] - n*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])))/(6
*d^4*i)
```

Maple [F]

$$\int \frac{(bgx+ag)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{dix+ci}\,dx$$

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
[Out] int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, a
lgorithm="fricas")
[Out] integral((A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a
^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a
```

```
^3*g^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2
*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log(e*((b*x + a)/(d*x + c))^n))
/(d*i*x + c*i), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, a
lgorithm="maxima")
[Out] 3*A^2*a^2*b*g^3*(x/(d*i) - c*log(d*x + c)/(d^2*i)) - 1/6*A^2*b^3*g^3*(6*c^3
*log(d*x + c)/(d^4*i) - (2*d^2*x^3 - 3*c*d*x^2 + 6*c^2*x)/(d^3*i)) + 3/2*A^
2*a*b^2*g^3*(2*c^2*log(d*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A^2*a^
3*g^3*log(d*i*x + c*i)/(d*i) + 1/6*(2*B^2*b^3*d^3*g^3*x^3 - 3*(b^3*c*d^2*g^
3 - 3*a*b^2*d^3*g^3)*B^2*x^2 + 6*(b^3*c^2*d*g^3 - 3*a*b^2*c*d^2*g^3 + 3*a^2
*b*d^3*g^3)*B^2*x - 6*(b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3
- a^3*d^3*g^3)*B^2*log(d*x + c))*log((d*x + c)^n)^2/(d^4*i) - integrate(-1/
3*(3*B^2*a^3*d^3*g^3*log(e)^2 + 6*A*B*a^3*d^3*g^3*log(e) + 3*(B^2*b^3*d^3*g
^3*log(e)^2 + 2*A*B*b^3*d^3*g^3*log(e))*x^3 + 9*(B^2*a*b^2*d^3*g^3*log(e)^2
 + 2*A*B*a*b^2*d^3*g^3*log(e))*x^2 + 3*(B^2*b^3*d^3*g^3*x^3 + 3*B^2*a*b^2*d
^3*g^3*x^2 + 3*B^2*a^2*b*d^3*g^3*x + B^2*a^3*d^3*g^3)*log((b*x + a)^n)^2 +
9*(B^2*a^2*b*d^3*g^3*log(e)^2 + 2*A*B*a^2*b*d^3*g^3*log(e))*x + 6*(B^2*a^3*
d^3*g^3*log(e) + A*B*a^3*d^3*g^3 + (B^2*b^3*d^3*g^3*log(e) + A*B*b^3*d^3*g^
3)*x^3 + 3*(B^2*a*b^2*d^3*g^3*log(e) + A*B*a*b^2*d^3*g^3)*x^2 + 3*(B^2*a^2*
b*d^3*g^3*log(e) + A*B*a^2*b*d^3*g^3)*x)*log((b*x + a)^n) - (6*B^2*a^3*d^3*
g^3*log(e) + 6*A*B*a^3*d^3*g^3 + 2*(3*A*B*b^3*d^3*g^3 + (g^3*n + 3*g^3*log(
e))*B^2*b^3*d^3)*x^3 - 6*(b^3*c^3*g^3*n - 3*a*b^2*c^2*d*g^3*n + 3*a^2*b*c*d
^2*g^3*n - a^3*d^3*g^3*n)*B^2*log(d*x + c) + 3*(6*A*B*a*b^2*d^3*g^3 - (b^3*
c*d^2*g^3*n - 3*(g^3*n + 2*g^3*log(e))*a*b^2*d^3)*B^2)*x^2 + 6*(3*A*B*a^2*b
*d^3*g^3 + (b^3*c^2*d*g^3*n - 3*a*b^2*c*d^2*g^3*n + 3*(g^3*n + g^3*log(e))*
a^2*b*d^3)*B^2)*x + 6*(B^2*b^3*d^3*g^3*x^3 + 3*B^2*a*b^2*d^3*g^3*x^2 + 3*B^
2*a^2*b*d^3*g^3*x + B^2*a^3*d^3*g^3)*log((b*x + a)^n))*log((d*x + c)^n))/(d
^4*i*x + c*d^3*i), x)
```

Giac [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, a
lgorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x +
 c*i), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=\int \frac{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

```
[In] int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x
),x)
```

```
[Out] int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x
), x)
```

3.187 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 573

$$\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx \\
&= -\frac{B(bc-ad)g^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d^2i} \\
&\quad -\frac{2(bc-ad)g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i} \\
&\quad +\frac{b^2g^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^3i} \\
&\quad -\frac{4B(bc-ad)^2g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i} \\
&\quad -\frac{(bc-ad)^2g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i}+\frac{B^2(bc-ad)^2g^2n^2\log(c+dx)}{d^3i} \\
&\quad +\frac{B(bc-ad)^2g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{d^3i} \\
&\quad -\frac{4B^2(bc-ad)^2g^2n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i} \\
&\quad -\frac{2B(bc-ad)^2g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i} \\
&\quad -\frac{B^2(bc-ad)^2g^2n^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{d^3i}+\frac{2B^2(bc-ad)^2g^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}
\end{aligned}$$

```
[Out] -B*(-a*d+b*c)*g^2*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^2/i-2*(-a*d+b
*c)*g^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^2/i+1/2*b^2*g^2*(d*x+c)
^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^3/i-4*B*(-a*d+b*c)^2*g^2*n*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d^3/i-(-a*d+b*c)^2*g^2*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b*c)/b/(d*x+c))/d^3/i+B^2*(-a*d+b*c)
^2*g^2*n^2*ln(d*x+c)/d^3/i+B*(-a*d+b*c)^2*g^2*n*(A+B*ln(e*((b*x+a)/(d*x+c))
^n))*ln(1-b*(d*x+c)/d/(b*x+a))/d^3/i-4*B^2*(-a*d+b*c)^2*g^2*n^2*polylog(2,d
*(b*x+a)/b/(d*x+c))/d^3/i-2*B*(-a*d+b*c)^2*g^2*n*(A+B*ln(e*((b*x+a)/(d*x+c)
)^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i-B^2*(-a*d+b*c)^2*g^2*n^2*polylog
(2,b*(d*x+c)/d/(b*x+a))/d^3/i+2*B^2*(-a*d+b*c)^2*g^2*n^2*polylog(3,d*(b*x+a
)/b/(d*x+c))/d^3/i
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2561, 2395, 2356, 2389, 2379, 2438, 2351, 31, 2355, 2354, 2421, 6724}

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$$
$$= \frac{b^2g^2(c+dx)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d^3i} - \frac{2Bg^2n(bc-ad)^2 \operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^3i} - \frac{g^2(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^3i} - \frac{4Bg^2n(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^3i} + \frac{Bg^2n(bc-ad)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^3i} - \frac{2g^2(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^2i} - \frac{Bg^2n(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^2i} - \frac{4B^2g^2n^2(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i} - \frac{B^2g^2n^2(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{d^3i} + \frac{2B^2g^2n^2(bc-ad)^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i} + \frac{B^2g^2n^2(bc-ad)^2\log(c+dx)}{d^3i}$$

```
[In] Int[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*x
),x]
```

```
[Out] -((B*(b*c - a*d)*g^2*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(d
^2*i)) - (2*(b*c - a*d)*g^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2)/(d^2*i) + (b^2*g^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2)/(2*d^3*i) - (4*B*(b*c - a*d)^2*g^2*n*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])*Log[(b*c - a*d)/(b*(c + d*x))])/(d^3*i) - ((b*c - a*d)^2*g^2*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])^2*Log[(b*c - a*d)/(b*(c + d*x))])/(d^3*i) + (B
^2*(b*c - a*d)^2*g^2*n^2*Log[c + d*x])/(d^3*i) + (B*(b*c - a*d)^2*g^2*n*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(
d^3*i) - (4*B^2*(b*c - a*d)^2*g^2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x)
)])/(d^3*i) - (2*B*(b*c - a*d)^2*g^2*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^3*i) - (B^2*(b*c - a*d)^2*g^
2*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(d^3*i) + (2*B^2*(b*c - a*d)
^2*g^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(d^3*i)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
```

```
NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)^2g^2)\,\text{Subst}\left(\int \frac{x^2(A+B\log(ex^n))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{i}$$

$$= \frac{((bc-ad)^2g^2)\,\text{Subst}\left(\int \left(\frac{b^2(A+B\log(ex^n))^2}{d^2(b-dx)^3} - \frac{2b(A+B\log(ex^n))^2}{d^2(b-dx)^2} + \frac{(A+B\log(ex^n))^2}{d^2(b-dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{i}$$

$$= \frac{((bc-ad)^2g^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i} - \frac{(2b(bc-ad)^2g^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i} + \frac{(b^2(bc-ad)^2g^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i}$$

$$= -\frac{2(bc-ad)g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i} + \frac{b^2g^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^3i} - \frac{(bc-ad)^2g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i} + \frac{(2B(bc-ad)^2g^2n)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i} - \frac{(b^2B(bc-ad)^2g^2n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i} + \frac{(4B(bc-ad)^2g^2n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i}$$

$$
\begin{aligned}
&= -\frac{2(bc-ad)g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i} \\
&+ \frac{b^2g^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^3i} \\
&- \frac{4B(bc-ad)^2g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i} \\
&- \frac{(bc-ad)^2g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i} \\
&- \frac{2B(bc-ad)^2g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i} \\
&- \frac{(bB(bc-ad)^2g^2n)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i} \\
&- \frac{(bB(bc-ad)^2g^2n)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^2i} \\
&+ \frac{(2B^2(bc-ad)^2g^2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i} \\
&+ \frac{(4B^2(bc-ad)^2g^2n^2)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i}
\end{aligned}
$$

$$= -\frac{B(bc-ad)g^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d^2i}$$
$$-\frac{2(bc-ad)g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i}$$
$$+\frac{b^2g^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^3i}$$
$$-\frac{4B(bc-ad)^2g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i}$$
$$-\frac{(bc-ad)^2g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i}$$
$$+\frac{B(bc-ad)^2g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{d^3i}$$
$$-\frac{4B^2(bc-ad)^2g^2n^2\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}$$
$$-\frac{2B(bc-ad)^2g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\mathrm{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}$$
$$+\frac{2B^2(bc-ad)^2g^2n^2\mathrm{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}$$
$$-\frac{(B^2(bc-ad)^2g^2n^2)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{b}{dx}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i}$$
$$+\frac{(B^2(bc-ad)^2g^2n^2)\,\mathrm{Subst}\left(\int\frac{1}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^2i}$$

$$= -\frac{B(bc-ad)g^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d^2i} - \frac{2(bc-ad)g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i} + \frac{b^2g^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^3i} - \frac{4B(bc-ad)^2g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i} - \frac{(bc-ad)^2g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i} + \frac{B^2(bc-ad)^2g^2n^2\log(c+dx)}{d^3i} + \frac{B(bc-ad)^2g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{d^3i} - \frac{4B^2(bc-ad)^2g^2n^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i} - \frac{2B(bc-ad)^2g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i} - \frac{B^2(bc-ad)^2g^2n^2\text{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{d^3i} + \frac{2B^2(bc-ad)^2g^2n^2\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 763, normalized size of antiderivative = 1.33

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \frac{g^2\left(-2bd(bc-ad)x\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + d^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + 2A^2(bc-ad)^2\log(c\right.}{}$$

```
[In] Integrate[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i +
 d*i*x),x]
```

```
[Out] (g^2*(-2*b*d*(b*c - a*d)*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + d^2*(
a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 2*A^2*(b*c - a*d)^2*L
og[c + d*x] - 4*A*B*(b*c - a*d)^2*Log[e*((a + b*x)/(c + d*x))^n]*Log[(b*c -
 a*d)/(b*c + b*d*x)] - 2*B^2*(b*c - a*d)^2*Log[e*((a + b*x)/(c + d*x))^n]^2
*Log[(b*c - a*d)/(b*c + b*d*x)] + 2*A*B*(b*c - a*d)^2*n*(Log[(b*c - a*d)/(b
*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c - a*d)/(b*c +
b*d*x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 2*B*(b*c - a*d)*n*(2*
```

```
a*d*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b*c*(A + B*Log[
e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - a*B*d*n*(Log[a + b*x]*(Log[a + b
*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c
) + a*d)]) + b*B*c*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*
Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - B*(b*c - a*d)*n*
(2*A*b*d*x + 2*B*d*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 2*B*(b*c - a*
d)*n*Log[c + d*x] - 2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Lo
g[c + d*x] + B*(b*c - a*d)*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c
+ d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + 4*B^2*(b
*c - a*d)^2*n*(-(Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[2, (d*(a + b*x))/(b
*(c + d*x))]) + n*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])))/(2*d^3*i)
```

Maple [F]

$$\int \frac{(bgx+ag)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{dix+ci}\,dx$$

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

```
[Out] int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, a
lgorithm="fricas")
```

```
[Out] integral((A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^
2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*
B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^
n))/(d*i*x + c*i), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, a
lgorithm="maxima")
```

```
[Out] 2*A^2*a*b*g^2*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + 1/2*A^2*b^2*g^2*(2*c^2*l
og(d*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A^2*a^2*g^2*log(d*i*x + c*
i)/(d*i) + 1/2*(B^2*b^2*d^2*g^2*x^2 - 2*(b^2*c*d*g^2 - 2*a*b*d^2*g^2)*B^2*x
 + 2*(b^2*c^2*g^2 - 2*a*b*c*d*g^2 + a^2*d^2*g^2)*B^2*log(d*x + c))*log((d*x
 + c)^n)^2/(d^3*i) - integrate(-(B^2*a^2*d^2*g^2*log(e)^2 + 2*A*B*a^2*d^2*g
^2*log(e) + (B^2*b^2*d^2*g^2*log(e)^2 + 2*A*B*b^2*d^2*g^2*log(e))*x^2 + (B^
2*b^2*d^2*g^2*x^2 + 2*B^2*a*b*d^2*g^2*x + B^2*a^2*d^2*g^2)*log((b*x + a)^n)
^2 + 2*(B^2*a*b*d^2*g^2*log(e)^2 + 2*A*B*a*b*d^2*g^2*log(e))*x + 2*(B^2*a^2
*d^2*g^2*log(e) + A*B*a^2*d^2*g^2 + (B^2*b^2*d^2*g^2*log(e) + A*B*b^2*d^2*g
^2)*x^2 + 2*(B^2*a*b*d^2*g^2*log(e) + A*B*a*b*d^2*g^2)*x)*log((b*x + a)^n)
- (2*B^2*a^2*d^2*g^2*log(e) + 2*A*B*a^2*d^2*g^2 + 2*(b^2*c^2*g^2*n - 2*a*b*
c*d*g^2*n + a^2*d^2*g^2*n)*B^2*log(d*x + c) + (2*A*B*b^2*d^2*g^2 + (g^2*n +
 2*g^2*log(e))*B^2*b^2*d^2)*x^2 + 2*(2*A*B*a*b*d^2*g^2 - (b^2*c*d*g^2*n - 2
*(g^2*n + g^2*log(e))*a*b*d^2)*B^2)*x + 2*(B^2*b^2*d^2*g^2*x^2 + 2*B^2*a*b*
d^2*g^2*x + B^2*a^2*d^2*g^2)*log((b*x + a)^n))*log((d*x + c)^n))/(d^3*i*x +
 c*d^2*i), x)
```

Giac [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, a
lgorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x +
 c*i), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=\int \frac{\left(a\,g+b\,g\,x\right)^2\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

```
[In] int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x
),x)
```

```
[Out] int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x
), x)
```

3.188 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 303

$$\begin{aligned}
&\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx\\
&=\frac{g(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{di}\\
&\quad+\frac{2B(bc-ad)gn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i}\\
&\quad+\frac{(bc-ad)g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i}\\
&\quad+\frac{2B^2(bc-ad)gn^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}\\
&\quad+\frac{2B(bc-ad)gn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}\\
&\quad-\frac{2B^2(bc-ad)gn^2\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}
\end{aligned}$$

```
[Out] g*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d/i+2*B*(-a*d+b*c)*g*n*(A+B*ln(
e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d^2/i+(-a*d+b*c)*g*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b*c)/b/(d*x+c))/d^2/i+2*B^2*(-a*d+b*c)*
g*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/d^2/i+2*B*(-a*d+b*c)*g*n*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^2/i-2*B^2*(-a*d+b*c)*g*
n^2*polylog(3,d*(b*x+a)/b/(d*x+c))/d^2/i
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {2561, 2395, 2355, 2354, 2438, 2421, 6724}

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$$
$$=\frac{2Bgn(bc-ad)\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^2i}+\frac{2Bgn(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^2i}+\frac{g(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^2i}+\frac{g(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{di}+\frac{2B^2gn^2(bc-ad)\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}-\frac{2B^2gn^2(bc-ad)\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}$$

```
[In] Int[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*x),
x]
```

```
[Out] (g*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d*i) + (2*B*(b*c -
a*d)*g*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x
))])/(d^2*i) + ((b*c - a*d)*g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[
(b*c - a*d)/(b*(c + d*x))])/(d^2*i) + (2*B^2*(b*c - a*d)*g*n^2*PolyLog[2, (
d*(a + b*x))/(b*(c + d*x))])/(d^2*i) + (2*B*(b*c - a*d)*g*n*(A + B*Log[e*((
a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^2*i) -
(2*B^2*(b*c - a*d)*g*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(d^2*i)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{((bc-ad)g)\text{Subst}\left(\int \frac{x(A+B\log(ex^n))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{i} \\
&= \frac{((bc-ad)g)\text{Subst}\left(\int \left(\frac{b(A+B\log(ex^n))^2}{d(-b+dx)^2}+\frac{(A+B\log(ex^n))^2}{d(-b+dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{i} \\
&= \frac{((bc-ad)g)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{-b+dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{di} \\
&\quad + \frac{(b(bc-ad)g)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(-b+dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{di}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{g(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{di} \\
&+ \frac{(bc-ad)g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i} \\
&- \frac{(2B(bc-ad)gn)\text{Subst}\left(\int\frac{(A+B\log(ex^n))\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^2i} \\
&+ \frac{(2B(bc-ad)gn)\text{Subst}\left(\int\frac{A+B\log(ex^n)}{-b+dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{di} \\
&= \frac{g(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{di} \\
&+ \frac{2B(bc-ad)gn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i} \\
&+ \frac{(bc-ad)g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i} \\
&+ \frac{2B(bc-ad)gn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i} \\
&- \frac{(2B^2(bc-ad)gn^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^2i} \\
&- \frac{(2B^2(bc-ad)gn^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^2i} \\
&= \frac{g(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{di}+\frac{2B(bc-ad)gn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i} \\
&+ \frac{(bc-ad)g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i}+\frac{2B^2(bc-ad)gn^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i} \\
&+ \frac{2B(bc-ad)gn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}-\frac{2B^2(bc-ad)gn^2\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1385 vs. 2(303) = 606.

Time = 0.56 (sec) , antiderivative size = 1385, normalized size of antiderivative = 4.57

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$$

$$= \frac{g\Big(3bdx\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-Bn\log\left(\frac{a+bx}{c+dx}\right)\right)^2-3(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-Bn\log\left(\frac{a+bx}{c+dx}\right)\right)^2\log}{}$$

```
[In] Integrate[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d
*i*x),x]
```

```
[Out] (g*(3*b*d*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c +
d*x)])^2 - 3*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a
 + b*x)/(c + d*x)])^2*Log[c + d*x] - 3*a*B*d*n*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])*(Log[c/d + x]^2 + 2*(Log[a/b + x]
 - Log[c/d + x] - Log[(a + b*x)/(c + d*x)])*Log[c + d*x] - 2*(Log[a/b + x]*
Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))
 - 3*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x
)])*(-2*d*(a + b*x)*(-1 + Log[a/b + x]) + 2*b*(c + d*x)*(-1 + Log[c/d + x])
 - b*c*Log[c/d + x]^2 + 2*b*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c
 + d*x)])*(d*x - c*Log[c + d*x]) + 2*b*c*(Log[a/b + x]*Log[(b*(c + d*x))/(b
*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])) + a*B^2*d*n^2*(Log[
c/d + x]^3 + 3*Log[c/d + x]^2*(-Log[a/b + x] + Log[(d*(a + b*x))/(-(b*c) +
a*d)]) + 3*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])^2*Log[
c + d*x] + 3*Log[a/b + x]^2*Log[(b*(c + d*x))/(b*c - a*d)] + 6*Log[a/b + x]
*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 3*(Log[a/b + x] - Log[c/d + x]
- Log[(a + b*x)/(c + d*x)])*(Log[c/d + x]^2 - 2*(Log[a/b + x]*Log[(b*(c + d
*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])) + 6*Log[c/d
+ x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 6*PolyLog[3, (d*(a + b*x))/(-(
b*c) + a*d)] - 6*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]) + B^2*n^2*(3*d*(2*b
*x - 2*(a + b*x)*Log[a/b + x] + (a + b*x)*Log[a/b + x]^2) - b*c*Log[c/d + x
]^3 + 3*b*(2*d*x - 2*(c + d*x)*Log[c/d + x] + (c + d*x)*Log[c/d + x]^2) + 3
*b*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])^2*(d*x - c*Log
[c + d*x]) - 6*(a*d + 2*b*d*x - b*d*x*Log[c/d + x] - b*c*Log[c + d*x] + Log
[a/b + x]*(-(d*(a + b*x)) + d*(a + b*x)*Log[c/d + x] + (b*c - a*d)*Log[(b*(
c + d*x))/(b*c - a*d)]) + (b*c - a*d)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*
d)]) + 3*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)])*(-2*d*(a
+ b*x)*(-1 + Log[a/b + x]) + 2*b*(c + d*x)*(-1 + Log[c/d + x]) - b*c*Log[c/
d + x]^2 + 2*b*c*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2,
(d*(a + b*x))/(-(b*c) + a*d)])) - 3*b*c*(Log[a/b + x]^2*Log[(b*(c + d*x))/(
b*c - a*d)] + 2*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 2*P
olyLog[3, (d*(a + b*x))/(-(b*c) + a*d)]) + 3*b*c*(Log[c/d + x]^2*(Log[a/b +
```

```
x] - Log[(d*(a + b*x))/(-(b*c) + a*d)]) - 2*Log[c/d + x]*PolyLog[2, (b*(c
+ d*x))/(b*c - a*d)] + 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]))))/(3*d^2*i
)
```

Maple [F]

$$\int \frac{(bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{dix+ci}dx$$

```
[In] int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
[Out] int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

Fricas [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{ci+dix}\,dx=\int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{dix+ci}\,dx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, alg
orithm="fricas")
[Out] integral((A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log(e*((b*x + a)/(d*x
 + c))^n)^2 + 2*(A*B*b*g*x + A*B*a*g)*log(e*((b*x + a)/(d*x + c))^n))/(d*i*
x + c*i), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{ci+dix}\,dx$$
$$=\frac{g\left(\int \frac{A^2a}{c+dx}\,dx+\int \frac{A^2bx}{c+dx}\,dx+\int \frac{B^2a\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)^{2}}{c+dx}\,dx+\int \frac{2ABa\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}{c+dx}\,dx+\int \frac{B^2bx\log\left(e\left(\frac{a}{c+dx}+\right.\right.}{c+dx}\right.}{i}$$

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i),x)
[Out] g*(Integral(A**2*a/(c + d*x), x) + Integral(A**2*b*x/(c + d*x), x) + Integr
al(B**2*a*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c + d*x), x) + Integr
al(2*A*B*a*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x) + Integral
(B**2*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c + d*x), x) + Integr
al(2*A*B*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x))/i
```

Maxima [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, alg
orithm="maxima")
```

```
[Out] A^2*b*g*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + A^2*a*g*log(d*i*x + c*i)/(d*i)
 + (B^2*b*d*g*x - (b*c*g - a*d*g)*B^2*log(d*x + c))*log((d*x + c)^n)^2/(d^2
*i) - integrate(-(B^2*a*d*g*log(e)^2 + 2*A*B*a*d*g*log(e) + (B^2*b*d*g*x +
B^2*a*d*g)*log((b*x + a)^n)^2 + (B^2*b*d*g*log(e)^2 + 2*A*B*b*d*g*log(e))*x
 + 2*(B^2*a*d*g*log(e) + A*B*a*d*g + (B^2*b*d*g*log(e) + A*B*b*d*g)*x)*log(
(b*x + a)^n) - 2*(B^2*a*d*g*log(e) + A*B*a*d*g - (b*c*g*n - a*d*g*n)*B^2*lo
g(d*x + c) + ((g*n + g*log(e))*B^2*b*d + A*B*b*d*g)*x + (B^2*b*d*g*x + B^2*
a*d*g)*log((b*x + a)^n))*log((d*x + c)^n))/(d^2*i*x + c*d*i), x)
```

Giac [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, alg
orithm="giac")
```

```
[Out] integrate((b*g*x + a*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x + c
*i), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

```
[In] int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x),
x)
```

```
[Out] int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x),
 x)
```

3.189 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 137

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{di}-\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{di}+\frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{di}$$

```
[Out] -(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b*c)/b/(d*x+c))/d/i-2*B*n*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/d/i+2*B^2*n^2*pol
ylog(3,d*(b*x+a)/b/(d*x+c))/d/i
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2551, 2354, 2421, 6724}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=-\frac{2Bn\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{di}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{di}+\frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{di}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*i + d*i*x),x]
```

```
[Out] -(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[(b*c - a*d)/(b*(c + d*x))])
/(d*i)) - (2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a +
b*x))/(b*(c + d*x))])/(d*i) + (2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d
*x))])/(d*i)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{i}$$

$$= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{di} + \frac{(2Bn)\text{Subst}\left(\int \frac{(A+B\log(ex^n))\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{di}$$

$$
\begin{aligned}
&= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{di} \\
&\quad -\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{di} \\
&\quad +\frac{(2B^2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{di} \\
&= -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{di} \\
&\quad -\frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{di}+\frac{2B^2n^2\operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{di}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.96

$$
\int\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx
= \frac{A^2\log(c+dx)+2ABn\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log\left(\frac{bc-ad}{bc+bdx}\right)-2AB\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{bc+bdx}\right)-B^2\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{}
$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*i + d*i*x),x]
```

```
[Out] (A^2*Log[c + d*x] + 2*A*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[(b*c - a*
d)/(b*c + b*d*x)] - 2*A*B*Log[e*((a + b*x)/(c + d*x))^n]*Log[(b*c - a*d)/(b
*c + b*d*x)] - B^2*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*c +
b*d*x)] + A*B*n*Log[(b*c - a*d)/(b*c + b*d*x)]^2 - 2*B^2*n*Log[e*((a + b*x)
/(c + d*x))^n]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 2*A*B*n*PolyLog[2,
 (b*(c + d*x))/(b*c - a*d)] + 2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*
x))])/(d*i)
```

Maple [F]

$$
\int\frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{dix+ci}dx
$$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, algorithm="fric
as")
```

```
[Out] integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*
x + c))^n) + A^2)/(d*i*x + c*i), x)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \frac{\int \frac{A^2}{c+dx}\,dx+\int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c+dx}\,dx+\int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx}{i}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i),x)
```

```
[Out] (Integral(A**2/(c + d*x), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)**2/(c + d*x), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d
*x))**n)/(c + d*x), x))/i
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, algorithm="maxi
ma")
```

```
[Out] B^2*log(d*x + c)*log((d*x + c)^n)^2/(d*i) + A^2*log(d*i*x + c*i)/(d*i) - in
tegrate(-(B^2*log((b*x + a)^n)^2 + B^2*log(e)^2 + 2*A*B*log(e) + 2*(B^2*log
(e) + A*B)*log((b*x + a)^n) - 2*(B^2*n*log(d*x + c) + B^2*log((b*x + a)^n)
+ B^2*log(e) + A*B)*log((d*x + c)^n))/(d*i*x + c*i), x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, algorithm="giac
")

[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x + c*i), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=\int \frac{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*i + d*i*x),x)

[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*i + d*i*x), x)

3.190 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 50

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx=\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)gin}$$

```
[Out] 1/3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b*c)/g/i/n
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2561, 2339, 30}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx=\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3Bgin(bc-ad)}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)*(c*i + d*i*x)),
x]
```

```
[Out] (A + B*Log[e*((a + b*x)/(c + d*x))^n])^3/(3*B*(b*c - a*d)*g*i*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
```

```
x]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)gi}\\ &= \frac{\text{Subst}\left(\int x^2\,dx, x, A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)gin}\\ &= \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)gin}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.80

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = \frac{3A^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3AB\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+B^2\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bcgin-3adgin}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)*(c*i + d*
i*x)),x]
```

```
[Out] (3*A^2*Log[e*((a + b*x)/(c + d*x))^n] + 3*A*B*Log[e*((a + b*x)/(c + d*x))^n
]^2 + B^2*Log[e*((a + b*x)/(c + d*x))^n]^3)/(3*b*c*g*i*n - 3*a*d*g*i*n)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(48) = 96.

Time = 1.44 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.36

method	result	size
parallelrisch	$-\frac{B^{2} a^{2} c^{2} \ln\left(e\left(\frac{b x+a}{d x+c}\right)^{n}\right)^{3}+3 A B\, a^{2} c^{2} \ln\left(e\left(\frac{b x+a}{d x+c}\right)^{n}\right)^{2}+3 A^{2} \ln\left(e\left(\frac{b x+a}{d x+c}\right)^{n}\right) a^{2} c^{2}}{3 i g\, c^{2} a^{2} n (a d-c b)}$	118
default	$\frac{A^{2}\left(\frac{\ln(d x+c)}{a d-c b}-\frac{\ln(b x+a)}{a d-c b}\right)}{g i}-\frac{B^{2} \ln\left(e\left(\frac{b x+a}{d x+c}\right)^{n}\right)^{3}}{3 g i n (a d-c b)}-\frac{A B \ln\left(e\left(\frac{b x+a}{d x+c}\right)^{n}\right)^{2}}{g i n (a d-c b)}$	135
parts	$\frac{A^{2}\left(\frac{\ln(d x+c)}{a d-c b}-\frac{\ln(b x+a)}{a d-c b}\right)}{g i}-\frac{B^{2} \ln\left(e\left(\frac{b x+a}{d x+c}\right)^{n}\right)^{3}}{3 g i n (a d-c b)}-\frac{A B \ln\left(e\left(\frac{b x+a}{d x+c}\right)^{n}\right)^{2}}{g i n (a d-c b)}$	135

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i),x,method=_RET
URNVERBOSE)
```

```
[Out] -1/3*(B^2*a^2*c^2*ln(e*((b*x+a)/(d*x+c))^n)^3+3*A*B*a^2*c^2*ln(e*((b*x+a)/(
d*x+c))^n)^2+3*A^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*c^2)/i/g/c^2/a^2/n/(a*d-b*
c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(48) = 96.

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.98

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = \frac{B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^3+3\,B^2\log\left(e\right)^2\log\left(\frac{bx+a}{dx+c}\right)+3\,ABn\log\left(\frac{bx+a}{dx+c}\right)^2+3\,A^2\log\left(\frac{bx+a}{dx+c}\right)+3\left(B^2n\log\left(\frac{bx+a}{dx+c}\right)^2+2\right.}{3\,(bc-ad)gi}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i),x, alg
orithm="fricas")
```

```
[Out] 1/3*(B^2*n^2*log((b*x + a)/(d*x + c))^3 + 3*B^2*log(e)^2*log((b*x + a)/(d*x
 + c)) + 3*A*B*n*log((b*x + a)/(d*x + c))^2 + 3*A^2*log((b*x + a)/(d*x + c)
) + 3*(B^2*n*log((b*x + a)/(d*x + c))^2 + 2*A*B*log((b*x + a)/(d*x + c)))*l
og(e))/((b*c - a*d)*g*i)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = \frac{\int \frac{A^2}{ac+adx+bcx+bdx^2}\,dx+\int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{ac+adx+bcx+bdx^2}\,dx+\int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{ac+adx+bcx+bdx^2}\,dx}{gi}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)/(d*i*x+c*i),x)
```

```
[Out] (Integral(A**2/(a*c + a*d*x + b*c*x + b*d*x**2), x) + Integral(B**2*log(e*(
a/(c + d*x) + b*x/(c + d*x))**n)**2/(a*c + a*d*x + b*c*x + b*d*x**2), x) +
Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a*c + a*d*x + b*c*x
 + b*d*x**2), x))/(g*i)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(48) = 96$.

Time = 0.22 (sec) , antiderivative size = 407, normalized size of antiderivative = 8.14

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = B^2\left(\frac{\log(bx+a)}{(bc-ad)gi}-\frac{\log(dx+c)}{(bc-ad)gi}\right)\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)^2 + 2\,AB\left(\frac{\log(bx+a)}{(bc-ad)gi}-\frac{\log(dx+c)}{(bc-ad)gi}\right)\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) + \frac{1}{3}\left(\frac{\left(\log(bx+a)^3-3\,\log(bx+a)^2\log(dx+c)+3\,\log(bx+a)\log(dx+c)^2-\log(dx+c)^3\right)n^2}{bcgi-adgi} - \frac{3\,(\cdots}{\cdots}\right. - \frac{\left(\log(bx+a)^2-2\,\log(bx+a)\log(dx+c)+\log(dx+c)^2\right)ABn}{bcgi-adgi} + A^2\left(\frac{\log(bx+a)}{(bc-ad)gi}-\frac{\log(dx+c)}{(bc-ad)gi}\right)$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i),x, alg
orithm="maxima")
```

```
[Out] B^2*(log(b*x + a)/((b*c - a*d)*g*i) - log(d*x + c)/((b*c - a*d)*g*i))*log(e
*(b*x/(d*x + c) + a/(d*x + c))^n)^2 + 2*A*B*(log(b*x + a)/((b*c - a*d)*g*i)
 - log(d*x + c)/((b*c - a*d)*g*i))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) +
 1/3*((log(b*x + a)^3 - 3*log(b*x + a)^2*log(d*x + c) + 3*log(b*x + a)*log(
d*x + c)^2 - log(d*x + c)^3)*n^2/(b*c*g*i - a*d*g*i) - 3*(log(b*x + a)^2 -
```

```
2*log(b*x + a)*log(d*x + c) + log(d*x + c)^2)*n*log(e*(b*x/(d*x + c) + a/(d
*x + c))^n)/(b*c*g*i - a*d*g*i))*B^2 - (log(b*x + a)^2 - 2*log(b*x + a)*log
(d*x + c) + log(d*x + c)^2)*A*B*n/(b*c*g*i - a*d*g*i) + A^2*(log(b*x + a)/(
(b*c - a*d)*g*i) - log(d*x + c)/((b*c - a*d)*g*i))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(48) = 96$.

Time = 0.55 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.34

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = \frac{\left(B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^3+3\,B^2n\log(e)\log\left(\frac{bx+a}{dx+c}\right)^2+3\,B^2\log(e)^2\log\left(\frac{bx+a}{dx+c}\right)+3\,ABn\log\left(\frac{bx+a}{dx+c}\right)^2+6\,AB\log(e)\right.}{3\,gi}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i),x, alg
orithm="giac")
```

```
[Out] 1/3*(B^2*n^2*log((b*x + a)/(d*x + c))^3 + 3*B^2*n*log(e)*log((b*x + a)/(d*x
 + c))^2 + 3*B^2*log(e)^2*log((b*x + a)/(d*x + c)) + 3*A*B*n*log((b*x + a)/
(d*x + c))^2 + 6*A*B*log(e)*log((b*x + a)/(d*x + c)) + 3*A^2*log((b*x + a)/
(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(g*i)
```

Mupad [B] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.44

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = -\frac{\frac{B^2\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^3}{3}+A\,B\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{g\,i\,n\,(a\,d-b\,c)} + \frac{A^2\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{g\,i\,(a\,d-b\,c)}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)*(c*i + d*i*x)),
x)
```

```
[Out] (A^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(g*i*(a*d - b*c)) -
 ((B^2*log(e*((a + b*x)/(c + d*x))^n)^3)/3 + A*B*log(e*((a + b*x)/(c + d*x)
)^n)^2)/(g*i*n*(a*d - b*c))
```

3.191 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 199

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{2bB^2n^2(c+dx)}{(bc-ad)^2g^2i(a+bx)} - \frac{2bBn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2g^2i(a+bx)} - \frac{b(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2g^2i(a+bx)} - \frac{d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^2g^2in}$$

```
[Out] -2*b*B^2*n^2*(d*x+c)/(-a*d+b*c)^2/g^2/i/(b*x+a)-2*b*B*n*(d*x+c)*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))/(-a*d+b*c)^2/g^2/i/(b*x+a)-b*(d*x+c)*(A+B*ln(e*((b*x+a
)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^2/i/(b*x+a)-1/3*d*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))^3/B/(-a*d+b*c)^2/g^2/i/n
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used

= {2561, 2395, 2342, 2341, 2339, 30}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{d\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3Bg^2in(bc-ad)^2} - \frac{b(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^2i(a+bx)(bc-ad)^2} - \frac{2bBn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^2i(a+bx)(bc-ad)^2} - \frac{2bB^2n^2(c+dx)}{g^2i(a+bx)(bc-ad)^2}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^2*(c*i + d*i*x)
),x]
```

```
[Out] (-2*b*B^2*n^2*(c + d*x))/((b*c - a*d)^2*g^2*i*(a + b*x)) - (2*b*B*n*(c + d*
x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^2*g^2*i*(a + b*x))
- (b*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*c - a*d)^2*g^2
*i*(a + b*x)) - (d*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(3*B*(b*c - a*
d)^2*g^2*i*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 g^2 i} \\
&= \frac{\text{Subst}\left(\int \left(\frac{b(A+B\log(ex^n))^2}{x^2} - \frac{d(A+B\log(ex^n))^2}{x}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 g^2 i} \\
&= \frac{b\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 g^2 i} - \frac{d\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 g^2 i} \\
&= -\frac{b(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2 g^2 i(a+bx)} - \frac{d\text{Subst}\left(\int x^2\,dx, x, A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)^2 g^2 in} \\
&\quad + \frac{(2bBn)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 g^2 i} \\
&= -\frac{2bB^2n^2(c+dx)}{(bc-ad)^2 g^2 i(a+bx)} - \frac{2bBn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2 g^2 i(a+bx)} \\
&\quad - \frac{b(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2 g^2 i(a+bx)} - \frac{d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^2 g^2 in}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 793 vs. 2(199) = 398.

Time = 0.45 (sec) , antiderivative size = 793, normalized size of antiderivative = 3.98

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$$
$$= -\frac{B^2dn^2\log^3\left(\frac{a+bx}{c+dx}\right)}{3(bc-ad)^2g^2i} + \frac{2Bn\log\left(\frac{a+bx}{c+dx}\right)\left(A+Bn+B\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)\right)}{(-bc+ad)g^2i(a+bx)}$$
$$+ \frac{\log^2\left(\frac{a+bx}{c+dx}\right)\left(-aABdn-bB^2cn^2-AbBdnx-bB^2dn^2x-aB^2dn\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)-bB^2d\cdots\right)}{(-bc+ad)^2g^2i(a+bx)}$$
$$+ \frac{-A^2-2ABn-2B^2n^2-2AB\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)-2B^2n\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)-\cdots}{(bc-ad)g^2i(a+bx)}$$
$$- \frac{d\log(a+bx)\left(A^2+2ABn+2B^2n^2+2AB\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)+2B^2n\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\cdots\right.\right.}{(bc-ad)^2g^2i}$$
$$+ \frac{d\left(A^2+2ABn+2B^2n^2+2AB\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)+2B^2n\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)\cdots\right.}{(bc-ad)^2g^2i}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^2*(c*i +
d*i*x)),x]
```

```
[Out] -1/3*(B^2*d*n^2*Log[(a + b*x)/(c + d*x)]^3)/((b*c - a*d)^2*g^2*i) + (2*B*n*
Log[(a + b*x)/(c + d*x)]*(A + B*n + B*(Log[e*((a + b*x)/(c + d*x))^n] - n*L
og[(a + b*x)/(c + d*x)])))/((-(b*c) + a*d)*g^2*i*(a + b*x)) + (Log[(a + b*x
)/(c + d*x)]^2*(-(a*A*B*d*n) - b*B^2*c*n^2 - A*b*B*d*n*x - b*B^2*d*n^2*x -
a*B^2*d*n*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) - b
*B^2*d*n*x*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])))/
((-(b*c) + a*d)^2*g^2*i*(a + b*x)) + (-A^2 - 2*A*B*n - 2*B^2*n^2 - 2*A*B*(L
og[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) - 2*B^2*n*(Log[
e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) - B^2*(Log[e*((a +
 b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])^2)/((b*c - a*d)*g^2*i*(a
+ b*x)) - (d*Log[a + b*x]*(A^2 + 2*A*B*n + 2*B^2*n^2 + 2*A*B*(Log[e*((a + b
*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) + 2*B^2*n*(Log[e*((a + b*x)
/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) + B^2*(Log[e*((a + b*x)/(c + d
*x))^n] - n*Log[(a + b*x)/(c + d*x)])^2))/((b*c - a*d)^2*g^2*i) + (d*(A^2 +
 2*A*B*n + 2*B^2*n^2 + 2*A*B*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b
*x)/(c + d*x)]) + 2*B^2*n*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)
/(c + d*x)]) + B^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d
*x)])^2)*Log[c + d*x])/((b*c - a*d)^2*g^2*i)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(197) = 394$.

Time = 4.86 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.56

method	result
parallelrisch	$-\frac{-6B^2ab^3d^3n^3+6B^2b^4cd^2n^3-3A^2ab^3d^3n+3A^2b^4cd^2n+B^2x\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^3b^4d^3+B^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^3ab^3d^3+3A^2x\ln\left(\right.}{}$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x,method=_R
ETURNVERBOSE)
```

```
[Out] -1/3*(-6*B^2*a*b^3*d^3*n^3+6*B^2*b^4*c*d^2*n^3-3*A^2*a*b^3*d^3*n+3*A^2*b^4*
c*d^2*n+B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*b^4*d^3+B^2*ln(e*((b*x+a)/(d*x+c)
)^n)^3*a*b^3*d^3+3*A^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^3+3*A^2*ln(e*((b*x
+a)/(d*x+c))^n)*a*b^3*d^3+6*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^3*n+6*A*B
*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^2*n-6*A*B*a*b^3*d^3*n^2+6*A*B*b^4*c*d^2*
n^2+3*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*d^3*n+6*B^2*x*ln(e*((b*x+a)/(d*
x+c))^n)*b^4*d^3*n^2+3*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*d^3+3*B^2*ln(e
*((b*x+a)/(d*x+c))^n)^2*b^4*c*d^2*n+6*B^2*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d
^2*n^2+3*A*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^3*d^3)/i/g^2/(b*x+a)/n/(a^2*d^
2-2*a*b*c*d+b^2*c^2)/b^3/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(197) = 394$.

Time = 0.34 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.15

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{3A^2bc-3A^2ad+(B^2bdn^2x+B^2adn^2)\log\left(\frac{bx+a}{dx+c}\right)^3+6(B^2bc-B^2ad)n^2+3\left(B^2bc-B^2ad+(B^2bdx\right.}{}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x, a
lgorithm="fricas")
```

```
[Out] -1/3*(3*A^2*b*c - 3*A^2*a*d + (B^2*b*d*n^2*x + B^2*a*d*n^2)*log((b*x + a)/(
d*x + c))^3 + 6*(B^2*b*c - B^2*a*d)*n^2 + 3*(B^2*b*c - B^2*a*d + (B^2*b*d*x
 + B^2*a*d)*log((b*x + a)/(d*x + c)))*log(e)^2 + 3*(B^2*b*c*n^2 + A*B*a*d*n
 + (B^2*b*d*n^2 + A*B*b*d*n)*x)*log((b*x + a)/(d*x + c))^2 + 6*(A*B*b*c - A
*B*a*d)*n + 3*(2*A*B*b*c - 2*A*B*a*d + (B^2*b*d*n*x + B^2*a*d*n)*log((b*x +
 a)/(d*x + c))^2 + 2*(B^2*b*c - B^2*a*d)*n + 2*(B^2*b*c*n + A*B*a*d + (B^2*
b*d*n + A*B*b*d)*x)*log((b*x + a)/(d*x + c)))*log(e) + 3*(2*B^2*b*c*n^2 + 2
*A*B*b*c*n + A^2*a*d + (2*B^2*b*d*n^2 + 2*A*B*b*d*n + A^2*b*d)*x)*log((b*x
```

```
+ a)/(d*x + c)))/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^2*i*x + (a*b^2*c^2
- 2*a^2*b*c*d + a^3*d^2)*g^2*i)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**2/(d*i*x+c*i),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1018 vs. $2(197) = 394$.

Time = 0.27 (sec) , antiderivative size = 1018, normalized size of antiderivative = 5.12

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx =$$
$$-B^2\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i} + \frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i} - \frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)\log\left(e\right. \ldots$$
$$-2\,AB\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i} + \frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i} - \frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)\log\left(\right. \ldots$$
$$-\frac{1}{3}\left(\frac{\left((bdx+ad)\log(bx+a)^3 - (bdx+ad)\log(dx+c)^3 - 3\,(bdx+ad)\log(bx+a)^2 - 3\,(bdx+ad-(bc \ldots\right.}{ab \ldots}\right.$$
$$+\frac{\left((bdx+ad)\log(bx+a)^2 + (bdx+ad)\log(dx+c)^2 - 2\,bc + 2\,ad - 2\,(bdx+ad)\log(bx+a) + 2\,(bdx+ \ldots\right.}{ab^2c^2g^2i - 2\,a^2bcdg^2i + a^3d^2g^2i + (b^3c^2g^2i - 2\,ab^2cdg^2i + a^2bd \ldots}$$
$$-A^2\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i} + \frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i} - \frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x, a
lgorithm="maxima")
[Out] -B^2*(1/((b^2*c - a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/
((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*g^2*i))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2 - 2*A*B*(1/
((b^2*c - a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/((b^2*c^
2 - 2*a*b*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*g^2*i))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1/3*(((b*d*x + a*d)
*log(b*x + a)^3 - (b*d*x + a*d)*log(d*x + c)^3 - 3*(b*d*x + a*d)*log(b*x +
a)^2 - 3*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a))*log(d*x + c)^2 + 6*b*c
```

```
- 6*a*d + 6*(b*d*x + a*d)*log(b*x + a) - 3*(2*b*d*x + (b*d*x + a*d)*log(b*x
 + a)^2 + 2*a*d - 2*(b*d*x + a*d)*log(b*x + a))*log(d*x + c))*n^2/(a*b^2*c^
2*g^2*i - 2*a^2*b*c*d*g^2*i + a^3*d^2*g^2*i + (b^3*c^2*g^2*i - 2*a*b^2*c*d*
g^2*i + a^2*b*d^2*g^2*i)*x) - 3*((b*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*
d)*log(d*x + c)^2 - 2*b*c + 2*a*d - 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x
 + a*d - (b*d*x + a*d)*log(b*x + a))*log(d*x + c))*n*log(e*(b*x/(d*x + c) +
 a/(d*x + c))^n)/(a*b^2*c^2*g^2*i - 2*a^2*b*c*d*g^2*i + a^3*d^2*g^2*i + (b^
3*c^2*g^2*i - 2*a*b^2*c*d*g^2*i + a^2*b*d^2*g^2*i)*x))*B^2 + ((b*d*x + a*d)
*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d - 2*(b*d*x +
 a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a))*log(d*x +
 c))*A*B*n/(a*b^2*c^2*g^2*i - 2*a^2*b*c*d*g^2*i + a^3*d^2*g^2*i + (b^3*c^2*
g^2*i - 2*a*b^2*c*d*g^2*i + a^2*b*d^2*g^2*i)*x) - A^2*(1/((b^2*c - a*b*d)*g
^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/((b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g^2*i))
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2(dix+ci)}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x, a
lgorithm="giac")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/((b*g*x + a*g)^2*(d*i*x
+ c*i)), x)
```

Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.81

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$$
$$=\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{(a\,d-b\,c)\,\left(a\,g^2\,i+b\,g^2\,i\,x\right)}-\frac{B\,d\,(A+B\,n)}{g^2\,i\,n\,{(a\,d-b\,c)}^2}\right)+\frac{A^2+2\,A\,B\,n+2\,B^2\,n^2}{(a\,d-b\,c)\,\left(a\,g^2\,i+b\,g^2\,i\,x\right)}+\frac{2\,B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\,(A+B\,n)}{(a\,d-b\,c)\,\left(a\,g^2\,i+b\,g^2\,i\,x\right)}-\frac{B^2\,d\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)^3}{3\,g^2\,i\,n\,{(a\,d-b\,c)}^2}+\frac{d\,\mathrm{atan}\left(\frac{d\,\left(2\,b\,d\,x+\frac{a^2\,d^2\,g^2\,i-b^2\,c^2\,g^2\,i}{g^2\,i\,(a\,d-b\,c)}\right)\,\left(A^2+2\,A\,B\,n+2\,B^2\,n^2\right)\,1{}\mathrm{i}}{(a\,d-b\,c)\,\left(d\,A^2+2\,d\,A\,B\,n+2\,d\,B^2\,n^2\right)}\right)\,\left(A^2+2\,A\,B\,n+2\,B^2\,n^2\right)\,2{}\mathrm{i}}{g^2\,i\,{(a\,d-b\,c)}^2}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^2*(c*i + d*i*x)
),x)
```

```
[Out] log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/((a*d - b*c)*(a*g^2*i + b*g^2*i*x)) -
 (B*d*(A + B*n))/(g^2*i*n*(a*d - b*c)^2)) + (A^2 + 2*B^2*n^2 + 2*A*B*n)/((a
*d - b*c)*(a*g^2*i + b*g^2*i*x)) + (2*B*log(e*((a + b*x)/(c + d*x))^n)*(A +
 B*n))/((a*d - b*c)*(a*g^2*i + b*g^2*i*x)) + (d*atan((d*(2*b*d*x + (a^2*d^2
*g^2*i - b^2*c^2*g^2*i)/(g^2*i*(a*d - b*c)))*(A^2 + 2*B^2*n^2 + 2*A*B*n)*1i
)/((a*d - b*c)*(A^2*d + 2*B^2*d*n^2 + 2*A*B*d*n)))*(A^2 + 2*B^2*n^2 + 2*A*B
*n)*2i)/(g^2*i*(a*d - b*c)^2) - (B^2*d*log(e*((a + b*x)/(c + d*x))^n)^3)/(3
*g^2*i*n*(a*d - b*c)^2)
```

3.192 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 369

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = \frac{4bB^2dn^2(c+dx)}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2B^2n^2(c+dx)^2}{4(bc-ad)^3g^3i(a+bx)^2} + \frac{4bBdn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2Bn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2} + \frac{2bd(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^3g^3i(a+bx)^2} + \frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^3g^3in}$$

```
[Out] 4*b*B^2*d*n^2*(d*x+c)/(-a*d+b*c)^3/g^3/i/(b*x+a)-1/4*b^2*B^2*n^2*(d*x+c)^2/
(-a*d+b*c)^3/g^3/i/(b*x+a)^2+4*b*B*d*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^
n))/(-a*d+b*c)^3/g^3/i/(b*x+a)-1/2*b^2*B*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*
x+c))^n))/(-a*d+b*c)^3/g^3/i/(b*x+a)^2+2*b*d*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*
x+c))^n))^2/(-a*d+b*c)^3/g^3/i/(b*x+a)-1/2*b^2*(d*x+c)^2*(A+B*ln(e*((b*x+a)
/(d*x+c))^n))^2/(-a*d+b*c)^3/g^3/i/(b*x+a)^2+1/3*d^2*(A+B*ln(e*((b*x+a)/(d*
x+c))^n))^3/B/(-a*d+b*c)^3/g^3/i/n
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2561, 2395, 2342, 2341, 2339, 30}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = -\frac{b^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g^3i(a+bx)^2(bc-ad)^3} - \frac{b^2Bn(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^3i(a+bx)^2(bc-ad)^3} + \frac{d^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3Bg^3in(bc-ad)^3} + \frac{2bd(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^3i(a+bx)(bc-ad)^3} + \frac{4bBdn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^3i(a+bx)(bc-ad)^3} - \frac{b^2B^2n^2(c+dx)^2}{4g^3i(a+bx)^2(bc-ad)^3} + \frac{4bB^2dn^2(c+dx)}{g^3i(a+bx)(bc-ad)^3}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^3*(c*i + d*i*x)
),x]
```

```
[Out] (4*b*B^2*d*n^2*(c + d*x))/((b*c - a*d)^3*g^3*i*(a + b*x)) - (b^2*B^2*n^2*(c
 + d*x)^2)/(4*(b*c - a*d)^3*g^3*i*(a + b*x)^2) + (4*b*B*d*n*(c + d*x)*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^3*g^3*i*(a + b*x)) - (b^2*B
*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*c - a*d)^3*g^3
*i*(a + b*x)^2) + (2*b*d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
)/((b*c - a*d)^3*g^3*i*(a + b*x)) - (b^2*(c + d*x)^2*(A + B*Log[e*((a + b*x
)/(c + d*x))^n])^2)/(2*(b*c - a*d)^3*g^3*i*(a + b*x)^2) + (d^2*(A + B*Log[e
*((a + b*x)/(c + d*x))^n])^3)/(3*B*(b*c - a*d)^3*g^3*i*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i} \\
&= \frac{\text{Subst}\left(\int \left(\frac{b^2(A+B\log(ex^n))^2}{x^3} - \frac{2bd(A+B\log(ex^n))^2}{x^2} + \frac{d^2(A+B\log(ex^n))^2}{x}\right) dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i} \\
&= \frac{b^2\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i} - \frac{(2bd)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i} \\
&\quad + \frac{d^2\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i}
\end{aligned}$$

$$= \frac{2bd(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^3g^3i(a+bx)^2} + \frac{d^2\text{Subst}\left(\int x^2\,dx, x, A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)^3g^3in} + \frac{(b^2Bn)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i} - \frac{(4bBdn)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i}$$

$$= \frac{4bB^2dn^2(c+dx)}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2B^2n^2(c+dx)^2}{4(bc-ad)^3g^3i(a+bx)^2} + \frac{4bBdn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2Bn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2} + \frac{2bd(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^3i(a+bx)} - \frac{b^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^3g^3i(a+bx)^2} + \frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^3g^3in}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 975 vs. $2(369) = 738$.

Time = 0.77 (sec) , antiderivative size = 975, normalized size of antiderivative = 2.64

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = \frac{4B^2d^2n^2(a+bx)^2\log^3\left(\frac{a+bx}{c+dx}\right) + 6Bn\log^2\left(\frac{a+bx}{c+dx}\right)\left(2a^2Ad^2 - b^2Bc^2n + 4abBcdn + 4aAbd^2x + 2b^2Bcdnx + \right.}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^3*(c*i + 
d*i*x)),x]
```

```
[Out] (4*B^2*d^2*n^2*(a + b*x)^2*Log[(a + b*x)/(c + d*x)]^3 + 6*B*n*Log[(a + b*x)
/(c + d*x)]^2*(2*a^2*A*d^2 - b^2*B*c^2*n + 4*a*b*B*c*d*n + 4*a*A*b*d^2*x + 
2*b^2*B*c*d*n*x + 4*a*b*B*d^2*n*x + 2*A*b^2*d^2*x^2 + 3*b^2*B*d^2*n*x^2 + 2
*B*d^2*(a + b*x)^2*Log[e*((a + b*x)/(c + d*x))^n] - 2*B*d^2*n*(a + b*x)^2*L
og[(a + b*x)/(c + d*x)]) - 6*B*(b*c - a*d)*n*Log[(a + b*x)/(c + d*x)]*(2*A*
b*c - 6*a*A*d + b*B*c*n - 7*a*B*d*n - 4*A*b*d*x - 6*b*B*d*n*x + 2*B*(-3*a*d
 + b*(c - 2*d*x))*Log[e*((a + b*x)/(c + d*x))^n] + 2*B*n*(-(b*c) + 3*a*d +
2*b*d*x)*Log[(a + b*x)/(c + d*x)]) - 3*(b*c - a*d)^2*(2*A^2 + 2*A*B*n + B^2
*n^2 + 2*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 2*B*n*(2*A + B*n)*Log[(a +
b*x)/(c + d*x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 2*B*Log[e*((a + b*
```

```
x)/(c + d*x))^n]*(2*A + B*n - 2*B*n*Log[(a + b*x)/(c + d*x)])) + 6*d*(b*c -
 a*d)*(a + b*x)*(2*A^2 + 6*A*B*n + 7*B^2*n^2 + 2*B^2*Log[e*((a + b*x)/(c +
d*x))^n]^2 - 2*B*n*(2*A + 3*B*n)*Log[(a + b*x)/(c + d*x)] + 2*B^2*n^2*Log[(
a + b*x)/(c + d*x)]^2 + 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(2*A + 3*B*n - 2
*B*n*Log[(a + b*x)/(c + d*x)])) + 6*d^2*(a + b*x)^2*Log[a + b*x]*(2*A^2 + 6
*A*B*n + 7*B^2*n^2 + 2*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 2*B*n*(2*A +
3*B*n)*Log[(a + b*x)/(c + d*x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 2*
B*Log[e*((a + b*x)/(c + d*x))^n]*(2*A + 3*B*n - 2*B*n*Log[(a + b*x)/(c + d*
x)])) - 6*d^2*(a + b*x)^2*(2*A^2 + 6*A*B*n + 7*B^2*n^2 + 2*B^2*Log[e*((a +
b*x)/(c + d*x))^n]^2 - 2*B*n*(2*A + 3*B*n)*Log[(a + b*x)/(c + d*x)] + 2*B^2
*n^2*Log[(a + b*x)/(c + d*x)]^2 + 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(2*A +
 3*B*n - 2*B*n*Log[(a + b*x)/(c + d*x)]))*Log[c + d*x])/(12*(b*c - a*d)^3*g
^3*i*(a + b*x)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1258 vs. $2(361) = 722$.

Time = 11.70 (sec) , antiderivative size = 1259, normalized size of antiderivative = 3.41

method	result	size
parallelrisch	Expression too large to display	1259

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x,method=_R
ETURNVERBOSE)
```

```
[Out] -1/12*(24*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^2*c^3*d*n+12*B^2*x*ln(e*((b
*x+a)/(d*x+c))^n)^2*a^4*b^2*c^3*d*n+36*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^
4*b^2*c^2*d^2*n+48*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b*c^2*d^2*n+48*B^2*x
*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b*c^2*d^2*n^2+36*B^2*x*ln(e*((b*x+a)/(d*x+c)
)^n)*a^4*b^2*c^3*d*n^2+24*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^5*b*c^2*d^2+4
8*A*B*x*a^5*b*c^2*d^2*n^2-60*A*B*x*a^4*b^2*c^3*d*n^2+48*A*B*ln(e*((b*x+a)/(
d*x+c))^n)*a^5*b*c^3*d*n+18*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b^2*c^2
*d^2*n+42*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^2*c^2*d^2*n^2+12*A*B*x^2*
ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b^2*c^2*d^2+42*A*B*x^2*a^4*b^2*c^2*d^2*n^2-
48*A*B*x^2*a^3*b^3*c^3*d*n^2+24*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^5*b*c^2
*d^2*n+4*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a^4*b^2*c^2*d^2+45*B^2*x^2*a^4
*b^2*c^2*d^2*n^3-48*B^2*x^2*a^3*b^3*c^3*d*n^3+6*A*B*x^2*a^2*b^4*c^4*n^2+8*B
^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*a^5*b*c^2*d^2+48*B^2*x*a^5*b*c^2*d^2*n^3-5
4*B^2*x*a^4*b^2*c^3*d*n^3+12*A^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^2*c^2*
d^2+18*A^2*x^2*a^4*b^2*c^2*d^2*n-24*A^2*x^2*a^3*b^3*c^3*d*n+12*A*B*x*a^3*b^
3*c^4*n^2+24*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^5*b*c^3*d*n+48*B^2*ln(e*((b*
x+a)/(d*x+c))^n)*a^5*b*c^3*d*n^2+24*A^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b*c
^2*d^2+24*A^2*x*a^5*b*c^2*d^2*n-36*A^2*x*a^4*b^2*c^3*d*n-12*A*B*ln(e*((b*x+
a)/(d*x+c))^n)*a^4*b^2*c^4*n+3*B^2*x^2*a^2*b^4*c^4*n^3+6*B^2*x*a^3*b^3*c^4*
n^3+6*A^2*x^2*a^2*b^4*c^4*n-6*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b^2*c^4*n
```

-6*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^2*c^4*n^2+12*A^2*x*a^3*b^3*c^4*n+12*A*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*c^2*d^2+4*B^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a^6*c^2*d^2+12*A^2*ln(e*((b*x+a)/(d*x+c))^n)*a^6*c^2*d^2)/i/g^3/(b*x+a)^2/a^4/c^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/n

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1068 vs. $2(361) = 722$.

Time = 0.35 (sec) , antiderivative size = 1068, normalized size of antiderivative = 2.89

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = -\frac{6A^2b^2c^2-24A^2abcd+18A^2a^2d^2-4\left(B^2b^2d^2n^2x^2+2B^2abd^2n^2x+B^2a^2d^2n^2\right)\log\left(\frac{bx+a}{dx+c}\right)^3+3\left(B^2b^2c^2\right.}{}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x, a
lgorithm="fricas")
```

```
[Out] -1/12*(6*A^2*b^2*c^2 - 24*A^2*a*b*c*d + 18*A^2*a^2*d^2 - 4*(B^2*b^2*d^2*n^2
*x^2 + 2*B^2*a*b*d^2*n^2*x + B^2*a^2*d^2*n^2)*log((b*x + a)/(d*x + c))^3 +
3*(B^2*b^2*c^2 - 16*B^2*a*b*c*d + 15*B^2*a^2*d^2)*n^2 + 6*(B^2*b^2*c^2 - 4*
B^2*a*b*c*d + 3*B^2*a^2*d^2 - 2*(B^2*b^2*c*d - B^2*a*b*d^2)*x - 2*(B^2*b^2*
d^2*x^2 + 2*B^2*a*b*d^2*x + B^2*a^2*d^2)*log((b*x + a)/(d*x + c)))*log(e)^2
 - 6*(2*A*B*a^2*d^2*n - (B^2*b^2*c^2 - 4*B^2*a*b*c*d)*n^2 + (3*B^2*b^2*d^2*
n^2 + 2*A*B*b^2*d^2*n)*x^2 + 2*(2*A*B*a*b*d^2*n + (B^2*b^2*c*d + 2*B^2*a*b*
d^2)*n^2)*x)*log((b*x + a)/(d*x + c))^2 + 6*(A*B*b^2*c^2 - 8*A*B*a*b*c*d +
7*A*B*a^2*d^2)*n - 6*(2*A^2*b^2*c*d - 2*A^2*a*b*d^2 + 7*(B^2*b^2*c*d - B^2*
a*b*d^2)*n^2 + 6*(A*B*b^2*c*d - A*B*a*b*d^2)*n)*x + 6*(2*A*B*b^2*c^2 - 8*A*
B*a*b*c*d + 6*A*B*a^2*d^2 - 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*a*b*d^2*n*x + B^2*
a^2*d^2*n)*log((b*x + a)/(d*x + c))^2 + (B^2*b^2*c^2 - 8*B^2*a*b*c*d + 7*B^
2*a^2*d^2)*n - 2*(2*A*B*b^2*c*d - 2*A*B*a*b*d^2 + 3*(B^2*b^2*c*d - B^2*a*b*
d^2)*n)*x - 2*(2*A*B*a^2*d^2 + (3*B^2*b^2*d^2*n + 2*A*B*b^2*d^2)*x^2 - (B^2
*b^2*c^2 - 4*B^2*a*b*c*d)*n + 2*(2*A*B*a*b*d^2 + (B^2*b^2*c*d + 2*B^2*a*b*d
^2)*n)*x)*log((b*x + a)/(d*x + c)))*log(e) - 6*(2*A^2*a^2*d^2 - (B^2*b^2*c^
2 - 8*B^2*a*b*c*d)*n^2 + (7*B^2*b^2*d^2*n^2 + 6*A*B*b^2*d^2*n + 2*A^2*b^2*d
^2)*x^2 - 2*(A*B*b^2*c^2 - 4*A*B*a*b*c*d)*n + 2*(2*A^2*a*b*d^2 + (3*B^2*b^2
*c*d + 4*B^2*a*b*d^2)*n^2 + 2*(A*B*b^2*c*d + 2*A*B*a*b*d^2)*n)*x)*log((b*x
+ a)/(d*x + c)))/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)
*g^3*i*x^2 + 2*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*
g^3*i*x + (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*g^3*i)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**3/(d*i*x+c*i),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2126 vs. 2(361) = 722.

Time = 0.34 (sec) , antiderivative size = 2126, normalized size of antiderivative = 5.76

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x, a
lgorithm="maxima")
[Out] 1/2*B^2*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3
*i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i*x + (a^2*b^2*c^2 -
 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c)/((b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i))*log(e*(b*x/(d*x + c) + a/(d*x +
 c))^n)^2 + A*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*
d^2)*g^3*i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i*x + (a^2*b
^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c)/((b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i))*log(e*(b*x/(d*x + c) +
a/(d*x + c))^n) - 1/12*((3*b^2*c^2 - 48*a*b*c*d + 45*a^2*d^2 - 4*(b^2*d^2*x
^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^3 + 4*(b^2*d^2*x^2 + 2*a*b*d^2*x +
 a^2*d^2)*log(d*x + c)^3 + 18*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x
 + a)^2 + 6*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a
*b*d^2*x + a^2*d^2)*log(b*x + a))*log(d*x + c)^2 - 42*(b^2*c*d - a*b*d^2)*x
 - 42*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 6*(7*b^2*d^2*x^2
 + 14*a*b*d^2*x + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b
*x + a)^2 - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))*log(d*x +
 c))*n^2/(a^2*b^3*c^3*g^3*i - 3*a^3*b^2*c^2*d*g^3*i + 3*a^4*b*c*d^2*g^3*i -
 a^5*d^3*g^3*i + (b^5*c^3*g^3*i - 3*a*b^4*c^2*d*g^3*i + 3*a^2*b^3*c*d^2*g^3
*i - a^3*b^2*d^3*g^3*i)*x^2 + 2*(a*b^4*c^3*g^3*i - 3*a^2*b^3*c^2*d*g^3*i +
3*a^3*b^2*c*d^2*g^3*i - a^4*b*d^3*g^3*i)*x) + 6*(b^2*c^2 - 8*a*b*c*d + 7*a^
2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^2 + 2*(b^2*d^2
```

```
*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2)*x - 6*
(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*x^2 + 6*a
*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)
)*log(d*x + c))*n*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(a^2*b^3*c^3*g^3*i
 - 3*a^3*b^2*c^2*d*g^3*i + 3*a^4*b*c*d^2*g^3*i - a^5*d^3*g^3*i + (b^5*c^3*g
^3*i - 3*a*b^4*c^2*d*g^3*i + 3*a^2*b^3*c*d^2*g^3*i - a^3*b^2*d^3*g^3*i)*x^2
 + 2*(a*b^4*c^3*g^3*i - 3*a^2*b^3*c^2*d*g^3*i + 3*a^3*b^2*c*d^2*g^3*i - a^4
*b*d^3*g^3*i)*x))*B^2 - 1/2*(b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x
^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x +
 a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b
*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2
 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))*log(d*x + c))*A*B*
n/(a^2*b^3*c^3*g^3*i - 3*a^3*b^2*c^2*d*g^3*i + 3*a^4*b*c*d^2*g^3*i - a^5*d^
3*g^3*i + (b^5*c^3*g^3*i - 3*a*b^4*c^2*d*g^3*i + 3*a^2*b^3*c*d^2*g^3*i - a^
3*b^2*d^3*g^3*i)*x^2 + 2*(a*b^4*c^3*g^3*i - 3*a^2*b^3*c^2*d*g^3*i + 3*a^3*b
^2*c*d^2*g^3*i - a^4*b*d^3*g^3*i)*x) + 1/2*A^2*((2*b*d*x - b*c + 3*a*d)/((b
^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3*i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*
d + a^3*b*d^2)*g^3*i*x + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d
^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i)
 - 2*d^2*log(d*x + c)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
g^3*i))
```

Giac [A] (verification not implemented)

none

Time = 277.82 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.53

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = -\frac{1}{4}\left(\frac{2\,(dx+c)^2B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(bx+a)^2g^3i} + \frac{2\,(B^2n^2+2\,B^2n\log(e)+2\,ABn)(dx+c)^2\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)^2g^3i} + \frac{(B^2n^2+2\,B\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x, a
lgorithm="giac")
```

```
[Out] -1/4*(2*(d*x + c)^2*B^2*n^2*log((b*x + a)/(d*x + c))^2/((b*x + a)^2*g^3*i)
+ 2*(B^2*n^2 + 2*B^2*n*log(e) + 2*A*B*n)*(d*x + c)^2*log((b*x + a)/(d*x + c
))/((b*x + a)^2*g^3*i) + (B^2*n^2 + 2*B^2*n*log(e) + 2*B^2*log(e)^2 + 2*A*B
*n + 4*A*B*log(e) + 2*A^2)*(d*x + c)^2/((b*x + a)^2*g^3*i))*(b*c/(b*c - a*d
)^2 - a*d/(b*c - a*d)^2)^2
```

Mupad [B] (verification not implemented)

Time = 5.25 (sec) , antiderivative size = 1011, normalized size of antiderivative = 2.74

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx$$

$$=\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B^2\,n}{x^2\,\left(b^3\,c\,g^3\,i-a\,b^2\,d\,g^3\,i\right)+x\,\left(2\,a\,b^2\,c\,g^3\,i-2\,a^2\,b\,d\,g^3\,i\right)-a^3\,d\,g^3\,i+a^2\,b\,c\,g^3\,i}\right.$$
$$-\frac{d^2\,\left(3\,n\,B^2+2\,A\,B\right)\,\left(\frac{a\,g^3\,i\,n\,(a\,d-b\,c)^2}{2\,d}+\frac{g^3\,i\,n\,(a\,d-b\,c)^2\,(2\,a\,d-b\,c)}{2\,d^2}+\frac{b\,g^3\,i\,n\,x\,(a\,d-b\,c)^2}{d}\right)}{g^3\,i\,n\,\left(a\,d-b\,c\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(x^2\,\left(b^3\,c\,g^3\,i-a\,b^2\,d\,g^3\,i\right)+x\,\left(2\,a\,b^2\,c\,g^3\,i-2\,a^2\,b\,d\,g^3\,i\right)-a^3\cdots\right.}$$
$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{d^2\,\left(3\,n\,B^2+2\,A\,B\right)}{2\,g^3\,i\,n\,\left(a\,d-b\,c\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right.$$
$$\left.-\frac{B^2\,d^2\,\left(\frac{g^3\,i\,n\,(a\,d-b\,c)\,(2\,a\,d-b\,c)}{2\,d^2}+\frac{a\,g^3\,i\,n\,(a\,d-b\,c)}{2\,d}+\frac{b\,g^3\,i\,n\,x\,(a\,d-b\,c)}{d}\right)}{g^3\,i\,n\,\left(a\,d-b\,c\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(i\,a^2\,g^3+2\,i\,a\,b\,g^3\,x+i\,b^2\,g^3\,x^2\right)}\right)$$
$$-\frac{\frac{6\,A^2\,a\,d-2\,A^2\,b\,c+15\,B^2\,a\,d\,n^2-B^2\,b\,c\,n^2+14\,A\,B\,a\,d\,n-2\,A\,B\,b\,c\,n}{2\,(a\,d-b\,c)}+\frac{x\,\left(2\,b\,d\,A^2+6\,b\,d\,A\,B\,n+7\,b\,d\,B^2\,n^2\right)}{a\,d-b\,c}}{x^2\,\left(2\,b^3\,c\,g^3\,i-2\,a\,b^2\,d\,g^3\,i\right)+x\,\left(4\,a\,b^2\,c\,g^3\,i-4\,a^2\,b\,d\,g^3\,i\right)-2\,a^3\,d\,g^3\,i+2\,a^2\,b\,c\,g^3\,i}$$
$$-\frac{B^2\,d^2\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^3}{3\,g^3\,i\,n\,\left(a\,d-b\,c\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}$$
$$+\frac{d^2\,\mathrm{atan}\left(\frac{d^2\,\left(\frac{i\,a^3\,d^3\,g^3-i\,a^2\,b\,c\,d^2\,g^3-i\,a\,b^2\,c^2\,d\,g^3+i\,b^3\,c^3\,g^3}{i\,a^2\,d^2\,g^3-2\,i\,a\,b\,c\,d\,g^3+i\,b^2\,c^2\,g^3}+2\,b\,d\,x\right)\,\left(A^2+3\,A\,B\,n+\frac{7\,B^2\,n^2}{2}\right)\,\left(i\,a^2\,d^2\,g^3-2\,i\,a\,b\,c\,d\,g^3+i\,b^2\,c^2\,g^3\right)\,2\mathrm{i}}{g^3\,i\,(a\,d-b\,c)^3\,\left(2\,A^2\,d^2+6\,A\,B\,d^2\,n+7\,B^2\,d^2\,n^2\right)}\right)\,\left(A\cdots\right.}{g^3\,i\,\left(a\,d-b\,c\right)^3}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^3*(c*i + d*i*x)
),x)
```

```
[Out] log(e*((a + b*x)/(c + d*x))^n)*((B^2*n)/(x^2*(b^3*c*g^3*i - a*b^2*d*g^3*i)
+ x*(2*a*b^2*c*g^3*i - 2*a^2*b*d*g^3*i) - a^3*d*g^3*i + a^2*b*c*g^3*i) - (d
^2*(3*B^2*n + 2*A*B)*((a*g^3*i*n*(a*d - b*c)^2)/(2*d) + (g^3*i*n*(a*d - b*c
)^2*(2*a*d - b*c))/(2*d^2) + (b*g^3*i*n*x*(a*d - b*c)^2)/d))/(g^3*i*n*(a*d
- b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(x^2*(b^3*c*g^3*i - a*b^2*d*g^3*i) +
 x*(2*a*b^2*c*g^3*i - 2*a^2*b*d*g^3*i) - a^3*d*g^3*i + a^2*b*c*g^3*i))) - l
og(e*((a + b*x)/(c + d*x))^n)^2*((d^2*(3*B^2*n + 2*A*B))/(2*g^3*i*n*(a*d -
b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (B^2*d^2*((g^3*i*n*(a*d - b*c)*(2*a
*d - b*c))/(2*d^2) + (a*g^3*i*n*(a*d - b*c))/(2*d) + (b*g^3*i*n*x*(a*d - b*
c))/d))/(g^3*i*n*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*g^3*i + b
^2*g^3*i*x^2 + 2*a*b*g^3*i*x))) - ((6*A^2*a*d - 2*A^2*b*c + 15*B^2*a*d*n^2
- B^2*b*c*n^2 + 14*A*B*a*d*n - 2*A*B*b*c*n)/(2*(a*d - b*c)) + (x*(2*A^2*b*d
 + 7*B^2*b*d*n^2 + 6*A*B*b*d*n))/(a*d - b*c))/(x^2*(2*b^3*c*g^3*i - 2*a*b^2
```

```
*d*g^3*i) + x*(4*a*b^2*c*g^3*i - 4*a^2*b*d*g^3*i) - 2*a^3*d*g^3*i + 2*a^2*b
*c*g^3*i) + (d^2*atan((d^2*((a^3*d^3*g^3*i + b^3*c^3*g^3*i - a*b^2*c^2*d*g^
3*i - a^2*b*c*d^2*g^3*i)/(a^2*d^2*g^3*i + b^2*c^2*g^3*i - 2*a*b*c*d*g^3*i)
+ 2*b*d*x)*(A^2 + (7*B^2*n^2)/2 + 3*A*B*n)*(a^2*d^2*g^3*i + b^2*c^2*g^3*i -
 2*a*b*c*d*g^3*i)*2i)/(g^3*i*(a*d - b*c)^3*(2*A^2*d^2 + 7*B^2*d^2*n^2 + 6*A
*B*d^2*n)))*(A^2 + (7*B^2*n^2)/2 + 3*A*B*n)*2i)/(g^3*i*(a*d - b*c)^3) - (B^
2*d^2*log(e*((a + b*x)/(c + d*x))^n)^3)/(3*g^3*i*n*(a*d - b*c)*(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d))
```

3.193 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 543

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx =& -\frac{6bB^2d^2n^2(c+dx)}{(bc-ad)^4g^4i(a+bx)}+\frac{3b^2B^2dn^2(c+dx)^2}{4(bc-ad)^4g^4i(a+bx)^2}\\&-\frac{2b^3B^2n^2(c+dx)^3}{27(bc-ad)^4g^4i(a+bx)^3}\\&-\frac{6bBd^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^4i(a+bx)}\\&+\frac{3b^2Bdn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2}\\&-\frac{2b^3Bn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^4g^4i(a+bx)^3}\\&-\frac{3bd^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^4i(a+bx)}\\&+\frac{3b^2d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^4g^4i(a+bx)^2}\\&-\frac{b^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^4g^4i(a+bx)^3}\\&-\frac{d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^4g^4in}\end{aligned}$$

```
[Out] -6*b*B^2*d^2*n^2*(d*x+c)/(-a*d+b*c)^4/g^4/i/(b*x+a)+3/4*b^2*B^2*d*n^2*(d*x+
c)^2/(-a*d+b*c)^4/g^4/i/(b*x+a)^2-2/27*b^3*B^2*n^2*(d*x+c)^3/(-a*d+b*c)^4/g
^4/i/(b*x+a)^3-6*b*B*d^2*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*
```

```
c)^4/g^4/i/(b*x+a)+3/2*b^2*B*d*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/
(-a*d+b*c)^4/g^4/i/(b*x+a)^2-2/9*b^3*B*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+
c))^n))/(-a*d+b*c)^4/g^4/i/(b*x+a)^3-3*b*d^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*
x+c))^n))^2/(-a*d+b*c)^4/g^4/i/(b*x+a)+3/2*b^2*d*(d*x+c)^2*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^4/i/(b*x+a)^2-1/3*b^3*(d*x+c)^3*(A+B*ln(e*
((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^4/i/(b*x+a)^3-1/3*d^3*(A+B*ln(e*((b*
x+a)/(d*x+c))^n))^3/B/(-a*d+b*c)^4/g^4/i/n
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2561, 2395, 2342, 2341, 2339, 30}

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx=&-\frac{b^3(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g^4i(a+bx)^3(bc-ad)^4}\\&-\frac{2b^3Bn(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9g^4i(a+bx)^3(bc-ad)^4}\\&+\frac{3b^2d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g^4i(a+bx)^2(bc-ad)^4}\\&+\frac{3b^2Bdn(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^4i(a+bx)^2(bc-ad)^4}\\&-\frac{d^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3Bg^4in(bc-ad)^4}\\&-\frac{3bd^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^4i(a+bx)(bc-ad)^4}\\&-\frac{6bBd^2n(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^4i(a+bx)(bc-ad)^4}\\&-\frac{2b^3B^2n^2(c+dx)^3}{27g^4i(a+bx)^3(bc-ad)^4}\\&+\frac{3b^2B^2dn^2(c+dx)^2}{4g^4i(a+bx)^2(bc-ad)^4}-\frac{6bB^2d^2n^2(c+dx)}{g^4i(a+bx)(bc-ad)^4}\end{aligned}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^4*(c*i + d*i*x)
),x]
```

```
[Out] (-6*b*B^2*d^2*n^2*(c + d*x))/((b*c - a*d)^4*g^4*i*(a + b*x)) + (3*b^2*B^2*d
*n^2*(c + d*x)^2)/(4*(b*c - a*d)^4*g^4*i*(a + b*x)^2) - (2*b^3*B^2*n^2*(c +
 d*x)^3)/(27*(b*c - a*d)^4*g^4*i*(a + b*x)^3) - (6*b*B*d^2*n*(c + d*x)*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^4*g^4*i*(a + b*x)) + (3*b^
2*B*d*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*c - a*d)^
```

```
4*g^4*i*(a + b*x)^2) - (2*b^3*B*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(9*(b*c - a*d)^4*g^4*i*(a + b*x)^3) - (3*b*d^2*(c + d*x)*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*c - a*d)^4*g^4*i*(a + b*x)) + (3*b^2
*d*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(b*c - a*d)^4*g
^4*i*(a + b*x)^2) - (b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2)/(3*(b*c - a*d)^4*g^4*i*(a + b*x)^3) - (d^3*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n])^3)/(3*B*(b*c - a*d)^4*g^4*i*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
```

```
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)^3(A+B\log(ex^n))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^3(A+B\log(ex^n))^2}{x^4} - \frac{3b^2d(A+B\log(ex^n))^2}{x^3} + \frac{3bd^2(A+B\log(ex^n))^2}{x^2} - \frac{d^3(A+B\log(ex^n))^2}{x}\right)dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i}$$

$$= \frac{b^3\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i} - \frac{(3b^2d)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i} + \frac{(3bd^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i} - \frac{d^3\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i}$$

$$= -\frac{3bd^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^4i(a+bx)} + \frac{3b^2d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^4g^4i(a+bx)^2} - \frac{b^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^4g^4i(a+bx)^3} - \frac{d^3\text{Subst}\left(\int x^2\,dx, x, A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)^4g^4in} + \frac{(2b^3Bn)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bc-ad)^4g^4i} - \frac{(3b^2Bdn)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i} + \frac{(6bBd^2n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i}$$

$$
\begin{aligned}
&= -\frac{6bB^2d^2n^2(c+dx)}{(bc-ad)^4g^4i(a+bx)} + \frac{3b^2B^2dn^2(c+dx)^2}{4(bc-ad)^4g^4i(a+bx)^2} \\
&\quad - \frac{2b^3B^2n^2(c+dx)^3}{27(bc-ad)^4g^4i(a+bx)^3} - \frac{6bBd^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^4i(a+bx)} \\
&\quad + \frac{3b^2Bdn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2} \\
&\quad - \frac{2b^3Bn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^4g^4i(a+bx)^3} \\
&\quad - \frac{3bd^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^4i(a+bx)} + \frac{3b^2d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^4g^4i(a+bx)^2} \\
&\quad - \frac{b^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^4g^4i(a+bx)^3} - \frac{d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^4g^4in}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1295 vs. $2(543) = 1086$.

Time = 1.00 (sec) , antiderivative size = 1295, normalized size of antiderivative = 2.38

$$
\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = -\frac{36B^2d^3n^2(a+bx)^3\log^3\left(\frac{a+bx}{c+dx}\right)+18Bn\log^2\left(\frac{a+bx}{c+dx}\right)\left(6a^3Ad^3+2b^3Bc^3n-9ab^2Bc^2dn+18a^2bBcd^2n+\right.}{}
$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^4*(c*i +
d*i*x)),x]
```

```
[Out] -1/108*(36*B^2*d^3*n^2*(a + b*x)^3*Log[(a + b*x)/(c + d*x)]^3 + 18*B*n*Log[
(a + b*x)/(c + d*x)]^2*(6*a^3*A*d^3 + 2*b^3*B*c^3*n - 9*a*b^2*B*c^2*d*n + 1
8*a^2*b*B*c*d^2*n + 18*a^2*A*b*d^3*x - 3*b^3*B*c^2*d*n*x + 18*a*b^2*B*c*d^2
*n*x + 18*a^2*b*B*d^3*n*x + 18*a*A*b^2*d^3*x^2 + 6*b^3*B*c*d^2*n*x^2 + 27*a
*b^2*B*d^3*n*x^2 + 6*A*b^3*d^3*x^3 + 11*b^3*B*d^3*n*x^3 + 6*B*d^3*(a + b*x)
^3*Log[e*((a + b*x)/(c + d*x))^n] - 6*B*d^3*n*(a + b*x)^3*Log[(a + b*x)/(c
 + d*x)]) - 3*d*(b*c - a*d)^2*(a + b*x)*(18*A^2 + 30*A*B*n + 19*B^2*n^2 + 18
*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(6*A + 5*B*n)*Log[(a + b*x)/(
c + d*x)] + 18*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 6*B*Log[e*((a + b*x)/(c
 + d*x))^n]*(6*A + 5*B*n - 6*B*n*Log[(a + b*x)/(c + d*x)])) + 6*d^2*(b*c -
a*d)*(a + b*x)^2*(18*A^2 + 66*A*B*n + 85*B^2*n^2 + 18*B^2*Log[e*((a + b*x)/
(c + d*x))^n]^2 - 6*B*n*(6*A + 11*B*n)*Log[(a + b*x)/(c + d*x)] + 18*B^2*n^
2*Log[(a + b*x)/(c + d*x)]^2 + 6*B*Log[e*((a + b*x)/(c + d*x))^n]*(6*A + 11
*B*n - 6*B*n*Log[(a + b*x)/(c + d*x)])) + 6*d^3*(a + b*x)^3*Log[a + b*x]*(1
8*A^2 + 66*A*B*n + 85*B^2*n^2 + 18*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 6
*B*n*(6*A + 11*B*n)*Log[(a + b*x)/(c + d*x)] + 18*B^2*n^2*Log[(a + b*x)/(c
```

```
+ d*x)]^2 + 6*B*Log[e*((a + b*x)/(c + d*x))^n]*(6*A + 11*B*n - 6*B*n*Log[(a
 + b*x)/(c + d*x)])) + 4*(b*c - a*d)^3*(9*A^2 + 6*A*B*n + 2*B^2*n^2 + 9*B^2
*Log[e*((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(3*A + B*n)*Log[(a + b*x)/(c + d*
x)] + 9*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 6*B*Log[e*((a + b*x)/(c + d*x)
)^n]*(3*A + B*n - 3*B*n*Log[(a + b*x)/(c + d*x)])) + 6*B*(b*c - a*d)*n*Log[
(a + b*x)/(c + d*x)]*(3*d*(-(b*c) + a*d)*(a + b*x)*(6*A + 5*B*n + 6*B*Log[e
*((a + b*x)/(c + d*x))^n] - 6*B*n*Log[(a + b*x)/(c + d*x)]) + 6*d^2*(a + b*
x)^2*(6*A + 11*B*n + 6*B*Log[e*((a + b*x)/(c + d*x))^n] - 6*B*n*Log[(a + b*
x)/(c + d*x)]) + 4*(b*c - a*d)^2*(3*A + B*n + 3*B*Log[e*((a + b*x)/(c + d*x
))^n] - 3*B*n*Log[(a + b*x)/(c + d*x)])) - 6*d^3*(a + b*x)^3*(18*A^2 + 66*A
*B*n + 85*B^2*n^2 + 18*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(6*A +
11*B*n)*Log[(a + b*x)/(c + d*x)] + 18*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 +
6*B*Log[e*((a + b*x)/(c + d*x))^n]*(6*A + 11*B*n - 6*B*n*Log[(a + b*x)/(c +
 d*x)]))*Log[c + d*x])/((b*c - a*d)^4*g^4*i*(a + b*x)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2215 vs. 2(529) = 1058.

Time = 25.75 (sec) , antiderivative size = 2216, normalized size of antiderivative = 4.08

method	result	size
parallelrisch	Expression too large to display	2216

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x,method=_R
ETURNVERBOSE)
```

```
[Out] -1/108*(-8*B^2*x^3*a^2*b^6*c^5*n^3-24*B^2*x^2*a^3*b^5*c^5*n^3-36*A^2*x^3*a^
2*b^6*c^5*n-24*B^2*x*a^4*b^4*c^5*n^3-108*A^2*x^2*a^3*b^5*c^5*n+36*B^2*ln(e*
((b*x+a)/(d*x+c))^n)^2*a^5*b^3*c^5*n+24*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b
^3*c^5*n^2-108*A^2*x*a^4*b^4*c^5*n+108*A*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a^8*
c^2*d^3+396*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^3*c^2*d^3*n+972*A*B*x^2
*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^2*d^3*n+216*A*B*x^2*ln(e*((b*x+a)/(d*x
+c))^n)*a^5*b^3*c^3*d^2*n+648*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c^2*d^3
*n+648*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^3*d^2*n-108*A*B*x*ln(e*((b
*x+a)/(d*x+c))^n)*a^5*b^3*c^4*d*n+198*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*a
^5*b^3*c^2*d^3*n+510*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^3*c^2*d^3*n^2+
108*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*a^5*b^3*c^2*d^3+510*A*B*x^3*a^5*b^3
*c^2*d^3*n^2-648*A*B*x^3*a^4*b^4*c^3*d^2*n^2+162*A*B*x^3*a^3*b^5*c^4*d*n^2+
486*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b^2*c^2*d^3*n+108*B^2*x^2*ln(e*
((b*x+a)/(d*x+c))^n)^2*a^5*b^3*c^3*d^2*n+1134*B^2*x^2*ln(e*((b*x+a)/(d*x+c)
)^n)*a^6*b^2*c^2*d^3*n^2+396*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^3*c^3*
d^2*n^2+324*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b^2*c^2*d^3+1134*A*B*x^
2*a^6*b^2*c^2*d^3*n^2-1548*A*B*x^2*a^5*b^3*c^3*d^2*n^2+486*A*B*x^2*a^4*b^4*
c^4*d*n^2+324*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^7*b*c^2*d^3*n+324*B^2*x*l
n(e*((b*x+a)/(d*x+c))^n)^2*a^6*b^2*c^3*d^2*n-54*B^2*x*ln(e*((b*x+a)/(d*x+c)
```

```
)^n)^2*a^5*b^3*c^4*d*n+648*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c^2*d^3*n^
2+972*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^3*d^2*n^2-90*B^2*x*ln(e*((b
*x+a)/(d*x+c))^n)*a^5*b^3*c^4*d*n^2+324*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a
^7*b*c^2*d^3+108*A^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^3*c^2*d^3+198*A^2*
x^3*a^5*b^3*c^2*d^3*n-324*A^2*x^3*a^4*b^4*c^3*d^2*n+162*A^2*x^3*a^3*b^5*c^4
*d*n-72*A*B*x^2*a^3*b^5*c^5*n^2+108*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*a^7*b
*c^2*d^3+648*B^2*x*a^7*b*c^2*d^3*n^3-810*B^2*x*a^6*b^2*c^3*d^2*n^3+186*B^2*
x*a^5*b^3*c^4*d*n^3+324*A^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^2*d^3+4
86*A^2*x^2*a^6*b^2*c^2*d^3*n-864*A^2*x^2*a^5*b^3*c^3*d^2*n+486*A^2*x^2*a^4*
b^4*c^4*d*n-72*A*B*x*a^4*b^4*c^5*n^2+324*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^
7*b*c^3*d^2*n-162*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b^2*c^4*d*n+648*B^2*l
n(e*((b*x+a)/(d*x+c))^n)*a^7*b*c^3*d^2*n^2-162*B^2*ln(e*((b*x+a)/(d*x+c))^n
)*a^6*b^2*c^4*d*n^2+324*A^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c^2*d^3+324*A
^2*x*a^7*b*c^2*d^3*n-648*A^2*x*a^6*b^2*c^3*d^2*n+432*A^2*x*a^5*b^3*c^4*d*n+
72*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^3*c^5*n+36*B^2*ln(e*((b*x+a)/(d*x+c)
)^n)^3*a^8*c^2*d^3+108*A^2*ln(e*((b*x+a)/(d*x+c))^n)*a^8*c^2*d^3+648*A*B*x*
a^7*b*c^2*d^3*n^2-972*A*B*x*a^6*b^2*c^3*d^2*n^2+396*A*B*x*a^5*b^3*c^4*d*n^2
+648*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c^3*d^2*n-324*A*B*ln(e*((b*x+a)/(d
*x+c))^n)*a^6*b^2*c^4*d*n+36*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^3*a^5*b^3*c^
2*d^3+575*B^2*x^3*a^5*b^3*c^2*d^3*n^3-648*B^2*x^3*a^4*b^4*c^3*d^2*n^3+81*B^
2*x^3*a^3*b^5*c^4*d*n^3-24*A*B*x^3*a^2*b^6*c^5*n^2+108*B^2*x^2*ln(e*((b*x+a
)/(d*x+c))^n)^3*a^6*b^2*c^2*d^3+1215*B^2*x^2*a^6*b^2*c^2*d^3*n^3-1434*B^2*x
^2*a^5*b^3*c^3*d^2*n^3+243*B^2*x^2*a^4*b^4*c^4*d*n^3)/i/g^4/(b*x+a)^3/(a*d-
b*c)^2/n/c^2/a^5/(a^2*d^2-2*a*b*c*d+b^2*c^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1910 vs. $2(529) = 1058$.

Time = 0.42 (sec) , antiderivative size = 1910, normalized size of antiderivative = 3.52

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x, a
lgorithm="fricas")
```

```
[Out] -1/108*(36*A^2*b^3*c^3 - 162*A^2*a*b^2*c^2*d + 324*A^2*a^2*b*c*d^2 - 198*A^
2*a^3*d^3 + 36*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*a*b^2*d^3*n^2*x^2 + 3*B^2*a^2*b
*d^3*n^2*x + B^2*a^3*d^3*n^2)*log((b*x + a)/(d*x + c))^3 + (8*B^2*b^3*c^3 -
 81*B^2*a*b^2*c^2*d + 648*B^2*a^2*b*c*d^2 - 575*B^2*a^3*d^3)*n^2 + 6*(18*A^
2*b^3*c*d^2 - 18*A^2*a*b^2*d^3 + 85*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n^2 + 6
6*(A*B*b^3*c*d^2 - A*B*a*b^2*d^3)*n)*x^2 + 18*(2*B^2*b^3*c^3 - 9*B^2*a*b^2*
c^2*d + 18*B^2*a^2*b*c*d^2 - 11*B^2*a^3*d^3 + 6*(B^2*b^3*c*d^2 - B^2*a*b^2*
d^3)*x^2 - 3*(B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 + 5*B^2*a^2*b*d^3)*x + 6*(B
```

```
^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*a^3*d^3)*log
((b*x + a)/(d*x + c)))*log(e)^2 + 18*(6*A*B*a^3*d^3*n + (11*B^2*b^3*d^3*n^2
 + 6*A*B*b^3*d^3*n)*x^3 + (2*B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 18*B^2*a^2*b
*c*d^2)*n^2 + 3*(6*A*B*a*b^2*d^3*n + (2*B^2*b^3*c*d^2 + 9*B^2*a*b^2*d^3)*n^
2)*x^2 + 3*(6*A*B*a^2*b*d^3*n - (B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 - 6*B^2*
a^2*b*d^3)*n^2)*x)*log((b*x + a)/(d*x + c))^2 + 6*(4*A*B*b^3*c^3 - 27*A*B*a
*b^2*c^2*d + 108*A*B*a^2*b*c*d^2 - 85*A*B*a^3*d^3)*n - 3*(18*A^2*b^3*c^2*d
- 108*A^2*a*b^2*c*d^2 + 90*A^2*a^2*b*d^3 + (19*B^2*b^3*c^2*d - 378*B^2*a*b^
2*c*d^2 + 359*B^2*a^2*b*d^3)*n^2 + 6*(5*A*B*b^3*c^2*d - 54*A*B*a*b^2*c*d^2
+ 49*A*B*a^2*b*d^3)*n)*x + 6*(12*A*B*b^3*c^3 - 54*A*B*a*b^2*c^2*d + 108*A*B
*a^2*b*c*d^2 - 66*A*B*a^3*d^3 + 6*(6*A*B*b^3*c*d^2 - 6*A*B*a*b^2*d^3 + 11*(
B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n)*x^2 + 18*(B^2*b^3*d^3*n*x^3 + 3*B^2*a*b^2
*d^3*n*x^2 + 3*B^2*a^2*b*d^3*n*x + B^2*a^3*d^3*n)*log((b*x + a)/(d*x + c))^
2 + (4*B^2*b^3*c^3 - 27*B^2*a*b^2*c^2*d + 108*B^2*a^2*b*c*d^2 - 85*B^2*a^3*
d^3)*n - 3*(6*A*B*b^3*c^2*d - 36*A*B*a*b^2*c*d^2 + 30*A*B*a^2*b*d^3 + (5*B^
2*b^3*c^2*d - 54*B^2*a*b^2*c*d^2 + 49*B^2*a^2*b*d^3)*n)*x + 6*(6*A*B*a^3*d^
3 + (11*B^2*b^3*d^3*n + 6*A*B*b^3*d^3)*x^3 + 3*(6*A*B*a*b^2*d^3 + (2*B^2*b^
3*c*d^2 + 9*B^2*a*b^2*d^3)*n)*x^2 + (2*B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 18
*B^2*a^2*b*c*d^2)*n + 3*(6*A*B*a^2*b*d^3 - (B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d
^2 - 6*B^2*a^2*b*d^3)*n)*x)*log((b*x + a)/(d*x + c)))*log(e) + 6*(18*A^2*a^
3*d^3 + (85*B^2*b^3*d^3*n^2 + 66*A*B*b^3*d^3*n + 18*A^2*b^3*d^3)*x^3 + (4*B
^2*b^3*c^3 - 27*B^2*a*b^2*c^2*d + 108*B^2*a^2*b*c*d^2)*n^2 + 3*(18*A^2*a*b^
2*d^3 + (22*B^2*b^3*c*d^2 + 63*B^2*a*b^2*d^3)*n^2 + 6*(2*A*B*b^3*c*d^2 + 9*
A*B*a*b^2*d^3)*n)*x^2 + 6*(2*A*B*b^3*c^3 - 9*A*B*a*b^2*c^2*d + 18*A*B*a^2*b
*c*d^2)*n + 3*(18*A^2*a^2*b*d^3 - (5*B^2*b^3*c^2*d - 54*B^2*a*b^2*c*d^2 - 3
6*B^2*a^2*b*d^3)*n^2 - 6*(A*B*b^3*c^2*d - 6*A*B*a*b^2*c*d^2 - 6*A*B*a^2*b*d
^3)*n)*x)*log((b*x + a)/(d*x + c)))/((b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c
^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4)*g^4*i*x^3 + 3*(a*b^6*c^4 - 4*a^2*b^
5*c^3*d + 6*a^3*b^4*c^2*d^2 - 4*a^4*b^3*c*d^3 + a^5*b^2*d^4)*g^4*i*x^2 + 3*
(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 6*a^4*b^3*c^2*d^2 - 4*a^5*b^2*c*d^3 + a^6*
b*d^4)*g^4*i*x + (a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6
*b*c*d^3 + a^7*d^4)*g^4*i)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**4/(d*i*x+c*i),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3445 vs. 2(529) = 1058.

Time = 0.46 (sec) , antiderivative size = 3445, normalized size of antiderivative = 6.34

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x, a
lgorithm="maxima")
```

```
[Out] -1/6*B^2*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
 - 5*a*b*d^2)*x)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*
g^4*i*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)
*g^4*i*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3
)*g^4*i*x + (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*g^4*i
) + 6*d^3*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^
3*b*c*d^3 + a^4*d^4)*g^4*i) - 6*d^3*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d
 + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i))*log(e*(b*x/(d*x + c)
 + a/(d*x + c))^n)^2 - 1/3*A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11
*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4
*c*d^2 - a^3*b^3*d^3)*g^4*i*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^
3*c*d^2 - a^4*b^2*d^3)*g^4*i*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4
*b^2*c*d^2 - a^5*b*d^3)*g^4*i*x + (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*
c*d^2 - a^6*d^3)*g^4*i) + 6*d^3*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*
a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i) - 6*d^3*log(d*x + c)/((b^
4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i)
)*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1/108*((8*b^3*c^3 - 81*a*b^2*c^2
*d + 648*a^2*b*c*d^2 - 575*a^3*d^3 + 36*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*
a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^3 - 36*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 +
 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^3 + 510*(b^3*c*d^2 - a*b^2*d^3)*x^2
 - 198*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a
)^2 - 18*(11*b^3*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 -
 6*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*
log(d*x + c)^2 - 3*(19*b^3*c^2*d - 378*a*b^2*c*d^2 + 359*a^2*b*d^3)*x + 510
*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6
*(85*b^3*d^3*x^3 + 255*a*b^2*d^3*x^2 + 255*a^2*b*d^3*x + 85*a^3*d^3 + 18*(b
^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 66
*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*lo
g(d*x + c))*n^2/(a^3*b^4*c^4*g^4*i - 4*a^4*b^3*c^3*d*g^4*i + 6*a^5*b^2*c^2*
d^2*g^4*i - 4*a^6*b*c*d^3*g^4*i + a^7*d^4*g^4*i + (b^7*c^4*g^4*i - 4*a*b^6*
c^3*d*g^4*i + 6*a^2*b^5*c^2*d^2*g^4*i - 4*a^3*b^4*c*d^3*g^4*i + a^4*b^3*d^4
*g^4*i)*x^3 + 3*(a*b^6*c^4*g^4*i - 4*a^2*b^5*c^3*d*g^4*i + 6*a^3*b^4*c^2*d^
2*g^4*i - 4*a^4*b^3*c*d^3*g^4*i + a^5*b^2*d^4*g^4*i)*x^2 + 3*(a^2*b^5*c^4*g
^4*i - 4*a^3*b^4*c^3*d*g^4*i + 6*a^4*b^3*c^2*d^2*g^4*i - 4*a^5*b^2*c*d^3*g^
```

```
4*i + a^6*b*d^4*g^4*i)*x) + 6*(4*b^3*c^3 - 27*a*b^2*c^2*d + 108*a^2*b*c*d^2
 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*
d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18*(b^3*d^3*x^3 + 3*a*b
^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 - 3*(5*b^3*c^2*d - 54*
a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b
*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*
a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x
 + a^3*d^3)*log(b*x + a))*log(d*x + c))*n*log(e*(b*x/(d*x + c) + a/(d*x + c
))^n)/(a^3*b^4*c^4*g^4*i - 4*a^4*b^3*c^3*d*g^4*i + 6*a^5*b^2*c^2*d^2*g^4*i
- 4*a^6*b*c*d^3*g^4*i + a^7*d^4*g^4*i + (b^7*c^4*g^4*i - 4*a*b^6*c^3*d*g^4*
i + 6*a^2*b^5*c^2*d^2*g^4*i - 4*a^3*b^4*c*d^3*g^4*i + a^4*b^3*d^4*g^4*i)*x^
3 + 3*(a*b^6*c^4*g^4*i - 4*a^2*b^5*c^3*d*g^4*i + 6*a^3*b^4*c^2*d^2*g^4*i -
4*a^4*b^3*c*d^3*g^4*i + a^5*b^2*d^4*g^4*i)*x^2 + 3*(a^2*b^5*c^4*g^4*i - 4*a
^3*b^4*c^3*d*g^4*i + 6*a^4*b^3*c^2*d^2*g^4*i - 4*a^5*b^2*c*d^3*g^4*i + a^6*
b*d^4*g^4*i)*x))*B^2 - 1/18*(4*b^3*c^3 - 27*a*b^2*c^2*d + 108*a^2*b*c*d^2 -
 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^
3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2
*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 - 3*(5*b^3*c^2*d - 54*a*
b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d
^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^
2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x +
 a^3*d^3)*log(b*x + a))*log(d*x + c))*A*B*n/(a^3*b^4*c^4*g^4*i - 4*a^4*b^3*
c^3*d*g^4*i + 6*a^5*b^2*c^2*d^2*g^4*i - 4*a^6*b*c*d^3*g^4*i + a^7*d^4*g^4*i
 + (b^7*c^4*g^4*i - 4*a*b^6*c^3*d*g^4*i + 6*a^2*b^5*c^2*d^2*g^4*i - 4*a^3*b
^4*c*d^3*g^4*i + a^4*b^3*d^4*g^4*i)*x^3 + 3*(a*b^6*c^4*g^4*i - 4*a^2*b^5*c^
3*d*g^4*i + 6*a^3*b^4*c^2*d^2*g^4*i - 4*a^4*b^3*c*d^3*g^4*i + a^5*b^2*d^4*g
^4*i)*x^2 + 3*(a^2*b^5*c^4*g^4*i - 4*a^3*b^4*c^3*d*g^4*i + 6*a^4*b^3*c^2*d^
2*g^4*i - 4*a^5*b^2*c*d^3*g^4*i + a^6*b*d^4*g^4*i)*x) - 1/6*A^2*((6*b^2*d^2
*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^
6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4*i*x^3 + 3*(a*b^5
*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*g^4*i*x^2 + 3*(a^2*
b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*g^4*i*x + (a^3*b^3
*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*g^4*i) + 6*d^3*log(b*x +
a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*g^4*i) - 6*d^3*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
- 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i))
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x, a
lgorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 7.20 (sec) , antiderivative size = 1921, normalized size of antiderivative = 3.54

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx=\text{Too large to display}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^4*(c*i + d*i*x)
),x)
```

```
[Out] ((198*A^2*a^2*d^2 + 36*A^2*b^2*c^2 + 575*B^2*a^2*d^2*n^2 + 8*B^2*b^2*c^2*n^
2 - 126*A^2*a*b*c*d + 510*A*B*a^2*d^2*n + 24*A*B*b^2*c^2*n - 73*B^2*a*b*c*d
*n^2 - 138*A*B*a*b*c*d*n)/(6*(a*d - b*c)) + (x^2*(18*A^2*b^2*d^2 + 85*B^2*b
^2*d^2*n^2 + 66*A*B*b^2*d^2*n))/(a*d - b*c) + (x*(90*A^2*a*b*d^2 - 18*A^2*b
^2*c*d + 359*B^2*a*b*d^2*n^2 - 19*B^2*b^2*c*d*n^2 + 294*A*B*a*b*d^2*n - 30*
A*B*b^2*c*d*n))/(2*(a*d - b*c)))/(x*(54*a^4*b*d^2*g^4*i + 54*a^2*b^3*c^2*g^
4*i - 108*a^3*b^2*c*d*g^4*i) + x^2*(54*a*b^4*c^2*g^4*i + 54*a^3*b^2*d^2*g^4
*i - 108*a^2*b^3*c*d*g^4*i) + x^3*(18*b^5*c^2*g^4*i + 18*a^2*b^3*d^2*g^4*i
- 36*a*b^4*c*d*g^4*i) + 18*a^5*d^2*g^4*i + 18*a^3*b^2*c^2*g^4*i - 36*a^4*b*
c*d*g^4*i) - log(e*((a + b*x)/(c + d*x))^n)^2*((d^3*(11*B^2*n + 6*A*B))/(6*
g^4*i*n*(a*d - b*c)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) -
(B^2*d^3*(x*(b*((g^4*i*n*(a*d - b*c)*(3*a*d - b*c))/(6*d^2) + (a*g^4*i*n*(a
*d - b*c))/(3*d)) + (2*a*b*g^4*i*n*(a*d - b*c))/(3*d) + (b*g^4*i*n*(a*d - b
*c)*(3*a*d - b*c))/(3*d^2)) + a*((g^4*i*n*(a*d - b*c)*(3*a*d - b*c))/(6*d^2
) + (a*g^4*i*n*(a*d - b*c))/(3*d)) + (g^4*i*n*(a*d - b*c)*(3*a^2*d^2 + b^2*
c^2 - 3*a*b*c*d))/(3*d^3) + (b^2*g^4*i*n*x^2*(a*d - b*c))/d))/(g^4*i*n*(a*d
 - b*c)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^3*g^4*i + b^
3*g^4*i*x^3 + 3*a^2*b*g^4*i*x + 3*a*b^2*g^4*i*x^2))) - log(e*((a + b*x)/(c
+ d*x))^n)*((6*B^2*a*d*n - 3*B^2*b*c*n + 3*B^2*b*d*n*x)/(x*(9*a^4*b*d^2*g^4
*i + 9*a^2*b^3*c^2*g^4*i - 18*a^3*b^2*c*d*g^4*i) + x^2*(9*a*b^4*c^2*g^4*i +
 9*a^3*b^2*d^2*g^4*i - 18*a^2*b^3*c*d*g^4*i) + x^3*(3*b^5*c^2*g^4*i + 3*a^2
*b^3*d^2*g^4*i - 6*a*b^4*c*d*g^4*i) + 3*a^5*d^2*g^4*i + 3*a^3*b^2*c^2*g^4*i
 - 6*a^4*b*c*d*g^4*i) - (d^3*(11*B^2*n + 6*A*B)*(x*(b*((a*g^4*i*n*(a*d - b*
c)^3)/d + (g^4*i*n*(a*d - b*c)^3*(3*a*d - b*c))/(2*d^2)) + (2*a*b*g^4*i*n*(
```

```
a*d - b*c)^3)/d + (b*g^4*i*n*(a*d - b*c)^3*(3*a*d - b*c))/d^2) + a*((a*g^4*
i*n*(a*d - b*c)^3)/d + (g^4*i*n*(a*d - b*c)^3*(3*a*d - b*c))/(2*d^2)) + (g^
4*i*n*(a*d - b*c)^3*(3*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/d^3 + (3*b^2*g^4*i*n
*x^2*(a*d - b*c)^3)/d))/(3*g^4*i*n*(a*d - b*c)*(a^3*d^3 - b^3*c^3 + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2)*(x*(9*a^4*b*d^2*g^4*i + 9*a^2*b^3*c^2*g^4*i - 18*a^
3*b^2*c*d*g^4*i) + x^2*(9*a*b^4*c^2*g^4*i + 9*a^3*b^2*d^2*g^4*i - 18*a^2*b^
3*c*d*g^4*i) + x^3*(3*b^5*c^2*g^4*i + 3*a^2*b^3*d^2*g^4*i - 6*a*b^4*c*d*g^4
*i) + 3*a^5*d^2*g^4*i + 3*a^3*b^2*c^2*g^4*i - 6*a^4*b*c*d*g^4*i))) + (d^3*a
tan((d^3*(A^2 + (85*B^2*n^2)/18 + (11*A*B*n)/3)*(18*a^4*d^4*g^4*i - 18*b^4*
c^4*g^4*i + 36*a*b^3*c^3*d*g^4*i - 36*a^3*b*c*d^3*g^4*i)*1i)/(g^4*i*(a*d -
b*c)^4*(18*A^2*d^3 + 85*B^2*d^3*n^2 + 66*A*B*d^3*n)) + (b*d^4*x*(A^2 + (85*
B^2*n^2)/18 + (11*A*B*n)/3)*(a^3*d^3*g^4*i - b^3*c^3*g^4*i + 3*a*b^2*c^2*d*
g^4*i - 3*a^2*b*c*d^2*g^4*i)*36i)/(g^4*i*(a*d - b*c)^4*(18*A^2*d^3 + 85*B^2
*d^3*n^2 + 66*A*B*d^3*n)))*(A^2 + (85*B^2*n^2)/18 + (11*A*B*n)/3)*2i)/(g^4*
i*(a*d - b*c)^4) - (B^2*d^3*log(e*((a + b*x)/(c + d*x))^n)^3)/(3*g^4*i*n*(a
*d - b*c)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))
```

3.194 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 770

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$$
$$=\frac{2AB(bc-ad)^2g^3n(a+bx)}{d^3i^2(c+dx)}-\frac{2B^2(bc-ad)^2g^3n^2(a+bx)}{d^3i^2(c+dx)}$$
$$+\frac{2B^2(bc-ad)^2g^3n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^3i^2(c+dx)}$$
$$-\frac{bB(bc-ad)g^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d^3i^2}$$
$$-\frac{3b(bc-ad)g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^2}$$
$$-\frac{(bc-ad)^2g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^2(c+dx)}$$
$$+\frac{b^3g^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^4i^2}$$
$$-\frac{6bB(bc-ad)^2g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^2}$$
$$-\frac{3b(bc-ad)^2g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^2}+\frac{bB^2(bc-ad)^2g^3n^2\log(c+dx)}{d^4i^2}$$
$$+\frac{bB(bc-ad)^2g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{d^4i^2}$$
$$-\frac{6bB^2(bc-ad)^2g^3n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}$$
$$-\frac{6bB(bc-ad)^2g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}$$
$$-\frac{bB^2(bc-ad)^2g^3n^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{d^4i^2}+\frac{6bB^2(bc-ad)^2g^3n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}$$

```
[Out] 2*A*B*(-a*d+b*c)^2*g^3*n*(b*x+a)/d^3/i^2/(d*x+c)-2*B^2*(-a*d+b*c)^2*g^3*n^2
*(b*x+a)/d^3/i^2/(d*x+c)+2*B^2*(-a*d+b*c)^2*g^3*n*(b*x+a)*ln(e*((b*x+a)/(d*
x+c))^n)/d^3/i^2/(d*x+c)-b*B*(-a*d+b*c)*g^3*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d
*x+c))^n))/d^3/i^2-3*b*(-a*d+b*c)*g^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n
))^2/d^3/i^2-(-a*d+b*c)^2*g^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^3
/i^2/(d*x+c)+1/2*b^3*g^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^4/i^
2-6*b*B*(-a*d+b*c)^2*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/
(d*x+c))/d^4/i^2-3*b*(-a*d+b*c)^2*g^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln(
```

```
(-a*d+b*c)/b/(d*x+c))/d^4/i^2+b*B^2*(-a*d+b*c)^2*g^3*n^2*ln(d*x+c)/d^4/i^2+
b*B*(-a*d+b*c)^2*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*
x+a))/d^4/i^2-6*b*B^2*(-a*d+b*c)^2*g^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/d
^4/i^2-6*b*B*(-a*d+b*c)^2*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,d
*(b*x+a)/b/(d*x+c))/d^4/i^2-b*B^2*(-a*d+b*c)^2*g^3*n^2*polylog(2,b*(d*x+c)/
d/(b*x+a))/d^4/i^2+6*b*B^2*(-a*d+b*c)^2*g^3*n^2*polylog(3,d*(b*x+a)/b/(d*x+
c))/d^4/i^2
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 770, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.311$, Rules used = {2561, 2395, 2333, 2332, 2356, 2389, 2379, 2438, 2351, 31, 2355, 2354, 2421, 6724}

$$\begin{aligned}
&\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}dx \\
&= \frac{b^3g^3(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d^4i^2} \\
&- \frac{6bBg^3n(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^4i^2} \\
&- \frac{3bg^3(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^4i^2} \\
&- \frac{6bBg^3n(bc-ad)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^4i^2} \\
&+ \frac{bBg^3n(bc-ad)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^4i^2} \\
&- \frac{3bg^3(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^3i^2} \\
&- \frac{g^3(a+bx)(bc-ad)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^3i^2(c+dx)} \\
&- \frac{bBg^3n(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^3i^2} + \frac{2ABg^3n(a+bx)(bc-ad)^2}{d^3i^2(c+dx)} \\
&- \frac{6bB^2g^3n^2(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2} - \frac{bB^2g^3n^2(bc-ad)^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{d^4i^2} \\
&+ \frac{6bB^2g^3n^2(bc-ad)^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2} + \frac{bB^2g^3n^2(bc-ad)^2\log(c+dx)}{d^4i^2} \\
&+ \frac{2B^2g^3n(a+bx)(bc-ad)^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^3i^2(c+dx)} - \frac{2B^2g^3n^2(a+bx)(bc-ad)^2}{d^3i^2(c+dx)}
\end{aligned}$$

```
[In] Int[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*x
)^2,x]
```

```
[Out] (2*A*B*(b*c - a*d)^2*g^3*n*(a + b*x))/(d^3*i^2*(c + d*x)) - (2*B^2*(b*c - a
*d)^2*g^3*n^2*(a + b*x))/(d^3*i^2*(c + d*x)) + (2*B^2*(b*c - a*d)^2*g^3*n*(
a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(d^3*i^2*(c + d*x)) - (b*B*(b*c -
a*d)*g^3*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(d^3*i^2) - (3
*b*(b*c - a*d)*g^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d^3
*i^2) - ((b*c - a*d)^2*g^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2)/(d^3*i^2*(c + d*x)) + (b^3*g^3*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n])^2)/(2*d^4*i^2) - (6*b*B*(b*c - a*d)^2*g^3*n*(A + B*Log[e*((a + b
*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(d^4*i^2) - (3*b*(b*c -
a*d)^2*g^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[(b*c - a*d)/(b*(c +
 d*x))])/(d^4*i^2) + (b*B^2*(b*c - a*d)^2*g^3*n^2*Log[c + d*x])/(d^4*i^2) +
 (b*B*(b*c - a*d)^2*g^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b
*(c + d*x))/(d*(a + b*x))])/(d^4*i^2) - (6*b*B^2*(b*c - a*d)^2*g^3*n^2*Poly
Log[2, (d*(a + b*x))/(b*(c + d*x))])/(d^4*i^2) - (6*b*B*(b*c - a*d)^2*g^3*n
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*
x))])/(d^4*i^2) - (b*B^2*(b*c - a*d)^2*g^3*n^2*PolyLog[2, (b*(c + d*x))/(d*
(a + b*x))])/(d^4*i^2) + (6*b*B^2*(b*c - a*d)^2*g^3*n^2*PolyLog[3, (d*(a +
b*x))/(b*(c + d*x))])/(d^4*i^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
```

```
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\left((bc-ad)^2g^3\right)\text{Subst}\left(\int \frac{x^3(A+B\log(ex^n))^2}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{i^2}$$

$$= \frac{\left((bc-ad)^2g^3\right)\text{Subst}\left(\int \left(-\frac{(A+B\log(ex^n))^2}{d^3}+\frac{b^3(A+B\log(ex^n))^2}{d^3(b-dx)^3}-\frac{3b^2(A+B\log(ex^n))^2}{d^3(b-dx)^2}+\frac{3b(A+B\log(ex^n))^2}{d^3(b-dx)}\right)\,dx}{i^2}$$

$$= -\frac{\left((bc-ad)^2g^3\right)\text{Subst}\left(\int (A+B\log(ex^n))^2\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i^2} + \frac{\left(3b(bc-ad)^2g^3\right)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i^2} - \frac{\left(3b^2(bc-ad)^2g^3\right)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i^2} + \frac{\left(b^3(bc-ad)^2g^3\right)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(b-dx)^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i^2}$$

$$= -\frac{3b(bc-ad)g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^2} - \frac{(bc-ad)^2g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^2(c+dx)} + \frac{b^3g^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^4i^2} - \frac{3b(bc-ad)^2g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^2} + \frac{(6bB(bc-ad)^2g^3n)\,\text{Subst}\left(\int\frac{(A+B\log(ex^n))\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^4i^2} - \frac{(b^3B(bc-ad)^2g^3n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^4i^2} + \frac{(2B(bc-ad)^2g^3n)\,\text{Subst}\left(\int(A+B\log(ex^n))\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i^2} + \frac{(6bB(bc-ad)^2g^3n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{b-dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i^2}$$

$$= \frac{2AB(bc-ad)^2g^3n(a+bx)}{d^3i^2(c+dx)} - \frac{3b(bc-ad)g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^2}$$

$$- \frac{(bc-ad)^2g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^2(c+dx)}$$

$$+ \frac{b^3g^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^4i^2}$$

$$- \frac{6bB(bc-ad)^2g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^2}$$

$$- \frac{3b(bc-ad)^2g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^2}$$

$$- \frac{6bB(bc-ad)^2g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}$$

$$- \frac{(b^2B(bc-ad)^2g^3n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{x(b-dx)}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4i^2}$$

$$- \frac{(b^2B(bc-ad)^2g^3n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^2}$$

$$+ \frac{(2B^2(bc-ad)^2g^3n)\,\text{Subst}\left(\int\log\left(ex^n\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^2}$$

$$+ \frac{(6bB^2(bc-ad)^2g^3n^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4i^2}$$

$$+ \frac{(6bB^2(bc-ad)^2g^3n^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4i^2}$$

$$= \frac{2AB(bc-ad)^2 g^3 n(a+bx)}{d^3 i^2 (c+dx)} - \frac{2B^2 (bc-ad)^2 g^3 n^2 (a+bx)}{d^3 i^2 (c+dx)}$$
$$+ \frac{2B^2 (bc-ad)^2 g^3 n(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^3 i^2 (c+dx)}$$
$$- \frac{bB(bc-ad) g^3 n(a+bx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d^3 i^2}$$
$$- \frac{3b(bc-ad) g^3 (a+bx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3 i^2}$$
$$- \frac{(bc-ad)^2 g^3 (a+bx) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3 i^2 (c+dx)}$$
$$+ \frac{b^3 g^3 (c+dx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^4 i^2}$$
$$- \frac{6bB(bc-ad)^2 g^3 n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4 i^2}$$
$$- \frac{3b(bc-ad)^2 g^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4 i^2}$$
$$+ \frac{bB(bc-ad)^2 g^3 n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{d^4 i^2}$$
$$- \frac{6bB^2 (bc-ad)^2 g^3 n^2 \text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4 i^2}$$
$$- \frac{6bB(bc-ad)^2 g^3 n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4 i^2}$$
$$+ \frac{6bB^2 (bc-ad)^2 g^3 n^2 \text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4 i^2}$$
$$- \frac{(bB^2 (bc-ad)^2 g^3 n^2) \text{Subst}\left(\int \frac{\log\left(1-\frac{b}{dx}\right)}{x} \, dx, x, \frac{a+bx}{c+dx}\right)}{d^4 i^2}$$
$$+ \frac{(bB^2 (bc-ad)^2 g^3 n^2) \text{Subst}\left(\int \frac{1}{b-dx} \, dx, x, \frac{a+bx}{c+dx}\right)}{d^3 i^2}$$

$$
\begin{aligned}
&= \frac{2AB(bc-ad)^2g^3n(a+bx)}{d^3i^2(c+dx)} - \frac{2B^2(bc-ad)^2g^3n^2(a+bx)}{d^3i^2(c+dx)} \\
&+ \frac{2B^2(bc-ad)^2g^3n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^3i^2(c+dx)} \\
&- \frac{bB(bc-ad)g^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d^3i^2} \\
&- \frac{3b(bc-ad)g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^2} \\
&- \frac{(bc-ad)^2g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^2(c+dx)} \\
&+ \frac{b^3g^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^4i^2} \\
&- \frac{6bB(bc-ad)^2g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^2} \\
&- \frac{3b(bc-ad)^2g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^2} \\
&+ \frac{bB^2(bc-ad)^2g^3n^2\log(c+dx)}{d^4i^2} \\
&+ \frac{bB(bc-ad)^2g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{d^4i^2} \\
&- \frac{6bB^2(bc-ad)^2g^3n^2\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2} \\
&- \frac{6bB(bc-ad)^2g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2} \\
&- \frac{bB^2(bc-ad)^2g^3n^2\operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{d^4i^2} + \frac{6bB^2(bc-ad)^2g^3n^2\operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5850 vs. $2(770) = 1540$.

Time = 6.98 (sec) , antiderivative size = 5850, normalized size of antiderivative = 7.60

$$
\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i +
 d*i*x)^2,x]
```

```
[Out] Result too large to show
```

Maple [F]

$$\int \frac{(bgx+ag)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(dix+ci)^2}dx$$

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x)
[Out] int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x,
 algorithm="fricas")
[Out] integral((A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a
^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a
^3*g^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2
*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log(e*((b*x + a)/(d*x + c))^n))
/(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**2
,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x,
 algorithm="maxima")
```

[Out] 2*A*B*a^3*g^3*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*i^2) - b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) + 1/2*(2*c^3/(d^5*i^2*x + c*d^4*i^2) + 6*c^2*log(d*x + c)/(d^4*i^2) + (d*x^2 - 4*c*x)/(d^3*i^2))*A^2*b^3*g^3 - 3*A^2*a*b^2*(c^2/(d^4*i^2*x + c*d^3*i^2) - x/(d^2*i^2) + 2*c*log(d*x + c)/(d^3*i^2))*g^3 + 3*A^2*a^2*b*g^3*(c/(d^3*i^2*x + c*d^2*i^2) + log(d*x + c)/(d^2*i^2)) - 2*A*B*a^3*g^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^2*i^2*x + c*d*i^2) - A^2*a^3*g^3/(d^2*i^2*x + c*d*i^2) + 1/2*(B^2*b^3*d^3*g^3*x^3 - 3*(b^3*c*d^2*g^3 - 2*a*b^2*d^3*g^3)*B^2*x^2 - 2*(2*b^3*c^2*d*g^3 - 3*a*b^2*c*d^2*g^3)*B^2*x + 2*(b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 - a^3*d^3*g^3)*B^2 + 6*((b^3*c^2*d*g^3 - 2*a*b^2*c*d^2*g^3 + a^2*b*d^3*g^3)*B^2*x + (b^3*c^3*g^3 - 2*a*b^2*c^2*d*g^3 + a^2*b*c*d^2*g^3)*B^2)*log(d*x + c))*log((d*x + c)^n)^2/(d^5*i^2*x + c*d^4*i^2) - integrate(-(B^2*a^3*d^3*g^3*log(e)^2 + (B^2*b^3*d^3*g^3*log(e)^2 + 2*A*B*b^3*d^3*g^3*log(e))*x^3 + 3*(B^2*a*b^2*d^3*g^3*log(e)^2 + 2*A*B*a*b^2*d^3*g^3*log(e))*x^2 + (B^2*b^3*d^3*g^3*x^3 + 3*B^2*a*b^2*d^3*g^3*x^2 + 3*B^2*a^2*b*d^3*g^3*x + B^2*a^3*d^3*g^3)*log((b*x + a)^n)^2 + 3*(B^2*a^2*b*d^3*g^3*log(e)^2 + 2*A*B*a^2*b*d^3*g^3*log(e))*x + 2*(B^2*a^3*d^3*g^3*log(e) + (B^2*b^3*d^3*g^3*log(e) + A*B*b^3*d^3*g^3)*x^3 + 3*(B^2*a*b^2*d^3*g^3*log(e) + A*B*a*b^2*d^3*g^3)*x^2 + 3*(B^2*a^2*b*d^3*g^3*log(e) + A*B*a^2*b*d^3*g^3)*x)*log((b*x + a)^n) - ((2*A*B*b^3*d^3*g^3 + (g^3*n + 2*g^3*log(e))*B^2*b^3*d^3)*x^3 + 2*(b^3*c^3*g^3*n - 3*a*b^2*c^2*d*g^3*n + 3*a^2*b*c*d^2*g^3*n - (g^3*n - g^3*log(e))*a^3*d^3)*B^2 + 3*(2*A*B*a*b^2*d^3*g^3 - (b^3*c*d^2*g^3*n - 2*(g^3*n + g^3*log(e))*a*b^2*d^3)*B^2)*x^2 + 2*(3*A*B*a^2*b*d^3*g^3 - (2*b^3*c^2*d*g^3*n - 3*a*b^2*c*d^2*g^3*n - 3*a^2*b*d^3*g^3*log(e))*B^2)*x + 6*((b^3*c^2*d*g^3*n - 2*a*b^2*c*d^2*g^3*n + a^2*b*d^3*g^3*n)*B^2*x + (b^3*c^3*g^3*n - 2*a*b^2*c^2*d*g^3*n + a^2*b*c*d^2*g^3*n)*B^2)*log(d*x + c) + 2*(B^2*b^3*d^3*g^3*x^3 + 3*B^2*a*b^2*d^3*g^3*x^2 + 3*B^2*a^2*b*d^3*g^3*x + B^2*a^3*d^3*g^3)*log((b*x + a)^n))*log((d*x + c)^n))/(d^5*i^2*x^2 + 2*c*d^4*i^2*x + c^2*d^3*i^2), x)

Giac [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}\,dx$$

[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x, algorithm="giac")

[Out] integrate((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x + c*i)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{\left(a\,g+b\,g\,x\right)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{\left(c\,i+d\,i\,x\right)^2}\,dx$$

```
[In] int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x
)^2,x)
```

```
[Out] int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x
)^2, x)
```

3.195 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 500

$$
\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx \\
&= -\frac{2AB(bc-ad)g^2n(a+bx)}{d^2i^2(c+dx)}+\frac{2B^2(bc-ad)g^2n^2(a+bx)}{d^2i^2(c+dx)} \\
&\quad -\frac{2B^2(bc-ad)g^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^2i^2(c+dx)}+\frac{bg^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i^2} \\
&\quad +\frac{(bc-ad)g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i^2(c+dx)} \\
&\quad +\frac{2bB(bc-ad)g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^2} \\
&\quad +\frac{2b(bc-ad)g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^2} \\
&\quad +\frac{2bB^2(bc-ad)g^2n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2} \\
&\quad +\frac{4bB(bc-ad)g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2} \\
&\quad -\frac{4bB^2(bc-ad)g^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2}
\end{aligned}
$$

```
[Out] -2*A*B*(-a*d+b*c)*g^2*n*(b*x+a)/d^2/i^2/(d*x+c)+2*B^2*(-a*d+b*c)*g^2*n^2*(b
*x+a)/d^2/i^2/(d*x+c)-2*B^2*(-a*d+b*c)*g^2*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))
```

```
^n)/d^2/i^2/(d*x+c)+b*g^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^2/i^2
+(-a*d+b*c)*g^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^2/i^2/(d*x+c)+2
*b*B*(-a*d+b*c)*g^2*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+
c))/d^3/i^2+2*b*(-a*d+b*c)*g^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b
*c)/b/(d*x+c))/d^3/i^2+2*b*B^2*(-a*d+b*c)*g^2*n^2*polylog(2,d*(b*x+a)/b/(d*
x+c))/d^3/i^2+4*b*B*(-a*d+b*c)*g^2*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*polylo
g(2,d*(b*x+a)/b/(d*x+c))/d^3/i^2-4*b*B^2*(-a*d+b*c)*g^2*n^2*polylog(3,d*(b*
x+a)/b/(d*x+c))/d^3/i^2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2561, 2395, 2333, 2332, 2355, 2354, 2438, 2421, 6724}

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2} \, dx$$
$$= \frac{4bBg^2n(bc-ad)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^3i^2} + \frac{2bg^2(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^3i^2} + \frac{2bBg^2n(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^3i^2} + \frac{bg^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^2i^2} + \frac{g^2(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^2i^2(c+dx)} - \frac{2ABg^2n(a+bx)(bc-ad)}{d^2i^2(c+dx)} + \frac{2bB^2g^2n^2(bc-ad)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2} - \frac{4bB^2g^2n^2(bc-ad)\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2} - \frac{2B^2g^2n(a+bx)(bc-ad)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^2i^2(c+dx)} + \frac{2B^2g^2n^2(a+bx)(bc-ad)}{d^2i^2(c+dx)}$$

```
[In] Int[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*x
)^2,x]
```

```
[Out] (-2*A*B*(b*c - a*d)*g^2*n*(a + b*x))/(d^2*i^2*(c + d*x)) + (2*B^2*(b*c - a*
d)*g^2*n^2*(a + b*x))/(d^2*i^2*(c + d*x)) - (2*B^2*(b*c - a*d)*g^2*n*(a + b
*x)*Log[e*((a + b*x)/(c + d*x))^n])/(d^2*i^2*(c + d*x)) + (b*g^2*(a + b*x)*
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d^2*i^2) + ((b*c - a*d)*g^2*(a +
 b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d^2*i^2*(c + d*x)) + (2*b*
B*(b*c - a*d)*g^2*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/
```

```
(b*(c + d*x))])/(d^3*i^2) + (2*b*(b*c - a*d)*g^2*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^2*Log[(b*c - a*d)/(b*(c + d*x))])/(d^3*i^2) + (2*b*B^2*(b*c -
a*d)*g^2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^3*i^2) + (4*b*B*(b
*c - a*d)*g^2*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b
*x))/(b*(c + d*x))])/(d^3*i^2) - (4*b*B^2*(b*c - a*d)*g^2*n^2*PolyLog[3, (d
*(a + b*x))/(b*(c + d*x))])/(d^3*i^2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)g^2)\,\text{Subst}\left(\int \frac{x^2(A+B\log(ex^n))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{i^2}$$

$$= \frac{((bc-ad)g^2)\,\text{Subst}\left(\int \left(\frac{(A+B\log(ex^n))^2}{d^2}+\frac{b^2(A+B\log(ex^n))^2}{d^2(-b+dx)^2}+\frac{2b(A+B\log(ex^n))^2}{d^2(-b+dx)}\right)dx, x, \frac{a+bx}{c+dx}\right)}{i^2}$$

$$= \frac{((bc-ad)g^2)\,\text{Subst}\left(\int (A+B\log(ex^n))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^2} + \frac{(2b(bc-ad)g^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{-b+dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^2} + \frac{(b^2(bc-ad)g^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(-b+dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^2}$$

$$= \frac{bg^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i^2} + \frac{(bc-ad)g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i^2(c+dx)} + \frac{2b(bc-ad)g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^2} - \frac{(4bB(bc-ad)g^2n)\,\text{Subst}\left(\int\frac{(A+B\log(ex^n))\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i^2} - \frac{(2B(bc-ad)g^2n)\,\text{Subst}\left(\int(A+B\log(ex^n))\,dx,x,\frac{a+bx}{c+dx}\right)}{d^2i^2} + \frac{(2bB(bc-ad)g^2n)\,\text{Subst}\left(\int\frac{A+B\log(ex^n)}{-b+dx}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^2i^2}$$

$$= -\frac{2AB(bc-ad)g^2n(a+bx)}{d^2i^2(c+dx)} + \frac{bg^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i^2} + \frac{(bc-ad)g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i^2(c+dx)} + \frac{2bB(bc-ad)g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^2} + \frac{2b(bc-ad)g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^2} + \frac{4bB(bc-ad)g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2} - \frac{(2B^2(bc-ad)g^2n)\,\text{Subst}\left(\int\log(ex^n)\,dx,x,\frac{a+bx}{c+dx}\right)}{d^2i^2} - \frac{(2bB^2(bc-ad)g^2n^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i^2} - \frac{(4bB^2(bc-ad)g^2n^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i^2}$$

$$= -\frac{2AB(bc-ad)g^2n(a+bx)}{d^2i^2(c+dx)} + \frac{2B^2(bc-ad)g^2n^2(a+bx)}{d^2i^2(c+dx)} - \frac{2B^2(bc-ad)g^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^2i^2(c+dx)} + \frac{bg^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i^2} + \frac{(bc-ad)g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i^2(c+dx)} + \frac{2bB(bc-ad)g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^2} + \frac{2b(bc-ad)g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^2} + \frac{2bB^2(bc-ad)g^2n^2\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2} + \frac{4bB(bc-ad)g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2} - \frac{4bB^2(bc-ad)g^2n^2\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2859 vs. $2(500) = 1000$.

Time = 3.97 (sec) , antiderivative size = 2859, normalized size of antiderivative = 5.72

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i +
 d*i*x)^2,x]
```

```
[Out] (g^2*(3*b^2*d*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(
c + d*x)])^2 - (3*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n
*Log[(a + b*x)/(c + d*x)])^2)/(c + d*x) - 6*b*(b*c - a*d)*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2*Log[c + d*x] + (6*a^
2*B*d^2*n*(-A - B*Log[e*((a + b*x)/(c + d*x))^n] + B*n*Log[(a + b*x)/(c + d
*x)])*(b*c - a*d + b*(c + d*x)*Log[a/b + x] + (-(b*c) + a*d)*Log[(a + b*x)/
(c + d*x)] - b*c*Log[(b*(c + d*x))/(b*c - a*d)] - b*d*x*Log[(b*(c + d*x))/(
b*c - a*d)]))/((-(b*c) + a*d)*(c + d*x)) + 6*a*b*B*d*n*(A + B*Log[e*((a + b
*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])*(-Log[c/d + x]^2 + 2*Log[
c/d + x]*Log[c + d*x] + 2*(-(c/(c + d*x)) + (b*c*Log[a + b*x])/(-(b*c) + a*
d) + (b*c*Log[c + d*x])/(b*c - a*d) - Log[a/b + x]*Log[c + d*x] + Log[(a +
b*x)/(c + d*x)]*(c/(c + d*x) + Log[c + d*x]) + Log[a/b + x]*Log[(b*(c + d*x
```

))/(b*c - a*d)]) + 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 6*b^2*B*n*
(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])*(d*(a
/b + x)*(-1 + Log[a/b + x]) - (c^2*Log[a/b + x])/(c + d*x) - (c + d*x)*(-1
+ Log[c/d + x]) + c*Log[c/d + x]^2 + (c^2*(1 + Log[c/d + x]))/(c + d*x) + (
b*c^2*(Log[a + b*x] - Log[c + d*x]))/(b*c - a*d) + (-Log[a/b + x] + Log[c/d
 + x] + Log[(a + b*x)/(c + d*x)])*(d*x - c^2/(c + d*x) - 2*c*Log[c + d*x])
- 2*c*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x
))/(-(b*c) + a*d)])) - (3*a^2*B^2*d^2*n^2*(2*b*c - 2*a*d + 2*b*(c + d*x)*Lo
g[a + b*x] - 2*(b*c - a*d)*Log[(a + b*x)/(c + d*x)] - 2*b*(c + d*x)*Log[a +
 b*x]*Log[(a + b*x)/(c + d*x)] + (b*c - a*d)*Log[(a + b*x)/(c + d*x)]^2 - 2
b(c + d*x)*Log[c + d*x] - 2*b*(c + d*x)*Log[(a + b*x)/(c + d*x)]*Log[(b*c
 - a*d)/(b*c + b*d*x)] + b*(c + d*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b
*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b
*(c + d*x)*(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a
*d)] + Log[(b*c - a*d)/(b*c + b*d*x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c -
a*d)])))/((b*c - a*d)*(c + d*x)) + b^2*B^2*n^2*(6*d*x + (3*d*(2*b*x - 2*(a
+ b*x)*Log[a/b + x] + (a + b*x)*Log[a/b + x]^2))/b - 6*(c + d*x)*Log[c/d +
x] + 3*(c + d*x)*Log[c/d + x]^2 - 2*c*Log[c/d + x]^3 - (3*c^2*(2 + 2*Log[c/
d + x] + Log[c/d + x]^2))/(c + d*x) + 3*(-Log[a/b + x] + Log[c/d + x] + Log
[(a + b*x)/(c + d*x)])^2*(d*x - c^2/(c + d*x) - 2*c*Log[c + d*x]) - (6*(a*d
 + 2*b*d*x - b*d*x*Log[c/d + x] - b*c*Log[c + d*x] + Log[a/b + x]*(-(d*(a +
 b*x)) + d*(a + b*x)*Log[c/d + x] + (b*c - a*d)*Log[(b*(c + d*x))/(b*c - a*
d)]) + (b*c - a*d)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/b + (3*c^2*(-
(d*(a + b*x)*Log[a/b + x]^2) + 2*b*(c + d*x)*Log[a/b + x]*Log[(b*(c + d*x))
/(b*c - a*d)] + 2*b*(c + d*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/((
-(b*c) + a*d)*(c + d*x)) + 6*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/
(c + d*x)])*(d*(a/b + x)*(-1 + Log[a/b + x]) - (c^2*Log[a/b + x])/(c + d*x)
 - (c + d*x)*(-1 + Log[c/d + x]) + c*Log[c/d + x]^2 + (c^2*(1 + Log[c/d + x
]))/(c + d*x) + (b*c^2*(Log[a + b*x] - Log[c + d*x]))/(b*c - a*d) - 2*c*(Lo
g[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c
) + a*d)])) + 3*c^2*((2*Log[a/b + x]*(1 + Log[c/d + x]))/(c + d*x) + (b*(Lo
g[c/d + x]^2 - 2*Log[a + b*x] - 2*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) +
a*d)] + 2*Log[c + d*x]))/(b*c - a*d) - (2*b*PolyLog[2, (b*(c + d*x))/(b*c -
 a*d)])/(b*c - a*d)) - 6*c*(Log[a/b + x]^2*Log[(b*(c + d*x))/(b*c - a*d)] +
 2*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 2*PolyLog[3, (d*
(a + b*x))/(-(b*c) + a*d)]) + 6*c*(Log[c/d + x]^2*(Log[a/b + x] - Log[(d*(a
 + b*x))/(-(b*c) + a*d)]) - 2*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c -
a*d)] + 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])) + (2*a*b*B^2*d*n^2*((b*c
- a*d)*(c + d*x)*Log[c/d + x]^3 + 3*c*(b*c - a*d)*(2 + 2*Log[c/d + x] + Log
[c/d + x]^2) + 3*(b*c - a*d)*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/
(c + d*x)])^2*(c + (c + d*x)*Log[c + d*x]) + 3*c*Log[a/b + x]*(-(d*(a + b*x
)*Log[a/b + x]) + 2*b*(c + d*x)*Log[(b*(c + d*x))/(b*c - a*d)]) + 6*b*c*(c
+ d*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 3*(Log[a/b + x] - Log[c/d
 + x] - Log[(a + b*x)/(c + d*x)])*(-2*c*(b*c - a*d)*Log[a/b + x] + (b*c - a
d)(c + d*x)*Log[c/d + x]^2 + 2*c*(b*c - a*d)*(1 + Log[c/d + x]) + 2*b*c*(

```
c + d*x)*(Log[a + b*x] - Log[c + d*x]) - 2*(b*c - a*d)*(c + d*x)*(Log[a/b +
 x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d
)])) + 3*(b*c - a*d)*(c + d*x)*(Log[a/b + x]^2*Log[(b*(c + d*x))/(b*c - a*d
)] + 2*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 2*PolyLog[3,
 (d*(a + b*x))/(-(b*c) + a*d)]) - 3*(c*(2*(b*c - a*d)*Log[a/b + x]*(1 + Log
[c/d + x]) + b*(c + d*x)*(Log[c/d + x]^2 - 2*Log[a + b*x] - 2*Log[c/d + x]*
Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[c + d*x]) - 2*b*(c + d*x)*PolyLog
[2, (b*(c + d*x))/(b*c - a*d)]) + (b*c - a*d)*(c + d*x)*(Log[c/d + x]^2*(Lo
g[a/b + x] - Log[(d*(a + b*x))/(-(b*c) + a*d)]) - 2*Log[c/d + x]*PolyLog[2,
 (b*(c + d*x))/(b*c - a*d)] + 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]))))/(
(b*c - a*d)*(c + d*x))))/(3*d^3*i^2)
```

Maple [F]

$$\int \frac{(bgx+ag)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(dix+ci)^2}dx$$

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x)
[Out] int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x,
 algorithm="fricas")
[Out] integral((A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^
2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*
B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^
n))/(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**2
,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x,
 algorithm="maxima")
```

```
[Out] 2*A*B*a^2*g^2*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*
i^2) - b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) - A^2*b^2*(c^2/(d^4*i^2*x + c*
d^3*i^2) - x/(d^2*i^2) + 2*c*log(d*x + c)/(d^3*i^2))*g^2 + 2*A^2*a*b*g^2*(c
/(d^3*i^2*x + c*d^2*i^2) + log(d*x + c)/(d^2*i^2)) - 2*A*B*a^2*g^2*log(e*(b
*x/(d*x + c) + a/(d*x + c))^n)/(d^2*i^2*x + c*d*i^2) - A^2*a^2*g^2/(d^2*i^2
*x + c*d*i^2) + (B^2*b^2*d^2*g^2*x^2 + B^2*b^2*c*d*g^2*x - (b^2*c^2*g^2 - 2
*a*b*c*d*g^2 + a^2*d^2*g^2)*B^2 - 2*((b^2*c*d*g^2 - a*b*d^2*g^2)*B^2*x + (b
^2*c^2*g^2 - a*b*c*d*g^2)*B^2)*log(d*x + c))*log((d*x + c)^n)^2/(d^4*i^2*x
+ c*d^3*i^2) - integrate(-(B^2*a^2*d^2*g^2*log(e)^2 + (B^2*b^2*d^2*g^2*log(
e)^2 + 2*A*B*b^2*d^2*g^2*log(e))*x^2 + (B^2*b^2*d^2*g^2*x^2 + 2*B^2*a*b*d^2
*g^2*x + B^2*a^2*d^2*g^2)*log((b*x + a)^n)^2 + 2*(B^2*a*b*d^2*g^2*log(e)^2
+ 2*A*B*a*b*d^2*g^2*log(e))*x + 2*(B^2*a^2*d^2*g^2*log(e) + (B^2*b^2*d^2*g^
2*log(e) + A*B*b^2*d^2*g^2)*x^2 + 2*(B^2*a*b*d^2*g^2*log(e) + A*B*a*b*d^2*g
^2)*x)*log((b*x + a)^n) + 2*((b^2*c^2*g^2*n - 2*a*b*c*d*g^2*n + (g^2*n - g^
2*log(e))*a^2*d^2)*B^2 - (A*B*b^2*d^2*g^2 + (g^2*n + g^2*log(e))*B^2*b^2*d^
2)*x^2 - (2*A*B*a*b*d^2*g^2 + (b^2*c*d*g^2*n + 2*a*b*d^2*g^2*log(e))*B^2)*x
 + 2*((b^2*c*d*g^2*n - a*b*d^2*g^2*n)*B^2*x + (b^2*c^2*g^2*n - a*b*c*d*g^2*
n)*B^2)*log(d*x + c) - (B^2*b^2*d^2*g^2*x^2 + 2*B^2*a*b*d^2*g^2*x + B^2*a^2
*d^2*g^2)*log((b*x + a)^n))*log((d*x + c)^n))/(d^4*i^2*x^2 + 2*c*d^3*i^2*x
+ c^2*d^2*i^2), x)
```

Giac [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x,
 algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x +
 c*i)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{\left(a\,g+b\,g\,x\right)^2\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{\left(c\,i+d\,i\,x\right)^2}\,dx$$

```
[In] int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x
)^2,x)
```

```
[Out] int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x
)^2, x)
```

3.196 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 282

$$\begin{aligned}
&\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx \\
&= \frac{2ABgn(a+bx)}{di^2(c+dx)}-\frac{2B^2gn^2(a+bx)}{di^2(c+dx)}+\frac{2B^2gn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{di^2(c+dx)} \\
&\quad-\frac{g(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{di^2(c+dx)}-\frac{bg\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i^2} \\
&\quad-\frac{2bBgn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2}+\frac{2bB^2gn^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2}
\end{aligned}$$

```
[Out] 2*A*B*g*n*(b*x+a)/d/i^2/(d*x+c)-2*B^2*g*n^2*(b*x+a)/d/i^2/(d*x+c)+2*B^2*g*n
*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/d/i^2/(d*x+c)-g*(b*x+a)*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))^2/d/i^2/(d*x+c)-b*g*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-
a*d+b*c)/b/(d*x+c))/d^2/i^2-2*b*B*g*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*polyl
og(2,d*(b*x+a)/b/(d*x+c))/d^2/i^2+2*b*B^2*g*n^2*polylog(3,d*(b*x+a)/b/(d*x+
c))/d^2/i^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {2561, 2395, 2333, 2332, 2354, 2421, 6724}

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$$
$$= -\frac{2bBgn\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^2i^2} - \frac{bg\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^2i^2} - \frac{g(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{di^2(c+dx)} + \frac{2ABgn(a+bx)}{di^2(c+dx)} + \frac{2bB^2gn^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2} + \frac{2B^2gn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{di^2(c+dx)} - \frac{2B^2gn^2(a+bx)}{di^2(c+dx)}$$

```
[In] Int[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*x)^
2,x]
```

```
[Out] (2*A*B*g*n*(a + b*x))/(d*i^2*(c + d*x)) - (2*B^2*g*n^2*(a + b*x))/(d*i^2*(c
 + d*x)) + (2*B^2*g*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(d*i^2*(c +
 d*x)) - (g*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d*i^2*(c +
 d*x)) - (b*g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[(b*c - a*d)/(b*(
c + d*x))])/(d^2*i^2) - (2*b*B*g*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*P
olyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^2*i^2) + (2*b*B^2*g*n^2*PolyLog[
3, (d*(a + b*x))/(b*(c + d*x))])/(d^2*i^2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
```

```
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g\text{Subst}\left(\int \frac{x(A+B\log(ex^n))^2}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{i^2}\\
&= \frac{g\text{Subst}\left(\int \left(-\frac{(A+B\log(ex^n))^2}{d}-\frac{b(A+B\log(ex^n))^2}{d(-b+dx)}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{i^2}\\
&= -\frac{g\text{Subst}\left(\int (A+B\log(ex^n))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{di^2}-\frac{(bg)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{-b+dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{di^2}
\end{aligned}$$

$$= -\frac{g(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{di^2(c+dx)} - \frac{bg\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i^2} + \frac{(2bBgn)\text{Subst}\left(\int\frac{(A+B\log(ex^n))\log\left(1-\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^2i^2} + \frac{(2Bgn)\text{Subst}\left(\int(A+B\log(ex^n))\,dx,x,\frac{a+bx}{c+dx}\right)}{di^2}$$

$$= \frac{2ABgn(a+bx)}{di^2(c+dx)} - \frac{g(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{di^2(c+dx)} - \frac{bg\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i^2} - \frac{2bBgn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2} + \frac{(2B^2gn)\text{Subst}\left(\int\log(ex^n)\,dx,x,\frac{a+bx}{c+dx}\right)}{di^2} + \frac{(2bB^2gn^2)\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^2i^2}$$

$$= \frac{2ABgn(a+bx)}{di^2(c+dx)} - \frac{2B^2gn^2(a+bx)}{di^2(c+dx)} + \frac{2B^2gn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{di^2(c+dx)} - \frac{g(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{di^2(c+dx)} - \frac{bg\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i^2} - \frac{2bBgn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2} + \frac{2bB^2gn^2\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1570 vs. $2(282) = 564$.

Time = 1.01 (sec) , antiderivative size = 1570, normalized size of antiderivative = 5.57

$$\int\frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$$

$$= \frac{g\left(\frac{3(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-Bn\log\left(\frac{a+bx}{c+dx}\right)\right)^2}{c+dx} + 3b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-Bn\log\left(\frac{a+bx}{c+dx}\right)\right)^2\log(c+dx) + \frac{6a}{}\right.}{}$$

```
[In] Integrate[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d
*i*x)^2,x]
```

```
[Out] (g*((3*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x
)/(c + d*x)])^2)/(c + d*x) + 3*b*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*
n*Log[(a + b*x)/(c + d*x)])^2*Log[c + d*x] + (6*a*B*d*n*(-A - B*Log[e*((a +
 b*x)/(c + d*x))^n] + B*n*Log[(a + b*x)/(c + d*x)])*(b*c - a*d + b*(c + d*x
)*Log[a/b + x] + (-(b*c) + a*d)*Log[(a + b*x)/(c + d*x)] - b*c*Log[(b*(c +
d*x))/(b*c - a*d)] - b*d*x*Log[(b*(c + d*x))/(b*c - a*d)]))/((-(b*c) + a*d)
*(c + d*x)) + 3*b*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a +
b*x)/(c + d*x)])*(-Log[c/d + x]^2 + 2*Log[c/d + x]*Log[c + d*x] + 2*(-(c/(c
 + d*x)) + (b*c*Log[a + b*x])/(-(b*c) + a*d) + (b*c*Log[c + d*x])/(b*c - a*
d) - Log[a/b + x]*Log[c + d*x] + Log[(a + b*x)/(c + d*x)]*(c/(c + d*x) + Lo
g[c + d*x]) + Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)]) + 2*PolyLog[2, (
d*(a + b*x))/(-(b*c) + a*d)]) - (3*a*B^2*d*n^2*(2*b*c - 2*a*d + 2*b*(c + d*
x)*Log[a + b*x] - 2*(b*c - a*d)*Log[(a + b*x)/(c + d*x)] - 2*b*(c + d*x)*Lo
g[a + b*x]*Log[(a + b*x)/(c + d*x)] + (b*c - a*d)*Log[(a + b*x)/(c + d*x)]^
2 - 2*b*(c + d*x)*Log[c + d*x] - 2*b*(c + d*x)*Log[(a + b*x)/(c + d*x)]*Log
[(b*c - a*d)/(b*c + b*d*x)] + b*(c + d*x)*(Log[a + b*x]*(Log[a + b*x] - 2*L
og[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]
) + b*(c + d*x)*(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c
) + a*d)] + Log[(b*c - a*d)/(b*c + b*d*x)]) - 2*PolyLog[2, (b*(c + d*x))/(b
*c - a*d)])))/((b*c - a*d)*(c + d*x)) + (b*B^2*n^2*((b*c - a*d)*(c + d*x)*L
og[c/d + x]^3 + 3*c*(b*c - a*d)*(2 + 2*Log[c/d + x] + Log[c/d + x]^2) + 3*(
b*c - a*d)*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])^2*(c +
 (c + d*x)*Log[c + d*x]) + 3*c*Log[a/b + x]*(-(d*(a + b*x)*Log[a/b + x]) +
2*b*(c + d*x)*Log[(b*(c + d*x))/(b*c - a*d)]) + 6*b*c*(c + d*x)*PolyLog[2,
(d*(a + b*x))/(-(b*c) + a*d)] + 3*(Log[a/b + x] - Log[c/d + x] - Log[(a + b
*x)/(c + d*x)])*(-2*c*(b*c - a*d)*Log[a/b + x] + (b*c - a*d)*(c + d*x)*Log[
c/d + x]^2 + 2*c*(b*c - a*d)*(1 + Log[c/d + x]) + 2*b*c*(c + d*x)*(Log[a +
b*x] - Log[c + d*x]) - 2*(b*c - a*d)*(c + d*x)*(Log[a/b + x]*Log[(b*(c + d*
x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])) + 3*(b*c - a*
d)*(c + d*x)*(Log[a/b + x]^2*Log[(b*(c + d*x))/(b*c - a*d)] + 2*Log[a/b + x
]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 2*PolyLog[3, (d*(a + b*x))/(-(
b*c) + a*d)]) - 3*(c*(2*(b*c - a*d)*Log[a/b + x]*(1 + Log[c/d + x]) + b*(c
+ d*x)*(Log[c/d + x]^2 - 2*Log[a + b*x] - 2*Log[c/d + x]*Log[(d*(a + b*x))/
(-(b*c) + a*d)] + 2*Log[c + d*x]) - 2*b*(c + d*x)*PolyLog[2, (b*(c + d*x))/
(b*c - a*d)]) + (b*c - a*d)*(c + d*x)*(Log[c/d + x]^2*(Log[a/b + x] - Log[(
d*(a + b*x))/(-(b*c) + a*d)]) - 2*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*
c - a*d)] + 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]))))/((b*c - a*d)*(c + d
*x))))/(3*d^2*i^2)
```

Maple [F]

$$\int \frac{(bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{(dix+ci)^{2}}dx$$

```
[In] int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x)
[Out] int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{(ci+dix)^{2}}\,dx=\int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{(dix+ci)^{2}}\,dx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x, a
lgorithm="fricas")
[Out] integral((A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log(e*((b*x + a)/(d*x
 + c))^n)^2 + 2*(A*B*b*g*x + A*B*a*g)*log(e*((b*x + a)/(d*x + c))^n))/(d^2*
i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{(ci+dix)^{2}}\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{(ci+dix)^{2}}\,dx=\int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{(dix+ci)^{2}}\,dx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x, a
lgorithm="maxima")
[Out] 2*A*B*a*g*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*i^2)
 - b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) + A^2*b*g*(c/(d^3*i^2*x + c*d^2*i^
2) + log(d*x + c)/(d^2*i^2)) - 2*A*B*a*g*log(e*(b*x/(d*x + c) + a/(d*x + c)
```

```
)^n)/(d^2*i^2*x + c*d*i^2) - A^2*a*g/(d^2*i^2*x + c*d*i^2) + ((b*c*g - a*d*
g)*B^2 + (B^2*b*d*g*x + B^2*b*c*g)*log(d*x + c))*log((d*x + c)^n)^2/(d^3*i^
2*x + c*d^2*i^2) - integrate(-(B^2*a*d*g*log(e)^2 + (B^2*b*d*g*x + B^2*a*d*
g)*log((b*x + a)^n)^2 + (B^2*b*d*g*log(e)^2 + 2*A*B*b*d*g*log(e))*x + 2*(B^
2*a*d*g*log(e) + (B^2*b*d*g*log(e) + A*B*b*d*g)*x)*log((b*x + a)^n) - 2*((b
*c*g*n - (g*n - g*log(e))*a*d)*B^2 + (B^2*b*d*g*log(e) + A*B*b*d*g)*x + (B^
2*b*d*g*n*x + B^2*b*c*g*n)*log(d*x + c) + (B^2*b*d*g*x + B^2*a*d*g)*log((b*
x + a)^n))*log((d*x + c)^n))/(d^3*i^2*x^2 + 2*c*d^2*i^2*x + c^2*d*i^2), x)
```

Giac [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x, a
lgorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x + c
*i)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx=\int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(c\,i+d\,i\,x)^2}\,dx$$

```
[In] int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x)^
2,x)
```

```
[Out] int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x)^
2, x)
```

3.197 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 163

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = -\frac{2ABn(a+bx)}{(bc-ad)i^2(c+dx)}+\frac{2B^2n^2(a+bx)}{(bc-ad)i^2(c+dx)} - \frac{2B^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)i^2(c+dx)} + \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(c+dx)}$$

```
[Out] -2*A*B*n*(b*x+a)/(-a*d+b*c)/i^2/(d*x+c)+2*B^2*n^2*(b*x+a)/(-a*d+b*c)/i^2/(d
*x+c)-2*B^2*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/i^2/(d*x+c)+(b*x
+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/i^2/(d*x+c)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2551, 2333, 2332}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{i^2(c+dx)(bc-ad)} - \frac{2ABn(a+bx)}{i^2(c+dx)(bc-ad)} - \frac{2B^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{i^2(c+dx)(bc-ad)} + \frac{2B^2n^2(a+bx)}{i^2(c+dx)(bc-ad)}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*i + d*i*x)^2,x]
```

```
[Out] (-2*A*B*n*(a + b*x))/((b*c - a*d)*i^2*(c + d*x)) + (2*B^2*n^2*(a + b*x))/((
b*c - a*d)*i^2*(c + d*x)) - (2*B^2*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^
n])/((b*c - a*d)*i^2*(c + d*x)) + ((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])^2)/((b*c - a*d)*i^2*(c + d*x))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (A+B\log(ex^n))^2\, dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}\\
&= \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(c+dx)} - \frac{(2Bn)\text{Subst}\left(\int (A+B\log(ex^n))\, dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}\\
&= -\frac{2ABn(a+bx)}{(bc-ad)i^2(c+dx)} + \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(c+dx)}\\
&\quad - \frac{(2B^2n)\,\text{Subst}\left(\int \log(ex^n)\, dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}\\
&= -\frac{2ABn(a+bx)}{(bc-ad)i^2(c+dx)} + \frac{2B^2n^2(a+bx)}{(bc-ad)i^2(c+dx)}\\
&\quad - \frac{2B^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)i^2(c+dx)} + \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(c+dx)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.03

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$$
$$= \frac{-\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(2(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2b(c+dx)\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2b(c+dx)\left(A+B\right.}{}}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*i + d*i*x)^2,x]
```

```
[Out] (-(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]) + 2*b*(c + d*x)*Log[a + b*x]*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n]) - 2*b*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)*Log[c + d*x] - 2*B*n*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*
Log[c + d*x]) - b*B*n*(c + d*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c +
 d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*n*
(c + d*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x
] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(d*i^2*(c + d*x
))
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.80

method	result
parallelrisch	$-\frac{2B^2ab\,d^3n^3-2B^2b^2c\,d^2n^3+A^2ab\,d^3n-A^2b^2c\,d^2n+2ABx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^2d^3n+2AB\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)ab\,d^3n-2ABab\,d^3n^2}{i^2(dx-}$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] -(2*B^2*a*b*d^3*n^3-2*B^2*b^2*c*d^2*n^3+A^2*a*b*d^3*n-A^2*b^2*c*d^2*n+2*A*B
*x*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^3*n+2*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b*
d^3*n-2*A*B*a*b*d^3*n^2+2*A*B*b^2*c*d^2*n^2+B^2*x*ln(e*((b*x+a)/(d*x+c))^n)
^2*b^2*d^3*n-2*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^3*n^2+B^2*ln(e*((b*x+a
)/(d*x+c))^n)^2*a*b*d^3*n-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^3*n^2)/i^2/
(d*x+c)/b/d^3/n/(a*d-b*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.61

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx =$$
$$-\frac{A^2bc-A^2ad+2\,(B^2bc-B^2ad)n^2+(B^2bc-B^2ad)\log\left(e\right)^2-(B^2bdn^2x+B^2adn^2)\log\left(\frac{bx+a}{dx+c}\right)^2-2\,(AB\cdots}{}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x, algorithm="fr
icas")
```

```
[Out] -(A^2*b*c - A^2*a*d + 2*(B^2*b*c - B^2*a*d)*n^2 + (B^2*b*c - B^2*a*d)*log(e
)^2 - (B^2*b*d*n^2*x + B^2*a*d*n^2)*log((b*x + a)/(d*x + c))^2 - 2*(A*B*b*c
 - A*B*a*d)*n + 2*(A*B*b*c - A*B*a*d - (B^2*b*c - B^2*a*d)*n - (B^2*b*d*n*x
 + B^2*a*d*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*(B^2*a*d*n^2 - A*B*a*d*n
 + (B^2*b*d*n^2 - A*B*b*d*n)*x)*log((b*x + a)/(d*x + c)))/((b*c*d^2 - a*d^3
)*i^2*x + (b*c^2*d - a*c*d^2)*i^2)
```

Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \frac{\int \frac{A^2}{c^2+2cdx+d^2x^2}\,dx+\int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^2+2cdx+d^2x^2}\,dx+\int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^2+2cdx+d^2x^2}\,dx}{i^2}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**2,x)
```

```
[Out] (Integral(A**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(B**2*log(e*(a/(c
 + d*x) + b*x/(c + d*x))**n)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral
(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**2 + 2*c*d*x + d**2*x**2)
, x))/i**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(163) = 326.

Time = 0.21 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.63

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = 2\,ABn\left(\frac{1}{d^2i^2x+cdi^2}+\frac{b\log(bx+a)}{(bcd-ad^2)i^2}-\frac{b\log(dx+c)}{(bcd-ad^2)i^2}\right) + \left(2\,n\left(\frac{1}{d^2i^2x+cdi^2}+\frac{b\log(bx+a)}{(bcd-ad^2)i^2}-\frac{b\log(dx+c)}{(bcd-ad^2)i^2}\right)\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)-\frac{((bdx+bc)\log}{} \right. - \frac{B^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)^2}{d^2i^2x+cdi^2} - \frac{2\,AB\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{d^2i^2x+cdi^2} - \frac{A^2}{d^2i^2x+cdi^2}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x, algorithm="ma
xima")
```

```
[Out] 2*A*B*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*i^2) - b
*log(d*x + c)/((b*c*d - a*d^2)*i^2)) + (2*n*(1/(d^2*i^2*x + c*d*i^2) + b*lo
g(b*x + a)/((b*c*d - a*d^2)*i^2) - b*log(d*x + c)/((b*c*d - a*d^2)*i^2))*lo
g(e*(b*x/(d*x + c) + a/(d*x + c))^n) - ((b*d*x + b*c)*log(b*x + a)^2 + (b*d
*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*a*d + 2*(b*d*x + b*c)*log(b*x + a) - 2
*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a))*log(d*x + c))*n^2/(b*c^2*d*i^2
- a*c*d^2*i^2 + (b*c*d^2*i^2 - a*d^3*i^2)*x))*B^2 - B^2*log(e*(b*x/(d*x + c
) + a/(d*x + c))^n)^2/(d^2*i^2*x + c*d*i^2) - 2*A*B*log(e*(b*x/(d*x + c) +
a/(d*x + c))^n)/(d^2*i^2*x + c*d*i^2) - A^2/(d^2*i^2*x + c*d*i^2)
```

Giac [A] (verification not implemented)

none

Time = 0.99 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.07

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \left(\frac{(bx+a)B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(dx+c)i^2} - \frac{2\left(B^2n^2-B^2n\log(e)-ABn\right)(bx+a)\log\left(\frac{bx+a}{dx+c}\right)}{(dx+c)i^2} + \frac{\left(2\,B^2n^2-2\,B^2n\log(\right.}{}\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x, algorithm="gi
ac")
```

```
[Out] ((b*x + a)*B^2*n^2*log((b*x + a)/(d*x + c))^2/((d*x + c)*i^2) - 2*(B^2*n^2
- B^2*n*log(e) - A*B*n)*(b*x + a)*log((b*x + a)/(d*x + c))/((d*x + c)*i^2)
+ (2*B^2*n^2 - 2*B^2*n*log(e) + B^2*log(e)^2 - 2*A*B*n + 2*A*B*log(e) + A^2
)*(b*x + a)/((d*x + c)*i^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.45

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(\frac{2\,B^2\,n}{x\,d^2\,i^2+c\,d\,i^2}-\frac{2\,A\,B}{x\,d^2\,i^2+c\,d\,i^2}\right) - \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{d\,\left(c\,i^2+d\,i^2\,x\right)} + \frac{B^2\,b}{d\,i^2\,\left(a\,d-b\,c\right)}\right) - \frac{A^2-2\,A\,B\,n+2\,B^2\,n^2}{x\,d^2\,i^2+c\,d\,i^2} + \frac{B\,b\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{a\,d^2\,i^2+b\,c\,d\,i^2}{d\,i^2}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,\left(A-B\,n\right)\,4\mathrm{i}}{d\,i^2\,\left(a\,d-b\,c\right)}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*i + d*i*x)^2,x)
```

```
[Out] log(e*((a + b*x)/(c + d*x))^n)*((2*B^2*n)/(d^2*i^2*x + c*d*i^2) - (2*A*B)/(
d^2*i^2*x + c*d*i^2)) - log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(d*(c*i^2 + d
*i^2*x)) + (B^2*b)/(d*i^2*(a*d - b*c))) - (A^2 + 2*B^2*n^2 - 2*A*B*n)/(d^2*
i^2*x + c*d*i^2) + (B*b*n*atan(((2*b*d*x + (a*d^2*i^2 + b*c*d*i^2)/(d*i^2))
*1i)/(a*d - b*c))*(A - B*n)*4i)/(d*i^2*(a*d - b*c))
```

3.198 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 231

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx = \frac{2ABdn(a+bx)}{(bc-ad)^2gi^2(c+dx)}-\frac{2B^2dn^2(a+bx)}{(bc-ad)^2gi^2(c+dx)}+\frac{2B^2dn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^2gi^2(c+dx)}-\frac{d(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2gi^2(c+dx)}+\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^2gi^2n}$$

```
[Out] 2*A*B*d*n*(b*x+a)/(-a*d+b*c)^2/g/i^2/(d*x+c)-2*B^2*d*n^2*(b*x+a)/(-a*d+b*c)
^2/g/i^2/(d*x+c)+2*B^2*d*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^2/g
/i^2/(d*x+c)-d*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g/i^2
/(d*x+c)+1/3*b*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b*c)^2/g/i^2/n
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used

$= \{2561, 2388, 2339, 30, 2333, 2332\}$

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx = \frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3Bgi^2n(bc-ad)^2} - \frac{d(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{gi^2(c+dx)(bc-ad)^2} + \frac{2ABdn(a+bx)}{gi^2(c+dx)(bc-ad)^2} + \frac{2B^2dn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{gi^2(c+dx)(bc-ad)^2} - \frac{2B^2dn^2(a+bx)}{gi^2(c+dx)(bc-ad)^2}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)*(c*i + d*i*x)^2
),x]
```

```
[Out] (2*A*B*d*n*(a + b*x))/((b*c - a*d)^2*g*i^2*(c + d*x)) - (2*B^2*d*n^2*(a + b
*x))/((b*c - a*d)^2*g*i^2*(c + d*x)) + (2*B^2*d*n*(a + b*x)*Log[e*((a + b*x
)/(c + d*x))^n])/((b*c - a*d)^2*g*i^2*(c + d*x)) - (d*(a + b*x)*(A + B*Log[
e*((a + b*x)/(c + d*x))^n])^2)/((b*c - a*d)^2*g*i^2*(c + d*x)) + (b*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])^3)/(3*B*(b*c - a*d)^2*g*i^2*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
```

```
x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex^n))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2gi^2}\\
&= \frac{b\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2gi^2} - \frac{d\text{Subst}\left(\int (A+B\log(ex^n))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2gi^2}\\
&= -\frac{d(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2gi^2(c+dx)} + \frac{b\text{Subst}\left(\int x^2\,dx, x, A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)^2gi^2n}\\
&\quad + \frac{(2Bdn)\text{Subst}\left(\int (A+B\log(ex^n))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2gi^2}\\
&= \frac{2ABdn(a+bx)}{(bc-ad)^2gi^2(c+dx)} - \frac{d(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2gi^2(c+dx)}\\
&\quad + \frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^2gi^2n} + \frac{(2B^2dn)\,\text{Subst}\left(\int \log(ex^n)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2gi^2}\\
&= \frac{2ABdn(a+bx)}{(bc-ad)^2gi^2(c+dx)} - \frac{2B^2dn^2(a+bx)}{(bc-ad)^2gi^2(c+dx)} + \frac{2B^2dn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^2gi^2(c+dx)}\\
&\quad - \frac{d(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2gi^2(c+dx)} + \frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^2gi^2n}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 789 vs. 2(231) = 462.

Time = 0.40 (sec) , antiderivative size = 789, normalized size of antiderivative = 3.42

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \frac{bB^2n^2\log^3\left(\frac{a+bx}{c+dx}\right)}{3(bc-ad)^2gi^2} - \frac{2Bn\log\left(\frac{a+bx}{c+dx}\right)\left(-A+Bn-B\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)\right)}{(bc-ad)gi^2(c+dx)}$$
$$+ \frac{\log^2\left(\frac{a+bx}{c+dx}\right)\left(AbBcn-aB^2dn^2+AbBdnx-bB^2dn^2x+bB^2cn\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)+bB^2dnx\right.}{(bc-ad)^2gi^2(c+dx)}$$
$$+ \frac{A^2-2ABn+2B^2n^2+2AB\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)-2B^2n\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)+B}{(bc-ad)gi^2(c+dx)}$$
$$+ \frac{b\log(a+bx)\left(A^2-2ABn+2B^2n^2+2AB\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)-2B^2n\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\right.\right.}{(bc-ad)^2gi^2}$$
$$- \frac{b\left(A^2-2ABn+2B^2n^2+2AB\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)-2B^2n\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)\right.}{(bc-ad)^2gi^2}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)*(c*i + d*
i*x)^2),x]
```

```
[Out] (b*B^2*n^2*Log[(a + b*x)/(c + d*x)]^3)/(3*(b*c - a*d)^2*g*i^2) - (2*B*n*Log
[(a + b*x)/(c + d*x)]*(-A + B*n - B*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log
[(a + b*x)/(c + d*x)])))/((b*c - a*d)*g*i^2*(c + d*x)) + (Log[(a + b*x)/(c
+ d*x)]^2*(A*b*B*c*n - a*B^2*d*n^2 + A*b*B*d*n*x - b*B^2*d*n^2*x + b*B^2*c*
n*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) + b*B^2*d*n
*x*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])))/((b*c -
a*d)^2*g*i^2*(c + d*x)) + (A^2 - 2*A*B*n + 2*B^2*n^2 + 2*A*B*(Log[e*((a + b
*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) - 2*B^2*n*(Log[e*((a + b*x)
/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) + B^2*(Log[e*((a + b*x)/(c + d
*x))^n] - n*Log[(a + b*x)/(c + d*x)])^2)/((b*c - a*d)*g*i^2*(c + d*x)) + (b
*Log[a + b*x]*(A^2 - 2*A*B*n + 2*B^2*n^2 + 2*A*B*(Log[e*((a + b*x)/(c + d*x
))^n] - n*Log[(a + b*x)/(c + d*x)]) - 2*B^2*n*(Log[e*((a + b*x)/(c + d*x))^
n] - n*Log[(a + b*x)/(c + d*x)]) + B^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*
Log[(a + b*x)/(c + d*x)])^2))/((b*c - a*d)^2*g*i^2) - (b*(A^2 - 2*A*B*n + 2
*B^2*n^2 + 2*A*B*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x
)]) - 2*B^2*n*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])
 + B^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])^2)*Log
[c + d*x])/((b*c - a*d)^2*g*i^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. 2(229) = 458.

Time = 5.06 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.20

method	result
parallelrisch	$\frac{-6B^2 a b^2 d^4 n^3+6B^2 b^3 c d^3 n^3-3A^2 a b^2 d^4 n+3A^2 b^3 c d^3 n+B^2 x \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^3 b^3 d^4+B^2 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^3 b^3 c d^3+3A^2 x \ln\left(e\left(\right.\right.}{}$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/3*(-6*B^2*a*b^2*d^4*n^3+6*B^2*b^3*c*d^3*n^3-3*A^2*a*b^2*d^4*n+3*A^2*b^3*c
*d^3*n+B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*b^3*d^4+B^2*ln(e*((b*x+a)/(d*x+c))
^n)^3*b^3*c*d^3+3*A^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^4+3*A^2*ln(e*((b*x+
a)/(d*x+c))^n)*b^3*c*d^3-6*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^4*n-6*A*B*
ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*d^4*n+6*A*B*a*b^2*d^4*n^2-6*A*B*b^3*c*d^3*n
^2-3*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*d^4*n+6*B^2*x*ln(e*((b*x+a)/(d*x
+c))^n)*b^3*d^4*n^2+3*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*d^4-3*B^2*ln(e*
((b*x+a)/(d*x+c))^n)^2*a*b^2*d^4*n+6*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*d^
4*n^2+3*A*B*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*c*d^3)/i^2/g/(d*x+c)/(a^2*d^2-2
*a*b*c*d+b^2*c^2)/b^2/d^3/n
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.87

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx = \frac{3A^2bc-3A^2ad+(B^2bdn^2x+B^2bcn^2)\log\left(\frac{bx+a}{dx+c}\right)^3+6(B^2bc-B^2ad)n^2+3\left(B^2bc-B^2ad+(B^2bdx+\right.}{}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x, a
lgorithm="fricas")
```

```
[Out] 1/3*(3*A^2*b*c - 3*A^2*a*d + (B^2*b*d*n^2*x + B^2*b*c*n^2)*log((b*x + a)/(d
*x + c))^3 + 6*(B^2*b*c - B^2*a*d)*n^2 + 3*(B^2*b*c - B^2*a*d + (B^2*b*d*x
+ B^2*b*c)*log((b*x + a)/(d*x + c)))*log(e)^2 - 3*(B^2*a*d*n^2 - A*B*b*c*n
+ (B^2*b*d*n^2 - A*B*b*d*n)*x)*log((b*x + a)/(d*x + c))^2 - 6*(A*B*b*c - A*
B*a*d)*n + 3*(2*A*B*b*c - 2*A*B*a*d + (B^2*b*d*n*x + B^2*b*c*n)*log((b*x +
a)/(d*x + c))^2 - 2*(B^2*b*c - B^2*a*d)*n - 2*(B^2*a*d*n - A*B*b*c + (B^2*b
*d*n - A*B*b*d)*x)*log((b*x + a)/(d*x + c)))*log(e) + 3*(2*B^2*a*d*n^2 - 2*
A*B*a*d*n + A^2*b*c + (2*B^2*b*d*n^2 - 2*A*B*b*d*n + A^2*b*d)*x)*log((b*x +
```

```
a)/(d*x + c)))/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g*i^2*x + (b^2*c^3 - 2
*a*b*c^2*d + a^2*c*d^2)*g*i^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)/(d*i*x+c*i)**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1014 vs. $2(229) = 458$.

Time = 0.26 (sec) , antiderivative size = 1014, normalized size of antiderivative = 4.39

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= B^2\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2}+\frac{b\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}-\frac{b\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}\right)\log\left(e\left(\frac{}{d}\right.\right.$$
$$+2\,AB\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2}+\frac{b\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}-\frac{b\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}\right)\log\left(\right.$$
$$+\frac{1}{3}\left(\frac{\left((bdx+bc)\log(bx+a)^3-(bdx+bc)\log(dx+c)^3+3\,(bdx+bc)\log(bx+a)^2+3\,(bdx+bc+(bdx\right.}{b^2c^3g}\right.$$
$$-\frac{\left((bdx+bc)\log(bx+a)^2+(bdx+bc)\log(dx+c)^2+2\,bc-2\,ad+2\,(bdx+bc)\log(bx+a)-2\,(bdx+b\right.}{b^2c^3gi^2-2\,abc^2dgi^2+a^2cd^2gi^2+(b^2c^2dgi^2-2\,abcd^2gi^2+a^2d^3}$$
$$+A^2\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2}+\frac{b\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}-\frac{b\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}\right)$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x, a
lgorithm="maxima")
[Out] B^2*(1/((b*c*d - a*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/(
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*g*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2 + 2*A*B*(1/(
(b*c*d - a*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/((b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*g*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*(((b*d*x + b*c)*
log(b*x + a)^3 - (b*d*x + b*c)*log(d*x + c)^3 + 3*(b*d*x + b*c)*log(b*x + a
)^2 + 3*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a))*log(d*x + c)^2 + 6*b*c -
```

```
6*a*d + 6*(b*d*x + b*c)*log(b*x + a) - 3*(2*b*d*x + (b*d*x + b*c)*log(b*x
+ a)^2 + 2*b*c + 2*(b*d*x + b*c)*log(b*x + a))*log(d*x + c))*n^2/(b^2*c^3*g
*i^2 - 2*a*b*c^2*d*g*i^2 + a^2*c*d^2*g*i^2 + (b^2*c^2*d*g*i^2 - 2*a*b*c*d^2
*g*i^2 + a^2*d^3*g*i^2)*x) - 3*((b*d*x + b*c)*log(b*x + a)^2 + (b*d*x + b*c
)*log(d*x + c)^2 + 2*b*c - 2*a*d + 2*(b*d*x + b*c)*log(b*x + a) - 2*(b*d*x
+ b*c + (b*d*x + b*c)*log(b*x + a))*log(d*x + c))*n*log(e*(b*x/(d*x + c) +
a/(d*x + c))^n)/(b^2*c^3*g*i^2 - 2*a*b*c^2*d*g*i^2 + a^2*c*d^2*g*i^2 + (b^2
*c^2*d*g*i^2 - 2*a*b*c*d^2*g*i^2 + a^2*d^3*g*i^2)*x))*B^2 - ((b*d*x + b*c)*
log(b*x + a)^2 + (b*d*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*a*d + 2*(b*d*x +
b*c)*log(b*x + a) - 2*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a))*log(d*x +
c))*A*B*n/(b^2*c^3*g*i^2 - 2*a*b*c^2*d*g*i^2 + a^2*c*d^2*g*i^2 + (b^2*c^2*d
*g*i^2 - 2*a*b*c*d^2*g*i^2 + a^2*d^3*g*i^2)*x) + A^2*(1/((b*c*d - a*d^2)*g*
i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/((b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g*i^2))
```

Giac [A] (verification not implemented)

none

Time = 0.90 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.59

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$
$$=\frac{1}{3}\left(\frac{B^2bn^2\log\left(\frac{bx+a}{dx+c}\right)^3}{bcgi^2-adgi^2}-3\left(\frac{(bx+a)B^2dn^2}{(bcgi^2-adgi^2)(dx+c)}-\frac{B^2bn\log(e)+ABbn}{bcgi^2-adgi^2}\right)\log\left(\frac{bx+a}{dx+c}\right)^2+\frac{3\left(B^2b\log\right.}{}\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x, a
lgorithm="giac")
```

```
[Out] 1/3*(B^2*b*n^2*log((b*x + a)/(d*x + c))^3/(b*c*g*i^2 - a*d*g*i^2) - 3*((b*x
 + a)*B^2*d*n^2/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)) - (B^2*b*n*log(e) + A*B
*b*n)/(b*c*g*i^2 - a*d*g*i^2))*log((b*x + a)/(d*x + c))^2 + 3*(B^2*b*log(e)
^2 + 2*A*B*b*log(e) + A^2*b)*log((b*x + a)/(d*x + c))/(b*c*g*i^2 - a*d*g*i^
2) + 6*(B^2*d*n^2 - B^2*d*n*log(e) - A*B*d*n)*(b*x + a)*log((b*x + a)/(d*x
+ c))/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)) - 3*(2*B^2*d*n^2 - 2*B^2*d*n*log(
e) + B^2*d*log(e)^2 - 2*A*B*d*n + 2*A*B*d*log(e) + A^2*d)*(b*x + a)/((b*c*g
*i^2 - a*d*g*i^2)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.58

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$
$$=\frac{B^2\,b\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^3}{3\,g\,i^2\,n\,(a\,d-b\,c)^2}-\frac{A^2-2\,A\,B\,n+2\,B^2\,n^2}{(a\,d-b\,c)\,\left(c\,g\,i^2+d\,g\,i^2\,x\right)}-\frac{2\,B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,(A-B\,n)}{(a\,d-b\,c)\,\left(c\,g\,i^2+d\,g\,i^2\,x\right)}$$
$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{(a\,d-b\,c)\,\left(c\,g\,i^2+d\,g\,i^2\,x\right)}-\frac{B\,b\,(A-B\,n)}{g\,i^2\,n\,(a\,d-b\,c)^2}\right)$$
$$-\frac{b\,\mathrm{atan}\left(\frac{b\left(2\,b\,d\,x+\frac{a^2\,d^2\,g\,i^2-b^2\,c^2\,g\,i^2}{g\,i^2\,(a\,d-b\,c)}\right)\left(A^2-2\,A\,B\,n+2\,B^2\,n^2\right)\,1\mathrm{i}}{(a\,d-b\,c)\,\left(b\,A^2-2\,b\,A\,B\,n+2\,b\,B^2\,n^2\right)}\right)\left(A^2-2\,A\,B\,n+2\,B^2\,n^2\right)\,2\mathrm{i}}{g\,i^2\,(a\,d-b\,c)^2}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)*(c*i + d*i*x)^2
),x)
```

```
[Out] (B^2*b*log(e*((a + b*x)/(c + d*x))^n)^3)/(3*g*i^2*n*(a*d - b*c)^2) - (A^2 +
 2*B^2*n^2 - 2*A*B*n)/((a*d - b*c)*(c*g*i^2 + d*g*i^2*x)) - (2*B*log(e*((a
+ b*x)/(c + d*x))^n)*(A - B*n))/((a*d - b*c)*(c*g*i^2 + d*g*i^2*x)) - (b*at
an((b*(2*b*d*x + (a^2*d^2*g*i^2 - b^2*c^2*g*i^2)/(g*i^2*(a*d - b*c)))*(A^2
+ 2*B^2*n^2 - 2*A*B*n)*1i)/((a*d - b*c)*(A^2*b + 2*B^2*b*n^2 - 2*A*B*b*n)))
*(A^2 + 2*B^2*n^2 - 2*A*B*n)*2i)/(g*i^2*(a*d - b*c)^2) - log(e*((a + b*x)/(
c + d*x))^n)^2*(B^2/((a*d - b*c)*(c*g*i^2 + d*g*i^2*x)) - (B*b*(A - B*n))/(
g*i^2*n*(a*d - b*c)^2))
```

3.199 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 392

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx &= -\frac{2ABd^2n(a+bx)}{(bc-ad)^3g^2i^2(c+dx)}+\frac{2B^2d^2n^2(a+bx)}{(bc-ad)^3g^2i^2(c+dx)}\\
&\quad-\frac{2b^2B^2n^2(c+dx)}{(bc-ad)^3g^2i^2(a+bx)}-\frac{2B^2d^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^3g^2i^2(c+dx)}\\
&\quad-\frac{2b^2Bn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^2i^2(a+bx)}\\
&\quad+\frac{d^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^2i^2(c+dx)}\\
&\quad-\frac{b^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^2i^2(a+bx)}\\
&\quad-\frac{2bd\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^3g^2i^2n}\end{aligned}$$

```
[Out] -2*A*B*d^2*n*(b*x+a)/(-a*d+b*c)^3/g^2/i^2/(d*x+c)+2*B^2*d^2*n^2*(b*x+a)/(-a
*d+b*c)^3/g^2/i^2/(d*x+c)-2*b^2*B^2*n^2*(d*x+c)/(-a*d+b*c)^3/g^2/i^2/(b*x+a
)-2*B^2*d^2*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^3/g^2/i^2/(d*x+c
)-2*b^2*B*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^2/i^2/(b
*x+a)+d^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^2/i^2/(d
*x+c)-b^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^2/i^2/(b
*x+a)-2/3*b*d*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b*c)^3/g^2/i^2/n
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {2561, 2395, 2333, 2332, 2342, 2341, 2339, 30}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = -\frac{b^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^2i^2(a+bx)(bc-ad)^3} - \frac{2b^2Bn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^2i^2(a+bx)(bc-ad)^3} + \frac{d^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^2i^2(c+dx)(bc-ad)^3} - \frac{2ABd^2n(a+bx)}{g^2i^2(c+dx)(bc-ad)^3} - \frac{2bd\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3Bg^2i^2n(bc-ad)^3} - \frac{2b^2B^2n^2(c+dx)}{g^2i^2(a+bx)(bc-ad)^3} - \frac{2B^2d^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g^2i^2(c+dx)(bc-ad)^3} + \frac{2B^2d^2n^2(a+bx)}{g^2i^2(c+dx)(bc-ad)^3}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^2*(c*i + d*i*x)
^2),x]
```

```
[Out] (-2*A*B*d^2*n*(a + b*x))/((b*c - a*d)^3*g^2*i^2*(c + d*x)) + (2*B^2*d^2*n^2
*(a + b*x))/((b*c - a*d)^3*g^2*i^2*(c + d*x)) - (2*b^2*B^2*n^2*(c + d*x))/(
(b*c - a*d)^3*g^2*i^2*(a + b*x)) - (2*B^2*d^2*n*(a + b*x)*Log[e*((a + b*x)/
(c + d*x))^n])/((b*c - a*d)^3*g^2*i^2*(c + d*x)) - (2*b^2*B*n*(c + d*x)*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^3*g^2*i^2*(a + b*x)) + (d
^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*c - a*d)^3*g^2*i
^2*(c + d*x)) - (b^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((
b*c - a*d)^3*g^2*i^2*(a + b*x)) - (2*b*d*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])^3)/(3*B*(b*c - a*d)^3*g^2*i^2*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}$$

$$= \frac{\text{Subst}\left(\int \left(d^2(A+B\log(ex^n))^2+\frac{b^2(A+B\log(ex^n))^2}{x^2}-\frac{2bd(A+B\log(ex^n))^2}{x}\right)dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}$$

$$= \frac{b^2\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}-\frac{(2bd)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x}dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}+\frac{d^2\text{Subst}\left(\int (A+B\log(ex^n))^2\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}$$

$$= \frac{d^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^2i^2(c+dx)}-\frac{b^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^2i^2(a+bx)}-\frac{(2bd)\text{Subst}\left(\int x^2\,dx,x,A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)^3g^2i^2n}+\frac{(2b^2Bn)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}-\frac{(2Bd^2n)\,\text{Subst}\left(\int (A+B\log(ex^n))\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}$$

$$= -\frac{2ABd^2n(a+bx)}{(bc-ad)^3g^2i^2(c+dx)}-\frac{2b^2B^2n^2(c+dx)}{(bc-ad)^3g^2i^2(a+bx)}-\frac{2b^2Bn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^2i^2(a+bx)}+\frac{d^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^2i^2(c+dx)}-\frac{b^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^2i^2(a+bx)}-\frac{2bd\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^3g^2i^2n}-\frac{(2B^2d^2n)\,\text{Subst}\left(\int \log(ex^n)\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}$$

$$= -\frac{2ABd^2n(a+bx)}{(bc-ad)^3g^2i^2(c+dx)}+\frac{2B^2d^2n^2(a+bx)}{(bc-ad)^3g^2i^2(c+dx)}-\frac{2b^2B^2n^2(c+dx)}{(bc-ad)^3g^2i^2(a+bx)}-\frac{2B^2d^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^3g^2i^2(c+dx)}-\frac{2b^2Bn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^2i^2(a+bx)}+\frac{d^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^2i^2(c+dx)}-\frac{b^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^2i^2(a+bx)}-\frac{2bd\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^3g^2i^2n}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 870 vs. 2(392) = 784.

Time = 0.62 (sec) , antiderivative size = 870, normalized size of antiderivative = 2.22

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = -\frac{2bB^2dn^2(a+bx)(c+dx)\log^3\left(\frac{a+bx}{c+dx}\right)+3Bn\log^2\left(\frac{a+bx}{c+dx}\right)\left(2aAbcd+b^2Bc^2n-a^2Bd^2n+2Ab^2cdx+2a\right.}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^2*(c*i +
d*i*x)^2),x]
```

```
[Out] -1/3*(2*b*B^2*d*n^2*(a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^3 + 3*B*n*
Log[(a + b*x)/(c + d*x)]^2*(2*a*A*b*c*d + b^2*B*c^2*n - a^2*B*d^2*n + 2*A*b
^2*c*d*x + 2*a*A*b*d^2*x + 2*b^2*B*c*d*n*x - 2*a*b*B*d^2*n*x + 2*A*b^2*d^2*
x^2 + 2*b*B*d*(a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x))^n] - 2*b*B*d*
n*(a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]) + 6*B*(b*c - a*d)*n*Log[(a
+ b*x)/(c + d*x)]*(A*b*c + a*A*d + b*B*c*n - a*B*d*n + 2*A*b*d*x + B*(a*d +
 b*(c + 2*d*x))*Log[e*((a + b*x)/(c + d*x))^n] - B*n*(b*c + a*d + 2*b*d*x)*
Log[(a + b*x)/(c + d*x)]) + 6*b*d*(a + b*x)*(c + d*x)*Log[a + b*x]*(A^2 + 2
*B^2*n^2 + 2*A*B*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x
)]) + B^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])^2)
+ 3*b*(b*c - a*d)*(c + d*x)*(A^2 + 2*A*B*n + 2*B^2*n^2 + B^2*Log[e*((a + b*
x)/(c + d*x))^n]^2 - 2*B*n*(A + B*n)*Log[(a + b*x)/(c + d*x)] + B^2*n^2*Log
[(a + b*x)/(c + d*x)]^2 + 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(A + B*n - B*n
*Log[(a + b*x)/(c + d*x)])) + 3*d*(b*c - a*d)*(a + b*x)*(A^2 - 2*A*B*n + 2*
B^2*n^2 + B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 + 2*B*n*(-A + B*n)*Log[(a +
b*x)/(c + d*x)] + B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 - 2*B*Log[e*((a + b*x)
/(c + d*x))^n]*(-A + B*n + B*n*Log[(a + b*x)/(c + d*x)])) - 6*b*d*(a + b*x)
*(c + d*x)*(A^2 + 2*B^2*n^2 + 2*A*B*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log
[(a + b*x)/(c + d*x)]) + B^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b
*x)/(c + d*x)])^2)*Log[c + d*x])/((b*c - a*d)^3*g^2*i^2*(a + b*x)*(c + d*x)
)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1306 vs. 2(390) = 780.

Time = 9.72 (sec) , antiderivative size = 1307, normalized size of antiderivative = 3.33

method	result	size
parallelrisch	Expression too large to display	1307

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x,method=
_RETURNVERBOSE)
```

```
[Out] 1/3*(-12*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b*c^3*d^2*n+12*A*B*x*ln(e*((b*
x+a)/(d*x+c))^n)*a^3*b^2*c^4*d*n+12*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b
^2*c^3*d^2*n^2+6*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^2*c^3*d^2-6*A*B*
x^2*a^4*b*c^2*d^3*n^2+12*A*B*x^2*a^3*b^2*c^3*d^2*n^2-6*A*B*x^2*a^2*b^3*c^4*
d*n^2-6*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b*c^3*d^2*n+6*B^2*x*ln(e*((b*
x+a)/(d*x+c))^n)^2*a^3*b^2*c^4*d*n+12*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b
*c^3*d^2*n^2+12*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^4*d*n^2+6*A*B*x*l
n(e*((b*x+a)/(d*x+c))^n)^2*a^4*b*c^3*d^2+6*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^
2*a^3*b^2*c^4*d+6*A*B*x*a^4*b*c^3*d^2*n^2+6*A*B*x*a^3*b^2*c^4*d*n^2+6*B^2*x
^2*a^4*b*c^2*d^3*n^3-6*B^2*x^2*a^2*b^3*c^4*d*n^3+2*B^2*x*ln(e*((b*x+a)/(d*x
+c))^n)^3*a^4*b*c^3*d^2+2*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*a^3*b^2*c^4*d-6
*B^2*x*a^4*b*c^3*d^2*n^3+6*B^2*x*a^3*b^2*c^4*d*n^3+6*A^2*x^2*ln(e*((b*x+a)/
(d*x+c))^n)*a^3*b^2*c^3*d^2+3*A^2*x^2*a^4*b*c^2*d^3*n-3*A^2*x^2*a^2*b^3*c^4
*d*n-6*A*B*x*a^5*c^2*d^3*n^2-6*A*B*x*a^2*b^3*c^5*n^2+6*A^2*x*ln(e*((b*x+a)/
(d*x+c))^n)*a^4*b*c^3*d^2+6*A^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^4*d-3
*A^2*x*a^4*b*c^3*d^2*n+3*A^2*x*a^3*b^2*c^4*d*n+6*A*B*ln(e*((b*x+a)/(d*x+c))
^n)^2*a^4*b*c^4*d-6*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^5*c^3*d^2*n+6*A*B*ln(e*
((b*x+a)/(d*x+c))^n)*a^3*b^2*c^5*n+3*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^
2*c^5*n+6*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^5*c^3*d^2*n^2+6*B^2*ln(e*((b*x+a)
/(d*x+c))^n)*a^3*b^2*c^5*n^2+3*A^2*x*a^5*c^2*d^3*n-3*A^2*x*a^2*b^3*c^5*n+6*
A^2*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b*c^4*d+2*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^
n)^3*a^3*b^2*c^3*d^2+6*B^2*x*a^5*c^2*d^3*n^3-6*B^2*x*a^2*b^3*c^5*n^3+2*B^2*
ln(e*((b*x+a)/(d*x+c))^n)^3*a^4*b*c^4*d-3*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a
^5*c^3*d^2*n)/i^2/g^2/(d*x+c)/(b*x+a)/a^3/c^3/n/(a^3*d^3-3*a^2*b*c*d^2+3*a*
b^2*c^2*d-b^3*c^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. 2(390) = 780.

Time = 0.38 (sec) , antiderivative size = 983, normalized size of antiderivative = 2.51

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = -\frac{3\,A^2b^2c^2-3\,A^2a^2d^2+2\,(B^2b^2d^2n^2x^2+B^2abcdn^2+(B^2b^2cd+B^2abd^2)n^2x)\log\left(\frac{bx+a}{dx+c}\right)^3+6\,(B^2b^2c^2-B}{}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x,
 algorithm="fricas")
```

```
[Out] -1/3*(3*A^2*b^2*c^2 - 3*A^2*a^2*d^2 + 2*(B^2*b^2*d^2*n^2*x^2 + B^2*a*b*c*d*
n^2 + (B^2*b^2*c*d + B^2*a*b*d^2)*n^2*x)*log((b*x + a)/(d*x + c))^3 + 6*(B^
```

```
2*b^2*c^2 - B^2*a^2*d^2)*n^2 + 3*(B^2*b^2*c^2 - B^2*a^2*d^2 + 2*(B^2*b^2*c*
d - B^2*a*b*d^2)*x + 2*(B^2*b^2*d^2*x^2 + B^2*a*b*c*d + (B^2*b^2*c*d + B^2*
a*b*d^2)*x)*log((b*x + a)/(d*x + c)))*log(e)^2 + 3*(2*A*B*b^2*d^2*n*x^2 + 2
*A*B*a*b*c*d*n + (B^2*b^2*c^2 - B^2*a^2*d^2)*n^2 + 2*((B^2*b^2*c*d - B^2*a*
b*d^2)*n^2 + (A*B*b^2*c*d + A*B*a*b*d^2)*n)*x)*log((b*x + a)/(d*x + c))^2 +
 6*(A*B*b^2*c^2 - 2*A*B*a*b*c*d + A*B*a^2*d^2)*n + 6*(A^2*b^2*c*d - A^2*a*b
*d^2 + 2*(B^2*b^2*c*d - B^2*a*b*d^2)*n^2)*x + 6*(A*B*b^2*c^2 - A*B*a^2*d^2
+ (B^2*b^2*d^2*n*x^2 + B^2*a*b*c*d*n + (B^2*b^2*c*d + B^2*a*b*d^2)*n*x)*log
((b*x + a)/(d*x + c))^2 + (B^2*b^2*c^2 - 2*B^2*a*b*c*d + B^2*a^2*d^2)*n + 2
*(A*B*b^2*c*d - A*B*a*b*d^2)*x + (2*A*B*b^2*d^2*x^2 + 2*A*B*a*b*c*d + (B^2*
b^2*c^2 - B^2*a^2*d^2)*n + 2*(A*B*b^2*c*d + A*B*a*b*d^2 + (B^2*b^2*c*d - B^
2*a*b*d^2)*n)*x)*log((b*x + a)/(d*x + c)))*log(e) + 6*(A^2*a*b*c*d + (B^2*b
^2*c^2 + B^2*a^2*d^2)*n^2 + (2*B^2*b^2*d^2*n^2 + A^2*b^2*d^2)*x^2 + (A*B*b^
2*c^2 - A*B*a^2*d^2)*n + (A^2*b^2*c*d + A^2*a*b*d^2 + 2*(B^2*b^2*c*d + B^2*
a*b*d^2)*n^2 + 2*(A*B*b^2*c*d - A*B*a*b*d^2)*n)*x)*log((b*x + a)/(d*x + c))
)/((b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*g^2*i^2*x^2
+ (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*g^2*i^2*x + (a*b^3*c^
4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*g^2*i^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**2/(d*i*x+c*i)**2
,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2006 vs. 2(390) = 780.

Time = 0.31 (sec) , antiderivative size = 2006, normalized size of antiderivative = 5.12

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x,
 algorithm="maxima")
```

```
[Out] -B^2*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*g^2*i^
2*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*g^2*i^2*x + (a*b^2*
c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2) + 2*b*d*log(b*x + a)/((b^3*c^3 -
```

3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2) - 2*b*d*log(d*x + c)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2 - 2*A*B*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*g^2*i^2*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*g^2*i^2*x + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2) + 2*b*d*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2) - 2*b*d*log(d*x + c)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 2/3*((3*b^2*c^2 - 3*a^2*d^2 + (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(b*x + a)^3 + 3*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(b*x + a)*log(d*x + c)^2 - (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(d*x + c)^3 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(b*x + a) - 3*(2*b^2*d^2*x^2 + 2*a*b*c*d + (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(b*x + a)^2 + 2*(b^2*c*d + a*b*d^2)*x)*log(d*x + c))*n^2/(a*b^3*c^4*g^2*i^2 - 3*a^2*b^2*c^3*d*g^2*i^2 + 3*a^3*b*c^2*d^2*g^2*i^2 - a^4*c*d^3*g^2*i^2 + (b^4*c^3*d*g^2*i^2 - 3*a*b^3*c^2*d^2*g^2*i^2 + 3*a^2*b^2*c*d^3*g^2*i^2 - a^3*b*d^4*g^2*i^2)*x^2 + (b^4*c^4*g^2*i^2 - 2*a*b^3*c^3*d*g^2*i^2 + 2*a^3*b*c*d^3*g^2*i^2 - a^4*d^4*g^2*i^2)*x) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(b*x + a)*log(d*x + c) - (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(d*x + c)^2)*n*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(a*b^3*c^4*g^2*i^2 - 3*a^2*b^2*c^3*d*g^2*i^2 + 3*a^3*b*c^2*d^2*g^2*i^2 - a^4*c*d^3*g^2*i^2 + (b^4*c^3*d*g^2*i^2 - 3*a*b^3*c^2*d^2*g^2*i^2 + 3*a^2*b^2*c*d^3*g^2*i^2 - a^3*b*d^4*g^2*i^2)*x^2 + (b^4*c^4*g^2*i^2 - 2*a*b^3*c^3*d*g^2*i^2 + 2*a^3*b*c*d^3*g^2*i^2 - a^4*d^4*g^2*i^2)*x))*B^2 - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(b*x + a)*log(d*x + c) - (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(d*x + c)^2)*A*B*n/(a*b^3*c^4*g^2*i^2 - 3*a^2*b^2*c^3*d*g^2*i^2 + 3*a^3*b*c^2*d^2*g^2*i^2 - a^4*c*d^3*g^2*i^2 + (b^4*c^3*d*g^2*i^2 - 3*a*b^3*c^2*d^2*g^2*i^2 + 3*a^2*b^2*c*d^3*g^2*i^2 - a^3*b*d^4*g^2*i^2)*x^2 + (b^4*c^4*g^2*i^2 - 2*a*b^3*c^3*d*g^2*i^2 + 2*a^3*b*c*d^3*g^2*i^2 - a^4*d^4*g^2*i^2)*x) - A^2*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*g^2*i^2*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*g^2*i^2*x + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2) + 2*b*d*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2) - 2*b*d*log(d*x + c)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2))

Giac [A] (verification not implemented)

none

Time = 285.30 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.47

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = -\left(\frac{(dx+c)B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(bx+a)g^2i^2}+\frac{2\left(B^2n^2+B^2n\log(e)+ABn\right)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)g^2i^2}+\frac{\left(2B^2n^2+2B^2n\log\right.}{}\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x,
 algorithm="giac")
```

```
[Out] -((d*x + c)*B^2*n^2*log((b*x + a)/(d*x + c))^2/((b*x + a)*g^2*i^2) + 2*(B^2
*n^2 + B^2*n*log(e) + A*B*n)*(d*x + c)*log((b*x + a)/(d*x + c))/((b*x + a)*
g^2*i^2) + (2*B^2*n^2 + 2*B^2*n*log(e) + B^2*log(e)^2 + 2*A*B*n + 2*A*B*log
(e) + A^2)*(d*x + c)/((b*x + a)*g^2*i^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a
*d)^2)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = \int \frac{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}{{\left(a\,g+b\,g\,x\right)}^2\,{\left(c\,i+d\,i\,x\right)}^2}\,dx$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^2*(c*i + d*i*x)
^2),x)
```

```
[Out] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^2*(c*i + d*i*x)
^2), x)
```

3.200 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 560

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx &= \frac{2ABd^3n(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} - \frac{2B^2d^3n^2(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} \\ &+ \frac{6b^2B^2dn^2(c+dx)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3B^2n^2(c+dx)^2}{4(bc-ad)^4g^3i^2(a+bx)^2} \\ &+ \frac{2B^2d^3n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^4g^3i^2(c+dx)} \\ &+ \frac{6b^2Bdn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} \\ &- \frac{b^3Bn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} \\ &- \frac{d^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^3i^2(c+dx)} \\ &+ \frac{3b^2d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^3i^2(a+bx)} \\ &- \frac{b^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^4g^3i^2(a+bx)^2} \\ &+ \frac{bd^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{B(bc-ad)^4g^3i^2n}\end{aligned}$$

```
[Out] 2*A*B*d^3*n*(b*x+a)/(-a*d+b*c)^4/g^3/i^2/(d*x+c)-2*B^2*d^3*n^2*(b*x+a)/(-a*
d+b*c)^4/g^3/i^2/(d*x+c)+6*b^2*B^2*d*n^2*(d*x+c)/(-a*d+b*c)^4/g^3/i^2/(b*x+
a)-1/4*b^3*B^2*n^2*(d*x+c)^2/(-a*d+b*c)^4/g^3/i^2/(b*x+a)^2+2*B^2*d^3*n*(b*
```

```
x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^4/g^3/i^2/(d*x+c)+6*b^2*B*d*n*(d*
x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^3/i^2/(b*x+a)-1/2*b^3*B
*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^3/i^2/(b*x+a)^2
-d^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^3/i^2/(d*x+c)
+3*b^2*d*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^3/i^2/(b*
x+a)-1/2*b^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^3/i
^2/(b*x+a)^2+b*d^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b*c)^4/g^3/i^2
/n
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {2561, 2395, 2333, 2332, 2342, 2341, 2339, 30}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = -\frac{b^3(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g^3i^2(a+bx)^2(bc-ad)^4} - \frac{b^3Bn(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^3i^2(a+bx)^2(bc-ad)^4} + \frac{3b^2d(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^3i^2(a+bx)(bc-ad)^4} + \frac{6b^2Bdn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^3i^2(a+bx)(bc-ad)^4} - \frac{d^3(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^3i^2(c+dx)(bc-ad)^4} + \frac{2ABd^3n(a+bx)}{g^3i^2(c+dx)(bc-ad)^4} + \frac{bd^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{Bg^3i^2n(bc-ad)^4} - \frac{b^3B^2n^2(c+dx)^2}{4g^3i^2(a+bx)^2(bc-ad)^4} + \frac{6b^2B^2dn^2(c+dx)}{g^3i^2(a+bx)(bc-ad)^4} + \frac{2B^2d^3n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g^3i^2(c+dx)(bc-ad)^4} - \frac{2B^2d^3n^2(a+bx)}{g^3i^2(c+dx)(bc-ad)^4}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^3*(c*i + d*i*x)
^2),x]
```

```
[Out] (2*A*B*d^3*n*(a + b*x))/((b*c - a*d)^4*g^3*i^2*(c + d*x)) - (2*B^2*d^3*n^2*
(a + b*x))/((b*c - a*d)^4*g^3*i^2*(c + d*x)) + (6*b^2*B^2*d*n^2*(c + d*x))/
((b*c - a*d)^4*g^3*i^2*(a + b*x)) - (b^3*B^2*n^2*(c + d*x)^2)/(4*(b*c - a*d
)^4*g^3*i^2*(a + b*x)^2) + (2*B^2*d^3*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x
))^n])/((b*c - a*d)^4*g^3*i^2*(c + d*x)) + (6*b^2*B*d*n*(c + d*x)*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^4*g^3*i^2*(a + b*x)) - (b^3*B*n
*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*c - a*d)^4*g^3*i
```

```
^2*(a + b*x)^2) - (d^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/
((b*c - a*d)^4*g^3*i^2*(c + d*x)) + (3*b^2*d*(c + d*x)*(A + B*Log[e*((a + b
*x)/(c + d*x))^n])^2)/((b*c - a*d)^4*g^3*i^2*(a + b*x)) - (b^3*(c + d*x)^2*
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(b*c - a*d)^4*g^3*i^2*(a + b*x
)^2) + (b*d^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(B*(b*c - a*d)^4*g^
3*i^2*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
```

```
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)^3(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2}$$

$$= \frac{\text{Subst}\left(\int \left(-d^3(A+B\log(ex^n))^2 + \frac{b^3(A+B\log(ex^n))^2}{x^3} - \frac{3b^2d(A+B\log(ex^n))^2}{x^2} + \frac{3bd^2(A+B\log(ex^n))^2}{x}\right)\,dx, x}{(bc-ad)^4g^3i^2}$$

$$= \frac{b^3\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2} - \frac{(3b^2d)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2} + \frac{(3bd^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2} - \frac{d^3\text{Subst}\left(\int (A+B\log(ex^n))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2}$$

$$= -\frac{d^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^3i^2(c+dx)} + \frac{3b^2d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^4g^3i^2(a+bx)^2} + \frac{(3bd^2)\,\text{Subst}\left(\int x^2\,dx, x, A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)^4g^3i^2n} + \frac{(b^3Bn)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2} - \frac{(6b^2Bdn)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2} + \frac{(2Bd^3n)\,\text{Subst}\left(\int (A+B\log(ex^n))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2}$$

$$= \frac{2ABd^3n(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} + \frac{6b^2B^2dn^2(c+dx)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3B^2n^2(c+dx)^2}{4(bc-ad)^4g^3i^2(a+bx)^2} + \frac{6b^2Bdn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3Bn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} - \frac{d^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^3i^2(c+dx)} + \frac{3b^2d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^4g^3i^2(a+bx)^2} + \frac{bd^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{B(bc-ad)^4g^3i^2n} + \frac{(2B^2d^3n)\,\text{Subst}\left(\int\log\left(ex^n\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2}$$

$$= \frac{2ABd^3n(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} - \frac{2B^2d^3n^2(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} + \frac{6b^2B^2dn^2(c+dx)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3B^2n^2(c+dx)^2}{4(bc-ad)^4g^3i^2(a+bx)^2} + \frac{2B^2d^3n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^4g^3i^2(c+dx)} + \frac{6b^2Bdn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3Bn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} - \frac{d^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^3i^2(c+dx)} + \frac{3b^2d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^4g^3i^2(a+bx)^2} + \frac{bd^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{B(bc-ad)^4g^3i^2n}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1340 vs. $2(560) = 1120$.

Time = 0.93 (sec) , antiderivative size = 1340, normalized size of antiderivative = 2.39

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \frac{4bB^2d^2n^2(a+bx)^2(c+dx)\log^3\left(\frac{a+bx}{c+dx}\right) + 2Bn\log^2\left(\frac{a+bx}{c+dx}\right)\left(6a^2Abcd^2 - b^3Bc^3n + 6ab^2Bc^2dn - 2a^3Bd^3n + \right.}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^3*(c*i +
d*i*x)^2),x]
```

```
[Out] (4*b*B^2*d^2*n^2*(a + b*x)^2*(c + d*x)*Log[(a + b*x)/(c + d*x)]^3 + 2*B*n*L
og[(a + b*x)/(c + d*x)]^2*(6*a^2*A*b*c*d^2 - b^3*B*c^3*n + 6*a*b^2*B*c^2*d*
n - 2*a^3*B*d^3*n + 12*a*A*b^2*c*d^2*x + 6*a^2*A*b*d^3*x + 3*b^3*B*c^2*d*n*
```

```
x + 12*a*b^2*B*c*d^2*n*x - 6*a^2*b*B*d^3*n*x + 6*A*b^3*c*d^2*x^2 + 12*a*A*b
^2*d^3*x^2 + 9*b^3*B*c*d^2*n*x^2 + 6*A*b^3*d^3*x^3 + 3*b^3*B*d^3*n*x^3 + 6*
b*B*d^2*(a + b*x)^2*(c + d*x)*Log[e*((a + b*x)/(c + d*x))^n] - 6*b*B*d^2*n*
(a + b*x)^2*(c + d*x)*Log[(a + b*x)/(c + d*x)]) + 2*b*d*(b*c - a*d)*(a + b*
x)*(c + d*x)*(4*A^2 + 10*A*B*n + 11*B^2*n^2 + 4*B^2*Log[e*((a + b*x)/(c + d
*x))^n]^2 - 2*B*n*(4*A + 5*B*n)*Log[(a + b*x)/(c + d*x)] + 4*B^2*n^2*Log[(a
 + b*x)/(c + d*x)]^2 + 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(4*A + 5*B*n - 4*
B*n*Log[(a + b*x)/(c + d*x)])) - b*(b*c - a*d)^2*(c + d*x)*(2*A^2 + 2*A*B*n
 + B^2*n^2 + 2*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 2*B*n*(2*A + B*n)*Log
[(a + b*x)/(c + d*x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 2*B*Log[e*((
a + b*x)/(c + d*x))^n]*(2*A + B*n - 2*B*n*Log[(a + b*x)/(c + d*x)])) + 6*b*
d^2*(a + b*x)^2*(c + d*x)*Log[a + b*x]*(2*A^2 + 2*A*B*n + 5*B^2*n^2 + 2*B^2
*Log[e*((a + b*x)/(c + d*x))^n]^2 - 2*B*n*(2*A + B*n)*Log[(a + b*x)/(c + d*
x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 2*B*Log[e*((a + b*x)/(c + d*x)
)^n]*(2*A + B*n - 2*B*n*Log[(a + b*x)/(c + d*x)])) + 2*B*(b*c - a*d)*n*Log[
(a + b*x)/(c + d*x)]*(2*b*d*(a + b*x)*(c + d*x)*(4*A + 5*B*n + 4*B*Log[e*((
a + b*x)/(c + d*x))^n] - 4*B*n*Log[(a + b*x)/(c + d*x)]) - b*(b*c - a*d)*(c
 + d*x)*(2*A + B*n + 2*B*Log[e*((a + b*x)/(c + d*x))^n] - 2*B*n*Log[(a + b*
x)/(c + d*x)]) + 4*d^2*(a + b*x)^2*(A - B*n + B*Log[e*((a + b*x)/(c + d*x))
^n] - B*n*Log[(a + b*x)/(c + d*x)])) + 4*d^2*(b*c - a*d)*(a + b*x)^2*(A^2 -
 2*A*B*n + 2*B^2*n^2 + B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 + 2*B*n*(-A + B
*n)*Log[(a + b*x)/(c + d*x)] + B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 - 2*B*Log
[e*((a + b*x)/(c + d*x))^n]*(-A + B*n + B*n*Log[(a + b*x)/(c + d*x)])) - 6*
b*d^2*(a + b*x)^2*(c + d*x)*(2*A^2 + 2*A*B*n + 5*B^2*n^2 + 2*B^2*Log[e*((a
+ b*x)/(c + d*x))^n]^2 - 2*B*n*(2*A + B*n)*Log[(a + b*x)/(c + d*x)] + 2*B^2
*n^2*Log[(a + b*x)/(c + d*x)]^2 + 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(2*A +
 B*n - 2*B*n*Log[(a + b*x)/(c + d*x)]))*Log[c + d*x])/(4*(b*c - a*d)^4*g^3*
i^2*(a + b*x)^2*(c + d*x))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2000 vs. $2(554) = 1108$.

Time = 28.10 (sec) , antiderivative size = 2001, normalized size of antiderivative = 3.57

method	result	size
parallelrisch	Expression too large to display	2001

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x,method=
_RETURNVERBOSE)
```

```
[Out] 1/4*(24*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^6*c*d^5*n-15*B^2*a^2*b^5*c*d^
5*n^3+24*B^2*a*b^6*c^2*d^4*n^3+8*A*B*a^3*b^4*d^6*n^2-2*A*B*b^7*c^3*d^3*n^2-
6*A^2*a^2*b^5*c*d^5*n+12*A^2*a*b^6*c^2*d^4*n+48*A*B*x*ln(e*((b*x+a)/(d*x+c)
)^n)*a*b^6*c*d^5*n-30*A*B*a^2*b^5*c*d^5*n^2+24*A*B*a*b^6*c^2*d^4*n^2+12*A*B
*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^7*d^6*n+18*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^
```

```
n)^2*b^7*c*d^5*n+48*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*d^6*n^2+42*B^2*
x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c*d^5*n^2+24*A*B*x^2*ln(e*((b*x+a)/(d*x+c
))^n)^2*a*b^6*d^6+12*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^7*c*d^5-12*A*B*x
^2*a*b^6*d^6*n^2+12*A*B*x^2*b^7*c*d^5*n^2+8*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)
^3*a*b^6*c*d^5-12*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^5*d^6*n+6*B^2*x*l
n(e*((b*x+a)/(d*x+c))^n)^2*b^7*c^2*d^4*n+24*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)
*a^2*b^5*d^6*n^2+18*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c^2*d^4*n^2+4*B^2*x
^3*ln(e*((b*x+a)/(d*x+c))^n)^3*b^7*d^6+12*A^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)
*b^7*d^6-8*B^2*a^3*b^4*d^6*n^3-B^2*b^7*c^3*d^3*n^3-4*A^2*a^3*b^4*d^6*n-2*A^
2*b^7*c^3*d^3*n+18*B^2*x*a*b^6*c*d^5*n^3+12*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)
^2*a^2*b^5*d^6-6*A*B*x*a^2*b^5*d^6*n^2+18*A*B*x*b^7*c^2*d^4*n^2+12*B^2*ln(e
*((b*x+a)/(d*x+c))^n)^2*a*b^6*c^2*d^4*n+24*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*
b^6*c^2*d^4*n^2+24*A^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*c*d^5+12*A^2*x*a*b
^6*c*d^5*n+12*A*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^5*c*d^5-8*A*B*ln(e*((b*
x+a)/(d*x+c))^n)*a^3*b^4*d^6*n-4*A*B*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c^3*d^3*
n+48*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*c*d^5*n^2+24*A*B*x*ln(e*((b*x+a)
/(d*x+c))^n)^2*a*b^6*c*d^5-24*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^5*d^6*n
+12*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c^2*d^4*n-12*A*B*x*a*b^6*c*d^5*n^2+
24*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*c^2*d^4*n+36*A*B*x^2*ln(e*((b*x+a)/(
d*x+c))^n)*b^7*c*d^5*n+8*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a*b^6*d^6+4*B^
2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*b^7*c*d^5-30*B^2*x^2*a*b^6*d^6*n^3+30*B^2
*x^2*b^7*c*d^5*n^3+4*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*a^2*b^5*d^6-39*B^2*x
*a^2*b^5*d^6*n^3+21*B^2*x*b^7*c^2*d^4*n^3+24*A^2*x^2*ln(e*((b*x+a)/(d*x+c))
^n)*a*b^6*d^6+12*A^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c*d^5-12*A^2*x^2*a*b
^6*d^6*n+12*A^2*x^2*b^7*c*d^5*n+4*B^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a^2*b^5*c
*d^5-4*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^4*d^6*n-2*B^2*ln(e*((b*x+a)/(d
*x+c))^n)^2*b^7*c^3*d^3*n+8*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^4*d^6*n^2-2
*B^2*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c^3*d^3*n^2+12*A^2*x*ln(e*((b*x+a)/(d*x+
c))^n)*a^2*b^5*d^6-18*A^2*x*a^2*b^5*d^6*n+6*A^2*x*b^7*c^2*d^4*n+12*A^2*ln(e
*((b*x+a)/(d*x+c))^n)*a^2*b^5*c*d^5+6*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b
^7*d^6*n+30*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^7*d^6*n^2+12*A*B*x^3*ln(e*(
(b*x+a)/(d*x+c))^n)^2*b^7*d^6)/i^2/g^3/(d*x+c)/(b*x+a)^2/(a^3*d^3-3*a^2*b*c
*d^2+3*a*b^2*c^2*d-b^3*c^3)/(a*d-b*c)/b^4/d^3/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2052 vs. $2(554) = 1108$.

Time = 0.37 (sec) , antiderivative size = 2052, normalized size of antiderivative = 3.66

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x,
 algorithm="fricas")
```

[Out] -1/4*(2*A^2*b^3*c^3 - 12*A^2*a*b^2*c^2*d + 6*A^2*a^2*b*c*d^2 + 4*A^2*a^3*d^3 - 4*(B^2*b^3*d^3*n^2*x^3 + B^2*a^2*b*c*d^2*n^2 + (B^2*b^3*c*d^2 + 2*B^2*a*b^2*d^3)*n^2*x^2 + (2*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3)*n^2*x)*log((b*x + a)/(d*x + c))^3 + (B^2*b^3*c^3 - 24*B^2*a*b^2*c^2*d + 15*B^2*a^2*b*c*d^2 + 8*B^2*a^3*d^3)*n^2 - 6*(2*A^2*b^3*c*d^2 - 2*A^2*a*b^2*d^3 + 5*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n^2 + 2*(A*B*b^3*c*d^2 - A*B*a*b^2*d^3)*n)*x^2 + 2*(B^2*b^3*c^3 - 6*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2 + 2*B^2*a^3*d^3 - 6*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*x^2 - 3*(B^2*b^3*c^2*d + 2*B^2*a*b^2*c*d^2 - 3*B^2*a^2*b*d^3)*x - 6*(B^2*b^3*d^3*x^3 + B^2*a^2*b*c*d^2 + (B^2*b^3*c*d^2 + 2*B^2*a*b^2*d^3)*x^2 + (2*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3)*x)*log((b*x + a)/(d*x + c)))*log(e)^2 - 2*(6*A*B*a^2*b*c*d^2*n + 3*(B^2*b^3*d^3*n^2 + 2*A*B*b^3*d^3*n)*x^3 - (B^2*b^3*c^3 - 6*B^2*a*b^2*c^2*d + 2*B^2*a^3*d^3)*n^2 + 3*(3*B^2*b^3*c*d^2*n^2 + 2*(A*B*b^3*c*d^2 + 2*A*B*a*b^2*d^3)*n)*x^2 + 3*((B^2*b^3*c^2*d + 4*B^2*a*b^2*c*d^2 - 2*B^2*a^2*b*d^3)*n^2 + 2*(2*A*B*a*b^2*c*d^2 + A*B*a^2*b*d^3)*n)*x)*log((b*x + a)/(d*x + c))^2 + 2*(A*B*b^3*c^3 - 12*A*B*a*b^2*c^2*d + 15*A*B*a^2*b*c*d^2 - 4*A*B*a^3*d^3)*n - 3*(2*A^2*b^3*c^2*d + 4*A^2*a*b^2*c*d^2 - 6*A^2*a^2*b*d^3 + (7*B^2*b^3*c^2*d + 6*B^2*a*b^2*c*d^2 - 13*B^2*a^2*b*d^3)*n^2 + 2*(3*A*B*b^3*c^2*d - 2*A*B*a*b^2*c*d^2 - A*B*a^2*b*d^3)*n)*x + 2*(2*A*B*b^3*c^3 - 12*A*B*a*b^2*c^2*d + 6*A*B*a^2*b*c*d^2 + 4*A*B*a^3*d^3 - 6*(2*A*B*b^3*c*d^2 - 2*A*B*a*b^2*d^3 + (B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n)*x^2 - 6*(B^2*b^3*d^3*n*x^3 + B^2*a^2*b*c*d^2*n + (B^2*b^3*c*d^2 + 2*B^2*a*b^2*d^3)*n*x^2 + (2*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3)*n*x)*log((b*x + a)/(d*x + c))^2 + (B^2*b^3*c^3 - 12*B^2*a*b^2*c^2*d + 15*B^2*a^2*b*c*d^2 - 4*B^2*a^3*d^3)*n - 3*(2*A*B*b^3*c^2*d + 4*A*B*a*b^2*c*d^2 - 6*A*B*a^2*b*d^3 + (3*B^2*b^3*c^2*d - 2*B^2*a*b^2*c*d^2 - B^2*a^2*b*d^3)*n)*x - 2*(6*A*B*a^2*b*c*d^2 + 3*(B^2*b^3*d^3*n + 2*A*B*b^3*d^3)*x^3 + 3*(3*B^2*b^3*c*d^2*n + 2*A*B*b^3*c*d^2 + 4*A*B*a*b^2*d^3)*x^2 - (B^2*b^3*c^3 - 6*B^2*a*b^2*c^2*d + 2*B^2*a^3*d^3)*n + 3*(4*A*B*a*b^2*c*d^2 + 2*A*B*a^2*b*d^3 + (B^2*b^3*c^2*d + 4*B^2*a*b^2*c*d^2 - 2*B^2*a^2*b*d^3)*n)*x)*log((b*x + a)/(d*x + c)))*log(e) - 2*(6*A^2*a^2*b*c*d^2 + 3*(5*B^2*b^3*d^3*n^2 + 2*A*B*b^3*d^3*n + 2*A^2*b^3*d^3)*x^3 - (B^2*b^3*c^3 - 12*B^2*a*b^2*c^2*d - 4*B^2*a^3*d^3)*n^2 + 3*(6*A*B*b^3*c*d^2*n + 2*A^2*b^3*c*d^2 + 4*A^2*a*b^2*d^3 + (7*B^2*b^3*c*d^2 + 8*B^2*a*b^2*d^3)*n^2)*x^2 - 2*(A*B*b^3*c^3 - 6*A*B*a*b^2*c^2*d + 2*A*B*a^3*d^3)*n + 3*(4*A^2*a*b^2*c*d^2 + 2*A^2*a^2*b*d^3 + (3*B^2*b^3*c^2*d + 8*B^2*a*b^2*c*d^2 + 4*B^2*a^2*b*d^3)*n^2 + 2*(A*B*b^3*c^2*d + 4*A*B*a*b^2*c*d^2 - 2*A*B*a^2*b*d^3)*n)*x)*log((b*x + a)/(d*x + c)))/((b^6*c^4*d - 4*a*b^5*c^3*d^2 + 6*a^2*b^4*c^2*d^3 - 4*a^3*b^3*c*d^4 + a^4*b^2*d^5)*g^3*i^2*x^3 + (b^6*c^5 - 2*a*b^5*c^4*d - 2*a^2*b^4*c^3*d^2 + 8*a^3*b^3*c^2*d^3 - 7*a^4*b^2*c*d^4 + 2*a^5*b*d^5)*g^3*i^2*x^2 + (2*a*b^5*c^5 - 7*a^2*b^4*c^4*d + 8*a^3*b^3*c^3*d^2 - 2*a^4*b^2*c^2*d^3 - 2*a^5*b*c*d^4 + a^6*d^5)*g^3*i^2*x + (a^2*b^4*c^5 - 4*a^3*b^3*c^4*d + 6*a^4*b^2*c^3*d^2 - 4*a^5*b*c^2*d^3 + a^6*c*d^4)*g^3*i^2)

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**3/(d*i*x+c*i)**2
,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4198 vs. $2(554) = 1108$.

Time = 0.51 (sec) , antiderivative size = 4198, normalized size of antiderivative = 7.50

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x,
 algorithm="maxima")
```

```
[Out] 1/2*B^2*((6*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*
a*b*d^2)*x)/((b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*
g^3*i^2*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3
- 2*a^4*b*d^4)*g^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2
*d^2 + a^4*b*c*d^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3*a^3*b^2*c^3*d +
3*a^4*b*c^2*d^2 - a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x + a)/((b^4*c^4 - 4*
a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2) - 6*b*d
^2*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d
^3 + a^4*d^4)*g^3*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2 + A*B*((6*
b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a*b*d^2)*x)/
((b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*g^3*i^2*x^3
+ (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 - 2*a^4*b*d^
4)*g^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 + a^4*b
*c*d^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b*c^2*
d^2 - a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2) - 6*b*d^2*log(d*x +
 c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4
)*g^3*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1/4*((b^3*c^3 - 24*a*b
^2*c^2*d + 15*a^2*b*c*d^2 + 8*a^3*d^3 - 4*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3
*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a)^3 +
 4*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*
d^2 + a^2*b*d^3)*x)*log(d*x + c)^3 - 30*(b^3*c*d^2 - a*b^2*d^3)*x^2 + 6*(b^
```

3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a)^2 + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x - 2*(b^3*d^3*x^3 + a^2*b*c *d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b *x + a))*log(d*x + c)^2 - 3*(7*b^3*c^2*d + 6*a*b^2*c*d^2 - 13*a^2*b*d^3)*x - 30*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2* c*d^2 + a^2*b*d^3)*x)*log(b*x + a) + 6*(5*b^3*d^3*x^3 + 5*a^2*b*c*d^2 + 5*(b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + 2*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a)^2 + 5*(2*a*b ^2*c*d^2 + a^2*b*d^3)*x - 2*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b ^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a))*log(d*x + c))*n^ 2/(a^2*b^4*c^5*g^3*i^2 - 4*a^3*b^3*c^4*d*g^3*i^2 + 6*a^4*b^2*c^3*d^2*g^3*i^ 2 - 4*a^5*b*c^2*d^3*g^3*i^2 + a^6*c*d^4*g^3*i^2 + (b^6*c^4*d*g^3*i^2 - 4*a* b^5*c^3*d^2*g^3*i^2 + 6*a^2*b^4*c^2*d^3*g^3*i^2 - 4*a^3*b^3*c*d^4*g^3*i^2 + a^4*b^2*d^5*g^3*i^2)*x^3 + (b^6*c^5*g^3*i^2 - 2*a*b^5*c^4*d*g^3*i^2 - 2*a^ 2*b^4*c^3*d^2*g^3*i^2 + 8*a^3*b^3*c^2*d^3*g^3*i^2 - 7*a^4*b^2*c*d^4*g^3*i^2 + 2*a^5*b*d^5*g^3*i^2)*x^2 + (2*a*b^5*c^5*g^3*i^2 - 7*a^2*b^4*c^4*d*g^3*i^ 2 + 8*a^3*b^3*c^3*d^2*g^3*i^2 - 2*a^4*b^2*c^2*d^3*g^3*i^2 - 2*a^5*b*c*d^4*g ^3*i^2 + a^6*d^5*g^3*i^2)*x) + 2*(b^3*c^3 - 12*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3 - 6*(b^3*c*d^2 - a*b^2*d^3)*x^2 + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a)^2 + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a *b^2*c*d^2 + a^2*b*d^3)*x)*log(d*x + c)^2 - 3*(3*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b*d^3)*x - 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x ^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a) + 6*(b^3*d^3*x^3 + a^2*b*c *d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x - 2*(b ^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a))*log(d*x + c))*n*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(a^2*b^4*c^5*g^3*i^2 - 4*a^3*b^3*c^4*d*g^3*i^2 + 6*a^4*b^2*c^3*d^2 *g^3*i^2 - 4*a^5*b*c^2*d^3*g^3*i^2 + a^6*c*d^4*g^3*i^2 + (b^6*c^4*d*g^3*i^2 - 4*a*b^5*c^3*d^2*g^3*i^2 + 6*a^2*b^4*c^2*d^3*g^3*i^2 - 4*a^3*b^3*c*d^4*g^ 3*i^2 + a^4*b^2*d^5*g^3*i^2)*x^3 + (b^6*c^5*g^3*i^2 - 2*a*b^5*c^4*d*g^3*i^2 - 2*a^2*b^4*c^3*d^2*g^3*i^2 + 8*a^3*b^3*c^2*d^3*g^3*i^2 - 7*a^4*b^2*c*d^4* g^3*i^2 + 2*a^5*b*d^5*g^3*i^2)*x^2 + (2*a*b^5*c^5*g^3*i^2 - 7*a^2*b^4*c^4*d *g^3*i^2 + 8*a^3*b^3*c^3*d^2*g^3*i^2 - 2*a^4*b^2*c^2*d^3*g^3*i^2 - 2*a^5*b* c*d^4*g^3*i^2 + a^6*d^5*g^3*i^2)*x))*B^2 - 1/2*(b^3*c^3 - 12*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3 - 6*(b^3*c*d^2 - a*b^2*d^3)*x^2 + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a)^2 + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d ^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(d*x + c)^2 - 3*(3*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b*d^3)*x - 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a) + 6*(b^3*d^3 *x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b *d^3)*x - 2*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2 *a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a))*log(d*x + c))*A*B*n/(a^2*b^4*c^5

```
*g^3*i^2 - 4*a^3*b^3*c^4*d*g^3*i^2 + 6*a^4*b^2*c^3*d^2*g^3*i^2 - 4*a^5*b*c^
2*d^3*g^3*i^2 + a^6*c*d^4*g^3*i^2 + (b^6*c^4*d*g^3*i^2 - 4*a*b^5*c^3*d^2*g^
3*i^2 + 6*a^2*b^4*c^2*d^3*g^3*i^2 - 4*a^3*b^3*c*d^4*g^3*i^2 + a^4*b^2*d^5*g
^3*i^2)*x^3 + (b^6*c^5*g^3*i^2 - 2*a*b^5*c^4*d*g^3*i^2 - 2*a^2*b^4*c^3*d^2*
g^3*i^2 + 8*a^3*b^3*c^2*d^3*g^3*i^2 - 7*a^4*b^2*c*d^4*g^3*i^2 + 2*a^5*b*d^5
*g^3*i^2)*x^2 + (2*a*b^5*c^5*g^3*i^2 - 7*a^2*b^4*c^4*d*g^3*i^2 + 8*a^3*b^3*
c^3*d^2*g^3*i^2 - 2*a^4*b^2*c^2*d^3*g^3*i^2 - 2*a^5*b*c*d^4*g^3*i^2 + a^6*d
^5*g^3*i^2)*x) + 1/2*A^2*((6*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2
+ 3*(b^2*c*d + 3*a*b*d^2)*x)/((b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^
3 - a^3*b^2*d^4)*g^3*i^2*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 +
 5*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*g^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*
d + 3*a^3*b^2*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3
*a^3*b^2*c^3*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x +
a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*g^3*i^2) - 6*b*d^2*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*
d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2))
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x,
 algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 7.25 (sec) , antiderivative size = 1784, normalized size of antiderivative = 3.19

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx=\text{Too large to display}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^3*(c*i + d*i*x)
^2),x)
```

```
[Out] (B^2*b*d^2*log(e*((a + b*x)/(c + d*x))^n)^3)/(g^3*i^2*n*(a*d - b*c)^4) - lo
g(e*((a + b*x)/(c + d*x))^n)^2*(((B^2*(2*a*d + b*c))/(2*(a^2*d^2 + b^2*c^2
- 2*a*b*c*d)) + (3*B^2*b*d*x)/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(x*(a^2*
d*g^3*i^2 + 2*a*b*c*g^3*i^2) + x^2*(b^2*c*g^3*i^2 + 2*a*b*d*g^3*i^2) + a^2*
c*g^3*i^2 + b^2*d*g^3*i^2*x^3) - (3*B*b*d^2*(2*A + B*n))/(2*g^3*i^2*n*(a*d
- b*c)^4) + (3*B^2*b*d^2*(b*g^3*i^2*n*x^2*(a*d - b*c) + (a*c*g^3*i^2*n*(a*d
```

 - b*c))/d + (g^3*i^2*n*x*(a*d + b*c)*(a*d - b*c))/d))/(g^3*i^2*n*(a*d - b*c)^4*(x*(a^2*d*g^3*i^2 + 2*a*b*c*g^3*i^2) + x^2*(b^2*c*g^3*i^2 + 2*a*b*d*g^3*i^2) + a^2*c*g^3*i^2 + b^2*d*g^3*i^2*x^3))) - ((4*A^2*a^2*d^2 - 2*A^2*b^2*c^2 + 8*B^2*a^2*d^2*n^2 - B^2*b^2*c^2*n^2 + 10*A^2*a*b*c*d - 8*A*B*a^2*d^2*n - 2*A*B*b^2*c^2*n + 23*B^2*a*b*c*d*n^2 + 22*A*B*a*b*c*d*n)/(2*(a*d - b*c)) + (3*x^2*(2*A^2*b^2*d^2 + 5*B^2*b^2*d^2*n^2 + 2*A*B*b^2*d^2*n))/(a*d - b*c) + (3*x*(6*A^2*a*b*d^2 + 2*A^2*b^2*c*d + 13*B^2*a*b*d^2*n^2 + 7*B^2*b^2*c*d*n^2 + 2*A*B*a*b*d^2*n + 6*A*B*b^2*c*d*n))/(2*(a*d - b*c)))/(x*(2*a^4*d^3*g^3*i^2 + 4*a*b^3*c^3*g^3*i^2 - 6*a^2*b^2*c^2*d*g^3*i^2) + x^2*(2*b^4*c^3*g^3*i^2 + 4*a^3*b*d^3*g^3*i^2 - 6*a^2*b^2*c*d^2*g^3*i^2) + x^3*(2*a^2*b^2*d^3*g^3*i^2 + 2*b^4*c^2*d*g^3*i^2 - 4*a*b^3*c*d^2*g^3*i^2) + 2*a^2*b^2*c^3*g^3*i^2 + 2*a^4*c*d^2*g^3*i^2 - 4*a^3*b*c^2*d*g^3*i^2) - (b*d^2*atan((b*d^2*(2*A^2 + 5*B^2*n^2 + 2*A*B*n)*(2*a^4*d^4*g^3*i^2 - 2*b^4*c^4*g^3*i^2 + 4*a*b^3*c^3*d*g^3*i^2 - 4*a^3*b*c*d^3*g^3*i^2)*3i)/(2*g^3*i^2*(a*d - b*c)^4*(6*A^2*b*d^2 + 15*B^2*b*d^2*n^2 + 6*A*B*b*d^2*n)) + (b^2*d^3*x*(2*A^2 + 5*B^2*n^2 + 2*A*B*n)*(a^3*d^3*g^3*i^2 - b^3*c^3*g^3*i^2 + 3*a*b^2*c^2*d*g^3*i^2 - 3*a^2*b*c*d^2*g^3*i^2)*6i)/(g^3*i^2*(a*d - b*c)^4*(6*A^2*b*d^2 + 15*B^2*b*d^2*n^2 + 6*A*B*b*d^2*n)))*(2*A^2 + 5*B^2*n^2 + 2*A*B*n)*3i)/(g^3*i^2*(a*d - b*c)^4) - log(e*((a + b*x)/(c + d*x))^n)*(((B^2*b*c*n)/2 - 2*B^2*a*d*n - x*((3*B^2*b*d*n)/2 - 3*A*B*b*d) + 2*A*B*a*d + A*B*b*c)/(x*(a^4*d^3*g^3*i^2 + 2*a*b^3*c^3*g^3*i^2 - 3*a^2*b^2*c^2*d*g^3*i^2) + x^2*(b^4*c^3*g^3*i^2 + 2*a^3*b*d^3*g^3*i^2 - 3*a^2*b^2*c*d^2*g^3*i^2) + x^3*(a^2*b^2*d^3*g^3*i^2 + b^4*c^2*d*g^3*i^2 - 2*a*b^3*c*d^2*g^3*i^2) + a^2*b^2*c^3*g^3*i^2 + a^4*c*d^2*g^3*i^2 - 2*a^3*b*c^2*d*g^3*i^2) + (3*B*b*d^2*(2*A + B*n)*(b*g^3*i^2*n*x^2*(a*d - b*c)^3 + (g^3*i^2*n*x*(a*d + b*c)*(a*d - b*c)^3)/d + (a*c*g^3*i^2*n*(a*d - b*c)^3)/d))/(g^3*i^2*n*(a*d - b*c)^4*(x*(a^4*d^3*g^3*i^2 + 2*a*b^3*c^3*g^3*i^2 - 3*a^2*b^2*c^2*d*g^3*i^2) + x^2*(b^4*c^3*g^3*i^2 + 2*a^3*b*d^3*g^3*i^2 - 3*a^2*b^2*c*d^2*g^3*i^2) + x^3*(a^2*b^2*d^3*g^3*i^2 + b^4*c^2*d*g^3*i^2 - 2*a*b^3*c*d^2*g^3*i^2) + a^2*b^2*c^3*g^3*i^2 + a^4*c*d^2*g^3*i^2 - 2*a^3*b*c^2*d*g^3*i^2)))

3.201 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 729

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = -\frac{2ABd^4n(a+bx)}{(bc-ad)^5g^4i^2(c+dx)}+\frac{2B^2d^4n^2(a+bx)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{12b^2B^2d^2n^2(c+dx)}{(bc-ad)^5g^4i^2(a+bx)}+\frac{b^3B^2dn^2(c+dx)^2}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{2b^4B^2n^2(c+dx)^3}{27(bc-ad)^5g^4i^2(a+bx)^3} - \frac{2B^2d^4n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{12b^2Bd^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(a+bx)} + \frac{2b^3Bdn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{2b^4Bn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^5g^4i^2(a+bx)^3} + \frac{d^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^4i^2(c+dx)} - \frac{6b^2d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)} + \frac{2b^3d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{b^4(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^5g^4i^2(a+bx)^3} - \frac{4bd^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^5g^4i^2n}$$

```
[Out] -2*A*B*d^4*n*(b*x+a)/(-a*d+b*c)^5/g^4/i^2/(d*x+c)+2*B^2*d^4*n^2*(b*x+a)/(-a
*d+b*c)^5/g^4/i^2/(d*x+c)-12*b^2*B^2*d^2*n^2*(d*x+c)/(-a*d+b*c)^5/g^4/i^2/(
b*x+a)+b^3*B^2*d*n^2*(d*x+c)^2/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-2/27*b^4*B^2*
n^2*(d*x+c)^3/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^3-2*B^2*d^4*n*(b*x+a)*ln(e*((b*x
+a)/(d*x+c))^n)/(-a*d+b*c)^5/g^4/i^2/(d*x+c)-12*b^2*B*d^2*n*(d*x+c)*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^4/i^2/(b*x+a)+2*b^3*B*d*n*(d*x+c)^2
*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-2/9*b^4*B*n
*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^3+d
^4*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^5/g^4/i^2/(d*x+c)-6
*b^2*d^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^5/g^4/i^2/(b*
x+a)+2*b^3*d*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^5/g^4/i
```

```
^2/(b*x+a)^2-1/3*b^4*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)
^5/g^4/i^2/(b*x+a)^3-4/3*b*d^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b*
c)^5/g^4/i^2/n
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 729, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.178$, Rules used = {2561, 2395, 2333, 2332, 2342, 2341, 2339, 30}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = -\frac{b^4(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3g^4i^2(a+bx)^3(bc-ad)^5} - \frac{2b^4Bn(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9g^4i^2(a+bx)^3(bc-ad)^5} + \frac{2b^3d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^4i^2(a+bx)^2(bc-ad)^5} + \frac{2b^3Bdn(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^4i^2(a+bx)^2(bc-ad)^5} - \frac{6b^2d^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^4i^2(a+bx)(bc-ad)^5} - \frac{12b^2Bd^2n(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^4i^2(a+bx)(bc-ad)^5} + \frac{d^4(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^4i^2(c+dx)(bc-ad)^5} - \frac{2ABd^4n(a+bx)}{g^4i^2(c+dx)(bc-ad)^5} - \frac{4bd^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3Bg^4i^2n(bc-ad)^5} - \frac{2b^4B^2n^2(c+dx)^3}{27g^4i^2(a+bx)^3(bc-ad)^5} + \frac{b^3B^2dn^2(c+dx)^2}{g^4i^2(a+bx)^2(bc-ad)^5} - \frac{12b^2B^2d^2n^2(c+dx)}{g^4i^2(a+bx)(bc-ad)^5} - \frac{2B^2d^4n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g^4i^2(c+dx)(bc-ad)^5} + \frac{2B^2d^4n^2(a+bx)}{g^4i^2(c+dx)(bc-ad)^5}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^4*(c*i + d*i*x)
^2),x]

[Out] (-2*A*B*d^4*n*(a + b*x))/((b*c - a*d)^5*g^4*i^2*(c + d*x)) + (2*B^2*d^4*n^2
*(a + b*x))/((b*c - a*d)^5*g^4*i^2*(c + d*x)) - (12*b^2*B^2*d^2*n^2*(c + d*
x))/((b*c - a*d)^5*g^4*i^2*(a + b*x)) + (b^3*B^2*d*n^2*(c + d*x)^2)/((b*c -
 a*d)^5*g^4*i^2*(a + b*x)^2) - (2*b^4*B^2*n^2*(c + d*x)^3)/(27*(b*c - a*d)^
5*g^4*i^2*(a + b*x)^3) - (2*B^2*d^4*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))
```

```
^n])/((b*c - a*d)^5*g^4*i^2*(c + d*x)) - (12*b^2*B*d^2*n*(c + d*x)*(A + B*L
og[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^5*g^4*i^2*(a + b*x)) + (2*b^3*
B*d*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^5*g^
4*i^2*(a + b*x)^2) - (2*b^4*B*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]))/(9*(b*c - a*d)^5*g^4*i^2*(a + b*x)^3) + (d^4*(a + b*x)*(A + B*Log[
e*((a + b*x)/(c + d*x))^n])^2)/((b*c - a*d)^5*g^4*i^2*(c + d*x)) - (6*b^2*d
^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*c - a*d)^5*g^4*i
^2*(a + b*x)) + (2*b^3*d*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2)/((b*c - a*d)^5*g^4*i^2*(a + b*x)^2) - (b^4*(c + d*x)^3*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n])^2)/(3*(b*c - a*d)^5*g^4*i^2*(a + b*x)^3) - (4*b*d^3*
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(3*B*(b*c - a*d)^5*g^4*i^2*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)^4(A+B\log(ex^n))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5 g^4 i^2}$$

$$= \frac{\text{Subst}\left(\int \left(d^4(A+B\log(ex^n))^2 + \frac{b^4(A+B\log(ex^n))^2}{x^4} - \frac{4b^3d(A+B\log(ex^n))^2}{x^3} + \frac{6b^2d^2(A+B\log(ex^n))^2}{x^2} - \frac{4bd^3(A+\cdots}{\cdots}\right.\right.}{(bc-ad)^5 g^4 i^2}$$

$$= \frac{b^4\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5 g^4 i^2} - \frac{(4b^3d)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5 g^4 i^2}$$
$$+ \frac{(6b^2d^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5 g^4 i^2}$$
$$- \frac{(4bd^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5 g^4 i^2}$$
$$+ \frac{d^4\text{Subst}\left(\int (A+B\log(ex^n))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5 g^4 i^2}$$

$$
\begin{aligned}
&= \frac{d^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^4i^2(c+dx)} - \frac{6b^2d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)} \\
&+ \frac{2b^3d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{b^4(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^5g^4i^2(a+bx)^3} \\
&- \frac{(4bd^3)\operatorname{Subst}\left(\int x^2\,dx, x, A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)^5g^4i^2n} \\
&+ \frac{(2b^4Bn)\operatorname{Subst}\left(\int \frac{A+B\log(ex^n)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bc-ad)^5g^4i^2} \\
&- \frac{(4b^3Bdn)\operatorname{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2} \\
&+ \frac{(12b^2Bd^2n)\operatorname{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2} \\
&- \frac{(2Bd^4n)\operatorname{Subst}\left(\int (A+B\log(ex^n))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2ABd^4n(a+bx)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{12b^2B^2d^2n^2(c+dx)}{(bc-ad)^5g^4i^2(a+bx)} + \frac{b^3B^2dn^2(c+dx)^2}{(bc-ad)^5g^4i^2(a+bx)^2} \\
&- \frac{2b^4B^2n^2(c+dx)^3}{27(bc-ad)^5g^4i^2(a+bx)^3} - \frac{12b^2Bd^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(a+bx)} \\
&+ \frac{2b^3Bdn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{2b^4Bn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^5g^4i^2(a+bx)^3} \\
&+ \frac{d^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^4i^2(c+dx)} - \frac{6b^2d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)} \\
&+ \frac{2b^3d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{b^4(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^5g^4i^2(a+bx)^3} \\
&- \frac{4bd^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^5g^4i^2n} - \frac{(2B^2d^4n)\operatorname{Subst}\left(\int \log(ex^n)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2}
\end{aligned}
$$

$$= -\frac{2ABd^4n(a+bx)}{(bc-ad)^5g^4i^2(c+dx)} + \frac{2B^2d^4n^2(a+bx)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{12b^2B^2d^2n^2(c+dx)}{(bc-ad)^5g^4i^2(a+bx)} + \frac{b^3B^2dn^2(c+dx)^2}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{2b^4B^2n^2(c+dx)^3}{27(bc-ad)^5g^4i^2(a+bx)^3} - \frac{2B^2d^4n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{12b^2Bd^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(a+bx)} + \frac{2b^3Bdn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{2b^4Bn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^5g^4i^2(a+bx)^3} + \frac{d^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^4i^2(c+dx)} - \frac{6b^2d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)} + \frac{2b^3d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{b^4(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^5g^4i^2(a+bx)^3} - \frac{4bd^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^5g^4i^2n}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1695 vs. $2(729) = 1458$.

Time = 1.41 (sec) , antiderivative size = 1695, normalized size of antiderivative = 2.33

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = -\frac{36bB^2d^3n^2(a+bx)^3(c+dx)\log^3\left(\frac{a+bx}{c+dx}\right) + 9Bn\log^2\left(\frac{a+bx}{c+dx}\right)\left(12a^3Abcd^3 + b^4Bc^4n - 6ab^3Bc^3dn + 18a^2b^2\right.}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^4*(c*i + 
d*i*x)^2),x]
```

```
[Out] -1/27*(36*b*B^2*d^3*n^2*(a + b*x)^3*(c + d*x)*Log[(a + b*x)/(c + d*x)]^3 + 
9*B*n*Log[(a + b*x)/(c + d*x)]^2*(12*a^3*A*b*c*d^3 + b^4*B*c^4*n - 6*a*b^3*
B*c^3*d*n + 18*a^2*b^2*B*c^2*d^2*n - 3*a^4*B*d^4*n + 36*a^2*A*b^2*c*d^3*x +
 12*a^3*A*b*d^4*x - 2*b^4*B*c^3*d*n*x + 18*a*b^3*B*c^2*d^2*n*x + 36*a^2*b^2
*B*c*d^3*n*x - 12*a^3*b*B*d^4*n*x + 36*a*A*b^3*c*d^3*x^2 + 36*a^2*A*b^2*d^4
*x^2 + 6*b^4*B*c^2*d^2*n*x^2 + 54*a*b^3*B*c*d^3*n*x^2 + 12*A*b^4*c*d^3*x^3
+ 36*a*A*b^3*d^4*x^3 + 22*b^4*B*c*d^3*n*x^3 + 18*a*b^3*B*d^4*n*x^3 + 12*A*b
^4*d^4*x^4 + 10*b^4*B*d^4*n*x^4 + 12*b*B*d^3*(a + b*x)^3*(c + d*x)*Log[e*((
a + b*x)/(c + d*x))^n] - 12*b*B*d^3*n*(a + b*x)^3*(c + d*x)*Log[(a + b*x)/(
c + d*x)]) + 3*b*d^2*(b*c - a*d)*(a + b*x)^2*(c + d*x)*(27*A^2 + 78*A*B*n +
 92*B^2*n^2 + 27*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(9*A + 13*B*n
)*Log[(a + b*x)/(c + d*x)] + 27*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 6*B*Lo
g[e*((a + b*x)/(c + d*x))^n]*(9*A + 13*B*n - 9*B*n*Log[(a + b*x)/(c + d*x)]
```

```
)) + 6*b*d^3*(a + b*x)^3*(c + d*x)*Log[a + b*x]*(18*A^2 + 30*A*B*n + 55*B^2
*n^2 + 18*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(6*A + 5*B*n)*Log[(a
 + b*x)/(c + d*x)] + 18*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 6*B*Log[e*((a
+ b*x)/(c + d*x))^n]*(6*A + 5*B*n - 6*B*n*Log[(a + b*x)/(c + d*x)])) + b*(b
*c - a*d)^3*(c + d*x)*(9*A^2 + 6*A*B*n + 2*B^2*n^2 + 9*B^2*Log[e*((a + b*x)
/(c + d*x))^n]^2 - 6*B*n*(3*A + B*n)*Log[(a + b*x)/(c + d*x)] + 9*B^2*n^2*L
og[(a + b*x)/(c + d*x)]^2 + 6*B*Log[e*((a + b*x)/(c + d*x))^n]*(3*A + B*n -
 3*B*n*Log[(a + b*x)/(c + d*x)])) - 3*b*d*(b*c - a*d)^2*(a + b*x)*(c + d*x)
*(9*A^2 + 12*A*B*n + 7*B^2*n^2 + 9*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 6
*B*n*(3*A + 2*B*n)*Log[(a + b*x)/(c + d*x)] + 9*B^2*n^2*Log[(a + b*x)/(c +
d*x)]^2 + 6*B*Log[e*((a + b*x)/(c + d*x))^n]*(3*A + 2*B*n - 3*B*n*Log[(a +
b*x)/(c + d*x)])) + 6*B*(b*c - a*d)*n*Log[(a + b*x)/(c + d*x)]*(3*b*d^2*(a
+ b*x)^2*(c + d*x)*(9*A + 13*B*n + 9*B*Log[e*((a + b*x)/(c + d*x))^n] - 9*B
*n*Log[(a + b*x)/(c + d*x)]) + b*(b*c - a*d)^2*(c + d*x)*(3*A + B*n + 3*B*L
og[e*((a + b*x)/(c + d*x))^n] - 3*B*n*Log[(a + b*x)/(c + d*x)]) - 3*b*d*(b*
c - a*d)*(a + b*x)*(c + d*x)*(3*A + 2*B*n + 3*B*Log[e*((a + b*x)/(c + d*x))
^n] - 3*B*n*Log[(a + b*x)/(c + d*x)]) + 9*d^3*(a + b*x)^3*(A - B*n + B*Log[
e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])) + 27*d^3*(b*c -
 a*d)*(a + b*x)^3*(A^2 - 2*A*B*n + 2*B^2*n^2 + B^2*Log[e*((a + b*x)/(c + d*
x))^n]^2 + 2*B*n*(-A + B*n)*Log[(a + b*x)/(c + d*x)] + B^2*n^2*Log[(a + b*x
)/(c + d*x)]^2 - 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(-A + B*n + B*n*Log[(a
+ b*x)/(c + d*x)])) - 6*b*d^3*(a + b*x)^3*(c + d*x)*(18*A^2 + 30*A*B*n + 55
*B^2*n^2 + 18*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(6*A + 5*B*n)*Lo
g[(a + b*x)/(c + d*x)] + 18*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 6*B*Log[e*
((a + b*x)/(c + d*x))^n]*(6*A + 5*B*n - 6*B*n*Log[(a + b*x)/(c + d*x)]))*Lo
g[c + d*x])/((b*c - a*d)^5*g^4*i^2*(a + b*x)^3*(c + d*x))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3040 vs. $2(721) = 1442$.

Time = 44.82 (sec) , antiderivative size = 3041, normalized size of antiderivative = 4.17

method	result	size
parallelrisch	Expression too large to display	3041

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x,method=
_RETURNVERBOSE)
```

```
[Out] 1/27*(-342*A*B*x*a^2*b^7*c*d^6*n^2+36*B^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)^3*b
^9*d^7+108*A^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^9*d^7-54*B^2*a^4*b^5*d^7*n^3
+2*B^2*b^9*c^4*d^3*n^3-27*A^2*a^4*b^5*d^7*n+9*A^2*b^9*c^4*d^3*n+180*A*B*x^3
*b^9*c*d^6*n^2+108*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a*b^8*c*d^6+54*B^2*x
^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^9*c^2*d^5*n+648*B^2*x^2*ln(e*((b*x+a)/(d*x
+c))^n)*a^2*b^7*d^7*n^2+198*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^2*d^5*n
^2+324*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^7*c^2*d^5*n^2-54*B^2*ln(e*((b*x+
```

a)/(d*x+c))^n)*a*b^8*c^3*d^4*n^2+324*A^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^7*c*d^6+54*A^2*x*a^2*b^7*c*d^6*n+162*A^2*x*a*b^8*c^2*d^5*n+108*A*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^6*c*d^6-54*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^5*d^7*n+18*A*B*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^4*d^3*n-330*A*B*a^3*b^6*c*d^6*n^2+324*A*B*a^2*b^7*c^2*d^5*n^2-54*A*B*a*b^8*c^3*d^4*n^2+180*A*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^9*d^7*n+162*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^8*d^7*n+198*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b^9*c*d^6*n+810*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*d^7*n^2+510*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c*d^6*n^2+324*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^8*d^7+108*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b^9*c*d^6-180*A*B*x^3*a*b^8*d^7*n^2+972*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c*d^6*n+648*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^7*c*d^6*n+324*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c^2*d^5*n+480*B^2*x^2*a*b^8*c*d^6*n^3+324*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^7*d^7-342*A*B*x^2*a^2*b^7*d^7*n^2+198*A*B*x^2*b^9*c^2*d^5*n^2+108*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*a^2*b^7*c*d^6-108*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^6*d^7*n-18*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^9*c^3*d^4*n+216*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^6*d^7*n^2-30*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^3*d^4*n^2-87*B^2*x*a^2*b^7*c*d^6*n^3+567*B^2*x*a*b^8*c^2*d^5*n^3+324*A^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c*d^6+216*A^2*x^2*a*b^8*c*d^6*n+108*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^6*d^7-114*A*B*x*a^3*b^6*d^7*n^2-30*A*B*x*b^9*c^3*d^4*n^2+486*A*B*x*a*b^8*c^2*d^5*n^2+324*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^7*c^2*d^5*n-108*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c^3*d^4*n+324*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*d^7*n+396*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c*d^6*n+486*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^8*c*d^6*n+162*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^7*c^2*d^5*n-54*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^8*c^3*d^4*n+90*B^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)^2*b^9*d^7*n+330*B^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^9*d^7*n^2+108*A*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)^2*b^9*d^7+108*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^3*a*b^8*d^7+36*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^3*b^9*c*d^6-330*B^2*x^3*a*b^8*d^7*n^3+330*B^2*x^3*b^9*c*d^6*n^3+108*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a^2*b^7*d^7-735*B^2*x^2*a^2*b^7*d^7*n^3+255*B^2*x^2*b^9*c^2*d^5*n^3+324*A^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*d^7+108*A^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c*d^6-108*A^2*x^3*a*b^8*d^7*n+108*A^2*x^3*b^9*c*d^6*n+36*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*a^3*b^6*d^7-461*B^2*x*a^3*b^6*d^7*n^3-19*B^2*x*b^9*c^3*d^4*n^3+324*A^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^7*d^7-270*A^2*x^2*a^2*b^7*d^7*n+54*A^2*x^2*b^9*c^2*d^5*n+36*B^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a^3*b^6*c*d^6-27*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b^5*d^7*n+9*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^9*c^4*d^3*n-245*B^2*a^3*b^6*c*d^6*n^3+324*B^2*a^2*b^7*c^2*d^5*n^3-27*B^2*a*b^8*c^3*d^4*n^3+54*A*B*a^4*b^5*d^7*n^2+6*A*B*b^9*c^4*d^3*n^2-90*A^2*a^3*b^6*c*d^6*n+162*A^2*a^2*b^7*c^2*d^5*n-54*A^2*a*b^8*c^3*d^4*n+1134*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c*d^6*n^2+324*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^8*c*d^6+108*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^2*d^5*n+144*A*B*x^2*a*b^8*c*d^6*n^2+324*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^7*c*d^6*n+162*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^8*c^2*d^5*n+648*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^7*c*d^6*n^2+486*B^2*x*ln(e*((b*x+a)

```
/(d*x+c))^n)*a*b^8*c^2*d^5*n^2+324*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^
7*c*d^6-216*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^6*d^7*n-36*A*B*x*ln(e*((b
*x+a)/(d*x+c))^n)*b^9*c^3*d^4*n+54*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^5*d^
7*n^2+6*B^2*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^4*d^3*n^2+108*A^2*x*ln(e*((b*x+
a)/(d*x+c))^n)*a^3*b^6*d^7-198*A^2*x*a^3*b^6*d^7*n-18*A^2*x*b^9*c^3*d^4*n+1
08*A^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^6*c*d^6)/i^2/g^4/(d*x+c)/(b*x+a)^3/(
a*d-b*c)^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^5/d^3/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3183 vs. $2(721) = 1442$.

Time = 0.43 (sec) , antiderivative size = 3183, normalized size of antiderivative = 4.37

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x,
 algorithm="fricas")
```

```
[Out] -1/27*(9*A^2*b^4*c^4 - 54*A^2*a*b^3*c^3*d + 162*A^2*a^2*b^2*c^2*d^2 - 90*A^
2*a^3*b*c*d^3 - 27*A^2*a^4*d^4 + 6*(18*A^2*b^4*c*d^3 - 18*A^2*a*b^3*d^4 + 5
5*(B^2*b^4*c*d^3 - B^2*a*b^3*d^4)*n^2 + 30*(A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*
n)*x^3 + 36*(B^2*b^4*d^4*n^2*x^4 + B^2*a^3*b*c*d^3*n^2 + (B^2*b^4*c*d^3 + 3
*B^2*a*b^3*d^4)*n^2*x^3 + 3*(B^2*a*b^3*c*d^3 + B^2*a^2*b^2*d^4)*n^2*x^2 + (
3*B^2*a^2*b^2*c*d^3 + B^2*a^3*b*d^4)*n^2*x)*log((b*x + a)/(d*x + c))^3 + (2
*B^2*b^4*c^4 - 27*B^2*a*b^3*c^3*d + 324*B^2*a^2*b^2*c^2*d^2 - 245*B^2*a^3*b
*c*d^3 - 54*B^2*a^4*d^4)*n^2 + 3*(18*A^2*b^4*c^2*d^2 + 72*A^2*a*b^3*c*d^3 -
 90*A^2*a^2*b^2*d^4 + 5*(17*B^2*b^4*c^2*d^2 + 32*B^2*a*b^3*c*d^3 - 49*B^2*a
^2*b^2*d^4)*n^2 + 6*(11*A*B*b^4*c^2*d^2 + 8*A*B*a*b^3*c*d^3 - 19*A*B*a^2*b^
2*d^4)*n)*x^2 + 9*(B^2*b^4*c^4 - 6*B^2*a*b^3*c^3*d + 18*B^2*a^2*b^2*c^2*d^2
 - 10*B^2*a^3*b*c*d^3 - 3*B^2*a^4*d^4 + 12*(B^2*b^4*c*d^3 - B^2*a*b^3*d^4)*
x^3 + 6*(B^2*b^4*c^2*d^2 + 4*B^2*a*b^3*c*d^3 - 5*B^2*a^2*b^2*d^4)*x^2 - 2*(
B^2*b^4*c^3*d - 9*B^2*a*b^3*c^2*d^2 - 3*B^2*a^2*b^2*c*d^3 + 11*B^2*a^3*b*d^
4)*x + 12*(B^2*b^4*d^4*x^4 + B^2*a^3*b*c*d^3 + (B^2*b^4*c*d^3 + 3*B^2*a*b^3
*d^4)*x^3 + 3*(B^2*a*b^3*c*d^3 + B^2*a^2*b^2*d^4)*x^2 + (3*B^2*a^2*b^2*c*d^
3 + B^2*a^3*b*d^4)*x)*log((b*x + a)/(d*x + c)))*log(e)^2 + 9*(12*A*B*a^3*b*
c*d^3*n + 2*(5*B^2*b^4*d^4*n^2 + 6*A*B*b^4*d^4*n)*x^4 + 2*((11*B^2*b^4*c*d^
3 + 9*B^2*a*b^3*d^4)*n^2 + 6*(A*B*b^4*c*d^3 + 3*A*B*a*b^3*d^4)*n)*x^3 + (B^
2*b^4*c^4 - 6*B^2*a*b^3*c^3*d + 18*B^2*a^2*b^2*c^2*d^2 - 3*B^2*a^4*d^4)*n^2
 + 6*((B^2*b^4*c^2*d^2 + 9*B^2*a*b^3*c*d^3)*n^2 + 6*(A*B*a*b^3*c*d^3 + A*B*
a^2*b^2*d^4)*n)*x^2 - 2*((B^2*b^4*c^3*d - 9*B^2*a*b^3*c^2*d^2 - 18*B^2*a^2*
b^2*c*d^3 + 6*B^2*a^3*b*d^4)*n^2 - 6*(3*A*B*a^2*b^2*c*d^3 + A*B*a^3*b*d^4)*
n)*x)*log((b*x + a)/(d*x + c))^2 + 6*(A*B*b^4*c^4 - 9*A*B*a*b^3*c^3*d + 54*
A*B*a^2*b^2*c^2*d^2 - 55*A*B*a^3*b*c*d^3 + 9*A*B*a^4*d^4)*n - (18*A^2*b^4*c
```

```
^3*d - 162*A^2*a*b^3*c^2*d^2 - 54*A^2*a^2*b^2*c*d^3 + 198*A^2*a^3*b*d^4 + (
19*B^2*b^4*c^3*d - 567*B^2*a*b^3*c^2*d^2 + 87*B^2*a^2*b^2*c*d^3 + 461*B^2*a
^3*b*d^4)*n^2 + 6*(5*A*B*b^4*c^3*d - 81*A*B*a*b^3*c^2*d^2 + 57*A*B*a^2*b^2*
c*d^3 + 19*A*B*a^3*b*d^4)*n)*x + 6*(3*A*B*b^4*c^4 - 18*A*B*a*b^3*c^3*d + 54
*A*B*a^2*b^2*c^2*d^2 - 30*A*B*a^3*b*c*d^3 - 9*A*B*a^4*d^4 + 6*(6*A*B*b^4*c*
d^3 - 6*A*B*a*b^3*d^4 + 5*(B^2*b^4*c*d^3 - B^2*a*b^3*d^4)*n)*x^3 + 3*(6*A*B
*b^4*c^2*d^2 + 24*A*B*a*b^3*c*d^3 - 30*A*B*a^2*b^2*d^4 + (11*B^2*b^4*c^2*d^
2 + 8*B^2*a*b^3*c*d^3 - 19*B^2*a^2*b^2*d^4)*n)*x^2 + 18*(B^2*b^4*d^4*n*x^4
+ B^2*a^3*b*c*d^3*n + (B^2*b^4*c*d^3 + 3*B^2*a*b^3*d^4)*n*x^3 + 3*(B^2*a*b^
3*c*d^3 + B^2*a^2*b^2*d^4)*n*x^2 + (3*B^2*a^2*b^2*c*d^3 + B^2*a^3*b*d^4)*n*
x)*log((b*x + a)/(d*x + c))^2 + (B^2*b^4*c^4 - 9*B^2*a*b^3*c^3*d + 54*B^2*a
^2*b^2*c^2*d^2 - 55*B^2*a^3*b*c*d^3 + 9*B^2*a^4*d^4)*n - (6*A*B*b^4*c^3*d -
 54*A*B*a*b^3*c^2*d^2 - 18*A*B*a^2*b^2*c*d^3 + 66*A*B*a^3*b*d^4 + (5*B^2*b^
4*c^3*d - 81*B^2*a*b^3*c^2*d^2 + 57*B^2*a^2*b^2*c*d^3 + 19*B^2*a^3*b*d^4)*n
)*x + 3*(12*A*B*a^3*b*c*d^3 + 2*(5*B^2*b^4*d^4*n + 6*A*B*b^4*d^4)*x^4 + 2*(
6*A*B*b^4*c*d^3 + 18*A*B*a*b^3*d^4 + (11*B^2*b^4*c*d^3 + 9*B^2*a*b^3*d^4)*n
)*x^3 + 6*(6*A*B*a*b^3*c*d^3 + 6*A*B*a^2*b^2*d^4 + (B^2*b^4*c^2*d^2 + 9*B^2
*a*b^3*c*d^3)*n)*x^2 + (B^2*b^4*c^4 - 6*B^2*a*b^3*c^3*d + 18*B^2*a^2*b^2*c^
2*d^2 - 3*B^2*a^4*d^4)*n + 2*(18*A*B*a^2*b^2*c*d^3 + 6*A*B*a^3*b*d^4 - (B^2
*b^4*c^3*d - 9*B^2*a*b^3*c^2*d^2 - 18*B^2*a^2*b^2*c*d^3 + 6*B^2*a^3*b*d^4)*
n)*x)*log((b*x + a)/(d*x + c)))*log(e) + 6*(18*A^2*a^3*b*c*d^3 + (55*B^2*b^
4*d^4*n^2 + 30*A*B*b^4*d^4*n + 18*A^2*b^4*d^4)*x^4 + (18*A^2*b^4*c*d^3 + 54
*A^2*a*b^3*d^4 + 5*(17*B^2*b^4*c*d^3 + 27*B^2*a*b^3*d^4)*n^2 + 6*(11*A*B*b^
4*c*d^3 + 9*A*B*a*b^3*d^4)*n)*x^3 + (B^2*b^4*c^4 - 9*B^2*a*b^3*c^3*d + 54*B
^2*a^2*b^2*c^2*d^2 + 9*B^2*a^4*d^4)*n^2 + 3*(18*A^2*a*b^3*c*d^3 + 18*A^2*a^
2*b^2*d^4 + (11*B^2*b^4*c^2*d^2 + 63*B^2*a*b^3*c*d^3 + 36*B^2*a^2*b^2*d^4)*
n^2 + 6*(A*B*b^4*c^2*d^2 + 9*A*B*a*b^3*c*d^3)*n)*x^2 + 3*(A*B*b^4*c^4 - 6*A
*B*a*b^3*c^3*d + 18*A*B*a^2*b^2*c^2*d^2 - 3*A*B*a^4*d^4)*n + (54*A^2*a^2*b^
2*c*d^3 + 18*A^2*a^3*b*d^4 - (5*B^2*b^4*c^3*d - 81*B^2*a*b^3*c^2*d^2 - 108*
B^2*a^2*b^2*c*d^3 - 36*B^2*a^3*b*d^4)*n^2 - 6*(A*B*b^4*c^3*d - 9*A*B*a*b^3*
c^2*d^2 - 18*A*B*a^2*b^2*c*d^3 + 6*A*B*a^3*b*d^4)*n)*x)*log((b*x + a)/(d*x
+ c)))/((b^8*c^5*d - 5*a*b^7*c^4*d^2 + 10*a^2*b^6*c^3*d^3 - 10*a^3*b^5*c^2*
d^4 + 5*a^4*b^4*c*d^5 - a^5*b^3*d^6)*g^4*i^2*x^4 + (b^8*c^6 - 2*a*b^7*c^5*d
 - 5*a^2*b^6*c^4*d^2 + 20*a^3*b^5*c^3*d^3 - 25*a^4*b^4*c^2*d^4 + 14*a^5*b^3
*c*d^5 - 3*a^6*b^2*d^6)*g^4*i^2*x^3 + 3*(a*b^7*c^6 - 4*a^2*b^6*c^5*d + 5*a^
3*b^5*c^4*d^2 - 5*a^5*b^3*c^2*d^4 + 4*a^6*b^2*c*d^5 - a^7*b*d^6)*g^4*i^2*x^
2 + (3*a^2*b^6*c^6 - 14*a^3*b^5*c^5*d + 25*a^4*b^4*c^4*d^2 - 20*a^5*b^3*c^3
*d^3 + 5*a^6*b^2*c^2*d^4 + 2*a^7*b*c*d^5 - a^8*d^6)*g^4*i^2*x + (a^3*b^5*c^
6 - 5*a^4*b^4*c^5*d + 10*a^5*b^3*c^4*d^2 - 10*a^6*b^2*c^3*d^3 + 5*a^7*b*c^2
*d^4 - a^8*c*d^5)*g^4*i^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**4/(d*i*x+c*i)**2
,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6171 vs. $2(721) = 1442$.

Time = 0.68 (sec) , antiderivative size = 6171, normalized size of antiderivative = 8.47

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x,
 algorithm="maxima")
```

```
[Out] -1/3*B^2*((12*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*d + 13*a^2*b*c*d^2 + 3*a^
3*d^3 + 6*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*c^2*d - 8*a*b^2*c*d^2 - 11
*a^2*b*d^3)*x)/((b^7*c^4*d - 4*a*b^6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^
4*c*d^4 + a^4*b^3*d^5)*g^4*i^2*x^4 + (b^7*c^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3
*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a^5*b^2*d^5)*g^4*i^2*x^3 +
 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^4*b^3*c^2*d^3 - 3
*a^5*b^2*c*d^4 + a^6*b*d^5)*g^4*i^2*x^2 + (3*a^2*b^5*c^5 - 11*a^3*b^4*c^4*d
 + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)*g^4*i^2*
x + (a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 +
a^7*c*d^4)*g^4*i^2) + 12*b*d^3*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*
a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2) -
12*b*d^3*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a
^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2))*log(e*(b*x/(d*x + c) +
a/(d*x + c))^n)^2 - 2/3*A*B*((12*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*d + 13
*a^2*b*c*d^2 + 3*a^3*d^3 + 6*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*c^2*d -
 8*a*b^2*c*d^2 - 11*a^2*b*d^3)*x)/((b^7*c^4*d - 4*a*b^6*c^3*d^2 + 6*a^2*b^5
*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*g^4*i^2*x^4 + (b^7*c^5 - a*b^6*c^
4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a^5*b^2
*d^5)*g^4*i^2*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*
a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*g^4*i^2*x^2 + (3*a^2*b^5*c^5
 - 11*a^3*b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4
+ a^7*d^5)*g^4*i^2*x + (a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6*a^5*b^2*c^3*d^2 -
```

4*a^6*b*c^2*d^3 + a^7*c*d^4)*g^4*i^2) + 12*b*d^3*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2) - 12*b*d^3*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1/27*((2*b^4*c^4 - 27*a*b^3*c^3*d + 324*a^2*b^2*c^2*d^2 - 245*a^3*b*c*d^3 - 54*a^4*d^4 + 330*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 36*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a)^3 - 36*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(d*x + c)^3 + 15*(17*b^4*c^2*d^2 + 32*a*b^3*c*d^3 - 49*a^2*b^2*d^4)*x^2 - 90*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a)^2 - 18*(5*b^4*d^4*x^4 + 5*a^3*b*c*d^3 + 5*(b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 15*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 5*(3*a^2*b^2*c*d^3 + a^3*b*d^4)*x - 6*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a))*log(d*x + c)^2 - (19*b^4*c^3*d - 567*a*b^3*c^2*d^2 + 87*a^2*b^2*c*d^3 + 461*a^3*b*d^4)*x + 330*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a) - 6*(55*b^4*d^4*x^4 + 55*a^3*b*c*d^3 + 55*(b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 165*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 18*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a)^2 + 55*(3*a^2*b^2*c*d^3 + a^3*b*d^4)*x - 30*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a))*log(d*x + c))*n^2/(a^3*b^5*c^6*g^4*i^2 - 5*a^4*b^4*c^5*d*g^4*i^2 + 10*a^5*b^3*c^4*d^2*g^4*i^2 - 10*a^6*b^2*c^3*d^3*g^4*i^2 + 5*a^7*b*c^2*d^4*g^4*i^2 - a^8*c*d^5*g^4*i^2 + (b^8*c^5*d*g^4*i^2 - 5*a*b^7*c^4*d^2*g^4*i^2 + 10*a^2*b^6*c^3*d^3*g^4*i^2 - 10*a^3*b^5*c^2*d^4*g^4*i^2 + 5*a^4*b^4*c*d^5*g^4*i^2 - a^5*b^3*d^6*g^4*i^2)*x^4 + (b^8*c^6*g^4*i^2 - 2*a*b^7*c^5*d*g^4*i^2 - 5*a^2*b^6*c^4*d^2*g^4*i^2 + 20*a^3*b^5*c^3*d^3*g^4*i^2 - 25*a^4*b^4*c^2*d^4*g^4*i^2 + 14*a^5*b^3*c*d^5*g^4*i^2 - 3*a^6*b^2*d^6*g^4*i^2)*x^3 + 3*(a*b^7*c^6*g^4*i^2 - 4*a^2*b^6*c^5*d*g^4*i^2 + 5*a^3*b^5*c^4*d^2*g^4*i^2 - 5*a^5*b^3*c^2*d^4*g^4*i^2 + 4*a^6*b^2*c*d^5*g^4*i^2 - a^7*b*d^6*g^4*i^2)*x^2 + (3*a^2*b^6*c^6*g^4*i^2 - 14*a^3*b^5*c^5*d*g^4*i^2 + 25*a^4*b^4*c^4*d^2*g^4*i^2 - 20*a^5*b^3*c^3*d^3*g^4*i^2 + 5*a^6*b^2*c^2*d^4*g^4*i^2 + 2*a^7*b*c*d^5*g^4*i^2 - a^8*d^6*g^4*i^2)*x) + 6*(b^4*c^4 - 9*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 - 55*a^3*b*c*d^3 + 9*a^4*d^4 + 30*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 3*(11*b^4*c^2*d^2 + 8*a*b^3*c*d^3 - 19*a^2*b^2*d^4)*x^2 - 18*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a)^2 - 18*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(d*x + c)^2 - (5*b^4*c^3*d - 81*a*b^3*c^2*d^2 + 57*a^2*b^2*c*d^3 + 19*a^3*b*d^4)*x + 30*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*

c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c
*d^3 + a^3*b*d^4)*x)*log(b*x + a) - 6*(5*b^4*d^4*x^4 + 5*a^3*b*c*d^3 + 5*(b
^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 15*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 5*(3*a^2
*b^2*c*d^3 + a^3*b*d^4)*x - 6*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a
*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*
b*d^4)*x)*log(b*x + a))*log(d*x + c))*n*log(e*(b*x/(d*x + c) + a/(d*x + c))
^n)/(a^3*b^5*c^6*g^4*i^2 - 5*a^4*b^4*c^5*d*g^4*i^2 + 10*a^5*b^3*c^4*d^2*g^4
*i^2 - 10*a^6*b^2*c^3*d^3*g^4*i^2 + 5*a^7*b*c^2*d^4*g^4*i^2 - a^8*c*d^5*g^4
*i^2 + (b^8*c^5*d*g^4*i^2 - 5*a*b^7*c^4*d^2*g^4*i^2 + 10*a^2*b^6*c^3*d^3*g^
4*i^2 - 10*a^3*b^5*c^2*d^4*g^4*i^2 + 5*a^4*b^4*c*d^5*g^4*i^2 - a^5*b^3*d^6*
g^4*i^2)*x^4 + (b^8*c^6*g^4*i^2 - 2*a*b^7*c^5*d*g^4*i^2 - 5*a^2*b^6*c^4*d^2
*g^4*i^2 + 20*a^3*b^5*c^3*d^3*g^4*i^2 - 25*a^4*b^4*c^2*d^4*g^4*i^2 + 14*a^5
*b^3*c*d^5*g^4*i^2 - 3*a^6*b^2*d^6*g^4*i^2)*x^3 + 3*(a*b^7*c^6*g^4*i^2 - 4*
a^2*b^6*c^5*d*g^4*i^2 + 5*a^3*b^5*c^4*d^2*g^4*i^2 - 5*a^5*b^3*c^2*d^4*g^4*i
^2 + 4*a^6*b^2*c*d^5*g^4*i^2 - a^7*b*d^6*g^4*i^2)*x^2 + (3*a^2*b^6*c^6*g^4*
i^2 - 14*a^3*b^5*c^5*d*g^4*i^2 + 25*a^4*b^4*c^4*d^2*g^4*i^2 - 20*a^5*b^3*c^
3*d^3*g^4*i^2 + 5*a^6*b^2*c^2*d^4*g^4*i^2 + 2*a^7*b*c*d^5*g^4*i^2 - a^8*d^6
*g^4*i^2)*x))*B^2 - 2/9*(b^4*c^4 - 9*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 - 55*
a^3*b*c*d^3 + 9*a^4*d^4 + 30*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 3*(11*b^4*c^2*d^
2 + 8*a*b^3*c*d^3 - 19*a^2*b^2*d^4)*x^2 - 18*(b^4*d^4*x^4 + a^3*b*c*d^3 + (
b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b
^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a)^2 - 18*(b^4*d^4*x^4 + a^3*b*c*d^3 + (
b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b
^2*c*d^3 + a^3*b*d^4)*x)*log(d*x + c)^2 - (5*b^4*c^3*d - 81*a*b^3*c^2*d^2 +
57*a^2*b^2*c*d^3 + 19*a^3*b*d^4)*x + 30*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*
c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c
*d^3 + a^3*b*d^4)*x)*log(b*x + a) - 6*(5*b^4*d^4*x^4 + 5*a^3*b*c*d^3 + 5*(b
^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 15*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 5*(3*a^2
*b^2*c*d^3 + a^3*b*d^4)*x - 6*(b^4*d^4*x^4 + a^3*b*c*d^3 + (b^4*c*d^3 + 3*a
*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + (3*a^2*b^2*c*d^3 + a^3*
b*d^4)*x)*log(b*x + a))*log(d*x + c))*A*B*n/(a^3*b^5*c^6*g^4*i^2 - 5*a^4*b^
4*c^5*d*g^4*i^2 + 10*a^5*b^3*c^4*d^2*g^4*i^2 - 10*a^6*b^2*c^3*d^3*g^4*i^2 +
5*a^7*b*c^2*d^4*g^4*i^2 - a^8*c*d^5*g^4*i^2 + (b^8*c^5*d*g^4*i^2 - 5*a*b^7
*c^4*d^2*g^4*i^2 + 10*a^2*b^6*c^3*d^3*g^4*i^2 - 10*a^3*b^5*c^2*d^4*g^4*i^2
+ 5*a^4*b^4*c*d^5*g^4*i^2 - a^5*b^3*d^6*g^4*i^2)*x^4 + (b^8*c^6*g^4*i^2 - 2
*a*b^7*c^5*d*g^4*i^2 - 5*a^2*b^6*c^4*d^2*g^4*i^2 + 20*a^3*b^5*c^3*d^3*g^4*i
^2 - 25*a^4*b^4*c^2*d^4*g^4*i^2 + 14*a^5*b^3*c*d^5*g^4*i^2 - 3*a^6*b^2*d^6*
g^4*i^2)*x^3 + 3*(a*b^7*c^6*g^4*i^2 - 4*a^2*b^6*c^5*d*g^4*i^2 + 5*a^3*b^5*c
^4*d^2*g^4*i^2 - 5*a^5*b^3*c^2*d^4*g^4*i^2 + 4*a^6*b^2*c*d^5*g^4*i^2 - a^7*
b*d^6*g^4*i^2)*x^2 + (3*a^2*b^6*c^6*g^4*i^2 - 14*a^3*b^5*c^5*d*g^4*i^2 + 25
*a^4*b^4*c^4*d^2*g^4*i^2 - 20*a^5*b^3*c^3*d^3*g^4*i^2 + 5*a^6*b^2*c^2*d^4*g
^4*i^2 + 2*a^7*b*c*d^5*g^4*i^2 - a^8*d^6*g^4*i^2)*x) - 1/3*A^2*((12*b^3*d^3
*x^3 + b^3*c^3 - 5*a*b^2*c^2*d + 13*a^2*b*c*d^2 + 3*a^3*d^3 + 6*(b^3*c*d^2
+ 5*a*b^2*d^3)*x^2 - 2*(b^3*c^2*d - 8*a*b^2*c*d^2 - 11*a^2*b*d^3)*x)/((b^7*
c^4*d - 4*a*b^6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5

```
)*g^4*i^2*x^4 + (b^7*c^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2
*d^3 - 11*a^4*b^3*c*d^4 + 3*a^5*b^2*d^5)*g^4*i^2*x^3 + 3*(a*b^6*c^5 - 3*a^2
*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*
b*d^5)*g^4*i^2*x^2 + (3*a^2*b^5*c^5 - 11*a^3*b^4*c^4*d + 14*a^4*b^3*c^3*d^2
 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)*g^4*i^2*x + (a^3*b^4*c^5 - 4*
a^4*b^3*c^4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4)*g^4*i^2) +
 12*b*d^3*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*
a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2) - 12*b*d^3*log(d*x + c)
/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^
4*b*c*d^4 - a^5*d^5)*g^4*i^2))
```

Giac [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x,
 algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 8.35 (sec) , antiderivative size = 3157, normalized size of antiderivative = 4.33

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx=\text{Too large to display}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^4*(c*i + d*i*x)
^2),x)
```

```
[Out] log(e*((a + b*x)/(c + d*x))^n)*((x*(8*A*B*b^2*c*d - 8*A*B*a*b*d^2 + 12*B^2*
b*d*n*(a*d + b*c) + (16*B^2*a*b*d^2*n)/3 - (16*B^2*b^2*c*d*n)/3) - 6*A*B*a^
2*d^2 + 2*A*B*b^2*c^2 + 6*B^2*a^2*d^2*n + (2*B^2*b^2*c^2*n)/3 + 12*B^2*b^2*
d^2*n*x^2 + 4*A*B*a*b*c*d + (16*B^2*a*b*c*d*n)/3)/(x*(3*a^6*d^4*g^4*i^2 - 9
*a^2*b^4*c^4*g^4*i^2 + 24*a^3*b^3*c^3*d*g^4*i^2 - 18*a^4*b^2*c^2*d^2*g^4*i^
2) - x^2*(9*a*b^5*c^4*g^4*i^2 - 9*a^5*b*d^4*g^4*i^2 - 18*a^2*b^4*c^3*d*g^4*
i^2 + 18*a^4*b^2*c*d^3*g^4*i^2) - x^3*(3*b^6*c^4*g^4*i^2 - 9*a^4*b^2*d^4*g^
4*i^2 + 24*a^3*b^3*c*d^3*g^4*i^2 - 18*a^2*b^4*c^2*d^2*g^4*i^2) + x^4*(3*a^3
*b^3*d^4*g^4*i^2 - 3*b^6*c^3*d*g^4*i^2 + 9*a*b^5*c^2*d^2*g^4*i^2 - 9*a^2*b^
4*c*d^3*g^4*i^2) - 3*a^3*b^3*c^4*g^4*i^2 + 3*a^6*c*d^3*g^4*i^2 + 9*a^4*b^2*
c^3*d*g^4*i^2 - 9*a^5*b*c^2*d^2*g^4*i^2) - (4*d^3*(6*A*B*b + 5*B^2*b*n)*(x*
((a*d + b*c)*((3*a*g^4*i^2*n*(a*d - b*c)^4)/(2*d) + (3*g^4*i^2*n*(a*d - b*c
```

)^4*(2*a*d - b*c))/(2*d^2)) + (3*a*b*c*g^4*i^2*n*(a*d - b*c)^4)/d) + x^2*(b
d((3*a*g^4*i^2*n*(a*d - b*c)^4)/(2*d) + (3*g^4*i^2*n*(a*d - b*c)^4*(2*a*d
 - b*c))/(2*d^2)) + (3*b*g^4*i^2*n*(a*d + b*c)*(a*d - b*c)^4)/d) + a*c*((3*
a*g^4*i^2*n*(a*d - b*c)^4)/(2*d) + (3*g^4*i^2*n*(a*d - b*c)^4*(2*a*d - b*c)
)/(2*d^2)) + 3*b^2*g^4*i^2*n*x^3*(a*d - b*c)^4))/(3*g^4*i^2*n*(a*d - b*c)^3
*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(x*(3*a^6*d^4*g^4*i^2 - 9*a^2*b^4*c^4*g^4*
i^2 + 24*a^3*b^3*c^3*d*g^4*i^2 - 18*a^4*b^2*c^2*d^2*g^4*i^2) - x^2*(9*a*b^5
*c^4*g^4*i^2 - 9*a^5*b*d^4*g^4*i^2 - 18*a^2*b^4*c^3*d*g^4*i^2 + 18*a^4*b^2*
c*d^3*g^4*i^2) - x^3*(3*b^6*c^4*g^4*i^2 - 9*a^4*b^2*d^4*g^4*i^2 + 24*a^3*b^
3*c*d^3*g^4*i^2 - 18*a^2*b^4*c^2*d^2*g^4*i^2) + x^4*(3*a^3*b^3*d^4*g^4*i^2
 - 3*b^6*c^3*d*g^4*i^2 + 9*a*b^5*c^2*d^2*g^4*i^2 - 9*a^2*b^4*c*d^3*g^4*i^2)
 - 3*a^3*b^3*c^4*g^4*i^2 + 3*a^6*c*d^3*g^4*i^2 + 9*a^4*b^2*c^3*d*g^4*i^2 - 9
*a^5*b*c^2*d^2*g^4*i^2))) - ((27*A^2*a^3*d^3 + 9*A^2*b^3*c^3 + 54*B^2*a^3*d
^3*n^2 + 2*B^2*b^3*c^3*n^2 - 45*A^2*a*b^2*c^2*d + 117*A^2*a^2*b*c*d^2 - 54*
A*B*a^3*d^3*n + 6*A*B*b^3*c^3*n - 25*B^2*a*b^2*c^2*d*n^2 + 299*B^2*a^2*b*c*
d^2*n^2 - 48*A*B*a*b^2*c^2*d*n + 276*A*B*a^2*b*c*d^2*n)/(3*(a*d - b*c)) + (
x^2*(90*A^2*a*b^2*d^3 + 18*A^2*b^3*c*d^2 + 245*B^2*a*b^2*d^3*n^2 + 85*B^2*b
^3*c*d^2*n^2 + 114*A*B*a*b^2*d^3*n + 66*A*B*b^3*c*d^2*n))/(a*d - b*c) + (2*
x^3*(18*A^2*b^3*d^3 + 55*B^2*b^3*d^3*n^2 + 30*A*B*b^3*d^3*n))/(a*d - b*c) +
 (x*(198*A^2*a^2*b*d^3 - 18*A^2*b^3*c^2*d + 144*A^2*a*b^2*c*d^2 + 461*B^2*a
^2*b*d^3*n^2 - 19*B^2*b^3*c^2*d*n^2 + 548*B^2*a*b^2*c*d^2*n^2 + 114*A*B*a^2
*b*d^3*n - 30*A*B*b^3*c^2*d*n + 456*A*B*a*b^2*c*d^2*n))/(3*(a*d - b*c)))/(x
*(9*a^6*d^4*g^4*i^2 - 27*a^2*b^4*c^4*g^4*i^2 + 72*a^3*b^3*c^3*d*g^4*i^2 - 5
4*a^4*b^2*c^2*d^2*g^4*i^2) - x^2*(27*a*b^5*c^4*g^4*i^2 - 27*a^5*b*d^4*g^4*i
^2 - 54*a^2*b^4*c^3*d*g^4*i^2 + 54*a^4*b^2*c*d^3*g^4*i^2) - x^3*(9*b^6*c^4*
g^4*i^2 - 27*a^4*b^2*d^4*g^4*i^2 + 72*a^3*b^3*c*d^3*g^4*i^2 - 54*a^2*b^4*c^
2*d^2*g^4*i^2) + x^4*(9*a^3*b^3*d^4*g^4*i^2 - 9*b^6*c^3*d*g^4*i^2 + 27*a*b^
5*c^2*d^2*g^4*i^2 - 27*a^2*b^4*c*d^3*g^4*i^2) - 9*a^3*b^3*c^4*g^4*i^2 + 9*a
^6*c*d^3*g^4*i^2 + 27*a^4*b^2*c^3*d*g^4*i^2 - 27*a^5*b*c^2*d^2*g^4*i^2) - l
og(e*((a + b*x)/(c + d*x))^n)^2*(((B^2*(3*a*d + b*c))/(3*(a^2*d^2 + b^2*c^2
 - 2*a*b*c*d)) + (4*B^2*b*d*x)/(3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(x^3*(b
^3*c*g^4*i^2 + 3*a*b^2*d*g^4*i^2) + x^2*(3*a*b^2*c*g^4*i^2 + 3*a^2*b*d*g^4*
i^2) + x*(a^3*d*g^4*i^2 + 3*a^2*b*c*g^4*i^2) + a^3*c*g^4*i^2 + b^3*d*g^4*i^
2*x^4) - (2*d^3*(6*A*B*b + 5*B^2*b*n))/(3*g^4*i^2*n*(a*d - b*c)^3*(a^2*d^2
+ b^2*c^2 - 2*a*b*c*d)) + (4*B^2*b*d^3*(x*((a*d + b*c)*((a*g^4*i^2*n*(a*d -
 b*c))/(2*d) + (g^4*i^2*n*(a*d - b*c)*(2*a*d - b*c))/(2*d^2)) + (a*b*c*g^4*
i^2*n*(a*d - b*c))/d) + x^2*(b*d*((a*g^4*i^2*n*(a*d - b*c))/(2*d) + (g^4*i^
2*n*(a*d - b*c)*(2*a*d - b*c))/(2*d^2)) + (b*g^4*i^2*n*(a*d + b*c)*(a*d - b
*c))/d) + a*c*((a*g^4*i^2*n*(a*d - b*c))/(2*d) + (g^4*i^2*n*(a*d - b*c)*(2*
a*d - b*c))/(2*d^2)) + b^2*g^4*i^2*n*x^3*(a*d - b*c)))/(g^4*i^2*n*(a*d - b*
c)^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(x^3*(b^3*c*g^4*i^2 + 3*a*b^2*d*g^4*i^
2) + x^2*(3*a*b^2*c*g^4*i^2 + 3*a^2*b*d*g^4*i^2) + x*(a^3*d*g^4*i^2 + 3*a^2
*b*c*g^4*i^2) + a^3*c*g^4*i^2 + b^3*d*g^4*i^2*x^4))) - (b*d^3*atan((b*d^3*(
18*A^2 + 55*B^2*n^2 + 30*A*B*n)*(9*a^5*d^5*g^4*i^2 + 9*b^5*c^5*g^4*i^2 - 27
*a*b^4*c^4*d*g^4*i^2 - 27*a^4*b*c*d^4*g^4*i^2 + 18*a^2*b^3*c^3*d^2*g^4*i^2

```
+ 18*a^3*b^2*c^2*d^3*g^4*i^2)*2i)/(9*g^4*i^2*(a*d - b*c)^5*(36*A^2*b*d^3 +
110*B^2*b*d^3*n^2 + 60*A*B*b*d^3*n)) + (b^2*d^4*x*(18*A^2 + 55*B^2*n^2 + 30
*A*B*n)*(a^4*d^4*g^4*i^2 + b^4*c^4*g^4*i^2 - 4*a*b^3*c^3*d*g^4*i^2 - 4*a^3*
b*c*d^3*g^4*i^2 + 6*a^2*b^2*c^2*d^2*g^4*i^2)*4i)/(g^4*i^2*(a*d - b*c)^5*(36
*A^2*b*d^3 + 110*B^2*b*d^3*n^2 + 60*A*B*b*d^3*n)))*(18*A^2 + 55*B^2*n^2 + 3
0*A*B*n)*4i)/(9*g^4*i^2*(a*d - b*c)^5) + (4*B^2*b*d^3*log(e*((a + b*x)/(c +
 d*x))^n)^3)/(3*g^4*i^2*n*(a*d - b*c)^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
```

3.202 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 676

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{B^2(bc-ad)g^3n^2(a+bx)^2}{4d^2i^3(c+dx)^2} - \frac{4AbB(bc-ad)g^3n(a+bx)}{d^3i^3(c+dx)}$$
$$+ \frac{4bB^2(bc-ad)g^3n^2(a+bx)}{d^3i^3(c+dx)} - \frac{4bB^2(bc-ad)g^3n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^3i^3(c+dx)}$$
$$- \frac{B(bc-ad)g^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i^3(c+dx)^2}$$
$$+ \frac{b^2g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^3} + \frac{(bc-ad)g^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^2i^3(c+dx)^2}$$
$$+ \frac{2b(bc-ad)g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^3(c+dx)}$$
$$+ \frac{2b^2B(bc-ad)g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^3}$$
$$+ \frac{3b^2(bc-ad)g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^3}$$
$$+ \frac{2b^2B^2(bc-ad)g^3n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}$$
$$+ \frac{6b^2B(bc-ad)g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}$$
$$- \frac{6b^2B^2(bc-ad)g^3n^2\,\mathrm{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}$$

```
[Out] 1/4*B^2*(-a*d+b*c)*g^3*n^2*(b*x+a)^2/d^2/i^3/(d*x+c)^2-4*A*b*B*(-a*d+b*c)*g
^3*n*(b*x+a)/d^3/i^3/(d*x+c)+4*b*B^2*(-a*d+b*c)*g^3*n^2*(b*x+a)/d^3/i^3/(d*
x+c)-4*b*B^2*(-a*d+b*c)*g^3*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/d^3/i^3/(d*
x+c)-1/2*B*(-a*d+b*c)*g^3*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^2/i
^3/(d*x+c)^2+b^2*g^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^3/i^3+1/2*
(-a*d+b*c)*g^3*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^2/i^3/(d*x+c)^
2+2*b*(-a*d+b*c)*g^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^3/i^3/(d*x
+c)+2*b^2*B*(-a*d+b*c)*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/
b/(d*x+c))/d^4/i^3+3*b^2*(-a*d+b*c)*g^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*l
n((-a*d+b*c)/b/(d*x+c))/d^4/i^3+2*b^2*B^2*(-a*d+b*c)*g^3*n^2*polylog(2,d*(b
*x+a)/b/(d*x+c))/d^4/i^3+6*b^2*B*(-a*d+b*c)*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i^3-6*b^2*B^2*(-a*d+b*c)*g^3*n^2*
polylog(3,d*(b*x+a)/b/(d*x+c))/d^4/i^3
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 676, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.244, Rules used = {2561, 2395, 2333, 2332, 2342, 2341, 2355, 2354, 2438, 2421, 6724}

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{6b^2Bg^3n(bc-ad)\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^4i^3}$$
$$+\frac{3b^2g^3(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^4i^3}$$
$$+\frac{2b^2Bg^3n(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^4i^3}$$
$$+\frac{b^2g^3(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^3i^3}$$
$$+\frac{2bg^3(a+bx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^3i^3(c+dx)}$$
$$-\frac{4AbBg^3n(a+bx)(bc-ad)}{d^3i^3(c+dx)}+\frac{g^3(a+bx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d^2i^3(c+dx)^2}$$
$$-\frac{Bg^3n(a+bx)^2(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d^2i^3(c+dx)^2}$$
$$+\frac{2b^2B^2g^3n^2(bc-ad)\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}-\frac{6b^2B^2g^3n^2(bc-ad)\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}$$
$$-\frac{4bB^2g^3n(a+bx)(bc-ad)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^3i^3(c+dx)}$$
$$+\frac{4bB^2g^3n^2(a+bx)(bc-ad)}{d^3i^3(c+dx)}+\frac{B^2g^3n^2(a+bx)^2(bc-ad)}{4d^2i^3(c+dx)^2}$$

```
[In] Int[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*x
)^3,x]
```

```
[Out] (B^2*(b*c - a*d)*g^3*n^2*(a + b*x)^2)/(4*d^2*i^3*(c + d*x)^2) - (4*A*b*B*(b
*c - a*d)*g^3*n*(a + b*x))/(d^3*i^3*(c + d*x)) + (4*b*B^2*(b*c - a*d)*g^3*n
^2*(a + b*x))/(d^3*i^3*(c + d*x)) - (4*b*B^2*(b*c - a*d)*g^3*n*(a + b*x)*Lo
g[e*((a + b*x)/(c + d*x))^n])/(d^3*i^3*(c + d*x)) - (B*(b*c - a*d)*g^3*n*(a
 + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d^2*i^3*(c + d*x)^2) +
 (b^2*g^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d^3*i^3) + (
(b*c - a*d)*g^3*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*d^
2*i^3*(c + d*x)^2) + (2*b*(b*c - a*d)*g^3*(a + b*x)*(A + B*Log[e*((a + b*x)
```

/(c + d*x))^n])^2)/(d^3*i^3*(c + d*x)) + (2*b^2*B*(b*c - a*d)*g^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(d^4*i^3) + (3*b^2*(b*c - a*d)*g^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[(b*c - a*d)/(b*(c + d*x))])/(d^4*i^3) + (2*b^2*B^2*(b*c - a*d)*g^3*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^4*i^3) + (6*b^2*B*(b*c - a*d)*g^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^4*i^3) - (6*b^2*B^2*(b*c - a*d)*g^3*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(d^4*i^3)

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{((bc-ad)g^3)\,\text{Subst}\left(\int \frac{x^3(A+B\log(ex^n))^2}{(b-dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{i^3}$$

$$= \frac{((bc-ad)g^3)\,\text{Subst}\left(\int \left(\frac{2b(A+B\log(ex^n))^2}{d^3} + \frac{x(A+B\log(ex^n))^2}{d^2} + \frac{b^3(A+B\log(ex^n))^2}{d^3(-b+dx)^2} + \frac{3b^2(A+B\log(ex^n))^2}{d^3(-b+dx)}\right)\,dx\right.}{i^3}$$

$$= \frac{(2b(bc-ad)g^3)\,\text{Subst}\left(\int (A+B\log(ex^n))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^3} + \frac{(3b^2(bc-ad)g^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{-b+dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^3} + \frac{(b^3(bc-ad)g^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(-b+dx)^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^3} + \frac{((bc-ad)g^3)\,\text{Subst}\left(\int x(A+B\log(ex^n))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^3}$$

$$= \frac{b^2g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^3} + \frac{(bc-ad)g^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^2i^3(c+dx)^2} + \frac{2b(bc-ad)g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^3(c+dx)} + \frac{3b^2(bc-ad)g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^3} - \frac{(6b^2B(bc-ad)g^3n)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4i^3} - \frac{(4bB(bc-ad)g^3n)\,\text{Subst}\left(\int (A+B\log(ex^n))\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^3} + \frac{(2b^2B(bc-ad)g^3n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{-b+dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^3} - \frac{(B(bc-ad)g^3n)\,\text{Subst}\left(\int x(A+B\log(ex^n))\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^3}$$

$$
\begin{aligned}
&= \frac{B^2(bc-ad)g^3n^2(a+bx)^2}{4d^2i^3(c+dx)^2} - \frac{4AbB(bc-ad)g^3n(a+bx)}{d^3i^3(c+dx)} \\
&- \frac{B(bc-ad)g^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i^3(c+dx)^2} \\
&+ \frac{b^2g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^3} \\
&+ \frac{(bc-ad)g^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^2i^3(c+dx)^2} \\
&+ \frac{2b(bc-ad)g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^3(c+dx)} \\
&+ \frac{2b^2B(bc-ad)g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^3} \\
&+ \frac{3b^2(bc-ad)g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^3} \\
&+ \frac{6b^2B(bc-ad)g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3} \\
&- \frac{(4bB^2(bc-ad)g^3n)\operatorname{Subst}\left(\int\log\left(ex^n\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^3} \\
&- \frac{(2b^2B^2(bc-ad)g^3n^2)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4i^3} \\
&- \frac{(6b^2B^2(bc-ad)g^3n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^4i^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{B^2(bc-ad)g^3n^2(a+bx)^2}{4d^2i^3(c+dx)^2} - \frac{4AbB(bc-ad)g^3n(a+bx)}{d^3i^3(c+dx)} \\
&+ \frac{4bB^2(bc-ad)g^3n^2(a+bx)}{d^3i^3(c+dx)} - \frac{4bB^2(bc-ad)g^3n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^3i^3(c+dx)} \\
&- \frac{B(bc-ad)g^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i^3(c+dx)^2} \\
&+ \frac{b^2g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^3} \\
&+ \frac{(bc-ad)g^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^2i^3(c+dx)^2} \\
&+ \frac{2b(bc-ad)g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^3(c+dx)} \\
&+ \frac{2b^2B(bc-ad)g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^3} \\
&+ \frac{3b^2(bc-ad)g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^3} \\
&+ \frac{2b^2B^2(bc-ad)g^3n^2\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3} \\
&+ \frac{6b^2B(bc-ad)g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3} \\
&- \frac{6b^2B^2(bc-ad)g^3n^2\operatorname{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6878 vs. $2(676) = 1352$.

Time = 7.02 (sec) , antiderivative size = 6878, normalized size of antiderivative = 10.17

$$
\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i +
 d*i*x)^3,x]
[Out] Result too large to show
```

Maple [F]

$$\int \frac{(bgx+ag)^3 \left(A+B\ln \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(dix+ci)^3}dx$$

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x)
[Out] int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^3 \left(B\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^3}\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x,
 algorithm="fricas")
[Out] integral((A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a
^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a
^3*g^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2
*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log(e*((b*x + a)/(d*x + c))^n))
/(d^3*i^3*x^3 + 3*c*d^2*i^3*x^2 + 3*c^2*d*i^3*x + c^3*i^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**3
,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^3 \left(B\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^3}\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x,
 algorithm="maxima")
```

```
[Out] 3/2*A*B*a^2*b*g^3*n*((b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*x)/((b*c*d^4 -
a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^
3) + 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4
)*i^3) - 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2
*d^4)*i^3)) + 1/2*A*B*a^3*g^3*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)
*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b
^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x +
 c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) - 1/2*A^2*b^3*g^3*((6*c^2*d*
x + 5*c^3)/(d^6*i^3*x^2 + 2*c*d^5*i^3*x + c^2*d^4*i^3) - 2*x/(d^3*i^3) + 6*
c*log(d*x + c)/(d^4*i^3)) + 3/2*A^2*a*b^2*g^3*((4*c*d*x + 3*c^2)/(d^5*i^3*x
^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) + 2*log(d*x + c)/(d^3*i^3)) - 3*(2*d*x +
c)*A*B*a^2*b*g^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^4*i^3*x^2 + 2*c*
d^3*i^3*x + c^2*d^2*i^3) - 3/2*(2*d*x + c)*A^2*a^2*b*g^3/(d^4*i^3*x^2 + 2*c
*d^3*i^3*x + c^2*d^2*i^3) - A*B*a^3*g^3*log(e*(b*x/(d*x + c) + a/(d*x + c))
^n)/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - 1/2*A^2*a^3*g^3/(d^3*i^3*x^
2 + 2*c*d^2*i^3*x + c^2*d*i^3) + 1/2*(2*B^2*b^3*d^3*g^3*x^3 + 4*B^2*b^3*c*d
^2*g^3*x^2 - 2*(2*b^3*c^2*d*g^3 - 6*a*b^2*c*d^2*g^3 + 3*a^2*b*d^3*g^3)*B^2*
x - (5*b^3*c^3*g^3 - 9*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 + a^3*d^3*g^3)*B
^2 - 6*((b^3*c*d^2*g^3 - a*b^2*d^3*g^3)*B^2*x^2 + 2*(b^3*c^2*d*g^3 - a*b^2*
c*d^2*g^3)*B^2*x + (b^3*c^3*g^3 - a*b^2*c^2*d*g^3)*B^2)*log(d*x + c))*log((
d*x + c)^n)^2/(d^6*i^3*x^2 + 2*c*d^5*i^3*x + c^2*d^4*i^3) - integrate(-(3*B
^2*a^2*b*d^3*g^3*x*log(e)^2 + B^2*a^3*d^3*g^3*log(e)^2 + (B^2*b^3*d^3*g^3*l
og(e)^2 + 2*A*B*b^3*d^3*g^3*log(e))*x^3 + 3*(B^2*a*b^2*d^3*g^3*log(e)^2 + 2
*A*B*a*b^2*d^3*g^3*log(e))*x^2 + (B^2*b^3*d^3*g^3*x^3 + 3*B^2*a*b^2*d^3*g^3
*x^2 + 3*B^2*a^2*b*d^3*g^3*x + B^2*a^3*d^3*g^3)*log((b*x + a)^n)^2 + 2*(3*B
^2*a^2*b*d^3*g^3*x*log(e) + B^2*a^3*d^3*g^3*log(e) + (B^2*b^3*d^3*g^3*log(e
) + A*B*b^3*d^3*g^3)*x^3 + 3*(B^2*a*b^2*d^3*g^3*log(e) + A*B*a*b^2*d^3*g^3)
*x^2)*log((b*x + a)^n) + (2*(2*b^3*c^2*d*g^3*n - 6*a*b^2*c*d^2*g^3*n + 3*(g
^3*n - g^3*log(e))*a^2*b*d^3)*B^2*x - 2*(A*B*b^3*d^3*g^3 + (g^3*n + g^3*log
(e))*B^2*b^3*d^3)*x^3 + (5*b^3*c^3*g^3*n - 9*a*b^2*c^2*d*g^3*n + 3*a^2*b*c*
d^2*g^3*n + (g^3*n - 2*g^3*log(e))*a^3*d^3)*B^2 - 2*(3*A*B*a*b^2*d^3*g^3 +
(2*b^3*c*d^2*g^3*n + 3*a*b^2*d^3*g^3*log(e))*B^2)*x^2 + 6*((b^3*c*d^2*g^3*n
 - a*b^2*d^3*g^3*n)*B^2*x^2 + 2*(b^3*c^2*d*g^3*n - a*b^2*c*d^2*g^3*n)*B^2*x
 + (b^3*c^3*g^3*n - a*b^2*c^2*d*g^3*n)*B^2)*log(d*x + c) - 2*(B^2*b^3*d^3*g
^3*x^3 + 3*B^2*a*b^2*d^3*g^3*x^2 + 3*B^2*a^2*b*d^3*g^3*x + B^2*a^3*d^3*g^3)
*log((b*x + a)^n))*log((d*x + c)^n))/(d^6*i^3*x^3 + 3*c*d^5*i^3*x^2 + 3*c^2
*d^4*i^3*x + c^3*d^3*i^3), x)
```

Giac [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^3}\,dx$$

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x,
 algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x +
 c*i)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(a\,g+b\,g\,x)^3\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(c\,i+d\,i\,x)^3}\,dx$$

```
[In] int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x
)^3,x)
```

```
[Out] int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x
)^3, x)
```

3.203 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 441

$$\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx \\
&= -\frac{B^2g^2n^2(a+bx)^2}{4di^3(c+dx)^2}+\frac{2AbBg^2n(a+bx)}{d^2i^3(c+dx)}-\frac{2bB^2g^2n^2(a+bx)}{d^2i^3(c+dx)} \\
&\quad+\frac{2bB^2g^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^2i^3(c+dx)}+\frac{Bg^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di^3(c+dx)^2} \\
&\quad-\frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2di^3(c+dx)^2}-\frac{bg^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i^3(c+dx)} \\
&\quad-\frac{b^2g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^3} \\
&\quad-\frac{2b^2Bg^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3} \\
&\quad+\frac{2b^2B^2g^2n^2\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3}
\end{aligned}$$

```
[Out] -1/4*B^2*g^2*n^2*(b*x+a)^2/d/i^3/(d*x+c)^2+2*A*b*B*g^2*n*(b*x+a)/d^2/i^3/(d
*x+c)-2*b*B^2*g^2*n^2*(b*x+a)/d^2/i^3/(d*x+c)+2*b*B^2*g^2*n*(b*x+a)*ln(e*((
b*x+a)/(d*x+c))^n)/d^2/i^3/(d*x+c)+1/2*B*g^2*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)
/(d*x+c))^n))/d/i^3/(d*x+c)^2-1/2*g^2*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))
^n))^2/d/i^3/(d*x+c)^2-b*g^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^2/
i^3/(d*x+c)-b^2*g^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b*c)/b/(d*x+
c))/d^3/i^3-2*b^2*B*g^2*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,d*(b*x+
```

```
a)/b/(d*x+c))/d^3/i^3+2*b^2*B^2*g^2*n^2*polylog(3,d*(b*x+a)/b/(d*x+c))/d^3/
i^3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2561, 2395, 2333, 2332, 2342, 2341, 2354, 2421, 6724}

$$\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx \\
&= -\frac{2b^2Bg^2n\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^3i^3} \\
&\quad -\frac{b^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^3i^3}-\frac{bg^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^2i^3(c+dx)} \\
&\quad +\frac{2AbBg^2n(a+bx)}{d^2i^3(c+dx)}-\frac{g^2(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2di^3(c+dx)^2} \\
&\quad +\frac{Bg^2n(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2di^3(c+dx)^2}+\frac{2b^2B^2g^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3} \\
&\quad +\frac{2bB^2g^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^2i^3(c+dx)}-\frac{2bB^2g^2n^2(a+bx)}{d^2i^3(c+dx)}-\frac{B^2g^2n^2(a+bx)^2}{4di^3(c+dx)^2}
\end{aligned}$$

```
[In] Int[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*x
)^3,x]
```

```
[Out] -1/4*(B^2*g^2*n^2*(a + b*x)^2)/(d*i^3*(c + d*x)^2) + (2*A*b*B*g^2*n*(a + b*
x))/(d^2*i^3*(c + d*x)) - (2*b*B^2*g^2*n^2*(a + b*x))/(d^2*i^3*(c + d*x)) +
 (2*b*B^2*g^2*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(d^2*i^3*(c + d*x
)) + (B*g^2*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d*i^3*
(c + d*x)^2) - (g^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(
2*d*i^3*(c + d*x)^2) - (b*g^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2)/(d^2*i^3*(c + d*x)) - (b^2*g^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2*Log[(b*c - a*d)/(b*(c + d*x))])/(d^3*i^3) - (2*b^2*B*g^2*n*(A + B*Log[e
*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(d^3*i^
3) + (2*b^2*B^2*g^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(d^3*i^3)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
```

```
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g^2\text{Subst}\left(\int \frac{x^2(A+B\log(ex^n))^2}{b-dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{i^3}\\
&= \frac{g^2\text{Subst}\left(\int \left(-\frac{b(A+B\log(ex^n))^2}{d^2}-\frac{x(A+B\log(ex^n))^2}{d}-\frac{b^2(A+B\log(ex^n))^2}{d^2(-b+dx)}\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{i^3}\\
&= -\frac{(bg^2)\,\text{Subst}\left(\int (A+B\log(ex^n))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^3}\\
&\quad-\frac{(b^2g^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{-b+dx}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^3}\\
&\quad-\frac{g^2\text{Subst}\left(\int x(A+B\log(ex^n))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{di^3}\\
&= -\frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2di^3(c+dx)^2}-\frac{bg^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i^3(c+dx)}\\
&\quad-\frac{b^2g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^3}\\
&\quad+\frac{(2b^2Bg^2n)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))\log\left(1-\frac{dx}{b}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{d^3i^3}\\
&\quad+\frac{(2bBg^2n)\,\text{Subst}\left(\int (A+B\log(ex^n))\,dx, x, \frac{a+bx}{c+dx}\right)}{d^2i^3}\\
&\quad+\frac{(Bg^2n)\,\text{Subst}\left(\int x(A+B\log(ex^n))\,dx, x, \frac{a+bx}{c+dx}\right)}{di^3}
\end{aligned}$$

$$= -\frac{B^2g^2n^2(a+bx)^2}{4di^3(c+dx)^2} + \frac{2AbBg^2n(a+bx)}{d^2i^3(c+dx)} + \frac{Bg^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di^3(c+dx)^2} - \frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2di^3(c+dx)^2} - \frac{bg^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i^3(c+dx)} - \frac{b^2g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^3} - \frac{2b^2Bg^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3} + \frac{(2bB^2g^2n)\,\text{Subst}\left(\int\log(ex^n)\,dx,x,\frac{a+bx}{c+dx}\right)}{d^2i^3} + \frac{(2b^2B^2g^2n^2)\,\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{dx}{b}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{d^3i^3}$$

$$= -\frac{B^2g^2n^2(a+bx)^2}{4di^3(c+dx)^2} + \frac{2AbBg^2n(a+bx)}{d^2i^3(c+dx)} - \frac{2bB^2g^2n^2(a+bx)}{d^2i^3(c+dx)} + \frac{2bB^2g^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^2i^3(c+dx)} + \frac{Bg^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di^3(c+dx)^2} - \frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2di^3(c+dx)^2} - \frac{bg^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i^3(c+dx)} - \frac{b^2g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^3} - \frac{2b^2Bg^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{Li}_2\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3} + \frac{2b^2B^2g^2n^2\text{Li}_3\left(\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3622 vs. $2(441) = 882$.

Time = 4.78 (sec) , antiderivative size = 3622, normalized size of antiderivative = 8.21

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i +
 d*i*x)^3,x]
```

```
[Out] (g^2*((-6*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a
+ b*x)/(c + d*x)])^2)/(c + d*x)^2 + (24*b*(b*c - a*d)*(A + B*Log[e*((a + b*
x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2)/(c + d*x) + 12*b^2*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2*Log[c +
```

d*x] + (12*a*b*B*d*n*(-A - B*Log[e*((a + b*x)/(c + d*x))^n] + B*n*Log[(a + b*x)/(c + d*x)])*(-(b^2*c^3) + 4*a*b*c^2*d - 3*a^2*c*d^2 - 2*b^2*c^2*d*x + 6*a*b*c*d^2*x - 4*a^2*d^3*x - 2*b*(b*c - 2*a*d)*(c + d*x)^2*Log[a + b*x] + 2*(b*c - a*d)^2*(c + 2*d*x)*Log[(a + b*x)/(c + d*x)] + 2*b^2*c^3*Log[c + d*x] - 4*a*b*c^2*d*Log[c + d*x] + 4*b^2*c^2*d*x*Log[c + d*x] - 8*a*b*c*d^2*x*Log[c + d*x] + 2*b^2*c*d^2*x^2*Log[c + d*x] - 4*a*b*d^3*x^2*Log[c + d*x]))/((b*c - a*d)^2*(c + d*x)^2) + (6*a^2*B*d^2*n*(-A - B*Log[e*((a + b*x)/(c + d*x))^n] + B*n*Log[(a + b*x)/(c + d*x)])*(-(b^2*c^2) + 4*a*b*c*d - a^2*d^2 + 2*b^2*c*d*x + 2*a*b*d^2*x + 2*b^2*d^2*x^2 - 2*b^2*(c + d*x)^2*Log[a/b + x] + 2*(b*c - a*d)^2*Log[(a + b*x)/(c + d*x)] + 2*b^2*c^2*Log[(b*(c + d*x))/(b*c - a*d)] + 4*b^2*c*d*x*Log[(b*(c + d*x))/(b*c - a*d)] + 2*b^2*d^2*x^2*Log[(b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)^2*(c + d*x)^2) + 6*b^2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])*(-2*Log[c/d + x]^2 - (8*c*(1 + Log[c/d + x]))/(c + d*x) + (c^2*(1 + 2*Log[c/d + x]))/(c + d*x)^2 + 8*c*(Log[a/b + x]/(c + d*x) + (b*(Log[a + b*x] - Log[c + d*x]))/(-(b*c) + a*d)) + 2*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])*((c*(3*c + 4*d*x))/(c + d*x)^2 + 2*Log[c + d*x]) + (2*c^2*(-Log[a/b + x] + (b*(c + d*x)*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]))/(b*c - a*d)^2))/(c + d*x)^2 + 4*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])) + (3*a^2*B^2*d^2*n^2*(-(b*c - a*d)^2 - 6*b*(b*c - a*d)*(c + d*x) - 6*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[a + b*x]^2 + 2*(b*c - a*d)^2*Log[(a + b*x)/(c + d*x)] + 4*b*(b*c - a*d)*(c + d*x)*Log[(a + b*x)/(c + d*x)] + 4*b^2*(c + d*x)^2*Log[a + b*x]*Log[(a + b*x)/(c + d*x)] - 2*(b*c - a*d)^2*Log[(a + b*x)/(c + d*x)]^2 + 6*b^2*(c + d*x)^2*Log[c + d*x] + 4*b^2*(c + d*x)^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 4*b^2*(c + d*x)^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[(b*c - a*d)/(b*c + b*d*x)] + 4*b^2*(c + d*x)^2*Log[(a + b*x)/(c + d*x)]*Log[(b*c - a*d)/(b*c + b*d*x)] - 2*b^2*(c + d*x)^2*Log[(b*c - a*d)/(b*c + b*d*x)]^2 + 4*b^2*(c + d*x)^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 4*b^2*(c + d*x)^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)^2*(c + d*x)^2) + (6*a*b*B^2*d*n^2*(-4*(b*c - a*d)^2*(c + d*x)*(2 + 2*Log[c/d + x] + Log[c/d + x]^2) + c*(b*c - a*d)^2*(1 + 2*Log[c/d + x] + 2*Log[c/d + x]^2) - 2*(b*c - a*d)^2*(c + 2*d*x)*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])^2 - 2*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)])*(4*(b*c - a*d)^2*(c + d*x)*(1 + Log[c/d + x]) - c*(b*c - a*d)^2*(1 + 2*Log[c/d + x]) - 4*(b*c - a*d)*(c + d*x)*((b*c - a*d)*Log[a/b + x] - b*(c + d*x)*(Log[a + b*x] - Log[c + d*x])) + 2*c*((b*c - a*d)^2*Log[a/b + x] - b*(c + d*x)*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]))) + 4*(b*c - a*d)*(c + d*x)*(Log[a/b + x]*(d*(a + b*x)*Log[a/b + x] - 2*b*(c + d*x)*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*b*(c + d*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*c*((b*c - a*d)^2*Log[a/b + x]^2 - b*(c + d*x)*(b*(c + d*x)*Log[a/b + x]^2 - 2*b*(c + d*x)*(Log[a + b*x] - Log[c + d*x]) - 2*Log[a/b + x]*(-(b*c) + a*d + b*(c + d*x)*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*b*(c + d*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])) + 4*(b*c - a*d)*(c + d*x)*(2*(b*c - a*d)*Log[a/b + x]*(1 + Log[c/d + x])

```
+ b*(c + d*x)*(Log[c/d + x]^2 - 2*Log[a + b*x] - 2*Log[c/d + x]*Log[(d*(a +
 b*x))/(-(b*c) + a*d)] + 2*Log[c + d*x]) - 2*b*(c + d*x)*PolyLog[2, (b*(c +
 d*x))/(b*c - a*d)]) + 2*c*(b*(b*c - a*d)*(c + d*x) - (b*c - a*d)^2*Log[a/b
 + x]*(1 + 2*Log[c/d + x]) + b^2*(c + d*x)^2*Log[a + b*x] - b^2*(c + d*x)^2
*Log[c + d*x] - b*(c + d*x)*(b*(c + d*x)*Log[c/d + x]^2 - 2*(b*c - a*d)*(1
+ Log[c/d + x]) - 2*b*(c + d*x)*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a
*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))))/((b*c - a*d)^2*(c + d*x)^
2) + b^2*B^2*n^2*(4*Log[c/d + x]^3 + (24*c*(2 + 2*Log[c/d + x] + Log[c/d +
x]^2))/(c + d*x) - (3*c^2*(1 + 2*Log[c/d + x] + 2*Log[c/d + x]^2))/(c + d*x
)^2 + (6*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])^2*(c*(3*
c + 4*d*x) + 2*(c + d*x)^2*Log[c + d*x]))/(c + d*x)^2 + (24*c*(-(d*(a + b*x
)*Log[a/b + x]^2) + 2*b*(c + d*x)*Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d
)] + 2*b*(c + d*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/((b*c - a*d)*
(c + d*x)) + 6*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])*((
-2*c^2*Log[a/b + x])/(c + d*x)^2 + (8*c*Log[a/b + x])/(c + d*x) - 2*Log[c/d
 + x]^2 - (8*c*(1 + Log[c/d + x]))/(c + d*x) + (c^2*(1 + 2*Log[c/d + x]))/(
c + d*x)^2 + (8*b*c*(-Log[a + b*x] + Log[c + d*x]))/(b*c - a*d) + (2*b*c^2*
(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]))/((b*c -
a*d)^2*(c + d*x)) + 4*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLo
g[2, (d*(a + b*x))/(-(b*c) + a*d)])) + (6*c^2*(-Log[a/b + x]^2 + (b*(c + d*
x)*(b*(c + d*x)*Log[a/b + x]^2 - 2*b*(c + d*x)*(Log[a + b*x] - Log[c + d*x]
) - 2*Log[a/b + x]*(-(b*c) + a*d + b*(c + d*x)*Log[(b*(c + d*x))/(b*c - a*d
)]) - 2*b*(c + d*x)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/(b*c - a*d)^
2))/(c + d*x)^2 + 12*(Log[a/b + x]^2*Log[(b*(c + d*x))/(b*c - a*d)] + 2*Log
[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 2*PolyLog[3, (d*(a + b
*x))/(-(b*c) + a*d)]) + 6*(2*Log[c/d + x]^2*(-Log[a/b + x] + Log[(d*(a + b*
x))/(-(b*c) + a*d)]) + 4*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]
 + (4*c*(2*(b*c - a*d)*Log[a/b + x]*(1 + Log[c/d + x]) + b*(c + d*x)*(Log[c
/d + x]^2 - 2*Log[a + b*x] - 2*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d
)] + 2*Log[c + d*x]) - 2*b*(c + d*x)*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
)/((-(b*c) + a*d)*(c + d*x)) + (c^2*(-(b*(b*c - a*d)*(c + d*x)) + (b*c - a*
d)^2*Log[a/b + x]*(1 + 2*Log[c/d + x]) - b^2*(c + d*x)^2*Log[a + b*x] + b^2
*(c + d*x)^2*Log[c + d*x] + b*(c + d*x)*(b*(c + d*x)*Log[c/d + x]^2 - 2*(b*
c - a*d)*(1 + Log[c/d + x]) - 2*b*(c + d*x)*(Log[c/d + x]*Log[(d*(a + b*x))
/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))))/((b*c - a*d)^2
*(c + d*x)^2) - 4*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]))))/(12*d^3*i^3)
```

Maple [F]

$$\int \frac{(bgx+ag)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(dix+ci)^3}dx$$

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x)
[Out] int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x)
```

Fricas [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^3}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x,
 algorithm="fricas")
[Out] integral((A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^
2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*
B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^
n))/(d^3*i^3*x^3 + 3*c*d^2*i^3*x^2 + 3*c^2*d*i^3*x + c^3*i^3), x)
```

Sympy [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{g^2\left(\int \frac{A^2a^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{A^2b^2x^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{B^2a^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{2ABa^2\log\left(e\left(\frac{a}{c+dx}\right.\right.}{c^3+3c^2dx+3cd^2}\right.}{}$$

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**3
,x)
[Out] g**2*(Integral(A**2*a**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x
) + Integral(A**2*b**2*x**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3)
, x) + Integral(B**2*a**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c**3
+ 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*a**2*log(e*(
a/(c + d*x) + b*x/(c + d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*
x**3), x) + Integral(2*A**2*a*b*x/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3
*x**3), x) + Integral(B**2*b**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n
)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*b
**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d
```

```
**2*x**2 + d**3*x**3), x) + Integral(2*B**2*a*b*x*log(e*(a/(c + d*x) + b*x/
(c + d*x))**n)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Int
egral(4*A*B*a*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**3 + 3*c**2*d*
x + 3*c*d**2*x**2 + d**3*x**3), x))/i**3
```

Maxima [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^3}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x,
 algorithm="maxima")
```

```
[Out] A*B*a*b*g^2*n*((b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*x)/((b*c*d^4 - a*d^5)
*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^3) + 2
*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*i^3)
 - 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*
i^3)) + 1/2*A*B*a^2*g^2*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x
^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b^2*log
(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((
b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) + 1/2*A^2*b^2*g^2*((4*c*d*x + 3*c^
2)/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) + 2*log(d*x + c)/(d^3*i^3))
- 2*(2*d*x + c)*A*B*a*b*g^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^4*i^3
*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - (2*d*x + c)*A^2*a*b*g^2/(d^4*i^3*x^2
+ 2*c*d^3*i^3*x + c^2*d^2*i^3) - A*B*a^2*g^2*log(e*(b*x/(d*x + c) + a/(d*x
+ c))^n)/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - 1/2*A^2*a^2*g^2/(d^3*i
^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) + 1/2*(4*(b^2*c*d*g^2 - a*b*d^2*g^2)*B^
2*x + (3*b^2*c^2*g^2 - 2*a*b*c*d*g^2 - a^2*d^2*g^2)*B^2 + 2*(B^2*b^2*d^2*g^
2*x^2 + 2*B^2*b^2*c*d*g^2*x + B^2*b^2*c^2*g^2)*log(d*x + c))*log((d*x + c)^
n)^2/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) - integrate(-(2*B^2*a*b*d^
2*g^2*x*log(e)^2 + B^2*a^2*d^2*g^2*log(e)^2 + (B^2*b^2*d^2*g^2*log(e)^2 + 2
*A*B*b^2*d^2*g^2*log(e))*x^2 + (B^2*b^2*d^2*g^2*x^2 + 2*B^2*a*b*d^2*g^2*x +
 B^2*a^2*d^2*g^2)*log((b*x + a)^n)^2 + 2*(2*B^2*a*b*d^2*g^2*x*log(e) + B^2*
a^2*d^2*g^2*log(e) + (B^2*b^2*d^2*g^2*log(e) + A*B*b^2*d^2*g^2)*x^2)*log((b
*x + a)^n) - (4*(b^2*c*d*g^2*n - (g^2*n - g^2*log(e))*a*b*d^2)*B^2*x + (3*b
^2*c^2*g^2*n - 2*a*b*c*d*g^2*n - (g^2*n - 2*g^2*log(e))*a^2*d^2)*B^2 + 2*(B
^2*b^2*d^2*g^2*log(e) + A*B*b^2*d^2*g^2)*x^2 + 2*(B^2*b^2*d^2*g^2*n*x^2 + 2
*B^2*b^2*c*d*g^2*n*x + B^2*b^2*c^2*g^2*n)*log(d*x + c) + 2*(B^2*b^2*d^2*g^2
*x^2 + 2*B^2*a*b*d^2*g^2*x + B^2*a^2*d^2*g^2)*log((b*x + a)^n))*log((d*x +
c)^n))/(d^5*i^3*x^3 + 3*c*d^4*i^3*x^2 + 3*c^2*d^3*i^3*x + c^3*d^2*i^3), x)
```

Giac [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^3}\,dx$$

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x,
 algorithm="giac")
[Out] integrate((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x +
 c*i)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(c\,i+d\,i\,x)^3}\,dx$$

```
[In] int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x
)^3,x)
[Out] int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x
)^3, x)
```

3.204 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 151

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \frac{B^2gn^2(a+bx)^2}{4(bc-ad)i^3(c+dx)^2} - \frac{Bgn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)i^3(c+dx)^2} + \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)i^3(c+dx)^2}$$

```
[Out] 1/4*B^2*g*n^2*(b*x+a)^2/(-a*d+b*c)/i^3/(d*x+c)^2-1/2*B*g*n*(b*x+a)^2*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/i^3/(d*x+c)^2+1/2*g*(b*x+a)^2*(A+B*ln(
e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/i^3/(d*x+c)^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.070, Rules used = {2561, 2342, 2341}

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \frac{g(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2i^3(c+dx)^2(bc-ad)} - \frac{Bgn(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2i^3(c+dx)^2(bc-ad)} + \frac{B^2gn^2(a+bx)^2}{4i^3(c+dx)^2(bc-ad)}$$

```
[In] Int[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*x)^
3,x]
```

```
[Out] (B^2*g*n^2*(a + b*x)^2)/(4*(b*c - a*d)*i^3*(c + d*x)^2) - (B*g*n*(a + b*x)^
2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*c - a*d)*i^3*(c + d*x)^2) +
 (g*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(b*c - a*d)*i^
3*(c + d*x)^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{g\text{Subst}\left(\int x(A+B\log(ex^n))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^3}\\
&= \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)i^3(c+dx)^2} - \frac{(Bgn)\text{Subst}\left(\int x(A+B\log(ex^n))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^3}\\
&= \frac{B^2gn^2(a+bx)^2}{4(bc-ad)i^3(c+dx)^2} - \frac{Bgn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)i^3(c+dx)^2}\\
&\quad + \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)i^3(c+dx)^2}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.56 (sec) , antiderivative size = 803, normalized size of antiderivative = 5.32

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{g\Big(2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-4b(bc-ad)(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+4bBn(c+dx)\Big(2}{}$$

```
[In] Integrate[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d
*i*x)^3,x]
```

```
[Out] (g*(2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - 4*b*(b*c - a
*d)*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 4*b*B*n*(c + d*x)*
(2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*b*(c + d*x)*Log[a
 + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b*(c + d*x)*(A + B*Log[e
*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 2*B*n*(b*c - a*d + b*(c + d*x)*Lo
g[a + b*x] - b*(c + d*x)*Log[c + d*x]) - b*B*n*(c + d*x)*(Log[a + b*x]*(Log
[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/
(-(b*c) + a*d)]) + b*B*n*(c + d*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] -
Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - B*
n*(2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*b*(b*c - a*d)
*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*b^2*(c + d*x)^2*Log[a
 + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 4*b^2*(c + d*x)^2*(A + B*L
og[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 4*b*B*n*(c + d*x)*(b*c - a*d
+ b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]) - B*n*((b*c - a*d)^2
 + 2*b*(b*c - a*d)*(c + d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c +
d*x)^2*Log[c + d*x]) - 2*b^2*B*n*(c + d*x)^2*(Log[a + b*x]*(Log[a + b*x] -
2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*
d)]) + 2*b^2*B*n*(c + d*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c
+ d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))))/(4*d^2*(
b*c - a*d)*i^3*(c + d*x)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(145) = 290.

Time = 4.68 (sec) , antiderivative size = 530, normalized size of antiderivative = 3.51

method	result
parallelrisch	$-\frac{8ABx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)ab^2d^4gn+4ABxb^3cd^3gn^2+4AB\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^2bd^4gn+4ABx^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3d^4gn+4B^2x\ln\left(e\right.}{}$

```
[In] int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x,method=_R
ETURNVERBOSE)
```

```
[Out] -1/4*(8*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*d^4*g*n+4*A*B*x*b^3*c*d^3*g*n
^2+4*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b*d^4*g*n+4*A*B*x^2*ln(e*((b*x+a)/(d
*x+c))^n)*b^3*d^4*g*n+4*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^2*d^4*g*n-4*B
^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*d^4*g*n^2-4*A*B*x*a*b^2*d^4*g*n^2+B^2*
a^2*b*d^4*g*n^3-B^2*b^3*c^2*d^2*g*n^3+2*A^2*a^2*b*d^4*g*n-2*A^2*b^3*c^2*d^2
*g*n-2*A*B*a^2*b*d^4*g*n^2+2*A*B*b^3*c^2*d^2*g*n^2+2*B^2*x^2*ln(e*((b*x+a)/
(d*x+c))^n)^2*b^3*d^4*g*n-2*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^4*g*n^2
+2*B^2*x*a*b^2*d^4*g*n^3-2*B^2*x*b^3*c*d^3*g*n^3+2*B^2*ln(e*((b*x+a)/(d*x+c
))^n)^2*a^2*b*d^4*g*n-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b*d^4*g*n^2+4*A^2
*x*a*b^2*d^4*g*n-4*A^2*x*b^3*c*d^3*g*n)/i^3/(d*x+c)^2/b/d^4/n/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(145) = 290$.

Time = 0.35 (sec) , antiderivative size = 600, normalized size of antiderivative = 3.97

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = -\frac{(B^2b^2c^2-B^2a^2d^2)gn^2-2\left(ABb^2c^2-ABa^2d^2\right)gn+2\left(2\left(B^2b^2cd-B^2abd^2\right)gx+\left(B^2b^2c^2-B^2a^2d^2\right)g\right)}{}$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x, a
lgorithm="fricas")
```

```
[Out] -1/4*((B^2*b^2*c^2 - B^2*a^2*d^2)*g*n^2 - 2*(A*B*b^2*c^2 - A*B*a^2*d^2)*g*n
 + 2*(2*(B^2*b^2*c*d - B^2*a*b*d^2)*g*x + (B^2*b^2*c^2 - B^2*a^2*d^2)*g)*lo
g(e)^2 - 2*(B^2*b^2*d^2*g*n^2*x^2 + 2*B^2*a*b*d^2*g*n^2*x + B^2*a^2*d^2*g*n
^2)*log((b*x + a)/(d*x + c))^2 + 2*(A^2*b^2*c^2 - A^2*a^2*d^2)*g + 2*((B^2*
b^2*c*d - B^2*a*b*d^2)*g*n^2 - 2*(A*B*b^2*c*d - A*B*a*b*d^2)*g*n + 2*(A^2*b
^2*c*d - A^2*a*b*d^2)*g)*x - 2*((B^2*b^2*c^2 - B^2*a^2*d^2)*g*n - 2*(A*B*b^
2*c^2 - A*B*a^2*d^2)*g + 2*((B^2*b^2*c*d - B^2*a*b*d^2)*g*n - 2*(A*B*b^2*c*
d - A*B*a*b*d^2)*g)*x + 2*(B^2*b^2*d^2*g*n*x^2 + 2*B^2*a*b*d^2*g*n*x + B^2*
a^2*d^2*g*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*(B^2*a^2*d^2*g*n^2 - 2*A*
B*a^2*d^2*g*n + (B^2*b^2*d^2*g*n^2 - 2*A*B*b^2*d^2*g*n)*x^2 + 2*(B^2*a*b*d^
2*g*n^2 - 2*A*B*a*b*d^2*g*n)*x)*log((b*x + a)/(d*x + c)))/((b*c*d^4 - a*d^5
)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^3)
```

Sympy [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{g\left(\int \frac{A^2a}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{A^2bx}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{B^2a\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{2ABa\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+}\right.\right.}{c^3+3c^2dx+3cd^2x^2+c}\right.}{i^3}$$

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**3,x)
```

```
[Out] g*(Integral(A**2*a/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + In
tegral(A**2*b*x/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integ
ral(B**2*a*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c**3 + 3*c**2*d*x +
3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*a*log(e*(a/(c + d*x) + b*x/
(c + d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(B**2*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c**3 + 3*c**2*d*x +
 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*b*x*log(e*(a/(c + d*x) + b
*x/(c + d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/i**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1995 vs. $2(145) = 290$.

Time = 0.31 (sec) , antiderivative size = 1995, normalized size of antiderivative = 13.21

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x, a
lgorithm="maxima")
```

```
[Out] 1/2*A*B*b*g*n*((b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*x)/((b*c*d^4 - a*d^5)
*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^3) + 2
*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*i^3)
 - 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*
i^3)) + 1/2*A*B*a*g*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2 +
 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b^2*log(b*x
 + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*
c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) - 1/2*(2*d*x + c)*B^2*b*g*log(e*(b*x/(
d*x + c) + a/(d*x + c))^n)^2/(d^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) +
1/4*(2*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2
 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b^2*log(b*x + a)/((b^2*c
^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b
```

```
*c*d^2 + a^2*d^3)*i^3))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - (7*b^2*c^2
 - 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x +
a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x + c)^2 + 6*(b^2*c*d
- a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a) - 2*(3*
b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*
c^2)*log(b*x + a))*log(d*x + c))*n^2/(b^2*c^4*d*i^3 - 2*a*b*c^3*d^2*i^3 + a
^2*c^2*d^3*i^3 + (b^2*c^2*d^3*i^3 - 2*a*b*c*d^4*i^3 + a^2*d^5*i^3)*x^2 + 2*
(b^2*c^3*d^2*i^3 - 2*a*b*c^2*d^3*i^3 + a^2*c*d^4*i^3)*x))*B^2*a*g + 1/4*(2*
n*((b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*x)/((b*c*d^4 - a*d^5)*i^3*x^2 + 2
*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^3) + 2*(b^2*c - 2*
a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*i^3) - 2*(b^2*c
- 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*i^3))*log(e*
(b*x/(d*x + c) + a/(d*x + c))^n) - (b^2*c^3 - 8*a*b*c^2*d + 7*a^2*c*d^2 + 2
*(b^2*c^3 - 2*a*b*c^2*d + (b^2*c*d^2 - 2*a*b*d^3)*x^2 + 2*(b^2*c^2*d - 2*a*
b*c*d^2)*x)*log(b*x + a)^2 + 2*(b^2*c^3 - 2*a*b*c^2*d + (b^2*c*d^2 - 2*a*b*
d^3)*x^2 + 2*(b^2*c^2*d - 2*a*b*c*d^2)*x)*log(d*x + c)^2 + 2*(b^2*c^2*d - 5
*a*b*c*d^2 + 4*a^2*d^3)*x + 2*(b^2*c^3 - 4*a*b*c^2*d + (b^2*c*d^2 - 4*a*b*d
^3)*x^2 + 2*(b^2*c^2*d - 4*a*b*c*d^2)*x)*log(b*x + a) - 2*(b^2*c^3 - 4*a*b*
c^2*d + (b^2*c*d^2 - 4*a*b*d^3)*x^2 + 2*(b^2*c^2*d - 4*a*b*c*d^2)*x + 2*(b^
2*c^3 - 2*a*b*c^2*d + (b^2*c*d^2 - 2*a*b*d^3)*x^2 + 2*(b^2*c^2*d - 2*a*b*c*
d^2)*x)*log(b*x + a))*log(d*x + c))*n^2/(b^2*c^4*d^2*i^3 - 2*a*b*c^3*d^3*i^
3 + a^2*c^2*d^4*i^3 + (b^2*c^2*d^4*i^3 - 2*a*b*c*d^5*i^3 + a^2*d^6*i^3)*x^2
 + 2*(b^2*c^3*d^3*i^3 - 2*a*b*c^2*d^4*i^3 + a^2*c*d^5*i^3)*x))*B^2*b*g - (2
*d*x + c)*A*B*b*g*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^4*i^3*x^2 + 2*c
*d^3*i^3*x + c^2*d^2*i^3) - 1/2*B^2*a*g*log(e*(b*x/(d*x + c) + a/(d*x + c))
^n)^2/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - 1/2*(2*d*x + c)*A^2*b*g/(
d^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - A*B*a*g*log(e*(b*x/(d*x + c) +
 a/(d*x + c))^n)/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - 1/2*A^2*a*g/(d
^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3)
```

Giac [A] (verification not implemented)

none

Time = 1.91 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.29

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(\frac{2\,(bx+a)^2B^2gn^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(dx+c)^2i^3}-\frac{2\,(B^2gn^2-2\,B^2gn\log(e)-2\,ABgn)(bx+a)^2\log\left(\frac{bx+a}{dx+c}\right)}{(dx+c)^2i^3}+\frac{(B^2gn^2\right.$$

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x, a
lgorithm="giac")
```

```
[Out] 1/4*(2*(b*x + a)^2*B^2*g*n^2*log((b*x + a)/(d*x + c))^2/((d*x + c)^2*i^3) -
 2*(B^2*g*n^2 - 2*B^2*g*n*log(e) - 2*A*B*g*n)*(b*x + a)^2*log((b*x + a)/(d*
x + c))/((d*x + c)^2*i^3) + (B^2*g*n^2 - 2*B^2*g*n*log(e) + 2*B^2*g*log(e)^
2 - 2*A*B*g*n + 4*A*B*g*log(e) + 2*A^2*g)*(b*x + a)^2/((d*x + c)^2*i^3))*(b
*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 565, normalized size of antiderivative = 3.74

$$
\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx
$$

$$
\begin{aligned}
&= -\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{\frac{B^2\,a\,g}{2\,d}+\frac{B^2\,b\,g\,x}{d}+\frac{B^2\,b\,c\,g}{2\,d^2}}{c^2\,i^3+2\,c\,d\,i^3\,x+d^2\,i^3\,x^2}+\frac{B^2\,b^2\,g}{2\,d^2\,i^3\,\left(a\,d-b\,c\right)}\right) \\
&\quad - \frac{x\left(2\,b\,d\,g\,A^2-2\,b\,d\,g\,A\,B\,n+b\,d\,g\,B^2\,n^2\right)+A^2\,a\,d\,g+A^2\,b\,c\,g+\frac{B^2\,a\,d\,g\,n^2}{2}+\frac{B^2\,b\,c\,g\,n^2}{2}-A\,B\,a\,d\,g\,n-A\,B}{2\,c^2\,d^2\,i^3+4\,c\,d^3\,i^3\,x+2\,d^4\,i^3\,x^2} \\
&\quad - \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{A\,B\,a\,d\,g+A\,B\,b\,c\,g-B^2\,a\,d\,g\,n+B^2\,b\,c\,g\,n+2\,A\,B\,b\,d\,g\,x}{c^2\,d^2\,i^3+2\,c\,d^3\,i^3\,x+d^4\,i^3\,x^2}\right. \\
&\qquad \left. -\frac{B^2\,b^2\,g\left(\frac{c\,d^2\,i^3\,n\,\left(a\,d-b\,c\right)}{2\,b}+\frac{d^3\,i^3\,n\,x\,\left(a\,d-b\,c\right)}{b}-\frac{d^2\,i^3\,n\,\left(a\,d-b\,c\right)\,\left(a\,d-2\,b\,c\right)}{2\,b^2}\right)}{d^2\,i^3\,\left(a\,d-b\,c\right)\,\left(c^2\,d^2\,i^3+2\,c\,d^3\,i^3\,x+d^4\,i^3\,x^2\right)}\right) \\
&\quad - \frac{B\,b^2\,g\,n\,\mathrm{atan}\left(\frac{B\,b^2\,g\,n\,\left(2\,A-B\,n\right)\,\left(\frac{a\,d^3\,i^3+b\,c\,d^2\,i^3}{d^2\,i^3}+2\,b\,d\,x\right)\,1{}\mathrm{i}}{\left(a\,d-b\,c\right)\,\left(B^2\,b^2\,g\,n^2-2\,A\,B\,b^2\,g\,n\right)}\right)\,\left(2\,A-B\,n\right)\,1{}\mathrm{i}}{d^2\,i^3\,\left(a\,d-b\,c\right)}
\end{aligned}
$$

```
[In] int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x)^
3,x)
```

```
[Out] - log(e*((a + b*x)/(c + d*x))^n)^2*(((B^2*a*g)/(2*d) + (B^2*b*g*x)/d + (B^2
*b*c*g)/(2*d^2))/(c^2*i^3 + d^2*i^3*x^2 + 2*c*d*i^3*x) + (B^2*b^2*g)/(2*d^2
*i^3*(a*d - b*c))) - (x*(2*A^2*b*d*g + B^2*b*d*g*n^2 - 2*A*B*b*d*g*n) + A^2
*a*d*g + A^2*b*c*g + (B^2*a*d*g*n^2)/2 + (B^2*b*c*g*n^2)/2 - A*B*a*d*g*n -
A*B*b*c*g*n)/(2*c^2*d^2*i^3 + 2*d^4*i^3*x^2 + 4*c*d^3*i^3*x) - log(e*((a +
b*x)/(c + d*x))^n)*((A*B*a*d*g + A*B*b*c*g - B^2*a*d*g*n + B^2*b*c*g*n + 2*
A*B*b*d*g*x)/(c^2*d^2*i^3 + d^4*i^3*x^2 + 2*c*d^3*i^3*x) - (B^2*b^2*g*((c*d
^2*i^3*n*(a*d - b*c))/(2*b) + (d^3*i^3*n*x*(a*d - b*c))/b - (d^2*i^3*n*(a*d
 - b*c)*(a*d - 2*b*c))/(2*b^2)))/(d^2*i^3*(a*d - b*c)*(c^2*d^2*i^3 + d^4*i^
3*x^2 + 2*c*d^3*i^3*x))) - (B*b^2*g*n*atan((B*b^2*g*n*(2*A - B*n)*((a*d^3*i
^3 + b*c*d^2*i^3)/(d^2*i^3) + 2*b*d*x)*1i)/((a*d - b*c)*(B^2*b^2*g*n^2 - 2*
A*B*b^2*g*n)))*(2*A - B*n)*1i)/(d^2*i^3*(a*d - b*c))
```

3.205 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 317

$$\begin{aligned}
\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx &= -\frac{B^2dn^2(a+bx)^2}{4(bc-ad)^2i^3(c+dx)^2} - \frac{2AbBn(a+bx)}{(bc-ad)^2i^3(c+dx)} \\
&\quad + \frac{2bB^2n^2(a+bx)}{(bc-ad)^2i^3(c+dx)} - \frac{2bB^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^2i^3(c+dx)} \\
&\quad + \frac{Bdn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2i^3(c+dx)^2} \\
&\quad - \frac{d(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^2i^3(c+dx)^2} \\
&\quad + \frac{b(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2i^3(c+dx)}
\end{aligned}$$

```
[Out] -1/4*B^2*d*n^2*(b*x+a)^2/(-a*d+b*c)^2/i^3/(d*x+c)^2-2*A*b*B*n*(b*x+a)/(-a*d
+b*c)^2/i^3/(d*x+c)+2*b*B^2*n^2*(b*x+a)/(-a*d+b*c)^2/i^3/(d*x+c)-2*b*B^2*n*
(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^2/i^3/(d*x+c)+1/2*B*d*n*(b*x+a
)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^2/i^3/(d*x+c)^2-1/2*d*(b*x+a
)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/i^3/(d*x+c)^2+b*(b*x+a)*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/i^3/(d*x+c)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2551, 2367, 2333, 2332, 2342, 2341}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \frac{Bdn(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2i^3(c+dx)^2(bc-ad)^2} + \frac{b(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{i^3(c+dx)(bc-ad)^2} - \frac{d(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2i^3(c+dx)^2(bc-ad)^2} - \frac{2AbBn(a+bx)}{i^3(c+dx)(bc-ad)^2} - \frac{2bB^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{i^3(c+dx)(bc-ad)^2} + \frac{2bB^2n^2(a+bx)}{i^3(c+dx)(bc-ad)^2} - \frac{B^2dn^2(a+bx)^2}{4i^3(c+dx)^2(bc-ad)^2}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*i + d*i*x)^3,x]
```

```
[Out] -1/4*(B^2*d*n^2*(a + b*x)^2)/((b*c - a*d)^2*i^3*(c + d*x)^2) - (2*A*b*B*n*(
a + b*x))/((b*c - a*d)^2*i^3*(c + d*x)) + (2*b*B^2*n^2*(a + b*x))/((b*c - a
*d)^2*i^3*(c + d*x)) - (2*b*B^2*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])
/((b*c - a*d)^2*i^3*(c + d*x)) + (B*d*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/(2*(b*c - a*d)^2*i^3*(c + d*x)^2) - (d*(a + b*x)^2*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(b*c - a*d)^2*i^3*(c + d*x)^2) + (b*(
a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*c - a*d)^2*i^3*(c +
d*x))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (b-dx)\left(A+B\log\left(ex^n\right)\right)^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 i^3}\\
&= \frac{\text{Subst}\left(\int \left(b(A+B\log\left(ex^n\right))^2-dx(A+B\log\left(ex^n\right))^2\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 i^3}\\
&= \frac{b\text{Subst}\left(\int (A+B\log\left(ex^n\right))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 i^3}-\frac{d\text{Subst}\left(\int x(A+B\log\left(ex^n\right))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 i^3}\\
&= -\frac{d(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^2 i^3(c+dx)^2}+\frac{b(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2 i^3(c+dx)}\\
&\quad -\frac{(2bBn)\text{Subst}\left(\int (A+B\log\left(ex^n\right))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 i^3}\\
&\quad +\frac{(Bdn)\text{Subst}\left(\int x(A+B\log\left(ex^n\right))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2 i^3}
\end{aligned}$$

$$= -\frac{B^2dn^2(a+bx)^2}{4(bc-ad)^2i^3(c+dx)^2} - \frac{2AbBn(a+bx)}{(bc-ad)^2i^3(c+dx)} + \frac{Bdn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2i^3(c+dx)^2} - \frac{d(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^2i^3(c+dx)^2} + \frac{b(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2i^3(c+dx)} - \frac{(2bB^2n)\,\text{Subst}\left(\int\log\left(ex^n\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^2i^3}$$

$$= -\frac{B^2dn^2(a+bx)^2}{4(bc-ad)^2i^3(c+dx)^2} - \frac{2AbBn(a+bx)}{(bc-ad)^2i^3(c+dx)} + \frac{2bB^2n^2(a+bx)}{(bc-ad)^2i^3(c+dx)} - \frac{2bB^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^2i^3(c+dx)} + \frac{Bdn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2i^3(c+dx)^2} - \frac{d(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^2i^3(c+dx)^2} + \frac{b(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2i^3(c+dx)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.22 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.46

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$

$$= \frac{-2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + \frac{Bn\left(2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4b(bc-ad)(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4b^2(c+dx)^2\log(a}{}}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*i + d*i*x)^3,x]
```

```
[Out] (-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)^2*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]) + 4*b*(b*c - a*d)*(c + d*x)*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]) + 4*b^2*(c + d*x)^2*Log[a + b*x]*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n]) - 4*b^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])*Log[c + d*x] - 4*b*B*n*(c + d*x)*(b*c - a*d + b*(c + d*x)*Log[a + b*x
] - b*(c + d*x)*Log[c + d*x]) - B*n*((b*c - a*d)^2 + 2*b*(b*c - a*d)*(c + d
*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c + d*x]) - 2*
b^2*B*n*(c + d*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c
- a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*b^2*B*n*(c + d*x
)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*
PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(4*d*i^3*(c + d*x)^
2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(311) = 622$.

Time = 4.86 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.12

method	result
parallelrisch	$-\frac{-8B^2ab^2cd^4n^3-2AB\,a^2bd^5n^2-6AB\,b^3c^2d^3n^2-4A^2ab^2cd^4n+2A^2b^3c^2d^3n+2A^2a^2bd^5n+7B^2b^3c^2d^3n^3+B^2a^2bd^5n^3-4}{}$

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/4*(-8*B^2*a*b^2*c*d^4*n^3-2*A*B*a^2*b*d^5*n^2-6*A*B*b^3*c^2*d^3*n^2-4*A^
2*a*b^2*c*d^4*n+2*A^2*b^3*c^2*d^3*n+2*A^2*a^2*b*d^5*n+7*B^2*b^3*c^2*d^3*n^3
+B^2*a^2*b*d^5*n^3-4*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^5*n-4*B^2*x*ln
(e*((b*x+a)/(d*x+c))^n)^2*b^3*c*d^4*n+4*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b
^2*d^5*n^2+8*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d^4*n^2+4*A*B*x*a*b^2*d^
5*n^2-4*A*B*x*b^3*c*d^4*n^2-4*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^2*c*d^4*n
+8*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*c*d^4*n^2+4*A*B*ln(e*((b*x+a)/(d*x+c
))^n)*a^2*b*d^5*n+8*A*B*a*b^2*c*d^4*n^2-8*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b
^3*c*d^4*n-8*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*c*d^4*n-2*B^2*x^2*ln(e*((b
*x+a)/(d*x+c))^n)^2*b^3*d^5*n+6*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^5*n
^2-6*B^2*x*a*b^2*d^5*n^3+6*B^2*x*b^3*c*d^4*n^3+2*B^2*ln(e*((b*x+a)/(d*x+c))
^n)^2*a^2*b*d^5*n-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b*d^5*n^2)/i^3/(d*x+c
)^2/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/d^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(311) = 622$.

Time = 0.33 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.06

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = -\frac{2\,A^2b^2c^2-4\,A^2abcd+2\,A^2a^2d^2+(7\,B^2b^2c^2-8\,B^2abcd+B^2a^2d^2)n^2+2\,(B^2b^2c^2-2\,B^2abcd+B^2a^2d}{}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x, algorithm="fr
icas")
```

```
[Out] -1/4*(2*A^2*b^2*c^2 - 4*A^2*a*b*c*d + 2*A^2*a^2*d^2 + (7*B^2*b^2*c^2 - 8*B^
2*a*b*c*d + B^2*a^2*d^2)*n^2 + 2*(B^2*b^2*c^2 - 2*B^2*a*b*c*d + B^2*a^2*d^2
)*log(e)^2 - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^2*b^2*c*d*n^2*x + (2*B^2*a*b*c*d
- B^2*a^2*d^2)*n^2)*log((b*x + a)/(d*x + c))^2 - 2*(3*A*B*b^2*c^2 - 4*A*B*a
*b*c*d + A*B*a^2*d^2)*n + 2*(3*(B^2*b^2*c*d - B^2*a*b*d^2)*n^2 - 2*(A*B*b^2
```

```
*c*d - A*B*a*b*d^2)*n)*x + 2*(2*A*B*b^2*c^2 - 4*A*B*a*b*c*d + 2*A*B*a^2*d^2
 - 2*(B^2*b^2*c*d - B^2*a*b*d^2)*n*x - (3*B^2*b^2*c^2 - 4*B^2*a*b*c*d + B^2
*a^2*d^2)*n - 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*b^2*c*d*n*x + (2*B^2*a*b*c*d - B
^2*a^2*d^2)*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*((4*B^2*a*b*c*d - B^2*a
^2*d^2)*n^2 + (3*B^2*b^2*d^2*n^2 - 2*A*B*b^2*d^2*n)*x^2 - 2*(2*A*B*a*b*c*d
- A*B*a^2*d^2)*n - 2*(2*A*B*b^2*c*d*n - (2*B^2*b^2*c*d + B^2*a*b*d^2)*n^2)*
x)*log((b*x + a)/(d*x + c)))/((b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*i^3*x^2
 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*i^3*x + (b^2*c^4*d - 2*a*b*c
^3*d^2 + a^2*c^2*d^3)*i^3)
```

Sympy [F]

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \frac{\int \frac{A^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{i^3}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**3,x)
```

```
[Out] (Integral(A**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c**3 + 3*c**2*d*x + 3*c
*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/i**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. 2(311) = 622.

Time = 0.24 (sec) , antiderivative size = 861, normalized size of antiderivative = 2.72

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \frac{1}{2}ABn\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)i^3x^2+2\,(bc^2d^2-acd^3)i^3x+(bc^3d-ac^2d^2)i^3} + \frac{2\,b^2\log(bx+a)}{(b^2c^2d-2\,abcd^2+a^2d^3)i^3} - \frac{2\,b}{(b^2c^2d-}\right. + \frac{1}{4}\left(2\,n\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)i^3x^2+2\,(bc^2d^2-acd^3)i^3x+(bc^3d-ac^2d^2)i^3} + \frac{2\,b^2\log(bx+a)}{(b^2c^2d-2\,abcd^2+a^2d^3)i^3} - \frac{}{(b^2c^2}\right.\right. - \frac{B^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)^2}{2\,(d^3i^3x^2+2\,cd^2i^3x+c^2di^3)} - \frac{AB\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{d^3i^3x^2+2\,cd^2i^3x+c^2di^3} - \frac{A^2}{2\,(d^3i^3x^2+2\,cd^2i^3x+c^2di^3)}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x, algorithm="ma
xima")
```

```
[Out] 1/2*A*B*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^
2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b^2*log(b*x + a)/((b^2*
c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*
b*c*d^2 + a^2*d^3)*i^3)) + 1/4*(2*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*
d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) +
 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d
*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3))*log(e*(b*x/(d*x + c) + a
/(d*x + c))^n) - (7*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^2 + 2*b^2*
c*d*x + b^2*c^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*l
og(d*x + c)^2 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2)*log(b*x + a) - 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*(b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x + c))*n^2/(b^2*c^4*d*
i^3 - 2*a*b*c^3*d^2*i^3 + a^2*c^2*d^3*i^3 + (b^2*c^2*d^3*i^3 - 2*a*b*c*d^4*
i^3 + a^2*d^5*i^3)*x^2 + 2*(b^2*c^3*d^2*i^3 - 2*a*b*c^2*d^3*i^3 + a^2*c*d^4
*i^3)*x))*B^2 - 1/2*B^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2/(d^3*i^3*x
^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - A*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
)/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - 1/2*A^2/(d^3*i^3*x^2 + 2*c*d^
2*i^3*x + c^2*d*i^3)
```

Giac [A] (verification not implemented)

none

Time = 1.09 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.28

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(2\left(\frac{2\,(bx+a)B^2bn^2}{(bci^3-adi^3)(dx+c)}-\frac{(bx+a)^2B^2dn^2}{(bci^3-adi^3)(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)^2+2\left(\frac{(B^2dn^2-2\,B^2dn\log(e)-}{(bci^3-adi^3)(d}\right.\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x, algorithm="gi
ac")
```

```
[Out] 1/4*(2*(2*(b*x + a)*B^2*b*n^2/((b*c*i^3 - a*d*i^3)*(d*x + c)) - (b*x + a)^2
*B^2*d*n^2/((b*c*i^3 - a*d*i^3)*(d*x + c)^2))*log((b*x + a)/(d*x + c))^2 +
2*((B^2*d*n^2 - 2*B^2*d*n*log(e) - 2*A*B*d*n)*(b*x + a)^2/((b*c*i^3 - a*d*i
^3)*(d*x + c)^2) - 4*(B^2*b*n^2 - B^2*b*n*log(e) - A*B*b*n)*(b*x + a)/((b*c
*i^3 - a*d*i^3)*(d*x + c)))*log((b*x + a)/(d*x + c)) - (B^2*d*n^2 - 2*B^2*d
*n*log(e) + 2*B^2*d*log(e)^2 - 2*A*B*d*n + 4*A*B*d*log(e) + 2*A^2*d)*(b*x +
 a)^2/((b*c*i^3 - a*d*i^3)*(d*x + c)^2) + 4*(2*B^2*b*n^2 - 2*B^2*b*n*log(e)
 + B^2*b*log(e)^2 - 2*A*B*b*n + 2*A*B*b*log(e) + A^2*b)*(b*x + a)/((b*c*i^3
 - a*d*i^3)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 3.17 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.59

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= -\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{2\,d\,\left(c^2\,i^3+2\,c\,d\,i^3\,x+d^2\,i^3\,x^2\right)} - \frac{B^2\,b^2}{2\,d\,i^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right) - \frac{\frac{2\,A^2\,a\,d-2\,A^2\,b\,c+B^2\,a\,d\,n^2-7\,B^2\,b\,c\,n^2-2\,A\,B\,a\,d\,n+6\,A\,B\,b\,c\,n}{2\,(a\,d-b\,c)} - \frac{b\,x\,\left(3\,B^2\,d\,n^2-2\,A\,B\,d\,n\right)}{a\,d-b\,c}}{2\,c^2\,d\,i^3+4\,c\,d^2\,i^3\,x+2\,d^3\,i^3\,x^2} - \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{A\,B}{c^2\,d\,i^3+2\,c\,d^2\,i^3\,x+d^3\,i^3\,x^2} + \frac{B^2\,b^2\,\left(\frac{d^2\,i^3\,n\,x\,(a\,d-b\,c)}{b} - \frac{d\,i^3\,n\,(a\,d-b\,c)\,(a\,d-2\,b\,c)}{2\,b^2} + \frac{c\,d\,i^3\,n\,(a\,d-b\,c)}{2\,b}\right)}{d\,i^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(c^2\,d\,i^3+2\,c\,d^2\,i^3\,x+d^3\,i^3\,x^2\right)}\right) - \frac{B\,b^2\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{2\,a^2\,d^3\,i^3-2\,b^2\,c^2\,d\,i^3}{2\,d\,i^3\,(a\,d-b\,c)}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,\left(2\,A-3\,B\,n\right)\,1\mathrm{i}}{d\,i^3\,{\left(a\,d-b\,c\right)}^2}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*i + d*i*x)^3,x)
```

```
[Out] - log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(2*d*(c^2*i^3 + d^2*i^3*x^2 + 2*c*d
*i^3*x)) - (B^2*b^2)/(2*d*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2*a
*d - 2*A^2*b*c + B^2*a*d*n^2 - 7*B^2*b*c*n^2 - 2*A*B*a*d*n + 6*A*B*b*c*n)/(
2*(a*d - b*c)) - (b*x*(3*B^2*d*n^2 - 2*A*B*d*n))/(a*d - b*c))/(2*c^2*d*i^3
+ 2*d^3*i^3*x^2 + 4*c*d^2*i^3*x) - log(e*((a + b*x)/(c + d*x))^n)*((A*B)/(c
^2*d*i^3 + d^3*i^3*x^2 + 2*c*d^2*i^3*x) + (B^2*b^2*((d^2*i^3*n*x*(a*d - b*c
))/b - (d*i^3*n*(a*d - b*c)*(a*d - 2*b*c))/(2*b^2) + (c*d*i^3*n*(a*d - b*c)
)/(2*b)))/(d*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(c^2*d*i^3 + d^3*i^3*x^2 +
 2*c*d^2*i^3*x))) - (B*b^2*n*atan(((2*b*d*x + (2*a^2*d^3*i^3 - 2*b^2*c^2*d*
i^3)/(2*d*i^3*(a*d - b*c)))*1i)/(a*d - b*c))*(2*A - 3*B*n)*1i)/(d*i^3*(a*d
- b*c)^2)
```

3.206 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 402

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx = \frac{B^2d^2n^2(a+bx)^2}{4(bc-ad)^3gi^3(c+dx)^2}+\frac{4AbBdn(a+bx)}{(bc-ad)^3gi^3(c+dx)} - \frac{4bB^2dn^2(a+bx)}{(bc-ad)^3gi^3(c+dx)}+\frac{4bB^2dn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^3gi^3(c+dx)} - \frac{Bd^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3gi^3(c+dx)^2} + \frac{d^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^3gi^3(c+dx)^2} - \frac{2bd(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3gi^3(c+dx)} + \frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^3gi^3n}$$

```
[Out] 1/4*B^2*d^2*n^2*(b*x+a)^2/(-a*d+b*c)^3/g/i^3/(d*x+c)^2+4*A*b*B*d*n*(b*x+a)/
(-a*d+b*c)^3/g/i^3/(d*x+c)-4*b*B^2*d*n^2*(b*x+a)/(-a*d+b*c)^3/g/i^3/(d*x+c)
+4*b*B^2*d*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^3/g/i^3/(d*x+c)-1
/2*B*d^2*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g/i^3/(d*
x+c)^2+1/2*d^2*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g/i
^3/(d*x+c)^2-2*b*d*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g
/i^3/(d*x+c)+1/3*b^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b*c)^3/g/i^3
/n
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2561, 2388, 2339, 30, 2333, 2332, 2367, 2342, 2341}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx = \frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3Bgi^3n(bc-ad)^3} + \frac{d^2(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2gi^3(c+dx)^2(bc-ad)^3} - \frac{Bd^2n(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2gi^3(c+dx)^2(bc-ad)^3} - \frac{2bd(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{gi^3(c+dx)(bc-ad)^3} + \frac{4AbBdn(a+bx)}{gi^3(c+dx)(bc-ad)^3} + \frac{B^2d^2n^2(a+bx)^2}{4gi^3(c+dx)^2(bc-ad)^3} + \frac{4bB^2dn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{gi^3(c+dx)(bc-ad)^3} - \frac{4bB^2dn^2(a+bx)}{gi^3(c+dx)(bc-ad)^3}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)*(c*i + d*i*x)^3
),x]
```

```
[Out] (B^2*d^2*n^2*(a + b*x)^2)/(4*(b*c - a*d)^3*g*i^3*(c + d*x)^2) + (4*A*b*B*d*
n*(a + b*x))/((b*c - a*d)^3*g*i^3*(c + d*x)) - (4*b*B^2*d*n^2*(a + b*x))/((
b*c - a*d)^3*g*i^3*(c + d*x)) + (4*b*B^2*d*n*(a + b*x)*Log[e*((a + b*x)/(c
+ d*x))^n])/((b*c - a*d)^3*g*i^3*(c + d*x)) - (B*d^2*n*(a + b*x)^2*(A + B*L
og[e*((a + b*x)/(c + d*x))^n]))/(2*(b*c - a*d)^3*g*i^3*(c + d*x)^2) + (d^2*
(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(b*c - a*d)^3*g*i^
3*(c + d*x)^2) - (2*b*d*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)
/((b*c - a*d)^3*g*i^3*(c + d*x)) + (b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^3)/(3*B*(b*c - a*d)^3*g*i^3*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)^2(A+B\log(ex^n))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3}$$

$$= \frac{b\text{Subst}\left(\int \frac{(b-dx)(A+B\log(ex^n))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3} - \frac{d\text{Subst}\left(\int (b-dx)\left(A+B\log\left(ex^n\right)\right)^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3}$$

$$= \frac{b^2\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3} - \frac{d\text{Subst}\left(\int \left(b(A+B\log\left(ex^n\right))^2 - dx(A+B\log\left(ex^n\right))^2\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3} - \frac{(bd)\text{Subst}\left(\int \left(A+B\log\left(ex^n\right)\right)^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3}$$

$$= -\frac{bd(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3gi^3(c+dx)} - \frac{(bd)\text{Subst}\left(\int \left(A+B\log\left(ex^n\right)\right)^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3} + \frac{d^2\text{Subst}\left(\int x(A+B\log\left(ex^n\right))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3} + \frac{b^2\text{Subst}\left(\int x^2\,dx, x, A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)^3gi^3n} + \frac{(2bBdn)\text{Subst}\left(\int \left(A+B\log\left(ex^n\right)\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3}$$

$$= \frac{2AbBdn(a+bx)}{(bc-ad)^3gi^3(c+dx)} + \frac{d^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^3gi^3(c+dx)^2} - \frac{2bd(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3gi^3(c+dx)} + \frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^3gi^3n} + \frac{(2bBdn)\text{Subst}\left(\int \left(A+B\log\left(ex^n\right)\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3} + \frac{\left(2bB^2dn\right)\text{Subst}\left(\int \log\left(ex^n\right)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3} - \frac{\left(Bd^2n\right)\text{Subst}\left(\int x(A+B\log\left(ex^n\right))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3}$$

$$
\begin{aligned}
&= \frac{B^2d^2n^2(a+bx)^2}{4(bc-ad)^3gi^3(c+dx)^2} + \frac{4AbBdn(a+bx)}{(bc-ad)^3gi^3(c+dx)} - \frac{2bB^2dn^2(a+bx)}{(bc-ad)^3gi^3(c+dx)} \\
&+ \frac{2bB^2dn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^3gi^3(c+dx)} - \frac{Bd^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3gi^3(c+dx)^2} \\
&+ \frac{d^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^3gi^3(c+dx)^2} - \frac{2bd(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3gi^3(c+dx)} \\
&+ \frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^3gi^3n} + \frac{(2bB^2dn)\,\text{Subst}\left(\int \log(ex^n)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3} \\
&= \frac{B^2d^2n^2(a+bx)^2}{4(bc-ad)^3gi^3(c+dx)^2} + \frac{4AbBdn(a+bx)}{(bc-ad)^3gi^3(c+dx)} \\
&- \frac{4bB^2dn^2(a+bx)}{(bc-ad)^3gi^3(c+dx)} + \frac{4bB^2dn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^3gi^3(c+dx)} \\
&- \frac{Bd^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3gi^3(c+dx)^2} + \frac{d^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^3gi^3(c+dx)^2} \\
&- \frac{2bd(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3gi^3(c+dx)} + \frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^3gi^3n}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 971 vs. $2(402) = 804$.

Time = 0.52 (sec) , antiderivative size = 971, normalized size of antiderivative = 2.42

$$
\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx
$$

$$
= \frac{4b^2B^2n^2\log^3\left(\frac{a+bx}{c+dx}\right) - \frac{6Bn\log^2\left(\frac{a+bx}{c+dx}\right)\left(-2Ab^2c^2+4abBcdn-a^2Bd^2n-4Ab^2cdx+4b^2Bcdnx+2abBd^2nx-2Ab^2d^2x^2+3b^2Bd^2nx^2-2b^2\right.}{(c+dx)^2}}{}
$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)*(c*i + d*
i*x)^3),x]
```

```
[Out] (4*b^2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^3 - (6*B*n*Log[(a + b*x)/(c + d*x)]
^2*(-2*A*b^2*c^2 + 4*a*b*B*c*d*n - a^2*B*d^2*n - 4*A*b^2*c*d*x + 4*b^2*B*c*
d*n*x + 2*a*b*B*d^2*n*x - 2*A*b^2*d^2*x^2 + 3*b^2*B*d^2*n*x^2 - 2*b^2*B*(c
+ d*x)^2*Log[e*((a + b*x)/(c + d*x))^n] + 2*b^2*B*n*(c + d*x)^2*Log[(a + b*
x)/(c + d*x)]))/(c + d*x)^2 - (6*B*(b*c - a*d)*n*Log[(a + b*x)/(c + d*x)]*(
-6*A*b*c + 2*a*A*d + 7*b*B*c*n - a*B*d*n - 4*A*b*d*x + 6*b*B*d*n*x + 2*B*(-
3*b*c + a*d - 2*b*d*x)*Log[e*((a + b*x)/(c + d*x))^n] + 2*B*n*(3*b*c - a*d
+ 2*b*d*x)*Log[(a + b*x)/(c + d*x)]))/(c + d*x)^2 + (3*(b*c - a*d)^2*(2*A^2
 - 2*A*B*n + B^2*n^2 + 2*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 + 2*B*n*(-2*A
 + B*n)*Log[(a + b*x)/(c + d*x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 - 2
```

```
*B*Log[e*((a + b*x)/(c + d*x))^n]*(-2*A + B*n + 2*B*n*Log[(a + b*x)/(c + d*
x)])))/(c + d*x)^2 + (6*b*(b*c - a*d)*(2*A^2 - 6*A*B*n + 7*B^2*n^2 + 2*B^2*
Log[e*((a + b*x)/(c + d*x))^n]^2 + 2*B*n*(-2*A + 3*B*n)*Log[(a + b*x)/(c +
d*x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 - 2*B*Log[e*((a + b*x)/(c + d*
x))^n]*(-2*A + 3*B*n + 2*B*n*Log[(a + b*x)/(c + d*x)])))/(c + d*x) + 6*b^2*
Log[a + b*x]*(2*A^2 - 6*A*B*n + 7*B^2*n^2 + 2*B^2*Log[e*((a + b*x)/(c + d*x
))^n]^2 + 2*B*n*(-2*A + 3*B*n)*Log[(a + b*x)/(c + d*x)] + 2*B^2*n^2*Log[(a
+ b*x)/(c + d*x)]^2 - 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(-2*A + 3*B*n + 2*
B*n*Log[(a + b*x)/(c + d*x)])) - 6*b^2*(2*A^2 - 6*A*B*n + 7*B^2*n^2 + 2*B^2
*Log[e*((a + b*x)/(c + d*x))^n]^2 + 2*B*n*(-2*A + 3*B*n)*Log[(a + b*x)/(c +
 d*x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 - 2*B*Log[e*((a + b*x)/(c + d
*x))^n]*(-2*A + 3*B*n + 2*B*n*Log[(a + b*x)/(c + d*x)]))*Log[c + d*x])/(12*
(b*c - a*d)^3*g*i^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1235 vs. 2(394) = 788.

Time = 11.42 (sec) , antiderivative size = 1236, normalized size of antiderivative = 3.07

method	result	size
parallelrisch	Expression too large to display	1236

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x,method=_R
ETURNVERBOSE)
```

```
[Out] -1/12*(4*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a^2*b^2*c^4*d^2+48*B^2*x^2*a^3
*b*c^3*d^3*n^3-45*B^2*x^2*a^2*b^2*c^4*d^2*n^3+6*A*B*x^2*a^4*c^2*d^4*n^2+8*B
^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*a^2*b^2*c^5*d+54*B^2*x*a^3*b*c^4*d^2*n^3-4
8*B^2*x*a^2*b^2*c^5*d*n^3+12*A^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^4*
d^2+24*A^2*x^2*a^3*b*c^3*d^3*n-36*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2
*c^4*d^2*n-24*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b*c^4*d^2*n-48*A*B*x*ln(e
*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^5*d*n+4*B^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a^2
*b^2*c^6+12*A^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^6-24*B^2*x*ln(e*((b*x+a
)/(d*x+c))^n)^2*a^2*b^2*c^5*d*n+36*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b*c^
4*d^2*n^2+48*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^5*d*n^2+24*A*B*x*ln(
e*((b*x+a)/(d*x+c))^n)^2*a^2*b^2*c^5*d-60*A*B*x*a^3*b*c^4*d^2*n^2+48*A*B*x*
a^2*b^2*c^5*d*n^2-48*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b*c^5*d*n-18*B^2*x^2
*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^2*c^4*d^2*n+42*B^2*x^2*ln(e*((b*x+a)/(d*
x+c))^n)*a^2*b^2*c^4*d^2*n^2+12*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^2
*c^4*d^2-48*A*B*x^2*a^3*b*c^3*d^3*n^2+42*A*B*x^2*a^2*b^2*c^4*d^2*n^2-12*B^2
*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b*c^4*d^2*n-3*B^2*x^2*a^4*c^2*d^4*n^3-6*
B^2*x*a^4*c^3*d^3*n^3-6*A^2*x^2*a^4*c^2*d^4*n+6*B^2*ln(e*((b*x+a)/(d*x+c))^
n)^2*a^4*c^4*d^2*n-6*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^4*c^4*d^2*n^2-12*A^2*x
*a^4*c^3*d^3*n+12*A*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^2*c^6-18*A^2*x^2*a^
2*b^2*c^4*d^2*n+12*A*B*x*a^4*c^3*d^3*n^2-24*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2
```

```
*a^3*b*c^5*d*n+48*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b*c^5*d*n^2+24*A^2*x*ln
(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^5*d+36*A^2*x*a^3*b*c^4*d^2*n-24*A^2*x*a^2
*b^2*c^5*d*n+12*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^4*c^4*d^2*n)/i^3/g/(d*x+c)^
2/(a*d-b*c)^3/a^2/c^4/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. 2(394) = 788.

Time = 0.32 (sec) , antiderivative size = 1076, normalized size of antiderivative = 2.68

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$$
$$= \frac{18\,A^2b^2c^2-24\,A^2abcd+6\,A^2a^2d^2+4\left(B^2b^2d^2n^2x^2+2\,B^2b^2cdn^2x+B^2b^2c^2n^2\right)\log\left(\frac{bx+a}{dx+c}\right)^3+3\left(15\,B^2b^2c^2\right.}{}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x, a
lgorithm="fricas")
```

```
[Out] 1/12*(18*A^2*b^2*c^2 - 24*A^2*a*b*c*d + 6*A^2*a^2*d^2 + 4*(B^2*b^2*d^2*n^2*
x^2 + 2*B^2*b^2*c*d*n^2*x + B^2*b^2*c^2*n^2)*log((b*x + a)/(d*x + c))^3 + 3
*(15*B^2*b^2*c^2 - 16*B^2*a*b*c*d + B^2*a^2*d^2)*n^2 + 6*(3*B^2*b^2*c^2 - 4
*B^2*a*b*c*d + B^2*a^2*d^2 + 2*(B^2*b^2*c*d - B^2*a*b*d^2)*x + 2*(B^2*b^2*d
^2*x^2 + 2*B^2*b^2*c*d*x + B^2*b^2*c^2)*log((b*x + a)/(d*x + c)))*log(e)^2
+ 6*(2*A*B*b^2*c^2*n - (4*B^2*a*b*c*d - B^2*a^2*d^2)*n^2 - (3*B^2*b^2*d^2*n
^2 - 2*A*B*b^2*d^2*n)*x^2 + 2*(2*A*B*b^2*c*d*n - (2*B^2*b^2*c*d + B^2*a*b*d
^2)*n^2)*x)*log((b*x + a)/(d*x + c))^2 - 6*(7*A*B*b^2*c^2 - 8*A*B*a*b*c*d +
 A*B*a^2*d^2)*n + 6*(2*A^2*b^2*c*d - 2*A^2*a*b*d^2 + 7*(B^2*b^2*c*d - B^2*a
*b*d^2)*n^2 - 6*(A*B*b^2*c*d - A*B*a*b*d^2)*n)*x + 6*(6*A*B*b^2*c^2 - 8*A*B
*a*b*c*d + 2*A*B*a^2*d^2 + 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*b^2*c*d*n*x + B^2*b
^2*c^2*n)*log((b*x + a)/(d*x + c))^2 - (7*B^2*b^2*c^2 - 8*B^2*a*b*c*d + B^2
*a^2*d^2)*n + 2*(2*A*B*b^2*c*d - 2*A*B*a*b*d^2 - 3*(B^2*b^2*c*d - B^2*a*b*d
^2)*n)*x + 2*(2*A*B*b^2*c^2 - (3*B^2*b^2*d^2*n - 2*A*B*b^2*d^2)*x^2 - (4*B^
2*a*b*c*d - B^2*a^2*d^2)*n + 2*(2*A*B*b^2*c*d - (2*B^2*b^2*c*d + B^2*a*b*d^
2)*n)*x)*log((b*x + a)/(d*x + c)))*log(e) + 6*(2*A^2*b^2*c^2 + (8*B^2*a*b*c
*d - B^2*a^2*d^2)*n^2 + (7*B^2*b^2*d^2*n^2 - 6*A*B*b^2*d^2*n + 2*A^2*b^2*d^
2)*x^2 - 2*(4*A*B*a*b*c*d - A*B*a^2*d^2)*n + 2*(2*A^2*b^2*c*d + (4*B^2*b^2*
c*d + 3*B^2*a*b*d^2)*n^2 - 2*(2*A*B*b^2*c*d + A*B*a*b*d^2)*n)*x)*log((b*x +
 a)/(d*x + c)))/((b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*
g*i^3*x^2 + 2*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*g
*i^3*x + (b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*g*i^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)/(d*i*x+c*i)**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2126 vs. $2(394) = 788$.

Time = 0.33 (sec) , antiderivative size = 2126, normalized size of antiderivative = 5.29

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x, a
lgorithm="maxima")
[Out] 1/2*B^2*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*g*i
^3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3*x + (b^2*c^4 - 2*a
*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c)/((b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3))*log(e*(b*x/(d*x + c) + a/(d*x +
 c))^n)^2 + A*B*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*
d^4)*g*i^3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3*x + (b^2*c
^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c)/((b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3))*log(e*(b*x/(d*x + c) +
a/(d*x + c))^n) + 1/12*((45*b^2*c^2 - 48*a*b*c*d + 3*a^2*d^2 + 4*(b^2*d^2*x
^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a)^3 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*log(d*x + c)^3 + 18*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x
 + a)^2 + 6*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x + c)^2 + 42*(b^2*c*d - a*b*d^2)*x
 + 42*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a) - 6*(7*b^2*d^2*x^2
 + 14*b^2*c*d*x + 7*b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b
*x + a)^2 + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x +
 c))*n^2/(b^3*c^5*g*i^3 - 3*a*b^2*c^4*d*g*i^3 + 3*a^2*b*c^3*d^2*g*i^3 - a^3
*c^2*d^3*g*i^3 + (b^3*c^3*d^2*g*i^3 - 3*a*b^2*c^2*d^3*g*i^3 + 3*a^2*b*c*d^4
*g*i^3 - a^3*d^5*g*i^3)*x^2 + 2*(b^3*c^4*d*g*i^3 - 3*a*b^2*c^3*d^2*g*i^3 +
3*a^2*b*c^2*d^3*g*i^3 - a^3*c*d^4*g*i^3)*x) - 6*(7*b^2*c^2 - 8*a*b*c*d + a^
2*d^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a)^2 + 2*(b^2*d^2
```

```
*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x + c)^2 + 6*(b^2*c*d - a*b*d^2)*x + 6*
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a) - 2*(3*b^2*d^2*x^2 + 6*b
^2*c*d*x + 3*b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a)
)*log(d*x + c))*n*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^3*c^5*g*i^3 - 3
*a*b^2*c^4*d*g*i^3 + 3*a^2*b*c^3*d^2*g*i^3 - a^3*c^2*d^3*g*i^3 + (b^3*c^3*d
^2*g*i^3 - 3*a*b^2*c^2*d^3*g*i^3 + 3*a^2*b*c*d^4*g*i^3 - a^3*d^5*g*i^3)*x^2
 + 2*(b^3*c^4*d*g*i^3 - 3*a*b^2*c^3*d^2*g*i^3 + 3*a^2*b*c^2*d^3*g*i^3 - a^3
*c*d^4*g*i^3)*x))*B^2 - 1/2*(7*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x
^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*log(d*x + c)^2 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*log(b*x + a) - 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2
 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x + c))*A*B*
n/(b^3*c^5*g*i^3 - 3*a*b^2*c^4*d*g*i^3 + 3*a^2*b*c^3*d^2*g*i^3 - a^3*c^2*d^
3*g*i^3 + (b^3*c^3*d^2*g*i^3 - 3*a*b^2*c^2*d^3*g*i^3 + 3*a^2*b*c*d^4*g*i^3
- a^3*d^5*g*i^3)*x^2 + 2*(b^3*c^4*d*g*i^3 - 3*a*b^2*c^3*d^2*g*i^3 + 3*a^2*b
*c^2*d^3*g*i^3 - a^3*c*d^4*g*i^3)*x) + 1/2*A^2*((2*b*d*x + 3*b*c - a*d)/((b
^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*g*i^3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^
2 + a^2*c*d^3)*g*i^3*x + (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b
^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3)
 - 2*b^2*log(d*x + c)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
g*i^3))
```

Giac [A] (verification not implemented)

none

Time = 1.19 (sec) , antiderivative size = 747, normalized size of antiderivative = 1.86

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$$
$$=\frac{1}{12}\left(\frac{4\,B^2b^2n^2\log\left(\frac{bx+a}{dx+c}\right)^3}{b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3}-6\left(\frac{4\,(bx+a)B^2bdn^2}{(b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3)(dx+c)}-\frac{(bx+a)^2}{(b^2c^2gi^3-2\,abcdgi^3}\right.\right.$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x, a
lgorithm="giac")
```

```
[Out] 1/12*(4*B^2*b^2*n^2*log((b*x + a)/(d*x + c))^3/(b^2*c^2*g*i^3 - 2*a*b*c*d*g
*i^3 + a^2*d^2*g*i^3) - 6*(4*(b*x + a)*B^2*b*d*n^2/((b^2*c^2*g*i^3 - 2*a*b*
c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)) - (b*x + a)^2*B^2*d^2*n^2/((b^2*c^2*g
*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)^2) - 2*(B^2*b^2*n*log(e)
+ A*B*b^2*n)/(b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3))*log((b*x +
a)/(d*x + c))^2 - 6*((B^2*d^2*n^2 - 2*B^2*d^2*n*log(e) - 2*A*B*d^2*n)*(b*x
+ a)^2/((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)^2) - 8*
(B^2*b*d*n^2 - B^2*b*d*n*log(e) - A*B*b*d*n)*(b*x + a)/((b^2*c^2*g*i^3 - 2*
```

```
a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)))*log((b*x + a)/(d*x + c)) + 12*(B
^2*b^2*log(e)^2 + 2*A*B*b^2*log(e) + A^2*b^2)*log((b*x + a)/(d*x + c))/(b^2
*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3) + 3*(B^2*d^2*n^2 - 2*B^2*d^2*
n*log(e) + 2*B^2*d^2*log(e)^2 - 2*A*B*d^2*n + 4*A*B*d^2*log(e) + 2*A^2*d^2)
*(b*x + a)^2/((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)^2
) - 24*(2*B^2*b*d*n^2 - 2*B^2*b*d*n*log(e) + B^2*b*d*log(e)^2 - 2*A*B*b*d*n
 + 2*A*B*b*d*log(e) + A^2*b*d)*(b*x + a)/((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3
+ a^2*d^2*g*i^3)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

Mupad [B] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 1007, normalized size of antiderivative = 2.50

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$$
$$= \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{b^2\,(3\,B^2\,n-2\,A\,B)}{2\,g\,i^3\,n\,(a\,d-b\,c)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\frac{B^2\,b^2\left(\frac{c\,g\,i^3\,n\,(a\,d-b\,c)}{2\,b}-\frac{g\,i^3\,n\,(a\,d-b\,c)\,(a\,d-2\,b\,c)}{2\,b^2}+\frac{d\,g\,i^3\,n\,x\,(a\,d-b\,c)}{b}\right)}{g\,i^3\,n\,(a\,d-b\,c)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)\,(g\,c^2\,i^3+2\,g\,c\,d\,i^3\,x+g\,d^2\,i^3\,x^2)}\right)$$
$$-\frac{\frac{2\,A^2\,a\,d-6\,A^2\,b\,c+B^2\,a\,d\,n^2-15\,B^2\,b\,c\,n^2-2\,A\,B\,a\,d\,n+14\,A\,B\,b\,c\,n}{2\,(a\,d-b\,c)}-\frac{x\,(2\,b\,d\,A^2-6\,b\,d\,A\,B\,n+7\,b\,d\,B^2\,n^2)}{a\,d-b\,c}}{x^2\,(2\,a\,d^3\,g\,i^3-2\,b\,c\,d^2\,g\,i^3)+x\,(4\,a\,c\,d^2\,g\,i^3-4\,b\,c^2\,d\,g\,i^3)-2\,b\,c^3\,g\,i^3+2\,a\,c^2\,d\,g\,i^3}$$
$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B^2\,n}{x^2\,(a\,d^3\,g\,i^3-b\,c\,d^2\,g\,i^3)+x\,(2\,a\,c\,d^2\,g\,i^3-2\,b\,c^2\,d\,g\,i^3)-b\,c^3\,g\,i^3+a\,c^2\,d\,g\,i^3}\right.$$
$$+\frac{b^2\,(3\,B^2\,n-2\,A\,B)\left(\frac{c\,g\,i^3\,n\,(a\,d-b\,c)^2}{2\,b}-\frac{g\,i^3\,n\,(a\,d-b\,c)^2\,(a\,d-2\,b\,c)}{2\,b^2}+\frac{d\,g\,i^3\,n\,x\,(a\,d-b\,c)^2}{b}\right)}{g\,i^3\,n\,(a\,d-b\,c)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)\,(x^2\,(a\,d^3\,g\,i^3-b\,c\,d^2\,g\,i^3)+x\,(2\,a\,c\,d^2\,g\,i^3-2\,b\,c^2\,d\,g\,i^3)-b\,c^3}$$
$$-\frac{B^2\,b^2\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^3}{3\,g\,i^3\,n\,(a\,d-b\,c)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}$$
$$+\frac{b^2\,\mathrm{atan}\left(\frac{b^2\left(\frac{g\,a^3\,d^3\,i^3-g\,a^2\,b\,c\,d^2\,i^3-g\,a\,b^2\,c^2\,d\,i^3+g\,b^3\,c^3\,i^3}{g\,a^2\,d^2\,i^3-2\,g\,a\,b\,c\,d\,i^3+g\,b^2\,c^2\,i^3}+2\,b\,d\,x\right)\left(A^2-3\,A\,B\,n+\frac{7\,B^2\,n^2}{2}\right)(g\,a^2\,d^2\,i^3-2\,g\,a\,b\,c\,d\,i^3+g\,b^2\,c^2\,i^3)\,2\mathrm{i}}{g\,i^3\,(a\,d-b\,c)^3\,(2\,A^2\,b^2-6\,A\,B\,b^2\,n+7\,B^2\,b^2\,n^2)}\right)\left(A^2\right.}{g\,i^3\,(a\,d-b\,c)^3}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)*(c*i + d*i*x)^3
),x)
```

```
[Out] log(e*((a + b*x)/(c + d*x))^n)^2*((b^2*(3*B^2*n - 2*A*B))/(2*g*i^3*n*(a*d -
 b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B^2*b^2*((c*g*i^3*n*(a*d - b*c))/
(2*b) - (g*i^3*n*(a*d - b*c)*(a*d - 2*b*c))/(2*b^2) + (d*g*i^3*n*x*(a*d - b
```

```
*c))/b))/(g*i^3*n*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(c^2*g*i^3 +
d^2*g*i^3*x^2 + 2*c*d*g*i^3*x))) - ((2*A^2*a*d - 6*A^2*b*c + B^2*a*d*n^2 -
15*B^2*b*c*n^2 - 2*A*B*a*d*n + 14*A*B*b*c*n)/(2*(a*d - b*c)) - (x*(2*A^2*b*
d + 7*B^2*b*d*n^2 - 6*A*B*b*d*n))/(a*d - b*c))/(x^2*(2*a*d^3*g*i^3 - 2*b*c*
d^2*g*i^3) + x*(4*a*c*d^2*g*i^3 - 4*b*c^2*d*g*i^3) - 2*b*c^3*g*i^3 + 2*a*c^
2*d*g*i^3) - log(e*((a + b*x)/(c + d*x))^n)*((B^2*n)/(x^2*(a*d^3*g*i^3 - b*
c*d^2*g*i^3) + x*(2*a*c*d^2*g*i^3 - 2*b*c^2*d*g*i^3) - b*c^3*g*i^3 + a*c^2*
d*g*i^3) + (b^2*(3*B^2*n - 2*A*B)*((c*g*i^3*n*(a*d - b*c)^2)/(2*b) - (g*i^3
*n*(a*d - b*c)^2*(a*d - 2*b*c))/(2*b^2) + (d*g*i^3*n*x*(a*d - b*c)^2)/b))/(
g*i^3*n*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(x^2*(a*d^3*g*i^3 - b*c
*d^2*g*i^3) + x*(2*a*c*d^2*g*i^3 - 2*b*c^2*d*g*i^3) - b*c^3*g*i^3 + a*c^2*d
*g*i^3))) + (b^2*atan((b^2*((a^3*d^3*g*i^3 + b^3*c^3*g*i^3 - a*b^2*c^2*d*g*
i^3 - a^2*b*c*d^2*g*i^3)/(a^2*d^2*g*i^3 + b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3)
+ 2*b*d*x)*(A^2 + (7*B^2*n^2)/2 - 3*A*B*n)*(a^2*d^2*g*i^3 + b^2*c^2*g*i^3 -
 2*a*b*c*d*g*i^3)*2i)/(g*i^3*(a*d - b*c)^3*(2*A^2*b^2 + 7*B^2*b^2*n^2 - 6*A
*B*b^2*n)))*(A^2 + (7*B^2*n^2)/2 - 3*A*B*n)*2i)/(g*i^3*(a*d - b*c)^3) - (B^
2*b^2*log(e*((a + b*x)/(c + d*x))^n)^3)/(3*g*i^3*n*(a*d - b*c)*(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d))
```

3.207 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 562

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx &= -\frac{B^2d^3n^2(a+bx)^2}{4(bc-ad)^4g^2i^3(c+dx)^2}-\frac{6AbBd^2n(a+bx)}{(bc-ad)^4g^2i^3(c+dx)}\\
&\quad+\frac{6bB^2d^2n^2(a+bx)}{(bc-ad)^4g^2i^3(c+dx)}-\frac{2b^3B^2n^2(c+dx)}{(bc-ad)^4g^2i^3(a+bx)}\\
&\quad-\frac{6bB^2d^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^4g^2i^3(c+dx)}\\
&\quad+\frac{Bd^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^2i^3(c+dx)^2}\\
&\quad-\frac{2b^3Bn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^2i^3(a+bx)}\\
&\quad-\frac{d^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^4g^2i^3(c+dx)^2}\\
&\quad+\frac{3bd^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^2i^3(c+dx)}\\
&\quad-\frac{b^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^2i^3(a+bx)}\\
&\quad-\frac{b^2d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{B(bc-ad)^4g^2i^3n}\end{aligned}$$

```
[Out] -1/4*B^2*d^3*n^2*(b*x+a)^2/(-a*d+b*c)^4/g^2/i^3/(d*x+c)^2-6*A*b*B*d^2*n*(b*
x+a)/(-a*d+b*c)^4/g^2/i^3/(d*x+c)+6*b*B^2*d^2*n^2*(b*x+a)/(-a*d+b*c)^4/g^2/
i^3/(d*x+c)-2*b^3*B^2*n^2*(d*x+c)/(-a*d+b*c)^4/g^2/i^3/(b*x+a)-6*b*B^2*d^2*
```

```
n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^4/g^2/i^3/(d*x+c)+1/2*B*d^3*
n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^2/i^3/(d*x+c)^2-
2*b^3*B*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^2/i^3/(b*x
+a)-1/2*d^3*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^2/i^
3/(d*x+c)^2+3*b*d^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/
g^2/i^3/(d*x+c)-b^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/
g^2/i^3/(b*x+a)-b^2*d*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b*c)^4/g^2/
i^3/n
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.178$, Rules used = {2561, 2395, 2333, 2332, 2342, 2341, 2339, 30}

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = -\frac{b^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^2i^3(a+bx)(bc-ad)^4} - \frac{2b^3Bn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^2i^3(a+bx)(bc-ad)^4} - \frac{b^2d\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{Bg^2i^3n(bc-ad)^4} - \frac{d^3(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g^2i^3(c+dx)^2(bc-ad)^4} + \frac{Bd^3n(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^2i^3(c+dx)^2(bc-ad)^4} + \frac{3bd^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^2i^3(c+dx)(bc-ad)^4} - \frac{6AbBd^2n(a+bx)}{g^2i^3(c+dx)(bc-ad)^4} - \frac{2b^3B^2n^2(c+dx)}{g^2i^3(a+bx)(bc-ad)^4} - \frac{B^2d^3n^2(a+bx)^2}{4g^2i^3(c+dx)^2(bc-ad)^4} - \frac{6bB^2d^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g^2i^3(c+dx)(bc-ad)^4} + \frac{6bB^2d^2n^2(a+bx)}{g^2i^3(c+dx)(bc-ad)^4}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^2*(c*i + d*i*x)
^3),x]
```

```
[Out] -1/4*(B^2*d^3*n^2*(a + b*x)^2)/((b*c - a*d)^4*g^2*i^3*(c + d*x)^2) - (6*A*b
*B*d^2*n*(a + b*x))/((b*c - a*d)^4*g^2*i^3*(c + d*x)) + (6*b*B^2*d^2*n^2*(a
```

```
 + b*x))/((b*c - a*d)^4*g^2*i^3*(c + d*x)) - (2*b^3*B^2*n^2*(c + d*x))/((b*
c - a*d)^4*g^2*i^3*(a + b*x)) - (6*b*B^2*d^2*n*(a + b*x)*Log[e*((a + b*x)/(
c + d*x))^n])/((b*c - a*d)^4*g^2*i^3*(c + d*x)) + (B*d^3*n*(a + b*x)^2*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*c - a*d)^4*g^2*i^3*(c + d*x)^2) -
 (2*b^3*B*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^
4*g^2*i^3*(a + b*x)) - (d^3*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2)/(2*(b*c - a*d)^4*g^2*i^3*(c + d*x)^2) + (3*b*d^2*(a + b*x)*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n])^2)/((b*c - a*d)^4*g^2*i^3*(c + d*x)) - (b^3*(
c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*c - a*d)^4*g^2*i^3*(
a + b*x)) - (b^2*d*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(B*(b*c - a*d)
^4*g^2*i^3*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)^3(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}$$

$$= \frac{\text{Subst}\left(\int \left(3bd^2(A+B\log(ex^n))^2 + \frac{b^3(A+B\log(ex^n))^2}{x^2} - \frac{3b^2d(A+B\log(ex^n))^2}{x} - d^3x(A+B\log(ex^n))^2\right)\right.}{(bc-ad)^4g^2i^3}$$

$$= \frac{b^3\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3} - \frac{(3b^2d)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}$$
$$+ \frac{(3bd^2)\,\text{Subst}\left(\int (A+B\log(ex^n))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}$$
$$- \frac{d^3\text{Subst}\left(\int x(A+B\log(ex^n))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}$$

$$=-\frac{d^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^4g^2i^3(c+dx)^2}+\frac{3bd^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^2i^3(c+dx)}$$
$$-\frac{b^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^2i^3(a+bx)}$$
$$-\frac{(3b^2d)\operatorname{Subst}\left(\int x^2\,dx,x,A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)^4g^2i^3n}$$
$$+\frac{(2b^3Bn)\operatorname{Subst}\left(\int\frac{A+B\log(ex^n)}{x^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}$$
$$-\frac{(6bBd^2n)\operatorname{Subst}\left(\int(A+B\log(ex^n))\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}$$
$$+\frac{(Bd^3n)\operatorname{Subst}\left(\int x(A+B\log(ex^n))\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}$$

$$=-\frac{B^2d^3n^2(a+bx)^2}{4(bc-ad)^4g^2i^3(c+dx)^2}-\frac{6AbBd^2n(a+bx)}{(bc-ad)^4g^2i^3(c+dx)}$$
$$-\frac{2b^3B^2n^2(c+dx)}{(bc-ad)^4g^2i^3(a+bx)}+\frac{Bd^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^2i^3(c+dx)^2}$$
$$-\frac{2b^3Bn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^2i^3(a+bx)}-\frac{d^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^4g^2i^3(c+dx)^2}$$
$$+\frac{3bd^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^2i^3(c+dx)}-\frac{b^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^2i^3(a+bx)}$$
$$-\frac{b^2d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{B(bc-ad)^4g^2i^3n}-\frac{(6bB^2d^2n)\operatorname{Subst}\left(\int\log(ex^n)\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3}$$

$$=-\frac{B^2d^3n^2(a+bx)^2}{4(bc-ad)^4g^2i^3(c+dx)^2}-\frac{6AbBd^2n(a+bx)}{(bc-ad)^4g^2i^3(c+dx)}+\frac{6bB^2d^2n^2(a+bx)}{(bc-ad)^4g^2i^3(c+dx)}$$
$$-\frac{2b^3B^2n^2(c+dx)}{(bc-ad)^4g^2i^3(a+bx)}-\frac{6bB^2d^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^4g^2i^3(c+dx)}$$
$$+\frac{Bd^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^2i^3(c+dx)^2}-\frac{2b^3Bn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^2i^3(a+bx)}$$
$$-\frac{d^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^4g^2i^3(c+dx)^2}+\frac{3bd^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^2i^3(c+dx)}$$
$$-\frac{b^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^2i^3(a+bx)}-\frac{b^2d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{B(bc-ad)^4g^2i^3n}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1334 vs. 2(562) = 1124.

Time = 0.89 (sec) , antiderivative size = 1334, normalized size of antiderivative = 2.37

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = -\frac{4b^2B^2dn^2(a+bx)(c+dx)^2\log^3\left(\frac{a+bx}{c+dx}\right)+2Bn\log^2\left(\frac{a+bx}{c+dx}\right)\left(6aAb^2c^2d+2b^3Bc^3n-6a^2bBcd^2n+a^3Bd^3\right.}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^2*(c*i +
d*i*x)^3),x]
```

```
[Out] -1/4*(4*b^2*B^2*d*n^2*(a + b*x)*(c + d*x)^2*Log[(a + b*x)/(c + d*x)]^3 + 2*
B*n*Log[(a + b*x)/(c + d*x)]^2*(6*a*A*b^2*c^2*d + 2*b^3*B*c^3*n - 6*a^2*b*B
*c*d^2*n + a^3*B*d^3*n + 6*A*b^3*c^2*d*x + 12*a*A*b^2*c*d^2*x + 6*b^3*B*c^2
*d*n*x - 12*a*b^2*B*c*d^2*n*x - 3*a^2*b*B*d^3*n*x + 12*A*b^3*c*d^2*x^2 + 6*
a*A*b^2*d^3*x^2 - 9*a*b^2*B*d^3*n*x^2 + 6*A*b^3*d^3*x^3 - 3*b^3*B*d^3*n*x^3
 + 6*b^2*B*d*(a + b*x)*(c + d*x)^2*Log[e*((a + b*x)/(c + d*x))^n] - 6*b^2*B
*d*n*(a + b*x)*(c + d*x)^2*Log[(a + b*x)/(c + d*x)]) + 4*b^2*(b*c - a*d)*(c
 + d*x)^2*(A^2 + 2*A*B*n + 2*B^2*n^2 + B^2*Log[e*((a + b*x)/(c + d*x))^n]^2
 - 2*B*n*(A + B*n)*Log[(a + b*x)/(c + d*x)] + B^2*n^2*Log[(a + b*x)/(c + d*
x)]^2 + 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(A + B*n - B*n*Log[(a + b*x)/(c
+ d*x)])) + 2*B*(b*c - a*d)*n*Log[(a + b*x)/(c + d*x)]*(2*b*d*(a + b*x)*(c
+ d*x)*(4*A - 5*B*n + 4*B*Log[e*((a + b*x)/(c + d*x))^n] - 4*B*n*Log[(a + b
*x)/(c + d*x)]) + d*(b*c - a*d)*(a + b*x)*(2*A - B*n + 2*B*Log[e*((a + b*x)
/(c + d*x))^n] - 2*B*n*Log[(a + b*x)/(c + d*x)]) + 4*b^2*(c + d*x)^2*(A + B
*n + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])) + d*
(b*c - a*d)^2*(a + b*x)*(2*A^2 - 2*A*B*n + B^2*n^2 + 2*B^2*Log[e*((a + b*x)
/(c + d*x))^n]^2 + 2*B*n*(-2*A + B*n)*Log[(a + b*x)/(c + d*x)] + 2*B^2*n^2*
Log[(a + b*x)/(c + d*x)]^2 - 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(-2*A + B*n
 + 2*B*n*Log[(a + b*x)/(c + d*x)])) + 6*b^2*d*(a + b*x)*(c + d*x)^2*Log[a +
 b*x]*(2*A^2 - 2*A*B*n + 5*B^2*n^2 + 2*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2
 + 2*B*n*(-2*A + B*n)*Log[(a + b*x)/(c + d*x)] + 2*B^2*n^2*Log[(a + b*x)/(c
 + d*x)]^2 - 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(-2*A + B*n + 2*B*n*Log[(a
+ b*x)/(c + d*x)])) + 2*b*d*(b*c - a*d)*(a + b*x)*(c + d*x)*(4*A^2 - 10*A*B
*n + 11*B^2*n^2 + 4*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 + 2*B*n*(-4*A + 5*
B*n)*Log[(a + b*x)/(c + d*x)] + 4*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 - 2*B*
Log[e*((a + b*x)/(c + d*x))^n]*(-4*A + 5*B*n + 4*B*n*Log[(a + b*x)/(c + d*x
)])) - 6*b^2*d*(a + b*x)*(c + d*x)^2*(2*A^2 - 2*A*B*n + 5*B^2*n^2 + 2*B^2*L
og[e*((a + b*x)/(c + d*x))^n]^2 + 2*B*n*(-2*A + B*n)*Log[(a + b*x)/(c + d*x
)] + 2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 - 2*B*Log[e*((a + b*x)/(c + d*x))
^n]*(-2*A + B*n + 2*B*n*Log[(a + b*x)/(c + d*x)]))*Log[c + d*x])/((b*c - a*
d)^4*g^2*i^3*(a + b*x)*(c + d*x)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1999 vs. 2(556) = 1112.

Time = 28.47 (sec) , antiderivative size = 2000, normalized size of antiderivative = 3.56

method	result	size
parallelrisch	Expression too large to display	2000

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x,method=
_RETURNVERBOSE)
```

```
[Out] -1/4*(-36*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*d^7*n-48*A*B*x*ln(e*((b*x
+a)/(d*x+c))^n)*a*b^5*c*d^6*n+4*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^3*b^6*d^7
+12*A^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^6*d^7+24*A*B*a^2*b^4*c*d^6*n^2-30*A
*B*a*b^5*c^2*d^5*n^2+B^2*a^3*b^3*d^7*n^3+8*B^2*b^6*c^3*d^4*n^3+2*A^2*a^3*b^
3*d^7*n+4*A^2*b^6*c^3*d^4*n-12*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^6*d^7*n-
18*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^5*d^7*n+42*B^2*x^2*ln(e*((b*x+a)
/(d*x+c))^n)*a*b^5*d^7*n^2+48*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c*d^6*n
^2+12*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^5*d^7+24*A*B*x^2*ln(e*((b*x+a
)/(d*x+c))^n)^2*b^6*c*d^6+12*A*B*x^2*a*b^5*d^7*n^2-12*A*B*x^2*b^6*c*d^6*n^2
+8*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*a*b^5*c*d^6-6*B^2*x*ln(e*((b*x+a)/(d*x
+c))^n)^2*a^2*b^4*d^7*n+12*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^6*c^2*d^5*n+
18*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*d^7*n^2+24*B^2*x*ln(e*((b*x+a)/(
d*x+c))^n)*b^6*c^2*d^5*n^2-18*B^2*x*a*b^5*c*d^6*n^3+12*A*B*x*ln(e*((b*x+a)/
(d*x+c))^n)^2*b^6*c^2*d^5+18*A*B*x*a^2*b^4*d^7*n^2-6*A*B*x*b^6*c^2*d^5*n^2-
12*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^4*c*d^6*n+24*B^2*ln(e*((b*x+a)/(d*
x+c))^n)*a^2*b^4*c*d^6*n^2+24*A^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c*d^6-1
2*A^2*x*a*b^5*c*d^6*n+12*A*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^5*c^2*d^5+4*A*
B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^3*d^7*n+8*A*B*ln(e*((b*x+a)/(d*x+c))^n)*b
^6*c^3*d^4*n-6*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b^6*d^7*n+30*B^2*x^3*ln(
e*((b*x+a)/(d*x+c))^n)*b^6*d^7*n^2+12*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b
^6*d^7+4*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a*b^5*d^7+8*B^2*x^2*ln(e*((b*x
+a)/(d*x+c))^n)^3*b^6*c*d^6-30*B^2*x^2*a*b^5*d^7*n^3+30*B^2*x^2*b^6*c*d^6*n
^3+4*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*b^6*c^2*d^5-21*B^2*x*a^2*b^4*d^7*n^3
+39*B^2*x*b^6*c^2*d^5*n^3+12*A^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*d^7+24
*A^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c*d^6-12*A^2*x^2*a*b^5*d^7*n+12*A^2*
x^2*b^6*c*d^6*n+4*B^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a*b^5*c^2*d^5+2*B^2*ln(e*
((b*x+a)/(d*x+c))^n)^2*a^3*b^3*d^7*n+4*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^6*
c^3*d^4*n-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^3*d^7*n^2+8*B^2*ln(e*((b*x+
a)/(d*x+c))^n)*b^6*c^3*d^4*n^2+12*A^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c^2*d
^5-6*A^2*x*a^2*b^4*d^7*n+18*A^2*x*b^6*c^2*d^5*n+12*A^2*ln(e*((b*x+a)/(d*x+c
))^n)*a*b^5*c^2*d^5-24*B^2*a^2*b^4*c*d^6*n^3+15*B^2*a*b^5*c^2*d^5*n^3-2*A*B
*a^3*b^3*d^7*n^2+8*A*B*b^6*c^3*d^4*n^2-12*A^2*a^2*b^4*c*d^6*n+6*A^2*a*b^5*c
^2*d^5*n-24*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^5*c*d^6*n+48*B^2*x*ln(e*(
(b*x+a)/(d*x+c))^n)*a*b^5*c*d^6*n^2+24*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*
```

```
b^5*c*d^6-12*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*d^7*n+24*A*B*x*ln(e*((
b*x+a)/(d*x+c))^n)*b^6*c^2*d^5*n-12*A*B*x*a*b^5*c*d^6*n^2-24*A*B*ln(e*((b*x
+a)/(d*x+c))^n)*a^2*b^4*c*d^6*n)/i^3/g^2/(d*x+c)^2/(b*x+a)/(a^3*d^3-3*a^2*b
*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(a*d-b*c)/b^3/d^4/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2057 vs. $2(556) = 1112$.

Time = 0.35 (sec) , antiderivative size = 2057, normalized size of antiderivative = 3.66

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x,
 algorithm="fricas")
```

```
[Out] -1/4*(4*A^2*b^3*c^3 + 6*A^2*a*b^2*c^2*d - 12*A^2*a^2*b*c*d^2 + 2*A^2*a^3*d^
3 + 4*(B^2*b^3*d^3*n^2*x^3 + B^2*a*b^2*c^2*d*n^2 + (2*B^2*b^3*c*d^2 + B^2*a
*b^2*d^3)*n^2*x^2 + (B^2*b^3*c^2*d + 2*B^2*a*b^2*c*d^2)*n^2*x)*log((b*x + a
)/(d*x + c))^3 + (8*B^2*b^3*c^3 + 15*B^2*a*b^2*c^2*d - 24*B^2*a^2*b*c*d^2 +
 B^2*a^3*d^3)*n^2 + 6*(2*A^2*b^3*c*d^2 - 2*A^2*a*b^2*d^3 + 5*(B^2*b^3*c*d^2
 - B^2*a*b^2*d^3)*n^2 - 2*(A*B*b^3*c*d^2 - A*B*a*b^2*d^3)*n)*x^2 + 2*(2*B^2
*b^3*c^3 + 3*B^2*a*b^2*c^2*d - 6*B^2*a^2*b*c*d^2 + B^2*a^3*d^3 + 6*(B^2*b^3
*c*d^2 - B^2*a*b^2*d^3)*x^2 + 3*(3*B^2*b^3*c^2*d - 2*B^2*a*b^2*c*d^2 - B^2*
a^2*b*d^3)*x + 6*(B^2*b^3*d^3*x^3 + B^2*a*b^2*c^2*d + (2*B^2*b^3*c*d^2 + B^
2*a*b^2*d^3)*x^2 + (B^2*b^3*c^2*d + 2*B^2*a*b^2*c*d^2)*x)*log((b*x + a)/(d*
x + c)))*log(e)^2 + 2*(6*A*B*a*b^2*c^2*d*n - 3*(B^2*b^3*d^3*n^2 - 2*A*B*b^3
*d^3*n)*x^3 + (2*B^2*b^3*c^3 - 6*B^2*a^2*b*c*d^2 + B^2*a^3*d^3)*n^2 - 3*(3*
B^2*a*b^2*d^3*n^2 - 2*(2*A*B*b^3*c*d^2 + A*B*a*b^2*d^3)*n)*x^2 + 3*((2*B^2*
b^3*c^2*d - 4*B^2*a*b^2*c*d^2 - B^2*a^2*b*d^3)*n^2 + 2*(A*B*b^3*c^2*d + 2*A
*B*a*b^2*c*d^2)*n)*x)*log((b*x + a)/(d*x + c))^2 + 2*(4*A*B*b^3*c^3 - 15*A*
B*a*b^2*c^2*d + 12*A*B*a^2*b*c*d^2 - A*B*a^3*d^3)*n + 3*(6*A^2*b^3*c^2*d -
4*A^2*a*b^2*c*d^2 - 2*A^2*a^2*b*d^3 + (13*B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2
 - 7*B^2*a^2*b*d^3)*n^2 - 2*(A*B*b^3*c^2*d + 2*A*B*a*b^2*c*d^2 - 3*A*B*a^2*
b*d^3)*n)*x + 2*(4*A*B*b^3*c^3 + 6*A*B*a*b^2*c^2*d - 12*A*B*a^2*b*c*d^2 + 2
*A*B*a^3*d^3 + 6*(2*A*B*b^3*c*d^2 - 2*A*B*a*b^2*d^3 - (B^2*b^3*c*d^2 - B^2*
a*b^2*d^3)*n)*x^2 + 6*(B^2*b^3*d^3*n*x^3 + B^2*a*b^2*c^2*d*n + (2*B^2*b^3*c
*d^2 + B^2*a*b^2*d^3)*n*x^2 + (B^2*b^3*c^2*d + 2*B^2*a*b^2*c*d^2)*n*x)*log(
(b*x + a)/(d*x + c))^2 + (4*B^2*b^3*c^3 - 15*B^2*a*b^2*c^2*d + 12*B^2*a^2*b
*c*d^2 - B^2*a^3*d^3)*n + 3*(6*A*B*b^3*c^2*d - 4*A*B*a*b^2*c*d^2 - 2*A*B*a^
2*b*d^3 - (B^2*b^3*c^2*d + 2*B^2*a*b^2*c*d^2 - 3*B^2*a^2*b*d^3)*n)*x + 2*(6
*A*B*a*b^2*c^2*d - 3*(B^2*b^3*d^3*n - 2*A*B*b^3*d^3)*x^3 - 3*(3*B^2*a*b^2*d
^3*n - 4*A*B*b^3*c*d^2 - 2*A*B*a*b^2*d^3)*x^2 + (2*B^2*b^3*c^3 - 6*B^2*a^2*
b*c*d^2 + B^2*a^3*d^3)*n + 3*(2*A*B*b^3*c^2*d + 4*A*B*a*b^2*c*d^2 + (2*B^2*
```

```
b^3*c^2*d - 4*B^2*a*b^2*c*d^2 - B^2*a^2*b*d^3)*n)*x)*log((b*x + a)/(d*x + c
)))*log(e) + 2*(6*A^2*a*b^2*c^2*d + 3*(5*B^2*b^3*d^3*n^2 - 2*A*B*b^3*d^3*n
+ 2*A^2*b^3*d^3)*x^3 + (4*B^2*b^3*c^3 + 12*B^2*a^2*b*c*d^2 - B^2*a^3*d^3)*n
^2 - 3*(6*A*B*a*b^2*d^3*n - 4*A^2*b^3*c*d^2 - 2*A^2*a*b^2*d^3 - (8*B^2*b^3*
c*d^2 + 7*B^2*a*b^2*d^3)*n^2)*x^2 + 2*(2*A*B*b^3*c^3 - 6*A*B*a^2*b*c*d^2 +
A*B*a^3*d^3)*n + 3*(2*A^2*b^3*c^2*d + 4*A^2*a*b^2*c*d^2 + (4*B^2*b^3*c^2*d
+ 8*B^2*a*b^2*c*d^2 + 3*B^2*a^2*b*d^3)*n^2 + 2*(2*A*B*b^3*c^2*d - 4*A*B*a*b
^2*c*d^2 - A*B*a^2*b*d^3)*n)*x)*log((b*x + a)/(d*x + c)))/((b^5*c^4*d^2 - 4
*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b*d^6)*g^2*i^3*x
^3 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2*c^2*d^4
 - 2*a^4*b*c*d^5 + a^5*d^6)*g^2*i^3*x^2 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*a^2*
b^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*g^2*i^3*x
+ (a*b^4*c^6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*
c^2*d^4)*g^2*i^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**2/(d*i*x+c*i)**3
,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4199 vs. $2(556)=1112$.

Time = 0.52 (sec) , antiderivative size = 4199, normalized size of antiderivative = 7.47

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x,
 algorithm="maxima")
```

```
[Out] -1/2*B^2*((6*b^2*d^2*x^2 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d +
 a*b*d^2)*x)/((b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)
*g^2*i^3*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c
*d^4 - a^4*d^5)*g^2*i^3*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 +
5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*g^2*i^3*x + (a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3
*a^3*b*c^3*d^2 - a^4*c^2*d^3)*g^2*i^3) + 6*b^2*d*log(b*x + a)/((b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3) - 6*b^
```

2*d*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*
d^3 + a^4*d^4)*g^2*i^3))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2 - A*B*((6
*b^2*d^2*x^2 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a*b*d^2)*x)
/((b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*g^2*i^3*x^3
 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d
^5)*g^2*i^3*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*
d^3 - 2*a^4*c*d^4)*g^2*i^3*x + (a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d
^2 - a^4*c^2*d^3)*g^2*i^3) + 6*b^2*d*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d
 + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3) - 6*b^2*d*log(d*x
+ c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^
4)*g^2*i^3))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1/4*((8*b^3*c^3 + 15*
a*b^2*c^2*d - 24*a^2*b*c*d^2 + a^3*d^3 + 4*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*
b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(b*x + a)^3
- 4*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d
 + 2*a*b^2*c*d^2)*x)*log(d*x + c)^3 + 30*(b^3*c*d^2 - a*b^2*d^3)*x^2 + 6*(b
^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a
*b^2*c*d^2)*x)*log(b*x + a)^2 + 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2
 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x + 2*(b^3*d^3*x^3 + a*b^2*
c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(
b*x + a))*log(d*x + c)^2 + 3*(13*b^3*c^2*d - 6*a*b^2*c*d^2 - 7*a^2*b*d^3)*x
 + 30*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2
*d + 2*a*b^2*c*d^2)*x)*log(b*x + a) - 6*(5*b^3*d^3*x^3 + 5*a*b^2*c^2*d + 5*
(2*b^3*c*d^2 + a*b^2*d^3)*x^2 + 2*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2
 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(b*x + a)^2 + 5*(b^3*
c^2*d + 2*a*b^2*c*d^2)*x + 2*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*
b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(b*x + a))*log(d*x + c))*n
^2/(a*b^4*c^6*g^2*i^3 - 4*a^2*b^3*c^5*d*g^2*i^3 + 6*a^3*b^2*c^4*d^2*g^2*i^3
 - 4*a^4*b*c^3*d^3*g^2*i^3 + a^5*c^2*d^4*g^2*i^3 + (b^5*c^4*d^2*g^2*i^3 - 4
*a*b^4*c^3*d^3*g^2*i^3 + 6*a^2*b^3*c^2*d^4*g^2*i^3 - 4*a^3*b^2*c*d^5*g^2*i^
3 + a^4*b*d^6*g^2*i^3)*x^3 + (2*b^5*c^5*d*g^2*i^3 - 7*a*b^4*c^4*d^2*g^2*i^3
 + 8*a^2*b^3*c^3*d^3*g^2*i^3 - 2*a^3*b^2*c^2*d^4*g^2*i^3 - 2*a^4*b*c*d^5*g^
2*i^3 + a^5*d^6*g^2*i^3)*x^2 + (b^5*c^6*g^2*i^3 - 2*a*b^4*c^5*d*g^2*i^3 - 2
*a^2*b^3*c^4*d^2*g^2*i^3 + 8*a^3*b^2*c^3*d^3*g^2*i^3 - 7*a^4*b*c^2*d^4*g^2*
i^3 + 2*a^5*c*d^5*g^2*i^3)*x) + 2*(4*b^3*c^3 - 15*a*b^2*c^2*d + 12*a^2*b*c*
d^2 - a^3*d^3 - 6*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 6*(b^3*d^3*x^3 + a*b^2*c^2*
d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(b*x
+ a)^2 - 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^
3*c^2*d + 2*a*b^2*c*d^2)*x)*log(d*x + c)^2 - 3*(b^3*c^2*d + 2*a*b^2*c*d^2 -
 3*a^2*b*d^3)*x - 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*
x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(b*x + a) + 6*(b^3*d^3*x^3 + a*b^2*
c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x + 2*(
b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*
a*b^2*c*d^2)*x)*log(b*x + a))*log(d*x + c))*n*log(e*(b*x/(d*x + c) + a/(d*x
 + c))^n)/(a*b^4*c^6*g^2*i^3 - 4*a^2*b^3*c^5*d*g^2*i^3 + 6*a^3*b^2*c^4*d^2*
g^2*i^3 - 4*a^4*b*c^3*d^3*g^2*i^3 + a^5*c^2*d^4*g^2*i^3 + (b^5*c^4*d^2*g^2*

```
i^3 - 4*a*b^4*c^3*d^3*g^2*i^3 + 6*a^2*b^3*c^2*d^4*g^2*i^3 - 4*a^3*b^2*c*d^5
*g^2*i^3 + a^4*b*d^6*g^2*i^3)*x^3 + (2*b^5*c^5*d*g^2*i^3 - 7*a*b^4*c^4*d^2*
g^2*i^3 + 8*a^2*b^3*c^3*d^3*g^2*i^3 - 2*a^3*b^2*c^2*d^4*g^2*i^3 - 2*a^4*b*c
*d^5*g^2*i^3 + a^5*d^6*g^2*i^3)*x^2 + (b^5*c^6*g^2*i^3 - 2*a*b^4*c^5*d*g^2*
i^3 - 2*a^2*b^3*c^4*d^2*g^2*i^3 + 8*a^3*b^2*c^3*d^3*g^2*i^3 - 7*a^4*b*c^2*d
^4*g^2*i^3 + 2*a^5*c*d^5*g^2*i^3)*x))*B^2 - 1/2*(4*b^3*c^3 - 15*a*b^2*c^2*d
 + 12*a^2*b*c*d^2 - a^3*d^3 - 6*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 6*(b^3*d^3*x^
3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^
2)*x)*log(b*x + a)^2 - 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*
d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(d*x + c)^2 - 3*(b^3*c^2*d + 2
*a*b^2*c*d^2 - 3*a^2*b*d^3)*x - 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2
 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(b*x + a) + 6*(b^3*d^
3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*
c*d^2)*x + 2*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (
b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(b*x + a))*log(d*x + c))*A*B*n/(a*b^4*c^6*
g^2*i^3 - 4*a^2*b^3*c^5*d*g^2*i^3 + 6*a^3*b^2*c^4*d^2*g^2*i^3 - 4*a^4*b*c^3
*d^3*g^2*i^3 + a^5*c^2*d^4*g^2*i^3 + (b^5*c^4*d^2*g^2*i^3 - 4*a*b^4*c^3*d^3
*g^2*i^3 + 6*a^2*b^3*c^2*d^4*g^2*i^3 - 4*a^3*b^2*c*d^5*g^2*i^3 + a^4*b*d^6*
g^2*i^3)*x^3 + (2*b^5*c^5*d*g^2*i^3 - 7*a*b^4*c^4*d^2*g^2*i^3 + 8*a^2*b^3*c
^3*d^3*g^2*i^3 - 2*a^3*b^2*c^2*d^4*g^2*i^3 - 2*a^4*b*c*d^5*g^2*i^3 + a^5*d^
6*g^2*i^3)*x^2 + (b^5*c^6*g^2*i^3 - 2*a*b^4*c^5*d*g^2*i^3 - 2*a^2*b^3*c^4*d
^2*g^2*i^3 + 8*a^3*b^2*c^3*d^3*g^2*i^3 - 7*a^4*b*c^2*d^4*g^2*i^3 + 2*a^5*c*
d^5*g^2*i^3)*x) - 1/2*A^2*((6*b^2*d^2*x^2 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2
 + 3*(3*b^2*c*d + a*b*d^2)*x)/((b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c
*d^4 - a^3*b*d^5)*g^2*i^3*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*
c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*g^2*i^3*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a
^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*g^2*i^3*x + (a*b^3*c^5 - 3*
a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3)*g^2*i^3) + 6*b^2*d*log(b*x +
 a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4
)*g^2*i^3) - 6*b^2*d*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2
*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3))
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2(dix+ci)^3}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x,
 algorithm="giac")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/((b*g*x + a*g)^2*(d*i*x
+ c*i)^3), x)
```

Mupad [B] (verification not implemented)

Time = 7.09 (sec) , antiderivative size = 1785, normalized size of antiderivative = 3.18

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx=\text{Too large to display}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^2*(c*i + d*i*x)
^3),x)
[Out] ((4*A^2*b^2*c^2 - 2*A^2*a^2*d^2 - B^2*a^2*d^2*n^2 + 8*B^2*b^2*c^2*n^2 + 10*
A^2*a*b*c*d + 2*A*B*a^2*d^2*n + 8*A*B*b^2*c^2*n + 23*B^2*a*b*c*d*n^2 - 22*A
*B*a*b*c*d*n)/(2*(a*d - b*c)) + (3*x^2*(2*A^2*b^2*d^2 + 5*B^2*b^2*d^2*n^2 -
 2*A*B*b^2*d^2*n))/(a*d - b*c) + (3*x*(2*A^2*a*b*d^2 + 6*A^2*b^2*c*d + 7*B^
2*a*b*d^2*n^2 + 13*B^2*b^2*c*d*n^2 - 6*A*B*a*b*d^2*n - 2*A*B*b^2*c*d*n))/(2
*(a*d - b*c)))/(x*(2*b^3*c^4*g^2*i^3 + 4*a^3*c*d^3*g^2*i^3 - 6*a^2*b*c^2*d^
2*g^2*i^3) + x^2*(2*a^3*d^4*g^2*i^3 + 4*b^3*c^3*d*g^2*i^3 - 6*a*b^2*c^2*d^2
*g^2*i^3) + x^3*(2*b^3*c^2*d^2*g^2*i^3 + 2*a^2*b*d^4*g^2*i^3 - 4*a*b^2*c*d^
3*g^2*i^3) + 2*a^3*c^2*d^2*g^2*i^3 + 2*a*b^2*c^4*g^2*i^3 - 4*a^2*b*c^3*d*g^
2*i^3) - log(e*((a + b*x)/(c + d*x))^n)^2*(((B^2*(a*d + 2*b*c))/(2*(a^2*d^2
 + b^2*c^2 - 2*a*b*c*d)) + (3*B^2*b*d*x)/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)
))/(x*(b*c^2*g^2*i^3 + 2*a*c*d*g^2*i^3) + x^2*(a*d^2*g^2*i^3 + 2*b*c*d*g^2*
i^3) + a*c^2*g^2*i^3 + b*d^2*g^2*i^3*x^3) + (3*B*b^2*d*(2*A - B*n))/(2*g^2*
i^3*n*(a*d - b*c)^4) - (3*B^2*b^2*d*(d*g^2*i^3*n*x^2*(a*d - b*c) + (a*c*g^2
*i^3*n*(a*d - b*c))/b + (g^2*i^3*n*x*(a*d + b*c)*(a*d - b*c))/b))/(g^2*i^3*
n*(a*d - b*c)^4*(x*(b*c^2*g^2*i^3 + 2*a*c*d*g^2*i^3) + x^2*(a*d^2*g^2*i^3 +
 2*b*c*d*g^2*i^3) + a*c^2*g^2*i^3 + b*d^2*g^2*i^3*x^3))) - log(e*((a + b*x)
/(c + d*x))^n)*((x*((3*B^2*b*d*n)/2 + 3*A*B*b*d) - (B^2*a*d*n)/2 + 2*B^2*b*
c*n + A*B*a*d + 2*A*B*b*c)/(x*(b^3*c^4*g^2*i^3 + 2*a^3*c*d^3*g^2*i^3 - 3*a^
2*b*c^2*d^2*g^2*i^3) + x^2*(a^3*d^4*g^2*i^3 + 2*b^3*c^3*d*g^2*i^3 - 3*a*b^2
*c^2*d^2*g^2*i^3) + x^3*(b^3*c^2*d^2*g^2*i^3 + a^2*b*d^4*g^2*i^3 - 2*a*b^2*
c*d^3*g^2*i^3) + a^3*c^2*d^2*g^2*i^3 + a*b^2*c^4*g^2*i^3 - 2*a^2*b*c^3*d*g^
2*i^3) - (3*B*b^2*d*(2*A - B*n)*(d*g^2*i^3*n*x^2*(a*d - b*c)^3 + (g^2*i^3*n
*x*(a*d + b*c)*(a*d - b*c)^3)/b + (a*c*g^2*i^3*n*(a*d - b*c)^3)/b))/(g^2*i^
3*n*(a*d - b*c)^4*(x*(b^3*c^4*g^2*i^3 + 2*a^3*c*d^3*g^2*i^3 - 3*a^2*b*c^2*d
^2*g^2*i^3) + x^2*(a^3*d^4*g^2*i^3 + 2*b^3*c^3*d*g^2*i^3 - 3*a*b^2*c^2*d^2*
g^2*i^3) + x^3*(b^3*c^2*d^2*g^2*i^3 + a^2*b*d^4*g^2*i^3 - 2*a*b^2*c*d^3*g^2
*i^3) + a^3*c^2*d^2*g^2*i^3 + a*b^2*c^4*g^2*i^3 - 2*a^2*b*c^3*d*g^2*i^3)))
+ (b^2*d*atan((b^2*d*(2*A^2 + 5*B^2*n^2 - 2*A*B*n)*(2*a^4*d^4*g^2*i^3 - 2*b
^4*c^4*g^2*i^3 + 4*a*b^3*c^3*d*g^2*i^3 - 4*a^3*b*c*d^3*g^2*i^3)*3i)/(2*g^2*
i^3*(a*d - b*c)^4*(6*A^2*b^2*d + 15*B^2*b^2*d*n^2 - 6*A*B*b^2*d*n)) + (b^3*
d^2*x*(2*A^2 + 5*B^2*n^2 - 2*A*B*n)*(a^3*d^3*g^2*i^3 - b^3*c^3*g^2*i^3 + 3*
a*b^2*c^2*d*g^2*i^3 - 3*a^2*b*c*d^2*g^2*i^3)*6i)/(g^2*i^3*(a*d - b*c)^4*(6*
A^2*b^2*d + 15*B^2*b^2*d*n^2 - 6*A*B*b^2*d*n)))*(2*A^2 + 5*B^2*n^2 - 2*A*B*
n)*3i)/(g^2*i^3*(a*d - b*c)^4) - (B^2*b^2*d*log(e*((a + b*x)/(c + d*x))^n)^
3)/(g^2*i^3*n*(a*d - b*c)^4)
```

3.208 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 732

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \frac{B^2d^4n^2(a+bx)^2}{4(bc-ad)^5g^3i^3(c+dx)^2}+\frac{8AbBd^3n(a+bx)}{(bc-ad)^5g^3i^3(c+dx)} - \frac{8bB^2d^3n^2(a+bx)}{(bc-ad)^5g^3i^3(c+dx)}+\frac{8b^3B^2dn^2(c+dx)}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4B^2n^2(c+dx)^2}{4(bc-ad)^5g^3i^3(a+bx)^2} + \frac{8bB^2d^3n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^5g^3i^3(c+dx)} - \frac{Bd^4n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^5g^3i^3(c+dx)^2} + \frac{8b^3Bdn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4Bn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^5g^3i^3(a+bx)^2} + \frac{d^4(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^5g^3i^3(c+dx)^2} - \frac{4bd^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^3i^3(c+dx)} + \frac{4b^3d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^5g^3i^3(a+bx)^2} + \frac{2b^2d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{B(bc-ad)^5g^3i^3n}$$

```
[Out] 1/4*B^2*d^4*n^2*(b*x+a)^2/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2+8*A*b*B*d^3*n*(b*x
+a)/(-a*d+b*c)^5/g^3/i^3/(d*x+c)-8*b*B^2*d^3*n^2*(b*x+a)/(-a*d+b*c)^5/g^3/i
^3/(d*x+c)+8*b^3*B^2*d*n^2*(d*x+c)/(-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/4*b^4*B^2
*n^2*(d*x+c)^2/(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2+8*b*B^2*d^3*n*(b*x+a)*ln(e*((
b*x+a)/(d*x+c))^n)/(-a*d+b*c)^5/g^3/i^3/(d*x+c)-1/2*B*d^4*n*(b*x+a)^2*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2+8*b^3*B*d*n*(d*x+
c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/2*b^4*B*n
*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2+1
/2*d^4*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^5/g^3/i^3/(d*
x+c)^2-4*b*d^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^5/g^3/i
^3/(d*x+c)+4*b^3*d*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^5/g
```

```
^3/i^3/(b*x+a)-1/2*b^4*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*
c)^5/g^3/i^3/(b*x+a)^2+2*b^2*d^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+
b*c)^5/g^3/i^3/n
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 732, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.178$, Rules used = {2561, 2395, 2333, 2332, 2342, 2341, 2339, 30}

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx=&-\frac{b^4(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g^3i^3(a+bx)^2(bc-ad)^5}\\&-\frac{b^4Bn(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^3i^3(a+bx)^2(bc-ad)^5}\\&+\frac{4b^3d(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^3i^3(a+bx)(bc-ad)^5}\\&+\frac{8b^3Bdn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^3i^3(a+bx)(bc-ad)^5}\\&+\frac{2b^2d^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{Bg^3i^3n(bc-ad)^5}\\&+\frac{d^4(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g^3i^3(c+dx)^2(bc-ad)^5}\\&-\frac{Bd^4n(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^3i^3(c+dx)^2(bc-ad)^5}\\&-\frac{4bd^3(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^3i^3(c+dx)(bc-ad)^5}\\&+\frac{8AbBd^3n(a+bx)}{g^3i^3(c+dx)(bc-ad)^5}-\frac{b^4B^2n^2(c+dx)^2}{4g^3i^3(a+bx)^2(bc-ad)^5}\\&+\frac{8b^3B^2dn^2(c+dx)}{g^3i^3(a+bx)(bc-ad)^5}+\frac{B^2d^4n^2(a+bx)^2}{4g^3i^3(c+dx)^2(bc-ad)^5}\\&+\frac{8bB^2d^3n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g^3i^3(c+dx)(bc-ad)^5}\\&-\frac{8bB^2d^3n^2(a+bx)}{g^3i^3(c+dx)(bc-ad)^5}\end{aligned}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^3*(c*i + d*i*x)
^3),x]
```

```
[Out] (B^2*d^4*n^2*(a + b*x)^2)/(4*(b*c - a*d)^5*g^3*i^3*(c + d*x)^2) + (8*A*b*B*
d^3*n*(a + b*x))/((b*c - a*d)^5*g^3*i^3*(c + d*x)) - (8*b*B^2*d^3*n^2*(a +
```

```
b*x))/((b*c - a*d)^5*g^3*i^3*(c + d*x)) + (8*b^3*B^2*d*n^2*(c + d*x))/((b*c
 - a*d)^5*g^3*i^3*(a + b*x)) - (b^4*B^2*n^2*(c + d*x)^2)/(4*(b*c - a*d)^5*g
^3*i^3*(a + b*x)^2) + (8*b*B^2*d^3*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^
n])/((b*c - a*d)^5*g^3*i^3*(c + d*x)) - (B*d^4*n*(a + b*x)^2*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]))/(2*(b*c - a*d)^5*g^3*i^3*(c + d*x)^2) + (8*b^3*B*
d*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^5*g^3*i^
3*(a + b*x)) - (b^4*B*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
/(2*(b*c - a*d)^5*g^3*i^3*(a + b*x)^2) + (d^4*(a + b*x)^2*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n])^2)/(2*(b*c - a*d)^5*g^3*i^3*(c + d*x)^2) - (4*b*d^3*(
a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*c - a*d)^5*g^3*i^3*(
c + d*x)) + (4*b^3*d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((
b*c - a*d)^5*g^3*i^3*(a + b*x)) - (b^4*(c + d*x)^2*(A + B*Log[e*((a + b*x)/
(c + d*x))^n])^2)/(2*(b*c - a*d)^5*g^3*i^3*(a + b*x)^2) + (2*b^2*d^2*(A + B
*Log[e*((a + b*x)/(c + d*x))^n])^3)/(B*(b*c - a*d)^5*g^3*i^3*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
```

```
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(b-dx)^4(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}$$

$$= \frac{\text{Subst}\left(\int \left(-4bd^3(A+B\log(ex^n))^2 + \frac{b^4(A+B\log(ex^n))^2}{x^3} - \frac{4b^3d(A+B\log(ex^n))^2}{x^2} + \frac{6b^2d^2(A+B\log(ex^n))^2}{x} + d^4x\right.\right.}{(bc-ad)^5g^3i^3}$$

$$= \frac{b^4\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3} - \frac{(4b^3d)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3} + \frac{(6b^2d^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3} - \frac{(4bd^3)\,\text{Subst}\left(\int (A+B\log(ex^n))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3} + \frac{d^4\text{Subst}\left(\int x(A+B\log(ex^n))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}$$

$$= \frac{d^4(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^5g^3i^3(c+dx)^2} - \frac{4bd^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^3i^3(c+dx)} + \frac{4b^3d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^5g^3i^3(a+bx)^2} + \frac{(6b^2d^2)\,\text{Subst}\left(\int x^2\,dx, x, A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)^5g^3i^3n} + \frac{(b^4Bn)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3} - \frac{(8b^3Bdn)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3} + \frac{(8bBd^3n)\,\text{Subst}\left(\int (A+B\log(ex^n))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3} - \frac{(Bd^4n)\,\text{Subst}\left(\int x(A+B\log(ex^n))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}$$

$$= \frac{B^2d^4n^2(a+bx)^2}{4(bc-ad)^5g^3i^3(c+dx)^2} + \frac{8AbBd^3n(a+bx)}{(bc-ad)^5g^3i^3(c+dx)} + \frac{8b^3B^2dn^2(c+dx)}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4B^2n^2(c+dx)^2}{4(bc-ad)^5g^3i^3(a+bx)^2} - \frac{Bd^4n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^5g^3i^3(c+dx)^2} + \frac{8b^3Bdn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4Bn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^5g^3i^3(a+bx)^2} + \frac{d^4(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^5g^3i^3(c+dx)^2} - \frac{4bd^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^3i^3(c+dx)} + \frac{4b^3d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^5g^3i^3(a+bx)^2} + \frac{2b^2d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{B(bc-ad)^5g^3i^3n} + \frac{(8bB^2d^3n)\,\text{Subst}\left(\int \log(ex^n)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}$$

$$= \frac{B^2d^4n^2(a+bx)^2}{4(bc-ad)^5g^3i^3(c+dx)^2} + \frac{8AbBd^3n(a+bx)}{(bc-ad)^5g^3i^3(c+dx)} - \frac{8bB^2d^3n^2(a+bx)}{(bc-ad)^5g^3i^3(c+dx)} + \frac{8b^3B^2dn^2(c+dx)}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4B^2n^2(c+dx)^2}{4(bc-ad)^5g^3i^3(a+bx)^2} + \frac{8bB^2d^3n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^5g^3i^3(c+dx)} - \frac{Bd^4n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^5g^3i^3(c+dx)^2} + \frac{8b^3Bdn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4Bn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^5g^3i^3(a+bx)^2} + \frac{d^4(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^5g^3i^3(c+dx)^2} - \frac{4bd^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^3i^3(c+dx)} + \frac{4b^3d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^5g^3i^3(a+bx)^2} + \frac{2b^2d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{B(bc-ad)^5g^3i^3n}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1653 vs. 2(732) = 1464.

Time = 1.17 (sec) , antiderivative size = 1653, normalized size of antiderivative = 2.26

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \frac{8b^2B^2d^2n^2(a+bx)^2(c+dx)^2\log^3\left(\frac{a+bx}{c+dx}\right) + 2Bn\log^2\left(\frac{a+bx}{c+dx}\right)\left(12a^2Ab^2c^2d^2 - b^4Bc^4n + 8ab^3Bc^3dn - 8a^3bB\right.}{}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^3*(c*i +
d*i*x)^3),x]
```

```
[Out] (8*b^2*B^2*d^2*n^2*(a + b*x)^2*(c + d*x)^2*Log[(a + b*x)/(c + d*x)]^3 + 2*B
*n*Log[(a + b*x)/(c + d*x)]^2*(12*a^2*A*b^2*c^2*d^2 - b^4*B*c^4*n + 8*a*b^3
*B*c^3*d*n - 8*a^3*b*B*c*d^3*n + a^4*B*d^4*n + 24*a*A*b^3*c^2*d^2*x + 24*a^
2*A*b^2*c*d^3*x + 4*b^4*B*c^3*d*n*x + 24*a*b^3*B*c^2*d^2*n*x - 24*a^2*b^2*B
*c*d^3*n*x - 4*a^3*b*B*d^4*n*x + 12*A*b^4*c^2*d^2*x^2 + 48*a*A*b^3*c*d^3*x^
2 + 12*a^2*A*b^2*d^4*x^2 + 18*b^4*B*c^2*d^2*n*x^2 - 18*a^2*b^2*B*d^4*n*x^2
+ 24*A*b^4*c*d^3*x^3 + 24*a*A*b^3*d^4*x^3 + 12*b^4*B*c*d^3*n*x^3 - 12*a*b^3
*B*d^4*n*x^3 + 12*A*b^4*d^4*x^4 + 12*b^2*B*d^2*(a + b*x)^2*(c + d*x)^2*Log[
e*((a + b*x)/(c + d*x))^n] - 12*b^2*B*d^2*n*(a + b*x)^2*(c + d*x)^2*Log[(a
+ b*x)/(c + d*x)]) + 12*b^2*d^2*(a + b*x)^2*(c + d*x)^2*Log[a + b*x]*(2*A^2
 + 5*B^2*n^2 + 4*A*B*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c +
 d*x)]) + 2*B^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)
])^2) + 2*b^2*d*(b*c - a*d)*(a + b*x)*(c + d*x)^2*(6*A^2 + 14*A*B*n + 15*B^
```

```
2*n^2 + 6*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 2*B*n*(6*A + 7*B*n)*Log[(a
 + b*x)/(c + d*x)] + 6*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 2*B*Log[e*((a +
 b*x)/(c + d*x))^n]*(6*A + 7*B*n - 6*B*n*Log[(a + b*x)/(c + d*x)])) - b^2*(
b*c - a*d)^2*(c + d*x)^2*(2*A^2 + 2*A*B*n + B^2*n^2 + 2*B^2*Log[e*((a + b*x
)/(c + d*x))^n]^2 - 2*B*n*(2*A + B*n)*Log[(a + b*x)/(c + d*x)] + 2*B^2*n^2*
Log[(a + b*x)/(c + d*x)]^2 + 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(2*A + B*n
- 2*B*n*Log[(a + b*x)/(c + d*x)])) + 2*B*(b*c - a*d)*n*Log[(a + b*x)/(c + d
*x)]*(2*b*d^2*(a + b*x)^2*(c + d*x)*(6*A - 7*B*n + 6*B*Log[e*((a + b*x)/(c
+ d*x))^n] - 6*B*n*Log[(a + b*x)/(c + d*x)]) + 2*b^2*d*(a + b*x)*(c + d*x)^
2*(6*A + 7*B*n + 6*B*Log[e*((a + b*x)/(c + d*x))^n] - 6*B*n*Log[(a + b*x)/(
c + d*x)]) + d^2*(b*c - a*d)*(a + b*x)^2*(2*A - B*n + 2*B*Log[e*((a + b*x)/
(c + d*x))^n] - 2*B*n*Log[(a + b*x)/(c + d*x)]) - b^2*(b*c - a*d)*(c + d*x)
^2*(2*A + B*n + 2*B*Log[e*((a + b*x)/(c + d*x))^n] - 2*B*n*Log[(a + b*x)/(c
 + d*x)])) + d^2*(b*c - a*d)^2*(a + b*x)^2*(2*A^2 - 2*A*B*n + B^2*n^2 + 2*B
^2*Log[e*((a + b*x)/(c + d*x))^n]^2 + 2*B*n*(-2*A + B*n)*Log[(a + b*x)/(c +
 d*x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 - 2*B*Log[e*((a + b*x)/(c + d
*x))^n]*(-2*A + B*n + 2*B*n*Log[(a + b*x)/(c + d*x)])) + 2*b*d^2*(b*c - a*d
)*(a + b*x)^2*(c + d*x)*(6*A^2 - 14*A*B*n + 15*B^2*n^2 + 6*B^2*Log[e*((a +
b*x)/(c + d*x))^n]^2 + 2*B*n*(-6*A + 7*B*n)*Log[(a + b*x)/(c + d*x)] + 6*B^
2*n^2*Log[(a + b*x)/(c + d*x)]^2 - 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(-6*A
 + 7*B*n + 6*B*n*Log[(a + b*x)/(c + d*x)])) - 12*b^2*d^2*(a + b*x)^2*(c + d
*x)^2*(2*A^2 + 5*B^2*n^2 + 4*A*B*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a
 + b*x)/(c + d*x)]) + 2*B^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*
x)/(c + d*x)])^2)*Log[c + d*x])/(4*(b*c - a*d)^5*g^3*i^3*(a + b*x)^2*(c + d
*x)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3005 vs. $2(720) = 1440$.

Time = 54.53 (sec) , antiderivative size = 3006, normalized size of antiderivative = 4.11

method	result	size
parallelrisch	Expression too large to display	3006

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x,method=
_RETURNVERBOSE)
```

```
[Out] -1/4*(-32*B^2*a^3*b^5*c*d^7*n^3+32*B^2*a*b^7*c^3*d^5*n^3-2*A*B*a^4*b^4*d^8*
n^2-2*A*B*b^8*c^4*d^4*n^2-16*A^2*a^3*b^5*c*d^7*n+16*A^2*a*b^7*c^3*d^5*n+16*
B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^3*b^8*c*d^7-60*B^2*x^3*a*b^7*d^8*n^3+60*B
^2*x^3*b^8*c*d^7*n^3+8*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a^2*b^6*d^8+8*B^
2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*b^8*c^2*d^6-90*B^2*x^2*a^2*b^6*d^8*n^3+90
*B^2*x^2*b^8*c^2*d^6*n^3+48*A^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^7*d^8+48*
A^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^8*c*d^7-24*A^2*x^3*a*b^7*d^8*n+24*A^2*x
^3*b^8*c*d^7*n-28*B^2*x*a^3*b^5*d^8*n^3+28*B^2*x*b^8*c^3*d^5*n^3+24*A^2*x^2
```

```
*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^6*d^8+24*A^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)
*b^8*c^2*d^6-36*A^2*x^2*a^2*b^6*d^8*n+36*A^2*x^2*b^8*c^2*d^6*n+8*B^2*ln(e*(
(b*x+a)/(d*x+c))^n)^3*a^2*b^6*c^2*d^6+2*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4
*b^4*d^8*n-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^8*c^4*d^4*n-2*B^2*ln(e*((b*x
+a)/(d*x+c))^n)*a^4*b^4*d^8*n^2-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)*b^8*c^4*d^4
*n^2-8*A^2*x*a^3*b^5*d^8*n+8*A^2*x*b^8*c^3*d^5*n+24*A^2*ln(e*((b*x+a)/(d*x+
c))^n)*a^2*b^6*c^2*d^6+60*B^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^8*d^8*n^2+24*
A*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)^2*b^8*d^8+16*B^2*x^3*ln(e*((b*x+a)/(d*x+c
))^n)^3*a*b^7*d^8-96*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^6*c*d^7*n+96*A*B
*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^7*c^2*d^6*n-48*A*B*x^3*ln(e*((b*x+a)/(d*x+
c))^n)*a*b^7*d^8*n+48*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^8*c*d^7*n+192*B^2
*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^7*c*d^7*n^2+96*A*B*x^2*ln(e*((b*x+a)/(d*
x+c))^n)^2*a*b^7*c*d^7-72*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^6*d^8*n+7
2*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^8*c^2*d^6*n-48*A*B*x^2*a*b^7*c*d^7*n^
2-48*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^6*c*d^7*n+48*B^2*x*ln(e*((b*x+
a)/(d*x+c))^n)^2*a*b^7*c^2*d^6*n+96*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^6
*c*d^7*n^2+96*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^7*c^2*d^6*n^2+48*A*B*x*ln
(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^6*c*d^7+48*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^
2*a*b^7*c^2*d^6-16*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^5*d^8*n+16*A*B*x*l
n(e*((b*x+a)/(d*x+c))^n)*b^8*c^3*d^5*n-24*A*B*x*a^2*b^6*c*d^7*n^2-24*A*B*x*
a*b^7*c^2*d^6*n^2-32*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^5*c*d^7*n+8*B^2*x^
4*ln(e*((b*x+a)/(d*x+c))^n)^3*b^8*d^8+24*A^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)*
b^8*d^8-24*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^7*d^8*n+24*B^2*x^3*ln(e*
((b*x+a)/(d*x+c))^n)^2*b^8*c*d^7*n+120*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*
b^7*d^8*n^2+120*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^8*c*d^7*n^2+48*A*B*x^3*
ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^7*d^8+48*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^
2*b^8*c*d^7+32*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a*b^7*c*d^7-36*B^2*x^2*l
n(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^6*d^8*n+36*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^
n)^2*b^8*c^2*d^6*n+84*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^6*d^8*n^2+84*
B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^8*c^2*d^6*n^2+24*A*B*x^2*ln(e*((b*x+a)/
(d*x+c))^n)^2*a^2*b^6*d^8+24*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^8*c^2*d^
6+24*A*B*x^2*a^2*b^6*d^8*n^2+24*A*B*x^2*b^8*c^2*d^6*n^2+16*B^2*x*ln(e*((b*x
+a)/(d*x+c))^n)^3*a^2*b^6*c*d^7+16*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*a*b^7*
c^2*d^6-8*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^5*d^8*n+8*B^2*x*ln(e*((b*
x+a)/(d*x+c))^n)^2*b^8*c^3*d^5*n+24*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^5
*d^8*n^2+24*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^8*c^3*d^5*n^2-96*B^2*x*a^2*b^
6*c*d^7*n^3+96*B^2*x*a*b^7*c^2*d^6*n^3+96*A^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)
*a*b^7*c*d^7+24*A*B*x*a^3*b^5*d^8*n^2+B^2*a^4*b^4*d^8*n^3-B^2*b^8*c^4*d^4*n
^3+2*A^2*a^4*b^4*d^8*n-2*A^2*b^8*c^4*d^4*n+32*A*B*ln(e*((b*x+a)/(d*x+c))^n)
*a*b^7*c^3*d^5*n+32*A*B*a^3*b^5*c*d^7*n^2-60*A*B*a^2*b^6*c^2*d^6*n^2+32*A*B
*a*b^7*c^3*d^5*n^2+24*A*B*x*b^8*c^3*d^5*n^2-16*B^2*ln(e*((b*x+a)/(d*x+c))^n
)^2*a^3*b^5*c*d^7*n+16*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^7*c^3*d^5*n+32*B
^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^5*c*d^7*n^2+32*B^2*ln(e*((b*x+a)/(d*x+c)
)^n)*a*b^7*c^3*d^5*n^2+48*A^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^6*c*d^7+48*
A^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^7*c^2*d^6-48*A^2*x*a^2*b^6*c*d^7*n+48*A
```

```
^2*x*a*b^7*c^2*d^6*n+24*A*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^6*c^2*d^6+4*A
*B*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^4*d^8*n-4*A*B*ln(e*((b*x+a)/(d*x+c))^n)*
b^8*c^4*d^4*n)/i^3/g^3/(d*x+c)^2/(b*x+a)^2/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2
*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/d^4/b^4/n/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3062 vs. $2(720) = 1440$.

Time = 0.39 (sec) , antiderivative size = 3062, normalized size of antiderivative = 4.18

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x,
 algorithm="fricas")
```

```
[Out] -1/4*(2*A^2*b^4*c^4 - 16*A^2*a*b^3*c^3*d + 16*A^2*a^3*b*c*d^3 - 2*A^2*a^4*d
^4 - 12*(2*A^2*b^4*c*d^3 - 2*A^2*a*b^3*d^4 + 5*(B^2*b^4*c*d^3 - B^2*a*b^3*d
^4)*n^2)*x^3 - 8*(B^2*b^4*d^4*n^2*x^4 + B^2*a^2*b^2*c^2*d^2*n^2 + 2*(B^2*b^
4*c*d^3 + B^2*a*b^3*d^4)*n^2*x^3 + (B^2*b^4*c^2*d^2 + 4*B^2*a*b^3*c*d^3 + B
^2*a^2*b^2*d^4)*n^2*x^2 + 2*(B^2*a*b^3*c^2*d^2 + B^2*a^2*b^2*c*d^3)*n^2*x)*
log((b*x + a)/(d*x + c))^3 + (B^2*b^4*c^4 - 32*B^2*a*b^3*c^3*d + 32*B^2*a^3
*b*c*d^3 - B^2*a^4*d^4)*n^2 - 6*(6*A^2*b^4*c^2*d^2 - 6*A^2*a^2*b^2*d^4 + 15
*(B^2*b^4*c^2*d^2 - B^2*a^2*b^2*d^4)*n^2 + 4*(A*B*b^4*c^2*d^2 - 2*A*B*a*b^3
*c*d^3 + A*B*a^2*b^2*d^4)*n)*x^2 + 2*(B^2*b^4*c^4 - 8*B^2*a*b^3*c^3*d + 8*B
^2*a^3*b*c*d^3 - B^2*a^4*d^4 - 12*(B^2*b^4*c*d^3 - B^2*a*b^3*d^4)*x^3 - 18*
(B^2*b^4*c^2*d^2 - B^2*a^2*b^2*d^4)*x^2 - 4*(B^2*b^4*c^3*d + 6*B^2*a*b^3*c^
2*d^2 - 6*B^2*a^2*b^2*c*d^3 - B^2*a^3*b*d^4)*x - 12*(B^2*b^4*d^4*x^4 + B^2*
a^2*b^2*c^2*d^2 + 2*(B^2*b^4*c*d^3 + B^2*a*b^3*d^4)*x^3 + (B^2*b^4*c^2*d^2
+ 4*B^2*a*b^3*c*d^3 + B^2*a^2*b^2*d^4)*x^2 + 2*(B^2*a*b^3*c^2*d^2 + B^2*a^2
*b^2*c*d^3)*x)*log((b*x + a)/(d*x + c)))*log(e)^2 - 2*(12*A*B*b^4*d^4*n*x^4
 + 12*A*B*a^2*b^2*c^2*d^2*n + 12*((B^2*b^4*c*d^3 - B^2*a*b^3*d^4)*n^2 + 2*(
A*B*b^4*c*d^3 + A*B*a*b^3*d^4)*n)*x^3 - (B^2*b^4*c^4 - 8*B^2*a*b^3*c^3*d +
8*B^2*a^3*b*c*d^3 - B^2*a^4*d^4)*n^2 + 6*(3*(B^2*b^4*c^2*d^2 - B^2*a^2*b^2*
d^4)*n^2 + 2*(A*B*b^4*c^2*d^2 + 4*A*B*a*b^3*c*d^3 + A*B*a^2*b^2*d^4)*n)*x^2
 + 4*((B^2*b^4*c^3*d + 6*B^2*a*b^3*c^2*d^2 - 6*B^2*a^2*b^2*c*d^3 - B^2*a^3*
b*d^4)*n^2 + 6*(A*B*a*b^3*c^2*d^2 + A*B*a^2*b^2*c*d^3)*n)*x)*log((b*x + a)/
(d*x + c))^2 + 2*(A*B*b^4*c^4 - 16*A*B*a*b^3*c^3*d + 30*A*B*a^2*b^2*c^2*d^2
 - 16*A*B*a^3*b*c*d^3 + A*B*a^4*d^4)*n - 4*(2*A^2*b^4*c^3*d + 12*A^2*a*b^3*
c^2*d^2 - 12*A^2*a^2*b^2*c*d^3 - 2*A^2*a^3*b*d^4 + (7*B^2*b^4*c^3*d + 24*B^
2*a*b^3*c^2*d^2 - 24*B^2*a^2*b^2*c*d^3 - 7*B^2*a^3*b*d^4)*n^2 + 6*(A*B*b^4*
c^3*d - A*B*a*b^3*c^2*d^2 - A*B*a^2*b^2*c*d^3 + A*B*a^3*b*d^4)*n)*x + 2*(2*
A*B*b^4*c^4 - 16*A*B*a*b^3*c^3*d + 16*A*B*a^3*b*c*d^3 - 2*A*B*a^4*d^4 - 24*
(A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*x^3 - 12*(3*A*B*b^4*c^2*d^2 - 3*A*B*a^2*b^2
```

```
*d^4 + (B^2*b^4*c^2*d^2 - 2*B^2*a*b^3*c*d^3 + B^2*a^2*b^2*d^4)*n)*x^2 - 12*
(B^2*b^4*d^4*n*x^4 + B^2*a^2*b^2*c^2*d^2*n + 2*(B^2*b^4*c*d^3 + B^2*a*b^3*d
^4)*n*x^3 + (B^2*b^4*c^2*d^2 + 4*B^2*a*b^3*c*d^3 + B^2*a^2*b^2*d^4)*n*x^2 +
 2*(B^2*a*b^3*c^2*d^2 + B^2*a^2*b^2*c*d^3)*n*x)*log((b*x + a)/(d*x + c))^2
+ (B^2*b^4*c^4 - 16*B^2*a*b^3*c^3*d + 30*B^2*a^2*b^2*c^2*d^2 - 16*B^2*a^3*b
*c*d^3 + B^2*a^4*d^4)*n - 4*(2*A*B*b^4*c^3*d + 12*A*B*a*b^3*c^2*d^2 - 12*A*
B*a^2*b^2*c*d^3 - 2*A*B*a^3*b*d^4 + 3*(B^2*b^4*c^3*d - B^2*a*b^3*c^2*d^2 -
B^2*a^2*b^2*c*d^3 + B^2*a^3*b*d^4)*n)*x - 2*(12*A*B*b^4*d^4*x^4 + 12*A*B*a^
2*b^2*c^2*d^2 + 12*(2*A*B*b^4*c*d^3 + 2*A*B*a*b^3*d^4 + (B^2*b^4*c*d^3 - B^
2*a*b^3*d^4)*n)*x^3 + 6*(2*A*B*b^4*c^2*d^2 + 8*A*B*a*b^3*c*d^3 + 2*A*B*a^2*
b^2*d^4 + 3*(B^2*b^4*c^2*d^2 - B^2*a^2*b^2*d^4)*n)*x^2 - (B^2*b^4*c^4 - 8*B
^2*a*b^3*c^3*d + 8*B^2*a^3*b*c*d^3 - B^2*a^4*d^4)*n + 4*(6*A*B*a*b^3*c^2*d^
2 + 6*A*B*a^2*b^2*c*d^3 + (B^2*b^4*c^3*d + 6*B^2*a*b^3*c^2*d^2 - 6*B^2*a^2*
b^2*c*d^3 - B^2*a^3*b*d^4)*n)*x)*log((b*x + a)/(d*x + c)))*log(e) - 2*(12*A
^2*a^2*b^2*c^2*d^2 + 6*(5*B^2*b^4*d^4*n^2 + 2*A^2*b^4*d^4)*x^4 + 12*(2*A^2*
b^4*c*d^3 + 2*A^2*a*b^3*d^4 + 5*(B^2*b^4*c*d^3 + B^2*a*b^3*d^4)*n^2 + 2*(A*
B*b^4*c*d^3 - A*B*a*b^3*d^4)*n)*x^3 - (B^2*b^4*c^4 - 16*B^2*a*b^3*c^3*d - 1
6*B^2*a^3*b*c*d^3 + B^2*a^4*d^4)*n^2 + 6*(2*A^2*b^4*c^2*d^2 + 8*A^2*a*b^3*c
*d^3 + 2*A^2*a^2*b^2*d^4 + (7*B^2*b^4*c^2*d^2 + 16*B^2*a*b^3*c*d^3 + 7*B^2*
a^2*b^2*d^4)*n^2 + 6*(A*B*b^4*c^2*d^2 - A*B*a^2*b^2*d^4)*n)*x^2 - 2*(A*B*b^
4*c^4 - 8*A*B*a*b^3*c^3*d + 8*A*B*a^3*b*c*d^3 - A*B*a^4*d^4)*n + 4*(6*A^2*a
*b^3*c^2*d^2 + 6*A^2*a^2*b^2*c*d^3 + 3*(B^2*b^4*c^3*d + 4*B^2*a*b^3*c^2*d^2
 + 4*B^2*a^2*b^2*c*d^3 + B^2*a^3*b*d^4)*n^2 + 2*(A*B*b^4*c^3*d + 6*A*B*a*b^
3*c^2*d^2 - 6*A*B*a^2*b^2*c*d^3 - A*B*a^3*b*d^4)*n)*x)*log((b*x + a)/(d*x +
 c)))/((b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^2
*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^7)*g^3*i^3*x^4 + 2*(b^7*c^6*d - 4*a*b^6*
c^5*d^2 + 5*a^2*b^5*c^4*d^3 - 5*a^4*b^3*c^2*d^5 + 4*a^5*b^2*c*d^6 - a^6*b*d
^7)*g^3*i^3*x^3 + (b^7*c^7 - a*b^6*c^6*d - 9*a^2*b^5*c^5*d^2 + 25*a^3*b^4*c
^4*d^3 - 25*a^4*b^3*c^3*d^4 + 9*a^5*b^2*c^2*d^5 + a^6*b*c*d^6 - a^7*d^7)*g^
3*i^3*x^2 + 2*(a*b^6*c^7 - 4*a^2*b^5*c^6*d + 5*a^3*b^4*c^5*d^2 - 5*a^5*b^2*
c^3*d^4 + 4*a^6*b*c^2*d^5 - a^7*c*d^6)*g^3*i^3*x + (a^2*b^5*c^7 - 5*a^3*b^4
*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^
2*d^5)*g^3*i^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^3(ci + dix)^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**3/(d*i*x+c*i)**3
,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5594 vs. 2(720) = 1440.

Time = 0.66 (sec) , antiderivative size = 5594, normalized size of antiderivative = 7.64

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx=\text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x,
 algorithm="maxima")
```

```
[Out] 1/2*B^2*((12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d^
3 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 + a^2*b*d
^3)*x)/((b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^
5 + a^4*b^2*d^6)*g^3*i^3*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4*c
^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*g^3*i^3*x^3 + (b^
6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d^
6)*g^3*i^3*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4
*b^2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*g^3*i^3*x + (a^2*b^4*c^6 - 4*a^
3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4)*g^3*i^3) +
 12*b^2*d^2*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 1
0*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3) - 12*b^2*d^2*log(d*x
+ c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 +
5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2
 + A*B*((12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d^3
 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 + a^2*b*d^
3)*x)/((b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5
 + a^4*b^2*d^6)*g^3*i^3*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4*c^
3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*g^3*i^3*x^3 + (b^6
*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d^6
)*g^3*i^3*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*
b^2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*g^3*i^3*x + (a^2*b^4*c^6 - 4*a^3
*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4)*g^3*i^3) +
12*b^2*d^2*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10
*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3) - 12*b^2*d^2*log(d*x +
 c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5
*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) -
1/4*((b^4*c^4 - 32*a*b^3*c^3*d + 32*a^3*b*c*d^3 - a^4*d^4 - 60*(b^4*c*d^3 -
 a*b^3*d^4)*x^3 - 8*(b^4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d
^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^
2 + a^2*b^2*c*d^3)*x)*log(b*x + a)^3 - 24*(b^4*d^4*x^4 + a^2*b^2*c^2*d^2 +
2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)
*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a)*log(d*x + c)^2 + 8
*(b^4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*
d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*
```

x)*log(d*x + c)^3 - 90*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^2 - 4*(7*b^4*c^3*d + 2
4*a*b^3*c^2*d^2 - 24*a^2*b^2*c*d^3 - 7*a^3*b*d^4)*x - 60*(b^4*d^4*x^4 + a^2
*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3
 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a) + 1
2*(5*b^4*d^4*x^4 + 5*a^2*b^2*c^2*d^2 + 10*(b^4*c*d^3 + a*b^3*d^4)*x^3 + 5*(
b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(b^4*d^4*x^4 + a^2*b^2*c
^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2
*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a)^2 + 10*(a
*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(d*x + c))*n^2/(a^2*b^5*c^7*g^3*i^3 - 5
*a^3*b^4*c^6*d*g^3*i^3 + 10*a^4*b^3*c^5*d^2*g^3*i^3 - 10*a^5*b^2*c^4*d^3*g^
3*i^3 + 5*a^6*b*c^3*d^4*g^3*i^3 - a^7*c^2*d^5*g^3*i^3 + (b^7*c^5*d^2*g^3*i^
3 - 5*a*b^6*c^4*d^3*g^3*i^3 + 10*a^2*b^5*c^3*d^4*g^3*i^3 - 10*a^3*b^4*c^2*d
^5*g^3*i^3 + 5*a^4*b^3*c*d^6*g^3*i^3 - a^5*b^2*d^7*g^3*i^3)*x^4 + 2*(b^7*c^
6*d*g^3*i^3 - 4*a*b^6*c^5*d^2*g^3*i^3 + 5*a^2*b^5*c^4*d^3*g^3*i^3 - 5*a^4*b
^3*c^2*d^5*g^3*i^3 + 4*a^5*b^2*c*d^6*g^3*i^3 - a^6*b*d^7*g^3*i^3)*x^3 + (b^
7*c^7*g^3*i^3 - a*b^6*c^6*d*g^3*i^3 - 9*a^2*b^5*c^5*d^2*g^3*i^3 + 25*a^3*b^
4*c^4*d^3*g^3*i^3 - 25*a^4*b^3*c^3*d^4*g^3*i^3 + 9*a^5*b^2*c^2*d^5*g^3*i^3
 + a^6*b*c*d^6*g^3*i^3 - a^7*d^7*g^3*i^3)*x^2 + 2*(a*b^6*c^7*g^3*i^3 - 4*a^2
*b^5*c^6*d*g^3*i^3 + 5*a^3*b^4*c^5*d^2*g^3*i^3 - 5*a^5*b^2*c^3*d^4*g^3*i^3
 + 4*a^6*b*c^2*d^5*g^3*i^3 - a^7*c*d^6*g^3*i^3)*x) + 2*(b^4*c^4 - 16*a*b^3*c
^3*d + 30*a^2*b^2*c^2*d^2 - 16*a^3*b*c*d^3 + a^4*d^4 - 12*(b^4*c^2*d^2 - 2*
a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 12*(b^4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4
*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 +
 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a)^2 - 24*(b^4*d^4*x^4 + a^
2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^
3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a)*lo
g(d*x + c) + 12*(b^4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*
x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 +
a^2*b^2*c*d^3)*x)*log(d*x + c)^2 - 12*(b^4*c^3*d - a*b^3*c^2*d^2 - a^2*b^2*
c*d^3 + a^3*b*d^4)*x)*n*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(a^2*b^5*c^7
*g^3*i^3 - 5*a^3*b^4*c^6*d*g^3*i^3 + 10*a^4*b^3*c^5*d^2*g^3*i^3 - 10*a^5*b^
2*c^4*d^3*g^3*i^3 + 5*a^6*b*c^3*d^4*g^3*i^3 - a^7*c^2*d^5*g^3*i^3 + (b^7*c^
5*d^2*g^3*i^3 - 5*a*b^6*c^4*d^3*g^3*i^3 + 10*a^2*b^5*c^3*d^4*g^3*i^3 - 10*a
^3*b^4*c^2*d^5*g^3*i^3 + 5*a^4*b^3*c*d^6*g^3*i^3 - a^5*b^2*d^7*g^3*i^3)*x^4
 + 2*(b^7*c^6*d*g^3*i^3 - 4*a*b^6*c^5*d^2*g^3*i^3 + 5*a^2*b^5*c^4*d^3*g^3*i
^3 - 5*a^4*b^3*c^2*d^5*g^3*i^3 + 4*a^5*b^2*c*d^6*g^3*i^3 - a^6*b*d^7*g^3*i^
3)*x^3 + (b^7*c^7*g^3*i^3 - a*b^6*c^6*d*g^3*i^3 - 9*a^2*b^5*c^5*d^2*g^3*i^3
 + 25*a^3*b^4*c^4*d^3*g^3*i^3 - 25*a^4*b^3*c^3*d^4*g^3*i^3 + 9*a^5*b^2*c^2*
d^5*g^3*i^3 + a^6*b*c*d^6*g^3*i^3 - a^7*d^7*g^3*i^3)*x^2 + 2*(a*b^6*c^7*g^3
*i^3 - 4*a^2*b^5*c^6*d*g^3*i^3 + 5*a^3*b^4*c^5*d^2*g^3*i^3 - 5*a^5*b^2*c^3*
d^4*g^3*i^3 + 4*a^6*b*c^2*d^5*g^3*i^3 - a^7*c*d^6*g^3*i^3)*x))*B^2 - 1/2*(b
^4*c^4 - 16*a*b^3*c^3*d + 30*a^2*b^2*c^2*d^2 - 16*a^3*b*c*d^3 + a^4*d^4 - 1
2*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 12*(b^4*d^4*x^4 + a^2*b
^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 +
 a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a)^2 - 2

```
4*(b^4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2
*d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)
*x)*log(b*x + a)*log(d*x + c) + 12*(b^4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*
c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 +
2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(d*x + c)^2 - 12*(b^4*c^3*d - a*b^3
*c^2*d^2 - a^2*b^2*c*d^3 + a^3*b*d^4)*x)*A*B*n/(a^2*b^5*c^7*g^3*i^3 - 5*a^3
*b^4*c^6*d*g^3*i^3 + 10*a^4*b^3*c^5*d^2*g^3*i^3 - 10*a^5*b^2*c^4*d^3*g^3*i^
3 + 5*a^6*b*c^3*d^4*g^3*i^3 - a^7*c^2*d^5*g^3*i^3 + (b^7*c^5*d^2*g^3*i^3 -
5*a*b^6*c^4*d^3*g^3*i^3 + 10*a^2*b^5*c^3*d^4*g^3*i^3 - 10*a^3*b^4*c^2*d^5*g
^3*i^3 + 5*a^4*b^3*c*d^6*g^3*i^3 - a^5*b^2*d^7*g^3*i^3)*x^4 + 2*(b^7*c^6*d*
g^3*i^3 - 4*a*b^6*c^5*d^2*g^3*i^3 + 5*a^2*b^5*c^4*d^3*g^3*i^3 - 5*a^4*b^3*c
^2*d^5*g^3*i^3 + 4*a^5*b^2*c*d^6*g^3*i^3 - a^6*b*d^7*g^3*i^3)*x^3 + (b^7*c^
7*g^3*i^3 - a*b^6*c^6*d*g^3*i^3 - 9*a^2*b^5*c^5*d^2*g^3*i^3 + 25*a^3*b^4*c^
4*d^3*g^3*i^3 - 25*a^4*b^3*c^3*d^4*g^3*i^3 + 9*a^5*b^2*c^2*d^5*g^3*i^3 + a^
6*b*c*d^6*g^3*i^3 - a^7*d^7*g^3*i^3)*x^2 + 2*(a*b^6*c^7*g^3*i^3 - 4*a^2*b^5
*c^6*d*g^3*i^3 + 5*a^3*b^4*c^5*d^2*g^3*i^3 - 5*a^5*b^2*c^3*d^4*g^3*i^3 + 4*
a^6*b*c^2*d^5*g^3*i^3 - a^7*c*d^6*g^3*i^3)*x) + 1/2*A^2*((12*b^3*d^3*x^3 -
b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d^3 + 18*(b^3*c*d^2 + a*b^2*d
^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 + a^2*b*d^3)*x)/((b^6*c^4*d^2 - 4*a*
b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5 + a^4*b^2*d^6)*g^3*i^3*x^
4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4
- 3*a^4*b^2*c*d^5 + a^5*b*d^6)*g^3*i^3*x^3 + (b^6*c^6 - 9*a^2*b^4*c^4*d^2 +
 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d^6)*g^3*i^3*x^2 + 2*(a*b^5*c
^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b^2*c^3*d^3 - 3*a^5*b*c^2*
d^4 + a^6*c*d^5)*g^3*i^3*x + (a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4
*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4)*g^3*i^3) + 12*b^2*d^2*log(b*x + a)/((
b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b
*c*d^4 - a^5*d^5)*g^3*i^3) - 12*b^2*d^2*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^
4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^
3*i^3))
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^3(dix+ci)^3}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x,
 algorithm="giac")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/((b*g*x + a*g)^3*(d*i*x
+ c*i)^3), x)
```

Mupad [B] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 2419, normalized size of antiderivative = 3.30

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^3*(c*i + d*i*x)
^3),x)
```

```
[Out] ((3*x^2*(6*A^2*a*b^2*d^3 + 6*A^2*b^3*c*d^2 + 15*B^2*a*b^2*d^3*n^2 + 15*B^2*
b^3*c*d^2*n^2 - 4*A*B*a*b^2*d^3*n + 4*A*B*b^3*c*d^2*n))/(a*d - b*c) - (2*A^
2*a^3*d^3 + 2*A^2*b^3*c^3 + B^2*a^3*d^3*n^2 + B^2*b^3*c^3*n^2 - 14*A^2*a*b^
2*c^2*d - 14*A^2*a^2*b*c*d^2 - 2*A*B*a^3*d^3*n + 2*A*B*b^3*c^3*n - 31*B^2*a
*b^2*c^2*d*n^2 - 31*B^2*a^2*b*c*d^2*n^2 - 30*A*B*a*b^2*c^2*d*n + 30*A*B*a^2
*b*c*d^2*n)/(2*(a*d - b*c)) + (2*x*(2*A^2*a^2*b*d^3 + 2*A^2*b^3*c^2*d + 14*
A^2*a*b^2*c*d^2 + 7*B^2*a^2*b*d^3*n^2 + 7*B^2*b^3*c^2*d*n^2 + 31*B^2*a*b^2*
c*d^2*n^2 - 6*A*B*a^2*b*d^3*n + 6*A*B*b^3*c^2*d*n))/(a*d - b*c) + (6*x^3*(2
*A^2*b^3*d^3 + 5*B^2*b^3*d^3*n^2))/(a*d - b*c))/(x^4*(2*a^3*b^2*d^5*g^3*i^3
 - 2*b^5*c^3*d^2*g^3*i^3 + 6*a*b^4*c^2*d^3*g^3*i^3 - 6*a^2*b^3*c*d^4*g^3*i^
3) - x*(4*a*b^4*c^5*g^3*i^3 - 4*a^5*c*d^4*g^3*i^3 - 8*a^2*b^3*c^4*d*g^3*i^3
 + 8*a^4*b*c^2*d^3*g^3*i^3) + x^3*(4*a^4*b*d^5*g^3*i^3 - 4*b^5*c^4*d*g^3*i^
3 + 8*a*b^4*c^3*d^2*g^3*i^3 - 8*a^3*b^2*c*d^4*g^3*i^3) + x^2*(2*a^5*d^5*g^3
*i^3 - 2*b^5*c^5*g^3*i^3 - 2*a*b^4*c^4*d*g^3*i^3 + 2*a^4*b*c*d^4*g^3*i^3 +
16*a^2*b^3*c^3*d^2*g^3*i^3 - 16*a^3*b^2*c^2*d^3*g^3*i^3) - 2*a^2*b^3*c^5*g^
3*i^3 + 2*a^5*c^2*d^3*g^3*i^3 + 6*a^3*b^2*c^4*d*g^3*i^3 - 6*a^4*b*c^3*d^2*g
^3*i^3) + log(e*((a + b*x)/(c + d*x))^n)^2*((x*((3*B^2*b*d*(a*d + b*c)^2)/(
a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2 - (B^2*b*d)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d
) + (6*B^2*a*b^2*c*d^2)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) - (B^2*(a*d + b*
c))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (6*B^2*b^3*d^3*x^3)/(a^2*d^2 + b^
2*c^2 - 2*a*b*c*d)^2 + (9*B^2*b^2*d^2*x^2*(a*d + b*c))/(a^2*d^2 + b^2*c^2 -
 2*a*b*c*d)^2 + (3*B^2*a*b*c*d*(a*d + b*c))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)
^2)/(x*(2*a*b*c^2*g^3*i^3 + 2*a^2*c*d*g^3*i^3) + x^3*(2*a*b*d^2*g^3*i^3 + 2
*b^2*c*d*g^3*i^3) + x^2*(a^2*d^2*g^3*i^3 + b^2*c^2*g^3*i^3 + 4*a*b*c*d*g^3*
i^3) + a^2*c^2*g^3*i^3 + b^2*d^2*g^3*i^3*x^4) - (6*A*B*b^2*d^2)/(g^3*i^3*n*
(a*d - b*c)^5)) + (log(e*((a + b*x)/(c + d*x))^n)*(x*((6*(a*d + b*c)*(A*B*a
*b*d^2 + A*B*b^2*c*d - B^2*a*b*d^2*n + B^2*b^2*c*d*n))/(a*d - b*c) - 2*A*B*
a*b*d^2 + 2*A*B*b^2*c*d + (12*A*B*a*b^2*c*d^2)/(a*d - b*c)) + x^2*((6*b*d*(
A*B*a*b*d^2 + A*B*b^2*c*d - B^2*a*b*d^2*n + B^2*b^2*c*d*n))/(a*d - b*c) + (
12*A*B*b^2*d^2*(a*d + b*c))/(a*d - b*c)) + (6*a*c*(A*B*a*b*d^2 + A*B*b^2*c*
d - B^2*a*b*d^2*n + B^2*b^2*c*d*n))/(a*d - b*c) - A*B*a^2*d^2 + A*B*b^2*c^2
 + (B^2*a^2*d^2*n)/2 + (B^2*b^2*c^2*n)/2 + (12*A*B*b^3*d^3*x^3)/(a*d - b*c)
 - B^2*a*b*c*d*n))/(x^4*(a^3*b^2*d^5*g^3*i^3 - b^5*c^3*d^2*g^3*i^3 + 3*a*b^
4*c^2*d^3*g^3*i^3 - 3*a^2*b^3*c*d^4*g^3*i^3) - x*(2*a*b^4*c^5*g^3*i^3 - 2*a
^5*c*d^4*g^3*i^3 - 4*a^2*b^3*c^4*d*g^3*i^3 + 4*a^4*b*c^2*d^3*g^3*i^3) + x^3
```

```
*(2*a^4*b*d^5*g^3*i^3 - 2*b^5*c^4*d*g^3*i^3 + 4*a*b^4*c^3*d^2*g^3*i^3 - 4*a
^3*b^2*c*d^4*g^3*i^3) + x^2*(a^5*d^5*g^3*i^3 - b^5*c^5*g^3*i^3 - a*b^4*c^4*
d*g^3*i^3 + a^4*b*c*d^4*g^3*i^3 + 8*a^2*b^3*c^3*d^2*g^3*i^3 - 8*a^3*b^2*c^2
*d^3*g^3*i^3) - a^2*b^3*c^5*g^3*i^3 + a^5*c^2*d^3*g^3*i^3 + 3*a^3*b^2*c^4*d
*g^3*i^3 - 3*a^4*b*c^3*d^2*g^3*i^3) + (b^2*d^2*atan((b^2*d^2*((a^5*d^5*g^3*
i^3 + b^5*c^5*g^3*i^3 - 3*a*b^4*c^4*d*g^3*i^3 - 3*a^4*b*c*d^4*g^3*i^3 + 2*a
^2*b^3*c^3*d^2*g^3*i^3 + 2*a^3*b^2*c^2*d^3*g^3*i^3)/(a^4*d^4*g^3*i^3 + b^4*
c^4*g^3*i^3 - 4*a*b^3*c^3*d*g^3*i^3 - 4*a^3*b*c*d^3*g^3*i^3 + 6*a^2*b^2*c^2
*d^2*g^3*i^3) + 2*b*d*x)*(2*A^2 + 5*B^2*n^2)*(a^4*d^4*g^3*i^3 + b^4*c^4*g^3
*i^3 - 4*a*b^3*c^3*d*g^3*i^3 - 4*a^3*b*c*d^3*g^3*i^3 + 6*a^2*b^2*c^2*d^2*g^
3*i^3)*3i)/(g^3*i^3*(6*A^2*b^2*d^2 + 15*B^2*b^2*d^2*n^2)*(a*d - b*c)^5))*(2
*A^2 + 5*B^2*n^2)*6i)/(g^3*i^3*(a*d - b*c)^5) - (2*B^2*b^2*d^2*log(e*((a +
b*x)/(c + d*x))^n)^3)/(g^3*i^3*n*(a*d - b*c)^5)
```

3.209 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 908

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = -\frac{B^2d^5n^2(a+bx)^2}{4(bc-ad)^6g^4i^3(c+dx)^2}-\frac{10AbBd^4n(a+bx)}{(bc-ad)^6g^4i^3(c+dx)}+\frac{10bB^2d^4n^2(a+bx)}{(bc-ad)^6g^4i^3(c+dx)}-\frac{20b^3B^2d^2n^2(c+dx)}{(bc-ad)^6g^4i^3(a+bx)}+\frac{5b^4B^2dn^2(c+dx)^2}{4(bc-ad)^6g^4i^3(a+bx)^2}-\frac{2b^5B^2n^2(c+dx)^3}{27(bc-ad)^6g^4i^3(a+bx)^3}-\frac{10bB^2d^4n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^6g^4i^3(c+dx)}+\frac{Bd^5n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2}-\frac{20b^3Bd^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(a+bx)}+\frac{5b^4Bdn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2}-\frac{2b^5Bn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^6g^4i^3(a+bx)^3}-\frac{d^5(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^6g^4i^3(c+dx)^2}+\frac{5bd^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^6g^4i^3(c+dx)}-\frac{10b^3d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^6g^4i^3(a+bx)}+\frac{5b^4d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^6g^4i^3(a+bx)^2}-\frac{b^5(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^6g^4i^3(a+bx)^3}-\frac{10b^2d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^6g^4i^3n}$$

```
[Out] -1/4*B^2*d^5*n^2*(b*x+a)^2/(-a*d+b*c)^6/g^4/i^3/(d*x+c)^2-10*A*b*B*d^4*n*(b
*x+a)/(-a*d+b*c)^6/g^4/i^3/(d*x+c)+10*b*B^2*d^4*n^2*(b*x+a)/(-a*d+b*c)^6/g^
4/i^3/(d*x+c)-20*b^3*B^2*d^2*n^2*(d*x+c)/(-a*d+b*c)^6/g^4/i^3/(b*x+a)+5/4*b
^4*B^2*d*n^2*(d*x+c)^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-2/27*b^5*B^2*n^2*(d*x
+c)^3/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^3-10*b*B^2*d^4*n*(b*x+a)*ln(e*((b*x+a)/(
d*x+c))^n)/(-a*d+b*c)^6/g^4/i^3/(d*x+c)+1/2*B*d^5*n*(b*x+a)^2*(A+B*ln(e*((b
```

```
*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(d*x+c)^2-20*b^3*B*d^2*n*(d*x+c)*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(b*x+a)+5/2*b^4*B*d*n*(d
*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-2/9*
b^5*B*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(b*x
+a)^3-1/2*d^5*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^6/g^4/
i^3/(d*x+c)^2+5*b*d^4*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^
6/g^4/i^3/(d*x+c)-10*b^3*d^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*
d+b*c)^6/g^4/i^3/(b*x+a)+5/2*b^4*d*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)
)^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-1/3*b^5*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*
x+c))^n))^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^3-10/3*b^2*d^3*(A+B*ln(e*((b*x+a)/
(d*x+c))^n))^3/B/(-a*d+b*c)^6/g^4/i^3/n
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 908, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.178$, Rules used

$= \{2561, 2395, 2333, 2332, 2342, 2341, 2339, 30\}$

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = -\frac{2B^2n^2(c+dx)^3b^5}{27(bc-ad)^6g^4i^3(a+bx)^3} - \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2b^5}{3(bc-ad)^6g^4i^3(a+bx)^3} - \frac{2Bn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^5}{9(bc-ad)^6g^4i^3(a+bx)^3} + \frac{5B^2dn^2(c+dx)^2b^4}{4(bc-ad)^6g^4i^3(a+bx)^2} + \frac{5d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2b^4}{2(bc-ad)^6g^4i^3(a+bx)^2} + \frac{5Bdn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^4}{2(bc-ad)^6g^4i^3(a+bx)^2} - \frac{10d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2b^3}{(bc-ad)^6g^4i^3(a+bx)} - \frac{20B^2d^2n^2(c+dx)b^3}{(bc-ad)^6g^4i^3(a+bx)} - \frac{20Bd^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{(bc-ad)^6g^4i^3(a+bx)} - \frac{10d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3b^2}{3B(bc-ad)^6g^4i^3n} + \frac{5d^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2b}{(bc-ad)^6g^4i^3(c+dx)} - \frac{10B^2d^4n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)b}{(bc-ad)^6g^4i^3(c+dx)} + \frac{10B^2d^4n^2(a+bx)b}{(bc-ad)^6g^4i^3(c+dx)} - \frac{10ABd^4n(a+bx)b}{(bc-ad)^6g^4i^3(c+dx)} - \frac{d^5(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^6g^4i^3(c+dx)^2} + \frac{Bd^5n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2} - \frac{B^2d^5n^2(a+bx)^2}{4(bc-ad)^6g^4i^3(c+dx)^2}$$

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^4*(c*i + d*i*x)
^3),x]
```

```
[Out] -1/4*(B^2*d^5*n^2*(a + b*x)^2)/((b*c - a*d)^6*g^4*i^3*(c + d*x)^2) - (10*A*
b*B*d^4*n*(a + b*x))/((b*c - a*d)^6*g^4*i^3*(c + d*x)) + (10*b*B^2*d^4*n^2*
```

```
(a + b*x))/((b*c - a*d)^6*g^4*i^3*(c + d*x)) - (20*b^3*B^2*d^2*n^2*(c + d*x
))/((b*c - a*d)^6*g^4*i^3*(a + b*x)) + (5*b^4*B^2*d*n^2*(c + d*x)^2)/(4*(b*
c - a*d)^6*g^4*i^3*(a + b*x)^2) - (2*b^5*B^2*n^2*(c + d*x)^3)/(27*(b*c - a*
d)^6*g^4*i^3*(a + b*x)^3) - (10*b*B^2*d^4*n*(a + b*x)*Log[e*((a + b*x)/(c +
 d*x))^n])/((b*c - a*d)^6*g^4*i^3*(c + d*x)) + (B*d^5*n*(a + b*x)^2*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(2*(b*c - a*d)^6*g^4*i^3*(c + d*x)^2) - (2
0*b^3*B*d^2*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d
)^6*g^4*i^3*(a + b*x)) + (5*b^4*B*d*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(
c + d*x))^n]))/(2*(b*c - a*d)^6*g^4*i^3*(a + b*x)^2) - (2*b^5*B*n*(c + d*x)
^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(b*c - a*d)^6*g^4*i^3*(a + b*
x)^3) - (d^5*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(b*c
- a*d)^6*g^4*i^3*(c + d*x)^2) + (5*b*d^4*(a + b*x)*(A + B*Log[e*((a + b*x)/
(c + d*x))^n])^2)/((b*c - a*d)^6*g^4*i^3*(c + d*x)) - (10*b^3*d^2*(c + d*x)
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*c - a*d)^6*g^4*i^3*(a + b*x)
) + (5*b^4*d*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(b*c
- a*d)^6*g^4*i^3*(a + b*x)^2) - (b^5*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^2)/(3*(b*c - a*d)^6*g^4*i^3*(a + b*x)^3) - (10*b^2*d^3*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])^3)/(3*B*(b*c - a*d)^6*g^4*i^3*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(b-dx)^5(A+B\log(ex^n))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3} \\
&= \frac{\text{Subst}\left(\int \left(5bd^4(A+B\log(ex^n))^2+\frac{b^5(A+B\log(ex^n))^2}{x^4}-\frac{5b^4d(A+B\log(ex^n))^2}{x^3}+\frac{10b^3d^2(A+B\log(ex^n))^2}{x^2}-10b\right.\right.}{(bc-ad)^6g^4i^3} \\
&= \frac{b^5\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}-\frac{(5b^4d)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3} \\
&\quad+\frac{(10b^3d^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3} \\
&\quad-\frac{(10b^2d^3)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3} \\
&\quad+\frac{(5bd^4)\,\text{Subst}\left(\int (A+B\log(ex^n))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3} \\
&\quad-\frac{d^5\text{Subst}\left(\int x(A+B\log(ex^n))^2\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}
\end{aligned}
$$

$$= -\frac{d^5(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^6g^4i^3(c+dx)^2} + \frac{5bd^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^6g^4i^3(c+dx)}$$
$$- \frac{10b^3d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^6g^4i^3(a+bx)}$$
$$+ \frac{5b^4d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^6g^4i^3(a+bx)^2} - \frac{b^5(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^6g^4i^3(a+bx)^3}$$
$$- \frac{(10b^2d^3)\,\text{Subst}\left(\int x^2\,dx, x, A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)^6g^4i^3n}$$
$$+ \frac{(2b^5Bn)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^4}\,dx, x, \frac{a+bx}{c+dx}\right)}{3(bc-ad)^6g^4i^3}$$
$$- \frac{(5b^4Bdn)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}$$
$$+ \frac{(20b^3Bd^2n)\,\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}$$
$$- \frac{(10bBd^4n)\,\text{Subst}\left(\int (A+B\log(ex^n))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}$$
$$+ \frac{(Bd^5n)\,\text{Subst}\left(\int x(A+B\log(ex^n))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}$$

$$= -\frac{B^2d^5n^2(a+bx)^2}{4(bc-ad)^6g^4i^3(c+dx)^2} - \frac{10AbBd^4n(a+bx)}{(bc-ad)^6g^4i^3(c+dx)}$$
$$- \frac{20b^3B^2d^2n^2(c+dx)}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4B^2dn^2(c+dx)^2}{4(bc-ad)^6g^4i^3(a+bx)^2}$$
$$- \frac{2b^5B^2n^2(c+dx)^3}{27(bc-ad)^6g^4i^3(a+bx)^3} + \frac{Bd^5n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2}$$
$$- \frac{20b^3Bd^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(a+bx)}$$
$$+ \frac{5b^4Bdn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2}$$
$$- \frac{2b^5Bn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^6g^4i^3(a+bx)^3} - \frac{d^5(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^6g^4i^3(c+dx)^2}$$
$$+ \frac{5bd^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^6g^4i^3(c+dx)} - \frac{10b^3d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^6g^4i^3(a+bx)}$$
$$+ \frac{5b^4d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^6g^4i^3(a+bx)^2} - \frac{b^5(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^6g^4i^3(a+bx)^3}$$
$$- \frac{10b^2d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^6g^4i^3n} - \frac{(10bB^2d^4n)\,\text{Subst}\left(\int \log(ex^n)\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}$$

$$= -\frac{B^2d^5n^2(a+bx)^2}{4(bc-ad)^6g^4i^3(c+dx)^2} - \frac{10AbBd^4n(a+bx)}{(bc-ad)^6g^4i^3(c+dx)} + \frac{10bB^2d^4n^2(a+bx)}{(bc-ad)^6g^4i^3(c+dx)} - \frac{20b^3B^2d^2n^2(c+dx)}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4B^2dn^2(c+dx)^2}{4(bc-ad)^6g^4i^3(a+bx)^2} - \frac{2b^5B^2n^2(c+dx)^3}{27(bc-ad)^6g^4i^3(a+bx)^3} - \frac{10bB^2d^4n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^6g^4i^3(c+dx)} + \frac{Bd^5n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2} - \frac{20b^3Bd^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4Bdn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2} - \frac{2b^5Bn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^6g^4i^3(a+bx)^3} - \frac{d^5(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^6g^4i^3(c+dx)^2} + \frac{5bd^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^6g^4i^3(c+dx)} - \frac{10b^3d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^6g^4i^3(a+bx)^2} - \frac{b^5(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^6g^4i^3(a+bx)^3} - \frac{10b^2d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^6g^4i^3n}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2138 vs. 2(908) = 1816.

Time = 1.90 (sec) , antiderivative size = 2138, normalized size of antiderivative = 2.35

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^4*(c*i +
d*i*x)^3),x]
```

```
[Out] -1/108*(360*b^2*B^2*d^3*n^2*(a + b*x)^3*(c + d*x)^2*Log[(a + b*x)/(c + d*x)
]^3 + 18*B*n*Log[(a + b*x)/(c + d*x)]^2*(60*a^3*A*b^2*c^2*d^3 + 2*b^5*B*c^5
*n - 15*a*b^4*B*c^4*d*n + 60*a^2*b^3*B*c^3*d^2*n - 30*a^4*b*B*c*d^4*n + 3*a
^5*B*d^5*n + 180*a^2*A*b^3*c^2*d^3*x + 120*a^3*A*b^2*c*d^4*x - 5*b^5*B*c^4*
d*n*x + 60*a*b^4*B*c^3*d^2*n*x + 180*a^2*b^3*B*c^2*d^3*n*x - 120*a^3*b^2*B*
c*d^4*n*x - 15*a^4*b*B*d^5*n*x + 180*a*A*b^4*c^2*d^3*x^2 + 360*a^2*A*b^3*c*
d^4*x^2 + 60*a^3*A*b^2*d^5*x^2 + 20*b^5*B*c^3*d^2*n*x^2 + 270*a*b^4*B*c^2*d
^3*n*x^2 - 90*a^3*b^2*B*d^5*n*x^2 + 60*A*b^5*c^2*d^3*x^3 + 360*a*A*b^4*c*d^
4*x^3 + 180*a^2*A*b^3*d^5*x^3 + 110*b^5*B*c^2*d^3*n*x^3 + 180*a*b^4*B*c*d^4
*n*x^3 - 90*a^2*b^3*B*d^5*n*x^3 + 120*A*b^5*c*d^4*x^4 + 180*a*A*b^4*d^5*x^4
 + 100*b^5*B*c*d^4*n*x^4 + 60*A*b^5*d^5*x^5 + 20*b^5*B*d^5*n*x^5 + 60*b^2*B
```

d^3(a + b*x)^3*(c + d*x)^2*Log[e*((a + b*x)/(c + d*x))^n] - 60*b^2*B*d^3*
n*(a + b*x)^3*(c + d*x)^2*Log[(a + b*x)/(c + d*x)]) + 6*b^2*d^2*(b*c - a*d)
*(a + b*x)^2*(c + d*x)^2*(108*A^2 + 282*A*B*n + 319*B^2*n^2 + 108*B^2*Log[e
*((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(36*A + 47*B*n)*Log[(a + b*x)/(c + d*x)
] + 108*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 6*B*Log[e*((a + b*x)/(c + d*x)
)^n]*(36*A + 47*B*n - 36*B*n*Log[(a + b*x)/(c + d*x)])) - 3*b^2*d*(b*c - a*
d)^2*(a + b*x)*(c + d*x)^2*(54*A^2 + 66*A*B*n + 37*B^2*n^2 + 54*B^2*Log[e*(
(a + b*x)/(c + d*x))^n]^2 - 6*B*n*(18*A + 11*B*n)*Log[(a + b*x)/(c + d*x)]
+ 54*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 6*B*Log[e*((a + b*x)/(c + d*x))^n
]*(18*A + 11*B*n - 18*B*n*Log[(a + b*x)/(c + d*x)])) + 4*b^2*(b*c - a*d)^3*
(c + d*x)^2*(9*A^2 + 6*A*B*n + 2*B^2*n^2 + 9*B^2*Log[e*((a + b*x)/(c + d*x)
)^n]^2 - 6*B*n*(3*A + B*n)*Log[(a + b*x)/(c + d*x)] + 9*B^2*n^2*Log[(a + b*
x)/(c + d*x)]^2 + 6*B*Log[e*((a + b*x)/(c + d*x))^n]*(3*A + B*n - 3*B*n*Log
[(a + b*x)/(c + d*x)])) + 60*b^2*d^3*(a + b*x)^3*(c + d*x)^2*Log[a + b*x]*(
18*A^2 + 12*A*B*n + 49*B^2*n^2 + 18*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 -
12*B*n*(3*A + B*n)*Log[(a + b*x)/(c + d*x)] + 18*B^2*n^2*Log[(a + b*x)/(c +
d*x)]^2 + 12*B*Log[e*((a + b*x)/(c + d*x))^n]*(3*A + B*n - 3*B*n*Log[(a +
b*x)/(c + d*x)])) + 27*d^3*(b*c - a*d)^2*(a + b*x)^3*(2*A^2 - 2*A*B*n + B^2
*n^2 + 2*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 + 2*B*n*(-2*A + B*n)*Log[(a +
b*x)/(c + d*x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 - 2*B*Log[e*((a + b
*x)/(c + d*x))^n]*(-2*A + B*n + 2*B*n*Log[(a + b*x)/(c + d*x)])) + 54*b*d^3
*(b*c - a*d)*(a + b*x)^3*(c + d*x)*(8*A^2 - 18*A*B*n + 19*B^2*n^2 + 8*B^2*L
og[e*((a + b*x)/(c + d*x))^n]^2 + 2*B*n*(-8*A + 9*B*n)*Log[(a + b*x)/(c + d
*x)] + 8*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 - 2*B*Log[e*((a + b*x)/(c + d*x
))^n]*(-8*A + 9*B*n + 8*B*n*Log[(a + b*x)/(c + d*x)])) + 6*B*(b*c - a*d)*n*
Log[(a + b*x)/(c + d*x)]*(18*b*d^3*(a + b*x)^3*(c + d*x)*(8*A - 9*B*n + 8*B
Log[e((a + b*x)/(c + d*x))^n] - 8*B*n*Log[(a + b*x)/(c + d*x)]) + 4*b^2*(
b*c - a*d)^2*(c + d*x)^2*(3*A + B*n + 3*B*Log[e*((a + b*x)/(c + d*x))^n] -
3*B*n*Log[(a + b*x)/(c + d*x)]) + 9*d^3*(b*c - a*d)*(a + b*x)^3*(2*A - B*n
+ 2*B*Log[e*((a + b*x)/(c + d*x))^n] - 2*B*n*Log[(a + b*x)/(c + d*x)]) - 3*
b^2*d*(b*c - a*d)*(a + b*x)*(c + d*x)^2*(18*A + 11*B*n + 18*B*(Log[e*((a +
b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])) + 6*b^2*d^2*(a + b*x)^2*(
c + d*x)^2*(36*A + 47*B*n + 36*B*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a
+ b*x)/(c + d*x)]))) - 60*b^2*d^3*(a + b*x)^3*(c + d*x)^2*(18*A^2 + 12*A*B
*n + 49*B^2*n^2 + 18*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 12*B*n*(3*A + B
*n)*Log[(a + b*x)/(c + d*x)] + 18*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 12*B
Log[e((a + b*x)/(c + d*x))^n]*(3*A + B*n - 3*B*n*Log[(a + b*x)/(c + d*x)]
))*Log[c + d*x])/((b*c - a*d)^6*g^4*i^3*(a + b*x)^3*(c + d*x)^2)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4485 vs. 2(888) = 1776.

Time = 205.16 (sec) , antiderivative size = 4486, normalized size of antiderivative = 4.94

method	result	size
parallelrisch	Expression too large to display	4486

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x,method=
_RETURNVERBOSE)
[Out] -1/108*(1080*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b^10*c^2*d^7-720*A*B*x^3*a
^2*b^8*d^9*n^2+2160*A*B*x^3*b^10*c^2*d^7*n^2+2160*B^2*x^2*ln(e*((b*x+a)/(d*
x+c))^n)^3*a^2*b^8*c*d^8+1080*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a*b^9*c^2
*d^7-1620*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^7*d^9*n+360*B^2*x^2*ln(
e*((b*x+a)/(d*x+c))^n)^2*b^10*c^3*d^6*n+3780*B^2*x^2*ln(e*((b*x+a)/(d*x+c))
^n)*a^3*b^7*d^9*n^2+1320*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^10*c^3*d^6*n^2
-5880*B^2*x^2*a^2*b^8*c*d^8*n^3+9210*B^2*x^2*a*b^9*c^2*d^7*n^3+6480*A^2*x^3
*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9*c*d^8+6600*B^2*x^4*ln(e*((b*x+a)/(d*x+c))^
n)*b^10*c*d^8*n^2+3240*A*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^9*d^9+16200*
B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9*c*d^8*n^2-540*B^2*ln(e*((b*x+a)/(d*
x+c))^n)^2*a^4*b^6*c*d^8*n+1080*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^8*c^3
*d^6*n-90*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^10*c^4*d^5*n+810*B^2*x*ln(e*(
(b*x+a)/(d*x+c))^n)*a^4*b^6*d^9*n^2-150*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^1
0*c^4*d^5*n^2-6280*B^2*x*a^3*b^7*c*d^8*n^3+1080*A^2*x^3*a*b^9*c*d^8*n+1080*
A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^7*d^9+840*A*B*x^2*a^3*b^7*d^9*n^2
+1320*A*B*x^2*b^10*c^3*d^6*n^2+720*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*a^3*b^
7*c*d^8+1080*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*a^2*b^8*c^2*d^7-270*B^2*x*ln
(e*((b*x+a)/(d*x+c))^n)^2*a^4*b^6*d^9*n+2160*A*B*x^4*ln(e*((b*x+a)/(d*x+c))
^n)^2*b^10*c*d^8-720*A*B*x^4*a*b^9*d^9*n^2+720*A*B*x^4*b^10*c*d^8*n^2+2160*
B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^3*a*b^9*c*d^8-1620*B^2*x^3*ln(e*((b*x+a)/
(d*x+c))^n)^2*a^2*b^8*d^9*n+1980*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b^10*c
^2*d^7*n+8100*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^8*d^9*n^2+5100*B^2*x^
3*ln(e*((b*x+a)/(d*x+c))^n)*b^10*c^2*d^7*n^2+2220*B^2*x^3*a*b^9*c*d^8*n^3+3
240*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^8*d^9-2880*A^2*x*a^3*b^7*c*d^
8*n+2160*A^2*x*a^2*b^8*c^2*d^7*n+1080*A^2*x*a*b^9*c^3*d^6*n+1080*A*B*ln(e*(
(b*x+a)/(d*x+c))^n)^2*a^3*b^7*c^2*d^7+108*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^5
*b^5*d^9*n+72*A*B*ln(e*((b*x+a)/(d*x+c))^n)*b^10*c^5*d^4*n+3540*B^2*x*a^2*b
^8*c^2*d^7*n^3+3780*B^2*x*a*b^9*c^3*d^6*n^3+6480*A^2*x^2*ln(e*((b*x+a)/(d*x
+c))^n)*a^2*b^8*c*d^8+3240*A^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9*c^2*d^7-
2160*A^2*x^2*a^2*b^8*c*d^8*n+3780*A^2*x^2*a*b^9*c^2*d^7*n+810*A*B*x*a^4*b^6
*d^9*n^2-150*A*B*x*b^10*c^4*d^5*n^2-270*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b
^9*c^4*d^5*n+1080*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^6*c*d^8*n^2+2160*B^2*
ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^8*c^3*d^6*n^2-270*B^2*ln(e*((b*x+a)/(d*x+c)
)^n)*a*b^9*c^4*d^5*n^2+2160*A^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^7*c*d^8+3
```

240*A^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^8*c^2*d^7-270*A*B*a*b^9*c^4*d^5*n
^2+1080*A*B*a^4*b^6*c*d^8*n^2-2940*A*B*a^3*b^7*c^2*d^7*n^2+2160*A*B*a^2*b^8
*c^3*d^6*n^2+720*A*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*b^10*d^9*n+1800*B^2*x^4*
ln(e*((b*x+a)/(d*x+c))^n)^2*b^10*c*d^8*n+8100*B^2*x^4*ln(e*((b*x+a)/(d*x+c)
)^n)*a*b^9*d^9*n^2+360*B^2*x^5*ln(e*((b*x+a)/(d*x+c))^n)^3*b^10*d^9+1080*A^
2*x^5*ln(e*((b*x+a)/(d*x+c))^n)*b^10*d^9-5030*B^2*x^2*a^3*b^7*d^9*n^3+1700*
B^2*x^2*b^10*c^3*d^6*n^3+3240*A^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^8*d^9
+1080*A^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^10*c^2*d^7-2700*A^2*x^3*a^2*b^8*d
^9*n+1620*A^2*x^3*b^10*c^2*d^7*n-945*B^2*x*a^4*b^6*d^9*n^3-95*B^2*x*b^10*c^
4*d^5*n^3+1080*A^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^7*d^9-1980*A^2*x^2*a
^3*b^7*d^9*n+360*A^2*x^2*b^10*c^3*d^6*n+360*B^2*ln(e*((b*x+a)/(d*x+c))^n)^3
*a^3*b^7*c^2*d^7+54*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^5*b^5*d^9*n+36*B^2*ln
(e*((b*x+a)/(d*x+c))^n)^2*b^10*c^5*d^4*n-54*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a
^5*b^5*d^9*n^2+24*B^2*ln(e*((b*x+a)/(d*x+c))^n)*b^10*c^5*d^4*n^2+6480*A*B*x
^3*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^9*c*d^8-3240*A*B*x^3*ln(e*((b*x+a)/(d*x+
c))^n)*a^2*b^8*d^9*n+3960*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^10*c^2*d^7*n-
1440*A*B*x^3*a*b^9*c*d^8*n^2+4860*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^9
*c^2*d^7*n+12960*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^8*c*d^8*n^2+11340*
B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9*c^2*d^7*n^2+6480*A*B*x^2*ln(e*((b*x
+a)/(d*x+c))^n)^2*a^2*b^8*c*d^8+3240*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*
b^9*c^2*d^7-3240*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^7*d^9*n+720*A*B*x^
2*ln(e*((b*x+a)/(d*x+c))^n)*b^10*c^3*d^6*n-4680*A*B*x^2*a^2*b^8*c*d^8*n^2+3
600*A*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^10*c*d^8*n+3240*B^2*x^3*ln(e*((b*x+
a)/(d*x+c))^n)^2*a*b^9*c*d^8*n+6480*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9
*c*d^8*n+9720*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9*c^2*d^7*n-4320*A*B*x*
ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^7*c*d^8*n+6480*A*B*x*ln(e*((b*x+a)/(d*x+c))
^n)*a^2*b^8*c^2*d^7*n+2160*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9*c^3*d^6*n-
1080*B^2*a^4*b^6*c*d^8*n^3-980*B^2*a^3*b^7*c^2*d^7*n^3+2160*B^2*a^2*b^8*c^3
*d^6*n^3-135*B^2*a*b^9*c^4*d^5*n^3-54*A*B*a^5*b^5*d^9*n^2+24*A*B*b^10*c^5*d
^4*n^2-540*A^2*a^4*b^6*c*d^8*n-360*A^2*a^3*b^7*c^2*d^7*n+1080*A^2*a^2*b^8*c
^3*d^6*n-270*A^2*a*b^9*c^4*d^5*n+27*B^2*a^5*b^5*d^9*n^3+8*B^2*b^10*c^5*d^4*
n^3+54*A^2*a^5*b^5*d^9*n+36*A^2*b^10*c^5*d^4*n+2520*A*B*x^2*a*b^9*c^2*d^7*n
^2-2160*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^7*c*d^8*n+3240*B^2*x*ln(e*(
(b*x+a)/(d*x+c))^n)^2*a^2*b^8*c^2*d^7*n+1080*B^2*x*ln(e*((b*x+a)/(d*x+c))^n
)^2*a*b^9*c^3*d^6*n+4320*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^7*c*d^8*n^2+
6480*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^8*c^2*d^7*n^2+3240*B^2*x*ln(e*((
b*x+a)/(d*x+c))^n)*a*b^9*c^3*d^6*n^2+2160*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2
*a^3*b^7*c*d^8+3240*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^8*c^2*d^7-540*A
*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^6*d^9*n-180*A*B*x*ln(e*((b*x+a)/(d*x+c
))^n)*b^10*c^4*d^5*n-1560*A*B*x*a^3*b^7*c*d^8*n^2-2340*A*B*x*a^2*b^8*c^2*d^
7*n^2+3240*A*B*x*a*b^9*c^3*d^6*n^2-1080*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b
^6*c*d^8*n+2160*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^8*c^3*d^6*n-540*A*B*ln(
e*((b*x+a)/(d*x+c))^n)*a*b^9*c^4*d^5*n-270*A^2*x*a^4*b^6*d^9*n-90*A^2*x*b^1
0*c^4*d^5*n+1080*A^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^7*c^2*d^7+360*B^2*x^5*
ln(e*((b*x+a)/(d*x+c))^n)^2*b^10*d^9*n+2940*B^2*x^5*ln(e*((b*x+a)/(d*x+c))^

```
n)*b^10*d^9*n^2+1080*A*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)^2*b^10*d^9+1080*B^2*
x^4*ln(e*((b*x+a)/(d*x+c))^n)^3*a*b^9*d^9+720*B^2*x^4*ln(e*((b*x+a)/(d*x+c)
)^n)^3*b^10*c*d^8-2940*B^2*x^4*a*b^9*d^9*n^3+2940*B^2*x^4*b^10*c*d^8*n^3+10
80*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^3*a^2*b^8*d^9+360*B^2*x^3*ln(e*((b*x+a
)/(d*x+c))^n)^3*b^10*c^2*d^7-6990*B^2*x^3*a^2*b^8*d^9*n^3+4770*B^2*x^3*b^10
*c^2*d^7*n^3+3240*A^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9*d^9+2160*A^2*x^4*
ln(e*((b*x+a)/(d*x+c))^n)*b^10*c*d^8-1080*A^2*x^4*a*b^9*d^9*n+1080*A^2*x^4*
b^10*c*d^8*n+360*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a^3*b^7*d^9)/i^3/g^4/(
d*x+c)^2/(b*x+a)^3/(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3
*d^2+5*a*b^4*c^4*d-b^5*c^5)/(a*d-b*c)/d^4/b^5/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4725 vs. 2(888) = 1776.

Time = 0.49 (sec) , antiderivative size = 4725, normalized size of antiderivative = 5.20

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x,
 algorithm="fricas")
```

```
[Out] -1/108*(36*A^2*b^5*c^5 - 270*A^2*a*b^4*c^4*d + 1080*A^2*a^2*b^3*c^3*d^2 - 3
60*A^2*a^3*b^2*c^2*d^3 - 540*A^2*a^4*b*c*d^4 + 54*A^2*a^5*d^5 + 60*(18*A^2*
b^5*c*d^4 - 18*A^2*a*b^4*d^5 + 49*(B^2*b^5*c*d^4 - B^2*a*b^4*d^5)*n^2 + 12*
(A*B*b^5*c*d^4 - A*B*a*b^4*d^5)*n)*x^4 + 30*(54*A^2*b^5*c^2*d^3 + 36*A^2*a*
b^4*c*d^4 - 90*A^2*a^2*b^3*d^5 + (159*B^2*b^5*c^2*d^3 + 74*B^2*a*b^4*c*d^4
- 233*B^2*a^2*b^3*d^5)*n^2 + 24*(3*A*B*b^5*c^2*d^3 - 2*A*B*a*b^4*c*d^4 - A*
B*a^2*b^3*d^5)*n)*x^3 + 360*(B^2*b^5*d^5*n^2*x^5 + B^2*a^3*b^2*c^2*d^3*n^2
+ (2*B^2*b^5*c*d^4 + 3*B^2*a*b^4*d^5)*n^2*x^4 + (B^2*b^5*c^2*d^3 + 6*B^2*a*
b^4*c*d^4 + 3*B^2*a^2*b^3*d^5)*n^2*x^3 + (3*B^2*a*b^4*c^2*d^3 + 6*B^2*a^2*b
^3*c*d^4 + B^2*a^3*b^2*d^5)*n^2*x^2 + (3*B^2*a^2*b^3*c^2*d^3 + 2*B^2*a^3*b^
2*c*d^4)*n^2*x)*log((b*x + a)/(d*x + c))^3 + (8*B^2*b^5*c^5 - 135*B^2*a*b^4
*c^4*d + 2160*B^2*a^2*b^3*c^3*d^2 - 980*B^2*a^3*b^2*c^2*d^3 - 1080*B^2*a^4*
b*c*d^4 + 27*B^2*a^5*d^5)*n^2 + 10*(36*A^2*b^5*c^3*d^2 + 378*A^2*a*b^4*c^2*
d^3 - 216*A^2*a^2*b^3*c*d^4 - 198*A^2*a^3*b^2*d^5 + (170*B^2*b^5*c^3*d^2 +
921*B^2*a*b^4*c^2*d^3 - 588*B^2*a^2*b^3*c*d^4 - 503*B^2*a^3*b^2*d^5)*n^2 +
12*(11*A*B*b^5*c^3*d^2 + 21*A*B*a*b^4*c^2*d^3 - 39*A*B*a^2*b^3*c*d^4 + 7*A*
B*a^3*b^2*d^5)*n)*x^2 + 18*(2*B^2*b^5*c^5 - 15*B^2*a*b^4*c^4*d + 60*B^2*a^2
*b^3*c^3*d^2 - 20*B^2*a^3*b^2*c^2*d^3 - 30*B^2*a^4*b*c*d^4 + 3*B^2*a^5*d^5
+ 60*(B^2*b^5*c*d^4 - B^2*a*b^4*d^5)*x^4 + 30*(3*B^2*b^5*c^2*d^3 + 2*B^2*a*
b^4*c*d^4 - 5*B^2*a^2*b^3*d^5)*x^3 + 10*(2*B^2*b^5*c^3*d^2 + 21*B^2*a*b^4*c
^2*d^3 - 12*B^2*a^2*b^3*c*d^4 - 11*B^2*a^3*b^2*d^5)*x^2 - 5*(B^2*b^5*c^4*d
- 12*B^2*a*b^4*c^3*d^2 - 24*B^2*a^2*b^3*c^2*d^3 + 32*B^2*a^3*b^2*c*d^4 + 3*
```

B^2*a^4*b*d^5)*x + 60*(B^2*b^5*d^5*x^5 + B^2*a^3*b^2*c^2*d^3 + (2*B^2*b^5*c
*d^4 + 3*B^2*a*b^4*d^5)*x^4 + (B^2*b^5*c^2*d^3 + 6*B^2*a*b^4*c*d^4 + 3*B^2*
a^2*b^3*d^5)*x^3 + (3*B^2*a*b^4*c^2*d^3 + 6*B^2*a^2*b^3*c*d^4 + B^2*a^3*b^2
*d^5)*x^2 + (3*B^2*a^2*b^3*c^2*d^3 + 2*B^2*a^3*b^2*c*d^4)*x)*log((b*x + a)/
(d*x + c)))*log(e)^2 + 18*(60*A*B*a^3*b^2*c^2*d^3*n + 20*(B^2*b^5*d^5*n^2 +
 3*A*B*b^5*d^5*n)*x^5 + 20*(5*B^2*b^5*c*d^4*n^2 + 3*(2*A*B*b^5*c*d^4 + 3*A*
B*a*b^4*d^5)*n)*x^4 + 10*((11*B^2*b^5*c^2*d^3 + 18*B^2*a*b^4*c*d^4 - 9*B^2*
a^2*b^3*d^5)*n^2 + 6*(A*B*b^5*c^2*d^3 + 6*A*B*a*b^4*c*d^4 + 3*A*B*a^2*b^3*d
^5)*n)*x^3 + (2*B^2*b^5*c^5 - 15*B^2*a*b^4*c^4*d + 60*B^2*a^2*b^3*c^3*d^2 -
 30*B^2*a^4*b*c*d^4 + 3*B^2*a^5*d^5)*n^2 + 10*((2*B^2*b^5*c^3*d^2 + 27*B^2*
a*b^4*c^2*d^3 - 9*B^2*a^3*b^2*d^5)*n^2 + 6*(3*A*B*a*b^4*c^2*d^3 + 6*A*B*a^2
*b^3*c*d^4 + A*B*a^3*b^2*d^5)*n)*x^2 - 5*((B^2*b^5*c^4*d - 12*B^2*a*b^4*c^3
*d^2 - 36*B^2*a^2*b^3*c^2*d^3 + 24*B^2*a^3*b^2*c*d^4 + 3*B^2*a^4*b*d^5)*n^2
 - 12*(3*A*B*a^2*b^3*c^2*d^3 + 2*A*B*a^3*b^2*c*d^4)*n)*x)*log((b*x + a)/(d*
x + c))^2 + 6*(4*A*B*b^5*c^5 - 45*A*B*a*b^4*c^4*d + 360*A*B*a^2*b^3*c^3*d^2
 - 490*A*B*a^3*b^2*c^2*d^3 + 180*A*B*a^4*b*c*d^4 - 9*A*B*a^5*d^5)*n - 5*(18
*A^2*b^5*c^4*d - 216*A^2*a*b^4*c^3*d^2 - 432*A^2*a^2*b^3*c^2*d^3 + 576*A^2*
a^3*b^2*c*d^4 + 54*A^2*a^4*b*d^5 + (19*B^2*b^5*c^4*d - 756*B^2*a*b^4*c^3*d^
2 - 708*B^2*a^2*b^3*c^2*d^3 + 1256*B^2*a^3*b^2*c*d^4 + 189*B^2*a^4*b*d^5)*n
^2 + 6*(5*A*B*b^5*c^4*d - 108*A*B*a*b^4*c^3*d^2 + 78*A*B*a^2*b^3*c^2*d^3 +
52*A*B*a^3*b^2*c*d^4 - 27*A*B*a^4*b*d^5)*n)*x + 6*(12*A*B*b^5*c^5 - 90*A*B*
a*b^4*c^4*d + 360*A*B*a^2*b^3*c^3*d^2 - 120*A*B*a^3*b^2*c^2*d^3 - 180*A*B*a
^4*b*c*d^4 + 18*A*B*a^5*d^5 + 120*(3*A*B*b^5*c*d^4 - 3*A*B*a*b^4*d^5 + (B^2
*b^5*c*d^4 - B^2*a*b^4*d^5)*n)*x^4 + 60*(9*A*B*b^5*c^2*d^3 + 6*A*B*a*b^4*c*
d^4 - 15*A*B*a^2*b^3*d^5 + 2*(3*B^2*b^5*c^2*d^3 - 2*B^2*a*b^4*c*d^4 - B^2*a
^2*b^3*d^5)*n)*x^3 + 20*(6*A*B*b^5*c^3*d^2 + 63*A*B*a*b^4*c^2*d^3 - 36*A*B*
a^2*b^3*c*d^4 - 33*A*B*a^3*b^2*d^5 + (11*B^2*b^5*c^3*d^2 + 21*B^2*a*b^4*c^2
*d^3 - 39*B^2*a^2*b^3*c*d^4 + 7*B^2*a^3*b^2*d^5)*n)*x^2 + 180*(B^2*b^5*d^5*
n*x^5 + B^2*a^3*b^2*c^2*d^3*n + (2*B^2*b^5*c*d^4 + 3*B^2*a*b^4*d^5)*n*x^4 +
 (B^2*b^5*c^2*d^3 + 6*B^2*a*b^4*c*d^4 + 3*B^2*a^2*b^3*d^5)*n*x^3 + (3*B^2*a
*b^4*c^2*d^3 + 6*B^2*a^2*b^3*c*d^4 + B^2*a^3*b^2*d^5)*n*x^2 + (3*B^2*a^2*b^
3*c^2*d^3 + 2*B^2*a^3*b^2*c*d^4)*n*x)*log((b*x + a)/(d*x + c))^2 + (4*B^2*b
^5*c^5 - 45*B^2*a*b^4*c^4*d + 360*B^2*a^2*b^3*c^3*d^2 - 490*B^2*a^3*b^2*c^2
*d^3 + 180*B^2*a^4*b*c*d^4 - 9*B^2*a^5*d^5)*n - 5*(6*A*B*b^5*c^4*d - 72*A*B
*a*b^4*c^3*d^2 - 144*A*B*a^2*b^3*c^2*d^3 + 192*A*B*a^3*b^2*c*d^4 + 18*A*B*a
^4*b*d^5 + (5*B^2*b^5*c^4*d - 108*B^2*a*b^4*c^3*d^2 + 78*B^2*a^2*b^3*c^2*d^
3 + 52*B^2*a^3*b^2*c*d^4 - 27*B^2*a^4*b*d^5)*n)*x + 6*(60*A*B*a^3*b^2*c^2*d
^3 + 20*(B^2*b^5*d^5*n + 3*A*B*b^5*d^5)*x^5 + 20*(5*B^2*b^5*c*d^4*n + 6*A*B
*b^5*c*d^4 + 9*A*B*a*b^4*d^5)*x^4 + 10*(6*A*B*b^5*c^2*d^3 + 36*A*B*a*b^4*c*
d^4 + 18*A*B*a^2*b^3*d^5 + (11*B^2*b^5*c^2*d^3 + 18*B^2*a*b^4*c*d^4 - 9*B^2
*a^2*b^3*d^5)*n)*x^3 + 10*(18*A*B*a*b^4*c^2*d^3 + 36*A*B*a^2*b^3*c*d^4 + 6*
A*B*a^3*b^2*d^5 + (2*B^2*b^5*c^3*d^2 + 27*B^2*a*b^4*c^2*d^3 - 9*B^2*a^3*b^2
*d^5)*n)*x^2 + (2*B^2*b^5*c^5 - 15*B^2*a*b^4*c^4*d + 60*B^2*a^2*b^3*c^3*d^2
 - 30*B^2*a^4*b*c*d^4 + 3*B^2*a^5*d^5)*n + 5*(36*A*B*a^2*b^3*c^2*d^3 + 24*A
*B*a^3*b^2*c*d^4 - (B^2*b^5*c^4*d - 12*B^2*a*b^4*c^3*d^2 - 36*B^2*a^2*b^3*c

```
^2*d^3 + 24*B^2*a^3*b^2*c*d^4 + 3*B^2*a^4*b*d^5)*n)*x)*log((b*x + a)/(d*x +
 c)))*log(e) + 6*(180*A^2*a^3*b^2*c^2*d^3 + 10*(49*B^2*b^5*d^5*n^2 + 12*A*B
*b^5*d^5*n + 18*A^2*b^5*d^5)*x^5 + 10*(60*A*B*b^5*c*d^4*n + 36*A^2*b^5*c*d^
4 + 54*A^2*a*b^4*d^5 + 5*(22*B^2*b^5*c*d^4 + 27*B^2*a*b^4*d^5)*n^2)*x^4 + 1
0*(18*A^2*b^5*c^2*d^3 + 108*A^2*a*b^4*c*d^4 + 54*A^2*a^2*b^3*d^5 + 5*(17*B^
2*b^5*c^2*d^3 + 54*B^2*a*b^4*c*d^4 + 27*B^2*a^2*b^3*d^5)*n^2 + 6*(11*A*B*b^
5*c^2*d^3 + 18*A*B*a*b^4*c*d^4 - 9*A*B*a^2*b^3*d^5)*n)*x^3 + (4*B^2*b^5*c^5
 - 45*B^2*a*b^4*c^4*d + 360*B^2*a^2*b^3*c^3*d^2 + 180*B^2*a^4*b*c*d^4 - 9*B
^2*a^5*d^5)*n^2 + 10*(54*A^2*a*b^4*c^2*d^3 + 108*A^2*a^2*b^3*c*d^4 + 18*A^2
*a^3*b^2*d^5 + (22*B^2*b^5*c^3*d^2 + 189*B^2*a*b^4*c^2*d^3 + 216*B^2*a^2*b^
3*c*d^4 + 63*B^2*a^3*b^2*d^5)*n^2 + 6*(2*A*B*b^5*c^3*d^2 + 27*A*B*a*b^4*c^2
*d^3 - 9*A*B*a^3*b^2*d^5)*n)*x^2 + 6*(2*A*B*b^5*c^5 - 15*A*B*a*b^4*c^4*d +
60*A*B*a^2*b^3*c^3*d^2 - 30*A*B*a^4*b*c*d^4 + 3*A*B*a^5*d^5)*n + 5*(108*A^2
*a^2*b^3*c^2*d^3 + 72*A^2*a^3*b^2*c*d^4 - (5*B^2*b^5*c^4*d - 108*B^2*a*b^4*
c^3*d^2 - 216*B^2*a^2*b^3*c^2*d^3 - 144*B^2*a^3*b^2*c*d^4 - 27*B^2*a^4*b*d^
5)*n^2 - 6*(A*B*b^5*c^4*d - 12*A*B*a*b^4*c^3*d^2 - 36*A*B*a^2*b^3*c^2*d^3 +
 24*A*B*a^3*b^2*c*d^4 + 3*A*B*a^4*b*d^5)*n)*x)*log((b*x + a)/(d*x + c)))/((
b^9*c^6*d^2 - 6*a*b^8*c^5*d^3 + 15*a^2*b^7*c^4*d^4 - 20*a^3*b^6*c^3*d^5 + 1
5*a^4*b^5*c^2*d^6 - 6*a^5*b^4*c*d^7 + a^6*b^3*d^8)*g^4*i^3*x^5 + (2*b^9*c^7
*d - 9*a*b^8*c^6*d^2 + 12*a^2*b^7*c^5*d^3 + 5*a^3*b^6*c^4*d^4 - 30*a^4*b^5*
c^3*d^5 + 33*a^5*b^4*c^2*d^6 - 16*a^6*b^3*c*d^7 + 3*a^7*b^2*d^8)*g^4*i^3*x^
4 + (b^9*c^8 - 18*a^2*b^7*c^6*d^2 + 52*a^3*b^6*c^5*d^3 - 60*a^4*b^5*c^4*d^4
 + 24*a^5*b^4*c^3*d^5 + 10*a^6*b^3*c^2*d^6 - 12*a^7*b^2*c*d^7 + 3*a^8*b*d^8
)*g^4*i^3*x^3 + (3*a*b^8*c^8 - 12*a^2*b^7*c^7*d + 10*a^3*b^6*c^6*d^2 + 24*a
^4*b^5*c^5*d^3 - 60*a^5*b^4*c^4*d^4 + 52*a^6*b^3*c^3*d^5 - 18*a^7*b^2*c^2*d
^6 + a^9*d^8)*g^4*i^3*x^2 + (3*a^2*b^7*c^8 - 16*a^3*b^6*c^7*d + 33*a^4*b^5*
c^6*d^2 - 30*a^5*b^4*c^5*d^3 + 5*a^6*b^3*c^4*d^4 + 12*a^7*b^2*c^3*d^5 - 9*a
^8*b*c^2*d^6 + 2*a^9*c*d^7)*g^4*i^3*x + (a^3*b^6*c^8 - 6*a^4*b^5*c^7*d + 15
*a^5*b^4*c^6*d^2 - 20*a^6*b^3*c^5*d^3 + 15*a^7*b^2*c^4*d^4 - 6*a^8*b*c^3*d^
5 + a^9*c^2*d^6)*g^4*i^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**4/(d*i*x+c*i)**3
,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9293 vs. 2(888) = 1776.

Time = 1.04 (sec) , antiderivative size = 9293, normalized size of antiderivative = 10.23

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x,
 algorithm="maxima")
```

```
[Out] -1/6*B^2*((60*b^4*d^4*x^4 + 2*b^4*c^4 - 13*a*b^3*c^3*d + 47*a^2*b^2*c^2*d^2
 + 27*a^3*b*c*d^3 - 3*a^4*d^4 + 30*(3*b^4*c*d^3 + 5*a*b^3*d^4)*x^3 + 10*(2*
b^4*c^2*d^2 + 23*a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2 - 5*(b^4*c^3*d - 11*a*b^
3*c^2*d^2 - 35*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x)/((b^8*c^5*d^2 - 5*a*b^7*c^4*
d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*b^3*d
^7)*g^4*i^3*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 + 10*a
^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7)*g^4
*i^3*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c^4*d^3
 - 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d^7)*g^
4*i^3*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a^4*b^4*c
^4*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a^8*d^7)*g
^4*i^3*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5*d^2 - 10*a^
5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*d^6)*g^4*i^3*
x + (a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*a^6*b^2*c^4*d^
3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5)*g^4*i^3) + 60*b^2*d^3*log(b*x + a)/((b^6
*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2
*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3) - 60*b^2*d^3*log(d*x + c)/((b^
6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^
2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3))*log(e*(b*x/(d*x + c) + a/(d*
x + c))^n)^2 - 1/3*A*B*((60*b^4*d^4*x^4 + 2*b^4*c^4 - 13*a*b^3*c^3*d + 47*a
^2*b^2*c^2*d^2 + 27*a^3*b*c*d^3 - 3*a^4*d^4 + 30*(3*b^4*c*d^3 + 5*a*b^3*d^4
)*x^3 + 10*(2*b^4*c^2*d^2 + 23*a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2 - 5*(b^4*c
^3*d - 11*a*b^3*c^2*d^2 - 35*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x)/((b^8*c^5*d^2
- 5*a*b^7*c^4*d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d
^6 - a^5*b^3*d^7)*g^4*i^3*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*
c^4*d^3 + 10*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^
6*b^2*d^7)*g^4*i^3*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a
^3*b^5*c^4*d^3 - 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3
*a^7*b*d^7)*g^4*i^3*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2
+ 25*a^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^
6 - a^8*d^7)*g^4*i^3*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c
^5*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c
*d^6)*g^4*i^3*x + (a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*
a^6*b^2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5)*g^4*i^3) + 60*b^2*d^3*log(
```

b*x + a)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^
3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3) - 60*b^2*d^3*log
(d*x + c)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d
^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3))*log(e*(b*x/(d*
x + c) + a/(d*x + c))^n) - 1/108*((8*b^5*c^5 - 135*a*b^4*c^4*d + 2160*a^2*b
^3*c^3*d^2 - 980*a^3*b^2*c^2*d^3 - 1080*a^4*b*c*d^4 + 27*a^5*d^5 + 2940*(b^
5*c*d^4 - a*b^4*d^5)*x^4 + 30*(159*b^5*c^2*d^3 + 74*a*b^4*c*d^4 - 233*a^2*b
^3*d^5)*x^3 + 360*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d
^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*
d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c
*d^4)*x)*log(b*x + a)^3 - 360*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4
 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (
3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 +
 2*a^3*b^2*c*d^4)*x)*log(d*x + c)^3 + 10*(170*b^5*c^3*d^2 + 921*a*b^4*c^2*d
^3 - 588*a^2*b^3*c*d^4 - 503*a^3*b^2*d^5)*x^2 - 360*(b^5*d^5*x^5 + a^3*b^2*
c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 +
3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2
+ (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a)^2 - 360*(b^5*d^5*x^
5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*
b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b
^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x - 3*(b^5*d^5*x^5 + a^
3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*
d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5
)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a))*log(d*x + c)
^2 - 5*(19*b^5*c^4*d - 756*a*b^4*c^3*d^2 - 708*a^2*b^3*c^2*d^3 + 1256*a^3*b
^2*c*d^4 + 189*a^4*b*d^5)*x + 2940*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*
c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^
3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*
d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a) - 60*(49*b^5*d^5*x^5 + 49*a^3*b^2*c^
2*d^3 + 49*(2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + 49*(b^5*c^2*d^3 + 6*a*b^4*c*d^
4 + 3*a^2*b^3*d^5)*x^3 + 49*(3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^
5)*x^2 + 18*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^
4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 +
6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*
x)*log(b*x + a)^2 + 49*(3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x - 12*(b^5*d^
5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 +
6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a
^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a))*lo
g(d*x + c))*n^2/(a^3*b^6*c^8*g^4*i^3 - 6*a^4*b^5*c^7*d*g^4*i^3 + 15*a^5*b^4
*c^6*d^2*g^4*i^3 - 20*a^6*b^3*c^5*d^3*g^4*i^3 + 15*a^7*b^2*c^4*d^4*g^4*i^3
- 6*a^8*b*c^3*d^5*g^4*i^3 + a^9*c^2*d^6*g^4*i^3 + (b^9*c^6*d^2*g^4*i^3 - 6*
a*b^8*c^5*d^3*g^4*i^3 + 15*a^2*b^7*c^4*d^4*g^4*i^3 - 20*a^3*b^6*c^3*d^5*g^4
*i^3 + 15*a^4*b^5*c^2*d^6*g^4*i^3 - 6*a^5*b^4*c*d^7*g^4*i^3 + a^6*b^3*d^8*g
^4*i^3)*x^5 + (2*b^9*c^7*d*g^4*i^3 - 9*a*b^8*c^6*d^2*g^4*i^3 + 12*a^2*b^7*c
^5*d^3*g^4*i^3 + 5*a^3*b^6*c^4*d^4*g^4*i^3 - 30*a^4*b^5*c^3*d^5*g^4*i^3 + 3

3*a^5*b^4*c^2*d^6*g^4*i^3 - 16*a^6*b^3*c*d^7*g^4*i^3 + 3*a^7*b^2*d^8*g^4*i^
3)*x^4 + (b^9*c^8*g^4*i^3 - 18*a^2*b^7*c^6*d^2*g^4*i^3 + 52*a^3*b^6*c^5*d^3
*g^4*i^3 - 60*a^4*b^5*c^4*d^4*g^4*i^3 + 24*a^5*b^4*c^3*d^5*g^4*i^3 + 10*a^6
*b^3*c^2*d^6*g^4*i^3 - 12*a^7*b^2*c*d^7*g^4*i^3 + 3*a^8*b*d^8*g^4*i^3)*x^3
+ (3*a*b^8*c^8*g^4*i^3 - 12*a^2*b^7*c^7*d*g^4*i^3 + 10*a^3*b^6*c^6*d^2*g^4*
i^3 + 24*a^4*b^5*c^5*d^3*g^4*i^3 - 60*a^5*b^4*c^4*d^4*g^4*i^3 + 52*a^6*b^3*
c^3*d^5*g^4*i^3 - 18*a^7*b^2*c^2*d^6*g^4*i^3 + a^9*d^8*g^4*i^3)*x^2 + (3*a^
2*b^7*c^8*g^4*i^3 - 16*a^3*b^6*c^7*d*g^4*i^3 + 33*a^4*b^5*c^6*d^2*g^4*i^3 -
30*a^5*b^4*c^5*d^3*g^4*i^3 + 5*a^6*b^3*c^4*d^4*g^4*i^3 + 12*a^7*b^2*c^3*d^
5*g^4*i^3 - 9*a^8*b*c^2*d^6*g^4*i^3 + 2*a^9*c*d^7*g^4*i^3)*x) + 6*(4*b^5*c^
5 - 45*a*b^4*c^4*d + 360*a^2*b^3*c^3*d^2 - 490*a^3*b^2*c^2*d^3 + 180*a^4*b*
c*d^4 - 9*a^5*d^5 + 120*(b^5*c*d^4 - a*b^4*d^5)*x^4 + 120*(3*b^5*c^2*d^3 -
2*a*b^4*c*d^4 - a^2*b^3*d^5)*x^3 + 20*(11*b^5*c^3*d^2 + 21*a*b^4*c^2*d^3 -
39*a^2*b^3*c*d^4 + 7*a^3*b^2*d^5)*x^2 - 180*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3
+ (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^
3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2
*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a)^2 - 180*(b^5*d^5*x^5 + a^3*
b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^
4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*
x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(d*x + c)^2 - 5*(5*b^5*c^
4*d - 108*a*b^4*c^3*d^2 + 78*a^2*b^3*c^2*d^3 + 52*a^3*b^2*c*d^4 - 27*a^4*b*
d^5)*x + 120*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x
^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 +
6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)
*x)*log(b*x + a) - 120*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*
b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4
*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*
b^2*c*d^4)*x - 3*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^
5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d
^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*
d^4)*x)*log(b*x + a))*log(d*x + c))*n*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
)/(a^3*b^6*c^8*g^4*i^3 - 6*a^4*b^5*c^7*d*g^4*i^3 + 15*a^5*b^4*c^6*d^2*g^4*i
^3 - 20*a^6*b^3*c^5*d^3*g^4*i^3 + 15*a^7*b^2*c^4*d^4*g^4*i^3 - 6*a^8*b*c^3*
d^5*g^4*i^3 + a^9*c^2*d^6*g^4*i^3 + (b^9*c^6*d^2*g^4*i^3 - 6*a*b^8*c^5*d^3*
g^4*i^3 + 15*a^2*b^7*c^4*d^4*g^4*i^3 - 20*a^3*b^6*c^3*d^5*g^4*i^3 + 15*a^4*
b^5*c^2*d^6*g^4*i^3 - 6*a^5*b^4*c*d^7*g^4*i^3 + a^6*b^3*d^8*g^4*i^3)*x^5 +
(2*b^9*c^7*d*g^4*i^3 - 9*a*b^8*c^6*d^2*g^4*i^3 + 12*a^2*b^7*c^5*d^3*g^4*i^3
+ 5*a^3*b^6*c^4*d^4*g^4*i^3 - 30*a^4*b^5*c^3*d^5*g^4*i^3 + 33*a^5*b^4*c^2*
d^6*g^4*i^3 - 16*a^6*b^3*c*d^7*g^4*i^3 + 3*a^7*b^2*d^8*g^4*i^3)*x^4 + (b^9*
c^8*g^4*i^3 - 18*a^2*b^7*c^6*d^2*g^4*i^3 + 52*a^3*b^6*c^5*d^3*g^4*i^3 - 60*
a^4*b^5*c^4*d^4*g^4*i^3 + 24*a^5*b^4*c^3*d^5*g^4*i^3 + 10*a^6*b^3*c^2*d^6*g
^4*i^3 - 12*a^7*b^2*c*d^7*g^4*i^3 + 3*a^8*b*d^8*g^4*i^3)*x^3 + (3*a*b^8*c^8
*g^4*i^3 - 12*a^2*b^7*c^7*d*g^4*i^3 + 10*a^3*b^6*c^6*d^2*g^4*i^3 + 24*a^4*b
^5*c^5*d^3*g^4*i^3 - 60*a^5*b^4*c^4*d^4*g^4*i^3 + 52*a^6*b^3*c^3*d^5*g^4*i^
3 - 18*a^7*b^2*c^2*d^6*g^4*i^3 + a^9*d^8*g^4*i^3)*x^2 + (3*a^2*b^7*c^8*g^4*

i^3 - 16*a^3*b^6*c^7*d*g^4*i^3 + 33*a^4*b^5*c^6*d^2*g^4*i^3 - 30*a^5*b^4*c^5*d^3*g^4*i^3 + 5*a^6*b^3*c^4*d^4*g^4*i^3 + 12*a^7*b^2*c^3*d^5*g^4*i^3 - 9*a^8*b*c^2*d^6*g^4*i^3 + 2*a^9*c*d^7*g^4*i^3)*x))*B^2 - 1/18*(4*b^5*c^5 - 45*a*b^4*c^4*d + 360*a^2*b^3*c^3*d^2 - 490*a^3*b^2*c^2*d^3 + 180*a^4*b*c*d^4 - 9*a^5*d^5 + 120*(b^5*c*d^4 - a*b^4*d^5)*x^4 + 120*(3*b^5*c^2*d^3 - 2*a*b^4*c*d^4 - a^2*b^3*d^5)*x^3 + 20*(11*b^5*c^3*d^2 + 21*a*b^4*c^2*d^3 - 39*a^2*b^3*c*d^4 + 7*a^3*b^2*d^5)*x^2 - 180*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a)^2 - 180*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(d*x + c)^2 - 5*(5*b^5*c^4*d - 108*a*b^4*c^3*d^2 + 78*a^2*b^3*c^2*d^3 + 52*a^3*b^2*c*d^4 - 27*a^4*b*d^5)*x + 120*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a) - 120*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x - 3*(b^5*d^5*x^5 + a^3*b^2*c^2*d^3 + (2*b^5*c*d^4 + 3*a*b^4*d^5)*x^4 + (b^5*c^2*d^3 + 6*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + (3*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^2 + (3*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4)*x)*log(b*x + a))*log(d*x + c))*A*B*n/(a^3*b^6*c^8*g^4*i^3 - 6*a^4*b^5*c^7*d*g^4*i^3 + 15*a^5*b^4*c^6*d^2*g^4*i^3 - 20*a^6*b^3*c^5*d^3*g^4*i^3 + 15*a^7*b^2*c^4*d^4*g^4*i^3 - 6*a^8*b*c^3*d^5*g^4*i^3 + a^9*c^2*d^6*g^4*i^3 + (b^9*c^6*d^2*g^4*i^3 - 6*a*b^8*c^5*d^3*g^4*i^3 + 15*a^2*b^7*c^4*d^4*g^4*i^3 - 20*a^3*b^6*c^3*d^5*g^4*i^3 + 15*a^4*b^5*c^2*d^6*g^4*i^3 - 6*a^5*b^4*c*d^7*g^4*i^3 + a^6*b^3*d^8*g^4*i^3)*x^5 + (2*b^9*c^7*d*g^4*i^3 - 9*a*b^8*c^6*d^2*g^4*i^3 + 12*a^2*b^7*c^5*d^3*g^4*i^3 + 5*a^3*b^6*c^4*d^4*g^4*i^3 - 30*a^4*b^5*c^3*d^5*g^4*i^3 + 33*a^5*b^4*c^2*d^6*g^4*i^3 - 16*a^6*b^3*c*d^7*g^4*i^3 + 3*a^7*b^2*d^8*g^4*i^3)*x^4 + (b^9*c^8*g^4*i^3 - 18*a^2*b^7*c^6*d^2*g^4*i^3 + 52*a^3*b^6*c^5*d^3*g^4*i^3 - 60*a^4*b^5*c^4*d^4*g^4*i^3 + 24*a^5*b^4*c^3*d^5*g^4*i^3 + 10*a^6*b^3*c^2*d^6*g^4*i^3 - 12*a^7*b^2*c*d^7*g^4*i^3 + 3*a^8*b*d^8*g^4*i^3)*x^3 + (3*a*b^8*c^8*g^4*i^3 - 12*a^2*b^7*c^7*d*g^4*i^3 + 10*a^3*b^6*c^6*d^2*g^4*i^3 + 24*a^4*b^5*c^5*d^3*g^4*i^3 - 60*a^5*b^4*c^4*d^4*g^4*i^3 + 52*a^6*b^3*c^3*d^5*g^4*i^3 - 18*a^7*b^2*c^2*d^6*g^4*i^3 + a^9*d^8*g^4*i^3)*x^2 + (3*a^2*b^7*c^8*g^4*i^3 - 16*a^3*b^6*c^7*d*g^4*i^3 + 33*a^4*b^5*c^6*d^2*g^4*i^3 - 30*a^5*b^4*c^5*d^3*g^4*i^3 + 5*a^6*b^3*c^4*d^4*g^4*i^3 + 12*a^7*b^2*c^3*d^5*g^4*i^3 - 9*a^8*b*c^2*d^6*g^4*i^3 + 2*a^9*c*d^7*g^4*i^3)*x) - 1/6*A^2*((60*b^4*d^4*x^4 + 2*b^4*c^4 - 13*a*b^3*c^3*d + 47*a^2*b^2*c^2*d^2 + 27*a^3*b*c*d^3 - 3*a^4*d^4 + 30*(3*b^4*c*d^3 + 5*a*b^3*d^4)*x^3 + 10*(2*b^4*c^2*d^2 + 23*a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2 - 5*(b^4*c^3*d - 11*a*b^3*c^2*d^2 - 35*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x)/((b^8*c^5*d^2 - 5*a*b^7*c^4*d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a

```
^5*b^3*d^7)*g^4*i^3*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^
3 + 10*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*
d^7)*g^4*i^3*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5
*c^4*d^3 - 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b
*d^7)*g^4*i^3*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a
^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a^
8*d^7)*g^4*i^3*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5*d^2
 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*d^6)*
g^4*i^3*x + (a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*a^6*b^
2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5)*g^4*i^3) + 60*b^2*d^3*log(b*x +
a)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15
*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3) - 60*b^2*d^3*log(d*x +
 c)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 1
5*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3))
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^4(dix+ci)^3}\,dx$$

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x,
 algorithm="giac")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/((b*g*x + a*g)^4*(d*i*x
+ c*i)^3), x)
```

Mupad [B] (verification not implemented)

Time = 10.11 (sec) , antiderivative size = 4649, normalized size of antiderivative = 5.12

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx=\text{Too large to display}$$

```
[In] int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^4*(c*i + d*i*x)
^3),x)
```

```
[Out] log(e*((a + b*x)/(c + d*x))^n)*((x*((a*d + b*c)*(20*A*B*a*b*d^2 + 10*A*B*b^
2*c*d - (70*B^2*a*b*d^2*n)/3 + (10*B^2*b^2*c*d*n)/3) + a*c*(30*A*B*b^2*d^2
- 20*B^2*b^2*d^2*n) + (5*B^2*a^2*b*d^3*n)/6 + (5*B^2*b^3*c^2*d*n)/6 - 5*A*B
*a^2*b*d^3 - 5*A*B*b^3*c^2*d + 10*A*B*a*b^2*c*d^2 - (5*B^2*a*b^2*c*d^2*n)/3
) + x^2*((a*d + b*c)*(30*A*B*b^2*d^2 - 20*B^2*b^2*d^2*n) + b*d*(20*A*B*a*b*
d^2 + 10*A*B*b^2*c*d - (70*B^2*a*b*d^2*n)/3 + (10*B^2*b^2*c*d*n)/3)) + a*c*
(20*A*B*a*b*d^2 + 10*A*B*b^2*c*d - (70*B^2*a*b*d^2*n)/3 + (10*B^2*b^2*c*d*n
```

)/3) - 3*A*B*a^3*d^3 - 2*A*B*b^3*c^3 + b*d*x^3*(30*A*B*b^2*d^2 - 20*B^2*b^2 *d^2*n) + (3*B^2*a^3*d^3*n)/2 - (2*B^2*b^3*c^3*n)/3 + A*B*a*b^2*c^2*d + 4*A *B*a^2*b*c*d^2 + (17*B^2*a*b^2*c^2*d*n)/6 - (11*B^2*a^2*b*c*d^2*n)/3)/(x^5* (3*a^4*b^3*d^6*g^4*i^3 + 3*b^7*c^4*d^2*g^4*i^3 - 12*a*b^6*c^3*d^3*g^4*i^3 - 12*a^3*b^4*c*d^5*g^4*i^3 + 18*a^2*b^5*c^2*d^4*g^4*i^3) + x*(9*a^2*b^5*c^6* g^4*i^3 + 6*a^7*c*d^5*g^4*i^3 - 30*a^3*b^4*c^5*d*g^4*i^3 - 15*a^6*b*c^2*d^4 *g^4*i^3 + 30*a^4*b^3*c^4*d^2*g^4*i^3) + x^2*(3*a^7*d^6*g^4*i^3 + 9*a*b^6*c ^6*g^4*i^3 + 6*a^6*b*c*d^5*g^4*i^3 - 18*a^2*b^5*c^5*d*g^4*i^3 - 15*a^3*b^4* c^4*d^2*g^4*i^3 + 60*a^4*b^3*c^3*d^3*g^4*i^3 - 45*a^5*b^2*c^2*d^4*g^4*i^3) + x^3*(3*b^7*c^6*g^4*i^3 + 9*a^6*b*d^6*g^4*i^3 + 6*a*b^6*c^5*d*g^4*i^3 - 18 *a^5*b^2*c*d^5*g^4*i^3 - 45*a^2*b^5*c^4*d^2*g^4*i^3 + 60*a^3*b^4*c^3*d^3*g^ 4*i^3 - 15*a^4*b^3*c^2*d^4*g^4*i^3) + x^4*(9*a^5*b^2*d^6*g^4*i^3 + 6*b^7*c^ 5*d*g^4*i^3 - 15*a*b^6*c^4*d^2*g^4*i^3 - 30*a^4*b^3*c*d^5*g^4*i^3 + 30*a^3* b^4*c^2*d^4*g^4*i^3) + 3*a^3*b^4*c^6*g^4*i^3 + 3*a^7*c^2*d^4*g^4*i^3 - 12*a ^4*b^3*c^5*d*g^4*i^3 - 12*a^6*b*c^3*d^3*g^4*i^3 + 18*a^5*b^2*c^4*d^2*g^4*i^ 3) + (20*B*b^2*d^3*(3*A + B*n)*(x^2*((3*g^4*i^3*n*(a*d + b*c)^2*(a*d - b*c) ^5)/d + 6*a*b*c*g^4*i^3*n*(a*d - b*c)^5) + 6*b*g^4*i^3*n*x^3*(a*d + b*c)*(a *d - b*c)^5 + 3*b^2*d*g^4*i^3*n*x^4*(a*d - b*c)^5 + (3*a^2*c^2*g^4*i^3*n*(a *d - b*c)^5)/d + (6*a*c*g^4*i^3*n*x*(a*d + b*c)*(a*d - b*c)^5)/d))/(3*g^4*i ^3*n*(a*d - b*c)^6*(x^5*(3*a^4*b^3*d^6*g^4*i^3 + 3*b^7*c^4*d^2*g^4*i^3 - 12 *a*b^6*c^3*d^3*g^4*i^3 - 12*a^3*b^4*c*d^5*g^4*i^3 + 18*a^2*b^5*c^2*d^4*g^4* i^3) + x*(9*a^2*b^5*c^6*g^4*i^3 + 6*a^7*c*d^5*g^4*i^3 - 30*a^3*b^4*c^5*d*g^ 4*i^3 - 15*a^6*b*c^2*d^4*g^4*i^3 + 30*a^4*b^3*c^4*d^2*g^4*i^3) + x^2*(3*a^7 *d^6*g^4*i^3 + 9*a*b^6*c^6*g^4*i^3 + 6*a^6*b*c*d^5*g^4*i^3 - 18*a^2*b^5*c^5 *d*g^4*i^3 - 15*a^3*b^4*c^4*d^2*g^4*i^3 + 60*a^4*b^3*c^3*d^3*g^4*i^3 - 45*a ^5*b^2*c^2*d^4*g^4*i^3) + x^3*(3*b^7*c^6*g^4*i^3 + 9*a^6*b*d^6*g^4*i^3 + 6* a*b^6*c^5*d*g^4*i^3 - 18*a^5*b^2*c*d^5*g^4*i^3 - 45*a^2*b^5*c^4*d^2*g^4*i^3 + 60*a^3*b^4*c^3*d^3*g^4*i^3 - 15*a^4*b^3*c^2*d^4*g^4*i^3) + x^4*(9*a^5*b^ 2*d^6*g^4*i^3 + 6*b^7*c^5*d*g^4*i^3 - 15*a*b^6*c^4*d^2*g^4*i^3 - 30*a^4*b^3 *c*d^5*g^4*i^3 + 30*a^3*b^4*c^2*d^4*g^4*i^3) + 3*a^3*b^4*c^6*g^4*i^3 + 3*a^ 7*c^2*d^4*g^4*i^3 - 12*a^4*b^3*c^5*d*g^4*i^3 - 12*a^6*b*c^3*d^3*g^4*i^3 + 1 8*a^5*b^2*c^4*d^2*g^4*i^3))) + log(e*((a + b*x)/(c + d*x))^n)^2*((x*((5*B^2 *(2*a*b*d^2 + b^2*c*d)*(a*d + b*c))/(3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) - (5*B^2*b*d)/(6*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (5*B^2*a*b^2*c*d^2)/(a^2 *d^2 + b^2*c^2 - 2*a*b*c*d)^2) + x^2*((5*B^2*b*d*(2*a*b*d^2 + b^2*c*d))/(3* (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) + (5*B^2*b^2*d^2*(a*d + b*c))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) - (B^2*(3*a*d + 2*b*c))/(6*(a^2*d^2 + b^2*c^2 - 2* a*b*c*d)) + (5*B^2*a*c*(2*a*b*d^2 + b^2*c*d))/(3*(a^2*d^2 + b^2*c^2 - 2*a*b *c*d)^2) + (5*B^2*b^3*d^3*x^3)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2)/(x*(2*a^3 *c*d*g^4*i^3 + 3*a^2*b*c^2*g^4*i^3) + x^2*(a^3*d^2*g^4*i^3 + 3*a*b^2*c^2*g^ 4*i^3 + 6*a^2*b*c*d*g^4*i^3) + x^3*(b^3*c^2*g^4*i^3 + 3*a^2*b*d^2*g^4*i^3 + 6*a*b^2*c*d*g^4*i^3) + x^4*(2*b^3*c*d*g^4*i^3 + 3*a*b^2*d^2*g^4*i^3) + a^3 *c^2*g^4*i^3 + b^3*d^2*g^4*i^3*x^5) - (10*B*b^2*d^3*(3*A + B*n))/(3*g^4*i^3 *n*(a*d - b*c)^6) + (10*B^2*b^2*d^3*(x^2*((g^4*i^3*n*(a*d + b*c)^2*(a*d - b *c))/d + 2*a*b*c*g^4*i^3*n*(a*d - b*c)) + b^2*d*g^4*i^3*n*x^4*(a*d - b*c) +

(a^2*c^2*g^4*i^3*n*(a*d - b*c))/d + 2*b*g^4*i^3*n*x^3*(a*d + b*c)*(a*d - b
*c) + (2*a*c*g^4*i^3*n*x*(a*d + b*c)*(a*d - b*c))/d))/(g^4*i^3*n*(a*d - b*c
)^6*(x*(2*a^3*c*d*g^4*i^3 + 3*a^2*b*c^2*g^4*i^3) + x^2*(a^3*d^2*g^4*i^3 + 3
*a*b^2*c^2*g^4*i^3 + 6*a^2*b*c*d*g^4*i^3) + x^3*(b^3*c^2*g^4*i^3 + 3*a^2*b*
d^2*g^4*i^3 + 6*a*b^2*c*d*g^4*i^3) + x^4*(2*b^3*c*d*g^4*i^3 + 3*a*b^2*d^2*g
^4*i^3) + a^3*c^2*g^4*i^3 + b^3*d^2*g^4*i^3*x^5))) + ((36*A^2*b^4*c^4 - 54*
A^2*a^4*d^4 - 27*B^2*a^4*d^4*n^2 + 8*B^2*b^4*c^4*n^2 + 846*A^2*a^2*b^2*c^2*
d^2 - 234*A^2*a*b^3*c^3*d + 486*A^2*a^3*b*c*d^3 + 54*A*B*a^4*d^4*n + 24*A*B
*b^4*c^4*n - 127*B^2*a*b^3*c^3*d*n^2 + 1053*B^2*a^3*b*c*d^3*n^2 + 2033*B^2*
a^2*b^2*c^2*d^2*n^2 + 1914*A*B*a^2*b^2*c^2*d^2*n - 246*A*B*a*b^3*c^3*d*n -
1026*A*B*a^3*b*c*d^3*n)/(6*(a*d - b*c)) + (5*x*(54*A^2*a^3*b*d^4 - 18*A^2*b
^4*c^3*d + 198*A^2*a*b^3*c^2*d^2 + 630*A^2*a^2*b^2*c*d^3 + 189*B^2*a^3*b*d^
4*n^2 - 19*B^2*b^4*c^3*d*n^2 - 162*A*B*a^3*b*d^4*n - 30*A*B*b^4*c^3*d*n + 7
37*B^2*a*b^3*c^2*d^2*n^2 + 1445*B^2*a^2*b^2*c*d^3*n^2 + 618*A*B*a*b^3*c^2*d
^2*n + 150*A*B*a^2*b^2*c*d^3*n))/(6*(a*d - b*c)) + (5*x^3*(90*A^2*a*b^3*d^4
+ 54*A^2*b^4*c*d^3 + 233*B^2*a*b^3*d^4*n^2 + 159*B^2*b^4*c*d^3*n^2 + 24*A*
B*a*b^3*d^4*n + 72*A*B*b^4*c*d^3*n))/(a*d - b*c) + (10*x^4*(18*A^2*b^4*d^4
+ 49*B^2*b^4*d^4*n^2 + 12*A*B*b^4*d^4*n))/(a*d - b*c) + (5*x^2*(198*A^2*a^2
*b^2*d^4 + 36*A^2*b^4*c^2*d^2 + 503*B^2*a^2*b^2*d^4*n^2 + 170*B^2*b^4*c^2*d
^2*n^2 + 414*A^2*a*b^3*c*d^3 + 1091*B^2*a*b^3*c*d^3*n^2 - 84*A*B*a^2*b^2*d^
4*n + 132*A*B*b^4*c^2*d^2*n + 384*A*B*a*b^3*c*d^3*n))/(3*(a*d - b*c)))/(x^5
*(18*a^4*b^3*d^6*g^4*i^3 + 18*b^7*c^4*d^2*g^4*i^3 - 72*a*b^6*c^3*d^3*g^4*i^
3 - 72*a^3*b^4*c*d^5*g^4*i^3 + 108*a^2*b^5*c^2*d^4*g^4*i^3) + x*(54*a^2*b^5
*c^6*g^4*i^3 + 36*a^7*c*d^5*g^4*i^3 - 180*a^3*b^4*c^5*d*g^4*i^3 - 90*a^6*b*
c^2*d^4*g^4*i^3 + 180*a^4*b^3*c^4*d^2*g^4*i^3) + x^2*(18*a^7*d^6*g^4*i^3 +
54*a*b^6*c^6*g^4*i^3 + 36*a^6*b*c*d^5*g^4*i^3 - 108*a^2*b^5*c^5*d*g^4*i^3 -
90*a^3*b^4*c^4*d^2*g^4*i^3 + 360*a^4*b^3*c^3*d^3*g^4*i^3 - 270*a^5*b^2*c^2
*d^4*g^4*i^3) + x^3*(18*b^7*c^6*g^4*i^3 + 54*a^6*b*d^6*g^4*i^3 + 36*a*b^6*c
^5*d*g^4*i^3 - 108*a^5*b^2*c*d^5*g^4*i^3 - 270*a^2*b^5*c^4*d^2*g^4*i^3 + 36
0*a^3*b^4*c^3*d^3*g^4*i^3 - 90*a^4*b^3*c^2*d^4*g^4*i^3) + x^4*(54*a^5*b^2*d
^6*g^4*i^3 + 36*b^7*c^5*d*g^4*i^3 - 90*a*b^6*c^4*d^2*g^4*i^3 - 180*a^4*b^3*
c*d^5*g^4*i^3 + 180*a^3*b^4*c^2*d^4*g^4*i^3) + 18*a^3*b^4*c^6*g^4*i^3 + 18*
a^7*c^2*d^4*g^4*i^3 - 72*a^4*b^3*c^5*d*g^4*i^3 - 72*a^6*b*c^3*d^3*g^4*i^3 +
108*a^5*b^2*c^4*d^2*g^4*i^3) + (b^2*d^3*atan((b^2*d^3*(18*A^2 + 49*B^2*n^2
+ 12*A*B*n)*(9*a^6*d^6*g^4*i^3 - 9*b^6*c^6*g^4*i^3 + 36*a*b^5*c^5*d*g^4*i^
3 - 36*a^5*b*c*d^5*g^4*i^3 - 45*a^2*b^4*c^4*d^2*g^4*i^3 + 45*a^4*b^2*c^2*d^
4*g^4*i^3)*5i)/(9*g^4*i^3*(a*d - b*c)^6*(90*A^2*b^2*d^3 + 245*B^2*b^2*d^3*n
^2 + 60*A*B*b^2*d^3*n)) + (b^3*d^4*x*(18*A^2 + 49*B^2*n^2 + 12*A*B*n)*(a^5*
d^5*g^4*i^3 - b^5*c^5*g^4*i^3 + 5*a*b^4*c^4*d*g^4*i^3 - 5*a^4*b*c*d^4*g^4*i
^3 - 10*a^2*b^3*c^3*d^2*g^4*i^3 + 10*a^3*b^2*c^2*d^3*g^4*i^3)*10i)/(g^4*i^3
*(a*d - b*c)^6*(90*A^2*b^2*d^3 + 245*B^2*b^2*d^3*n^2 + 60*A*B*b^2*d^3*n)))*
(18*A^2 + 49*B^2*n^2 + 12*A*B*n)*10i)/(9*g^4*i^3*(a*d - b*c)^6) - (10*B^2*b
^2*d^3*log(e*((a + b*x)/(c + d*x))^n)^3)/(3*g^4*i^3*n*(a*d - b*c)^6)

3.210 $\int (ag+bgx)^m(ci+dix)^{-2-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p$

Optimal result

Integrand size = 49, antiderivative size = 189

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$$
$$=\frac{e^{-\frac{A(1+m)}{Bn}}(a+bx)(g(a+bx))^m\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{1+m}{n}}(i(c+dx))^{-m}\Gamma\left(1+p,-\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)\left(A+B\right.}{(bc-ad)i^2(1+m)(c+dx)}$$

```
[Out] (b*x+a)*(g*(b*x+a))^m*GAMMA(p+1,-(1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)
*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^p/(-a*d+b*c)/exp(A*(1+m)/B/n)/i^2/(1+m)/((
e*((b*x+a)/(d*x+c))^n)^((1+m)/n))/(d*x+c)/((i*(d*x+c))^m)/((-(1+m)*(A+B*ln(
e*((b*x+a)/(d*x+c))^n))/B/n)^p)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2563, 2347, 2212}

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$$
$$=\frac{(a+bx)e^{-\frac{A(m+1)}{Bn}}(g(a+bx))^m(i(c+dx))^{-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^p\left(-\frac{(m+1)\left(B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{Bn}\right.}{i^2(m+1)(c+dx)(bc-ad)}$$

```
[In] Int[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^p,x]
```

```
[Out] ((a + b*x)*(g*(a + b*x))^m*Gamma[1 + p, -(((1 + m)*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]))/(B*n))]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p)/((b*c - a
*d)*E^((A*(1 + m))/(B*n))*i^2*(1 + m)*(e*((a + b*x)/(c + d*x))^n)^((1 + m)/
n)*(c + d*x)*(i*(c + d*x))^m*(-(((1 + m)*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]))/(B*n)))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2563

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m)), Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*x
)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x]
 && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
q + 2, 0]
```

Rubi steps

integral

$$= \frac{\left((g(a+bx))^m \left(\frac{a+bx}{c+dx}\right)^{-m} (i(c+dx))^{-m}\right) \text{Subst}\left(\int x^m (A+B\log(ex^n))^p \, dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}$$

$$= \frac{\left((a+bx)(g(a+bx))^m \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{1+m}{n}} (i(c+dx))^{-m}\right) \text{Subst}\left(\int e^{\frac{(1+m)x}{n}} (A+Bx)^p \, dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2 n(c+dx)}$$

$$= \frac{e^{-\frac{A(1+m)}{Bn}} (a+bx)(g(a+bx))^m \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{1+m}{n}} (i(c+dx))^{-m} \Gamma\left(1+p, -\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right) (A+B}{(bc-ad)i^2(1+m)(c+dx)}$$

Mathematica [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$$
$$=\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$$

```
[In] Integrate[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^p,x]
```

```
[Out] Integrate[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^p, x]
```

Maple [F]

$$\int (bgx+ag)^m\,(dix+ci)^{-2-m}\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^p dx$$

```
[In] int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^p,x)
```

```
[Out] int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^p,x)
```

Fricas [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$$
$$=\int (bgx+ag)^m(dix+ci)^{-m-2}\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^p\,dx$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n))
^p,x, algorithm="fricas")
```

```
[Out] integral((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)*(B*log(e*((b*x + a)/(d*x +
c))^n) + A)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))**n
))**p,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$$
$$=\int (bgx+ag)^m(dix+ci)^{-m-2}\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^p\,dx$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n))
^p,x, algorithm="maxima")
```

```
[Out] integrate((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)*(B*log(e*((b*x + a)/(d*x +
 c))^n) + A)^p, x)
```

Giac [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$$
$$=\int (bgx+ag)^m(dix+ci)^{-m-2}\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^p\,dx$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n))
^p,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)*(B*log(e*((b*x + a)/(d*x +
 c))^n) + A)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$$
$$=\int \frac{(a\,g+b\,g\,x)^m\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^p}{(c\,i+d\,i\,x)^{m+2}}\,dx$$

```
[In] int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^p)/(c*i + d*i*x
)^(m + 2),x)
[Out] int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^p)/(c*i + d*i*x
)^(m + 2), x)
```

3.211 $\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d$

Optimal result

Integrand size = 49, antiderivative size = 190

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p dx = -\frac{e^{\frac{A(1+m)}{Bn}}(a+bx)(g(a+bx))^{-2-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1+m}{n}}(i(c+dx))^{2+m}\Gamma\left(1+p,\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)(A+ \ldots}{(bc-ad)i^2(1+m)(c+dx)}$$

```
[Out] -exp(A*(1+m)/B/n)*(b*x+a)*(g*(b*x+a))^(-2-m)*(e*((b*x+a)/(d*x+c))^n)^((1+m)
/n)*(i*(d*x+c))^(2+m)*GAMMA(p+1,(1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))^p/(-a*d+b*c)/i^2/(1+m)/(d*x+c)/(((1+m)*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))/B/n)^p)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2563, 2347, 2212}

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p dx = -\frac{(a+bx)e^{\frac{A(m+1)}{Bn}}(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^p\left(\frac{(m+1)\left(B\log\left(e\left(\frac{a+}{c+}\ldots\right.\right.\right.}{Bn}\ldots\right.}{i^2(m+1)(c+dx)(bc-ad)}$$

```
[In] Int[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^p,x]
```

```
[Out] -((E^((A*(1 + m))/(B*n))*(a + b*x)*(g*(a + b*x))^(-2 - m)*(e*((a + b*x)/(c
+ d*x))^n)^((1 + m)/n)*(i*(c + d*x))^(2 + m)*Gamma[1 + p, ((1 + m)*(A + B*L
og[e*((a + b*x)/(c + d*x))^n]))/(B*n)]*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])^p)/((b*c - a*d)*i^2*(1 + m)*(c + d*x)*(((1 + m)*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]))/(B*n))^p))
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2563

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m)), Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*x
)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x]
 && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
q + 2, 0]
```

Rubi steps

integral

$$= \frac{\left((g(a+bx))^{-2-m}\left(\frac{a+bx}{c+dx}\right)^{2+m}(i(c+dx))^{2+m}\right)\text{Subst}\left(\int x^{-2-m}(A+B\log(ex^n))^p\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}$$

$$= \frac{\left((a+bx)(g(a+bx))^{-2-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{-1-m}{n}}(i(c+dx))^{2+m}\right)\text{Subst}\left(\int e^{\frac{(-1-m)x}{n}}(A+Bx)^p\,dx,x,\text{lo}\right.}{(bc-ad)i^2n(c+dx)}$$

$$= -\frac{e^{\frac{A(1+m)}{Bn}}(a+bx)(g(a+bx))^{-2-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1+m}{n}}(i(c+dx))^{2+m}\Gamma\left(1+p,\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{Bn}\right.}{(bc-ad)i^2(1+m)(c+dx)}$$

Mathematica [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$$
$$=\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$$

```
[In] Integrate[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^p,x]
[Out] Integrate[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^p, x]
```

Maple [F]

$$\int (bgx+ag)^{-2-m}(dix+ci)^m\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^p dx$$

```
[In] int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^p,x)
[Out] int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^p,x)
```

Fricas [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$$
$$=\int (bgx+ag)^{-m-2}(dix+ci)^m\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^p\,dx$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n))
^p,x, algorithm="fricas")
[Out] integral((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m*(B*log(e*((b*x + a)/(d*x +
c))^n) + A)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m*(A+B*ln(e*((b*x+a)/(d*x+c))**n
))**p,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\begin{aligned}&\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx\\&=\int (bgx+ag)^{-m-2}(dix+ci)^m\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^p\,dx\end{aligned}$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n))
^p,x, algorithm="maxima")
```

```
[Out] integrate((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m*(B*log(e*((b*x + a)/(d*x +
 c))^n) + A)^p, x)
```

Giac [F]

$$\begin{aligned}&\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx\\&=\int (bgx+ag)^{-m-2}(dix+ci)^m\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^p\,dx\end{aligned}$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n))
^p,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m*(B*log(e*((b*x + a)/(d*x +
 c))^n) + A)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$$
$$=\int \frac{(c\,i+d\,i\,x)^m\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^p}{(a\,g+b\,g\,x)^{m+2}}\,dx$$

```
[In] int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^p)/(a*g + b*g*x
)^(m + 2),x)
```

```
[Out] int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^p)/(a*g + b*g*x
)^(m + 2), x)
```

3.212 $\int (ag+bgx)^m(ci+dix)^{-2-m} \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3$

Optimal result

Integrand size = 49, antiderivative size = 292

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\,dx$$
$$= -\frac{6B^3n^3(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{(bc-ad)i^2(1+m)^4(c+dx)}$$
$$+\frac{6B^2n^2(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)^3(c+dx)}$$
$$-\frac{3Bn(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(1+m)^2(c+dx)}$$
$$+\frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(bc-ad)i^2(1+m)(c+dx)}$$

```
[Out] -6*B^3*n^3*(b*x+a)*(g*(b*x+a))^m/(-a*d+b*c)/i^2/(1+m)^4/(d*x+c)/((i*(d*x+c)
)^m)+6*B^2*n^2*(b*x+a)*(g*(b*x+a))^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+
b*c)/i^2/(1+m)^3/(d*x+c)/((i*(d*x+c))^m)-3*B*n*(b*x+a)*(g*(b*x+a))^m*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/i^2/(1+m)^2/(d*x+c)/((i*(d*x+c))^m)+
(b*x+a)*(g*(b*x+a))^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/(-a*d+b*c)/i^2/(1+m
)/(d*x+c)/((i*(d*x+c))^m)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2563, 2342, 2341}

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\,dx$$
$$=\frac{6B^2n^2(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{i^2(m+1)^3(c+dx)(bc-ad)}$$
$$+\frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{i^2(m+1)(c+dx)(bc-ad)}$$
$$-\frac{3Bn(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{i^2(m+1)^2(c+dx)(bc-ad)}$$
$$-\frac{6B^3n^3(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{i^2(m+1)^4(c+dx)(bc-ad)}$$

```
[In] Int[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^3,x]
```

```
[Out] (-6*B^3*n^3*(a + b*x)*(g*(a + b*x))^m)/((b*c - a*d)*i^2*(1 + m)^4*(c + d*x)
*(i*(c + d*x))^m) + (6*B^2*n^2*(a + b*x)*(g*(a + b*x))^m*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/((b*c - a*d)*i^2*(1 + m)^3*(c + d*x)*(i*(c + d*x))^m)
 - (3*B*n*(a + b*x)*(g*(a + b*x))^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2)/((b*c - a*d)*i^2*(1 + m)^2*(c + d*x)*(i*(c + d*x))^m) + ((a + b*x)*(g*(a
 + b*x))^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/((b*c - a*d)*i^2*(1 +
m)*(c + d*x)*(i*(c + d*x))^m)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2563

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
```

```
] :> Dist[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m)), Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*x
)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x]
 && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
q + 2, 0]
```

Rubi steps

integral

$$= \frac{\left((g(a+bx))^m \left(\frac{a+bx}{c+dx}\right)^{-m} (i(c+dx))^{-m}\right) \operatorname{Subst}\left(\int x^m (A+B\log(ex^n))^3\, dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}$$

$$= \frac{(a+bx)(g(a+bx))^m (i(c+dx))^{-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(bc-ad)i^2(1+m)(c+dx)} - \frac{\left(3Bn(g(a+bx))^m \left(\frac{a+bx}{c+dx}\right)^{-m} (i(c+dx))^{-m}\right) \operatorname{Subst}\left(\int x^m (A+B\log(ex^n))^2\, dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^2(1+m)}$$

$$= -\frac{3Bn(a+bx)(g(a+bx))^m (i(c+dx))^{-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(1+m)^2(c+dx)} + \frac{(a+bx)(g(a+bx))^m (i(c+dx))^{-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(bc-ad)i^2(1+m)(c+dx)} + \frac{\left(6B^2n^2(g(a+bx))^m \left(\frac{a+bx}{c+dx}\right)^{-m} (i(c+dx))^{-m}\right) \operatorname{Subst}\left(\int x^m (A+B\log(ex^n))\, dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^2(1+m)^2}$$

$$= -\frac{6B^3n^3(a+bx)(g(a+bx))^m (i(c+dx))^{-m}}{(bc-ad)i^2(1+m)^4(c+dx)} + \frac{6B^2n^2(a+bx)(g(a+bx))^m (i(c+dx))^{-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)^3(c+dx)} - \frac{3Bn(a+bx)(g(a+bx))^m (i(c+dx))^{-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(1+m)^2(c+dx)} + \frac{(a+bx)(g(a+bx))^m (i(c+dx))^{-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(bc-ad)i^2(1+m)(c+dx)}$$

Mathematica [A] (verified)

Time = 4.71 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.71

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\,dx = \frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-1-m}\left(A^3(1+m)^3-3A^2B(1+m)^2n+6AB^2(1+m)n^2-6B^3n^3+3B(1\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^3,x]
```

```
[Out] ((a + b*x)*(g*(a + b*x))^m*(i*(c + d*x))^(-1 - m)*(A^3*(1 + m)^3 - 3*A^2*B*
(1 + m)^2*n + 6*A*B^2*(1 + m)*n^2 - 6*B^3*n^3 + 3*B*(1 + m)*(A^2*(1 + m)^2
- 2*A*B*(1 + m)*n + 2*B^2*n^2)*Log[e*((a + b*x)/(c + d*x))^n] + 3*B^2*(1 +
m)^2*(A + A*m - B*n)*Log[e*((a + b*x)/(c + d*x))^n]^2 + B^3*(1 + m)^3*Log[e
*((a + b*x)/(c + d*x))^n]^3))/((b*c - a*d)*i*(1 + m)^4)
```

Maple [F]

$$\int (bgx+ag)^m\,(dix+ci)^{-2-m}\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^3dx$$

```
[In] int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

```
[Out] int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2680 vs. 2(292) = 584.

Time = 0.42 (sec) , antiderivative size = 2680, normalized size of antiderivative = 9.18

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n))
^3,x, algorithm="fricas")
```

```
[Out] (A^3*a*c*m^3 - 6*B^3*a*c*n^3 + 3*A^3*a*c*m^2 + 3*A^3*a*c*m + A^3*a*c + (B^3
*a*c*m^3 + 3*B^3*a*c*m^2 + 3*B^3*a*c*m + B^3*a*c + (B^3*b*d*m^3 + 3*B^3*b*d
*m^2 + 3*B^3*b*d*m + B^3*b*d)*x^2 + (B^3*b*c + B^3*a*d + (B^3*b*c + B^3*a*d
)*m^3 + 3*(B^3*b*c + B^3*a*d)*m^2 + 3*(B^3*b*c + B^3*a*d)*m)*x)*log(e)^3 +
((B^3*b*d*m^3 + 3*B^3*b*d*m^2 + 3*B^3*b*d*m + B^3*b*d)*n^3*x^2 + (B^3*b*c +
 B^3*a*d + (B^3*b*c + B^3*a*d)*m^3 + 3*(B^3*b*c + B^3*a*d)*m^2 + 3*(B^3*b*c
 + B^3*a*d)*m)*n^3*x + (B^3*a*c*m^3 + 3*B^3*a*c*m^2 + 3*B^3*a*c*m + B^3*a*c
```

)*n^3)*log((b*x + a)/(d*x + c))^3 + 6*(A*B^2*a*c*m + A*B^2*a*c)*n^2 + (A^3*
b*d*m^3 - 6*B^3*b*d*n^3 + 3*A^3*b*d*m^2 + 3*A^3*b*d*m + A^3*b*d + 6*(A*B^2*
b*d*m + A*B^2*b*d)*n^2 - 3*(A^2*B*b*d*m^2 + 2*A^2*B*b*d*m + A^2*B*b*d)*n)*x
^2 + 3*(A*B^2*a*c*m^3 + 3*A*B^2*a*c*m^2 + 3*A*B^2*a*c*m + A*B^2*a*c + (A*B^
2*b*d*m^3 + 3*A*B^2*b*d*m^2 + 3*A*B^2*b*d*m + A*B^2*b*d - (B^3*b*d*m^2 + 2*
B^3*b*d*m + B^3*b*d)*n)*x^2 - (B^3*a*c*m^2 + 2*B^3*a*c*m + B^3*a*c)*n + (A*
B^2*b*c + A*B^2*a*d + (A*B^2*b*c + A*B^2*a*d)*m^3 + 3*(A*B^2*b*c + A*B^2*a*
d)*m^2 + 3*(A*B^2*b*c + A*B^2*a*d)*m - (B^3*b*c + B^3*a*d + (B^3*b*c + B^3*
a*d)*m^2 + 2*(B^3*b*c + B^3*a*d)*m)*n)*x + ((B^3*b*d*m^3 + 3*B^3*b*d*m^2 +
3*B^3*b*d*m + B^3*b*d)*n*x^2 + (B^3*b*c + B^3*a*d + (B^3*b*c + B^3*a*d)*m^3
 + 3*(B^3*b*c + B^3*a*d)*m^2 + 3*(B^3*b*c + B^3*a*d)*m)*n*x + (B^3*a*c*m^3
+ 3*B^3*a*c*m^2 + 3*B^3*a*c*m + B^3*a*c)*n)*log((b*x + a)/(d*x + c)))*log(e
)^2 - 3*((B^3*a*c*m^2 + 2*B^3*a*c*m + B^3*a*c)*n^3 - (A*B^2*a*c*m^3 + 3*A*B
^2*a*c*m^2 + 3*A*B^2*a*c*m + A*B^2*a*c)*n^2 + ((B^3*b*d*m^2 + 2*B^3*b*d*m +
 B^3*b*d)*n^3 - (A*B^2*b*d*m^3 + 3*A*B^2*b*d*m^2 + 3*A*B^2*b*d*m + A*B^2*b*
d)*n^2)*x^2 + ((B^3*b*c + B^3*a*d + (B^3*b*c + B^3*a*d)*m^2 + 2*(B^3*b*c +
B^3*a*d)*m)*n^3 - (A*B^2*b*c + A*B^2*a*d + (A*B^2*b*c + A*B^2*a*d)*m^3 + 3*
(A*B^2*b*c + A*B^2*a*d)*m^2 + 3*(A*B^2*b*c + A*B^2*a*d)*m)*n^2)*x)*log((b*x
 + a)/(d*x + c))^2 - 3*(A^2*B*a*c*m^2 + 2*A^2*B*a*c*m + A^2*B*a*c)*n + (A^3
*b*c + A^3*a*d + (A^3*b*c + A^3*a*d)*m^3 - 6*(B^3*b*c + B^3*a*d)*n^3 + 3*(A
^3*b*c + A^3*a*d)*m^2 + 6*(A*B^2*b*c + A*B^2*a*d + (A*B^2*b*c + A*B^2*a*d)*
m)*n^2 + 3*(A^3*b*c + A^3*a*d)*m - 3*(A^2*B*b*c + A^2*B*a*d + (A^2*B*b*c +
A^2*B*a*d)*m^2 + 2*(A^2*B*b*c + A^2*B*a*d)*m)*n)*x + 3*(A^2*B*a*c*m^3 + 3*A
^2*B*a*c*m^2 + 3*A^2*B*a*c*m + A^2*B*a*c + 2*(B^3*a*c*m + B^3*a*c)*n^2 + (A
^2*B*b*d*m^3 + 3*A^2*B*b*d*m^2 + 3*A^2*B*b*d*m + A^2*B*b*d + 2*(B^3*b*d*m +
 B^3*b*d)*n^2 - 2*(A*B^2*b*d*m^2 + 2*A*B^2*b*d*m + A*B^2*b*d)*n)*x^2 + ((B^
3*b*d*m^3 + 3*B^3*b*d*m^2 + 3*B^3*b*d*m + B^3*b*d)*n^2*x^2 + (B^3*b*c + B^3
*a*d + (B^3*b*c + B^3*a*d)*m^3 + 3*(B^3*b*c + B^3*a*d)*m^2 + 3*(B^3*b*c + B
^3*a*d)*m)*n^2*x + (B^3*a*c*m^3 + 3*B^3*a*c*m^2 + 3*B^3*a*c*m + B^3*a*c)*n^
2)*log((b*x + a)/(d*x + c))^2 - 2*(A*B^2*a*c*m^2 + 2*A*B^2*a*c*m + A*B^2*a*
c)*n + (A^2*B*b*c + A^2*B*a*d + (A^2*B*b*c + A^2*B*a*d)*m^3 + 3*(A^2*B*b*c
+ A^2*B*a*d)*m^2 + 2*(B^3*b*c + B^3*a*d + (B^3*b*c + B^3*a*d)*m)*n^2 + 3*(A
^2*B*b*c + A^2*B*a*d)*m - 2*(A*B^2*b*c + A*B^2*a*d + (A*B^2*b*c + A*B^2*a*d
)*m^2 + 2*(A*B^2*b*c + A*B^2*a*d)*m)*n)*x - 2*((B^3*a*c*m^2 + 2*B^3*a*c*m +
 B^3*a*c)*n^2 + ((B^3*b*d*m^2 + 2*B^3*b*d*m + B^3*b*d)*n^2 - (A*B^2*b*d*m^3
 + 3*A*B^2*b*d*m^2 + 3*A*B^2*b*d*m + A*B^2*b*d)*n)*x^2 - (A*B^2*a*c*m^3 + 3
*A*B^2*a*c*m^2 + 3*A*B^2*a*c*m + A*B^2*a*c)*n + ((B^3*b*c + B^3*a*d + (B^3*
b*c + B^3*a*d)*m^2 + 2*(B^3*b*c + B^3*a*d)*m)*n^2 - (A*B^2*b*c + A*B^2*a*d
+ (A*B^2*b*c + A*B^2*a*d)*m^3 + 3*(A*B^2*b*c + A*B^2*a*d)*m^2 + 3*(A*B^2*b*
c + A*B^2*a*d)*m)*n)*x)*log((b*x + a)/(d*x + c)))*log(e) + 3*(2*(B^3*a*c*m
+ B^3*a*c)*n^3 - 2*(A*B^2*a*c*m^2 + 2*A*B^2*a*c*m + A*B^2*a*c)*n^2 + (2*(B^
3*b*d*m + B^3*b*d)*n^3 - 2*(A*B^2*b*d*m^2 + 2*A*B^2*b*d*m + A*B^2*b*d)*n^2
+ (A^2*B*b*d*m^3 + 3*A^2*B*b*d*m^2 + 3*A^2*B*b*d*m + A^2*B*b*d)*n)*x^2 + (A
^2*B*a*c*m^3 + 3*A^2*B*a*c*m^2 + 3*A^2*B*a*c*m + A^2*B*a*c)*n + (2*(B^3*b*c
 + B^3*a*d + (B^3*b*c + B^3*a*d)*m)*n^3 - 2*(A*B^2*b*c + A*B^2*a*d + (A*B^2

```
*b*c + A*B^2*a*d)*m^2 + 2*(A*B^2*b*c + A*B^2*a*d)*m)*n^2 + (A^2*B*b*c + A^2
*B*a*d + (A^2*B*b*c + A^2*B*a*d)*m^3 + 3*(A^2*B*b*c + A^2*B*a*d)*m^2 + 3*(A
^2*B*b*c + A^2*B*a*d)*m)*n)*x)*log((b*x + a)/(d*x + c)))*(b*g*x + a*g)^m*e^
(-(m + 2)*log(b*g*x + a*g) + (m + 2)*log((b*x + a)/(d*x + c)) - (m + 2)*log
(i/g))/((b*c - a*d)*m^4 + 4*(b*c - a*d)*m^3 + 6*(b*c - a*d)*m^2 + b*c - a*d
 + 4*(b*c - a*d)*m)
```

Sympy [F(-2)]

Exception generated.

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))**n
))**3,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\,dx$$
$$=\int\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3(bgx+ag)^m(dix+ci)^{-m-2}\,dx$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n))
^3,x, algorithm="maxima")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^3*(b*g*x + a*g)^m*(d*i*x +
 c*i)^(-m - 2), x)
```

Giac [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\,dx$$
$$=\int\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3(bgx+ag)^m(dix+ci)^{-m-2}\,dx$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n))
^3,x, algorithm="giac")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^3*(b*g*x + a*g)^m*(d*i*x +
 c*i)^(-m - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\,dx$$
$$=\int \frac{(a\,g+b\,g\,x)^m\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^3}{(c\,i+d\,i\,x)^{m+2}}\,dx$$

```
[In] int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^3)/(c*i + d*i*x
)^(m + 2),x)
[Out] int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^3)/(c*i + d*i*x
)^(m + 2), x)
```

3.213 $\int (ag+bgx)^m(ci+dix)^{-2-m} \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d$

Optimal result

Integrand size = 49, antiderivative size = 210

$$\begin{aligned}
&\int (ag+bgx)^m(ci+dix)^{-2-m} \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= \frac{2B^2n^2(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{(bc-ad)i^2(1+m)^3(c+dx)} \\
&\quad - \frac{2Bn(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)^2(c+dx)} \\
&\quad + \frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(1+m)(c+dx)}
\end{aligned}$$

```
[Out] 2*B^2*n^2*(b*x+a)*(g*(b*x+a))^m/(-a*d+b*c)/i^2/(1+m)^3/(d*x+c)/((i*(d*x+c))
^m)-2*B*n*(b*x+a)*(g*(b*x+a))^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/
i^2/(1+m)^2/(d*x+c)/((i*(d*x+c))^m)+(b*x+a)*(g*(b*x+a))^m*(A+B*ln(e*((b*x+a
)/(d*x+c))^n))^2/(-a*d+b*c)/i^2/(1+m)/(d*x+c)/((i*(d*x+c))^m)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used

$= \{2563, 2342, 2341\}$

$$\int (agx+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$

$$= \frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{i^2(m+1)(c+dx)(bc-ad)} - \frac{2Bn(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{i^2(m+1)^2(c+dx)(bc-ad)} + \frac{2B^2n^2(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{i^2(m+1)^3(c+dx)(bc-ad)}$$

```
[In] Int[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^2,x]
```

```
[Out] (2*B^2*n^2*(a + b*x)*(g*(a + b*x))^m)/((b*c - a*d)*i^2*(1 + m)^3*(c + d*x)*
(i*(c + d*x))^m) - (2*B*n*(a + b*x)*(g*(a + b*x))^m*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/((b*c - a*d)*i^2*(1 + m)^2*(c + d*x)*(i*(c + d*x))^m) + ((
a + b*x)*(g*(a + b*x))^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*c -
a*d)*i^2*(1 + m)*(c + d*x)*(i*(c + d*x))^m)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2563

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m)), Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*x
)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x]
 && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
q + 2, 0]
```

Rubi steps

integral

$$= \frac{\left((g(a+bx))^m \left(\frac{a+bx}{c+dx}\right)^{-m} (i(c+dx))^{-m}\right) \text{Subst}\left(\int x^m (A+B\log(ex^n))^2\, dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}$$

$$= \frac{(a+bx)(g(a+bx))^m (i(c+dx))^{-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(1+m)(c+dx)} - \frac{\left(2Bn(g(a+bx))^m \left(\frac{a+bx}{c+dx}\right)^{-m} (i(c+dx))^{-m}\right) \text{Subst}\left(\int x^m (A+B\log(ex^n))\, dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^2(1+m)}$$

$$= \frac{2B^2n^2(a+bx)(g(a+bx))^m (i(c+dx))^{-m}}{(bc-ad)i^2(1+m)^3(c+dx)} - \frac{2Bn(a+bx)(g(a+bx))^m (i(c+dx))^{-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)^2(c+dx)} + \frac{(a+bx)(g(a+bx))^m (i(c+dx))^{-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(1+m)(c+dx)}$$

Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.64

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \frac{(a+bx)(g(a+bx))^m (i(c+dx))^{-1-m} \left(A^2(1+m)^2 - 2AB(1+m)n + 2B^2n^2 + 2B(1+m)(A+Am-Bn\right.}{(bc-ad)i(1+m)^3}$$

```
[In] Integrate[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^2,x]
```

```
[Out] ((a + b*x)*(g*(a + b*x))^m*(i*(c + d*x))^(-1 - m)*(A^2*(1 + m)^2 - 2*A*B*(1
 + m)*n + 2*B^2*n^2 + 2*B*(1 + m)*(A + A*m - B*n)*Log[e*((a + b*x)/(c + d*x
))^n] + B^2*(1 + m)^2*Log[e*((a + b*x)/(c + d*x))^n]^2))/((b*c - a*d)*i*(1
+ m)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2619 vs. 2(210) = 420.

Time = 134.76 (sec) , antiderivative size = 2620, normalized size of antiderivative = 12.48

method	result	size
parallelrisch	Expression too large to display	2620

```
[In] int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x,me
thod=_RETURNVERBOSE)
```

```
[Out] -(2*A*B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*
d*m^2*n+4*A*B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*
a*b*d^2*m*n+4*A*B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))
^n)*b^2*c*d*m*n+2*A*B*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c
))^n)*a*b*c*d*m^2*n+4*A*B*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d
*x+c))^n)*a*b*c*d*m*n+B^2*x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a
)/(d*x+c))^n)^2*b^2*d^2*n-2*B^2*x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*(
(b*x+a)/(d*x+c))^n)*b^2*d^2*n^2+2*A^2*x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*
b^2*d^2*m*n+2*A*B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))
^n)*a*b*d^2*m^2*n+B^2*x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d
*x+c))^n)^2*b^2*d^2*m^2*n+2*B^2*x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*(
(b*x+a)/(d*x+c))^n)^2*b^2*d^2*m*n-2*B^2*x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m
)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*m*n^2-2*A*B*x^2*(g*(b*x+a))^m*(i*(d*x+c
))^(-2-m)*b^2*d^2*m*n^2+A^2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*d^2*m^2*
n+A^2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*b^2*c*d*m^2*n+2*A*B*x^2*(g*(b*x+a)
)^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*n+B^2*x*(g*(b*x+a)
)^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b*d^2*n+B^2*x*(g*(b*x+
a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*b^2*c*d*n-2*B^2*x*(g*(
b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*n^2-2*B^2*x*
(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*n^2+2*A^
2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*d^2*m*n+2*A^2*x*(g*(b*x+a))^m*(i*(
d*x+c))^(-2-m)*b^2*c*d*m*n+A^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*c*d*m^2
*n-2*A*B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*d^2*n^2-2*A*B*x*(g*(b*x+a))
^m*(i*(d*x+c))^(-2-m)*b^2*c*d*n^2+B^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e
*((b*x+a)/(d*x+c))^n)^2*a*b*c*d*n-2*B^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln
(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*n^2+2*A^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*
a*b*c*d*m*n-2*A*B*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*c*d*n^2+2*A*B*(g*(b*
x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*n+2*A*B*x^2*(g
*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*m^2*n+B^2*
x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b*d^2*m^2*
n+B^2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*b^2*c*
d*m^2*n+4*A*B*x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n
)*b^2*d^2*m*n+2*B^2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c
))^n)^2*a*b*d^2*m*n+2*B^2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/
```

```
(d*x+c))^n)^2*b^2*c*d*m*n-2*B^2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b
*x+a)/(d*x+c))^n)*a*b*d^2*m*n^2-2*B^2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln
(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*m*n^2+B^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*
ln(e*((b*x+a)/(d*x+c))^n)^2*a*b*c*d*m^2*n-2*A*B*x*(g*(b*x+a))^m*(i*(d*x+c))
^(-2-m)*a*b*d^2*m*n^2-2*A*B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*b^2*c*d*m*n^
2+2*B^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b*c*
d*m*n-2*B^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*
c*d*m*n^2+2*A*B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n
)*a*b*d^2*n+2*A*B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))
^n)*b^2*c*d*n-2*A*B*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*c*d*m*n^2-2*A*B*x^
2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*b^2*d^2*n^2+2*B^2*x*(g*(b*x+a))^m*(i*(d*
x+c))^(-2-m)*a*b*d^2*n^3+2*B^2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*b^2*c*d*n
^3+2*B^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*c*d*n^3+A^2*x*(g*(b*x+a))^m*(
i*(d*x+c))^(-2-m)*a*b*d^2*n+A^2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*b^2*c*d*
n+A^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*c*d*n+A^2*x^2*(g*(b*x+a))^m*(i*(
d*x+c))^(-2-m)*b^2*d^2*m^2*n+2*B^2*x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*b^2
*d^2*n^3+A^2*x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*b^2*d^2*n)/n/(a*d*m-b*c*m
+a*d-b*c)/(m^2+2*m+1)/b/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 991 vs. $2(210) = 420$.

Time = 0.39 (sec) , antiderivative size = 991, normalized size of antiderivative = 4.72

$$\int (ag + bgx)^m (ci + dix)^{-2-m} \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \frac{\left(A^2acm^2 + 2\,B^2acn^2 + 2\,A^2acm + A^2ac + (A^2bdm^2 + 2\,B^2bdn^2 + 2\,A^2bdm + A^2bd - 2\,(ABbdm + ABbd\right.}{}$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n))
^2,x, algorithm="fricas")
```

```
[Out] (A^2*a*c*m^2 + 2*B^2*a*c*n^2 + 2*A^2*a*c*m + A^2*a*c + (A^2*b*d*m^2 + 2*B^2
*b*d*n^2 + 2*A^2*b*d*m + A^2*b*d - 2*(A*B*b*d*m + A*B*b*d)*n)*x^2 + (B^2*a*
c*m^2 + 2*B^2*a*c*m + B^2*a*c + (B^2*b*d*m^2 + 2*B^2*b*d*m + B^2*b*d)*x^2 +
 (B^2*b*c + B^2*a*d + (B^2*b*c + B^2*a*d)*m^2 + 2*(B^2*b*c + B^2*a*d)*m)*x)
*log(e)^2 + ((B^2*b*d*m^2 + 2*B^2*b*d*m + B^2*b*d)*n^2*x^2 + (B^2*b*c + B^2
*a*d + (B^2*b*c + B^2*a*d)*m^2 + 2*(B^2*b*c + B^2*a*d)*m)*n^2*x + (B^2*a*c*
m^2 + 2*B^2*a*c*m + B^2*a*c)*n^2)*log((b*x + a)/(d*x + c))^2 - 2*(A*B*a*c*m
 + A*B*a*c)*n + (A^2*b*c + A^2*a*d + (A^2*b*c + A^2*a*d)*m^2 + 2*(B^2*b*c +
 B^2*a*d)*n^2 + 2*(A^2*b*c + A^2*a*d)*m - 2*(A*B*b*c + A*B*a*d + (A*B*b*c +
 A*B*a*d)*m)*n)*x + 2*(A*B*a*c*m^2 + 2*A*B*a*c*m + A*B*a*c + (A*B*b*d*m^2 +
 2*A*B*b*d*m + A*B*b*d - (B^2*b*d*m + B^2*b*d)*n)*x^2 - (B^2*a*c*m + B^2*a*
```

```
c)*n + (A*B*b*c + A*B*a*d + (A*B*b*c + A*B*a*d)*m^2 + 2*(A*B*b*c + A*B*a*d)
*m - (B^2*b*c + B^2*a*d + (B^2*b*c + B^2*a*d)*m)*n)*x + ((B^2*b*d*m^2 + 2*B
^2*b*d*m + B^2*b*d)*n*x^2 + (B^2*b*c + B^2*a*d + (B^2*b*c + B^2*a*d)*m^2 +
2*(B^2*b*c + B^2*a*d)*m)*n*x + (B^2*a*c*m^2 + 2*B^2*a*c*m + B^2*a*c)*n)*log
((b*x + a)/(d*x + c)))*log(e) - 2*((B^2*a*c*m + B^2*a*c)*n^2 + ((B^2*b*d*m
+ B^2*b*d)*n^2 - (A*B*b*d*m^2 + 2*A*B*b*d*m + A*B*b*d)*n)*x^2 - (A*B*a*c*m^
2 + 2*A*B*a*c*m + A*B*a*c)*n + ((B^2*b*c + B^2*a*d + (B^2*b*c + B^2*a*d)*m)
*n^2 - (A*B*b*c + A*B*a*d + (A*B*b*c + A*B*a*d)*m^2 + 2*(A*B*b*c + A*B*a*d)
*m)*n)*x)*log((b*x + a)/(d*x + c)))*(b*g*x + a*g)^m*e^(-(m + 2)*log(b*g*x +
 a*g) + (m + 2)*log((b*x + a)/(d*x + c)) - (m + 2)*log(i/g))/((b*c - a*d)*m
^3 + 3*(b*c - a*d)*m^2 + b*c - a*d + 3*(b*c - a*d)*m)
```

Sympy [F(-2)]

Exception generated.

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))**n
))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2(bgx+ag)^m(dix+ci)^{-m-2}\,dx$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n))
^2,x, algorithm="maxima")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2*(b*g*x + a*g)^m*(d*i*x +
 c*i)^(-m - 2), x)
```

Giac [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2(bgx+ag)^m(dix+ci)^{-m-2}\,dx$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n))
^2,x, algorithm="giac")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2*(b*g*x + a*g)^m*(d*i*x +
 c*i)^(-m - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int \frac{(a\,g+b\,g\,x)^m\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}{(c\,i+d\,i\,x)^{m+2}}\,dx$$

```
[In] int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x
)^(m + 2),x)
```

```
[Out] int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x
)^(m + 2), x)
```

3.214 $\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)$ d

Optimal result

Integrand size = 47, antiderivative size = 128

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = -\frac{Bn(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{(bc-ad)i^2(1+m)^2(c+dx)} + \frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)(c+dx)}$$

```
[Out] -B*n*(b*x+a)*(g*(b*x+a))^m/(-a*d+b*c)/i^2/(1+m)^2/(d*x+c)/((i*(d*x+c))^m)+(
b*x+a)*(g*(b*x+a))^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/i^2/(1+m)/(
d*x+c)/((i*(d*x+c))^m)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2563, 2341}

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{i^2(m+1)(c+dx)(bc-ad)} - \frac{Bn(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{i^2(m+1)^2(c+dx)(bc-ad)}$$

```
[In] Int[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]),x]
```

```
[Out] -((B*n*(a + b*x)*(g*(a + b*x))^m)/((b*c - a*d)*i^2*(1 + m)^2*(c + d*x)*(i*(
c + d*x))^m)) + ((a + b*x)*(g*(a + b*x))^m*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]))/((b*c - a*d)*i^2*(1 + m)*(c + d*x)*(i*(c + d*x))^m)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2563

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m)), Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*x
)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x]
 && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
q + 2, 0]
```

Rubi steps

$$\text{integral} = \frac{\left((g(a+bx))^m \left(\frac{a+bx}{c+dx}\right)^{-m} (i(c+dx))^{-m}\right) \text{Subst}\left(\int x^m(A+B\log(ex^n))\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}$$

$$= -\frac{Bn(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{(bc-ad)i^2(1+m)^2(c+dx)} + \frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)(c+dx)}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.61

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-1-m}\left(A+Am-Bn+B(1+m)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i(1+m)^2}$$

```
[In] Integrate[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]),x]
```

```
[Out] ((a + b*x)*(g*(a + b*x))^m*(i*(c + d*x))^(-1 - m)*(A + A*m - B*n + B*(1 + m
)*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)*i*(1 + m)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 816 vs. $2(128) = 256$.

Time = 32.12 (sec) , antiderivative size = 817, normalized size of antiderivative = 6.38

method	result
parallelrisch	$-\frac{Ax(g(bx+a))^m (i(dx+c))^{-2-m} ab\, d^2 mn + Ax(g(bx+a))^m (i(dx+c))^{-2-m} b^2 cdmn + Bx(g(bx+a))^m (i(dx+c))^{-2-m} \ln\left(e\left(\frac{bx}{dx}\right.\right.}{}$

```
[In] int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,meth
od=_RETURNVERBOSE)
```

```
[Out] -(A*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*d^2*m*n+A*x*(g*(b*x+a))^m*(i*(d*
x+c))^(-2-m)*b^2*c*d*m*n+B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)
/(d*x+c))^n)*a*b*d^2*n+B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(
d*x+c))^n)*b^2*c*d*n+A*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*c*d*m*n+B*(g*(b
*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*n+B*x^2*(g*(b
*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*m*n+B*x*(g*(b
*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*m*n+B*x*(g*(b
*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*m*n+B*(g*(b*x
+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*m*n+A*x^2*(g*(b
*x+a))^m*(i*(d*x+c))^(-2-m)*b^2*d^2*m*n-B*x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2
-m)*b^2*d^2*n^2+A*x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*b^2*d^2*n+B*x^2*(g*(
b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*n-B*x*(g*(b*
x+a))^m*(i*(d*x+c))^(-2-m)*a*b*d^2*n^2-B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)
*b^2*c*d*n^2+A*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*d^2*n+A*x*(g*(b*x+a))
^m*(i*(d*x+c))^(-2-m)*b^2*c*d*n-B*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*c*d*
n^2+A*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*c*d*n)/d/b/(a*d-b*c)/n/(1+m)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(128) = 256$.

Time = 0.32 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.14

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{(Aacm - Bacn + Aac + (Abdm - Bbdn + Abd)x^2 + (Abc + Aad + (Abc + Aad)m - (Bbc + Bad)n)x + \cdots}{}$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n))
,x, algorithm="fricas")
```

```
[Out] (A*a*c*m - B*a*c*n + A*a*c + (A*b*d*m - B*b*d*n + A*b*d)*x^2 + (A*b*c + A*a
*d + (A*b*c + A*a*d)*m - (B*b*c + B*a*d)*n)*x + (B*a*c*m + B*a*c + (B*b*d*m
```

```
 + B*b*d)*x^2 + (B*b*c + B*a*d + (B*b*c + B*a*d)*m)*x)*log(e) + ((B*b*d*m +
 B*b*d)*n*x^2 + (B*b*c + B*a*d + (B*b*c + B*a*d)*m)*n*x + (B*a*c*m + B*a*c)
*n)*log((b*x + a)/(d*x + c)))*(b*g*x + a*g)^m*e^(-(m + 2)*log(b*g*x + a*g)
+ (m + 2)*log((b*x + a)/(d*x + c)) - (m + 2)*log(i/g))/((b*c - a*d)*m^2 + b
*c - a*d + 2*(b*c - a*d)*m)
```

Sympy [F(-2)]

Exception generated.

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))**n
)),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)(bgx+ag)^m(dix+ci)^{-m-2}\,dx$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n))
,x, algorithm="maxima")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(b*g*x + a*g)^m*(d*i*x + c
*i)^(-m - 2), x)
```

Giac [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)(bgx+ag)^m(dix+ci)^{-m-2}\,dx$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n))
,x, algorithm="giac")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(b*g*x + a*g)^m*(d*i*x + c
*i)^(-m - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \int \frac{(a\,g+b\,g\,x)^m\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(c\,i+d\,i\,x)^{m+2}}\,dx$$

```
[In] int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)^
(m + 2),x)
[Out] int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)^
(m + 2), x)
```

3.215 $\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 125

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{e^{-\frac{A(1+m)}{Bn}}(a+bx)(g(a+bx))^m\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{1+m}{n}}(i(c+dx))^{-m}\,\text{ExpIntegralEi}\left(\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B(bc-ad)i^2n(c+dx)}$$

```
[Out] (b*x+a)*(g*(b*x+a))^m*Ei((1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B/(-a*d
+b*c)/exp(A*(1+m)/B/n)/i^2/n/((e*((b*x+a)/(d*x+c))^n)^((1+m)/n))/(d*x+c)/((
i*(d*x+c))^m)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2563, 2347, 2209}

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{(a+bx)e^{-\frac{A(m+1)}{Bn}}(g(a+bx))^m(i(c+dx))^{-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\,\text{ExpIntegralEi}\left(\frac{(m+1)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{Bi^2n(c+dx)(bc-ad)}$$

```
[In] Int[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)/(c + d
*x))^n]),x]
```

```
[Out] ((a + b*x)*(g*(a + b*x))^m*ExpIntegralEi[((1 + m)*(A + B*Log[e*((a + b*x)/(
c + d*x))^n]))/(B*n)])/(B*(b*c - a*d)*E^((A*(1 + m))/(B*n))*i^2*n*(e*((a +
b*x)/(c + d*x))^n)^((1 + m)/n)*(c + d*x)*(i*(c + d*x))^m)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2563

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m)), Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*x
)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x]
 && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
q + 2, 0]
```

Rubi steps

$$\text{integral} = \frac{\left((g(a+bx))^m \left(\frac{a+bx}{c+dx}\right)^{-m} (i(c+dx))^{-m}\right) \text{Subst}\left(\int \frac{x^m}{A+B\log(ex^n)}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}$$

$$= \frac{\left((a+bx)(g(a+bx))^m \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{1+m}{n}} (i(c+dx))^{-m}\right) \text{Subst}\left(\int \frac{e^{\frac{(1+m)x}{n}}}{A+Bx}\,dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2n(c+dx)}$$

$$= \frac{e^{-\frac{A(1+m)}{Bn}}(a+bx)(g(a+bx))^m \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{1+m}{n}} (i(c+dx))^{-m}\text{Ei}\left(\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B(bc-ad)i^2n(c+dx)}$$

Mathematica [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] Integrate[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)/
(c + d*x))^n]),x]
```

```
[Out] Integrate[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)/
(c + d*x))^n]), x]
```

Maple [F]

$$\int \frac{(bgx+ag)^m\,(dix+ci)^{-2-m}}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

```
[In] int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.78

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\frac{\operatorname{Ei}\left(\frac{(Bm+B)n\log\left(\frac{bx+a}{dx+c}\right)+Am+(Bm+B)\log(e)+A}{Bn}\right)e^{\left(-\frac{(Bm+2\,B)n\log\left(\frac{i}{g}\right)+Am+(Bm+B)\log(e)+A}{Bn}\right)}}{(Bbc-Bad)g^2n}$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n))
,x, algorithm="fricas")
```

```
[Out] Ei(((B*m + B)*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(e) + A)/(B*n
))*e^(-((B*m + 2*B)*n*log(i/g) + A*m + (B*m + B)*log(e) + A)/(B*n))/((B*b*c
 - B*a*d)*g^2*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))**n
)),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bgx+ag)^m(dix+ci)^{-m-2}}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n))
,x, algorithm="maxima")
[Out] integrate((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)/(B*log(e*((b*x + a)/(d*x +
 c))^n) + A), x)
```

Giac [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bgx+ag)^m(dix+ci)^{-m-2}}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n))
,x, algorithm="giac")
[Out] integrate((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)/(B*log(e*((b*x + a)/(d*x +
 c))^n) + A), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(a\,g+b\,g\,x)^m}{(c\,i+d\,i\,x)^{m+2}\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}\,dx$$

```
[In] int((a*g + b*g*x)^m/((c*i + d*i*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*x
))^n))),x)
```

```
[Out] int((a*g + b*g*x)^m/((c*i + d*i*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*x
))^n))), x)
```

3.216 $\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 206

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$
$$= \frac{e^{-\frac{A(1+m)}{Bn}}(1+m)(a+bx)(g(a+bx))^m\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{1+m}{n}}(i(c+dx))^{-m}\,\text{ExpIntegralEi}\left(\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{Bn}\right.}{B^2(bc-ad)i^2n^2(c+dx)}$$
$$-\frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

```
[Out] (1+m)*(b*x+a)*(g*(b*x+a))^m*Ei((1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B
^2/(-a*d+b*c)/exp(A*(1+m)/B/n)/i^2/n^2/((e*((b*x+a)/(d*x+c))^n)^((1+m)/n))/
(d*x+c)/((i*(d*x+c))^m)-(b*x+a)*(g*(b*x+a))^m/B/(-a*d+b*c)/i^2/n/(d*x+c)/((
i*(d*x+c))^m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.082$, Rules used

$= \{2563, 2343, 2347, 2209\}$

$$\int \frac{(ag+bgx)^m (ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

$$= \frac{(m+1)(a+bx)e^{-\frac{A(m+1)}{Bn}}(g(a+bx))^m(i(c+dx))^{-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\text{ExpIntegralEi}\left(\frac{(m+1)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B^2i^2n^2(c+dx)(bc-ad)} - \frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{Bi^2n(c+dx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}$$

```
[In] Int[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)/(c + d
*x))^n])^2,x]
```

```
[Out] ((1 + m)*(a + b*x)*(g*(a + b*x))^m*ExpIntegralEi[((1 + m)*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n]))/(B*n)])/(B^2*(b*c - a*d)*E^((A*(1 + m))/(B*n))*i^2*n
^2*(e*((a + b*x)/(c + d*x))^n)^((1 + m)/n)*(c + d*x)*(i*(c + d*x))^m) - ((a
 + b*x)*(g*(a + b*x))^m)/(B*(b*c - a*d)*i^2*n*(c + d*x)*(i*(c + d*x))^m*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2563

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m)), Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*x
)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x]
 && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
```

```
q + 2, 0]
```

Rubi steps

$$\text{integral} = \frac{\left((g(a+bx))^m \left(\frac{a+bx}{c+dx}\right)^{-m} (i(c+dx))^{-m}\right) \text{Subst}\left(\int \frac{x^m}{(A+B\log(ex^n))^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}$$

$$= -\frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} + \frac{\left((1+m)(g(a+bx))^m \left(\frac{a+bx}{c+dx}\right)^{-m} (i(c+dx))^{-m}\right) \text{Subst}\left(\int \frac{x^m}{A+B\log(ex^n)}\,dx, x, \frac{a+bx}{c+dx}\right)}{B(bc-ad)i^2n}$$

$$= -\frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} + \frac{\left((1+m)(a+bx)(g(a+bx))^m \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{1+m}{n}} (i(c+dx))^{-m}\right) \text{Subst}\left(\int \frac{e^{\frac{(1+m)x}{n}}}{A+Bx}\,dx, x, \log\left(e\left(\frac{a-}{c+}\right.\right.\right.}{B(bc-ad)i^2n^2(c+dx)}$$

$$= \frac{e^{-\frac{A(1+m)}{Bn}}(1+m)(a+bx)(g(a+bx))^m \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{1+m}{n}} (i(c+dx))^{-m}\text{Ei}\left(\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B^2(bc-ad)i^2n^2(c+dx)} - \frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

Mathematica [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Integrate[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)/
(c + d*x))^n])^2,x]
[Out] Integrate[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)/
(c + d*x))^n])^2, x]
```

Maple [F]

$$\int \frac{(bgx+ag)^m\,(dix+ci)^{-2-m}}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

```
[In] int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.42

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx =$$

$$-\frac{(Bbdg^2nx^2+Bacg^2n+(Bbc+Bad)g^2nx)(bgx+ag)^m e^{\left(-(m+2)\log(bgx+ag)+(m+2)\log\left(\frac{bx+a}{dx+c}\right)-(m+2)\log\left(\frac{i}{g}\right)\right)} -}{(B^3bc-B^3ad)g^2n^3\log\left(\frac{bx\cdot}{dx\cdot}\right.}$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n))
^2,x, algorithm="fricas")
```

```
[Out] -((B*b*d*g^2*n*x^2 + B*a*c*g^2*n + (B*b*c + B*a*d)*g^2*n*x)*(b*g*x + a*g)^m
*e^(-(m + 2)*log(b*g*x + a*g) + (m + 2)*log((b*x + a)/(d*x + c)) - (m + 2)*
log(i/g)) - ((B*m + B)*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(e)
+ A)*Ei(((B*m + B)*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(e) + A)
/(B*n))*e^(-((B*m + 2*B)*n*log(i/g) + A*m + (B*m + B)*log(e) + A)/(B*n)))/(
(B^3*b*c - B^3*a*d)*g^2*n^3*log((b*x + a)/(d*x + c)) + (B^3*b*c - B^3*a*d)*
g^2*n^2*log(e) + (A*B^2*b*c - A*B^2*a*d)*g^2*n^2)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))**n
))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(bgx+ag)^m(dix+ci)^{-m-2}}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n))
^2,x, algorithm="maxima")
```

```
[Out] -g^m*(m + 1)*integrate(-(b*x + a)^m/((B^2*d^2*i^(m + 2)*n*x^2 + 2*B^2*c*d*i
^(m + 2)*n*x + B^2*c^2*i^(m + 2)*n)*(d*x + c)^m*log((b*x + a)^n) - (B^2*d^2
*i^(m + 2)*n*x^2 + 2*B^2*c*d*i^(m + 2)*n*x + B^2*c^2*i^(m + 2)*n)*(d*x + c)
^m*log((d*x + c)^n) + (B^2*c^2*i^(m + 2)*n*log(e) + A*B*c^2*i^(m + 2)*n + (
B^2*d^2*i^(m + 2)*n*log(e) + A*B*d^2*i^(m + 2)*n)*x^2 + 2*(B^2*c*d*i^(m + 2
)*n*log(e) + A*B*c*d*i^(m + 2)*n)*x)*(d*x + c)^m), x) - (b*g^m*x + a*g^m)*(
b*x + a)^m/(((b*c*d*i^(m + 2)*n - a*d^2*i^(m + 2)*n)*B^2*x + (b*c^2*i^(m +
2)*n - a*c*d*i^(m + 2)*n)*B^2)*(d*x + c)^m*log((b*x + a)^n) - ((b*c*d*i^(m
+ 2)*n - a*d^2*i^(m + 2)*n)*B^2*x + (b*c^2*i^(m + 2)*n - a*c*d*i^(m + 2)*n)
*B^2)*(d*x + c)^m*log((d*x + c)^n) + ((b*c^2*i^(m + 2)*n - a*c*d*i^(m + 2)*
n)*A*B + (b*c^2*i^(m + 2)*n*log(e) - a*c*d*i^(m + 2)*n*log(e))*B^2 + ((b*c*
d*i^(m + 2)*n - a*d^2*i^(m + 2)*n)*A*B + (b*c*d*i^(m + 2)*n*log(e) - a*d^2*
i^(m + 2)*n*log(e))*B^2)*x)*(d*x + c)^m)
```

Giac [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(bgx+ag)^m(dix+ci)^{-m-2}}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n))
^2,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)/(B*log(e*((b*x + a)/(d*x +
 c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(a\,g+b\,g\,x)^m}{(c\,i+d\,i\,x)^{m+2}\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

```
[In] int((a*g + b*g*x)^m/((c*i + d*i*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*x
))^n))^2),x)
```

```
[Out] int((a*g + b*g*x)^m/((c*i + d*i*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*x
))^n))^2), x)
```

3.217 $\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 295

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx$$
$$= \frac{e^{-\frac{A(1+m)}{Bn}}(1+m)^2(a+bx)(g(a+bx))^m\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{1+m}{n}}(i(c+dx))^{-m}\operatorname{ExpIntegralEi}\left(\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{Bn}\right.}{2B^3(bc-ad)i^2n^3(c+dx)}$$
$$-\frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{2B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}$$
$$-\frac{(1+m)(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{2B^2(bc-ad)i^2n^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

```
[Out] 1/2*(1+m)^2*(b*x+a)*(g*(b*x+a))^m*Ei((1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/
B/n)/B^3/(-a*d+b*c)/exp(A*(1+m)/B/n)/i^2/n^3/((e*((b*x+a)/(d*x+c))^n)^((1+m
)/n))/(d*x+c)/((i*(d*x+c))^m)-1/2*(b*x+a)*(g*(b*x+a))^m/B/(-a*d+b*c)/i^2/n/
(d*x+c)/((i*(d*x+c))^m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2-1/2*(1+m)*(b*x+a)
*(g*(b*x+a))^m/B^2/(-a*d+b*c)/i^2/n^2/(d*x+c)/((i*(d*x+c))^m)/(A+B*ln(e*((b
*x+a)/(d*x+c))^n))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.082, Rules used = {2563, 2343, 2347, 2209}

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx$$
$$=\frac{(m+1)^2(a+bx)e^{-\frac{A(m+1)}{Bn}}(g(a+bx))^m(i(c+dx))^{-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\text{ExpIntegralEi}\left(\frac{(m+1)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{2B^3i^2n^3(c+dx)(bc-ad)}$$
$$-\frac{(m+1)(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{2B^2i^2n^2(c+dx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}$$
$$-\frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{2Bi^2n(c+dx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}$$

```
[In] Int[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)/(c + d
*x))^n])^3,x]
```

```
[Out] ((1 + m)^2*(a + b*x)*(g*(a + b*x))^m*ExpIntegralEi[((1 + m)*(A + B*Log[e*((
a + b*x)/(c + d*x))^n]))/(B*n)])/(2*B^3*(b*c - a*d)*E^((A*(1 + m))/(B*n))*i
^2*n^3*(e*((a + b*x)/(c + d*x))^n)^((1 + m)/n)*(c + d*x)*(i*(c + d*x))^m) -
 ((a + b*x)*(g*(a + b*x))^m)/(2*B*(b*c - a*d)*i^2*n*(c + d*x)*(i*(c + d*x))
^m*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2) - ((1 + m)*(a + b*x)*(g*(a + b
*x))^m)/(2*B^2*(b*c - a*d)*i^2*n^2*(c + d*x)*(i*(c + d*x))^m*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2563

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m)), Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*x
)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x]
 && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
q + 2, 0]
```

Rubi steps

$$\text{integral} = \frac{\left((g(a+bx))^m \left(\frac{a+bx}{c+dx}\right)^{-m} (i(c+dx))^{-m}\right) \text{Subst}\left(\int \frac{x^m}{(A+B\log(ex^n))^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}$$

$$= -\frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{2B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} + \frac{\left((1+m)(g(a+bx))^m \left(\frac{a+bx}{c+dx}\right)^{-m} (i(c+dx))^{-m}\right) \text{Subst}\left(\int \frac{x^m}{(A+B\log(ex^n))^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{2B(bc-ad)i^2n}$$

$$= -\frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{2B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} - \frac{(1+m)(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{2B^2(bc-ad)i^2n^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} + \frac{\left((1+m)^2(g(a+bx))^m \left(\frac{a+bx}{c+dx}\right)^{-m} (i(c+dx))^{-m}\right) \text{Subst}\left(\int \frac{x^m}{A+B\log(ex^n)}\,dx, x, \frac{a+bx}{c+dx}\right)}{2B^2(bc-ad)i^2n^2}$$

$$= -\frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{2B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} - \frac{(1+m)(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{2B^2(bc-ad)i^2n^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} + \frac{\left((1+m)^2(a+bx)(g(a+bx))^m \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{1+m}{n}} (i(c+dx))^{-m}\right) \text{Subst}\left(\int \frac{e^{\frac{(1+m)x}{n}}}{A+Bx}\,dx, x, \log\left(e\left(\frac{a+ \cdots}{c+ \cdots}\right.\right.\right.}{2B^2(bc-ad)i^2n^3(c+dx)}$$

$$= \frac{e^{-\frac{A(1+m)}{Bn}}(1+m)^2(a+bx)(g(a+bx))^m \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{1+m}{n}} (i(c+dx))^{-m}\text{Ei}\left(\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{2B^3(bc-ad)i^2n^3(c+dx)} - \frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{2B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} - \frac{(1+m)(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{2B^2(bc-ad)i^2n^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

Mathematica [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx=\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx$$

```
[In] Integrate[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)/
(c + d*x))^n])^3,x]
```

```
[Out] Integrate[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)/
(c + d*x))^n])^3, x]
```

Maple [F]

$$\int \frac{(bgx+ag)^m\,(dix+ci)^{-2-m}}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^3}dx$$

```
[In] int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

```
[Out] int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. 2(289) = 578.

Time = 0.34 (sec) , antiderivative size = 818, normalized size of antiderivative = 2.77

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx=$$

$$-\frac{(B^2acg^2n^2+(B^2bdg^2n^2+(ABbdg^2m+ABbdg^2)n)x^2+(ABacg^2m+ABacg^2)n+((B^2bc+B^2ad)g^2}{}$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n))
^3,x, algorithm="fricas")
```

```
[Out] -1/2*((B^2*a*c*g^2*n^2 + (B^2*b*d*g^2*n^2 + (A*B*b*d*g^2*m + A*B*b*d*g^2)*n
)*x^2 + (A*B*a*c*g^2*m + A*B*a*c*g^2)*n + ((B^2*b*c + B^2*a*d)*g^2*n^2 + ((
A*B*b*c + A*B*a*d)*g^2*m + (A*B*b*c + A*B*a*d)*g^2)*n)*x + ((B^2*b*d*g^2*m
+ B^2*b*d*g^2)*n*x^2 + ((B^2*b*c + B^2*a*d)*g^2*m + (B^2*b*c + B^2*a*d)*g^2
)*n*x + (B^2*a*c*g^2*m + B^2*a*c*g^2)*n)*log(e) + ((B^2*b*d*g^2*m + B^2*b*d
*g^2)*n^2*x^2 + ((B^2*b*c + B^2*a*d)*g^2*m + (B^2*b*c + B^2*a*d)*g^2)*n^2*x
 + (B^2*a*c*g^2*m + B^2*a*c*g^2)*n^2)*log((b*x + a)/(d*x + c)))*(b*g*x + a*
g)^m*e^(-(m + 2)*log(b*g*x + a*g) + (m + 2)*log((b*x + a)/(d*x + c)) - (m +
```

```
2)*log(i/g)) - ((B^2*m^2 + 2*B^2*m + B^2)*n^2*log((b*x + a)/(d*x + c))^2 +
 A^2*m^2 + 2*A^2*m + (B^2*m^2 + 2*B^2*m + B^2)*log(e)^2 + 2*(A*B*m^2 + 2*A*
B*m + A*B)*n*log((b*x + a)/(d*x + c)) + A^2 + 2*(A*B*m^2 + 2*A*B*m + (B^2*m
^2 + 2*B^2*m + B^2)*n*log((b*x + a)/(d*x + c)) + A*B)*log(e))*Ei(((B*m + B)
*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(e) + A)/(B*n))*e^(-((B*m
+ 2*B)*n*log(i/g) + A*m + (B*m + B)*log(e) + A)/(B*n)))/((B^5*b*c - B^5*a*d
)*g^2*n^5*log((b*x + a)/(d*x + c))^2 + (B^5*b*c - B^5*a*d)*g^2*n^3*log(e)^2
 + 2*(A*B^4*b*c - A*B^4*a*d)*g^2*n^4*log((b*x + a)/(d*x + c)) + (A^2*B^3*b*
c - A^2*B^3*a*d)*g^2*n^3 + 2*((B^5*b*c - B^5*a*d)*g^2*n^4*log((b*x + a)/(d*
x + c)) + (A*B^4*b*c - A*B^4*a*d)*g^2*n^3)*log(e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))**n
))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx = \int \frac{(bgx+ag)^m(dix+ci)^{-m-2}}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3}\,dx$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n))
^3,x, algorithm="maxima")
```

```
[Out] -(m^2 + 2*m + 1)*g^m*integrate(-1/2*(b*x + a)^m/((B^3*d^2*i^(m + 2)*n^2*x^2
 + 2*B^3*c*d*i^(m + 2)*n^2*x + B^3*c^2*i^(m + 2)*n^2)*(d*x + c)^m*log((b*x
+ a)^n) - (B^3*d^2*i^(m + 2)*n^2*x^2 + 2*B^3*c*d*i^(m + 2)*n^2*x + B^3*c^2*
i^(m + 2)*n^2)*(d*x + c)^m*log((d*x + c)^n) + (B^3*c^2*i^(m + 2)*n^2*log(e)
 + A*B^2*c^2*i^(m + 2)*n^2 + (B^3*d^2*i^(m + 2)*n^2*log(e) + A*B^2*d^2*i^(m
 + 2)*n^2)*x^2 + 2*(B^3*c*d*i^(m + 2)*n^2*log(e) + A*B^2*c*d*i^(m + 2)*n^2)
*x)*(d*x + c)^m), x) - 1/2*((B*b*g^m*(m + 1)*x + B*a*g^m*(m + 1))*(b*x + a)
^m*log((b*x + a)^n) - (B*b*g^m*(m + 1)*x + B*a*g^m*(m + 1))*(b*x + a)^m*log
((d*x + c)^n) + (A*a*g^m*(m + 1) + (g^m*(m + 1)*log(e) + g^m*n)*B*a + (A*b*
g^m*(m + 1) + (g^m*(m + 1)*log(e) + g^m*n)*B*b)*x)*(b*x + a)^m)/(((b*c*d*i^
(m + 2)*n^2 - a*d^2*i^(m + 2)*n^2)*B^4*x + (b*c^2*i^(m + 2)*n^2 - a*c*d*i^(
m + 2)*n^2)*B^4)*(d*x + c)^m*log((b*x + a)^n)^2 + ((b*c*d*i^(m + 2)*n^2 - a
*d^2*i^(m + 2)*n^2)*B^4*x + (b*c^2*i^(m + 2)*n^2 - a*c*d*i^(m + 2)*n^2)*B^4
```

```
)*(d*x + c)^m*log((d*x + c)^n)^2 + 2*((b*c^2*i^(m + 2)*n^2 - a*c*d*i^(m + 2
)*n^2)*A*B^3 + (b*c^2*i^(m + 2)*n^2*log(e) - a*c*d*i^(m + 2)*n^2*log(e))*B^
4 + ((b*c*d*i^(m + 2)*n^2 - a*d^2*i^(m + 2)*n^2)*A*B^3 + (b*c*d*i^(m + 2)*n
^2*log(e) - a*d^2*i^(m + 2)*n^2*log(e))*B^4)*x)*(d*x + c)^m*log((b*x + a)^n
) + ((b*c^2*i^(m + 2)*n^2 - a*c*d*i^(m + 2)*n^2)*A^2*B^2 + 2*(b*c^2*i^(m +
2)*n^2*log(e) - a*c*d*i^(m + 2)*n^2*log(e))*A*B^3 + (b*c^2*i^(m + 2)*n^2*lo
g(e)^2 - a*c*d*i^(m + 2)*n^2*log(e)^2)*B^4 + ((b*c*d*i^(m + 2)*n^2 - a*d^2*
i^(m + 2)*n^2)*A^2*B^2 + 2*(b*c*d*i^(m + 2)*n^2*log(e) - a*d^2*i^(m + 2)*n^
2*log(e))*A*B^3 + (b*c*d*i^(m + 2)*n^2*log(e)^2 - a*d^2*i^(m + 2)*n^2*log(e
)^2)*B^4)*x)*(d*x + c)^m - 2*(((b*c*d*i^(m + 2)*n^2 - a*d^2*i^(m + 2)*n^2)*
B^4*x + (b*c^2*i^(m + 2)*n^2 - a*c*d*i^(m + 2)*n^2)*B^4)*(d*x + c)^m*log((b
*x + a)^n) + ((b*c^2*i^(m + 2)*n^2 - a*c*d*i^(m + 2)*n^2)*A*B^3 + (b*c^2*i^
(m + 2)*n^2*log(e) - a*c*d*i^(m + 2)*n^2*log(e))*B^4 + ((b*c*d*i^(m + 2)*n^
2 - a*d^2*i^(m + 2)*n^2)*A*B^3 + (b*c*d*i^(m + 2)*n^2*log(e) - a*d^2*i^(m +
 2)*n^2*log(e))*B^4)*x)*(d*x + c)^m)*log((d*x + c)^n))
```

Giac [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx=\int \frac{(bgx+ag)^m(dix+ci)^{-m-2}}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3}\,dx$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n))
^3,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)/(B*log(e*((b*x + a)/(d*x +
 c))^n) + A)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx=\int \frac{(a\,g+b\,g\,x)^m}{(c\,i+d\,i\,x)^{m+2}\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^3}\,dx$$

```
[In] int((a*g + b*g*x)^m/((c*i + d*i*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*x
))^n))^3),x)
```

```
[Out] int((a*g + b*g*x)^m/((c*i + d*i*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*x
))^n))^3), x)
```

3.218 $\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 d$

Optimal result

Integrand size = 49, antiderivative size = 309

$$
\begin{aligned}
&\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx \\
&= -\frac{6B^3n^3(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{(bc-ad)i^2(1+m)^4(c+dx)} \\
&\quad - \frac{6B^2n^2(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)^3(c+dx)} \\
&\quad - \frac{3Bn(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(1+m)^2(c+dx)} \\
&\quad - \frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(bc-ad)i^2(1+m)(c+dx)}
\end{aligned}
$$

```
[Out] -6*B^3*n^3*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)/(-a*d+b*c)/i^2/(1+m
)^4/(d*x+c)-6*B^2*n^2*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)*(A+B*ln(
e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/i^2/(1+m)^3/(d*x+c)-3*B*n*(b*x+a)*(g*(b*
x+a))^(-2-m)*(i*(d*x+c))^(2+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)
/i^2/(1+m)^2/(d*x+c)-(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))^3/(-a*d+b*c)/i^2/(1+m)/(d*x+c)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2563, 2342, 2341}

$$\begin{aligned}&\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3dx\\&=-\frac{6B^2n^2(a+bx)(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{i^2(m+1)^3(c+dx)(bc-ad)}\\&\quad-\frac{(a+bx)(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{i^2(m+1)(c+dx)(bc-ad)}\\&\quad-\frac{3Bn(a+bx)(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{i^2(m+1)^2(c+dx)(bc-ad)}\\&\quad-\frac{6B^3n^3(a+bx)(g(a+bx))^{-m-2}(i(c+dx))^{m+2}}{i^2(m+1)^4(c+dx)(bc-ad)}\end{aligned}$$

```
[In] Int[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^3,x]
```

```
[Out] (-6*B^3*n^3*(a + b*x)*(g*(a + b*x))^(-2 - m)*(i*(c + d*x))^(2 + m))/((b*c -
 a*d)*i^2*(1 + m)^4*(c + d*x)) - (6*B^2*n^2*(a + b*x)*(g*(a + b*x))^(-2 - m
)*(i*(c + d*x))^(2 + m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d
)*i^2*(1 + m)^3*(c + d*x)) - (3*B*n*(a + b*x)*(g*(a + b*x))^(-2 - m)*(i*(c
+ d*x))^(2 + m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*c - a*d)*i^2*
(1 + m)^2*(c + d*x)) - ((a + b*x)*(g*(a + b*x))^(-2 - m)*(i*(c + d*x))^(2 +
 m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/((b*c - a*d)*i^2*(1 + m)*(c +
 d*x))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2563

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
```

```
] :> Dist[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m)), Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*x
)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x]
 && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
q + 2, 0]
```

Rubi steps

integral

$$= \frac{\left((g(a+bx))^{-2-m}\left(\frac{a+bx}{c+dx}\right)^{2+m}(i(c+dx))^{2+m}\right)\text{Subst}\left(\int x^{-2-m}(A+B\log(ex^n))^3\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}$$

$$= -\frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(bc-ad)i^2(1+m)(c+dx)} + \frac{\left(3Bn(g(a+bx))^{-2-m}\left(\frac{a+bx}{c+dx}\right)^{2+m}(i(c+dx))^{2+m}\right)\text{Subst}\left(\int x^{-2-m}(A+B\log(ex^n))^2\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)i^2(1+m)}$$

$$= -\frac{3Bn(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(1+m)^2(c+dx)} - \frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(bc-ad)i^2(1+m)(c+dx)} + \frac{\left(6B^2n^2(g(a+bx))^{-2-m}\left(\frac{a+bx}{c+dx}\right)^{2+m}(i(c+dx))^{2+m}\right)\text{Subst}\left(\int x^{-2-m}(A+B\log(ex^n))\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)i^2(1+m)^2}$$

$$= -\frac{6B^3n^3(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{(bc-ad)i^2(1+m)^4(c+dx)} - \frac{6B^2n^2(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)^3(c+dx)} - \frac{3Bn(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(1+m)^2(c+dx)} - \frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(bc-ad)i^2(1+m)(c+dx)}$$

Mathematica [A] (verified)

Time = 4.82 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.67

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx = -\frac{(g(a+bx))^{-1-m}(c+dx)(i(c+dx))^m\left(A^3(1+m)^3+3A^2B(1+m)^2n+6AB^2(1+m)n^2+6B^3n^3+3\right.}{}$$

```
[In] Integrate[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^3,x]
```

```
[Out] -((((g*(a + b*x))^(-1 - m)*(c + d*x)*(i*(c + d*x))^m*(A^3*(1 + m)^3 + 3*A^2*
B*(1 + m)^2*n + 6*A*B^2*(1 + m)*n^2 + 6*B^3*n^3 + 3*B*(1 + m)*(A^2*(1 + m)^
2 + 2*A*B*(1 + m)*n + 2*B^2*n^2)*Log[e*((a + b*x)/(c + d*x))^n] + 3*B^2*(1
+ m)^2*(A + A*m + B*n)*Log[e*((a + b*x)/(c + d*x))^n]^2 + B^3*(1 + m)^3*Log
[e*((a + b*x)/(c + d*x))^n]^3))/((b*c - a*d)*g*(1 + m)^4))
```

Maple [F]

$$\int (bgx+ag)^{-2-m}(dix+ci)^m\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^3 dx$$

```
[In] int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

```
[Out] int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2669 vs. 2(309) = 618.

Time = 0.44 (sec) , antiderivative size = 2669, normalized size of antiderivative = 8.64

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx = \text{Too large to display}$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n))
^3,x, algorithm="fricas")
```

```
[Out] -(A^3*a*c*m^3 + 6*B^3*a*c*n^3 + 3*A^3*a*c*m^2 + 3*A^3*a*c*m + A^3*a*c + (B^
3*a*c*m^3 + 3*B^3*a*c*m^2 + 3*B^3*a*c*m + B^3*a*c + (B^3*b*d*m^3 + 3*B^3*b*
d*m^2 + 3*B^3*b*d*m + B^3*b*d)*x^2 + (B^3*b*c + B^3*a*d + (B^3*b*c + B^3*a*
d)*m^3 + 3*(B^3*b*c + B^3*a*d)*m^2 + 3*(B^3*b*c + B^3*a*d)*m)*x)*log(e)^3 +
 ((B^3*b*d*m^3 + 3*B^3*b*d*m^2 + 3*B^3*b*d*m + B^3*b*d)*n^3*x^2 + (B^3*b*c
+ B^3*a*d + (B^3*b*c + B^3*a*d)*m^3 + 3*(B^3*b*c + B^3*a*d)*m^2 + 3*(B^3*b*
c + B^3*a*d)*m)*n^3*x + (B^3*a*c*m^3 + 3*B^3*a*c*m^2 + 3*B^3*a*c*m + B^3*a*
```

c)*n^3)*log((b*x + a)/(d*x + c))^3 + 6*(A*B^2*a*c*m + A*B^2*a*c)*n^2 + (A^3
*b*d*m^3 + 6*B^3*b*d*n^3 + 3*A^3*b*d*m^2 + 3*A^3*b*d*m + A^3*b*d + 6*(A*B^2
*b*d*m + A*B^2*b*d)*n^2 + 3*(A^2*B*b*d*m^2 + 2*A^2*B*b*d*m + A^2*B*b*d)*n)*
x^2 + 3*(A*B^2*a*c*m^3 + 3*A*B^2*a*c*m^2 + 3*A*B^2*a*c*m + A*B^2*a*c + (A*B
^2*b*d*m^3 + 3*A*B^2*b*d*m^2 + 3*A*B^2*b*d*m + A*B^2*b*d + (B^3*b*d*m^2 + 2
*B^3*b*d*m + B^3*b*d)*n)*x^2 + (B^3*a*c*m^2 + 2*B^3*a*c*m + B^3*a*c)*n + (A
*B^2*b*c + A*B^2*a*d + (A*B^2*b*c + A*B^2*a*d)*m^3 + 3*(A*B^2*b*c + A*B^2*a
*d)*m^2 + 3*(A*B^2*b*c + A*B^2*a*d)*m + (B^3*b*c + B^3*a*d + (B^3*b*c + B^3
*a*d)*m^2 + 2*(B^3*b*c + B^3*a*d)*m)*n)*x + ((B^3*b*d*m^3 + 3*B^3*b*d*m^2 +
3*B^3*b*d*m + B^3*b*d)*n*x^2 + (B^3*b*c + B^3*a*d + (B^3*b*c + B^3*a*d)*m^
3 + 3*(B^3*b*c + B^3*a*d)*m^2 + 3*(B^3*b*c + B^3*a*d)*m)*n*x + (B^3*a*c*m^3
+ 3*B^3*a*c*m^2 + 3*B^3*a*c*m + B^3*a*c)*n)*log((b*x + a)/(d*x + c)))*log(
e)^2 + 3*((B^3*a*c*m^2 + 2*B^3*a*c*m + B^3*a*c)*n^3 + (A*B^2*a*c*m^3 + 3*A*
B^2*a*c*m^2 + 3*A*B^2*a*c*m + A*B^2*a*c)*n^2 + ((B^3*b*d*m^2 + 2*B^3*b*d*m
+ B^3*b*d)*n^3 + (A*B^2*b*d*m^3 + 3*A*B^2*b*d*m^2 + 3*A*B^2*b*d*m + A*B^2*b
*d)*n^2)*x^2 + ((B^3*b*c + B^3*a*d + (B^3*b*c + B^3*a*d)*m^2 + 2*(B^3*b*c +
B^3*a*d)*m)*n^3 + (A*B^2*b*c + A*B^2*a*d + (A*B^2*b*c + A*B^2*a*d)*m^3 + 3
*(A*B^2*b*c + A*B^2*a*d)*m^2 + 3*(A*B^2*b*c + A*B^2*a*d)*m)*n^2)*x)*log((b*
x + a)/(d*x + c))^2 + 3*(A^2*B*a*c*m^2 + 2*A^2*B*a*c*m + A^2*B*a*c)*n + (A^
3*b*c + A^3*a*d + (A^3*b*c + A^3*a*d)*m^3 + 6*(B^3*b*c + B^3*a*d)*n^3 + 3*(
A^3*b*c + A^3*a*d)*m^2 + 6*(A*B^2*b*c + A*B^2*a*d + (A*B^2*b*c + A*B^2*a*d)
*m)*n^2 + 3*(A^3*b*c + A^3*a*d)*m + 3*(A^2*B*b*c + A^2*B*a*d + (A^2*B*b*c +
A^2*B*a*d)*m^2 + 2*(A^2*B*b*c + A^2*B*a*d)*m)*n)*x + 3*(A^2*B*a*c*m^3 + 3*
A^2*B*a*c*m^2 + 3*A^2*B*a*c*m + A^2*B*a*c + 2*(B^3*a*c*m + B^3*a*c)*n^2 + (
A^2*B*b*d*m^3 + 3*A^2*B*b*d*m^2 + 3*A^2*B*b*d*m + A^2*B*b*d + 2*(B^3*b*d*m
+ B^3*b*d)*n^2 + 2*(A*B^2*b*d*m^2 + 2*A*B^2*b*d*m + A*B^2*b*d)*n)*x^2 + ((B
^3*b*d*m^3 + 3*B^3*b*d*m^2 + 3*B^3*b*d*m + B^3*b*d)*n^2*x^2 + (B^3*b*c + B^
3*a*d + (B^3*b*c + B^3*a*d)*m^3 + 3*(B^3*b*c + B^3*a*d)*m^2 + 3*(B^3*b*c +
B^3*a*d)*m)*n^2*x + (B^3*a*c*m^3 + 3*B^3*a*c*m^2 + 3*B^3*a*c*m + B^3*a*c)*n
^2)*log((b*x + a)/(d*x + c))^2 + 2*(A*B^2*a*c*m^2 + 2*A*B^2*a*c*m + A*B^2*a
*c)*n + (A^2*B*b*c + A^2*B*a*d + (A^2*B*b*c + A^2*B*a*d)*m^3 + 3*(A^2*B*b*c
+ A^2*B*a*d)*m^2 + 2*(B^3*b*c + B^3*a*d + (B^3*b*c + B^3*a*d)*m)*n^2 + 3*(
A^2*B*b*c + A^2*B*a*d)*m + 2*(A*B^2*b*c + A*B^2*a*d + (A*B^2*b*c + A*B^2*a*
d)*m^2 + 2*(A*B^2*b*c + A*B^2*a*d)*m)*n)*x + 2*((B^3*a*c*m^2 + 2*B^3*a*c*m
+ B^3*a*c)*n^2 + ((B^3*b*d*m^2 + 2*B^3*b*d*m + B^3*b*d)*n^2 + (A*B^2*b*d*m^
3 + 3*A*B^2*b*d*m^2 + 3*A*B^2*b*d*m + A*B^2*b*d)*n)*x^2 + (A*B^2*a*c*m^3 +
3*A*B^2*a*c*m^2 + 3*A*B^2*a*c*m + A*B^2*a*c)*n + ((B^3*b*c + B^3*a*d + (B^3
*b*c + B^3*a*d)*m^2 + 2*(B^3*b*c + B^3*a*d)*m)*n^2 + (A*B^2*b*c + A*B^2*a*d
+ (A*B^2*b*c + A*B^2*a*d)*m^3 + 3*(A*B^2*b*c + A*B^2*a*d)*m^2 + 3*(A*B^2*b
*c + A*B^2*a*d)*m)*n)*x)*log((b*x + a)/(d*x + c)))*log(e) + 3*(2*(B^3*a*c*m
+ B^3*a*c)*n^3 + 2*(A*B^2*a*c*m^2 + 2*A*B^2*a*c*m + A*B^2*a*c)*n^2 + (2*(B
^3*b*d*m + B^3*b*d)*n^3 + 2*(A*B^2*b*d*m^2 + 2*A*B^2*b*d*m + A*B^2*b*d)*n^2
+ (A^2*B*b*d*m^3 + 3*A^2*B*b*d*m^2 + 3*A^2*B*b*d*m + A^2*B*b*d)*n)*x^2 + (
A^2*B*a*c*m^3 + 3*A^2*B*a*c*m^2 + 3*A^2*B*a*c*m + A^2*B*a*c)*n + (2*(B^3*b*
c + B^3*a*d + (B^3*b*c + B^3*a*d)*m)*n^3 + 2*(A*B^2*b*c + A*B^2*a*d + (A*B^

```
2*b*c + A*B^2*a*d)*m^2 + 2*(A*B^2*b*c + A*B^2*a*d)*m)*n^2 + (A^2*B*b*c + A^
2*B*a*d + (A^2*B*b*c + A^2*B*a*d)*m^3 + 3*(A^2*B*b*c + A^2*B*a*d)*m^2 + 3*(
A^2*B*b*c + A^2*B*a*d)*m)*n)*x)*log((b*x + a)/(d*x + c)))*(b*g*x + a*g)^(-m
 - 2)*e^(m*log(b*g*x + a*g) - m*log((b*x + a)/(d*x + c)) + m*log(i/g))/((b*
c - a*d)*m^4 + 4*(b*c - a*d)*m^3 + 6*(b*c - a*d)*m^2 + b*c - a*d + 4*(b*c -
 a*d)*m)
```

Sympy [F(-2)]

Exception generated.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m*(A+B*ln(e*((b*x+a)/(d*x+c))**n
))**3,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3 (bgx+ag)^{-m-2}(dix+ci)^m\,dx$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n))
^3,x, algorithm="maxima")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^3*(b*g*x + a*g)^(-m - 2)*(
d*i*x + c*i)^m, x)
```

Giac [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3 (bgx+ag)^{-m-2}(dix+ci)^m\,dx$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n))
^3,x, algorithm="giac")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^3*(b*g*x + a*g)^(-m - 2)*(
d*i*x + c*i)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx$$
$$=\int \frac{(c\,i+d\,i\,x)^m\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^3}{(a\,g+b\,g\,x)^{m+2}}\,dx$$

```
[In] int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^3)/(a*g + b*g*x
)^(m + 2),x)
[Out] int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^3)/(a*g + b*g*x
)^(m + 2), x)
```

3.219 $\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2$

Optimal result

Integrand size = 49, antiderivative size = 223

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = -\frac{2B^2n^2(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{(bc-ad)i^2(1+m)^3(c+dx)} - \frac{2Bn(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)^2(c+dx)} - \frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(1+m)(c+dx)}$$

```
[Out] -2*B^2*n^2*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)/(-a*d+b*c)/i^2/(1+m
)^3/(d*x+c)-2*B*n*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))/(-a*d+b*c)/i^2/(1+m)^2/(d*x+c)-(b*x+a)*(g*(b*x+a))^(-2-
m)*(i*(d*x+c))^(2+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/i^2/(1+m)
/(d*x+c)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used

= {2563, 2342, 2341}

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= -\frac{(a+bx)(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{i^2(m+1)(c+dx)(bc-ad)} - \frac{2Bn(a+bx)(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{i^2(m+1)^2(c+dx)(bc-ad)} - \frac{2B^2n^2(a+bx)(g(a+bx))^{-m-2}(i(c+dx))^{m+2}}{i^2(m+1)^3(c+dx)(bc-ad)}$$

```
[In] Int[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^2,x]
```

```
[Out] (-2*B^2*n^2*(a + b*x)*(g*(a + b*x))^(-2 - m)*(i*(c + d*x))^(2 + m))/((b*c -
 a*d)*i^2*(1 + m)^3*(c + d*x)) - (2*B*n*(a + b*x)*(g*(a + b*x))^(-2 - m)*(i
*(c + d*x))^(2 + m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)*i^
2*(1 + m)^2*(c + d*x)) - ((a + b*x)*(g*(a + b*x))^(-2 - m)*(i*(c + d*x))^(2
 + m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*c - a*d)*i^2*(1 + m)*(c
 + d*x))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2563

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m)), Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*x
)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x]
 && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
q + 2, 0]
```

Rubi steps

$$\text{integral} = \frac{\left((g(a+bx))^{-2-m}\left(\frac{a+bx}{c+dx}\right)^{2+m}(i(c+dx))^{2+m}\right)\text{Subst}\left(\int x^{-2-m}(A+B\log(ex^n))^2\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}$$

$$= -\frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(1+m)(c+dx)} + \frac{\left(2Bn(g(a+bx))^{-2-m}\left(\frac{a+bx}{c+dx}\right)^{2+m}(i(c+dx))^{2+m}\right)\text{Subst}\left(\int x^{-2-m}(A+B\log(ex^n))\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)i^2(1+m)}$$

$$= -\frac{2B^2n^2(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{(bc-ad)i^2(1+m)^3(c+dx)} - \frac{2Bn(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)^2(c+dx)} - \frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(1+m)(c+dx)}$$

Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.60

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = -\frac{(g(a+bx))^{-1-m}(c+dx)(i(c+dx))^m\left(A^2(1+m)^2+2AB(1+m)n+2B^2n^2+2B(1+m)(A+Am+\ldots\right.}{(bc-ad)g(1+m)^3}$$

```
[In] Integrate[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^2,x]
```

```
[Out] -(((g*(a + b*x))^(-1 - m)*(c + d*x)*(i*(c + d*x))^m*(A^2*(1 + m)^2 + 2*A*B*
(1 + m)*n + 2*B^2*n^2 + 2*B*(1 + m)*(A + A*m + B*n)*Log[e*((a + b*x)/(c + d
*x))^n] + B^2*(1 + m)^2*Log[e*((a + b*x)/(c + d*x))^n]^2))/((b*c - a*d)*g*(
1 + m)^3))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2618 vs. 2(223) = 446.

Time = 131.30 (sec) , antiderivative size = 2619, normalized size of antiderivative = 11.74

method	result	size
parallelrisch	Expression too large to display	2619

```
[In] int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x,me
thod=_RETURNVERBOSE)
```

```
[Out] (2*A*B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2
*n+2*A*B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c
*d*n+2*A*B*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*c*d*m*n^2+2*A*B*(i*(d*x+c))
^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*n+2*B^2*x*(i*(d*x+c
))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*m*n^2+2*B^2*x*(i*
(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*m*n^2+B^2*(
i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b*c*d*m^2*n+2
*A*B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*d^2*m*n^2+4*A*B*x*(i*(d*x+c))^m
*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*m*n+4*A*B*x*(i*(d*x+c
))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*m*n+2*A*B*(i*(d*x
+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*m^2*n+4*A*B*(i*
(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*m*n+B^2*x*(
i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b*d^2*n+B^2*x
*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*b^2*c*d*n+2*B
^2*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*n^2
+2*B^2*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d
*n^2+2*A^2*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*d^2*m*n+2*A^2*x*(i*(d*x+c
))^m*(g*(b*x+a))^(-2-m)*b^2*c*d*m*n+A^2*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*
b*c*d*m^2*n+2*A*B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*d^2*n^2+2*A*B*x*(i
*(d*x+c))^m*(g*(b*x+a))^(-2-m)*b^2*c*d*n^2+B^2*(i*(d*x+c))^m*(g*(b*x+a))^(-
2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b*c*d*n+2*B^2*(i*(d*x+c))^m*(g*(b*x+a))^
(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*n^2+2*A*B*x*(i*(d*x+c))^m*(g*(b*x+
a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*m^2*n+2*A*B*x*(i*(d*x+c))^m*(g
*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*m^2*n+A^2*x^2*(i*(d*x+c)
)^m*(g*(b*x+a))^(-2-m)*b^2*d^2*m^2*n+2*A^2*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)
*a*b*c*d*m*n+2*A*B*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*c*d*n^2+B^2*x^2*(i*
(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*b^2*d^2*m^2*n+2*B
^2*x^2*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*b^2*d^2
*m*n+2*B^2*x^2*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b
^2*d^2*m*n^2+2*A*B*x^2*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*b^2*d^2*m*n^2+A^2*x
*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*d^2*m^2*n+A^2*x*(i*(d*x+c))^m*(g*(b*x
+a))^(-2-m)*b^2*c*d*m^2*n+2*A*B*x^2*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*(
(b*x+a)/(d*x+c))^n)*b^2*d^2*n+2*A*B*x^2*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln
(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*m^2*n+B^2*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m
```

```
)*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b*d^2*m^2*n+B^2*x*(i*(d*x+c))^m*(g*(b*x+a))
^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*b^2*c*d*m^2*n+4*A*B*x^2*(i*(d*x+c))^m*(
g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*m*n+2*B^2*x*(i*(d*x+c))
^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b*d^2*m*n+2*B^2*x*(i*(d
*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*b^2*c*d*m*n+2*A*B*x
*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*b^2*c*d*m*n^2+2*B^2*(i*(d*x+c))^m*(g*(b*x
+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b*c*d*m*n+2*B^2*(i*(d*x+c))^m*(g*
(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*m*n^2+B^2*x^2*(i*(d*x+c))
^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*b^2*d^2*n+2*B^2*x^2*(i*(d
*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*n^2+2*A^2*x^2
*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*b^2*d^2*m*n+2*A*B*x^2*(i*(d*x+c))^m*(g*(b
*x+a))^(-2-m)*b^2*d^2*n^2+2*B^2*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*d^2*
n^3+2*B^2*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*b^2*c*d*n^3+2*B^2*(i*(d*x+c))^
m*(g*(b*x+a))^(-2-m)*a*b*c*d*n^3+A^2*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b
*d^2*n+A^2*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*b^2*c*d*n+A^2*(i*(d*x+c))^m*(
g*(b*x+a))^(-2-m)*a*b*c*d*n+2*B^2*x^2*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*b^2*
d^2*n^3+A^2*x^2*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*b^2*d^2*n)/n/(a*d*m-b*c*m+
a*d-b*c)/(m^2+2*m+1)/b/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(223) = 446$.

Time = 0.34 (sec) , antiderivative size = 983, normalized size of antiderivative = 4.41

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx =$$

$$-\frac{\left(A^2acm^2+2\,B^2acn^2+2\,A^2acm+A^2ac+\left(A^2bdm^2+2\,B^2bdn^2+2\,A^2bdm+A^2bd+2\left(ABbdm+A\right.\right.\right.}{}$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n))
^2,x, algorithm="fricas")
```

```
[Out] -(A^2*a*c*m^2 + 2*B^2*a*c*n^2 + 2*A^2*a*c*m + A^2*a*c + (A^2*b*d*m^2 + 2*B^
2*b*d*n^2 + 2*A^2*b*d*m + A^2*b*d + 2*(A*B*b*d*m + A*B*b*d)*n)*x^2 + (B^2*a
*c*m^2 + 2*B^2*a*c*m + B^2*a*c + (B^2*b*d*m^2 + 2*B^2*b*d*m + B^2*b*d)*x^2
+ (B^2*b*c + B^2*a*d + (B^2*b*c + B^2*a*d)*m^2 + 2*(B^2*b*c + B^2*a*d)*m)*x
)*log(e)^2 + ((B^2*b*d*m^2 + 2*B^2*b*d*m + B^2*b*d)*n^2*x^2 + (B^2*b*c + B^
2*a*d + (B^2*b*c + B^2*a*d)*m^2 + 2*(B^2*b*c + B^2*a*d)*m)*n^2*x + (B^2*a*c
*m^2 + 2*B^2*a*c*m + B^2*a*c)*n^2)*log((b*x + a)/(d*x + c))^2 + 2*(A*B*a*c*
m + A*B*a*c)*n + (A^2*b*c + A^2*a*d + (A^2*b*c + A^2*a*d)*m^2 + 2*(B^2*b*c
+ B^2*a*d)*n^2 + 2*(A^2*b*c + A^2*a*d)*m + 2*(A*B*b*c + A*B*a*d + (A*B*b*c
+ A*B*a*d)*m)*n)*x + 2*(A*B*a*c*m^2 + 2*A*B*a*c*m + A*B*a*c + (A*B*b*d*m^2
+ 2*A*B*b*d*m + A*B*b*d + (B^2*b*d*m + B^2*b*d)*n)*x^2 + (B^2*a*c*m + B^2*a
```

```
*c)*n + (A*B*b*c + A*B*a*d + (A*B*b*c + A*B*a*d)*m^2 + 2*(A*B*b*c + A*B*a*d
)*m + (B^2*b*c + B^2*a*d + (B^2*b*c + B^2*a*d)*m)*n)*x + ((B^2*b*d*m^2 + 2*
B^2*b*d*m + B^2*b*d)*n*x^2 + (B^2*b*c + B^2*a*d + (B^2*b*c + B^2*a*d)*m^2 +
 2*(B^2*b*c + B^2*a*d)*m)*n*x + (B^2*a*c*m^2 + 2*B^2*a*c*m + B^2*a*c)*n)*lo
g((b*x + a)/(d*x + c)))*log(e) + 2*((B^2*a*c*m + B^2*a*c)*n^2 + ((B^2*b*d*m
 + B^2*b*d)*n^2 + (A*B*b*d*m^2 + 2*A*B*b*d*m + A*B*b*d)*n)*x^2 + (A*B*a*c*m
^2 + 2*A*B*a*c*m + A*B*a*c)*n + ((B^2*b*c + B^2*a*d + (B^2*b*c + B^2*a*d)*m
)*n^2 + (A*B*b*c + A*B*a*d + (A*B*b*c + A*B*a*d)*m^2 + 2*(A*B*b*c + A*B*a*d
)*m)*n)*x)*log((b*x + a)/(d*x + c)))*(b*g*x + a*g)^(-m - 2)*e^(m*log(b*g*x
+ a*g) - m*log((b*x + a)/(d*x + c)) + m*log(i/g))/((b*c - a*d)*m^3 + 3*(b*c
 - a*d)*m^2 + b*c - a*d + 3*(b*c - a*d)*m)
```

Sympy [F(-2)]

Exception generated.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m*(A+B*ln(e*((b*x+a)/(d*x+c))**n
))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2(bgx+ag)^{-m-2}(dix+ci)^m\,dx$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n))
^2,x, algorithm="maxima")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2*(b*g*x + a*g)^(-m - 2)*(
d*i*x + c*i)^m, x)
```

Giac [F]

$$\int (ag + bgx)^{-2-m}(ci + dix)^m \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2 (bgx + ag)^{-m-2}(dix + ci)^m\, dx$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n))
^2,x, algorithm="giac")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2*(b*g*x + a*g)^(-m - 2)*(
d*i*x + c*i)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag + bgx)^{-2-m}(ci + dix)^m \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int \frac{(c\,i + d\,i\,x)^m \left(A + B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(a\,g + b\,g\,x)^{m+2}}\, dx$$

```
[In] int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x
)^(m + 2),x)
[Out] int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x
)^(m + 2), x)
```

3.220 $\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) d$

Optimal result

Integrand size = 47, antiderivative size = 137

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = -\frac{Bn(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{(bc-ad)i^2(1+m)^2(c+dx)} - \frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)(c+dx)}$$

```
[Out] -B*n*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)/(-a*d+b*c)/i^2/(1+m)^2/(d
*x+c)-(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)*(A+B*ln(e*((b*x+a)/(d*x+
c))^n))/(-a*d+b*c)/i^2/(1+m)/(d*x+c)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2563, 2341}

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = -\frac{(a+bx)(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{i^2(m+1)(c+dx)(bc-ad)} - \frac{Bn(a+bx)(g(a+bx))^{-m-2}(i(c+dx))^{m+2}}{i^2(m+1)^2(c+dx)(bc-ad)}$$

```
[In] Int[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]),x]
```

```
[Out] -((B*n*(a + b*x)*(g*(a + b*x))^(-2 - m)*(i*(c + d*x))^(2 + m))/((b*c - a*d)
*i^2*(1 + m)^2*(c + d*x))) - ((a + b*x)*(g*(a + b*x))^(-2 - m)*(i*(c + d*x)
)^(2 + m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)*i^2*(1 + m)*
(c + d*x))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2563

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m)), Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*x
)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x]
 && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
q + 2, 0]
```

Rubi steps

$$\text{integral} = \frac{\left((g(a+bx))^{-2-m}\left(\frac{a+bx}{c+dx}\right)^{2+m}(i(c+dx))^{2+m}\right)\text{Subst}\left(\int x^{-2-m}(A+B\log(ex^n))\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}$$

$$= -\frac{Bn(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{(bc-ad)i^2(1+m)^2(c+dx)} - \frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)(c+dx)}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.57

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = -\frac{(g(a+bx))^{-1-m}(c+dx)(i(c+dx))^m\left(A+Am+Bn+B(1+m)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)g(1+m)^2}$$

```
[In] Integrate[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]),x]
```

```
[Out] -(((g*(a + b*x))^(-1 - m)*(c + d*x)*(i*(c + d*x))^m*(A + A*m + B*n + B*(1 +
 m)*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)*g*(1 + m)^2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 820 vs. 2(137) = 274.

Time = 31.01 (sec) , antiderivative size = 821, normalized size of antiderivative = 5.99

method	result
parallelrisch	$\frac{Bx(i(dx+c))^m(g(bx+a))^{-2-m}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)ab\,d^2mn+Bx(i(dx+c))^m(g(bx+a))^{-2-m}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^2cdmn+B(i(dx+c))^{m}}{}$

```
[In] int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,meth
od=_RETURNVERBOSE)
```

```
[Out] (B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*m*n
+B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*m*n
+B*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*m*n+B
*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*n+A*(
i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*c*d*m*n+B*(i*(d*x+c))^m*(g*(b*x+a))^(-2
-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*n+A*x^2*(i*(d*x+c))^m*(g*(b*x+a))^(-2
-m)*b^2*d^2*n+B*x^2*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))
^n)*b^2*d^2*m*n+A*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*d^2*m*n+A*x*(i*(d*
x+c))^m*(g*(b*x+a))^(-2-m)*b^2*c*d*m*n+B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)
*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*n+B*x^2*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)
*b^2*d^2*n^2+A*x^2*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*b^2*d^2*m*n+B*x^2*(i*(d
*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*n+B*x*(i*(d*x
+c))^m*(g*(b*x+a))^(-2-m)*a*b*d^2*n^2+B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*
b^2*c*d*n^2+A*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*d^2*n+A*x*(i*(d*x+c))^
m*(g*(b*x+a))^(-2-m)*b^2*c*d*n+B*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*c*d*n
^2+A*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*c*d*n)/d/b/n/(a*d*m-b*c*m+a*d-b*c
)/(1+m)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.96

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx =$$

$$-\frac{(Aacm+Bacn+Aac+(Abdm+Bbdn+Abd)x^2+(Abc+Aad+(Abc+Aad)m+(Bbc+Bad)n)x+}{}$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n))
,x, algorithm="fricas")
```

```
[Out] -(A*a*c*m + B*a*c*n + A*a*c + (A*b*d*m + B*b*d*n + A*b*d)*x^2 + (A*b*c + A*
a*d + (A*b*c + A*a*d)*m + (B*b*c + B*a*d)*n)*x + (B*a*c*m + B*a*c + (B*b*d*
```

```
m + B*b*d)*x^2 + (B*b*c + B*a*d + (B*b*c + B*a*d)*m)*x)*log(e) + ((B*b*d*m
+ B*b*d)*n*x^2 + (B*b*c + B*a*d + (B*b*c + B*a*d)*m)*n*x + (B*a*c*m + B*a*c
)*n)*log((b*x + a)/(d*x + c)))*(b*g*x + a*g)^(-m - 2)*e^(m*log(b*g*x + a*g)
 - m*log((b*x + a)/(d*x + c)) + m*log(i/g))/((b*c - a*d)*m^2 + b*c - a*d +
2*(b*c - a*d)*m)
```

Sympy [F(-2)]

Exception generated.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m*(A+B*ln(e*((b*x+a)/(d*x+c))**n
)),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\int\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)(bgx+ag)^{-m-2}(dix+ci)^m\,dx$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n))
,x, algorithm="maxima")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(b*g*x + a*g)^(-m - 2)*(d*
i*x + c*i)^m, x)
```

Giac [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\int\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)(bgx+ag)^{-m-2}(dix+ci)^m\,dx$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n))
,x, algorithm="giac")
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(b*g*x + a*g)^(-m - 2)*(d*
i*x + c*i)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\int\frac{(c\,i+d\,i\,x)^m\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(a\,g+b\,g\,x)^{m+2}}\,dx$$

```
[In] int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^
(m + 2),x)
```

```
[Out] int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^
(m + 2), x)
```

3.221 $\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 128

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{e^{\frac{A(1+m)}{Bn}}(a+bx)(g(a+bx))^{-2-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1+m}{n}}(i(c+dx))^{2+m}\,\text{ExpIntegralEi}\left(-\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right.}{B(bc-ad)i^2n(c+dx)}$$

```
[Out] exp(A*(1+m)/B/n)*(b*x+a)*(g*(b*x+a))^(-2-m)*(e*((b*x+a)/(d*x+c))^n)^((1+m)/
n)*(i*(d*x+c))^(2+m)*Ei(-(1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B/(-a*d
+b*c)/i^2/n/(d*x+c)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2563, 2347, 2209}

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{(a+bx)e^{\frac{A(m+1)}{Bn}}(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\,\text{ExpIntegralEi}\left(-\frac{(m+1)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right.}{Bi^2n(c+dx)(bc-ad)}$$

```
[In] Int[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)/(c + d
*x))^n]),x]
```

```
[Out] (E^((A*(1 + m))/(B*n))*(a + b*x)*(g*(a + b*x))^(-2 - m)*(e*((a + b*x)/(c +
d*x))^n)^((1 + m)/n)*(i*(c + d*x))^(2 + m)*ExpIntegralEi[-(((1 + m)*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(B*n))])/(B*(b*c - a*d)*i^2*n*(c + d*x))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2563

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m)), Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*x
)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x]
 && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
q + 2, 0]
```

Rubi steps

$$\text{integral} = \frac{\left((g(a+bx))^{-2-m}\left(\frac{a+bx}{c+dx}\right)^{2+m}(i(c+dx))^{2+m}\right)\text{Subst}\left(\int\frac{x^{-2-m}}{A+B\log(ex^n)}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}$$

$$= \frac{\left((a+bx)(g(a+bx))^{-2-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{-1-m}{n}}(i(c+dx))^{2+m}\right)\text{Subst}\left(\int\frac{e^{\frac{(-1-m)x}{n}}}{A+Bx}\,dx,x,\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.\right.}{(bc-ad)i^2n(c+dx)}$$

$$= \frac{e^{\frac{A(1+m)}{Bn}}(a+bx)(g(a+bx))^{-2-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1+m}{n}}(i(c+dx))^{2+m}\text{Ei}\left(-\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B(bc-ad)i^2n(c+dx)}$$

Mathematica [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] Integrate[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)/
(c + d*x))^n]),x]
[Out] Integrate[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)/
(c + d*x))^n]), x]
```

Maple [F]

$$\int \frac{(bgx+ag)^{-2-m}\,(dix+ci)^m}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

```
[In] int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
[Out] int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.73

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\frac{\mathrm{Ei}\left(-\frac{(Bm+B)n\log\left(\frac{bx+a}{dx+c}\right)+Am+(Bm+B)\log(e)+A}{Bn}\right)e^{\left(\frac{Bmn\log\left(\frac{i}{g}\right)+Am+(Bm+B)\log(e)+A}{Bn}\right)}}{(Bbc-Bad)g^2n}$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n))
,x, algorithm="fricas")
[Out] Ei(-((B*m + B)*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(e) + A)/(B*
n))*e^((B*m*n*log(i/g) + A*m + (B*m + B)*log(e) + A)/(B*n))/((B*b*c - B*a*d
)*g^2*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m/(A+B*ln(e*((b*x+a)/(d*x+c))**n
)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bgx+ag)^{-m-2}(dix+ci)^m}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n))
,x, algorithm="maxima")
```

```
[Out] integrate((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m/(B*log(e*((b*x + a)/(d*x +
 c))^n) + A), x)
```

Giac [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bgx+ag)^{-m-2}(dix+ci)^m}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n))
,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m/(B*log(e*((b*x + a)/(d*x +
 c))^n) + A), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(c\,i+d\,i\,x)^m}{(a\,g+b\,g\,x)^{m+2}\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}\,dx$$

```
[In] int((c*i + d*i*x)^m/((a*g + b*g*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*x
))^n))),x)
```

```
[Out] int((c*i + d*i*x)^m/((a*g + b*g*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*x
))^n))), x)
```

3.222 $\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 214

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = -\frac{e^{\frac{A(1+m)}{Bn}}(1+m)(a+bx)(g(a+bx))^{-2-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1+m}{n}}(i(c+dx))^{2+m}\,\text{ExpIntegralEi}\left(-\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B^2(bc-ad)i^2n^2(c+dx)} - \frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

```
[Out] -exp(A*(1+m)/B/n)*(1+m)*(b*x+a)*(g*(b*x+a))^(-2-m)*(e*((b*x+a)/(d*x+c))^n)^
((1+m)/n)*(i*(d*x+c))^(2+m)*Ei(-(1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/
B^2/(-a*d+b*c)/i^2/n^2/(d*x+c)-(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)
/B/(-a*d+b*c)/i^2/n/(d*x+c)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.082$, Rules used

= {2563, 2343, 2347, 2209}

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = -\frac{(m+1)(a+bx)e^{\frac{A(m+1)}{Bn}}(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\text{ExpIntegralEi}\left(-\frac{(m+1)\left(A+B\log\left(\right.\right.}{Bn}\right.}{B^2i^2n^2(c+dx)(bc-ad)} - \frac{(a+bx)(g(a+bx))^{-m-2}(i(c+dx))^{m+2}}{Bi^2n(c+dx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}$$

```
[In] Int[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)/(c + d
*x))^n])^2,x]
```

```
[Out] -((E^((A*(1 + m))/(B*n))*(1 + m)*(a + b*x)*(g*(a + b*x))^(-2 - m)*(e*((a +
b*x)/(c + d*x))^n)^((1 + m)/n)*(i*(c + d*x))^(2 + m)*ExpIntegralEi[-(((1 +
m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n))])/(B^2*(b*c - a*d)*i^2*n^
2*(c + d*x))) - ((a + b*x)*(g*(a + b*x))^(-2 - m)*(i*(c + d*x))^(2 + m))/(B
*(b*c - a*d)*i^2*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2563

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m)), Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*x
)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x]
 && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
```

q + 2, 0]

Rubi steps

$$\text{integral} = \frac{\left((g(a+bx))^{-2-m}\left(\frac{a+bx}{c+dx}\right)^{2+m}(i(c+dx))^{2+m}\right)\text{Subst}\left(\int\frac{x^{-2-m}}{(A+B\log(ex^n))^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}$$

$$= -\frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} - \frac{\left((1+m)(g(a+bx))^{-2-m}\left(\frac{a+bx}{c+dx}\right)^{2+m}(i(c+dx))^{2+m}\right)\text{Subst}\left(\int\frac{x^{-2-m}}{A+B\log(ex^n)}\,dx,x,\frac{a+bx}{c+dx}\right)}{B(bc-ad)i^2n}$$

$$= -\frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} - \frac{\left((1+m)(a+bx)(g(a+bx))^{-2-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{-1-m}{n}}(i(c+dx))^{2+m}\right)\text{Subst}\left(\int\frac{e^{\frac{(-1-m)x}{n}}}{A+Bx}\,dx,x,\log\right.}{B(bc-ad)i^2n^2(c+dx)}$$

$$= -\frac{e^{\frac{A(1+m)}{Bn}}(1+m)(a+bx)(g(a+bx))^{-2-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1+m}{n}}(i(c+dx))^{2+m}\text{Ei}\left(-\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{Bn}\right.}{B^2(bc-ad)i^2n^2(c+dx)} - \frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

Mathematica [F]

$$\int\frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int\frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

```
[In] Integrate[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)/
(c + d*x))^n])^2,x]
```

```
[Out] Integrate[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)/
(c + d*x))^n])^2, x]
```

Maple [F]

$$\int \frac{(bgx+ag)^{-2-m}(dix+ci)^m}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

```
[In] int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
[Out] int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.33

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx =$$

$$-\frac{(Bbdg^2nx^2+Bacg^2n+(Bbc+Bad)g^2nx)(bgx+ag)^{-m-2}e^{\left(m\log(bgx+ag)-m\log\left(\frac{bx+a}{dx+c}\right)+m\log\left(\frac{i}{g}\right)\right)}+((Bm}{(B^3bc-B^3ad)g^2n^3\log\left(\frac{bx+a}{dx+c}\right)+}$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n))
^2,x, algorithm="fricas")
[Out] -((B*b*d*g^2*n*x^2 + B*a*c*g^2*n + (B*b*c + B*a*d)*g^2*n*x)*(b*g*x + a*g)^(
-m - 2)*e^(m*log(b*g*x + a*g) - m*log((b*x + a)/(d*x + c)) + m*log(i/g)) +
((B*m + B)*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(e) + A)*Ei(-((B
*m + B)*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(e) + A)/(B*n))*e^(
(B*m*n*log(i/g) + A*m + (B*m + B)*log(e) + A)/(B*n)))/((B^3*b*c - B^3*a*d)*
g^2*n^3*log((b*x + a)/(d*x + c)) + (B^3*b*c - B^3*a*d)*g^2*n^2*log(e) + (A*
B^2*b*c - A*B^2*a*d)*g^2*n^2)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m/(A+B*ln(e*((b*x+a)/(d*x+c))**n
))**2,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(bgx+ag)^{-m-2}(dix+ci)^m}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n))
^2,x, algorithm="maxima")
```

```
[Out] i^m*(m + 1)*integrate(-(d*x + c)^m/((B^2*b^2*g^(m + 2)*n*x^2 + 2*B^2*a*b*g^
(m + 2)*n*x + B^2*a^2*g^(m + 2)*n)*(b*x + a)^m*log((b*x + a)^n) - (B^2*b^2*
g^(m + 2)*n*x^2 + 2*B^2*a*b*g^(m + 2)*n*x + B^2*a^2*g^(m + 2)*n)*(b*x + a)^
m*log((d*x + c)^n) + (B^2*a^2*g^(m + 2)*n*log(e) + A*B*a^2*g^(m + 2)*n + (B
^2*b^2*g^(m + 2)*n*log(e) + A*B*b^2*g^(m + 2)*n)*x^2 + 2*(B^2*a*b*g^(m + 2)
*n*log(e) + A*B*a*b*g^(m + 2)*n)*x)*(b*x + a)^m), x) - (d*i^m*x + c*i^m)*(d
*x + c)^m/(((b^2*c*g^(m + 2)*n - a*b*d*g^(m + 2)*n)*B^2*x + (a*b*c*g^(m + 2
)*n - a^2*d*g^(m + 2)*n)*B^2)*(b*x + a)^m*log((b*x + a)^n) - ((b^2*c*g^(m +
 2)*n - a*b*d*g^(m + 2)*n)*B^2*x + (a*b*c*g^(m + 2)*n - a^2*d*g^(m + 2)*n)*
B^2)*(b*x + a)^m*log((d*x + c)^n) + ((a*b*c*g^(m + 2)*n - a^2*d*g^(m + 2)*n
)*A*B + (a*b*c*g^(m + 2)*n*log(e) - a^2*d*g^(m + 2)*n*log(e))*B^2 + ((b^2*c
*g^(m + 2)*n - a*b*d*g^(m + 2)*n)*A*B + (b^2*c*g^(m + 2)*n*log(e) - a*b*d*g
^(m + 2)*n*log(e))*B^2)*x)*(b*x + a)^m)
```

Giac [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(bgx+ag)^{-m-2}(dix+ci)^m}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n))
^2,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m/(B*log(e*((b*x + a)/(d*x +
 c))^n) + A)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(c\,i+d\,i\,x)^m}{(a\,g+b\,g\,x)^{m+2}\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

```
[In] int((c*i + d*i*x)^m/((a*g + b*g*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*x
))^n))^2),x)
```

```
[Out] int((c*i + d*i*x)^m/((a*g + b*g*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*x
))^n))^2), x)
```

3.223 $\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 306

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx = \frac{e^{\frac{A(1+m)}{Bn}}(1+m)^2(a+bx)(g(a+bx))^{-2-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1+m}{n}}(i(c+dx))^{2+m}\,\text{ExpIntegralEi}\left(-\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{2B^3(bc-ad)i^2n^3(c+dx)} - \frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{2B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} + \frac{(1+m)(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{2B^2(bc-ad)i^2n^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

```
[Out] 1/2*exp(A*(1+m)/B/n)*(1+m)^2*(b*x+a)*(g*(b*x+a))^(-2-m)*(e*((b*x+a)/(d*x+c)
)^n)^((1+m)/n)*(i*(d*x+c))^(2+m)*Ei(-(1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/
B/n)/B^3/(-a*d+b*c)/i^2/n^3/(d*x+c)-1/2*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+
c))^(2+m)/B/(-a*d+b*c)/i^2/n/(d*x+c)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2+1/2*
(1+m)*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)/B^2/(-a*d+b*c)/i^2/n^2/(
d*x+c)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.082, Rules used = {2563, 2343, 2347, 2209}

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx$$

$$=\frac{(m+1)^2(a+bx)e^{\frac{A(m+1)}{Bn}}(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\text{ExpIntegralEi}\left(-\frac{(m+1)\left(A+B\log\left(e\left(\cdots\right.\right.\right.}{Bn}\right.}{2B^3i^2n^3(c+dx)(bc-ad)}$$
$$+\frac{(m+1)(a+bx)(g(a+bx))^{-m-2}(i(c+dx))^{m+2}}{2B^2i^2n^2(c+dx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}$$
$$-\frac{(a+bx)(g(a+bx))^{-m-2}(i(c+dx))^{m+2}}{2Bi^2n(c+dx)(bc-ad)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}$$

```
[In] Int[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)/(c + d
*x))^n])^3,x]
```

```
[Out] (E^((A*(1 + m))/(B*n))*(1 + m)^2*(a + b*x)*(g*(a + b*x))^(-2 - m)*(e*((a +
b*x)/(c + d*x))^n)^((1 + m)/n)*(i*(c + d*x))^(2 + m)*ExpIntegralEi[-(((1 +
m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n))])/(2*B^3*(b*c - a*d)*i^2*
n^3*(c + d*x)) - ((a + b*x)*(g*(a + b*x))^(-2 - m)*(i*(c + d*x))^(2 + m))/(
2*B*(b*c - a*d)*i^2*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2) +
 ((1 + m)*(a + b*x)*(g*(a + b*x))^(-2 - m)*(i*(c + d*x))^(2 + m))/(2*B^2*(b
*c - a*d)*i^2*n^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2563

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m)), Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*x
)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x]
 && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
q + 2, 0]
```

Rubi steps

$$\text{integral} = \frac{\left((g(a+bx))^{-2-m}\left(\frac{a+bx}{c+dx}\right)^{2+m}(i(c+dx))^{2+m}\right)\text{Subst}\left(\int\frac{x^{-2-m}}{(A+B\log(ex^n))^3}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)i^2}$$

$$= -\frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{2B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} - \frac{\left((1+m)(g(a+bx))^{-2-m}\left(\frac{a+bx}{c+dx}\right)^{2+m}(i(c+dx))^{2+m}\right)\text{Subst}\left(\int\frac{x^{-2-m}}{(A+B\log(ex^n))^2}\,dx,x,\frac{a+bx}{c+dx}\right)}{2B(bc-ad)i^2n}$$

$$= -\frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{2B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} + \frac{(1+m)(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{2B^2(bc-ad)i^2n^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} + \frac{\left((1+m)^2(g(a+bx))^{-2-m}\left(\frac{a+bx}{c+dx}\right)^{2+m}(i(c+dx))^{2+m}\right)\text{Subst}\left(\int\frac{x^{-2-m}}{A+B\log(ex^n)}\,dx,x,\frac{a+bx}{c+dx}\right)}{2B^2(bc-ad)i^2n^2}$$

$$= -\frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{2B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} + \frac{(1+m)(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{2B^2(bc-ad)i^2n^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} + \frac{\left((1+m)^2(a+bx)(g(a+bx))^{-2-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{-1-m}{n}}(i(c+dx))^{2+m}\right)\text{Subst}\left(\int\frac{e^{\frac{(-1-m)x}{n}}}{A+Bx}\,dx,x,\text{l}\right.}{2B^2(bc-ad)i^2n^3(c+dx)}$$

$$= \frac{e^{\frac{A(1+m)}{Bn}}(1+m)^2(a+bx)(g(a+bx))^{-2-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1+m}{n}}(i(c+dx))^{2+m}\text{Ei}\left(-\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{Bn}\right.}{2B^3(bc-ad)i^2n^3(c+dx)} - \frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{2B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} + \frac{(1+m)(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{2B^2(bc-ad)i^2n^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

Mathematica [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx = \int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx$$

```
[In] Integrate[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)/
(c + d*x))^n])^3,x]
```

```
[Out] Integrate[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)/
(c + d*x))^n])^3, x]
```

Maple [F]

$$\int \frac{(bgx+ag)^{-2-m}\,(dix+ci)^m}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^3}dx$$

```
[In] int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

```
[Out] int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 815 vs. $2(299) = 598$.

Time = 0.39 (sec) , antiderivative size = 815, normalized size of antiderivative = 2.66

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx =$$

$$-\frac{(B^2acg^2n^2 + (B^2bdg^2n^2 - (ABbdg^2m + ABbdg^2)n)x^2 - (ABacg^2m + ABacg^2)n + ((B^2bc + B^2ad)g^2n}{}$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n))
^3,x, algorithm="fricas")
```

```
[Out] -1/2*((B^2*a*c*g^2*n^2 + (B^2*b*d*g^2*n^2 - (A*B*b*d*g^2*m + A*B*b*d*g^2)*n
)*x^2 - (A*B*a*c*g^2*m + A*B*a*c*g^2)*n + ((B^2*b*c + B^2*a*d)*g^2*n^2 - ((
A*B*b*c + A*B*a*d)*g^2*m + (A*B*b*c + A*B*a*d)*g^2)*n)*x - ((B^2*b*d*g^2*m
+ B^2*b*d*g^2)*n*x^2 + ((B^2*b*c + B^2*a*d)*g^2*m + (B^2*b*c + B^2*a*d)*g^2
)*n*x + (B^2*a*c*g^2*m + B^2*a*c*g^2)*n)*log(e) - ((B^2*b*d*g^2*m + B^2*b*d
*g^2)*n^2*x^2 + ((B^2*b*c + B^2*a*d)*g^2*m + (B^2*b*c + B^2*a*d)*g^2)*n^2*x
 + (B^2*a*c*g^2*m + B^2*a*c*g^2)*n^2)*log((b*x + a)/(d*x + c)))*(b*g*x + a*
g)^(-m - 2)*e^(m*log(b*g*x + a*g) - m*log((b*x + a)/(d*x + c)) + m*log(i/g)
```

```
) - ((B^2*m^2 + 2*B^2*m + B^2)*n^2*log((b*x + a)/(d*x + c))^2 + A^2*m^2 + 2
*A^2*m + (B^2*m^2 + 2*B^2*m + B^2)*log(e)^2 + 2*(A*B*m^2 + 2*A*B*m + A*B)*n
*log((b*x + a)/(d*x + c)) + A^2 + 2*(A*B*m^2 + 2*A*B*m + (B^2*m^2 + 2*B^2*m
 + B^2)*n*log((b*x + a)/(d*x + c)) + A*B)*log(e))*Ei(-((B*m + B)*n*log((b*x
 + a)/(d*x + c)) + A*m + (B*m + B)*log(e) + A)/(B*n))*e^((B*m*n*log(i/g) +
A*m + (B*m + B)*log(e) + A)/(B*n)))/((B^5*b*c - B^5*a*d)*g^2*n^5*log((b*x +
 a)/(d*x + c))^2 + (B^5*b*c - B^5*a*d)*g^2*n^3*log(e)^2 + 2*(A*B^4*b*c - A*
B^4*a*d)*g^2*n^4*log((b*x + a)/(d*x + c)) + (A^2*B^3*b*c - A^2*B^3*a*d)*g^2
*n^3 + 2*((B^5*b*c - B^5*a*d)*g^2*n^4*log((b*x + a)/(d*x + c)) + (A*B^4*b*c
 - A*B^4*a*d)*g^2*n^3)*log(e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m/(A+B*ln(e*((b*x+a)/(d*x+c))**n
))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx = \int \frac{(bgx+ag)^{-m-2}(dix+ci)^m}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3}\,dx$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n))
^3,x, algorithm="maxima")
```

```
[Out] -(m^2 + 2*m + 1)*i^m*integrate(-1/2*(d*x + c)^m/((B^3*b^2*g^(m + 2)*n^2*x^2
 + 2*B^3*a*b*g^(m + 2)*n^2*x + B^3*a^2*g^(m + 2)*n^2)*(b*x + a)^m*log((b*x
+ a)^n) - (B^3*b^2*g^(m + 2)*n^2*x^2 + 2*B^3*a*b*g^(m + 2)*n^2*x + B^3*a^2*
g^(m + 2)*n^2)*(b*x + a)^m*log((d*x + c)^n) + (B^3*a^2*g^(m + 2)*n^2*log(e)
 + A*B^2*a^2*g^(m + 2)*n^2 + (B^3*b^2*g^(m + 2)*n^2*log(e) + A*B^2*b^2*g^(m
 + 2)*n^2)*x^2 + 2*(B^3*a*b*g^(m + 2)*n^2*log(e) + A*B^2*a*b*g^(m + 2)*n^2)
*x)*(b*x + a)^m), x) + 1/2*((B*d*i^m*(m + 1)*x + B*c*i^m*(m + 1))*(d*x + c)
^m*log((b*x + a)^n) - (B*d*i^m*(m + 1)*x + B*c*i^m*(m + 1))*(d*x + c)^m*log
((d*x + c)^n) + (A*c*i^m*(m + 1) + (i^m*(m + 1)*log(e) - i^m*n)*B*c + (A*d*
i^m*(m + 1) + (i^m*(m + 1)*log(e) - i^m*n)*B*d)*x)*(d*x + c)^m)/(((b^2*c*g^
(m + 2)*n^2 - a*b*d*g^(m + 2)*n^2)*B^4*x + (a*b*c*g^(m + 2)*n^2 - a^2*d*g^(
m + 2)*n^2)*B^4)*(b*x + a)^m*log((b*x + a)^n)^2 + ((b^2*c*g^(m + 2)*n^2 - a
*b*d*g^(m + 2)*n^2)*B^4*x + (a*b*c*g^(m + 2)*n^2 - a^2*d*g^(m + 2)*n^2)*B^4
```

```
)*(b*x + a)^m*log((d*x + c)^n)^2 + 2*((a*b*c*g^(m + 2)*n^2 - a^2*d*g^(m + 2
)*n^2)*A*B^3 + (a*b*c*g^(m + 2)*n^2*log(e) - a^2*d*g^(m + 2)*n^2*log(e))*B^
4 + ((b^2*c*g^(m + 2)*n^2 - a*b*d*g^(m + 2)*n^2)*A*B^3 + (b^2*c*g^(m + 2)*n
^2*log(e) - a*b*d*g^(m + 2)*n^2*log(e))*B^4)*x)*(b*x + a)^m*log((b*x + a)^n
) + ((a*b*c*g^(m + 2)*n^2 - a^2*d*g^(m + 2)*n^2)*A^2*B^2 + 2*(a*b*c*g^(m +
2)*n^2*log(e) - a^2*d*g^(m + 2)*n^2*log(e))*A*B^3 + (a*b*c*g^(m + 2)*n^2*lo
g(e)^2 - a^2*d*g^(m + 2)*n^2*log(e)^2)*B^4 + ((b^2*c*g^(m + 2)*n^2 - a*b*d*
g^(m + 2)*n^2)*A^2*B^2 + 2*(b^2*c*g^(m + 2)*n^2*log(e) - a*b*d*g^(m + 2)*n^
2*log(e))*A*B^3 + (b^2*c*g^(m + 2)*n^2*log(e)^2 - a*b*d*g^(m + 2)*n^2*log(e
)^2)*B^4)*x)*(b*x + a)^m - 2*(((b^2*c*g^(m + 2)*n^2 - a*b*d*g^(m + 2)*n^2)*
B^4*x + (a*b*c*g^(m + 2)*n^2 - a^2*d*g^(m + 2)*n^2)*B^4)*(b*x + a)^m*log((b
*x + a)^n) + ((a*b*c*g^(m + 2)*n^2 - a^2*d*g^(m + 2)*n^2)*A*B^3 + (a*b*c*g^
(m + 2)*n^2*log(e) - a^2*d*g^(m + 2)*n^2*log(e))*B^4 + ((b^2*c*g^(m + 2)*n^
2 - a*b*d*g^(m + 2)*n^2)*A*B^3 + (b^2*c*g^(m + 2)*n^2*log(e) - a*b*d*g^(m +
 2)*n^2*log(e))*B^4)*x)*(b*x + a)^m)*log((d*x + c)^n))
```

Giac [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx=\int \frac{(bgx+ag)^{-m-2}(dix+ci)^m}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3}\,dx$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n))
^3,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m/(B*log(e*((b*x + a)/(d*x +
 c))^n) + A)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx=\int \frac{(c\,i+d\,i\,x)^m}{(a\,g+b\,g\,x)^{m+2}\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^3}\,dx$$

```
[In] int((c*i + d*i*x)^m/((a*g + b*g*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*x
))^n))^3),x)
```

```
[Out] int((c*i + d*i*x)^m/((a*g + b*g*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*x
))^n))^3), x)
```

3.224 $\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 41

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx=\frac{\log^{1+p}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n(1+p)}$$

```
[Out] ln(e*((b*x+a)/(d*x+c))^n)^(p+1)/(-a*d+b*c)/n/(p+1)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2561, 2339, 30}

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx=\frac{\log^{p+1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(p+1)(bc-ad)}$$

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]^p/((a + b*x)*(c + d*x)),x]
[Out] Log[e*((a + b*x)/(c + d*x))^n]^(1 + p)/((b*c - a*d)*n*(1 + p))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
```

```
x]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{\log^p(ex^n)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}\\ &= \frac{\text{Subst}\left(\int x^p\,dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)n}\\ &= \frac{\log^{1+p}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n(1+p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \frac{\log^{1+p}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bcn-adn)(1+p)}$$

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]^p/((a + b*x)*(c + d*x)),x]
[Out] Log[e*((a + b*x)/(c + d*x))^n]^(1 + p)/((b*c*n - a*d*n)*(1 + p))
```

Maple [A] (verified)

Time = 7.37 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$-\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^{p+1}}{n(ad-cb)(p+1)}$	43
default	$-\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^{p+1}}{n(ad-cb)(p+1)}$	43
parallelrisch	$-\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^p}{n(adp-bcp+ad-cb)}$	63

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)^p/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/n/(a*d-b*c)*ln(e*((b*x+a)/(d*x+c))^n)^(p+1)/(p+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \frac{\left(n\log\left(\frac{bx+a}{dx+c}\right)+\log(e)\right)\left(n\log\left(\frac{bx+a}{dx+c}\right)+\log(e)\right)^p}{(bc-ad)np+(bc-ad)n}$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^p/(b*x+a)/(d*x+c),x, algorithm="fricas
")
```

```
[Out] (n*log((b*x + a)/(d*x + c)) + log(e))*(n*log((b*x + a)/(d*x + c)) + log(e))
^p/((b*c - a*d)*n*p + (b*c - a*d)*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)**p/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^p}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^p/(b*x+a)/(d*x+c),x, algorithm="maxima
")
```

```
[Out] integrate(log(e*((b*x + a)/(d*x + c))^n)^p/((b*x + a)*(d*x + c)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \frac{\left(n\log\left(\frac{bx+a}{dx+c}\right)+\log\left(e\right)\right)^{p+1}}{(bcn-adn)(p+1)}$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^p/(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] (n*log((b*x + a)/(d*x + c)) + log(e))^(p + 1)/((b*c*n - a*d*n)*(p + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^p}{(a+b\,x)\,(c+d\,x)}\,dx$$

```
[In] int(log(e*((a + b*x)/(c + d*x))^n)^p/((a + b*x)*(c + d*x)),x)
[Out] int(log(e*((a + b*x)/(c + d*x))^n)^p/((a + b*x)*(c + d*x)), x)
```

3.225 $\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 41

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx = \frac{\log^{1+p}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n(1+p)}$$

```
[Out] ln(e*((b*x+a)/(d*x+c))^n)^(p+1)/(-a*d+b*c)/n/(p+1)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2574, 2561, 2339, 30}

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx = \frac{\log^{p+1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(p+1)(bc-ad)}$$

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]^p/(a*c + (b*c + a*d)*x + b*d*x^2),x]
[Out] Log[e*((a + b*x)/(c + d*x))^n]^(1 + p)/((b*c - a*d)*n*(1 + p))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
```

```
x]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 2574

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_) + (h_.)*(x_)^2)^(m_.), x_Symbol] :> Dist[h^
m/(b^m*d^m), Int[(a + b*x)^m*(c + d*x)^m*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, n, p}, x] && EqQ[b*d
*f - a*c*h, 0] && EqQ[b*d*g - h*(b*c + a*d), 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{\log^p(ex^n)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}\\ &= \frac{\text{Subst}\left(\int x^p\,dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)n}\\ &= \frac{\log^{1+p}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n(1+p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx = \frac{\log^{1+p}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bcn-adn)(1+p)}$$

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]^p/(a*c + (b*c + a*d)*x + b*d*x^2),
x]
```

```
[Out] Log[e*((a + b*x)/(c + d*x))^n]^(1 + p)/((b*c*n - a*d*n)*(1 + p))
```

Maple [A] (verified)

Time = 10.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.54

method	result	size
parallelrisch	$-\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{p}}{n(adp-bcp+ad-cb)}$	63

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)^p/(c*a+(a*d+b*c)*x+b*d*x^2),x,method=_RETURNV
ERBOSE)
```

```
[Out] -ln(e*((b*x+a)/(d*x+c))^n)*ln(e*((b*x+a)/(d*x+c))^n)^p/n/(a*d*p-b*c*p+a*d-b
*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx = \frac{\left(n\log\left(\frac{bx+a}{dx+c}\right)+\log(e)\right)\left(n\log\left(\frac{bx+a}{dx+c}\right)+\log(e)\right)^p}{(bc-ad)np+(bc-ad)n}$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^p/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorit
hm="fricas")
```

```
[Out] (n*log((b*x + a)/(d*x + c)) + log(e))*(n*log((b*x + a)/(d*x + c)) + log(e))
^p/((b*c - a*d)*n*p + (b*c - a*d)*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)**p/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx = \int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^p}{bdx^2+ac+(bc+ad)x}\,dx$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^p/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorit
hm="maxima")
```

```
[Out] integrate(log(e*((b*x + a)/(d*x + c))^n)^p/(b*d*x^2 + a*c + (b*c + a*d)*x),
 x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx = \frac{\left(n\log\left(\frac{bx+a}{dx+c}\right)+\log(e)\right)^{p+1}}{(bcn-adn)(p+1)}$$

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^p/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorit
hm="giac")
```

```
[Out] (n*log((b*x + a)/(d*x + c)) + log(e))^(p + 1)/((b*c*n - a*d*n)*(p + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx = \int \frac{\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^p}{b\,d\,x^2+\left(a\,d+b\,c\right)\,x+a\,c}\,dx$$

```
[In] int(log(e*((a + b*x)/(c + d*x))^n)^p/(a*c + x*(a*d + b*c) + b*d*x^2),x)
```

```
[Out] int(log(e*((a + b*x)/(c + d*x))^n)^p/(a*c + x*(a*d + b*c) + b*d*x^2), x)
```

3.226 $\int (ag+bgx)^m(ci+dix)^{-2-m} (A + B \log (e(a + bx)^n(c$

Optimal result

Integrand size = 50, antiderivative size = 193

$$\int (ag + bgx)^m(ci + dix)^{-2-m} \left(A + B \log \left(e(a + bx)^n(c + dx)^{-n}\right)\right)^p \, dx$$
$$= \frac{e^{-\frac{A(1+m)}{Bn}}(a + bx)(g(a + bx))^m(i(c + dx))^{-m} \left(e(a + bx)^n(c + dx)^{-n}\right)^{-\frac{1+m}{n}} \Gamma\left(1 + p, -\frac{(1+m)(A+B\log(e(a+bx)^n(}{Bn}\right.}{(bc - ad)i^2(1 + m)(c + }$$

```
[Out] (b*x+a)*(g*(b*x+a))^m*GAMMA(p+1,-(1+m)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/
n)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p/(-a*d+b*c)/exp(A*(1+m)/B/n)/i^2/(1+m
)/(d*x+c)/((i*(d*x+c))^m)/((e*(b*x+a)^n/((d*x+c)^n))^((1+m)/n))/((-(1+m)*(A
+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/n)^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2573, 2563, 2347, 2212}

$$\int (ag + bgx)^m(ci + dix)^{-2-m} \left(A + B \log \left(e(a + bx)^n(c + dx)^{-n}\right)\right)^p \, dx$$
$$= \frac{(a + bx)e^{-\frac{A(m+1)}{Bn}}(g(a + bx))^m(i(c + dx))^{-m} \left(e(a + bx)^n(c + dx)^{-n}\right)^{-\frac{m+1}{n}} \left(B \log \left(e(a + bx)^n(c + dx)^{-n}\right) + \right.}{i^2(m + 1)(c + dx)(bc - }$$

```
[In] Int[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[(e*(a + b*x)^n)/(c +
d*x)^n])^p,x]
```

```
[Out] ((a + b*x)*(g*(a + b*x))^m*Gamma[1 + p, -(((1 + m)*(A + B*Log[(e*(a + b*x)^
n)/(c + d*x)^n]))/(B*n))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p)/((b*c
```

```
 - a*d)*E^((A*(1 + m))/(B*n))*i^2*(1 + m)*(c + d*x)*(i*(c + d*x))^m*((e*(a
+ b*x)^n)/(c + d*x)^n)^((1 + m)/n)*(-(((1 + m)*(A + B*Log[(e*(a + b*x)^n)/(
c + d*x)^n]))/(B*n)))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2563

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m)), Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*x
)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x]
 && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
q + 2, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$=\text{Subst}\left(\frac{\left((g(a+bx))^m\left(\frac{a+bx}{c+dx}\right)^{-m}(i(c+dx))^{-m}\right)\text{Subst}\left(\int x^m(A+B\log(ex^n))^p\,dx,x,\frac{a+bx}{c+dx}\right)}{(bc-ad)i^2},e\left(\frac{a}{c}\right.\right.$$

$$\left.+bx)^n(c+dx)^{-n}\right)$$

$$=\text{Subst}\left(\frac{\left((a+bx)(g(a+bx))^m\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{1+m}{n}}(i(c+dx))^{-m}\right)\text{Subst}\left(\int e^{\frac{(1+m)x}{n}}(A+Bx)^p\,dx,x,\text{l}\right.}{(bc-ad)i^2n(c+dx)}\right.$$

$$\left.+bx)^n(c+dx)^{-n}\right)$$

$$=\frac{e^{-\frac{A(1+m)}{Bn}}(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(e(a+bx)^n(c+dx)^{-n}\right)^{-\frac{1+m}{n}}\Gamma\left(1+p,-\frac{(1+m)(A+B\log(}{}\right.}{(bc-ad)i^2(1+}$$

Mathematica [F]

$$\int(ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$
$$=\int(ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$

```
[In] Integrate[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[(e*(a + b*x)^n)
/(c + d*x)^n])^p,x]
[Out] Integrate[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[(e*(a + b*x)^n)
/(c + d*x)^n])^p, x]
```

Maple [F]

$$\int(bgx+ag)^m(dix+ci)^{-2-m}\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)^p dx$$

```
[In] int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p,x)
[Out] int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p,x)
```

Fricas [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$
$$=\int (bgx+ag)^m(dix+ci)^{-m-2}\left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^p\,dx$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)
))^p,x, algorithm="fricas")
```

```
[Out] integral((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)*(B*log((b*x + a)^n*e/(d*x +
 c)^n) + A)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)*(A+B*ln(e*(b*x+a)**n/((d*x+c)*
*n)))**p,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$
$$=\int (bgx+ag)^m(dix+ci)^{-m-2}\left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^p\,dx$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)
))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)*(B*log((b*x + a)^n*e/(d*x
+ c)^n) + A)^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$
$$= \text{Exception raised: RuntimeError}$$

```
[In] integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)
))^p,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0,0
,5,5,0,2,2,3,3,0,0,0,2]%%%}+%%%{-2,[0,0,5,4,1,3,1,3,3,0,0,0,2]%%%}+%%%{1,[0
,0,5,3,2,
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$
$$= \int \frac{(a\,g+b\,g\,x)^m\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^p}{(c\,i+d\,i\,x)^{m+2}}\,dx$$

```
[In] int(((a*g + b*g*x)^m*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p)/(c*i + d*i
*x)^(m + 2),x)
```

```
[Out] int(((a*g + b*g*x)^m*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p)/(c*i + d*i
*x)^(m + 2), x)
```

3.227 $\int (ag+bgx)^{-2-m}(ci+dix)^m (A+B\log(e(a+bx)^n(c$

Optimal result

Integrand size = 50, antiderivative size = 194

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx =$$

$$-\frac{e^{\frac{A(1+m)}{Bn}}(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(e(a+bx)^n(c+dx)^{-n}\right)^{\frac{1+m}{n}}\Gamma\left(1+p,\frac{(1+m)(A+B\log(e(a+bx)^n}{Bn}\right.}{(bc-ad)i^2(1+m)(c+}$$

```
[Out] -exp(A*(1+m)/B/n)*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)*(e*(b*x+a)^n
/((d*x+c)^n))^((1+m)/n)*GAMMA(p+1,(1+m)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B
/n)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p/(-a*d+b*c)/i^2/(1+m)/(d*x+c)/(((1+m
)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/n)^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2573, 2563, 2347, 2212}

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx =$$

$$-\frac{(a+bx)e^{\frac{A(m+1)}{Bn}}(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(e(a+bx)^n(c+dx)^{-n}\right)^{\frac{m+1}{n}}\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right.}{i^2(m+1)(c+dx)(bc-}$$

```
[In] Int[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c +
d*x)^n])^p,x]
```

```
[Out] -((E^((A*(1 + m))/(B*n))*(a + b*x)*(g*(a + b*x))^(-2 - m)*(i*(c + d*x))^(2
+ m)*((e*(a + b*x)^n)/(c + d*x)^n)^((1 + m)/n)*Gamma[1 + p, ((1 + m)*(A + B
```

```
*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(B*n)]*(A + B*Log[(e*(a + b*x)^n)/(c +
d*x)^n])^p)/((b*c - a*d)*i^2*(1 + m)*(c + d*x)*(((1 + m)*(A + B*Log[(e*(a +
 b*x)^n)/(c + d*x)^n]))/(B*n))^p))
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2563

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m)), Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*x
)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x]
 && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
q + 2, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\left((g(a+bx))^{-2-m}\left(\frac{a+bx}{c+dx}\right)^{2+m}(i(c+dx))^{2+m}\right)\text{Subst}\left(\int x^{-2-m}(A+B\log(ex^n))^p\,dx,x,\frac{a+bx}{c+dx}\right.}{(bc-ad)i^2}\right.$$

$$\left.+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\left((a+bx)(g(a+bx))^{-2-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{-1-m}{n}}(i(c+dx))^{2+m}\right)\text{Subst}\left(\int e^{\frac{(-1-m)x}{n}}(A+Bx)^p\,d\right.}{(bc-ad)i^2n(c+dx)}\right.$$

$$\left.+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{e^{\frac{A(1+m)}{Bn}}(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(e(a+bx)^n(c+dx)^{-n}\right)^{\frac{1+m}{n}}\Gamma\left(1+p,\frac{(1+m)(A+B\log}{}\right.}{(bc-ad)i^2(1+}$$

Mathematica [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$
$$= \int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$

```
[In] Integrate[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[(e*(a + b*x)^n)
/(c + d*x)^n])^p,x]
[Out] Integrate[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[(e*(a + b*x)^n)
/(c + d*x)^n])^p, x]
```

Maple [F]

$$\int (bgx+ag)^{-2-m}(dix+ci)^m\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)^p dx$$

```
[In] int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p,x)
[Out] int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p,x)
```

Fricas [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p \,dx$$
$$=\int (bgx+ag)^{-m-2}(dix+ci)^m\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^p \,dx$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)
))^p,x, algorithm="fricas")
```

```
[Out] integral((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m*(B*log((b*x + a)^n*e/(d*x +
 c)^n) + A)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p \,dx=\text{Timed out}$$

```
[In] integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m*(A+B*ln(e*(b*x+a)**n/((d*x+c)*
*n)))**p,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p \,dx$$
$$=\int (bgx+ag)^{-m-2}(dix+ci)^m\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^p \,dx$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)
))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m*(B*log((b*x + a)^n*e/(d*x
+ c)^n) + A)^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$
$$= \text{Exception raised: RuntimeError}$$

```
[In] integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)
))^p,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0,0
,5,5,0,2,2,3,3,0,0,0,2]%%%}+%%%{-2,[0,0,5,4,1,3,1,3,3,0,0,0,2]%%%}+%%%{1,[0
,0,5,3,2,
```

Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$
$$= \int \frac{(c\,i+d\,i\,x)^m\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^p}{(a\,g+b\,g\,x)^{m+2}}\,dx$$

```
[In] int(((c*i + d*i*x)^m*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p)/(a*g + b*g
*x)^(m + 2),x)
```

```
[Out] int(((c*i + d*i*x)^m*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p)/(a*g + b*g
*x)^(m + 2), x)
```

3.228 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 45

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)(c+dx)}\,dx = \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^4}{4B(bc-ad)n}$$

```
[Out] 1/4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^4/B/(-a*d+b*c)/n
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2573, 2561, 2339, 30}

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)(c+dx)}\,dx = \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^4}{4Bn(bc-ad)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/((a + b*x)*(c + d*x)),x]
[Out] (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^4/(4*B*(b*c - a*d)*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
```

```
x]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(a+bx)(c+dx)}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{(A+B\log(ex^n))^3}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int x^3\,dx, x, A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)n}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^4}{4B(bc-ad)n}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)(c+dx)}\,dx = \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^4}{4(bBcn-aBdn)}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/((a + b*x)*(c + d*x)),
x]
```

```
[Out] (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^4/(4*(b*B*c*n - a*B*d*n))
```

Maple [A] (verified)

Time = 33.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

method	result
derivativedivides	$-\frac{\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{4}}{4n(ad-cb)B}$
default	$-\frac{\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{4}}{4n(ad-cb)B}$
parallelrisch	$-\frac{B^{3}\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)^{4}b^{2}d^{2}+4AB^{2}\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)^{3}b^{2}d^{2}+6A^{2}B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)^{2}b^{2}d^{2}+4A}{4n\,b^{2}d^{2}(ad-cb)}$
parts	$\frac{A^{3}\ln(dx+c)}{ad-cb}-\frac{A^{3}\ln(bx+a)}{ad-cb}-\frac{B^{3}\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)^{4}}{4n(ad-cb)}-\frac{AB^{2}\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)^{3}}{n(ad-cb)}-\frac{3A^{2}B\ln\left(e(bx\right.}{2n(}$
risch	Expression too large to display

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)/(d*x+c),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/4/n/(a*d-b*c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^4/B
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(43) = 86.

Time = 0.32 (sec) , antiderivative size = 375, normalized size of antiderivative = 8.33

$$\int \frac{(A+B\log(e(a+bx)^{n}(c+dx)^{-n}))^{3}}{(a+bx)(c+dx)}\,dx$$
$$=\frac{B^{3}n^{3}\log(bx+a)^{4}+B^{3}n^{3}\log(dx+c)^{4}+4\left(B^{3}n^{2}\log(e)+AB^{2}n^{2}\right)\log(bx+a)^{3}-4\left(B^{3}n^{3}\log(bx+a)\right.}{}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)/(d*x+c),x, algorithm
="fricas")
```

```
[Out] 1/4*(B^3*n^3*log(b*x + a)^4 + B^3*n^3*log(d*x + c)^4 + 4*(B^3*n^2*log(e) +
A*B^2*n^2)*log(b*x + a)^3 - 4*(B^3*n^3*log(b*x + a) + B^3*n^2*log(e) + A*B^
2*n^2)*log(d*x + c)^3 + 6*(B^3*n*log(e)^2 + 2*A*B^2*n*log(e) + A^2*B*n)*log
(b*x + a)^2 + 6*(B^3*n^3*log(b*x + a)^2 + B^3*n*log(e)^2 + 2*A*B^2*n*log(e)
 + A^2*B*n + 2*(B^3*n^2*log(e) + A*B^2*n^2)*log(b*x + a))*log(d*x + c)^2 +
4*(B^3*log(e)^3 + 3*A*B^2*log(e)^2 + 3*A^2*B*log(e) + A^3)*log(b*x + a) - 4
*(B^3*n^3*log(b*x + a)^3 + B^3*log(e)^3 + 3*A*B^2*log(e)^2 + 3*A^2*B*log(e)
 + A^3 + 3*(B^3*n^2*log(e) + A*B^2*n^2)*log(b*x + a)^2 + 3*(B^3*n*log(e)^2
+ 2*A*B^2*n*log(e) + A^2*B*n)*log(b*x + a))*log(d*x + c))/(b*c - a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)(c+dx)}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)/(d*x+c),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 766 vs. 2(43) = 86.

Time = 0.25 (sec) , antiderivative size = 766, normalized size of antiderivative = 17.02

$$\begin{aligned}
&\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)(c+dx)}\,dx \\
&= B^3\left(\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3 \\
&\quad+3\,AB^2\left(\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 \\
&\quad+3\,A^2B\left(\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) \\
&\quad-\frac{1}{4}B^3\left(\frac{6\left(en\log(bx+a)^2-2\,en\log(bx+a)\log(dx+c)+en\log(dx+c)^2\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2}{(bc-ad)e}-\frac{4\left(e^2n^2\log(\right.}{}\right. \\
&\quad+A^3\left(\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}\right) \\
&\quad-AB^2\left(\frac{3\left(en\log(bx+a)^2-2\,en\log(bx+a)\log(dx+c)+en\log(dx+c)^2\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{(bc-ad)e}-\frac{e^2n^2\log(b}{}\right. \\
&\quad-\frac{3\left(en\log(bx+a)^2-2\,en\log(bx+a)\log(dx+c)+en\log(dx+c)^2\right)A^2B}{2\,(bc-ad)e}
\end{aligned}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)/(d*x+c),x, algorithm
="maxima")
[Out] B^3*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d))*log((b*x + a)^n*e
/(d*x + c)^n)^3 + 3*A*B^2*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a
*d))*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*A^2*B*(log(b*x + a)/(b*c - a*d) -
 log(d*x + c)/(b*c - a*d))*log((b*x + a)^n*e/(d*x + c)^n) - 1/4*B^3*(6*(e*n
*log(b*x + a)^2 - 2*e*n*log(b*x + a)*log(d*x + c) + e*n*log(d*x + c)^2)*log
```

```
((b*x + a)^n*e/(d*x + c)^n)^2/((b*c - a*d)*e) - (4*(e^2*n^2*log(b*x + a)^3
- 3*e^2*n^2*log(b*x + a)^2*log(d*x + c) + 3*e^2*n^2*log(b*x + a)*log(d*x +
c)^2 - e^2*n^2*log(d*x + c)^3)*log((b*x + a)^n*e/(d*x + c)^n)/((b*c - a*d)*
e) - (e^3*n^3*log(b*x + a)^4 - 4*e^3*n^3*log(b*x + a)^3*log(d*x + c) + 6*e^
3*n^3*log(b*x + a)^2*log(d*x + c)^2 - 4*e^3*n^3*log(b*x + a)*log(d*x + c)^3
 + e^3*n^3*log(d*x + c)^4)/((b*c - a*d)*e^2))/e) + A^3*(log(b*x + a)/(b*c -
 a*d) - log(d*x + c)/(b*c - a*d)) - A*B^2*(3*(e*n*log(b*x + a)^2 - 2*e*n*lo
g(b*x + a)*log(d*x + c) + e*n*log(d*x + c)^2)*log((b*x + a)^n*e/(d*x + c)^n
)/((b*c - a*d)*e) - (e^2*n^2*log(b*x + a)^3 - 3*e^2*n^2*log(b*x + a)^2*log(
d*x + c) + 3*e^2*n^2*log(b*x + a)*log(d*x + c)^2 - e^2*n^2*log(d*x + c)^3)/
((b*c - a*d)*e^2)) - 3/2*(e*n*log(b*x + a)^2 - 2*e*n*log(b*x + a)*log(d*x +
 c) + e*n*log(d*x + c)^2)*A^2*B/((b*c - a*d)*e)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)(c+dx)}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(bx+a)(dx+c)}\,dx$$

[In] `integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)/(d*x+c),x, algorithm ="giac")`

[Out] `integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/((b*x + a)*(d*x + c)), x )`

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.13

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)(c+dx)}\,dx = -\frac{\frac{3A^2B\ln\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)^2}{2}+AB^2\ln\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)^3+\frac{B^3\ln\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)^4}{4}}{n\,(a\,d-b\,c)} + \frac{A^3\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{a\,d-b\,c}$$

[In] `int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/((a + b*x)*(c + d*x)),x)`

[Out] `(A^3*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(a*d - b*c) - ((B^3 *log((e*(a + b*x)^n)/(c + d*x)^n)^4)/4 + (3*A^2*B*log((e*(a + b*x)^n)/(c + d*x)^n)^2)/2 + A*B^2*log((e*(a + b*x)^n)/(c + d*x)^n)^3)/(n*(a*d - b*c))`

3.229 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 45

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)(c+dx)}\,dx=\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{3B(bc-ad)n}$$

```
[Out] 1/3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/B/(-a*d+b*c)/n
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2573, 2561, 2339, 30}

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)(c+dx)}\,dx=\frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{3Bn(bc-ad)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/((a + b*x)*(c + d*x)),x]
[Out] (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(3*B*(b*c - a*d)*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
```

```
x]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)(c+dx)}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)\\
&= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)\\
&= \text{Subst}\left(\frac{\text{Subst}\left(\int x^2\,dx, x, A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)n}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)\\
&= \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3B(bc-ad)n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(a+bx)(c+dx)}\,dx = \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{3(bBcn-aBdn)}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/((a + b*x)*(c + d*x)),
x]
[Out] (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(3*(b*B*c*n - a*B*d*n))
```

Maple [A] (verified)

Time = 5.82 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$-\frac{\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{3}}{3n(ad-cb)B}$	44
default	$-\frac{\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{3}}{3n(ad-cb)B}$	44
parallelrisch	$-\frac{B^{2}\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)^{3}b^{2}d^{2}+3AB\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)^{2}b^{2}d^{2}+3A^{2}\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)b^{2}d^{2}}{3n\,b^{2}d^{2}(ad-cb)}$	115
parts	$\frac{A^{2}\ln(dx+c)}{ad-cb}-\frac{A^{2}\ln(bx+a)}{ad-cb}-\frac{B^{2}\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)^{3}}{3n(ad-cb)}-\frac{BA\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)^{2}}{n(ad-cb)}$	120
risch	Expression too large to display	11062

[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)

[Out] -1/3/n/(a*d-b*c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/B

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(43) = 86.

Time = 0.33 (sec) , antiderivative size = 188, normalized size of antiderivative = 4.18

$$\int \frac{(A+B\log(e(a+bx)^{n}(c+dx)^{-n}))^{2}}{(a+bx)(c+dx)}\,dx$$
$$=\frac{B^{2}n^{2}\log(bx+a)^{3}-B^{2}n^{2}\log(dx+c)^{3}+3\left(B^{2}n\log(e)+ABn\right)\log(bx+a)^{2}+3\left(B^{2}n^{2}\log(bx+a)+B^{2}\right.}{}$$

[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)/(d*x+c),x, algorithm="fricas")

[Out] 1/3*(B^2*n^2*log(b*x + a)^3 - B^2*n^2*log(d*x + c)^3 + 3*(B^2*n*log(e) + A*B*n)*log(b*x + a)^2 + 3*(B^2*n^2*log(b*x + a) + B^2*n*log(e) + A*B*n)*log(d*x + c)^2 + 3*(B^2*log(e)^2 + 2*A*B*log(e) + A^2)*log(b*x + a) - 3*(B^2*n^2*log(b*x + a)^2 + B^2*log(e)^2 + 2*A*B*log(e) + A^2 + 2*(B^2*n*log(e) + A*B*n)*log(b*x + a))*log(d*x + c))/(b*c - a*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)/(d*x+c),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(43) = 86.

Time = 0.22 (sec) , antiderivative size = 387, normalized size of antiderivative = 8.60

$$\begin{aligned}
&\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)(c+dx)}\,dx \\
&= B^2\left(\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 \\
&\quad + 2\,AB\left(\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) \\
&\quad + A^2\left(\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}\right) \\
&\quad - \frac{1}{3}B^2\left(\frac{3\left(en\log(bx+a)^2-2\,en\log(bx+a)\log(dx+c)+en\log(dx+c)^2\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{(bc-ad)e}-\frac{e^2n^2\log(\right. \\
&\quad - \frac{\left(en\log(bx+a)^2-2\,en\log(bx+a)\log(dx+c)+en\log(dx+c)^2\right)AB}{(bc-ad)e}
\end{aligned}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)/(d*x+c),x, algorithm
="maxima")
[Out] B^2*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d))*log((b*x + a)^n*e
/(d*x + c)^n)^2 + 2*A*B*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d
))*log((b*x + a)^n*e/(d*x + c)^n) + A^2*(log(b*x + a)/(b*c - a*d) - log(d*x
 + c)/(b*c - a*d)) - 1/3*B^2*(3*(e*n*log(b*x + a)^2 - 2*e*n*log(b*x + a)*lo
g(d*x + c) + e*n*log(d*x + c)^2)*log((b*x + a)^n*e/(d*x + c)^n)/((b*c - a*d
)*e) - (e^2*n^2*log(b*x + a)^3 - 3*e^2*n^2*log(b*x + a)^2*log(d*x + c) + 3*
e^2*n^2*log(b*x + a)*log(d*x + c)^2 - e^2*n^2*log(d*x + c)^3)/((b*c - a*d)*
e^2)) - (e*n*log(b*x + a)^2 - 2*e*n*log(b*x + a)*log(d*x + c) + e*n*log(d*x
 + c)^2)*A*B/((b*c - a*d)*e)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)(c+dx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)/(d*x+c),x, algorithm
="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/((b*x + a)*(d*x + c)), x
)
```

Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.22

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)(c+dx)}\,dx = -\frac{-6\mathrm{i}\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,A^2+3\,A\,B\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2+B^2\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^3}{3\,n\,(a\,d-b\,c)}$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/((a + b*x)*(c + d*x)),x)
```

```
[Out] -(B^2*log((e*(a + b*x)^n)/(c + d*x)^n)^3 + 3*A*B*log((e*(a + b*x)^n)/(c + d
*x)^n)^2 - A^2*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*6i)/(3*n*(a*d -
 b*c))
```

3.230 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 45

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx=\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2B(bc-ad)n}$$

```
[Out] 1/2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/B/(-a*d+b*c)/n
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2573, 2561, 2338}

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx=\frac{\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}{2Bn(bc-ad)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/((a + b*x)*(c + d*x)),x]
[Out] (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(2*B*(b*c - a*d)*n)
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
```

```
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^2}{2B(bc-ad)n}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx = \frac{(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right))^2}{2(bBcn-aBdn)}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/((a + b*x)*(c + d*x)),x]
[Out] (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(2*(b*B*c*n - a*B*d*n))
```

Maple [A] (verified)

Time = 3.84 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.38

method	result	size
derivativedivides	$-\frac{\frac{B \ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)^{2}}{2}+\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)A}{n(ad-cb)}$	62
default	$-\frac{\frac{B \ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)^{2}}{2}+\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)A}{n(ad-cb)}$	62
parts	$\frac{A\ln(dx+c)}{ad-cb}-\frac{\ln(bx+a)A}{ad-cb}-\frac{B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)^{2}}{2n(ad-cb)}$	76
parallelrisch	$-\frac{B a^{2}c^{2}\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)^{2}+2A\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)a^{2}c^{2}}{2c^{2}a^{2}n(ad-cb)}$	80
risch	Expression too large to display	1152

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)/(d*x+c),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/n/(a*d-b*c)*(1/2*B*ln(e*(b*x+a)^n/((d*x+c)^n))^2+ln(e*(b*x+a)^n/((d*x+c)
^n))*A)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.60

$$\int \frac{A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx = \frac{Bn\log\left(bx+a\right)^{2}+Bn\log\left(dx+c\right)^{2}+2\left(B\log\left(e\right)+A\right)\log\left(bx+a\right)-2\left(Bn\log\left(bx+a\right)+B\log\left(e\right)+A}{2\left(bc-ad\right)}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)/(d*x+c),x, algorithm="
fricas")
```

```
[Out] 1/2*(B*n*log(b*x + a)^2 + B*n*log(d*x + c)^2 + 2*(B*log(e) + A)*log(b*x + a
) - 2*(B*n*log(b*x + a) + B*log(e) + A)*log(d*x + c))/(b*c - a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(43) = 86$.

Time = 0.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.36

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx = B\left(\frac{\log\left(bx+a\right)}{bc-ad}-\frac{\log\left(dx+c\right)}{bc-ad}\right)\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\left(\frac{\log\left(bx+a\right)}{bc-ad}-\frac{\log\left(dx+c\right)}{bc-ad}\right) - \frac{\left(en\log\left(bx+a\right)^2-2\,en\log\left(bx+a\right)\log\left(dx+c\right)+en\log\left(dx+c\right)^2\right)B}{2\,(bc-ad)e}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)/(d*x+c),x, algorithm="
maxima")
```

```
[Out] B*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d))*log((b*x + a)^n*e/(
d*x + c)^n) + A*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d)) - 1/2
*(e*n*log(b*x + a)^2 - 2*e*n*log(b*x + a)*log(d*x + c) + e*n*log(d*x + c)^2
)*B/((b*c - a*d)*e)
```

Giac [F]

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)/(d*x+c),x, algorithm="
giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/((b*x + a)*(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.58

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx = -\frac{B\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2-A\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,4\mathrm{i}}{2\,n\,\left(a\,d-b\,c\right)}$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/((a + b*x)*(c + d*x)),x)
```

```
[Out] -(B*log((e*(a + b*x)^n)/(c + d*x)^n)^2 - A*n*atan((b*c*2i + b*d*x*2i)/(a*d
- b*c) + 1i)*4i)/(2*n*(a*d - b*c))
```

3.231 $\int \frac{1}{(a+bx)(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 41

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{B(bc-ad)n}$$

```
[Out] ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/(-a*d+b*c)/n
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2573, 2561, 2339, 29}

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{Bn(bc-ad)}$$

```
[In] Int[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
[Out] Log[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]]/(B*(b*c - a*d)*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{x(A+B\log(ex^n))}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)n}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{B(bc-ad)n}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bBcn-aBdn}$$

```
[In] Integrate[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),
x]
```

```
[Out] Log[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]]/(b*B*c*n - a*B*d*n)
```

Maple [A] (verified)

Time = 21.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

method	result
derivativedivides	$-\frac{\ln\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)}{n(ad-cb)B}$
default	$-\frac{\ln\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)}{n(ad-cb)B}$
parallelrisch	$-\frac{\ln\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)}{n(ad-cb)B}$
risch	$-\frac{\ln\left(\ln((dx+c)^n)-\frac{-iB\pi\ \mathrm{csgn}(ie)\ \mathrm{csgn}\left(i(bx+a)^n(dx+c)^{-n}\right)\ \mathrm{csgn}\left(ie(dx+c)^{-n}(bx+a)^n\right)+iB\pi\ \mathrm{csgn}(ie)\ \mathrm{csgn}\left(ie(dx+c)^{-n}(bx+a\right.}{}\right.}{}$

```
[In] int(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/n/(a*d-b*c)*ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(-Bn\log(bx+a)+Bn\log(dx+c)-B\log(e)-A\right)}{(Bbc-Bad)n}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm
="fricas")
```

```
[Out] log(-B*n*log(b*x + a) + B*n*log(d*x + c) - B*log(e) - A)/((B*b*c - B*a*d)*n
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(-\frac{B\log((bx+a)^n)-B\log((dx+c)^n)+B\log(e)+A}{B}\right)}{(bcn-adn)B}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm
="maxima")
```

```
[Out] log(-(B*log((b*x + a)^n) - B*log((d*x + c)^n) + B*log(e) + A)/B)/((b*c*n -
a*d*n)*B)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(Bn\log\left(bx+a\right)-Bn\log\left(dx+c\right)+B\log\left(e\right)+A\right)}{Bbcn-Badn}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm
="giac")
```

```
[Out] log(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)/(B*b*c*n - B*a*d*n)
```

Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx=-\frac{\ln\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)}{B\,a\,d\,n-B\,b\,c\,n}$$

```
[In] int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x)*(c + d*x)),x)
[Out] -log(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(B*a*d*n - B*b*c*n)
```

3.232 $\int \frac{1}{(a+bx)(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2} dx$

Optimal result

Integrand size = 40, antiderivative size = 43

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = -\frac{1}{B(bc-ad)n\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}$$

```
[Out] -1/B/(-a*d+b*c)/n/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2573, 2561, 2339, 30}

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = -\frac{1}{Bn(bc-ad)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}$$

```
[In] Int[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2),x]
[Out] -(1/(B*(b*c - a*d)*n*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{x(A+B\log(ex^n))^2}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)n}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{1}{B(bc-ad)n\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = -\frac{1}{(bBcn-aBdn)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}$$

```
[In] Integrate[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2
),x]
```

```
[Out] -(1/((b*B*c*n - a*B*d*n)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])))
```

Maple [A] (verified)

Time = 65.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{1}{\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)Bn(ad-cb)}$
default	$\frac{1}{\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)Bn(ad-cb)}$
parallelrisch	$\frac{1}{\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)Bn(ad-cb)}$
risch	$\overline{Bn(ad-cb)\left(2A+2B\ln(e)+2B\ln((bx+a)^n)-2B\ln((dx+c)^n)-iB\pi\ \mathrm{csgn}(i(bx+a)^n)\ \mathrm{csgn}\left(i(dx+c)^{-n}\right)\ \mathrm{csgn}\left(i(bx+a)^n(d\right.\right.}$

```
[In] int(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/n/(a*d-b*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.00

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = -\frac{1}{(B^2bc-B^2ad)n^2\log(bx+a)-(B^2bc-B^2ad)n^2\log(dx+c)+(B^2bc-B^2ad)n\log(e)+(ABbc-AB}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorit
hm="fricas")
```

[Out] -1/((B^2*b*c - B^2*a*d)*n^2*log(b*x + a) - (B^2*b*c - B^2*a*d)*n^2*log(d*x + c) + (B^2*b*c - B^2*a*d)*n*log(e) + (A*B*b*c - A*B*a*d)*n)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \text{Timed out}$$

[In] integrate(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.88

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = -\frac{1}{(bcn-adn)B^2\log\left((bx+a)^n\right)-(bcn-adn)B^2\log\left((dx+c)^n\right)+(bcn-adn)AB+(bcn\log(e)-adn$$

[In] integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxima")

[Out] -1/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(43) = 86.

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.30

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = -\frac{1}{B^2bcn^2\log(bx+a)-B^2adn^2\log(bx+a)-B^2bcn^2\log(dx+c)+B^2adn^2\log(dx+c)+B^2bcn\log(e$$

[In] integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="giac")

[Out] -1/(B^2*b*c*n^2*log(b*x + a) - B^2*a*d*n^2*log(b*x + a) - B^2*b*c*n^2*log(d*x + c) + B^2*a*d*n^2*log(d*x + c) + B^2*b*c*n*log(e) - B^2*a*d*n*log(e) + A*B*b*c*n - A*B*a*d*n)

Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \frac{1}{B\,n\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)\,\left(a\,d-b\,c\right)}$$

```
[In] int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)*(c + d*x)),x)
[Out] 1/(B*n*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a*d - b*c))
```

3.233 $\int \frac{1}{(a+bx)(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 45

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx = -\frac{1}{2B(bc-ad)n\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}$$

```
[Out] -1/2/B/(-a*d+b*c)/n/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2573, 2561, 2339, 30}

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx = -\frac{1}{2Bn(bc-ad)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}$$

```
[In] Int[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3),x]
[Out] -1/2*1/(B*(b*c - a*d)*n*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{x(A+B\log(ex^n))^3}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{x^3}\,dx, x, A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)n}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{1}{2B(bc-ad)n\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx$$
$$= -\frac{1}{2(bBcn-aBdn)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}$$

```
[In] Integrate[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3
),x]
```

```
[Out] -1/2*1/((b*B*c*n - a*B*d*n)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)
```

Maple [A] (verified)

Time = 163.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{1}{2\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)^2 Bn(ad-cb)}$
default	$\frac{1}{2\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)^2 Bn(ad-cb)}$
parallelrisch	$\frac{1}{2\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)^2 Bn(ad-cb)}$
risch	$\frac{}{Bn(ad-cb)\left(2A+2B\ln(e)+2B\ln((bx+a)^n)-2B\ln((dx+c)^n)-iB\pi\ \mathrm{csgn}(i(bx+a)^n)\,\mathrm{csgn}\left(i(dx+c)^{-n}\right)\mathrm{csgn}\left(i(bx+a)^n\right.\right.}$

```
[In] int(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/B/n/(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(43) = 86$.

Time = 0.33 (sec) , antiderivative size = 238, normalized size of antiderivative = 5.29

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx =$$
$$-\frac{}{2\left((B^3bc-B^3ad)n^3\log(bx+a)^2+(B^3bc-B^3ad)n^3\log(dx+c)^2+(B^3bc-B^3ad)n\log(e)^2+2\,(AB\right.}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorit
hm="fricas")
```

```
[Out] -1/2/((B^3*b*c - B^3*a*d)*n^3*log(b*x + a)^2 + (B^3*b*c - B^3*a*d)*n^3*log(
d*x + c)^2 + (B^3*b*c - B^3*a*d)*n*log(e)^2 + 2*(A*B^2*b*c - A*B^2*a*d)*n*l
og(e) + (A^2*B*b*c - A^2*B*a*d)*n + 2*((B^3*b*c - B^3*a*d)*n^2*log(e) + (A*
B^2*b*c - A*B^2*a*d)*n^2)*log(b*x + a) - 2*((B^3*b*c - B^3*a*d)*n^3*log(b*x
 + a) + (B^3*b*c - B^3*a*d)*n^2*log(e) + (A*B^2*b*c - A*B^2*a*d)*n^2)*log(d
*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(43) = 86.

Time = 0.36 (sec) , antiderivative size = 220, normalized size of antiderivative = 4.89

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx =$$

$$-\frac{}{2\left((bcn-adn)B^3\log\left((bx+a)^n\right)^2+(bcn-adn)B^3\log\left((dx+c)^n\right)^2+(bcn-adn)A^2B+2\left(bcn\log(e)\right.\right.}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorit
hm="maxima")
```

```
[Out] -1/2/((b*c*n - a*d*n)*B^3*log((b*x + a)^n)^2 + (b*c*n - a*d*n)*B^3*log((d*x
 + c)^n)^2 + (b*c*n - a*d*n)*A^2*B + 2*(b*c*n*log(e) - a*d*n*log(e))*A*B^2
+ (b*c*n*log(e)^2 - a*d*n*log(e)^2)*B^3 + 2*((b*c*n - a*d*n)*A*B^2 + (b*c*n
*log(e) - a*d*n*log(e))*B^3)*log((b*x + a)^n) - 2*((b*c*n - a*d*n)*B^3*log(
(b*x + a)^n) + (b*c*n - a*d*n)*A*B^2 + (b*c*n*log(e) - a*d*n*log(e))*B^3)*l
og((d*x + c)^n))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(43) = 86$.

Time = 0.29 (sec) , antiderivative size = 321, normalized size of antiderivative = 7.13

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx =$$

$$-\frac{}{2\left(B^3bcn^3\log\left(bx+a\right)^2-B^3adn^3\log\left(bx+a\right)^2-2\,B^3bcn^3\log\left(bx+a\right)\log\left(dx+c\right)+2\,B^3adn^3\log\left(bx\right.\right.}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorit
hm="giac")
```

```
[Out] -1/2/(B^3*b*c*n^3*log(b*x + a)^2 - B^3*a*d*n^3*log(b*x + a)^2 - 2*B^3*b*c*n
^3*log(b*x + a)*log(d*x + c) + 2*B^3*a*d*n^3*log(b*x + a)*log(d*x + c) + B^
3*b*c*n^3*log(d*x + c)^2 - B^3*a*d*n^3*log(d*x + c)^2 + 2*B^3*b*c*n^2*log(b
*x + a)*log(e) - 2*B^3*a*d*n^2*log(b*x + a)*log(e) - 2*B^3*b*c*n^2*log(d*x
+ c)*log(e) + 2*B^3*a*d*n^2*log(d*x + c)*log(e) + 2*A*B^2*b*c*n^2*log(b*x +
 a) - 2*A*B^2*a*d*n^2*log(b*x + a) - 2*A*B^2*b*c*n^2*log(d*x + c) + 2*A*B^2
*a*d*n^2*log(d*x + c) + B^3*b*c*n*log(e)^2 - B^3*a*d*n*log(e)^2 + 2*A*B^2*b
*c*n*log(e) - 2*A*B^2*a*d*n*log(e) + A^2*B*b*c*n - A^2*B*a*d*n)
```

Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.60

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx = \frac{1}{2\,B\,n\,\left(a\,d-b\,c\right)\,\left(A^2+2\,A\,B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)+B^2\,{\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)}^2\right)}$$

```
[In] int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)*(c + d*x)),x)
```

```
[Out] 1/(2*B*n*(a*d - b*c)*(B^2*log((e*(a + b*x)^n)/(c + d*x)^n)^2 + A^2 + 2*A*B*
log((e*(a + b*x)^n)/(c + d*x)^n)))
```

3.234 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 49

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(a+bx)(c+dx)}\,dx=\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^{1+p}}{B(bc-ad)n(1+p)}$$

```
[Out] (A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^(p+1)/B/(-a*d+b*c)/n/(p+1)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2573, 2561, 2339, 30}

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(a+bx)(c+dx)}\,dx=\frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^{p+1}}{Bn(p+1)(bc-ad)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p/((a + b*x)*(c + d*x)),x]
```

```
[Out] (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^(1 + p)/(B*(b*c - a*d)*n*(1 + p))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
```

```
x]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p}{(a+bx)(c+dx)}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)\\
&= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{(A+B\log(ex^n))^p}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)\\
&= \text{Subst}\left(\frac{\text{Subst}\left(\int x^p\,dx, x, A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)n}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)\\
&= \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^{1+p}}{B(bc-ad)n(1+p)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$
\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p}{(a+bx)(c+dx)}\,dx = \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^{1+p}}{(bBcn-aBdn)(1+p)}
$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p/((a + b*x)*(c + d*x)),
x]
```

```
[Out] (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^(1 + p)/((b*B*c*n - a*B*d*n)*(1 +
p))
```

Maple [A] (verified)

Time = 234.62 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$-\frac{\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{p+1}}{n(ad-cb)B(p+1)}$	51
default	$-\frac{\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{p+1}}{n(ad-cb)B(p+1)}$	51
parallelrisch	$-\frac{B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{p}a^{2}c^{2}+A\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{p}a^{2}c^{2}}{(adp-bcp+ad-cb)B\,a^{2}c^{2}n}$	120

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/n/(a*d-b*c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^(p+1)/B/(p+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.65

$$\int \frac{(A+B\log(e(a+bx)^{n}(c+dx)^{-n}))^{p}}{(a+bx)(c+dx)}\,dx = \frac{(Bn\log(bx+a)-Bn\log(dx+c)+B\log(e)+A)(Bn\log(bx+a)-Bn\log(dx+c)+B\log(e)+A)^{p}}{(Bbc-Bad)np+(Bbc-Bad)n}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] (B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)*(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)^p/((B*b*c - B*a*d)*n*p + (B*b*c - B*a*d)*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^{n}(c+dx)^{-n}))^{p}}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**p/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^p}{(a + bx)(c + dx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^p}{(bx + a)(dx + c)}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*x+a)/(d*x+c),x, algorithm
="maxima")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^p/((b*x + a)*(d*x + c)), x
)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^p}{(a + bx)(c + dx)}\,dx = \frac{(Bn\log(bx + a) - Bn\log(dx + c) + B\log(e) + A)^{p+1}}{(Bbcn - Badn)(p + 1)}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*x+a)/(d*x+c),x, algorithm
="giac")
```

```
[Out] (B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)^(p + 1)/((B*b*c*n - B*
a*d*n)*(p + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^p}{(a + bx)(c + dx)}\,dx = \int \frac{\left(A + B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^p}{(a + b\,x)\,(c + d\,x)}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p/((a + b*x)*(c + d*x)),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p/((a + b*x)*(c + d*x)), x)
```

3.235 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(af+bfx)(cg+dgx)} dx$

Optimal result

Integrand size = 46, antiderivative size = 55

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(af+bfx)(cg+dgx)} dx = \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^{1+p}}{B(bc-ad)fgn(1+p)}$$

```
[Out] (A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^(p+1)/B/(-a*d+b*c)/f/g/n/(p+1)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2573, 2561, 2339, 30}

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(af+bfx)(cg+dgx)} dx = \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^{p+1}}{Bfgn(p+1)(bc-ad)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p/((a*f + b*f*x)*(c*g + d*g*x)
),x]
```

```
[Out] (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^(1 + p)/(B*(b*c - a*d)*f*g*n*(1 +
p))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
```

```
x]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p}{(af+bfx)(cg+dgx)}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)\\
&= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{(A+B\log(ex^n))^p}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)fg}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)\\
&= \text{Subst}\left(\frac{\text{Subst}\left(\int x^p\,dx, x, A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)fgn}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)\\
&= \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^{1+p}}{B(bc-ad)fgn(1+p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p}{(af+bfx)(cg+dgx)}\,dx = \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^{1+p}}{(bBcfgn-aBdfgn)(1+p)}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p/((a*f + b*f*x)*(c*g +
d*g*x)),x]
```

```
[Out] (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^(1 + p)/((b*B*c*f*g*n - a*B*d*f*g*
n)*(1 + p))
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$-\frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{p+1}}{fgn\left(ad-cb\right)B\left(p+1\right)}$$

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*f*x+a*f)/(d*g*x+c*g),x)
[Out] -1/f/g/n/(a*d-b*c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^(p+1)/B/(p+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.55

$$\int\frac{(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right))^{p}}{(af+bfx)(cg+dgx)}\,dx$$
$$=\frac{(Bn\log\left(bx+a\right)-Bn\log\left(dx+c\right)+B\log\left(e\right)+A)(Bn\log\left(bx+a\right)-Bn\log\left(dx+c\right)+B\log\left(e\right)+A)^{p}}{(Bbc-Bad)fgnp+(Bbc-Bad)fgn}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*f*x+a*f)/(d*g*x+c*g),x, a
lgorithm="fricas")
[Out] (B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)*(B*n*log(b*x + a) - B*
n*log(d*x + c) + B*log(e) + A)^p/((B*b*c - B*a*d)*f*g*n*p + (B*b*c - B*a*d)
*f*g*n)
```

Sympy [F(-1)]

Timed out.

$$\int\frac{(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right))^{p}}{(af+bfx)(cg+dgx)}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**p/(b*f*x+a*f)/(d*g*x+c*g),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(af+bfx)(cg+dgx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^p}{(bfx+af)(dgx+cg)}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*f*x+a*f)/(d*g*x+c*g),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^p/((b*f*x + a*f)*(d*g*x +
c*g)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(af+bfx)(cg+dgx)}\,dx = \frac{(Bn\log(bx+a) - Bn\log(dx+c) + B\log(e) + A)^{p+1}}{(Bbcfgn - Badfgn)(p+1)}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*f*x+a*f)/(d*g*x+c*g),x, a
lgorithm="giac")
```

```
[Out] (B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)^(p + 1)/((B*b*c*f*g*n
- B*a*d*f*g*n)*(p + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(af+bfx)(cg+dgx)}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^p}{(a\,f+b\,f\,x)\,(c\,g+d\,g\,x)}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p/((a*f + b*f*x)*(c*g + d*g*x)
),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p/((a*f + b*f*x)*(c*g + d*g*x)
), x)
```

3.236 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{acf+(bc+ad)fx+bdfx^2}\,dx$

Optimal result

Integrand size = 50, antiderivative size = 52

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{acf+(bc+ad)fx+bdfx^2}\,dx=\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^{1+p}}{B(bc-ad)fn(1+p)}$$

```
[Out] (A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^(p+1)/B/(-a*d+b*c)/f/n/(p+1)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2573, 2574, 2561, 2339, 30}

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{acf+(bc+ad)fx+bdfx^2}\,dx=\frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^{p+1}}{Bfn(p+1)(bc-ad)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p/(a*c*f + (b*c + a*d)*f*x + b
*d*f*x^2),x]
```

```
[Out] (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^(1 + p)/(B*(b*c - a*d)*f*n*(1 + p)
)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
```

```
x]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 2574

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_) + (h_.)*(x_)^2)^(m_.), x_Symbol] :> Dist[h^
m/(b^m*d^m), Int[(a + b*x)^m*(c + d*x)^m*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, n, p}, x] && EqQ[b*d
*f - a*c*h, 0] && EqQ[b*d*g - h*(b*c + a*d), 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p}{acf+(bc+ad)fx+bdfx^2}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)\\
&= \text{Subst}\left(\frac{\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p}{(a+bx)(c+dx)}\,dx}{f}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)\\
&= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{(A+B\log(ex^n))^p}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)f}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)\\
&= \text{Subst}\left(\frac{\text{Subst}\left(\int x^p\,dx, x, A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)fn}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)\\
&= \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^{1+p}}{B(bc-ad)fn(1+p)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{acf+(bc+ad)fx+bdfx^2}\,dx=\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^{1+p}}{f(bBcn-aBdn)(1+p)}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p/(a*c*f + (b*c + a*d)*f
*x + b*d*f*x^2),x]
```

```
[Out] (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^(1 + p)/(f*(b*B*c*n - a*B*d*n)*(1
+ p))
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^n\,(dx+c)^{-n}\right)\right)^p}{acf+(ad+cb)\,fx+bdf\,x^2}dx$$

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p/(a*c*f+(a*d+b*c)*f*x+b*d*f*x^2),x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p/(a*c*f+(a*d+b*c)*f*x+b*d*f*x^2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.60

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{acf+(bc+ad)fx+bdfx^2}\,dx =\frac{(Bn\log(bx+a)-Bn\log(dx+c)+B\log(e)+A)(Bn\log(bx+a)-Bn\log(dx+c)+B\log(e)+A)^p}{(Bbc-Bad)fnp+(Bbc-Bad)fn}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(a*c*f+(a*d+b*c)*f*x+b*d*f*x
^2),x, algorithm="fricas")
```

```
[Out] (B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)*(B*n*log(b*x + a) - B*
n*log(d*x + c) + B*log(e) + A)^p/((B*b*c - B*a*d)*f*n*p + (B*b*c - B*a*d)*f
*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{acf+(bc+ad)fx+bdfx^2}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**p/(a*c*f+(a*d+b*c)*f*x+b*d*f
*x**2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{acf+(bc+ad)fx+bdfx^2}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^p}{bdfx^2+acf+(bc+ad)fx}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(a*c*f+(a*d+b*c)*f*x+b*d*f*x
^2),x, algorithm="maxima")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^p/(b*d*f*x^2 + a*c*f + (b*
c + a*d)*f*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{acf+(bc+ad)fx+bdfx^2}\,dx=\frac{(Bn\log(bx+a)-Bn\log(dx+c)+B\log(e)+A)^{p+1}}{(Bbcfn-Badfn)(p+1)}$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(a*c*f+(a*d+b*c)*f*x+b*d*f*x
^2),x, algorithm="giac")
```

```
[Out] (B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)^(p + 1)/((B*b*c*f*n -
B*a*d*f*n)*(p + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^p}{acf + (bc + ad)fx + bdfx^2} dx = \int \frac{\left(A + B\ln\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)\right)^p}{b\,d\,f\,x^2 + f\,(a\,d + b\,c)\,x + a\,c\,f} dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p/(a*c*f + f*x*(a*d + b*c) + b
*d*f*x^2),x)
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p/(a*c*f + f*x*(a*d + b*c) + b
*d*f*x^2), x)
```

3.237 $\int \frac{1}{(a+bx)(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 41

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{B(bc-ad)n}$$

```
[Out] ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/(-a*d+b*c)/n
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2573, 2561, 2339, 29}

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{Bn(bc-ad)}$$

```
[In] Int[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
```

```
[Out] Log[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]]/(B*(b*c - a*d)*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{x(A+B\log(ex^n))}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)n}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{B(bc-ad)n}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bBcn-aBdn}$$

```
[In] Integrate[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),
x]
```

```
[Out] Log[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]]/(b*B*c*n - a*B*d*n)
```

Maple [A] (verified)

Time = 20.47 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

method	result
derivativedivides	$-\frac{\ln\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)}{n(ad-cb)B}$
default	$-\frac{\ln\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)}{n(ad-cb)B}$
parallelrisch	$-\frac{\ln\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)}{n(ad-cb)B}$
risch	$-\frac{\ln\left(\ln((dx+c)^n)-\frac{-iB\pi\ \mathrm{csgn}(ie)\ \mathrm{csgn}\left(i(bx+a)^n(dx+c)^{-n}\right)\ \mathrm{csgn}\left(ie(dx+c)^{-n}(bx+a)^n\right)+iB\pi\ \mathrm{csgn}(ie)\ \mathrm{csgn}\left(ie(dx+c)^{-n}(bx+a\right.}{}\right.}{}$

```
[In] int(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/n/(a*d-b*c)*ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(-Bn\log(bx+a)+Bn\log(dx+c)-B\log(e)-A\right)}{(Bbc-Bad)n}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm
="fricas")
```

```
[Out] log(-B*n*log(b*x + a) + B*n*log(d*x + c) - B*log(e) - A)/((B*b*c - B*a*d)*n
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(-\frac{B\log\left((bx+a)^n\right)-B\log\left((dx+c)^n\right)+B\log(e)+A}{B}\right)}{(bcn-adn)B}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm
="maxima")
```

```
[Out] log(-(B*log((b*x + a)^n) - B*log((d*x + c)^n) + B*log(e) + A)/B)/((b*c*n -
a*d*n)*B)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(Bn\log\left(bx+a\right)-Bn\log\left(dx+c\right)+B\log\left(e\right)+A\right)}{Bbcn-Badn}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm
="giac")
```

```
[Out] log(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)/(B*b*c*n - B*a*d*n)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = -\frac{\ln\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)}{B\,a\,d\,n-B\,b\,c\,n}$$

```
[In] int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x)*(c + d*x)),x)
[Out] -log(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(B*a*d*n - B*b*c*n)
```

3.238 $\int \frac{1}{(af+bfx)(cg+dgx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 47

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{B(bc-ad)fgn}$$

```
[Out] ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/(-a*d+b*c)/f/g/n
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2573, 2561, 2339, 29}

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{Bfgn(bc-ad)}$$

```
[In] Int[1/((a*f + b*f*x)*(c*g + d*g*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])
),x]
```

```
[Out] Log[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]]/(B*(b*c - a*d)*f*g*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{x(A+B\log(ex^n))}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)fg}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)fgn}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{B(bc-ad)fgn}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bBcfgn-aBdfgn}$$

```
[In] Integrate[1/((a*f + b*f*x)*(c*g + d*g*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*
x)^n])),x]
```

```
[Out] Log[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]]/(b*B*c*f*g*n - a*B*d*f*g*n)
```

Maple [A] (verified)

Time = 66.62 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

method	result
default	$-\frac{\ln\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)}{fgn(ad-cb)B}$
parallelrisch	$-\frac{\ln\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)}{fgn(ad-cb)B}$
risch	$-\frac{\ln\left(\ln((dx+c)^n)-\frac{-iB\pi\ \mathrm{csgn}(ie)\ \mathrm{csgn}\left(i(bx+a)^n(dx+c)^{-n}\right)\ \mathrm{csgn}\left(ie(dx+c)^{-n}(bx+a)^n\right)+iB\pi\ \mathrm{csgn}(ie)\ \mathrm{csgn}\left(ie(dx+c)^{-n}(bx+a)^n\right)^2-iB\cdots}{}\right.}{}$

```
[In] int(1/(b*f*x+a*f)/(d*g*x+c*g)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_R
ETURNVERBOSE)
```

```
[Out] -1/f/g/n/(a*d-b*c)*ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(-Bn\log\left(bx+a\right)+Bn\log\left(dx+c\right)-B\log\left(e\right)-A\right)}{(Bbc-Bad)fgn}$$

```
[In] integrate(1/(b*f*x+a*f)/(d*g*x+c*g)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, a
lgorithm="fricas")
```

```
[Out] log(-B*n*log(b*x + a) + B*n*log(d*x + c) - B*log(e) - A)/((B*b*c - B*a*d)*f
*g*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*f*x+a*f)/(d*g*x+c*g)/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(-\frac{B\log((bx+a)^n)-B\log((dx+c)^n)+B\log(e)+A}{B}\right)}{(bcfgn-adfgn)B}$$

```
[In] integrate(1/(b*f*x+a*f)/(d*g*x+c*g)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, a
lgorithm="maxima")
```

```
[Out] log(-(B*log((b*x + a)^n) - B*log((d*x + c)^n) + B*log(e) + A)/B)/((b*c*f*g*
n - a*d*f*g*n)*B)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(Bn\log\left(bx+a\right)-Bn\log\left(dx+c\right)+B\log\left(e\right)+A\right)}{Bbcfgn-Badfgn}$$

```
[In] integrate(1/(b*f*x+a*f)/(d*g*x+c*g)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, a
lgorithm="giac")
```

```
[Out] log(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)/(B*b*c*f*g*n - B*a*
d*f*g*n)
```

Mupad [B] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = -\frac{\ln\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)}{B\,a\,d\,f\,g\,n-B\,b\,c\,f\,g\,n}$$

```
[In] int(1/((a*f + b*f*x)*(c*g + d*g*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))
),x)
```

```
[Out] -log(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(B*a*d*f*g*n - B*b*c*f*g*n)
```

3.239 $\int \frac{1}{(acf+(bc+ad)fx+bdfx^2)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$

Optimal result

Integrand size = 50, antiderivative size = 44

$$\int \frac{1}{(acf+(bc+ad)fx+bdfx^2)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{B(bc-ad)fn}$$

```
[Out] ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/(-a*d+b*c)/f/n
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2573, 2574, 2561, 2339, 29}

$$\int \frac{1}{(acf+(bc+ad)fx+bdfx^2)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{Bfn(bc-ad)}$$

```
[In] Int[1/((a*c*f + (b*c + a*d)*f*x + b*d*f*x^2)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
```

```
[Out] Log[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]]/(B*(b*c - a*d)*f*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 2574

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_) + (h_.)*(x_)^2)^(m_.), x_Symbol] :> Dist[h^
m/(b^m*d^m), Int[(a + b*x)^m*(c + d*x)^m*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, n, p}, x] && EqQ[b*d
*f - a*c*h, 0] && EqQ[b*d*g - h*(b*c + a*d), 0] && IntegerQ[m]
```

Rubi steps

integral

$$= \text{Subst}\left(\int \frac{1}{(acf+(bc+ad)fx+bdfx^2)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx}{f}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{x(A+B\log(ex^n))}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bc-ad)f}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{B(bc-ad)fn}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \frac{\log\left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{B(bc-ad)fn}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{1}{(acf + (bc+ad)fx + bdfx^2)\left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$
$$= \frac{\log\left(A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{f(bBcn - aBdn)}$$

```
[In] Integrate[1/((a*c*f + (b*c + a*d)*f*x + b*d*f*x^2)*(A + B*Log[(e*(a + b*x)^
n)/(c + d*x)^n])),x]
```

```
[Out] Log[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]]/(f*(b*B*c*n - a*B*d*n))
```

Maple [A] (verified)

Time = 55.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

method	result
parallelrisch	$-\frac{\ln\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)}{Bfn(ad-cb)}$
risch	$-\frac{\ln\left(\ln((dx+c)^n)-\frac{-iB\pi\ \text{csgn}(ie)\ \text{csgn}\left(i(bx+a)^n(dx+c)^{-n}\right)\ \text{csgn}\left(ie(dx+c)^{-n}(bx+a)^n\right)+iB\pi\ \text{csgn}(ie)\ \text{csgn}\left(ie(dx+c)^{-n}(bx+a)^n\right)^2-i}{}\right.}{}$

```
[In] int(1/(a*c*f+(a*d+b*c)*f*x+b*d*f*x^2)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/f/n/(a*d-b*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \frac{1}{(acf+(bc+ad)fx+bdfx^2)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(-Bn\log\left(bx+a\right)+Bn\log\left(dx+c\right)-B\log\left(e\right)-A\right)}{(Bbc-Bad)fn}$$

```
[In] integrate(1/(a*c*f+(a*d+b*c)*f*x+b*d*f*x^2)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n
))),x, algorithm="fricas")
```

```
[Out] log(-B*n*log(b*x + a) + B*n*log(d*x + c) - B*log(e) - A)/((B*b*c - B*a*d)*f
*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(acf+(bc+ad)fx+bdfx^2)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a*c*f+(a*d+b*c)*f*x+b*d*f*x**2)/(A+B*ln(e*(b*x+a)**n/((d*x+c)*
*n))),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{1}{(acf+(bc+ad)fx+bdfx^2)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(-\frac{B\log\left((bx+a)^n\right)-B\log\left((dx+c)^n\right)+B\log(e)+A}{B}\right)}{(bcfn-adfn)B}$$

```
[In] integrate(1/(a*c*f+(a*d+b*c)*f*x+b*d*f*x^2)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n
))),x, algorithm="maxima")
```

```
[Out] log(-(B*log((b*x + a)^n) - B*log((d*x + c)^n) + B*log(e) + A)/B)/((b*c*f*n
- a*d*f*n)*B)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{1}{(acf+(bc+ad)fx+bdfx^2)\,(A+B\log{(e(a+bx)^n(c+dx)^{-n})})}\,dx = \frac{\log{(Bn\log{(bx+a)}-Bn\log{(dx+c)}+B\log{(e)}+A)}}{Bbcfn-Badfn}$$

```
[In] integrate(1/(a*c*f+(a*d+b*c)*f*x+b*d*f*x^2)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n
))),x, algorithm="giac")
```

```
[Out] log(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)/(B*b*c*f*n - B*a*d*
f*n)
```

Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{1}{(acf+(bc+ad)fx+bdfx^2)\,(A+B\log{(e(a+bx)^n(c+dx)^{-n})})}\,dx = -\frac{\ln\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)}{B\,a\,d\,f\,n-B\,b\,c\,f\,n}$$

```
[In] int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a*c*f + f*x*(a*d + b*c) +
b*d*f*x^2)),x)
```

```
[Out] -log(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(B*a*d*f*n - B*b*c*f*n)
```

3.240 $\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log(e(a+bx)^n(c+dx)^{-n})}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 88

$$\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log(e(a+bx)^n(c+dx)^{-n})}\,dx = \frac{(a+bx)^{1+m}(c+dx)^{-1-m}\left(e(a+bx)^n(c+dx)^{-n}\right)^{-\frac{1+m}{n}}\text{ExpIntegralEi}\left(\frac{(1+m)\log(e(a+bx)^n(c+dx)^{-n})}{n}\right)}{(bc-ad)n}$$

```
[Out] (b*x+a)^(1+m)*(d*x+c)^(-1-m)*Ei((1+m)*ln(e*(b*x+a)^n/((d*x+c)^n))/n)/(-a*d+
b*c)/n/((e*(b*x+a)^n/((d*x+c)^n))^((1+m)/n))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2573, 2563, 2347, 2209}

$$\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log(e(a+bx)^n(c+dx)^{-n})}\,dx = \frac{(a+bx)^{m+1}(c+dx)^{-m-1}\left(e(a+bx)^n(c+dx)^{-n}\right)^{-\frac{m+1}{n}}\text{ExpIntegralEi}\left(\frac{(m+1)\log(e(a+bx)^n(c+dx)^{-n})}{n}\right)}{n(bc-ad)}$$

```
[In] Int[((a + b*x)^m*(c + d*x)^(-2 - m))/Log[(e*(a + b*x)^n)/(c + d*x)^n],x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(-1 - m)*ExpIntegralEi[((1 + m)*Log[(e*(a + b*
x)^n)/(c + d*x)^n])/n])/((b*c - a*d)*n*((e*(a + b*x)^n)/(c + d*x)^n)^((1 +
m)/n))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2563

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m)), Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*x
)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x]
 && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
q + 2, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$=\text{Subst}\left(\frac{\left((a+bx)^m\left(\frac{a+bx}{c+dx}\right)^{-m}(c+dx)^{-m}\right)\text{Subst}\left(\int \frac{x^m}{\log(ex^n)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\left((a+bx)^{1+m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{1+m}{n}}(c+dx)^{-1-m}\right)\text{Subst}\left(\int\frac{e^{\frac{(1+m)x}{n}}}{x}dx,x,\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)n},e\left(\frac{a}{c}\right.\right.$$

$$\left.+bx)^n(c+dx)^{-n}\right)$$

$$=\frac{(a+bx)^{1+m}(c+dx)^{-1-m}\left(e(a+bx)^n(c+dx)^{-n}\right)^{-\frac{1+m}{n}}\text{Ei}\left(\frac{(1+m)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{n}\right)}{(bc-ad)n}$$

Mathematica [F]

$$\int\frac{(a+bx)^m(c+dx)^{-2-m}}{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}dx=\int\frac{(a+bx)^m(c+dx)^{-2-m}}{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}dx$$

```
[In] Integrate[((a + b*x)^m*(c + d*x)^(-2 - m))/Log[(e*(a + b*x)^n)/(c + d*x)^n]
,x]
[Out] Integrate[((a + b*x)^m*(c + d*x)^(-2 - m))/Log[(e*(a + b*x)^n)/(c + d*x)^n]
, x]
```

Maple [F]

$$\int\frac{(bx+a)^m\,(dx+c)^{-2-m}}{\ln\left(e\,(bx+a)^n\,(dx+c)^{-n}\right)}dx$$

```
[In] int((b*x+a)^m*(d*x+c)^(-2-m)/ln(e*(b*x+a)^n/((d*x+c)^n)),x)
[Out] int((b*x+a)^m*(d*x+c)^(-2-m)/ln(e*(b*x+a)^n/((d*x+c)^n)),x)
```

Fricas [F]

$$\int\frac{(a+bx)^m(c+dx)^{-2-m}}{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}dx=\int\frac{(bx+a)^m(dx+c)^{-m-2}}{\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)}dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^(-2-m)/log(e*(b*x+a)^n/((d*x+c)^n)),x, algorith
m="fricas")
[Out] integral((b*x + a)^m*(d*x + c)^(-m - 2)/log((b*x + a)^n*e/(d*x + c)^n), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x+a)**m*(d*x+c)**(-2-m)/ln(e*(b*x+a)**n/((d*x+c)**n)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}\,dx = \int \frac{(bx+a)^m(dx+c)^{-m-2}}{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^(-2-m)/log(e*(b*x+a)^n/((d*x+c)^n)),x, algorith
m="maxima")
[Out] integrate((b*x + a)^m*(d*x + c)^(-m - 2)/log((b*x + a)^n*e/(d*x + c)^n), x)
```

Giac [F]

$$\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}\,dx = \int \frac{(bx+a)^m(dx+c)^{-m-2}}{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}\,dx$$

```
[In] integrate((b*x+a)^m*(d*x+c)^(-2-m)/log(e*(b*x+a)^n/((d*x+c)^n)),x, algorith
m="giac")
[Out] integrate((b*x + a)^m*(d*x + c)^(-m - 2)/log((b*x + a)^n*e/(d*x + c)^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}\,dx = \int \frac{(a+b\,x)^m}{\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\,(c+d\,x)^{m+2}}\,dx$$

```
[In] int((a + b*x)^m/(log((e*(a + b*x)^n)/(c + d*x)^n)*(c + d*x)^(m + 2)),x)
[Out] int((a + b*x)^m/(log((e*(a + b*x)^n)/(c + d*x)^n)*(c + d*x)^(m + 2)), x)
```

3.241 $\int \frac{(a+bx)^3}{(c+dx)^5 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$

Optimal result

Integrand size = 35, antiderivative size = 75

$$\int \frac{(a+bx)^3}{(c+dx)^5 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{(a+bx)^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-4/n} \text{ExpIntegralEi}\left(\frac{4\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)^4}$$

```
[Out] (b*x+a)^4*Ei(4*ln(e*((b*x+a)/(d*x+c))^n)/n)/(-a*d+b*c)/n/((e*((b*x+a)/(d*x+
c))^n)^(4/n))/(d*x+c)^4
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2561, 2347, 2209}

$$\int \frac{(a+bx)^3}{(c+dx)^5 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{(a+bx)^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-4/n} \text{ExpIntegralEi}\left(\frac{4\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(c+dx)^4(bc-ad)}$$

```
[In] Int[(a + b*x)^3/((c + d*x)^5*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] ((a + b*x)^4*ExpIntegralEi[(4*Log[e*((a + b*x)/(c + d*x))^n])/n])/((b*c - a
*d)*n*(e*((a + b*x)/(c + d*x))^n)^(4/n)*(c + d*x)^4)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^3}{\log(ex^n)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad} \\
&= \frac{\left((a+bx)^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-4/n}\right)\text{Subst}\left(\int \frac{e^{\frac{4x}{n}}}{x}\,dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)n(c+dx)^4} \\
&= \frac{(a+bx)^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-4/n}\text{Ei}\left(\frac{4\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^3}{(c+dx)^5\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{(a+bx)^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-4/n}\text{ExpIntegralEi}\left(\frac{4\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)^4}$$

```
[In] Integrate[(a + b*x)^3/((c + d*x)^5*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] ((a + b*x)^4*ExpIntegralEi[(4*Log[e*((a + b*x)/(c + d*x))^n])/n])/((b*c - a
*d)*n*(e*((a + b*x)/(c + d*x))^n)^(4/n)*(c + d*x)^4)
```

Maple [F]

$$\int \frac{(bx+a)^3}{(dx+c)^5 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

[In] int((b*x+a)^3/(d*x+c)^5/ln(e*((b*x+a)/(d*x+c))^n),x)

[Out] int((b*x+a)^3/(d*x+c)^5/ln(e*((b*x+a)/(d*x+c))^n),x)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx)^3}{(c+dx)^5 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\text{log\_integral}\left(\frac{(b^4x^4+4\,ab^3x^3+6\,a^2b^2x^2+4\,a^3bx+a^4)e^{\frac{4}{n}}}{d^4x^4+4\,cd^3x^3+6\,c^2d^2x^2+4\,c^3dx+c^4}\right)}{(bc-ad)e^{\frac{4}{n}}n}$$

[In] integrate((b*x+a)^3/(d*x+c)^5/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fricas")

[Out] log_integral((b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*e^(4/n)/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4))/((b*c - a*d)*e^(4/n)*n)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3}{(c+dx)^5 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Timed out}$$

[In] integrate((b*x+a)**3/(d*x+c)**5/ln(e*((b*x+a)/(d*x+c))**n),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a+bx)^3}{(c+dx)^5 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bx+a)^3}{(dx+c)^5 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

[In] integrate((b*x+a)^3/(d*x+c)^5/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxima")

[Out] integrate((b*x + a)^3/((d*x + c)^5*log(e*((b*x + a)/(d*x + c))^n)), x)

Giac [F]

$$\int \frac{(a+bx)^3}{(c+dx)^5\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bx+a)^3}{(dx+c)^5\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

```
[In] integrate((b*x+a)^3/(d*x+c)^5/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac
")
```

```
[Out] integrate((b*x + a)^3/((d*x + c)^5*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3}{(c+dx)^5\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(a+b\,x)^3}{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,(c+d\,x)^5}\,dx$$

```
[In] int((a + b*x)^3/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^5),x)
```

```
[Out] int((a + b*x)^3/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^5), x)
```

3.242 $\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$

Optimal result

Integrand size = 35, antiderivative size = 75

$$\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{(a+bx)^3 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-3/n} \text{ExpIntegralEi}\left(\frac{3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)^3}$$

```
[Out] (b*x+a)^3*Ei(3*ln(e*((b*x+a)/(d*x+c))^n)/n)/(-a*d+b*c)/n/((e*((b*x+a)/(d*x+
c))^n)^(3/n))/(d*x+c)^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2561, 2347, 2209}

$$\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{(a+bx)^3 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-3/n} \text{ExpIntegralEi}\left(\frac{3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(c+dx)^3(bc-ad)}$$

```
[In] Int[(a + b*x)^2/((c + d*x)^4*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] ((a + b*x)^3*ExpIntegralEi[(3*Log[e*((a + b*x)/(c + d*x))^n])/n])/((b*c - a
*d)*n*(e*((a + b*x)/(c + d*x))^n)^(3/n)*(c + d*x)^3)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{\log(ex^n)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}\\
&= \frac{\left((a+bx)^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-3/n}\right)\text{Subst}\left(\int \frac{e^{\frac{3x}{n}}}{x}\,dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)n(c+dx)^3}\\
&= \frac{(a+bx)^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-3/n}\text{Ei}\left(\frac{3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$
\int \frac{(a+bx)^2}{(c+dx)^4\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{(a+bx)^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-3/n}\text{ExpIntegralEi}\left(\frac{3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)^3}
$$

```
[In] Integrate[(a + b*x)^2/((c + d*x)^4*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] ((a + b*x)^3*ExpIntegralEi[(3*Log[e*((a + b*x)/(c + d*x))^n])/n])/((b*c - a
*d)*n*(e*((a + b*x)/(c + d*x))^n)^(3/n)*(c + d*x)^3)
```

Maple [F]

$$\int \frac{\left(bx + a\right)^{2}}{\left(dx + c\right)^{4} \ln \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)}dx$$

```
[In] int((b*x+a)^2/(d*x+c)^4/ln(e*((b*x+a)/(d*x+c))^n),x)
```

```
[Out] int((b*x+a)^2/(d*x+c)^4/ln(e*((b*x+a)/(d*x+c))^n),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.17

$$\int \frac{(a + bx)^2}{(c + dx)^4 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}\, dx = \frac{\text{log\_integral}\left(\frac{\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)e^{\frac{3}{n}}}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)}{(bc - ad)e^{\frac{3}{n}}n}$$

```
[In] integrate((b*x+a)^2/(d*x+c)^4/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fric
as")
```

```
[Out] log_integral((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*e^(3/n)/(d^3*x^3 + 3
*c*d^2*x^2 + 3*c^2*d*x + c^3))/((b*c - a*d)*e^(3/n)*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + bx)^2}{(c + dx)^4 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}\, dx = \text{Timed out}$$

```
[In] integrate((b*x+a)**2/(d*x+c)**4/ln(e*((b*x+a)/(d*x+c))**n),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a + bx)^2}{(c + dx)^4 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}\, dx = \int \frac{\left(bx + a\right)^{2}}{\left(dx + c\right)^{4} \log \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)}\, dx$$

```
[In] integrate((b*x+a)^2/(d*x+c)^4/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*x + a)^2/((d*x + c)^4*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Giac [F]

$$\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bx+a)^2}{(dx+c)^4 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

```
[In] integrate((b*x+a)^2/(d*x+c)^4/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac
")
[Out] integrate((b*x + a)^2/((d*x + c)^4*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(a+b\,x)^2}{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,(c+d\,x)^4}\,dx$$

```
[In] int((a + b*x)^2/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^4),x)
[Out] int((a + b*x)^2/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^4), x)
```

3.243 $\int \frac{a+bx}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$

Optimal result

Integrand size = 33, antiderivative size = 75

$$\int \frac{a+bx}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{(a+bx)^2 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n} \text{ExpIntegralEi}\left(\frac{2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)^2}$$

```
[Out] (b*x+a)^2*Ei(2*ln(e*((b*x+a)/(d*x+c))^n)/n)/(-a*d+b*c)/n/((e*((b*x+a)/(d*x+
c))^n)^(2/n))/(d*x+c)^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2561, 2347, 2209}

$$\int \frac{a+bx}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{(a+bx)^2 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n} \text{ExpIntegralEi}\left(\frac{2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(c+dx)^2(bc-ad)}$$

```
[In] Int[(a + b*x)/((c + d*x)^3*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] ((a + b*x)^2*ExpIntegralEi[(2*Log[e*((a + b*x)/(c + d*x))^n])/n])/((b*c - a
*d)*n*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x}{\log(ex^n)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}\\ &= \frac{\left((a+bx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{x}\,dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)n(c+dx)^2}\\ &= \frac{(a+bx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\text{Ei}\left(\frac{2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{(c+dx)^3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{(a+bx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)^2}$$

```
[In] Integrate[(a + b*x)/((c + d*x)^3*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] ((a + b*x)^2*ExpIntegralEi[(2*Log[e*((a + b*x)/(c + d*x))^n])/n])/((b*c - a
*d)*n*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2)
```

Maple [F]

$$\int \frac{bx+a}{(dx+c)^3 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

[In] int((b*x+a)/(d*x+c)^3/ln(e*((b*x+a)/(d*x+c))^n),x)

[Out] int((b*x+a)/(d*x+c)^3/ln(e*((b*x+a)/(d*x+c))^n),x)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

$$\int \frac{a+bx}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\text{log\_integral}\left(\frac{(b^2x^2+2\,abx+a^2)e^{\frac{2}{n}}}{d^2x^2+2\,cdx+c^2}\right)}{(bc-ad)e^{\frac{2}{n}}n}$$

[In] integrate((b*x+a)/(d*x+c)^3/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fricas
")

[Out] log_integral((b^2*x^2 + 2*a*b*x + a^2)*e^(2/n)/(d^2*x^2 + 2*c*d*x + c^2))/(
(b*c - a*d)*e^(2/n)*n)

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Timed out}$$

[In] integrate((b*x+a)/(d*x+c)**3/ln(e*((b*x+a)/(d*x+c))**n),x)

[Out] Timed out

Maxima [F]

$$\int \frac{a+bx}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{bx+a}{(dx+c)^3 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

[In] integrate((b*x+a)/(d*x+c)^3/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxima
")

[Out] integrate((b*x + a)/((d*x + c)^3*log(e*((b*x + a)/(d*x + c))^n)), x)

Giac [F]

$$\int \frac{a+bx}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{bx+a}{(dx+c)^3 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

```
[In] integrate((b*x+a)/(d*x+c)^3/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac")
[Out] integrate((b*x + a)/((d*x + c)^3*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{a+b\,x}{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,(c+d\,x)^3}\,dx$$

```
[In] int((a + b*x)/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^3),x)
[Out] int((a + b*x)/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^3), x)
```

3.244 $\int \frac{1}{(c+dx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$

Optimal result

Integrand size = 28, antiderivative size = 72

$$\int \frac{1}{(c+dx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n} \text{ExpIntegralEi}\left(\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)}$$

```
[Out] (b*x+a)*Ei(ln(e*((b*x+a)/(d*x+c))^n)/n)/(-a*d+b*c)/n/((e*((b*x+a)/(d*x+c))^
n)^(1/n))/(d*x+c)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2551, 2337, 2209}

$$\int \frac{1}{(c+dx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n} \text{ExpIntegralEi}\left(\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(c+dx)(bc-ad)}$$

```
[In] Int[1/((c + d*x)^2*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] ((a + b*x)*ExpIntegralEi[Log[e*((a + b*x)/(c + d*x))^n]/n])/((b*c - a*d)*n*
(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2551

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/d)^m, Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c -
 a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1
])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\log(ex^n)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}\\
&= \frac{\left((a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{x}\,dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)n(c+dx)}\\
&= \frac{(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{Ei}\left(\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$
\int \frac{1}{(c+dx)^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)}
$$

```
[In] Integrate[1/((c + d*x)^2*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] ((a + b*x)*ExpIntegralEi[Log[e*((a + b*x)/(c + d*x))^n]/n])/((b*c - a*d)*n*
(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x))
```

Maple [F]

$$\int \frac{1}{\left(dx+c\right)^{2} \ln \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)}dx$$

```
[In] int(1/(d*x+c)^2/ln(e*((b*x+a)/(d*x+c))^n),x)
[Out] int(1/(d*x+c)^2/ln(e*((b*x+a)/(d*x+c))^n),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int \frac{1}{(c+dx)^2 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)} \, dx = \frac{\text{log\_integral}\left(\frac{(bx+a)e^{\left(\frac{1}{n}\right)}}{dx+c}\right)}{(bc-ad)e^{\left(\frac{1}{n}\right)}n}$$

```
[In] integrate(1/(d*x+c)^2/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fricas")
[Out] log_integral((b*x + a)*e^(1/n)/(d*x + c))/((b*c - a*d)*e^(1/n)*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)} \, dx = \text{Timed out}$$

```
[In] integrate(1/(d*x+c)**2/ln(e*((b*x+a)/(d*x+c))**n),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(c+dx)^2 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)} \, dx = \int \frac{1}{\left(dx+c\right)^{2} \log \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)} \,dx$$

```
[In] integrate(1/(d*x+c)^2/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxima")
[Out] integrate(1/((d*x + c)^2*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{1}{(dx+c)^2 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

```
[In] integrate(1/(d*x+c)^2/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac")
[Out] integrate(1/((d*x + c)^2*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{1}{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,(c+d\,x)^2}\,dx$$

```
[In] int(1/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^2),x)
[Out] int(1/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^2), x)
```

3.245 $\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 33

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\frac{\log\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)n}$$

```
[Out] ln(ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/n
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2561, 2339, 29}

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\frac{\log\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{n(bc-ad)}$$

```
[In] Int[1/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] Log[Log[e*((a + b*x)/(c + d*x))^n]]/((b*c - a*d)*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x\log(ex^n)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)n}\\ &= \frac{\log\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = -\frac{\log\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(-bc+ad)n}$$

```
[In] Integrate[1/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] -(Log[Log[e*((a + b*x)/(c + d*x))^n]]/((-(b*c) + a*d)*n))
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$-\frac{\ln\left(\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{n(ad-cb)}$	35
default	$-\frac{\ln\left(\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{n(ad-cb)}$	35
parallelrisch	$-\frac{\ln\left(\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{n(ad-cb)}$	35

```
[In] int(1/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n),x,method=_RETURNVERBOSE)
```

[Out] -1/n/(a*d-b*c)*ln(ln(e*((b*x+a)/(d*x+c))^n))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\log\left(n\log\left(\frac{bx+a}{dx+c}\right)+\log(e)\right)}{(bc-ad)n}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fricas
")
```

```
[Out] log(n*log((b*x + a)/(d*x + c)) + log(e))/((b*c - a*d)*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(24) = 48.

Time = 56.71 (sec) , antiderivative size = 146, normalized size of antiderivative = 4.42

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \begin{cases} -\frac{1}{(bc+bdx)\log(e)} & \text{for } a=\frac{bc}{d}\wedge n=0 \\ -\frac{1}{bc\log\left(e\left(\frac{bc}{cd+d^2x}+\frac{bx}{c+dx}\right)^n\right)+bdx\log\left(e\left(\frac{bc}{cd+d^2x}+\frac{bx}{c+dx}\right)^n\right)} & \text{for } a=\frac{bc}{d} \\ \frac{-\frac{\log\left(\frac{a}{b}+x\right)}{ad-bc}+\frac{\log\left(\frac{c}{d}+x\right)}{ad-bc}}{\log(e)} & \text{for } n=0 \\ -\frac{\log\left(\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)}{adn-bcn} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))**n),x)
```

```
[Out] Piecewise((-1/((b*c + b*d*x)*log(e)), Eq(n, 0) & Eq(a, b*c/d)), (-1/(b*c*lo
g(e*(b*c/(c*d + d**2*x) + b*x/(c + d*x))**n) + b*d*x*log(e*(b*c/(c*d + d**2
*x) + b*x/(c + d*x))**n)), Eq(a, b*c/d)), ((-log(a/b + x)/(a*d - b*c) + log
(c/d + x)/(a*d - b*c))/log(e), Eq(n, 0)), (-log(log(e*(a/(c + d*x) + b*x/(c
 + d*x))**n))/(a*d*n - b*c*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\log\left(-\log\left((bx+a)^n\right)+\log\left((dx+c)^n\right)-\log\left(e\right)\right)}{bcn-adn}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxima
")
[Out] log(-log((b*x + a)^n) + log((d*x + c)^n) - log(e))/(b*c*n - a*d*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(33) = 66.

Time = 0.49 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.67

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$
$$= \frac{\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)\log\left(\frac{1}{4}\left(\pi(\operatorname{sgn}(bx+a)\operatorname{sgn}(dx+c)-1)n+\pi(\operatorname{sgn}(e)-1)\right)^2+\left(n\log\left(\frac{|bx+a|}{|dx+c|}\right)+\log(\right.\right.}{2\,n}$$

```
[In] integrate(1/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac")
[Out] 1/2*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)*log(1/4*(pi*(sgn(b*x + a)*sgn(d
*x + c) - 1)*n + pi*(sgn(e) - 1))^2 + (n*log(abs(b*x + a)/abs(d*x + c)) + l
og(abs(e)))^2)/n
```

Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = -\frac{\ln\left(\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{a\,d\,n-b\,c\,n}$$

```
[In] int(1/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)*(c + d*x)),x)
[Out] -log(log(e*((a + b*x)/(c + d*x))^n))/(a*d*n - b*c*n)
```

3.246 $\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 71

$$\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralEi}\left(-\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(a+bx)}$$

```
[Out] (e*((b*x+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei(-ln(e*((b*x+a)/(d*x+c))^n)/n)/(-a*
d+b*c)/n/(b*x+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2549, 2347, 2209}

$$\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(a+bx)(bc-ad)}$$

```
[In] Int[1/((a + b*x)^2*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] ((e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[-(Log[e*((a + b
*x)/(c + d*x))^n]/n)])/((b*c - a*d)*n*(a + b*x))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2549

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[(b*c - a*d)^(m +
1)*(g/b)^m, Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
(a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || LtQ
[m, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^2\log(ex^n)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}\\ &= \frac{\left(\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\right)\text{Subst}\left(\int \frac{e^{-\frac{x}{n}}}{x}\,dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)n(a+bx)}\\ &= \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\text{Ei}\left(-\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(a+bx)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)\,\text{ExpIntegralEi}\left(-\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(a+bx)}$$

```
[In] Integrate[1/((a + b*x)^2*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] ((e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[-(Log[e*((a + b
*x)/(c + d*x))^n]/n)])/((b*c - a*d)*n*(a + b*x))
```

Maple [F]

$$\int \frac{1}{(bx+a)^2 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

```
[In] int(1/(b*x+a)^2/ln(e*((b*x+a)/(d*x+c))^n),x)
[Out] int(1/(b*x+a)^2/ln(e*((b*x+a)/(d*x+c))^n),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{e^{\left(\frac{1}{n}\right)} \operatorname{log\_integral}\left(\frac{dx+c}{(bx+a)e^{\left(\frac{1}{n}\right)}}\right)}{(bc-ad)n}$$

```
[In] integrate(1/(b*x+a)^2/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fricas")
[Out] e^(1/n)*log_integral((d*x + c)/((b*x + a)*e^(1/n)))/((b*c - a*d)*n)
```

Sympy [F]

$$\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx$$

```
[In] integrate(1/(b*x+a)**2/ln(e*((b*x+a)/(d*x+c))**n),x)
[Out] Integral(1/((a + b*x)**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{1}{(bx+a)^2 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

```
[In] integrate(1/(b*x+a)^2/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^2*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{1}{(bx+a)^2 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

```
[In] integrate(1/(b*x+a)^2/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^2*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{1}{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,(a+b\,x)^2}\,dx$$

```
[In] int(1/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)^2),x)
[Out] int(1/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)^2), x)
```

3.247 $\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$

Optimal result

Integrand size = 33, antiderivative size = 75

$$\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n} (c+dx)^2 \text{ExpIntegralEi}\left(-\frac{2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(a+bx)^2}$$

```
[Out] (e*((b*x+a)/(d*x+c))^n)^(2/n)*(d*x+c)^2*Ei(-2*ln(e*((b*x+a)/(d*x+c))^n)/n)/
(-a*d+b*c)/n/(b*x+a)^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2561, 2347, 2209}

$$\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{(c+dx)^2 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n} \text{ExpIntegralEi}\left(-\frac{2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(a+bx)^2(bc-ad)}$$

```
[In] Int[(c + d*x)/((a + b*x)^3*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] ((e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2*ExpIntegralEi[(-2*Log[e*((a
+ b*x)/(c + d*x))^n])/n])/((b*c - a*d)*n*(a + b*x)^2)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^3\log(ex^n)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}\\
&= \frac{\left(\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)^2\right)\text{Subst}\left(\int \frac{e^{-\frac{2x}{n}}}{x}\,dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)n(a+bx)^2}\\
&= \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)^2\text{Ei}\left(-\frac{2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(a+bx)^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{c+dx}{(a+bx)^3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)^2\,\text{ExpIntegralEi}\left(-\frac{2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(a+bx)^2}$$

```
[In] Integrate[(c + d*x)/((a + b*x)^3*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] ((e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2*ExpIntegralEi[(-2*Log[e*((a
+ b*x)/(c + d*x))^n])/n])/((b*c - a*d)*n*(a + b*x)^2)
```

Maple [F]

$$\int \frac{dx + c}{(bx + a)^3 \ln \left(e \left(\frac{bx+a}{dx+c}\right)^n\right)} dx$$

[In] int((d*x+c)/(b*x+a)^3/ln(e*((b*x+a)/(d*x+c))^n),x)

[Out] int((d*x+c)/(b*x+a)^3/ln(e*((b*x+a)/(d*x+c))^n),x)

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

$$\int \frac{c + dx}{(a + bx)^3 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{e^{\frac{2}{n}} \operatorname{log\_integral}\left(\frac{d^2x^2+2\,cdx+c^2}{(b^2x^2+2\,abx+a^2)e^{\frac{2}{n}}}\right)}{(bc - ad)n}$$

[In] integrate((d*x+c)/(b*x+a)^3/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fricas
")

[Out] e^(2/n)*log_integral((d^2*x^2 + 2*c*d*x + c^2)/((b^2*x^2 + 2*a*b*x + a^2)*e
^(2/n)))/((b*c - a*d)*n)

Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx}{(a + bx)^3 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \text{Timed out}$$

[In] integrate((d*x+c)/(b*x+a)**3/ln(e*((b*x+a)/(d*x+c))**n),x)

[Out] Timed out

Maxima [F]

$$\int \frac{c + dx}{(a + bx)^3 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \int \frac{dx + c}{(bx + a)^3 \log \left(e \left(\frac{bx+a}{dx+c}\right)^n\right)} dx$$

[In] integrate((d*x+c)/(b*x+a)^3/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxima
")

[Out] integrate((d*x + c)/((b*x + a)^3*log(e*((b*x + a)/(d*x + c))^n)), x)

Giac [F]

$$\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{dx+c}{(bx+a)^3 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

```
[In] integrate((d*x+c)/(b*x+a)^3/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac")
[Out] integrate((d*x + c)/((b*x + a)^3*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{c+d\,x}{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,(a+b\,x)^3}\,dx$$

```
[In] int((c + d*x)/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)^3),x)
[Out] int((c + d*x)/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)^3), x)
```

3.248 $\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$

Optimal result

Integrand size = 35, antiderivative size = 75

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{3/n} (c+dx)^3 \operatorname{ExpIntegralEi}\left(-\frac{3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(a+bx)^3}$$

```
[Out] (e*((b*x+a)/(d*x+c))^n)^(3/n)*(d*x+c)^3*Ei(-3*ln(e*((b*x+a)/(d*x+c))^n)/n)/
(-a*d+b*c)/n/(b*x+a)^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2561, 2347, 2209}

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{(c+dx)^3 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{3/n} \operatorname{ExpIntegralEi}\left(-\frac{3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(a+bx)^3(bc-ad)}$$

```
[In] Int[(c + d*x)^2/((a + b*x)^4*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] ((e*((a + b*x)/(c + d*x))^n)^(3/n)*(c + d*x)^3*ExpIntegralEi[(-3*Log[e*((a
+ b*x)/(c + d*x))^n])/n])/((b*c - a*d)*n*(a + b*x)^3)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2561

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q, Subst[Int[x^m*((A + B*Lo
g[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Free
Q[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[b
*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^4\log(ex^n)}\,dx, x, \frac{a+bx}{c+dx}\right)}{bc-ad}\\ &= \frac{\left(\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{3/n}(c+dx)^3\right)\text{Subst}\left(\int \frac{e^{-\frac{3x}{n}}}{x}\,dx, x, \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)n(a+bx)^3}\\ &= \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{3/n}(c+dx)^3\text{Ei}\left(-\frac{3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(a+bx)^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^2}{(a+bx)^4\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{3/n}(c+dx)^3\,\text{ExpIntegralEi}\left(-\frac{3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(a+bx)^3}$$

```
[In] Integrate[(c + d*x)^2/((a + b*x)^4*Log[e*((a + b*x)/(c + d*x))^n]),x]
[Out] ((e*((a + b*x)/(c + d*x))^n)^(3/n)*(c + d*x)^3*ExpIntegralEi[(-3*Log[e*((a
+ b*x)/(c + d*x))^n])/n])/((b*c - a*d)*n*(a + b*x)^3)
```

Maple [F]

$$\int \frac{(dx+c)^2}{(bx+a)^4 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

[In] int((d*x+c)^2/(b*x+a)^4/ln(e*((b*x+a)/(d*x+c))^n),x)

[Out] int((d*x+c)^2/(b*x+a)^4/ln(e*((b*x+a)/(d*x+c))^n),x)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.17

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{e^{\frac{3}{n}}\,\mathrm{log\_integral}\left(\frac{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}{(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)e^{\frac{3}{n}}}\right)}{(bc-ad)n}$$

[In] integrate((d*x+c)^2/(b*x+a)^4/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fricas")

[Out] e^(3/n)*log_integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)/((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*e^(3/n)))/((b*c - a*d)*n)

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Timed out}$$

[In] integrate((d*x+c)**2/(b*x+a)**4/ln(e*((b*x+a)/(d*x+c))**n),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(dx+c)^2}{(bx+a)^4 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

[In] integrate((d*x+c)^2/(b*x+a)^4/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxima")

[Out] integrate((d*x + c)^2/((b*x + a)^4*log(e*((b*x + a)/(d*x + c))^n)), x)

Giac [F]

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(dx+c)^2}{(bx+a)^4 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

```
[In] integrate((d*x+c)^2/(b*x+a)^4/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac
")
[Out] integrate((d*x + c)^2/((b*x + a)^4*log(e*((b*x + a)/(d*x + c))^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(c+d\,x)^2}{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,(a+b\,x)^4}\,dx$$

```
[In] int((c + d*x)^2/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)^4),x)
[Out] int((c + d*x)^2/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)^4), x)
```

3.249 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^4}{(f+gx)(ah+bhx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 361

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^4}{(f+gx)(ah+bhx)}\,dx = -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^4\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{4Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\operatorname{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{12B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\operatorname{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{24B^3n^3(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(4,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{24B^4n^4\operatorname{PolyLog}\left(5,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}$$

```
[Out] -(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^4*ln(1-(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*
x+a))/(-a*g+b*f)/h+4*B*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3*polylog(2,(-a*
g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h+12*B^2*n^2*(A+B*ln(e*(b*x+a
)^n/((d*x+c)^n)))^2*polylog(3,(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+
b*f)/h+24*B^3*n^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(4,(-a*g+b*f)*(d
*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h+24*B^4*n^4*polylog(5,(-a*g+b*f)*(d*x
+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {2573, 2567, 12, 2379, 2421, 2430, 6724}

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^4}{(f+gx)(ah+bhx)}\,dx = \frac{24B^3n^3\,\text{PolyLog}\left(4,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{h(bf-ag)} + \frac{12B^2n^2\,\text{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{h(bf-ag)} + \frac{4Bn\,\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{h(bf-ag)} - \frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^4}{h(bf-ag)} + \frac{24B^4n^4\,\text{PolyLog}\left(5,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{h(bf-ag)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^4/((f + g*x)*(a*h + b*h*x)),x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^4*Log[1 - ((b*f - a*g)*(c + d*x
))/((d*f - c*g)*(a + b*x))])/((b*f - a*g)*h)) + (4*B*n*(A + B*Log[(e*(a + b
*x)^n)/(c + d*x)^n])^3*PolyLog[2, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a +
 b*x))])/((b*f - a*g)*h) + (12*B^2*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)
^n])^2*PolyLog[3, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])/((b*f -
 a*g)*h) + (24*B^3*n^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog[4,
((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])/((b*f - a*g)*h) + (24*B^4
*n^4*PolyLog[5, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])/((b*f - a
*g)*h)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2567

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[b*c - a*d, Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*(b*h - a*i - (
d*h - c*i)*x)^q*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, q] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^4}{(f+gx)(ah+bhx)}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left((-bc+ad)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^4}{(-bch+adh)x(bf-ag-(df-cg)x)}\,dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{(A+B\log(ex^n))^4}{x(bf-ag+(-df+cg)x)}\,dx, x, \frac{a+bx}{c+dx}\right)}{h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^4\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \text{Subst}\left(\frac{(4Bn)\text{Subst}\left(\int \frac{\log\left(1+\frac{bf-ag}{(-df+cg)x}\right)(A+B\log(ex^n))^3}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bf-ag)h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^4\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{4Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\,\text{Li}_2\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} - \text{Subst}\left(\frac{(12B^2n^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2\text{Li}_2\left(-\frac{bf-ag}{(-df+cg)x}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bf-ag)h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^4\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{4Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\operatorname{Li}_2\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{12B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\operatorname{Li}_3\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} - \operatorname{Subst}\left(\frac{(24B^3n^3)\operatorname{Subst}\left(\int\frac{(A+B\log(ex^n))\operatorname{Li}_3\left(-\frac{bf-ag}{(-df+cg)x}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bf-ag)h},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^4\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{4Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\operatorname{Li}_2\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{12B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\operatorname{Li}_3\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{24B^3n^3(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{Li}_4\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} - \operatorname{Subst}\left(\frac{(24B^4n^4)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_4\left(-\frac{bf-ag}{(-df+cg)x}\right)}{x}\,dx,x,\frac{a+bx}{c+dx}\right)}{(bf-ag)h},e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^4\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{4Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\operatorname{Li}_2\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{12B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\operatorname{Li}_3\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{24B^3n^3(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{Li}_4\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{24B^4n^4\operatorname{Li}_5\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}$$

Mathematica [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^4}{(f+gx)(ah+bhx)}\,dx = \int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^4}{(f+gx)(ah+bhx)}\,dx$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^4/((f + g*x)*(a*h + b*h*
x)),x]
[Out] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^4/((f + g*x)*(a*h + b*h*
x)), x]
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^n\,(dx+c)^{-n}\right)\right)^4}{(gx+f)\,(bhx+ah)}dx$$

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^4/(g*x+f)/(b*h*x+a*h),x)
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^4/(g*x+f)/(b*h*x+a*h),x)
```

Fricas [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^4}{(f+gx)(ah+bhx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^4}{(bhx+ah)(gx+f)}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^4/(g*x+f)/(b*h*x+a*h),x, algor
ithm="fricas")
[Out] integral((B^4*log((b*x + a)^n*e/(d*x + c)^n)^4 + 4*A*B^3*log((b*x + a)^n*e/
(d*x + c)^n)^3 + 6*A^2*B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 4*A^3*B*log((
b*x + a)^n*e/(d*x + c)^n) + A^4)/(b*g*h*x^2 + a*f*h + (b*f + a*g)*h*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^4}{(f+gx)(ah+bhx)}\,dx=\text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**4/(g*x+f)/(b*h*x+a*h),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^4}{(f+gx)(ah+bhx)}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^4}{(bhx+ah)(gx+f)}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^4/(g*x+f)/(b*h*x+a*h),x, algor
ithm="maxima")
```

```
[Out] A^4*(log(b*x + a)/((b*f - a*g)*h) - log(g*x + f)/((b*f - a*g)*h)) + integra
te((B^4*log((b*x + a)^n)^4 + B^4*log((d*x + c)^n)^4 + B^4*log(e)^4 + 4*A*B^
3*log(e)^3 + 6*A^2*B^2*log(e)^2 + 4*A^3*B*log(e) + 4*(B^4*log(e) + A*B^3)*l
og((b*x + a)^n)^3 - 4*(B^4*log((b*x + a)^n) + B^4*log(e) + A*B^3)*log((d*x
+ c)^n)^3 + 6*(B^4*log(e)^2 + 2*A*B^3*log(e) + A^2*B^2)*log((b*x + a)^n)^2
+ 6*(B^4*log((b*x + a)^n)^2 + B^4*log(e)^2 + 2*A*B^3*log(e) + A^2*B^2 + 2*(
B^4*log(e) + A*B^3)*log((b*x + a)^n))*log((d*x + c)^n)^2 + 4*(B^4*log(e)^3
+ 3*A*B^3*log(e)^2 + 3*A^2*B^2*log(e) + A^3*B)*log((b*x + a)^n) - 4*(B^4*lo
g((b*x + a)^n)^3 + B^4*log(e)^3 + 3*A*B^3*log(e)^2 + 3*A^2*B^2*log(e) + A^3
*B + 3*(B^4*log(e) + A*B^3)*log((b*x + a)^n)^2 + 3*(B^4*log(e)^2 + 2*A*B^3*
log(e) + A^2*B^2)*log((b*x + a)^n))*log((d*x + c)^n))/(b*g*h*x^2 + a*f*h +
(b*f*h + a*g*h)*x), x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^4}{(f+gx)(ah+bhx)}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^4}{(bhx+ah)(gx+f)}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^4/(g*x+f)/(b*h*x+a*h),x, algor
ithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^4/((b*h*x + a*h)*(g*x + f)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^4}{(f+gx)(ah+bhx)}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^4}{(f+g\,x)\,(a\,h+b\,h\,x)}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^4/((f + g*x)*(a*h + b*h*x)),x)
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^4/((f + g*x)*(a*h + b*h*x)), x
)
```

3.250 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(f+gx)(ah+bhx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 282

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(f+gx)(ah+bhx)}\,dx = -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{3Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\operatorname{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{6B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{6B^3n^3\operatorname{PolyLog}\left(4,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}$$

```
[Out] -(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3*ln(1-(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*
x+a))/(-a*g+b*f)/h+3*B*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*polylog(2,(-a*
g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h+6*B^2*n^2*(A+B*ln(e*(b*x+a)
^n/((d*x+c)^n)))*polylog(3,(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f
)/h+6*B^3*n^3*polylog(4,(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {2573, 2567, 12, 2379, 2421, 2430, 6724}

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(f+gx)(ah+bhx)}\,dx$$
$$=\frac{6B^2n^2\operatorname{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{h(bf-ag)}$$
$$+\frac{3Bn\operatorname{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{h(bf-ag)}$$
$$-\frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{h(bf-ag)}$$
$$+\frac{6B^3n^3\operatorname{PolyLog}\left(4,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{h(bf-ag)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/((f + g*x)*(a*h + b*h*x)),x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3*Log[1 - ((b*f - a*g)*(c + d*x
))/((d*f - c*g)*(a + b*x))])/((b*f - a*g)*h)) + (3*B*n*(A + B*Log[(e*(a + b
*x)^n)/(c + d*x)^n])^2*PolyLog[2, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a +
 b*x))])/((b*f - a*g)*h) + (6*B^2*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^
n])*PolyLog[3, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])/((b*f - a*
g)*h) + (6*B^3*n^3*PolyLog[4, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x
))])/((b*f - a*g)*h)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
```

```
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2567

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[b*c - a*d, Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*(b*h - a*i - (
d*h - c*i)*x)^q*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, q] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(f+gx)(ah+bhx)}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left((-bc+ad)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^3}{(-bch+adh)x(bf-ag-(df-cg)x)}\,dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{(A+B\log(ex^n))^3}{x(bf-ag+(-df+cg)x)}\,dx, x, \frac{a+bx}{c+dx}\right)}{h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \text{Subst}\left(\frac{(3Bn)\text{Subst}\left(\int \frac{\log\left(1+\frac{bf-ag}{(-df+cg)x}\right)(A+B\log(ex^n))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bf-ag)h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{3Bn\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\text{Li}_2\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} - \text{Subst}\left(\frac{(6B^2n^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))\text{Li}_2\left(-\frac{bf-ag}{(-df+cg)x}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bf-ag)h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{3Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\operatorname{Li}_2\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{6B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{Li}_3\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} - \operatorname{Subst}\left(\frac{(6B^3n^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(-\frac{bf-ag}{(-df+cg)x}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bf-ag)h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{3Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\operatorname{Li}_2\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{6B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{Li}_3\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{6B^3n^3\operatorname{Li}_4\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}$$

Mathematica [F]

$$\int\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(f+gx)(ah+bhx)}\,dx = \int\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(f+gx)(ah+bhx)}\,dx$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/((f + g*x)*(a*h + b*h*
x)),x]
```

```
[Out] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/((f + g*x)*(a*h + b*h*
x)), x]
```

Maple [F]

$$\int \frac{\left(A + B \ln \left(e(bx+a)^{n} (dx+c)^{-n}\right)\right)^{3}}{(gx+f)\,(bhx+ah)}dx$$

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(g*x+f)/(b*h*x+a*h),x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(g*x+f)/(b*h*x+a*h),x)
```

Fricas [F]

$$\int \frac{(A + B \log(e(a+bx)^{n}(c+dx)^{-n}))^{3}}{(f+gx)(ah+bhx)}\,dx = \int \frac{\left(B \log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right) + A\right)^{3}}{(bhx+ah)(gx+f)}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(g*x+f)/(b*h*x+a*h),x, algor
ithm="fricas")
```

```
[Out] integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e/
(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(b*g*h*x^2 +
 a*f*h + (b*f + a*g)*h*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B \log(e(a+bx)^{n}(c+dx)^{-n}))^{3}}{(f+gx)(ah+bhx)}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(g*x+f)/(b*h*x+a*h),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A + B \log(e(a+bx)^{n}(c+dx)^{-n}))^{3}}{(f+gx)(ah+bhx)}\,dx = \int \frac{\left(B \log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right) + A\right)^{3}}{(bhx+ah)(gx+f)}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(g*x+f)/(b*h*x+a*h),x, algor
ithm="maxima")
```

```
[Out] A^3*(log(b*x + a)/((b*f - a*g)*h) - log(g*x + f)/((b*f - a*g)*h)) - integra
te(-(B^3*log((b*x + a)^n)^3 - B^3*log((d*x + c)^n)^3 + B^3*log(e)^3 + 3*A*B
^2*log(e)^2 + 3*A^2*B*log(e) + 3*(B^3*log(e) + A*B^2)*log((b*x + a)^n)^2 +
```

```
3*(B^3*log((b*x + a)^n) + B^3*log(e) + A*B^2)*log((d*x + c)^n)^2 + 3*(B^3*l
og(e)^2 + 2*A*B^2*log(e) + A^2*B)*log((b*x + a)^n) - 3*(B^3*log((b*x + a)^n
)^2 + B^3*log(e)^2 + 2*A*B^2*log(e) + A^2*B + 2*(B^3*log(e) + A*B^2)*log((b
*x + a)^n))*log((d*x + c)^n))/(b*g*h*x^2 + a*f*h + (b*f*h + a*g*h)*x), x)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(f+gx)(ah+bhx)}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(bhx+ah)(gx+f)}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(g*x+f)/(b*h*x+a*h),x, algor
ithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/((b*h*x + a*h)*(g*x + f)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(f+gx)(ah+bhx)}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3}{(f+g\,x)\,(a\,h+b\,h\,x)}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/((f + g*x)*(a*h + b*h*x)),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/((f + g*x)*(a*h + b*h*x)), x
)
```

3.251 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(f+gx)(ah+bhx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 203

$$
\begin{aligned}
\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(f+gx)(ah+bhx)}\,dx
&= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} \\
&\quad+\frac{2Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} \\
&\quad+\frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}
\end{aligned}
$$

```
[Out] -(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*ln(1-(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*
x+a))/(-a*g+b*f)/h+2*B*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,(-a*g+
b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h+2*B^2*n^2*polylog(3,(-a*g+b*f
)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used

$= \{2573, 2567, 12, 2379, 2421, 6724\}$

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(f+gx)(ah+bhx)}\,dx$$
$$= \frac{2Bn\,\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{h(bf-ag)}$$
$$-\frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{h(bf-ag)}$$
$$+\frac{2B^2n^2\,\text{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{h(bf-ag)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/((f + g*x)*(a*h + b*h*x)),x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2*Log[1 - ((b*f - a*g)*(c + d*x
))/((d*f - c*g)*(a + b*x))])/((b*f - a*g)*h)) + (2*B*n*(A + B*Log[(e*(a + b
*x)^n)/(c + d*x)^n])*PolyLog[2, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b
*x))])/((b*f - a*g)*h) + (2*B^2*n^2*PolyLog[3, ((b*f - a*g)*(c + d*x))/((d*
f - c*g)*(a + b*x))])/((b*f - a*g)*h)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2567

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[b*c - a*d, Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*(b*h - a*i - (
d*h - c*i)*x)^q*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b
```

```
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, q] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)(ah+bhx)}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left((-bc+ad)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(-bch+adh)x(bf-ag-(df-cg)x)}\,dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x(bf-ag+(-df+cg)x)}\,dx, x, \frac{a+bx}{c+dx}\right)}{h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \text{Subst}\left(\frac{(2Bn)\text{Subst}\left(\int \frac{\log\left(1+\frac{bf-ag}{(-df+cg)x}\right)(A+B\log(ex^n))}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bf-ag)h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{2Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{Li}_2\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} - \operatorname{Subst}\left(\frac{(2B^2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{bf-ag}{(-df+cg)x}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bf-ag)h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{2Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{Li}_2\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{2B^2n^2\operatorname{Li}_3\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1415 vs. $2(203) = 406$.

Time = 0.78 (sec) , antiderivative size = 1415, normalized size of antiderivative = 6.97

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(f+gx)(ah+bhx)}\,dx$$
$$= \frac{3\log(a+bx)\left(A+B(-n\log(a+bx)+n\log(c+dx)+\log(e(a+bx)^n(c+dx)^{-n}))\right)^2 - 3(A+B(-n\log(a}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/((f + g*x)*(a*h + b*h*
x)),x]
```

```
[Out] (3*Log[a + b*x]*(A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*
x)^n)/(c + d*x)^n]))^2 - 3*(A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log
[(e*(a + b*x)^n)/(c + d*x)^n]))^2*Log[f + g*x] + 3*B*n*(A + B*(-(n*Log[a +
b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n]))*(Log[a + b*x]^2
 - 2*(Log[a + b*x]*Log[(b*(f + g*x))/(b*f - a*g)] + PolyLog[2, (g*(a + b*x)
)/(-(b*f) + a*g)])) - 6*A*B*n*(Log[c + d*x]*(Log[(d*(a + b*x))/(-(b*c) + a*
d)] - Log[(d*(f + g*x))/(d*f - c*g)]) + PolyLog[2, (b*(c + d*x))/(b*c - a*d
```

)] - PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)]) + 6*B^2*n*(n*Log[a + b*x] - n*Log[c + d*x] - Log[(e*(a + b*x)^n)/(c + d*x)^n])*(Log[c + d*x]*(Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[(d*(f + g*x))/(d*f - c*g)]) + PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)]) + B^2*n^2*(Log[a + b*x]^2*(Log[a + b*x] - 3*Log[(b*(f + g*x))/(b*f - a*g)]) - 6*Log[a + b*x]*PolyLog[2, (g*(a + b*x))/(-(b*f) + a*g)] + 6*PolyLog[3, (g*(a + b*x))/(-(b*f) + a*g)]) + 3*B^2*n^2*(Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x]^2 - Log[c + d*x]^2*Log[(d*(f + g*x))/(d*f - c*g)] + 2*Log[c + d*x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 2*Log[c + d*x]*PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)] - 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] + 2*PolyLog[3, (g*(c + d*x))/(-(d*f) + c*g)]) - 6*B^2*n^2*((Log[a + b*x]^2*(Log[c + d*x] - Log[(b*(c + d*x))/(b*c - a*d)]))/2 - Log[a + b*x]*Log[c + d*x]*Log[(b*(f + g*x))/(b*f - a*g)] - (Log[(g*(c + d*x))/(-(d*f) + c*g)]*(-2*Log[a + b*x] + Log[(g*(c + d*x))/(-(d*f) + c*g)])*(Log[(b*(f + g*x))/(b*f - a*g)] - Log[(d*(f + g*x))/(d*f - c*g)]))/2 + Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]*(Log[(b*(f + g*x))/(b*f - a*g)] - Log[(d*(f + g*x))/(d*f - c*g)]) - (Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]^2*(Log[(-(b*c) + a*d)/(d*(a + b*x))] + Log[(b*(f + g*x))/(b*f - a*g)] - Log[((-(b*c) + a*d)*(f + g*x))/((d*f - c*g)*(a + b*x))]))/2 - Log[a + b*x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - (Log[c + d*x] - Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])*PolyLog[2, (g*(a + b*x))/(-(b*f) + a*g)] - (Log[a + b*x] + Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])*PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)] - Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]*(PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] - PolyLog[2, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]) + PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[3, (g*(a + b*x))/(-(b*f) + a*g)] + PolyLog[3, (g*(c + d*x))/(-(d*f) + c*g)] + PolyLog[3, (b*(c + d*x))/(d*(a + b*x))] - PolyLog[3, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]))/(3*(b*f - a*g)*h)

Maple [F]

$$\int \frac{\left(A + B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{2}}{\left(gx+f\right)\left(bhx+ah\right)}dx$$

[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(g*x+f)/(b*h*x+a*h),x)

[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(g*x+f)/(b*h*x+a*h),x)

Fricas [F]

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(f+gx)(ah+bhx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{(bhx+ah)(gx+f)}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(g*x+f)/(b*h*x+a*h),x, algor
ithm="fricas")
```

```
[Out] integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(d
*x + c)^n) + A^2)/(b*g*h*x^2 + a*f*h + (b*f + a*g)*h*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(f+gx)(ah+bhx)}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(g*x+f)/(b*h*x+a*h),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(f+gx)(ah+bhx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{(bhx+ah)(gx+f)}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(g*x+f)/(b*h*x+a*h),x, algor
ithm="maxima")
```

```
[Out] A^2*(log(b*x + a)/((b*f - a*g)*h) - log(g*x + f)/((b*f - a*g)*h)) + integra
te((B^2*log((b*x + a)^n)^2 + B^2*log((d*x + c)^n)^2 + B^2*log(e)^2 + 2*A*B*
log(e) + 2*(B^2*log(e) + A*B)*log((b*x + a)^n) - 2*(B^2*log((b*x + a)^n) +
B^2*log(e) + A*B)*log((d*x + c)^n))/(b*g*h*x^2 + a*f*h + (b*f*h + a*g*h)*x)
, x)
```

Giac [F]

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^2}{(f + gx)(ah + bhx)}\, dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{(bhx + ah)(gx + f)}\, dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(g*x+f)/(b*h*x+a*h),x, algor
ithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/((b*h*x + a*h)*(g*x + f)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^2}{(f + gx)(ah + bhx)}\, dx = \int \frac{\left(A + B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2}{(f + g\,x)\,(a\,h + b\,h\,x)}\, dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/((f + g*x)*(a*h + b*h*x)),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/((f + g*x)*(a*h + b*h*x)), x
)
```

3.252 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 123

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx = -\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{Bn\,\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}$$

```
[Out] -(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+
a))/(-a*g+b*f)/h+B*n*polylog(2,(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g
+b*f)/h
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {2573, 2567, 12, 2379, 2438}

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx = \frac{Bn\,\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{h(bf-ag)} - \frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{h(bf-ag)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/((f + g*x)*(a*h + b*h*x)),x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*Log[1 - ((b*f - a*g)*(c + d*x))
/((d*f - c*g)*(a + b*x))])/((b*f - a*g)*h)) + (B*n*PolyLog[2, ((b*f - a*g)*
(c + d*x))/((d*f - c*g)*(a + b*x))])/((b*f - a*g)*h)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2567

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[b*c - a*d, Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*(b*h - a*i - (
d*h - c*i)*x)^q*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b
*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && NeQ
[b*c - a*d, 0] && IntegersQ[m, q] && IGtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)(ah+bhx)}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\Bigg((-bc + ad)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(-bch+adh)x(bf-ag-(df-cg)x)}\,dx, x, \frac{a+bx}{c+dx}\right), e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\Bigg)$$

$$= \text{Subst}\left(\frac{\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x(bf-ag+(-df+cg)x)}\,dx, x, \frac{a+bx}{c+dx}\right)}{h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \text{Subst}\left(\frac{(Bn)\text{Subst}\left(\int \frac{\log\left(1+\frac{bf-ag}{(-df+cg)x}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bf-ag)h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{Bn\text{Li}_2\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 304 vs. $2(123) = 246$.

Time = 0.26 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.47

$$\int \frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{(f+gx)(ah+bhx)}\,dx = -\frac{-2A\log(a+bx)+Bn\log^2(a+bx)-2Bn\log(a+bx)\log(c+dx)+2Bn\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log(c+dx)-2I}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/((f + g*x)*(a*h + b*h*x)
),x]
```

```
[Out] -1/2*(-2*A*Log[a + b*x] + B*n*Log[a + b*x]^2 - 2*B*n*Log[a + b*x]*Log[c + d
*x] + 2*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 2*B*Log[a + b*
x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*A*Log[f + g*x] - 2*B*n*Log[a + b*x]
*Log[f + g*x] + 2*B*n*Log[c + d*x]*Log[f + g*x] + 2*B*Log[(e*(a + b*x)^n)/(
c + d*x)^n]*Log[f + g*x] + 2*B*n*Log[a + b*x]*Log[(b*(f + g*x))/(b*f - a*g)
```

```
] - 2*B*n*Log[c + d*x]*Log[(d*(f + g*x))/(d*f - c*g)] + 2*B*n*PolyLog[2, (g
*(a + b*x))/(-(b*f) + a*g)] + 2*B*n*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] -
 2*B*n*PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)])/((b*f - a*g)*h)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 22.58 (sec) , antiderivative size = 1447, normalized size of antiderivative = 11.76

method	result	size
risch	Expression too large to display	1447

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(g*x+f)/(b*h*x+a*h),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn
(I*(b*x+a)^n/((d*x+c)^n))-1/2*I/h/(a*g-b*f)*ln(g*x+f)*B*Pi*csgn(I*e)*csgn(I
*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I/h/(a*g-b*f)*l
n(g*x+f)*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+
c)^n))+1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)
^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/h/(a*g-b*f)*ln(b*x+a)*A+1/h/(a*g-b*f
)*ln(g*x+f)*A+1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n)
)^3+1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/2*
I/h/(a*g-b*f)*ln(g*x+f)*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/2*I/h/(a*g-b
*f)*ln(g*x+f)*B*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/2*I/h/(a*g-b*f)*ln(b
*x+a)*B*Pi*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/2*I/h/(a*g-b*f)*ln
(b*x+a)*B*Pi*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-1/2*I/h/(a*g
-b*f)*ln(b*x+a)*B*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/
2*I/h/(a*g-b*f)*ln(g*x+f)*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x
+c)^n)*(b*x+a)^n)^2-1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*(b*x+a)^n/((d*x
+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I/h/(a*g-b*f)*ln(g*x+f)*B*Pi*
csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I/h/(a*g-b*f)*ln(g*x+f)*B*P
i*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I/h/(a*g-b*f)*ln(g*
x+f)*B*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-1/h*B*n/(a*g-
b*f)*ln(g*x+f)*ln((b*(g*x+f)+a*g-b*f)/(a*g-b*f))+1/h*B*n/(a*g-b*f)*ln(g*x+f
)*ln((d*(g*x+f)+c*g-d*f)/(c*g-d*f))-1/h*B*n/(a*g-b*f)*ln(b*x+a)*ln((-a*d+c*
b+d*(b*x+a))/(-a*d+b*c))-1/h/(a*g-b*f)*ln(b*x+a)*B*ln(e)+1/h/(a*g-b*f)*ln(g
*x+f)*B*ln(e)-1/h*B*ln((b*x+a)^n)/(a*g-b*f)*ln(b*x+a)+1/h*B/(a*g-b*f)*ln(g*
x+f)*ln((b*x+a)^n)-1/h*B*n/(a*g-b*f)*dilog((b*(g*x+f)+a*g-b*f)/(a*g-b*f))+1
/2/h*B*n/(a*g-b*f)*ln(b*x+a)^2+1/h*B*ln((d*x+c)^n)/(a*g-b*f)*ln(b*x+a)-1/h*
B/(a*g-b*f)*ln(g*x+f)*ln((d*x+c)^n)+1/h*B*n/(a*g-b*f)*dilog((d*(g*x+f)+c*g-
d*f)/(c*g-d*f))-1/h*B*n/(a*g-b*f)*dilog((-a*d+c*b+d*(b*x+a))/(-a*d+b*c))
```

Fricas [F]

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx = \int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A}{(bhx+ah)(gx+f)}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(g*x+f)/(b*h*x+a*h),x, algorit
hm="fricas")
```

```
[Out] integral((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(b*g*h*x^2 + a*f*h + (b*f +
 a*g)*h*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(g*x+f)/(b*h*x+a*h),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx = \int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A}{(bhx+ah)(gx+f)}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(g*x+f)/(b*h*x+a*h),x, algorit
hm="maxima")
```

```
[Out] A*(log(b*x + a)/((b*f - a*g)*h) - log(g*x + f)/((b*f - a*g)*h)) - B*integra
te(-(log((b*x + a)^n) - log((d*x + c)^n) + log(e))/(b*g*h*x^2 + a*f*h + (b*
f*h + a*g*h)*x), x)
```

Giac [F]

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx = \int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{(bhx+ah)(gx+f)}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(g*x+f)/(b*h*x+a*h),x, algorit
hm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/((b*h*x + a*h)*(g*x + f)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx = \int \frac{A + B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{(f+g\,x)\,(a\,h+b\,h\,x)}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/((f + g*x)*(a*h + b*h*x)),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/((f + g*x)*(a*h + b*h*x)), x)
```

3.253 $\int \frac{1}{(f+gx)(ah+bhx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 43

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \text{Subst}\left(\text{Int}\left(\frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right),e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

```
[Out] _eval(Unintegrable(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
,e*((b*x+a)/(d*x+c))^n = e*(b*x+a)^n/((d*x+c)^n))
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$

```
[In] Int[1/((f + g*x)*(a*h + b*h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
```

```
[Out] Defer[Subst][Defer[Int][1/((f + g*x)*(a*h + b*h*x)*(A + B*Log[e*((a + b*x)/
(c + d*x))^n])), x], e*((a + b*x)/(c + d*x))^n, (e*(a + b*x)^n)/(c + d*x)^n
]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$

```
[In] Integrate[1/((f + g*x)*(a*h + b*h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n
])),x]
[Out] Integrate[1/((f + g*x)*(a*h + b*h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n
])), x]
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)(bhx+ah)\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)}dx$$

```
[In] int(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
[Out] int(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.77

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(bhx+ah)(gx+f)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algor
ithm="fricas")
```

```
[Out] integral(1/(A*b*g*h*x^2 + A*a*f*h + (A*b*f + A*a*g)*h*x + (B*b*g*h*x^2 + B*
a*f*h + (B*b*f + B*a*g)*h*x)*log((b*x + a)^n*e/(d*x + c)^n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(bhx+ah)(gx+f)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algor
ithm="maxima")
```

```
[Out] integrate(1/((b*h*x + a*h)*(g*x + f)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)
), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(bhx+ah)(gx+f)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

```
[In] integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algor
ithm="giac")
```

```
[Out] integrate(1/((b*h*x + a*h)*(g*x + f)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)
), x)
```

Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(f+g\,x)\,\left(a\,h+b\,h\,x\right)\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)}\,dx$$

```
[In] int(1/((f + g*x)*(a*h + b*h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))),x)
[Out] int(1/((f + g*x)*(a*h + b*h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))), x
)
```

3.254 $\int \frac{1}{(f+gx)(ah+bhx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 43

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \text{Subst}\left(\text{Int}\left(\frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right),e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

```
[Out] _eval(Unintegrable(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,
x),e*((b*x+a)/(d*x+c))^n = e*(b*x+a)^n/((d*x+c)^n))
```

Rubi [N/A]

Not integrable

Time = 0.16 (sec), antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx$$

```
[In] Int[1/((f + g*x)*(a*h + b*h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2),
x]
```

```
[Out] Defer[Subst][Defer[Int][1/((f + g*x)*(a*h + b*h*x)*(A + B*Log[e*((a + b*x)/
(c + d*x))^n])^2), x], e*((a + b*x)/(c + d*x))^n, (e*(a + b*x)^n)/(c + d*x)
^n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx$$

```
[In] Integrate[1/((f + g*x)*(a*h + b*h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n
])^2),x]
```

```
[Out] Integrate[1/((f + g*x)*(a*h + b*h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n
])^2), x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)(bhx+ah)\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)^2}dx$$

```
[In] int(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

```
[Out] int(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.37

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \int \frac{1}{(bhx+ah)(gx+f)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, alg
orithm="fricas")
```

```
[Out] integral(1/(A^2*b*g*h*x^2 + A^2*a*f*h + (A^2*b*f + A^2*a*g)*h*x + (B^2*b*g*
h*x^2 + B^2*a*f*h + (B^2*b*f + B^2*a*g)*h*x)*log((b*x + a)^n*e/(d*x + c)^n)
^2 + 2*(A*B*b*g*h*x^2 + A*B*a*f*h + (A*B*b*f + A*B*a*g)*h*x)*log((b*x + a)^
n*e/(d*x + c)^n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

```
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 506, normalized size of antiderivative = 11.77

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \int \frac{1}{(bhx+ah)(gx+f)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, alg
orithm="maxima")
```

```
[Out] (d*f - c*g)*integrate(1/((b*c*f^2*h*n - a*d*f^2*h*n)*A*B + (b*c*f^2*h*n*log
(e) - a*d*f^2*h*n*log(e))*B^2 + ((b*c*g^2*h*n - a*d*g^2*h*n)*A*B + (b*c*g^2
*h*n*log(e) - a*d*g^2*h*n*log(e))*B^2)*x^2 + 2*((b*c*f*g*h*n - a*d*f*g*h*n)
*A*B + (b*c*f*g*h*n*log(e) - a*d*f*g*h*n*log(e))*B^2)*x + ((b*c*g^2*h*n - a
*d*g^2*h*n)*B^2*x^2 + 2*(b*c*f*g*h*n - a*d*f*g*h*n)*B^2*x + (b*c*f^2*h*n -
a*d*f^2*h*n)*B^2)*log((b*x + a)^n) - ((b*c*g^2*h*n - a*d*g^2*h*n)*B^2*x^2 +
 2*(b*c*f*g*h*n - a*d*f*g*h*n)*B^2*x + (b*c*f^2*h*n - a*d*f^2*h*n)*B^2)*log
((d*x + c)^n)), x) - (d*x + c)/((b*c*f*h*n - a*d*f*h*n)*A*B + (b*c*f*h*n*lo
g(e) - a*d*f*h*n*log(e))*B^2 + ((b*c*g*h*n - a*d*g*h*n)*A*B + (b*c*g*h*n*lo
g(e) - a*d*g*h*n*log(e))*B^2)*x + ((b*c*g*h*n - a*d*g*h*n)*B^2*x + (b*c*f*h
*n - a*d*f*h*n)*B^2)*log((b*x + a)^n) - ((b*c*g*h*n - a*d*g*h*n)*B^2*x + (b
*c*f*h*n - a*d*f*h*n)*B^2)*log((d*x + c)^n))
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \int \frac{1}{(bhx+ah)(gx+f)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, alg
orithm="giac")
```

```
[Out] integrate(1/((b*h*x + a*h)*(g*x + f)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)
^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \int \frac{1}{(f+g\,x)\,\left(a\,h+b\,h\,x\right)\,\left(A+B\,\ln\left(\frac{e\,{(a+b\,x)}^n}{{(c+d\,x)}^n}\right)\right)^2}\,dx$$

```
[In] int(1/((f + g*x)*(a*h + b*h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2),
x)
```

```
[Out] int(1/((f + g*x)*(a*h + b*h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2),
 x)
```

3.255 $\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 33

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx = -\frac{\text{PolyLog}\left(2,1-\frac{c+dx}{a+bx}\right)}{(bc-ad)h}$$

```
[Out] -polylog(2,1+(-d*x-c)/(b*x+a))/(-a*d+b*c)/h
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2565, 2352}

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx = -\frac{\text{PolyLog}\left(2,1-\frac{c+dx}{a+bx}\right)}{h(bc-ad)}$$

```
[In] Int[Log[(c + d*x)/(a + b*x)]/((a + b*x)*((a - c)*h + (b - d)*h*x)),x]
[Out] -(PolyLog[2, 1 - (c + d*x)/(a + b*x)]/((b*c - a*d)*h))
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2565

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Dist[(b*c - a*d)^(q + 1)*(i/d)^q, Subst[Int[(b*f - a*g - (d*f - c*g)*x
```

```
)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x
)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && NeQ[b*c - a*d,
0] && IntegersQ[m, q] && IGtQ[p, 0] && EqQ[d*h - c*i, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{\log(x)}{-c(b-d)h+(a-c)dh+(-b(a-c)h+a(b-d)h)x}\,dx, x, \frac{c+dx}{a+bx}\right)\\ &= -\frac{\text{Li}_2\left(1-\frac{c+dx}{a+bx}\right)}{(bc-ad)h}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 298 vs. 2(33) = 66.

Time = 0.13 (sec) , antiderivative size = 298, normalized size of antiderivative = 9.03

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx = \frac{-\log^2\left(\frac{-bc+ad}{d(a+bx)}\right)+2\log\left(\frac{(b-d)(a+bx)}{bc-ad}\right)\log(a-c+bx-dx)-2\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)-2\log(a-c+}{}$$

```
[In] Integrate[Log[(c + d*x)/(a + b*x)]/((a + b*x)*((a - c)*h + (b - d)*h*x)),x]
```

```
[Out] (-Log[(-(b*c) + a*d)/(d*(a + b*x))]^2 + 2*Log[((b - d)*(a + b*x))/(b*c - a*
d)]*Log[a - c + b*x - d*x] - 2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(b*(c
+ d*x))/(b*c - a*d)] - 2*Log[a - c + b*x - d*x]*Log[((b - d)*(c + d*x))/(b*
c - a*d)] + 2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(c + d*x)/(a + b*x)] +
2*Log[a - c + b*x - d*x]*Log[(c + d*x)/(a + b*x)] + 2*PolyLog[2, (d*(a + b*
x))/(-(b*c) + a*d)] + 2*PolyLog[2, -((b*(a - c + b*x - d*x))/(b*c - a*d))]
- 2*PolyLog[2, -((d*(-a + c - b*x + d*x))/(-(b*c) + a*d))])/((2*b*c - 2*a*d
)*h)
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{\operatorname{dilog}\left(-\frac{ad-cb}{b(bx+a)}+\frac{d}{b}\right)}{h(ad-cb)}$
default	$\frac{\operatorname{dilog}\left(-\frac{ad-cb}{b(bx+a)}+\frac{d}{b}\right)}{h(ad-cb)}$
risch	$\frac{\operatorname{dilog}\left(-\frac{ad-cb}{b(bx+a)}+\frac{d}{b}\right)}{h(ad-cb)}$
parts	$-\frac{\ln\left(\frac{dx+c}{bx+a}\right)\ln(bx-dx+a-c)b}{h(ad-cb)(b-d)}+\frac{\ln\left(\frac{dx+c}{bx+a}\right)\ln(bx-dx+a-c)d}{h(ad-cb)(b-d)}+\frac{\ln\left(\frac{dx+c}{bx+a}\right)\ln(bx+a)}{h(ad-cb)}-\frac{-\frac{\ln(bx+a)^2}{2(ad-cb)}+\frac{\operatorname{dilog}\left(\frac{-ad+cb+}{-ad}\right.}{ad-cb}}{}$

```
[In] int(ln((d*x+c)/(b*x+a))/(b*x+a)/((a-c)*h+(b-d)*h*x),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/h/(a*d-b*c)*dilog(-(a*d-b*c)/b/(b*x+a)+d/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx = -\frac{\operatorname{Li}_2\left(-\frac{dx+c}{bx+a}+1\right)}{(bc-ad)h}$$

```
[In] integrate(log((d*x+c)/(b*x+a))/(b*x+a)/((a-c)*h+(b-d)*h*x),x, algorithm="fr
icas")
```

```
[Out] -dilog(-(d*x + c)/(b*x + a) + 1)/((b*c - a*d)*h)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx = \text{Timed out}$$

```
[In] integrate(ln((d*x+c)/(b*x+a))/(b*x+a)/((a-c)*h+(b-d)*h*x),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(32) = 64$.

Time = 0.20 (sec) , antiderivative size = 357, normalized size of antiderivative = 10.82

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx = \left(\frac{\log\left(-(b-d)x-a+c\right)}{(bc-ad)h} - \frac{\log\left(bx+a\right)}{(bc-ad)h}\right)\log\left(\frac{dx+c}{bx+a}\right) + \frac{2\log\left(-(b-d)x-a+c\right)\log\left(bx+a\right)-\log\left(bx+a\right)^2}{2\,(bch-adh)} + \frac{\log\left(bx+a\right)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)}{bch-adh} - \frac{\log\left(bx+a\right)\log\left(-\frac{a(b-d)+(b^2-bd)x}{bc-ad}+1\right)+\mathrm{Li}_2\left(\frac{a(b-d)+(b^2-bd)x}{bc-ad}\right)}{bch-adh} - \frac{\log\left(-(b-d)x-a+c\right)\log\left(\frac{ad-cd+(bd-d^2)x}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{ad-cd+(bd-d^2)x}{bc-ad}\right)}{bch-adh}$$

```
[In] integrate(log((d*x+c)/(b*x+a))/(b*x+a)/((a-c)*h+(b-d)*h*x),x, algorithm="ma
xima")
```

```
[Out] (log(-(b - d)*x - a + c)/((b*c - a*d)*h) - log(b*x + a)/((b*c - a*d)*h))*lo
g((d*x + c)/(b*x + a)) + 1/2*(2*log(-(b - d)*x - a + c)*log(b*x + a) - log(
b*x + a)^2)/(b*c*h - a*d*h) + (log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) +
 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*c*h - a*d*h) - (log(b*x + a)*lo
g(-(a*(b - d) + (b^2 - b*d)*x)/(b*c - a*d) + 1) + dilog((a*(b - d) + (b^2 -
 b*d)*x)/(b*c - a*d)))/(b*c*h - a*d*h) - (log(-(b - d)*x - a + c)*log((a*d
- c*d + (b*d - d^2)*x)/(b*c - a*d) + 1) + dilog(-(a*d - c*d + (b*d - d^2)*x
)/(b*c - a*d)))/(b*c*h - a*d*h)
```

Giac [F]

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx = \int \frac{\log\left(\frac{dx+c}{bx+a}\right)}{((b-d)hx+(a-c)h)(bx+a)}\,dx$$

```
[In] integrate(log((d*x+c)/(b*x+a))/(b*x+a)/((a-c)*h+(b-d)*h*x),x, algorithm="gi
ac")
```

```
[Out] integrate(log((d*x + c)/(b*x + a))/(((b - d)*h*x + (a - c)*h)*(b*x + a)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx=\int \frac{\ln \left(\frac{c+d\,x}{a+b\,x}\right)}{\left(h\,\left(a-c\right)+h\,x\,\left(b-d\right)\right)\,\left(a+b\,x\right)} \,d x$$

```
[In] int(log((c + d*x)/(a + b*x))/((h*(a - c) + h*x*(b - d))*(a + b*x)),x)
[Out] int(log((c + d*x)/(a + b*x))/((h*(a - c) + h*x*(b - d))*(a + b*x)), x)
```

3.256 $\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 27

$$\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx=\frac{\text{PolyLog}\left(2,\frac{g(c+dx)}{a+bx}\right)}{bc-ad}$$

```
[Out] polylog(2,g*(d*x+c)/(b*x+a))/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2565, 2352}

$$\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx=\frac{\text{PolyLog}\left(2,\frac{g(c+dx)}{a+bx}\right)}{bc-ad}$$

```
[In] Int[Log[(a - c*g + (b - d*g)*x)/(a + b*x)]/((a + b*x)*(c + d*x)),x]
[Out] PolyLog[2, (g*(c + d*x))/(a + b*x)]/(b*c - a*d)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2565

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
```

```
] :> Dist[(b*c - a*d)^(q + 1)*(i/d)^q, Subst[Int[(b*f - a*g - (d*f - c*g)*x
)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x
)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && NeQ[b*c - a*d,
0] && IntegersQ[m, q] && IGtQ[p, 0] && EqQ[d*h - c*i, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\log(x)}{-d(a-cg)+c(b-dg)+(-bc+ad)x}\,dx, x, \frac{a-cg+(b-dg)x}{a+bx}\right)$$
$$= \frac{\text{Li}_2\left(\frac{g(c+dx)}{a+bx}\right)}{bc-ad}$$

Mathematica [B] (verified)

Leave count is larger than twice the leaf count of optimal. 375 vs. 2(27) = 54.

Time = 0.22 (sec) , antiderivative size = 375, normalized size of antiderivative = 13.89

$$\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$$
$$= \frac{-\log^2\left(\frac{a}{b}+x\right)+2\log\left(\frac{a}{b}+x\right)\log(a+bx)-2\log\left(\frac{a-cg}{b-dg}+x\right)\log(a+bx)+2\log\left(\frac{a-cg}{b-dg}+x\right)\log\left(\frac{(b-dg)(a+\cdots}{(bc-ad)g}\right.}{}$$

```
[In] Integrate[Log[(a - c*g + (b - d*g)*x)/(a + b*x)]/((a + b*x)*(c + d*x)),x]
```

```
[Out] (-Log[a/b + x]^2 + 2*Log[a/b + x]*Log[a + b*x] - 2*Log[(a - c*g)/(b - d*g)
+ x]*Log[a + b*x] + 2*Log[(a - c*g)/(b - d*g) + x]*Log[((b - d*g)*(a + b*x)
)/((b*c - a*d)*g)] - 2*Log[a/b + x]*Log[c + d*x] + 2*Log[(a - c*g)/(b - d*g
) + x]*Log[c + d*x] + 2*Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] - 2*Log
[(a - c*g)/(b - d*g) + x]*Log[((b - d*g)*(c + d*x))/(b*c - a*d)] + 2*Log[a
+ b*x]*Log[(a - c*g + b*x - d*g*x)/(a + b*x)] - 2*Log[c + d*x]*Log[(a - c*g
 + b*x - d*g*x)/(a + b*x)] + 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2
*PolyLog[2, -((b*(a - c*g + b*x - d*g*x))/((b*c - a*d)*g))] - 2*PolyLog[2,
-((d*(-a + c*g - b*x + d*g*x))/(-(b*c) + a*d))])/(2*b*c - 2*a*d)
```

Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

method	result
derivativedivides	$-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{ad-cb}$
default	$-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{ad-cb}$
risch	$-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{ad-cb}$
parts	$\frac{\ln\left(\frac{a-cg+(-dg+b)x}{bx+a}\right)\ln(dx+c)}{ad-cb}-\frac{\ln\left(\frac{a-cg+(-dg+b)x}{bx+a}\right)\ln(bx+a)}{ad-cb}-\frac{g\left(\frac{b\ln(bx+a)^2}{2g(ad-cb)}-\frac{b(-dg+b)\left(\frac{\operatorname{dilog}\left(\frac{(-dg+b)(bx+a)+}{adg-bcg}\right.}{-dg+b}\right.}{}\right.}{}$

```
[In] int(ln((a-c*g+(-d*g+b)*x)/(b*x+a))/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
[Out] -1/(a*d-b*c)*dilog((a*d*g-b*c*g)/b/(b*x+a)+(-d*g+b)/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx=\frac{\operatorname{Li}_2\left(\frac{cg+(dg-b)x-a}{bx+a}+1\right)}{bc-ad}$$

```
[In] integrate(log((a-c*g+(-d*g+b)*x)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="fri
cas")
[Out] dilog((c*g + (d*g - b)*x - a)/(b*x + a) + 1)/(b*c - a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx=\text{Timed out}$$

```
[In] integrate(ln((a-c*g+(-d*g+b)*x)/(b*x+a))/(b*x+a)/(d*x+c),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(26) = 52$.

Time = 0.20 (sec) , antiderivative size = 344, normalized size of antiderivative = 12.74

$$\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \left(\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}\right)\log\left(-\frac{cg+(dg-b)x-a}{bx+a}\right) + \frac{\log(bx+a)^2-2\log(bx+a)\log(dx+c)}{2(bc-ad)} - \frac{\log(bx+a)\log\left(\frac{(dg-b)a+(bdg-b^2)x}{bcg-adg}+1\right)+\mathrm{Li}_2\left(-\frac{(dg-b)a+(bdg-b^2)x}{bcg-adg}\right)}{bc-ad} + \frac{\log(dx+c)\log\left(\frac{cdg-bc+(d^2g-bd)x}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{cdg-bc+(d^2g-bd)x}{bc-ad}\right)}{bc-ad} + \frac{\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)}{bc-ad}$$

```
[In] integrate(log((a-c*g+(-d*g+b)*x)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="max
ima")
```

```
[Out] (log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d))*log(-(c*g + (d*g - b)
*x - a)/(b*x + a)) + 1/2*(log(b*x + a)^2 - 2*log(b*x + a)*log(d*x + c))/(b*
c - a*d) - (log(b*x + a)*log(((d*g - b)*a + (b*d*g - b^2)*x)/(b*c*g - a*d*g
) + 1) + dilog(-((d*g - b)*a + (b*d*g - b^2)*x)/(b*c*g - a*d*g)))/(b*c - a*
d) + (log(d*x + c)*log((c*d*g - b*c + (d^2*g - b*d)*x)/(b*c - a*d) + 1) + d
ilog(-(c*d*g - b*c + (d^2*g - b*d)*x)/(b*c - a*d)))/(b*c - a*d) + (log(b*x
+ a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d))
)/(b*c - a*d)
```

Giac [F]

$$\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(-\frac{cg+(dg-b)x-a}{bx+a}\right)}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(log((a-c*g+(-d*g+b)*x)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="gia
c")
```

```
[Out] integrate(log(-(c*g + (d*g - b)*x - a)/(b*x + a))/((b*x + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\ln\left(\frac{a-c\,g+x\,(b-d\,g)}{a+b\,x}\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

```
[In] int(log((a - c*g + x*(b - d*g))/(a + b*x))/((a + b*x)*(c + d*x)),x)
[Out] int(log((a - c*g + x*(b - d*g))/(a + b*x))/((a + b*x)*(c + d*x)), x)
```

3.257 $\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 27

$$\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \frac{\text{PolyLog}\left(2,\frac{g(c+dx)}{a+bx}\right)}{bc-ad}$$

```
[Out] polylog(2,g*(d*x+c)/(b*x+a))/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2597, 2565, 2352}

$$\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \frac{\text{PolyLog}\left(2,\frac{g(c+dx)}{a+bx}\right)}{bc-ad}$$

```
[In] Int[Log[1 - (g*(c + d*x))/(a + b*x)]/((a + b*x)*(c + d*x)),x]
```

```
[Out] PolyLog[2, (g*(c + d*x))/(a + b*x)]/(b*c - a*d)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2565

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
```

```
] :> Dist[(b*c - a*d)^(q + 1)*(i/d)^q, Subst[Int[(b*f - a*g - (d*f - c*g)*x
)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x
)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && NeQ[b*c - a*d,
0] && IntegersQ[m, q] && IGtQ[p, 0] && EqQ[d*h - c*i, 0]
```

Rule 2597

```
Int[Log[(e_.)*((f_.)*((g_) + (v_.)/(w_)))^(r_.)]^(s_.)*(u_.), x_Symbol] :>
Int[u*Log[e*(f*(ExpandToSum[v + g*w, x]/ExpandToSum[w, x]))^r]^s, x] /; Fre
eQ[{e, f, g, r, s}, x] && LinearQ[w, x] && (FreeQ[v, x] || LinearQ[v, x]) &
& AlgebraicFunctionQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx\\ &= \text{Subst}\left(\int \frac{\log(x)}{-d(a-cg)+c(b-dg)+(-bc+ad)x}\,dx, x, \frac{a-cg+(b-dg)x}{a+bx}\right)\\ &= \frac{\text{Li}_2\left(\frac{g(c+dx)}{a+bx}\right)}{bc-ad}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 375 vs. $2(27) = 54$.

Time = 0.17 (sec) , antiderivative size = 375, normalized size of antiderivative = 13.89

$$\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \frac{-\log^2\left(\frac{a}{b}+x\right)+2\log\left(\frac{a}{b}+x\right)\log(a+bx)-2\log\left(\frac{a-cg}{b-dg}+x\right)\log(a+bx)+2\log\left(\frac{a-cg}{b-dg}+x\right)\log\left(\frac{(b-dg)(a}{(bc-ad}\right.}{}$$

```
[In] Integrate[Log[1 - (g*(c + d*x))/(a + b*x)]/((a + b*x)*(c + d*x)),x]
```

```
[Out] (-Log[a/b + x]^2 + 2*Log[a/b + x]*Log[a + b*x] - 2*Log[(a - c*g)/(b - d*g)
+ x]*Log[a + b*x] + 2*Log[(a - c*g)/(b - d*g) + x]*Log[((b - d*g)*(a + b*x)
)/((b*c - a*d)*g)] - 2*Log[a/b + x]*Log[c + d*x] + 2*Log[(a - c*g)/(b - d*g
) + x]*Log[c + d*x] + 2*Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] - 2*Log
[(a - c*g)/(b - d*g) + x]*Log[((b - d*g)*(c + d*x))/(b*c - a*d)] + 2*Log[a
+ b*x]*Log[(a - c*g + b*x - d*g*x)/(a + b*x)] - 2*Log[c + d*x]*Log[(a - c*g
 + b*x - d*g*x)/(a + b*x)] + 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2
*PolyLog[2, -((b*(a - c*g + b*x - d*g*x))/((b*c - a*d)*g))] - 2*PolyLog[2,
-((d*(-a + c*g - b*x + d*g*x))/(-(b*c) + a*d))])/(2*b*c - 2*a*d)
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

method	result
derivativedivides	$-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{ad-cb}$
default	$-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{ad-cb}$
risch	$-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{ad-cb}$
parts	$\frac{\ln\left(1-\frac{g(dx+c)}{bx+a}\right)\ln(dx+c)}{ad-cb}-\frac{\ln\left(1-\frac{g(dx+c)}{bx+a}\right)\ln(bx+a)}{ad-cb}-\frac{g\left(\frac{b\ln(bx+a)^2}{2g(ad-cb)}-\frac{b(-dg+b)\left(\frac{\operatorname{dilog}\left(\frac{(-dg+b)(bx+a)+adg-bcg}{adg-bcg}\right)}{-dg+b}+\right.}{g(ad-c}\right.}{b}$

[In] int(ln(1-g*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)

[Out] -1/(a*d-b*c)*dilog((a*d*g-b*c*g)/b/(b*x+a)+(-d*g+b)/b)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx=\frac{\operatorname{Li}_2\left(\frac{cg+(dg-b)x-a}{bx+a}+1\right)}{bc-ad}$$

[In] integrate(log(1-g*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="fricas")

[Out] dilog((c*g + (d*g - b)*x - a)/(b*x + a) + 1)/(b*c - a*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx=\text{Timed out}$$

[In] integrate(ln(1-g*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(26) = 52$.

Time = 0.20 (sec) , antiderivative size = 336, normalized size of antiderivative = 12.44

$$\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \left(\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}\right)\log\left(-\frac{(dx+c)g}{bx+a}+1\right) + \frac{\log(bx+a)^2-2\,\log(bx+a)\log(dx+c)}{2\,(bc-ad)} - \frac{\log(bx+a)\log\left(\frac{(dg-b)a+(bdg-b^2)x}{bcg-adg}+1\right)+\mathrm{Li}_2\left(-\frac{(dg-b)a+(bdg-b^2)x}{bcg-adg}\right)}{bc-ad} + \frac{\log(dx+c)\log\left(\frac{cdg-bc+(d^2g-bd)x}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{cdg-bc+(d^2g-bd)x}{bc-ad}\right)}{bc-ad} + \frac{\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)}{bc-ad}$$

```
[In] integrate(log(1-g*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] (log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d))*log(-(d*x + c)*g/(b*x
 + a) + 1) + 1/2*(log(b*x + a)^2 - 2*log(b*x + a)*log(d*x + c))/(b*c - a*d)
 - (log(b*x + a)*log(((d*g - b)*a + (b*d*g - b^2)*x)/(b*c*g - a*d*g) + 1) +
 dilog(-((d*g - b)*a + (b*d*g - b^2)*x)/(b*c*g - a*d*g)))/(b*c - a*d) + (lo
g(d*x + c)*log((c*d*g - b*c + (d^2*g - b*d)*x)/(b*c - a*d) + 1) + dilog(-(c
*d*g - b*c + (d^2*g - b*d)*x)/(b*c - a*d)))/(b*c - a*d) + (log(b*x + a)*log
((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*c -
 a*d)
```

Giac [F]

$$\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(-\frac{(dx+c)g}{bx+a}+1\right)}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(log(1-g*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(log(-(d*x + c)*g/(b*x + a) + 1)/((b*x + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\ln\left(1-\frac{g\,(c+d\,x)}{a+b\,x}\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

```
[In] int(log(1 - (g*(c + d*x))/(a + b*x))/((a + b*x)*(c + d*x)),x)
[Out] int(log(1 - (g*(c + d*x))/(a + b*x))/((a + b*x)*(c + d*x)), x)
```

3.258 $\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 27

$$\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx=\frac{\text{PolyLog}\left(2,\frac{g(c+dx)}{a+bx}\right)}{bc-ad}$$

```
[Out] polylog(2,g*(d*x+c)/(b*x+a))/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2571, 2565, 2352}

$$\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx=\frac{\text{PolyLog}\left(2,\frac{g(c+dx)}{a+bx}\right)}{bc-ad}$$

```
[In] Int[Log[(a - c*g + b*x - d*g*x)/(a + b*x)]/((a + b*x)*(c + d*x)),x]
[Out] PolyLog[2, (g*(c + d*x))/(a + b*x)]/(b*c - a*d)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2565

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
```

```
] :> Dist[(b*c - a*d)^(q + 1)*(i/d)^q, Subst[Int[(b*f - a*g - (d*f - c*g)*x
)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x
)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && NeQ[b*c - a*d,
0] && IntegersQ[m, q] && IGtQ[p, 0] && EqQ[d*h - c*i, 0]
```

Rule 2571

```
Int[((A_.) + Log[(e_.)*((u_)/(v_))^(n_.)]*(B_.))^(p_.)*(w_)^(m_.)*(y_)^(q_.
), x_Symbol] :> Int[ExpandToSum[w, x]^m*ExpandToSum[y, x]^q*(A + B*Log[e*(E
xpandToSum[u, x]/ExpandToSum[v, x])^n])^p, x] /; FreeQ[{e, A, B, m, n, p, q
}, x] && LinearQ[{u, v, w, y}, x] &&  !LinearMatchQ[{u, v, w, y}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx\\ &= \text{Subst}\left(\int \frac{\log(x)}{-d(a-cg)+c(b-dg)+(-bc+ad)x}\,dx, x, \frac{a-cg+(b-dg)x}{a+bx}\right)\\ &= \frac{\text{Li}_2\left(\frac{g(c+dx)}{a+bx}\right)}{bc-ad}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 375 vs. 2(27) = 54.

Time = 0.01 (sec) , antiderivative size = 375, normalized size of antiderivative = 13.89

$$\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \frac{-\log^2\left(\frac{a}{b}+x\right)+2\log\left(\frac{a}{b}+x\right)\log(a+bx)-2\log\left(\frac{a-cg}{b-dg}+x\right)\log(a+bx)+2\log\left(\frac{a-cg}{b-dg}+x\right)\log\left(\frac{(b-dg)(a+}{(bc-ad)g}\right.}{}$$

```
[In] Integrate[Log[(a - c*g + b*x - d*g*x)/(a + b*x)]/((a + b*x)*(c + d*x)),x]
```

```
[Out] (-Log[a/b + x]^2 + 2*Log[a/b + x]*Log[a + b*x] - 2*Log[(a - c*g)/(b - d*g)
+ x]*Log[a + b*x] + 2*Log[(a - c*g)/(b - d*g) + x]*Log[((b - d*g)*(a + b*x)
)/((b*c - a*d)*g)] - 2*Log[a/b + x]*Log[c + d*x] + 2*Log[(a - c*g)/(b - d*g
) + x]*Log[c + d*x] + 2*Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] - 2*Log
[(a - c*g)/(b - d*g) + x]*Log[((b - d*g)*(c + d*x))/(b*c - a*d)] + 2*Log[a
+ b*x]*Log[(a - c*g + b*x - d*g*x)/(a + b*x)] - 2*Log[c + d*x]*Log[(a - c*g
 + b*x - d*g*x)/(a + b*x)] + 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2
*PolyLog[2, -((b*(a - c*g + b*x - d*g*x))/((b*c - a*d)*g))] - 2*PolyLog[2,
-((d*(-a + c*g - b*x + d*g*x))/(-(b*c) + a*d))])/(2*b*c - 2*a*d)
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

method	result
derivativedivides	$-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{ad-cb}$
default	$-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{ad-cb}$
risch	$-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{ad-cb}$
parts	$\frac{\ln\left(\frac{-dgx+bx-cg+a}{bx+a}\right)\ln(dx+c)}{ad-cb}-\frac{\ln\left(\frac{-dgx+bx-cg+a}{bx+a}\right)\ln(bx+a)}{ad-cb}-\frac{g\left(\frac{b\ln(bx+a)^2}{2g(ad-cb)}-\frac{b(-dg+b)\left(\frac{\operatorname{dilog}\left(\frac{(-dg+b)(bx+a)+a\cdots}{adg-bcg}\cdots\right)}{-dg+b}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$

[In] int(ln((-d*g*x+b*x-c*g+a)/(b*x+a))/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)

[Out] -1/(a*d-b*c)*dilog((a*d*g-b*c*g)/b/(b*x+a)+(-d*g+b)/b)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx=\frac{\operatorname{Li}_2\left(\frac{cg+(dg-b)x-a}{bx+a}+1\right)}{bc-ad}$$

[In] integrate(log((-d*g*x+b*x-c*g+a)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="fricas")

[Out] dilog((c*g + (d*g - b)*x - a)/(b*x + a) + 1)/(b*c - a*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx=\text{Timed out}$$

[In] integrate(ln((-d*g*x+b*x-c*g+a)/(b*x+a))/(b*x+a)/(d*x+c),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(26) = 52.

Time = 0.20 (sec) , antiderivative size = 343, normalized size of antiderivative = 12.70

$$\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \left(\frac{\log(bx+a)}{bc-ad} - \frac{\log(dx+c)}{bc-ad}\right)\log\left(-\frac{dgx+cg-bx-a}{bx+a}\right) + \frac{\log(bx+a)^2 - 2\log(bx+a)\log(dx+c)}{2(bc-ad)} - \frac{\log(bx+a)\log\left(\frac{(dg-b)a+(bdg-b^2)x}{bcg-adg}+1\right) + \mathrm{Li}_2\left(-\frac{(dg-b)a+(bdg-b^2)x}{bcg-adg}\right)}{bc-ad} + \frac{\log(dx+c)\log\left(\frac{cdg-bc+(d^2g-bd)x}{bc-ad}+1\right) + \mathrm{Li}_2\left(-\frac{cdg-bc+(d^2g-bd)x}{bc-ad}\right)}{bc-ad} + \frac{\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right) + \mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)}{bc-ad}$$

```
[In] integrate(log((-d*g*x+b*x-c*g+a)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="max
ima")
```

```
[Out] (log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d))*log(-(d*g*x + c*g - b
*x - a)/(b*x + a)) + 1/2*(log(b*x + a)^2 - 2*log(b*x + a)*log(d*x + c))/(b*
c - a*d) - (log(b*x + a)*log(((d*g - b)*a + (b*d*g - b^2)*x)/(b*c*g - a*d*g
) + 1) + dilog(-((d*g - b)*a + (b*d*g - b^2)*x)/(b*c*g - a*d*g)))/(b*c - a*
d) + (log(d*x + c)*log((c*d*g - b*c + (d^2*g - b*d)*x)/(b*c - a*d) + 1) + d
ilog(-(c*d*g - b*c + (d^2*g - b*d)*x)/(b*c - a*d)))/(b*c - a*d) + (log(b*x
+ a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d))
)/(b*c - a*d)
```

Giac [F]

$$\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(-\frac{dgx+cg-bx-a}{bx+a}\right)}{(bx+a)(dx+c)}\,dx$$

```
[In] integrate(log((-d*g*x+b*x-c*g+a)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="gia
c")
```

```
[Out] integrate(log(-(d*g*x + c*g - b*x - a)/(b*x + a))/((b*x + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\ln\left(\frac{a-c\,g+b\,x-d\,g\,x}{a+b\,x}\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

```
[In] int(log((a - c*g + b*x - d*g*x)/(a + b*x))/((a + b*x)*(c + d*x)),x)
[Out] int(log((a - c*g + b*x - d*g*x)/(a + b*x))/((a + b*x)*(c + d*x)), x)
```

3.259 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{afh+bghx^2+h(bfx+agx)}\,dx$

Optimal result

Integrand size = 51, antiderivative size = 282

$$\begin{aligned}\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{afh+bghx^2+h(bfx+agx)}\,dx &= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}\\ &+\frac{3Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\operatorname{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}\\ &+\frac{6B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}\\ &+\frac{6B^3n^3\operatorname{PolyLog}\left(4,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}\end{aligned}$$

```
[Out] -(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3*ln(1-(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*
x+a))/(-a*g+b*f)/h+3*B*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*polylog(2,(-a*
g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h+6*B^2*n^2*(A+B*ln(e*(b*x+a)
^n/((d*x+c)^n)))*polylog(3,(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f
)/h+6*B^3*n^3*polylog(4,(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.157$, Rules used = {2573, 2576, 3, 1607, 2379, 2421, 2430, 6724}

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{afh+bghx^2+h(bfx+agx)}\,dx$$
$$=\frac{6B^2n^2\,\text{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{h(bf-ag)}$$
$$+\frac{3Bn\,\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{h(bf-ag)}$$
$$-\frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{h(bf-ag)}$$
$$+\frac{6B^3n^3\,\text{PolyLog}\left(4,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{h(bf-ag)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a*f*h + b*g*h*x^2 + h*(b*f*
x + a*g*x)),x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3*Log[1 - ((b*f - a*g)*(c + d*x
))/((d*f - c*g)*(a + b*x))])/((b*f - a*g)*h)) + (3*B*n*(A + B*Log[(e*(a + b
*x)^n)/(c + d*x)^n])^2*PolyLog[2, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a +
 b*x))])/((b*f - a*g)*h) + (6*B^2*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^
n])*PolyLog[3, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])/((b*f - a*
g)*h) + (6*B^3*n^3*PolyLog[4, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x
))])/((b*f - a*g)*h)
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
```

```
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 2576

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Dist[b*c - a*d, Subst[Int[(b^2*f - a*b
*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^2*h
)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/(c
+ d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{afh+bghx^2+h(bfx+agx)}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\Bigg((bc - ad)\text{Subst}\Bigg(\int \frac{(A + B\log(ex^n))^3}{ab^2fh + a^2bgh - ab(bfh + agh) - (2abdfh + 2abcgh - bc(bfh + agh) - ad(bfh}$$

$$+ bx)^n(c + dx)^{-n}\Bigg)$$

$$= \text{Subst}\Bigg((bc - ad)\text{Subst}\Bigg(\int \frac{(A + B\log(ex^n))^3}{(-2abdfh - 2abcgh + bc(bfh + agh) + ad(bfh + agh))x + (ad^2fh + bc^2gh - cd}$$

$$+ bx)^n(c + dx)^{-n}\Bigg)$$

$$= \text{Subst}\Bigg((bc - ad)\text{Subst}\Bigg(\int \frac{(A + B\log(ex^n))^3}{x\,(-2abdfh - 2abcgh + bc(bfh + agh) + ad(bfh + agh) + (ad^2fh + bc^2gh - cd}$$

$$+ bx)^n(c + dx)^{-n}\Bigg)$$

$$= -\frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^3\log\left(1 - \frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf - ag)h}$$

$$+\text{Subst}\left(\frac{(3Bn)\text{Subst}\left(\int \frac{\log\left(1+\frac{-2abdfh-2abcgh+bc(bfh+agh)+ad(bfh+agh)}{\left(ad^2fh+bc^2gh-cd(bfh+agh)\right)x}\right)(A+B\log(ex^n))^2}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bf - ag)h}, e\left(\frac{a}{c}\right.\right.$$

$$+ bx)^n(c + dx)^{-n}\Bigg)$$

$$= -\frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3 \log\left(1 - \frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}$$
$$+ \frac{3Bn(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2 \operatorname{Li}_2\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}$$
$$-\text{Subst}\left(\frac{(6B^2n^2)\,\text{Subst}\left(\int \frac{(A+B\log(ex^n))\operatorname{Li}_2\left(-\frac{-2abdfh-2abcgh+bc(bfh+agh)+ad(bfh+agh)}{\left(ad^2fh+bc^2gh-cd(bfh+agh)\right)x}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bf-ag)h}, e\left(\frac{a}{c\cdots}\right.\right.$$
$$\left. + bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3 \log\left(1 - \frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}$$
$$+ \frac{3Bn(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2 \operatorname{Li}_2\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}$$
$$+ \frac{6B^2n^2(A + B\log(e(a+bx)^n(c+dx)^{-n})) \operatorname{Li}_3\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}$$
$$-\text{Subst}\left(\frac{(6B^3n^3)\,\text{Subst}\left(\int \frac{\operatorname{Li}_3\left(-\frac{-2abdfh-2abcgh+bc(bfh+agh)+ad(bfh+agh)}{\left(ad^2fh+bc^2gh-cd(bfh+agh)\right)x}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bf-ag)h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(\cdots\right.$$
$$\left. + bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3 \log\left(1 - \frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{3Bn(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2 \operatorname{Li}_2\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{6B^2n^2(A + B\log(e(a+bx)^n(c+dx)^{-n})) \operatorname{Li}_3\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{6B^3n^3\operatorname{Li}_4\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}$$

Mathematica [F]

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{afh + bghx^2 + h(bfx + agx)}\,dx = \int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{afh + bghx^2 + h(bfx + agx)}\,dx$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a*f*h + b*g*h*x^2 + h
*(b*f*x + a*g*x)),x]
```

```
[Out] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a*f*h + b*g*h*x^2 + h
*(b*f*x + a*g*x)), x]
```

Maple [F]

$$\int \frac{\left(A + B\ln\left(e(bx+a)^n\,(dx+c)^{-n}\right)\right)^3}{afh + bgh\,x^2 + h\,(agx + bfx)}dx$$

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x)),x
)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x)),x
)
```

Fricas [F]

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{afh + bghx^2 + h(bfx + agx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3}{bghx^2 + afh + (bfx + agx)h}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(a*f*h+b*g*h*x^2+h*(a*g*x+b*
f*x)),x, algorithm="fricas")
```

```
[Out] integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e/
(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(b*g*h*x^2 +
 a*f*h + (b*f + a*g)*h*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{afh+bghx^2+h(bfx+agx)}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(a*f*h+b*g*h*x**2+h*(a*g*x
+b*f*x)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{afh+bghx^2+h(bfx+agx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{bghx^2+afh+(bfx+agx)h}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(a*f*h+b*g*h*x^2+h*(a*g*x+b*
f*x)),x, algorithm="maxima")
```

```
[Out] A^3*(log(b*x + a)/((b*f - a*g)*h) - log(g*x + f)/((b*f - a*g)*h)) - integra
te(-(B^3*log((b*x + a)^n)^3 - B^3*log((d*x + c)^n)^3 + B^3*log(e)^3 + 3*A*B
^2*log(e)^2 + 3*A^2*B*log(e) + 3*(B^3*log(e) + A*B^2)*log((b*x + a)^n)^2 +
3*(B^3*log((b*x + a)^n) + B^3*log(e) + A*B^2)*log((d*x + c)^n)^2 + 3*(B^3*l
og(e)^2 + 2*A*B^2*log(e) + A^2*B)*log((b*x + a)^n) - 3*(B^3*log((b*x + a)^n
)^2 + B^3*log(e)^2 + 2*A*B^2*log(e) + A^2*B + 2*(B^3*log(e) + A*B^2)*log((b
*x + a)^n))*log((d*x + c)^n))/(b*g*h*x^2 + a*f*h + (b*f*h + a*g*h)*x), x)
```

Giac [F]

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{afh+bghx^2+h(bfx+agx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{bghx^2+afh+(bfx+agx)h}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(a*f*h+b*g*h*x^2+h*(a*g*x+b*
f*x)),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*g*h*x^2 + a*f*h + (b*
f*x + a*g*x)*h), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{afh+bghx^2+h(bfx+agx)}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3}{h\,\left(a\,g\,x+b\,f\,x\right)+a\,f\,h+b\,g\,h\,x^2}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(h*(a*g*x + b*f*x) + a*f*h +
 b*g*h*x^2),x)
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(h*(a*g*x + b*f*x) + a*f*h +
 b*g*h*x^2), x)
```

3.260 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{afh+bghx^2+h(bfx+agx)}\,dx$

Optimal result

Integrand size = 51, antiderivative size = 203

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{afh+bghx^2+h(bfx+agx)}\,dx = -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{2Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}$$

```
[Out] -(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*ln(1-(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*
x+a))/(-a*g+b*f)/h+2*B*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,(-a*g+
b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h+2*B^2*n^2*polylog(3,(-a*g+b*f
)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.137, Rules used

$= \{2573, 2576, 3, 1607, 2379, 2421, 6724\}$

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{afh+bghx^2+h(bfx+agx)}\,dx$$
$$=\frac{2Bn\,\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)}{h(bf-ag)}$$
$$-\frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{h(bf-ag)}$$
$$+\frac{2B^2n^2\,\text{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{h(bf-ag)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a*f*h + b*g*h*x^2 + h*(b*f*
x + a*g*x)),x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2*Log[1 - ((b*f - a*g)*(c + d*x
))/((d*f - c*g)*(a + b*x))])/((b*f - a*g)*h)) + (2*B*n*(A + B*Log[(e*(a + b
*x)^n)/(c + d*x)^n])*PolyLog[2, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b
*x))])/((b*f - a*g)*h) + (2*B^2*n^2*PolyLog[3, ((b*f - a*g)*(c + d*x))/((d*
f - c*g)*(a + b*x))])/((b*f - a*g)*h)
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
```

```
] && EqQ[d*e, 1]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 2576

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Dist[b*c - a*d, Subst[Int[(b^2*f - a*b
*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^2*h
)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/(c
+ d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{afh+bghx^2+h(bfx+agx)}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\Bigg((bc$$

$$-ad)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{ab^2fh+a^2bgh-ab(bfh+agh)-(2abdfh+2abcgh-bc(bfh+agh)-ad(bfh+\right.$$

$$\left.+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\Bigg((bc$$

$$-ad)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{(-2abdfh-2abcgh+bc(bfh+agh)+ad(bfh+agh))x+(ad^2fh+bc^2gh-cd(\right.$$

$$\left.+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\Bigg((bc$$

$$-ad)\text{Subst}\left(\int \frac{(A+B\log(ex^n))^2}{x\left(-2abdfh-2abcgh+bc(bfh+agh)+ad(bfh+agh)+(ad^2fh+bc^2gh-cd\right.}$$

$$+bx)^n(c+dx)^{-n}\Bigg)$$

$$= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}$$

$$+\text{Subst}\left(\frac{(2Bn)\text{Subst}\left(\int \frac{\log\left(1+\frac{-2abdfh-2abcgh+bc(bfh+agh)+ad(bfh+agh)}{\left(ad^2fh+bc^2gh-cd(bfh+agh)\right)x}\right)(A+B\log(ex^n))}{x}dx,x,\frac{a+bx}{c+dx}\right)}{(bf-ag)h},e\left(\frac{a+}{c+}\right.\right.$$

$$\left.+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{2Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{Li}_2\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} - \operatorname{Subst}\left(\frac{(2B^2n^2)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(-\frac{-2abdfh-2abcgh+bc(bfh+agh)+ad(bfh+agh)}{(ad^2fh+bc^2gh-cd(bfh+agh))x}\right)}{x}dx,x,\frac{a+bx}{c+dx}\right)}{(bf-ag)h},e\left(\frac{a+bx}{c+dx}\right)^n,e(c\right. \ldots \left. +bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{2Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{Li}_2\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{2B^2n^2\operatorname{Li}_3\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1415 vs. 2(203) = 406.

Time = 0.57 (sec) , antiderivative size = 1415, normalized size of antiderivative = 6.97

$$\int\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{afh+bghx^2+h(bfx+agx)}\,dx$$

$$= \frac{3\log(a+bx)\left(A+B(-n\log(a+bx)+n\log(c+dx)+\log(e(a+bx)^n(c+dx)^{-n}))\right)^2-3(A+B(-n\log(a \ldots}{\ldots}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a*f*h + b*g*h*x^2 + h
*(b*f*x + a*g*x)),x]
```

```
[Out] (3*Log[a + b*x]*(A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*
x)^n)/(c + d*x)^n]))^2 - 3*(A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log
```

```
[(e*(a + b*x)^n)/(c + d*x)^n]))^2*Log[f + g*x] + 3*B*n*(A + B*(-(n*Log[a +
b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n]))*(Log[a + b*x]^2
 - 2*(Log[a + b*x]*Log[(b*(f + g*x))/(b*f - a*g)] + PolyLog[2, (g*(a + b*x)
)/(-(b*f) + a*g)])) - 6*A*B*n*(Log[c + d*x]*(Log[(d*(a + b*x))/(-(b*c) + a*
d)] - Log[(d*(f + g*x))/(d*f - c*g)]) + PolyLog[2, (b*(c + d*x))/(b*c - a*d
)] - PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)]) + 6*B^2*n*(n*Log[a + b*x] -
n*Log[c + d*x] - Log[(e*(a + b*x)^n)/(c + d*x)^n])*(Log[c + d*x]*(Log[(d*(a
 + b*x))/(-(b*c) + a*d)] - Log[(d*(f + g*x))/(d*f - c*g)]) + PolyLog[2, (b*
(c + d*x))/(b*c - a*d)] - PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)]) + B^2*n
^2*(Log[a + b*x]^2*(Log[a + b*x] - 3*Log[(b*(f + g*x))/(b*f - a*g)]) - 6*Lo
g[a + b*x]*PolyLog[2, (g*(a + b*x))/(-(b*f) + a*g)] + 6*PolyLog[3, (g*(a +
b*x))/(-(b*f) + a*g)]) + 3*B^2*n^2*(Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c
 + d*x]^2 - Log[c + d*x]^2*Log[(d*(f + g*x))/(d*f - c*g)] + 2*Log[c + d*x]*
PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 2*Log[c + d*x]*PolyLog[2, (g*(c + d
*x))/(-(d*f) + c*g)] - 2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] + 2*PolyLog[
3, (g*(c + d*x))/(-(d*f) + c*g)]) - 6*B^2*n^2*((Log[a + b*x]^2*(Log[c + d*x
] - Log[(b*(c + d*x))/(b*c - a*d)]))/2 - Log[a + b*x]*Log[c + d*x]*Log[(b*(
f + g*x))/(b*f - a*g)] - (Log[(g*(c + d*x))/(-(d*f) + c*g)]*(-2*Log[a + b*x
] + Log[(g*(c + d*x))/(-(d*f) + c*g)])*(Log[(b*(f + g*x))/(b*f - a*g)] - Lo
g[(d*(f + g*x))/(d*f - c*g)]))/2 + Log[(g*(c + d*x))/(-(d*f) + c*g)]*Log[((
b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]*(Log[(b*(f + g*x))/(b*f - a*
g)] - Log[(d*(f + g*x))/(d*f - c*g)]) - (Log[((b*f - a*g)*(c + d*x))/((d*f
- c*g)*(a + b*x))]^2*(Log[(-(b*c) + a*d)/(d*(a + b*x))] + Log[(b*(f + g*x))
/(b*f - a*g)] - Log[((-(b*c) + a*d)*(f + g*x))/((d*f - c*g)*(a + b*x))]))/2
 - Log[a + b*x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - (Log[c + d*x] -
Log[((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])*PolyLog[2, (g*(a + b*
x))/(-(b*f) + a*g)] - (Log[a + b*x] + Log[((b*f - a*g)*(c + d*x))/((d*f - c
*g)*(a + b*x))])*PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)] - Log[((b*f - a*g
)*(c + d*x))/((d*f - c*g)*(a + b*x))]*(PolyLog[2, (b*(c + d*x))/(d*(a + b*x
))] - PolyLog[2, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]) + PolyLo
g[3, (d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[3, (g*(a + b*x))/(-(b*f) + a*g
)] + PolyLog[3, (g*(c + d*x))/(-(d*f) + c*g)] + PolyLog[3, (b*(c + d*x))/(d
*(a + b*x))] - PolyLog[3, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])
)/(3*(b*f - a*g)*h)
```

Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{2}}{afh+bgh\,x^{2}+h\left(agx+bfx\right)}dx$$

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x)),x
)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x)),x
)
```

Fricas [F]

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{afh + bghx^2 + h(bfx + agx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{bghx^2 + afh + (bfx + agx)h}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(a*f*h+b*g*h*x^2+h*(a*g*x+b*
f*x)),x, algorithm="fricas")
```

```
[Out] integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(d
*x + c)^n) + A^2)/(b*g*h*x^2 + a*f*h + (b*f + a*g)*h*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{afh + bghx^2 + h(bfx + agx)}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(a*f*h+b*g*h*x**2+h*(a*g*x
+b*f*x)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{afh + bghx^2 + h(bfx + agx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{bghx^2 + afh + (bfx + agx)h}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(a*f*h+b*g*h*x^2+h*(a*g*x+b*
f*x)),x, algorithm="maxima")
```

```
[Out] A^2*(log(b*x + a)/((b*f - a*g)*h) - log(g*x + f)/((b*f - a*g)*h)) + integra
te((B^2*log((b*x + a)^n)^2 + B^2*log((d*x + c)^n)^2 + B^2*log(e)^2 + 2*A*B*
log(e) + 2*(B^2*log(e) + A*B)*log((b*x + a)^n) - 2*(B^2*log((b*x + a)^n) +
B^2*log(e) + A*B)*log((d*x + c)^n))/(b*g*h*x^2 + a*f*h + (b*f*h + a*g*h)*x)
, x)
```

Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{afh+bghx^2+h(bfx+agx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{bghx^2+afh+(bfx+agx)h}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(a*f*h+b*g*h*x^2+h*(a*g*x+b*
f*x)),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*g*h*x^2 + a*f*h + (b*
f*x + a*g*x)*h), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{afh+bghx^2+h(bfx+agx)}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2}{h\,(a\,g\,x+b\,f\,x)+a\,f\,h+b\,g\,h\,x^2}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(h*(a*g*x + b*f*x) + a*f*h +
 b*g*h*x^2),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(h*(a*g*x + b*f*x) + a*f*h +
 b*g*h*x^2), x)
```

3.261 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{afh+bghx^2+h(bfx+agx)}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 123

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{afh+bghx^2+h(bfx+agx)}\,dx = -\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{Bn\,\mathrm{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}$$

```
[Out] -(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+
a))/(-a*g+b*f)/h+B*n*polylog(2,(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g
+b*f)/h
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {2573, 2576, 3, 1607, 2379, 2438}

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{afh+bghx^2+h(bfx+agx)}\,dx = \frac{Bn\,\mathrm{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{h(bf-ag)} - \frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}{h(bf-ag)}$$

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a*f*h + b*g*h*x^2 + h*(b*f*x
+ a*g*x)),x]
```

```
[Out] -(((A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*Log[1 - ((b*f - a*g)*(c + d*x))
/((d*f - c*g)*(a + b*x))])/((b*f - a*g)*h)) + (B*n*PolyLog[2, ((b*f - a*g)*
(c + d*x))/((d*f - c*g)*(a + b*x))])/((b*f - a*g)*h)
```

Rule 3

```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2573

```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

Rule 2576

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Dist[b*c - a*d, Subst[Int[(b^2*f - a*b
*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^2*h
)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/(c
+ d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{afh+bghx^2+h(bfx+agx)}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\Bigg((bc$$

$$-ad)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{ab^2fh+a^2bgh-ab(bfh+agh)-(2abdfh+2abcgh-bc(bfh+agh)-ad(bfh+\right.$$

$$\left.+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\Bigg((bc$$

$$-ad)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{(-2abdfh-2abcgh+bc(bfh+agh)+ad(bfh+agh))x+(ad^2fh+bc^2gh-cd(\right.$$

$$\left.+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\Bigg((bc$$

$$-ad)\text{Subst}\left(\int \frac{A+B\log(ex^n)}{x\,(-2abdfh-2abcgh+bc(bfh+agh)+ad(bfh+agh)+(ad^2fh+bc^2gh-cd(b\right.$$

$$\left.+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}$$

$$+\text{Subst}\left(\frac{(Bn)\text{Subst}\left(\int \frac{\log\left(1+\frac{-2abdfh-2abcgh+bc(bfh+agh)+ad(bfh+agh)}{\left(ad^2fh+bc^2gh-cd(bfh+agh)\right)x}\right)}{x}\,dx, x, \frac{a+bx}{c+dx}\right)}{(bf-ag)h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a\right.$$

$$\left.+bx)^n(c+dx)^{-n}\right)$$

$$= -\frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))\log\left(1 - \frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{Bn\text{Li}_2\left(\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 303 vs. 2(123) = 246.

Time = 0.18 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.46

$$\int \frac{A + B\log(e(a+bx)^n(c+dx)^{-n})}{afh + bghx^2 + h(bfx + agx)}\,dx = -\frac{-2A\log(a+bx) + Bn\log^2(a+bx) - 2Bn\log(a+bx)\log(c+dx) + 2Bn\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log(c+dx) - 2}{}$$

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a*f*h + b*g*h*x^2 + h*(
b*f*x + a*g*x)),x]
```

```
[Out] -((-2*A*Log[a + b*x] + B*n*Log[a + b*x]^2 - 2*B*n*Log[a + b*x]*Log[c + d*x]
 + 2*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 2*B*Log[a + b*x]*
Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*A*Log[f + g*x] - 2*B*n*Log[a + b*x]*Lo
g[f + g*x] + 2*B*n*Log[c + d*x]*Log[f + g*x] + 2*B*Log[(e*(a + b*x)^n)/(c +
 d*x)^n]*Log[f + g*x] + 2*B*n*Log[a + b*x]*Log[(b*(f + g*x))/(b*f - a*g)] -
 2*B*n*Log[c + d*x]*Log[(d*(f + g*x))/(d*f - c*g)] + 2*B*n*PolyLog[2, (g*(a
 + b*x))/(-(b*f) + a*g)] + 2*B*n*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 2*
B*n*PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)])/((2*b*f - 2*a*g)*h))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 25.90 (sec) , antiderivative size = 1447, normalized size of antiderivative = 11.76

method	result	size
risch	Expression too large to display	1447

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x)),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn
(I*(b*x+a)^n/((d*x+c)^n))-1/2*I/h/(a*g-b*f)*ln(g*x+f)*B*Pi*csgn(I*e)*csgn(I
*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I/h/(a*g-b*f)*l
n(g*x+f)*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+
c)^n))+1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)
^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/h/(a*g-b*f)*ln(b*x+a)*A+1/h/(a*g-b*f
)*ln(g*x+f)*A+1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n)
```

```
)^3+1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/2*
I/h/(a*g-b*f)*ln(g*x+f)*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/2*I/h/(a*g-b
*f)*ln(g*x+f)*B*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/2*I/h/(a*g-b*f)*ln(b
*x+a)*B*Pi*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/2*I/h/(a*g-b*f)*ln
(b*x+a)*B*Pi*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-1/2*I/h/(a*g
-b*f)*ln(b*x+a)*B*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/
2*I/h/(a*g-b*f)*ln(g*x+f)*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x
+c)^n)*(b*x+a)^n)^2-1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*(b*x+a)^n/((d*x
+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I/h/(a*g-b*f)*ln(g*x+f)*B*Pi*
csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I/h/(a*g-b*f)*ln(g*x+f)*B*P
i*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I/h/(a*g-b*f)*ln(g*
x+f)*B*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-1/h*B*n/(a*g-
b*f)*ln(g*x+f)*ln((b*(g*x+f)+a*g-b*f)/(a*g-b*f))+1/h*B*n/(a*g-b*f)*ln(g*x+f
)*ln((d*(g*x+f)+c*g-d*f)/(c*g-d*f))-1/h*B*n/(a*g-b*f)*ln(b*x+a)*ln((-a*d+c*
b+d*(b*x+a))/(-a*d+b*c))-1/h/(a*g-b*f)*ln(b*x+a)*B*ln(e)+1/h/(a*g-b*f)*ln(g
*x+f)*B*ln(e)-1/h*B*ln((b*x+a)^n)/(a*g-b*f)*ln(b*x+a)+1/h*B/(a*g-b*f)*ln(g*
x+f)*ln((b*x+a)^n)-1/h*B*n/(a*g-b*f)*dilog((b*(g*x+f)+a*g-b*f)/(a*g-b*f))+1
/2/h*B*n/(a*g-b*f)*ln(b*x+a)^2+1/h*B*ln((d*x+c)^n)/(a*g-b*f)*ln(b*x+a)-1/h*
B/(a*g-b*f)*ln(g*x+f)*ln((d*x+c)^n)+1/h*B*n/(a*g-b*f)*dilog((d*(g*x+f)+c*g-
d*f)/(c*g-d*f))-1/h*B*n/(a*g-b*f)*dilog((-a*d+c*b+d*(b*x+a))/(-a*d+b*c))
```

Fricas [F]

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{afh + bghx^2 + h(bfx+agx)}\,dx = \int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{bghx^2 + afh + (bfx+agx)h}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*
x)),x, algorithm="fricas")
```

```
[Out] integral((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(b*g*h*x^2 + a*f*h + (b*f +
 a*g)*h*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{afh + bghx^2 + h(bfx+agx)}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(a*f*h+b*g*h*x**2+h*(a*g*x+b*
f*x)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{afh + bghx^2 + h(bfx + agx)}\,dx = \int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{bghx^2 + afh + (bfx + agx)h}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*
x)),x, algorithm="maxima")
```

```
[Out] A*(log(b*x + a)/((b*f - a*g)*h) - log(g*x + f)/((b*f - a*g)*h)) - B*integra
te(-(log((b*x + a)^n) - log((d*x + c)^n) + log(e))/(b*g*h*x^2 + a*f*h + (b*
f*h + a*g*h)*x), x)
```

Giac [F]

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{afh + bghx^2 + h(bfx + agx)}\,dx = \int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{bghx^2 + afh + (bfx + agx)h}\,dx$$

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*
x)),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(b*g*h*x^2 + a*f*h + (b*f*
x + a*g*x)*h), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{afh + bghx^2 + h(bfx + agx)}\,dx = \int \frac{A + B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{h\,\left(a\,g\,x + b\,f\,x\right) + a\,f\,h + b\,g\,h\,x^2}\,dx$$

```
[In] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(h*(a*g*x + b*f*x) + a*f*h + b
*g*h*x^2),x)
```

```
[Out] int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(h*(a*g*x + b*f*x) + a*f*h + b
*g*h*x^2), x)
```

3.262 $\int \frac{1}{(afh+bghx^2+h(bfx+agx))(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$

Optimal result

Integrand size = 51, antiderivative size = 51

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\text{Subst}\left(\text{Int}\left(\frac{1}{(a+bx)(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right),e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)}{h}$$

```
[Out] _eval(Unintegrable(1/(b*x+a)/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x),e*(
(b*x+a)/(d*x+c))^n = e*(b*x+a)^n/((d*x+c)^n))/h
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$

```
[In] Int[1/((a*f*h + b*g*h*x^2 + h*(b*f*x + a*g*x))*(A + B*Log[(e*(a + b*x)^n)/(
c + d*x)^n])),x]
```

```
[Out] Defer[Subst][Defer[Int][1/((a + b*x)*(f + g*x)*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n])), x], e*((a + b*x)/(c + d*x))^n, (e*(a + b*x)^n)/(c + d*x)^n]/h
```

Rubi steps

integral

$$= \text{Subst}\left(\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\int \frac{1}{h(a+bx)(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\int \frac{1}{(a+bx)(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}\,dx}{h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx = \int \frac{1}{(afh+bghx^2+h(bfx+agx))\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$$

```
[In] Integrate[1/((a*f*h + b*g*h*x^2 + h*(b*f*x + a*g*x))*(A + B*Log[(e*(a + b*x
)^n)/(c + d*x)^n])),x]
[Out] Integrate[1/((a*f*h + b*g*h*x^2 + h*(b*f*x + a*g*x))*(A + B*Log[(e*(a + b*x
)^n)/(c + d*x)^n])), x]
```

Maple [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{(afh+bgh\,x^2+h\,(agx+bfx))\left(A+B\ln\left(e\,(bx+a)^n\,(dx+c)^{-n}\right)\right)}dx$$

```
[In] int(1/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x))/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x
)
[Out] int(1/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x))/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x
)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.49

$$\int \frac{1}{(afh + bghx^2 + h(bfx + agx))\left(A + B\log\left(e(a + bx)^n(c + dx)^{-n}\right)\right)}\,dx$$
$$= \int \frac{1}{(bghx^2 + afh + (bfx + agx)h)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)}\,dx$$

```
[In] integrate(1/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x))/(A+B*log(e*(b*x+a)^n/((d*x+c)
^n))),x, algorithm="fricas")
```

```
[Out] integral(1/(A*b*g*h*x^2 + A*a*f*h + (A*b*f + A*a*g)*h*x + (B*b*g*h*x^2 + B*
a*f*h + (B*b*f + B*a*g)*h*x)*log((b*x + a)^n*e/(d*x + c)^n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(afh + bghx^2 + h(bfx + agx))\left(A + B\log\left(e(a + bx)^n(c + dx)^{-n}\right)\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a*f*h+b*g*h*x**2+h*(a*g*x+b*f*x))/(A+B*ln(e*(b*x+a)**n/((d*x+c
)**n))),x)
```

```
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{1}{(afh + bghx^2 + h(bfx + agx))\left(A + B\log\left(e(a + bx)^n(c + dx)^{-n}\right)\right)}\,dx$$
$$= \int \frac{1}{(bghx^2 + afh + (bfx + agx)h)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)}\,dx$$

```
[In] integrate(1/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x))/(A+B*log(e*(b*x+a)^n/((d*x+c)
^n))),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*g*h*x^2 + a*f*h + (b*f*x + a*g*x)*h)*(B*log((b*x + a)^n*e/(
d*x + c)^n) + A)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{1}{(afh + bghx^2 + h(bfx + agx))\,(A + B\log{(e(a + bx)^n(c + dx)^{-n})})}\,dx$$
$$= \int \frac{1}{(bghx^2 + afh + (bfx + agx)h)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)}\,dx$$

```
[In] integrate(1/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x))/(A+B*log(e*(b*x+a)^n/((d*x+c)
^n))),x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*h*x^2 + a*f*h + (b*f*x + a*g*x)*h)*(B*log((b*x + a)^n*e/(
d*x + c)^n) + A)), x)
```

Mupad [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{1}{(afh + bghx^2 + h(bfx + agx))\,(A + B\log{(e(a + bx)^n(c + dx)^{-n})})}\,dx$$
$$= \int \frac{1}{\left(A + B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)\,\left(h\,\left(a\,g\,x + b\,f\,x\right) + a\,f\,h + b\,g\,h\,x^2\right)}\,dx$$

```
[In] int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(h*(a*g*x + b*f*x) + a*f*h
+ b*g*h*x^2)),x)
```

```
[Out] int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(h*(a*g*x + b*f*x) + a*f*h
+ b*g*h*x^2)), x)
```

3.263 $\int \frac{1}{(afh+bghx^2+h(bfx+agx))(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}\,dx$

Optimal result

Integrand size = 51, antiderivative size = 51

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \frac{\text{Subst}\left(\text{Int}\left(\frac{1}{(a+bx)(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right),e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)}{h}$$

```
[Out] _eval(Unintegrable(1/(b*x+a)/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x),e
*((b*x+a)/(d*x+c))^n = e*(b*x+a)^n/((d*x+c)^n))/h
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx$$

```
[In] Int[1/((a*f*h + b*g*h*x^2 + h*(b*f*x + a*g*x))*(A + B*Log[(e*(a + b*x)^n)/(
c + d*x)^n])^2),x]
```

```
[Out] Defer[Subst][Defer[Int][1/((a + b*x)*(f + g*x)*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n])^2), x], e*((a + b*x)/(c + d*x))^n, (e*(a + b*x)^n)/(c + d*x)^n]/
h
```

Rubi steps

integral

$$= \text{Subst}\left(\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\int \frac{1}{h(a+bx)(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

$$= \text{Subst}\left(\frac{\int \frac{1}{(a+bx)(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx}{h}, e\left(\frac{a+bx}{c+dx}\right)^n, e(a+bx)^n(c+dx)^{-n}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx$$

```
[In] Integrate[1/((a*f*h + b*g*h*x^2 + h*(b*f*x + a*g*x))*(A + B*Log[(e*(a + b*x
)^n)/(c + d*x)^n])^2),x]
[Out] Integrate[1/((a*f*h + b*g*h*x^2 + h*(b*f*x + a*g*x))*(A + B*Log[(e*(a + b*x
)^n)/(c + d*x)^n])^2), x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{(afh+bgh\,x^2+h\,(agx+bfx))\left(A+B\ln\left(e\,(bx+a)^n\,(dx+c)^{-n}\right)\right)^2}dx$$

```
[In] int(1/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x))/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2
,x)
[Out] int(1/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x))/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2
,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.84

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \int \frac{1}{(bghx^2+afh+(bfx+agx)h)\left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x))/(A+B*log(e*(b*x+a)^n/((d*x+c)
^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(A^2*b*g*h*x^2 + A^2*a*f*h + (A^2*b*f + A^2*a*g)*h*x + (B^2*b*g*
h*x^2 + B^2*a*f*h + (B^2*b*f + B^2*a*g)*h*x)*log((b*x + a)^n*e/(d*x + c)^n)
^2 + 2*(A*B*b*g*h*x^2 + A*B*a*f*h + (A*B*b*f + A*B*a*g)*h*x)*log((b*x + a)^
n*e/(d*x + c)^n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a*f*h+b*g*h*x**2+h*(a*g*x+b*f*x))/(A+B*ln(e*(b*x+a)**n/((d*x+c
)**n)))**2,x)
```

```
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 506, normalized size of antiderivative = 9.92

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \int \frac{1}{(bghx^2+afh+(bfx+agx)h)\left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x))/(A+B*log(e*(b*x+a)^n/((d*x+c)
^n)))^2,x, algorithm="maxima")
```

```
[Out] (d*f - c*g)*integrate(1/((b*c*f^2*h*n - a*d*f^2*h*n)*A*B + (b*c*f^2*h*n*log
(e) - a*d*f^2*h*n*log(e))*B^2 + ((b*c*g^2*h*n - a*d*g^2*h*n)*A*B + (b*c*g^2
*h*n*log(e) - a*d*g^2*h*n*log(e))*B^2)*x^2 + 2*((b*c*f*g*h*n - a*d*f*g*h*n)
*A*B + (b*c*f*g*h*n*log(e) - a*d*f*g*h*n*log(e))*B^2)*x + ((b*c*g^2*h*n - a
*d*g^2*h*n)*B^2*x^2 + 2*(b*c*f*g*h*n - a*d*f*g*h*n)*B^2*x + (b*c*f^2*h*n -
a*d*f^2*h*n)*B^2)*log((b*x + a)^n) - ((b*c*g^2*h*n - a*d*g^2*h*n)*B^2*x^2 +
 2*(b*c*f*g*h*n - a*d*f*g*h*n)*B^2*x + (b*c*f^2*h*n - a*d*f^2*h*n)*B^2)*log
((d*x + c)^n)), x) - (d*x + c)/((b*c*f*h*n - a*d*f*h*n)*A*B + (b*c*f*h*n*lo
g(e) - a*d*f*h*n*log(e))*B^2 + ((b*c*g*h*n - a*d*g*h*n)*A*B + (b*c*g*h*n*lo
g(e) - a*d*g*h*n*log(e))*B^2)*x + ((b*c*g*h*n - a*d*g*h*n)*B^2*x + (b*c*f*h
*n - a*d*f*h*n)*B^2)*log((b*x + a)^n) - ((b*c*g*h*n - a*d*g*h*n)*B^2*x + (b
*c*f*h*n - a*d*f*h*n)*B^2)*log((d*x + c)^n))
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \int \frac{1}{(bghx^2+afh+(bfx+agx)h)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}\,dx$$

```
[In] integrate(1/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x))/(A+B*log(e*(b*x+a)^n/((d*x+c)
^n)))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*h*x^2 + a*f*h + (b*f*x + a*g*x)*h)*(B*log((b*x + a)^n*e/(
d*x + c)^n) + A)^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \int \frac{1}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2\,\left(h\,\left(a\,g\,x+b\,f\,x\right)+a\,f\,h+b\,g\,h\,x^2\right)}\,dx$$

```
[In] int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(h*(a*g*x + b*f*x) + a*f*
h + b*g*h*x^2)),x)
```

```
[Out] int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(h*(a*g*x + b*f*x) + a*f*
h + b*g*h*x^2)), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                   end if
               else #result contains complex but optimal is not
                   if debug then
                           print("result contains complex but optimal is not");
                   fi;
                   return "C","Result contains complex when optimal does not.";
               fi;
           else # result do not contain complex
                # this assumes optimal do not as well. No check is needed here.
               if debug then
                      print("result do not contain complex, this assumes optimal do not as well")
               fi;
               if leaf_count_result<=2*leaf_count_optimal then
                   if debug then
                       print("leaf_count_result<=2*leaf_count_optimal");
                   fi;
                   return "A"," ";
               else
                   if debug then
                       print("leaf_count_result>2*leaf_count_optimal");
                   fi;
                   return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                    convert(leaf_count_result,string),"$ vs. $2(",
                                    convert(leaf_count_optimal,string),")=",convert(2*leaf_count
               fi;
           fi;
       else #ExpnType(result) > ExpnType(optimal)
           if debug then
               print("ExpnType(result) > ExpnType(optimal)");
           fi;
           return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
       fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-ty
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```